Pyrotechnic Shock Analysis Using Statistical Energy Analysis

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ABSTRACT

MIT Lincoln Laboratory (MIT/LL) has been developing a computational method to predict the maximum structural response due to a pyrotechnic shock input using Statistical Energy Analysis (SEA). It had been previously understood that since the pyrotechnic shock is not a steady state event, traditional SEA method may not applicable. A new analysis methodology effectively utilizes the traditional SEA modeling by employing an energy constraint using the Shock Response Spectrum (SRS) as the steady state input condition. Responses are recovered in the SRS and can be used to develop pyrotechnic shock design and test requirements for various subsystems of the model. Response accelerations in the time domain corresponding to the response spectra are estimated using the Peak Ratio (PR). The response waveform is obtained using the inverse transform of the product of the input acceleration and the transfer function of the SEA model. The most probable input acceleration may be obtained using the stochastic SRS decomposition method. Response waveforms can be further refined based on phase perturbations using the wave numbers of the SEA subsystems. A couple of validation examples are provided to demonstrate the new approach.

KEY WORDS: Peak Ratio, phase perturbation, pyrotechnic shock, shock response spectrum, Statistical Energy Analysis.

SYMBOLS

g = gravitational constant, 9.81 m/s^2
g_{pk} = temporal peak acceleration in gravitational constant
g_{rms} = root mean square acceleration in gravitational constant

INTRODUCTION

Launch vehicles and space systems use various types of pyrotechnic shock devices during their mission lives. While significant progress has been made in the last few decades, high fidelity simulation of structural responses due to a pyrotechnic shock input is still one of the challenging problems in space technologies. As noted by NASA[1] and ESA[2] references, the current shock prediction methods do not consistently produce successful results. Analysts frequently utilize the Finite Element Method (FEM) using the modal transient or direct integration methods to predict structural responses due to a pyrotechnic shock input. Unfortunately, there are many problems associated with this approach. First, pyrotechnic shock environments are broadband and typically analyzed at least to 10000 Hz. At low frequency, structural modes may be predicted using a deterministic method such as the FEM, however at high frequency structures typically produce modally dense responses with significant overlaps. Since the responses at high frequency typically converge to the mean value, an alternative method that can simulate an ensemble

average response would be better suited. Second, in order to use the FEM at high frequency, the model would require a large number of elements because it is typical to use at least 4 to 6 elements per each wavelength. Therefore, it becomes impractical to use the FEM for a full system analysis at high frequency. In addition to the large model size, high frequency responses would be severely influenced by the complex modal interactions among various structural members such as intervening structures, interfaces, junctions, etc. As a result, it becomes too complex to track the response deterministically at high frequency. Therefore, FEM modeling is typically applicable to a simple structure when the input is a pyrotechnic shock related event.

In this paper, Statistical Energy Analysis (SEA) is used as an alternative method to produce a response at high frequency. Since the SEA was originally developed to produce an ensemble average response at high frequency[3], it would be better suited for pyrotechnic shock response predictions. However, most of the times, the method is simplified for steady state problems and as a result, traditional SEA modeling may not be directly applicable to pyrotechnic shock problems. Manning[4] investigated a structural response of a simple model subjected to a transient input forcing function using the SEA modeling. In his approach, power balance equation was not simplified for steady state cases and the equation was solved at each time step with the steady state assumption for the coupling loss factors. This approach is frequently referenced as the transient SEA method in the literature. Dalton[5] used the FEM and SEA modeling to produce a response for a broad frequency band of interest. Input forcing function was estimated using the CTH Hydrocode developed by the Sandia National Laboratories[6]. This approach is often cited Virtual Mode Synthesis and Simulation (VMSS)[7] because it is based on curve fitting of the maximum magnitudes of imaginary single degrees of freedom system models to the Frequency Response Function (FRF) of the SEA model. Based on improved mid frequency predictions[8], Borello[9] developed a shock prediction method using the SEA modeling. In his approach, modal data (either from a FEM model or a test) are used with the SEA model to produce a response waveform with the random phase distribution. Input forcing functions simulating several types of pyrotechnic shock sources have been developed in the wave number domain as a result.

In the upcoming sections, a new pyrotechnic shock analysis methodology using the SEA modeling is introduced. A numerical example and a preliminary study of the complex validation model are provided to demonstrate the new approach.

PROPOSED PYROTECHNIC SHOCK ANALYSIS METHODOLOGY USING SEA

Figure 1 shows the overview of the proposed method using the SEA modeling. The new approach differs from the other methods in several categories. First, unlike the other methods, the proposed method does not require input forcing functions. Instead, the new approach uses the SRS as an energy constraint condition to the SEA model. The SRS is the maximum absolute temporal response of a single degree of freedom system due to an arbitrary base driven transient input at each prescribed frequency. For a given transient waveform, only one SRS is available. Alternatively, the SRS can be interpreted as the maximum response state of a system that does not change in time. This attribute allows the use of the traditional SEA modeling. By using the SRS as the input condition, coupling loss factors of the SEA model would simulate energy transfers among subsystems at the maximum response state. Since the pyrotechnic shock design and test requirements are provided in the SRS, this approach can be easily implemented as the

energy constraint condition at the interface of the SEA model to develop pyrotechnic shock design and test requirements for various subsystems of the model.
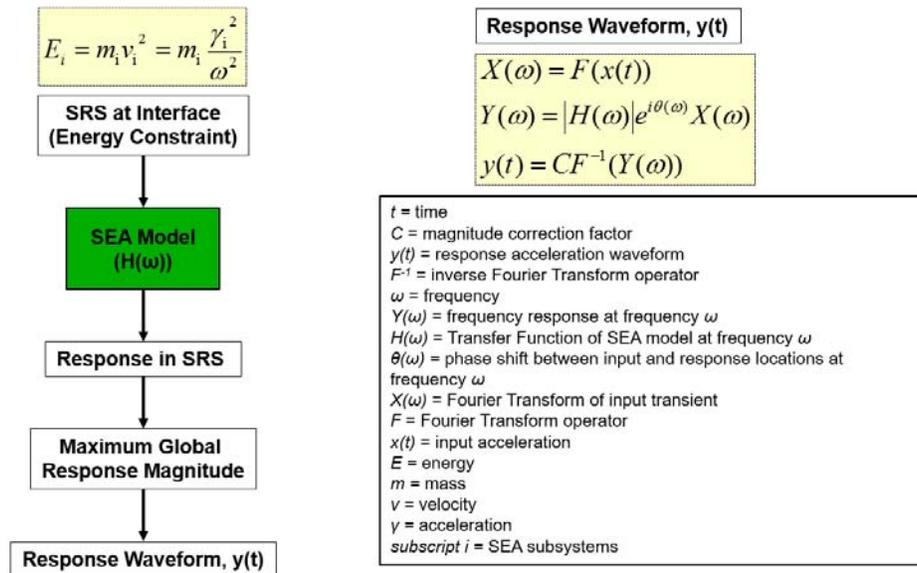

$$E_i = m_i v_i^2 = m_i \frac{\gamma_i^2}{\omega^2}$$

SRS at Interface (Energy Constraint)

SEA Model (H(ω))

Response in SRS

Maximum Global Response Magnitude

Response Waveform, y(t)

Response Waveform, y(t)

$$X(\omega) = F(x(t))$$
$$Y(\omega) = |H(\omega)|e^{i\theta(\omega)}X(\omega)$$
$$y(t) = CF^{-1}(Y(\omega))$$

t = time
C = magnitude correction factor
$y(t)$ = response acceleration waveform
F^{-1} = inverse Fourier Transform operator
ω = frequency
$Y(\omega)$ = frequency response at frequency ω
$H(\omega)$ = Transfer Function of SEA model at frequency ω
$\theta(\omega)$ = phase shift between input and response locations at frequency ω
$X(\omega)$ = Fourier Transform of input transient
F = Fourier Transform operator
$x(t)$ = input acceleration
E = energy
m = mass
v = velocity
γ = acceleration
subscript i = SEA subsystems

Figure 1: Overview of the proposed shock analysis method using SEA.

Second, since the SRS is used as the input, responses are recovered in the SRS as well. Estimated response acceleration in the time domain at each frequency can be developed using the Peak Ratio (PR)[10]. The PR describes the relationship between the input in the time domain and the response spectra in the frequency domain. In reality, the relationship between the input and the response can be described using the combination of the PR, Energy Ratio (ER), and the phase[10]. However, among the three variables, the PR is the most dominant, and in this paper, only the PR is used to estimate the response acceleration in the time domain at each frequency. Equations 1 and 2 describe the PR and the ER, respectively. The obtained response acceleration magnitudes are RSS'ed to produce the maximum global response magnitude.

$$PR = \frac{SRS}{g_{pk}} \tag{1}$$

$$ER = \frac{SRS}{g_{rms}} \tag{2}$$

Third, optionally, a response waveform may be estimated using the Transfer Function (TF) of the SEA model. Most of the times, the input acceleration is not available and in this case the SRS requirement may be decomposed into the most probable input accelerations using the stochastic SRS decomposition method[11]. Inverse Fourier transform of the product of the Fourier transform of the input acceleration and the TF of the SEA model produces the response waveform. Magnitude of the response waveform would likely be close to the maximum global response magnitude obtained from the SRS constraint condition. However, in case there is a difference, response waveform may be scaled to match the maximum global response magnitude obtained from the SRS energy constraint condition. In this proposed method, the response waveform

would carry the phase of the input acceleration to the response location. This would be a largely incorrect presumption and a conceptual mathematical model that could be used to simulate the phase change (between the source and the receiver) has been developed based on wave numbers of the SEA subsystems. Three types of waves including flexural, longitudinal, and shear could be used in the formulation to perturb the phase of the response waveform. The conceptual model also allows a weighting could be added to the perturbation based on modal energy or total energy of the SEA model. Figure 2 shows the mathematical concept of the phase perturbation method.

$$e^{i\vartheta(\omega)} = \prod_{j\_ss=1}^{n\_subsys} e^{i\phi_{j\_ss}(\omega)}$$

$$\phi_{j\_ss}(\omega) = Phase\left(\sum_{j\_mt=1}^{n\_modetype} W_{j\_mt,j\_ss}(\omega)e^{-ik_{j\_mt,j\_ss}(\omega)\ell_{j\_mt,j\_ss}}\right)$$

ω = frequency
$\theta(\omega)$ = phase shift between input and response locations at frequency ω
n_subsys = number of subsystems in SEA model
j_ss = jth subsystem in SEA model
n_modetype = number of different types of wave modes
j_mt = jth type of wave mode
W = weighting factor used to represent relative differences in acceleration amplitude due to contributions from different types of wave modes
k = wave number
ℓ = distance wave travels through subsystems

Figure 2: Conceptual phase perturbation mathematical model.

NUMERICAL EXAMPLE

Figure 3 shows a numerical validation model comprised of five beam and three plate SEA subsystems. The SRS input was applied as the energy constraint condition normal to the Small Side Plate subsystem. Figure 3 also shows the comparable FEM model developed with 48 CBAR elements for the Beam, 23104 QUAD4 elements for each Large Plate, 24 CBAR elements for each Small Beam, and 1742 QUAD4 elements for the Small Side Plate. Figure 4 shows the transient input acceleration applied in normal direction at the center of the Small Side Plate for the modal transient analysis. Information of the validation model can be found in Table 1.

Table 1: Information of the validation model.

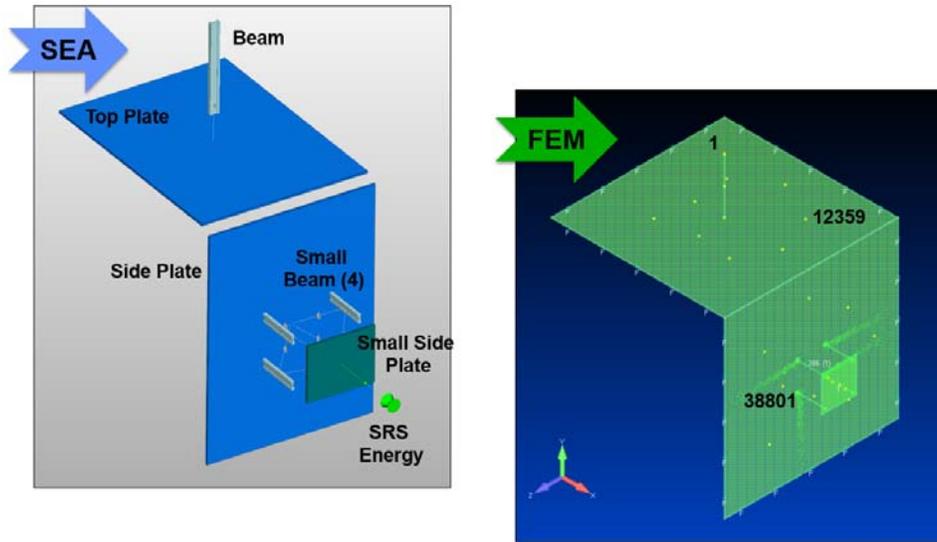|	Material	Size	Mass	Boundary Condition	Damping
Beam	Aluminum	0.036m X 0.036m X 0.609m	2.121Kg	Free top Clamped to plate	Q=10
Small Beam (4)	Aluminum	0.036m X 0.036m X 0.305m	1.061Kg	Clamped	Q=10
Large Plate (2)	Aluminum	1.926m X 1.926m X 0.010m	95.400Kg	Clamped	Q=10
Small Side Plate	Aluminum	0.385m X 0.385m X 0.010m	3.816Kg	Clamped to four beams	Q=10

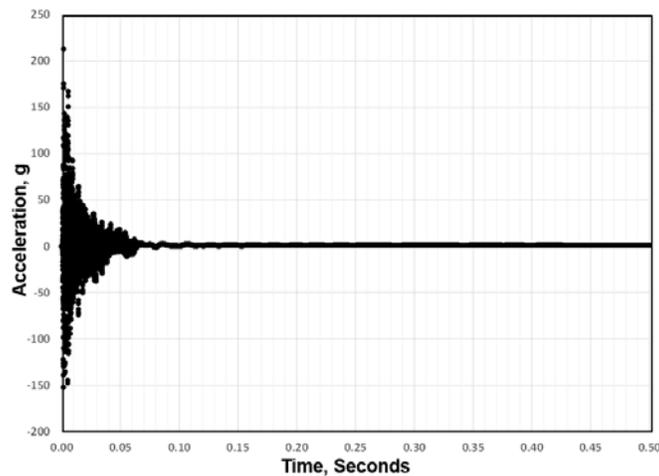**Figure 3:** Numerical validation model (SEA and FEM).

Figure 4: Pyrotechnic shock transient input acceleration.

Figure 5 shows the response measured at Node 12359 of the Top Plate using the modal transient method. Temporal peak acceleration values of 3.4g and -3.6g have been observed from the response. Figure 6 shows responses of 3 SEA subsystems recovered in the SRS with the SRS input applied as the energy constraint normal to the Small Side Plate subsystem. The SRS input has been derived from the transient input acceleration in Figure 4. The response spectra in Figure 6 are divided by the 3 PR values (3, 5, 7) in Figure 7 to produce the estimated response acceleration in the time domain at each frequency. The maximum global response magnitude is obtained by RSS'ing the response acceleration magnitudes. Table 2 summarizes the result for each SEA subsystem. For example, for the Top Plate, the modal transient method produced 3.4g and -3.6g temporal peak accelerations. Using PR=3, maximum global response magnitude of 9.0g was obtained. PR=5 produced 5.4g and PR=7 3.9g which is almost identical to the value obtained from the modal transient method. It has been noted that PR=3 would produce

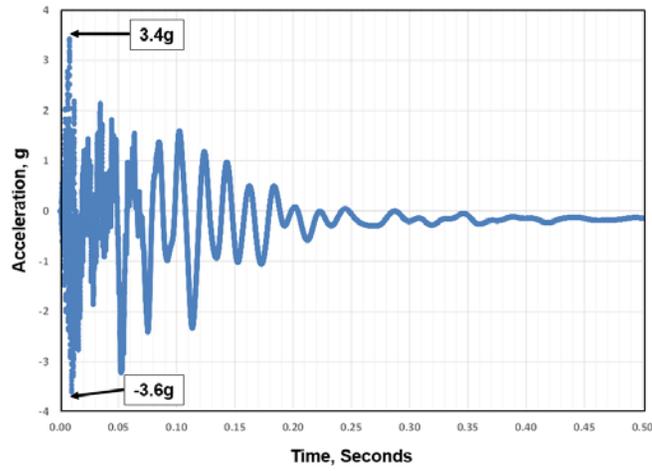conservative results enveloping temporal peak accelerations obtained from the modal transient method.

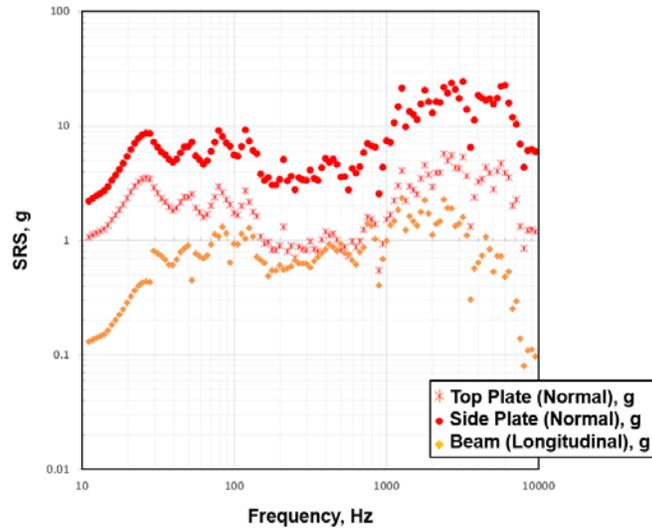**Figure 5:** Response of the Top Plate measured at Node 12359 using modal transient method.

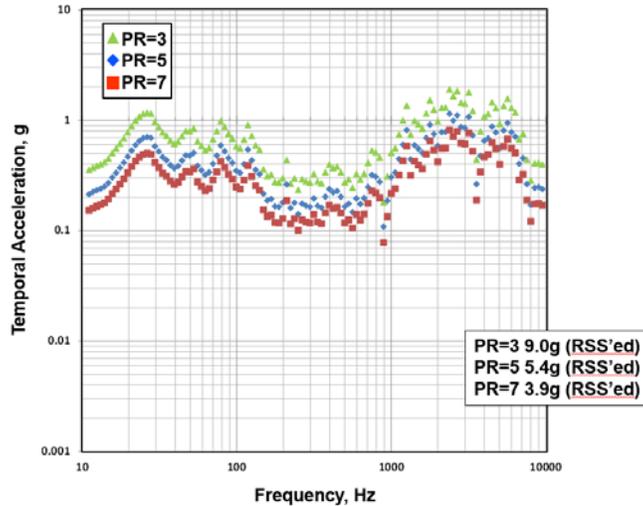**Figure 6:** Responses of the 3 SEA subsystems recovered in the SRS.

Figure 7: Estimated response acceleration magnitudes in the time domain (Top Plate).

Table 2: Maximum global response magnitudes for SEA subsystems.
Green shows the perfect match with the modal transient.

	SEA (PR=7)	SEA (PR=5)	SEA (PR=3)	FEM
Maximum Side Plate Global Response Magnitude, g	15.7g	22.0g	36.6g	22.7g, -12.6g
Maximum Top Plate Global Response Magnitude, g	3.9g	5.4g	9.0g	3.4g, -3.6g
Maximum Beam Global Response Magnitude, g	1.5g	2.1g	3.4g	2.6g, -2.7g

Using the TF of the SEA model and the input acceleration, response waveforms have been calculated for each SEA subsystem. Table 3 shows the maximum global response magnitudes obtained from the response waveforms. The maximum global response magnitude of the response waveform is simply the maximum value of the waveform, therefore it can be directly compared with the maximum global response magnitude obtained from the SRS energy constraint input condition. The maximum global response magnitudes of the response waveforms were lower than those from the SRS energy constraint condition in Table 2. For example, for the Top Plate, the SRS constraint condition produced the result ($3.9g$) matching the result from the modal transient method with PR=7. However, the response waveform produced the maximum global response magnitudes of $2.8g$ and $-4.7g$. If necessary, they could be further refined using the Magnitude Correction Factor, C=1.39. The response waveforms were further refined using the phase perturbations based on bending wave numbers of the Small Side Plate, Side Plate, and Top Plate subsystems, and longitudinal wave numbers of the Beam subsystem. It seems that the phase perturbation may have improved the result for the Top Plate subsystem, however no meaningful improvements were found from the Side Plate and the Beam subsystems from the maximum global response magnitude perspective.

Table 3: Maximum global response magnitudes using response waveforms and phase perturbations.

	SEA SRS Constraint	SEA (Response Waveform)	SEA (Phase Perturbed)	FEM	Magnitude Correction Factor, C
Maximum Side Plate Global Response Magnitude, g	22.0g	13.0g, -17.2g	13.4g, -19.0g	22.7g, -12.6g	1.69
Maximum Top Plate Global Response Magnitude, g	3.9g	2.8g, -4.7g	3.6g, -3.5g	3.4g, -3.6g	1.39
Maximum Beam Global Response Magnitude, g	2.1g	1.4g, -1.6g	1.4g, -1.6g	2.6g, -2.7g	1.50

Figure 8 shows the response waveform of the Side Plate subsystem compared to the modal transient result recovered at Node 38801. The maximum global response magnitude was corrected with C=1.69 and the phase perturbation was applied without an energy weighting. It appears that the response waveform follows the modal transient result reasonably well, except at 0.05 seconds, the SEA result appears to be opposite to the modal transient result. This may simply be related to the formulation of the phase perturbation methods. Figure 9 shows the response waveform of the Beam subsystem compared to the modal transient result recovered at Node 1. The maximum global response magnitude was corrected with C=1.50 and no phase perturbation was applied. The SEA result showed a good agreement with the modal transient result, except the modal transient method was able to produce the "ringing" of the beam at 0.05 seconds. The proposed method is based on producing maximum responses using the SRS as the input condition and the response waveforms are only "attached" to the maximum global response magnitudes, the response waveforms would not be able to simulate local modes such as this "ringing" mode. However, in general, the proposed method was able to produce a time signature matching reasonably well with the modal transient result for each subsystem.
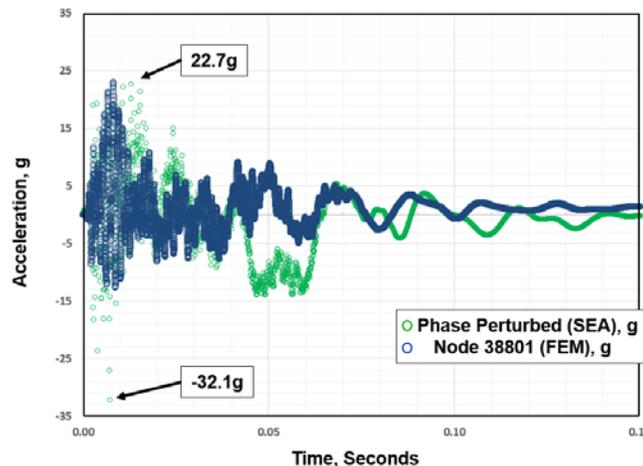

Figure 8: Response waveform of the Side Plate subsystem with C=1.69 and phase perturbation.
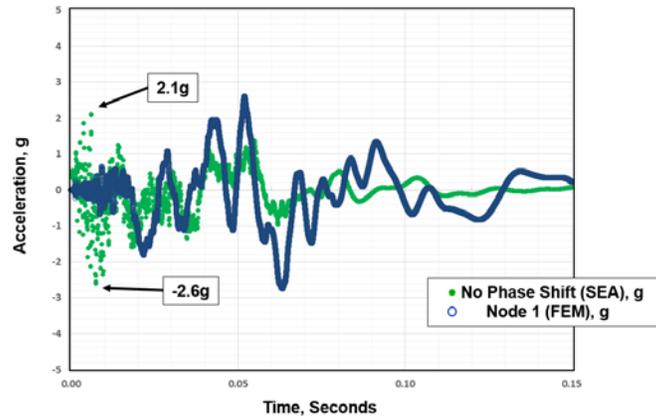
Figure 9: Response waveform of the Beam subsystem with C=1.50 and no phase perturbation.

COMPLEX VALIDATION MODEL

In order to validate the proposed pyrotechnic shock prediction method, a complex validation model was developed. Figure 10 shows the complex validation model undergoing pyrotechnic shock tests using the Universal Pyrotechnic Shock Simulator[12]. Figure 10 also shows the SEA model of the complex validation model. In this paper, only the preliminary results of the correlation study are provided. More than 50 pyrotechnic shock tests have been conducted to produce transmissibility functions between the input and the responses. The model is made of aluminum and consists of plate and beam like elements with complex geometries and various junctions including welding and multiple fastener combinations. During pyrotechnic shock tests, the model was instrumented with more than 15 Dytran shock accelerometers to measure response time histories from each structural member. Dataflex-1000A by DSPCon was used to measure the time histories with 100,000 Hz sample rate. Figure 11 shows the SRS input developed from the temporal accelerations measured at the system interface (Ch1 and Ch2). The SRS input was applied as the energy constraint condition at the interface subsystem of the SEA model. Figures 12 and 13 show the response spectra recovered at Ch5 and Ch13, respectively. It appears that the SEA results are well correlated with the test data and the predicted SRS levels from the SEA model successfully envelope the test data.
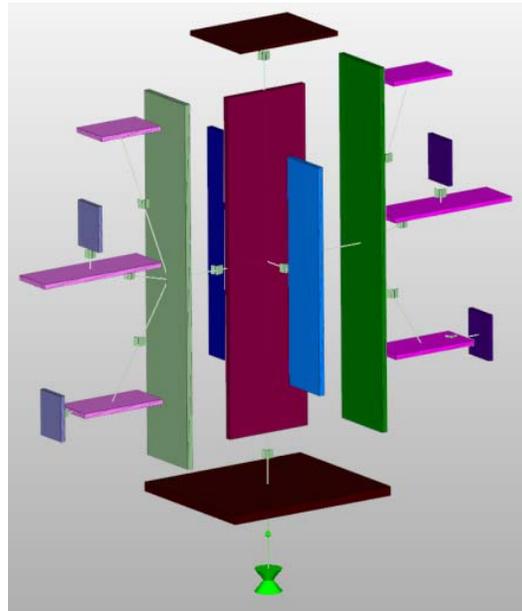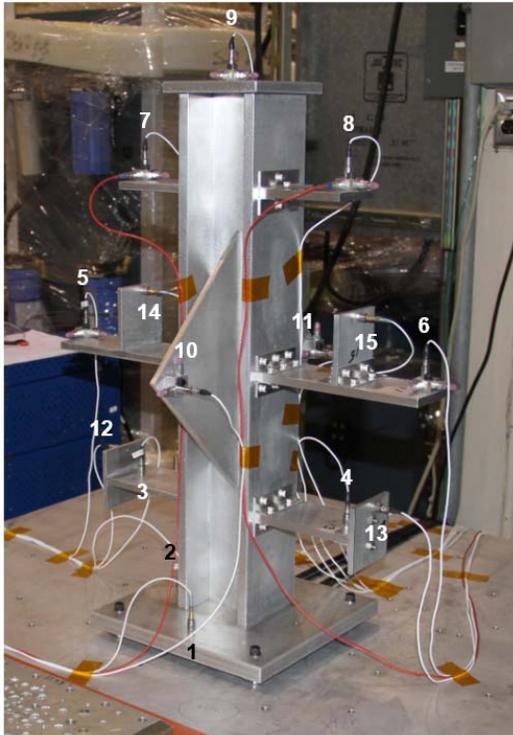
Figure 10: Complex validation model fully instrumented for pyrotechnic shock tests (L) and the SEA model (R). More than 50 pyrotechnic shock tests were conducted to develop transmissibility functions between the input and the responses. Maxi-Max SRS has been calculated from Ch1 and Ch2. Green hourglass represents the SRS energy constraint input condition to the interface SEA subsystem. SEA+ by InterAC/LMS was used for the SEA modeling.
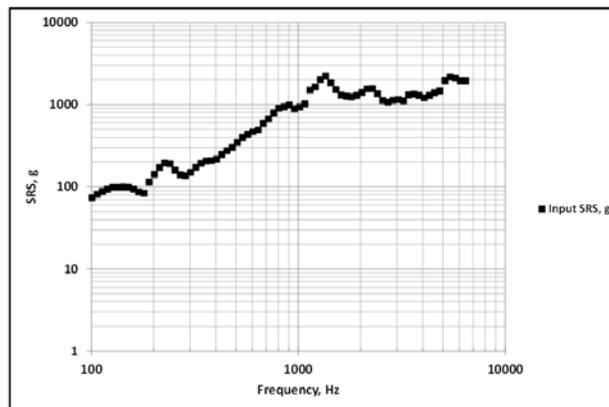

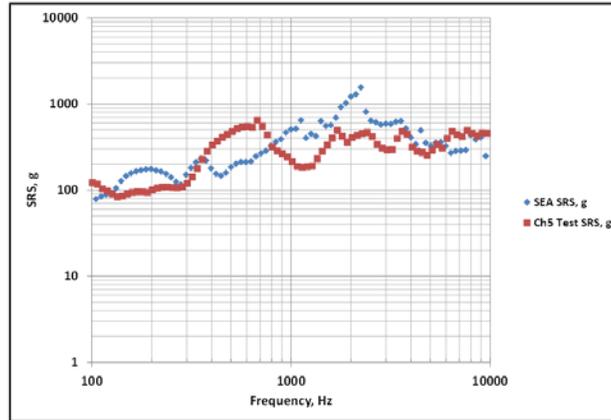**Figure 11:** Maxi-Max SRS input (Q=10) calculated from Ch1 and Ch2.

Figure 12: Comparison of the response spectra recovered from the proposed SEA method and from the test data (Ch5).
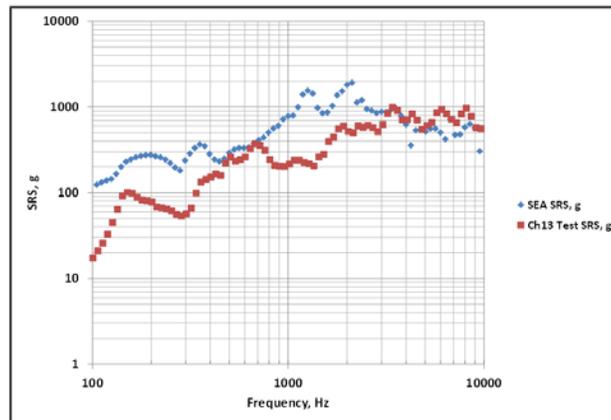

Figure 13: Comparison of the response spectra recovered from the proposed SEA method and from the test data (Ch13).

CONCLUSIONS AND RECOMMENDATIONS

The proposed pyrotechnic shock analysis method using the traditional SEA modeling produced an excellent correlation with the modal transient analysis for the numerical validation model. Using the SRS as an energy constraint condition, the proposed method was able to estimate the maximum global response magnitudes in the time domain and produced response waveforms well correlated with the modal transient results. In addition, preliminary study indicated a good correlation with the test data measured from the complex validation model. Currently, the test data from the complex validation model are being investigated for each structural member to further validate the proposed method. The proposed method may be used to produce pyrotechnic shock design and test requirements for components and subsystems due to the SRS requirement at the system interface. The results could be further refined with response waveform predictions based on inverse transform of the product of the input acceleration and the TF of the SEA model. The response waveform may be used to estimate various engineering variables such as stress and

velocity, and could be used as an input acceleration to shock susceptible components for the component level modal transient analysis.

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BIOGRAPHY

Dr. James Ho-Jin Hwang has over 15 years of experience in the field of dynamic environments and currently works as a subject matter expert in the area of high frequency dynamics and environmental testing at MIT Lincoln Laboratory. Prior to joining MIT LL, Dr. Hwang was a technical lead at Space Exploration Technologies, Jet Propulsion Laboratory, and Raytheon Aircraft Company. Dr. Hwang's area of technical expertise includes simulation of high frequency dynamics using Statistical Energy Analysis (SEA) and environmental tests including high fidelity pyrotechnic shock tests and force limited random vibration tests of complex space structures. He holds a Ph.D. in Aerospace Engineering from the University of Kansas.