



**NAVAL
POSTGRADUATE
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MONTEREY, CALIFORNIA

THESIS

**MULTICOMMODITY LOGISTICAL SUPPORT IN AN
ANTI-ACCESS, AREA DENIAL ENVIRONMENT**

by

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March 2018

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REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE March 2018	3. REPORT TYPE AND DATES COVERED Master's thesis		
4. TITLE AND SUBTITLE MULTICOMMODITY LOGISTICAL SUPPORT IN AN ANTI-ACCESS, AREA DENIAL ENVIRONMENT			5. FUNDING NUMBERS	
6. AUTHOR(S) Jonathan M. Krenz				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A			10. SPONSORING / MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. IRB number ___N/A___.				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release. Distribution is unlimited.			12b. DISTRIBUTION CODE	
<p>As countries around the world develop long-range anti-ship ballistic missiles (ASBMs), the current method of replenishing warships at sea may no longer be viable. These long-range ASBMs can be used to target combat logistic force (CLF) ships, resulting in the degradation of the warships' mission without targeting of the warships. Currently, the U.S. Navy has approximately 30 ships in its CLF fleet. The destruction or damage to a few of these ships would have a devastating impact on Navy surface operations.</p> <p>B. D. Colburn's 2015 thesis, <i>Preserving Logistical Support for Deployed Battle Groups in an Anti-access, Area Denial (A2AD) Environment</i>, developed an optimization model to consider the use of "mini-CLF" ships to shuttle fuel to deployed Carrier Task Force (CTF)/Cruiser/Destroyer (CRUDES) surface action groups (SAGs) and forward-deployed units while at sea. In practice, however, multiple commodities such as fuel, stores, and ammunition are required to keep ships operating during peacetime and wartime situations. This thesis extends Colburn's model to include all of these commodities and exercises the model on four case studies.</p>				
14. SUBJECT TERMS combat logistics force, CLF, shuttle ship, logistics planning factors, underway replenishment, distributed lethality, distributed logistics, anti-access, area denial, naval logistics, optimization, multicommodity, DL-RASM			15. NUMBER OF PAGES 89	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UU	

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**MULTICOMMODITY LOGISTICAL SUPPORT IN AN ANTI-ACCESS, AREA
DENIAL ENVIRONMENT**

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MASTER OF SCIENCE IN OPERATIONS RESEARCH

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ABSTRACT

As countries around the world develop long-range anti-ship ballistic missiles (ASBMs), the current method of replenishing warships at sea may no longer be viable. These long-range ASBMs can be used to target combat logistic force (CLF) ships, resulting in the degradation of the warships' mission without targeting of the warships. Currently, the U.S. Navy has approximately 30 ships in its CLF fleet. The destruction or damage to a few of these ships would have a devastating impact on Navy surface operations.

B. D. Colburn's 2015 thesis, *Preserving Logistical Support for Deployed Battle Groups in an Anti-access, Area Denial (A2AD) Environment*, developed an optimization model to consider the use of "mini-CLF" ships to shuttle fuel to deployed Carrier Task Force (CTF)/Cruiser/Destroyer (CRUDES) surface action groups (SAGs) and forward-deployed units while at sea. In practice, however, multiple commodities such as fuel, stores, and ammunition are required to keep ships operating during peacetime and wartime situations. This thesis extends Colburn's model to include all of these commodities and exercises the model on four case studies.

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LIST OF ACRONYMS AND ABBREVIATIONS

A2	anti-access
AFP	adaptive force package
AO	area of operations
ARL	aft RAS lane
ASBM	anti-ship ballistic missile
bbls	barrels
CG	guided-missile cruisers
CLF	combat logistic force
CRUDES	cruiser/destroyer
CTF	carrier task force
CVN	aircraft carrier, nuclear
DDG	guided-missile destroyer
DF-21D	Dong-Feng-21D
DFM	diesel fuel, marine
DL-RASM	Dual Lane Replenishment At-Sea Model
FRL	forward RAS lane
JP5	jet propellant fuel, type 5
LHD	amphibious assault ship (multipurpose)
MC-DL-RASM	Multicommodity Dual Lane Replenishment at Sea Model
RAS	replenishment at sea
RASP	Replenishment at Sea Planner
SAG	surface action group
T-AKE	auxiliary dry cargo and ammunition ship
T-AO	fleet oiler
T-AOE	fast combat support ship

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EXECUTIVE SUMMARY

As countries around the world develop long range anti-ship ballistic missiles (ASBMs), the current method of replenishing warships at sea may no longer be viable. Long-range ASBMs can be used to target combat logistic force (CLF) ships, resulting in the degradation of the warships' mission without targeting of the warships. Currently, the United States Navy has approximately 30 ships in its CLF fleet. The destruction or damage to a few of these ships would have a devastating impact on United States Navy surface operations.

Colburn (2015) developed an optimization model to examine the use of "mini-CLF" shuttle ships to move fuel to deployed Carrier Task Force (CTF)/Cruiser/Destroyer (CRUDES) surface action groups (SAGs) and forward deployed units while at sea. In practice, however, multiple commodities including fuel, stores, and ammunition are required to keep ships operating during peacetime and wartime situations. This thesis extends Colburn's model to include all of these commodities and exercises the model on four case studies.

Our case studies use port availability to model various strategic situations, including peacetime, "rising tensions," and wartime. We also explore different shuttle characteristics, including the number of shuttles and their speeds, and we utilize a rolling horizon approach. In each case study, we measure the amount of time each surface action group spends on station, as well as the amount of time each commodity is below its prescribed safety level. Our results indicate that although shuttle characteristics are important, we must also consider the CLFs' ability to provide fuel to the shuttles as well as warships. For example, our results indicate that in a wartime setting, a single T-AO cannot sustain two SAGs in the region we consider. We also find that implementing a rolling horizon in the model has a significant impact on the resulting solution. We illustrate how a planning horizon that is too short can result in highly suboptimal solutions. These results can guide future model users.

Reference

Colburn, B. D. (2015). *Preserving logistical support for deployed battle groups in an anti-access, area denial (A2AD) environment*. (Master's thesis). Naval Postgraduate School, Monterey, CA. Retrieved from <https://calhoun.nps.edu/handle/10945/47240>

ACKNOWLEDGMENTS

This thesis would not have been possible without the help and encouragement of too many people to list by name.

To all of my classmates at NPS, thank you for making my time at NPS enjoyable.

Professor Carlyle, without your help I would still be attempting to learn how to write this model in pyomo.

CAPT (ret) Kline, thank you for letting my group work on this concept during our Joint Campaign Analysis class.

Professor Craparo, I can't give you enough credit for helping me to get this project finished. Not only did you pique my interest in doing an optimization thesis early in my time at NPS, but you also gladly helped me find a topic.

Without the unconditional love and constant giggles of my daughters, Isabelle and Evalynn, I would not have finished. Thank you for making me stop working on homework and instead spend time playing games and reading your books.

Without my wife, Mirinda, I wouldn't even be here. I can't put into words all the help and encouragement you have given me over the last 13+ years. I wouldn't have completed this thesis without you taking care of our wonderful children, and making sure I stop and enjoy life.

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I. INTRODUCTION

As United States Navy leadership continues to develop the “distributed lethality” and “distributed maritime operations” concepts of operations, the way in which the Navy will replenish its surface action groups (SAG) is still an open question. Staff and students at Naval Postgraduate School have worked for several years to answer this question. As Atkinson, Kress and Szechtman note about uniquely configured SAGs called adaptive force packages (AFB):

Distributed Lethality is an operational concept that embodies significant logistic implications. The existing logistic system, supporting carrier battle group, where the logistic tail is an integral part of the tactical force—the shuttle-delivery ship setup—will clearly be inappropriate when the force structure is fragmented into small AFPs. Attaching a logistic tail to each such AFP is neither economically viable nor operationally feasible. A new logistic structure is required that adequately responds to the new naval force layout. (Atkinson, Kress, & Szechtman, 2017, p. 15)

This thesis contributes to this important line of research by continuing work initiated by Colburn (2015) which explores alternative logistic force architectures.

A. BACKGROUND

Since World War II, the U.S. Navy has conducted underway replenishments by using large vessels designed for alongside refueling. With the introduction of long range anti-ship ballistic missiles (ASBMs) into the battle space, however, this method of replenishment may no longer be viable. In particular, China’s development of the Dong-Feng-21D (DF-21D) ASBM, has created areas in the South China Sea, East China Sea, and Yellow Sea where the risk of operating current combat logistic force (CLF) vessels is too great. CLFs possess minimal self-defense capabilities, and the loss of only a few CLF vessels could cripple naval operations worldwide. Colburn (2015) suggests a novel concept to reduce the risk to CLFs while maintain adequate logistic support in theater:

- With the goal of preserving the CLF’s ability to perform its mission while not subjecting it to an ASBM threat, we consider the possibility of utilizing a “mini-CLF” to shuttle fuel between CLFs

operating in a safe environment and warships operating in a threat zone. (Colburn, 2015, p. 2)

Colburn's initial analysis considered the feasibility of this approach to deliver a single liquid fuel commodity to warships. This thesis extends Colburn's work to multiple commodities, analyzing the required capacities of Diesel Fuel Marine (DFM); jet propellant fuel, type 5 (JP5); stores; and ammunition required to support SAG operations.

B. OBJECTIVES

This thesis extends Colburn's work on the use of a mini-CLF shuttle ship to support operations in an anti-access (A2) environment. We examine the breakdown of DFM, JP5, stores and ammunition capacity needed for the mini-CLF ship to effectively operate as a shuttle between current CLF ships, auxiliary dry cargo and ammunition ships (T-AKEs) fleet oilers (T-AOs), and deployed SAGs. To accomplish this, we present the Multicommodity Dual Lane Replenishment at Sea Model (MC-DL-RASM), which extends the Dual Lane Replenishment At-Sea Model (DL-RASM) developed by Colburn (2015) to model all four commodity types.

C. SCOPE, LIMITATIONS, AND MODEL ASSUMPTIONS

1. Scope

This thesis focuses on the 7th Fleet Area of Operations (AO).

2. Limitations

Our model only accounts for routine supply transfers and does not address high-priority parts. It does not account for resupplying vertical launch system or other large ordnances. Our scenarios involve fixed numbers of ships, with no ships added or removed from the system and no external demands placed on any of the CLF ships.

3. Assumptions

Assumptions made for the model and scenario development will be discussed in the appropriate sections.

D. CONTRIBUTIONS

This thesis serves as a proof of concept for the Multicommodity Dual Lane Replenishment at Sea model (MC-DL-RASM) and provides guidance on its implementation and operation.

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II. LITERATURE REVIEW

Our literature review consists of a summary of the literature review from Colburn (2015) and a review of new literature since its publication in 2015.

A. PREVIOUSLY CITED LITERATURE

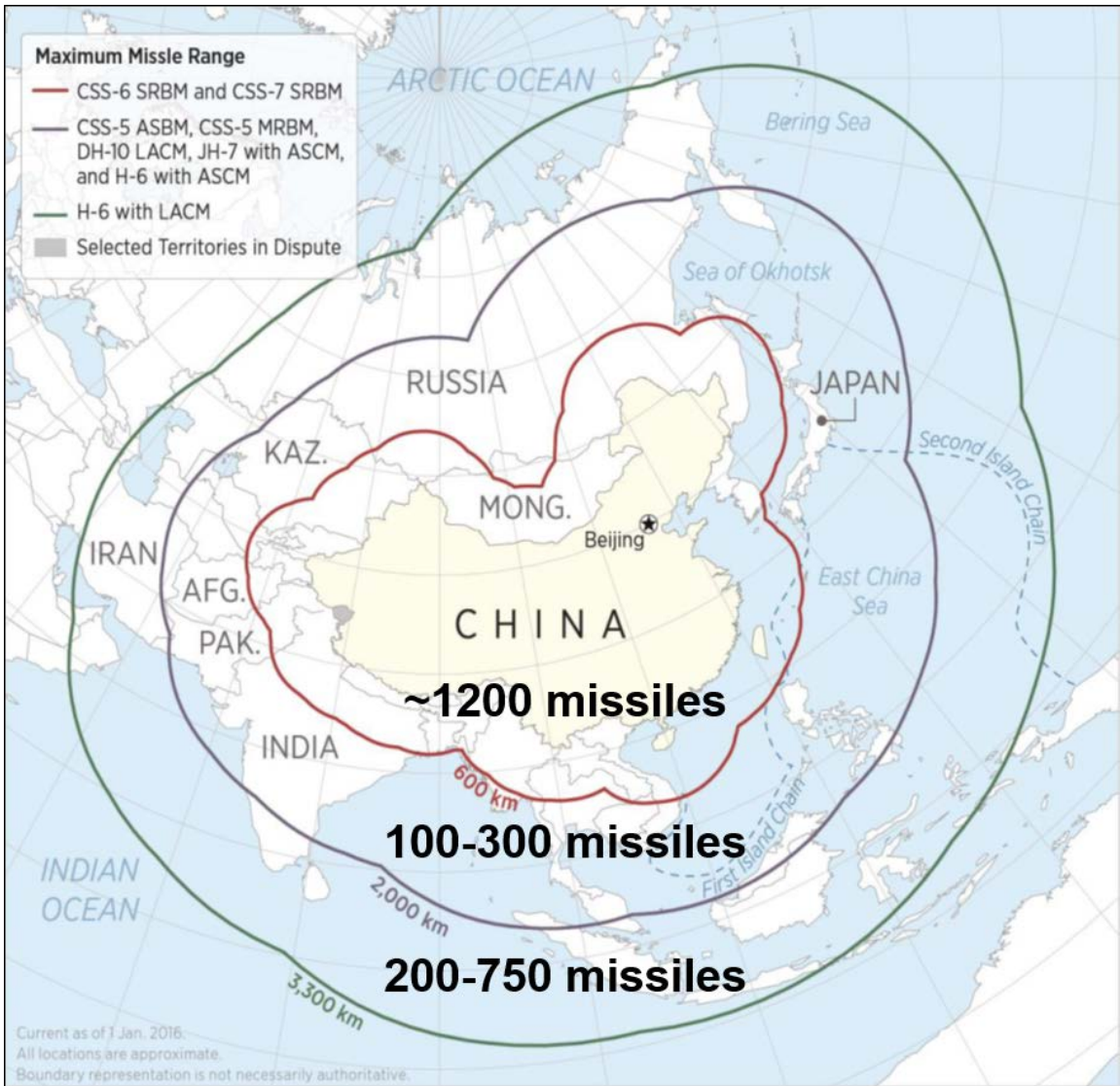
Colburn (2015) provides a more comprehensive review of the following works, which we briefly summarize for convenience.

Givens (2002) and Borden (2001) examine the configuration, loading, and employment of current and proposed CLF ships in order to support deployed units.

CLF planner is an optimizations tools developed by Brown and Carlyle (2008) and Brown, Carlyle, Kelton, Kline, and Salmeron (2009). CLF planner is used as an aid in the decision making at the strategic and operational levels.

Colburn (2015) describes the Replenishment at Sea Planner (RASP) as an aid to the everyday planning of CLF employment to efficiently resupply deployed units in a given region based on the current situation. RASP has been highly successful in its implementation in 5th Fleet and 7th Fleet (Brown, DeGrange, Price, & Rowe, 2018).

Ross and Harmon (2012) examine the development of the A2 environment through the use of Wayne Hughes' missile salvo equations (Hughes, 2000) specifically looking at the South China Sea and how the US can counter the growing threat from China in this area. The authors utilize a 1500 km range for the DF-21D (CSS-5). Figure 1 depicts increased missile ranges and data compiled from the "Annual Report to Congress: Military and Security Developments Involving the People's Republic of China 2016" (Department of Defense, 2016), and from the "U.S.-P.R.C Military Scorecard" (Heginbotham et al., 2015).



This graphic from Department of Defense [DOD] (2016) has been adapted with numbers from Heginbotham et al. (2015).

Figure 1. Range and inventory numbers for Chinese anti-ship cruise missiles.

In his 2009 paper, “New Navy Fighting Machine,” Hughes describes the impact of changes to the composition of the U.S. Navy to support operations in littoral waters. Colburn specifically points out that “this fleet composition result in approximately 216 ships requiring at-sea support, a considerable increase over today’s number (status of the Navy, 2015)” (Colburn, 2015, p. 11).

B. CURRENT LITERATURE

“Distributed logistics” is the term for the logistical support of the “distributed lethality” or “distributed maritime operation” ideas. Coburn (2015) develops the Dual Lane Replenishment At-Sea Model (DL-RASM), and he uses this model to determine the effectiveness of a fleet of shuttle ships to support both a carrier task force (CTF) and a surface action group (SAG). Colburn’s analysis shows that speed is not a critical factor in the ability of the shuttle ship to support the CTF or SAG, and “that a moderate number of shuttles is required to support peacetime operations” (Colburn, 2015, p. 55).

The work of Atkinson et al. (2017) furthers this concept by examining the possibility of using a CLF ship as a “gas station” to resupply deployed units. In their model the Adaptive Force Package (AFP) travels out of the combat zone to meet with a CLF ship to be resupplied. Their results show that for only one AFP the capacity of the CLF ship is a critical factor in determining the amount of time the AFP spends on station. When increased to look at multiple AFPs the critical factor becomes the size of the AFPs operating in the area (Atkinson et al., 2017).

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III. MODEL AND SCENARIO DEVELOPMENT

This chapter introduces the Multicommodity Dual Lane Replenishment at Sea Model (MC-DL-RASM). The original Dual Lane Replenishment At-Sea Model (DL-RASM) was developed by Colburn (2015). Like DL-RASM (Colburn, 2015), MC-DL-RASM (the model present in this thesis) is formulated as a deterministic discrete-time mixed integer network-based linear program. Given a set of shuttles and CLF ships, both models optimize their ability to provide logistical support over a specific time horizon to deployed warships by developing the movement and resupply schedule for each of the warships, shuttles, and CLF ships. Both models also apply penalties when ships' consumable or deliverable inventories fall below prescribed safety levels; rewards are applied for the amount of time the warships spend on station and the amount of supplies transmitted through the network. The network nodes in both models consist of forward RAS lanes (FRLs), aft RAS lanes (ARLs), and ports (Colburn, 2015). The key difference between MC-DL-RASM and DL-RASM is the handling of multiple commodities by MC-DL-RASM. This is reflected in the commodity index, l , which appears on a variety of decision variables and constraints in MC-DL-RASM and does not appear in DL-RASM (Colburn, 2015).

A. MULTICOMMODITY DUAL LANE, REPLENISHMENT AT SEA MODEL FORMULATION (MC-DL-RASM)

We now provide the mathematical formulation for MC-DL-RASM. Due to the substantial degree of overlap with DL-RASM's formulation (Colburn, 2015), we indicate new model elements using red text.

1. Assumptions

Colburn (2015) made the following assumptions in the development of DL-RASM "in order to produce a realistic mathematical model and accompanying input data" (p. 14); we make these assumptions as well.

- All ships in a [SAG] must travel together as a [group], unless a ship encounters a fuel shortage. If this occurs, that ship may separate from the [SAG] in order to obtain fuel. To enforce the requirement that all ships in a [SAG] remain together, we required that each ship be collocated with its [SAG] carrier unless low on fuel.
- Warships have a single [storage capacity for each commodity (DFM, JP5, stores, and ammunition).] Shuttles and CLFs have two [capacities for each of the commodities]: one containing [useable quantities] and one containing [deliverable quantities]. Each shuttle can transfer [commodities] from its deliverable [capacity] into the [useable capacity] of a warship or into its own [useable capacity]. Each CLF ship can transfer [commodities] from its deliverable [capacity] into the [useable capacity] of a shuttle, the deliverable [capacity] of a shuttle or into its own [useable capacity].
- For a RAS event to occur, [all ships involved in the RAS must be] collocated at a FRL or ARL for [the required number of time periods].
- During peace-time operations, RAS events only occur during daylight time periods. This is representative of actual operational restrictions currently practiced during peacetime operations.
- The combat fleet, shuttles and CLF fleet do not suffer any losses during the scenario. [No new ships arrive during the planning horizon.] That is, all ships remain available throughout the planning horizon.
- Each warship, shuttle, and CLF ship consumes [commodities at a constant] rate [during each time period unless the ship is located at a port node].
- Once a ship enters port, it must remain in port for a prescribed number of time periods.
- Requirements outside of the AO and scenario do not impact the availability of assets. (Colburn, 2015, p. 14-15)

2. Indices and Sets [Approximate Cardinality]¹

$c \in C$	CLF ships [2]
$n, i, j \in N$	nodes [~ 30]
$p \in P \subseteq N$	port nodes [6]
$r \in R \subseteq N$	RAS lane nodes [10]
$s \in S$	shuttle ships [6]
$t, tp \in T$	time periods [~ 180]
$t \in D \subseteq T$	daylight time periods [~ 90]
$w, wp \in W$	warship [10]
$(i, j) \in ac \subseteq NxN$	arc (i, j) can be traversed by CLF ships
$(i, j) \in as \subseteq NxN$	arc (i, j) can be traversed by shuttle ships
$(i, j) \in aw \subseteq NxN$	arc (i, j) can be traversed by warships
$(w, wp) \in carrier \subset WxW$	warship w carrier is warship wp
$(t, tp, c, i, n) \in dC \subset TxTxXxNxN$	if CLF ship c departs node i at time t , it will arrive at node n at time tp
$(t, tp, s, i, n) \in dS \subset TxTxXxNxN$	if shuttle ship s departs node i at time t , it will arrive at node n at time tp
$(t, tp, w, i, n) \in dW \subset TxTxXxNxN$	if warship w departs node i at time t , it will arrive at node n at time tp
$l \in L$	commodity type [4]

¹ We indicate new model elements not present in Colburn (2015) using red text.

3. Parameters [Units]²

$berths_p$	berths at port p [berths]
$capacity_{w,l}$	capacity of warship w for each commodity l [barrels, stons]
$capacity_{s,l}$	capacity of shuttle s for each commodity l [barrels, stons]
$capacity_{c,l}$	capacity of CLF c for each commodity l [barrels, stons]
$safetyLevel_{w,l}$	percentage of $capacity_{w,l}$ below which warship w incurs a penalty [percentage]
$safetyLevel_{s,l}$	percentage of $capacity_{s,l}$ below which shuttle s incurs a penalty [percentage]
$safetyLevel_{c,l}$	percentage of $capacity_{c,l}$ below which CLF ship c incurs a penalty [percentage]
$capDel_{s,l}$	capacity of deliverable commodity l for shuttle s [stons, barrels]
$capDel_{c,l}$	capacity of deliverable commodity l for CLF ship c [stons, barrels]
ε_l	small reward for each unit of commodity l picked up in port or transferred via RAS: intended to mitigate end-of-horizon effects [reward/(ton or barrel)]
$F_{w,l}$	commodity l used per time period by warship w [various]
$F_{s,l}$	commodity l used per time period by shuttle s [various]
$F_{c,l}$	commodity l used per time period by CLF ship c [various]
$inport_w$	time periods required for warship w to remain in port on each port visit [periods]
$inport_s$	time periods required for shuttle s to remain in port on each port visit [periods]
$inport_c$	time periods required for CLF ship c to remain in port on each port visit [periods]
$nrpp$	number of ships that can begin RASing per time period, for each shuttle and CLF [ships]
$penalty_{w,l}$	per-unit commodity l shortage penalty for warship w [penalty/(ton, barrel)]

² We indicate new model elements not present in Colburn (2015) using red text.

$penalty_{s,l}$	per-unit commodity l shortage penalty for shuttle s [penalty/(ton , barrel)]
$penalty_{c,l}$	per-unit commodity l shortage penalty for CLF ship c [penalty/(ton , barrel)]
$reward$	reward for on-station time [reward/(ship*period)]
$safetyDel_{s,l}$	percentage of $capDel_{s,l}$ below which shuttle s incurs a penalty [percentage]
$safetyDel_{c,l}$	percentage of $capDel_{s,l}$ below which CLF ship c incurs a penalty [percentage]
$simult_c$	number of shuttles that can simultaneously RAS from a single CLF [ship]
$simult_s$	number of warships that can simultaneously ras from a single shuttle [ship]
$TC_{c,i,j}$	time periods required for CLF ship c to transit from location i to location j [periods]
$TS_{s,i,j}$	time periods required for shuttle ship s to transit from location i to location j [periods]
$TW_{w,i,j}$	time periods required for warship w to transit from location i to location j [periods]
$TR_{w,s}$	time periods required to RAS ship [periods]

4. Decision Variables³

a. Binary Decision Variable

$FDIP_{w,p}^t$	1 if warship w enters port p at time t [binary]
$FDIP_{s,p}^t$	1 if shuttle s enters port p at time t [binary]
$FDIP_{c,p}^t$	1 if CLF ship c enters port p at time t [binary]
$LOWF_w^t$	1 if warship w 's fuel is below safety stock at time t [binary]
$REVUC_{s,c,r}^t$	1 if at start time t shuttle s collocated with CLF ship c in RAS lane r long enough to fill shuttle s [binary]
$REVUW_{w,s,r}^t$	1 if at start time t warship w collocated with shuttle ship s in RAS lane r long enough to fillwarship w [binary]

³ We indicate new model elements not present in Colburn (2015) using red text.

$YW_{w,i,j}^t$	1 if at time t warship w departs location i for location j [binary]
$YC_{c,i,j}^t$	1 if at time t CLF ship c departs location i for location j [binary]
$YS_{s,i,j}^t$	1 if at time t shuttle ship s departs location i for location j [binary]

b. Nonnegative Decision Variables

$HOLD_{s,l}^t$	deliverable commodity l inventory at start of time period t for shuttle s [barrels, tons]
$HOLD_{c,l}^t$	deliverable commodity l inventory at start of time period t for CLF ship c [barrels, tons]
$HOLDINV_{s,l}^t$	commodity l transferred from deliverable capacity of shuttle s to $INV_{s,l}^t$ of shuttle s during time t [barrels, tons]
$HOLDINV_{c,l}^t$	commodity l transferred from deliverable capacity of shuttle c to $INV_{c,l}^t$ of shuttle c during time t [barrels, tons]
$INPB_{w,l}^t$	useable commodity l replenished in port during time period t for warship w [barrels, tons]
$INPB_{s,l}^t$	useable commodity l replenished in port during time period t for shuttle s [barrels, tons]
$INPB_{c,l}^t$	useable commodity l replenished in port during time period t for CLF ship c [barrels, tons]
$INPD_{s,l}^t$	deliverable commodity l replenished in port at time t for shuttle s [barrels, tons]
$INPD_{c,l}^t$	deliverable commodity l replenished in port at time t for CLF ship c [barrels, tons]
$RASB_{s,c,l}^t$	useable commodity l transferred to shuttle s from CLF c at time t [barrels, tons]
$RASD_{s,c,l}^t$	deliverable commodity l transferred to shuttle s from CLF c at time t [barrels, tons]
$RASW_{w,s,l}^t$	useable commodity l transferred to warship w from shuttle s at time t [barrels, tons]
$SHORTUSE_{w,l}^t$	amount below safety stock of useable commodity l at time period t for warship w [barrels, tons]

- $SHORTUSE_{s,l}^t$ amount below safety stock of useable commodity l
at time period t for shuttle s [barrels, tons]
- $SHORTUSE_{c,l}^t$ amount below safety stock of useable commodity l
at time period t for CLF ship c [barrels, tons]
- $SHORTDel_{s,l}^t$ amount below safety stock of deliverable commodity l at time period t
for shuttle s [barrels, tons]
- $SHORTDel_{c,l}^t$ amount below safety stock of deliverable commodity l at time period t
for CLF ship c [barrels, tons]

c. Free Variables

- $INV_{w,l}^t$ useable commodity l inventory at start of time period t
for warship w [barrels, tons]
- $INV_{s,l}^t$ useable commodity l inventory at start of time period t
for shuttle s [barrels, tons]
- $INV_{c,l}^t$ useable commodity l inventory at start of time period t
for CLF ship c [barrels, tons]

5. Formulation⁴

$$\text{Max} \sum_{t,w,(n,f) \in aw} \text{reward} * YW_{w,n,f}^t - \sum_{w,t,l} \text{penalty}_{w,l} * \text{SHORTUSE}_{w,l}^t \quad (1)$$

$$- \sum_{s,t,l} \text{penalty}_{s,l} * (\text{SHORTUSE}_{s,l}^t + \text{SHORTDel}_{s,l}^t)$$

$$- \sum_{c,t,l} \text{penalty}_{c,l} * (\text{SHORTUSE}_{c,l}^t + \text{SHORTDel}_{c,l}^t)$$

$$- 2 * \sum_{t,w,s,r} \text{reward} * \text{REVUW}_{t,w,s,r} + \sum_l \varepsilon_l * \sum_{t \in D,s} \left(\sum_w \text{RASW}_{w,s,l}^t + \sum_c (\text{RASB}_{s,c,l}^t + \text{RASD}_{s,c,l}^t) \right)$$

$$+ \sum_l \varepsilon_l * \sum_t \left(\sum_w \text{INPB}_{w,l}^t + \sum_s (\text{INPB}_{s,l}^t + \text{INPD}_{s,l}^t) + \sum_c (\text{INPB}_{c,l}^t + \text{INPD}_{c,l}^t) \right)$$

$$\text{s.t.} \quad \sum_{j:(n,j) \in aw} YW_{w,n,j}^t = \sum_{tp,i:(tp,t,w,i,n) \in dW} YW_{w,i,n}^t \quad \forall w, n, t : t > 1; \exists i, j : (i, n) \in aw \text{ and } (n, j) \in aw \quad (2)$$

$$\sum_{j:(n,j) \in as} YS_{s,n,j}^t = \sum_{tp,i:(tp,t,s,i,n) \in dS} YS_{s,i,n}^t \quad \forall s, n, t : t > 1; \exists i, j : (i, n) \in as \text{ and } (n, j) \in as \quad (3)$$

$$\sum_{j:(n,j) \in ac} YC_{c,n,j}^t = \sum_{tp,i:(tp,t,c,i,n) \in dC} YC_{c,i,n}^t \quad \forall c, n, t : t > 1; \exists i, j : (i, n) \in ac \text{ and } (n, j) \in ac \quad (4)$$

$$\sum_{(n,j) \in aw} YW_{w,n,j}^{t=1} = 1 \quad \forall w \quad (5)$$

$$\sum_{(n,j) \in as} YS_{s,n,j}^{t=1} = 1 \quad \forall s \quad (6)$$

$$\sum_{(n,j) \in ac} YC_{c,n,j}^{t=1} = 1 \quad \forall c \quad (7)$$

$$\sum_{tp,w,n:(tp,t,w,n,p) \in dW} YW_{w,n,p}^{tp} + \sum_{tp,s,n:(tp,t,s,n,p) \in dS} YS_{s,n,p}^{tp} + \sum_{tp,c,n:(tp,t,c,n,p) \in dC} YC_{c,n,p}^{tp} \leq \text{berths}_p \quad \forall p, t \quad (8)$$

$$\text{INV}_{w,l}^t = \text{INV}_{w,l}^{t-1} + \text{INPB}_{w,l}^{t-1} - F_{w,l} * \left(1 - \sum_{tp,n,p:(tp,t,w,n,p) \in dW} YW_{w,n,p}^{tp} \right) + \sum_s \text{RASW}_{w,s,l}^{t-1} \quad \forall w, t > 1, l \quad (9)$$

$$\text{INVS}_{s,l}^t = \text{INVS}_{s,l}^{t-1} + \text{HOLDINV}_{s,l}^{t-1} - F_{s,l} * \left(1 - \sum_{tp,n,p:(tp,t,s,n,p) \in dS} YS_{s,n,p}^{tp} \right) + \text{INPB}_{s,l}^{t-1} + \sum_c \text{RASD}_{s,c,l}^{t-1} \quad \forall s, t > 1, l \quad (10)$$

⁴ We indicate new model elements not present in Colburn (2015) using red text.

$$INVc_{c,l}^t = INVc_{c,l}^{t-1} + HOLDINV_{c,l}^{t-1} - F_{c,l} * \left(1 - \sum_{tp,n,p:(tp,t,c,n,p) \in dC} YC_{c,n,p}^{tp} \right) \quad (11)$$

$$+ INPB_{c,l}^{t-1} \quad \forall c, t > 1, l$$

$$INPB_{w,l}^t \leq capacity_{w,l} \sum_{tp,n,p:(tp,t,w,n,p) \in dW} YW_{w,n,p}^{tp} \quad \forall w, t, l \quad (12)$$

$$INPB_{s,l}^t \leq capacity_{s,l} \sum_{tp,n,p:(tp,t,s,n,p) \in dS} YS_{s,n,p}^{tp} \quad \forall s, t, l \quad (13)$$

$$INPB_{c,l}^t \leq capacity_{c,l} \sum_{tp,n,p:(tp,t,c,n,p) \in dC} YC_{c,n,p}^{tp} \quad \forall c, t, l \quad (14)$$

$$INPD_{s,l}^t \leq capDel_{s,l} \sum_{tp,n,p:(tp,t,s,n,p) \in dS} YS_{s,n,p}^{tp} \quad \forall s, t, l \quad (15)$$

$$INPD_{c,l}^t \leq capDel_{c,l} \sum_{tp,n,p:(tp,t,c,n,p) \in dC} YC_{c,n,p}^{tp} \quad \forall c, t, l \quad (16)$$

$$HOLD_{s,l}^t = HOLD_{s,l}^{t-1} - HOLDINV_{s,l}^{t-1} + INPD_{s,l}^{t-1} - \sum_w RASW_{w,s,l}^{t-1} \quad (17)$$

$$+ \sum_c RASD_{s,c,l}^{t-1} \quad \forall s, t > 1, l$$

$$HOLD_{c,l}^t = HOLD_{c,l}^{t-1} - HOLDINV_{c,l}^{t-1} + INPD_{c,l}^{t-1} - \sum_s RASB_{s,c,l}^{t-1} \quad (18)$$

$$+ \sum_s RASD_{s,c,l}^{t-1} \quad \forall c, t > 1, l$$

$$\sum_w REVUW_{w,s,r}^t \leq nrpp \quad \forall t \in D, s, r \quad (19)$$

$$\sum_s REVUW_{w,s,r}^t \leq nrpp \quad \forall t \in D, w, r \quad (20)$$

$$\sum_{w, tp \in D: t - TR_w < tp \leq t} REVUW_{w,s,r}^{tp} \leq simult_s * \sum_{tp,n:(tp,t,s,n,r) \in dS} YS_{s,n,r}^t \quad \forall t \in D, s, r \quad (21)$$

$$\sum_s REVUC_{s,c,r}^t \leq nrpp \quad \forall t \in D, c, r \quad (22)$$

$$\sum_c REVUC_{s,c,r}^t \leq nrpp \quad \forall t \in D, s, r \quad (23)$$

$$\sum_{s, tp \in D: t - TR_s < tp \leq t} REVUC_{s,c,r}^{tp} \leq simult_c * \sum_{tp,n:(tp,t,c,n,r) \in dC} YC_{c,n,r}^t \quad \forall t \in D, c, r \quad (24)$$

$$REVUW_{w,s,r}^t \leq \sum_{tp,n,:(tp,t,w,n,r) \in dW} YW_{w,n,r}^{tp} \quad (25)$$

$$\forall w, s, r, t \in D, tp \in D: t \leq tp < t + TR_w$$

$$REVUW_{w,s,r}^t \leq \sum_{tp,n,:(tp,t,s,n,r) \in dS} YS_{s,n,r}^{tp} \quad (26)$$

$$\forall w, s, r, t \in D, tp \in D: t \leq tp < t + TR_w$$

$$REVUC_{s,c,r}^t \leq \sum_{tp,n:,(tp,sp,s,n,r) \in dS} YS_{s,n,r}^{tp} \quad (27)$$

$$\forall s, c, r, t \in D, tp \in D : t \leq tp < t + TR_s$$

$$REVUC_{s,c,r}^t \leq \sum_{tp,n:,(tp,sp,c,n,r) \in dC} YC_{c,n,r}^{tp} \quad (28)$$

$$\forall s, c, r, t \in D, tp \in D : t \leq tp < t + TR_s$$

$$RASW_{w,s,l}^t \leq \sum_r \min(capDel_{s,l}, capacity_{w,l}) * REVUW_{w,s,r}^{t-TR_w+1} \quad (29)$$

$$\forall s, w, t \in D, l$$

$$RASB_{s,c,l}^t + RASD_{s,c,l}^t \leq \sum_r \left(\min(capDel_{c,l}, (capDel_{s,l} + capacity_{s,l})) \right) * REVUC_{s,c,r}^{t-TR_c+1} \quad (30)$$

$$\forall c, s, t \in D, l$$

$$SHORTUSE_{w,l}^t \geq capacity_{w,l} * safetyLevel_{w,l} - INVw_{w,l}^t \quad \forall w, t, l \quad (31)$$

$$SHORTUSE_{s,l}^t \geq capacity_{s,l} * safetyLevel_{s,l} - INVw_{s,l}^t \quad \forall s, t, l \quad (32)$$

$$SHORTUSE_{c,l}^t \geq capacity_{c,l} * safetyLevel_{c,l} - INVw_{c,l}^t \quad \forall c, t, l \quad (33)$$

$$SHORTDel_{s,l}^t \geq safetyDel_{s,l} * capDel_{s,l} - HOLD_{s,l}^t \quad \forall s, t, l \quad (34)$$

$$SHORTDel_{c,l}^t \geq safetyDel_{c,l} * capDel_{c,l} - HOLD_{c,l}^t \quad \forall c, t, l \quad (35)$$

$$LOWF_w^t \leq 1 - \frac{INVw_{w,DFM'}^t - safetyLevel_{w,DFM'} * capacity_{w,DFM'}}{capacity_{w,DFM'} - safetyLevel_{w,DFM'} * capacity_{w,DFM'}} \quad \forall w, t \quad (36)$$

$$YW_{w,j,n}^t \geq YW_{wp,j,n}^t - LOWF_w^t \quad (37)$$

$$\forall w, wp, t, j, n : (w, wp) \in carrier, (j, n) \in aw$$

$$FDIP_{w,p}^t = \sum_{(tp,i):i \neq p, (tp,t,w,i,p) \in dW} YW_{w,i,p}^t \quad (38)$$

$$\forall w, p, t : \exists i : (i, p) \in aw$$

$$FDIP_{s,p}^t = \sum_{(tp,i):i \neq p, (tp,t,s,i,p) \in dS} YS_{s,i,p}^t \quad (39)$$

$$\forall s, p, t : \exists i : (i, p) \in as$$

$$FDIP_{c,p}^t = \sum_{(tp,i):i \neq p, (tp,t,c,i,p) \in dC} YC_{c,i,p}^t \quad (40)$$

$$\forall c, p, t : \exists i : (i, p) \in ac$$

$$YW_{w,p,p}^{tp} \geq FDIP_{w,p}^t \quad (41)$$

$$\forall w, p, t, tp : t < tp < inport_w, \exists i : (i, p) \in aw$$

$$YS_{s,p,p}^{tp} \geq FDIP_{s,p}^t \quad (42)$$

$$\forall s, p, t, tp : t < tp < inport_s, \exists i : (i, p) \in as$$

$$YC_{c,p}^{tp} \geq FDIP_{c,p}^t \quad (43)$$

$$\forall c, p, t, tp : t < tp < inport_c, \exists i : (i, p) \in ac$$

$$YW_{w,i,j}^t \in \{0,1\} \quad \forall t, w, i, j \quad (44)$$

$$YS_{s,i,j}^t \in \{0,1\} \quad \forall t, s, i, j \quad (45)$$

$$YC_{c,i,j}^t \in \{0,1\} \quad \forall t, c, i, j \quad (46)$$

$$REVUW_{w,s,r}^t \in \{0,1\} \quad \forall t, w, s, r \quad (47)$$

$$REVUC_{c,s,r}^t \in \{0,1\} \quad \forall t, c, s, r \quad (48)$$

$$LOWF_w^t \in \{0,1\} \quad \forall t, w \quad (49)$$

$$FDIP_{w,p}^t \in \{0,1\} \quad \forall t, w, p \quad (50)$$

$$FDIP_{s,p}^t \in \{0,1\} \quad \forall t, s, p \quad (51)$$

$$FDIP_{c,p}^t \in \{0,1\} \quad \forall t, c, p \quad (52)$$

$$0 \leq HOLD_{s,l}^t \leq capDel_{s,l} \quad \forall t, s, l \quad (53)$$

$$0 \leq HOLD_{c,l}^t \leq capDel_{c,l} \quad \forall t, c, l \quad (54)$$

$$0 \leq HOLDINV_{s,l}^t \leq capacity_{s,l} \quad \forall t, s, l \quad (55)$$

$$0 \leq HOLDINV_{c,l}^t \leq capacity_{c,l} \quad \forall t, c, l \quad (56)$$

$$0 \leq RASW_{w,s,l}^t \leq capacity_{w,l} \quad \forall t, w, s, l \quad (57)$$

$$0 \leq RASB_{s,c,l}^t \leq capacity_{s,l} \quad \forall t, s, c, l \quad (58)$$

$$0 \leq RASD_{s,c,l}^t \leq capDel_{s,l} \quad \forall t, s, c, l \quad (59)$$

$$0 \leq INPB_{w,l}^t \leq capacity_{w,l} \quad \forall t, w, l \quad (60)$$

$$0 \leq INPB_{s,l}^t \leq capacity_{s,l} \quad \forall t, s, l \quad (61)$$

$$0 \leq INPB_{c,l}^t \leq capacity_{c,l} \quad \forall t, c, l \quad (62)$$

$$0 \leq INPD_{s,l}^t \leq capacity_{s,l} \quad \forall t, s, l \quad (63)$$

$$0 \leq INPD_{c,l}^t \leq capacity_{c,l} \quad \forall t, c, l \quad (64)$$

$$0 \leq SHORTUSE_{w,l}^t \leq safetyLevel_{w,l} * capacity_{w,l} \quad \forall t, w, l \quad (65)$$

$$0 \leq SHORTUSE_{s,l}^t \leq safetyLevel_{s,l} * capacity_{s,l} \quad \forall t, s, l \quad (66)$$

$$0 \leq SHORTUSE_{c,l}^t \leq safetyLevel_{c,l} * capacity_{c,l} \quad \forall t, c, l \quad (67)$$

$$0 \leq SHORTDel_{s,l}^t \leq safetyDel_{s,l} * capDel_{s,l} \quad \forall t, s, l \quad (68)$$

$$0 \leq SHORTDel_{c,l}^t \leq safetyDel_{c,l} * capDel_{c,l} \quad \forall t, c, l \quad (69)$$

$$INVw_{w,l}^t \leq capacity_{w,l} \quad \forall w,t,l \quad (70)$$

$$INVw_{s,l}^t \leq capacity_{s,l} \quad \forall s,t,l \quad (71)$$

$$INVw_{c,l}^t \leq capacity_{c,l} \quad \forall c,t,l \quad (72)$$

6. Discussion

Below is Colburn's (2015, pp. 22–23) discussion of the function of each of the equation from his formulation, updated to describe the function of the equations in the MC-DL-RASM.

The objective function (1) rewards hours spent on-station [not conducting a replenishment at sea] and penalizes any [commodity] shortages. It also includes a small reward for all [commodities] transfers; this reward helps to mitigate end-of-horizon effects in which ships' inventories are depleted. Constraint sets (2), (3), and (4) require that each warship, shuttle, and CLF ship, respectively, must leave a node in a period if and only if it entered a node in the same period; in other words, these constraints enforce balance of flow in the ship routing network. Constraint sets (5), (6), and (7) require that each ship begin in a single location. Constraint set [(8)] ensures that the number of ships visiting a port in any time period does not exceed that port's available berths. Constraint sets [(9), (10),] and (11) calculate the [useable commodity] inventory for warships, shuttles, and CLFs, respectively, based on each ship's prior inventory, current consumption, and any internal transfers, RAS transfers, and port transfers that occur. Constraints sets [(12), (13),] (14), (15), and (16) require that each ship be located at a port node in order to take on [consumable] or deliverable [commodities] from a port. Constraint sets [(17) and (18)] calculate shuttle and CLF deliverable [commodities] inventories, respectively, based on each ship's prior inventory and any internal transfers, RAS transfers, and port transfers that occur. Constraint sets [(19), (20), (22), and (23)] limit the number of RAS events that can be initiated at each ship in a single time period, while constraint sets [(21)] and [(24)] limit the total number of RAS events ongoing in a single time period for each shuttle and CLF. Constraints sets [(25), (26),] (27) and (28) ensure that all relevant ships are present at a RAS lane in order for a RAS event to occur. For instance, in order for a shuttle to replenish a warship, both the shuttle and the warship must be located at the same RAS lane during the time periods in which the RAS event is to occur, and likewise if a CLF is to replenish a shuttle. Constraint sets (29) and (30) ensure that [commodities] are only transferred between ships if the ships have successfully rendezvoused according to the binary *REVUW* and *REVUC* decision variable set in constraints sets [(25), (26),] (27), and

(28). Constraint sets [(31), (32), and (33)] calculate [consumable commodities] shortages, while constraint sets [(34)] and [(35)] calculate deliverable [commodities] shortages. Constraint set [(36)] determines the value of a binary decision that indicates whether a warship's burnable [DFM] level is below its safety fuel level; this decision variable is used in constraint set [(37)] to allow the ship to depart from its [SAG] if it is low on [DFM], and otherwise to require it to remain with its carrier. Constraint sets [(38), (39)] and (40) determine the value of a binary decision variable indicating whether each warship, shuttle and CLF ship respectively, begins a port visit in period t . Following the beginning of a port visit, constraint sets [(41), (42),] and (43) require that each ship to remain in port for its required number of time periods. Constraint sets [(44) to (72)] define decision variable domains and bound. (Colburn, 2015, pp. 22–23)

B. SCENARIO DEVELOPMENT

We use the same 7th Fleet scenario as Colburn (2015) for testing MC-DL-RASM. Our supply network consists of 16 nodes: 7 FRL nodes, 3 ARL nodes, and 6 port nodes. This network is depicted in Figures 2–5.

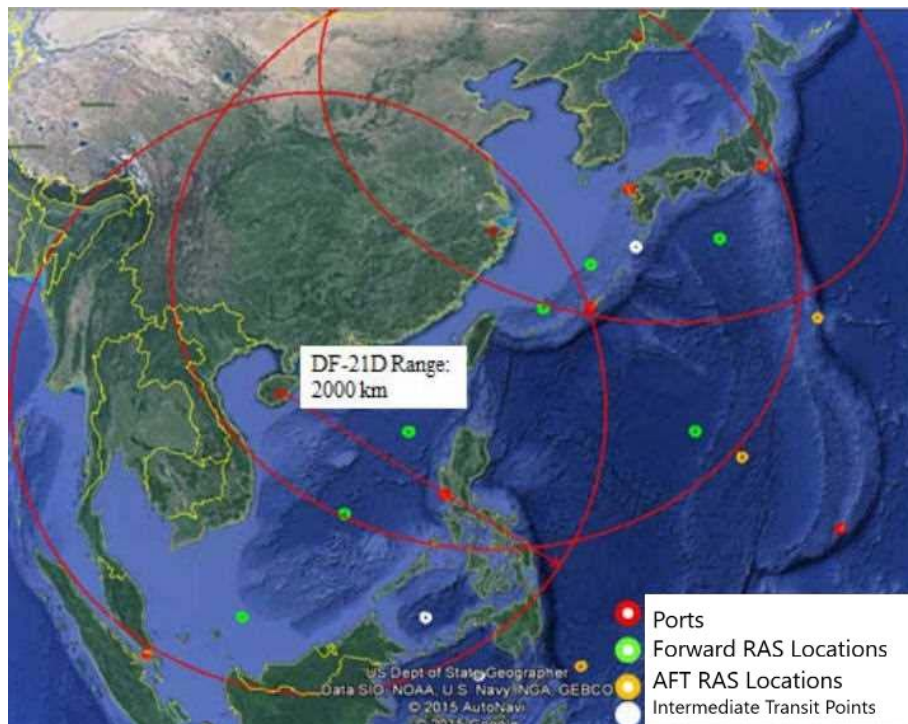


Figure 2. 7th Fleet network nodes and A2 threat area.
Source: Colburn (2015).

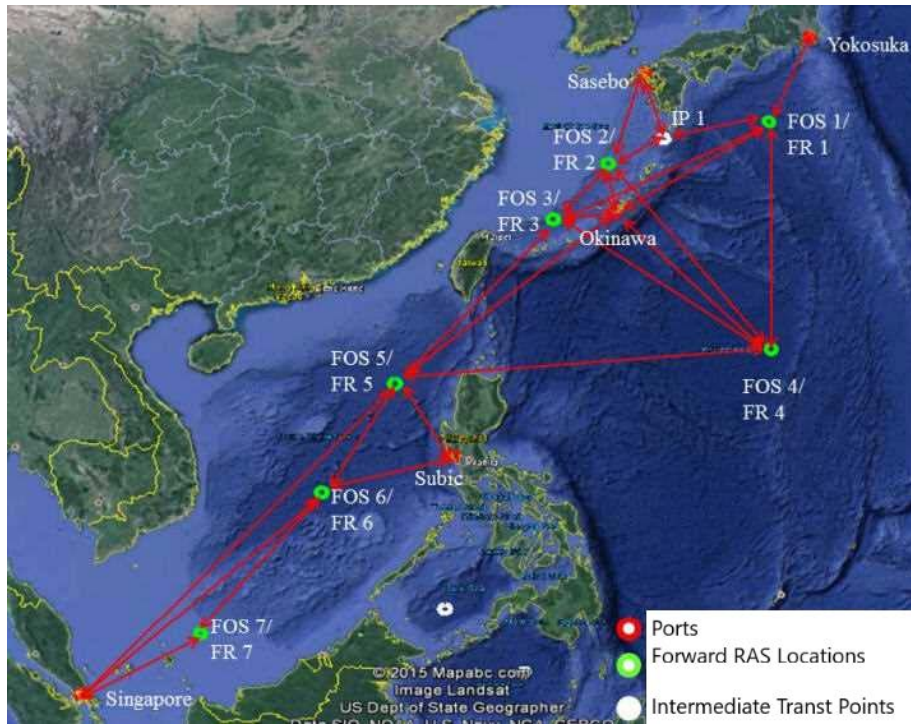


Figure 3. Warship node and arc network.
Source: Colburn (2015).

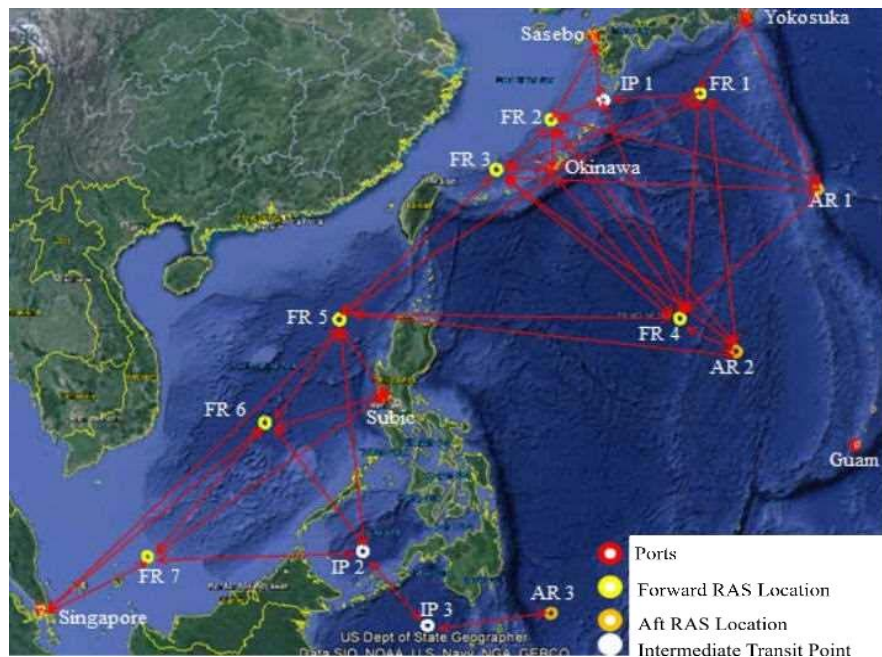


Figure 4. Shuttle node and arc network.
Source: Colburn (2015).

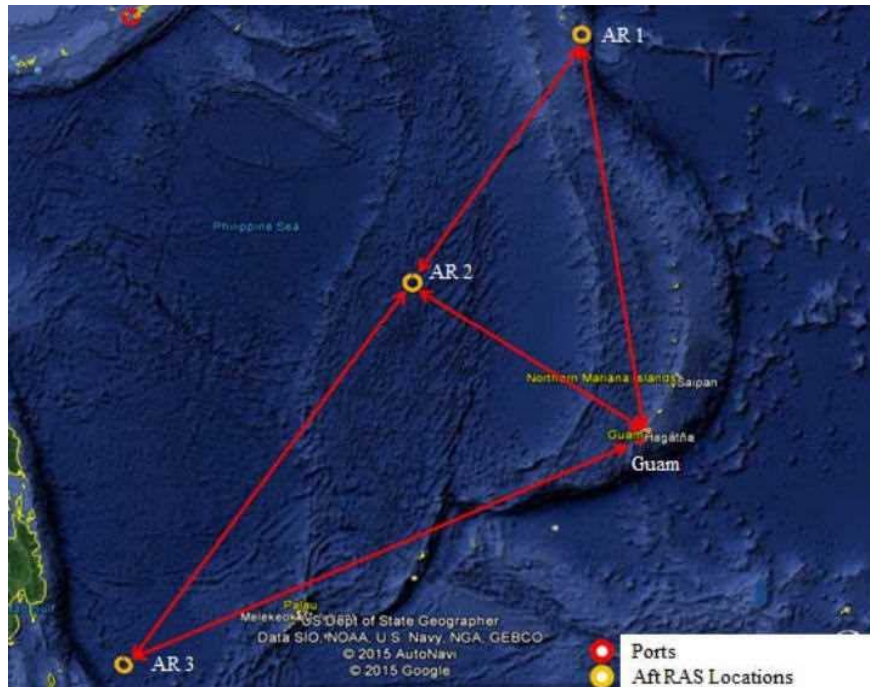


Figure 5. CLF node and arc network.
Source: Colburn (2015).

1. Assumptions

Colburn (2015) made the following assumptions in the development of his scenarios. We verify that these assumptions still hold for our scenarios.

- The CLF fleet consists of only T-AO and T-AKE class ships (one each). This also represents the typical CLF support package assigned to a deployed CTF/CRUDES SAG.
- All warships begin the scenario with fuel at 85% of capacity. This represents a fleet already performing sustained operations.
- All warships and CLF ships transit through the network at 15 knots (kts).
- We consider [two] distinct shuttle speeds in our scenarios.
- Warships, shuttles, and CLF ships occupy nodes as shown in Figures [3, 4, and 5].
- The DF-21D threat keeps large ships from safely operating in the South China Sea [as shown in Figure 2]. Large ships include the aircraft carrier, nuclear (CVN), amphibious assault ship [(multipurpose)] (LHD), auxiliary

dry cargo and ammunition ship (T-AKE), and fleet oiler (T-AO). This drives the design requirement for the production of a smaller shuttle ship to more safely operate within the DF-21D threat range.

- A deployed [SAG] requires resupply to continue operations at the [FRLs]. This drives the requirement for commodities to be transported through the network of arcs and nodes for consumption by the deployed [SAG]. (Colburn, 2015 pp. 3-4, 27-28)

2. Fleet Composition

Two SAGs constitute our combat forces. Our nominal SAG consists of three guided-missile destroyers (DDGs) and two guided-missile cruisers (CGs). Logistical support is provided by one T-AO and one T-AKE. We vary the number of “mini-CLF” shuttles from four to six as part of our sensitivity analysis. The designations for the ships in each SAG are shown in Table 1. DDG_1 to DDG_6 refer to DDGs, while CG_1 to CG_4 refer to CGs.

Table 1. Ship designations by SAG.

SAG 1	SAG 2
DDG_1	DDG_4
DDG_2	DDG_5
DDG_3	DDG_6
CG_1	CG_3
CG_2	CG_4

Whereas Colburn’s scenario used a CTF and a SAG as combat forces (Colburn, 2015), we replace the CTF with a SAG. This is done to meet the intent of the restriction on large ships operating in the South China Sea. The size of shuttle ship required to support a CTF is substantially different from that required to support a SAG. Table 2 shows the difference in deliverable capacities for a shuttle ship supporting a SAG and a CTF; detailed calculations of these numbers appear in the Appendix.

Table 2. Shuttle capacities by commodity type for SAG or CTF resupplying every 4 to 5 days performing normal operations.

Commodities	SAG	CTF
DFM	22,000 bbls	22,000 bbls
JP5	850 bbls	26,000 bbls
Stores	75 ton	400 ton
Ammunition	15 ton	150 ton

3. SAG Schedule

We use the same operating schedule as Colburn (2015). The schedule is designed to represent “show of force” operations. Both SAGs start in their assigned operating area and are required to be at various waypoints no later than the time periods listed in Table 3.

Table 3. Waypoint schedule. Adapted from Colburn (2015).

Time Period	Battle Group	Location
1	SAG 1	FWD_RAS_4
1	SAG 2	FWD_RAS_5
42	SAG 2	FWD_RAS_6
60	SAG 1	FWD_RAS_2
90	SAG 2	FWD_RAS_3
120	SAG 1	FWD_RAS_5
150	SAG 1	FWD_RAS_6
168	SAG 2	FWD_RAS_1

4. Planning Factors

We developed appropriate planning factors using the same method as Colburn (2015), and verified them against the NWP 4-01.2 (CNO, 2007). These planning factors are listed in Tables 4–6.

Table 4. Usage rates for each ship (4-hour time period).

Ship Class	DFM (bbls/period)	JP5 (bbls/period)	Stores (ton/period)	Ammunition (ton/period)
DDG	108.0	4.25	0.300	0.03
CG	126.2	4.25	0.300	0.10
T-AKE	160.0	1.70	0.167	0
T-AO	160.0	1.70	0.167	0
Shuttle	160.0	1.70	0.167	0

Table 5. Consumable capacity for each ship.
Adapted from NWP 4-01.2 (CNO, 2007).

Ship Class	DFM (bbls)	JP5(bbls)	Stores (ton)	Ammunition (ton)
DDG	10,518	475	55	48
CG	15032	475	68	94
T-AO	14453	505	20	0
T-AKE	31494	593	20	0
Shuttle	11905	500	20	0

Table 6. Deliverable capacity for each ship. Adapted from NWP 4-01.2 (CNO, 2007).

Ship Class	DFM (bbls)	JP5 (bbls)	Stores (ton)	Ammunition (ton)
T-AO	90,000	90,000	220	0
T-AKE	17,000	7,000	1,300	4,900

a. Ports

The number of ports will be varied by scenario in Chapter IV.

b. Penalties and Rewards

Table 7 lists the rewards and penalty factors for our model.

Table 7. Penalty and reward values. Adapted from Colburn (2015).

<i>reward</i> (reward/ship * period)	ϵ (reward/barrel) (reward/ton)	$penalty_{warship,l}$ (penalty/barrel) (penalty/ton)	$penalty_{shuttle,l}$ (penalty/barrel) (penalty/ton)	$penalty_{CLF,l}$ (penalty/barrel) (penalty/ton)
100	0.00001	100	10	1

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IV. ANALYSIS

We now exercise MC-DL-RASM on a set of problem instances in order to examine its performance and gain insight into the effectiveness of the mini-CLF concept in a multicommodity setting. For each problem instance, we record several figures of merit: the amount of time warships spend on station, the amount of time warships' inventories are below prescribed safety levels, and the lowest level each commodity reaches on each ship. We also compare the time spent on station to a baseline value that is calculated by allowing the warships to operate without resupplying. While this baseline could never be achieved in practice, it provides a bound on the level of performance that could be achieved by any resupply strategy.

We consider three strategic situations: Peacetime, Rising Tensions, and Wartime. We assume that during strategic situations other than Peacetime, our usage of foreign ports may be restricted or completely curtailed. Thus, each strategic situation is associated with a different number of berths available in the ports, as shown in Table 8.

Table 8. Available berths in each port for each strategic situation.

Ports	Peacetime	Rising Tensions	Wartime
Guam	4	4	4
Yokosuka	8	1	0
Sasebo	2	1	0
Okinawa	2	1	0
Subic Bay	2	0	0
Singapore	2	0	0

We implement MC-DL-RASM using the Pyomo (Hart, Laird, Woodruff, Hackebeil, Nicholson, & Sirola, 2017), (Hart, Watson, & Woodruff, 2011) package of the Python language (version 3.6.0, Python Software Foundation, 2001) with CPLEX Interactive Optimizer version 12.7.1.0 (IBM) as our solver. We use a rolling time horizon

approach to model imperfect information in real operations, as well as for computational efficiency. Additionally, we used an optimization gap of 10%. Using these settings, a typical problem instance has 277,903 rows and 199,511 columns and requires 49.5 hours to solve. We now examine the performance achieved with various shuttle configurations, including the number of shuttles available and their speed, as well as different strategic situation, as shown in Table 9.

Table 9. Planning factors for each scenario.

Scenario Number	Strategic Situation	Number of Shuttles	Shuttle Speed (kts)	Planning Horizon (periods per iteration)	Execution Horizon (periods)
1	Rising Tensions	4	15	60	30
2	Peacetime	4	20	60	30
3	Wartime	4	15	90	30
4	Rising Tensions	6	20	90	30

A. SCENARIO 1: RISING TENSIONS, 4 SHUTTLES AVAILABLE, SPEED 15 KTS

We first consider Scenario 1. In this excursion, the shuttles and CLF ships begin with their useable and deliverable inventories filled to capacity. Although this is an unlikely starting inventory for ships already in theater, it provides a point of comparison for future scenarios. We consider a total time horizon of 180 4-hour periods (30 days). For our rolling horizon implementation, we use a planning horizon of 60 time periods and an execution horizon of 30 time periods. This means that on rolling horizon iteration 1, we optimize over time periods 1 to 60. On iteration 2, we fix the decision variables for time periods 1 to 30 and optimize over time periods 31 to 90. On iteration 3, we fix the decision variables for time periods 1 to 60 and optimize over time periods 61 to 120. On iteration 4, we fix the decision variables for time periods 1 to 90 and optimize over time periods 91 to 150. Finally, on iteration 5, we fix the decision variables for time periods 1 to 120 and optimize over time periods 121 to 180.

MC-DL-RASM’s results indicate that the CLFs and shuttles can sustain the warships in SAG 1 on station for 77.2% to 77.8% of the time total time horizon. This is 3.9% to 4.4% less time than the baseline, with the entire discrepancy accounted for by RAS events. SAG 2 is on station for 75.6% to 76.7% of the time. This is 3.9% to 5.0% less time than the baseline, and again the remaining time is spent performing RAS events. These numbers are summarized in Tables 10–11.

Table 10. Percent of time performing different operations for SAG 1 (Scenario 1).

Ship	On Station	Transiting	RASing
CG_1	77.8 %	18.3 %	3.9 %
CG_2	77.8 %	18.3 %	3.9 %
DDG_1	77.8 %	18.3 %	3.9 %
DDG_2	77.8 %	18.3 %	3.9 %
DDG_3	77.2 %	18.3 %	4.4 %
Baseline	81.7 %	18.3 %	0%

Table 11. Percent of time performing different operations for SAG 2 (Scenario 1).

Ship	On Station	Transiting	RASing
CG_3	75.6 %	19.4 %	5.0 %
CG_4	76.7 %	19.4 %	3.9 %
DDG_4	75.6 %	19.4 %	5.0 %
DDG_5	76.1 %	19.4 %	4.4 %
DDG_6	76.7 %	19.4 %	3.9 %
Baseline	80.6 %	19.4 %	0%

Two warships’ DFM inventories fall below the 70% safety level. CG_1’s DFM inventory falls below the 70% safety level in time period 75 and reaches its lowest level (65.0%) in time period 80. DDG_1’s DFM inventory falls below the 70% safety level in time period 69, with a level of 69.0%.

All of the CGs' JP5 inventories fall below the 70% safety level. CG_1 spends the most time under the 70% safety level. It is below the safety level from time period 73 to time period 80 and again in time period 134. The lowest level CG_1 reaches is 63.3% in time period 80. CG_2 spends one time period under the 70% safety level: in time period 62 its inventory level falls to 69.6%. CG_3 spends four time periods under the 70% safety level; time periods 97, 98, 164, and 165. In both time periods 98 and 165 its inventory level reaches 68.7%. CG_4 spends two time periods under the 70% safety level, time periods 158 and 159. In time period 159 its inventory level reaches 68.7%. These levels are depicted in Figures 6–13.

No ship's inventory of stores or ammunition reaches its safety level in this excursion. Table 12 summarizes the percent of time that each warship commodity level falls below the safety level.

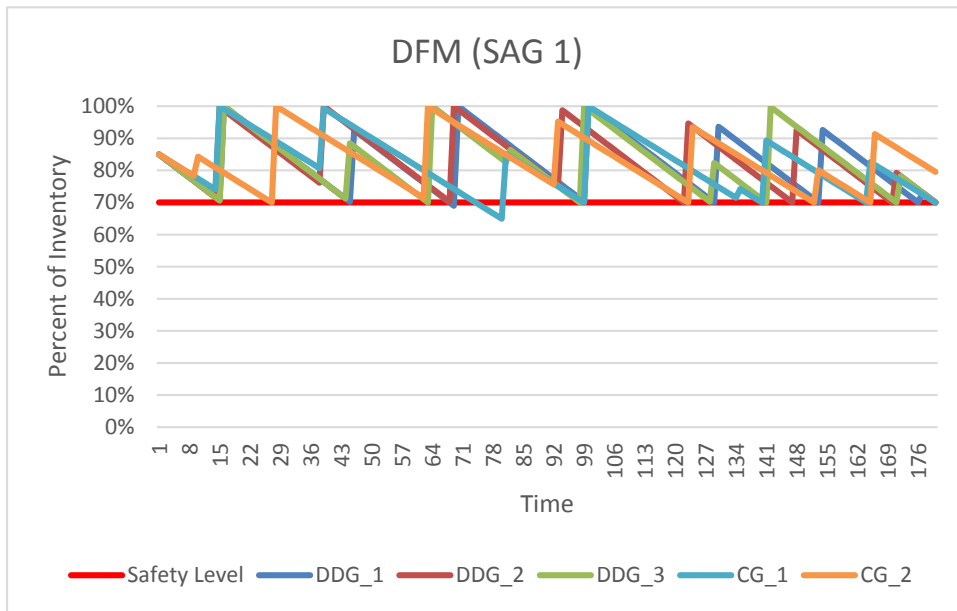


Figure 6. DFM inventory for SAG 1 (Scenario 1).

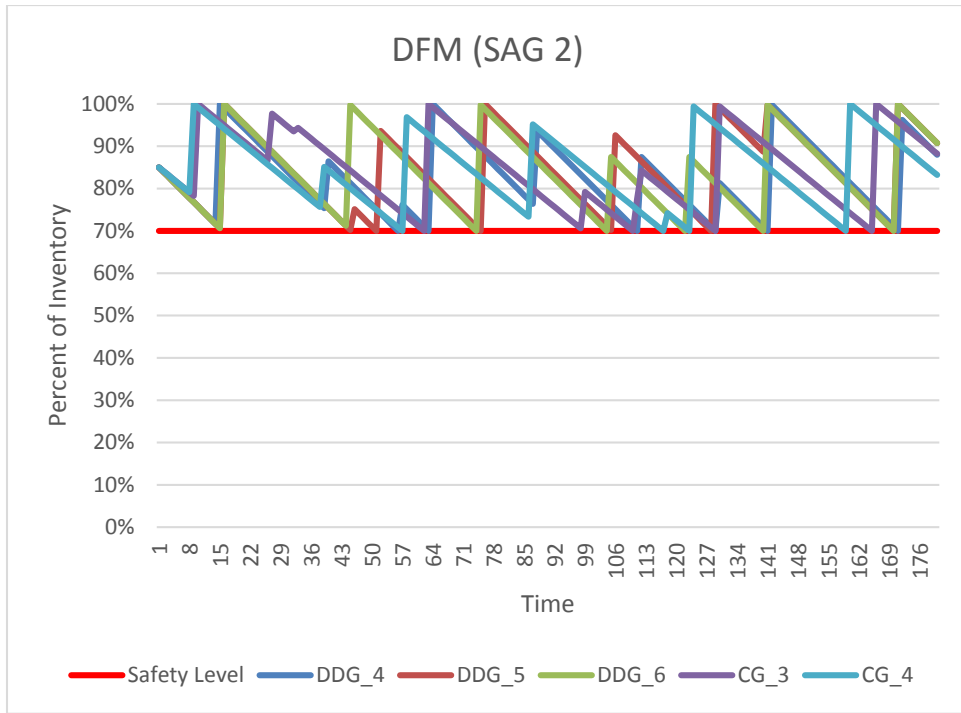


Figure 7. DFM inventory for SAG 2 (Scenario 1).

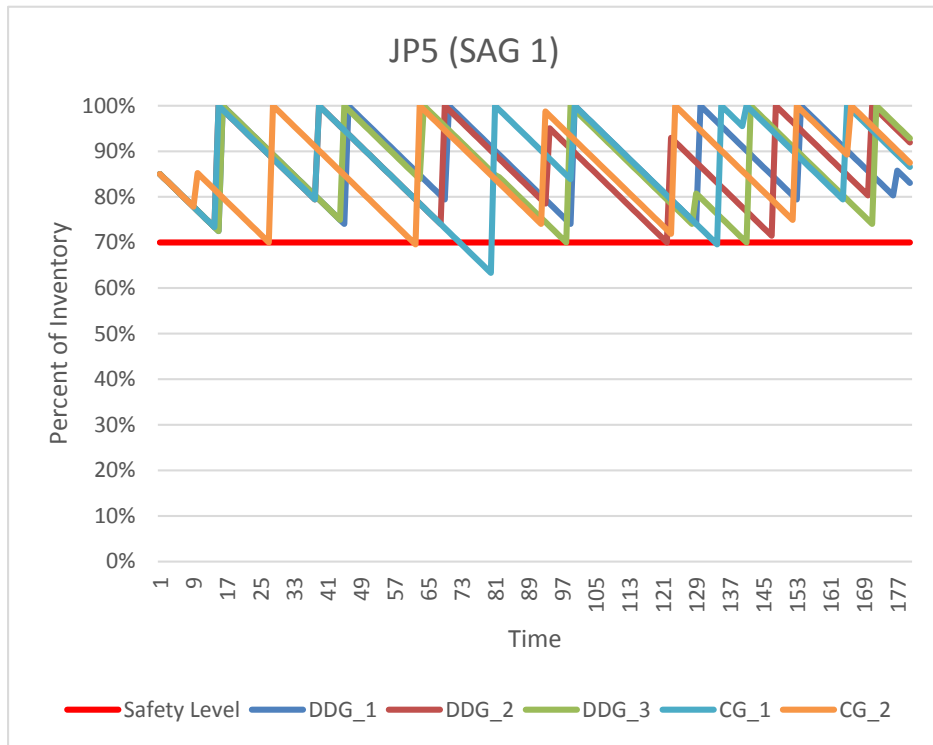


Figure 8. JP5 inventory for SAG 1 (Scenario 1).

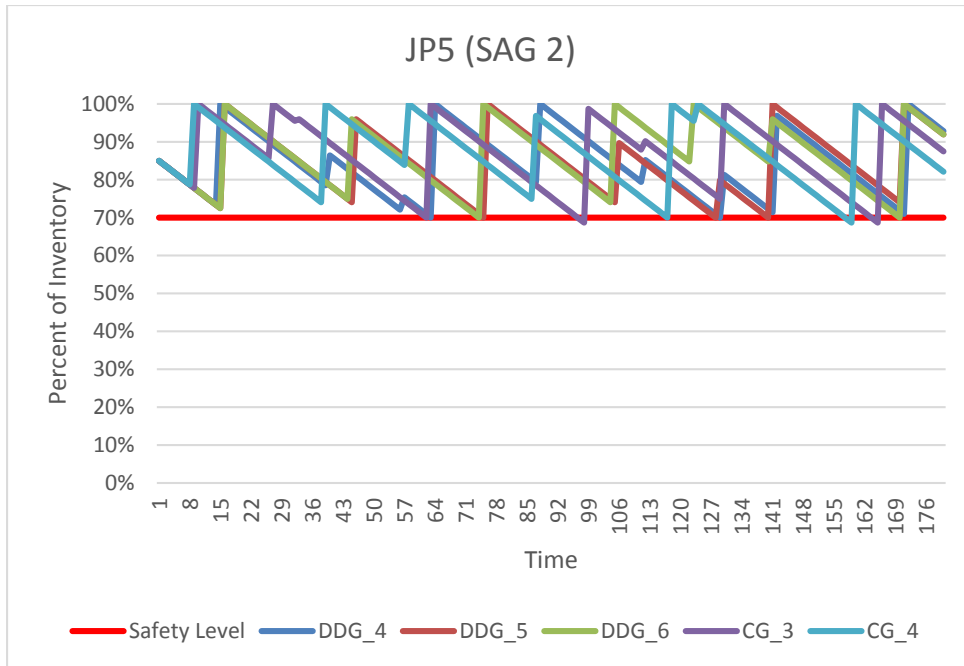


Figure 9. JP5 inventory for SAG 2 (Scenario 1).



Figure 10. Stores inventory for SAG 1 (Scenario 1).

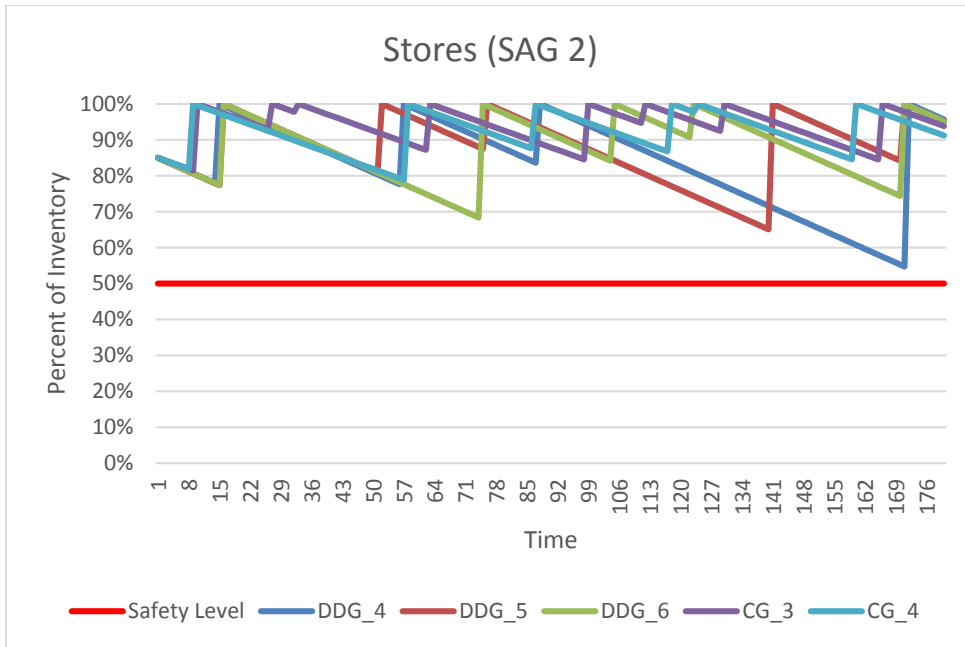


Figure 11. Stores inventory for SAG 2 (Scenario 1).

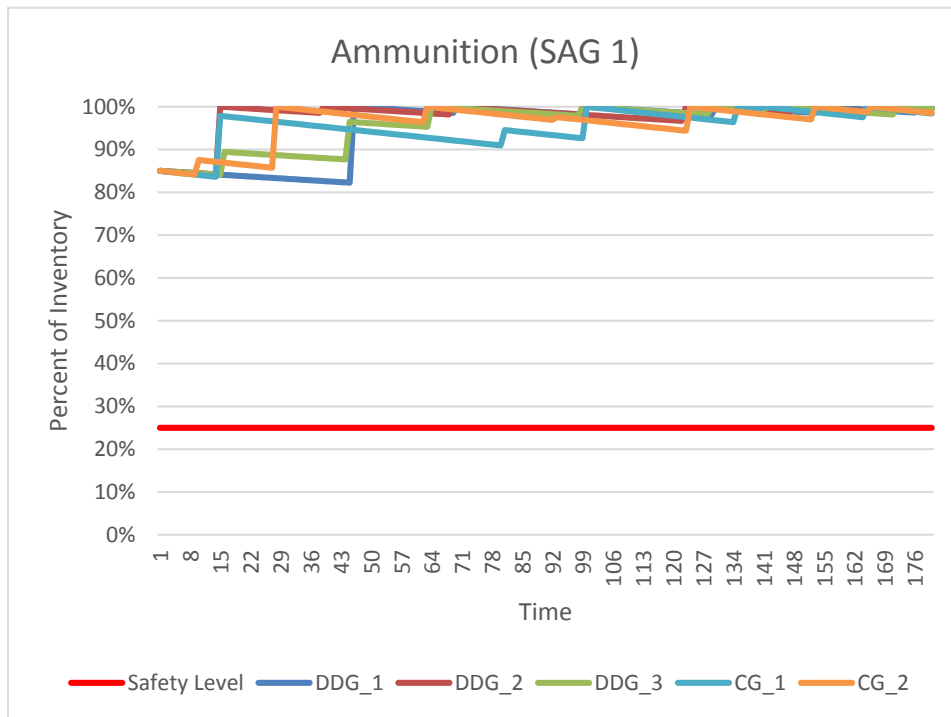


Figure 12. Ammunition inventory for SAG 1 (Scenario 1).

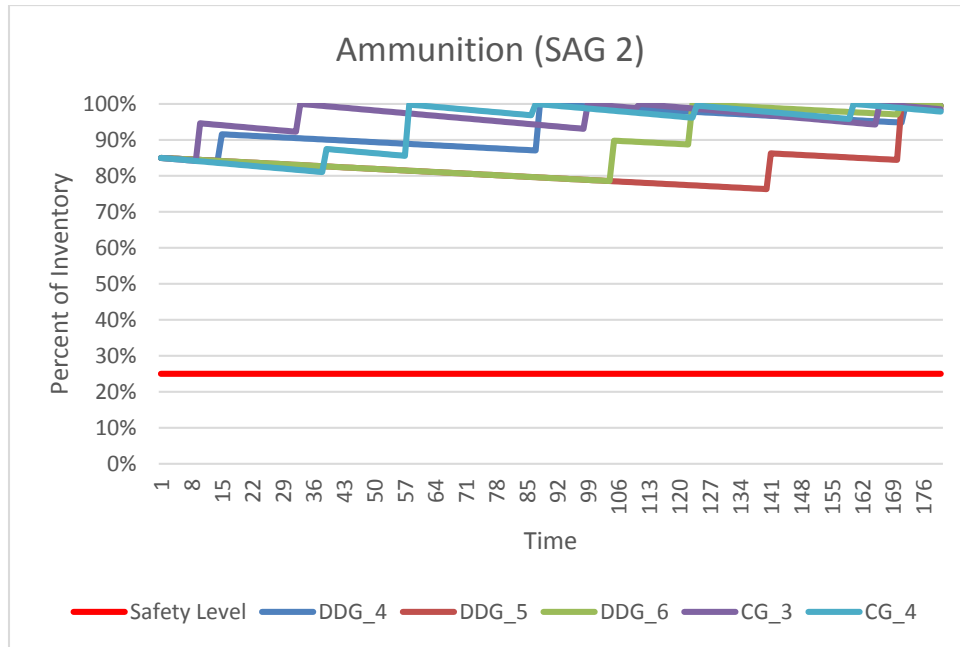


Figure 13. Ammunition inventory for SAG 2 (Scenario 1).

Table 12. Percent of time below safety level for each commodity and warship (Scenario 1).

Ship	DFM	JP5	Stores	Ammunition
CG_1	3%	5%	0%	0%
CG_2	0%	1%	0%	0%
CG_3	0%	2%	0%	0%
CG_4	0%	1%	0%	0%
DDG_1	1%	0%	0%	0%
DDG_2	0%	0%	0%	0%
DDG_3	0%	0%	0%	0%
DDG_4	0%	0%	0%	0%
DDG_5	0%	0%	0%	0%
DDG_6	0%	0%	0%	0%

B. SCENARIO 2: PEACETIME, 4 SHUTTLES AVAILABLE, SPEED 20 KTS

We now consider Scenario 2. As in the previous excursion, the shuttles and CLF ships begin with full useable and deliverable inventories. We consider a total time horizon of 180 4-hour periods (30 days), using the same rolling horizon method as Scenario 1.

We find that the shuttles and CLF ships can sustain the warships in SAG 1 on station for 76.7% to 77.8% of the time. This is 3.9% to 5.0% less time than the baseline, with the entire discrepancy accounted for by RAS events. SAG 2 is on station for 43.9% to 58.9% of the time. This is 21.7% to 36.7% less time than the baseline, with only 1.7% to 3.9% of this account for by RAS events. The rest of this time is accounted for by SAG 2 transiting to and visiting Yokosuka to resupply. These results are summarized in Tables 13–14.

It is interesting to compare the results of the current scenario with those of the previous scenario. In the current scenario we utilize the same number of shuttles as in the previous scenario, and the shuttles can transit at faster speeds. Additionally, more berths are available in the ports under the current Peacetime strategic situation. Thus, we would expect to achieve better performance in the current scenario than in the previous one. In actuality, we observe that SAG 1 achieves a comparable amount of time on station as in the previous scenario, while the warships of SAG 2 spend more time off station as four of the five warships visit Yokosuka to resupply. This behavior occurs due to the rolling time horizon implementation and is also apparent in the objective values achieved in each scenario. In this framework, decisions made early in the overall time horizon can have unanticipated consequences in later iterations. Thus, we recommend that future work include a “final solve” that encompasses the overall 180-period time horizon and uses the rolling horizon solution as an initial solution.

Table 13. Percent of time performing different operations for SAG 1 (Scenario 2).

Ship	On Station	Transiting	RASing
CG_1	77.8 %	18.3 %	3.9 %
CG_2	76.7 %	18.3 %	5.0 %
DDG_1	77.8 %	18.3 %	3.9 %
DDG_2	77.8 %	18.3 %	3.9 %
DDG_3	77.2 %	18.3 %	4.4 %
Baseline	81.7 %	18.3 %	0.0 %

Table 14. Percent of time performing different operations for SAG 2 (Scenario 2).

Ship	On Station	Transiting	RASing	In Port
CG_3	43.9 %	26.7 %	1.7 %	27.8 %
CG_4	46.1 %	23.9 %	2.8 %	27.2 %
DDG_4	46.1 %	51.1 %	2.8 %	28.3 %
DDG_5	45.6 %	52.2 %	2.2 %	28.9 %
DDG_6	58.9 %	37.2 %	3.9 %	0.0 %
Baseline	81.7 %	18.3 %	0.0 %	0.0 %

All but two warships' DFM inventories fall below the 70% safety level; only CG_1 and DDG_2 have DFM inventories that remain above the 70% safety level. For SAG 1, CG_2 and DDG_3's DFM inventories only fall below the 70% safety level during one time period each, in time periods 93 and 98, respectively. The lowest inventory level for each is 69.8% and 69.0%. DDG_1 spends the most time under the 70% safety level of all the ships in SAG 1. It is below the safety level for a total of six time periods: periods 88 to 92 and 99. The lowest level DDG_1 reaches is 65.1% in time period 92. For SAG 2 CG_3 spends seven time periods under the 70% safety level. It is below the safety level in time periods 106 to 112 and reaches its lowest level (65.6%) in time period 112. CG_4 spends six time periods under the 70% safety level. It is below the safety level during time periods 106 to 111 and reaches its lowest level (65.8%) in time period 111. DDG_4 spends eleven time

periods under the 70% safety level. It is below the safety level in time periods 100 to 110 and reaches its lowest level (59.5%) in time period 110. DDG_5 spends twelve time periods under the 70% safety level. It is below the safety level in time periods 99 to 110 and reaches its lowest level (58.7%) in time period 110. DDG_6 spends eight time periods under the 70% safety level. It is below the safety level in time periods 158 to 165 and reaches its lowest level (61.8%) in time period 165.

All of the warships' JP5 inventories fall below the 70% safety level, except those of CG_4 and DDG_2. In SAG 1, CG_1's, DDG_1's and DDG_3's inventories only fall below the 70% safety level for one or two time periods. DDG_1 falls below the safety level in time period 92. The lowest level DDG_1 reaches is 69.6% in time period 92. CG_1 falls below safety level in time period 98, with 69.1% inventory. DDG_3 spends two time periods (97 and 98) under the 70% safety level, with a minimum level of 68.2%. CG_2 spends time periods 91 to 93 under the 70% safety level, and its lowest level is 67.8% in period 93. In SAG 2, DDG_4 spends nine time periods under the 70% safety level. It is below the safety level in time periods 102 to 110 and reaches its lowest level (62.8%) in time period 110. DDG_5 spends time periods 98 to 110 under the 70% safety level and reaches its lowest level (59.7%) in time period 110. DDG_6 spends three time periods under the 70% safety level. It is below the safety level in time periods 86, 87, and 165. It reaches its lowest level (68.2%) in time period 87. CG_3 spends thirteen time periods under the 70% safety level. It is below the safety level in time periods 50, 51, 104 to 112, 133, and 138. It reaches its lowest level (63.3%) in time period 112. All inventory levels are depicted in Figures 14–21. Table 15 summarizes the percent of time that each warship commodity level falls below the safety level.

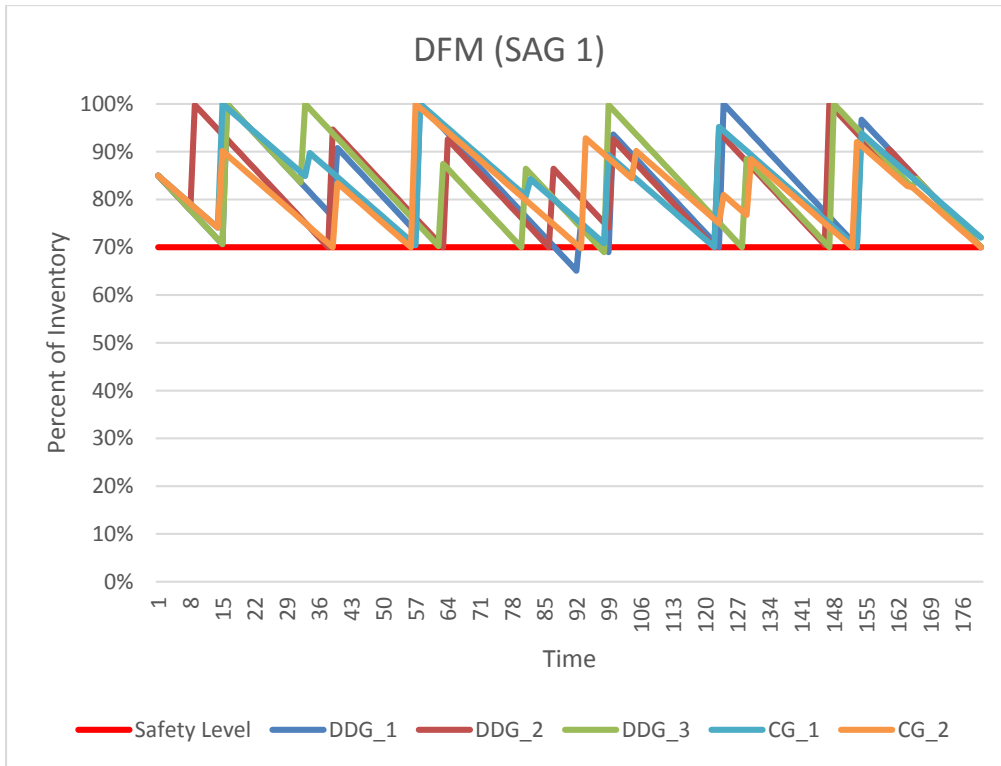


Figure 14. DFM inventory for SAG 1 (Scenario 2).

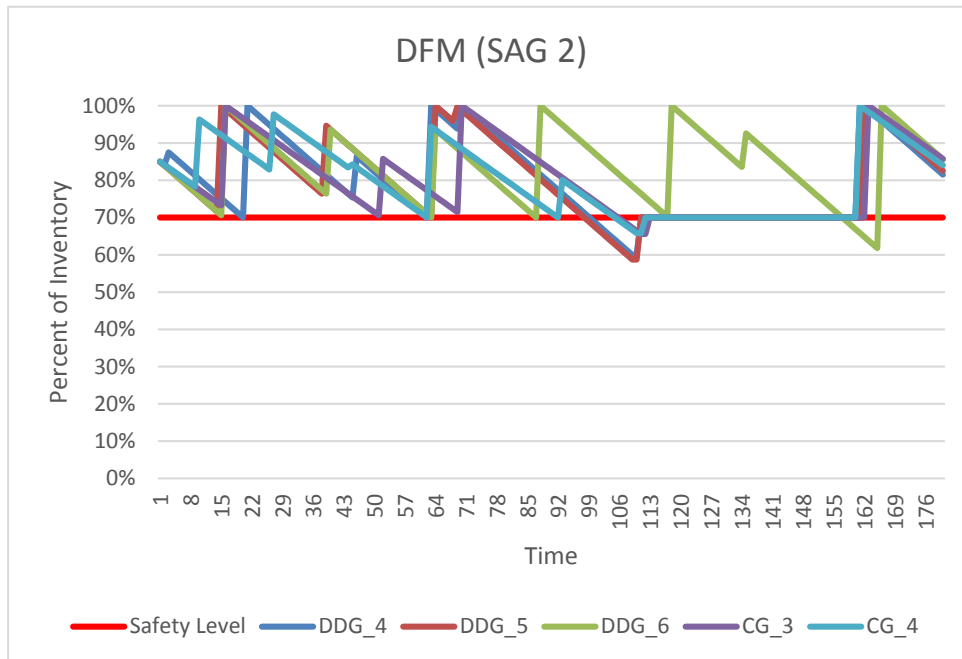


Figure 15. DFM inventory for SAG 2 (Scenario 2).



Figure 16. JP5 inventory for SAG 1 (Scenario 2).

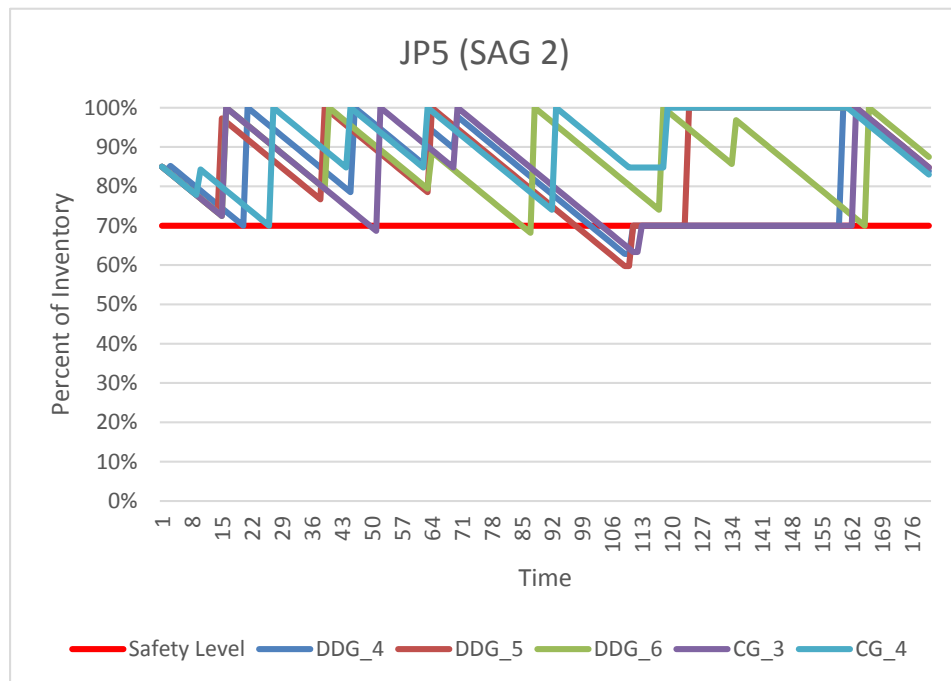


Figure 17. JP5 inventory for SAG 2 (Scenario 2).



Figure 18. Stores inventory for SAG 1 (Scenario 2).

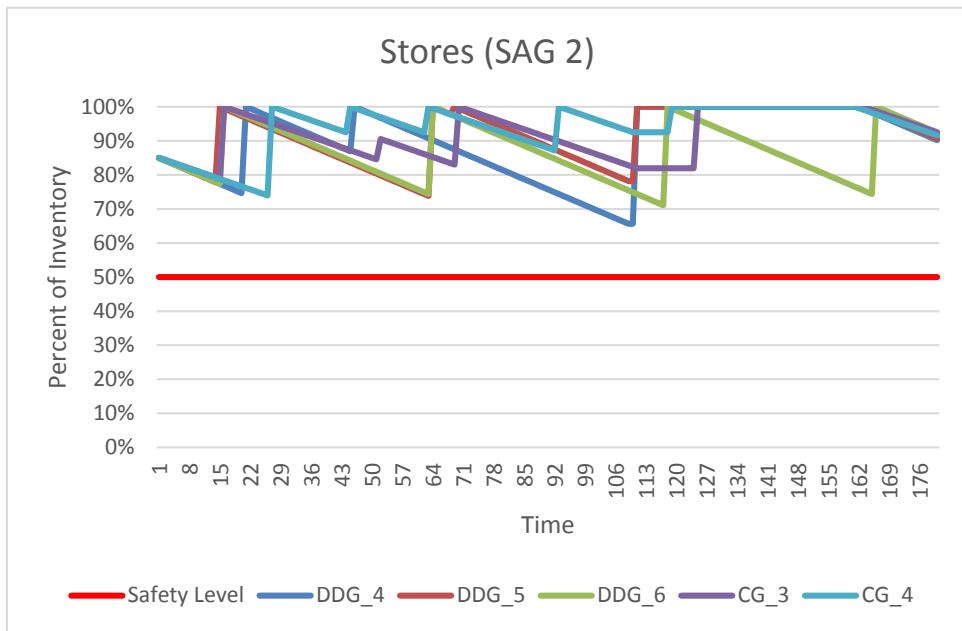


Figure 19. Stores inventory for SAG 2 (Scenario 2).

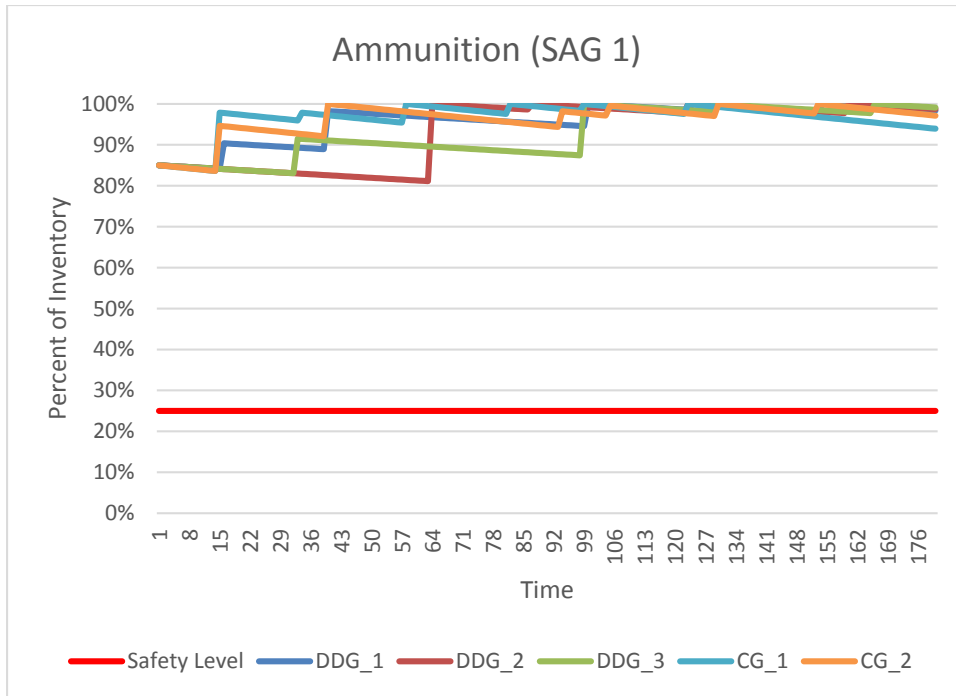


Figure 20. Ammunition inventory for SAG 1 (Scenario 2).

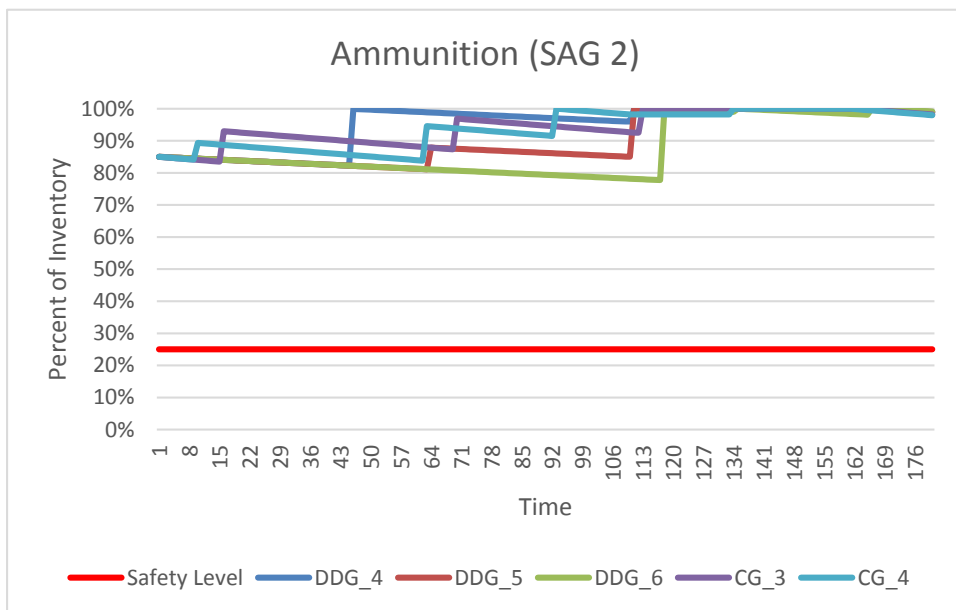


Figure 21. Ammunition inventory for SAG 2 (Scenario 2).

Table 15. Percent of time below safety level for each commodity and warship (Scenario 2).

Ship	DFM	JP5	Stores	Ammunition
CG_1	0%	1%	0%	0%
CG_2	1%	2%	0%	0%
CG_3	4%	7%	0%	0%
CG_4	3%	0%	0%	0%
DDG_1	3%	1%	0%	0%
DDG_2	3%	0%	0%	0%
DDG_3	1%	1%	0%	0%
DDG_4	6%	5%	0%	0%
DDG_5	7%	7%	0%	0%
DDG_6	4%	2%	0%	0%

C. SCENARIO 3: WARTIME, 4 SHUTTLES AVAILABLE, SPEED 15 KTS

We now consider Scenario 3. In contrast to the previous two excursions, we now initialize the shuttle and CLF ships with 80% useable and deliverable inventories. These are realistic starting conditions to model ships that have been underway for a period of time prior to the start of the model run. We consider a total time horizon of 150 4-hour periods (25 days). On iteration 1, we optimize over time periods 1 to 90. On iteration 2, we fix the decision variables for time periods 1 to 30 and optimize over time periods 31 to 120. On iteration 3 we fix the decision variables for time periods 37 to 60 and optimize over time periods 61 to 150.

With these settings, we find that the shuttles’ burnable and deliverable DFM inventories and the CLF ships’ deliverable DFM inventories all reach zero in time period 150 (day 25). These inventory levels are depicted in Figures 22–24.

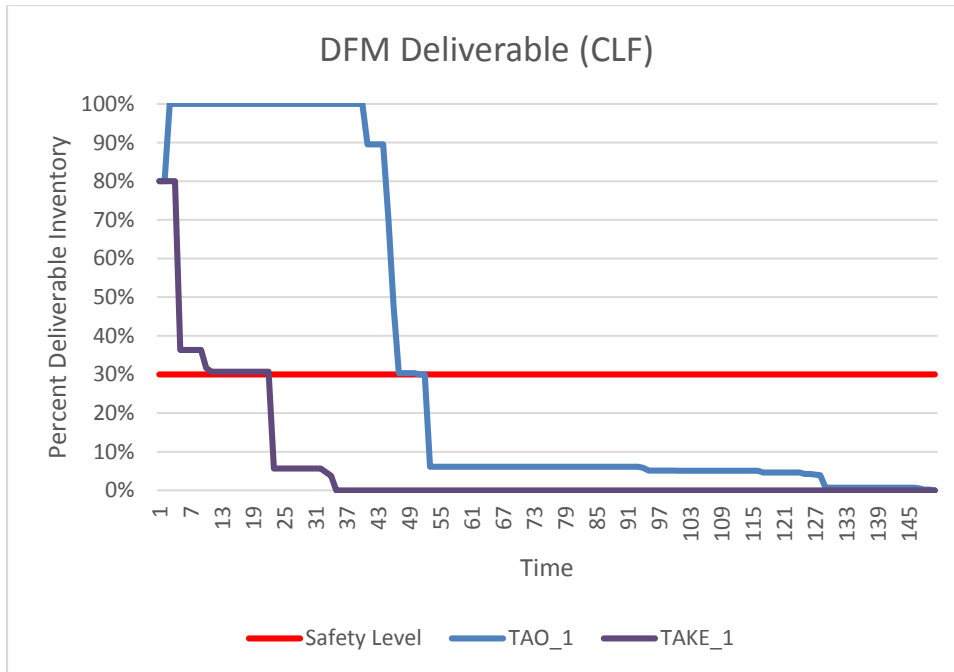


Figure 22. CLF DFM deliverable inventory (Scenario 3).

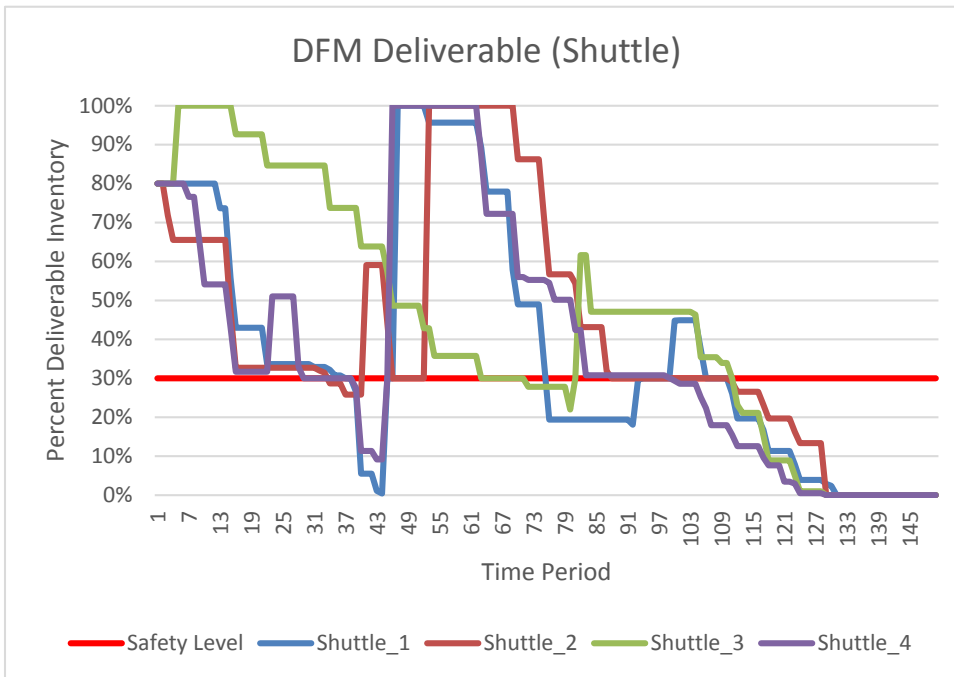


Figure 23. Shuttle DFM deliverable inventory (Scenario 3).

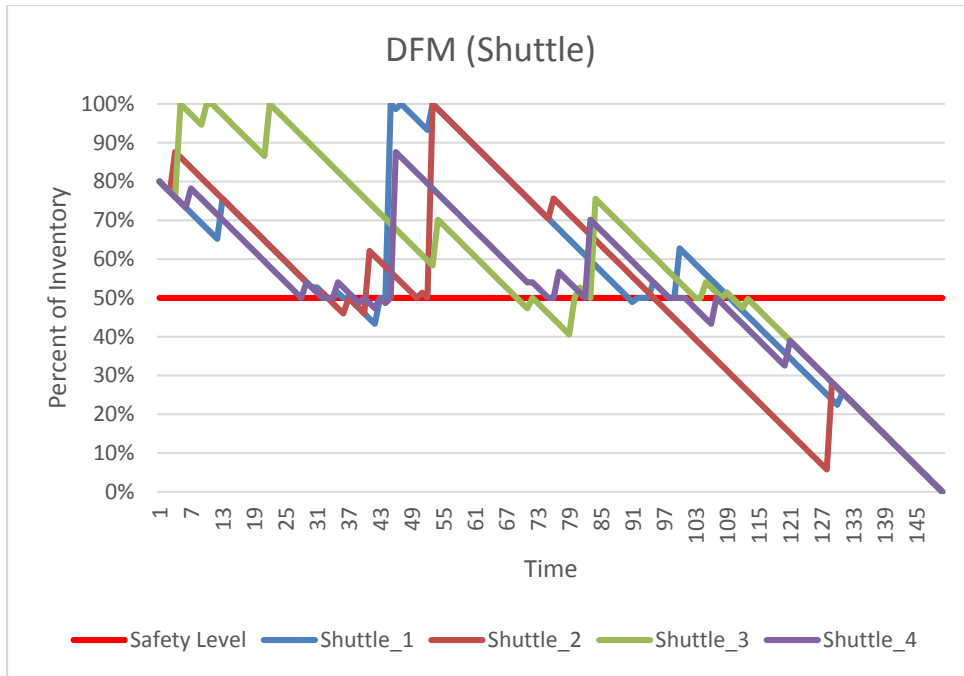


Figure 24. Shuttle DFM consumable inventory (Scenario 3).

This is a very interesting result, and a significant contrast with our previous excursions. To understand it, we now take a closer look at the network configuration and its implications for long-term operations. In practice, the time required for a T-AO to make the round trip between one of the ARLs and Guam would be at least 38 time periods and at most 56 time periods, depending on the ARL. During this time the shuttles would be unable to get additional supplies. Moreover, our results indicate that the CLF ships spend 63.3% of their time at ARL 2, and all RAS events between CLF ships and shuttle ships take place at ARL 2. The round-trip time for a CLF ship to visit Guam from ARL 2 is 38 time periods. The shuttles can make the trip from ARL 2 to FRL 5 (the furthest point directly served from ARL 2) in 19 time periods. Furthermore, we would need one shuttle's worth of DFM to be able to resupply all warships in a SAG after the CLF ship returned to ARL 2. When we account for the amount of DFM used by both SAGs and the four shuttles, it would take 1.04 T-AOs in order to resupply them. Thus, conducting sustained operations with one T-AO is not feasible.

Prior to the shuttles running out of DFM, the warships of SAG 1 were on station for 72.0% to 74.0% of the time, which is 4.0% to 6.0% less than the baseline. All of this

difference is accounted for by RAS events. SAG 2 was on station for 69.3% to 72.7% of the time which is 12.0% to 15.3% less than the baseline. Of this difference, 4.0% to 7.3% is accounted for by RAS events. The rest is accounted for by the fact that SAG 2 performed an additional move from FRL 3 to FRL 4 in order to reduce the distance the shuttle had to traverse in order to resupply the SAG. The baseline for SAG 1 is 78%, and 84.7% for SAG 2. These numbers are summarized in Tables 16–17.

Table 16. Percent of time performing different operations for SAG 1 (Scenario 3).

Ship	On Station	Transiting	RASing
CG_1	73.3 %	22.0 %	4.7 %
CG_2	74.0 %	22.0 %	4.0 %
DDG_1	72.7 %	22.0 %	5.3 %
DDG_2	72.7 %	22.0 %	5.3 %
DDG_3	72.0 %	22.0 %	6.0 %
Baseline	78.0%	22.0 %	0.0 %

Table 17. Percent of time performing different operations for SAG 2 (Scenario 3).

Ship	On Station	Transiting	RASing
CG_3	70.0 %	23.3 %	6.7 %
CG_4	71.3 %	23.3 %	5.3 %
DDG_4	71.3 %	23.3 %	5.3 %
DDG_5	69.3 %	23.3 %	7.3 %
DDG_6	72.7 %	23.3 %	4.0 %
Baseline	84.7 %	15.3 %	0.0 %

All of the warships' DFM inventories fall below the 70% safety level for the remaining time periods beginning in time period 129 or 130. Both DDG_3 and DDG_6 reach low levels of 48% in time period 150. The JP5 inventory levels for the warships in SAG 1 reach the safety level in time period 150, and only the CG_3 and DDG_5 from SAG 2 do not reach the safety level prior to time period 150. The stores and ammunition inventory levels of the SAGs never reach their safety levels. These levels are depicted in

Figures 25–32. Tables 18–19 summarize the percent of time that each warship commodity level falls below the safety level.

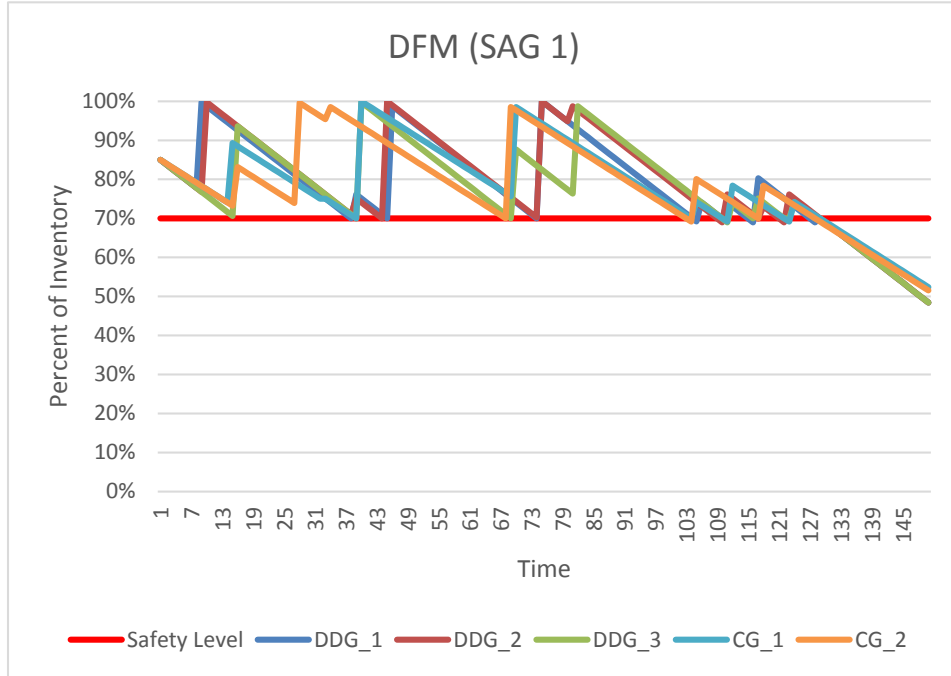


Figure 25. DFM inventory for SAG 1 (Scenario 3).

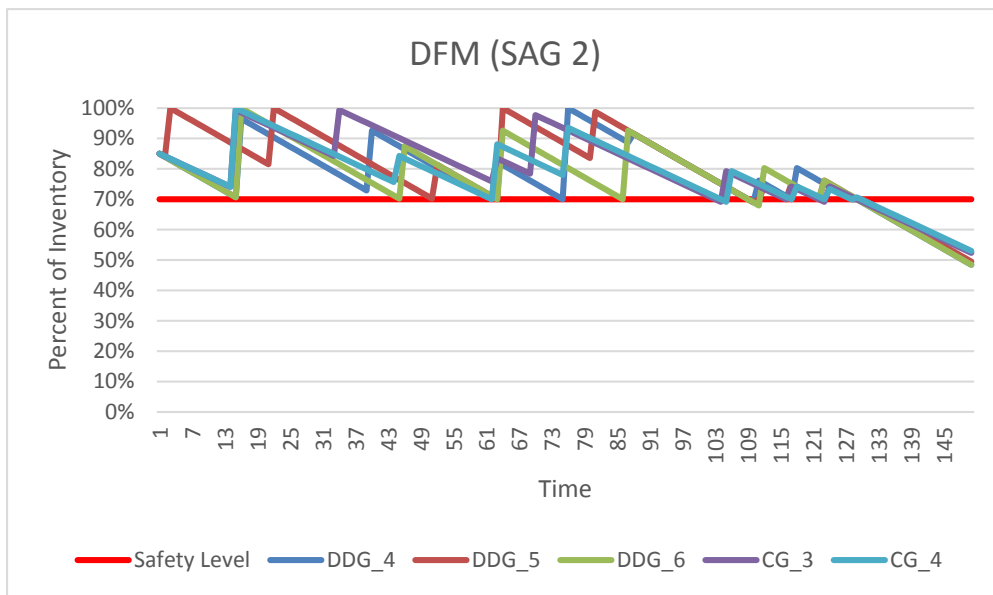


Figure 26. DFM inventory for SAG 2 (Scenario 3).

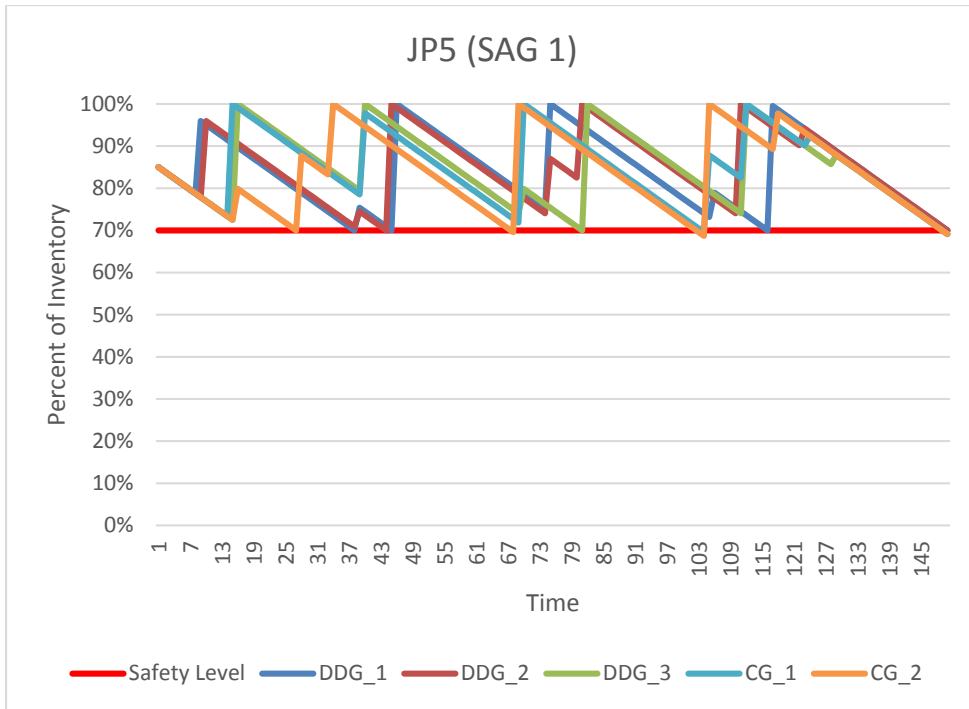


Figure 27. JP5 inventory for SAG 1 (Scenario 3).



Figure 28. JP5 inventory for SAG 2 (Scenario 3).



Figure 29. Stores inventory for SAG 1 (Scenario 3).

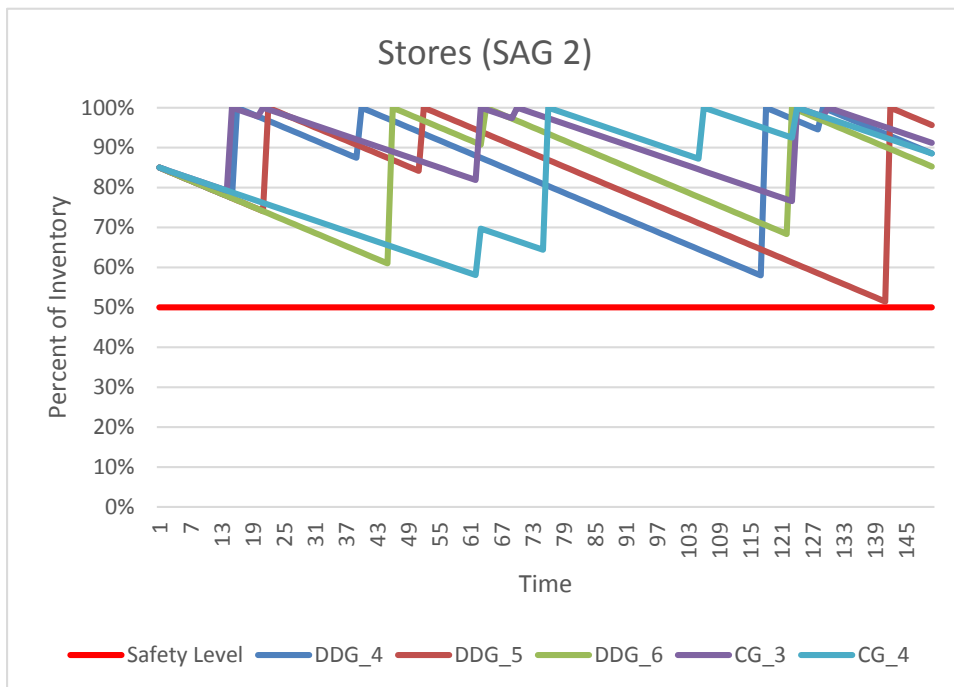


Figure 30. Stores inventory for SAG 2 (Scenario 3).

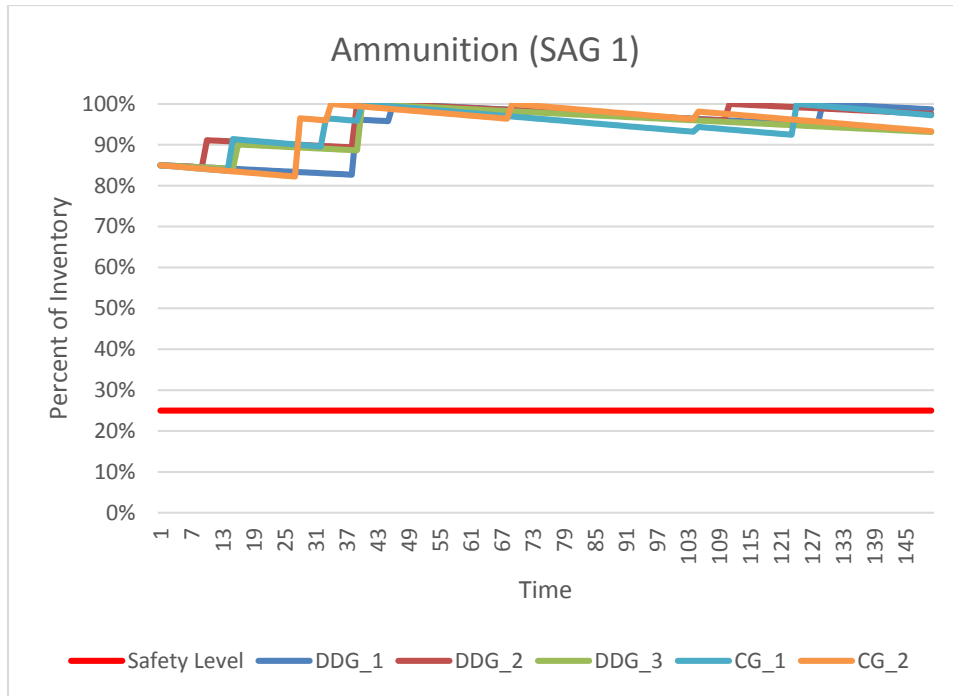


Figure 31. Ammunition inventory for SAG 1 (Scenario 3).

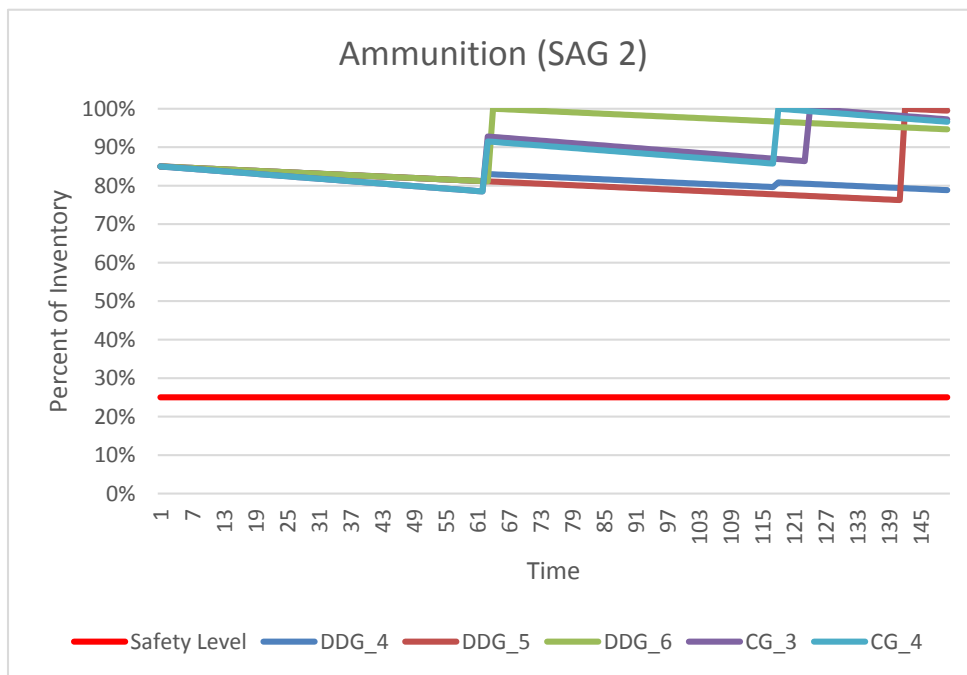


Figure 32. Ammunition inventory for SAG 2 (Scenario 3).

Table 18. Percent of time below safety level for each commodity and warship (Scenario 3, 150 time periods).

Ship	DFM	JP5	Stores	Ammunition
CG_1	15%	1%	0%	0%
CG_2	15%	3%	0%	0%
CG_3	15%	1%	0%	0%
CG_4	15%	1%	0%	0%
DDG_1	16%	1%	0%	0%
DDG_2	15%	0%	0%	0%
DDG_3	15%	1%	0%	0%
DDG_4	15%	2%	0%	0%
DDG_5	14%	1%	0%	0%
DDG_6	15%	1%	0%	0%

Table 19. Percent of time below safety level for each commodity and warship (Scenario 3, 120 time periods).

Ship	DFM	JP5	Stores	Ammunition
CG_1	1%	1%	0%	0%
CG_2	1%	3%	0%	0%
CG_3	1%	1%	0%	0%
CG_4	2%	1%	0%	0%
DDG_1	2%	1%	0%	0%
DDG_2	1%	0%	0%	0%
DDG_3	1%	0%	0%	0%
DDG_4	1%	2%	0%	0%
DDG_5	1%	1%	0%	0%
DDG_6	2%	0%	0%	0%

D. SCENARIO 4: RISING TENSIONS, 6 SHUTTLES AVAILABLE, SPEED 20 KTS

For our final scenario, we consider Scenario 4. We initialize the shuttle and CLF ships with 80% useable and deliverable inventories, reflecting ships that have been

underway for a period to time prior to the start of the model run. We consider a total time horizon of 150 4-hour periods, (25 days), using the same rolling horizon method as Scenario 3.

Our results indicate that the shuttles and CLFs can sustain the warships in SAG 1 on station for 72.7% to 73.3% of the time. This is 4.7% to 5.3% less time than the baseline, with all of the discrepancy accounted for by RAS events. SAG 2 is on station for 70.7% to 72.7% of the time. This is 12.0% to 14.0% less time than the baseline, with 4.0% to 6.0% of the difference accounted for by RAS events. The rest of the time is accounted for by the optimality gap and the warships moving to FRL 1 in time period 106. The baseline for SAG 1 is 78.0%, and for SAG 2 it is 84.7%. These numbers are summarized in Tables 20–21.

Table 20. Percent of time performing different operations for SAG 1 (Scenario 4).

Ship	On Station	Transiting	RASing
CG_1	72.7 %	22.0 %	5.3 %
CG_2	73.3 %	22.0 %	4.7 %
DDG_1	72.7 %	22.0 %	5.3 %
DDG_2	72.7 %	22.0 %	5.3 %
DDG_3	72.7 %	22.0 %	4.7 %
Baseline	78.0 %	22.0 %	0.0 %

Table 21. Percent of time performing different operations for SAG 2 (Scenario 4).

Ship	On Station	Transiting	RASing
CG_3	72.0 %	23.3 %	4.7 %
CG_4	72.0 %	23.3 %	4.7 %
DDG_4	70.7 %	23.3 %	6.0 %
DDG_5	71.3 %	23.3 %	5.3 %
DDG_6	72.7 %	23.3 %	4.0 %
Baseline	84.7%	15.3 %	0.0 %

The DFM inventories of all warships except CG_2 fall below the 70% safety level. From SAG 1, CG_1 only falls below the 70% safety level in time periods 105 and 150, and

its lowest inventory level is 69.2%. DDG_1 and DDG_3 spend the most time of SAG 1 under the 70% safety level, with four time periods each. DDG_1 is below the safety level in time periods 98 and 148 to 150, while DDG_3 is below the safety level in time periods 105 and 148 to 150. Each reaches its lowest level (66.9%) in time period 150. DDG_2 spends time periods 75, 139, and 140 under the 70% safety level and its lowest level is 67.9% in time period 140. From SAG 2, DDG_6 spends the most time below the safety level. DDG_6 is below the safety level for fifteen time periods (16 to 20, 51 to 57, 91, 92, and 150). It reaches its lowest inventory level in time period 57 of 63.0%. CG_4 spends nine time periods under the safety level: periods 64 to 68 and 147 to 150. Its lowest inventory level is 66.4% in time period 68. DDG_5 spends seven time periods under the safety level: periods 66 to 68, 81, 97, 98 and 150. Its lowest inventory level is 66.9% in time period 68. DDG_4 is below the safety level in time periods 68, 69, 104, and 135. Its lowest inventory level is 67.9% in time period 69. CG_3 spends time periods 67, 68 and 150 under the safety level and reaches its lowest inventory level (68.3%) in time period 68.

DDG_6 and CG_4 are the only warships in SAG 2 whose JP5 inventory falls below the 70% safety level. Their lowest inventory levels are 67.8% and 64.2% in time periods 57 and 68, respectively.

No ships in SAG 1 or SAG 2 reach the safety level for stores or ammunition. All inventory levels are depicted in Figures 33–40. Table 22 summarizes the percent of time that each warship commodity level falls below the safety level.

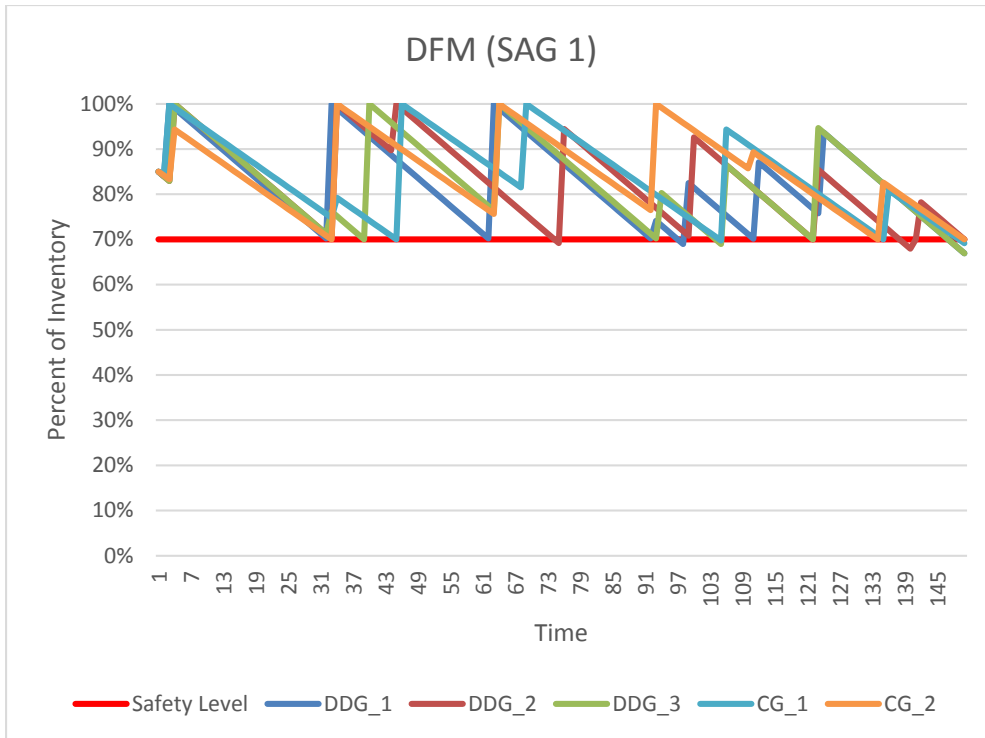


Figure 33. DFM inventory level for SAG 1 (Scenario 4).

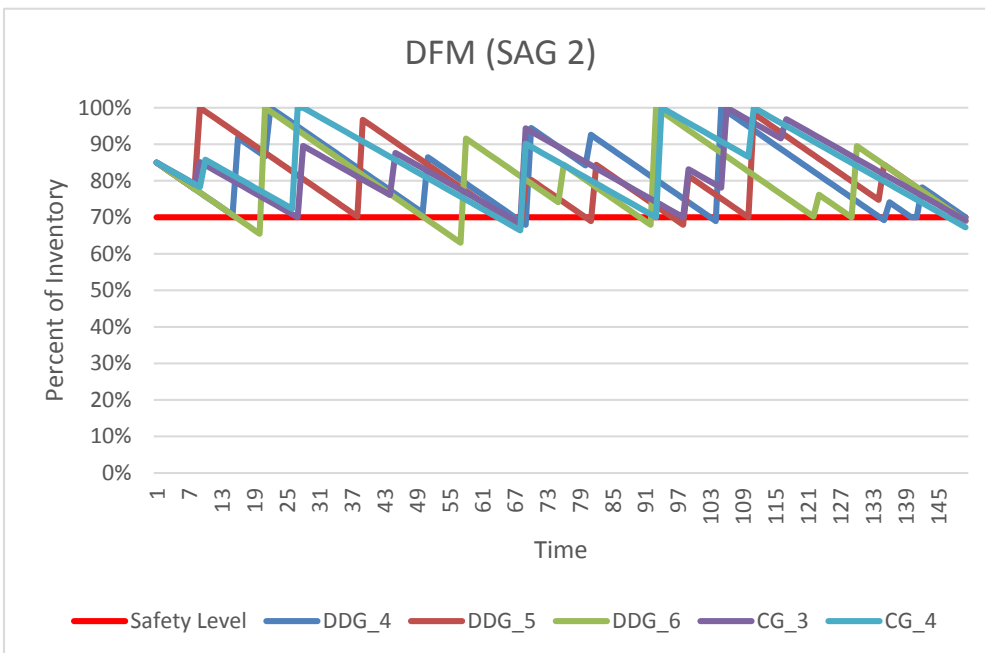


Figure 34. DFM inventory for SAG 2 (Scenario 4).

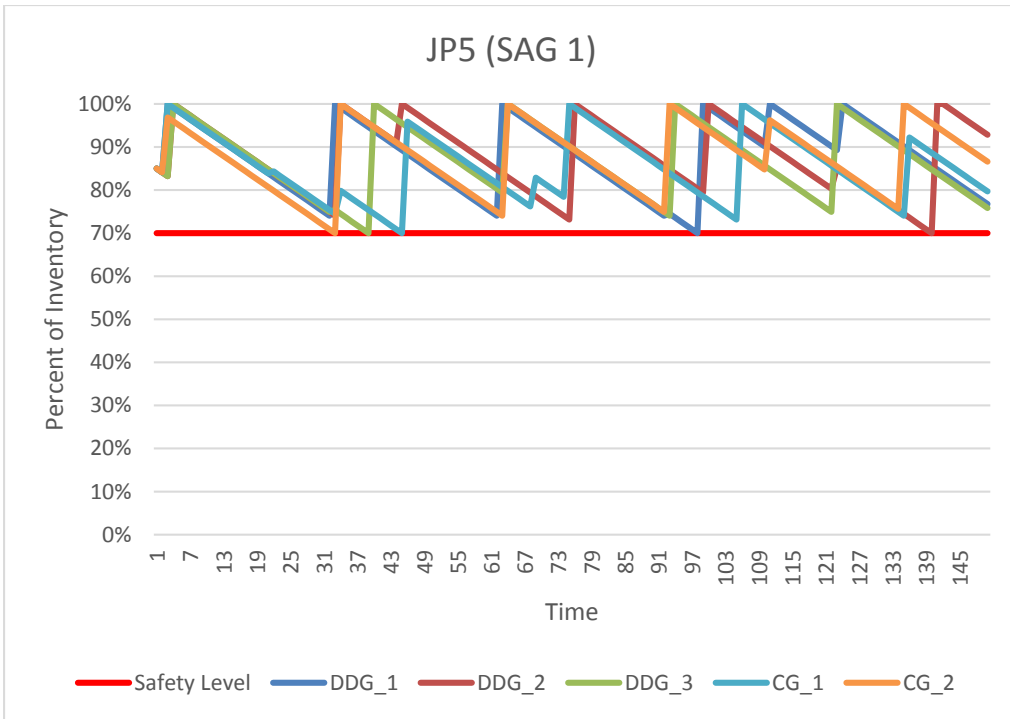


Figure 35. JP5 inventory level for SAG 1 (Scenario 4).

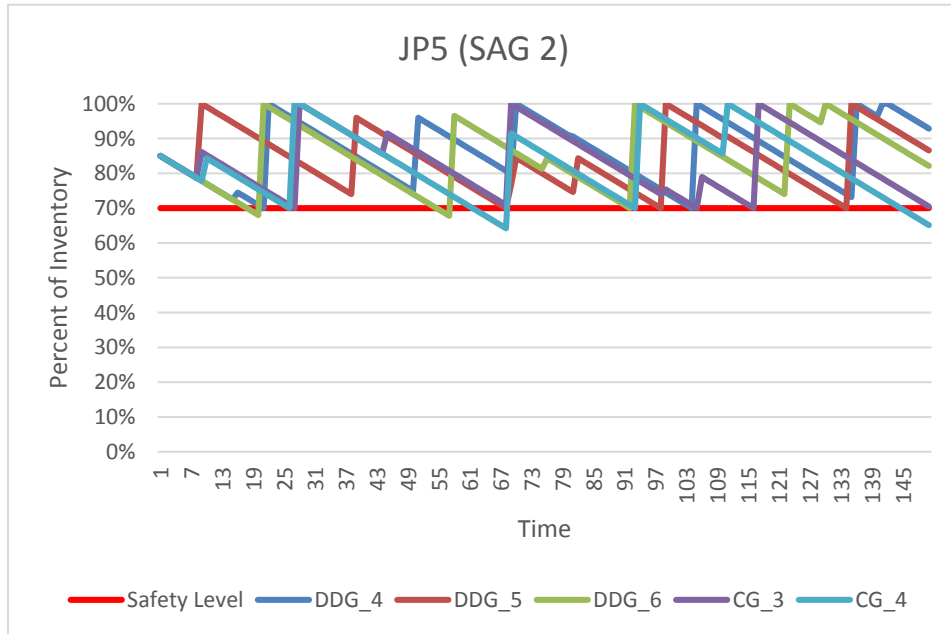


Figure 36. JP5 inventory level for SAG 2 (Scenario 4).



Figure 37. Stores inventory level for SAG 1 (Scenario 4).

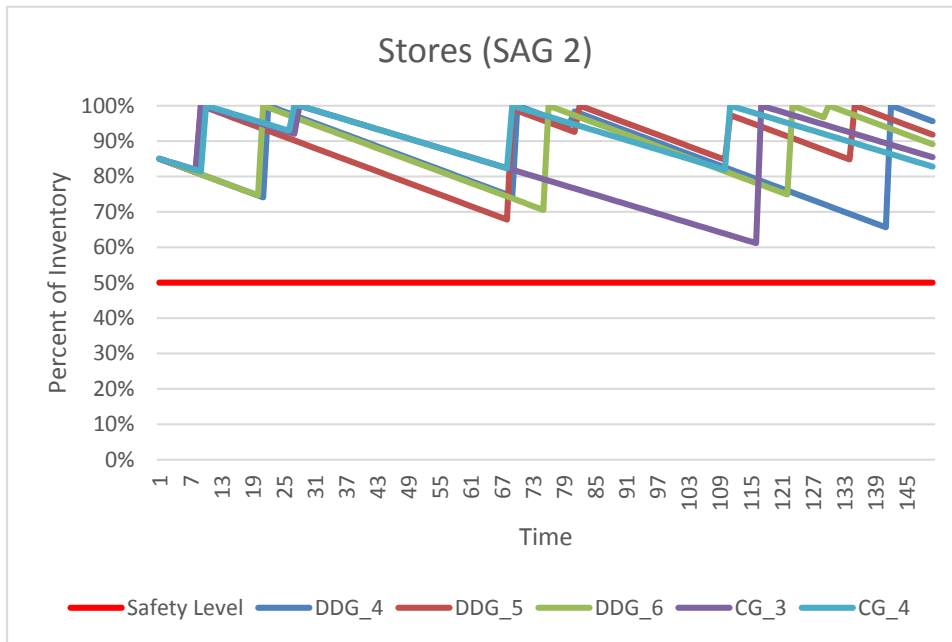


Figure 38. Stores inventory level for SAG 2 (Scenario 4).

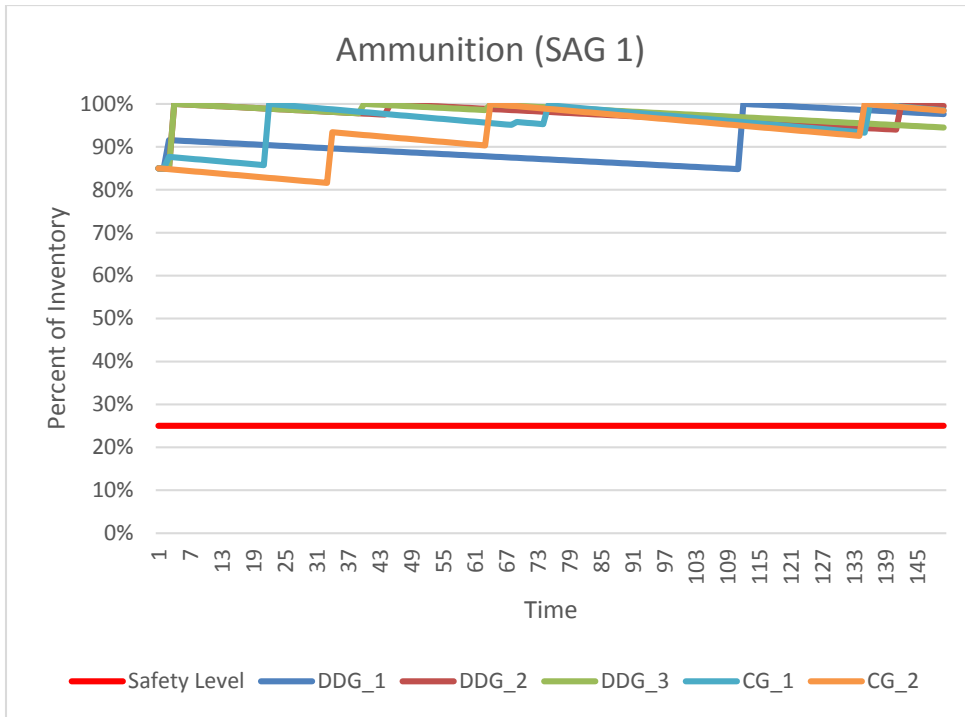


Figure 39. Ammunition level for SAG 1 (Scenario 4).

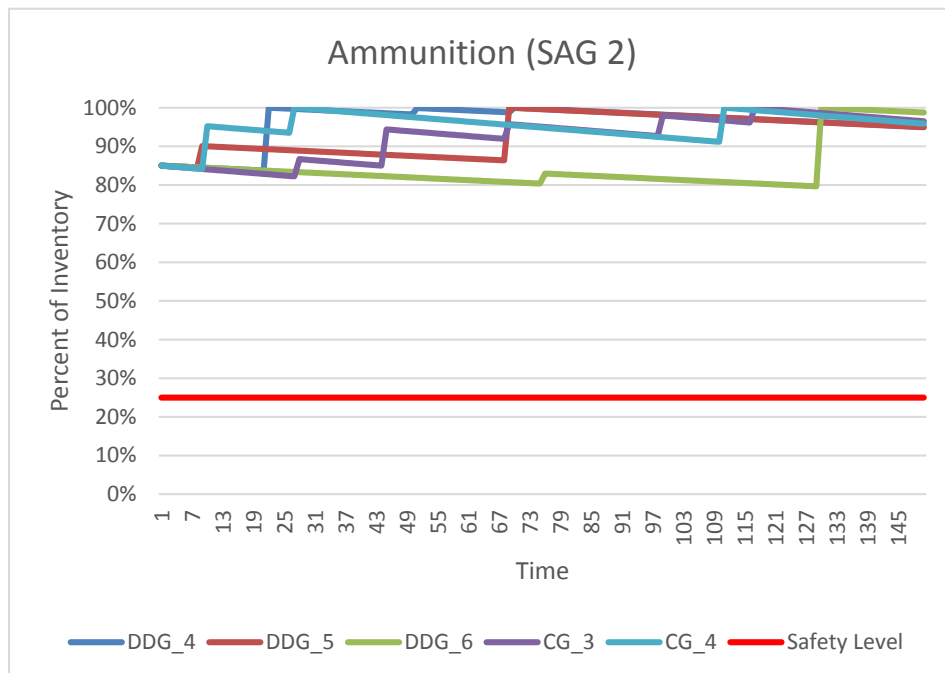


Figure 40. Ammunition level for SAG 2 (Scenario 4).

Table 22. Percent of time below safety level for each commodity and warship (Scenario 4).

Ship	DFM	JP5	Stores	Ammunition
CG_1	1%	0%	0%	0%
CG_2	0%	0%	0%	0%
CG_3	2%	0%	0%	0%
CG_4	6%	9%	0%	0%
DDG_1	3%	0%	0%	0%
DDG_2	2%	0%	0%	0%
DDG_3	3%	0%	0%	0%
DDG_4	3%	0%	0%	0%
DDG_5	5%	0%	0%	0%
DDG_6	10%	4%	0%	0%

E. DISCUSSION

We obtain a number of insights from our four scenarios. The first two scenarios demonstrate an important aspect of the rolling horizon optimization framework. Although we have more resources available in the second scenario, theoretically allowing better solutions, we observe that decisions made in early iterations of the rolling horizon sequence can result in inferior solutions in subsequent iterations. This important phenomenon should not go unnoticed by practitioners considering a rolling horizon approach.

We also note that in a rolling horizon framework, it is important to choose a long enough planning horizon to model any decisions we would like the model to be capable of making. With this in mind, and given that it would take a CLF ship up to 56 time periods to move from an ARL to Guam and back, we extend the planning horizon from 60 time periods to 90 time periods in the third excursion. Although a 60-period horizon would have allowed the movement to take place, the model would not have seen any benefit from making this decision. We also change the shuttle and CLF starting inventories to reflect them having been operating at sea for some time.

After making these changes we examine scenario 3. From these results we observe that a primary limitation is the number of T-AOs in the scenario. Table 19 shows that prior to the last 30 time periods, the warships' DFM inventories only fall below the safety level for a few time periods and by a small amount (1-2%). Had a second T-AO arrived between time periods 47 and 90, the shuttles would have been able to maintain the warships' DFM inventory levels. These results also show that the T-AKE is ill-suited for supporting this type of operation. The T-AKE has large quantities of stores and ammunition but smaller quantities of DFM and JP5. A fast combat support ship (T-AOE) could be a better support vessel for this type of operation as it brings comparable quantities of stores and ammunition as the T-AKE, but an additional 71,000 bbls of DFM and 61,000 bbls of JP5 (CNO, 2007).

It is also interesting to compare our first and fourth scenarios. Both involve a Rising Tensions strategic situation, but the fourth involves more shuttles traveling at faster speeds, and it uses a longer planning horizon as well as an 80% starting inventory (vice 100% for the first scenario). Interestingly, in the fourth scenario, the warships spend more time below the safety level for DFM and JP5. We speculate that this is due in part to the combination of a lower starting inventory and more shuttle ships. Ostensibly, more shuttles should result in better performance. But it is important to remember that while shuttles are delivering fuel, they are also burning it. When we reduce the starting inventory of the shuttles and CLF ships from 100% to 80% and increase the number of shuttles from four to six, we reduce the starting amount of deliverable DFM from 195,000 bbls to 191,200 bbls and the starting amount of deliverable JP5 from 100,400 bbls to 81,680 bbls. At the same time, we increase the usage rate of the system by 320 bbls of DFM per time period and 3.40 bbls of JP5 per time period. This decrease in starting inventory levels and increase in usage per time period reduces the effectiveness of the system, and this reduction is not completely mitigated by the increase in speed and number of shuttles.

V. CONCLUSIONS, RECOMMENDATIONS AND FOLLOW-ON WORK

A. CONCLUSION

We confirm that MC-DL-RASM produces reasonable results in the four scenarios we examined. We observe that the rolling horizon approach works best with a planning horizon of at least 79 time periods due to the length of time it would take a CLF ship to make the trip from ARL 3 to Guam and back to ARL 3 (54 time periods), then fill up a shuttle (2 time periods), and have it transit to FRL 7 (22 time periods) and resupply a warship (1 time period per warship). In fact, even this horizon is a bit optimistic when we take into account the fact that RAS events can only occur during daylight time periods.

We also observe that in each scenario, the shuttles will RAS with the CLF ships if possible rather than using a port to resupply. This could be an indication that having a port available in the AO is not as important to the maintaining the warships on station as having a constant supply of CLF ships.

B. POTENTIAL FOLLOW-ON WORK

More work remains to be done in the following areas:

1. Vary Shuttle Characteristics

Although we have performed a useful initial analysis, more work still needs to be done to determine the optimal capacity for each commodity and to determine the required number of shuttles and their transit speeds.

2. Other Scenarios

Although we have considered a set of timely and important scenarios, it is straightforward to construct different scenarios to exercise MC-DL-RASM and study the effectiveness of the min-CLF shuttle concept. These scenarios can model different geographical regions, or different port configurations in the 7th fleet region. Additional work can also be done to determine which ARL locations best support operations during

the Wartime scenario. By moving the ARL closer or further from Guam, we change the cycle time for the CLF ships and the shuttle ships. The CLF ships supporting these operations could also be varied to determine whether the addition of a second T-AO or T-AKE or if exchanging the T-AKE for a T-AOE would improve outcomes.

3. Different Warship Configurations

As new warships begin being incorporated into the Navy, their usage rates and numbers will need to be incorporated into the different scenarios to ensure that the shuttle ships are effective in supporting their operations.

4. Faster Solve Time and Longer Time Horizons

MC-DL-RASM currently requires 10-14 days to solve a 25-day scenario. Options for reducing this time should be explored. If the solve time is reduced enough, future research could evaluate the effectiveness of this method of resupply over months instead of the current 25-day time horizon.

APPENDIX. CALCULATIONS FOR DETERMINING SHUTTLE SIZE

Below are the calculations we use to determine the capacity of a shuttle ship if it were to support a CTF or SAG. The capacities were determined by finding the quantity of each commodity the ship could use if it started at 100% inventory and used to its safety level. That amount was divided by the usage rate per time period to determine the number of time periods this amount would cover. This gave us the minimum number of time periods before one of the ships would be below a safety level.

	<u>DDG</u>	
DFM	$10,518 \text{ bbls} * (1 - 0.7) = 3,155.4 \text{ bbls}$	
	$\frac{3,155.4 \text{ bbls}}{108 \text{ bbls}}$	$= 29.216 \text{ periods}$
	period	
JP5	$475 \text{ bbls} * (1 - 0.7) = 142.5 \text{ bbl}$	
	$\frac{142.5 \text{ bbls}}{4.25 \text{ bbls}}$	$= 33.529 \text{ periods}$
	period	
stores	$55 \text{ tons} * (1 - 0.5) = 27.5 \text{ tons}$	
	$\frac{27.5 \text{ tons}}{0.3 \text{ tons}}$	$= 91.666 \text{ periods}$
	period	
ammunition	$48 \text{ tons} * (1 - 0.25) = 36 \text{ tons}$	
	$\frac{36 \text{ tons}}{0.03 \text{ tons}}$	$= 1,200 \text{ periods}$
	periods	

$$\text{DFM } 15,032 \text{ bbls} * \frac{\text{CG}}{(1-0.7)} = 4,509.6 \text{ bbls}$$

$$\frac{4,509.6 \text{ bbls}}{126.2 \text{ bbls}} = 35.733 \text{ periods}$$

period

$$\text{JP5 } 475 \text{ bbls} * (1-0.7) = 142.5 \text{ bbl}$$

$$\frac{142.5 \text{ bbls}}{4.25 \text{ bbls}} = 33.529 \text{ periods}$$

period

$$\text{stores } 68 \text{ tons} * (1-0.5) = 34 \text{ tons}$$

$$\frac{34 \text{ tons}}{0.3 \text{ tons}} = 113.333 \text{ periods}$$

period

$$\text{ammunition } 94 \text{ tons} * (1-0.25) = 70.5 \text{ tons}$$

$$\frac{70.5 \text{ tons}}{0.03 \text{ tons}} = 705 \text{ periods}$$

periods

CVN

DFM

$$\text{JP5 } 74,642 \text{ bbls} * (1-0.7) = 22,392.6 \text{ bbls}$$

$$\frac{22,392.6 \text{ bbls}}{666.6 \text{ bbls}} = 33.592 \text{ periods}$$

period

$$\text{stores } 1,710 \text{ tons} * (1-0.5) = 855 \text{ tons}$$

$$\frac{855 \text{ tons}}{8.8 \text{ tons}} = 97.159 \text{ periods}$$

period

$$\text{ammunition } 1,765 \text{ tons} * (1-0.25) = 1,323.75 \text{ tons}$$

$$\frac{1,323.75 \text{ tons}}{3.3 \text{ tons}} = 401.136 \text{ periods}$$

periods

We took the minimum number of time periods (29) and multiplied it by the usage rate for each commodity to determine how much each ship in the SAG or CTF would require between RAS events. We then multiplied this by the number of each ship type in the SAG or CTF.

DDG

$$\begin{aligned} \text{DFM} & \frac{108 \text{ bbls}}{\text{period}} * 29 \text{ periods} = 3,132 \text{ bbls} * 3 = 9,396 \text{ bbls} \\ \text{JP5} & \frac{4.25 \text{ bbls}}{\text{period}} * 29 \text{ periods} = 123.25 \text{ bbls} * 3 = 369.75 \text{ bbls} \\ \text{stores} & \frac{0.3 \text{ tons}}{\text{period}} * 29 \text{ periods} = 8.7 \text{ tons} * 3 = 26.1 \text{ tons} \\ \text{ammunition} & \frac{0.03 \text{ tons}}{\text{periods}} * 29 \text{ periods} = 0.87 \text{ tons} * 3 = 2.61 \text{ tons} \end{aligned}$$

CG

$$\begin{aligned} \text{DFM} & \frac{126.2 \text{ bbls}}{\text{period}} * 29 \text{ periods} = 3,659.8 \text{ bbls} * 2 = 7,319.6 \text{ bbls} \\ \text{JP5} & \frac{4.25 \text{ bbls}}{\text{period}} * 29 \text{ periods} = 123.25 \text{ bbls} * 2 = 246.5 \text{ bbls} \\ \text{stores} & \frac{0.3 \text{ tons}}{\text{period}} * 29 \text{ periods} = 8.7 \text{ tons} * 2 = 17.4 \text{ tons} \\ \text{ammunition} & \frac{0.1 \text{ tons}}{\text{period}} * 29 \text{ periods} = 2.9 \text{ tons} * 2 = 5.8 \text{ tons} \end{aligned}$$

CVN

DFM

$$\begin{aligned} \text{JP5} & \frac{666.6 \text{ bbls}}{\text{period}} * 29 \text{ periods} = 19,331.4 \text{ bbls} \\ \text{stores} & \frac{8.8 \text{ tons}}{\text{period}} * 29 \text{ periods} = 255.2 \text{ tons} \\ \text{ammunition} & \frac{3.3 \text{ tons}}{\text{period}} * 29 \text{ periods} = 95.7 \text{ tons} \end{aligned}$$

For each commodity we then added up the required amounts for a SAG or CTF and added 30% as the safety level for the deliverable quantities.

SAG

$$\begin{aligned} \text{DFM} & (9,396 \text{ bbls} + 7,319.6 \text{ bbls}) * 1.30 = 21,730.28 \text{ bbls} \approx 22,000 \text{ bbls} \\ \text{JP5} & (369.75 \text{ bbls} + 246.5 \text{ bbls}) * 1.30 = 801.125 \text{ bbls} \approx 850 \text{ bbls} \\ \text{stores} & (26.1 \text{ tons} + 17.4 \text{ tons}) * 1.30 = 56.55 \text{ tons} \approx 75 \text{ tons} \\ \text{ammunition} & (2.61 \text{ tons} + 5.8 \text{ tons}) * 1.30 = 10.933 \text{ tons} \approx 15 \text{ tons} \end{aligned}$$

CTF

DFM $(9,396 \text{ bbls} + 7,319.6 \text{ bbls}) * 1.30 = 21,730.28 \text{ bbls} \approx 22,000 \text{ bbls}$

JP5 $(369.75 \text{ bbls} + 246.5 \text{ bbls} + 19,331.4 \text{ bbls}) * 1.30 = 25,931.945 \text{ bbls} \approx 26,000 \text{ bbls}$

stores $(26.1 \text{ tons} + 17.4 \text{ tons} + 255.2 \text{ tons}) * 1.30 = 388.31 \text{ tons} \approx 400 \text{ tons}$

ammunition $(2.61 \text{ tons} + 5.8 \text{ tons} + 95.7 \text{ tons}) * 1.30 = 135.343 \text{ tons} \approx 150 \text{ tons}$

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