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A Model for Microcontroller Functionality Upset Induced by External Pulsed Electromagnetic Irradiation

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A Model for Microcontroller Functionality Upset Induced by External Pulsed Electromagnetic Irradiation

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0. Abstract

We present a model that provides predictions for the occurrence of disruptive deviations of signal line activity in—and consequent malfunction of—a microcontroller (μC) subjected to external irradiation by a narrowband electromagnetic (EM) pulse. In our model, the state of a μC is completely specified by giving, for each of its relevant signal lines, the signal pulse train (SPT) time history on the line during any fixed but arbitrary time window of interest. The occurrence of such disruptive deviations is observed experimentally to behave stochastically in at least some EM pulse frequency regimes—for example, in the radio frequency (RF) regime—and our model provides predictions for the probability of such disruptions based upon all relevant characteristics of the EM pulse and of the μC SPT's. In the present paper we focus our attention on signals traversing a single μC signal line.

I. Introduction

In this paper, we are interested in the following situation. Initially, an electromagnetic (EM) pulse impinges upon a microcontroller (μC) in which at least one signal line is active, this activity resulting from (1) a single software application executing on the μC or from (2) other dynamic actions of the μC—"system tasks"—e.g., system clock pulse generation, program counter increment, instruction fetch, or the hardware-implemented portion of interrupt handling. This pulse electromagnetically couples to the μC at one or more of its spatial locations (e.g., at μC control or data lines or at register or memory structures or at power distribution system components). This coupling quantitatively depends upon the hardware details of the μC and may cause sufficient deviations of the normal time-dependent signal pulse trains (SPT's) on some of the signal lines, during some fixed but arbitrary finite time window $[0, T_E)$ of interest, to disrupt proper μC functionality (i.e., proper software application task or system task execution) sometime during that window. We term this disruption μC "*upset*". This upset is observed experimentally to occur stochastically in the radio frequency (RF) regime—and we present here a signal-centric model that provides predictions of the probability, P^{upset}, of such upset based upon all relevant characteristics of the EM pulse and of the μC SPT's.

There has been significant research into the effects of RF signals at the device level (Richardson 1979, Firestone 2004, Estep 2010, 2011, Estep et al 2011), as well as at the circuit and chip level (Laurin et al 1995, Liu & Ho 1995). More recently the 2001-2006 RF Effects MURI (Granatstein 2007) provided significant advances in understanding and predicting RF effects (Firestone et al 2006, Hemmady et al 2005, Yang & Kollman 2006, Wang et al 2006). Building on this work to develop a system-level model for RF effects on a full digital system such as a PC is daunting, in part because of the vast number of transistors that make up a modern CPU or memory module, but also because a PC consists of multiple printed circuit boards each containing many integrated circuits, discrete components and interconnects. These boards are connected together, housed in a metal box and connected to various peripheral devices. For these reasons a microcontroller, being a simple but complete computer on a single chip and thus representing an intermediate level of complexity between an individual CMOS device and a complete digital system, is an ideal device to use as a test-bed in attempting to understand and predict RF effects on digital electronic systems (Taylor 2011, Clarke et al 2011, Clarke et al 2012, Henderson et al 2012). Earlier research on quantifying the immunity of microcontrollers to ESD pulses (voltage spikes) (Vick & Habiger 1997, Wendsche et al 1999) indicated that the susceptibility varied with the particular instruction (or micro-instruction) being executed when the ESD pulse was applied. This suggested that carefully timed pulses relative to clock signals could yield useful information about the underlying mechanisms of RF interference, and provided much of the motivation for the current work.

The remainder of this paper proceeds as follows. In Section II we present the conceptual structure upon which our model rests, discussing the elements of a completely specified *bridge* between the application/system tasks executing on the μC and the resulting *normal* SPT's (i.e., those in the absence of an externally impingent EM pulse) on the μC lines corresponding to the execution of those tasks. We also discuss *non-normal* SPT's (i.e., those in the presence of an externally impingent EM pulse) on the μC lines resulting from the normal SPT's *via* exposure of the μC to an EM pulse. In addition, we discuss the approximations that we make in our treatment. In Section III, we consider the interaction between an *idealized*, normal SPT, $\pi^{\mathcal{N}}$, being carried on a *single signal line*, and a single EM pulse, π^{EM}, carefully characterizing for our purposes both of these entities as well as their interaction itself. This leads in Section IV, to an expression for the probability of a π^{EM}-induced disruptive deviation of $\pi^{\mathcal{N}}$. Sections III and IV are concerned with only a single signal line. Those results may be extended to an arbitrary finite number, L, of signal lines carrying an L-tuple of SPT's interacting with one and the same EM pulse, but we defer our consideration of multi-signal-lines to a subsequent paper. Finally, since *in practice* it will be rarely known with certainty which particular SPT of those possible is on the line during $[0, T_E)$, we consider, in Section V, random (stochastic) mixes of possible SPT's on a single signal line. In Section VI we conclude.

II. Conceptual Overview

In this section we discuss the conceptual structure of our model as well as the modeling approximations that we make. The goal here is to build a conceptual bridge between the executing application/system tasks and the SPT's resulting therefrom, both in the normal and non-normal cases. We understand that some of the theoretical entities we construct may not be presently readily obtainable but there is nevertheless some merit in laying down the logical structure of our model.

II.1 μC Tasks

As indicated in the Introduction, we are interested in the upset of a μC when a it is executing a (*single*) software application concurrently with system tasks during some time interval $[0, T_E)$ of interest. For us, an application is a finite set of assembly language (AL) instructions, *together with* the specification of the order of execution of—i.e., of the *time sequence* of—those AL instructions. In general, a given application in fact comprises many possible AL instruction execution sequences, which sequences we term *realizations*, as a consequence of the possibility of branching. An application having no possibility of branching (including no possibility of interrupts) has only one realization. We consider an interrupt request signal and the subsequent hardware-implemented interrupt handling response thereto to be system tasks but, in contrast, we consider the execution of the stored-in-memory interrupt servicing AL instructions themselves to be part of application execution. More generally, we use the term "system task" to refer to a μC action that requires no AL instructions in order to execute (i.e., is implemented in hardware) and we use the term "application task" to refer to a μC action that does require AL instructions in order to execute.

To facilitate further discussion, we introduce some constructs that we will subsequently employ to describe μC tasks. To begin, we label each software application that may execute on the μC by its own "application label", generically $\mathscr{a}$, and denote by $\mathscr{A}$ the finite set of all application labels arising from all those μC applications, so that $\mathscr{a} \in \mathscr{A}$. (The null application, i.e., no application actively executing, is denoted by the unique label $\mathscr{a} = 0$.) We denote by $r_1^{\mathscr{a}}, r_2^{\mathscr{a}}, \ldots, r_{N^{\mathscr{a}}}^{\mathscr{a}}$, $N^{\mathscr{a}} \geq 1$ if $\mathscr{a} \neq 0$, the finite number of all possible realizations of application $\mathscr{a}$, where each $r_j^{\mathscr{a}}$ is a finite sequence—

$r_j^{\mathscr{a}} = <\iota_{j,1}^{\mathscr{a}}, \iota_{j,2}^{\mathscr{a}}, \ldots, \iota_{j,N_j^{\mathscr{a}}}^{\mathscr{a}}>$, $N_j^{\mathscr{a}} \geq 1$ if $\mathscr{a} \neq 0$—in the set $\mathscr{I} = \{\iota_1, \ldots, \iota_{N_{\mathscr{I}}}\}$, $N_{\mathscr{I}} \geq 1$, of all assembly language instructions available to the μC; and we write (when $\mathscr{a} \neq 0$) $\mathscr{R}_{\mathscr{a}} = \{r_1^{\mathscr{a}}, r_2^{\mathscr{a}}, \ldots, r_{N^{\mathscr{a}}}^{\mathscr{a}}\} \subseteq \bigcup_{N=1}^{N_{\max}^{\mathrm{ALI}}[0,\, T_E)} \mathscr{I}^N$ (and we set $\mathscr{R}_0 = \varnothing$). Here $N_{\max}^{\mathrm{ALI}}[0, T_E)$ is the maximum number of AL instructions executable in $[0, T_E)$, namely $N_{\max}^{\mathrm{ALI}}[0, T_E) \leq [\![T_E/\tau_{c,1/2}]\!] + 1$, where $\tau_c > 0$ is the system clock period with $\tau_{c,1/2} = \tau_c/2$ and where $[\![\bullet]\!]$ denotes the greatest integer function, with $\tau_{c,1/2}$ being the lower limit of our time scale resolution. Now each instruction $\iota_{j,\ell}^{\mathscr{a}}$ executes on the μC during some time interval $[(\tau_{j,\ell}^{\mathscr{a}})_0, (\tau_{j,\ell}^{\mathscr{a}})_f)$ for which $(\tau_{j,\ell}^{\mathscr{a}})_f - (\tau_{j,\ell}^{\mathscr{a}})_0$ is, under normal execution, an integral multiple of $\tau_{c,1/2}$. Because some AL instructions require more clock periods to fully execute than do other AL instructions, it is necessary for our purposes to append, to each AL instruction in the $\mathcal{R}_j^{a}$ sequence, the initial and final time instants of the time interval in which the instruction executes. We thus specify a finite sequence of instructions executing during [0, T_E) as the finite sequence of ordered pairs $[\mathcal{R}_j^{a}]^\# = <<\iota_{j,\ell}^{a}, [(\tau_{j,\ell}^{a})_0, (\tau_{j,\ell}^{a})_f)>>_{\ell=1}^{N_j^{a}}$ where $\iota_{j,\ell}^{a} \in \mathcal{I}$ and $[(\tau_{j,\ell}^{a})_0, (\tau_{j,\ell}^{a})_f) \subseteq [0, T_E)$; and we write as well $\mathcal{R}_{a}^\# = \{[\mathcal{R}_1^{a}]^\#, [\mathcal{R}_2^{a}]^\#, \ldots, [\mathcal{R}_{N^{a}}^{a}]^\#\} \subseteq$

$\mathbf{U}_{N=1}^{N_{\max}^{\mathrm{ALI}}} [\mathcal{I} \times [0, T_E)]^N$ (and $\mathcal{R}_0^\# = \varnothing$). Note that $(\tau_{j,\ell}^{a})_f - (\tau_{j,\ell}^{a})_0 = n_{j,\ell}^{a}\tau_{c,1/2}$ where $n_{j,\ell}^{a} \in$ $\{1, 2, \ldots, N_{\max}^{\mathrm{ALI\text{-}CYC}}\}$, with $N_{\max}^{\mathrm{ALI\text{-}CYC}}$ being the maximum number of clock ½-cycles required by any AL instruction (e.g., $N_{\max}^{\mathrm{ALI\text{-}CYC}} = 1, 2, 4, 8, 16, 24$, depending upon the particular μC).

Next, we write the finite set of all system tasks as $\Theta = \{\theta_0, \theta_1, \ldots, \theta_{N_\Theta}\}$, $N_\Theta \geq 1$ (with θ_0 denoting the null system task, i.e., no system task executing), and any sequence of system tasks from Θ as $\mathcal{s}_k = <\theta_{k,1}, \theta_{k,2}, \ldots, \theta_{k,N_k^{s}}>$, $N_k^{s} \geq 1$; further, we write

$\mathcal{S} = \{\mathcal{s}_1, \ldots, \mathcal{s}_{N_\mathcal{S}}\}$, $N_\mathcal{S} \geq 1$, for the finite set of all such sequences, with $\mathcal{S} \subseteq \mathbf{U}_{N=1}^{N_{\max}^{\mathrm{ST}}[0, T_E)} \Theta^N$, where $N_{\max}^{\mathrm{ST}}[0, T_E)$ is the maximum number of system tasks executable in [0, T_E). (Note that it is not in general true that $N_{\max}^{\mathrm{ST}}[0, T_E) \leq [\![T_E/\tau_{c,1/2}]\!] + 1$ since several system tasks may execute simultaneously during any given clock ½-cycle.) As in the case of AL instructions, each system task $\theta_{k,m}$ executes on the μC during some time interval $[(\tau_{k,m}^{s})_0, (\tau_{k,m}^{s})_f)$ for which $(\tau_{k,m}^{s})_f - (\tau_{k,m}^{s})_0$ is, under normal execution, an integral multiple of $\tau_{c,1/2}$ (or is taken by us to be so since $\tau_{c,1/2}$ is the lower limit of our time scale resolution); further, since interrupts in general occur irregularly in time, it is necessary for our purposes to append, to each system task $\theta_{k,m}$ in the $\mathcal{s}_k$ sequence, the initial and final time instants of the time interval in which it executes. We thus specify a finite sequence of system tasks executing during $[0, T_E)$ as the finite sequence of ordered pairs $[\mathscr{s}_k]^{\#} = \langle\langle \theta_{k,m}, [(\tau^{\mathscr{s}}_{k,m})_0, (\tau^{\mathscr{s}}_{k,m})_f) \rangle\rangle_{m=1}^{N^{\mathscr{s}}_k}$ where $\theta_{k,m} \in \Theta$ and $[(\tau^{\mathscr{s}}_{k,m})_0, (\tau^{\mathscr{s}}_{k,m})_f) \subseteq$ $[0, T_E)$; and we write as well $\mathscr{S}^{\#} = \{[\mathscr{s}_1]^{\#}, [\mathscr{s}_2]^{\#}, \ldots, [\mathscr{s}_{N_{\mathscr{S}}}]^{\#}\} \subseteq \mathbf{U}_{N=1}^{N^{ST}_{max}[0,T_E)} [\Theta \times [0, T_E)]^N$. Note that $(\tau^{\mathscr{s}}_{k,m})_f - (\tau^{\mathscr{s}}_{k,m})_0 = n^{\mathscr{s}}_{k,m}\tau_{c,1/2}$, where $\{1, 2, \ldots, N^{\text{ST-CYC}}_{max}\}$ with $N^{\text{ST-CYC}}_{max}$ being the maximum number of clock ½-cycles required by any individual system task (typically $n^{\mathscr{s}}_{k,m} = 1$). Further, in addition to possible overlap in time of system tasks, i.e., $[(\tau^{\mathscr{s}}_{k,m})_0, (\tau^{\mathscr{s}}_{k,m})_f) \cap [(\tau^{\mathscr{s}}_{k,m'})_0, (\tau^{\mathscr{s}}_{k,m'})_f) \neq \emptyset$ for some $m, m' \in \{1, \ldots, N^{\mathscr{s}}_k\}$ with $m \neq m'$, it is also possible that an AL instruction and a system task may execute simultaneously at the ½-cycle time scale (e.g., in the case of a pre-fetch capability in some μC′s) so that $[(\tau^{\mathscr{a}}_{j,\ell})_0, (\tau^{\mathscr{a}}_{j,\ell})_f) \cap [(\tau^{\mathscr{s}}_{k,m})_0, (\tau^{\mathscr{s}}_{k,m})_f) \neq \emptyset$ for some $\ell = 1, \ldots, N^{\mathscr{a}}_j$ and $m = 1, \ldots, N^{\mathscr{s}}_k$.

We observe that the system task sequence fundamentally differs from the AL instruction sequence in that the latter is "quasi-deterministic" in time while the former—if it contains interrupts—may be irregular in time. That is, in the absence of interrupts, the AL instruction execution sequence timing is predictable for any given application realization while, on the other hand—for either the active or quiescent state of any executing realization of some application—the interrupt sequence timing is in general not predictable since, for example, external interrupts are activated by conditions appearing irregularly on devices external to the μC (which devices are monitored by the μC's input pins). This renders as non-deterministically predictable the particular realization, $\mathscr{r}^{\mathscr{a}}$, of an application $\mathscr{a}$, that is actually executed in $[0, T_E)$ in any particular instance of the execution of $\mathscr{a}$, since $\mathscr{r}^{\mathscr{a}}$ itself comprises the interrupt-servicing AL instructions; hence the inference from any particular $\mathscr{r}^{\mathscr{a}}$ instruction stream of the induced SPT's on the μC signal lines as a result of the execution of $\mathscr{a}$ also suffers from this unpredictability. For us, the statement that application $\mathscr{a}$ is executing during $[0, T_E)$ is tantamount to the statement that a particular realization of $\mathscr{a}$, say $\mathscr{r}^{\mathscr{a}}_{j^*} \in \mathscr{R}_{\mathscr{a}}$, is executing during $[0, T_E)$. However, to analyze the μC SPT behavior during execution of $\mathscr{a}$, we must resort to analyzing some statistical mix of $\mathscr{r}^{\mathscr{a}}$'s that are *possibly* executing during that $[0, T_E)$, hence some statistical mix of SPT's that are *possibly* executing during that $[0, T_E)$. We address this state-of-affairs in Section V where we discuss random (i.e., stochastic) collections of SPT's.

As per the above considerations, we may indicate any possible application-$\mathscr{a}$/system task combination executing on the μC during $[0, T_E)$ by the pair $<[\mathscr{r}^{\mathscr{a}}]^{\#}, [\mathscr{s}]^{\#}> \in \mathscr{R}^{\#}_{\mathscr{a}} \times \mathscr{S}^{\#}$.

There is however an additional factor that must be taken into account: the time-dependent data (numbers) occupying the various μC registers and memory locations. These values, being represented by binary 0's and 1's, determine which data signal lines are low and which are high at any particular instant, i.e., the SPT's on these data lines. These values arise from user inputs to the application, or from inputs to the application by the aforementioned external devices connected to the μC, or as values computed *via* arithmetic manipulations by the application. In principle these data may be represented by a time sequence of the data contained in each register and memory location used by the application. If we label the registers used by realization $\mathscr{r}^{\mathscr{a}}_j$ of

application $\mathscr{a}$ as $\Re = \{\rho_1, \rho_2, \ldots, \rho_{N_\rho(\mathscr{r}^{\mathscr{a}}_j)}\}$, $N_\rho(\mathscr{r}^{\mathscr{a}}_j) \geq 1$ if $\mathscr{a} \neq 0$, and if we label the memory locations used by application $\mathscr{a}$ as $\mathcal{M} = \{\mu_1, \mu_2, \ldots, \mu_{N_\mu(\mathscr{r}^{\mathscr{a}}_j)}\}$, $N_\mu(\mathscr{r}^{\mathscr{a}}_j) \geq 1$ if $\mathscr{a} \neq 0$, then we may represent the data history as $< <d_{\rho_p}(n\tau_{c,1/2})>_{p=1}^{N_\rho(\mathscr{r}^{\mathscr{a}}_j)}, <d_{\mu_q}(n\tau_{c,1/2})>_{q=1}^{N_\mu(\mathscr{r}^{\mathscr{a}}_j)} >_{n=0}^{N^{CYC}[0,\ T_E)}$ where $d_{\rho_p}(n\tau_{c,1/2})$ is the value of the data in register ρ_p at time $n\tau_{c,1/2}$ (the beginning of ½-cycle n), and similarly for $d_{\mu_q}(n\tau_{c,1/2})$, and $N^{CYC}[0, T_E) = [\![T_E/\tau_{c,1/2}]\!]$. Alternatively, we may represent the data histories of the registers and memory locations in a fashion that mimics the representation method that we used for $[\mathcal{h}^{a}]^{\#}$ and $[\mathcal{s}]^{\#}$. To this end we note that since, for any given register, ρ_p, there are only a finite number of instants at which the data value in that register actually changes, say at instants $0 < \tau_1^{\rho_p} < \ldots < \tau_{N_{\rho_p}} < T_E$ (with $\tau_0^{\rho_p} = 0$ and $\tau_{N_{\rho_p}+1} = T_E$) then we may write the time history of the data in register ρ_p as the sequence $<<d_{\rho_p}(\tau_{u-1}^{\rho_p}), [\tau_{u-1}^{\rho_p}, \tau_u^{\rho_p})>>_{u=1}^{N_{\rho_p}+1}$ where $d_{\rho_p}(\tau_u^{\rho_p}) \neq d_{\rho_p}(\tau_{u-1}^{\rho_p})$ and if

$t, t' \in [\tau_{u-1}^{\rho_p}, \tau_u^{\rho_p})$ then $d_{\rho_p}(t) = d_{\rho_p}(t')$. We then write

$d_{\mathfrak{R}}^{\#}(\mathcal{h}_j^{a}) = <<<d_{\rho_p}(\tau_{u-1}^{\rho_p}), [\tau_{u-1}^{\rho_p}, \tau_u^{\rho_p})>>_{u=1}^{N_{\rho_p}+1}>_{p=1}^{N_\rho(\mathcal{h}_j^{a})}$ for the histories of all the registers employed by $\mathcal{h}_j^{a}$. Analogously, for the memory locations employed by $\mathcal{h}_j^{a}$ we write $d_{\mathcal{M}}^{\#}(\mathcal{h}_j^{a})$ = $<<<d_{\mu_q}(\tau_{v-1}^{\mu_q}), [\tau_{v-1}^{\mu_q}, \tau_v^{\mu_q})>>_{v=1}^{N_{\mu_q}+1}>_{q=1}^{N_\mu(\mathcal{h}_j^{a})}$; and we then write $d_{\mathfrak{R},\mathcal{M}}^{\#}(\mathcal{h}_j^{a}) = <d_{\mathfrak{R}}^{\#}(\mathcal{h}_j^{a}), d_{\mathcal{M}}^{\#}(\mathcal{h}_j^{a})>$. Finally, we denote the set of all possible data histories of the form $d_{\mathfrak{R},\mathcal{M}}(\mathcal{h}_j^{a})$ as $\mathfrak{D}_{\mathcal{h}_j^{a}}^{\#}$ so that

$d_{\mathfrak{R},\mathcal{M}}^{\#}(\mathcal{h}_j^{a}) \in \mathfrak{D}_{\mathcal{h}_j^{a}}^{\#}$.

We may now indicate any possible application-a/system task combination executing on the μC during $[0, T_E)$ by the triple $\mathcal{T}_{a} = <[\mathcal{h}^{a}]^{\#}, [\mathcal{s}]^{\#}, d_{\mathfrak{R},\mathcal{M}}^{\#}(\mathcal{h}^{a})> \in \mathcal{R}_{a}^{\#} \times \mathcal{S}^{\#} \times \mathfrak{D}_{\mathcal{h}^{a}}^{\#}$, which triple specifies collectively an AL instruction stream, a system task stream, and streams of all applicable register and memory location contents, with all of these streams having timing information precisely specified. Since the three components of $\mathcal{T}_{a}$ are each precisely specified then the SPT's resulting therefrom on all μC signal and data lines are then in principle also precisely inferable. This conversion from $\mathcal{T}_{a}$ to the resulting induced SPT's is *the bridge* to which we have alluded previously. However, since having such precise knowledge is not realistic—because, for example, of the randomness in the timing of the interrupts of $[\mathscr{s}]^{\#}$ as well as the randomness in $d^{\#}_{\mathfrak{R},\,\mathcal{M}}(\mathfrak{r}^{a})$ of the values and timing of the data sent to the μC by external devices connected to it. Nevertheless, in order to proceed we will assume initially that a (precisely specified as above) $\mathcal{T}_{a}$ is given so that its associated resulting SPT's on all signal lines are known precisely as well (although we will not provide a prescription for the explicit SPT's so resulting). For our purposes it is sufficient to know that such a bridge—between a precisely described $\mathcal{T}_{a}$ and its associated induced SPT's—exists. We remind the reader that we will address, in Section V, this uncertainty as to which $\mathcal{T}_{a}$ is actually executing during any particular execution run of application $a \in \mathcal{A}$.

II.2 μC SPT's

We are now prepared to describe the normal μC execution of task $\mathcal{T}_{a}$ as well as the disruption of the execution of $\mathcal{T}_{a}$ by an EM pulse. In light of our aforementioned focus, we will do so from a signal-centric point of view. Consider then an application/system task $\mathcal{T}_{a}$ active during $[0, T_E)$. In what follows we will also be interested in some non-void "local" time sub-window $[T_0, T_f) \subseteq [0, T_E)$ that will designate the total time extent of π^{EM}. From our signal-centric point of view, the task $\mathcal{T}_{a}$ may be represented *via* a μC *state* that is completely specified by giving, for each relevant signal line of the μC, the SPT time history on the line during $[0, T_E)$ (say referenced to the line midpoint for definiteness) hence also during $[T_0, T_f)$. That is, if there are L such signal lines, labeled by $\ell = 1, \ldots, L$, then the μC state for the time window $[0, T_E)$ is given by a finite sequence $<\pi_\ell>_{\ell=1}^{L}$ where, for *normal* μC functionality, $\pi_\ell \equiv \pi_\ell(\mathcal{T}_{a})$: $[0, T_E) \rightarrow [V_{\pi,\,low}, V_{\pi,\,high}]$ is a continuous function giving the

SPT time-history for signal line ℓ, with $V_{\pi,\ low}$ representing "line low", $V_{\pi,\ high}$ representing "line high", and the interval ($V_{\pi,\ low}$, $V_{\pi,\ high}$) accommodating periods of line transition from low to high or *vice-versa* (and the range of $\pi_\ell(\mathcal{T}_a)$ has been idealized). For *non-normal* μC functionality, the range of continuous function $\pi_\ell \equiv$

$\pi_\ell(\mathcal{T}_a;\ \pi^{EM})$ on domain $[0,\ T_E)$ may extend beyond $[V_{\pi,\ low},\ V_{\pi,\ high}]$ but is still bounded. In general, it is sufficient to consider these SPT's—both normal and non-normal—as members of the Cartesian product $\mathcal{X}^L_B([0,\ T_E)) \equiv \mathbf{X}^L_{\ell=1}\ C_B([0,\ T_E))$, where $C_B([0,\ T_E))$ is the set of (real-valued) continuous functions on $[0,\ T_E)$ that are uniformly bounded there by $B > 0$ (i.e., $[\forall f \in C_B([0,\ T_E))][\|f\|_\infty \leq B]$), with the specific value of B being unimportant here except that B > max $\{|V_{\pi,\ low}|,\ |V_{\pi,\ high}|\}$ (using ">" rather than "=" for the condition on B in order to accommodate non-normal SPT's). Actually, since there are (almost certainly) $<\pi_\ell>^L_{\ell=1} \in \mathcal{X}^L_B([0,\ T_E))$ that, for physical reasons, never occur either as normal or as non-normal μC states, we then denote by $\Pi_L([0,\ T_E)) \subsetneqq \mathcal{X}^L_B([0,\ T_E))$ the set of all $<\pi_\ell>^L_{\ell=1} \in \mathcal{X}^L_B([0,\ T_E))$ that may in fact occur as μC states (either normal or non-normal). We point out that a state $<\pi_\ell(\mathcal{T}_a)>^L_{\ell=1}$ for normal μC functionality corresponding to given task $\mathcal{T}_a$ is in general not unique: Actually, because small but nonzero timing and amplitude variations in SPT's are allowed *by design* without impacting normal μC functionality, each $\pi_\ell(\mathcal{T}_a)$, $\ell = 1,\ldots,\ L$, should be replaced with an "equivalence class" of such $\pi_\ell(\mathcal{T}_a)$'s and, likewise, $<\pi_\ell(\mathcal{T}_a)>^L_{\ell=1}$ should be replaced with an equivalence class, say $\mathcal{C}\ell^L_{normal}[\mathcal{T}_a]$, of such $<\pi_\ell(\mathcal{T}_a)>^L_{\ell=1}$'s where, when $<\pi_\ell(\mathcal{T}_a)>^L_{\ell=1} \neq <\pi'_\ell(\mathcal{T}_a)>^L_{\ell=1}$, then $<\pi_\ell(\mathcal{T}_a)>^L_{\ell=1} \sim <\pi'_\ell(\mathcal{T}_a)>^L_{\ell=1}$ iff they both correspond to the successful μC execution of task $\mathcal{T}_a$. (Also, since the members of any given normal class differ only by intended "micro" design tolerances and not by any "macro" differences [e.g., totally differing line signal levels in any given clock

½-period], we could assume that $\mathcal{C}\ell^{L}_{normal}[\mathcal{T}_a] \cap \mathcal{C}\ell^{L}_{normal}[\mathcal{T}_{a'}] = \varnothing$ when $\mathcal{T}_a \neq \mathcal{T}_{a'}$ to support the use of the term equivalence class; for then ~ is a *bona fide* equivalence relation on the set $\mathbf{U}_{a \in \mathcal{A}}\, \mathcal{C}\ell^{L}_{normal}[\mathcal{T}_a]$.) We will, however, avoid equivalence classes for the most part in our main development in Section III; rather, with any given task $\mathcal{T}_a$ executing normally we will associate a *unique, idealized* (as per Section III) state representative of its equivalence class $\mathcal{C}\ell^{L}_{normal}[\mathcal{T}_a]$, namely,

$<\pi^{\mathcal{N}}_{\ell}(\mathcal{T}_a)>^{L}_{\ell=1}$.

Consider next the non-normal states that may occur when the μC is subject to an external EM pulse π^{EM}. To that end, for μC task $\mathcal{T}_a$ represented in normal execution by idealized state $<\pi^{\mathcal{N}}_{\ell}(\mathcal{T}_a)>^{L}_{\ell=1}$, let $<\pi_{\ell}(\mathcal{T}_a; \pi^{EM})>^{L}_{\ell=1} \in \Pi_L([0,\ T_E))$ represent any possible μC state achievable when the μC is subjected to external EM pulse π^{EM}. In general, the $<\pi_{\ell}(\mathcal{T}_a; \pi^{EM})>$'s differ from $<\pi^{\mathcal{N}}_{\ell}(\mathcal{T}_a)>$ in that the former engender timing and amplitude variations of the latter. In contrast to the normal case, there are no compelling natural equivalence classes for the non-normal states $<\pi_{\ell}(\mathcal{T}_a; \pi^{EM})>^{L}_{\ell=1}$. Now if state $<\pi_{\ell}(\mathcal{T}_a; \pi^{EM})>^{L}_{\ell=1} \notin \mathcal{C}\ell_{L,\ normal}[\mathcal{T}_a]$ then this state is said to be ***deviant with respect to (wrt) task*** $\mathcal{T}_a$ and written for emphasis as $<\pi^{\mathcal{D}}_{\ell}(\mathcal{T}_a; \pi^{EM})>^{L}_{\ell=1}$; further, this deviant state is termed, in addition, *disruptive wrt task* $\mathcal{T}_a$ if it corresponds to mal-execution of that task; and it is termed *non-disruptive wrt task* $\mathcal{T}_a$ otherwise. In this connection, there might be deviant states that—despite not being elements of $\mathcal{C}\ell^{L}_{normal}[\mathcal{T}_a]$—nevertheless do not correspond to mal-execution of task $\mathcal{T}_a$ and are thus not disruptive. To capture the above considerations in our formulation, we define the following subsets of $\Pi_L([0,T_E))\backslash \mathcal{C\ell}^L_{normal}[\mathcal{T}_a]$:

$\Pi^{\mathcal{D}}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM})$

$\equiv \{<\pi_\ell(\mathcal{T}_a;\pi^{EM})>^L_{\ell=1} \in \Pi_L([0,\ T_E))\backslash \mathcal{C\ell}^L_{normal}[\mathcal{T}_a] \mid <\pi_\ell(\mathcal{T}_a;\pi^{EM})>^L_{\ell=1}$ is a deviant state wrt task $\mathcal{T}_a\}$,

$\Pi^{\mathcal{D},\ ND}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM})$

$\equiv \{<\pi^{\mathcal{D}}_\ell(\mathcal{T}_a;\pi^{EM})>^L_{\ell=1} \in \Pi^{\mathcal{D}}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM}) \mid <\pi^{\mathcal{D}}_\ell(\mathcal{T}_a;\pi^{EM})>^L_{\ell=1}$ is a non-disruptively deviant state wrt task $\mathcal{T}_a\}$,

$\Pi^{\mathcal{D},\ D}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM})$

$\equiv \{<\pi^{\mathcal{D}}_\ell(\mathcal{T}_a;\pi^{EM})>^L_{\ell=1} \in \Pi^{\mathcal{D}}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM}) \mid <\pi^{\mathcal{D}}_\ell(\mathcal{T}_a;\pi^{EM})>^L_{\ell=1}$ is disruptively deviant state wrt task $\mathcal{T}_a\}$,

for which $\Pi^{\mathcal{D}}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM})$ is the disjoint union

$$\Pi^{\mathcal{D}}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM}) = \Pi^{\mathcal{D},\ ND}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM}) \cup \Pi^{\mathcal{D},\ D}_L(\mathcal{T}_a;\ [0,T_E);\ \pi^{EM}).$$

Also, we denote by $\Pi_L(\mathcal{T}_a;\ [0,\ T_E))$ the disjoint union

$$\Pi_L(\mathcal{T}_a;\ [0,\ T_E)) = \mathcal{C\ell}^L_{normal}[\mathcal{T}_a] \cup \Pi^{\mathcal{D},\ ND}_L(\mathcal{T}_a;\ [0,\ T_E);\ \pi^{EM}) \cup \Pi^{\mathcal{D},\ D}_L(\mathcal{T}_a;\ [0,\ T_E)\pi^{EM}).$$

II.3 Probability of μC Upset, P^{upset}

We may now more carefully state in terms of μC states (i.e., collections of SPT's on the μC lines) what we mean by μC upset probability for a given $\mathcal{T}_a$. Consider then the following thought experiment. A fixed μC is executing a *known*, *fixed* $\mathcal{T}_a$—hence known $<\pi^{\mathcal{N}}_\ell(\mathcal{T}_a)>^L_{\ell=1}$—and is irradiated by a fixed π^{EM} acting over fixed time interval $[T_0,\ T_f) \subseteq$

$[0, T_E)$, with T_0 being some fixed instant after $\mathcal{T}_a$ initiation (and with the EM pulse being strong enough to potentially cause μC upset but not so strong as to cause μC damage); in this case we also write $\pi^{EM}[T_0, T_f]$. Repeated trials of this procedure are conducted and the outcome state $<\pi_\ell(\mathcal{T}_a; \pi^{EM}[T_0, T_f])>_{\ell=1}^{L} \in \Pi_L(\mathcal{T}_a; [0, T_E))$ is observed and classified as to which of the three disjoint subsets of $\Pi_L(\mathcal{T}_a; [0, T_E))$ it belongs. According to our comments in the Introduction, we take this outcome to be probabilistic (our EM pulse being presumed to be in a frequency regime in which this is so) so that the resulting state $<\pi_\ell(\mathcal{T}_a; \pi^{EM}[T_0, T_f])>_{\ell=1}^{L}$ and the subset to which it belongs varies from trial to trial. This probabilistic experiment then has outcome set

$$\Omega \equiv \{\mathcal{C}\ell^{L}_{normal}[\mathcal{T}_a],\ \Pi_L^{\mathcal{D},\ ND}(\mathcal{T}_a;\ [0, T_E);\ \pi^{EM}[T_0, T_f]),\ \Pi_L^{\mathcal{D},\ D}(\mathcal{T}_a;\ [0, T_E);\ \pi^{EM}[T_0, T_f])\}$$

and we seek to theoretically determine the value of $\text{Prob}(\Pi_L^{\mathcal{D},\ D}(\mathcal{T}_a;\ [0, T_E);\ \pi^{EM}[T_0, T_f]))$, i.e., the fraction of all trials in which $<\pi_\ell(\mathcal{T}_a; \pi^{EM}[T_0, T_f])>_{\ell=1}^{L}$ is disruptively deviant; this number will then be declared to be equal to P^{upset} for task $\mathcal{T}_a$ subject to $\pi^{EM}[T_0, T_f]$:

$$P^{upset}(\mathcal{T}_a; \pi^{EM}[T_0, T_f]) \equiv \text{Prob}(\Pi_L^{\mathcal{D},\ D}(\mathcal{T}_a;\ [0, T_E);\ \pi^{EM}[T_0, T_f])).$$

We may also think of $P^{upset}(\mathcal{T}_a; \pi^{EM}[T_0, T_f])$ as (being the same as) the probability of the disruptive deviation of idealized, normal SPT $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ subject to $\pi^{EM}[T_0, T_f]$:

$$P^{upset}(\mathcal{T}_a; \pi^{EM}[T_0, T_f]) \equiv \text{Prob}_{\mathcal{D},\ D}(<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L};\ \pi^{EM}[T_0, T_f]).$$

In order then to theoretically determine $\text{Prob}(\Pi_L^{\mathcal{D},\ D}(\mathcal{T}_a;\ [0, T_E);\ \pi^{EM}[T_0, T_f]))$, we construct a model in which we provide, for each possible idealized, normal SPT $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$, the EM-pulse-dependent probability of a disruptive deviation of $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ by any given EM pulse π^{EM}. Note that our probability assignment will be a function of only $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ and not depend upon the resulting state $<\pi_\ell(\mathcal{T}_a; \pi^{\mathrm{EM}})>_{\ell=1}^{L}$. This approach frees us from the extremely challenging (and indeed probably currently unachievable) burden of precisely determining and specifying any particular EM-induced deviant state (disruptive or not): we need only possess $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ (which, if required in detail, is in principle specifiable—but in practice at best arduously so—for $\mathcal{T}_a$ from knowledge of the μC design).

We will proceed by initially reducing consideration of the full collection $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ of SPT's on all L ("important") μC signal lines to consideration of a single such signal line. We say that $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ *suffers a disruptive deviation* (i.e., that execution of $\mathcal{T}_a$ is upset) iff there is at least one $\ell = 1, \ldots, L$ such that $\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)$ suffers a disruptive deviation (i.e., corresponds to mal-execution of $\mathcal{T}_a$) and we then address the determination of the probability $\mathrm{Prob}_{\mathcal{D},D}(\pi_\ell^{\mathcal{N}}(\mathcal{T}_a);\ \pi^{\mathrm{EM}})$ that any single SPT $\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)$ suffers a disruptive deviation induced by π^{EM}. Of course, one can then infer the probability disruptive deviation of $<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ by π^{EM}, i.e., $\mathrm{Prob}_{\mathcal{D},D}(<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L};\ \pi^{\mathrm{EM}})$, from

$$\mathrm{Prob}_{\mathcal{D},D}(<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L};\ \pi^{\mathrm{EM}}) = 1 - \Big\{\mathrm{Prob}\Big([\pi_1^{\mathcal{N}}(\mathcal{T}_a) \text{ does not suffer disruptive deviation } \textit{via } \pi^{\mathrm{EM}}]\ \&\ [\pi_2^{\mathcal{N}}(\mathcal{T}_a) \text{ does not suffer disruptive deviation } \textit{via } \pi^{\mathrm{EM}}] \ldots \& \ldots [\pi_L^{\mathcal{N}}(\mathcal{T}_a) \text{ does not suffer disruptive deviation } \textit{via } \pi^{\mathrm{EM}}]\Big)\Big\}$$

(which in general is equal to $1 - \prod_{\ell=1}^{L} [1 - \mathrm{Prob}_{\mathcal{D},D}(<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L};\ \pi^{\mathrm{EM}})]$ *only* when the individual single line SPT upsets are independent); but, because of the need to make mathematically precise the notional arguments just presented, as well as to discuss subtleties that arise, we defer the computation of such $\mathrm{Prob}_{\mathcal{D},D}(<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L};\ \pi^{\mathrm{EM}})$ to a subsequent paper. *Thus the model in this paper provides the probability of disruptive* *deviation of any particular idealized, normal SPT,* $\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)$, *as induced by any EM pulse,* π^{EM}*, acting over time interval* $[T_0, T_f) \subseteq [0, T_E)$.

As a final point in this section, we remind the reader that we have earlier briefly discussed the uncertainty associated with knowing which $\mathcal{T}_a$ is actually executing when it is stated that "application a is executing" and have alluded to treating that stochastically (in Section V). That stochasticity is, however, completely different from that discussed in the previous paragraph in which the actually executing $\mathcal{T}_a$ is taken to be fully known but in which the upset of $\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)$ is stochastic.

III. SPT ($\pi^{\mathcal{N}}$) and EM Pulse (π^{EM}) Specification and Interaction

As indicated in the previous section, we are interested in determining $\mathrm{Prob}_{\mathcal{D},D}(\pi_\ell^{\mathcal{N}}(\mathcal{T}_a);\ \pi^{\mathrm{EM}})$ when $\mathcal{T}_a$ is presumed known, hence $(<\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)>_{\ell=1}^{L}$ being known as well. We will, however, more generally, determine $\mathrm{Prob}_{\mathcal{D},D}(\pi^{\mathcal{N}};\ \pi^{\mathrm{EM}})$ for *every* possible idealized, normal SPT $\pi^{\mathcal{N}}$ in $[0, T_E)$. Since for $\ell = 1, 2, \ldots, L$, $\pi_\ell^{\mathcal{N}}(\mathcal{T}_a)$ is one of these $\pi^{\mathcal{N}}$'s then $\mathrm{Prob}_{\mathcal{D},D}(\pi_\ell^{\mathcal{N}}(\mathcal{T}_a);\ \pi^{\mathrm{EM}})$ will follow from our more general results. To this end, we carefully specify in this section our idealized SPT's, $\pi^{\mathcal{N}}$, and our EM pulses, π^{EM}, as well as characterize the overlap in time between them. We consider a μC with a fundamental clock period of $\tau_c > 0$ and denote $\tau_{c,1/2} \equiv \tau_c/2$; the latter specifies our basic time scale. We thus consider the global time interval $[0, T_E)$ to be partitioned into $N \geq 1$ *basic time segments* of length $\tau_{c,1/2}$ according to

$$[0, T_E) = \mathbf{U}_{n=0}^{N-1}\, [n\tau_{c,1/2}, (n+1)\,\tau_{c,1/2}), \tag{1}$$

where we stipulate without practical consequence that T_E is an exact multiple of $\tau_{c,1/2}$; i.e., $T_E = N\tau_{c,1/2}$. We will specify our SPT, $\pi^{\mathcal{N}}$, in terms of the partition's basic time segments.

III.1. Individual SPT Elements

We consider any idealized SPT, $\pi^{\mathcal{N}}$ in $[0, T_E)$, to be constructible—in a graph of signal magnitude *vs*. time—as a concatenation of shortly-to-be-specified basic elements. Our basic elements are of two distinct classes, which two classes we term as "*primitive SPT elements*" and as "½-*period-segmented SPT elements*". There are four differing elements—"types"—in the former class and eight differing element types in the latter class and in this section we precisely specify all these.

Consider firstly the primitive SPT elements. They are of the following types: (A) *signal-high*, having constant value $-\infty < V_{\pi,\ high} < \infty$ during some positive length time interval (where $V_{\pi,\ high}$ is frequently taken to be positive); (B) *signal-low*, having constant value $-\infty < V_{\pi,\ low} < V_{\pi,\ high}$ during some positive length time interval (where $V_{\pi,\ low}$ is frequently be taken to be zero); (C) *signal-low-to-high-transition*, a straight line from termination of a signal-low element to commencement—at some positive time beyond that termination—of the very next-in-time signal-high element, and with that straight line having *finite*, *positive* slope $[(V_{\pi,\ high} - V_{\pi,\ low})/2)]/\alpha\tau_{c,1/2}$, with $\alpha > 0$; and (D) *signal-high-to-low-transition*, a straight line from termination of a signal-high element to commencement—at some positive time beyond that termination—of the very next-in-time signal-low element, and with that straight line having *finite*, *negative* slope

$-[(V_{\pi,\ high} - V_{\pi,\ low})/2)]/\beta\tau_{c,1/2}$, with $\beta > 0$. In practice, it usually occurs that α, $\beta << 1$; less stringently we stipulate only that $0 < \alpha + \beta < 1$ (so that $0 < \alpha < 1$ and $0 < \beta < 1$). Examples of primitive SPT elements are illustrated in the SPT of Fig. 1. Note that primitive SPT elements of Types A and B may (partially) occupy more than a single basic time segment while elements of Type C and D always (partially) occupy more than a single basic time segment.

Consider next the ½-period-segmented SPT elements. Now in general, the portion of any SPT occurring during any single basic time segment $[n\tau_{c,1/2}, (n + 1)\tau_{c,1/2})$ is constructible by concatenation of *portions of* several of the above primitive SPT element types. We illustrate in Fig. 2 all eight possibilities (types) for the structure of an SPT during (without loss of generality) the zeroth basic time segment ($n = 0$ in Eq. (1)); and we refer to these eight as "½-*period-segmented SPT elements*". As an example of the constructability of a ½-period-segmented SPT element from primitive SPT elements, note that the ½-period-segmented SPT element of Type-4 may be regarded as the concatenation of a primitive element of Type-A with half of each of primitive elements of Type-C and Type-D.

The reason we have introduced both primitive elements and ½-period-segmented elements is as follows. While the latter are more natural for describing an SPT in time, it nevertheless seems to us that the former are more natural for assigning disruptive deviation probabilities. Our ultimate method of assigning disruptive deviation probabilities to single ½-period-segmented SPT elements (and therefrom to an *N*-element SPT) will be to first assign disruptive deviation probabilities to *basic* (see Section IV.2.1) primitive SPT elements and therefrom compute the disruptive deviation probabilities for the ½-period-segmented SPT elements (Section IV.2). However, in what follows we will also initially take the simpler approach of assigning disruptive deviation probabilities directly to the ½-period-segmented SPT elements themselves (Section IV.1).

To proceed, we label the eight ½-period-segmented SPT elements in Fig. 2 by s_j, $j = 1, \ldots, 8$, as indicated specifically in that figure. In fact, each such s_j is a mapping from $[0, \tau_{c,1/2})$ into $[V_{\pi,\,low}, V_{\pi,\,high}]$. Explicitly, denoting $\Delta V_\pi \equiv V_{\pi,\,high} - V_{\pi,\,low} > 0$ and $V_{\pi,\,mid} \equiv (V_{\pi,\,high} + V_{\pi,\,low})/2$ we have the following:

$$s_1(t) = V_{\pi,\,high} \qquad t \in [0, \tau_{c,1/2}) \tag{2}$$

$$s_2(t) = \begin{cases} (\Delta V_\pi/\alpha\tau_c)\,t + V_{\pi,\,mid} & \text{if } t \in [0, \alpha\tau_{c,1/2}) \\ V_{\pi,\,high} & \text{if } t \in [\alpha\tau_{c,1/2}, \tau_{c,1/2}) \,, \end{cases} \tag{3}$$

$$s_3(t) = \begin{cases} V_{\pi,\,high} & \text{if } t \in [0, (1-\beta)\tau_{c,1/2}) \\ -(\Delta V_\pi/\beta\tau_c)\,t + (\Delta V_\pi/2\beta) + V_{\pi,\,mid} & \text{if } t \in [(1-\beta)\tau_{c,1/2}, \tau_{c,1/2}), \end{cases} \tag{4}$$

$$s_4(t) = \begin{cases} (\Delta V_\pi/\alpha\tau_c)\,t + V_{\pi,\,mid} & \text{if } t \in [0,\ \alpha\tau_{c,1/2}) \\ V_{\pi,\,high} & \text{if } t \in [\alpha\tau_{c,1/2},\ (1-\beta)\tau_{c,1/2}) \\ -(\Delta V_\pi/\beta\tau_c)\,t + (\Delta V_\pi/2\beta) + V_{\pi,\,mid} & \text{if } t \in [(1-\beta)\tau_{c,1/2},\ \tau_{c,1/2}), \end{cases} \tag{5}$$

$$s_5(t) \equiv V_{\pi,\,low} \qquad t \in [0,\ \tau_{c,1/2}) \tag{6}$$

$$s_6(t) = \begin{cases} -(\Delta V_\pi/\beta\tau_c)\,t + V_{\pi,\,mid} & \text{if } t \in [0,\ \beta\tau_{c,1/2}) \\ V_{\pi,\,low} & \text{if } t \in [\beta\tau_{c,1/2},\ \tau_{c,1/2})\ , \end{cases} \tag{7}$$

$$s_7(t) = \begin{cases} V_{\pi,\,low} & \text{if } t \in [0,\ (1-\alpha)\tau_{c,1/2}) \\ (\Delta V_\pi/\alpha\tau_c)\,t - (\Delta V_\pi/2\alpha) + V_{\pi,\,mid} & \text{if t} \in [(1-\alpha)\tau_{c,1/2},\ \tau_{c,1/2}), \end{cases} \tag{8}$$

$$s_8(t) = \begin{cases} -(\Delta V_\pi/\beta\tau_c)\,t + V_{\pi,\,mid} & \text{if } t \in [0,\ \beta\tau_{c,1/2}) \\ V_{\pi,\,low} & \text{if } t \in [\beta\tau_{c,1/2},\ (1-\alpha)\tau_{c,1/2}) \\ (\Delta V_\pi/\alpha\tau_c)\,t - (\Delta V_\pi/2\alpha) + V_{\pi,\,mid} & \text{if } t \in [(1-\alpha)\tau_{c,1/2},\ \tau_{c,1/2}). \end{cases} \tag{9}$$

It is convenient to extend the domain of each of these ½-period-segmented SPT elements to all of $[0,\ T_E)$; we take

$$s_j^0(t) \equiv \begin{cases} s_j(t) & \text{if } t \in [0,\ \tau_{c,1/2}) \\ 0 & \text{if } t \in [0,\ T_E) \setminus [0,\ \tau_{c,1/2}) \end{cases} \qquad (j = 1,\ldots,8) \tag{10}$$

and refer to these as well (with no confusion generally resulting from doing so) as

½-period-segmented SPT elements. (The possibility that $V_{\pi,\, low}$ or $V_{\pi,\, high}$ may have value zero is purely coincidental in relation to the possible value 0 taken on by $s_j^0(t)$ in $[0,\, T_E) \setminus [0,\, \tau_{c,1/2})$.) Further, to describe the SPT in basic time segments other than the zeroth one, we define, for $n = 1,\ldots,\, N - 1$ and $t \in [0,\, T_E)$, the n^{th}-time-segment ½-period-segmented SPT elements

$$s_j^n(t) \equiv \begin{cases} s_j^0(t - n\tau_{c,1/2}) & \text{if } t \in [n\tau_{c,1/2},\, (n+1)\,\tau_{c,1/2}) \\ 0 & \text{if } t \in [0,\, T_E) \setminus [n\tau_{c,1/2},\, (n+1)\tau_{c,1/2}); \end{cases} \qquad (j = 1,\ldots,\, 8) \qquad (11)$$

and, for each $n = 0,\ldots,\, N - 1$, we denote the set of all such ½-period-segmented SPT elements as

$$S_1^n \equiv \{s_j^n\}_{j=1}^8\,. \qquad (12)$$

III.2. <u>Consecutive ½-Period-Segmented SPT Elements: Arbitrary Idealized SPT's</u>

We now use the individual ½-period-segmented SPT elements of the previous subsection to construct an arbitrary N-element SPT having domain $[0,\, T_E)$ partitioned into basic time segments as per Eq. (1), with this N-element SPT consisting of a sequence of N consecutive-in-time single SPT elements, where the nth such single element ($n = 0,\ldots,\, N - 1$) of the sequence belongs to S_1^n of Eq. (12). In Fig. 3, we show an 8-element SPT (which, incidentally consists of all eight ½-period-segmented SPT element types). In general, any such consecutive-N-element SPT can be written as a sum, over $n = 0,\ldots,\, N - 1$, of individual SPT elements from the S_1^n's; to wit, as a sum $\sum_{n=0}^{N-1} s_{j_n}^n$ where $s_{j_n}^n \in S_1^n$ with $j_n \in \{1,\, \ldots,\, 8\}$. On the other hand, it is not the case that every sum of this form is an SPT: While in any given basic time segment (indexed by n) each of the eight ½-period-segmented elements ($s_{j_n}^n$ for $j_n = 1,\ldots,\, 8$) is possible, nevertheless the possible *pairs*, $<s_{j_n}^n,\, s_{j_{n+1}}^{n+1}>$, of ½-period-segmented elements occurring in consecutive time segments are constrained by the requirement that the SPT be a continuous function in $[n\tau_{c,1/2},\, (n + 2)\tau_{c,1/2})$ and differentiable at $t = (n + 1)\tau_{c,1/2}$ (but clearly need not be differentiable in all of $[n\tau_{c,1/2},\, (n + 2)\tau_{c,1/2})$). The required differentiability at $t = (n + 1)\tau_{c,1/2}$ serves to exclude any pair whose sum is continuous yet does not occur as part of any SPT, e.g., type-2 following type-3, or type-2 following type-4, or type-4 following type-3. It is straightforward to verify (from Fig. 2, for example) that given any $n = 0,\ldots, N-1$ and any $s^n_{j_n} \in S^n_1$ ($j_n \in \{1, \ldots, 8\}$) there are then precisely two members of S^{n+1}_1 for which the "two-element" SPT $[s^n_{j_n} + s^{n+1}_{j_{n+1}}]\restriction[n\tau_{c,1/2}, (n+2)\tau_{c,1/2})$ is *admissible* in the above sense. For example, if $j_n = 1$ then we must have $j_{n+1} \in \{1, 3\}$. Furthermore, this relationship is independent of n: if

$n' \neq n$ and $j_{n'} = 1$ then $j_{n'+1} \in \{1, 3\}$; and this n-independence holds as well for

$j_n = 2,\ldots, 8$. Thus, in general, for each $j = 1, \ldots, 8$ we may write $(j)^+_1$ and $(j)^+_2$ for those two indices for which—and only for which—the "two-element" SPT's $[s^0_j + s^1_{(j)^+_1}]\restriction[0, 2\tau_{c,1/2})$ and $[s^0_j + s^1_{(j)^+_2}]\restriction[0, 2\tau_{c,1/2})$ are admissible (hence so are

$[s^n_{j_n} + s^{n+1}_{(j_n)^+_1}]\restriction[n\tau_{c,1/2}, (n+2)\tau_{c,1/2})$ and $[s^n_{j_n} + s^{n+1}_{(j_n)^+_2}]\restriction[n\tau_{c,1/2}, (n+2)\tau_{c,1/2})$ admissible for all $n = 0, \ldots, N-1$). We also write

$$J^+ \equiv \{ <j, j^+> \mid j = 1,\ldots, 8 \text{ and } j^+ = (j)^+_1 \text{ or } j^+ = (j)^+_2\}. \tag{13}$$

We may now characterize all idealized, "normally executing" N-element SPT's in $[0, T_E)$ as the members of the set

$$\Pi^{\mathcal{N}}_N[0, T_E] \equiv \{ \pi^{\mathcal{N}} \mid \pi^{\mathcal{N}} = \textstyle\sum_{n=0}^{N-1} s^n_{j_n} \text{ with } (\forall n = 0, \ldots, N-1)(< j_n, j_{n+1} > \in J^+)\} \tag{14}$$

where

$$\pi^{\mathcal{N}}: \mathbf{U}_{n=0}^{N-1} [n\tau_{c,1/2}, (n+1)\tau_{c,1/2}) \rightarrow [V_{\pi,\,low}, V_{\pi,\,high}]. \tag{15}$$

The members of $\Pi^{\mathcal{N}}_N[0, T_E]$ may each have one of only eight possible individual ½-period-segmented SPT elements occupying $[0, \tau_{c,1/2})$ and for each of these eight there are two possible individual ½-period-segmented SPT elements occupying each of

$[n\tau_{c,1/2}, (n+1)\tau_{c,1/2})$, $n = 1,\ldots, N-1$; hence $\#(\Pi^{\mathcal{N}}_N) = 8 \bullet 2^{N-1} = 2^{N+2}$. Further, since each $\pi^{\mathcal{N}} \in \Pi^{\mathcal{N}}_N[0, T_E]$ is completely characterized by its associated sequence

$\langle j_0, j_1, \ldots, j_{N-1} \rangle$ (when α, β, $V_{\pi,\, low}$, and $V_{\pi,\, high}$, are fixed) then, when necessary, we will denote that SPT by $\pi^{\mathcal{N}}[j_0, j_1, \ldots, j_{N-1}]$.

III.3. <u>EM Pulses</u>

An EM pulse, π^{EM}, will be taken to be a rectangular-envelope-modulated monochromatic sinusoidal wave of frequency $\omega > 0$, having amplitude $V_{\mathrm{EM}} > 0$ during time interval $[T_0, T_f) \subseteq [0, T_E)$ (with $T_f \geq T_0$) and amplitude zero during $[0, T_E) \setminus [T_0, T_f)$. Explicitly,

$$\pi^{\mathrm{EM}}(t) \equiv \begin{cases} V_{\mathrm{EM}} \sin[\omega(t - T_0)] & \text{if } t \in [T_0, T_f) \\ 0 & \text{if } t \in [0, T_E) \setminus [T_0, T_f), \end{cases} \tag{16}$$

where we stipulate without practical consequence that $T_f - T_0$ is an integral multiple of π/ω so that π^{EM} is continuous at both T_0 and T_f (we attribute no significance to whether such multiple is odd or even). We will also make use of the upper-half envelope of π^{EM}, namely,

$$\pi_+^{\mathrm{EM}}(t) \equiv \begin{cases} V_{\mathrm{EM}} & \text{if } t \in [T_0, T_f) \\ 0 & \text{if } t \in [0, T_E) \setminus [T_0, T_f). \end{cases} \tag{17}$$

Further, when necessary, we will write both π^{EM} and π_+^{EM} more explicitly as, respectively, $\pi^{\mathrm{EM}}[T_0, T_f, V_{\mathrm{EM}}, \omega]$ and $\pi_+^{\mathrm{EM}}[T_0, T_f, V_{\mathrm{EM}}]$.

III.4. <u>Overlap in Time of $\pi^{\mathcal{N}}$ and π^{EM}</u>

Given $\pi_+^{\mathrm{EM}}[T_0, T_f, V_{\mathrm{EM}}]$, we will be interested, for use in the remainder of the paper, in characterizing the temporal overlap of $\pi_+^{\mathrm{EM}}[T_0, T_f, V_{\mathrm{EM}}]$, when $T_f > T_0$, and any $\pi^{\mathcal{N}} \in \Pi_N^{\mathcal{N}}[0, T_E]$, which overlap we indicate symbolically also as $\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}$; i.e., $\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}} \equiv [T_0, T_f)$. To this end, we define

$$n_{min}[T_0, T_f] \equiv \min\{ n \in \{0, \dots, N-1\} \mid [n\tau_{c,1/2}, (n+1)\,\tau_{c,1/2}) \cap [T_0, T_f) \neq \varnothing \} \tag{18}$$

and

$$n_{max}[T_0, T_f] \equiv \max\{ n \in \{0, \dots, N-1\} \mid [n\tau_{c,1/2}, (n+1)\,\tau_{c,1/2}) \cap [T_0, T_f) \neq \varnothing \} \tag{19}$$

where, of course, $N - 1 \geq n_{max} \geq n_{min} \geq 0$. Then

$$\begin{aligned}[T_0, T_f) = {} & \{ [n_{min}\tau_{c,1/2}, (n_{min}+1)\tau_{c,1/2}) \cap [T_0, T_f) \} \\ & \cup \bigcup_{n=n_{min}+1}^{n=n_{max}-1} [n\tau_{c,1/2}, (n+1)\,\tau_{c,1/2}) \\ & \cup \{ [n_{max}\tau_{c,1/2}, (n_{max}+1)\tau_{c,1/2}) \cap [T_0, T_f) \},\end{aligned} \tag{20}$$

since $[n\tau_{c,1/2}, (n+1)\tau_{c,1/2}) \cap [T_0, T_f) = [n\tau_{c,1/2}, (n+1)\tau_{c,1/2})$ for $n = n_{min} + 1, \dots, n_{max} - 1$. However, in general, $[n_{min}\tau_{c,1/2}, (n_{min}+1)\tau_{c,1/2}) \cap [T_0, T_f) \subsetneqq [T_0, T_f)$ and $[n_{max}\tau_{c,1/2}, (n_{max}+1)\tau_{c,1/2}) \cap [T_0, T_f) \subsetneqq [T_0, T_f)$. In other words, the overlap in time of $\pi_+^{EM}[T_0, T_f, V_{EM}]$ and any $\pi^{\mathcal{N}} \in \Pi_N^{\mathcal{N}}[0, T_E]$ is specified by the collection of $n_{max} - n_{min} - 1$ "complete" time segments contained inside the middle ("big union") term in Eq. (20) together with the two generally "partial" time sub-segments given by the first and last terms in that equation. Our interest in so specifying this overlap derives from the circumstance that in the next section we will ascribe EM-pulse disruption probabilities to each of the eight possible ½-period-segmented SPT elements and therefrom infer an EM-pulse disruption probability for any entire N-element SPT (i.e., any $\pi^{\mathcal{N}} \in \Pi_N^{\mathcal{N}}[0, T_E]$) constructed from those ½-period-segmented SPT elements. Each such ½-period-segmented SPT element of course temporally "occupies" a complete basic time segment; but, because of the possibility that the leading and trailing ends of $\pi^{\mathcal{N}} \cap \pi^{EM}$ each only partially occupy a basic time segment, we must in addition ascribe EM-pulse disruption probabilities to all possible partial ½-period-segmented SPT elements formable from any of the eight complete ½-period-segmented SPT elements. In this connection, it will also be useful to denote

$$\rrbracket \pi^{\mathcal{N}} \cap \pi^{EM} \llbracket \equiv [n_{min}\tau_{c,1/2}, (n_{max}+1)\tau_{c,1/2}), \tag{21}$$

this quantity being the minimal *extension* of the interval $[T_0, T_f)$ to a union of an integer number of *complete* basic time segments, where the interval $[T_0, T_f) = \pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}$ is not in general such a union but rather properly contained inside $]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![$.

IV. Probability of a π^{EM}-Induced Disruptive Deviation of $\pi^{\mathcal{N}}$

In this section we formulate our expression for the probability of a π^{EM}-induced disruptive deviation of any single signal line carrying any of the possible SPT's, $\pi^{\mathcal{N}} \in \Pi_N^{\mathcal{N}}[0, T_E]$. We firstly consider disruptive deviation probability assignments to each of the individual ½-period-segmented SPT elements s_j^n and do so from two points of view: (1) direct assignment to each of the eight ½-period-segmented SPT elements and (2) indirect assignment to each of the eight ½-period-segmented SPT elements *via* direct assignment of disruptive deviation probabilities to each of the four primitive SPT elements. We then consider, secondly, disruption probability assignment to a single, arbitrary *N*-element SPT built from the s_j^n's. The connection between these two entities is this: Given an SPT composed of a specified set of ½-period-segmented SPT elements then we say that the SPT suffers a disruptive deviation iff at least one of its elements suffers a disruptive deviation. Thirdly, we consider (in Section V) the disruptive deviation probability assignment to a single signal line in case the actual SPT on the line—being an SPT from the collection, $\Pi_N^{\mathcal{N}}$, of all possible *N*-element SPT's that may be carried on the line—is known only stochastically.

IV.1. Individual ½-Period-Segmented SPT Elements: Direct Probability Assignment

IV.1.1. *Full Overlap of π^{EM} and s_j^n*

Consider firstly any EM pulse $\pi^{\mathrm{EM}}[0, T_f, V_{\mathrm{EM}}, \omega]$ such that $[T_0, T_f) \cap [0,\tau_{c,1/2}) = [0,\tau_{c,1/2})$, so that $[T_0, T_f) \supseteq [0,\tau_{c,1/2})$ (and $T_f \geq \tau_{c,1/2}$), and further consider ½-period-segmented SPT elements $s_j^0 \in S_1^0$, $j = 1,\ldots, 8$. In this case we denote by $p^0_{j,\,full}[V_{\mathrm{EM}}, \omega] \in [0, 1]$ the probability that element s_j^0 suffers a disruptive deviation induced by π^{EM}. (We have suppressed the implicit dependence of $p^0_{j,\,full}$ upon $\tau_{c,1/2}$, $V_{\pi,\,low}$, $V_{\pi,\,high}$, α, and β.) *Note* *that we are considering this probability to be independent of T_f as long as T_f* $\geq \tau_{c,1/2}$: The differences in the probabilities of π^{EM}–induced disruptive deviations of multi-segment SPT′s for differing such π^{EM}′s—the π^{EM}′s differences arising from the differences in their T_f′s (when all of their T_f′s are strictly greater than $\tau_{c,1/2}$)—will be manifested in our model in the ½-period-segmented SPT elements beyond $n = 0$.

Next, for each $n = 1, \ldots, N - 1$, consider any EM pulse $\pi^{EM}[T_0, T_f, V_{EM}, \omega]$ such that $[T_0, T_f) \cap [n\tau_{c,1/2}, (n+1)\tau_{c,1/2}) = [n\tau_{c,1/2}, (n+1)\tau_{c,1/2})$, so that $[T_0, T_f) \supseteq$

$[n\tau_{c,1/2}, (n+1)\tau_{c,1/2})$ (and $T_0 \leq n\tau_{c,1/2}$ and $T_f \geq (n+1)\tau_{c,1/2}$), and further consider ½-period-segmented SPT elements $s_j^n \in S_n^0$, $j = 1, \ldots, 8$. In this case, we denote by $p_{j,\,full}^n[V_{EM}, \omega]$ the probability that element s_j^n suffers a disruptive deviation induced by π^{EM} where, once again*, we are considering this probability to be independent of* T_0 *and* T_f *as long as* $T_0 \leq n\tau_{c,1/2}$ and $T_f \geq (n+1)\tau_{c,1/2}$: The differences in the probabilities of π^{EM} disruptions of multi-segment SPT′s for differing such π^{EM}′s will be manifested in our model in the ½-period-segmented SPT elements previous to and subsequent to the n^{th}. Further, *we assume* that

$$p_{j,\,full}^n[V_{EM}, \omega] = p_{j,\,full}^0[V_{EM}, \omega], \qquad j = 1, \ldots, 8;\ n = 1, \ldots, N-1; \tag{22}$$

i.e., *for each one of the eight ½-period-segmented elements, its probability of* suffering a disruptive deviation *is independent of where that element occurs in the N-element SPT*, *as long as the element is completely encompassed by* π^{EM}, this probability depending rather only upon which one of the possible eight the element actually is. We term this property "*modularity*"; it appears to us to be a necessary assumption if we are to model the upset of all members of the collection of all possible SPT's in $[0, T_E)$ without treating each such member as an individual having no commonality with any other member of the collection with respect to response to an EM excitation.

IV.1.2. *Partial Overlap of π^{EM} and s_j^n*

Suppose next, contrary to the first paragraph of Section IV.1.1, that $\pi^{EM}[T_0, T_f, V_{EM}, \omega]$ is such that $[T_0, T_f) \cap [0, \tau_{c,1/2}) \subsetneqq [0, \tau_{c,1/2})$ and $[T_0, T_f) \cap [0, \tau_{c,1/2}) \neq \emptyset$, with

$$\gamma_0(T_0, T_f) \equiv \mathit{Length}\{[T_0, T_f) \cap [0, \tau_{c,1/2})\}/\tau_{c,1/2}; \tag{23}$$

note $0 < \gamma_0(T_0, T_f) < 1$. In this case, we assume that the probability that element s_j^0 suffers a disruptive deviation induced by π^{EM} is given by

$$p^0_{j,\,partial}[V_{EM}, \omega; T_0, T_f] \equiv \mathcal{G}(\gamma_0(T_0, T_f), p^0_{j,\,full}[V_{EM}, \omega])\, p^0_{j,\,full}[V_{EM}, \omega], \tag{24}$$

where $\mathcal{G}: [0, 1]^2 \rightarrow [0, 1]$ is some yet-to-be-specified function of γ_0 and $p^0_{j,\,full}$ that is non-decreasing in γ_0 for each possible fixed $p^0_{j,\,full} \in [0, 1]$ and which further satisfies

$$(\forall p^0_{j,\,full} \in [0, 1])(\mathcal{G}(0, p^0_{j,\,full}) = 0 \text{ and } \mathcal{G}(1, p^0_{j,\,full}) = 1). \tag{25}$$

This non-decreasing behavior of $\mathcal{G}$ in γ_0 seems reasonable to us but may in fact not obtain in reality; nevertheless, we adopt this behavior in our present model until experiment dictates otherwise. Note that Eq. (24) stipulates that—as far as the interval dependency of $\mathcal{G}$ is concerned—it is only the *length* of the interval $[T_0, T_f) \cap [0, \tau_{c,1/2})$ that determines $p^0_{j,\,partial}$ and not the specific location of this interval inside of $[0, \tau_{c,1/2})$. (We will modify this assumption later on, in Section IV.2—for the present it will suffice for our purpose.) Also note that the only dependency of $\mathcal{G}$ upon j is *via* $p^0_{\mathrm{j},\,full}$—the functional form of $\mathcal{G}$ is the same for each $j = 1, ..., 8$ (i.e., $\mathcal{G}$ does not carry a subscript "j"). Further, the inclusion of values 0 and 1 for γ_0, in addition to those allowed values of γ_0 specified immediately following Eq. (23), will allow us to incorporate the limiting cases of $[T_0, T_f) \cap [0, \tau_{c,1/2}) = \varnothing$ and $[T_0, T_f) \cap [0, \tau_{c,1/2}) = [0, \tau_{c,1/2})$, which cases we will employ shortly in explicitly determining $\mathcal{G}$.

Similarly, for each $n = 1,\ldots, N - 1$, consider any EM pulse $\pi^{EM}[T_0, T_f, V_{EM}, \omega]$ such that $[T_0, T_f) \cap [n\tau_{c,1/2}, (n + 1)\tau_{c,1/2}) \subsetneqq [n\tau_{c,1/2}, (n + 1)\tau_{c,1/2})$ and $[T_0, T_f) \cap [n\tau_{c,1/2}, (n + 1)\tau_{c,1/2}) \neq \varnothing$, with

$$\gamma_n(T_0, T_f) \equiv \mathit{Length}\{[T_0, T_f) \cap [n\tau_{c,1/2}, (n + 1)\tau_{c,1/2})\}/\tau_{c,1/2}; \tag{26}$$

and

$$0 < \gamma_n(T_0, T_f) < 1 \qquad (n = 1,\ldots, N-1). \tag{27}$$

Then we assume that the probability that element s_j^n suffers a disruptive deviation induced by π^{EM} is given by

$$\begin{aligned} p^n_{j,\,partial}[V_{\mathrm{EM}}, \omega;\, T_0, T_f] &\equiv \mathcal{G}(\gamma_n(T_0, T_f), p^n_{j,\,full}[V_{\mathrm{EM}}, \omega])\, p^n_{j,\,full}[V_{\mathrm{EM}}, \omega]) \\ &= \mathcal{G}(\gamma_n(T_0, T_f), p^0_{j,\,full}[V_{\mathrm{EM}}, \omega])\, p^0_{j,\,full}[V_{\mathrm{EM}}, \omega] \end{aligned} \tag{28}$$

where we have used here one and the same function $\mathcal{G}$ for all $n = 1,\ldots, N-1$, that function also being the one that we used for $n = 0$. We emphasize that we have thus taken the functional form of $\mathcal{G}$ to be independent of both j and n. Also, as in the $n = 0$ case, we allow in addition $\gamma_n = 0, 1$.

We now address the determination of the function $\mathcal{G}$. For the purpose of this discussion, we will suppress the subscript "j" and superscript "n" on $p^n_{j,\,full}$ and $p^n_{j,\,partial}$, as well as the subscript "n" on $\gamma_n(T_0, T_f)$, and also suppress all their arguments; we merely write p_{full} , $p_{partial}$, and γ for these, with n and j understood to be fixed but arbitrary in their allowed ranges. Also, for proper non-void subinterval $I = [T_0, T_f) \cap [n\tau_{c,1/2}, (n+1)\tau_{c,1/2})$ of basic time segment $[n\tau_{c,1/2}, (n+1)\tau_{c,1/2})$—with the former being a subinterval whose $p_{partial}$ is that of interest and whose length-fraction is γ, and with the latter having disruption probability p_{full} when fully encompassed by π^{EM}—we write

$$p_{partial}(\gamma, p_{full}) = \mathcal{G}(\gamma, p_{full})\, p_{full}\,. \tag{29}$$

(Recall, as indicated earlier, that two different subintervals inside the *same* basic time segment [so that both of these subintervals are referenced to the same p_{full}] and having the same γ also have the same $p_{partial}(\gamma, p_{full})$.) Further, for $I^{\sim} = [n\tau_{c,1/2}, (n+1)\tau_{c,1/2}) \setminus I$, with $I^{\sim}$ having length-fraction $1-\gamma$, we take its $p_{partial}$ as

$$p_{partial}(1-\gamma, p_{full}) = \mathcal{G}(1-\gamma, p_{full})\, p_{full}\,, \tag{30}$$

despite the fact that $I^{\sim}$ may sometimes not be an interval but rather the union of two intervals—we still assign a $p_{partial}$ to $I^{\sim}$ based solely upon its total length. This is in keeping with our implied basic assumption that $p_{partial}$ for a (measurable) subset of a *given* basic time segment (with the latter's p_{full} hence also given) depends solely upon the length of that subset—and nothing else.

To proceed with the determination of $\mathcal{G}$, consider a random experiment in which ½-period-segmented SPT element s_j^n—which element *when fully encompassed* by $\pi^{EM}[T_0, T_f]$ (suppressing V_{EM} and ω here as well) exhibits upset probability p_{full}—is in fact irradiated by EM pulse $\pi^{EM}[T_0, T_f]$ having $[T_0, T_f) = [n_{max}\tau_{c,1/2}, (n_{max}+1)\tau_{c,1/2})$. With some specified I of interest as per above and, in particular, having length fraction γ, one observes and records the subset—I or $I^{\sim}$—during which a disruptive deviation of the SPT element s_j^n *first* takes place, making allowance as well for the additional possibility that no disruptive deviation of the SPT element s_j^n takes place at all—which outcome is signified by $\mathcal{N}$ (i.e., no disruptive deviation). For this experiment, the outcome set is

$\Omega \equiv \{I, I^{\sim}, \mathcal{N}\}$ with probability measure (on 2^{Ω}—the power set of Ω) specified by $P(\{I\}) = p_I$, $P(\{I^{\sim}\}) = p_{I^{\sim}}$, and $P(\{\mathcal{N}\}) = 1 - (p_I + p_{I^{\sim}})$. Now in terms of the quantities $p_{partial}$ and p_{full} above we must have

$$p_I + (1 - p_I)p_{I^{\sim}} = p_{full} = p_{I^{\sim}} + (1 - p_{I^{\sim}})p_I \tag{31a}$$

$$p_I = p_{partial}(\gamma, p_{full}) \quad \text{and} \quad p_{I^{\sim}} = p_{partial}(1 - \gamma, p_{full}). \tag{31b}$$

The first equality in Eq. (31a) reflects the following: The fraction of experimental trials in which a disruption takes place in s_j^n—namely p_{full}—is equal to the fraction of trials in which the outcome is I—namely p_I—plus the fraction of trials in which the outcome is not I but is $I^{\sim}$—namely $(1 - p_I)p_{I^{\sim}}$; further, as expressed by the second equality in Eq. (31a), this sum is invariant under the interchange of I and $I^{\sim}$. We will use this reasoning again in the sequel. Continuing, we use Eqs. (29) and (30) in conjunction with Eqs. (31) to find

$$\mathcal{G}(\gamma, p_{full}) + \mathcal{G}(1-\gamma, p_{full}) - p_{full}\,\mathcal{G}(\gamma, p_{full})\mathcal{G}(1-\gamma, p_{full}) = 1. \tag{32}$$

Abbreviating $p \equiv p_{full}$, we thus seek solutions $\mathcal{G}$: $[0, 1]^2 \to [0, 1]$ of

$$\mathcal{G}(\gamma, p) + \mathcal{G}(1-\gamma, p) - p\mathcal{G}(\gamma, p)\mathcal{G}(1-\gamma, p) = 1 \tag{33}$$

for all γ, $p \in [0, 1]$, subject to the conditions of Eq. (25), namely,

$$(\forall p \in [0, 1])(\mathcal{G}(0, p) = 0 \text{ and } \mathcal{G}(1, p) = 1). \tag{34}$$

Note that Eq. (33) demands that

$$p\mathcal{G}^2(½, p) - 2\mathcal{G}(½, p) + 1 = 0 \tag{35}$$

for all $p \in [0, 1]$ and that this equation has one and only one of its solutions for $\mathcal{G}$ having value in [0,1], namely the solution

$$g_{½}(p) \equiv \begin{cases} [1-(1-p)^{1/2}]/p & \text{if } p \in (0, 1] \\ ½ & \text{if } p = 0. \end{cases} \tag{36}$$

We now exhibit a solution to Eqs. (33) + (34). (This solution is not unique—this will become evident shortly.) We have

$$\mathcal{G}(\delta, p) = \begin{cases} [1-(1-p)^{\delta}]/p & \text{if } p \in (0, 1) \ \& \ \delta \in [0, 1] \\ \delta & \text{if } p = 0 \ \& \ \delta \in [0, 1] \\ 0 & \text{if } p = 1 \ \& \ \delta = 0 \\ 1 & \text{if } p = 1 \ \& \ \delta \in (0, 1] \end{cases} \tag{37}$$

where we substitute $\delta = \gamma$ or $\delta = 1 - \gamma$ as applicable. (Eq. (37) may be written more succinctly but for clarity we have chosen not to do so.) Using Eq. (34) along with the observation that $(\partial\mathcal{G}/\partial\delta)(\delta, p) > 0$ for $p \in (0, 1)$ and $\delta \in [0, 1]$, we see that indeed $\mathcal{G}(\delta, p)$

$\in [0, 1]$ in $[0, 1]^2$. Further, $\mathcal{G}(\delta, p)$ is continuous in $[0, 1]^2 \backslash \{<0, 1>\}$ in each of its variables (we use the notation $\langle\, ,\, \rangle$ for ordered pair) since for all $\delta \in [0, 1]$ we have $\lim_{p\to 0^+} \{[1 - (1 - p)^\delta]/p\} = \delta = \mathcal{G}(\delta, 0)$ and

$$\lim_{p\to 1^-} \{[1 - (1 - p)^\delta]/p\} = \begin{cases} 0 & \text{if } \delta = 0 \\ 1 & \text{if } \delta \in (0, 1] \end{cases} = \mathcal{G}(\delta, 1) \qquad (38)$$

while, on the other hand, $\lim_{\delta\to 0^+} \mathcal{G}(\delta, 1) = 1 \neq \mathcal{G}(0, 1)$. The expression for $p_{partial}$ resulting *via* Eq. (29) from $\mathcal{G}$ given in Eq. (37), this former equation now being properly and completely rewritten (in view of Eq. (22), hence Eq. (28)) as

$$p^n_{j,\,partial}(\gamma_n(T_0, T_f), p^n_{j,\,full}) = \mathcal{G}(\gamma_n(T_0, T_f), p^0_{j,\,full})\, p^0_{j,\,full}\,, \qquad (39)$$

is

$$p^n_{j,\,partial}(\gamma_n(T_0, T_f), p^n_{j,\,full}) = \begin{cases} 1 - (1 - p^0_{j,\,full})^{\gamma_n} & \text{if } p^0_{j,\,full} \in [0, 1) \;\&\; \gamma_n \in [0, 1] \\ 0 & \text{if } p^0_{j,\,full} = 1 \;\&\; \gamma_n = 0 \\ 1 & \text{if } p^0_{j,\,full} = 1 \;\&\; \gamma_n \in (0, 1] \end{cases} \qquad (40)$$

for $j = 1,\ldots, 8$ and $n = 0,\ldots, N - 1$.

We now comment on our choice of solution $\mathcal{G}(\delta, p)$ for $p = 0$ and $p = 1$. Our choice of solution $\mathcal{G}(\delta, p)$ for $p = 0$ was guided by the mathematical considerations of preserving (a) the continuity of $\mathcal{G}$ in p at $p = 0$ for each $\delta \in [0, 1]$ as well as ensuring (b) the continuity of $\mathcal{G}(\delta, 0)$ in $[0, 1]$, and was reinforced by the circumstance that this choice causes no conceptual difficulties: it maintains $p_{partial}(\gamma, p_{full}) = 0$ for all $\gamma \in [0, 1]$ when $p_{full} = 0$. Indeed, there are other choices for the solution $\mathcal{G}(\delta, 0)$ which also cause no conceptual difficulties in the above sense and which maintain the continuity in (b) above but which destroy the continuity in (a) above. An example of this (and there are many others) follows by setting $\mathcal{G}(\delta, 0) = \mathcal{H}_\alpha(\delta)$ where, for any fixed $\alpha \in [0, ½)$, $\mathcal{H}_\alpha$ is the continuous function in [0, 1] given by

$$\mathcal{H}_\alpha(\delta) = \begin{cases} 0 & \text{if } \delta \in [0, \alpha) \\ (\delta - \alpha)/(1 - 2\alpha) & \text{if } \delta \in [\alpha, 1 - \alpha] \\ 1 & \text{if } \delta \in (1 - \alpha, 1]. \end{cases} \tag{41}$$

Of course the solution to Eqs. (33) + (34) given by modifying the second line of Eq. (37) to $\mathcal{G}(\delta, 0) = \mathcal{H}_\alpha(\delta)$ lacks the continuity of $\mathcal{G}$ in p at $p = 0$ for all $\delta \in [0, ½) \cup (½, 1]$. In any case, our actual choice of $\mathcal{G}(\delta, p)$ for $p = 0$ is the simplest one, hence (for us) the most natural.

Our choice of solution $\mathcal{G}(\delta, p)$ for $p = 1$ was also guided by the consideration of preserving the continuity of $\mathcal{G}$ in p at $p = 1$ for each $\delta \in [0, 1]$, but this choice necessarily leaves $\mathcal{G}(\delta, 1)$ discontinuous at $\delta = 0$ (see Eq. (38)). More seriously, this choice for $\mathcal{G}(\delta, 1)$ implies that $p_{partial}(\gamma, p_{full}) = p_{full}$ for *all* $\gamma \in (0, 1]$ when $p_{full} = 1$; but it is not at all clear that this state-of-affairs is not physically valid, as follows. We observe that governing Eq. (33), which is based upon the eminently reasonable Eq. (31a), becomes, when $p = 1$ ($= p_{full}$),

$$\mathcal{G}(\gamma, 1) + \mathcal{G}(1 - \gamma, 1) - \mathcal{G}(\gamma, 1)\mathcal{G}(1 - \gamma, 1) = 1. \tag{42}$$

If we now set $x = \mathcal{G}(\gamma, 1)$ and $y = \mathcal{G}(1 - \gamma, 1)$ and consider the function $f(x, y) = x + y - xy$ for $\langle x, y\rangle \in [0, 1]^2$, it is straightforward to see that $0 \le f(x, y) \le 1$ and that $f(x, y) = 1$ iff $\langle x, y\rangle \in (\{1\} \times [0,1]) \cup ([0,1] \times \{1\})$. In other words, to satisfy Eq. (42) we must have, for any γ, either $\mathcal{G}(\gamma, 1) = 1$ & $\mathcal{G}(1 - \gamma, 1) \in [0, 1]$, or $\mathcal{G}(1 - \gamma, 1) = 1$ and $\mathcal{G}(\gamma, 1) \in [0, 1]$; i.e., either $p_{partial}(\gamma, 1) = \mathcal{G}(\gamma, 1) \bullet 1 = 1$ & $p_{partial}(1 - \gamma, 1) = \mathcal{G}(1 - \gamma, 1) \bullet 1 \in [0, 1]$, or $p_{partial}(1 - \gamma, 1) = 1$ & $p_{partial}(\gamma, 1) \in [0, 1]$—*there is no way to satisfy Eq. (42) with both* $p_{partial}(\gamma, 1)$ *and* $p_{partial}(1 - \gamma, 1)$ *being less than 1*. So, if we accept that $\mathcal{G}(\gamma, 1)$ is non-decreasing in γ—as assumed following Eq. (24)—then we must have $\mathcal{G}(\gamma, 1) = 1$ for $\gamma \in [½, 1]$. Alternatively, we observe from Eq. (34) that we must have $\mathcal{G}(0, 1) = 0$ and

$\mathcal{G}(1, 1) = 1$, and from Eq. (36) that we must have $\mathcal{G}(½, 1) = 1$, so the requirement that $\mathcal{G}(\gamma, p)$ be nondecreasing in γ for all $p \in [0, 1]$ then dictates that $\mathcal{G}(\gamma, 1) = 1$ for $\gamma \in [½, 1]$. So at least for $\gamma \in [½, 1]$ it must hold that $p_{partial}(\gamma, p_{full}) = \mathcal{G}(\gamma, p_{full})\, p_{full} = \mathcal{G}(\gamma, 1) \bullet 1 = 1$.

We have just seen that $\mathcal{G}(\delta, 1) = 1$ for $\delta \in [½, 1]$. We next consider $\delta \in (0, ½)$. For such δ *we have chosen* $\mathcal{G}(\delta, 1) = 1$ as well, which choice implies the above behavior as well for $\gamma \in (0, ½)$, namely $p_{partial}(\gamma, p_{full}) = 1$. This behavior is perhaps now—in light of the previous argument for $\gamma \in [½, 1]$—at least plausible. It is nevertheless worthwhile to explore the alternatives to our choice of $\mathcal{G}(\delta, 1)$. We categorize the possibilities as follows: either $\mathcal{G}(\bullet, 1)$ is not continuous in(0, 1] or $\mathcal{G}(\bullet, 1)$ is continuous in (0, 1]. In the first case, one such family of solutions for $\mathcal{G}(\delta, 1)$ is given by setting $\mathcal{G}(\delta, 1) = \mathcal{J}_\alpha(\delta)$ where, for any fixed $\alpha \in [0, ½)$, $\mathcal{J}_\alpha$ is the function given by

$$\mathcal{J}_\alpha(\delta) = \begin{cases} 0 & \text{if } \delta \in [0, \alpha) \\ 1 & \text{if } \delta \in [\alpha, 1] \end{cases} \tag{43}$$

and therefore

$$p^{\mathcal{J}_\alpha}_{partial}(\gamma, 1) = \begin{cases} 0 & \text{if } \gamma \in [0, \alpha) \\ 1 & \text{if } \gamma \in [\alpha, 1]. \end{cases} \tag{44}$$

This solution $p^{\mathcal{J}_\alpha}_{partial}(\gamma, 1)$ exhibits a *threshold behavior*: If $\gamma < \alpha$ then $p^{\mathcal{J}_\alpha}_{partial}(\gamma, 1) = 0$ while if $\gamma \geq \alpha$ then $p^{\mathcal{J}_\alpha}_{partial}(\gamma, 1) = p_{full}$. The validity or non-validity of this behavior—and consequent appropriateness of this solution—may easily be determined experimentally. In the second case, in which $\mathcal{G}(\bullet, 1)$ is continuous in (0, 1], it turns out that in fact the *only* solution for $\mathcal{G}(\bullet, 1)$ is the one we have already chosen in Eq. (37). We now demonstrate this claim; namely:

> Let $\mathcal{G}(\delta, 1)$ satisfy Eq. (42) with $\mathcal{G}(0, 1) = 0$ and $\mathcal{G}(1, 1) = 1$ and let $\mathcal{G}(\bullet, 1)$ be non-decreasing in [0, 1] and continuous in (0, 1]; further, let $\mathcal{G}(\delta, 1) = 1$ for (*)

$\delta \in [½, 1]$. Then $\mathcal{G}(\delta, 1) = 1$ for $\delta \in (0,1]$.

To prove this result ($*$) we first make explicit a natural *extension* of Eq. (29) from basic time segments to any subinterval of a basic time segment, to wit:

If $B \equiv [n\tau_{c,1/2}, (n+1)\tau_{c,1/2})$ is any basic time segment having associated probability $p_{full} \equiv p_{full}(B)$ and if intervals J' and J are such that $J' \subseteq J \subseteq B$, with $\gamma' = \mathit{Length}\{J'\}/\tau_{c,1/2}$ and $\gamma = \mathit{Length}\{J\}/\tau_{c,1/2}$ and $p_{full}[\gamma] \equiv p_{partial}(\gamma, p_{full}(B))$, then we take

$$p_{partial}(\gamma', p_{full}(B)) = p_{partial}(\gamma'/\gamma, p_{full}[\gamma]) \tag{45}$$

since the argument γ' inside the $p_{partial}$ on the LHS of Eq. (45) refers to a sub-interval of B of length $\gamma'\tau_{c,1/2}$ and the argument γ'/γ inside the $p_{partial}$ on the RHS of Eq. (45) refers to a subinterval of B of length $(\gamma'/\gamma)\bullet\gamma\tau_{c,1/2} = \gamma'\tau_{c,1/2}$ as well. Further, we take

$$p_{partial}(\gamma'/\gamma, p_{full}[\gamma]) = \mathcal{G}(\gamma'/\gamma, p_{full}[\gamma])p_{full}[\gamma]. \tag{46}$$

The proof of ($*$) is now as follows. Let $\Delta \equiv \{\, \delta \in (0, ½] \mid \mathcal{G}(\delta, 1) = 1 \,\}$; then $½ \in \Delta$ and, denoting $\delta^* \equiv \inf \Delta$, then $\delta^* \in [0, ½]$. Now $(\delta^*, ½] \subseteq \Delta^{(\dagger)}$: If not then

$(\exists\delta^{\#} \in (\delta^*, ½])(\mathcal{G}(\delta^{\#}, 1) < 1)^{(\dagger\dagger)}$ so that $(\forall\delta \in \Delta)(\delta^{\#} < \delta)^{(\dagger\dagger\dagger)}$ [for if not (†††) then $(\exists\delta \in \Delta)(\delta^{\#} \geq \delta)$ so that $\mathcal{G}(\delta^{\#}, 1) \geq \mathcal{G}(\delta, 1) = 1$, a contradiction of (††)]; hence, by (†††), $\delta^{\#}$ is a lower bound for Δ and therefore $\delta^* \geq \delta^{\#} > 0$, a contradiction of $\delta^{\#} > \delta^*$ in (††). Now if $\delta^* = 0$ then we are done since in that case we have by (†) that $\Delta \subseteq (0, ½]$, hence $\Delta = (0, ½]$. So suppose that $\delta^* > 0$. Since $\mathcal{G}(\bullet, 1)$ is continuous in (0, 1] then $\mathcal{G}(\delta^*, 1) = \lim_{\delta\to(\delta^*)^+} \mathcal{G}(\delta, 1) = 1$, since $(\forall\delta > \delta^*)(\mathcal{G}(\delta, 1) = 1$ by (†); so $p_{partial}(\delta^*, p_{full}) = \mathcal{G}(\delta^*, 1)p_{full} = 1\bullet1 = 1$. Denote $p_{full}[\delta^*] \equiv p_{partial}(\delta^*, p_{full})$ so that $p_{full}[\delta^*] = 1$. Now $\delta^*/2 > 0$ and

$$\mathcal{G}(\delta^*/2, 1) = \mathcal{G}(\delta^*/2, 1)\bullet 1 = \mathcal{G}(\delta^*/2, p_{full})p_{full} = p_{partial}(\delta^*/2, p_{full}) = p_{partial}((\delta^*/2)/\delta^*, p_{full}[\delta^*]) \tag{47}$$

$$= p_{partial}(1/2, p_{full}[\delta^*]) = \mathcal{G}(½, p_{full}[\delta^*])p_{full}[\delta^*] = \mathcal{G}(½, 1) \bullet 1 = \mathcal{G}(½, 1) = 1$$

since $½ \in \Delta$, where the fourth equality in Eq. (47) follows from Eq. (45) with $\gamma' = \delta^*/2$ and $\gamma = \delta^*$ while the fifth equality in Eq. (47) follows from Eq. (46). Now from Eq. (47) we see that $\delta^*/2 \in \Delta$; but since $\delta^*/2 < \delta^*$ then $\delta^* \neq \inf \Delta$, a contradiction. So it is not true that $\delta^* > 0$ and we are done.♦

Finally, another more global example of an alternative solution to Eqs. (33) + (34), along the lines of the discontinuous solution $\mathcal{G}(\delta, 1)$ presented in Eq. (43), is:

$$\mathcal{G}(\delta, p) = \begin{cases} 0 & \text{if } \delta \in [0, ½) \\ g_{½}(p) & \text{if } \delta = ½ \qquad (p \in [0, 1]). \\ 1 & \text{if } \delta \in (½, 1] \end{cases} \tag{48}$$

Using $\mathcal{G}(\delta, p)$ we have, for all $p_{full} \in [0, 1]$,

$$p^*_{partial}(\gamma, p_{full}) = \begin{cases} 0 & \text{if } \gamma \in [0, ½) \\ g_{½}(p_{full})p_{full} & \text{if } \gamma = ½ \\ p_{full} & \text{if } \gamma \in (½, 1]. \end{cases} \tag{49}$$

The question arises as to where the basic $p^n_{j,\,full}[V_{EM}, \omega]$'s *are to come from: At this point they must be obtained externally to our model.* They may be obtained experimentally or predicted *via* additional modeling which has to date not yet been accomplished. However, once these $p^n_{j,\,full}[V_{EM}, \omega]$'s are available, then the $p^n_{j,\,partial}[V_{EM}, \omega]$'s may be computed *via* Eq. (40).

IV.2. <u>Individual ½-Period-Segmented SPT Elements: Indirect Probability Assignment</u>

As mentioned previously in Section III.1, we feel that the primitive SPT elements are the natural entities to which disruptive deviation probabilities should be assigned; disruptive deviation probabilities for the ½-period-segmented SPT elements may then be computed from these. In this section we present this approach. An additional benefit of this approach is that it allows us to overcome the objection (alluded to parenthetically following Eq. (25)) that the formulation of Eqs. (24) and (28) for $p_{partial}$ (and its elaboration in Eq. (39)) fails to take into account that the ½-period-segmented SPT elements s_j^n have—except for j =1, 5—internal structure (Eqs. (2) – (9)) and therefore

$p_{j,\,partial}^n$ should be based not merely upon a single γ_n for the entire time segment but rather upon as many γ_n′s for the segment as there are segment features. For example, s_4^n should use three γ_n′s—one each for the rising, constant, and falling time sub-segments—to compute its $p_{j,\,partial}^n$ in accordance with whether the partial overlap of $\pi^{\mathrm{EM}}[T_0,\, T_f,\, V_{\mathrm{EM}},\, \omega]$ with s_4^n occurs during the rising portion, the constant portion, or the falling portion of that segment or, more generally, during some combination of two or three of these. We present these enhanced results as well.

The reader may, upon first reading of this paper, skip the details in the present section and safely move on to Section IV.3; for all that is necessary to proceed thereto is that each ½-period-segmented SPT element have a π^{EM}-dependent disruptive deviation probability assigned to it and that has been already been accomplished *via* Eqs. (22) and (40).

IV.2.1. ½-*Period-Segmented SPT Elements from Primitive SPT Elements*

In this section, we derive expressions for the ½-period-segmented SPT elements in terms of the primitive SPT elements. It is sufficient to consider in detail only the zeroth ½-period-segmented SPT elements s_j^0; results for s_j^n, n = 1,…, N – 1, then follow immediately from Eqs. (11) and (22).

Consider firstly the four *basic* primitive SPT element types (A, B C, D), which we label by σ_ν, ν = A, B, C, D, and which are given specifically by

$$\sigma_{\mathrm{A}}(t) = V_{\pi,\,high} \qquad t \in [0,\, \tau_{c,1/2}) \tag{50}$$

$$\sigma_{\mathrm{B}}(t) = V_{\pi,\,low} \qquad t \in [0,\, \tau_{c,1/2}) \tag{51}$$

$$\sigma_{\mathrm{C}}(t) = (\Delta V_\pi/\alpha\tau_c)\,t + V_{\pi,\,mid} \qquad t \in [-\alpha\tau_{c,1/2},\, \alpha\tau_{c,1/2}) \tag{52}$$

$$\sigma_{\mathrm{D}}(t) = -\,(\Delta V_\pi/\beta\tau_c)\,t + V_{\pi,\,mid} \qquad t \in [-\beta\tau_{c,1/2},\, \beta\tau_{c,1/2}). \tag{53}$$

A glance at Fig. 2 depicting the s_j^0's reveals that each s_j^0 consists of portions of—but not the entirety of (except in the cases of s_1^0 and s_5^0)—several σ_ν's. These portions, which we refer to as *partial* primitive SPT elements, are as follows:

$$\sigma_{A+} \equiv \sigma_A \restriction [\alpha\tau_{c,1/2}, \tau_{c,1/2}) \tag{54}$$

$$\sigma_{A-} \equiv \sigma_A \restriction [0, (1-\beta)\tau_{c,1/2}) \tag{55}$$

$$\sigma_{A\pm} \equiv \sigma_A \restriction [\alpha\tau_{c,1/2}, (1-\beta)\tau_{c,1/2}) \tag{56}$$

$$\sigma_{B+} \equiv \sigma_B \restriction [\beta\tau_{c,1/2}, \tau_{c,1/2}) \tag{57}$$

$$\sigma_{B-} \equiv \sigma_B \restriction [0, (1-\alpha)\tau_{c,1/2}) \tag{58}$$

$$\sigma_{B\pm} \equiv \sigma_B \restriction [\beta\tau_{c,1/2}, (1-\alpha)\tau_{c,1/2}) \tag{59}$$

$$\sigma_{C+} \equiv \sigma_C \restriction [0, \alpha\tau_{c,1/2}) \tag{60}$$

$$\hat{\sigma}_{C-} \equiv \sigma_C \restriction [-\alpha\tau_{c,1/2}, 0) \tag{61}$$

$$\sigma_{D+} \equiv \sigma_D \restriction [0, \beta\tau_{c,1/2}) \tag{62}$$

$$\hat{\sigma}_{D-} \equiv \sigma_D \restriction [-\beta\tau_{c,1/2}, 0); \tag{63}$$

further, it is not $\hat{\sigma}_{C-}$ and $\hat{\sigma}_{D-}$ that are directly needed but rather their time translates

$$\sigma_{C-}(t) = \hat{\sigma}_{C-}(t - \tau_{c,1/2}) \qquad t \in [(1-\alpha)\tau_{c,1/2}, \tau_{c,1/2}) \tag{64}$$

and

$$\sigma_{D-}(t) = \hat{\sigma}_{D-}(t - \tau_{c,1/2}) \qquad t \in [(1-\beta)\tau_{c,1/2}, \tau_{c,1/2}). \tag{65}$$

We denote by σ_μ, $\mu \in \{A+, A-, A\pm, B+, B-, B\pm, C+, C-, D+, D-\} \equiv \mathcal{M}$, any of the various partial primitive SPT elements indicated in Eqs. (54) – (65) (but not in Eqs. (61) and (63)) and, further, by I_μ the respective restriction interval associated with σ_μ (e.g., $I_{C-} = [(1-\alpha)\tau_{c,1/2}, \tau_{c,1/2})$). We have written the σ_μ's in the above as restrictions to subintervals of the full domains of definition of the basic primitive SPT elements in order to emphasize that the disruptive deviation probabilities to be assigned, in the next subsection, to the σ_μ's for $\mu \in \mathcal{M}$ are in fact $p_{partial}(\gamma, p_{full})$'s in the sense of the previous subsection, where p_{full} is that of the "parent" basic primitive SPT element $\sigma_{\nu(\mu)}$ of σ_μ (e.g., σ_A is the parent of each of

σ_{A+}, σ_{A-}, and $\sigma_{A\pm}$ so that if μ = A+, A– or A$\pm$ then $\nu(\mu)$ = A) and γ is the relative fractional length given by

$$\gamma = \tilde{\gamma}_\mu \equiv \text{Length}\{I_\mu\}/\text{Length}\{\text{“parent” } \sigma_{\nu(\mu)}\}. \tag{66}$$

In this connection, we also observe that if we denote $\sigma^*_C(t) \equiv \sigma_C(t - \tau_{c,1/2})$ for $t \in$

$[(1-\alpha)\tau_{c,1/2}, (1+\alpha)\tau_{c,1/2})$, and $\sigma^*_D(t) \equiv \sigma_D(t - \tau_{c,1/2})$ for $t \in [(1-\beta)\tau_{c,1/2}, (1+\beta)\tau_{c,1/2})$, then in terms of these time translates we have

$$\sigma_{C-} = \sigma^*_C \restriction [(1-\alpha)\tau_{c,1/2}, \tau_{c,1/2}) \qquad \text{and} \qquad \sigma_{D-} = \sigma^*_D \restriction [(1-\beta)\tau_{c,1/2}, \tau_{c,1/2}). \tag{67}$$

This observation will be relevant in assigning, in the next subsection, disruptive deviation probabilities to σ_{C-} and σ_{D-}, for we will continue to assume for primitive SPT elements (both full and partial)—as we have previously assumed for ½-period-segmented SPT elements (see Eq. (22))—the property of time translation invariance (or what we have previously termed “modularity”) of disruption probabilities.

We denote by σ^0_μ the extension of σ_μ to $[0, \tau_{c,1/2})$, i.e.,

$$\sigma^0_\mu(t) \equiv \begin{cases} \sigma_\mu(t) & \text{if } t \in I_\mu \\ 0 & \text{if } t \in [0, T_E) \setminus I_\mu. \end{cases} \qquad (\mu \in \mathcal{M}) \tag{68}$$

Since σ_A and σ_B in their entirety are required to construct ½-period-segmented SPT elements s_1^0 and s_5^0, we also allow the index μ to take on values A and B (but not C or D), with $I_A = [0, \tau_{c,1/2}) = I_B$; and we write $\mathcal{M}^+ \equiv \mathcal{M} \cup \{A, B\}$. Thus the specification $\mu \in \mathcal{M}$ in Eq. (68) is to be replaced by $\mu \in \mathcal{M}^+$. Note that $\sigma^0_A = \sigma_A$ and $\sigma^0_B = \sigma_B$.

At this stage, all ½-period-segmented SPT elements in $[0, \tau_{c,1/2})$ may be constructed as sums of the (full or partial) primitive SPT elements of Eq. (68) (with $\mu \in \mathcal{M}^+$). To wit, for all $t \in [0, \tau_{c,1/2})$ we have:

$$s_1^0 = \sigma^0_A \qquad s_2^0 = \sigma^0_{C+} + \sigma^0_{A+} \qquad s_3^0 = \sigma^0_{A-} + \sigma^0_{D-} \qquad s_4^0 = \sigma^0_{C+} + \sigma^0_{A\pm} + \sigma^0_{D-} \tag{69}$$

$$s_5^0 = \sigma_B^0 \qquad s_6^0 = \sigma_{D+}^0 + \sigma_{B+}^0 \qquad s_7^0 = \sigma_{B-}^0 + \sigma_{C-}^0 \qquad s_8^0 = \sigma_{D+}^0 + \sigma_{B\pm}^0 + \sigma_{C-}^0 .$$

We denote, for $j = 1, \ldots, 8$,

$$\overset{+}{\mathcal{M}}{}_j^0 \equiv \{\mu \in \mathcal{M}^+ \mid \sigma_\mu^0 \text{ occurs in the sum for } s_j^0\} \tag{70}$$

so, for example, $\overset{+}{\mathcal{M}}{}_j^0 = \{\text{A}\pm, \text{C}+, \text{D}-\}$; also

$$s_j^0 = \sum_{\mu \in \overset{+}{\mathcal{M}}{}_j^0} \sigma_\mu^0. \tag{71}$$

IV.2.2. *Primitive SPT Element Disruptive Deviation Probabilities*

We next compute disruptive deviation probabilities for the partial primitive SPT elements based upon disruptive deviation probabilities for the basic primitive SPT elements.

To begin, we denote by $p_{\nu,\, full}[V_{\text{EM}}, \omega]$, ν = A, B, C, D, the probability that basic SPT element σ_ν suffers a disruptive deviation when it, or a time translate of it, occurs as part of an N-element SPT and when, in addition, it is fully encompassed by some $\pi^{\text{EM}}[T_0, T_f, V_{\text{EM}}, \omega]$. As previously assumed for ½-period-segmented SPT elements, we again assume modularity for these basic primitive SPT elements: The probability of a disruptive deviation of each one of the four basic SPT primitive elements is independent of where that primitive element occurs in the N-element SPT, as long as that primitive element is completely encompassed by π^{EM}. *And once again, the $p_{\nu,\, full}[V_{EM}, \omega]$'s must be obtained externally and made available to our model*; but note that

$$p_{\text{A},\, full}[V_{\text{EM}}, \omega] = p^0_{1,\, full}[V_{\text{EM}}, \omega] \qquad \text{and} \qquad p_{\text{B},\, full}[V_{\text{EM}}, \omega] = p^0_{5,\, full}[V_{\text{EM}}, \omega]. \tag{72}$$

Also, consistently with our previous notational convention, we denote for convenience $p^0_{\text{A},\, full}[V_{\text{EM}}, \omega] \equiv p_{\text{A},\, full}[V_{\text{EM}}, \omega]$ and $p^0_{\text{B},\, full}[V_{\text{EM}}, \omega] \equiv p_{\text{B},\, full}[V_{\text{EM}}, \omega]$.

In addition to the four probabilities $p_{\nu,\, full}[V_{\text{EM}}, \omega]$ above, we also need expressions for the disruptive deviation probabilities, $p^0_\mu[V_{EM}, \omega]$, of the σ^0_μ, $\mu \in \mathcal{M}$. Consider first μ = A+: Since the parent of $\sigma_{\text{A}+}$ is σ_A then from Eq. (66) we have $\tilde{\gamma}_{\text{A}+} = 1 - \alpha$, thus, using Eq. (40),

$$p^0_{A+}[V_{EM}, \omega] = p_{partial}(1-\alpha, p^0_{A,\,full}) = 1-(1-p^0_{A,\,full})^{1-\alpha} \quad \text{for } p^0_{A,\,full} \in [0, 1] \;\;\&\;\; \alpha \in (0, 1) \tag{73}$$

(where we have taken into account that α = 0, 1 is precluded by our initial definition [in Section III.1] of the type-C SPT primitive element). Similarly, we have $\tilde{\gamma}_{A-} = 1-\beta$ and

$$p^0_{A-}[V_{EM}, \omega] = p_{partial}(1-\beta, p^0_{A,\,full}) = 1-(1-p^0_{A,\,full})^{1-\beta} \quad \text{for } p^0_{A,\,full} \in [0, 1] \;\;\&\;\; \beta \in (0, 1) \tag{74}$$

(also where β = 0, 1 was precluded earlier). Next we have $\tilde{\gamma}_{A\pm} = 1-(\alpha+\beta)$ and

$$p^0_{A\pm}[V_{EM}, \omega] = p_{partial}(1-(\alpha+\beta), p^0_{A,\,full}) = 1-(1-p^0_{A,\,full})^{1-(\alpha+\beta)}$$
$$\text{for } p^0_{A,\,full} \in [0, 1] \;\;\&\;\; \alpha+\beta \in (0, 1) \tag{75}$$

(since $\alpha + \beta$ = 0, 1 has also been precluded previously). The expressions for $p^0_{B+}[V_{EM}, \omega]$, $p^0_{B-}[V_{EM}, \omega]$, and $p^0_{B\pm}[V_{EM}, \omega]$ follow directly from those for $p^0_{A+}[V_{EM}, \omega]$, $p^0_{A-}[V_{EM}, \omega]$, and $p^0_{A\pm}[V_{EM}, \omega]$, respectively, *via* the replacement everywhere in the latter three expressions of $p_{A,\,full}$ by $p_{B,\,full}$ as well as the interchange everywhere of α and β since $\tilde{\gamma}_{B+} = 1-\beta$, $\tilde{\gamma}_{B-} = 1-\alpha$, and $\tilde{\gamma}_{B\pm} = 1-(\alpha+\beta)$. Finally, since $\tilde{\gamma}_{C+} = ½ = \tilde{\gamma}_{C-}$, with the parents of σ_{C+} and σ_{C-} being σ_C or time translate σ^*_C respectively; and $\tilde{\gamma}_{D+} = ½ = \tilde{\gamma}_{D-}$, with parents of σ_{D+} and σ_{D-} being σ_D or σ^*_D respectively; then we have from Eq. (40) that

$$\begin{aligned} p^0_{C+}[V_{EM}, \omega] = p^0_{C-}[V_{EM}, \omega] &= 1-(1-p_{C,\,full})^{1/2} \text{ if } p_{C,\,full} \in [0, 1], \\ p^0_{D+}[V_{EM}, \omega] = p^0_{D-}[V_{EM}, \omega] &= 1-(1-p_{D,\,full})^{1/2} \text{ if } p_{D,\,full} \in [0, 1]. \end{aligned} \tag{76}$$

The computation of the disruptive deviation probabilities for the partial primitive SPT elements is now complete.

IV.2.3. *½-Period-Segmented SPT Disruptive Deviation Probabilities: Full Overlap of π^{EM} and s^n_j*

We now compute the disruptive deviation probabilities, $p^0_{j,\,full}[V_{EM}, \omega]$, $j = 1, \ldots, 8$, for the ½-period-segmented SPT elements, s^0_j, using the disruptive deviation probabilities $p^0_\mu[V_{EM}, \omega]$, $\mu \in \mathcal{M}^+$, of the primitive SPT elements σ^0_μ, $\mu \in \mathcal{M}^+$, under the condition, as in Section IV.1.1, that there is full overlap of π^{EM} and s^0_j. We suppress the argument $[V_{EM}, \omega]$ of p^0_μ. Using Eq. (69) along with reasoning analogous to that presented directly following Eqs. (31) we find

$$p^0_{1,\,full} = p^0_{A,\,full} \tag{77}$$

$$p^0_{2,\,full} = p^0_{C+} + (1 - p^0_{C+})\, p^0_{A+} \tag{78}$$

$$p^0_{3,\,full} = p^0_{A-} + (1 - p^0_{A-})\, p^0_{D-} \tag{79}$$

$$p^0_{4,\,full} = p^0_{C+} + (1 - p^0_{C+})\, p^0_{A\pm} + (1 - p^0_{C+})(1 - p^0_{A\pm})\, p^0_{D-} \tag{80}$$

$$p^0_{5,\,full} = p^0_{B,\,full} \tag{81}$$

$$p^0_{6,\,full} = p^0_{D+} + (1 - p^0_{D+})\, p^0_{B+} \tag{82}$$

$$p^0_{7,\,full} = p^0_{B-} + (1 - p^0_{B-})\, p^0_{C-} \tag{83}$$

$$p^0_{8,\,full} = p^0_{D+} + (1 - p^0_{D+})\, p^0_{B\pm} + (1 - p^0_{D+})(1 - p^0_{B\pm})\, p^0_{C-} \tag{84}$$

IV.2.4. *½-Period-Segmented SPT Disruptive Deviation Probabilities: Partial Overlap of π^{EM} and s^n_j*

Finally, we compute the disruptive deviation probabilities, $p^0_{j,\,partial}[V_{EM}, \omega]$, $j = 1, \ldots, 8$, for the ½-period-segmented SPT elements s^0_j, using the disruptive deviation probabilities, $p^0_\mu[V_{EM}, \omega]$, $\mu \in \mathcal{M}^+$, of the primitive SPT elements σ^0_μ, $\mu \in \mathcal{M}^+$, under the condition, as in Section IV.1.2, that there is partial overlap of $\pi^{EM}[T_0, T_f, V_{EM}, \omega]$ and s^0_j. We first define (see Eq. (23)), for $\mu \in \mathcal{M}$,

$$\gamma_{0,\,\mu}(T_0, T_f) \equiv Length\{[T_0, T_f) \cap I_\mu\}/Length\{I_\mu\}. \tag{85}$$

Then—still using the assumption that $p_{partial}$ depends only upon γ (however, see below)—we have

$$p^0_{1,\,partial} = p_{partial}(\gamma_{0,\,\mathrm{A}}, p^0_{\mathrm{A},\,full}) \tag{86}$$

$$p^0_{2,\,partial} = p_{partial}(\gamma_{0,\,\mathrm{C+}}, p^0_{\mathrm{C+}}) + [1 - p_{partial}(\gamma_{0,\,\mathrm{C+}}, p^0_{\mathrm{C+}})]p_{partial}(\gamma_{0,\,\mathrm{A+}}, p^0_{\mathrm{A+}}) \tag{87}$$

$$p^0_{3,\,partial} = p_{partial}(\gamma_{0,\,\mathrm{A-}}, p^0_{\mathrm{A-}}) + [1 - p_{partial}(\gamma_{0,\,\mathrm{A-}}, p^0_{\mathrm{A-}})]p_{partial}(\gamma_{0,\,\mathrm{D-}}, p^0_{\mathrm{D-}}) \tag{88}$$

$$\begin{aligned} p^0_{4,\,partial} = {} & p_{partial}(\gamma_{0,\,\mathrm{C+}}, p^0_{\mathrm{C+}}) + [1 - p_{partial}(\gamma_{0,\,\mathrm{C+}}, p^0_{\mathrm{C+}})]p_{partial}(\gamma_{0,\,\mathrm{A\pm}}, p^0_{\mathrm{A\pm}}) \\ & + [1 - p_{partial}(\gamma_{0,\,\mathrm{C+}}, p^0_{\mathrm{C+}})]\,[1 - p_{partial}(\gamma_{0,\,\mathrm{A\pm}}, p^0_{\mathrm{A\pm}})]\,p_{partial}(\gamma_{0,\,\mathrm{D-}}, p^0_{\mathrm{D-}}) \end{aligned} \tag{89}$$

$$p^0_{5,\,partial} = p_{partial}(\gamma_{0,\,\mathrm{B}}, p^0_{\mathrm{B},\,full}) \tag{90}$$

$$p^0_{6,\,partial} = p_{partial}(\gamma_{0,\,\mathrm{D+}}, p^0_{\mathrm{D+}}) + [1 - p_{partial}(\gamma_{0,\,\mathrm{D+}}, p^0_{\mathrm{D+}})]p_{partial}(\gamma_{0,\,\mathrm{B+}}, p^0_{\mathrm{B+}}) \tag{91}$$

$$p^0_{7,\,partial} = p_{partial}(\gamma_{0,\,\mathrm{B-}}, p^0_{\mathrm{B-}}) + [1 - p_{partial}(\gamma_{0,\,\mathrm{B-}}, p^0_{\mathrm{B-}})]p_{partial}(\gamma_{0,\,\mathrm{C-}}, p^0_{\mathrm{C-}}) \tag{92}$$

$$\begin{aligned} p^0_{8,\,partial} = {} & p_{partial}(\gamma_{0,\,\mathrm{D+}}, p^0_{\mathrm{D+}}) + [1 - p_{partial}(\gamma_{0,\,\mathrm{D+}}, p^0_{\mathrm{D+}})]p_{partial}(\gamma_{0,\,\mathrm{B\pm}}, p^0_{\mathrm{B\pm}}) \\ & + [1 - p_{partial}(\gamma_{0,\,\mathrm{D+}}, p^0_{\mathrm{D+}})]\,[1 - p_{partial}(\gamma_{0,\,\mathrm{B\pm}}, p^0_{\mathrm{B\pm}})]\,p_{partial}(\gamma_{0,\,\mathrm{C-}}, p^0_{\mathrm{C-}}). \end{aligned} \tag{93}$$

In summary, we have computed, for $j = 1, \ldots, 8$, both $p^0_{j,\,full}[V_{\mathrm{EM}}, \omega]$—*via* Eqs. (77) through (84)—and $p^0_{j,\,partial}\,[V_{\mathrm{EM}}, \omega; \{\gamma_{0,\,\mu}(T_0, T_f)\}_{\mu \in \mathcal{M}^{+0}_j}]$—*via* Eqs. (86) through (93)—from basic primitive SPT elements σ_ν, ν = A, B, C, D, with the aid of the intermediary partial primitive SPT element disruptive deviation probabilities $p^0_\mu[V_{EM}, \omega]$, $\mu \in \mathcal{M}$.

The assumption that $p_{partial}(\gamma_{0,\,\mu}, p^0_{\mu,\,full})$ depends only upon its two listed arguments may be simplistic in principle in case μ = C+, C–, D+, D– since—on the one hand—it is reasonable to conjecture that the (notationally suppressed) voltage dependencies of the various $p_{partial}$′s (as well as of the various p_{full}′s) are functions of the *ratio* $V_{\mathrm{EM}}/V_{\mathrm{signal\ pulse}}$

and—on the other hand—it is the case that the partial primitive SPT elements associated with the above four μ′s do not have constant-in-time signal voltage levels. Indeed, for each of these four partial primitive SPT elements, the $V_{\text{signal pulse}}$ voltage range encompassed during any sub-intervals thereof depends not only upon the length $\gamma_{0,\,\mu}(T_0, T_f)$ of the sub-interval but also upon the *position* of that sub-interval within the element. Since differing such intervals may be completely characterized, hence distinguished, by giving—in addition to their length—also the location of the interval's center point, say $\tau_{0,\,\mu}(T_0, T_f) \equiv \textit{Midpoint of}\ \{[T_0, T_f) \cap I_\mu\}$, then $\tau_{0,\,\mu}$ may augment the argument list of $p_{partial}(\gamma_{0,\,\mu}, p^0_{\mu,\,full})$ to yield $p^{\#}_{partial}(\gamma_{0,\,\mu}, p^0_{\mu,\,full})$ and thus provide an "anchor" to specify interval-location-dependent quantities, e.g., $V_{\text{EM}}/V_{\text{signal pulse}}$ at $\tau_{0,\,\mu}$, upon which $p^{\#}_{partial}$ may be declared to depend in some model; of course this dependence must further be specified. Eqs. (87) – (89) and (91) – (93) may then be modified by replacing, on their RHS′s, $p_{partial}(\gamma_{0,\,\mu}, p^0_{\mu,\,full})$, μ = C+, C–, D+, D–, with $p^{\#}_{partial}(\gamma_{0,\,\mu}, p^0_{\mu,\,full})$. We will not pursue this refinement further in this paper and take Eqs. (86) – (93) as they stand as our model for $p^0_{j,\,partial}[V_{\text{EM}}, \omega]$, $j = 1, \ldots, 8$.

IV.3. Arbitrary *N*-element SPT

In the previous sections we have shown how to build an *N*-segment SPT from ½-period-segmented SPT elements and, further, how to assign (in two ways) disruptive deviation probabilities to these elements. We now combine these two to compute the disruptive deviation probability for the entire idealized, normal, *N*-segment SPT $\pi^{\mathcal{N}}$. To this end, and with notation as in Section III, fix $\pi^{\text{EM}}[T_0, T_f, V_{\text{EM}}, \omega]$ and consider any

$\pi^{\mathcal{N}}[j_0, j_1,\ldots, j_{N-1}] \in \Pi^{\mathcal{N}}_N[0, T_E]$, with $j_n \in \{1,\ldots,8\}$ for $n = 0,\ldots, N-1$, and write

$\pi^{\mathcal{N}} = \sum_{n=0}^{N-1} s^n_{j_n}$ with $(\forall n = 0, \ldots, N-1)(\langle j_n, j_{n+1}\rangle \in J^+)$ as in Eq. (14). Now consider the following conceptual random experiment: At t = 0, irradiate with π^{EM} a single signal line carrying $\pi^{\mathcal{N}}$ and observe the *first* time segment in which a disruptive deviation of the SPT—*via* disruptive deviation of its ½-period-segmented SPT element in that time segment—takes place (which disruptive deviation can be identified by the mal-execution of the μC task, $\mathcal{T}_a$, that is executing and to which $\pi^{\mathcal{N}}$ belongs). In this situation, it is not all of $\pi^{\mathcal{N}}$ that is of interest but rather $\pi^{\mathcal{N}}\restriction]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![$. The possible outcomes of the random experiment can be given by the members of the set

$$\Omega_N^{[T_0,\, T_f)} \equiv \{\, n_{min}[T_0,\, T_f],\, n_{min}+ 1,\ldots,\, n_{max}- 1,\, n_{max}[T_0,\, T_f],\, n_{max}+ 1\}, \tag{94}$$

with the outcome "n" representing the situation, when $n \leq n_{max}$, that the first time segment in which a disruptive deviation of the SPT takes place is n and with the outcome "n_{max}+ 1" representing the situation in which *no* disruptive deviation takes place in any time segment occurring during $[T_0,\, T_f)$. The set of events associated with $\Omega_N^{[T_0,\, T_f)}$ is of course taken to be $2^{\Omega_N^{[T_0,\, T_f)}}$ (the power set of $\Omega_N^{[T_0,\, T_f)}$). We now specify a probability measure, denoted $\mathcal{P}_N^{[T_0,\, T_f)}$, on $(\Omega_N^{[T_0,\, T_f)},\, 2^{\Omega_N^{[T_0,\, T_f)}})$. First we take, for $n = n_{min},\ldots,\, n_{max}$,

$$\mathcal{P}_N^{[T_0,\, T_f)}(\{n\}) = \; p_{j_n}^{n}[V_{\mathrm{EM}},\, \omega;\, T_0,\, T_f] \bullet \prod\nolimits_{q\,=\,n_{min}}^{n\,-\,1}(1 - p_{j_q}^{q}\,[V_{\mathrm{EM}},\, \omega;\, T_0,\, T_f]) \tag{95}$$

where

$$p_j^r[V_{EM},\, \omega;\, T_0,\, T_f] = \begin{cases} p_{j,\,full}^{r}[V_{\mathrm{EM}},\, \omega] & \text{if } r = n_{min}+ 1,\ldots,\, n_{max}- 1 \\ \\ p_{j,\,partial}^{r}[V_{\mathrm{EM}},\, \omega;\, T_0,\, T_f] & \text{if } r = n_{min} \text{ or } n_{max} \end{cases} \tag{96}$$

and possibly $p_{j,\,partial}^{r}[V_{\mathrm{EM}},\, \omega;\, T_0,\, T_f] = p_{j,\,full}^{r}[V_{\mathrm{EM}},\, \omega]$ when $r = n_{min}$ or n_{max}; clearly,

$0 \leq \mathcal{P}_N^{[T_0,\, T_f)}(\{n\}) \leq 1$. Note that $p_{j,\,partial}^{r}$ may be that from either of the two alternative formulations given in Section IV.2. It is straightforward to show that

$$\sum\nolimits_{q\,=\,n_{min}}^{n_{max}} \mathcal{P}_N^{[T_0,\, T_f)}(\{n\}) = 1 - \prod\nolimits_{q\,=\,n_{min}}^{n_{max}} (1 - p_{j_q}^{q}) \leq 1 \tag{97}$$

and we therefore have

$$\mathcal{P}_N^{[T_0,\, T_f)}\,(\{n_{max}+ 1\}) = \prod\nolimits_{q\,=\,n_{min}}^{n_{max}} (1 - p_{j_q}^{q}), \tag{98}$$

as expected. Since $\{\{n\}\}_{n=n_{min}}^{n_{max}+1}$ is a partition of $\Omega_N^{[T_0,\,T_f)}$ we may then extend $\mathcal{P}_N^{[T_0,\,T_f)}$ to all of $2^{\Omega_N^{[T_0,\,T_f)}}$ by $\mathcal{P}_N^{[T_0,\,T_f)}(A) = \sum_{n\in A} \mathcal{P}_N^{[T_0,\,T_f)}(\{n\})$ for $A \in 2^{\Omega_N^{[T_0,\,T_f)}}$; then $\mathcal{P}_N^{[T_0,\,T_f)}$, being manifestly countably additive, is thus a probability measure. Our probability space for observing the occurrence of a disruptive deviation induced by $\pi^{\mathrm{EM}}[T_0, T_f, V_{EM}, \omega]$ upon a single signal line carrying a specific but arbitrary $\pi^{\mathcal{N}}[j_0, j_1,\ldots, j_{N-1}]\restriction]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![\in \Pi_N^{\mathcal{N}}[0, T_E]\restriction]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![$ is then $(\Omega_N^{[T_0,\,T_f)}, 2^{\Omega_N^{[T_0,\,T_f)}}, \mathcal{P}_N^{[T_0,\,T_f)})$, where we have denoted by $\Pi_N^{\mathcal{N}}[0, T_E]\restriction]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![$ the set

$$\Pi_N^{\mathcal{N}}[0, T_E]\restriction]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![\equiv \{\pi^{\mathcal{N}}\restriction]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![\mid \pi^{\mathcal{N}} \in \Pi_N^{\mathcal{N}}[0, T_E]\}. \tag{99}$$

The event "SPT $\pi^{\mathcal{N}}$ suffers a disruptive deviation" is $\mathcal{E}_{\mathcal{D},D} = \Omega_N^{[T_0,\,T_f)} \setminus \{n_{max}+1\}$ with, according to Eq. (98),

$$\mathcal{P}_N^{[T_0,\,T_f)}(\mathcal{E}_{\mathcal{D},D}) = 1 - \prod_{q=n_{min}}^{n_{max}} (1 - p_{j_q}^q[V_{\mathrm{EM}}, \omega; T_0, T_f]) \leq 1. \tag{100}$$

This is our desired final expression for the probability $\mathcal{P}_N^{[T_0,\,T_f)}(\mathcal{E}_{\mathcal{D},D})$ that SPT $\pi^{\mathcal{N}}$ suffers a disruptive deviation when subject to π^{EM}.

It is interesting to observe that the result given in Eq. (100) can be obtained in an alternative fashion using an $(n_{max} - n_{min} + 1)$-factor product space formulation in which a factor in the product appears for each of the $(n_{max} - n_{min} + 1)$ ½-period-segmented elements of the SPT $\pi^{\mathcal{N}}$ restricted to $]\!]\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}[\![$ and in which the two possible outcomes achievable for each individual ½-period-segmented element—a disruptive deviation of that element occurs or no disruptive deviation of that element occurs—are taken to be independent between elements. That is, if for $n = n_{min},\ldots, n_{max}$ we take $\mathcal{O}_n = \{d, \sim d\}$, where d is the outcome for $s_{j_n}^n$ that $s_{j_n}^n$ suffers a disruptive deviation, with $\mathscr{P}_n(\{d\}) = p_{j_n}^n$, and $\mathscr{P}_n(\{\sim d\}) = 1 - p_{j_n}^n$, and set

$$\mathcal{O}_N^{[T_0,\ T_f)} \equiv \mathbf{X}_{n=n_{min}}^{n_{max}} \mathcal{O}_n \tag{101}$$

and

$$\mathcal{P}_N^{[T_0,\ T_f)}(\mathbf{X}_{n=n_{min}}^{n_{max}} A_n) = \mathbf{X}_{n=n_{min}}^{n_{max}} \mathcal{P}_n(A_n) \quad \text{for} \quad A_n \in 2^{\mathcal{O}_n}, \tag{102}$$

then in this setting the event “SPT $\pi^{\mathcal{N}}$ does not suffer a disruptive deviation” is given by $\mathcal{E}_{\sim(\mathcal{D},\,D)} \equiv \{<\sim d, \sim d, \ldots, \sim d>\} \in 2^{\mathcal{O}_N^{[T_0,\ T_f)}}$ with probability $\mathcal{P}_N^{[T_0,\ T_f)}(\mathcal{E}_{\sim(\mathcal{D},\,D)}) = \prod_{q=n_{min}}^{n_{max}}(1 - p_{j_q}^q)$, which probability is consistent with the result in Eq. (100). More generally, the event $\{n\}$ in the first formulation corresponds to the event of the second formulation given by

$$\begin{aligned}\mathcal{E}_{(\mathcal{D},\,D),\ n} \equiv \{<\sim d, \ldots, \sim d, d, o, \ldots, o> \in \mathcal{O}_N^{[T_0,\ T_f)} \mid \sim d \text{ occurs in the first } n-1 \text{ entries} \\ \& \ o \in \{d, \sim d\}\} \\ = (\mathbf{X}_{q=n_{min}}^{n-1} \{\sim d\}) \times \{d\} \times (\mathbf{X}_{q=\mathrm{n}+1}^{N} \mathcal{O}_q);\end{aligned} \tag{103}$$

hence,

$$\mathcal{P}_N^{[T_0,\ T_f)}(\mathcal{E}_{(\mathcal{D},\,D),\ n}) = (\textstyle\prod_{q=n_{min}}^{n-1} [1 - p_{j_q}^q]) \bullet p_{j_n}^n \bullet (\prod_{q=\mathrm{n}+1}^{N} 1) = \mathcal{P}_N^{[T_0,\ T_f)}(\{n\}), \tag{104}$$

where the final equality follows from Eq. (95).

The reason we have not used this second, product space formulation is because many of the outcomes in $\mathcal{O}_N^{[T_0,\ T_f)}$ are not appropriate for our physical context. For example, an outcome in $\mathcal{O}_N^{[T_0,\ T_f)}$ of the type $<d, \sim d, \ldots, \sim d>$—in which a “no disruptive deviation” outcome must be observed for every individual ½-period-segmented SPT element subsequent to the first, with each such subsequent element following an individual element (the first element) for which a “yes disruptive deviation” outcome has occurred—may not be an observable outcome at all since the SPT may cease to exist after its first element is affected; further, the product formulation in general ascribes a *positive* probability to such outcomes (similarly to Eq. (104)). In order to avoid considering such perhaps unobservable outcomes, we have chosen our outcomes to be of the type given in our first formulation, namely $\{n\}$, in which observation of elements temporally beyond the first affected one is not required.

V. A Random Collection of SPT's

In the above, we formulated a probabilistic model for the disruptive deviation of an arbitrary but fixed $\pi^{\mathcal{N}} \in \Pi^{\mathcal{N}}_{N}[0, T_E]$ that is being carried on a given signal line when that line is exposed to EM disturbance $\pi^{\mathrm{EM}}[T_0, T_f, V_{\mathrm{EM}}, \omega]$. However, as discussed in the Introduction to this paper, in order to determine the probability of upset by π^{EM} of a μC executing a particular application task $\boldsymbol{a} \in \mathcal{A}$, we must resort to analyzing a stochastic mix of μC tasks $\mathcal{T}_{\boldsymbol{a}} = <[\mathfrak{r}^{\boldsymbol{a}}]^{\#}, [\boldsymbol{s}]^{\#}, d^{\#}_{\mathfrak{R}, \mathcal{M}}(\mathfrak{r}^{\boldsymbol{a}})> \in \mathcal{R}^{\#}_{\boldsymbol{a}} \times \mathcal{S}^{\#} \times \mathfrak{D}^{\#}_{\mathfrak{r}^{\boldsymbol{a}}}$, i.e., to analyzing—for *each* of the μC signal lines—a stochastic mix of $\pi^{\mathcal{N}}$'s on that signal line. Since in this paper we have been considering only a single such signal line, then we must therefore consider analyzing a stochastic mix of $\pi^{\mathcal{N}}$'s on that signal line. Thus we next want to allow the possibility that the specific SPT on that line during π^{EM}—that SPT being any one of the possibilities $\pi^{\mathcal{N}}\upharpoonright\rrbracket\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}\llbracket \in \Pi^{\mathcal{N}}_{N}[0, T_E]\upharpoonright\rrbracket\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}\llbracket$—is determined stochastically (for *example*, each of the possibilities for $\pi^{\mathcal{N}}\upharpoonright\rrbracket\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}\llbracket$ on the line is equally likely.) We may easily write the expression for the disruptive deviation probability in this case as a superposition of the SPT results in the previous Section IV.3.

To that end, we first abbreviate the notations for $\pi^{\mathcal{N}}\upharpoonright\rrbracket\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}\llbracket$ and $\Pi^{\mathcal{N}}_{N}[0, T_E]\upharpoonright\rrbracket\pi^{\mathcal{N}} \cap \pi^{\mathrm{EM}}\llbracket$ to $\pi^{\mathcal{N}}\upharpoonright$ and $\Pi^{\mathcal{N}}_{N}[0, T_E]\upharpoonright$ respectively. Next, we index the members $\pi^{\mathcal{N}}\upharpoonright$ of $\Pi^{\mathcal{N}}_{N}[0, T_E]\upharpoonright$, writing $\pi^{\mathcal{N},s}\upharpoonright$, $s = 1, \ldots, \#(\Pi^{\mathcal{N}}_{N}[0, T_E]\upharpoonright) \equiv S$ for them, where $\#(\Pi^{\mathcal{N}}_{N}[0, T_E]\upharpoonright) = 2^{n_{max} - n_{min} + 3}$. We then assign probabilities $\hat{\mathcal{P}}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\pi^{\mathcal{N},s}\upharpoonright)$, $s = 1, \ldots, S$, such that

$$\sum_{s=1}^{S} \hat{\mathcal{P}}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\pi^{\mathcal{N},s}\upharpoonright) = 1. \tag{105}$$

This assignment reflects our knowledge of the probabilities of occurrence of the various possible $\pi^{\mathcal{N}}\upharpoonright$'s on the signal line, which knowledge results from a random experiment conducted on the line in the absence of any EM disturbances and for which the outcome set is $\Pi^{\mathcal{N}}_{N}[0,\ T_E]\upharpoonright$ or, alternatively, from some model of the relative frequencies of occurrence of all possible $\pi^{\mathcal{N}}\upharpoonright$'s on the signal line. For example, equal probabilities for all of the $\pi^{\mathcal{N}}\upharpoonright$'s yields $\hat{\mathcal{P}}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\pi^{\mathcal{N},s}\upharpoonright) = 1/S$, $s = 1, \ldots, S$. Next, the outcome set for a different random experiment—an experiment to determine the probability of disruptive deviation on the stochastically populated line—is simply $\Omega_{\Pi^{\mathcal{N}}_{N}\upharpoonright} \equiv \{D, \sim D\}$, where "$D$" is the outcome that the signal line suffers a disruptive deviation (as opposed to the previously used outcome "d" to signify that a single ½-period-segmented SPT element suffers a disruptive deviation). Finally, we assign a probability, $\mathcal{P}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\{D\})$, to $\{D\}$. Using Eq. (100) to express $\mathcal{P}_{N}^{[T_0,\ T_f)}(\mathcal{E}_{\mathcal{D},\ D})$ for any N-element SPT, with this symbol augmented in notation by a subscript "s" to indicate any particular SPT $\pi^{\mathcal{N},s}\upharpoonright$ to which it refers (i.e., we write $\mathcal{P}_{N,s}^{[T_0,\ T_f)}(\mathcal{E}_{\mathcal{D},\ D})$), we then take

$$\mathcal{P}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\{D\}) = \sum_{s=1}^{S}[\mathcal{P}_{N,s}^{[T_0,\ T_f)}(\mathcal{E}_{\mathcal{D},\ D}) \bullet \hat{\mathcal{P}}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\pi^{\mathcal{N},s}\upharpoonright)] \leq 1 \tag{106}$$

(and of course $\mathcal{P}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\{\sim D\}) = 1 - \mathcal{P}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\{D\})$).

This is our desired final expression for the probability $\mathcal{P}_{\Pi^{\mathcal{N}}_{N}\upharpoonright}(\{D\})$ of disruptive deviation of an SPT on the signal line in the stochastic mix case.

VI. Conclusion

We have presented a model that provides predictions of the occurrence of disruptive deviations of signal line activity in a μC executing an application task when the μC is subjected to external irradiation by an EM pulse impinging upon it, modeling this situation from a signal-centric point of view. We have specified the set of all possible idealized, normal, N-element SPT's that may be carried on any signal line of the μC and have described as well the interaction of any such SPT with a rectangular-envelope-modulated monochromatic sinusoidal pulse influencing the SPT. This interaction may result in a disruptive deviation of any SPT on the line present during the EM pulse time window and, consequently, in disruption of μC task execution—an “upset”. Upsets occur stochastically and we have provided expressions for the probability of such upsets, both for any specified possible SPT on a single signal line as well as for stochastic collections of such SPT’s on that single line. (Results for collections of such signal lines [with a complete μC being a very large such collection] will be presented in a subsequent paper.) The disruptive deviation probability for an *N*-element SPT is given in terms of disruptive deviation probabilities for each of its *N* elements; these element disruptive deviation probabilities are not calculated in the model but rather must be provided externally, either by being obtained experimentally or *via* additional modeling performed beyond the present model and which has to date not yet been accomplished. Further, the disruption probability associated with any temporal sub-interval of any ½-period-segmented SPT element, which subinterval is also completely overlapped by the EM pulse time window, is taken most simply to be a function of the length of that sub-interval as well as of the disruptive deviation probability of the complete ½-period-segmented SPT element in which it resides. At a more detailed level, the sub-interval’s disruption probability takes into account not only its length but also the particular structure of the host ½-period-segmented SPT element itself.

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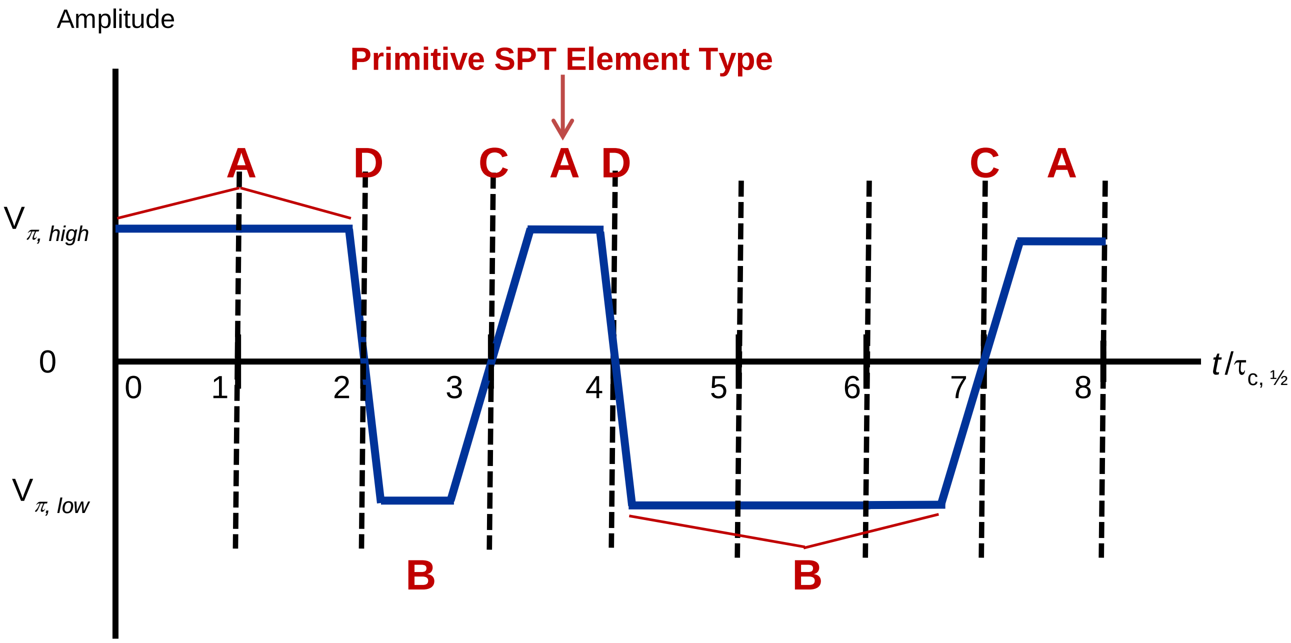

Figure 1. Examples of primitive SPT elements.

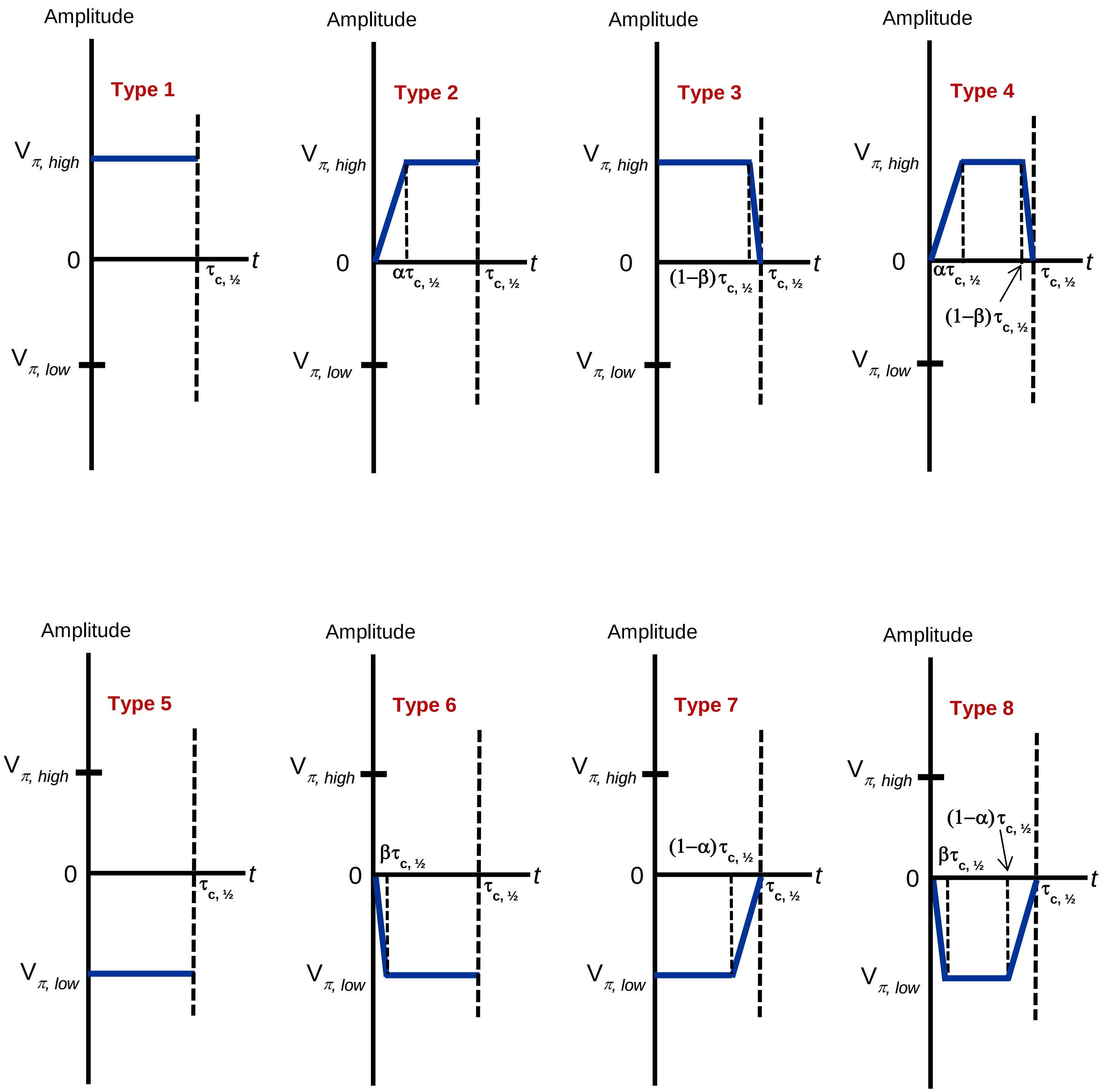

Figure 2. The eight ½-period-segmented SPT element types (n = 0).

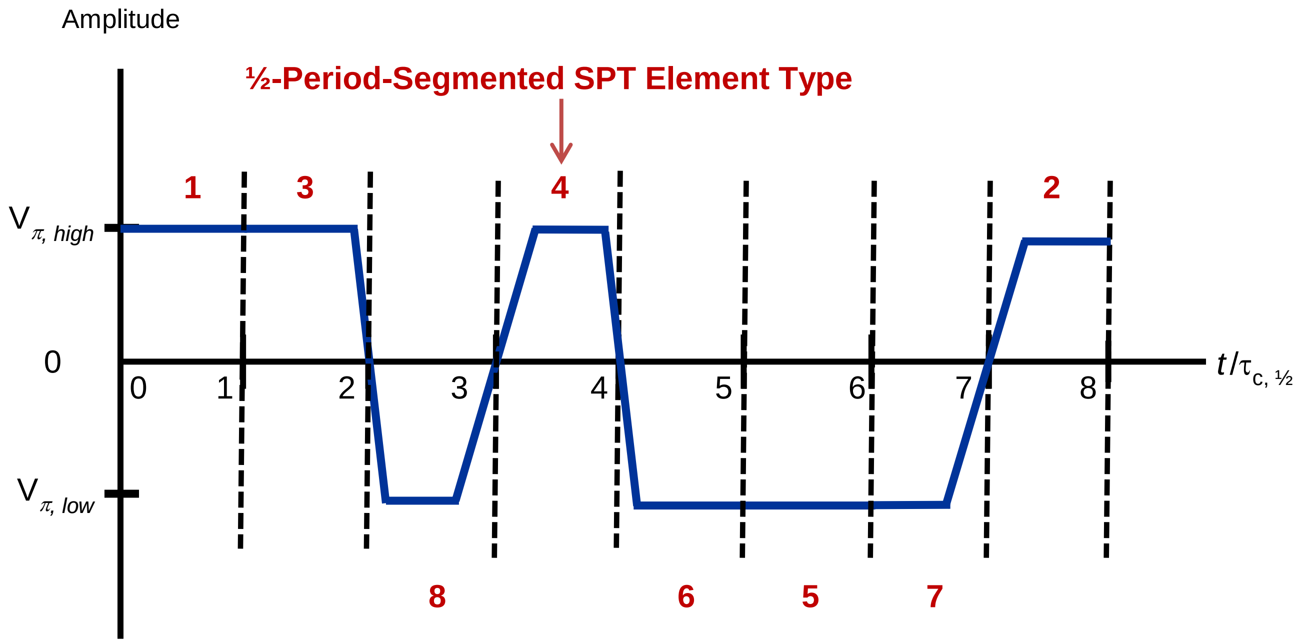

Figure 3. An eight-segment SPT constructed from ½-period-segmented SPT elements.

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