REPORT DOCUMENTATION PAGE

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED *(From - To)*
25 April 2017	Briefing Charts	01 April 2017 - 30 April 2017

4. TITLE AND SUBTITLE
Multi-scale and multi-physics simulations using the multi-fluid plasma model

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Eder Sousa

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER
Q1AM

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Research Laboratory (AFMC)
AFRL/RQRS
1 Ara Drive
Edwards AFB, CA 93524-7013

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Research Laboratory (AFMC)
AFRL/RQR
4 Draco Drive
Edwards AFB, CA 93524-7160

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-RQ-ED-VG-2017-071

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release; Distribution Unlimited. PA Clearance Number: 17211 Clearance Date: 24 April 2017

13. SUPPLEMENTARY NOTES
For presentation at Dartmouth University for an Invited Lecture
Prepared in collaboration with ERC. The U.S. Government is joint author of the work and has the right to use, modify, reproduce, release, perform, display, or disclose the work.

14. ABSTRACT
Viewgraph/Briefing Charts

15. SUBJECT TERMS
N/A

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Robert Martin
					19b. TELEPHONE NUMBER *(Include area code)*
Unclassified	Unclassified	Unclassified	SAR	31	N/A

MULTI-SCALE AND MULTI-PHYSICS SIMULATIONS USING THE MULTI-FLUID PLASMA MODEL

Éder M. Sousa

ERC INC., IN-SPACE PROPULSION BRANCH (RQRS)
AIR FORCE RESEARCH LABORATORY
EDWARDS AIR FORCE BASE, CA
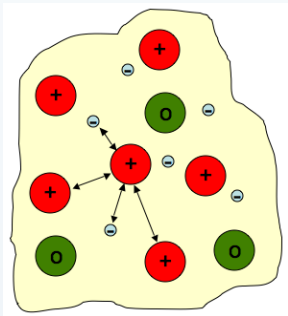
UMass Dartmouth April 25th, 2017

U.S. AIR FORCE

1. Plasma
2. Multi-Fluid Plasma Model
   - Advantages
   - Limits of the model
3. Numerics: Blended Finite Element Method
   - Discontinuous Galerkin for ion/neutrals
   - Continuous Galerkin for electrons/fields
   - Initial tests
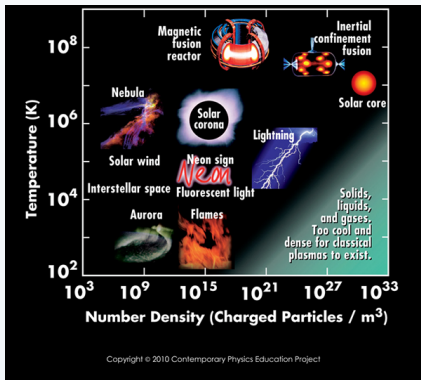
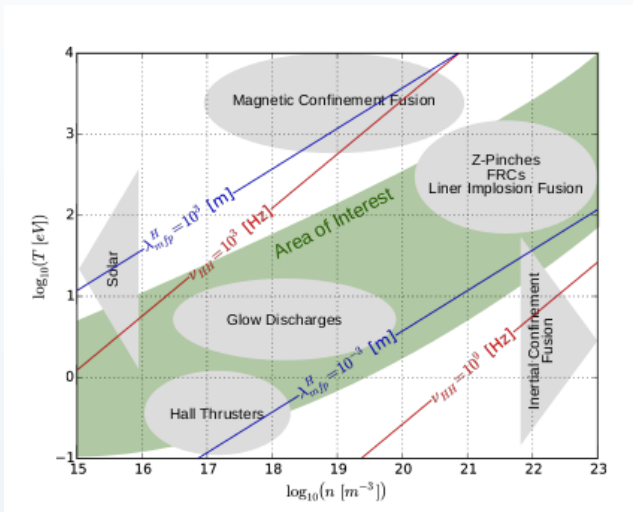Plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.

Credit: particleincell.com

- "99% of matter in the universe is in the state of plasma"

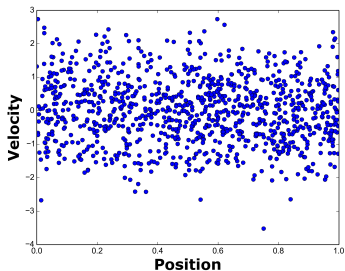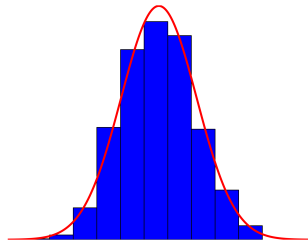- 3-Dimensions + 3-Velocities
- Evolve the particles position and velocity
- e.g. Particle-In-Cell models

- Ensemble average of particles distribution, $f_s(\mathbf{x}, \mathbf{v}, t)$
- Evolve the distribution function
- e.g. Vlasov-Maxwell models

- The Boltzmann eqn:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left. \frac{\partial f_s}{\partial t} \right|_c$$

- Take the 0th, 1st, 2nd moments of the Boltzmann Eqn.

$$m_s \int \mathbf{v}^n \frac{\partial f_s}{\partial t} d\mathbf{v} + m_s \int \mathbf{v}^{n+1} \cdot \frac{\partial f_s}{\partial \mathbf{x}} d\mathbf{v} + q_s \int \mathbf{v}^n \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} d\mathbf{v} = m_s \int \mathbf{v}^n \left. \frac{\partial f_s}{\partial t} \right|_c d\mathbf{v}$$

- Each moment of the Boltzmann eqn gives an equation for the moment variable, and introduces the next higher moment variable
- This process can go on indefinitely

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = \left. \frac{\partial \rho_s}{\partial t} \right|_\Gamma$$
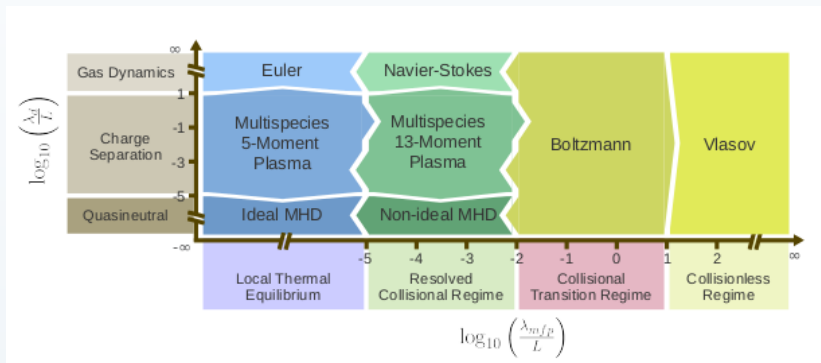
$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + p_s \mathbf{I} + \Pi_s) = \frac{\rho_s q_s}{m_s} (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) - \sum_{s*} \mathbf{R}_{s,s*} + \left. \frac{\partial \rho_s \mathbf{u}_s}{\partial t} \right|_\Gamma$$

$$\frac{\partial \varepsilon_s}{\partial t} + \nabla \cdot (((\varepsilon_s + p_s) \mathbf{I} + \Pi_s) \cdot \mathbf{u}_s + \mathbf{h}_s) = \frac{\rho_s q_s}{m_s} \mathbf{u}_s \cdot \mathbf{E} + \sum_{s*} Q_{s,s*} + \left. \frac{\partial \varepsilon_s}{\partial t} \right|_\Gamma$$

- System is truncated by relating higher moment variables to the lower ones
- The fluids are coupled to each other and to the electromagnetic fields through Maxwell's equations and interaction source terms.

Sean Miller, PhD dissertation, University of Washington (2016)

$$\text{Kinetic} \xrightarrow{\text{LTE, velocity moments}} \text{MFPM} \xrightarrow{\epsilon_o \to 0, \quad m_e \to 0} \text{MHD}$$

IDEAL MHD MODEL IS VALID WHEN:

- High collisionality, $\tau_{ii}/\tau \ll 1$
- Small Larmor radius, $r_{Li}/L \ll 1$
- Low Resistivity, $\left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{r_{Li}}{L}\right)^2 \frac{\tau}{\tau_{ii}} \ll 1$

MULTI-FLUID PLASMA MODEL

- Less computationally expensive than kinetic models
- Multi-fluid effects become relevant at small spacial and temporal scales
- Finite electron mass and speed-of-light effects are included
- There is charge separation is modeled
- Displacement current effects are resolved in the MFPM

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{x}} = \mathbf{S}$$

- The source Jacobian $\frac{\partial \mathbf{S}}{\partial \mathbf{Q}}$ has imaginary eigenvalues
- The equation system has dispersive sources
- The dispersion is physical (may be difficult to distinguish from numerical dispersion)
- This dispersion is due to plasma waves that result from ion and electron plasma interactions with electromagnetic fields
- An ideal numerical method for the MFPM should:
  - be high-order accurate
  - couple the flux and the sources
  - capture shocks
  - not impose strict time-step

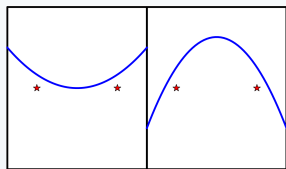Srinivasan et al, CCP 10 (2011)

- Solution to the electron and EM fields is smooth and does not shock
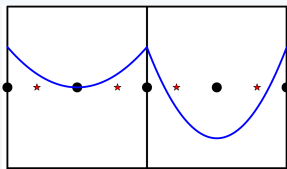
- Continuous Galerkin
- Electron fluid and EM fields

$$\mathbf{Q} = \sum_i \mathbf{q}_i v_i$$

- Discontinuous Galerkin
- Multiple ion and neutral fluids

$$\mathbf{Q} = \sum_i \mathbf{c}_i v_i$$

- For this implementation the balance law form is cast as

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{Q}} \cdot \frac{\partial \mathbf{Q}}{\partial \mathbf{x}} = \mathbf{S} + \kappa \nabla^2 \mathbf{Q}_d$$

- Lagrange polynomials are used for basis functions, v_r

$$\int_{\Omega} v_r \frac{\partial \mathbf{Q}}{\partial t} dV = \mathcal{L}_r(\mathbf{Q}) = \int_{\Omega} v_r \mathbf{S} dV - \int_{\Omega} v_r \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{Q}} \cdot \frac{\partial \mathbf{Q}}{\partial \mathbf{x}} dV + \kappa \int_{\Omega} v_r \nabla^2 \mathbf{Q}_d dV$$

- θ-method time integration

$$\mathcal{R}(\mathbf{Q}^n) = \overleftrightarrow{\mathbf{M}} \frac{\mathbf{Q}^{n+1} - \mathbf{Q}^n}{dt} - \theta \mathcal{L}_r(\mathbf{Q}^{n+1}) - (1 - \theta)\mathcal{L}_r(\mathbf{Q}^n) = 0$$

- $\theta = 0.5$ is used for 2nd order accuracy

$$\overleftrightarrow{\mathbf{J}}(\mathbf{Q}^n) = \frac{\partial \mathcal{R}(\mathbf{Q}^n)}{\partial \mathbf{Q}^n}, \quad \overleftrightarrow{\mathbf{J}}(\mathbf{Q}^n)\Delta \mathbf{Q} = -\mathcal{R}(\mathbf{Q}^n), \quad \mathbf{Q}^{n+1} = \mathbf{Q}^n + \Delta \mathbf{Q}$$

Reddy, An Introduction to the Finite Element Method (2006)

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{x}} = \mathbf{S}$$

- Legendre polynomials are used for basis functions, v_p
- The hyperbolic equation is multiplied by the basis function,

$$\int_\Omega v_p \frac{\partial \mathbf{Q}}{\partial t} dV = \mathcal{L}_p(\mathbf{Q}) = \int_\Omega v_p \mathbf{S} dV - \oint_{\partial \Omega} v_p \overleftrightarrow{\mathbf{F}} \cdot d\mathbf{A} + \int_\Omega \overleftrightarrow{\mathbf{F}} \cdot \nabla v_p dV$$

- Explicit Runge-Kutta time integration
- $CFL = c\Delta t/\Delta x \leq 1/(2p - 1)$, p is the polynomial order

$$\mathbf{Q}^* = \mathbf{Q}^n + \Delta t \mathcal{L}_p(\mathbf{Q}^n),$$

$$\mathbf{Q}^{n+1} = \frac{1}{2}\mathbf{Q}^* + \frac{1}{2}\mathbf{Q}^n + \frac{1}{2}\Delta t \mathcal{L}_p(\mathbf{Q}^*).$$

Loverich et al, CCP 9 (2006)

$$\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} = 0, \qquad Q(x,0) = e^{-10(x-8)^2}, \qquad ||\Delta Q||_2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{Q} - Q_i)^2}$$

Spatial Convergence
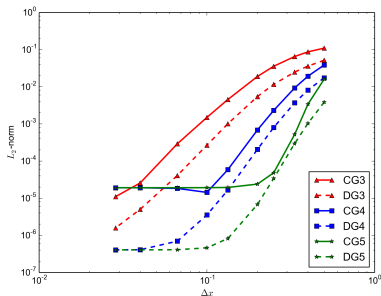
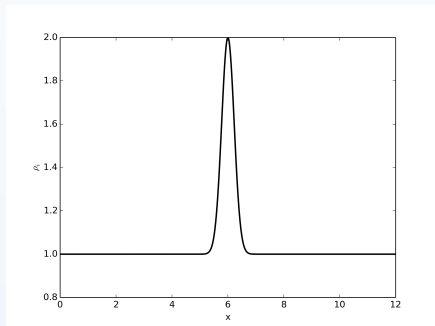**Temporal Convergence**
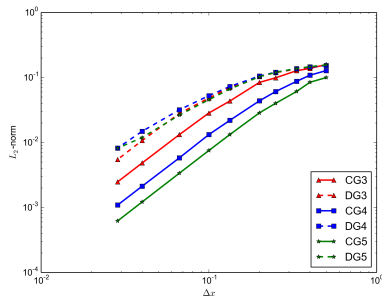
- Simulations at fixed time-step
- Simulations at fixed CFL=1

- 1D soliton is a two-fluid plasma problem
- The solution is smooth, therefore artificial dissipation can be small
- The simulation uses 512 second-order elements
- $B_z = 1.0$, $T_e = T_i = 0.01$, $\mathbf{u}_i = \mathbf{u}_e = \mathbf{0}$
- $n_e = n_i = 1.0 + e^{-10(x-6)^2}$

Baboolal, Math. and Comp. Sim. 55 (2001)

$$\frac{m_i}{m_e} = 1836, \; \frac{c}{c_{si}} = 1000\sqrt{2}, \text{ FV 5000 cells}$$
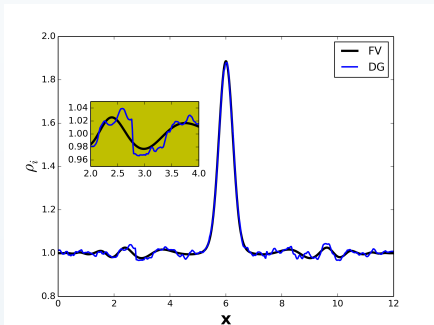
- DG Solution is very dispersive
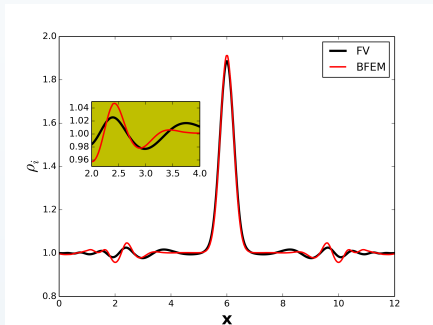
- BFEM is less dissipative than the converged solution

Hakim et al, JCP 219 (2006)

case	m_i/m_e	c/c_{si}	DG time(s)	BFEM time (s)	BFEM cost over DG
1	25	$10/\sqrt{2}$	0.32	37.7	+11681%
2	100	$10/\sqrt{2}$	1.28	37.7	+2845%
3	500	$10/\sqrt{2}$	6.82	37.7	+452.8%
4	1000	$10/\sqrt{2}$	12.4	38.2	+208.1%
5	1836	$10/\sqrt{2}$	23.5	40.4	+71.91%
6	3672	$10/\sqrt{2}$	47.2	39.2	-16.95%
7	3672	$100/\sqrt{2}$	520	265	-49.04%
8	3672	$1000/\sqrt{2}$	5274	2735	-48.14%

- Per time-step explicit DG is faster than BFEM, but it requires many more time-steps
- BFEM is more efficient only when time-step are considerably larger than explicit DG
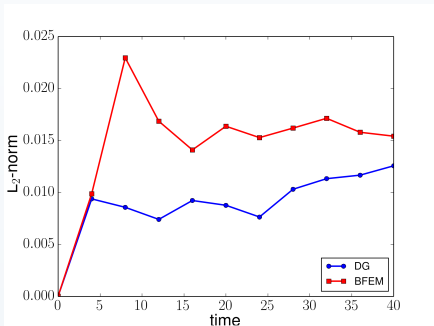
$m_i/m_e = 1836$ $m_i/m_e = 1$

- The BFEM seems to be less accurate than the DG implementation ($\sim 50\%$)
- When the mass ratio is one, the two methods have the same level of accuracy
- The discrepancy is due to the fact that the semi-implicit BFEM does not resolve the plasma frequency in this problem
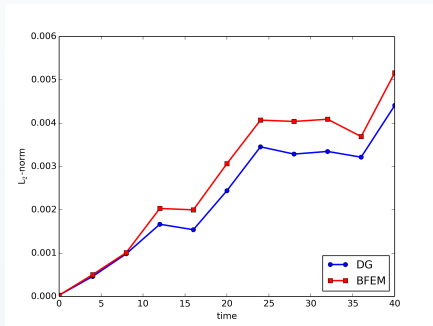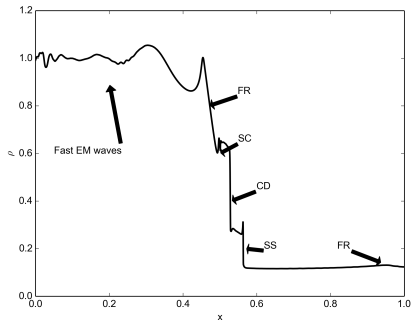
- Fast rarefaction wave (FR), a slow compound wave (SC), a contact discontinuity (CD), a slow shock (SS), and another fast rarefaction wave (FR)
- The problem exhibits limits of MHD and multi-fluid behavior by changing the Larmor radius, r_L
  - MHD: $r_L \to 0$
  - Multi-fluid: $r_L \sim L$

Brio and Wu, JCP 75 (1988)

- t=0.05/ω_{ci}, c/c$_{si}$=110, m$_i$/m$_e$=1836
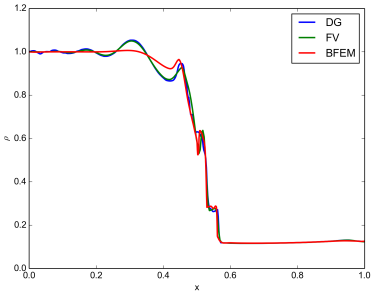
mass density
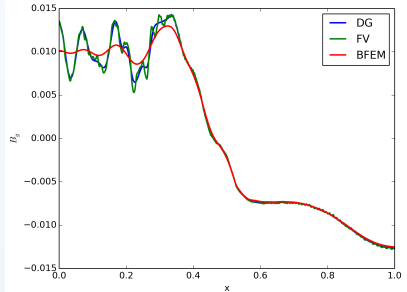
Magnetic field (y-comp.)

- The main features of the problem are captured by all three methods
- BFEM does not properly resolve the fast electromagnetic waves which require accurately resolving the electron dynamics

$$\Delta t_{max} = \min\left(\frac{\Delta x}{c_{se}}, \frac{\Delta x}{c_{si}}, \frac{\Delta x}{c}, \frac{0.1}{\omega_{ce}}, \frac{0.1}{\omega_{ci}}, \frac{0.1}{\omega_{pe}}, \frac{0.1}{\omega_{pi}}\right)$$

- Δt_{max} corresponds to the maximum value allowed for explicit methods based on the CFL condition

- $\Delta t = 42.9\Delta t_{max}$ is the maximum time step allowed by the BFEM due to ion dynamics
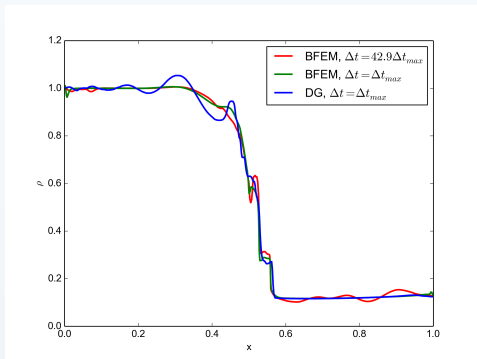
- Varying the artificial dissipation on the electron fluid, κ_e
- Wave-like behavior of the problem is better resolved
- Amplitude of the compound wave increases
- Right fast rarefaction wave is not visible
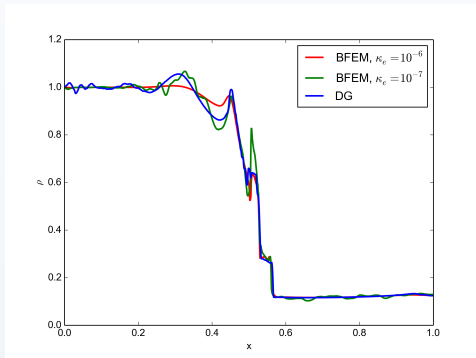
- Varying the artificial dissipation on the EM-field, κ_{EM}
- There is better agreement with the DG solution
- This reinforces the point that the wave-like behavior arises from the interaction of the electron fluid with the electromagnetic fields
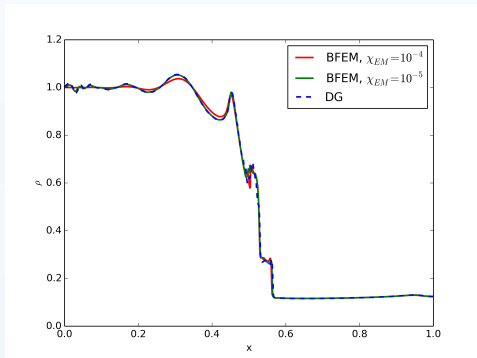
- The blended finite element method (BFEM) is presented
  - DG spatial discretization with explicit Runge-Kutta (i^+, n)
  - CG spatial discretization with implicit Crank-Nicolson (e^-, fileds)
  - DG captures shocks and discontinuities
  - CG is efficient and robust for smooth solutions
- Physics-based decomposition of the algorithm yields numerical solutions that resolve the desired timescales
- DG method takes less computational time to advance the solution by one time-step, however Δt is much smaller than that of the BFEM
- Computational cost savings using the BFEM will only occur for relatively large implicit time-steps compared to explicit time-steps

Sousa and Shumlak, JCP 326 (2016) 56-75

Thank you.

- Modeling each particle velocity and position is not practical.
- Instead an average is performed to give a statistical description.
- Calculate the number of particles per unit volume having approximately the velocity $\mathbf{v}$ near the position $\mathbf{x}$ and at time t, distribution function $f(\mathbf{v}, \mathbf{x}, \mathbf{t})$

$$\rho_s = m_s \int f_s(\mathbf{v}) d\mathbf{v}$$

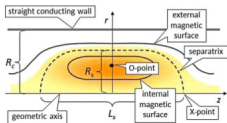$$\rho_s \mathbf{u}_s = m_s \int \mathbf{v} f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbb{P}_s = \mathbf{P}_s = m_s \int \mathbf{w}\mathbf{w} f_s(\mathbf{v}) d\mathbf{v}, \quad p_s = \frac{1}{3} m_s \int w^2 f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbf{H}_s = m_s \int \mathbf{w}\mathbf{w}\mathbf{w} f_s(\mathbf{v}) d\mathbf{v}, \quad \mathbf{h}_s = \frac{1}{2} m_s \int w^2 \mathbf{w} f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbf{w} = \mathbf{v} - \mathbf{u}_s$$

- Compact toroid, no central column
- Simple geometry and B-field configuration
- High power density (high plasma beta)
- Highly movable

- A Rotating Magnetic Field (RMF) is used to form the FRC
- Strong magnetization of the electrons to the RMF produces a current
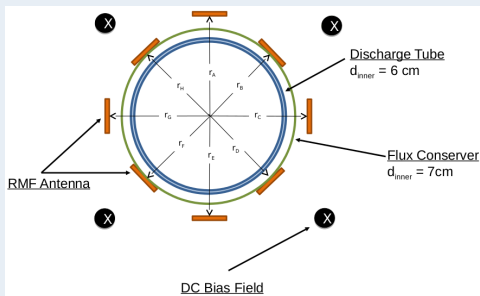- To conserve the total flux the B-field at the center reverses
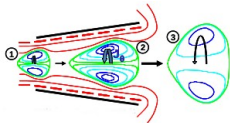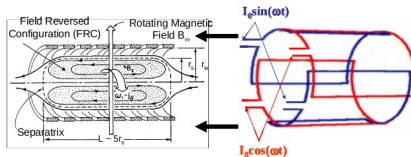
- 1 - Formation of high density FRC
- 2 - Acceleration of the FRC by Lorentz force
- 3 - FRC expands, converting thermal energy to directed energy

Advantages:
- Electrodeless and the plasmoid propellant is magnetically isolated from the walls
- Propellant is completely uncoupled from the driving and confining fields
- High plasma temperatures and densities significantly reduce ionization losses

- The formation of FRC using a rotating magnetic field (RMF)
- $\omega_{ci} < \omega < \omega_{ce}$
- $\nu_{ei} \ll \omega_{ce}$

- RMF BC

$$I_{RMF} = I_t \cos(\omega t + \phi)$$
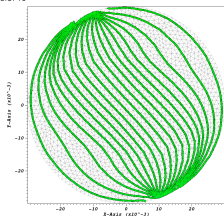$$I_t = I_o(1 - e^{-t/\tau})$$

- $\phi_{A,B} = 0, \phi_{E,F} = \pi$
- $\phi_{C,D} = \pi/2,$
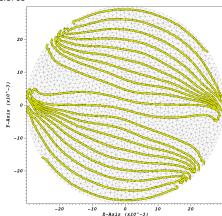 $\phi_{G,H} = -\pi/2$
- Bias Field is constant at t=0s

- Flux Conserver BC

$$B_z(a,t) = \frac{b^2 B_a(0)}{b^2 - a^2} - \frac{1}{\pi(b^2 - a^2)} \int_0^a r dr \int_0^{2\pi} d\theta B_z(r,\theta,t)$$

- Initialization: B_ω=90G, B_z=50G, ω=5MHz
- Solve Maxwell's eqns. with divergence constraints
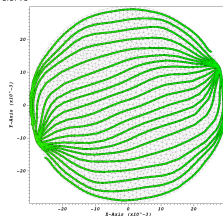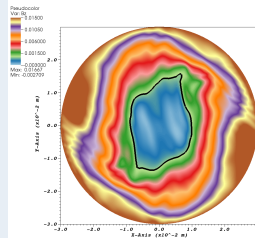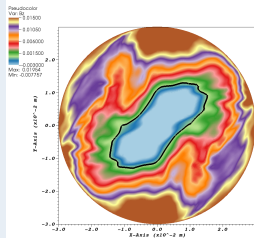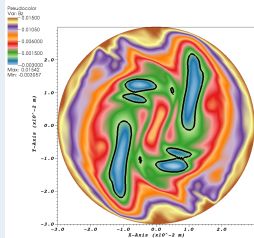
FRC FORMATION SIMULATIONS.

- Initialization: $T_i = T_e = 30\text{eV}$, $n_i = n_e = 10^{19}\text{m}^{-3}$, $m_i/m_e = 1836$
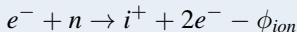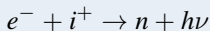- Solve Multi-Fluid eqns.
- B_z is plotted

$t = 250ns$

$t = 500ns$

$t = 1500ns$

Neutral-Plasma Model

- Interaction of multi-fluid plasma with gas dynamics neutral fluid
  - electron-impact ionization

  $$e^- + n \rightarrow i^+ + 2e^- - \phi_{ion}$$

  - radiative recombination

  $$e^- + i^+ \rightarrow n + h\nu$$
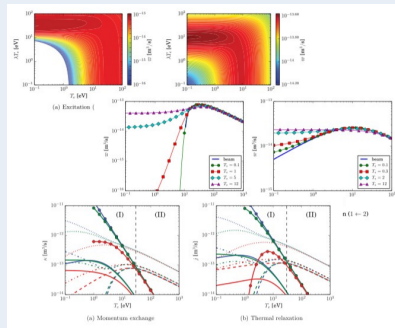
  - resonant charge exchange

  $$i^+ + n \rightarrow n + i^+$$
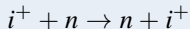
Meier and Shumlak, PoP 19 072508 (2012)

Collisional-Radiative Model

- Excitation/De-excitation rates

Le and Cambier, PoP 22 093512 (2015)