Scheduling and Topology Design in Networks with Directional Antennas

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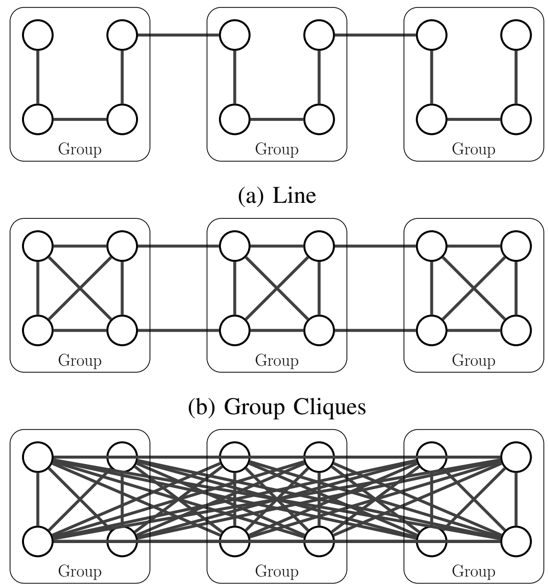
***Abstract*—In multihop wireless networks equipped with directional antennas, network controllers must choose which pairs of nodes should communicate in order to establish a topology over which traffic can be sent. Additionally, because of interference constraints, conflicting transmitters must be scheduled to transmit in time-separated intervals. In this work, we examine the interacting effects of topology design and transmission scheduling in wireless networks, in particular focusing on networks where nodes are divided into geographically localized groups. Herein, it is shown that in order to maximize network throughput, transmission schedules should be carefully chosen to match the topology design and traffic patterns. Specifically, we find that commonly used, suboptimal schedules can lead to greatly reduced network throughput. Results for both unicast and multicast traffic are examined, and it is found that the type of traffic can significantly impact the performance of varying topology and scheduling solutions.**

I. Introduction

We consider throughput optimization in multihop wireless networks where the users in the network are divided into geographically localized groups. In this setting, each group of users maximizes their local information exchange while also diffusing global information to the entire network. This setup may arise in disaster relief, humanitarian assistance, and military applications where users are naturally partitioned into teams. In these settings, communication often cannot rely upon pre-built infrastructure such as cellular networks, and instead mobile multihop networks may be deployed [1] . One emerging interest in these domains is the use of directional antennas at radio nodes. Directional antennas can greatly extend the range of communications between groups, improve network throughput in densely populated areas, and reduce power consumption [2].

In order to effectively communicate, directional antennas must locate and track neighboring nodes in the environment in order to steer antenna beams towards one another and establish communication links [3]. This introduces the problem of topology control where a network controller must choose which node pairs should form connections. The set of chosen pairs composes a topology over which network traffic may be routed.



(a) Line

(b) Group Cliques

(c) Clique

Fig. 1: Topologies

Topology control for wireless networks with directional antennas has been previously studied from both theoretical and applied perspectives. In [4] and [5] the impact of topology design on network connectivity was analyzed and heuristic algorithms were proposed. Likewise, algorithms for generating topologies with low inter-link interference were explored in [6], and the interaction between the node-degree of a topology and path-stretch was examined in [7]. Additionally, the topology control problem has also been studied in specific application domains such as: local area and mesh networks [8], [9], facilities networks [10], millimeter wave [11], optical wireless [12], and airborne communications [13]. The use of directional antennas in emergency response networks was recently studied in [14] and [15].

This work examines the topology control problem in group-based wireless networks that use directional antennas. In this application, network controllers should focus on constructing

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topologies that both support the high traffic demand that is internal to each group while simultaneously allowing groups to disseminate smaller, network-wide traffic loads amongst themselves. Naturally, one might suspect that dense network topologies that have a large number of edges in the topology subset should support high network throughput. However, herein we show that if other design considerations are not taken into account, the gains of deploying denser topologies may be greatly reduced.

For instance, if each node has only one radio processing chain, then it may only transmit to or receive from one other node at a time (a constraint that has been shown to well model the interference behavior of networks with highly-directional antennas [16]). Therefore, conflicting links must be scheduled during separate time intervals. We show that commonly used, suboptimal schedules may erase many of the gains of denser topologies and therefore such topologies may require optimal transmission scheduling. Additionally, we examine the effect of both unicast and multicast traffic on a topology's performance, finding that the type of traffic can greatly impact the topology design problem. Policies for maximizing the multicast and broadcast throughput of a network have been previously considered in the literature [17]–[19]. Herein, as part of our contribution, we derive a novel method for analyzing the multicast throughput of a topology under reasonable routing restrictions.

The rest of the paper is organized as follows. The system model and problem assumptions are defined in Section II. In Section III, we examine the impact of transmission scheduling on the topology control problem for a single group. In Section IV, a method for characterizing the multicast throughput of a topology spanning multiple groups is formulated. We then use this method in Section V to analyze the performance of several topology designs for both unicast and multicast traffic. We conclude the paper with some final remarks.

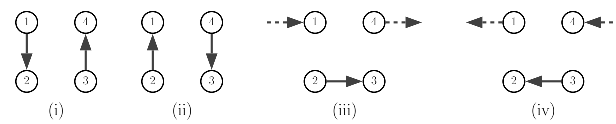
II. System model

We consider a wireless network modeled as a weighted graph $G = (N, E)$ consisting of nodes $i \in N$ and directed edges $(i, j) \in E$. Each edge has capacity c_{ij} indicating the total amount of traffic that can be carried over the edge. At any time a node in the network can either transmit to or receive from at most one other node in the network. Due to the use of highly directional antennas, we assume that concurrent transmissions on any pair of edges (i, j) and (i', j') for i, i', j, j' unique, do not interfere. This edge activation and interference model is termed primary interference and it influences the set of edges that can be activated simultaneously in a schedule.

We consider communication networks consisting of groups or teams of equal size M; i.e., N is a multiple of M. We are specifically interested in scenarios where the edge capacity for a pair of nodes in the same group is greater than or equal to the edge capacity for a pair of nodes in different groups, and in scenarios where more network traffic is sent within groups than between groups. However, our techniques for computing throughput are not restricted to these cases and can be applied in more general scenarios.

The traffic model is as follows. There is a set C of flows or commodities in the network, where each commodity $c \in C$ is specified by a source node $s_c \in N$ and a set of destination nodes $D_c \subseteq N$. The source node injects the commodity's data into the network at a rate λ_c and every bit of injected data must be received by every destination $d_c \in D_c$. Note that this is a general model of multicast traffic, and that the special cases $|D_c| = 1$ and $|D_c| = N - 1$ correspond to unicast and broadcast traffic, respectively.

The topology of the network dictates the set of edges E and we have considered a wide variety of topologies in our analysis. Here we focus on the three shown in Fig. 1. These include the line topology; a topology we term "Group Cliques," where the M nodes in each group are fully connected and each node maintains up to one edge with adjacent groups; and a fully connected clique topology.



(a) Activations for line network.

(i) 1/4	(ii) 1/4	(iii) 1/4	(iv) 1/4

(b) Uniform schedule

(i) 3/14	(ii) 3/14	(iii) 2/7	(iv) 2/7

(c) Optimal schedule for all-all unicast traffic

Fig. 2: Example schedules for a line network (cf. Fig 1a) with a group of $M = 4$ nodes. Edge activations (i)–(iv) shown in (a); dashed arrows indicate activations with nodes outside of the local group. Timeslots for (b) uniform and (c) optimal schedules represented by boxes labeled with associated edge activation and slot duration.

For each topology, a variety of schedules are possible. A schedule consists of a sequence of timeslots, where in each timeslot a set of edges that satisfy the primary interference constraint is activated. We assume that a schedule is generated before packets are sent through the network, and that the same schedule is repeated through the duration of network use. Example schedules for a four-node network with a line topology are shown in Fig. 2. We consider two classes of schedules: uniform schedules and optimal schedules. Both classes of schedules have the property that all network edges in the topology are activated at least once. For a uniform schedule, all edges in the network are activated for the same portion of time. As shown in Fig. 2b, there are four slots of equal length, and each edge is activated once, so each edge is activated $1/4$ of the time. For optimal schedules, edges in the network may be activated for different portions of time, and the amount of time allocated to each edge is based on the traffic demand on that edge, which in turn is dictated by the traffic model and routing. As shown in Fig. 2c, some edges are activated $3/14$ of the time, while others are activated $2/7$ of the time. In terms of maximizing throughput in a multi-user network, uniform schedules and optimal schedules represent two extremes in performance. While it may provide lower throughput, a uniform schedule can be easier to compute and to apply than an optimal schedule.

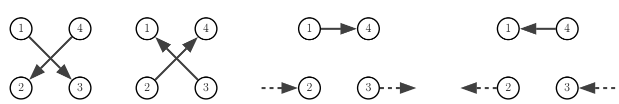
III. Throughput regions for one group

This section considers interactions between scheduling policies and topology, and the effect this has on the network throughput region. The throughput region is the set of all arrival rates that can be supported by the network, accounting for tradeoffs in feasible flow for all combinations of sources and destinations. We construct throughput regions for a network consisting of one group of $M = 4$ nodes connected as (a) a linear topology or (b) a group clique topology, as shown in Fig. 3. Results for a full clique topology will depend on the total number of nodes in the network, and will follow the trends of the group clique. For simplicity, we assume all links to have unit rate.

(a) Linear (b) Group Clique

Fig. 3: Connectivity in a four node group. Solid lines show connections within the group; dashed lines show connections to other groups.

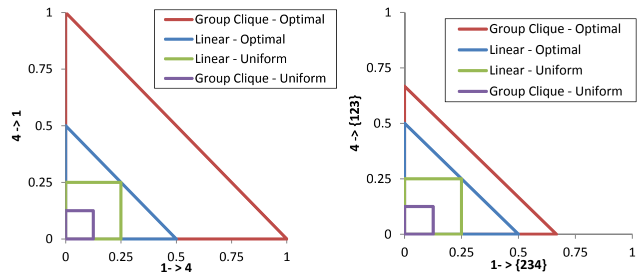
Recall that an edge activation is the set of links that are active at a given point in time. The number of activations needed by a topology is significant to the performance of a uniform schedule, since the schedule must cycle through all activations in the set. For the line topology, four activations are required for a uniform schedule (e.g. activations (i)–(iv) in Fig. 2a), yielding a maximum throughput of $1/4$ for any flow. For the group clique topology, however, eight activations are required for a uniform schedule (e.g. activations (i)–(iv) in Fig. 2a and activations (v)–(viii) in Fig. 4), decreasing maximum throughput to $1/8$ for any flow. Even more activations are required for the full clique topology with a uniform schedule, where maximum throughput decreases to $\frac{1}{2(N-1)}$ since there must be an activation for each of $N-1$ outgoing edges and $N-1$ incoming edges.



(v) (vi) (vii) (viii)

Fig. 4: Activations (v)–(viii) for group clique topology, in addition to activations (i)–(iv) from Fig. 2a.

The full throughput region for a four node network would include all $4 \cdot 3 = 12$ possible unicast traffic flows and all $4 \cdot (2^3 - 1 - 3) = 16$ non-degenerate multicast traffic flows. Thus, the full throughput region would be a 28-dimensional polytope. For simplicity we restrict our discussion to 2-dimensional slices of these regions. We show the throughput tradeoff between two unicast flows in Fig. 5a, and for two multicast flows in Fig. 5b.



(a) Unicast (b) Multicast

Fig. 5: Throughput regions for example flows.

For unicast traffic, we consider flows in both directions between nodes 1 and 4. We annotate the flow in the upward direction as $1 \rightarrow 4$ and the flow in the downward direction as $4 \rightarrow 1$. For the linear topology, as noted above, each uniform schedule is active $1/4$ of the time. It may be seen that while traffic multihops through the linear topology, the two flows do not conflict with each other and so each flow may achieve a maximum throughput of $1/4$ simultaneously. By using an optimal schedule, the $1 \rightarrow 4$ traffic flow can be maximized to achieve a throughput of $1/2$ by using the linear activations (i) and (iii) from Fig. 2a for $1/2$ time each. Likewise, the $4 \rightarrow 1$ traffic flow can be maximized to achieve a throughput of $1/2$ by alternating between the second and fourth linear activations. For the group clique topology, there is a link directly connecting nodes 1 and 4 in each direction. Since each of these links is active only $1/8$ of the time with a uniform schedule, each flow can achieve a maximum throughput of $1/8$ (again simultaneously). An optimal schedule could simply activate the direct link 100% of the time, thus either flow could individually achieve a rate of 1. Either optimal schedule can time share between the two traffic flows, thus the throughput region is convex across those peak throughput rates. Here we have assumed that unicast traffic is delivered only along a single path for the group clique topology; if multipath routing is allowed instead, then maximum per-flow throughput with a uniform schedule can increase from $1/8$ to $3/8$.

For multicast traffic, we consider a flow from node 1 to nodes 2, 3, and 4, annotated as $1 \rightarrow \{2, 3, 4\}$, and a flow from node 4 to nodes 1, 2, and 3, annotated as $4 \rightarrow \{1, 2, 3\}$. For the linear topology, the multicast throughput regions are identical to the unicast throughput regions, since the unicast traffic for flows $1 \rightarrow 4$ and $4 \rightarrow 1$ must already pass through intermediate nodes 2 and 3. For the group clique topology, the throughput region for the uniform schedule is again upper bounded by $1/8$ since each of the direct links is active $1/8$ of the time. For the optimal schedule, either individual multicast flow requires

3 link activations to deliver a message; since there are 2 link activations per schedule, we can deliver 2 messages every 3 time slots for a maximum throughput of $2/3$. Again, if both multicast flows are simultaneously active, the flows can time share between the activations needed for maximum throughput to achieve the convex region as shown.

We have observed that the additional connectivity in the group clique topology can actually decrease maximum throughput when a uniform schedule is used. This is because resources must be divided amongst every link in the network, even though some links may be underutilized in a non-uniform traffic flow. Thus, adding connectivity alone is not sufficient for improving throughput; the addition of new links should be combined with an optimal scheduler to realize an increase in the supportable throughput region.

IV. Max-min multicast throughput for arbitrary networks

We now describe an upper bound on the throughput of an arbitrary network that carries multicast traffic and supports optimal scheduling. The notion of throughput that we consider is *concurrent or max-min* throughput denoted by ρ: given a set of traffic flows C and demand rates λ_c for each, we will aim to maximize the scaling of the desired demand that can be simultaneously achieved by all flows. In other words, the max-min throughput quantifies how much traffic can be supported for the worst among all flows, while all other flows are able to achieve at least as much as ρ times their demand rate.

First we describe how optimal scheduling will be represented when computing network throughput. Let y_{ij} denote the total flow rate, over all commodities, across edge (i,j) and $\vec{y} = \{y_{ij}\}$ the set of all such flow rates. A well known result due to [20] is that $\vec{y}$ may be feasibly scheduled under the primary interference model (i.e., there exists a time slotted set of transmissions that may meet the demand of $\vec{y}$ without violating the primary interference constraint) only if

$$\sum_{j:(i,j)\in E} \frac{y_{ij}}{c_{ij}} + \sum_{j:(j,i)\in E} \frac{y_{ji}}{c_{ji}} \leq 1, \quad \forall i \in N. \tag{1}$$

Note that this is only a necessary condition. A sufficient condition for feasible scheduling may be obtained by replacing 1 on the left hand side of (1) with $\frac{2}{3}$ [20]. We will use (1) to represent the scheduling constraint while computing network throughput. Since (1) is a necessary condition, this implies that the throughput we compute will be an upper bound.

For multicast traffic, information must be routed over a set of trees, where each tree t_c in G is rooted at source s_c and spans the set of all destinations D_c. In this work we consider *trees that only span the source and destinations as admissible routes*. Thus, $s_c \in t_c$ and $d_c \in t_c$, $\forall d_c \in D_c$, but for all other nodes $i \notin \{s_c, D_c\}$, we assume $i \notin t_c$. We define T_c as the set of all trees that meet this condition for commodity c.

We are now ready to specify the multicast problem. We introduce variables q_{t_c} that specify the rate at which commodity c is routed over tree t_c. The problem formulation follows.

$$\max \rho$$
s.t.:
$$\sum_{t_c \in T_c} q_{t_c} = \rho\lambda_c, \quad \forall c \in C \tag{2a}$$
$$\sum_{j\in N}\sum_{c\in C}\sum_{\substack{t_c\in T_c:\\(i,j)\in t_c}} \frac{q_{t_c}}{c_{i,j}} + \sum_{j\in N}\sum_{c\in C}\sum_{\substack{t_c\in T_c:\\(j,i)\in t_c}} \frac{q_{t_c}}{c_{j,i}} \leq 1,$$
$$\forall i \in N, \quad q_{t_c} \geq 0, \quad \forall c \in C, \forall t_c \in T_c \tag{2b}$$

Note that constraint (2a) requires that for each commodity, the total flow sent over all trees in T_c delivers demand $\rho\lambda_c$, where ρ is maximized. Constraint (2b) is the same as (1) rewritten for the multicast tree formulation.

The number of variables in (2) in general grows exponentially with the number of nodes in a commodity $|D_c|$, since the number of trees spanning the destinations grows exponentially. In this section we will show that there exists a linear programming formulation equivalent to (2) that does not require the enumeration of the trees. First, let x_{ij}^c denote the flow rate for commodity c on edge (i,j). Additionally let f_{ij}^{c,d_c} denote the constituent of x_{ij}^c that is intended for destination $d_c \in D_c$. Consider the following linear program.

$$\max \rho$$
s.t.:
$$\sum_{j:(i,j)\in E} f_{ij}^{c,d_c} - \sum_{j:(j,i)\in E} f_{ji}^{c,d_c} = \begin{cases} \rho\lambda_c, & i = s_c \\ -\rho\lambda_c, & i = t_c \\ 0, & \text{else} \end{cases},$$
$$\forall i \in N, \forall c \in C, \forall d_c \in D_c \tag{3a}$$
$$x_{ij}^c \geq f_{ij}^{c,d_c}, \quad \forall (i,j) \in E, \forall c \in C, \forall d_c \in D_c \tag{3b}$$
$$y_{ij} = \sum_{c\in C} x_{ij}^c, \quad \forall c \in C, \forall (i,j) \in E \tag{3c}$$
$$\sum_{j:(i,j)\in E} \frac{y_{ij}}{c_{ij}} + \sum_{j:(j,i)\in E} \frac{y_{ji}}{c_{ji}} \leq 1, \quad \forall (i,j) \in E \tag{3d}$$
$$f_{ij}^{c,d_c} = 0, \quad \substack{\forall c\in C, \forall d_c \in D_c, \forall (i,j)\in E \\ \text{such that } i \notin \{s_c, D_c\} \text{ or } j \notin \{s_c, D_c\}} \tag{3e}$$

Here, (3a) is a flow-conservation constraint applied to each destination of each flow, constraints (3b) and (3c) follow from the definition of the flow variables, and (3d) is the scheduling constraint. The final constraint (3e) restricts flow to the set of trees T_c.

The linear program above does not require the enumeration of multicast trees, which makes it computationally feasible. Moreover, it can be shown that for our group-based communication model where commodities and routes are restricted so that traffic only traverses nodes that participate in the commodity, the result of the linear program above is equivalent to the result obtained through enumeration of all trees. This result is stated in the following theorem. The theorem assumes channel reciprocity, i.e., $\forall (i,j) \in E$, $c_{i,j} = c_{j,i}$.

Theorem 1. *There exists a solution to* (3) *obtaining an objective* ρ^* *if and only if there exists a solution to* (2) *obtaining an objective* ρ^*.

Proof: The complete proof is omitted for brevity. Herein we provide the intuition.

If. Suppose there exists a solution to (2) that obtains an objective ρ^*. Then we may construct a solution to (3) by setting

$$x_{ij}^c = \sum_{t_c \in T_c : (i,j) \in t_c} q_{t_c}.$$

By construction, constraints (3c), (3d), and (3e) are immediately met. Now, from (2a), for each destination $d_c \in D_c$, there exists a set of simple paths from source s_c to d_c such that the sum of flow along the paths equals $\rho^* \lambda_c$ and the sum of flow along any edge (i,j) is no greater that x_{ij}^c. Thus, variables f_{ij}^{c,d_c} may be found such that constraints (3a) and (3b) are met as well.

Only if. Suppose there exists a solution to (3) that obtains an objective ρ^*. Then, for each commodity $c \in C$, variables x_{ij}^c define a subgraph spanning only nodes $i \in \{s_c, D_c\}$. By (3a) and (3b), for each $d_c \in D_c$ there exists variables obeying conservation of flow over this subgraph and the capacity constraints x_{ij}^c such that a rate of flow $\rho^* \lambda_c$ may be supported between s_c and d_c. Because of this, it may be shown that we may partition this subgraph into a set of spanning trees that can support a rate of flow $\rho^* \lambda_c$ between s_c and all $d_c \in D_c$. Doing so gives variables q_{t_c} that meet constraint (2a). Since x_{ij}^c meets conditions (3c), (3d), and (3e), the constructed variables q_{t_c} are a feasible solution to (2). ■

V. Results and discussion

In this sections we present results from computing the upper bound on network throughput for unicast, multicast and broadcast traffic models. The model considered in these numerical results is as follows. We have up to six groups of $M = 4$ nodes each. For edge capacities, we consider two different models. First, we consider the case of *uniform edge capacities* and set $c_{ij} = 16$Mbps on all edges in the network. Next we consider *non-uniform edge capacities.* In this model, the edges within a group have capacity 16Mbps, while edges between nodes in adjacent groups have capacity 8Mbps. For edges connecting nodes in groups separated by one or more groups (which occur only in the clique topology), the edge capacity depends on distance: edges connecting groups separated by one other group have capacity 4Mbps, edges connecting groups separated by two other groups have capacity 2Mbps, and so on.

For unicast traffic, we assume that each node in the network sources one flow of equal demand to all other nodes in the network; in other words, there are $N(N-1)$ unicast commodities of equal demand $\lambda_c = 1$. For broadcast traffic, we assume that each node sources one commodity, and that commodity is destined to all other nodes in the network; so there are N broadcast commodities, and each commodity has $N-1$ destinations. Finally, for multicast traffic, we consider a group-based model. Each node sources a total of two commodities and 1 unit of demand: 0.7 units of demand for a commodity that is destined to the three other nodes in the same group as the source; and 0.3 units of demand for a commodity that is destined to all nodes in the same group and nodes in one adjacent group. This multicast traffic model represents localized information diffusion.

We compute an upper bound network throughput using the approach outlined in Section IV. For unicast traffic, the constraint (3e), which restricts traffic to only traverse nodes that participate in the flow, is dropped. Additionally, while the linear program in Section IV is used to compute results for an optimal schedule, we add an additional constraint to compute results for a uniform schedule. The additional constraint takes the form,

$$y_{ij} \leq \frac{1}{R} c_{ij}, \quad \forall (i,j) \in E \tag{4}$$

where R is a constant and depends on the topology. For a line topology, $R = 4$, while for the Group Cliques topology, $R = 8$.

The results are presented in Figs. 6 and 7. For all traffic models, the line topology and uniform schedule provide the lowest throughput, while the clique topology and optimal schedule provide the highest throughput. The relative performance of other topology and schedule combinations varies with the traffic model. For unicast traffic, the throughput improvement due to a more densely connected topology ranges between a factor of two and ten, and increases with the number of nodes. In this case, the optimal scheduling approach offers little benefit over uniform schedules, while a more densely connected topology can offer significant improvement. By contrast, for multicast and broadcast traffic, the throughput improvement offered by a densely connected topology is limited: it's approximately fixed with the number of nodes to about a factor of two. Here, the use of optimal scheduling provides greater benefit than a densely connected topology.

In addition to the throughput benefits offered by topology and schedule design, these results show the merits and drawbacks of specific schemes. Notably, a more densely connected topology does not necessarily provide higher throughput than a more sparsely connected topology. Rather, their relative performance depends critically on the type of scheduling used. Specifically, the Group Cliques topology with a uniform schedule performs worse than the Line topology with an optimal schedule, for multicast and broadcast traffic. This conclusion underscores the interdependence between topology and schedule design.

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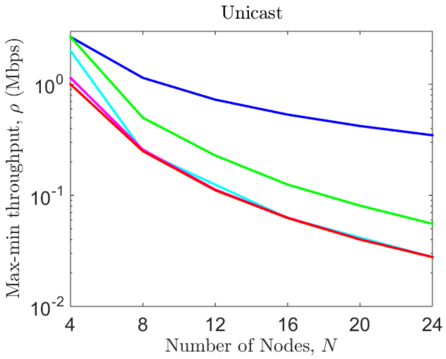
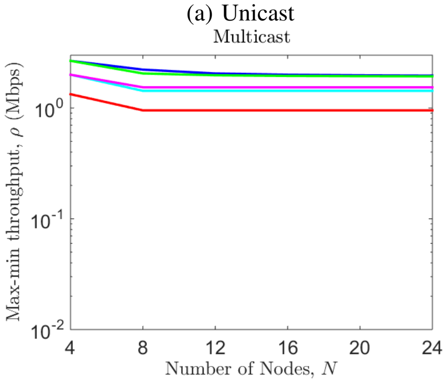
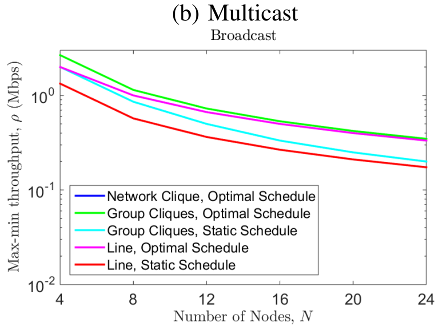


(a) Unicast

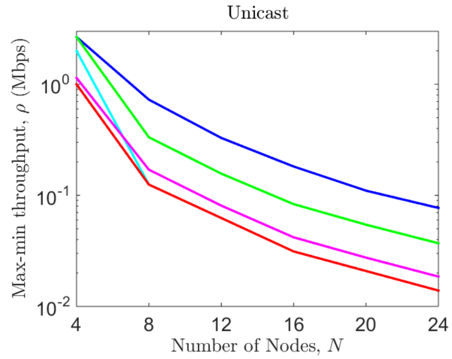


(b) Multicast

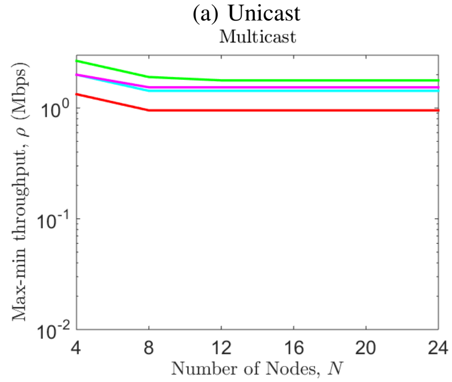


(c) Broadcast

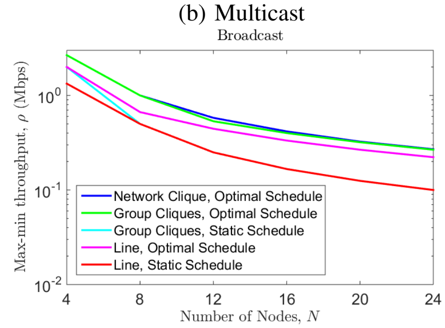
Fig. 6: Max-min throughput ρ versus number of nodes for uniform edge capacities with $c_{ij} = 16$Mbps



(a) Unicast



(b) Multicast



(c) Broadcast

Fig. 7: Max-min throughput ρ versus number of nodes for non-uniform edge capacities

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