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Research In The Measurements And Theory Of Plasmoids And Their Applications To Missiles And Satellite Technology Progress Report 1, AD0203743 - 11 Sep 1958; 88ABW-2017-5445, Cleared on November 1, 2017 Progress Report 2, AD0213968 - 13 Mar 1959; 88ABW-2017-5446, Cleared on November 1, 2017 Progress Report 3, AD1038383 - 11 Jun 1959; 88ABW-2017-5447, Cleared on November 1, 2017

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ARF 1121-4 First Annual Report)



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"RESEARCH IN THE MEASUREMENTS AND THEORY OF PLASMOIDS AND THEIR APPLICATIONS TO MISSILES AND SATELLITE TECHNOLOGY"

Dale J. DeGeeter

Wright Air Development Center, Wright-Patterson Air Force Base, Ohio

> Contract No. AF 33(616)-5791 (Task No. 70854)

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ARF 1121-4 (First Annual Report)

on

"RESEARCH IN THE MEASUREMENTS AND THEORY OF PLASMOIDS AND THEIR APPLICATIONS TO MISSILES AND SATELLITE TECHNOLOGY"

Contract No. AF 33(616)-5791 (Task No. 70854)

to

Aeronautical Research Laboratory, WCLJH, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio

Covering the period of June 1, 1958 to May 31, 1959

June 11, 1959

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ABSTRACT

Finite plasmas, termed plasmoids, have been produced and studied in this laboratory. A convenient and flexible experimental apparatus has been designed and constructed for producing the phenomena. An image converter light shutter unit is near completion which will allow examination of the plasmoid shape. Microwave equipment is being added to the system for measurements on the electron density and internal magnetic fields in the plasmoids. Time-exposure photographs have been made under various conditions of magnetic field and background gas pressure and show strong dependence of plasmoid characteristics on these parameters. Concurrent theoretical investigations of the stability of cylindrical plasma configurations have been made complementing the experimental phase. The simplest configuration whereby the internal magnetic field of the cylinder is "force-free" has been found incompatable due to the inability to meet boundary conditions. A more complex situation is now being treated where the stability or instability is controlled by interactions between the magnetic fields and fluid motions within the plasmoid.

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APPENDIX

I.	Hydromagnetic Study of Rotating Cylindrical Plasmas.
II.	Construction of the Experimental Apparatus.
III.	Experimental Results.
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V	Design and Application of the image converter.

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RESEARCH IN THE MEASUREMENTS AND THEORY OF PLASMOIDS AND THEIR APPLICATIONS TO MISSILES AND SATELLITE TECHNOLOGY

I. INTRODUCTION

The existence of relatively stable configurations of ionized gas, or plasma, formed under certain conditions in a laboratory vacuum chamber, was first reported by Bostick,¹ who adopted the name "plasmoids" for these plasma entities. Using his original plasma gun in a rather poor vacuum in the absonce of an externally applied magnetic field, he was able to obtain Kerr-cell photographs (0,5 μ sec exposure) of the formation of a plasmoid which appears to be toroidal in shape and which is projected across the vacuum space at speeds up to 2 x 10⁷ cm/sec. Applying a magnetic field perpendicular to the plane of the electrodes reduces the translational speed of the plasmoid only by a factor of about 1/2, while the plasmoid elongates along the direction of the field into a cylindrical "broomstick-shaped" plasmoid.

Further investigations^{1,2} have shown a remarkable tendency of these plasmoids to maintain their integrity in a collision between two or more plasmoids, and very interesting multiple-source configurations have been observed. Many other investigations have been made, including a measurement³ of the velocity of the plasmoids as a function of the direction and magnitude of an applied magnetic field up to about 6 kilogauss, of the source voltage up to about 15 kv, and other circumstances. However, much of the behavior of ¹ W. H. Bostick, Phys. Rev., <u>104</u>, 292 (1956). ² W. H. Bostick, Phys. Rev. <u>106</u>, 404 (1957). ³ E. G. Harris, R. B. Theus, and W. H. Bostick, Phys. Rev., <u>105</u>, 46 (1957).

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plasmoids is not yet clearly understood, and there remains a fruitful area for further investigation of the properties of such plasma configurations.

The project reported herein consists of an experimental and analytical study of plasma configurations that can be produced at the low pressure existing at high altitudes. The experimental program is concerned with the production and study of plasma clouds under free space conditions. The analytical program is a mathematical investigation of the stability of such structures starting from the basic hydromagnetic equations. In addition, suggestions of what practical use might be made of plasmoids with respect to high altitude vehicles will be made. This annual report outlines the progress and planning of this project for the interval June 1, 1958 to May 31, 1959.

II. THEORETICAL INVESTIGATIONS

The early reports of the remarkable behavior of plasmoids suggested the possibility that they might be configurations of plasma which would be completely stable in the absence of loss mechanisms such as the presence of background gas, and it was decided to investigate the stability of various cylindrical plasma configurations in the presence of a uniform external field.

The simplest case one might postulate would be a static configuration in which the internal magnetic field of the cylinder is "force-free" (i.e. the currents are parallel to the field, so that the Lorentz force vanishes). The stability of an infinitely long cylinder with a force-free field "with constant \propto " has been investigated by Trehan.⁴

Recent work by Woltjer and Chandrasekhar⁵ shows that the field "with

(1958).
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'L. Woltjer, Astrophys. J. <u>128</u>, 384 (1958); Proc. Nat. Acad. Sci. (U.S.A.) <u>山</u>, 489 (1958); <u>山</u>, 833 (1958); S. Chandrasekhar and L. Woltjer, Proc. N.A.S. <u>山</u>, 285 (1958); see also S. Chandrasekhar, Proc. N.A.S. <u>山</u>, 842 (1958).

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constant \propto " is probably the only stable type of force-free field; but this type of field cannot meet the boundary conditions corresponding to a cylinder of finite length embedded in a uniform external field. Therefore it was decided to consider a more complex situation, viz. one in which the stability or instability is controlled by interactions between the magnetic fields and fluid motions within the plasmoids.

A summary of the problems considered and results obtained during the first year of the project will be found in Appendix I. It is not yet clear whether it will be possible to complete the study of the compressible plasma. Allowing for the compressibility of the plasma introduces many mathematical complications into the problem; but if these can be overcome, the results should be more directly applicable to plasmoids than those resulting from an analysis of the incompressible case. It is interesting to note, however, that the work of Tayler (reference 3 of Appendix I).shows that for non-rotating cylindrical plasmas the conditions for stability are independent of the compressibility; and it would be interesting to see whether rotation of the plasma changes this situation. It is felt that it should, because of the difficulty with balancing the centrifugal force, as discussed in Appendix I.

During the second year of the project we expect to consider some of the points raised in Appendix I, e.g. the existence of a variational principle, the feasibility of the use of high speed computing machinery, the radial dependence of the angular velocity and internal fields of the plasma, and the problem of an induced electric field. We hope to be able to complete the study of the compressible plasma, but if this proves impractical we shall look for new results in the incompressible case.

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III. EXPERIMENTAL DESCRIPTION

The basic experimental unit has been completed and is being used for the work in progress. Fig. 1 shows the completed apparatus and Appendix II contains a detailed description. Minor additions and modifications are being continued for specific experiments. The two primary additions in progress at the moment are an image converter light shutter and a microwave beam device. They are discussed in Appendices V and IV respectively.

The delivery of an RCA type C73435B image converter tube has been scheduled for June 8, 1959. With the reception of this, the final mounting of the tube can be made including preparations for static tests. The electronic circuitry has been otherwise completed and is shown in Appendix V. The design is basically that of a unit in operation at Los Alamos Scientific Laboratory. Several modifications were introduced to facilitate adaption to our planned operation and the unit described in Appendix V is the final design. The addition of the image converter will give us a powerful research tool for stopping the motion of the plasmoids and rendering a better understanding of their nature.

The microwave beam apparatus was completed and assembled as shown in Appendix IV. Since the transmitting and receiving horns are in a vacuum region, thin glass windows were installed in the waveguide. However, considerable difficulty was encountered in obtaining a vacuum seal with this arrangement and finally a mylar window was sealed and compressed between the horn and waveguide. The resulting combination was eventually rendered vacuum tight and transparent to the microwave beam. The unit is ready to be installed in the chamber when a supporting structure in construction is completed and an installation problem is overcome. The unit was placed in operation in

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Fig. 1. Final Apparatus

the position of Fig. 2 of Appendix IV and a check of the components was made. The operation was satisfactory and the fundamental frequency was checked with the frequency meter.

The pictures shown and described in Appendix III are time-exposure photographs of plasmoids. This relatively rapid and easy method of observing the plasmoids provides information on the influence of the several experimental parameters on plasmoids. Additional quantitative investigations can then be carried on in the regions of experimental interest as indicated by these results.

Time-exposure photographs have been taken varying the magnetic field strength holding the background pressure, plasmoid source current, and other parameters constant. Additional photographs have been taken with the pressure at various fixed values and the results discussed in Appendix III. Photographs are now being made to investigate other areas and confirm the results already recorded.

The addition of a critical damping resistance in the source discharge circuit has rendered the discharge essentially undirectional. The peak current and plasmoid energy have been reduced accordingly, but previous difficulties have necessitated the addition to get a single plasmoid for observation. Probes were introduced into the path of the plasmoids and the signal across a 50 ohm resistance to ground displayed on the scope. It was hoped that the interaction of the plasmoid with the probe could be observed and a qualitative analysis made for the present report. No intensive probe study was considered due to the conflicting evidence encountered by previous, investigators.⁶

⁶ W. H. Bostick - private communication

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The results of the probe measurements were inconclusive and not worthy of any additional time expenditure. The signal was swamped by at least two extraneous noise sources and the problem of reducing these effects caused the measurements to be abandoned. The first of the disruptive signals was introduced by the photoemission of the probe due to the intense ultraviolet radiation from the arc at the currents involved. This could be reduced by either reducing the current in the arc or placing the probe at a greater distance from the arc. Neither of these alternatives were acceptable since our interest lies in higher energies and any vacuum chamber modifications are impractical at the moment. The other unwanted signal seems to come from pickup by the probe of the radiation from the triggered-gap switch and the arc. Shielding could have reduced this but the results were not worth the effort under the conditions present. Consequently, the probe measurements were discontinued and other more promising ideas pursued.

A photomultiplier telescope arrangement was made for observation of the plasmoids. Preliminary measurements were made on the risetime of the unit and then the combination was placed in position to record the passage of the plasmoid. In general, as discussed in Appendix II, the measurements indicated additional magnetic and electrostatic shielding was necessary before final measurements could be made. A new configuration, as described in Appendix II, has been completed and will be put into operation in the near future.

IV. EXPERIMENTAL RESULTS

Of the results described at length in Appendix III, several points are of particular interest. The time-exposure photographs confirmed the presence of the plasmoids and showed the effects of varying several of the experimental

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parameters. The effect of increasing the magnetic field was clearly shown in the increased pinching of the arc and plasma. By varying the background pressure from 10^{-5} to 10^{-2} mm. Hg. the plasmoid went from a straight trajectory in the high vacuum region to a curved trajectory at about 10^{-3} mm. Hg. to being stopped before traversing the vacuum chamber in a poor vacuum. The interaction of the plasmoid with probes confirmed the affinity to maintain itself in a collision and consequently its possible stability.

The elongation of the plasmoid along the magnetic field lines has been shown in photographs taken with a mirror providing this third dimension. It has also been confirmed by pictures of the recombination at the vacuum chamber wall. The plasmoids havebeen fired at thin mylar sheet for possible deposition of charge. No net charge was observed after developing and the results were inconclusive.

V. FUTURE WORK

The photographic recording of plasmoids with smear photographs will be continued and supplemented by the addition of the image converter light shutter. The smear photographs will be continued to get a qualitative indication of the behavior of plasmoids over wide variations in the various experimental parameters that have been studied to date. The image converter will undergo static tests upon receiving the tube about the middle of June. It will then be adapted to the apparatus and prepared for the photographing of the plasmoid in motion across the vacuum chamber. Here the major problem will consist of providing adequate shielding to reduce the effects of external radiation to an acceptable level. The design of the unit has included special precautions against the radiation problem but under actual operating conditions adjustments and necessary modifications will be made. A working unit under

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final operating conditions is expected within the next several months.

The introduction of our microwave experiments will be simultaneously in progress. The integration of the equipment into the system has been started and will be completed shortly. Several modifications to allow the horn installation and introduce an absorbing material will follow preparing the unit for initial operations. The initial microwave experiments will then follow as outlined in Appendix IV. The interaction of electromagnetic energy with plasmoids will be of interest both in obtaining information on the plasmoids and in determining what effect the plasmoid has on the incident microwave beam. The first experiment planned is to measure the magnetic field associated with the plasmoid by microwave absorption at the gyroresonance frequency of the electrons corresponding to the microwave frequency. If the magnetic field is decaying in time with passage through the microwave beam, the absorption at this frequency can be recorded provided the field is nearly uniform in the plasmoid. A non-uniform field would cause a spreading of the resonant peak and make measurements more difficult. In addition, with the knowledge of the plasmoid shape provided by the image converter, the investigations can be extended to measure such quantities as electron density, recombination coefficients, collision frequency, and propagation characteristics in the ionized gas.

Optical measurements with the photomultiplier telescope combination will be continued to provide the velocity of the plasmoids necessary for the image converter and microwave measurements.

VI. LOGBOOK REFERENCES

The data of this project are recorded in ARF Logbook Nos. C-8026, C-8036, C-8319, C-8326, C-8388, C-8456, C-8531, C-1025, C-8538, C-8820,

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C-8821, C-9088, C-1041, C-1025, and C-8821.

VII. CONTRIBUTING PERSONNEL

The theoretical investigations are being carried out by Val W. Pratt, D. J. DeGeeter, J. T. Jones, and R. L. Watkins are responsible for design and construction of the apparatus and the performing of experimental investigations.

Respectfully submitted,

ARMOUR RESEARCH FOUNDATION of Illinois Institute of Technology

Dale J. Defecter, Ass't. Physicist, Plasma and Electron Physics

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APPROVED BY:

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APPENDIX I

HYDROMAGNETIC STUDY OF ROTATING CYLINDRICAL PLASMAS

The equations of motion in the hydromagnetic approximation (ignoring viscosity and electrical resistivity) are:

$$\frac{\partial \mathbf{v}}{\partial t} + \rho(\vec{\mathbf{v}} \cdot \nabla) \vec{\mathbf{v}} = \frac{1}{\mu \pi} (\vec{B} \cdot \nabla) \vec{B}(-\operatorname{grad})(p + \frac{B^2}{8\pi}), \quad (1)$$

$$\frac{\partial \rho}{\partial t} = - \operatorname{div} \rho \vec{v}, \qquad (2)$$

$$\operatorname{curl} \vec{B} = \frac{\mu \pi \vec{J}}{c}, \qquad (3)$$

$$\operatorname{div} \vec{P} = 0$$

$$\operatorname{curl} \vec{E} = -\frac{1}{c} \quad \frac{\partial \vec{B}}{\partial t} \quad (5)$$

div
$$\vec{E} = 4\pi \epsilon$$
 (6)
 $\vec{E} + \frac{\vec{v} \times \vec{B}}{c} = 0$, (7)

and either

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$$\begin{cases} \operatorname{div} \vec{v} = 0 & (\text{incompressible}) \\ \text{or} \\ \frac{1}{2} & \frac{\mathrm{dp}}{\mathrm{dt}} = \frac{\sqrt{2}}{2} & \frac{\mathrm{dp}}{\mathrm{dt}} \\ \end{array}$$
(8) (8) (9)

where p, ρ , and δ' are respectively the pressure, density, and ratio of specific heats of the plasma; \vec{E} and \vec{B} are respectively the electric and magnetic fields, \vec{j} the electric current, ϵ the net electric charge density, and $\vec{\nabla}$ the fluid velocity field.

The first problem to be considered was to find stationary solutions of the hydromagnetic equations for the imcompressible case. Equations (5) and (7)

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can be combined to obtain

$$-\frac{1}{c} \frac{\partial \vec{B}}{\partial t} = \text{curl} \vec{E} = -\frac{1}{c} \text{curl} (\vec{v} \times \vec{B}),$$

$$\frac{\partial B}{\partial t} = \operatorname{curl}\left(\vec{v} \times \vec{B}\right) . \tag{10}$$

and Eq. (1) can be rewritten in the form

$$\frac{\partial \vec{v}}{\partial t} = \frac{1}{\mu n \rho} \operatorname{curl} \vec{B} \times \vec{B} - \operatorname{curl} \vec{v} \times \vec{v} - \operatorname{grad} \left(\frac{p}{\rho} + \frac{v^2}{2}\right) \qquad (11)$$

by applying the vector identity

$$(\vec{A} \cdot \nabla) \vec{A} = \operatorname{curl} \vec{A} \times \vec{A} + \operatorname{grad} \left(\frac{1}{2} \mid \vec{A} \right)$$

to both \vec{v} and \vec{B} . (The density ρ is assumed uniform). Now making the substitutions $\vec{h} \equiv \vec{B}/\sqrt{4\pi\rho}$ and $\varpi = (\frac{p}{\rho} + \frac{1}{2} |\vec{v}|^2)$, and setting time derivatives equal to zero, we have the requirements for the existence of a stationary solutions

$$\operatorname{curl}\left(\vec{\mathbf{v}} \times \vec{\mathbf{h}}\right) = 0 , \qquad (12)$$

$$\operatorname{curl} \vec{h} \times \vec{h} - \operatorname{curl} \vec{v} \times \vec{v} - \operatorname{grad} w = 0$$
, (13)

If we now assume that in our cylindrical coordinate system (r, Θ, z) , $\vec{h} = (0, h_0, h_z)$ and $\vec{v} = (0, v_0, 0)$, then $\vec{v} \times \vec{h} = (v_0, h_z, 0, 0)$; and if we assume that \vec{v} and \vec{h} can depend upon r only, then Eq. (12) is trivially satisfied and yields no information. Looking now at Eq. (13),

 $D \equiv \frac{d}{dr}$ and $D^* \equiv (\frac{d}{dr} + \frac{1}{r})$. Thus

where

curl
$$\vec{h} \times \vec{h} = (-h_z Dh_z - h_Q D^* h_Q, 0, 0)$$

and

$$\operatorname{curl} \vec{v} \times \vec{v} = (-v_0 D^* v_0, 0, 0)_{\mathfrak{s}}$$

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and since

$$\operatorname{grad} \boldsymbol{\varpi} = (D \boldsymbol{\varpi}, 0, 0)$$

Eq. (13) becomes

$$\mathbf{D} \quad \mathbf{\widehat{o}} = -\mathbf{h}_{\mathbf{z}} \quad \mathbf{D}\mathbf{h}_{\mathbf{z}} - \mathbf{h}_{\mathbf{\theta}} \quad \mathbf{D}^{\mathbf{x}}\mathbf{h}_{\mathbf{\theta}} + \mathbf{v}_{\mathbf{\theta}} \quad \mathbf{D}^{\mathbf{x}}\mathbf{v}_{\mathbf{\theta}}$$

$$= -\frac{1}{2} D(h_{z}^{2} + h_{\theta}^{2} - v_{\theta}^{2}) - \frac{1}{r} (h_{\theta}^{2} - v_{\theta}^{2})$$

and finally

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$$D \varpi = \frac{1}{r} (v^2 - h_0^2) + \frac{1}{2} D(v^2 - h^2) , \qquad (14)$$

as our only restriction upon possible stationary solutions. In the special case in which the force term (grad $\frac{p}{\rho}$) vanishes, we have $D \varpi = \frac{1}{2} D (v^2)$, and Eq. (14) reduces to

$$h_0^2 = v^2 - \frac{r}{2} D(h^2)$$
, (15)

which is a two - parameter family of solutions, since the only additional restriction is that $h^2 = h_Q^2 + h_Z^2$.

If we allow the variables of the problem to depend on z as well as r we obtain the conditions of Chandrasekhar;¹ he describes each of the axisymmetric vector fields in terms of two scalars, representing the "poloidal" (P,U) and "toroidal" (T, V) parts of the fields:

$$\vec{h} = -r \frac{\partial P}{\partial z} \vec{l}_r + r T \vec{l}_Q + \frac{1}{r} \frac{\partial}{\partial r} (r^2 P) \vec{l}_z, \quad (16)$$

$$\vec{v} = -r \frac{\partial U}{\partial z} \vec{l}_r + r V \vec{l}_Q + \frac{1}{r} \frac{\partial}{\partial r} (r^2 U) \vec{l}_Z , \qquad (17)$$

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where \vec{l}_r , \vec{l}_q , \vec{l}_z are unit vectors along the three principal directions of the coordinate system. With \vec{h} and \vec{v} in this form, Eqs. (4) and (8) are automatically satisfied, and symmetry about the z-axis is ensured by stipulating

¹ S. Chandrasekhar, Astrophys. J. <u>124</u>, 232 (1956).

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that P, T, U, V be independent of the aximith Θ_{\bullet} For solutions of Eqs. (12) and (13) Chandrasekhar finds the following relationships:

$$\frac{\partial (\mathbf{r}^2 \mathbf{P}, \mathbf{r}^2 \mathbf{U})}{\partial (\mathbf{z}, \mathbf{r})} = 0 , \qquad (18)$$

$$\frac{\partial(\mathbf{T}, \mathbf{r}^2 \mathbf{U})}{\partial(\mathbf{z}, \mathbf{r})} - \frac{\partial(\mathbf{V}, \mathbf{r}^2 \mathbf{P})}{\partial(\mathbf{z}, \mathbf{r})} = 0 \quad , \tag{19}$$

$$\frac{\partial (\mathbf{r}^2 \mathbf{T}, \mathbf{r}^2 \mathbf{P})}{\partial (\mathbf{z}, \mathbf{r})} - \frac{\partial (\mathbf{r}^2 \mathbf{V}, \mathbf{r}^2 \mathbf{U})}{\partial (\mathbf{z}, \mathbf{r})} = 0 , \qquad (20)$$

$$\frac{\partial (\Delta_5^{\mathrm{P}}, r^{2})}{\partial (z, r)} + \frac{\partial (\Delta_5^{\mathrm{U}}, r^{2})}{\partial (z, r)} = r \frac{\partial (r^{2})}{\partial z} - r \frac{\partial (v^{2})}{\partial z}, \quad (21)$$

where the Jacobians have the conventional meaning

$$\frac{\partial (r^2 P, r^2 U)}{\partial (z, r)} = \frac{\partial (r^2 P)}{\partial z} \frac{\partial (r^2 U)}{\partial r} - \frac{\partial (r^2 P)}{\partial r} \frac{\partial (r^2 U)}{\partial z}$$

and Δ_5 is the five-dimensional axisymmetric Laplacian operator

$$\Delta_5 = \frac{\partial^2}{\partial r^2} + \frac{3}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

These results (18) to (21) are, however, too general to be of much use for the present problem; the only new information we have found from these equations is a nonlinear differential equation connecting U and P for the case in which T = V = 0. This was presented in Progress Report No. 2, and will not be reproduced here, since (as was pointed out in that report) this case is unlikely to occur in cylindrical configurations.

In Progress Report No. 2 we presented the linearized perturbation equations (equations 17 to 24% of that Report) appropriate to the examination of the stability of a rotating cylinder of incompressible plasma, and pointed out that Dungey² had shown such a system to be unstable for the case of rigid

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²² J. W. Dungey, <u>Cosmic</u> <u>Electrodynamics</u> (Cambridge University Press, 1958) pp 79-82.

rotation (Ω = const.) and uniform longitudinal field (H = H_Z = const.). There are of course many other interesting possibilities which have not been investigated, and we may return to this problem at some later time; but for the present it has been set aside in the hopes of being able to solve the problem of a compressible plasma, which would have more direct applicability to plasmoids.

For the compressible case we begin with Eqs. (1) to (7), (9), and (10). The terms $(\vec{v} \cdot \nabla) \vec{v}$ and $(\vec{B} \cdot \nabla) \vec{B}$ in Eq. (1) must be interpreted properly in cylindrical coordinates:

$$\begin{bmatrix} (\vec{\nabla} \cdot \nabla) \vec{\nabla} \end{bmatrix}_{\mathbf{r}} = \vec{\nabla} \cdot (\text{grad } \mathbf{v}_{\mathbf{r}}) - \frac{\mathbf{Q}}{\mathbf{r}}$$

$$\begin{bmatrix} (\vec{\nabla} \cdot \nabla) \vec{\nabla} \end{bmatrix}_{\mathbf{Q}} = \vec{\nabla} \cdot (\text{grad } \mathbf{v}_{\mathbf{Q}}) + \frac{\mathbf{v}_{\mathbf{Q}}\mathbf{v}_{\mathbf{r}}}{\mathbf{r}} , \qquad (22)$$

$$\begin{bmatrix} (\vec{\nabla} \cdot \nabla) \vec{\nabla} \end{bmatrix}_{\mathbf{Z}} = \vec{\nabla} \cdot (\text{grad } \mathbf{v}_{\mathbf{Z}}) ,$$

where the extra terms in the \mathbf{F} and $\mathbf{\Theta}$ components correspond respectively to centripetal and Coriolis accelerations. For a steady state of pure rotation, $\vec{v} = (0, r \ \Omega, 0), (\vec{v} \cdot \nabla) \vec{v} = -r \ \Omega^2 \vec{1}_r$, where $\vec{1}_r$ denotes a unit vector in the radial direction. The $(\vec{B} \cdot \nabla) \vec{B}$ term gives no contribution from the magnetic field inside the plasma, where there is no r- nor $\mathbf{\Theta}$ -component, and the z-component depends only on r.

For the steady state within the plasma, Eq. (1) reduces to

$$-\rho_{0} r \Omega^{2} = -\frac{\partial}{\partial r} \left(p_{0} + \frac{B_{0}}{8\pi} \right) , \qquad (23)$$

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where the zero subscripts here denote the steady-state values. It is easily seen from equation (10) that we now cannot assume that p_0 , ρ_0 , B_0 , and Λ are all constants independent of r, for there would then be nothing to balance the centrifugal force on the left side of the equation. In the incompressible

case it is possible to keep ρ_0 and B_0 constant and divide p_0 into a constant term plus a term whose gradient balances the centrifugal force; the latter term can then be absorbed implicitly into the "generalized pressure" perturbation term and the problem carried through as though p_0 , ρ_0 , B_0 , and were all constant.

In a compressible fluid, however, one cannot have p varying and ρ constant except by maintaining a temperature gradient in the fluid, which appears to be impossible in the plasmas under consideration. It would of course be possible to consider ρ varying with p in such a way that their ratio remained constant; but in view of the very short relaxation times of most plasmas, it appears more reasonable physically to assume that p_o and ρ_o both remain constant, so that B_o must vary in such a way as to balance the centrifugal force.

Tayler³ has investigated in some detail the stability of a non-rotating compressible plasma, and we shall use mostly his notation here, to facilitate comparison where possible between the two cases. We assume that the initial velocity field $\vec{\nabla}_{0} = (0, r \Lambda_{0} 0)$, the vacuum magnetic field $\vec{B}_{0}^{V} = (0, B_{00} \frac{r_{0}}{r}, B_{00} b_{0})$, and the plasma magnetic field at the boundary $\vec{B}_{0}^{P}(r_{0}) = (0, 0, B_{00} b_{1})$, so that b_{1} and b_{0} denote the ratios of the z-components of the interior and exterior fields respectively to the Θ -component of the vacuum field at the boundary $r = r_{0}$. In Progress Report No. 3 we assumed Ω constant, as well as p_{0} and ρ_{0} , so that $\left|B_{0}^{p}(r)\right|^{2} = B_{00}^{-2} b_{1}^{-2} = \lim_{n \to 0} \rho_{0} \Omega^{2} (r_{0}^{-2} - r^{2})$. (24)

³ R. J. Tayler, Proc. Phys. Soc. (London) <u>B70</u>, 1049-1063 (1957).

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Then, according to Eq. (3), there must be within the plasma an azimuthal current j_0 , given by

$$\mathbf{j}_{o} = -\frac{\mathbf{c}}{4\mathbf{t}} \frac{\mathbf{d}}{\mathbf{d}\mathbf{r}} \frac{\mathbf{b}}{\mathbf{c}} \qquad (25)$$

(26)

We assume that the interior and exterior fields are balanced by an additional current $\vec{j}_0^* = (0, j_{00}, j_{02})$ which flows on the surface of the plasma. The boundary conditions are that $(p + \frac{B^2}{8\pi})$ and the normal component of \vec{B} shall each be continuous at the boundary. Thus the system will be in equilibrium if

$$j_{0,2}^{*} = \frac{c_{0,2}^{B}}{4\pi} (b_{1} - b_{e}) ,$$

$$j_{0,2}^{*} = \frac{c_{0,2}^{B}}{4\pi} ,$$

$$p_{0} = \frac{B_{0,0}^{2}}{4\pi} (1 + b_{e}^{2} - b_{1}^{2}) .$$

To examine the stability of the equilibrium, we linearize the equations of motion and examine the behavior of the system in the presence of small perturbations. With the equations linearized, any small perturbation may be represented as a superposition of fundamental modes. Therefore, we assume that any variable q takes the form

$$q = q_0 + q_1 \exp \left[i \left(m\varphi + kz\right) + \omega t\right], \qquad (27)$$

where q_0 is the equilibrium value and $q_{1} = q_1$ (r) is a first-order perturbation; we will neglect all second-order terms.

In the vacuum no current can flow, so Eq. (3) yields curl $\vec{B}_1^v = 0$; therefore the perturbation field can be derived from a magnetostatic potential W so that

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$$\vec{B}_1 = \text{grad} \quad \forall \quad \bullet \qquad (28)$$

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Further, from Eq. (14) we see that ψ must satisfy Laplace's equation

$$\nabla^2 \Psi = o, \qquad (29)$$

the solution of which

$$\Psi = C_{1,m} (kr) + C_{2,m} (kr), \qquad (30)$$

where I_m and K_m are the modified Bessel functions of the first and second kinds⁴ and C_1 and C_2 are constants to be determined from the boundary conditions. The components of \overline{B}_1^v then are

$$B_{\perp r}^{\mathbf{v}} = \mathbf{k} C_{\perp} K_{m}^{\mathbf{i}} (\mathbf{k}\mathbf{r}) + \mathbf{k} C_{2} I_{m}^{\mathbf{i}} (\mathbf{k}\mathbf{r}),$$

$$B_{\perp \Theta}^{\mathbf{v}} = \frac{\mathbf{i}\mathbf{m}}{\mathbf{r}} C_{\perp} K_{m} (\mathbf{k}\mathbf{r}) + \frac{\mathbf{i}\mathbf{m}}{\mathbf{r}} C_{2} I_{m} (\mathbf{k}\mathbf{r}), \qquad (31)$$

$$B_{\perp \Theta}^{\mathbf{v}} = \mathbf{i} \mathbf{k} C_{\perp} K (\mathbf{k}\mathbf{r}) + \mathbf{i} \mathbf{k} C_{\infty} I (\mathbf{k}\mathbf{r}),$$

⁻22 ⁻m

where a prime denotes differentiation with respect to the argument. From the requirement that B must be continuous at the rigid conducting boundary we see that

$$B_{lr}^{v}$$
 (R_o) = 0, which yields

$$\frac{C_{\parallel}}{C_{22}} = - \frac{I'_{m} (k R_{0})}{K'_{m} (k R_{0})}$$
(32)

as one relation between the constants C_{1} , C_{2} .

Linearizing Eq. (10) for the plasma in accordance with Eq. (27) yields the three component equations:

$$\omega B_{lr} = i m (\Delta B_{lr} + i k B_{o} v_{lr})$$

$$\omega B_{lQ} = i k r (\Delta B_{lz} + i k B_{o} v_{lQ} + (\Delta B_{lr} + r) (\Delta B_{lr}))$$

$$\omega B_{lz} = -B_{o} D^{*} v_{lr} + v_{lr} D_{c} B_{o} + (im (\Delta B_{lz}) - \frac{im}{r_{c}} B_{o} v_{lQ})$$
(33)

⁴ The notation agrees with that of G. N. Watson, <u>A Treatise on the Theory of</u> <u>Bessel Functions</u> (Cambridge, University Press, Second edition 1944).

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The solutions for the components of \vec{B}_1 in terms of the components of \vec{v}_1 are:

(34)

(35)

(37)

$$B_{lr} = \frac{i k B_{o}}{\sigma} v_{lr},$$

$$B_{l\Theta} = \frac{i k B_{o}}{\sigma} v_{l\Theta},$$

$$B_{lz} = \frac{B_{o}}{\sigma} (i k v_{lz} - div \vec{v}_{l}) - \frac{v_{lr}}{\sigma} D B_{o},$$

where $\sigma \equiv \omega + im \Omega$. From Eq. (2),

$$\rho_{1} = - \rho_{0} \operatorname{div} \vec{v}_{1} - \operatorname{im} \rho_{1} ,$$

$$\rho_{1} = - \frac{\rho_{0}}{\sigma} \operatorname{div} \vec{v}_{1} .$$

Equation (9) now gives

$$\frac{\omega p_{1}}{p_{0}} = \frac{\sqrt[3]{\omega \rho_{1}}}{\rho_{0}} \qquad (36)$$

$$p_{1} = c_{s}^{2} \quad \rho_{1} = -\frac{\rho_{0} c_{s}^{2}}{\sigma} \quad \text{div} \vec{\nabla}_{1} \quad (36)$$

$$\sqrt[3]{p_{0}}$$

where $c_s^2 \equiv \frac{\delta p_o}{\rho_o}$, so that c_s is the speed of sound in the unperturbed plasma. The current due to perturbations in the magnetic field is given by Eq. (3):

$$j_{1r} = \frac{c}{\mu \pi} \left[\frac{im}{r} B_{1r} - i k B_{1Q} \right]$$

$$j_{1Q} = \frac{c}{\mu \pi} \left[i k B_{1r} - D B_{1r} \right] ,$$

$$j_{1z} = \frac{c}{\mu \pi} \left[D^* B_{1Q} - \frac{im}{r} B_{1r} \right] .$$

Equation (1) can be written in the form

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \nabla)\vec{v} = - \operatorname{grad} p + \frac{\vec{J} \times \vec{B}}{c}, \quad (38)$$

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which yields the linearized components

$$\rho_{o} \sigma v_{lr} = 2 \rho_{o} \Omega v_{lo} - \rho_{l} r \Omega^{2} = -Dp_{l} + \frac{1}{c} (j_{o} B_{lz} + j_{lo} B_{o}),$$

$$\rho_{o} \sigma v_{lo} + 2 \rho_{o} \Omega v_{lr} = - \frac{im}{r} p_{l} + \frac{1}{c} (-j_{lr} B_{o}), \qquad (39)$$

$$\rho_0 \sigma v_{1z} = -i k p_{1} + \frac{1}{c} (-j_0 B_{1r})$$
.
When we substitute the expressions obtained above for ρ_1 , p_{1} , j_0 , \vec{j}_1 and \vec{B}_1 , the three equations (39) become

$$v_{lr} - 2 \frac{\Omega}{\sigma} v_{l0} = -\frac{i}{k} Dv_{ls} + \frac{r \Omega^2}{\sigma^2} \left[div \vec{v}_1 - 2 i k v_{ls} \right] + \frac{B_o^2}{lm \rho_o^2} \left[D(div \vec{v}_1) - i k Dv_{ls} - k^2 v_{lr} \right], \qquad (40a)$$
$$v_{l0} + 2 \frac{\Omega}{\sigma} v_{lr} = \frac{im}{r} \left[\frac{c_s^2}{\sigma^2} div \vec{v}_1 + \frac{r \Omega^2}{\sigma^2} v_{lr} \right]$$

$$+ \frac{1}{4\pi\rho_0 \sigma^2} \left[\frac{\mathrm{im}}{\mathrm{r}} (\mathrm{div} \, \vec{v}_1 - \mathrm{i} \, \mathrm{k} \, \mathrm{v}_{12}) - \mathrm{k}^2 \, \mathrm{v}_{10} \right] \qquad (40b)$$

$$\mathbf{v}_{1z} = \frac{\mathbf{i} \mathbf{k} \mathbf{c}^{2}}{\sigma^{2}} \operatorname{div} \vec{v}_{1} + \frac{\mathbf{i} \mathbf{k} \mathbf{r} \mathbf{\Omega}^{2}}{\sigma^{2}} \mathbf{v}_{1r}$$
(40c)

In principle, these equations can be reduced by elimination to a single differential equation for v_{lr} by the following procedures Equation (40c) can be solved for div \vec{v}_{l} in terms of v_{lz} and v_{lr} ; if this result is substituted into Eq. (40a) we have a relation involving v_{lr} , Dv_{lr} , v_{lQ} , v_{lz} and Dv_{lz} . Equations (40b) and (40c) can then each be solved for v_{lQ} and v_{lz} in turn, and the results equated to give an expression for the other in terms of v_{lr} and Dv_{lr} . The expression for v_{lz} must be differentiated to find Dv_{lz} in terms of v_{lr} and its derivatives; and this result, together with the expressions for v_{lQ} and v_{lz} can then be substituted into Eq. (40a), yielding a second order

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differential equation for v_{lr} subject to the appropriate boundary conditions at the perturbed plasma surface. The solution for v_{lr} could then be used to find v_{l0} and v_{lz} .

In practice it is extremely difficult to solve Eqs. (40) by this method, as the coefficients involved are quite complicated: e.g. the coefficient of Dv_{1z} in Eq. (40a), even when reduced to its lowest form, contains over 20 terms, including one in r^{10} , in its numerator (assuming Eq. 24). Therefore we will be unable to get a solution in closed form, and can at best hope to get an approximate series solution. Professor Chandrasekhar has suggested that, since only an approximate solution is possible, it would be best to be solve the problem by a variational method, if a variational principle for the problem can be found. (It must be remembered that the solution of Eqs. (40) would still be only an intermediate step in the solution of the problem). On the other hand, it may be feasible to solve Eqs. (40 a,b,c) simultaneously for many values of the parameters by the use of a high-speed digital computer.

Another problem which has received attention during the last quarter concerns the validity or applicability of the assumptions underlying Eq. (24). As mentioned above, the assumption of uniform ρ and p seems justified; but the assumption of constant Λ seems somewhat artificial. The rigid rotation of a plasma apparently has been produced experimentally⁵ but there is no <u>a priori</u> reason to believe that this situation occurs in plasmoids. If cylindrical plasmoids have a rotation, it is probably due to the presence of circulating currents in the toroidal plasma at the time it breaks away from the source. As mentioned above, the plasma probably has a very short

⁵ V. G. Stepanov, V. F. Zakharchenko, and V. S. Bezel', Soviet Physics JETP <u>7</u>, 353 (1958).

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relaxation time; and even if it was toroidal when fired, a redistribution should quickly take place to equalize the pressure and density across the plasma cross-section, thus changing the internal angular momentum distribution. It is interesting to note in this connection that the double probe trace obtained by Bostick⁶ could possibly be explained more easily by an internal magnetic field variation (e.g. as in Eq. 24 or 42) than by an actual density variation (provided the plasma conductivity is high enough.)

An alternative to Eq. (24) might be the configuration found by Boyer <u>et al</u>? in an analysis of the Ixion machine. In this machine a radial electric field is applied to a cylinder of plasma with a longitudinal magnetic field; and the $\vec{E} \times \vec{B}$ drift⁸ drives the plasma in the azimuthal direction, i.e. rotates it about its axis.

In the analysis of reference 7, it was assumed only that the particle density is uniform; then with no assumption about Ω , the form of B_o is determined from the fact that the electric field must vary as 1/r. From our present standpoint a simple argument might run thus: the plasma drift is given by⁸

$$\vec{v} = \frac{c\vec{E} \times \vec{B}}{B^2}$$
(41)

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W. H. Bostick, Phys. Rev. 104, 292 (1956); see Figs. 7 and 8.

K. Boyer, J. E. Hammel, C. L. Longmire, D. Nagle, F. L. Ribe, and W. B. Riesenfeld, AEC Report No. P-15-20 (TID-7558, p. 140); Second U. N. International Conference on the Peaceful Uses of Atomic Energy, (Geneva, 1958), paper no. A/CONF. 15//P/2383.

For a discussion of plasma drifts, see H. Alfvén, <u>Cosmical Electrodynamics</u> (Oxford University Press, 1950), Chapter 2. Eq. (41) can, however, be be derived from Eq. (7) above.

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Let $\vec{E} = (K_1/r, 0, 0)$, $\vec{B} = (0, 0, B_0)$ and $\vec{\forall} = (0, r \ \Omega, 0)$; then from Eqs. (41) and (23) with ρ_0 and p_0 constant $(K_1 - K_5$ are various constants),

$$\mathbf{r} \cdot \Omega = \frac{\mathrm{d}\mathbf{R}_{\mathrm{II}}}{\mathrm{r}\mathbf{B}_{\mathrm{o}}} = \frac{\mathrm{R}_{2}}{\mathrm{r}\mathbf{B}_{\mathrm{o}}} \cdot$$
$$\mathbf{r} \cdot \Omega^{2} = \frac{\mathrm{K}_{2}^{2}}{\mathrm{r}^{3}\mathrm{B}_{\mathrm{o}}^{2}} = \frac{1}{8\pi\rho_{\mathrm{o}}} \frac{\mathrm{d}}{\mathrm{d}\mathbf{r}} (\mathrm{B}_{\mathrm{o}}^{2}) \cdot$$
$$\mathrm{B}_{\mathrm{o}}^{3} \frac{\mathrm{d}\mathbf{B}_{\mathrm{o}}}{\mathrm{d}\mathbf{r}} = \frac{\mathrm{K}_{3}}{\mathrm{r}^{3}} \cdot$$
$$\mathrm{B}_{\mathrm{o}}^{14} = \mathrm{K}_{4} - \frac{\mathrm{K}_{5}}{\mathrm{r}^{2}} \cdot$$

(42)

Of course reference 7 gives the values of the constants in terms of the parameters of their problem, but the point here is that a field variation of the type given by Eq. (42) is not only possible but perhaps more probable than that of Eq. (24) for our present problem. By Eq. (7), a plasma rotating in a magnetic field will induce in itself a radial electric field; considered from the laboratory frame of reference, it then appears that Eq. (6) presents a problem of charge separation unless E varies as 1/r. It seems probable, however, that no such difficulty exists in the frame of reference of the rotating plasma, the velocity being just sufficient to transform the field away. This contention seems to be borne out by experiments on the Homopolar thermonuclear machine,⁹ in which a transient radial current is drawn while the plasma is being accelerated, but after reaching equilibrium "the driving electric field vanishes in the plasma rest frame, and the radial current ceases."

9 O. A. Anderson, W. R. Baker, A. Bratenahl, H. P. Furth, J. Ise, Jr., W. B. Kunkel, and J. M. Stone, Second U. N. International Conference on the Peaceful Uses of Atomic Energy (Geneva, 1958), paper no. A/CONF. 15/P/373.

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Appendix I

APPENDIX II

CONSTRUCTION OF THE EXPERIMENTAL APPARATUS

by

Dale J. DeGeeter

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APPENDIX II

CONSTRUCTION OF THE EXPERIMENTAL APPARATUS

A. MAGNETIC FIELD

A capacitor and solenoid combination has been constructed for supplying the magnetic field. Considerable effort was expended in bringing the operation to an acceptable level. A major problem was encountered in the operation of the triggered-gap switch in the magnetic field circuit.

The original switch, as described in a previous report, worked very well for the initial firings. Almost 100 percent operation was achieved for the first firings in the 500 to 4,000 volt region on the field capacitors. Hewever, as the energy delivered increased, the copper became badly pitted from the energy dissipation at the surface of the electrodes. Molybdemum inserts were then made for the electrodes and a considerable improvement was noted. At above 2,500 volts on the capacitor bank the molybdemum also pitted but cleaning after several firings restores it.

In addition, breakdown between the central trigger wire and its surrounding electrode also occurred at the voltages necessary for satisfactory operation. The breakdown was traced to the considerable voltage across the 6BK4 tube at the operating current and the sputtered material retained on the teflon insulator. The problem was solved by the insertion of a side trigger in the gap.

A search coil was used to calibrate the magnetic field. Fig. 1 shows several of the waveforms observed. It should be pointed out that this is not a plot of the magnetic field since the signal from the coil pickup represents the rate of change of flux with time. This presents no special problem for after the calibration is made only the time is needed for computing the

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magnetic field. A graph has been made plotting average values of the field against time and it will only be necessary to know when the plasmoid was fired to determine the magnetic field to an acceptable accuracy.

The method of arriving at the magnetic field curve for the various capacitor bank voltages can be illustrated using the voltage values from the curves in Fig. 1. The period of the oscillation can be read directly as 6 msec. The amplitude of the magnetic field can be calculated by substitution into the equation:

$\nabla = nAB\omega$

where n = mumber of turns, A = average area of the loops, B = magneticfield, and $\omega = angular$ velocity. For air the magnetic field amplitudes at 3,000 and 4,000 volts are then calculated as:

$$H_{3000} \approx \frac{6.0 \text{ x } 10^{41} (10^{4})}{140 (6.28 \text{ x } 167)} \approx 4100^{+200} \text{ gas}$$

$$^{\rm H}$$
 4000 $\approx \frac{7.8}{6.0}$ (4100) $\approx 5300^{+300}$ gauss

These values correspond to the expected values from the coil design and are within the desired accuracy. The magnetic field can then be easily found at a time t, by evaluating the sin ωt , for the corresponding amplitude.

A check was then made on the magnetic field uniformity across the diameter of the chamber at the center of the solenoid. The variation was undetectable with the search coil oscilloscope combination and the field was established as being essentially homogeneous across the plasmoid chamber.

B. PLASMOID SOURCE ASSEMBLY

The button plasmoid source has been installed and connected as shown in Fig. 1. The capacitor with the 3-ball triggered gap switch is being used

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Fig. 1 The above traces show signals from a pickup coil in the magnetic field. The sweep speed was 1 m sec per cm, with time going from right to left, for all the traces. The vertical deflections and magnetic field capacitor bank voltages were as follows: a) 5 volts per cm and 4,000 volts, b) 10 volts per cm and 4,000 volts, c) 5 volts/cm and 3,000 volts, and d) 10 volts per cm and 4,000 volts.

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for the button supply. The small capacitor is part of the pulse circuit to the center electrode and does not form a part of the actual discharge circuit. The leads from the capacitor are kept as close together as possible to minimize the inductance of the circuit. The terminating leads into the source are parallel copper strips with a surface for the leads from the button source to clamp to. These strips are wrapped with electrical tape to withstand the repulsive forces between the conductors. In addition, the copper leads were laminated to be flexible enough to rotate the source without uncoupling.

The button source has been fired and measurements made on the resulting discharge. Fig. 3 shows several oscilloscope traces of the voltage induced in a coil pickup from the discharge circuit. The quality of the pictures was limited by the oscilloscope operation at the high frequency encountered. A measurement of the curve over five cycles indicates there is an average period of about 0.3 μ sec. The current is considerably underdamped as was expected from impedance considerations. Again neglecting the slight damping, an approximation on the current through the source can be made by substituting appropriate values into the equation,

$$i_{max} = V \sqrt{\frac{C}{L}}$$
,

Where V = initial capacitor voltage, C = capacitance of the capacitor, and L = inductance of the circuit. The first two of these are known quantities and the third can be calculated from the equation,

$$L = \frac{T^2}{4\pi^2 C}$$

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Fig.2 The above traces show signals from a pickup coil in the plasmoid discharge circuit. The sweep speed was 0.2 μ sec per cm, with time going from right to left.

If a discharge through the button source with 25 KV on the capacitors is considered the current amplitude can be calculated as,

$$i_{max} = V \sqrt{\frac{C}{L}} = \frac{2\pi VC}{T}$$

 $i_{max} = \frac{2\pi (25,000 \times .025 \times 10^{-6})}{.30 \times 10^{-6}} \approx 13,000 \text{ amperes}$

The triggered-gap switch used in the plasmoid source circuit has been found to operate very well. In the course of placing the switch in operation two critical points were noted. First, the distance between electrodes had to be precisely set to obtain a proper breakdown sequence between the spheres, and secondly, a small resistance had to be included in the pulse electrode lead to damp out oscillations occurring in this circuit during the discharge. In addition it has been found that the circuit is a good transmitter of RF energy which causes some interference with other measuring equipment. Some of this has been eliminated, however, a shield will probably be added in the near future to reduce this undesirable characteristic.

C. ELECTRONIC CIRCUITRY

A change has been made in the method of triggering the time delay due to the presence of jitter in the low voltage switch triggering the magnetic field. It was not practical to continue work on removing the jitter since an alternate method was readily available. Instead of starting the time delay from the pushbutton triggering the magnetic field, a pickup coil on the magnetic field circuit will be used to accomplish the time delay triggering. Then, if there is jitter in the switch, it will not affect the time interval between the magnetic field and plasmoid firing.

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The time interval can then be set and duplicated on subsequent firings to the accuracy required.

D. PHOTOMULTIPLIER TELESCOPE

The use of a photomultiplier telescope combination was reported by Finklestein¹ for velocity measurements of plasmoids. A similar arrangement has been constructed for this project. The initial photomultiplier used was available from laboratory equipment and no special design was included for operating in the stray fields present. Some exploratory tests were made with regard to magnetic field effects and operation was found to be satisfactory only at points well away from the flux at the end of the solenoid. The necessary distance was ascertained and the photomultiplier was placed in the acceptable position. The source was then discharged and its effect noted on the photomultiplier output. The pickup by the photomultiplier was excessive and consequently a new arrangement was designed whereby the complete tube and amplifier are in a metal container with coaxial leads to all components external to the container. Preliminary tests have indicated that the attenuation is several orders of magnitude better. This redesigned unit will be installed shortly for application to the velocity measurements.

D. Finkelstein, G. A. Sawyer, and T. F. Stratton, Physics of Fluids 1, 188 (1958).

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APPENDIX III

EXPERIMENTAL RESULTS

by Dale J. DeGeeter

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EXPERIMENTAL RESULTS

In the course of the final developmental work on the plasmoid apparatus many preliminary firings were made and several were recorded by photographic methods. Figs. 1, 2, 3, and 4 are time-exposure pictures of plasmoids taken with a conventional photographic camera.

Considerable experience was gained in photographing the plasmoids. The first photographs failed to show anything beside the arc from the source. This was not unexpected and with constant improvement of photographic techniques and film a set of photographs of good quality was obtained. The final improvement was made by using Kodak Royal-X panchromatic film. This is the fastest commercial film made by Kodak and its application caused considerable improvement in the pictures. It appears that the photographing of this extremely high speed event at the low light levels involved could be within the reciprocity failure region of the preceeding films used.

There was considerable reflection within the container detracting from the photograph quality. To reduce this the glass container was given an "Aquadag" suspension coating and the metal parts were covered with black neoprene rubber. The picture of the camera, which can be seen supperimposed in the background, was introduced by the reflection from the glass plate on top of the vacuum chamber. If necessary, this image can be removed by masking.

A discussion of the photographs is especially interesting when one notes the differences between several of the plasmoids of Fig. 1. The

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Fig. 2 Plasma Discharge without a Magnetic Field



Fig. 3 Straight Plasmoid

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Fig. 4 Curved Plasmoid

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plasmoids were fired across the chamber with a geometry as sketched below.



The pressure in the chamber was about 10^{-3} mm and a magnetic field of differing magnitudes was present for each firing. A probe is directly opposite the discharge and the interaction can be seen in the pictures. The amplitude of the current through the source was about 14,000 amperes. Figures 1(d), 1(e), and 1(f) show the plasmoid traveling in a relatively straight trajectory. The difference in pinching of the plasma indicates that 1(d) was fired at a low magnetic field strength. Our observations of the discharge without the magnetic field as in Fig. 2 confirm the increased contracting action in the arc with increasing field as reported by Bostick.¹ Fig. 2 is an enlargement of one of the straight plasmoids.

It is interesting to note the interaction of this plasmoid with the probe. Streams of particles scattered from the probe are observed especially when the plasmoid trajectory tends to graze the probe on one side. It even appears that there is an elastic type scattering present. On the other hand, if the collision is head on into the probe the scattering takes on more of

W. H. Bostick, University of California Radiation Laboratory Report UCRL-4595 (1956) "Experimental Study of Ionized Matter Projected Across a Magnetic Field".

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an inelastic appearance. This amazing tendency to maintain itself again points up the possibility of this being a stable configuration.

The occurrence of plasma with considerably curved trajectories has been recorded. Fig. 3 is an enlargement of one of the curved plasmas. Again the curved trajectory has been reported by Bostick² with pictures similar to those in Fig. 1.

The experimental evidence encountered so far indicates that the curved plasmoid is formed in a region of pressures near 10⁻³ mm Hg. Additional work is being performed at the moment to determine the extent of this range. An analysis of this phenomenum will be made after the exact conditions are known to establish a theoretical explanation for the curvature. Pictures have been taken with the magnetic field direction reversed and the curvature was observed in the opposite direction. An explanation for this has been proposed by W. H. Bostick³ but there are several points that must be checked before the mechanism can be completely accepted. Essentially the contribution of the electrons produced by photoionization of the residual gas at this pressure can give rise to currents drived by the vector $\vec{E} = -\vec{v} \times \frac{H}{z}$. These currents could then act like an electromagnetic brake, reducing \overline{v} and \overline{E} progressively. This progressive reduction in E can conceivably give rise to a deflection of \overline{v} into a spiral trajectory. The above explanation has not been verified experimentally as yet since a third dimensional view must be recorded on the film. It is anticipated that the addition of a mirror will shed light on the proposed spiral trajectory, A verification of the decreasing

³ W. H. Bostick, Phys. Rev. <u>104</u>, 292 (1956).

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⁴ W. H. Bostick, University of California Radiation Laboratory Report UCRL-4595 (1956) "Experimental Study of Ionized Matter Projected Across a.Magnetic Field".

v will be possible when the image converter has been placed in operation and the plasmoid motion can be stopped. Additional hydromagnetic mechanisms will be sought to explain the curvature for the conditions being presently determined.

The effect of increasing pressure on the plasmoids has been recorded in the many photographs taken to date. The progression is from a straight plasmoid at good vacuums to the curved plasmoid at about 10^{-3} mm Hg to a plasmoid that is stopped before crossing the chamber at a poor vacuum. Although the exact limits of the region of curvature are not known it appears that it is fairly narrow. There is probably considerable error in the pressure readings in the poor vacuum range since thermocouple gauge readings are poor in this region of about 5-10 microns. However, the plasmoids have been photographed on several different occasions in this pressure region and all show the stopping of the plasmoid. Whereas at 10^{-3} mm of Hg the mean free paths for the electron, ion, molecule collisions are several times the chamber dimensions, here we are in a region where the collision effect is appreciable. The plasmoid is essentially stopped and broken up before traversal of the 9" diameter of the chamber is accomplished.

Photographs have been taken with a mirror in such a position as to get the elongation of the plasmoids along the magnetic field lines. The elongation is quite pronounced and rapid as indicated from the resulting photographs. However the mirror was not large enough to show the actual outline of the plasmoid in this direction. Later pictures have confirmed the elongation in the axial direction by noting the recombination at the wall of the container in relation to a marking system locating the central section of the cylindrical glass housing. The photographs of the elongation

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will be included in a later report as soon as the remaining view has been recorded and the material completely analyzed.

Plasmoids have been fired at thin mylar sheet to investigate the possibility of a net charge being associated with the plasmoid. If this were the case, we could possibly see the effect of the charge by spraying the mylar with "Zerox" developer after the mylar was bombarded with one or more plasmoids. The bombarded mylar was processed but there was no detectable effect for up to ten firings. The experiment was repeated and no charge was observed again. It must be emphasized that the results obtained do not necessarily prove there was no net charge of sufficient magnitude to detect by this process, since the process has been found to be uncertain under many conditions of bombardment similar to this. The experiment was primarily designed for detecting a positive result and the absence of the same left any interpretation of the results uncertain.

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<u>APPENDIX IV</u> <u>MEASUREMENT OF THE MAGNETIC FIELD IN A PLASMOID</u> <u>BY MEANS OF GYRORESONANCE WITH MICROWAVE BEAMS</u> by R. Lee Watkins

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APPENDIX IV

MEASUREMENT OF THE MAGNETIC FIELD IN A PLASMOID BY MEANS OF GYRORESONANCE WITH MICROWAVE BEAMS

By the application of the phenomena of gyroresonance of the electrons in an ionized gas there is the possibility of determining the magnitude of the magnetic field that exists within the plasmoid as it travels through space.

As is well known electrons in a plasma which is in a magnetic field will oscillate with a frequency given by,

(1)

where

is the charge to mass ratio of the electron,

H is the magnitude of the magnetic field,

 $\omega_{\rm b} = -\frac{\rm eH}{\rm mc}$

c is the velocity of light, and

ω is the angular frequency of oscillation.

An electromagnetic wave propagating through the medium will experience an absorption as well as a phase change when its frequency is equal to the frequency ω_{b} . The magnetic field H may be calculated therefore if the resonant frequency is known. If a plasmoid has the shape postulated by Bostick¹ with internal currents flowing thereby producing a magnetic field it would seem logical that electrons in their field would gyrate. The interactions between these gyrating electrons to microwaves will produce an absorption of the microwaves which can be measured by reflection or transmission experiments.

W. H. Bostick, Phys. Rev. 104, 292 (1956).

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The experimental arrangement is shown in Fig. 1. The plasmoid will be fired across a continuous microwave beam and at the point in time when an absorption is observed at the receiving horn one knows that the magnetic field in the plasmoid has gone through a specified value. One has assumed in the above case that the plasmoid will have a larger magnetic field at the time it is fired and that it passes through the gyroresonance frequency while it is in the microwave beam. It is also assumed that the magnetic field is reasonably uniform over the plasmoid. If the magnetic field is not reasonably uniform over its area then a continuous function will be observed at the receiving horn which is an indication of the inhomogeneity of the magnetic field.

A qualitative check was made of the radiation field associated with the microwave transmitting horn as shown in Fig. 2. The receiving horn as shown to the right in Fig. 2 was moved and the current plotted for the various positions. While the data obtained in such a manner was qualitative and of little value for the final calibration in the system, it did serve to confirm the design and construction of the horns in addition to providing some idea of the radiation characteristics. The resulting plots of the receiving horn current vs. sidewise displacement are roughly sketched below for three values of the separation distance between the horns.



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Fig. 2. Microwave Equipment

DESIGN AND APPLICATION OF THE IMAGE CONVERTER by Dale J. DeGeeter ARMOUR RESEARCH FOUNDATION OF ILLINOIS INSTITUTE OF TECHNOLOGY Appendix V

APPENDIX V

APPENDIX V

DESIGN AND APPLICATION OF THE IMAGE CONVERTER

The dynamic progress in image converter technology has resulted in the development of tubes of considerably improved performance readily adaptable to the present program as a light shutter for stopping the motion of the plasmoids at the extreme speeds involved. It is felt by many that the Kerr Cell has been surpassed in efficiency and is rapidly falling behind in low light level work such as ours. At the moment the principle advantages seem to be in the gain possible in the tube, its relatively easy triggering, and its excellent ratio of open to closed sensitivity, namely almost infinity. It suffers somewhat from lack of resolution, but this isn't to important in our application. The tube that will be initially used is the RCA type C73435B.

The electronic equipment necessary for the image converter has been essentially completed and is shown in Fig. 1. The fabrication of the pulse network with the tube in a single housing will be completed on arrival of the tube. The electronic equipment circuits including the pulse network are illustrated in Figs. 2, 3, 4, and 5. The power supplies are essentially those of a unit in operation at Los Alamos Scientific Laboratory. The pulse network and tube unit has been designed by Dr. P. D. Southgate of our laboratory.

The tube features a control grid for low voltage gating of the beam current. The sequence for operation of the tube is as follows: At cut-off the gating grid is held at from 90 to 110 volts negative. To open the shutter a positive square wave pulse of from 170 to 190 volts is applied to the gating grid for the exposure time required. Exposure times as short as

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 10^{-0} seconds are possible with this method well within the requirements of our application.

The low energy secondary electrons emitted from the photocathode are focused and then accelerated by the 15 kv. anode voltage. When factors such as photocathode and phosphor efficiency are introduced the resulting conversion gain at 4400 A° is a minimum of 12. This increase in output energy over input energy is one of the most important advantages of the image converter for low light level work.

A tube under development by I.T.T. laboratories and a Yerkes Observatory group led by Dr. W. A. Hiltner has been of particular interest to us since a gain of about 50 is possible over the conventional tube. The possibility of adapting such a tube to our unit is being investigated to provide us with an installation to our knowledge at least an order of magnitude better than the standard system used by most investigators. The increased efficiency is accomplished by eliminating the inefficient method of photographing with a camera the image on the phosphor screen of the tube and instead making a contact photograph on a special thin phosphored mica screen. The new tube will be in operation shortly at Yerkes Observatory and arrangements have been made to inspect the final installation. We naturally expect a small loss of resolution due to the finite thickness of the mica, however, this would not appreciably reduce the value to us as a research tool.

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Fig. 1 Image Converter Electronics

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FIG. 2 - IMAGE TUBE PULSE NETWORK.











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