# Cooperative Interference Alignment for the Multiple Access Channel

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***Abstract*—Interference alignment (IA) has emerged as a promising technique for the interference channel that guarantees a constant degree-of-freedom (DoF) gain for each user. Although successive interference cancellation (SIC) is known to achieve the sum capacity of the MAC channel, the DoF allocation to the users depends on the decoding order, e.g., the first few users get zero DoF, while the last users being decoded see full DoF gain. For a large number of users $K$, randomization between decoding order introduces significant latency in the system. One the other hand, orthogonal access schemes, e.g., TDMA/FDMA, waste significant resources for large number of users. To combat these problems, we propose a novel technique for the $K$-user multiple access (MAC) channel with $N$ receivers, called Cooperative Interference Alignment (C-IA) and derive bounds on the DoF for each user. Assuming $\frac{K}{2} \leq N \leq K$, we show that C-IA is sum-DoF optimal, i.e, the sum rate scaling is as $R_\Sigma(SNR) = N \log_2(\mathrm{SNR}) + o(\log_2(\mathrm{SNR}))$ and simultaneously achieves a constant DoF-per-user, i.e., $R_k(\mathrm{SNR}) = d_k \log_2(\mathrm{SNR}) + o(\log_2(\mathrm{SNR}))$ where $d_k \in \{1/2, 1\}$, *without* randomization between different decoding strategies and *without* the use of orthogonal access schemes.**

***Index Terms*—Interference Alignment (IA), Successive Interference Cancellation (SIC), Degrees of Freedom (DoF), Linear Precoding, Cooperative Communications.**

## I. Introduction

Conventional wireless networks were previously thought to be interference-limited, where interference is mainly caused by mobile and base station signals. Interference suppression increases capacity, spectral efficiency and possibly coverage. In the seminal paper of Cadambe & Jafar [1], interference alignment (IA) was proposed as a communication technique for the interference channel and was shown to achieve half the interference-free capacity for any number of users $K$ at high SNR. This fundamental result showed that wireless networks are not interference-limited as long as the transmitters perform precoding of their signals to align the aggregate inter-user interference to half the signaling space at each receiver, and the desired signal in the remaining half of the space.

There has been prior work studying the impact of transmitter and receiver cooperation on the degrees of freedom of interference channels [2], [3]. In this paper, transmitter cooperation will not be considered, but full receiver cooperation will be. The receivers can be thought of as being connected through a backhaul link, thus being capable of joint decoding of the $K$ users' messages. This is applicable in uplink transmissions in cellular communications, where mobiles transmit independent messages to a base station. A similar problem was studied in [4], where a novel technique based on interference alignment and interference cancellation (IAC) with receiver cooperation was proposed to combat the antennas-per-access point throughput limit in MIMO LANs. Gollakota et. al [4] analytically showed that IAC almost doubles the throughput of MIMO LANs and also provided experimental validation. Related work includes the development of network MIMO or virtual MIMO, where multiple transmitters transmit signals simultaneously and receivers cooperate for joint decoding. In practice, this can be realized by utilizing a high-speed backhaul link connecting the receivers. In network MIMO, coordination of transmission and reception of signals at multiple access points allows for significantly higher spectral efficiencies [5], [6], [7].

Orthogonal access schemes, e.g., TDMA/FDMA, have low transmitter and receiver complexity and achieve a constant DoF $1/K$ per user by splitting the available time (or frequency) resources amongst the users. However, as the number of users grow the DoF per user vanishes. Methods where all users transmit information at the same time and frequency space also exist, but the receiver complexity significantly increases. While standard successive interference cancellation and decoding (SIC) achieves the sum capacity of the Gaussian MAC channel, this approach yields zero degrees of freedom for the first $K - N$ users being decoded and full degrees of freedom for the remaining users [8]. Randomization on the decoding order of the $K$ user signals can be used to guarantee a nonzero DoF on average, but increases latency significantly since there are $K!$ possible orderings. Furthermore, in practice, the first few user messages being decoded will most likely result in errors when all transmitters are strong, which through the process of successive interference cancellation cause artificial interference to the remaining users. In contrast, our approach first aligns the interference from other strong users in appropriately-designed subspaces and the desired user signal in the remaining subspace, thus leading to a fair allocation of degrees of freedom among users and does not suffer from decoding errors that may occur with high probability when decoding a strong user in the presence of multiuser interference.

In this paper, we propose a cooperative communication scheme based on IA and SIC that can recover the desired information with optimal sum-DoF and constant DoF per user

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without randomization and linear complexity scaling at the receiver as a function of $K$. We stress that while IA is used as an intermediate stage in our method, our technique and formulation is different from that of standard IA [1] for the interference channel.

## II. Problem Formulation

We consider the $K$-user multiple access channel with $N$ receivers as shown in Fig. 1. All nodes are equipped with a single antenna. The received signal at the $l$th receiver is modeled as:

$$y^{(l)}(t) = \sum_{j=1}^{K} h^{(l,j)}(t) x^{(j)}(t) + z^{(l)}(t) \quad (1)$$

where $h^{(l,j)}(t)$ is the channel fading coefficient between the $j$th transmitter and the $l$th receiver for the $t$th time slot. The term $x^{(j)}(t)$ denotes the transmitted signal from the $j$th user's transmit antenna. The term $z^{(l)}(t)$ denotes complex additive white Gaussian noise at the $l$th receiver, i.e., $z^{(l)}(t) \sim \mathcal{N}_c(0, \sigma^2)$. It is assumed that all noise terms are independent identically distributed zero-mean complex Gaussian random variables with unit variance. As in [1], we assume a fully connected network with all channels drawn i.i.d. from continuous distributions, and assume causal and globally available channel state information (CSI).

The communication problem is stated as follows. Each of the $K$ transmitters wants to communicate an independent message $\mathcal{M}_k$ to the receiver, where $k$ indexes users, over a block length $T$. It is assumed that $\mathcal{M}_k$ is uniformly distributed over a set with size $2^{TR_k(P)}$, where there is a power constraint $P$ at the transmitters. Let $\mathcal{H}^t = \{h^{(k,j)}(s), g^{(k,l)}(s) : 1 \le k \le K, 1 \le l \le L\}_{s=1}^{t}$ with $\mathcal{H}^0 =$ and $\mathcal{Y}^t = \{y^{(1)}(s), \ldots, y^{(N)}(s)\}_{s=1}^{t}$. Then, a coding scheme for block length $T$ consists of $K$ encoding functions $f_k^{(T)} = \{f_{k,t}^{(T)}\}_{t=1}^{T}, k = 1, \ldots, K$ such that

$$x^{(k)}(t) = f_{k,t}^{(T)}(\mathcal{M}_k, \mathcal{H}^t), t = 1, \ldots, T$$

where $\mathbb{E}[|x^{(k)}(t)|^2] \le P$, and a decoding function such that

$$\hat{\mathcal{M}} = (\hat{\mathcal{M}}_1, \ldots, \hat{\mathcal{M}}_K) = g^{(T)}(\mathcal{Y}^T, \mathcal{H}^T).$$

We write $|W_k(P)|$ to denote the size of the message set. For codewords covering $T$ channel uses, the rate $R_k(P) = \frac{\log |W_k(P)|}{T}$ is achievable for each user $k$ if the probability of error for all messages can be simultaneously made arbitrarily close to zero for large enough $T$. A rate tuple $(R_1(P), \ldots, R_K(P))$ is achievable if there exists a sequence of coding schemes such that $\mathbb{P}\left(\bigcup_{k=1}^{K} \{\mathcal{M}_k \ne \hat{\mathcal{M}}_k\}\right)$ converges to zero as $T \to \infty$. The capacity region $\mathcal{C}(P)$ of the multiple access channel is the set of all achievable rate tuples $\mathbf{R}(P) = (R_1(P), \ldots, R_K(P))$.

### A. Degrees-of-Freedom (DoF)

We adopt the same definition for the degrees of freedom region as in [1]. We define the degrees of freedom region $\mathcal{D}$

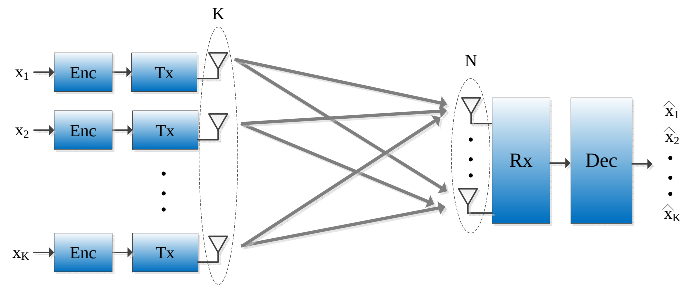


Fig. 1: Block diagram of $K$-user multiple access channel with $N$ receiver nodes.

for the $K$-user multiple access channel as follows:

$$\mathcal{D} = \Bigg\{ (d_1, \ldots, d_K) \in \mathbb{R}_+^K : \forall (w_1, \ldots, w_K) \in \mathbb{R}_+^K, \\ \sum_{k=1}^{K} w_k d_k \le \limsup_{P \to \infty} \left[ \frac{\sup_{\mathbf{R}(P) \in \mathcal{C}(P)} \sum_{k=1}^{K} w_k R_k(P)}{\log P} \right] \Bigg\}$$

It can be shown that the DoF region $\mathcal{D}$ is a convex [9] and closed [10] set. The total degrees of freedom (sum-DoF) is defined as $d_\Sigma := \max_{\mathcal{D}} \left\{ \sum_{k=1}^{K} d_k \right\}$.

## III. Multiple Access Channel (MAC) Capacity & Degrees of Freedom

The received signal at the $N$-antenna receiver is given by:

$$\mathbf{y} = \mathbf{H}\bar{\mathbf{x}} + \mathbf{z} \quad (2)$$

where $\mathbf{H}$ is the $N \times K$ channel matrix and $\mathbf{z}$ is complex white Gaussian noise with covariance $\sigma^2 \mathbf{I}_N$. The sum-capacity for the fast-fading Gaussian MAC channel is given by [8]:

$$C_{\text{MAC}} = \mathbb{E}\left[\log_2 \det\left(\mathbf{I}_N + P\mathbf{H}\mathbf{H}^* \mathbf{K}_{\mathbf{z}}^{-1}\right)\right] \quad (3)$$

We can rewrite the rate expression as $R_{MAC}(P) = \log_2 \det\left(\mathbf{I}_N + \frac{P}{\sigma^2}\mathbf{H}\mathbf{H}^*\right)$. Since $\text{rank}(\mathbf{H}\mathbf{H}^*) = N$ a.s., the sum-DoF available in the MAC channel can be calculated as $d_{\text{MAC}} = \lim_{P \to \infty} \frac{R_{MAC}(P)}{\log_2 P} = N$.

## IV. Cooperative Interference Alignment and Degrees of Freedom

Define $L \triangleq K - N$ as the dimension of the nullspace of the channel matrix $\mathbf{H} \in \mathbb{C}^{N \times K}$. In this section, we present the fundamental result of the paper assuming $L \le \frac{K}{2}$ [1].

**Theorem 1.** *The following bounds on the degrees of freedom (DoF) for the MAC channel are achievable with C-IA, given* $L \le \frac{K}{2}$,

$$d_\Sigma = \max_{\boldsymbol{d} \in \mathcal{D}} \sum_{k=1}^{K} d_k \ge N \quad (4)$$

*In addition, the DoF achievable per user satisfies*

$$\frac{1}{2} \le d_k \le 1, \forall k \quad (5)$$

[1]We conjecture that a similar method can be applied and performance guarantees can be derived for the case $L > \frac{K}{2}$, but this is beyond the scope of this paper.

The achievability proof is based on a cooperative interference alignment (C-IA) algorithm, under which the receivers cooperate to decode the $K$ information streams. This method consists of three main phases:

- Phase 1: Partial matrix diagonalization, via linear transformation with $\mathbf{R}_{pmd}$.
- Phase 2: Interference alignment design to recover $L$ source streams $\{x^{(k)}\}_{k=N+1}^{K}$.
- Phase 3: Successive interference cancellation to recover the remaining $N$ source streams $\{x^{(k)}\}_{k=1}^{N}$.

Let $M = M_n = (n+1)^D + n^D$. We show that the points $(d_1(n), \ldots, d_L(n), d_{L+1}(n), \ldots, d_N(n), d_{N+1}(n), \ldots, d_K(n)) = \left(\frac{(n+1)^D}{M_n}, \ldots, \frac{n^D}{M_n}, 1, \ldots, 1, \frac{(n+1)^D}{M_n}, \ldots, \frac{n^D}{M_n}\right)$ lies in the DoF region $\mathcal{D}$ for some $D \in \mathbb{N}$ (which depends on $L$ only). As the block length $n$ grows, this sequence converges to $(\frac{1}{2}, \ldots, \frac{1}{2}, 1, \ldots, 1, \frac{1}{2}, \ldots, \frac{1}{2})$, which implies that the bounds (4) and (5) hold, thus proving Theorem 1.

We consider communication over a symbol extension of the original channel as in [1]. Collecting the received signal at the $i$th receiver over $M = M_n$ time slots, we obtain from model (1):

$$\mathbf{y}^{(i)}(t) = \sum_{j=1}^{K} \bar{\mathbf{H}}^{(i,j)}(t) \underbrace{\mathbf{V}^{(j)}\mathbf{x}^{(j)}(t)}_{=\bar{\mathbf{x}}^{(j)}(t)} + \mathbf{z}^{(i)}(t) \qquad (6)$$

where the channel matrices corresponding to $M$ symbol extensions are $\bar{\mathbf{H}}^{(i,j)}(t) = \mathrm{diag}\left(\left\{h^{(i,j)}((t-1)M+l)\right\}_{l=1}^{M}\right)$. The signals at the $N$ receivers can be collected for receiver processing in a vector $\mathbf{y} \in \mathbb{C}^{NM}$ given by:

$$\mathbf{y}(t) \triangleq \begin{bmatrix} \mathbf{y}^{(1)}(t) \\ \vdots \\ \mathbf{y}^{(N)}(t) \end{bmatrix} = \bar{\mathbf{H}}(t)\bar{\mathbf{x}}(t) + \mathbf{z}(t) \qquad (7)$$

where $\bar{\mathbf{x}}(t) = \left[\bar{\mathbf{x}}^{(1)}(t)^T, \ldots, \bar{\mathbf{x}}^{(K)}(t)^T\right]^T$, $\mathbf{z}(t) = \left[\mathbf{z}^{(1)}(t)^T, \ldots, \mathbf{z}^{(K)}(t)^T\right]^T$ and the channel matrices are

$$\bar{\mathbf{H}}(t) = \begin{bmatrix} \bar{\mathbf{H}}^{(1,1)}(t) & \ldots & \bar{\mathbf{H}}^{(1,K)}(t) \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{H}}^{(N,1)}(t) & \ldots & \bar{\mathbf{H}}^{(N,K)}(t) \end{bmatrix}$$

To simplify the notation, we often suppress the time index $t$ in the sequel.

### A. Phase 1: Partial Matrix Diagonalization

Using a linear transformation $\mathbf{R}_{pmd} \in \mathbb{C}^{NM \times NM}$, $\mathbf{y}$ in (7) can be expressed in a different basis as

$$\check{\mathbf{y}} = \mathbf{R}_{pmd}\mathbf{y}$$

$$\equiv \left[\begin{array}{ccc|ccc} \mathbf{T}_1 & \ldots & \mathbf{0}_M & \mathbf{G}^{(1,1)} & \ldots & \mathbf{G}^{(1,L)} \\ \mathbf{0}_M & \ldots & \mathbf{0}_M & \mathbf{G}^{(2,1)} & \ldots & \mathbf{G}^{(2,L)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_M & \ldots & \mathbf{T}_N & \mathbf{G}^{(N,1)} & \ldots & \mathbf{G}^{(N,L)} \end{array}\right] \left[\begin{array}{c} \bar{\mathbf{x}}^{(1)} \\ \vdots \\ \bar{\mathbf{x}}^{(N)} \\ \hline \bar{\mathbf{x}}^{(N+1)} \\ \vdots \\ \bar{\mathbf{x}}^{(K)} \end{array}\right] \qquad (8)$$

corresponding to a matrix-type diagonalized form. In fact, $\mathbf{R}_{pmd}$ can be chosen such that $\mathbf{T}_m = \mathbf{T}$ for all $m = 1, \ldots, N$. One choice of $\mathbf{R}_{pmd}$ that leads to partial matrix diagonalization in (8) is given by partial channel matrix inversion. Consider the partitioning of the channel matrix $\mathbf{H}$ given by $\mathbf{H} = [\mathbf{H}_1|\mathbf{H}_2]$, where $\mathbf{H}_1$ has size $NM \times NM$, and choose $\mathbf{R}_{pmd} = (\mathbf{H}_1)^{-1}$. Then, we obtain

$$\check{\mathbf{y}} = \mathbf{R}_{pmd}\mathbf{y} = [\mathbf{I}_{NM}|(\mathbf{H}_1)^{-1}\mathbf{H}_2]\bar{\mathbf{x}} + \check{\mathbf{z}}$$

where $\check{\mathbf{z}} = (\mathbf{H}_1)^{-1}\mathbf{z}$. Thus, with this choice $\mathbf{T}_m = \mathbf{I}_M$ for all $m = 1, \ldots, N$. Recursive algorithms for generating $\mathbf{R}_{pmd}$ can be obtained by implementing the matrix inverse $(\tilde{\mathbf{H}}_1)^{-1}$ by exploiting its structure. For instance, in our case, each $M \times M$ submatrix of $\mathbf{H}$ is a diagonal matrix with nonzero elements almost surely, implying that the matrix $\mathbf{R}_{pmd} = (\mathbf{H}_1)^{-1}$ will also have the same structure.

### B. Phase 2: Interference Alignment for Recovery of L Source Streams

Each subvector of $\check{\mathbf{y}}$ in (8) takes the following form:

$$\check{\mathbf{y}}^{(k)} \equiv \mathbf{T}_k\mathbf{V}^{(k)}\mathbf{x}^{(k)} + \sum_{l=1}^{L} \mathbf{G}^{(k,l)}\tilde{\mathbf{V}}^{(l)}\mathbf{x}^{(N+l)}, k = 1, \ldots, N$$

where we suppress the additive noise term using '$\equiv$', and we re-defined $\mathbf{V}^{(N+l)} =: \tilde{\mathbf{V}}^{(l)}$. Thus, each subvector contains only interference from $L$ source streams $\{\mathbf{x}^{(k)}\}_{k=N+1}^{K}$. The task of this phase is to use interference alignment to recover these $L$ source streams. The precoding matrix $\tilde{\mathbf{V}}^{(1)} \in \mathbb{C}^{M \times (n+1)^D}$ will serve as a pivot, the precoding matrices $\{\tilde{\mathbf{V}}^{(l)}\}_{l=2}^{L}$ are of size $M \times n^D$, $\{\mathbf{V}^{(k)}\}_{k=1}^{L-1}$ are of size $M \times (n+1)^D$, $\mathbf{V}^{(L)}$ is $M \times n^D$, and $\{\mathbf{V}^{(k)}\}_{k=L+1}^{D}$ are of size $M \times M$. Define $\Delta = (n+1)^D - n^D$.

We will sequentially decode the $L$ streams $\mathbf{x}^{(K)}, \ldots, \mathbf{x}^{(N+1)}$, and after we decode each stream, we subtract its interference from all $\check{\mathbf{y}}^{(k)}$ to increase capacity. This interference subtraction step simplifies the interference alignment design by reducing the number of constraints that need to be simultaneously satisfied. Let $S_i$ denote the index of the variable being recovered, i.e., if $\mathbf{x}_{(N+l)}$ is being recovered at stage $i$, then $S_i = \{l\}$.

We start this process from $k = 1$ and aim to decode $\mathbf{x}^{(K)} = \mathbf{x}^{(N+L)} \in \mathbb{C}^{(n+1)^N}$ by aligning all interference in a subspace of dimension $(n+1)^D$ which is linearly independent from the subspace of dimension $n^D$ that $\mathbf{x}^{(N+L)}$ lives in. This allows a simple linear receiver, e.g. zero-forcing, to be used to decode $\mathbf{x}^{(N+L)}$ with DoF $d_K = d_{N+L} = \frac{n^D}{M}$. In the first stage, we decode $\mathbf{x}^{(N+L)}$, so $S_1 = \{L\}$.

$$\check{\mathbf{y}}^{(1)}(1) := \check{\mathbf{y}}^{(1)} = \mathbf{G}^{(1,L)}\tilde{\mathbf{V}}^{(L)}\mathbf{x}^{(N+L)} + \left\{\mathbf{T}_1\mathbf{V}^{(1)}\mathbf{x}^{(1)} + \sum_{\substack{l'=1 \\ l' \notin S_1}}^{L} \mathbf{G}^{(1,l')}\tilde{\mathbf{V}}^{(l')}\mathbf{x}^{(N+l')}\right\}$$

To align the signals in the brackets into the same subspace of dimension $(n+1)^D$, we require the following set of constraints:

$$\mathbf{T}_1\mathbf{V}^{(1)} = \mathbf{G}^{(1,1)}\tilde{\mathbf{V}}^{(1)} = \cdots = \mathbf{G}^{(1,L-1)}\tilde{\mathbf{V}}^{(L-1)} \qquad (9)$$

Pivot Column

$$\begin{bmatrix} \check{\mathbf{y}}^{(1)} \\ \check{\mathbf{y}}^{(2)} \\ \vdots \\ \check{\mathbf{y}}^{(L-1)} \\ \check{\mathbf{y}}^{(L)} \\ \hline \check{\mathbf{y}}^{(L+1)} \\ \vdots \\ \check{\mathbf{y}}^{(N)} \end{bmatrix} \equiv \left[\begin{array}{cccccccc|ccccc} \mathbf{T}_1 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{G}^{(1,1)} & \mathbf{G}^{(1,2)} & \dots & \mathbf{G}^{(1,L-1)} & \mathbf{G}^{(1,L)} \\ \mathbf{0} & \mathbf{T}_2 & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{G}^{(2,1)} & \mathbf{G}^{(2,2)} & \dots & \mathbf{G}^{(2,L-1)} & \mathbf{G}^{(2,L)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{T}_{L-1} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{G}^{(L-1,1)} & \mathbf{G}^{(L-1,2)} & \dots & \mathbf{G}^{(L-1,L-1)} & \mathbf{G}^{(L-1,L)} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{T}_L & \mathbf{0} & \dots & \mathbf{0} & \mathbf{G}^{(L,1)} & \mathbf{G}^{(L,2)} & \dots & \mathbf{G}^{(L,L-1)} & \mathbf{G}^{(L,L)} \\ \hline \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{T}_{L+1} & \dots & \mathbf{0} & \mathbf{G}^{(L+1,1)} & \mathbf{G}^{(L+1,2)} & \dots & \mathbf{G}^{(L+1,L-1)} & \mathbf{G}^{(L+1,L)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{T}_N & \mathbf{G}^{(N,1)} & \mathbf{G}^{(N,2)} & \dots & \mathbf{G}^{(N,L-1)} & \mathbf{G}^{(N,L)} \end{array}\right] \begin{bmatrix} \bar{\mathbf{x}}^{(1)} \\ \bar{\mathbf{x}}^{(2)} \\ \vdots \\ \bar{\mathbf{x}}^{(N)} \\ \hline \bar{\mathbf{x}}^{(N+1)} \\ \vdots \\ \bar{\mathbf{x}}^{(K)} \end{bmatrix}$$

Fig. 2: Interference Alignment Phase of C-IA algorithm. The symbol streams $\mathbf{x}^{(K)}, \dots, \mathbf{x}^{(N+1)}$ are sequentially decoded from $\check{\mathbf{y}}^{(1)}, \dots, \check{\mathbf{y}}^{(L)}$ respectively by aligning the interference (magenta) into one subspace that is linearly independent from the desired signal (blue) subspace. After decoding each stream, its decoded interference is subtracted from all equations as shown in the crossed-out regions. In each equation, all the variables corresponding to the magenta effective channel matrices are aligned in interference subspace when decoding the variable associated with the blue effective channel matrix.

We note that since $\mathbf{V}^{(1)}$ only appears in the subvector $\check{\mathbf{y}}^{(1)}$ and not on any other subvectors $\{\check{\mathbf{y}}^{(l)}\}_{l\neq 1}$, so the design of the precoding matrix $\mathbf{V}^{(1)}$ is decoupled from the design of $\{\tilde{\mathbf{V}}^{(l)}\}_{l=1}^{L}$. It can be computed as $\mathbf{V}^{(1)} = \mathbf{T}_1^{-1}\mathbf{G}^{(1,1)}\tilde{\mathbf{V}}^{(1)}$. Once (9) is satisfied, we can rewrite:

$$\check{\mathbf{y}}^{(1)} \equiv \mathbf{G}^{(1,L)}\tilde{\mathbf{V}}^{(L)}\mathbf{x}^{(N+1)} + \mathbf{T}_1\mathbf{V}^{(1)}\left(\mathbf{x}^{(1)} + \sum_{\substack{l=1\\ l\notin S_1}}^{L} \mathbf{x}^{(N+l)}\right)$$

Assuming the subspaces $\text{span}(\mathbf{G}^{(1,L)}\tilde{\mathbf{V}}^{(L)})$ and $\text{span}(\mathbf{T}_1\mathbf{V}^{(1)})$ are linearly independent a.s., then $\mathbf{x}^{(K)}$ can be decoded with DoF $d_K = \frac{n^D}{M}$ using a linear receiver that nulls out the interference.

Next, subtract the decoded interference associated with $\mathbf{x}^{(K)}$ from all $\check{\mathbf{y}}^{(k)}(1)$ to form $\check{\mathbf{y}}^{(k)}(2)$, and consider $k = 2$. To decode $\mathbf{x}^{(K-1)} = \mathbf{x}^{(N+L-1)} \in \mathbb{C}^{n^D}$ via a linear receiver, let $S_2 = \{L-1\}$ and write:

$$\begin{aligned} \check{\mathbf{y}}^{(2)}(2) &= \check{\mathbf{y}}^{(2)}(1) - \mathbf{G}^{(2,L)}\tilde{\mathbf{V}}^{(L)}\mathbf{x}^{(N+L)} \\ &\equiv \mathbf{T}_2\mathbf{V}^{(2)}\mathbf{x}^{(2)} + \sum_{\substack{l=1\\ l\notin S_1}}^{L} \mathbf{G}^{(2,l)}\tilde{\mathbf{V}}^{(l)}\mathbf{x}^{(N+l)} \\ &= \mathbf{G}^{(2,L-1)}\tilde{\mathbf{V}}^{(L-1)}\mathbf{x}^{(N+L-1)} \\ &\quad + \left\{\mathbf{T}_2\mathbf{V}^{(2)}\mathbf{x}^{(2)} + \sum_{\substack{l=1\\ l\notin S_1\cup S_2}}^{L} \mathbf{G}^{(2,l)}\tilde{\mathbf{V}}^{(l)}\mathbf{x}^{(N+l)}\right\} \end{aligned}$$

To align the signals in the brackets above into the same interference subspace of dimension $(n+1)^D$, we require:

$$\begin{aligned} \text{span}(\mathbf{G}^{(2,l)}\tilde{\mathbf{V}}^{(l)}) &\subseteq \text{span}(\mathbf{G}^{(2,1)}\tilde{\mathbf{V}}^{(1)}), \\ &\qquad \forall l \in \{1,\dots,L\}\backslash\{S_1 \cup S_2\} \\ \mathbf{T}_2\mathbf{V}^{(2)} &= \mathbf{G}^{(2,1)}\tilde{\mathbf{V}}^{(1)} \end{aligned}$$

Proceeding sequentially, for the $k$th stage we aim to decode $\mathbf{x}^{(K-k+1)}$ and we obtain:

$$\begin{aligned} \check{\mathbf{y}}^{(k)}(k) &= \check{\mathbf{y}}^{(k)}(k-1) - \sum_{\substack{l=1\\ l\in\bigcup_{i=1}^{n-1} S_i}}^{L} \mathbf{G}^{(k,l)}\tilde{\mathbf{V}}^{(l)}\mathbf{x}^{(N+l)} \\ &\equiv \mathbf{T}_k\mathbf{V}_k\mathbf{x}^{(k)} + \sum_{\substack{l=1\\ l\notin\bigcup_{i=1}^{n-1} S_i}}^{L} \mathbf{G}^{(k,l)}\tilde{\mathbf{V}}^{(l)}\mathbf{x}^{(N+l)} \\ &= \mathbf{G}^{(k,S_k)}\tilde{\mathbf{V}}^{(S_k)}\mathbf{x}^{(N+S_k)} \\ &\quad + \left\{\mathbf{T}_k\mathbf{V}_k\mathbf{x}^{(k)} + \sum_{\substack{l=1\\ l\notin\bigcup_{i=1}^{n} S_i}}^{L} \mathbf{G}^{(k,l)}\tilde{\mathbf{V}}^{(l)}\mathbf{x}^{(N+l)}\right\} \end{aligned}$$

where $S_k = \{L-k+1\}$. To align the signals in the brackets above in the same interference subspace we require:

$$\begin{aligned} \text{span}(\mathbf{G}^{(k,l)}\tilde{\mathbf{V}}^{(l)}) &\subseteq \text{span}(\mathbf{G}^{(k,1)}\tilde{\mathbf{V}}^{(1)}), \\ &\qquad \forall l \in \{1,\dots,L\}\backslash\left\{\bigcup_{i=1}^{k} S_i\right\} \\ \text{span}(\mathbf{T}_k\mathbf{V}^{(k)}) &\subseteq \text{span}(\mathbf{G}^{(k,1)}\tilde{\mathbf{V}}^{(1)}) \end{aligned}$$

In general, the interference alignment (IA) constraints can be summarized as:

For each $k = 1,\dots,L-2$:

$$\text{span}(\mathbf{G}^{(k,l)}\tilde{\mathbf{V}}^{(l)}) \subseteq \text{span}(\mathbf{G}^{(k,1)}\tilde{\mathbf{V}}^{(1)}), \forall l \in \{2,\dots,L-k\}. \tag{10}$$

We can reformulate the constraints by first simplifying the constraints involved for $k = 1$. We set

$$\mathbf{G}^{(1,2)}\tilde{\mathbf{V}}^{(2)} = \mathbf{G}^{(1,3)}\tilde{\mathbf{V}}^{(3)} = \dots = \mathbf{G}^{(1,L-1)}\tilde{\mathbf{V}}^{(L-1)} \tag{11}$$

where all these matrices have size $M \times n^D$. Once $\tilde{\mathbf{V}}^{(2)}$ is obtained, we obtain the remaining precoders via (11):

$$\tilde{\mathbf{V}}^{(l)} = (\mathbf{G}^{(1,l)})^{-1}\mathbf{G}^{(1,2)}\tilde{\mathbf{V}}^{(2)} \tag{12}$$

Plugging (12) into (10), we reformulate the IA constraints as:

For each $k = 1, \ldots, L-2$:

$$\text{span}(\mathbf{T}_{k,l}\tilde{\mathbf{V}}^{(2)}) \subseteq \text{span}(\tilde{\mathbf{V}}^{(1)}), \forall l \in \{2, \ldots, L-k\}. \quad (13)$$

where $\mathbf{T}_{k,l} := (\mathbf{G}^{(k,1)})^{-1}\mathbf{G}^{(k,l)}(\mathbf{G}^{(1,l)})^{-1}\mathbf{G}^{(1,2)}$. According to (13), there are $D+1 = \frac{(L-2)(L-1)}{2}$ IA constraints that need to be satisfied. Thus, the complexity scaling is only $\Theta(\frac{L^2}{2})$. This is a consequence of the partial matrix diagonalization phase of the C-IA method; otherwise the IA complexity scaling would be $\Theta(KN)$, which can be quite significant for a large number of users $K$.

Similarly as in Cadambe and Jafar [1], we may choose the columns of $\tilde{\mathbf{V}}^{(2)}$ and $\tilde{\mathbf{V}}^{(1)}$ using a product-based construction, i.e., from the sets $\mathcal{A}_{n-1} = \left\{\left(\prod_{(k,l)\in\Gamma_L}(\mathbf{T}_{k,l})^{\alpha_{k,l}}\right)\mathbf{w} : \alpha_{k,l} \in \{0, 1, \ldots, n-1\}\right\}$ and $\mathcal{A}_n = \left\{\left(\prod_{(k,l)\in\Gamma_L}(\mathbf{T}_{k,l})^{\alpha_{k,l}}\right)\mathbf{w} : \alpha_{k,l} \in \{0, 1, \ldots, n\}\right\}$, respectively, where $\Gamma_L := \{(k,l) : k = 1, \ldots, L-2, l = 2, \ldots, L-k\}$ and $\mathbf{w} = \mathbf{1} \in \mathbb{C}^M$ is the all-one vector. Note that the cardinality of the index set $\Gamma_L$ is $|\Gamma_L| = D+1$. We remark that $L > 2$ is implicitly assumed here in order to have the index set $\Gamma_L$ non-empty. For $L = 1$ or 2, the IA conditions become trivial.

Once the precoding matrices $\{\mathbf{V}^{(k)}\}_{k=N+1}^{K} = \{\tilde{\mathbf{V}}^{(l)}\}_{l=1}^{L}$ are chosen to satisfy the constraints (13) (and equivalently (10)), the remaining precoding matrices $\{\mathbf{V}^{(k)}\}_{k=1}^{N}$ can be obtained through a simple inversion:

$$\begin{aligned}
&\mathbf{V}^{(k)} = \mathbf{T}_k^{-1}\mathbf{G}^{(k,1)}\tilde{\mathbf{V}}^{(1)}, k = 1, \ldots, L-1\\
&\mathbf{V}^{(L)} = \mathbf{T}_L^{-1}\mathbf{G}^{(L,2)}\tilde{\mathbf{V}}^{(2)}\\
&\mathbf{V}^{(k)} = \text{i.i.d. random } M \times M \text{ matrix}, k = L+1, \ldots, N
\end{aligned}$$

Linear independence of the signal and interference subspaces can be guaranteed using the arguments presented in [1]. Thus, the signal streams $\{\mathbf{x}^{(k)}\}_{k=N+1}^{K}$ can be completely decoded by zero-forcing the interference with the following DoF guarantees:

$$d_K(n) = \cdots = d_{N+2}(n) = \frac{n^D}{M}, \quad d_{N+1}(n) = \frac{(n+1)^D}{M} \quad (14)$$

### C. Phase 3: Successive Interference Cancellation for Recovery of $K-L$ Source Streams

After subtracting the decoded interference corresponding to the symbol streams $\{\mathbf{x}^{(k)}\}_{k=N+1}^{K}$ from $\{\check{\mathbf{y}}^{(k)}\}_{k=1}^{N}$ (see (8)), we obtain for $k = 1, \ldots, N$:

$$\check{\mathbf{y}}^{(k)} - \sum_{l=1}^{L}\mathbf{G}^{(k,l)}\mathbf{V}^{(N+l)}\mathbf{x}^{(N+l)} \equiv \mathbf{T}_k\mathbf{V}^{(k)}\mathbf{x}^{(k)}$$

Since the elements of the diagonal matrix $\mathbf{T}_k$ are nonzero almost surely, we can decode $\{\mathbf{x}^{(k)}\}_{k=1}^{N}$ with DoF equal to $\frac{\text{rank}(\mathbf{V}^{(k)})}{M}$. Recalling the dimensions of the precoders, this implies that

$$d_1(n) = \cdots = d_{L-1}(n) = \frac{(n+1)^D}{M}, \quad d_L(n) = \frac{n^D}{M} \quad (15)$$

Combining (14) with (15), it follows that $d_l(n) + d_{K-l+1}(n) = 1$ for $l = 1, \ldots, L$. Furthermore, we have $d_{L+1}(n) = \cdots = d_N(n) = 1$. This implies $\max_{d\in\mathcal{D}}\sum_{k=1}^{K} d_k \geq K-L$ and $1 \geq \lim_{n\to\infty} d_k(n) \geq \lim_{n\to\infty}\frac{n^D}{(n+1)^D+n^D} = \frac{1}{2}$ for all $k$. Thus, the DoF bounds in (4) and (5) hold. This phase concludes the C-IA decoding algorithm.

## V. Conclusion

We propose a novel communication technique for the MAC channel, cooperative interference alignment (C-IA), which combines the ideas of interference alignment and successive interference cancellation. We prove that C-IA is sum-DoF optimal and derive bounds on the per-user DoF gain, showing that under mild conditions, nonzero DoF are achievable for all users simultaneously. An interesting open problem is extending the DoF analysis for delayed CSI.

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