Mitigation of Nonlinear Spurious Products using Least Mean-Square (LMS)

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***Abstract*—This paper analyzes the performance of an iterative solution for the nonlinear correction of a digital array receiver channel. The iterative least mean-square (LMS) algorithm is compared to weighted least-squares (WLS) and its ability to adapt to changes in system temperature, and therefore nonlinear characteristics, is shown. Lastly, a digital array is simulated with nonlinear channels that represent a real system. The resulting intermodulation distortion (IMD) is then mitigated using the LMS correction method.**

***Keywords*—phased array; nonlinear correction; intermodulation distortion; least mean-square; dynamic range.**

I. Introduction

With the increasing desire for low cost radar systems, specifically digital phased arrays, with each element requiring its own Tx/Rx channel, we are forced to use components with a lower third-order interception point (IP3). This paired with the possibility for interferers, both in-band and out-of-band, driving the receivers into compression and creating intermodulation distortion (IMD), has lead to a great need for digital nonlinear correction [1], [2]. Digital pre-distortion, used to correct nonlinearities in the Tx channel, was done in [3], for example, making use of a cross-memory polynomial model. Post-distortion methods, correcting distortion in the Rx channel, were given in [1] and [4], where the authors used weighted least-squares (WLS) to apply a memory polynomial model to the distorted data for receiver IMD correction, in [5], using the least mean-square (LMS) algorithm to apply a static power series based correction, and in [6], where LMS was used to correct both RF and baseband nonlinearities.

The use of the post-distortion method allows for correction in situations where receiver distortion is unavoidable, specifically when interferers drive the system into compression. This paper demonstrates cases where the LMS method proves advantageous over WLS in its ability to iteratively adapt the weights to changes in the system, such as temperature or frequency changes due to the use of tunable components. It will also be demonstrated that the memory polynomial, though it may not be optimal in all cases, can characterize the nonlinear aspects of the system in both power, frequency, and combinations of the two. Lastly, it will be shown that using an iterative solution for IMD correction on a digital array provides a much higher increase in dynamic range when compared to the use of a static solution.

Typically, characterization of the nonlinearities of a system is done by placing two tones and their respective IMD somewhere in-band. The disadvantages of this method are that it only characterizes a specific part of the bandwidth and only at a specific power level. The use of a frequency and power diverse waveform, such as band-limited white Gaussian noise (WGN), to characterize the system is much more effective as it results in many more frequencies and a multitude of power levels being simultaneously tested.

In this paper, the use of LMS to apply a memory polynomial-based nonlinear correction to data gathered from a linear auxiliary channel and a low-cost-nonlinear channel is presented. The ability of LMS to quickly adapt to changes in the temperature of the system, compared to that of the computationally costly WLS, is also shown. Section II demonstrates the framework of the LMS algorithm. Next, Section III will show the results of digital nonlinear correction from both WLS and LMS methods. Section IV shows the nonlinear characterization of a receiver channel and the use of this characterization in a MATLAB software suite designed for digital array simulations. Section V then shows the results of the the LMS correction on a simulation of a 12 element linear array with nonlinear receiver channels. Lastly, the Conclusion will summarize the work discussed in this paper and give examples of future work.

II. LMS Algorithm

The LMS algorithm attempts to minimize the mean-square error (MSE), the cost function, of the desired signal $d(n)$ and the corrected signal $y(n)$.

$$MSE = \frac{1}{N}\sum_{n=1}^{N}[d(n) - y(n)]^2 \tag{1}$$

The cost function is at a minimum when its first derivative is zero and its second derivative is positive. The gradient of the cost function $J[\mathbf{w}(n)]$ is given by

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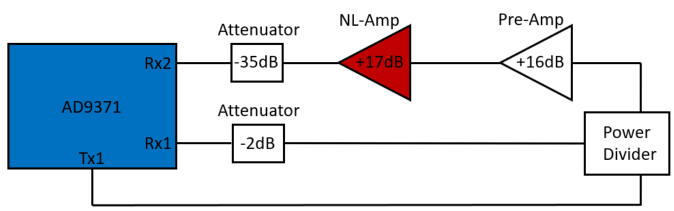

Fig. 1. The testbed setup; Rx1 is the auxilliary channel; Rx2 is the nonlinear channel, which is distorted by the nonlinear amplifier.

$$\begin{aligned}\nabla J[\mathbf{w}(n)] &= -2d(n)\mathbf{x}(n) + 2\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{w}(n) \\ &= -2e(n)\mathbf{x}(n)\end{aligned} \tag{2}$$

where $\mathbf{w}(n)$ are the weights used to correct the distorted signal $\mathbf{x}(n)$. The weights are calculated iteratively, where the next weight is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\nabla J[\mathbf{w}(n)], \tag{3}$$

where μ is the step size. The weights remain unchanged and are at their optimum values when the error is minimized.

In order to apply LMS to nonlinear correction we adapt it to a memory polynomial model, given as

$$y(n) = \sum_{p=0}^{P-1}\sum_{m=0}^{M-1} w_{pm}x(n-m)|x(n-m)|^{2p}, \tag{4}$$

where w_{pm} is the weight in the p^{th} row and the m^{th} column of the LMS weight matrix $\mathbf{w}$. The memory polynomial model allows for correction in P power levels and M orders, yielding a total of $P \times M$ coefficients to characterize the nonlinear system.

III. Channel Correction Results

We gathered data from an Analog Devices AD9371 transceiver, using the Tx module to transmit the 20MHz WGN waveform at 2.7GHz. This was then fed into a power divider, with one output feeding into the auxiliary channel and the other output into the pre-amp, a MiniCircuits XRL-3500+, followed by the nonlinear amplifier, a MiniCircuits ZJL-3G+. Both of the Rx channels were fitted with attenuators so that their respective signals had nearly the same amplitude, and lengths of cables were adjusted so that there was negligible time-delay between the two channels. We then calibrated and corrected the data with both WLS and LMS, each using six power terms (up to the 11th order) and five time delays (memory terms).

The system was first calibrated using the WLS method suggested in [1]. The calibrated coefficients were then applied to the collected data and the results are shown in Fig. 2. This resulted in about 10dB of correction in the IMD products.

The system was then calibrated and corrected using the LMS method. This correction produced more than 20dB of

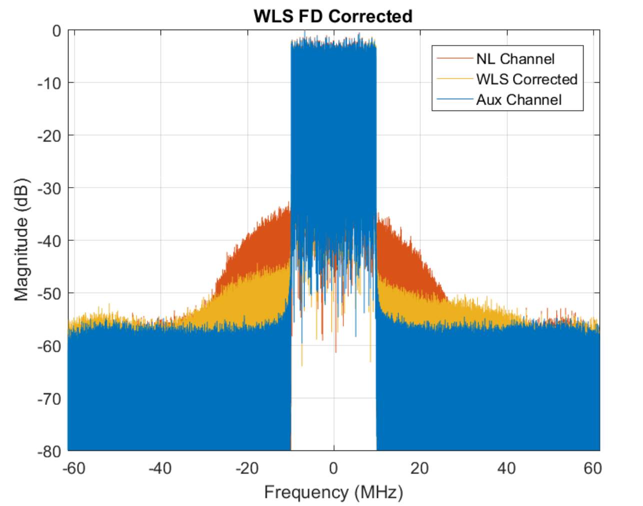

Fig. 2. WLS correction, yellow, of the nonlinear channel, orange, with IMD mitigation of about 10dB.

IMD mitigation, while increasing the noise floor about 10dB, seen in Fig. 3.

We then heated the nonlinear amplifier from 30°C to 60°C and gathered the new data. The nonlinear characteristics of the amplifier change with temperature and this can be seen in Figures 4 and 5, where the effectiveness of the previously calibrated coefficients decreased.

In order to continue to mitigate the IMD products as much as possible, we would need to re-calibrate the nonlinear system. Since LMS is an iterative algorithm, it is much more computationally efficient at calculating new coefficients, especially because this is the same system and its nonlinear characteristics have only changed slightly. We then ran the LMS calibration for a few iterations until it converged to new weights. The mitigation achieved by these new coefficients can

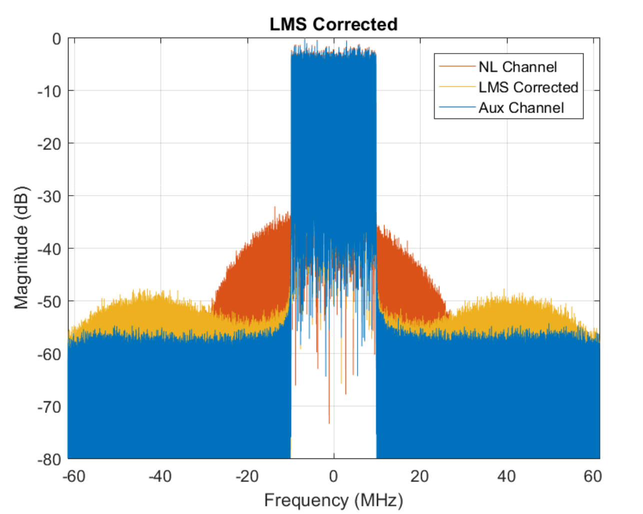

Fig. 3. LMS correction, yellow, of the nonlinear channel, orange, with IMD mitigation of more than 15dB.

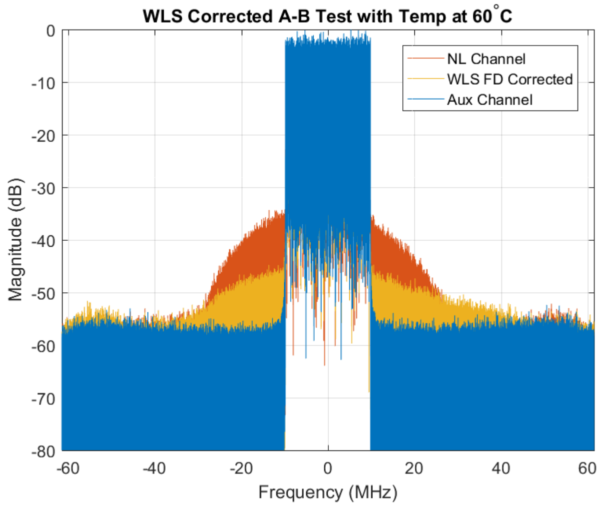

Fig. 4. WLS correction, yellow, of the heated nonlinear channel, orange, using the coefficients of the 30°C channel with slightly less effective correction.

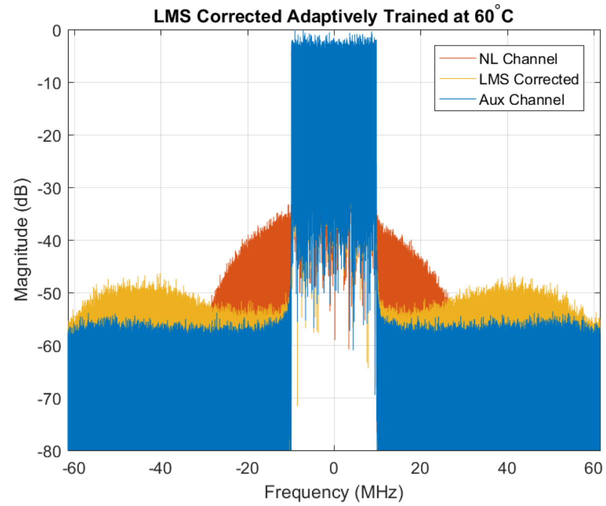

Fig. 6. LMS correction, yellow, of the heated nonlinear channel, orange, using the adaptively trained coefficients with greater than 15dB of IMD mitigation.

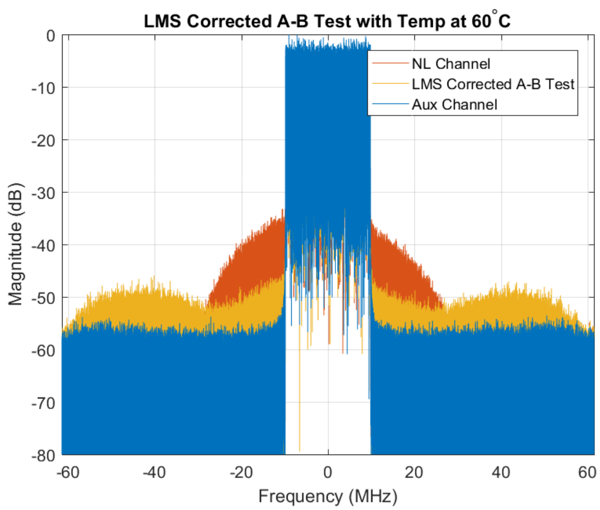

Fig. 5. LMS correction, yellow, of the heated nonlinear channel, orange, using the coefficients of the 30°C channel with about 5dB less of IMD suppression.

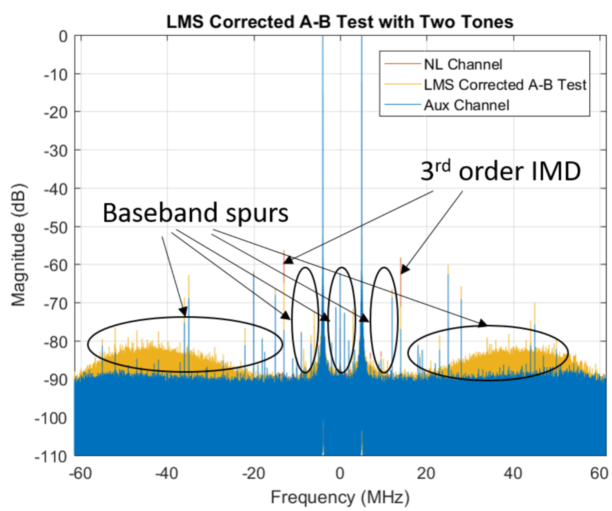

Fig. 7. LMS correction, yellow, of the nonlinear channel, orange, with two tone input and resulting third-order IMD. There are also many baseband spurious products both receiver channels.

be seen in Fig. 6.

Another example of the performance of the LMS algorithm is to apply the previously trained weights, used to produce the correction in Fig. 3, to a two tone test on the same system, shown in Fig. 7. Complex baseband tones of -4MHz and 5MHz were pushed through the nonlinear system, producing third-order IMD about 57dB down from the normalized power level. With the application of these previously calibrated weights we were able to achieve 7.5dB and 13dB of mitigation of the 3rd order IMD. The baseband spurious products seen in Fig. 7 were not corrected in this paper, but have been mentioned in [4] and corrected in [6].

Finally, for nonlinear systems, it is interesting to look at the power-in vs power-out curve as it provides information about the compression point of the system, observed in the knee, and the memory effects, seen in the smearing of the points. Fig. 8 shows the normalized power-in vs power-out curve for the data from Fig. 3. The LMS method does a good job of correcting the memory effects at the lower power levels by removing the smearing, and in correcting the power nonlinearities by removing the knee and straightening up the curve.

IV. Nonlinear Array System Modeler

The Nonlinear Array System Modeler (NASM), used in [7], allows us to simulate digital phased array systems. NASM is a MATLAB software suite designed to simulate an entire digital array, from the signal source to the antenna element to the digital beamformer. Nonlinear coefficients can be specified to produce specific nonlinearities or to represent a real channel. These coefficient can be calculated from a real system in the same way the correction was done in the previous section.

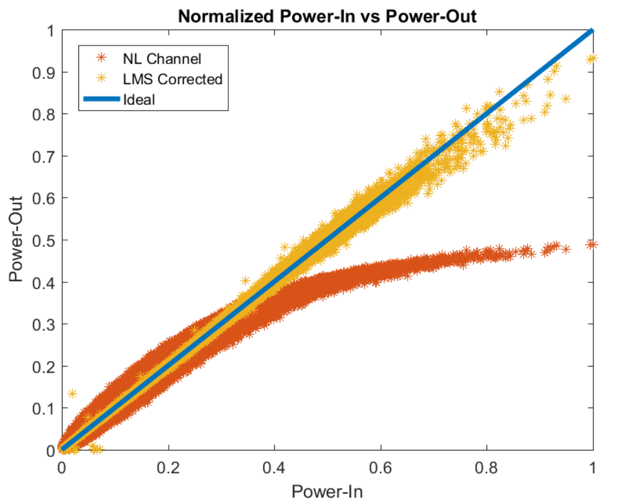

Fig. 8. The normalized power-in vs power-out curve of the LMS correction in yellow, the nonlinear channel in orange and the ideal linear channel response in blue.

In order to calculate the nonlinear coefficients of the tested system, the desired signal is replaced with the nonlinear signal. The LMS-based correction from the previous section was then run to determine the nonlinear coefficients to be used in NASM. Fig. 9 shows that the nonlinear coefficients produce the same nonlinearities as the real system and Fig. 10 shows that the normalized power-in power-out curve of the calculated coefficients follows that of the real data. The coefficients of the system for both temperatures were calculated. These coefficients are then used in the next section to simulate a 12 element linear array with IMD caused by two interferers.

V. Array Correction Results

Having demonstrated the performance of the LMS-based correction on a single channel, we then want to evaluate it's effectiveness on an array. Using NASM and the calculated nonlinear coefficients, we simulated a 12 element linear array with two-tone input, each tone from a different direction. The two third-order IMD correlate to predictable angles given by equations in [8], used by the authors of [4].

The array was simulated with at a frequency of 2.7GHz with the two tones, at baseband, having frequencies of 11MHz and 17MHz and directions of $15°$ and $-11°$, respectively. The third-order IMD were at baseband frequencies of 5MHz and 23MHz with correlated directions of $45.27°$ and $-39.68°$, respectively.

The first array simulation, shown in Fig. 11, used coefficients from the 30°C channel. The channels were corrected using the LMS-based correction method, decorrelating the third-order IMD by 23.47dB and 24.24dB. The second and third arrays were simulated using the coefficients from the $60°$C channel. The first of these two simulations was corrected using the correction coefficients from the first array simulation, shown in Fig. 12. The decorrelation of the third-order IMD in this simulation were only 10.61dB and 11.73dB, more than an order of magnitude less than the correction when the coefficients were used on the channels they were calibrated on.

The correction coefficients were then iteratively calibrated, as in Section III, by running the LMS for a few iterations on the new data allowing it to adapt the values to the new channel characteristics. Fig. 13 shows that the adaptive weights decorrelated the IMD by 19.94dB and 19.68dB. Using the iterative calibration solution to correct the nonlinear IMD of an array provided 9dB more mitigation when compared to the use of static coefficients.

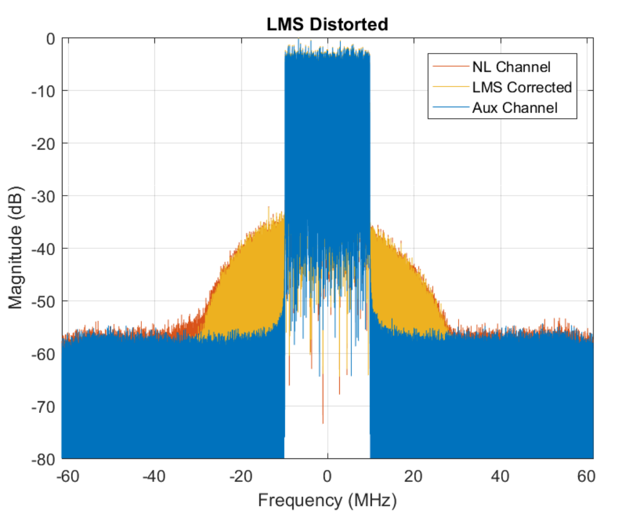

Fig. 9. The LMS trained nonlinear distortion, yellow, of the linear channel, blue, closely matches the nonlinear channel, orange.

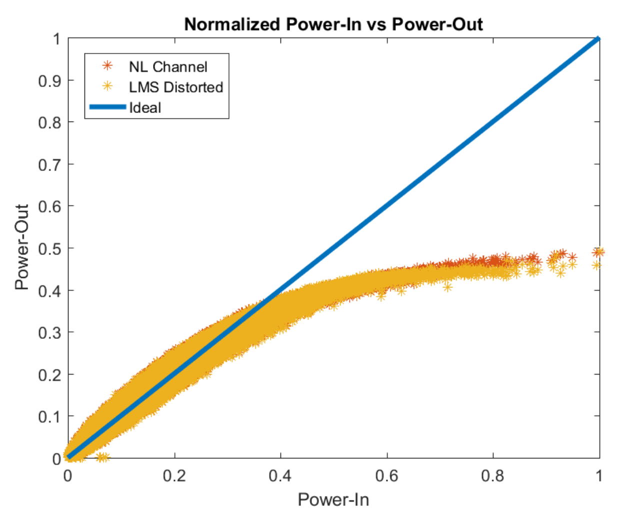

Fig. 10. The power-in vs power-out curve of the LMS trained nonlinear distortion, yellow, follows that of the nonlinear channel, orange.

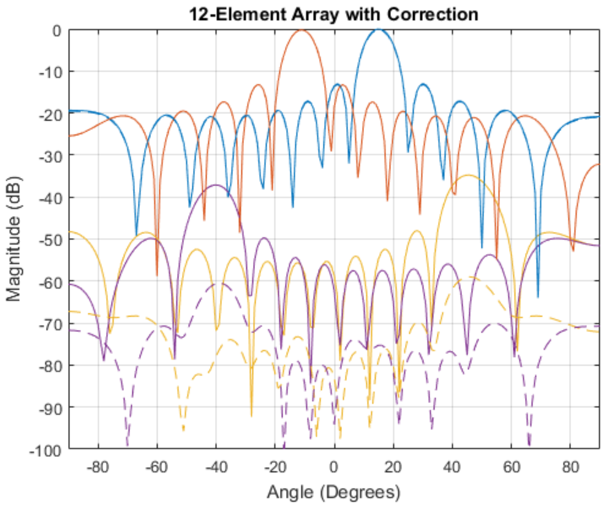

Fig. 11. Digital beamforming of a simulated 12-element array with nonlinear receive channels, solid lines, and its nonlinear correction, dashed lines. The two input tones are at baseband frequencies of 11MHz (blue) and 17MHz (orange) with third-order IMD at 5MHz (yellow) and 23MHz (purple). The correction shows the decorrelation of the third-order spurs.

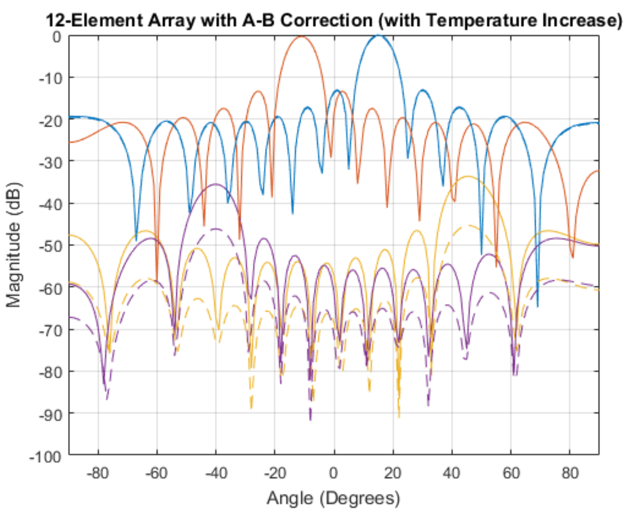

Fig. 12. Digital beamforming of a simulated 12-element array with heated nonlinear receive channels, solid lines, and its nonlinear correction using the coefficients of the non-heated channels, dashed lines. The two input tones are at baseband frequencies of 11MHz (blue) and 17MHz (orange) with third-order IMD at 5MHz (yellow) and 23MHz (purple). The correction shows less decorrelation of the third-order spurs.

Conclusion

The ability of the LMS algorithm to linearize signals that have been pushed into compression, paired with its ability to adapt to changes in the system make it a very desirable way to correct nonlinearities in low cost systems. It was also shown that having an iterative solution, when correcting IMD in a digital phased array, provides a much better correction than when using static coefficients. The next step would be to show how the LMS algorithm performs with the use of tunable components, which will greatly change the memory

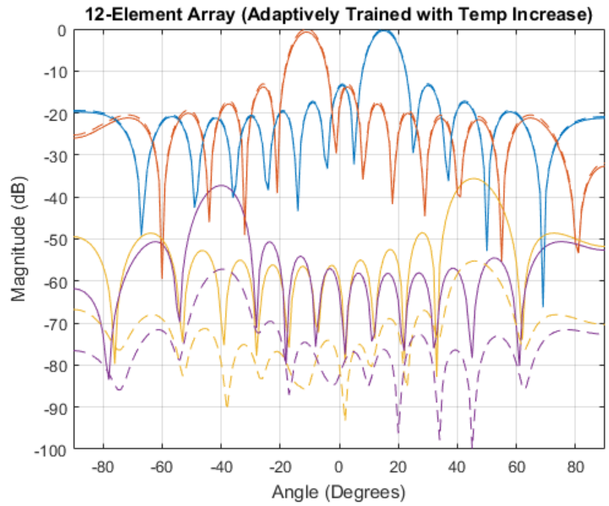

Fig. 13. Digital beamforming of a simulated 12-element array with heated nonlinear receive channels, solid lines, and its nonlinear correction using adaptively trained coefficients, dashed lines. The two input tones are at baseband frequencies of 11MHz (blue) and 17MHz (orange) with third-order IMD at 5MHz (yellow) and 23MHz (purple). The correction shows an increase in the decorrelation of the third-order spurs, as opposed to when the static correction was used.

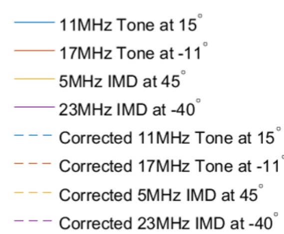

Fig. 14. Legend for Figures 11, 12, and 13

effects of the system. In general, it is still an open research topic to determine the most optimal manner for training receiver correction to be able to handle the widest variety of nonlinearities.

Acknowledgment

This work was supported by the Defense Advanced Research Projects Agency (DARPA) under grant no. D15A00090. The authors would also like to thank B. James for designing the NASM software suite.

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