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14. ABSTRACT
We compare Wiener chaos and stochastic collocation methods for linear advection-reaction-diffusion stochastic PDEs with multiplicative white noise. Wiener chaos solutions/expantions could be viewed as stochastic Fourier series. Both methods are constructed as a recursive multistage algorithms for long-time integration. We derived error estimates for both methods and compare their numerical performance.
Main theoretical and experimental advances include:
1.Introduction of a number of effective approaches to numerical analysis of of stochastic PDEs.

15. SUBJECT TERMS
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Report Title

Final Report: 3.4.1 Nonlinear Stochastic PDEs: Analysis and Approximations

ABSTRACT

We compare Wiener chaos and stochastic collocation methods for linear advection-reaction-diffusion stochastic PDEs with multiplicative white noise. Wiener chaos solutions/expantions could be viewed as stochastic Fourier series. Both methods are constructed as a recursive multistage algorithms for long-time integration. We derived error estimates for both methods and compare their numerical performance. Main theoretical and experimental advances include:

1.Introduction of a number of effective approaches to numerical analysis of of stochastic PDEs.

2.Development of Wiener Chaos approach to the analysis of nonlinear partial differential equations in random environment.

3.Implementation of effective classes of numerical algorithms related to stochastic PDEs of parabolic and elliptic types.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
05/22/2016	14.00	B. Rozovskii, M.V. Tretyakov, Z.Zhang, G.E. Karniadakis. A Multistage Wiener Chaos Expansion Method for Stochastic Advection-Diffusion-Reaction Equations, SIAM J Scientific Computing, (03 2012): 914. doi:
05/23/2016	15.00	X. Wang, Boris Rozovskii. The Wick-Malliavin Approximation on Elliptic Problems with Long-Normal Random Coefficients, SIAM J Scientific Computing, (10 2013): 2370. doi:
05/23/2016	16.00	Z. Zhang, M.V. Trrtykov, B. Rozovskii, G.E. Karniadakis. A Recursive Sparse Grid Collocation Methd for Differential Equations with White Noise, SIAM J Scientific Computing, (08 2014): 1652. doi:
05/23/2016	17.00	X. Wang, D. Nenturi, R. Mikulevicius, B. Rozovskii, G.E. Karniadakis. Wick- MAlliaviamn approximation to non-linear stochastic partial differential equations:analysis and simulations, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, (05 2014): 0. doi:
05/23/2016	18.00	M. Zheng, B. Rozovsky, G.E. Karniadakis. Adaptive Wick-Malliavian Approximation to Nonlinear SPDEs with Discrete Random Variables, SIAM J Scientific Computing, (08 2015): 1872. doi:
05/23/2016	20.00	R. Mikulevicius, B. Rozovskii. On Distribution free Skorokhod-Malliavian Calculus, Stochastic And Partial Differential Equations: Analysis and Computations, (06 2016): 319. doi:
05/23/2016	19.00	Z. Zhang, M.V. Tretyakov, B. Rozovskii, G.E. Karniadakis. Wiener Chaos Versus Stochastic Collocation Methods for Linear Advection-Diffusion-Reaction Equations with Multiplicative White Noise, SIAM J Numerical Analsis, (01 2015): 153. doi:

TOTAL: **7**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received		Paper
08/27/2015	11.00	Boris Rozovsky, George E Karniadakis, Michael V. Treytykov, Zhongqiang Zhang. Wiener Chaos Versus Stochastic Collocation Methods for Linea Advection-Diffusion-reaction Equations with Multiplicative White Noise , SIAM J. Numer. Anal Vol 53, No 1, pp.153-183, 2015, (2015): 0. doi:
12/01/2014	10.00	Z. Zhang, M.V. Trettyakov, B. Rozovskii, G.E. Karniadakis. A recursive sparse grid collocation method for differential equatins with white noise, SIAM Journal of Scientific Computing, (05 2013): 0. doi:

TOTAL: **2**

Number of Papers published in non peer-reviewed journals:

(c) Presentations

1) SPDE's and Applications-IX Levico-Terme, Italy, January 5-11,   2014
2) 10th AIMS Conference, Madrid, Spain, July 2014
3) Computational Math, July 29-31, Arligton, VA
4) Math Colloquium at WPI, October 2014
5) Math Colloquium at Princeton Univ- March 2015
6) Math Colloquium at Stanford University- May, 2015
7) Workshop at Brown University "Deterministic and Stochastic Partial Differential Equations" (organizer)- November 2015

Number of Presentations: 7.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

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Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
08/27/2015	13.00	Z.Zhang, B Rozovsky, G Karniadakis. Strong and Weak Convergence Order of Finite Element Methods for Stochastic PDEs with Spatial White Noise, J Of Numer Math (05 2015)
09/09/2013	3.00	. Limitations od sparse grid collocation method for differential equations with white noise, ( )
09/10/2013	4.00	Zhongqiang Zhang , Boris Rozovskii, George Karniadakis. Wiener chaos vs stochastic collocation methods for linear advection-diffusion-reaction-equations with multiplicative white noise., ( )
09/10/2013	5.00	G. Bal,, S. Kaligotla,, B. L. Rozovskii. On Homogenization of Elliptic SPDEs with Oscilating Coefficients, ( )
09/10/2013	6.00	. Adaptive Wiener-Askey Wick-Malliavin approximation to nonlinear stochastic differential equation with multiplicative discrete random variables, ( )
10/24/2014	7.00	S. Lototsky, B. Rozovsky. Stochastic Partial Differential Equations, (09 2015)
10/24/2014	9.00	B. Rozovsky, R. Mikulevicius. On Distribution Free Skorokhod-Malliavin Calculus, Probability Theory and Related Fields (06 2014)

TOTAL: **7**

Number of Manuscripts:

Books

Received	Book
05/23/2016 21.00	B. Rozovsky, S. Lototsky. Stochastic Partial Differential Equations, Germany: Springer, (10 2016)
08/27/2015 12.00	B Rozovsky, S Lototsky. Stochastic Partial Differential Equation, Germany: Springer, (12 2015)

TOTAL: **2**

Received	Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Boris Rozovsky	0.36	
FTE Equivalent:	**0.36**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

During 2012-2013 we developed Wick-Mallivian calculus for non-adapted systems of Stochastic PDE's.
In 2013-2014 I have been working in three areas:
(a) Stochastic partial differential equation
(b) Distribution-free Malliavian calculus
(c) Numerical methods for stochastic PDEs
In area (a) I have completed the main part of the monograph "Stochastic Partial Differential Equations" (to be published by Springer). In area (b) I have developed (in collaboration with Prof. Mikulevicius) a new version of Malliavian calculus. The main advantage of this new methodology is that it is universal. It applies to all types of randomness. In contrast, all previous results on this subject were limited to Gaussian or Levy randomness. In area (c) I have been concentrating on fast computational methodologies for numerical approximations of various types of stochastic PDEs.

In 2014-2015 I have been working on the overlap of three important areas of applied mathematics: stochastic partial differential equations, Malliavian calculus, and numerical approximations of stochastic PDE's including:
(1) Numerical methods in stochastic PDEs, in particular, convergence rate for finite element approximations for semi-linear systems;
(2) Applications of Malliavian calculus to analysis and approximations of adapted and non-adapted versions of stochastic PDEs.
(3) Development of distribution free approach to stochastic PDEs. The results obtained in this area turned out to be universal for all linear stochastic PDEs
(4) In addition, I have completed (in collaboration with Prof. S. Lototsy) the text book "Stochastic Partial Differential Equations" to be published by Springer.

The developed theory and related numerical methods contribute to accelerated analysis and testing of complex high dimensional systems.

Technology Transfer

ARO Grant #61840-MA

Nonlinear Stochastic PDEs: Analysis and Approximations

(2012-2015)

Recent Achievements. So far the theory and numerical methodology for solving stochastic PDEs or ODEs have dealt almost exclusively with Gaussian or Poisson/L´evy randomness. However, in various applications, practitioners are often forced to deal with a *variety of types of random perturbations*

Subject of Research: The main subject of the research supported by the Grant is a universal approach to solving linear and nonlinear stochastic PDEs driven by *various* types of random perturbations and their mixtures. More specifically, our research (related to the Grant) concentrates on stochastic PDEs with **polynomial nonlinearity** (e.g. stochastic Navier-Stokes, Burgers, etc.) driven by **arbitrary noise**.

Importance of the Subject: Stochastic partial differential equations (SPDEs) is an interdisciplinary area at the overlap of stochastic processes (random fields) and partial differential equations. Stochastic hydrodynamics, quantum physics, interacting particle systems, nonlinear filtering, and theory of super-processes have influenced the development of SPDEs at the overlap of stochastic processes (random fields) and partial differential equations. It is probably safe to say that in the last three decades SPDEs has been one of the most dynamic areas of stochastic analysis.

Mathematical objective: Development of the mathematically rigorous and numerically efficient methodology for solving complex stochastic partial differential equations of interest for ARO.

Task: Development of the universal approach for mathematically rigorous and numerically efficient methodology for solving complex stochastic partial differential equations of interest for ARO.

This task was solved in the paper "On distribution-free Skorokhod-Malliavin Calculus" in the journal "Stochastic and Partial Differential Equations. Analysis and Computations" Vol.4, Number 2, June 2016, Springer.

The fundamental fact proven in this paper is that for linear SPDEs with ANY reasonable randomness, the solution can be represented explicitly via the lower triangular set of

DETERMINISTIC PDEs which could be solved sequentially. Moreover, this deterministic system of SPDEs could be solved sequentially.

Development of new powerful methodologies for solving SPDE. A good example of such methodology is Polynomial Stochastic Chaos Expansion. This new technique could be viewed as stochastic Fourier expansion. It separates randomness noise in the system from the deterministic propagator (typically, a deterministic PDE)

In 2012-2013 I was working in the following areas:

(a) Stochastic analysis and numerical methods
(b) SPDEs
(c) Homogenization and uncertainty quantification for turbulent flows.
In particular, I have been actively working on a numerical approximations of SPDEs, Wick-Malliavian approximations, stochastic homogenization, theoretical approximation of polynomial nonlinearities in SPDEs.

Accomplishments: Developed Wick-Malliavian calculus for non-adapted systems of SPDEs.

In 2013-2014 I have been working in three areas:
(a) Stochastic partial differential equations
(b) Distribution-free Malliavin calculus
(c) Numerical methods for stochastic PDEs:

Accomplishments:
1. In area (a) I have completed the monograph "*Stochastic Partial Differential Equations*" (to be published by Springer in 2017)

2. In area (b) I have developed (in collaboration with Prof. Mikulevicius) a new version of Malliavin calculus. The main advantage of this new methodology is that it is *universal*. It applies to all types of randomness. In contrast, all previous results on this subject were limited to Gaussian or Levy randomness.

3. In area (c) I have been concentrating on fast computational methodologies for numerical approximations of various types of stochastic PDEs.

In 2014-2015

Accomplishments

Developed: A Wiener Chaos approach to analysis of nonlinear partial differential equations in random environment.

Introduced: A number of effective approaches to numerical analysis of several important types of stochastic PDEs.

Proposed and implemented: Several effective classes of numerical algorithms for practical implementation of the related algorithms.

The main problem in analysis and computations of solutions for SPDEs is that the solutions vary depending on chance. The randomness might appear in initial and boundary conditions, forcing, variability of coefficients. This results in formidable difficulties in solving SPDEs (numerically as well as theoretically).

To deal with these (quite formidable difficulties) in my research supported by the ARO (Grant 61840-MA) I have developed a completely new and very efficient methodology for treating SPDEs . This methodology is referred now as Polynomial (or Wiener) Chaos. Roughly speaking it is a "stochastic Fourier series expansion" which separates random and deterministic.
The deterministic part (finding the deterministic coefficients) can be solved off line. Combining this coefficients with the randomness involved in the equation becomes almost trivial and amazingly simple from the computational as well as theoretical points of view.

In fact, the aforementioned methodology allows to compute all statistical moments of the solution (mean, variance, and higher statistical moments of the solution off line) EXPLICITLY and do not worry about randomness:
NO NEED FOR MONTE CARLO!!!!

Overall Summary

Significance:
SPDEs is a new and fast developing area. However, practically, all research on stochastic PDEs was limited to SPDEs driven by Wiener (Gaussian random variables) process or Levi (exponential/ Poisson random variables) process.

These limitations are troublesome, because there are many other important types of randomness, including: uniform, gamma, beta, binomial hypergeometric, and other distributions.

To overcome this serious shortcoming we introduced and investigated an extension of our previous work to the settings with arbitrary randomness.

Methodology used

Our main assumption is very simple and at the same time very general: we will consider stochastic PDEs perturbed by a sequence of arbitrary uncorrelated random variables (ξ_1,ξ_2,...).

The above setting constitute the so called "distribution free" paradigm. Our task is to develop a universal stochastic analysis which includes all types of randomness mentioned above and beyond.

This methodology and related numerical algorithms were implemented for solving the main types of stochastic PDEs, including stochastic Boussinesq equation, Burgers equation, Navier-Stokes equation, Eiler equation, Cahn-Hillard equation, Camassa-Hillard equation, etc

The technical approach:

The technical approach of this proposal was four-fold :

a) Introduction of stochastic polynomial chaos (SPC) approach for various classes of randomness;

b) Development of SPC-type approximations for solving fundamental nonlinear and linear SPDEs.

d) Building numerical methods and algorithms based on the SPC approach.

e) Construction and implementation of "distribution free/independent") SPC.

The SPC approach is essentially a stochastic nonlinear "Fourier series" approach.

Major Result.

The following result (established in our research supported by ARO is a distribution free (DF) version of the celebrated Cameron-Martin Theorem. It holds under very general assumptions:

Major Theorem :There exists an explicit orthogonal system $\{R_n\}$ where

n=$(n_1, n_2, \ldots)$ $so\ that\ for\ any\ random\ function\ f(\xi_1, \xi_2, \ldots),$

E|f(ξ_1,ξ_2,..)|^2=$\sum_{n\geq 0} f_n R_n$ and $f_n = \boldsymbol{E}[f_n R_n]$/n!

where n $= (n_1, n_2, \ldots)\ and\ |n| = \sum_i n_i < \infty, \sum_i {f_n}^2$ n!=**E**$[f^2] < \infty$

How one solves stochastic PDEs in the distribution free setting?

Consider a general stationary/"elliptic" SPDE

$$\text{Au}+(Mu)\dot{R} = g \qquad (E1)$$

Then a solution to this equation is given by $\sum_n u_n$, where

$u_n\ solve\ the\ following\ lower-\ triangular\ system$ $\text{A}u_0 = Eg$ if |n|=0

$\text{A}u_n + \sum_{n\geq 1} M_n u_{n-\varepsilon_n}(t) = g_n\ \ if\ |n| > 0.$ (BL)

$Then$ solution to system (E1) is given by

$$\text{u}=\sum_{n\geq 0} u_n R_n.$$

The system (BL) is often referred to as the propagator. The term "propagator" emphasizes that this deterministic system propagates randomness involved in the related SPDE. Similarly to system (E1) one can solve non-stationary "parabolic" SPDE

$$\dot{v}\text{(t)=Av+(Mv)} \qquad \text{(E2)}$$

CONCLUSION:

The developed theory and related numerical methods contribute to accelerated analysis and testing of complex high dimensional systems.

Selected Publications (in the last 5 years)

Book: S. Lototsky and B.L. Rozovsky, Stochastic Partial Differential Equations, Springer, to appear in 2017.

Papers

R. Mikulevicius and B. Rozovsky and , "On distribution free Skorokhod- Malliavin Calculus", J. Stochastic Partial Differential Equations: Analysis and Computations, v. 4 pp 319-360, 2016

Z. Zhang, B. Rozovskii, G.E. Karniadakis, " Strong and weak convergence order of finite element methods for stochastic PDEs with spatial white noise," Numer.Math, 134, 61-89 2016

Z. Zhang, M.V. Tretyakov, B. Rozovskii and G.E. Karniadakis, "A recursive sparse grid collocation method for differential equations with white noise," SIAM J. Sci. Comput. 36(4), A1652-A1677, 2014.

Z. Zhang, B. Rosvoskii, M.V. Tretyakov and G.E. Karniadakis, "A multi-stage Wiener chaos expansion method for stochastic advection-diffusion-reaction equations," SIAM J. Sci. Comput., 34(2), A914-A936, 2012.

Z. Zhang, M. V. Tretyakov, B. Rozovskii, and G. E. Karniadakis. Wiener chaos vs stochastic collocation methods for linear advection-diffusion equations with multiplicative white noise. SIAM J. Numer. Anal., 53(1): 153-183, 2015

M. Zheng, B. Rozovsky and G.E. Karniadakis, "Adaptive Wick-Malliavin approximation to nonlinear SPDEs with discrete random variables," SIAM J. Sci. Comput., 37 (4), A1872-A1890, 2015.

D. Venturi, X. Wan, R. Mikulevicius, B. L. Rozovsky, G. E. Karniadakis, "Wick-Malliavin approximations to nonlinear stochastic partial differential equations: analysis and simulations". Proceedings of the Royal Society, 2013

S. Lototsky, B. Rozovsky, and D. Selesi. "On Generalized Malliavin Calculus". Stochastic Analyses and Applications, 122, pp 808-843, 2012.

R. Mikulevicius and B. Rozovsky, "On unbiased stochastic Navier-Stokes equation", Probab. Theory Related Fields, 154, pp. 787-834, 2012.

Number of presentations (related to this grant) -7:

1. SPDE's and Applications-IX Levico-Terme, Italy, January 5-11, 2014
2) 10th AIMS Conference, Madrid, Spain, July 2014
3) Computational Math, July 29-31, Arligton, VA
4) Math Colloquium at WPI, October 2014
5) Math Colloquium at Princeton Univ- March 2015
6) Math Colloquium at Stanford University- May, 2015
7) Workshop at Brown University "Deterministic and Stochastic Partial Differential Equations" (organizer)- November 2015

Stochastic Navier Stokes Equation

Stochastic Navier Stokes (SNS): a Navier Stokes Equation with randomness generated by any or all of the following random sources: forcing, initial conditions, boundary, etc.

Example: Evolution of NS with random initial condition forcing:

$$\begin{cases} \partial_t \boldsymbol{u} = v\Delta \boldsymbol{u} - (\boldsymbol{u}, \boldsymbol{\nabla})u + \boldsymbol{f}(t,x) - \boldsymbol{\nabla} p(t,x) \\ \quad +\boldsymbol{g}(t,x)W(t), x \in D, div\, \boldsymbol{u}(t,x) = 0 \\ \boldsymbol{u}(0,x) = \boldsymbol{u}_0\ \left(x, \mathfrak{z}(M)\right), \boldsymbol{u}_{|\partial D} = \varphi\left(x, \mathfrak{z}(M)\right), \end{cases}$$

Where **W**(t)=$\sum_{k\geq 1} m_k$(t)$\mathfrak{z}_k$ *is a Brownian motion*, $\mathfrak{z}$(M)=($\mathfrak{z}_1, \mathfrak{z}_2, \dots \mathfrak{z}_M$), $M \leq \infty$, *is a Gaussian vector*, $\boldsymbol{f}(t, \boldsymbol{x})$ *and* **g**(t, **x**) are appropriate deterministic functions.

Randomness generates uncertainty. How to quantify?

Wick-Mallivian Expansion

■ $Wick\ product\ is\ defined\ by\ h_n \lozenge\ h_1 = h_{n+1}. Malliavin\ derivative$ (inverse to Wick product) is defined by D$h_n = h_{n-1}$.

■ $An\ important\ formula: if\ f = f(\mathfrak{z})$ and g=g($\mathfrak{z}$) have second moment, then

$$fg=\sum_{p\geq 0}\frac{1}{p!}(D^p f \lozenge D^p g) \qquad (\mathbf{2})$$

Where ◊ is the so called Wick product and D is the Malliavin derivative (Mikulevicius and Rozovsky., Probability Theory and Random Fields, 2011)

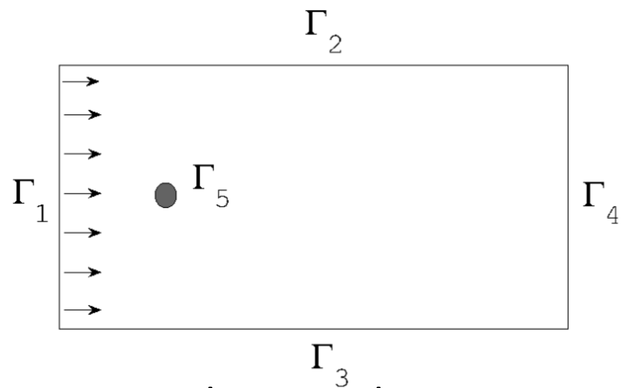
$Formula\ (\mathbf{2}) implies\ that\ the\ nonlinear\ terms\ in\ SNF$

$$\mathbf{u}\nabla\boldsymbol{u} \approx \sum_{p=0}^{Q}\frac{1}{p!}\frac{1}{p!}(D^p\boldsymbol{u} \lozenge D^p\nabla\boldsymbol{u})$$

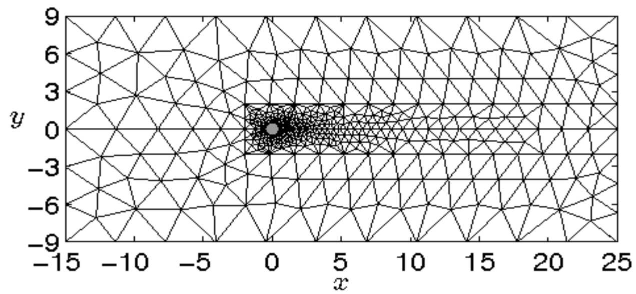
is a reasonable approximation to the nonlinear term in SNS.

■ Fact: The lower triangular approximation is equivalent to “zero order” approximation $\mathbf{u}\nabla\boldsymbol{u} \approx \boldsymbol{u} \lozenge \nabla\boldsymbol{u}$

Computational Mesh



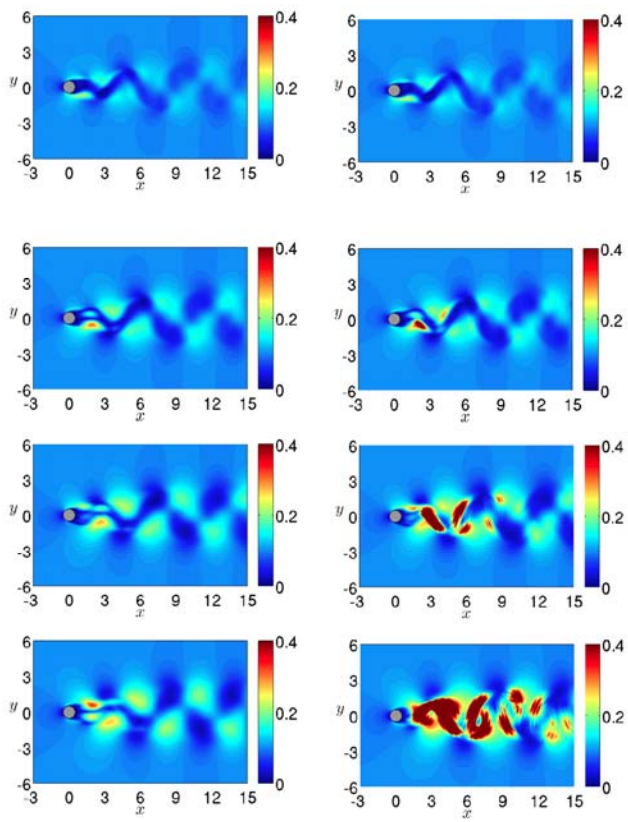
Boundary conditions



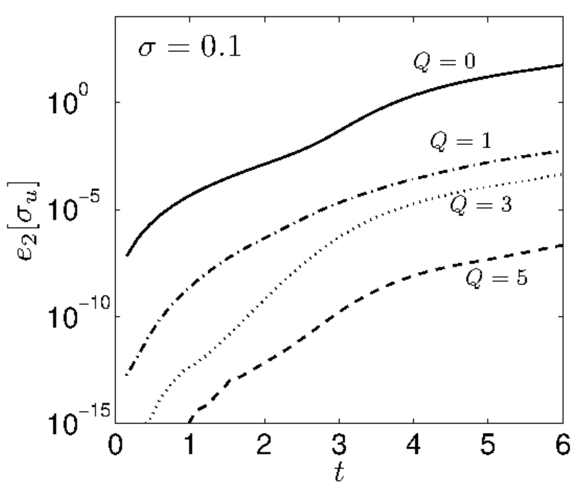
Computational mesh

Standard Deviation of the Streamwise Velosity

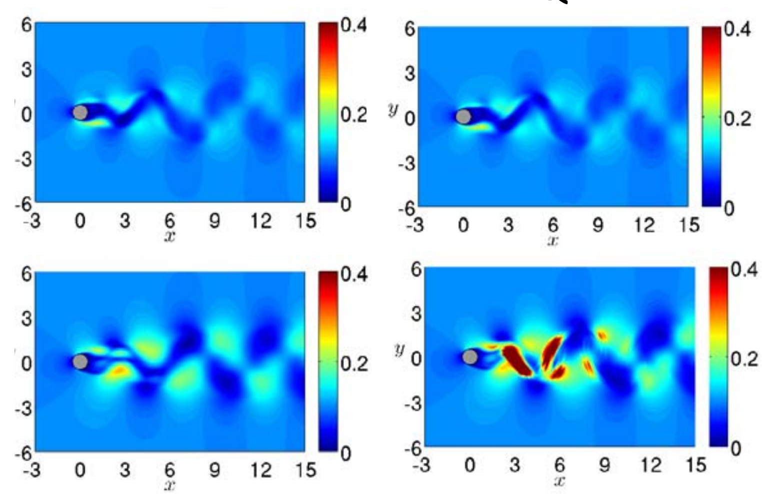
$Q = \infty$ $\qquad$ $Q = 1$



Convergence of Wick-Malliavin Approximations



Q=∞ Q=1



FUTURE RESEARCH
(in collaboration with Dr. Chi-Wang Shu)

The theory and applications of stochastic analysis and stochastic PDEs is well developed for Gaussian and Poisson processes. It appears that these results can be extended to a large class of stochastic systems perturbed by arbitrary noise. In fact, P. A. Meyer, one of the founders of stochastic analysis, noted: “The first and very important point is that, in the construction of multiple integrals, the Gaussian character of the process never appears"

With this in mind, we propose a reasonably general approach to analysis and computations of stochastic PDEs driven by “arbitrary” noise. The above setting is often called "distribution free" paradigm. Our task is to develop a version of Malliavin-Skorohod calculus in the distribution free setting and to apply this methodology to analysis and computations of linear and non-linear stochastic PDEs.