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OPERATIONAL PLANNING FOR THEATER ANTI-SUBMARINE WARFARE

by

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Anti-submarine warfare (ASW) operational planning has always been a complicated process, involving the assignment of limited resources across multiple mission areas over an extended period of time. With the emergence of more advanced adversary submarine capabilities, the need to plan for this underwater threat has become even more important.

We modify an existing optimization model of the ASW operational planning problem to add the ability to have multiple search platforms of several types, allow more complicated synergistic interactions between them, and then test it inside a realistic scenario to develop a tool to aid ASW planners. The scenario includes three types of ASW platforms: ships, submarines, and aircraft working out of two operating bases in the Western Pacific with six notional but geologically realistic mission areas. An optimal solution assigns combinations of multiple platforms to the mission areas to reach and maintain threshold levels, or the minimum probability of detection required, set by the commander. The model measures time and effort put forth by these platforms to achieve desired levels of achievement measured in terms of a weighted "percent clearance."

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OPERATIONAL PLANNING FOR THEATER ANTI-SUBMARINE WARFARE

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requirements for the degree of

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Anti-submarine warfare (ASW) operational planning has always been a complicated process, involving the assignment of limited resources across multiple mission areas over an extended period of time. With the emergence of more advanced adversary submarine capabilities, the need to plan for this underwater threat has become even more important.

We modify an existing optimization model of the ASW operational planning problem to add the ability to have multiple search platforms of several types, allow more complicated synergistic interactions between them, and then test it inside a realistic scenario to develop a tool to aid ASW planners. The scenario includes three types of ASW platforms: ships, submarines, and aircraft working out of two operating bases in the Western Pacific with six notional but geologically realistic mission areas. An optimal solution assigns combinations of multiple platforms to the mission areas to reach and maintain threshold levels, or the minimum probability of detection required, set by the commander. The model measures time and effort put forth by these platforms to achieve desired levels of achievement measured in terms of a weighted "percent clearance."

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LIST OF ACRONYMS AND ABBREVIATIONS

ACTUV	ASW Continuous Trail Unmanned Vessel
ASSET	ASW System Evaluation Tool
ASW	Anti-Submarine Warfare
ASWP	Anti-Submarine Warfare Planner
CZ	Convergence Zone
METOC	Meteorological and Oceanographic
MPRA	Maritime Patrol and Reconnaissance Aircraft
NMP	Navy Mission Planner
NOP	Navy Operational Planner
NOP–USW	Navy Operational Planner Undersea Warfare
NPP	Navy Planning Process
NWP	Navy Warfare Publication
P_d	Probability of Detection
RF	Radio Frequency
SURTASS	Surveillance Towed Array Sensor System
USV	Unmanned Surface Vehicle
UUV	Unmanned Undersea Vehicle

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EXECUTIVE SUMMARY

Operational planning has always been a complicated process, involving the assignment of limited resources across multiple mission sets over a period of time. With the emergence of adversary submarine capabilities, the need to plan for this underwater threat has become even more important. Computer-based aids are still not used to their maximum potential and planning continues to be done by hand consuming many man-hours while subject to human error.

The process for developing a mathematics based decision tool to aid operational planning began at the Naval Postgraduate School with the Navy Mission Planner (NMP). The NMP is a decision support tool for Navy ships, which rapidly selects employment schedules across the different mission requirements in a fixed-time horizon. After many improvements to the NMP model, the Navy Operational Planner (NOP) was developed to allocate multiple ships across multiple missions in order to accomplish the assigned missions as quickly as possible to a certain level of completion. Navy Operational Planner–Undersea Warfare (NOP–USW) adapts the concepts introduced by the NOP and applies them to multiple different platforms performing one mission set.

We modify the model developed in NOP–USW to create the Anti-Submarine Warfare Planner (ASWP). The model optimizes the time and effort required by the various platforms in the different mission areas to perform ASW. It attempts to search in each mission area so as to attain, and then maintain, a *threshold* level of the conditional *probability of detection*, given there is a submarine, in each area using formulas from the theory of random search. It allocates search platforms to missions based on mission priorities, or *values*, where, in any given time period, missions with higher values in that period are more likely to have platforms assigned (if there are not enough platforms available to get all of the missions to their threshold accomplishment simultaneously). The model attempts to reach these threshold levels, which are set by the commander, as quickly as possible by assigning the different combinations of available platforms. Once the effort applied to a mission area reaches the threshold level, the model's objective function is designed to reward solutions that maintain the achievement level at or above that threshold.

Although any number of ASW systems may be created in the model, the scenario for this thesis includes three types of ASW platforms: ships, submarines, and aircraft working out of two operating bases; Okinawa, Japan, and Luzon Island, the Philippines; and six notional but geologically realistic mission areas. The schedule included in the scenario is divided into 28 equal time periods with each time period differing in platform availability, whether a mission area is active, and the mission value associated with each area. Data inputs to the model include each platform's sensor sweep width, search speed, and the size of each mission area. These inputs are entered into the Random Search equation to produce performance values, translating into achievement.

This research further develops an Anti-Submarine Warfare planning aid, ASWP, that can be used by ASW planners and commanders. We modify an existing model, developed as NOP–USW, to allow for multiple platforms of each type and to model the synergistic effects between platforms working together. We test our new model against a more realistic scenario with dynamic mission values. An optimal solution to our model provides planners and commanders with valuable information in assigning platforms to mission areas in the most optimal way. The information comes in the form of achievement, a measure of the conditional probability that we would detect a target if one was there, which is a surrogate for the probability that there is no target in a region given that we have not yet detected one. This is important information, other than actually finding the submarine, to a commander and high value assets who want to know where to operate with a low likelihood of encountering a submarine threat.

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I. INTRODUCTION

A. OPERATIONAL PLANNING

Naval operational planners have the difficult task of allocating resources or platforms to accomplish various missions in different operating areas. The task is challenging due to limited resource availability, dynamic mission requirements, and the different capabilities of each type of platform. The addition of unmanned vehicles to either side of a conflict and the growing threat of cyber warfare add additional levels of complexity to the operational planning process. Despite the increasing difficulty and complexity of the problems faced by operational planners, computer-based decision aids are not currently utilized to their maximum potential in the planning process. White boards, paper charts, and simple spreadsheets are frequently the primary tools in the operational planning process. This way of planning continues to consume man-hours, is subject to errors, and does not offer any reasonable way to perform risk-reward analyses of different potential courses of action.

Anti-submarine warfare is a particularly demanding mission set to perform well. A single submarine is a very deadly platform that can eliminate high value targets, perform long-range missile strikes across countries, and deter a navy's fleet from entering a body of water. Stealth is a submarine's greatest strength, which makes searching, localizing, and tracking a submarine so difficult. With the United States' biggest rivals pushing to improve their submarine capabilities, the task of tracking their submarines is getting even harder. Naval forces can have trouble searching and tracking one submarine, let alone multiple submarines in different regions or mission areas.

B. LITERATURE REVIEW

The current doctrine used by the Department of Defense (DOD) for operational planning is Joint Publication 5-0 (JP 5-0) (Joint Chiefs of Staff, 2011). The Navy has taken JP 5-0, and refined the information into its own doctrine for operational planning in Navy Warfare Publication (NWP) 5-01: Navy Planning. Featured in NWP 5-01 is the

Navy Planning Process (NPP), the process to be used by commanders and staffs to plan naval operations. This research, along with prior work in this area, attempts to speed up this process and make the planning process easier, more thorough, and more robust.

1. Navy Mission Planner

The Navy Mission Planner (NMP) is a decision support tool for Navy ships that rapidly selects employment schedules for multiple ships across different mission requirements in a fixed time horizon (Dugan, 2007). Dugan says that NMP "gives decision makers the ability to adjust courses of action by manipulating the time horizon, optimality criterion, mission values, mission dependencies, and ships available, and provides valuable insight into which missions will and, more importantly, will not be covered for any set of mission priorities." Follow-on work by Silva (2009) tested NMP on a large-scale scenario over a long period of time, and Hallman (2009) added logistical considerations to the planning process.

2. Navy Operational Planner

The Navy Operational Planner (NOP) took the research done by NMP and applied it to operational planning (Deleon, 2015). Deleon states, "Navy Operational Planner advises how to allocate multiple ships to multiple missions in order accomplish missions to a prescribed level of completion as quickly as possible, to allow a transition to the next phase of a larger operation such as a war, or a large-scale humanitarian aid and disaster relief operation." The NOP began as a model for Mine Warfare and was tailored for other warfare areas. The NOP introduces the concept of level-of-effort as a way to track mission completion through the cumulative effort of multiple ships over time.

3. Navy Operational Planner—Undersea Warfare

NOP–Undersea Warfare (NOP–USW) adapts the work done with NMP and NOP to apply the concept of level-of-effort to the ASW spectrum (Molina, 2016). NOP–USW models ASW using standard formulas for random search and applies them to multiple platforms searching in the same area. Each different type of platform has different detection rates, which made the concept of interchanging the numbers of platforms and ship-days of effort not possible. It also models the possibility of “degradation” of effort if an area is left unattended for any significant period of time. This research will extend Molina’s model and is discussed in the following section.

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II. OPERATIONAL PLANNING—THEATER ASW

A. DESCRIPTION

In this thesis we modify the model developed in NOP–USW to create the Anti-Submarine Warfare Planner (ASWP), a multi-period, multi-platform operational planning model that allows for multiple platforms of each platform type, a richer set of feasible combinations of platforms that can work on a single mission, and a standard way to model beneficial (or detrimental) effects of platforms coordinating their efforts. We then test ASWP on a more realistic scenario than in the prior work and discuss the resulting plan.

The model optimizes the time and effort required by the various platforms in the different mission areas to perform ASW. It attempts to search in each mission area so as to attain, and then maintain, a *threshold* level of the conditional *probability of detection*, given there is a submarine, in each area using formulas from the theory of random search. It allocates search platforms to missions based on mission priorities, or *values*, where, in any given time period, missions with higher values in that period are more likely to have platforms assigned (if there are not enough platforms available to get all of the missions to their threshold accomplishment simultaneously). The model attempts to reach these threshold levels, which are set by the commander, as quickly as possible by assigning the different combinations of available platforms. Once the effort applied to a mission area reaches the threshold level, the model's objective function is designed to reward solutions that maintain an achievement level corresponding to a probability of detection at or above that threshold.

1. Random Search

There are many different tactics employed to conduct search on submarines, and the most appropriate tactic is determined by the particulars of a specific situation. This thesis uses the Random Search Model for the surface ships and submarines, and the Maritime Patrol and Reconnaissance Aircraft (MPRA) derived version of random search. The classic presentation of random search assumes the target is stationary, searcher's track is randomly and uniformly distributed over the search area, and no search effort falls outside the search area. Some of the reasons we use random search are that it is "mathematically easy, and provides a lower bound on the effectiveness of searches that attempt to be systematic, but are actually randomized by navigation errors, environmental uncertainties, and target motion" (Washburn, 2016).

Random search requires three inputs not including time: the searching platform sensor's sweep width, its operating speed, and the size of the search area. To determine the probability of detection (P_d) we use the equation $P_d = 1 - e^{-\gamma t}$, where t is the amount of time spent searching, and $\gamma = \frac{wv}{A}$ is the *detection rate* for the platform, determined by w, the sweep width of the platform, v, the search velocity of the platform, and A, the surface area of the region to be searched. These equations are applied to each platform type and the P_d values are calculated for each discrete time period. ASWP is a multi-period planning model that allows for different combinations of platforms to search an area in different time periods, and allows the probability of detection in an area to degrade if no platforms are assigned in that period. If this is the case the Probability of detection curve may not be smooth, or even monotonic.

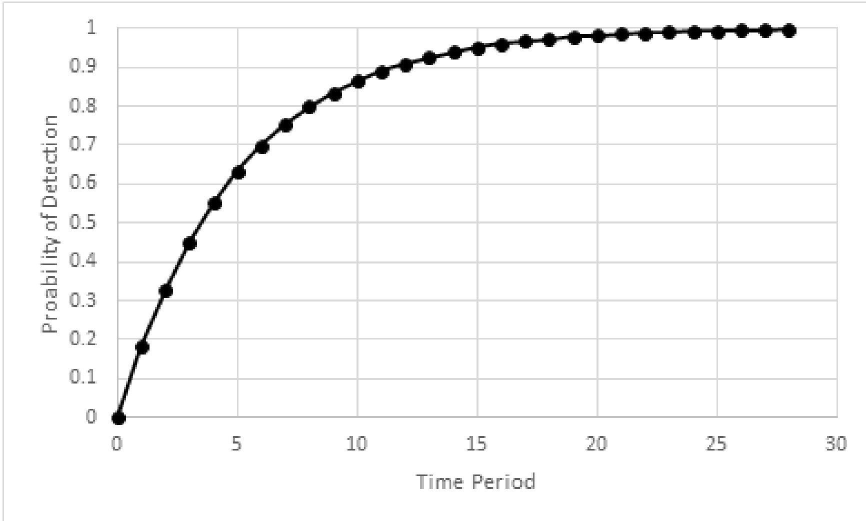
Our model uses the mission values to create a weighted combination of probabilities of detection over the mission areas. Maximizing total weighted probability of detection over a long time horizon and over multiple mission areas may not necessarily maximize the probability of detection of a submarine in any particular area. However, with all other things being held equal, if we improve the probability of detecting a target if one exists, this translates directly into a higher confidence that no target exists if we do *not* detect anything. By applying sufficient effort across well-chosen mission areas, the model can tell a commander where safe waters will be established by the suggested ASW operations.

2. Achievement

We cannot easily represent every possible probability of detection in a given search area; the resulting model would have nonlinearities that are difficult to handle. As in Molina (2016), we circumvent this difficulty by defining a finite number of *achievement levels*, indexed by k, each of which represents a particular probability of detection, a_k. These levels are chosen to represent the entire range of probabilities from 0.0 to 1.0. Reaching achievement levels corresponds to higher probabilities of detection, and increasing this probability of detection assures us that, if we have not yet detected a submarine, the probability there is a submarine inside the mission area is low. The term *threshold* refers to the minimum probability of detection required by a commander to comfortably state that a mission area is likely to be clear of any enemy submarines. Given the nature of random search and, in particular, its application to ASW, we can never know with certainty that a submarine is or is not there unless it is detected. Each mission has a nonzero *value* representing its importance relative to the other missions, and this value is also determined by a commander. Higher-valued missions will be rewarded more for threshold achievement and maintenance than lower-valued missions and therefore will have search assets assigned preferentially.

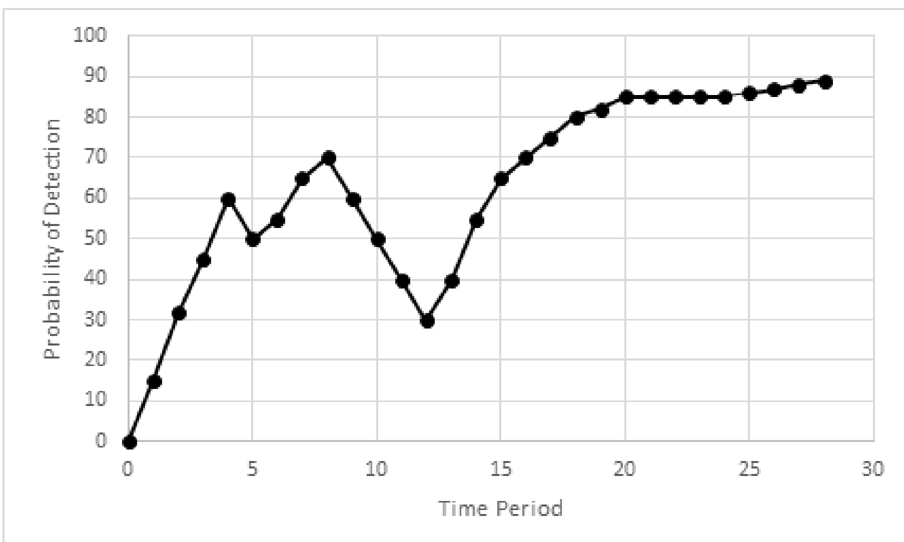
We will use the term *performance curve* to describe the trajectory of probabilities of detection produced by each combination of platforms with their performance capabilities entered in the random search equation. Figure 2 illustrates the performance curve of a single ship searching one mission area. Figure 3 illustrates the performance curve of a single mission where achievement is gained or lost as different combinations are assigned, or, in the cases in which the performance curve drops, no platforms are assigned.

Given all of this information, we can calculate the combinations of prior achievement levels and the combinations of platforms required in the previous time period to get to any particular achievement level in the current time period. We use the term *predecessors* of k, denoted PRE_k, to describe this set.



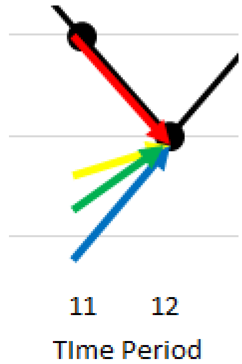
The performance curve increases probability by large amounts in the earlier time periods and much more effort is required later on. At time period 5, performance is at approximately 0.63 and the next 5 time periods P_d increases by only 0.23 to 0.87 almost a third the rate of increase.

Figure 1. Performance Curve for a Single Ship Searching a Single Mission Area over 28 Time Periods.



Performance curve resulting from varying combinations of platforms being assigned over time. Probability of detection increases over the first 4 time periods, drops in the 5th, jumps up again in the 6th, and drops for 4 straight periods. Starting from the 13th time period, the P_d is on a steady increase, reaching approximately .83 by time period 24.

Figure 2. Performance Curve for a Single Mission over 28 Time Periods



This zoomed-in look at two time periods from Figure 3 shows the possible transitions. The different colored arrows represent the different combination of platforms that could be applied to the previous time period to reach the next time period's probability of detection. In this example, P_d decreased (red arrow) because there were no platforms active in the mission area.

Figure 3. Single Transition of Achievement Level from One Time Period to Another.

3. New Additions

New additions to the model and scenario include the allowance of ASW aircraft to search alone, and the addition of synergistic effects. Previous work has allowed MPRA to only be used as a supplementary platform to a ship or submarine. Synergistic effects give a small bump to the growth of achievement when multiple platforms work in the same mission area.

a. Allowing Aircraft to Search Alone

MPRA are a long ranged ASW capable platform designed to drop multiple sono-buoys into the water at numerous depths to detect sound. Through the sensors onboard the aircraft, operators will monitor the sound emitted by the submarine to determine its course, speed and depth. A single P-8 can monitor up to 64 sono-buoys at one time and in this thesis that capability is utilized for every P-8. Any mention of sono-buoys in this research will be referring to only passive or active sono-buoys that produce direct path contact.

NOP–USW used the MPRA or P-8 as a supplementary platform to a ship or sub and never assigned to search alone. This thesis allows the MPRA to search alone and has a slightly different detection rate equation based on ASW Systems Evaluation Tool (ASSET). More details on ASSET is in Shaffer's 1991 thesis, "Evaluation of the MPA Detection and Allocation Models Utilized by the ASW Systems Evaluation Tool (ASSET)." Shaffer's thesis describes the detection rate for MPRA as $\gamma_p = \frac{n w_p u}{A}$, where n is the number of buoys dropped in the search area, w_p is the effective sweep width of each sono-buoy in nautical miles (nm), u is the velocity of the target in knots, and A is the area of the mission search region, in nm^2 (1991). This will be used in this thesis for P-8's as the detection rate in the random search equation. To be consistent with the random search theory that the target is stationary, u should be set to 0, but this would make the detection rate equal to 0, therefore we have set u to 1.

b. *Synergistic Effects*

It is difficult to numerically calculate and determine how much more effective the search effort is between multiple platforms working together. In this scenario, when platforms are working in the same mission area, we add synergistic effects to attempt to model the positives of communication between platforms to the overall search effort. For each combination c of platforms, we add or subtract a percentage bonus, β_c, to each platform's sweep width when working in the same mission area. This added bonus to sweep width encourages the model to add multiple synergistic platforms to mission areas to achieve and maintain the threshold level quicker, and can discourage the use of two platforms that might impede each other's progress. Platforms working in the same mission area as a submarine do not receive any added synergistic effects due to the fact communication with a submarine will most likely not happen. In general, for any combination of platforms, c, our probability of detection becomes: $P_d = 1 - e^{-(1+\beta_c)t\sum \gamma_p}$, where the sum is taken over the individual platforms in the combination.

c. *Mission Values*

NOP–USW maintained a constant priority value for the mission areas throughout the time periods. This is research will change priority values applied to each mission per time period to model a commander's change in preferences based on the information given to them during previous missions.

B. MODEL

The following model has been modified from NOP–USW (Molina, 2016, pp. 13–16). New features include specifying the number of platforms of each type in each combination, the number of platforms of each type available, and the ability to assign multiple platforms of each type to various missions. Because we expect multiple assets of each platform type to be available, the number used should reflect the expected number of platforms available to be on-station at any given time. The actual number of platforms might be larger than this, but the available assets data should account for downtime, logistical requirements, travel to and from operational mission areas, etc. Because of this assumption, we have removed the logistical constraints from the model in Molina (2016); these constraints made the model significantly harder to solve, for very little gain in planning realism.

1. Sets and Indices [Cardinality]

$t \in T$	Time periods in planning horizon [~28]
$\boldsymbol{m} \in \boldsymbol{M}$	Missions [~6]
$\boldsymbol{k} \in \boldsymbol{K}$	Discrete levels of achievement [~100]
$\boldsymbol{p} \in \boldsymbol{P}$	Platform types [~3]
$\boldsymbol{c} \in \boldsymbol{C}$	Distinct combinations of platforms [~10]
$p \in CP_c$	Platform types *p* in combination *c*
$(k', c, m) \in PRE_k$	Set of tuples that result in (are "predecessors" to) achievement level *k*.
$p \in TP_t$	Platforms *p* available in period *t*

2. Derived Set

$c \in TC_t$	Combinations c available in period t $(t,c) \in TC \quad \Leftrightarrow \quad (t,p) \in TP \ \forall \ p \in c$

3. Data [Units]

a_k	Numerical value of achievement level k [0.0-1.0]
$val_{t,m}$	Priority value of mission m in period t [1-5]
$thresh_m$	P_d threshold for accomplishing mission m [0.0-1.0]
$avail_{p,t}$	Number of platforms of type p available in period t
$req_{p,\mathrm{c}}$	Number of platforms of type p required in combination c

4. Variables [Units]

$ASGND_{t,m,p}$	Number of platforms of type p assigned to mission m at time t [Integer]
$CACT_{t,m,c}$	Combination of platforms c is chosen for mission m in t [Binary]
$KACH_{\mathrm{t},m,k}$	Achievement level k is feasible at time t for mission m [Binary]
$KCACT_{k,c,t,m}$	Mission m achievement level is at or above level k in t and combination c is applied to mission m in t [Binary]
$DONE_{t,m}$	Mission m achievement level meets or exceeds its threshold in time t [Binary]
$MACT_{t,m}$	Mission m has assets assigned at time t [Binary]

5. Formulation ASWP

$$\max \quad \sum_{t,m} val_{t,m}\left(DONE_{t,m} + \sum_{k}(0.1)\,a_k KACH_{t,m,k} + (0.01)\,MACT_{t,m}\right) \qquad (\text{M0})$$

$$s.t. \quad KCACT_{t,k,m,c} \le KACH_{t,m,k} \qquad \forall k,t,m,c \in TC_t \qquad (\text{M1})$$

$$KACH_{t,m,k} \le \sum_{(k',c):(k',c,m)\in PRE_k} KCACT_{t-1,m,k',c} \qquad \forall k,t>1,m \qquad (\text{M2})$$

$$\sum_{k} KACH_{t,m,k} = 1 \qquad \forall t,m \qquad (\text{M3})$$

$$KCACT_{t,c,m,k} \le CACT_{t,m,c} \qquad \forall k,t,m,c \in TC_t \qquad (\text{M4})$$

$$req_{p,c} CACT_{t,m,c} \le ASGND_{t,m,p} \qquad \forall t,m,c \in TC_t,\ p \in TP_t \qquad (\text{M5})$$

$$\sum_{c\in TC_t} CACT_{t,m,c} \le 1 \qquad \forall t,m \qquad (\text{M6})$$

$$MACT_{t,m} \le \sum_{c\in TC_t} CACT_{t,m,c} \qquad \forall t,m \qquad (\text{M7})$$

$$\sum_{m} ASGND_{t,m,p} \le avail_{p,t} \qquad \forall t,\ p \in TP_t \qquad (\text{M8})$$

$$ASGND_{t,m,p} \le req_{p,c} \sum_{c\in TC_t} CACT_{t,m,c} \qquad \forall t,m,\ p \in TP_t \qquad (\text{M9})$$

$$\sum_{(k',c):(k',c,m)\in PRE_k} KCACT_{t,m,k',c} \le 1 \qquad \forall k,t,m \qquad (\text{M10})$$

$$DONE_{t,m} \le \sum_{k:a_k \ge thresh_m} KACH_{t,m,k} \qquad \forall t,m \qquad (\text{M11})$$

$$DONE_{t,m} \in \{0,1\} \qquad \forall t,m$$

$$MACT_{t,m} \in \{0,1\} \qquad \forall t,m$$

$$KCACT_{t,k,m,c} \in \{0,1\} \qquad \forall t,m,k,c \in TC_t$$

$$KACH_{t,m,k} \in \{0,1\} \qquad \forall t,m,k$$

$$CACT_{t,m,c} \in \{0,1\} \qquad \forall t,m,c \in TC_t$$

$$ASGND_{t,m,p} \ge 0, Integer \qquad \forall t,m,\ p \in TP_t$$

6. Explanation

a. Objective Function

The objective function, (M0), calculates a total reward based on the value of each mission as a weight on the sum over all time periods of three terms, in decreasing order of importance and, therefore, decreasing sizes of coefficients: 1) whether the mission is above the threshold level of achievement in that period, 2) what its actual achievement level is in that period, and 3) whether it is actively pursued in that period by any non-empty combination of platforms.

b. Achievement Constraints

Each constraint (M1) prevents a combination from getting credit for an achievement level on a mission in a particular time period unless the corresponding mission is actually at that achievement level in that time period. Each constraint (M2) allows an achievement level to be activated for a mission in a time period only if an appropriate combination in the predecessor of that achievement level is active in the prior time period. Each constraint (M3) requires that exactly one achievement value is active at every time period. Each constraint (M4) allows a combination to achieve a given level for a mission in a time period only if it is active on that mission in that period. Each constraint (M5) states that a combination can only be active for a mission if the required number of platforms of each type are assigned to that mission. Each constraint (M6) says there can be only one combination of platforms assigned to a mission. Each constraint (M7) states that a mission will be considered active only if a combination is active on it. Each constraint (M8) restricts the total number of platforms of each type to the number of platforms available. Each constraint (M9) limits the number of platforms of a given type assigned to a mission cannot exceed the number of platforms required for the combination chosen. Each constraint (M10) states that only one predecessor of k value can be active at any time. Each constraint (M11) says that a mission can be considered to have achieved its threshold level only if it has reached an accomplishment level at or above the threshold.

c. Implementation

We formulated our model in Pyomo, a Python package for optimization modeling developed at Sandia National Labs (Hart, Laird, Watson, & Woodruff, 2012) and we used CPLEX (IBM, 2017) to solve the resulting model. We ran all of our models on a Lenovo ThinkPad P50 with an Intel Xeon CPU with 8 cores running at 2.70 GHz, and with 64 GB RAM available. Each model solved in less than 30 minutes.

d. Calculation of PRE_k

The sets PRE_k are the core of our model, and represent all of the possible transitions between (discrete) probability of detection values based on combinations of platforms being assigned to mission areas. For any particular achievement level, k, there is an associated probability of detection at that level, pd_k. The first achievement level corresponds to pd_k=0.0, and the last achievement level has pd_k=1.0. With these values in hand we can use a modified version of the multi-platform search equation, $P_d = 1 - e^{-(1+\beta_c)t\sum req_{p,c}\gamma_{p,m}}$, where we now include the number of platforms of each type in combination c, $req_{p,c}$, and the detection rates based on the platform properties and the surface area, A_m, of each mission region, to determine these transitions.

Specifically, if we have a fixed duration for each time period, τ, (in our examples, $\tau = 6$ hours) we calculate for each starting achievement level, k', the change in P_d after searching in a particular mission area for one time period with a particular combination, and then use this to calculate the resulting achievement level k as the one with the closest value of pd_k to this resulting value. Once we calculate all of these transitions, (one for each combination of k', c, and m), we build for each achievement level k the combinations (k',c,m) that result in achievement level k.

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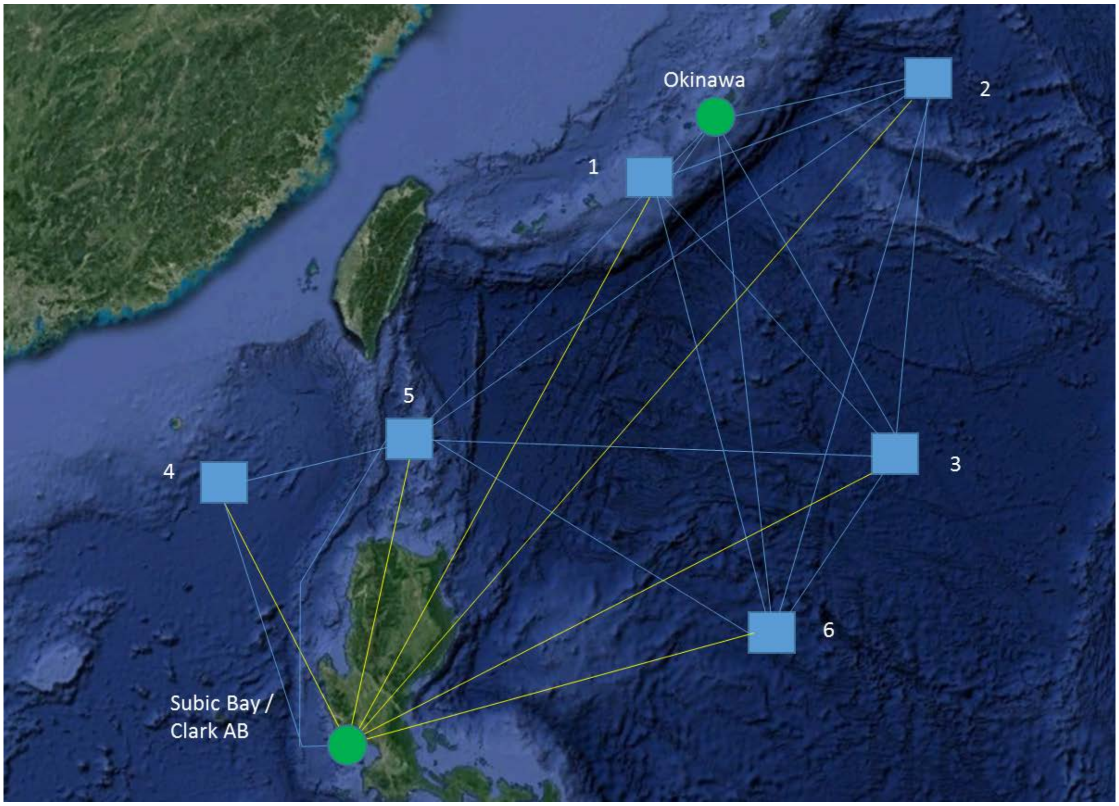
III. RESULTS

A. SCENARIO

The scenario is modeled after a potential real-world problem where operational planners need to assign multiple platforms to multiple ASW missions in multiple regions. The scenario is set in the western Pacific with two operational bases in Okinawa, Japan, and Luzon Island, the Philippines. Included with the two bases are six notional mission areas that have the potential for enemy submarines. Figure 5 shows the map of scenario with the two regions, mission areas, and routes taken to reach each area. We utilize three different types of platforms in this research; submarines modeled after our nuclear attack submarines or "sub," surface ships modeled after any ASW capable destroyer or cruiser referred to as "ship," and MPRA or P-8 Poseidon referred to as "p8." The scenario schedule includes platform availability, whether each mission area is active, and mission area values that change over time to model the potential dynamic nature of ASW and military operations in general. We send the platforms to each mission area to search for an enemy submarine or determine with confidence that a submarine is not there. Each mission area is given the size of $3600\,\text{nm}^2$. Time is divided into periods where each time period is equivalent to six hours and the scenario includes 28 time periods or one full week. NOP–USW included water depth in the mission area to model contiguous zone (CZ) which vastly increases the sweep width of each platform in that area. We will assume all oceanic conditions of each mission area to be constant and not include the potential effects of CZ. CZ, bottom bounce, and other underwater propagation paths are a possibility in the mission areas included in this scenario, but not always present therefore not included. We will only deal with sweep widths produced by direct path contact.



Centered near the Philippine Sea, the two green circles represent the two operating bases. Blue squares represent the mission areas, and the blue lines represent shortest distant routes by the ships and submarines. The yellow lines out of Clark AB in the Philippines show us the shortest route for aircraft involve flying over land and will have short distances than the ships and submarines.

Figure 4. Map of the Scenario. Adapted from Google Maps.

1. Data

The data for the scenario was generated by hand in Microsoft Excel. The numbers for the data are inputs into the model, and can be changed in any manner in order to make the scenario more realistic.

a. Schedule

The values for the schedules include the number of each platform available during each time period; if the specific mission area requires searching; and the mission value provided by the commander on each mission. Table 1 provides the complete schedule for the Okinawa missions.

Table 1. Schedule for Okinawa

Time Period	Number Available			Mission Area			Priority Value		
	ship	**sub**	**p8**	**1**	**2**	**3**	**1**	**2**	**3**
1	2	1	3	1	1	0	4	4	2
2	2	1	3	1	1	0	4	3	1
3	2	1	3	1	1	0	4	3	1
4	2	1	3	1	1	0	4	1	2
5	2	1	2	1	1	1	2	2	2
6	2	1	2	1	1	1	3	4	2
7	2	1	2	1	1	1	3	3	3
8	2	1	2	1	1	1	3	1	4
9	2	1	3	1	0	1	1	1	1
10	2	1	3	1	0	1	4	4	3
11	2	1	3	1	0	1	4	2	1
12	2	1	3	1	0	1	4	2	3
13	2	0	3	0	0	1	3	4	1
14	2	0	3	0	0	1	2	2	3
15	2	0	3	0	0	1	4	2	1
16	2	0	3	0	0	1	3	4	4
17	1	0	2	1	1	0	4	2	3
18	1	0	2	1	1	0	3	4	1
19	1	0	2	1	1	0	3	4	2
20	1	0	2	1	1	0	4	3	4
21	2	1	3	0	1	0	2	4	2
22	2	1	3	0	1	0	4	3	3
23	2	1	3	0	1	0	2	3	4
24	2	1	3	0	1	0	1	3	2
25	2	1	3	1	0	1	4	2	4
26	2	1	3	1	0	1	1	1	3
27	2	1	3	1	0	1	2	1	3
28	2	1	3	1	0	1	2	2	1

b. Platforms

Table 3 shows the data inputs for the platform into the model, and is constant for all platforms of each type. The sweep widths are based on direct path contact and assumes constant oceanic conditions in all mission areas.

Table 2. Platform Data for all Platform Types

	ship	sub	p8
Transit Speed (kts)	20	15	300
Operating Speed (kts)	15	10	240
Sensor Sweep Width (nm)	2	2.5	0.5
Endurance (time periods)	1.67	56	56
Down Time (time periods)	8	8	4

c. Combinations

Table 4 shows the possible combinations of platforms that can be assigned to each mission. The synergistic bonus provides a percentage increase to the sweep width of each platform in the combination. Each additional ship in a combination will provide a 10% bonus and additional P-8's will provide a 5% bonus. P-8's provide a smaller percentage increase because of the Radio Frequency (RF) channel limitation of the sono-buoys in the water. Each P-8 is capable of monitoring 64 sono-buoys at one time, but there can only be a maximum of 100 sono-buoys in the water of a given search area. Each sono-buoy takes up a RF channel and the sono-buoys' RF range is 1 to 100. Submarines do not offer any synergistic bonuses. Table 4 also shows the various combination bonuses applied to our search equation. For example, when two ships are working together in the same mission area, we use combination *c*='c9', and we see that there is a 10% bonus, or $\beta_c = 0.10$, to each platform's sweep width. The detection rate for each ship will then be $\gamma_1 = \frac{1.1 w_{ship} v_{ship}}{A}$ and $\gamma_2 = \frac{1.1 w_{ship} v_{ship}}{A}$, yielding $P_d = 1 - e^{-(\gamma_1 + \gamma_2) 1.1 t}$.

Table 3. Possible Combinations of Platforms Assigned to Missions

combo	ship	sub	p8	percent bonus
c0	0	0	0	0
c1	1	0	0	0
c2	0	1	0	0
c3	0	0	1	0
c4	1	1	0	0
c5	1	0	1	10
c6	0	1	1	0
c7	1	1	1	10
c8	1	1	2	15
c9	2	0	0	10
c10	2	0	1	15
c11	1	0	2	15
c12	2	0	2	20

d. Other Types of Search

Random search is used in NOP–USW and this thesis but it is not the most realistic type of search to be used by these platforms. Other types of search include Exhaustive Search and Barrier Search. Exhaustive Search, also known as "mowing the lawn," is the concept of sweeping the entire area starting at one end and ending at the other. This type of search will sweep the entire area for enemy submarines but is extremely time consuming, and the paths of the platforms are extremely predictable. MPRA cannot utilize exhaustive search due to the limited number of buoys that can be processed and maintained on the P-8's.

Barrier search is the concept of searching a small strip of area and continuously going back and forth in a line. Barrier search is useful in chokepoints where we can anticipate an enemy submarine passing through with high probability. It is not very useful in the open ocean where commanders want mission areas cleared of submarines. Despite the flaws of these two search types, they can produce probabilities of detection (P_d 's) or performance that can be used in the model. For the purposes of this research we use Random Search, but any other type of search can be incorporated by changing the calculation of the set Pre(k).

B. MODEL RESULTS

1. Platform Assignments

The optimal schedule computed by the model for Okinawa is shown in Table 5. The combination of platforms assigned for each time period affects the achievement level for the following time period. Data for the Philippines is available for this scenario, but we have focused on the Okinawa missions for the test case.

Table 4. Optimal Platform Schedule for Okinawa

Time Period	Mission 1		Mission 2		Mission 3	
	Combo	P_d	Combo	P_d	Combo	P_d
t1	c7	0	c3	0	c5	0
t2	c8	0.2808	c5	0.1891	c0	0.2374
t3	c8	0.4901	c5	0.3900	c0	0.1047
t4	c8	0.6398	c5	0.5384	c0	0.0378
t5	c7	0.7439	c3	0.6488	c1	0
t6	c7	0.8159	c3	0.7145	c1	0.0378
t7	c7	0.8671	c3	0.7648	c1	0.0726
t8	c5	0.9031	c4	0.8099	c3	0.1047
t9	c1	0.9271	c3	0.8278	c8	0.2596
t10	c1	0.9317	c3	0.8617	c8	0.4771
t11	c1	0.9363	c3	0.8880	c8	0.6307
t12	c3	0.9409	c3	0.9080	c10	0.7368
t13	c5	0.9498	c3	0.9224	c5	0.8037
t14	c3	0.9629	c10	0.9363	c3	0.8507
t15	c5	0.9714	c3	0.9542	c5	0.8777
t16	c3	0.9798	c5	0.9629	c5	0.9080
t17	c3	0.9839	c1	0.9714	c3	0.9317
t18	c3	0.9880	c3	0.9714	c1	0.9454
t19	c1	0.9920	c3	0.9756	c3	0.9498
t20	c1	0.9920	c3	0.9798	c3	0.9586
t21	c2	0.9920	c3	0.9839	c10	0.9672
t22	c7	0.9920	c3	0.9880	c3	0.9756
t23	c1	0.9960	c8	0.9920	c3	0.9798
t24	c2	0.9960	c3	0.9960	c11	0.9839
t25	c1	0.9960	c3	0.9960	c7	0.9880
t26	c3	0.9960	c1	0.9960	c8	0.9920
t27	c10	0.9960	c2	0.9960	c3	0.9960
t28	c0	0.9960	c0	0.9960	c0	0.9960

2. Mission Achievement

Figure 6 shows us the progression of achievement levels for the first three missions over the 28 time periods. Missions 1 and 2 are given the priority over mission 3 during the first time periods by the model. Mission 1 reaches and maintains above the threshold by time period 6, while mission 2 requires 8 time periods to get above that achievement level. Mission 3 reaches the threshold at time period 13 after the other two missions are maintaining achievement levels above the threshold.

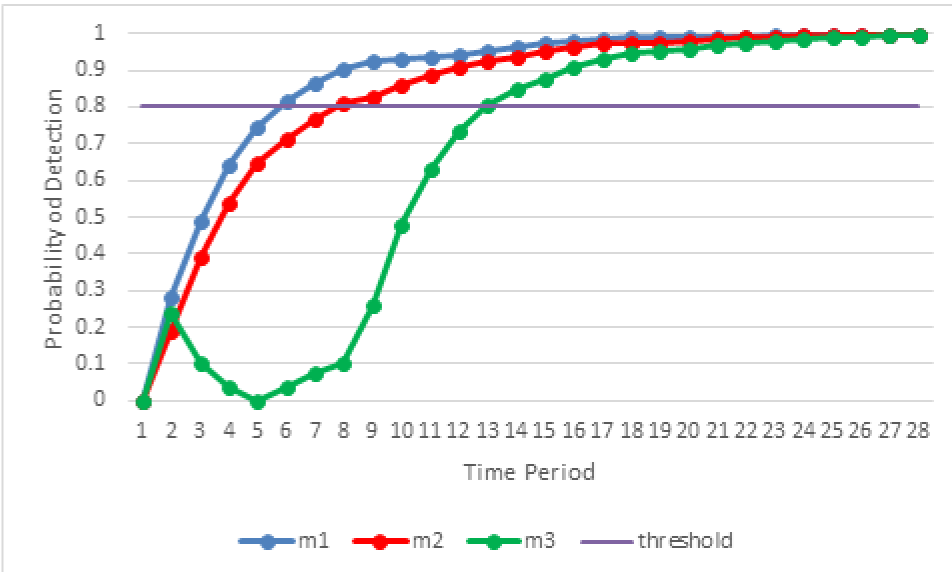


Figure 5. Achievement Level Progression for Missions 1,2,3

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IV. CONCLUSIONS

A. SUMMARY

This research further develops an Anti-Submarine Warfare planning aid, ASWP that can be used by ASW planners and commanders. We modify an existing model, developed as NOP–USW, to allow for multiple platforms of each type and to model the synergistic effects between platforms working together. We test our new model against a more realistic scenario with dynamic mission values. An optimal solution to our model provides planners and commanders with valuable information in assigning platforms to mission areas in the most optimal way. This is important information, other than actually finding the submarine, to a commander and high value assets who want to know where to operate with a low likelihood of encountering a submarine threat.

This research has advanced this model towards being used as an ASW planning aid by decision makers. We continue to make progress by testing the models limits, adding new and more complex model features, and continually increasing the realism, in terms of scope and scale, of our operational scenarios.

B. FOLLOW-ON WORK

Navy Mission Planner and Navy Operational Planner have laid the groundwork for operational planning tools where multiple platforms are assigned to multiple missions in multiple regions to reach a certain achievement sufficient to meet the commander's needs. Future research can take the concept and apply it to a different type of warfare or further enhance the realism of work already done.

1. Improvements to the Model/Scenario

a. Run Time

The NOP–USW model has a large run time mainly due to the high number of variables involved and resolution of the achievement level curve. We reduced the run time by removing the logistical constraint from the model, but further research could still reduce the computational burden by adjusting the models constraints or acceptance of results that are less than perfect. Minor changes or tweaks to the model could reduce the length it takes the model to compute results.

b. More Testing

A more robust and realistic scenario is applied to research done in NOP–USW but just the beginning of potential work to be done. Variations to the number of mission areas, number of platforms, and size of each area can alter how each combination of platforms perform when assigned. Adding different search types to the platforms can produce different results and give a level of flexibility to the platforms when searching. Random search is used in this research, but it is not the best option when searching near a chokepoint. Barrier search might be a better option and allowing the model to make that decision would require additional information fed into the model along with changes to the formulation. Fine tuning the capabilities of the different types of platforms will help make the scenario and model more realistic.

2. Additions to the Model/Scenario

a. Additional Platforms

In this research, we added ASW aircraft to search alone and not used as a supplementary platform. There is still room for the addition of other ASW platforms to make the scenario and model more realistic and a better planning aid for ASW decision makers or planners. Possible expansion could be set for ASW helicopters launched off surface ships, Surveillance Towed Array Sensor System (SURTASS) ships which can detect submarines from long distances, and even unmanned underwater vehicles (UUV) or unmanned surface vehicles (USV) such as the ASW Continuous Trail Unmanned Vessel (ACTUV). Addition of these platforms only require platform capabilities data as an input to the model and enlarge the possible combinations of platforms the model can assign.

b. Environmental Effects

Factors that greatly affect the detection ranges of a sensor include the body of water and time of year which affect the temperature, salinity, and pressure of the water thus changing the sound velocity profile. The model currently does not factor in weather or oceanographic conditions that can potentially affect the different sweep widths of each platform. These conditions are set to constant for ease of calculations to an already big model with slow run times. Work done in conjunction with the meteorological and oceanographic (METOC) department could help enhance the validity of the model. The addition of environmental data into the model will be very challenging, but greatly validate the realism and application of the model to operational planning for ASW.

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