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14. ABSTRACT The objective of our research is to develop a set of mathematical tools that can help solve design and analysis issues for dynamics of large-scale interconnected system of systems. These tools are a combination of operator-theoretic, geometric dynamical systems methods and graph-theoretic methods. In the past year we have developed methods for use of Koopman operator theory to understanding data from large sensor collections and applied it in a variety of ways to energy-related problems.
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15. SUBJECT TERMS Koopman operator analysis, Energy systems
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# RPPR Final Report

as of 07-Nov-2017

Agency Code:

Proposal Number: 60713EG

Agreement Number: W911NF-11-1-0511

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**Report Date:** 31-Dec-2013

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**Final Report** for Period Beginning 01-Oct-2011 and Ending 30-Sep-2016

**Title:** RESEARCH AREA 1. Mechanical Sciences 1.3.1: Dynamics of System of Systems and Applications to Net Zero Energy Facilities

**Begin Performance Period:** 01-Oct-2011

**End Performance Period:** 31-Mar-2017

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Submitted By: Igor Mezic

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**STEM Degrees:**

**STEM Participants:** 2

**Major Goals:** The objective of our research is to develop a set of mathematical tools that can help solve design and analysis

issues for dynamics of large-scale interconnected system of systems. These tools are a combination of operator-theoretic, geometric dynamical systems methods and graph-theoretic methods. We extend the analysis methods based on Koopman operator properties to include stable and unstable foliations and develop methods in Generalized Laplace Analysis to do so for a large class of nonlinear interconnected dynamical systems.

We will couple these operator-theoretic methods to graph theoretic analysis that will rely on spectral analysis of the interconnection graph associated with a dynamical system at hand. The coupling is enabled by the fact that both approaches are spectral. The new theory is able to treat discrete and continuous, stochastic and deterministic, and hybrid systems within a single framework. We pursue a research strategy to develop effective algorithms for energy efficiency in such complex dynamical systems and test them on an existing testbed site at the University of California, Santa Barbara, that is fully equipped with sensors and enables us to pursue detailed analyses of the real-world effectiveness of such algorithms.

Accomplishments Under

**Accomplishments:** 1) Comparison of different dynamical systems or a dynamical system with data is one of the core issues in

both dynamical systems and control theory. The fruitful approaches are particularly hard to come by for systems and data that show nonlinear behavior and non-Gaussian noise characteristics. We developed a theory that utilizes spectral theory of linear operators (composition, or Koopman) to provide the methodology, that was originally formulated for measure-preserving systems. We extended it to capture dissipative and finite-time dynamics. The approach combines a version of ergodic partition theory with Hardy space theory in observable space (rather than in time).

2) We applied the operator-theoretic viewpoint to a class of non-smooth dynamical systems that are exposed to event-triggered state resets. The considered benchmark problem is that of a pendulum which receives a downward kick under certain fixed angles. The pendulum is modeled as a hybrid automaton and is analyzed from both a geometric perspective and the formalism carried out by Koopman operator theory. A connection is drawn between these two interpretations of a dynamical system by means of establishing a link between the spectral properties of the Koopman operator and the geometric properties in the state-space.

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3) We developed the connections between the principle eigenfunctions of the Koopman operator associated with a diffeomorphism of a Banach space with an asymptotically stable, hyperbolic fixed point and the existence of a topological conjugacy between the diffeomorphism and its linearization around the fixed point. As long as the principle eigenvalues satisfy the non-resonance conditions of normal form theory, the principle eigenfunctions generate a commutative algebra of polynomials from which a sequence of coordinate transformations that linearize the diffeomorphism can be generated.

We answered the following questions.

(1) Is there a sub-algebra of  $C(X)$  of which every element has an expansion into eigenfunctions of Koopman operator associated with the linearization at equilibrium?

(2) Similarly, is there a sub-algebra of  $C(X)$  of which every element has an expansion into eigenfunctions of Koopman operator of the nonlinear system?

(3) If the above two sub-algebras exist, what is their relation?

Answering (1), the so-called principle eigenfunction can be used to generate an algebra of observables (called the principle algebra) which have expansions in eigenfunctions. This algebra can be pulled back to an algebra of observables for the nonlinear dynamics as long as a topological conjugacy exists. This conjugacy can actually be constructed from change of variables transformations that are formed from tensor products of the principle algebra with the Banach space (question (3)). The pullback algebra, formed via the topological conjugacy, also has spectral expansions since eigenfunctions are preserved when composing with topological conjugacies (question (2)).

4) Proposed a novel operator-theoretic framework to study global stability of nonlinear systems.

Based on the spectral properties of the so-called Koopman operator, our approach can be regarded as a natural extension of classic linear stability analysis to nonlinear systems. The main results establish the (necessary and sufficient) relationship between the existence of specific eigenfunctions of the Koopman operator and the global stability property of hyperbolic fixed points and limit cycles. These results are complemented with numerical methods which are used to estimate the region of attraction of the fixed point or to prove in a systematic way global stability of the attractor within a given region of the state space.

5) Demonstrated that the Koopman eigenfunctions and eigenvalues define a set of intrinsic coordinates, which serve as a natural framework for fusing measurements obtained from heterogeneous collections of sensors in systems governed by nonlinear evolution laws. These measurements can be nonlinear, but must, in principle, be rich enough to allow the state to be reconstructed. We approximate the associated Koopman operator using extended dynamic mode decomposition, so the method only requires time series of data for each set of measurements, and a single set of "joint" measurements, which are known to correspond to the same underlying state. We apply this procedure to the FitzHugh-Nagumo PDE, and fuse measurements taken at a single point with principal-component measurements.

6) Studied extreme phase sensitivity in systems with fractal isochrons. Sensitivity to initial conditions is usually associated with chaotic dynamics and strange attractors. However, even systems with (quasi)periodic dynamics can exhibit it. In this context we report on the fractal properties of the isochrons of some continuous-time asymptotically periodic systems. We define a global measure of phase sensitivity that we call the phase sensitivity coefficient and show that it is an invariant of the system related to the capacity dimension of the isochrons. Similar results are also obtained with discrete-time systems. As an illustration of the framework, we compute the phase sensitivity coefficient for popular models of bursting neurons, suggesting that some elliptic bursting neurons are characterized by isochrons of high fractal dimensions and exhibit a very sensitive (unreliable) phase response.

7) Developed a systematic approach to zoning and model reduction for energy systems in buildings using Koopman Mode Analysis. As the scope of building design and construction increases and building systems become more integrated, the use of building energy models has become increasingly widespread in evaluating and predicting building performance. Despite the growing sophistication of building modeling tools, errors can arise from approximations that are made by a practitioner during model creation. We examined the process of model zoning, i. e., how the volume of a building is divided into regions where properties are assumed to be uniform. Zoning is performed during model creation to decrease model complexity. However, accuracy reduces when dissimilar regions of a building are defined by a single zone. In this paper, a systematic approach to creating zoning approximations is introduced. Utilizing the Koopman operator, the time-series output produced by a building simulation can be decomposed into spatial modes which capture the thermal behavior of a building at different time-scales. Identification of spatial structures within these modes forms a framework for the creation of simplified models of varying levels of granularity. In this paper, a detailed model is analyzed, and model accuracy is studied as coarser building representations are created using the introduced method.

8) Applied the concepts above to substantially reduce energy use in a building on UCSB campus. In this work, an

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energy audit case study of a LEED silver university building is performed by decomposing building data into spatial-temporal modes known as Koopman modes. Koopman modes are influenced by heat loads, from internal and external sources, that a building is subject to, and analysis of these modes provides greater understanding of a building's thermal behavior. In the presented application of this framework, examples of energy waste are identified including faulty equipment operation, unnecessary equipment usage, and HVAC operating conditions requiring high energy to maintain. Through addressing the issues discovered, energy usage of the case study building is reduced by

13% with no side effects seen in building comfort. Koopman mode analysis has previously been used to study the thermal behavior of building models, but has seen only limited implementation using actual building data.

9) Applied stability analysis using Koopman operator theory to power grid stability issues. We analyzed nonlinear dynamics and stability in a network of fixed-speed induction generators.

**Training Opportunities:** Students and postdoctoral fellows participating in this work have unique opportunity to do interdisciplinary work between mathematics of dynamical systems and practical applications in energy systems.

**Results Dissemination:** 2012 "A New Systems Analysis Framework", Mechanical Engineering, MIT  
2012 "Analysis of Fluid Flows via Spectral Properties of Koopman Operator", APS DFD meeting, San Diego.

2013 "Koopman Operator Methods: An Overview", SIAM DS meeting, Snowbird, UT.

2013 "System-Level Tools for Whole Building Analyses", Intelligent Building Operations Workshop, University of Colorado Boulder.

2014 "Energy Management in Buildings and Grid using Koopman Operator Methods", UC Riverside.

2014 "Koopman Operator Methods and Control", Control of PDE's, Paris (Invited Plenary Lecture).  
2015 "On Applications of the Spectral Theory of the Koopman Operator in Dynamical Systems and Control Theory" Conference on Control and Decision, Osaka, Japan.

2016 "Theory and Application of Koopman Operator Methods" Hughes Research Laboratories.

• 2016 "Strongly Coupled Oscillators: A Koopman-Theoretic Approach", Network Frontier Workshop, Northwestern University.

• 2016 "Koopman Operator Methods: Theory and Applications", Mathematisches Forschungsinstitut Oberwolfach, Germany, Workshop on Applied Koopmanism.

• 2016 "Koopman Operator Theory in Fluid Mechanics" Department of Mathematics, University of Wisconsin, Madison.

• 2016 "Koopman Operator Theory in Fluid Mechanics" AEM Seminar at University of Minnesota.

2016 "Spectral Expansions, A Schrödinger-Type Formalism and Observable Wavefunctions in Dynamical Systems", International Symposium on NOLTA, Yugawara, Japan.

2017 "Extensions of the Koopman Operator Theory", SIAM Conference on Applications of Dynamical Systems, Snowbird, Utah.

**Honors and Awards:** The PI has been elected Fellow of the American Physical Society in 2015  
The PI has been elected Fellow of the SIAM in 2017

**Protocol Activity Status:**

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**Technology Transfer:** The PI has established continuing collaboration with the Army Research Lab, specifically Dr. Bryan Glaz.

US Patents awarded and licensed:

	PAT. NO.		Title
1	9,563,182	Full-Text	Building analysis systems and methods
2	9,043,163	Full-Text	Systems and methods for analyzing building operations sensor data

US Patent applications licensed:

	PUB. APP. NO.	Title
1	20170146965	BUILDING ANALYSIS SYSTEMS AND METHODS
2	20160203036	MACHINE LEARNING-BASED FAULT DETECTION SYSTEM
3	20160084889	SYSTEM AND METHOD FOR STABILITY MONITORING, ANALYSIS AND CONTROL OF ELECTRIC POWER SYSTEMS
4	20160012340	TEMPERATURE-BASED ESTIMATION OF BUILDING OCCUPANCY STATES
5	20150371151	ENERGY INFRASTRUCTURE SENSOR DATA RECTIFICATION USING REGRESSION MODELS
6	20150212119	SYSTEMS AND METHODS FOR ANALYZING BUILDING OPERATIONS SENSOR DATA

### PARTICIPANTS:

**Participant Type:** Graduate Student (research assistant)

**Participant:** Allan Avila

**Person Months Worked:** 1.00

**Funding Support:**

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

**Participant Type:** Graduate Student (research assistant)

**Participant:** Nithin Govindarajan

**Person Months Worked:** 2.00

**Funding Support:**

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

### ARTICLES:



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Publication Identifier: 10.1063/1.4772195

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**Article Title:** Applied Koopmanism

**Authors:**

**Keywords:** Dynamical Systems, Koopman operator Analysis

**Abstract:** A majority of methods from dynamical systems analysis, especially those in applied settings, rely on Poincaré's geometric picture that focuses on "dynamics of states." While this picture has fueled our field for a century, it has shown difficulties in handling high-dimensional, ill-described, and uncertain systems, which are more and more common in engineered systems design and analysis of "big data" measurements. This overview article presents an alternative framework for dynamical systems, based on the "dynamics of observables" picture. We present an overview of several approaches to studying dynamical systems using the Koopman operator, which holds promise to resolve these issues. The dynamics are analyzed by looking at evolutions of functions on the state space, rather than directly at state space trajectories. The evolution can be understood by expanding the function into a basis of eigenfunctions of the Koopman operator. The first approach is based on the Koopman modes (KMs), wh

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Publication Identifier: 10.1016/j.physd.2013.06.004

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**Article Title:** Isostables, isochrons, and Koopman spectrum for the action–angle representation of stable fixed point dynamics

**Authors:**

**Keywords:** Isostables, Isochrons, Koopman Operator Spectrum,

**Abstract:** For asymptotically periodic systems, a powerful (phase) reduction of the dynamics is obtained by computing the so-called isochrons, i.e. the sets of points that converge toward the same trajectory on the limit cycle. Motivated by the analysis of excitable systems, a similar reduction has been attempted for non-periodic systems admitting a stable fixed point. In this case, the isochrons can still be defined but they do not capture the asymptotic behavior of the trajectories. Instead, the sets of interest—that we call "isostables"—are defined in literature as the sets of points that converge toward the same trajectory on a stable slow manifold of the fixed point. However, it turns out that this definition of the isostables holds only for systems with slow-fast dynamics. Also, efficient methods for computing the isostables are missing. The present paper provides a general framework for the definition and the computation of the isostables of stable fixed points, which is based on the spect

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**Article Title:** Analysis of Fluid Flows via Spectral Properties of the Koopman Operator

**Authors:**

**Keywords:** Koopman mode expansion, dynamic mode decomposition, global modes, Arnoldi algorithm

**Abstract:** This article reviews theory and applications of Koopman modes in fluid mechanics. Koopman mode decomposition is based on the surprising fact, discovered in Meziřc (2005), that normal modes of linear oscillations have their natural analogs—Koopman modes—in the context of nonlinear dynamics. To pursue this analogy, one must change the representation of the system from the state-space representation to the dynamics governed by the linear Koopman operator on an infinite-dimensional space of observables. Whereas Koopman in his original paper dealt only with measure-preserving transformations, the discussion here is predominantly on dissipative systems arising from Navier-Stokes evolution. The analysis is based on spectral properties of the Koopman operator. Aspects of point and continuous parts of the spectrum are discussed. The point spectrum corresponds to isolated frequencies of oscillation present in the fluid flow, and also to growth rates of stable and unstable modes. The continuous p

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**Article Title:** Global Isochrons and Phase Sensitivity of Bursting Neurons

**Authors:**

**Keywords:** isochrons, bursting, phase response, Koopman operator

**Abstract:** Phase sensitivity analysis is a powerful method for studying (asymptotically periodic) bursting neuron models. One popular way of capturing phase sensitivity is through the computation of isochrons—subsets of the state space that each converge to the same trajectory on the limit cycle. However, the computation of isochrons is notoriously difficult, especially for bursting neuron models. In [W. E. Sherwood and J. Guckenheimer, SIAM J. Appl. Dyn. Syst., 9 (2010), pp. 659–703], the phase sensitivity of the bursting Hindmarsh–Rose model is studied through the use of singular perturbation theory: cross sections of the isochrons of the full system are approximated by those of fast subsystems. In this paper, we complement the previous study, providing a detailed phase sensitivity analysis of the full (three-dimensional) system, including computations of the full (twodimensional) isochrons. To our knowledge, this is the first such computation for a bursting neuron model. This was made possible

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**Paper Title:** On Comparison of Dynamics of Dissipative and Finite-Time Systems Using Koopman Operator Methods  
**Authors:** Igor Mezic  
Acknowledged Federal Support: **Y**

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Conference Location: Purdue University  
**Paper Title:** Improving HVAC Performance Through Spatiotemporal Analysis of Building Thermal Behavior  
**Authors:** Michael Vincent Georgescu, Igor Mezic  
Acknowledged Federal Support: **Y**

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**Publication Type:** Thesis or Dissertation  
**Institution:**  
Date Received: 09-Oct-2015 Completion Date:  
**Title:** Analysis of Systems in Buildings using Spectral Koopman Operator Methods  
**Authors:**  
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**INVENTIONS:**

**Intellectual Property Type:** Invention  
**Invention Title:** BUILDING SENSOR DATA RECTIFICATION USING MACHINE LEARNING METHODS  
**Description:**  
**Inventors:** Igor Mezic, Michael Georgescu  
Employer Name: UCSB  
Employer Address: 5070 Mes Rd, Santa Barbara, Ca, 91301  
Confirmatory Instrument:  
**Intellectual Property Type:** Invention  
**Invention Title:** METHOD FOR INSTABILITY MONITORING, ANALYSIS AND CONTROL OF ELECTRIC POWER SYSTEMS  
**Description:**  
**Inventors:** Igor Mezic, Yoshihiko Susuki  
Employer Name: UCSB  
Employer Address: Department of Electrical Engineering, Kyoto, Ja,  
Confirmatory Instrument:  
**Intellectual Property Type:** Invention  
**Invention Title:** Temperature-based estimation of occupancy states  
**Description:**  
**Inventors:** Igor Mezic, Michael Georgescu  
Employer Name: UCSB  
Employer Address: 5070 Mesa Rd, Santa Barbara, CA, 93106  
Confirmatory Instrument:

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”Nothing to report in the uploaded pdf (see accomplishments)”