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Composite Structure with Origami Core

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This project is aimed to create lightweight sandwich plates and shells with an origami core. These structures are similar to commonly used honeycomb sandwich structures except that the cores are replaced by a corrugated structure generated by folding sheet materials using origami techniques. The most noticeable advantages of the origami cores are that, first of all, they have an open cell design which eliminates humidity inclusion in the closed cells of a honeycomb; secondly, they can be formed to naturally fit even doubly curved shapes without distortion; and last but not least, their geometry can be designed to obtained the required structural performances. Through extensive analysis and experiments, we have successfully demonstrated that a suitable selection of core geometries can lead to significant increases in the structural performance, making such composites ideal for aircraft structures. Our research has paved the way for much broader utilization of such structures in aeronautics and aerospace industries.

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COMPOSITE STRUCTURE WITH ORIGAMI CORE

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Summary:

This project is aimed to create lightweight sandwich plates and shells with an origami core. These structures are similar to commonly used honeycomb sandwich structures except that the cores are replaced by a corrugated structure generated by folding sheet materials using origami techniques. The most noticeable advantages of the origami cores are that, first of all, they have an open cell design which eliminates humidity inclusion in the closed cells of a honeycomb; secondly, they can be formed to naturally fit even doubly curved shapes without distortion; and last but not least, their geometry can be designed to obtained the required structural performances. Through extensive analysis and experiments, we have successfully demonstrated that a suitable selection of core geometries can lead to significant increases in the structural performance, making such composites ideal for aircraft structures. Our research has paved the way for much broader utilization of such structures in aeronautics and aerospace industries.

Introduction:

Lightweight composites are widely used in aeronautics and aerospace industries. Among them, the composite plates and shells with a honeycomb core are most common because they exhibit the highest stiffness-to-density ratio. However, the geometry of honeycomb results in some inherited disadvantages including humidity inclusion in the closed cells and cell distortion when used to form curved shells, leading to a certain amount of deterioration in structural properties. A recent advance in research of lightweight composites is to use a core structure known as the foldcore to replace the honeycomb. The foldcore is essentially formed by folding a sheet material using origami. The foldcore has gained recognition due to its open channel design, abundant number of design parameters, so that it can be designed to meet the performance requirements, and its ability to naturally form curved profiles by rotating panels with respect to pre-determined creases.

In spite of the advantages listed above, the past research on the foldcore focused primarily on plate structures formed by a simple origami pattern known as the 'Miura-ori'. The reason for this is because the standard Miura-ori is known to fold into a flat profile (ideal as core of a composite plate) without bending or stretching the panels surrounding by the creases. Little work was done on formation of other profiles using the modified Miura-ori, or other origami patterns, let alone a detailed assessment of their structural performances and manufacture route.

In this project, we set to address these issues. We first treat the creation of a foldcore as a rigid origami process (the panels surrounding by creases are not allowed to bend or stretch except free rotation about pre-determined creases) to ensure that the formation of the origami core induces neither residual stresses nor deformation in the panels in order to maintain material property. To achieve this goal, we establish a precise analytical kinematic model that governs the formation of the core structures. Then we use these cores to build sandwich structures of both flat and curved profiles. When doing so, we ensure that these structures have a stiffness comparable to that of a honeycomb counterpart. Furthermore, we utilize the abundant design parameters of the origami patterns to optimize their structural performances. One of the key performance indicators being considered in this study is the energy absorption capability when the structures are subjected to impact. Through extensive numerical simulations, we obtain composite structures with mechanical performances that surpass that of the honeycomb counterpart. Then we manufacture prototypes and experimentally tested them to validate the analysis.

Through this rigorous approach, we have successfully demonstrated that the foldcore sandwich structures are more suitable for structures with a curved profile, and they can be designed to achieve the same stiffness over density ratio as that of honeycomb core, but with superior energy absorption capability over the honeycomb composites.

Our findings have led to a significant number of research articles in prestigious peer-reviewed journals. One of the highlights is a research paper in Science (*Chen et. al. 2015, also attached in Appendix a*). This work was widely reported in public media such as Forbes and Nature (*see Appendices b and c*). We hope that our work can serve as a catalyst for the uptake of origami structures by the aeronautics and aerospace industries.

Theory, Analysis and Experiment:

<u>Kinematic modelling and shape forming</u>

While forming the foldcores, we believe that it is most important to keep the material properties unaffected by the residual stresses or deformation during the forming process. This is achieved by making sure that folding from a sheet material to a core structure is rigid origami. We accomplished this using thorough and precise kinematic modelling. Cases where the sheet had both negligible thickness and finite thickness were considered. In the first case, we modelled the origami as a set of inter-linked spherical linkages using the mechanism theory. Precise motions of origami were identified. In the second case, we identified a link between thick panel origami and spatial linkages, and subsequently imposed kinematic relationships upon the thick panel folding. It was discovered that the motions in two cases could be made kinematically equivalent, i.e., they could have the identical motion paths provided certain geometrical designs were adopted (*Chen et. al. 2015, Chen et. al. 2016*). The significance of this approach is that it enables thick sheets to be folded in precisely the same manner as that for the thin sheets. Thus a vast number of origami patterns, which are primarily designed for a paper of zero thickness, can be used to form core structures. More importantly, the approach can also be applied to fold other rigid thick structures such as roof and solar panels in space.

Shape forming

The Miura-ori is one of the simplest and most amazing origami pattern. Its original form consists of a tessellation of parallelograms allowing a sheet to be folded into a structure with a flat profile. We obtained a set of modified Miura-ori that folds a flat sheet into a cylindrical profile (*Gattas et. al. 2013*). A design method was proposed in which the geometrical parameters of the Miura-ori could be adjusted so that it folds to a given cylindrical shape (*Zhou et. al. 2015a*). For other shapes such as the doubly curved and conical ones, we showed that they could also be formed by trimming the Miura-ori, resulting in a pattern that is flat-foldable (it could be compactly folded flat, but not developable). We applied this approach to produce cores with conical (*Gattas and You, 2015*) and hemispherical profiles (*Gattas et al. 2013*, *Moss et al. 2016*). Moreover, adopting piece-wise crease lines, we extended the Miura-ori to generate patterns with curved creases (*Ho and You 2015, Gattas and You 2014b*). The folding of all of these patterns is completely rigid origami.

It is also known that other patterns can be used to generate cores with profiles other than those given above. Our focus here was on the tubular core structure folded from the waterbomb pattern. The precise rigid origami motion of this pattern was worked out (*Chen et. al. 2016*, *Ma et. al. 2016, manuscript in preparation*). Hence, we are able to produce foldcores in any given shape.

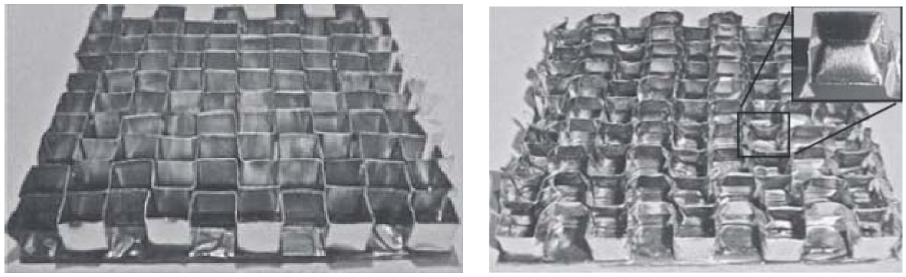
Mechanical behaviour of sandwich shells with foldcores

Equipped with the shape forming technique, we designed, built and tested a large number of sandwich prototypes using various materials and formation processes (*Gattas and You 2015*, *Zhou et. al. 2015b*, Zang et. al. 2016). The geometrical parameters of the patterns were optimised with the objective to obtain structures with the highest energy absorption capability when subjected to an impact. We chose this particular structural performance target to test the structures' suitability as an alternative to honeycomb based sandwich structures.

Fig. 1. A core made from the Miura-ori with curved creases.

In general, our sandwich structures have better energy absorption performance than that of sandwich structures with a honeycomb core, especially when straight creases were replaced by curved ones, Fig. 1. An ideal energy absorption structure has to have good load uniformity, i.e., there is no sudden drop of load resistance ability after first failure. The curved creases act as stiffeners to prevent local buckling from occurring after first failure, which in turn, maintains the load resistant ability (*Gattas and You 2014b*). We also examined sandwich shells made from foldcores other than the Miura-ori and found that an appropriate design of core structures could greatly influence the energy absorption capability. For example, a core formed with a pre-folded diamond strip, Fig. 2, could increase the energy absorption ability per unit weight by 23% and 92%, respectively, in comparison with a simple cube core and standard Miura-type foldcore (*Fathers et. al. 2015*).

The structural performance results were obtained by numerical simulation, validated by experiments. All experiments were conducted under quasi-static impact loadings using a standard Instron test machine. The test results matched those of numerical simulation. As predicted in the simulation, the strain rate effect was insignificant for quasi-static loadings.

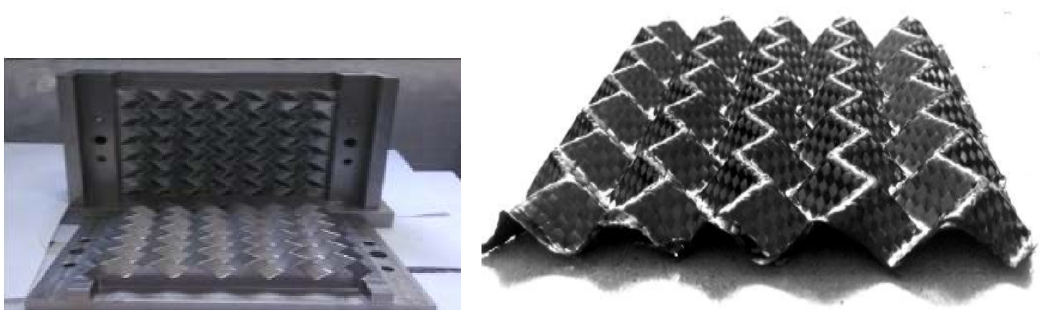


(a) (b)

Fig. 2. (a) A conventional cube core and (b) a pre-folded cube core.

<u>Manufacture of foldcores</u>

A number of ways to effectively manufacturing the foldcores were explored, including stamping, thermal forming, vacuum forming and digital fabrication using a CNC router. Sheet materials ranging from aluminum, polymer, peek, CFRP to Kevlar fiber reinforced composite were used. The straightforward stamping is best suited for making small specimen with metal sheet, but it could lead to significant residual stresses (though this can be dealt with by an annealing process) and cannot be applied to large samples. Thermal forming (hot press with a pair of moulds) is better suited for polymers and reinforced plastics, Fig. 3. Experiments on the resulting prototypes yielded results that matched those of numerical simulations, indicating that the residual stresses were at a level that did not influence the performance of the prototypes (*Jiang et. al. 2015*).



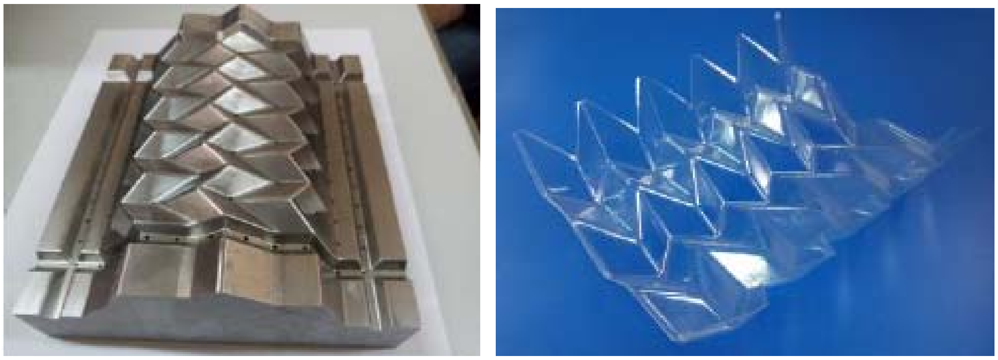
(a) (b)

Fig. 3. (a) Hot pressing moulds, and (b) CFRP foldcore made from it.

The best forming method is the vacuum forming because this method requires only one mould, allowing the sheet material to shrink into the mould shape without significantly stretching it, Fig. 4. We successfully applied this method to form foldcores with both flat and curved profiles (*Zhao et. al. 2015*, *Wang et. al. 2015*). The residual stresses were insignificant and the method seemed to be suitable for making large samples.

It is important to point out that in all manufacture routes above, no unexpected winkling was observed. This was because all the foldcores produced were based a rigid origami patterns that were developable, and thus they could be folded from a flat sheet. This reinforces our belief that foldcores must be designed using rigid origami patterns.

Finally, we should stress that digital fabrication using a CNC router is also an effective way to make origami structures, but it is best for making large scale structures (*Gattas and You 2016*).



(a) (b)

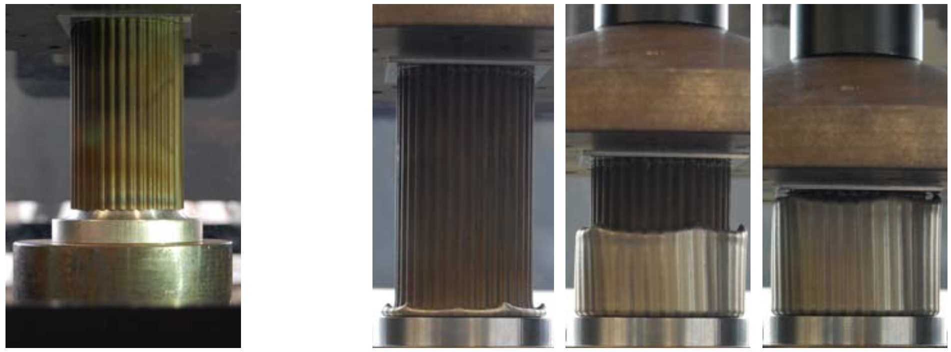
Fig. 4. (a) Vacuum forming mould, and (b) a core with a cylindrical profile produced using the vacuum forming.

Extended work into other origami structures

In addition to sandwich structures with foldcores, we also examined the possibility of utilizing origami to design standalone energy absorption thin-walled structures. We examined the performances of two types of tubes under quasi-static axial impact. One family of tubes adopted a multi-angled cross-section with 90° corners, Fig. 5. The large number of corners increases significantly the energy absorbed when the structure fails because of a noticeable increase of plastic hinge lines. The other family of tubes has a corrugated circular section, Fig. 6. An inducer was placed at the top of the tube when being pressed in order to invert the tube inside out. For a tube with circular section, this particular mode of failure has been proven to consume the largest amount energy. However, the failure mode only occur for relative thick tubes (larger thickness over diameter ratio) as the buckling mode will appear prior to the inversion mode for thinner tubes. The introduction of corrugation overcomes this problem. The presence of the corrugation prevents the buckling failure from occurring even when the tube is made from very thin ductile sheet material. We observed in both our simulation and experiments that, with the aid of an inducer, the tube inverted inside out under axial compression. This inversion process required significant amount of energy, making it one of the best device for the purpose of energy absorption.

(a) (b)

Fig. 5. (a) A tube with corrugated cross-section with straight corners; and (b) its failure mode under axial loading.



(a) (b)

Fig. 6. (a) A corrugated tube; and (b) its inversion under axial compression.

Results and Discussion:

The completion of this project has led to a number of significant findings, including a thorough and precise model for kinematic behavior of foldcores made from both thin and thick sheets, a clear approach to design foldcores of any shape, and a number of tried and tested manufacture routes. We have proven through examples that origami can provide good solutions to many engineering problems. The execution of the project has led us to the following conclusions.

- The sandwich structures with foldcores stand out as a good alternative to existing lightweight structures. They can be advantageous over other sandwich structures not only because they have an open cell design, but also due to the fact that the patterns have sufficient number of geometrical parameters that can be optimized to hit required structural performance target.
- The foldcores can be manufactured accurately without residual stresses, provided they are designed using a rigid origami patterns. The formation (motions) of the cores can be analytically modelled for both thin and thick sheet materials.
- Foldcores of any shape may be formed with a suitably chosen tessellation rigid origami pattern. The pattern ensures even distribution of tessellation units without distortion.
- Even for conventional thin-walled structures, origami folds can be introduced by pre-folding the surface of a structure to initiate a particular failure mode. These pre-folds act as an inducer to trigger structural performances that cannot be obtained otherwise in conventionally shaped structures.

We also identified a number of drawbacks for foldcores, which are listed as follows.

- The simplest Miura-ori can only accurately produce foldcores with a cylindrical profile. For doubly curved shapes, non-developable pattern needs to be used, which may lead to manufacture difficulty.
- A pattern can only be used for a specific target shape. One or a pair of moulds need to be made to manufacture a specific foldcore.
- 3D printing could be a way to produce more complex foldcores. However, the existing 3D printing machines makes the process less cost-effective and it is rather restrictive in terms of material selections.

During this research we have also explored and successfully created morphing origami structures (*Gattas and You 2015*). In addition to build morphing structures, we believe that the morphing nature of the origami structures may also be utilized in the manufacture process of the foldcores. For instance, a core could be made first in an initial configuration that is easy to produce with one of the methods mentioned in this research, and then we can fold the core to its final configuration along its motion path. We have yet to integrate this technique into our manufacturing methods.

Despite the focus of this project being on sandwich composite structures, some of the findings can be extended to other origami structures. For example, the kinematic approach, which has never been extensively deployed to model origami in the past, has been proven to be extremely effective to model origami. It can be used to deal with origami of a flat sheet, or a 3D origami object that may not be developable. It is the tool to model rigid origami including origami morphing structures and origami robots. Additionally, the ability to create foldcores with different structural properties, which has been extensively investigated in this project, also offers possibility to design functionally grated meta-material whose composition and structure could gradually change over its volume, resulting in changes to the properties of the material. We have submitted a proposal to the AFOSR to pursue research in this direction.

List of Publications and Significant Collaborations that resulted from your AOARD supported project:

a) Papers published in peer-reviewed journals

Chen Y, Feng H, Ma J, Peng R, You Z, Symmetric waterbomb origami. Proceedings of the Royal Society. A 472 20150846, **2016**
(doi: dx.doi.org/10.1098/rspa.2015.0846)

Summary: *The traditional waterbomb origami, produced from a pattern consisting of a series of vertices where six creases meet, is one of the most widely used origami patterns. From a rigid origami viewpoint, it generally has multiple degrees of freedom, but when the pattern is folded symmetrically, the mobility reduces to one. Through a thorough kinematic investigation, we found that the pattern can be used to fold thick panels, and the folding process is also kinematically equivalent to the origami of zero-thickness sheets.*
(One of the figures in this article has been chosen as cover image of the journal, Fig. 7)

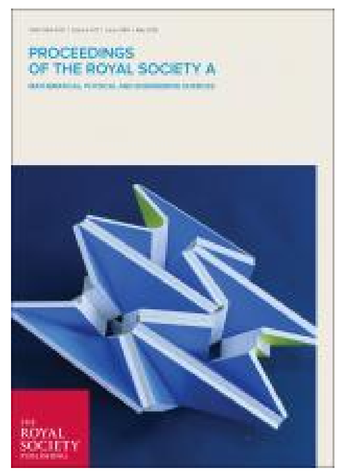


Fig. 7. The cover image of the journal.

Gattas J M and You Z, Design and digital fabrication of folded sandwich structures, Automation in Construction, 63, 79-87, **2016** (doi:10.1016/j.autcon.2015.12.002)
Summary: *This paper presents a design-to-fabrication process for folded sandwich structures that comprises surface to pattern conversion, manufacture rationalisation, and integral connection superposition. Folded sandwich structures are shown to possess a tessellated, origami-like structural form in which building component parameters are inherently dependant upon building surface parameters. Structural forms can therefore be designed with a minimum number of unique parts and with simultaneous consideration of surface and component constraints. Prototypes are also presented to demonstrate how the method can be applied generally for the digital fabrication of developable 3D surfaces with a known crease pattern.*

Zang S, Zhou X, Wang H and You Z, Foldcores made of thermoplastic materials: experimental study and finite element analysis, Thin-Walled Structures, 100, 170–179, **2016**

(doi:10.1016/j.tws.2015.12.017)
Summary: *This paper presents an experimental and numerical study on thermoplastic foldcores. A manufacturing process involving folding with dies and shape-setting by heat-treatment was established. Both PET and PEEK foldcore specimens were successfully manufactured using this process and tested in compression.*

Zhou X, Wang H and You Z, Design of 3D origami structures based on a vertex approach, Proceedings of Royal Society A, 471(2181), **2015a** (doi: dx.doi.org/10.1098/rspa.2015.0407)
Summary: *This paper presents a new method for the design of three-dimensional (3D) origami structures suitable for engineering use. Using input point sets specified, respectively, in a Cartesian coordinate system, the proposed method generates the coordinates of the vertices of a folded origami structure, whose fold lines are then defined by straight line segments each connecting two adjacent vertices. It is mathematically guaranteed that the origami structures obtained by this method are developable. Moreover, an algorithm to simulate the unfolding process from designed 3D configurations to planar crease patterns is provided. The validity and versatility of the proposed method are demonstrated through several numerical examples ranging from Miura-Ori to cylinder and curved-crease designs. Furthermore, it is shown that the proposed method can be used to design origami structures to support two given surfaces.*

Zhou X, Zang S, Wang H and You Z, Geometric design and mechanical properties of cylindrical foldcore sandwich structures, Thin-Walled Structures, 89, 116–130, **2015b** (doi:10.1016/j.tws.2014.12.017)
Summary: *While flat foldcore sandwich structures have been intensively studied in the literature, there lacks a general design tool to create foldcores for a given cylindrical sandwich structure and the mechanical properties of such structures have not been well investigated. In this paper, a geometrical design protocol for foldcores that will fit into the space between the external and internal walls of a given cylindrical sandwich structure is developed based on the vertex method. A parametric study on the mechanical properties of several selected cylindrical foldcore models and a honeycomb core model virtually tested in axial compression, internal pressure and radial crush using the finite element method is performed. It is shown that foldcores outperform the honeycomb core model in axial compression and radial crush but have lower radial stiffness when subjected to internal pressure. The design protocol together with the virtual test results can serve as a useful tool for researchers to design cylindrical foldcore sandwich structures for many potential applications including but not limited to aircraft fuselage, submarine shell and other pressurized cylinders.*

Fathers R, Gattas J M and You Z, Quasi-static crushing of eggbox, cube, and modified cube foldcore sandwich structures, International Journal of Mechanical Sciences, 101-102, 421-428, **2015** (doi:10.1016/j.ijmecsci.2015.08.013)
Summary: The paper explores a range of kirigami-inspired folded core structures for use in sandwich panels. Focus has been on assessing the energy-absorption capabilities of the cores, specifically on benchmarking core performance against the widely studied Miura-ori folded core. Four core architectures were investigated. Two cores are based on cube and eggbox known tessellated kirigami patterns. Two cores, the cube-strip and the diamond strip, are developed from geometric modifications of the cube tessellation. The cube strip is generated by removing face portions of the cube pattern that contribute little to energy absorption, effectively making a cellular square tube configuration. The diamond strip introduced a pre-folded origami pattern into the core which has been shown in previous research to substantially increase square tube energy absorption. The performance of each core is assessed under quasi-static loading with experimental and numerical analyses. The non-optimised diamond strip cube strip core offered a 41% increase in

average force compared to the best-performing curved-crease Miura-type foldcore previously reported and a 92% improvement over the standard Miura-type foldcore.

Chen Y, Peng R and You Z, Origami of thick panels, Science, 349(6246), 396-400, **2015** (doi: 10.1126/science.aab2870)
Summary: Origami patterns, including the rigid origami patterns in which flat inflexible sheets are joined by creases, are primarily created for zero-thickness sheets. In order to apply them to fold structures such as roofs, solar panels, and space mirrors, for which thickness cannot be disregarded, various methods have been suggested. However, they generally involve adding materials to or offsetting panels away from the idealized sheet without altering the kinematic model used to simulate folding.We develop a comprehensive kinematic synthesis for rigid origami of thick panels that differs from the existing kinematic model but is capable of reproducing motions identical to that of zero-thickness origami.The approach, proven to be effective for typical origami, can be readily applied to fold real engineering structures.

Gattas, J M and You, Z. Geometric assembly of rigid-foldable morphing sandwich structures. Engineering Structures, 94, 149-159, **2015** (doi:10.1016/j.engstruct.2015.03.019)
Summary: *Morphing plate-based sandwich mechanisms consist of three layers: an inner core designed to achieve a particular deployed geometric envelope and two outer faces designed to preserve core rigid-foldability from flat-folded to a deployed sandwich form. This paper examines rigid-foldable morphing sandwich mechanisms based on the Miura rigid origami pattern. An alternative mechanism is developed that has improved stability and locking capability compared with the existing mechanism reported previously. These improvements are demonstrated with steel prototypes. The alternative mechanism is then extended to form a family of new morphing sandwich structures, including a fan-shaped mechanism, and single-curved cylindrical and conical mechanisms. Each are derived by substituting the base Miura core pattern with a Miura-derivative pattern, and attaching faces that have compatible rigid-foldability and avoid self-intersection during deployment. Morphing mechanisms and geometric derivations are validated with physical prototypes.*

You Z, Folding structures out of plate materials, Science 345(6197), 623-645, **2014**
(doi: 10.1126/science.1257841)
Summary: *This paper is a review on existing research in origami structures. It also raised a few topics for future research in origami engineering.*

Gattas J M and You Z, Miura-Base Rigid Origami: Parametrisations of Curved-Crease Geometries, Journal of Mechanical Design, 136(12), MD-13-1509, **2014a**
(doi: 10.1115/1.4028532)
Summary: *Curved-crease (CC) origami differs from prismatic, or straight-crease origami, in that the folded surface of the pattern is bent during the folding process. Limited studies on the mechanical performance of such geometries have been conducted, in part because of the difficulty in parametrizing and modeling the pattern geometry. This paper presents a new method for generating and parametrizing rigid-foldable, CC geometries from Miura-derivative prismatic base patterns. The two stages of the method, the ellipse creation stage and rigid subdivision stage, are first demonstrated on a Miura-base pattern to generate a CC Miura pattern. It is shown that a single additional parameter to that required for the straight-crease pattern is sufficient to completely define the CC variant. The process is then applied to tapered Miura, Arc, Arc-Miura, and piecewise patterns to generate CC variants of each.*

Zhou X, Wang H and You Z, Mechanical properties of Miura-based folded cores under quasi-static loads, Thin-Walled Structures, 82, 296-310, **2014** (doi:10.1016/j.tws.2014.05.001)
Summary: *This paper presents a parametric study on the mechanical properties of a variety of Miura-based folded core models virtually tested in quasi-static compression, shear and bending using the finite element method. It is found that the folded core models with curved fold lines exhibit the best mechanical performances in compression and shear while the multiple layered models outperform the other folded core models in bending. Furthermore, the folded core models are compared to a honeycomb core model with the same density and height. In this case, it is shown that the honeycomb core has the best performance in compression while the folded cores have comparable or even better performances in the shear and bending cases. The virtual test results reported in this paper can provide researchers with a general guideline to design the most suitable folded core structure for given applications.*

Gattas J M and You Z, The Behaviour of Curved-Crease Foldcores under Quasi-Static Impact Loads, International Journal of Solids and Structures, 53, 80-91, **2014b**
(doi:10.1016/j.ijsolstr.2014.10.019)
Summary: *The primary aim of this paper was to manufacture aluminium curved-crease foldcores and assess their behaviour under quasi-static compressive loads, relative to existing straight-crease foldcores and a honeycomb cores. Four foldcore types, standard, indented, and two curved-crease foldcore tessellations, were constructed with comparable density and height to a commercial honeycomb core. An experimental and numerical study of foldcore performance under quasi-static crush loads showed that all foldcore types were highly sensitive to geometric imperfections, and that curved-crease foldcores had significantly higher energy-absorption capability than straight-crease foldcores. Validated numerical methods were used in a comprehensive parametric study on curved-crease foldcore geometry, with two main findings. First, it was seen that altering the curved-crease foldcore tessellation did not provide significant energy-absorption capability beyond that achievable with direct changes to the core aspect ratio. Second, an optimum configuration of the curved-crease foldcore was found which appeared to offer a comparable out-of-plane strength, energy-absorption under quasi-static compressive loads, and stiffness to a honeycomb core. A brief numerical investigation into low-velocity impact loading showed that curved-crease foldcores were the only foldcore type that saw a substantial inertial strengthening under dynamic loading, although not to as large an extent as honeycomb.*

Gattas J M, Wu W and You Z, Miura-Base Rigid Origami: Parametrisations of First-Level Derivative and Piecewise Geometries, Journal of Mechanical Design, 135(11), **2013**
(doi:10.1115/1.4025380)
Summary: *Miura and Miura-derivative rigid origami patterns are increasingly used for engineering and architectural applications. However, geometric modelling approaches used in existing studies are generally haphazard, with pattern identifications and parameterizations varying widely. Consequently, relationships between Miura-derivative patterns are poorly understood, and widespread application of rigid patterns to the design of folded plate structures is hindered. This paper explores the relationship between the Miura pattern, selected because it is a commonly used rigid origami pattern, and first-level derivative patterns, generated by altering a single characteristic of the Miura pattern. Five alterable characteristics are identified in this paper: crease orientation, crease alignment, developability, flat-foldability, and rectilinearity. A consistent parameterization is presented for five derivative patterns created by modifying each characteristic, with physical prototypes constructed for geometry validation. It is also shown how the consistent parameterization allows first-level derivative geometries to be combined into complex piecewise geometries.*

b) Papers published in peer-reviewed books

Ho J and You Z, Thin-walled deployable grid structures. Origami^6 II: technology, art and education (Edited by Miura K et. al.), American Mathematical Society, 439-445, **2015** (ISBN 978-1-4704-1876-2)

Gattas J M and You Z, Structural engineering applications of morphing sandwich structures. Origami^6 II: technology, art and education, (Edited by Miura K et. al.), American Mathematical Society, 421-430, **2015** (ISBN 978-1-4704-1876-2)

c) Papers published in non-peer-reviewed journals or in referred conference proceedings

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Moss E, Gattas J M and You Z, Flat-foldable Miura-ori for doubly curved surfaces, **2016**

f) Thesis

Gattas J M, Quasi-Static Impact of Foldcore Sandwich Panels, D. Phil. Dissertation, Univerity of Oxford, **2013**

g) Interactions with industry or with Air Force Research Laboratory scientists or significant collaborations that resulted from this work.

The PI took part in all review meetings organized by the AFOSR program manager. These meetings were held in one of the Air Force bases (Albuquerque NM 2014, Fort Walton Beach FL 2015 and Dayton OH 2016), and thus there were always many participants from the Air Force Research Institutions. The PI has had many useful and constructive communications with them. In particular, the PI communicates his research work with Dr James J. Joo of Air Force Research Laboratory, Wright-Patterson AFB, OH with possibilities of utilizing topological optimization to design foldcores.

Other matters for reporting

In the budget of the project, we planned to hire a post-doctoral researcher for one year. Dr Gattas, the PI's formal research student, started working on the project since April 2013 but only officially became a post-doctoral researcher from August 2013 after obtaining his D. Phil. (PhD). This was because he was a student under full scholarship prior to completion of his degree. He subsequently left after four months to take up a lectureship at University of Queensland, Australia. Dr Ma was hired as a replacement, but again he left for an associate professorship in China after being in post for six months. This has resulted in an under-spend in staff cost.

The budget also included a large sum for producing prototypes for experiments. The PI managed to find a group of collaborators at Dalian University of Technology, China, who specialize in fabrication and forming of sheet materials. Through joint research, the group helped to produce large number of specimen including five sets of moulds (for making polymer and fibre reinforced foldcores), and ten inversion tubes (by wire-erosion cutters) at no cost to the project. This has resulted in a significant savings in consumables.

The budget surplus has been returned to the AFOSR by University of Oxford.

Appendices

a) Research article published in **Science**

REPORTS

APPLIED ORIGAMI

Origami of thick panels

Yan Chen,[1] Rui Peng,[1] Zhong You[2]*

Origami patterns, including the rigid origami patterns in which flat inflexible sheets are joined by creases, are primarily created for zero-thickness sheets. In order to apply them to fold structures such as roofs, solar panels, and space mirrors, for which thickness cannot be disregarded, various methods have been suggested. However, they generally involve adding materials to or offsetting panels away from the idealized sheet without altering the kinematic model used to simulate folding. We develop a comprehensive kinematic synthesis for rigid origami of thick panels that differs from the existing kinematic model but is capable of reproducing motions identical to that of zero-thickness origami. The approach, proven to be effective for typical origami, can be readily applied to fold real engineering structures.

Origami is the art of folding essentially two-dimensional materials such as paper into three-dimensional objects. It has recently gained popularity among scientists and engineers because the technique can be used to create shape-changing structures. Rigid origami is a subset of origami that considers rigid objects connected by hinges. This type of origami has probably the greatest application potential in engineering structures (*1*, *2*)—ranging from solar panels (*3*), space mirrors, and aircraft wings, to robots (*4*)—because most materials used in these applications are relatively rigid. To date, all kinematic modeling of rigid origami treats the paper as having zero thickness. At each vertex where creases, or fold lines, meet, the origami is considered as a spherical linkage, where creases act as revolute joints (R) and the paper bounded by the creases act as links. A rigid origami pattern is therefore a combination of many such linkages. For panels of nonzero thickness, various techniques have been suggested to use the same kinematic model, which include adding tapered materials to the plane of zero-thickness (*5*) or offsetting panels away from the planes defined by the adjacent creases (*6*). The tapered material technique has been used to fold a thick panel based on the Miura-ori, whereas the offset one has been used for folding a square-twist origami pattern. However, these methods often result in surfaces that are either not entirely flat or with voids to allow folding. Two exceptions to this are the technique introduced by Hoberman to fold the symmetric Miura-ori (*7*), and the technique by De Temmerman for the diamond origami pattern (*8*). Not all of the fold lines meet at a point in either of these methods, and thus, the vertices no longer exist. This indicates that their folding cannot be simply treated as the motion of spherical linkage assemblies. This has led us to question what a rigorous equivalent kinematic model of this type of origami should be, and whether the model can be generalized and applied so as to create origami for thick materials.

Rigid origami patterns, designed universally for a zero-thickness sheet, are made from creases intersecting at vertices. However, these patterns cannot be directly applied to a panel of nonzero thickness, given that all of the fold lines cannot be placed on the same face of the panel because the subpanels would collide during folding. For this reason, some fold lines must be placed on the top face whereas others must be placed on the bottom face of a panel, forming an assembly in which the fold lines are neither concurrent nor intersecting at a vertex. As a result, the kinematic model of the spherical linkage around each vertex used for a zero-thickness sheet has now been replaced by a loop of rigid bodies (panels) connected by a set of revolute joints (fold lines) that are placed a distance apart because of the thickness. Its foldability depends on two conditions: (i) Each loop of connected rigid bodies must be a mechanical linkage, and (ii) the assembly of these linkages retains mobility so that it can be folded.

Most practically used patterns—such as the Miura-ori, square-twist, diamond, and waterbomb patterns—have four, five, or six creases intersecting at a vertex, and thus, the corresponding closed kinematic chains for thick panels, when foldable, are spatial $4R$, $5R$, and $6R$ linkages. These linkages belong to a specific family of spatial linkages often referred to as the overconstrained linkages because the Kutzbach criterion yields a mobility value of less than one (*9*). The existence of mobility is due to specific geometries that the linkages possess. In this work, we first found the kinematically equivalent spatial linkages to the single-vertex origami patterns made from four, five, and six creases, respectively, and then extended this to multiple vertex patterns by ensuring that the motion of the assembly of these linkages matches that of the zero-thickness pattern. Using this approach, origami patterns can still be designed based on a zero-thickness rigid sheet, and these patterns can consequently be synthesized for a thick rigid panel. We assume that thick panels are rigid with nonzero thickness and that fold lines, equivalent to creases in zero-thickness paper and revolute joints in linkages, can only be placed on the faces of a thick panel, and no fold lines along the depth of the panels are permitted.

A partially folded single-vertex four-crease origami pattern of a zero-thickness rigid sheet is shown in Fig. 1A. This pattern is the basic element in many well-known origami forms, such as the Miura-ori and the square-twist pattern. Four creases divide the sheet into four portions, with sector angles α_{12}, α_{23}, α_{34}, and α_{41}, respectively, and the sum of these angles equals 2π. To be flat foldable—the folded origami can be pressed flat eventually—$\alpha_{12} + \alpha_{34} = \alpha_{23} + \alpha_{41} = \pi$ must be satisfied (*10*).

The kinematic motion of this origami can be modeled as a spherical $4R$ linkage. It has a single degree of freedom, and the relationship among four dihedral angles φ_1, φ_2, φ_3, and φ_4 can be obtained analytically (*11*)

$$\frac{\tan\frac{\varphi_2}{2}}{\tan\frac{\varphi_1}{2}} = -\frac{\sin\frac{\alpha_{12}-\alpha_{23}}{2}}{\sin\frac{\alpha_{12}+\alpha_{23}}{2}},\ \varphi_1 = \varphi_3 \text{ and } \varphi_2 = \varphi_4 \quad (1)$$

Now consider its thick-panel counterpart (Fig. 1B), in which the panel is partitioned by the same set of sector angles into four subpanels. Place one fold line on the top face of subpanels, three on bottom faces. a_{12}, a_{23}, a_{34}, and a_{41} are distances between adjacent fold lines, respectively, which effectively represent the thicknesses of the subpanels.

Kinematically, this assembly is no longer a spherical linkage. It must be a spatial $4R$ linkage if it is capable of motion. The only spatial $4R$ linkage is the Bennett linkage, a century-old mechanism in which the axes of revolute joints neither meet nor are parallel (*12*). The existence of mobility is due to the special geometry conditions (*13*), which are

$$\alpha_{12} + \alpha_{34} = \alpha_{23} + \alpha_{41} = \pi \quad (2)$$

$$a_{12} = a_{34},\ a_{23} = a_{41} \quad (3)$$

and

$$\frac{a_{12}}{a_{23}} = \frac{\sin\alpha_{12}}{\sin\alpha_{23}} \quad (4)$$

Equation 2 matches the flat foldable condition for a zero-thickness sheet, whereas Eqs. 3 and 4 are conditions governing the placement of fold lines. The Bennett linkage has a single degree of freedom. We can prove that the relationships among the dihedral angles are identical to those in Eq. 1, except that φ_1, φ_2, φ_3, and φ_4 are replaced by φ_1^{Be}, φ_2^{Be}, φ_3^{Be}, and φ_4^{Be}, respectively. Its motion is therefore identical to that of the spherical $4R$ linkage throughout the entire folding process (fig. S4). The spherical $4R$ linkage and this Bennett linkage are kinematically equivalent as a result. The folding process of a zero-thickness rigid origami and its thick-panel counterpart is

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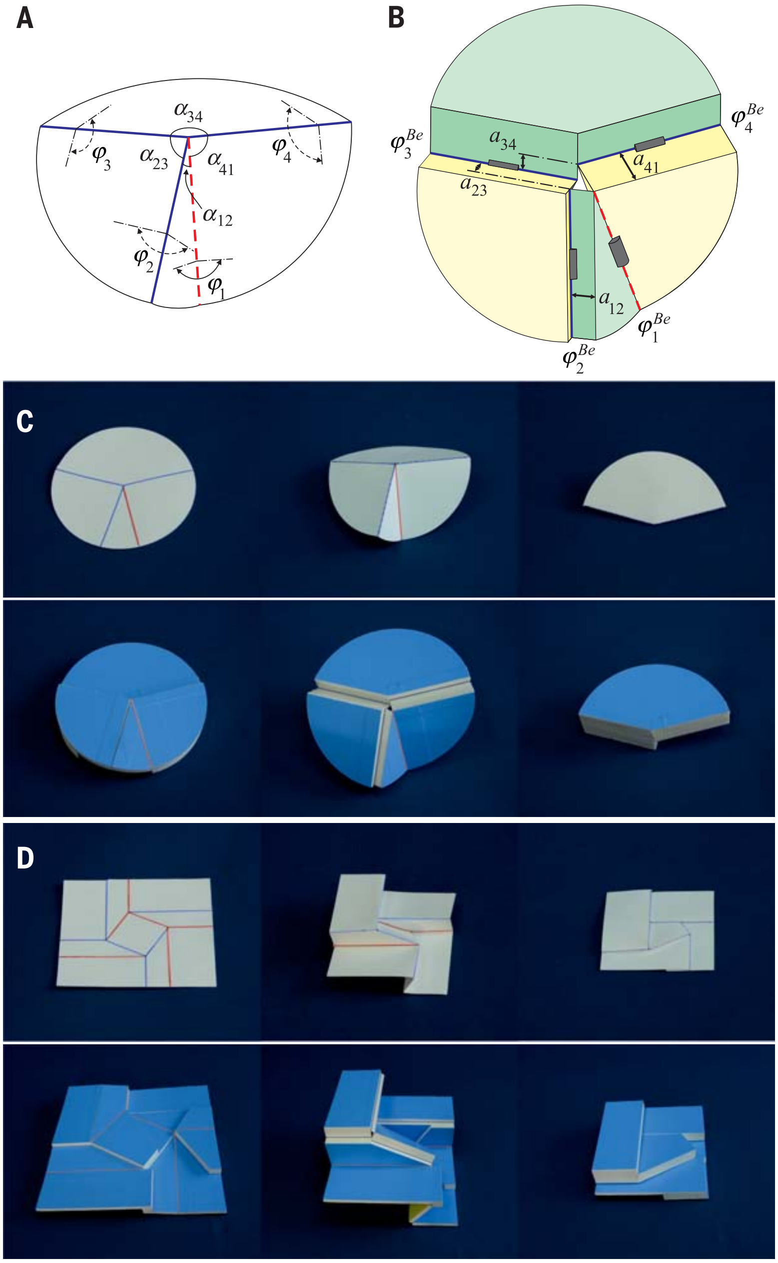


Fig. 1. Four-crease single- and multiple-vertex origami. (**A**) A partially folded single-vertex four-crease rigid origami for a zero-thickness sheet. Blue solid and red dash lines represent mountain and valley creases, respectively. αs and φs are sector and dihedral angles, respectively. (**B**) The thick-panel counterpart with the sector angles identical to those in (A), but the fold lines no longer meet at a point (vertex). *a*s are distances between adjacent fold lines, and φ^{Be}s are dihedral angles. (**C**) Folding sequence of a model made of zero-thickness sheet and its thick-panel counterpart. The sector angles for both models are π/6, π/2, 5π/6, and π/2. (**D**) Folding sequence of a zero-thickness square-twist origami and its thick-panel counterpart based on the Bennett linkage. The pattern consists of four identical four-crease vertices arranged in rotational symmetry. The sector angles for each vertex are the same as those for the single-vertex model in (C).

demonstrated in Fig. 1C and movie S1. Both have four fold lines, with the same sector angles.

The four-crease thick-panel origami can be readily applied to find the thick-panel equivalence of multiple-vertex rigid origami. For example, the square-twist pattern with rotational symmetry: First, the Bennett linkage can be applied to each vertex of the pattern, preserving the sector angle of each subpanel. We then merge the fold lines that are shared by two adjacent Bennett linkages. This is possible because there is rotational symmetry in the square-twist pattern, leading to the exact same amount of rotation for the combined fold lines. The folding process of a zero-thickness rigid panel and its thick counterpart by using the square-twist pattern are shown in Fig. 1D, fig. S4, and movie S1.

Similarly, we can also create a folding scheme of a thick panel using the Miura-ori, as done by Hoberman (*7*). However, Hoberman only reported the symmetrical case in which $\alpha_{12} = \alpha_{23}$ and $\alpha_{34} = \alpha_{41}$, which is a special case of Eq. 2.

The Bennett linkage requires sector angles to satisfy Eq. 2 that equates to the condition for flat foldability. Because the Bennett linkage is the only known spatial 4*R* linkage, it can be concluded that for the four-crease rigid origami, only flat foldable patterns can have thick-panel equivalents.

The method for devising folding of four-crease patterns can be extended to the single-vertex five-crease rigid origami case. Many single-vertex five-crease origami patterns for a zero-thickness sheet exist, and here, we consider a particular one that has been used to make boxes (Fig. 2A). In this pattern, the creases divide the sheet into five pieces, with sector angles αs in which $\alpha_{51} = \alpha_{12}$, $\alpha_{23} = \alpha_{45} = \frac{\pi}{2}$, and $\alpha_{34} = \pi - 2\alpha_{12}$. It can be modeled as a spherical 5*R* linkage. In general, this linkage has two degrees of freedom, but we restrict its motion by preserving symmetry—that is, during folding, by letting the dihedral angles satisfy

$$\varphi_5 = \varphi_2 \text{ and } \varphi_4 = \varphi_3 \quad (5)$$

We then consider its nonzero thickness counterpart (Fig. 2B), in which the thick panel is apportioned by exactly the same set of sector angles. The fold lines are then placed either on top or bottom of the panel surfaces, and the distances between a pair of neighboring fold lines are *a*s. It has become a spatial 5*R* linkage.

There are a number of 5*R* overconstrained linkages. The arrangement of the sector angles makes it possible to be a Myard linkage (*14*) because it has the same angle conditions as those for this particular linkage. In addition, the Myard linkage requires

$$a_{12} = a_{51}, a_{23} = a_{45}, \text{ and } a_{34} = 0 \quad (6)$$

and

$$\frac{a_{12}}{a_{23}} = \frac{\sin\alpha_{12}}{\sin\alpha_{23}} \quad (7)$$

The thick-panel assembly will have one degree of freedom if the arrangement of fold lines satisfies Eqs. 6 and 7 because it is now a Myard linkage. Furthermore, the proof in the supplementary text shows that the motion of this linkage is identical to that of the spherical 5*R* linkage when Eq. 5 is imposed. This folding scheme has been used to fold a box (Fig. 2C and movie S1).

The Myard linkage has only one degree of freedom, whereas a spherical 5*R* linkage commonly has two. It is possible to find a kinematic match only if one of the degrees of freedom of the latter is frozen, which is achieved in the above example by imposing symmetry. The same strategy is used for the six-crease example discussed next.

One of the typical six-crease origami patterns is the diamond pattern. The zero-thickness pattern (Fig. 3A) has sector angles $\alpha_{12} = \alpha_{34} = \alpha_{45} = \alpha_{61}$ and $\alpha_{23} = \alpha_{56} = \pi - 2\alpha_{12}$. This spherical linkage has three degrees of freedom in general. However, if

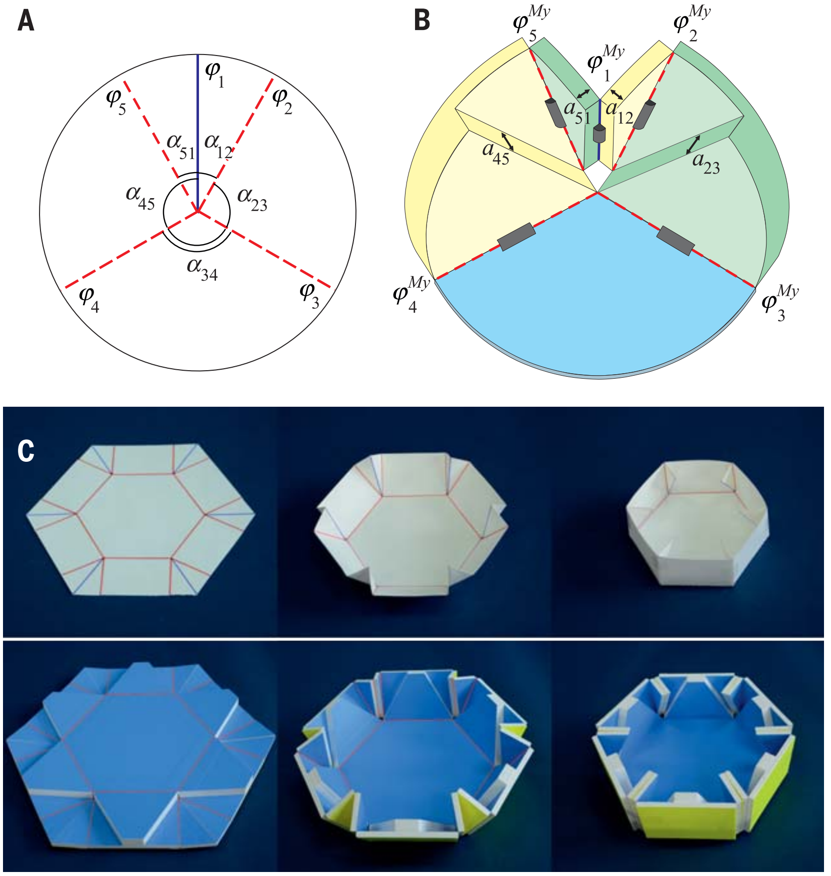


Fig. 2. Five-crease single- and multiple-vertex origami. (**A**) A single-vertex five-crease rigid origami pattern for a zero-thickness sheet. (**B**) The thick-panel counterpart with sector angles identical to those in (A), but the fold lines no longer meet at a point (vertex). (**C**) Folding sequence of a zero-thickness rigid origami with five-crease vertices and its thick-panel counterpart based on the Myard linkage. The pattern has six five-crease vertices arranged in rotational symmetry. The sector angles at each vertex are π/6, π/2, 2π/3, π/2, and π/6.

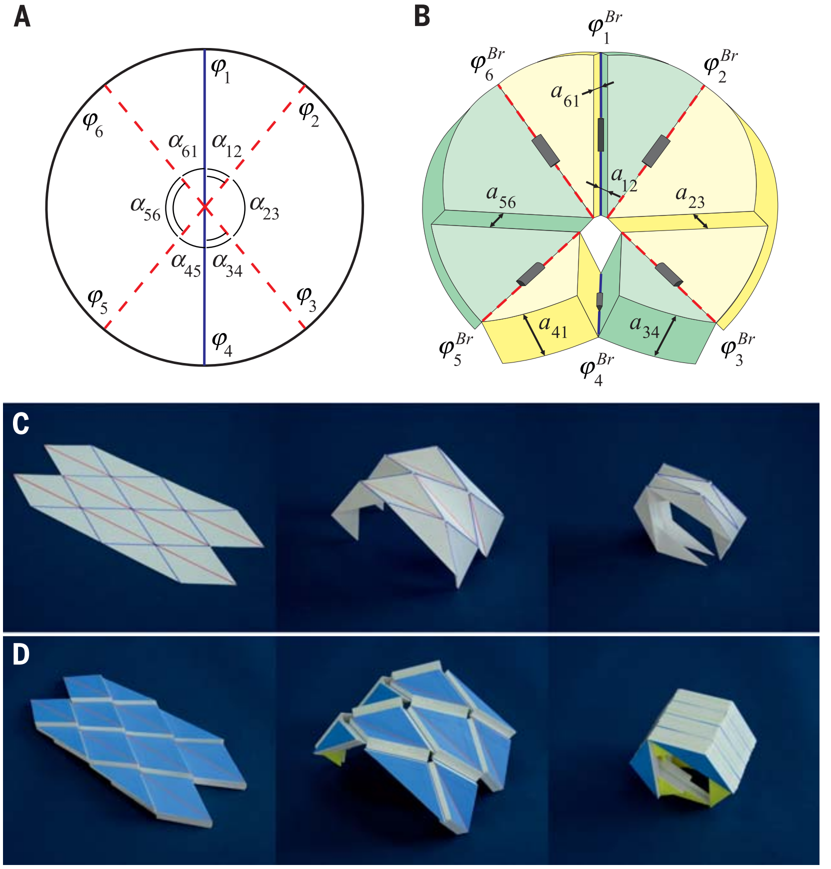
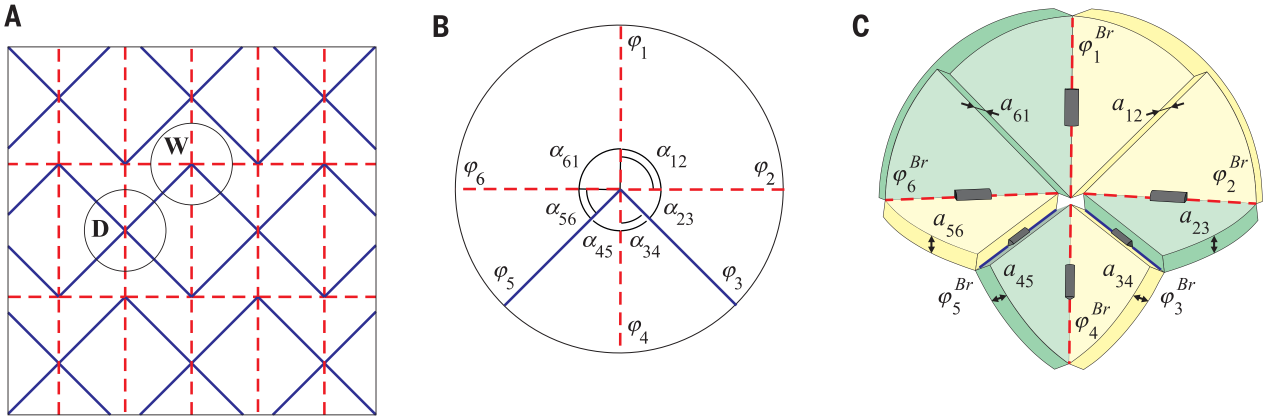


Fig. 3. The diamond origami pattern. (**A**) A single-vertex six-crease diamond pattern for a zero-thickness sheet. (**B**) The thick-panel counterpart with sector angles identical to those in (A), but the fold lines no longer meet at a point (vertex). (**C**) Folding sequence of a zero-thickness origami model of the diamond pattern and its thick-panel counterpart based on the plane-symmetric Bricard linkage. All the vertices are identical. The sector angles around each vertex are π/6, 2π/3, π/6, π/6, 2π/3, and π/6.



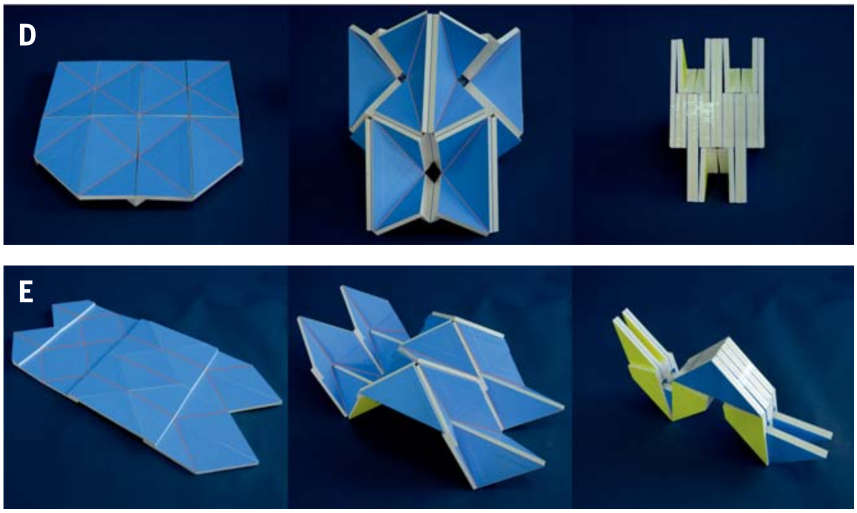


Fig. 4. Origami using the waterbomb pattern for zero- and nonzero-thickness panels and thick-panel origami of a pattern with a mixture of vertices with four and six creases. (**A**) The traditional waterbomb pattern for zero-thickness sheet. Two types of vertices are marked as **D** and **W**. (**B**) Origami pattern around vertex **W**. (**C**) Its thick-panel counterpart. (**D**) Folding sequence of the thick-panel waterbomb pattern. Around type **D** vertex, the sector angles are $\pi/4$, $\pi/2$, $\pi/4$, $\pi/4$, $\pi/2$, and $\pi/4$, whereas they are $\pi/2$, $\pi/4$, $\pi/4$, $\pi/4$, $\pi/4$, and $\pi/2$ around type **W** vertex. (**E**) Folding sequence of a thick-panel origami derived from an origami pattern with a mixture of both four- and six-crease vertices. It contains both the Bennett and Bricard linkages.

we confine its motions to the symmetric case by letting

$$\varphi_1 = \varphi_4 \text{ and } \varphi_2 = \varphi_3 = \varphi_5 = \varphi_6 \quad (8)$$

it reduces to a linkage with a single degree of freedom.

The corresponding thick-panel pattern is given in Fig. 3B, in which all sector angles are identical to those of the zero-thickness pattern. The distances between axes of fold lines are marked in the diagram.

The *6R* assembly of Fig. 3B reassembles the plane-symmetric Bricard linkage (*15*) with line- and plane-symmetric behavior. The geometrical conditions of the Bricard linkage for the distances between neighboring axes of the rotational joints are

$$a_{12} = a_{61}, a_{23} = a_{56}, \text{and } a_{34} = a_{45} \quad (9)$$

In addition, to achieve compact folding, there must be

$$a_{12} + a_{23} = a_{34}, \quad (10)$$

which is obvious by considering the completely packaged configuration.

The kinematic motion of this Bricard linkage again matches that of the spherical *6R* linkage of the zero-thickness model (fig. S8). This enables us to make a thick-panel origami arch using the diamond pattern. The folding sequences of both zero- and nonzero-thickness models are shown in Fig. 3C and movie S1. This agrees with results reported by De Temmerman (*8*), but a complete mathematic proof is presented here.

Another typical six-crease origami pattern is the traditional waterbomb pattern (*16*), whose zero-thickness origami pattern is shown in Fig. 4A. Unlike the diamond pattern, there are two types of vertices, marked as **D** and **W**. **D** is a special case of the diamond pattern for which Eqs. 9 and 10 can be used to syntheses thick panel origami. **W**, specific to the waterbomb pattern, is enlarged in Fig. 4B, in which the sector angles are

$$\alpha_{12} = \alpha_{61} = \frac{\pi}{2} \text{ and}$$

$$\alpha_{23} = \alpha_{34} = \alpha_{45} = \alpha_{56} = \frac{\pi}{4} \quad (11)$$

For **W**, we can show that the corresponding Bricard linkage is a plane-symmetric one in which the thicknesses of subpanels must satisfy

$$a_{23} = a_{56}, a_{34} = a_{45} = \mu a_{23}, \text{and}$$

$$a_{12} = a_{61} = (1 + \mu) a_{23} \quad (12)$$

in order to achieve compact folding. μ is a constant. When $\mu = 1$, the motion of this Bricard linkage is kinematically equivalent to that of the spherical linkage at **W**, as shown in Fig. 4C.

Considering the common folds appearing in both Bricard linkages as we did with the square-twist pattern described previously, the waterbomb pattern for the thick panel can be obtained. The folding sequence of nonzero thickness model is shown in Fig. 4D and movie S1.

If $\mu \neq 1$, the motion of the corresponding Bricard linkage differs from that of the spherical linkage at **W**, although both are flat foldable and can expand flat, which is demonstrated by the relationship between φ_1 (φ_1^{Br}) and φ_2 (φ_2^{Br}) in fig. S14. This indicates that the thick-panel origami can be devised by the mechanism theory alone without referring to its parent rigid origami pattern.

We have developed a comprehensive kinematic model for rigid origami of panels with nonzero thickness. This is done by identifying a spatial linkage model that is kinematically equivalent to the rigid origami of a zero-thickness sheet. In other words, the motion of the spatial linkage mimics that of the spherical linkage commonly used to model rigid origami. To achieve this, we identified a spatial linkage that has the angular conditions for arrangement of fold lines identical to that of the spherical linkage and then proved analytically that their motions are precisely alike.

The thick-panel counterparts to four-, five-, and six-crease vertex origami patterns are overconstrained spatial linkages. The number of such linkages is rather limited (*17*). It is relatively straightforward for four-crease origami patterns because only one spatial 4*R* linkage exists. However, five- and six-crease single-vertex patterns commonly comprise two or three degrees of freedom, whereas their corresponding spatial overconstrained linkages have only one mobility degree of freedom. In these cases, equivalence can only be accomplished through reducing the degrees of freedom of the former by symmetry or other means. This may be beneficial for practical applications because the folding of thick panels can be more easily controlled owing to their single degree of freedom. Moreover, the synthesis can also be used for origami patterns consisting of a mixture of vertices with various creases. The folding sequence of a thick-panel origami based on a pattern with both four- and six-crease vertices is shown in Fig. 4E and movie S1.

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SUPPLEMENTARY MATERIALS

www.sciencemag.org/content/349/6246/396/suppl/DC1
Materials and Methods
Supplementary Text
Figs. S1 to S14
Movie S1

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STRETCHY ELECTRONICS

Hierarchically buckled sheath-core fibers for superelastic electronics, sensors, and muscles

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Superelastic conducting fibers with improved properties and functionalities are needed for diverse applications. Here we report the fabrication of highly stretchable (up to 1320%) sheath-core conducting fibers created by wrapping carbon nanotube sheets oriented in the fiber direction on stretched rubber fiber cores. The resulting structure exhibited distinct short- and long-period sheath buckling that occurred reversibly out of phase in the axial and belt directions, enabling a resistance change of less than 5% for a 1000% stretch. By including other rubber and carbon nanotube sheath layers, we demonstrated strain sensors generating an 860% capacitance change and electrically powered torsional muscles operating reversibly by a coupled tension-to-torsion actuation mechanism. Using theory, we quantitatively explain the complementary effects of an increase in muscle length and a large positive Poisson's ratio on torsional actuation and electronic properties.

Highly elastic electrical conductors are needed for stretchable electronic circuits, pacemaker leads, light-emitting displays, batteries, supercapacitors, and strain sensors (*1*). For such purposes, conducting elastomers have been fabricated by incorporating conducting particles in rubber (*2*–*5*) or by attaching sheets of conducting nanofibers (*6*–*9*), graphene sheets (*10*, *11*), or coiled or serpentine conductors to a rubber sheet or fiber (*12*–*17*). Although reversible strains exceeding 500% have been demonstrated, the quality factor (*Q*, the percent strain divided by the percent resistance change) has been below three for such large strains (*17*–*20*). Elastomeric conductors with very low quality factors are useful as strain sensors, but the other applications noted above would benefit from the realization of very high quality factors. The availability of conducting fibers that can be stretched to great extents without significantly changing conductivity could enable the deployment of superelastic fibers as artificial muscles, electronic interconnects, supercapacitors, or light-emitting elements.

We replaced the frequently used laminate of a carbon nanotube (CNT) sheet wrapped on a stretched rubber sheet with a multilayer CNT sheath wrapped on a rubber fiber core (*21*, *22*). We enabled additional functions by including other rubber and CNT sheath layers. The conducting sheaths were derived from highly oriented multiwalled CNT aerogel sheets, which were drawn from CNT forests (*21*). Three basic configurations were deployed: NTS_m@fiber, rubber@NTS_m@fiber, and NTS_n@rubber@NTS_m@fiber. NTS_m@fiber denotes m carbon nanotube sheet (NTS) layers deposited on top of a rubber fiber core, rubber@NTS_m@fiber is a rubber-coated NTS_m@fiber, and NTS_n@rubber@NTS_m@fiber indicates an NTS_n sheath (where n is the number of NTS layers) on a rubber@NTS_m@fiber core.

The rubber fiber core was highly stretched (typically to 1400% strain) during the wrapping of NTS layers, and the CNT orientation was parallel to the rubber fiber direction (Fig. 1A). For the preparation of rubber@NTS_m@fibers, the outermost rubber coating was applied while the rubber core was fully stretched, whereas for the preparation of NTS_n@rubber@NTS_m@fibers, the thicker rubber layer used as a dielectric was deposited on an NTS_m@fiber that was not stretched (*22*). The parallel orientations of CNT fibers and the rubber core, the substantial strain applied during sheath wrapping, and the use of a large m resulted in the observed hierarchical two-dimensional buckling and corresponding

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b) Report in **Forbes** on our research work

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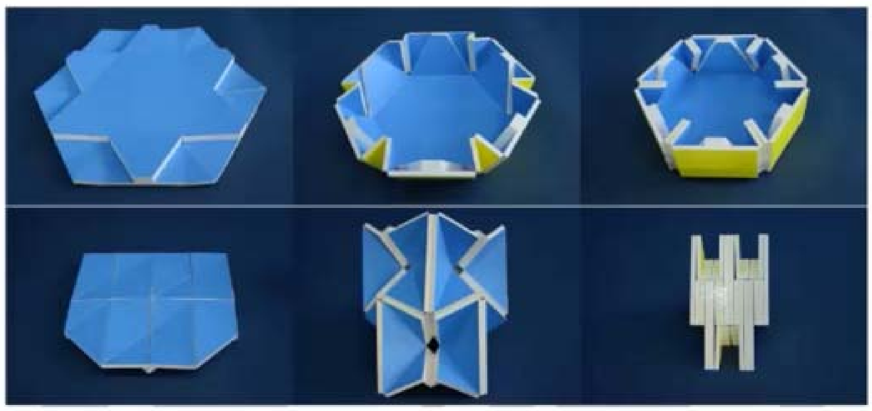
How Ancient Origami Techniques Could Help Engineers Fold In 3-D

Hilary Brueck
CONTRIBUTOR
I write about the ingredients for modern life.
FULL BIO



Traditional paper origami patterns have provided a blueprint for engineers to fold up structures like solar panels, airplane wings, and roofs. (Credit: Chen et al. / Cambridge University archive)

Folding is going rogue.

The action that's been reserved for thin, flexible materials is about to get a boost.

Researchers studying the ancient art of origami have figured out some new ways to make rigid, thick structures fold up and move out.

Author Zhong You, an engineering professor at Oxford, and one of the authors of the new study released in Science says the 3-D folding technique is all about having "a single point where creases meet."

Using that single point, and some very precise hinge placement, structures can fold both lengthwise and widthwise, making it easier to fold up and move materials like solar panels and wings, without compromising on how strong they'll be when they're full size again.

And that's just one of the new ways engineers are discovering origami can help them fold.

c) Report in **Nature** on our research work

NATURE | RESEARCH HIGHLIGHTS

ENGINEERING

Origami for thick materials

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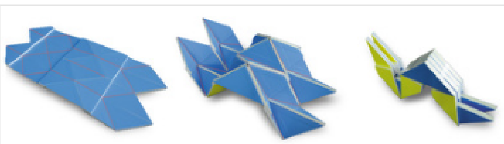
Subject terms: Engineering

Origami patterns designed for thin pieces of paper can be extended to thicker materials as well.

Previous attempts to fold 3D materials required adding layers of material or changing their geometry. To avoid this, Zhong You at the University of Oxford, UK, and his colleagues developed a method of assembling thick materials so that the hinges where they meet move in a limited number of ways. The researchers showed how carefully choosing the placement of the hinges and creases allows the structures to move and fold (**pictured**) in identical ways to origami patterns that use 2D materials.



Science/AAAS

The method could eventually improve the construction of foldable structures such as solar panels or aircraft wings, the authors report.

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