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TWO BEAM ENERGY EXCHANGE IN HYBRID LIQUID CRYSTAL CELLS WITH PHOTOREFRACTIVE FIELD CONTROLLED BOUNDARY CONDITIONS (POSTPRINT)

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Two beam energy exchange in hybrid liquid crystal cells with photorefractive field controlled boundary conditions

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We develop a theory describing energy gain when two light beams intersect in a hybrid nematic liquid crystal (LC) cell with photorefractive crystalline substrates. A periodic space-charge field induced by interfering light beams in the photorefractive substrates penetrates into the LC layer and reorients the director. We account for two main mechanisms of the LC director reorientation: the interaction of the photorefractive field with the LC flexopolarization and the director easy axis at the cell boundaries. It is shown that the resulting director grating is a sum of two in-phase gratings: the flexoelectric effect driven grating and the boundary-driven grating. Each light beam diffracts from the induced gratings leading to an energy exchange between beams. We evaluate the signal beam gain coefficient and analyze its dependence on the director anchoring energy and the magnitude of the director easy axis modulation. © *2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).* [http://dx.doi.org/10.1063/1.4962936]

I. INTRODUCTION

Energy transfer between light beams due to the photorefractive effect in solid inorganic crystals is a well-known effect.[1] Significant photorefractive-like interactions have been also observed in photoconductive liquid crystal cells, where charge separation of the photo-generated negative and positive ions is governed by the different diffusion constant of the ions.[2–6] In hybrid organic-inorganic photorefractives a LC sample is placed adjacent to a solid photorefractive layer or between two solid photorefractive layers. Incident intersecting coherent light beams generate space charges in the inorganic photorefractive layers. The space charges create a spatially modulated electric field (i.e. space-charge field), which penetrates into the adjacent LC layer, causing a director-modulation-induced grating of the LC permittivity. Both incident light beams propagate across the LC sample and diffract on the grating. Due to the beams coupling on the grating, one of the beams (small signal beam) is amplified. For the LC systems, very strong two-beam energy transfer between two coupled beams has been observed with gain coefficient values more than two orders of magnitude larger than those in solid inorganic photorefractive crystals.[7–13]

Until recently it has only been possible to operate in the Raman-Nath regime, for which the sample thickness is less than the grating thickness. In this case the coupled beams generate multiple order diffracted beams that leads to limited technological applicability of the effect.[13] However, in papers[10,14] it has been shown that inorganic photorefractive crystals can support efficient space-charge field generation in samples with thicknesses greater than the grating thickness. It allowed the Bragg regime to be reached, where only first order diffracted beams are generated.[10–13]

In discussing the formation of a director grating in hybrid organic-inorganic photorefractives, the authors of papers[15,16] supposed that the light-induced space-charge electric field penetrating

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from photorefractive substrates into LC couples with the director through the LC static dielectric anisotropy. However, this supposition predicts the maximal energy transfer at grating spacings comparable with the LC cell thickness, which contradicts experimental results[10–12] showing that this maximum occurs when the ratio of the grating spacing to cell thickness is rather small. The authors of paper[17] proposed that director grating formation in hybrid organic-inorganic photorefractives is governed by the interaction of the space-charge field with the LC flexoelectric polarization, rather than by static dielectric anisotropy coupling. Together with the additional assumption that the magnitude of the director grating is a non-linear function of the space-charge field, it allowed for a description of the experimental results obtained for both nematic[17] and cholesteric LC cells.[18,19]

Spatial director distribution in the LC cell depends strongly on director boundary conditions at the cell substrates, in particular, director pre-tilt angle and anchoring energy at the substrates. A series of methods have been developed to control the director boundary conditions, for example, irradiating alignment layers with ion beams,[20] doping the LC cells with nanoparticles,[21] or forming polymer structures on a substrate surface via electrostatic force[22,23] or UV light field.[24–26] In this present paper, we speculate that the photorefractive space-charge field may control the anchoring of the LC director at the cell substrates, and therefore affect the grating formation. We study the influence of the LC director anchoring energy and the easy axis direction on energy transfer between light beams incident on the hybrid cell.

The paper is organized as follows. In Sec. II we introduce a model of the hybrid nematic cell placed in the interference pattern of two incident light beams, and derive and solve equations for the LC director profile subject to the space-charge electric field. In Sec. III we discuss light propagation in the LC cell, derive expressions for the exponential gain coefficient, and analyze the influence of parameters characterizing boundary conditions on the gain coefficient. In Sec. IV we present some brief conclusions.

II. DIRECTOR SPATIAL PROFILE

The hybrid cell consists of flexoelectric nematic LC, bounded by the planes $z = -L/2$ and $z = L/2$ and placed between two plane-parallel transparent photorefractive layers. The cell is illuminated by two intersecting coherent light beams $\mathbf{E}_1 = A_1(z)\,\mathbf{e}_1 \exp(i\mathbf{k}_1\mathbf{r} - i\omega t)$ and $\mathbf{E}_2 = A_2(z)\,\mathbf{e}_2 \exp(i\mathbf{k}_2\mathbf{r} - i\omega t)$. The photorefractive substrates and LC possess non-linear properties that require $A_1(z)$, $A_2(z)$ to change as a function of position. The bisector of the beams is directed along the z-axis, the wave vectors $\mathbf{k}_1, \mathbf{k}_2$ and the polarization vectors $\mathbf{e}_1, \mathbf{e}_2$ of the beams lie in the xz-plane. The incident beams produce a light intensity interference pattern

$$I(z) = (I_1 + I_2)\left[1 + \frac{1}{2}\left(m(z)\exp(iqx) + c.c.\right)\right], \tag{1}$$

where the modulation parameter $m(z) = 2\cos(2\delta)\,A_1(z)\,A_2^*(z)/(I_1 + I_2)$, 2δ is the angle between the two incident beams in the photorefractive medium, $I_1 = A_1 A_1^*$, $I_2 = A_2 A_2^*$ are the intensities of incident beams, and $q = k_{1x} - k_{2x} = 2k\sin\delta \approx 2k\delta$ is the wave number of the intensity pattern.

The light intensity pattern (1) in the photorefractive substrates induces a space-charge electric field modulated along the x-axis with period equal to $2\pi/q$, which penetrates into the nematic LC. We will consider only small perturbations to the LC director profile in response to the electric field. In this case one can neglect the feedback of the LC director reorientation onto the electric field inside the LC slab. Then, the electric field inside the LC layer is given by the following expressions:[17]

$$E_x = E_{0x}\exp(iqx) + c.c, \quad E_z = E_{0z}\exp(iqx) + c.c.,$$
$$E_{0x} = q\left(\frac{\Phi_1 + \Phi_2}{4}\frac{\cosh(\tilde{q}z)}{\cosh(\tilde{q}L/2)} + \frac{\Phi_2 - \Phi_1}{4}\frac{\sinh(\tilde{q}z)}{\sinh(\tilde{q}L/2)}\right), \tag{2}$$
$$E_{0z} = -i\tilde{q}\left(\frac{\Phi_1 + \Phi_2}{4}\frac{\sinh(\tilde{q}z)}{\cosh(\tilde{q}L/2)} + \frac{\Phi_2 - \Phi_1}{4}\frac{\cosh(\tilde{q}z)}{\sinh(\tilde{q}L/2)}\right),$$

where

$$\Phi_{1,2} = \frac{E_{sc}}{q} m \left(\mp L/2 \right), \quad \tilde{q} = q \sqrt{\frac{\tilde{\varepsilon}_{||}}{\tilde{\varepsilon}_\perp}}. \tag{3}$$

In (3) $\tilde{\varepsilon}_{||}$ and $\tilde{\varepsilon}_\perp$ are the low frequency components of the LC dielectric tensor along and perpendicular to the LC director, E_{sc} is a magnitude of the photorefractive space-charge field, which in a diffusion-dominated case takes the following form:[27]

$$E_{sc}(q) = \frac{iE_d}{1 + \frac{E_d}{E_q}}, \quad E_d = q \frac{k_b T}{e}, \quad E_q = \left(1 - \frac{N_a}{N_d} \right) \frac{eN_a}{\varepsilon_0 \varepsilon_{Ph} q}, \tag{4}$$

where E_d is the diffusion field, E_q is the saturation field, N_a and N_d are respectively the acceptor and donor impurity densities, ε_{Ph} is the dielectric permittivity of photorefractive material, e is the electron charge, and k_b is the Boltzmann constant.

Denoting the director by **n**, the equilibrium director spatial profile can be found by minimizing the total free energy functional of the flexoelectric nematic cell defined by:

$$F = F_{el} + F_l + F_E + F_{fl} + F_S, \tag{5}$$

where

$$F_{el} = \frac{1}{2} \int \left[K_{11} \left(\nabla \cdot \mathbf{n} \right)^2 + K_{22} \left(\mathbf{n} \cdot \nabla \times \mathbf{n} \right)^2 + K_{33} \left(\mathbf{n} \times \nabla \times \mathbf{n} \right)^2 \right] dV, \quad F_l = -\frac{\varepsilon_0 \varepsilon_a}{4} \int \left(\mathbf{n} \cdot \mathbf{E}_{hv} \right)^2 dV,$$

$$F_E = -\frac{1}{2} \int \left(\mathbf{D} \cdot \mathbf{E} \right)^2 dV, \quad F_{fl} = -\int \left(\mathbf{P}_f \cdot \mathbf{E} \right) dV, \quad F_S = -\frac{1}{2} W \sum_{i=1}^{2} \int \left(\mathbf{n} \cdot \mathbf{d}_i \right)^2 dS_i. \tag{6}$$

In eqs. (5)-(6) F_{el} is the bulk elastic energy of a distorted nematic LC layer, F_l is the contribution of the light beam field-LC interaction, F_E is the contribution of the dc-electric field created in the LC cell by the photorefractive substrate layers, F_{fl} is the contribution of the interaction of the dc-electric field with the LC flexoelectric polarization; F_S is the surface term describing the interaction of the director with the LC cell substrates in the Rapini-Papoular approach,[28] P_f is the flexopolarization defined by the expression, $\mathbf{P}_f = e_1 \mathbf{n} \nabla \cdot \mathbf{n} + e_3 (\nabla \times \mathbf{n} \times \mathbf{n})$,[29] K_{11}, K_{22}, K_{33} are the elastic constants, e_1, e_3 are the flexocoefficients, $\mathbf{E}_{hv}$ is the electric vector of the light field in the nematic LC, ε_a is the anisotropy of the LC dielectric permittivity at optical frequencies, W is the director anchoring energy with the cell substrates, $d_i = (\cos \theta_i, 0, \sin \theta_i)$ is the unit vector of the director easy axis at the cell substrates.

Some terms in eq. (5) will be neglected in what follows. We suppose the optical frequency LC dielectric anisotropy $\varepsilon_a << 1$, implying that we can neglect the light field contribution F_l. As was shown in Ref. 17 the LC dielectric anisotropy term F_E is small in comparison with the LC flexopolarization term F_{fl} and can also be neglected. For simplicity, we also suppose the one-constant approximation, $K_{11} = K_{22} = K_{33} = K$.

As the director is confined to the xz-plane, the director spatial profile in the nematic cell can be defined in terms of the angle $\vartheta(x, z)$ between the director **n** and the x-axis, $\mathbf{n} = (\cos \vartheta(x, z), 0, \sin \vartheta(x, z))$. Taking into account expression (2) for the photorefractive field acting on the director, we can seek $\vartheta(x, z)$ in the form

$$\vartheta(x, z) = \theta_0(z) + \left[\theta(z) \exp(iqx) + c.c. \right]. \tag{6a}$$

It should be noted, that in eq. (6a) we have omitted the higher harmonics of the LC director field. These harmonics do not satisfy the phase-matching condition requiring the grating wave vector to be equal the difference of the wave vectors components of the incident beams, $k_{1x} - k_{2x}$, and therefore, the higher harmonics give a negligible contribution to the beam coupling and energy exchange between the light beams.[27]

To obtain an aligning layer the LC cell substrates are often covered with a polymer film. Polymer films have flexible side chains which specify the easy axis direction for the LC director at the substrate. If the chains possess electric dipoles, the easy axis direction may be affected by the photorefractive

field applied to the LC cell. Restricting ourselves by this case we present the LC director easy axis angle at the cell substrates as

$$\theta_i = \theta_{0i} + \left[\theta'_{0i} \exp\left(iqx\right) + c.c. \right], \tag{6b}$$

where the first term, θ_{0i}, denotes a director pre-tilt angle and the second term describes the easy axis direction change induced by the photorefractive field. We further assume that angles θ'_{01}, θ'_{02} are proportional to the photorefractive field magnitude at the substrates, i.e. they can be described by expression

$$\theta'_{01,2} = \frac{1}{2}\alpha_{1,2}\frac{E_{sc}(q)}{|E_{sc,\max}|}m\left(\mp L/2\right), \tag{7}$$

where α_1 and α_2 are fitting parameters characterizing the mobility of the director easy axis on the substrates $z = -L/2$ and $z = L/2$, respectively; $E_{sc,max}$, a maximal value of $E_{sc}(q)$, is introduced to make parameters $\alpha_{1,2}$ dimensionless.

Minimizing the free energy functional (5), we obtain the linearized Euler-Lagrange equations for the director angles $\theta(z)$ and $\theta_0(z)$:

$$\frac{\partial^2\theta(z)}{\partial z^2} - q^2\theta(z) = \frac{e_{11}+e_{33}}{K}\left(\frac{\partial E_{0z}}{\partial z} - iqE_{0x}\right)\theta_0(z) + \frac{e_{11}}{K}\frac{\partial E_{0x}}{\partial z} + iq\frac{e_{33}}{K}E_{0z}, \tag{8}$$

$$\frac{\partial^2\theta_0}{\partial z^2} = 0, \tag{9}$$

and the boundary conditions to eqs. (8), (9)

$$\left[\frac{\partial\theta(z)}{\partial z} - \frac{e_{11}}{K}E_{0x} - \frac{e_{11}+e_{33}}{K}\theta_0(z)E_{0z} - \frac{W}{K}[\theta'_{02} - \theta(z)]\right]_{z=L/2} = 0$$
$$\left[\frac{\partial\theta(z)}{\partial z} - \frac{e_{11}}{K}E_{0x} - \frac{e_{11}+e_{33}}{K}\theta_0(z)E_{0z} + \frac{W}{K}[\theta'_{01} - \theta(z)]\right]_{z=-L/2} = 0 \tag{10}$$

$$\left[\frac{\partial\theta_0(z)}{\partial z} - \frac{W}{K}[\theta_{02} - \theta_0(z)]\right]_{z=L/2} = 0$$
$$\left[\frac{\partial\theta_0(z)}{\partial z} + \frac{W}{K}[\theta_{01} - \theta_0(z)]\right]_{z=-L/2} = 0 \tag{11}$$

Eqs. (8), (9) were first obtained in our paper[17] for the case of an infinitely strong director anchoring ($W = \infty$) and in the absence of an easy axis modulation by the photorefractive field. We note that under the experimental conditions in hybrid cells (see, for example, Refs. 10–12) the condition $qL \gg 1$ is usually holds and the angle $\theta_0(z)$ has an order of magnitude of about 0.1 allowing us to neglect the higher order terms in $\theta_0(z)$. The equations below for the signal beam amplitude contain the product $\theta(z)\cdot\theta_0(z)$ (see eq. (17)). Therefore, limiting ourselves by small $\theta_0(z)$ and $qL \gg 1$, in the solution $\theta(z)$ to eq. (8) we neglect terms of the higher order in e^{-qL} and small terms proportional to $\theta_0(z)$. Then, the analytical solution to eqs. (8), (9), subjected to these restrictions, is given by:

$$\theta(z) = \frac{1}{2}E_{sc}(q)\left[b(z)m\left(-L/2\right) + c(z)m\left(L/2\right)\right],$$

$$b(z) = [-r_1 + (\tilde{q} + \frac{w}{L})\frac{r\tilde{q}}{\tilde{q}^2 - q^2}]\frac{e^{-q(z+L/2)}}{q + \frac{w}{L}} - \frac{r\tilde{q}e^{-\tilde{q}(z+L/2)}}{\tilde{q}^2 - q^2} + \frac{2\alpha_1}{|E_{sc,\max}|}\frac{w}{L}\frac{e^{-q(z+L/2)}}{q + \frac{w}{L}}, \tag{12}$$

$$c(z) = [r_1 - (\tilde{q} + \frac{w}{L})\frac{r\tilde{q}}{\tilde{q}^2 - q^2}]\frac{e^{q(z-L/2)}}{q + \frac{w}{L}} + \frac{r\tilde{q}e^{\tilde{q}(z-L/2)}}{\tilde{q}^2 - q^2} + \frac{2\alpha_2}{|E_{sc,\max}|}\frac{w}{L}\frac{e^{q(z-L/2)}}{q + \frac{w}{L}}$$

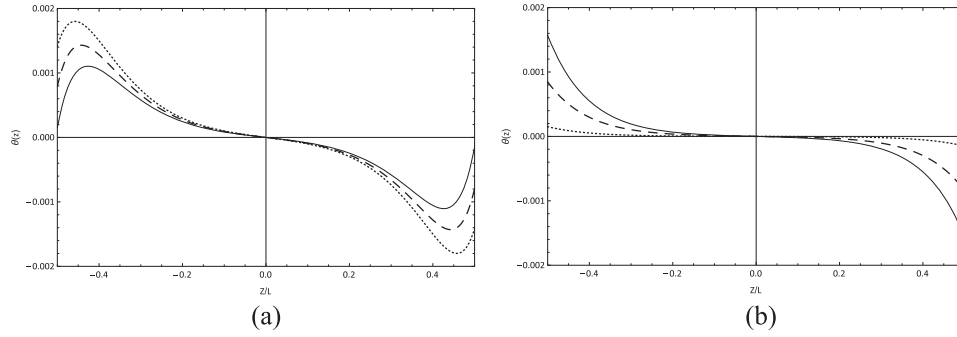$$\theta_0(z) = s + \frac{p}{1 + 2/w}z, \tag{13}$$

FIG. 1. Magnitudes of flexoelectric (a) and boundary-driven (b) gratings in the LC cell versus the director anchoring energy w: $w = 1$ - dotted, 10 - dashed, 10^2 - solid. The modulation parameters $m(-L/2) = m(L/2) = 0.1$, parameters $\alpha_1 = 0.02$, $\alpha_2 = -0.02$, cell thickness $L = 5\,\mu m$, and director grating spacing $\Lambda = 2\pi/q = 3\,\mu m$.

where $s = \dfrac{\theta_{01} + \theta_{02}}{2}$, $p = \dfrac{\theta_{02} - \theta_{01}}{L}$, $w = WL/K$, $r = \dfrac{e_{11} + e_{33}}{K}$, $r_1 = \dfrac{e_{11}}{K}$.

As it is seen from eq. (12), the director spatial profile is a result of the summation of two in-phase gratings induced in the LC: 1) the "flexoelectric" grating (terms proportional to flexoelectric parameters r and r_1) arising due to the photorefractive field coupling with the LC flexopolarization, and 2) the "boundary-driven" grating (terms with parameters α_1 and α_2) arising due to the director easy axis modulation caused by the photorefractive space-charge field.

In Fig.1 we show the spatial distribution of the magnitudes of flexoelectric and boundary-driven gratings at different values of the dimensionless director anchoring energy $w = WL/K$. For numerical calculations we use parameters of the nematic LC mixture TL205 from the paper:[10] low frequency dielectric constants $\tilde{\varepsilon}_\parallel = 9.1$, $\tilde{\varepsilon}_\perp = 4.1$, dielectric permittivity of the photorefractive layers $\varepsilon_{Ph} \approx 200$, and LC director pre-tilt angles $\theta_{01} = 12^0$, $\theta_{02} = -12^0$. To evaluate $E_{sc}(q)$, we assume following[10] that the ratio of the acceptor to donor impurity densities is very small, i.e. $N_d \gg N_a$, with $N_a \approx 3.8 \cdot 10^{21}\,m^{-3}$. The ratios of flexoelectric to elastic moduli in the absence of the photorefractive field $\frac{e_{11}}{K}$ and $\frac{e_{33}}{K}$ are not known for TL205. However, these ratios have been measured in other LC systems[30,31] and a value of order of magnitude $\sim 1 Cm^{-1}N^{-1}$ may be regarded as typical. As an example, we take $\frac{e_{11}}{K} = 1$, $\frac{e_{33}}{K} = 2$, but note that the values of these parameters only influence the magnitude of the effect and not any other functional properties.

It is seen from Figs.1a and 1b, the magnitudes of the flexoelectric and boundary-driven gratings have opposite dependence on the director anchoring energy: the flexoelectric grating magnitude decreases, while the boundary-driven grating magnitude increases with an increase of the director anchoring energy w. It reflects the fact that the director anchoring prevents the director deviations in a cell bulk caused by an interaction of the photorefractive field with the LC flexopolarization and enhances the director deviations induced by the LC cell boundaries.

III. BEAM COUPLING AND GAIN COEFFICIENT

Light beams incident on the hybrid cell propagate across the LC cell with the director grating obtained in Sec. II. The electric field of the light beams has the following form:

$$\mathbf{E}_{h\nu} = A_1(z)\mathbf{e}_1 \exp(i\mathbf{k}_1\mathbf{r} - i\omega t) + A_2(z)\mathbf{e}_2 \exp(i\mathbf{k}_2\mathbf{r} - i\omega t), \tag{14}$$

and satisfies the wave equation

$$\left[\nabla(\nabla \cdot) - \nabla^2\right]\mathbf{E}_{h\nu} - \frac{\omega^2}{c^2}\hat{\varepsilon}(x,z)\mathbf{E}_{h\nu} = 0, \tag{15}$$

where the LC dielectric tensor depends on the director components as $\varepsilon_{ij} = \varepsilon_\perp \delta_{ij} + \varepsilon_a n_i n_j$.[29] Substituting into this equation the director components expressed in terms of the angle $\vartheta(x,z)$ given by eq. (6a) we can rewrite the dielectric tensor in the following way

$$\hat{\varepsilon}(x,z) = \hat{\varepsilon}_1 + \hat{\varepsilon}_2(z) + [\hat{\varepsilon}_3(z)\exp(iqx) + c.c.]. \tag{16}$$

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The first term in equation (16) corresponds to a uniaxial homogeneous medium tilted at the angle θ_{01} with respect to the x-axis, the second term takes into account the inhomogeneity of the director distribution in the LC cell induced by the initial director pre-tilt at the cell substrates, and the third term describes the change of the dielectric tensor due to the periodic modulation of the director with a period $2\pi/q$. Expressions for $\hat{\varepsilon}_1$, $\hat{\varepsilon}_2(z)$ are presented in paper,[17] eq. (16a) below defines the matrix $\hat{\varepsilon}_3(z)$:

$$\hat{\varepsilon}_3(z) = \varepsilon_a \theta(z) \begin{pmatrix} -2\theta_0(z) & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2\theta_0(z) \end{pmatrix}. \tag{16a}$$

The coupling between the light waves in eq. (15) arises due to the term $\hat{\varepsilon}_3(z) \exp(iqx) + c.c.$ describing the dielectric permittivity grating. We follow a procedure analogous to that first outlined by Kogelnik[32] to obtain a system of coupled equations for the electric field magnitudes $A_1(z)$ and $A_2(z)$. It involves supposition that $A_1(z)$ and $A_2(z)$ vary slowly across the cell. We define beam 1 to be the signal and beam 2 to be the pump, we also adopt the Undepleted Pump Approximation,[27] for which the magnitude of the pump amplitude $|A_2| >> |A_1|$ may be regarded as constant. In this case we obtain an equation for $A_1(z)$ and its solution, as in paper,[17] allowing us to write the signal beam magnitude at the exit substrate, $A_1(L/2)$, as follows:[17]

$$A_1(L/2) = A_1(-L/2) - i\frac{\varepsilon_a}{k_{1z}}\frac{\omega^2}{c^2}A_2 \int_{-L/2}^{L/2} \theta_0(z)\,\theta(z)\,dz \tag{17}$$

Substituting expression for $\theta(z)$ from eq. (12) into eq. (17), and recalling that in the Undepleted Pump Approximation $m(z) \approx 2\cos(2\delta)A_1(z)/A_2$, we derive the following expression for the signal beam gain, $G = A_1(L/2)/A_1(-L/2)$, caused by the LC layer:

$$G = \frac{1 + a_1}{1 - a_2}, \tag{18}$$

where

$$a_1 = -i\frac{\varepsilon_a}{k_{1z}}\frac{\omega^2}{c^2}E_{sc}(q)\cos(2\delta)\int_{-L/2}^{L/2} \theta_0(z)\,b(z)dz,$$

$$a_2 = -i\frac{\varepsilon_a}{k_{1z}}\frac{\omega^2}{c^2}E_{sc}(q)\cos(2\delta)\int_{-L/2}^{L/2} \theta_0(z)\,c(z)dz, \tag{19}$$

The integrals in eqs. (19) can now be evaluated by substituting $b(z)$, $c(z)$ from eq. (12) and $\theta_0(z)$ from eq. (13). We express the result in terms of the exponential gain coefficient:

$$\Gamma = \frac{1}{L}\ln|G| = \frac{1}{L}\ln\left|\frac{1 + A(A_1 + B_1 - B_2)}{1 - A(A_2 + B_1 + B_2)}\right|, \tag{20}$$

where

$$A = \frac{\omega^2}{c^2}\frac{\cos(2\delta)}{k_{1z}}\frac{n_e - n_o}{2q(q + \frac{w}{L})}|E_{sc}|, \quad A_1 = \frac{2w}{L}\frac{\alpha_1}{E_{sc,\max}}[s - \frac{p}{1 + 2/w}(\frac{L}{2} - \frac{1}{q})],$$

$$A_2 = \frac{2w}{L}\frac{\alpha_2}{E_{sc,\max}}[s + \frac{p}{1 + 2/w}(\frac{L}{2} - \frac{1}{q})],$$

$$B_1 = \frac{p}{1 + 2/w}[r_1(\frac{L}{2} - \frac{1}{q}) + r\frac{q(q + \frac{w}{L})(\frac{L}{2} - \frac{1}{\tilde{q}}) - \tilde{q}(\tilde{q} + \frac{w}{L})(\frac{L}{2} - \frac{1}{q})}{\tilde{q}^2 - q^2}],$$

$$B_2 = s[r_1 - (1 + \frac{w/L}{\tilde{q} + q})r]. \tag{21}$$

Here n_o and n_e are the LC ordinary and extraordinary refractive indices, respectively.
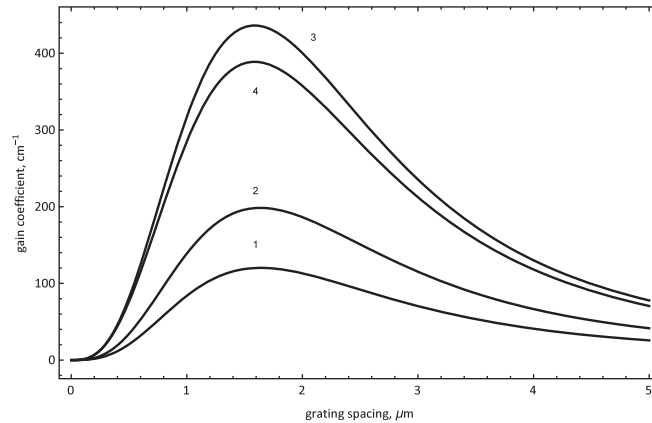
FIG. 2. Gain coefficient versus grating spacing for the flexoelectric grating at different strengths of anchoring: $w=0.5$ (1), 1 (2), 10 (3), 10^2 (4). The cell thickness is $L = 5\,\mu m$.

In order to evaluate the gain coefficient we take the laser wavelength in air $\lambda = 532$ nm and refractive indices of the LC mixture TL 205 $n_0 = 1.527$, $n_e = 1.744$.[10] Following[17] we also replace the "bare" flexoelectric coefficients occurring in eq. (21) by "effective" flexoelectric coefficients $e_{ii} = e_{ii}^0 \left(1 + \mu q^2 \, |E_{0sc}(q)|^2\right)$. These modified flexoelectric coefficients take into account the approximate effects of the flexoelectric LC component separation under the inhomogeneous photorefractive field. It allows us to bring the theory developed in Ref. 17 into agreement with the experimental results[10] using a single fitting parameter $\mu = 2 \cdot 10^{-21} \, J^{-2} C^2 m^4$. It is worth noting, that as it was shown in Ref. 17, the quadratic $|E_{0sc}(q)|^2$ term in the effective flexoelectric coefficients e_{ii} dominates to the extent that the beam coupling in the LC mixture TL 205 becomes cubic in $E_{0sc}(q)$. Using for our numerical calculations below the parameters of the LC TL 205 we also adopt the same value of the fitting parameter μ (which, generally speaking, may be different for different LCs).

In Fig.2 the gain coefficient Γ versus the grating spacing $\Lambda = 2\pi/q$ is plotted for different values of the director anchoring energy w if only the flexoelectric grating is written. It reaches its maximum at a grating spacing much less than the cell thickness in accordance with results obtained in paper[17] for the case of absolutely rigid director anchoring. A non-monotonic dependence of the gain coefficient on the anchoring energy w is obtained. It is also illustrated in Fig. 3, where the dependence of the gain coefficient on the anchoring energy is plotted for a grating spacing $\Lambda = 2\mu m$. The gain coefficient increases with an increase w if approximately $w < 10$ and decreases if $w > 10$.
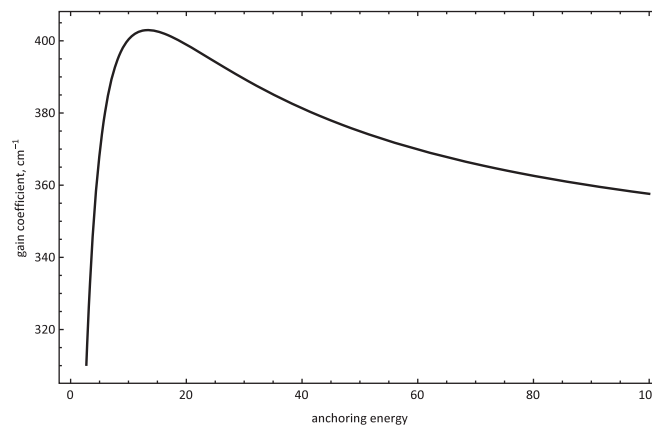

FIG. 3. Gain coefficient versus anchoring energy w for the flexoelectric grating; grating spacing $\Lambda = 2\mu m$.
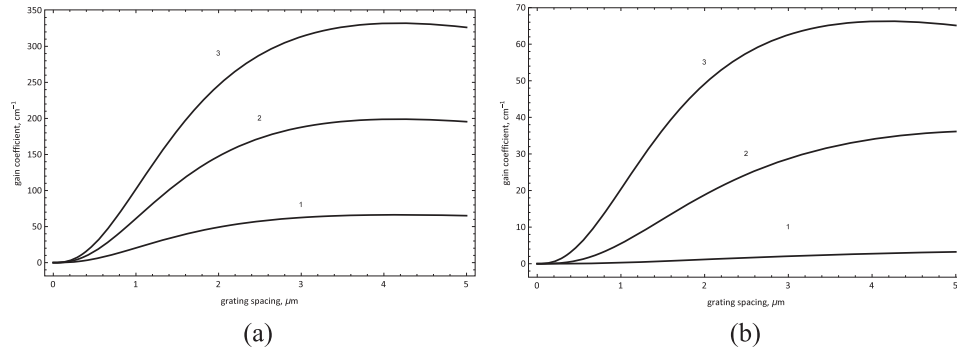
FIG. 4. Gain coefficient versus grating spacing for the boundary-driven grating: (a) different induced angles $\theta'_{01,2}$ in (7): $\alpha_{1,2} = \pm 0.1(1),\ \pm 0\,3(2),\ \pm 0.5(3)$; $w = 10^2$; (b) different anchoring energies: $w=1(1),\ 10\,(2),\ 10^2\,(3)$; $\alpha_{1,2} = \pm 0.1$.
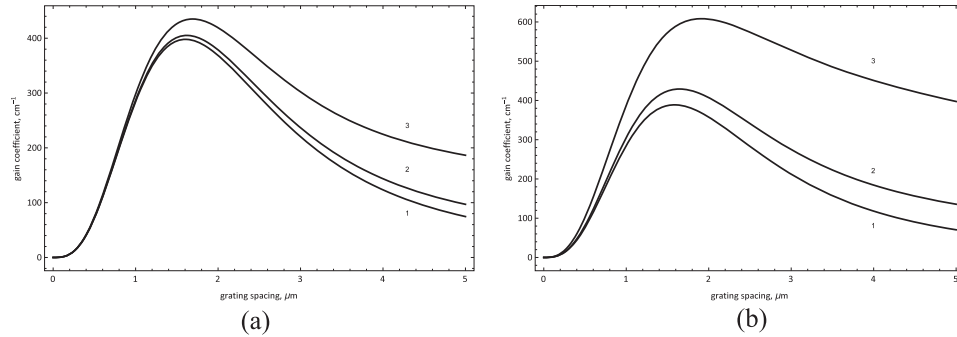

FIG. 5. Gain coefficient versus grating spacing for the total grating: $\alpha_{1,2} = 0\,(1),\ \pm 0\,.1(2),\ \pm 0.5(3)$; w=5 – (a), 10^2 – (b).

It is seen from the formula (16a) that the magnitude of the dielectric permittivity grating is proportional to the product $\theta_0(z) \cdot \theta(z)$, where the angle $\theta(z)$ describes the magnitude of the director grating and the angle $\theta_0(z)$ describes the director deviation in the cell caused by the director pre-tilt at the cell boundaries. For the flexoelectric grating these quantities have opposite dependence on the anchoring energy w providing non-monotonic dependence of their product: $\theta(z)$ decreases, while $\theta_0(z)$ defined by eq. (13) increases with an increase of w.

In Fig. 4 we show dependence of the gain coefficient on the grating spacing when there is only the pure boundary-driven grating induced by the director easy axis modulation. Fig. 4a presents the gain coefficient versus grating spacing at different values of the parameters $\alpha_{1,2}$ determining the magnitude of the easy axis modulation angle, but at fixed value of the anchoring energy w. In Fig. 4b the gain coefficient versus grating spacing is shown at different values of the anchoring energy w, but at fixed values of the parameters $\alpha_{1,2}$. For this grating the quantity $\theta(z)$ increases with an increase of w and $\alpha_{1,2}$, while $\theta_0(z)$ increases with an increase of w and does not depend on $\alpha_{1,2}$. As a result, as it is seen from Figs. 4a, 4b, the gain coefficient increases with an increase of both $\alpha_{1,2}$ and w.

The gain coefficient of the total grating versus grating spacing is shown in Fig. 5 for different values of parameters $\alpha_{1,2}$ characterizing the magnitude of the boundary-driven grating at two values of the anchoring energy w. It increases with the magnitude of the boundary-driven grating. This increase is more significant for grating spacings close to the cell thickness and at high values of the anchoring energy.

IV. CONCLUSIONS

Two interfering light beams incident onto an organic-inorganic hybrid nematic cell with photore-fractive substrates intersect and produce a space-charge field in the substrates. The spatially periodic space-charge field penetrates into the nematic cell and influences the LC director by two main ways:

interacting with the LC flexopolarization and reorienting the director easy axis at the cell boundaries. Thus, the director periodic modulation (director grating) arising in the cell is a sum of two in-phase gratings, a flexoelectric effect driven grating and a boundary-driven grating.

The magnitude of the flexoelectric effect driven grating depends linearly on ratio of the flexo-electric coefficients to the elastic constant, and decreases with a director anchoring energy increase. The magnitude of the boundary-driven grating is proportional to parameters $\alpha_{1,2}$ characterizing the magnitude of the director easy axis deviation under the photorefractive field, and increases with an increase of the director anchoring energy.

The director grating gives rise to the dielectric permittivity grating. Each light beam diffracts from the induced grating leading to an energy exchange between the beams. As a result, the amplitude of the small signal beam increases depending on the grating spacing and contribution from flexoelectric and boundary-driven gratings. If only the flexoelectric grating is present the gain coefficient depends non-monotonically on the anchoring energy w, it increases with an increase w at (approximately) $w < 10$ and decreases at $w > 10$. In the case of boundary-driven grating the gain coefficient increases with an increase of both anchoring energy and parameters $\alpha_{1,2}$. As a result, the gain coefficient of the total grating also increases with increase of the anchoring energy and the parameters $\alpha_{1,2}$, especially for the grating spacing close to the cell thickness and at high values of the anchoring energy.

ACKNOWLEDGMENTS

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