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To Whom It May Concern,

Enclosed please find the Final Technical Report with SF 298 for Dr. Erin E. Hackett's ONR grant entitled *Physics-based Inverse Problem to Deduce Marine Atmospheric Boundary Layer Parameters*.

If you have any questions regarding the submission of this report, please reach out to our office at your convenience at (843) 349-2978 or via email at OSPRS@coastal.edu. Should you have specific programmatic questions, Dr. Hackett is also available to discuss at your request.

Sincerely,

Tiffany N. Perry
Grants Administration Coordinator

Enclosures

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14. ABSTRACT This report describes research results related to the development and implementation of an inverse problem approach for determining atmospheric vertical refractivity structure. The results encompass three main components: (i) the evaluation of simplified refractivity models, (ii) evaluation of the sensitivity of EM propagation to environmental parameters, and (iii) incorporation of results from studies (i) and (ii) to develop and optimize the inversion approach. This basic research provides a step towards a semi-empirical approach for determining refractivity in an operational environment in order to provide the fleet with an "EM forecast," which improves their maritime domain awareness.					
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Physics-Based Inverse Problem To Deduce Marine Atmospheric Boundary Layer Parameters

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LONG-TERM GOALS

Understanding of and predictability of the propagation of electromagnetic waves (EM) in various environments is important for the Navy in order to determine and evaluate topside signature susceptibility. Physics-based models of EM propagation through the marine atmospheric boundary layer (MABL) have been developed and advanced over the last several decades. When all required parameters of the environment are defined at sufficient resolution, then the models provide a sufficiently accurate prediction of the electromagnetic fields in complex atmospheric conditions and over complex sea surfaces; however, limitations of these simulations are driven by a lack of sufficient high-resolution data for properly characterizing the environment. The needed atmospheric conditions, specifically the atmospheric refractivity, are difficult to measure because the lower atmosphere is turbulent and influenced by complex air-sea exchange processes. These limitations greatly restrict the utility of these EM propagation modeling and simulation (M&S) tools. One of the primary deficiencies is the lack of characterization of the MABL, particularly in the very near surface region where the sea surface also plays a non-trivial role in the propagation. To properly and definitively characterize the MABL, high-resolution measurements (in both space and time) of temperature, humidity, and pressure (among other variables), over large spatial areas is required. This objective is generally not feasible (nor practical) in a dynamic and evolving atmosphere, particularly in the regions of highest interest. As a result, surface layer models, such as those based on Monin-Obukhov (MO) similarity theory, are used in conjunction with a few bulk environmental measurements; however, differences between measured and simulated propagation are observed when using such theories to predict atmospheric refractivity for input into EM propagation simulations. Numerical weather prediction simulations are generally too coarse in resolution to use without merging it with a surface layer model based on MO theory.

The long-term goal of this research is to develop and implement a point-to-point inverse problem approach to deducing MABL parameters for use in physics-based EM propagation models. This long-term goal uses EM data with separated receivers and transmitters along with physics-based EM propagation models to inversely determine MABL parameters by optimizing the match between EM predictions and the measured EM data. This basic research provides a step towards a semi-empirical approach to determining MABL parameters in an operational environment in order to provide the fleet with an "EM forecast," which improves their maritime domain awareness.

OBJECTIVES

The objective of this research is to develop, implement, and evaluate a preliminary/idealistic inverse problem approach for determining atmospheric refractivity using synthetic EM measurements and an EM propagation simulation. Specifically, the following sub-objectives enable this objective to be achieved:

- (i) Evaluate simplified parametric refractivity models for use in the inversion problem to determine adequacy of such models for representing measured refractivity profiles.
- (ii) Study the forward problem to identify the environmental parameters that influence the simulated propagation most.
- (iii) Using the information determined from (i) and (ii), implement an idealized inversion approach. The approach targets evaporation ducts as these are the most common type of duct in the marine environment.
- (iv) Evaluate the posedness of the inverse approach.

APPROACH

This research is motivated by the extensive use of data assimilation techniques in weather prediction and other geophysics problems, including ocean acoustics. Complex models in geoscience generally contain a number of parameters and initial conditions that are not well defined. In such geophysical problems, the accuracy of complex models is increased by incorporating data to adjust these parameters to improve the comparison between numerical predictions and measured data. In recent years, inverse approaches have been gaining favor as a practical way to determine environmental parameters for EM propagation applications because direct measurements of the environment are too cumbersome to perform at the frequency and resolution that would be required for evaluating susceptibility in an evolving and dynamic environment, and numerical models are too computationally expensive for the required resolution and domain size.

A forward model uses physics-based models, which depend on several parameters to make a prediction:

$$y_{predicted} = F(x) \quad (1)$$

where F is the operator which relates the input parameters, x , to the prediction, $y_{predicted}$. In an inverse problem, the problem is reversed such that a set of observations, y_{obs} , is used to deduce the parameters, x .

This approach is essentially an optimization problem that aims to minimize a cost or objective function:

$$J = \|y_{obs} - y_{predicted}\|^2 \quad (2)$$

The cost function is minimized by adjusting the model parameters, x .

This effort uses the Variable Terrian Radio Parabolic Equation (VTRPE) simulation (Ryan, 1991) as the operator (F), which depends on a number of environmental parameters, some of which are not well known, especially the atmospheric refractivity profile. Synthetic observations are used in the cost function, which evaluates the difference between the VTRPE predictions of the EM field and the synthetic EM field observations. The environmental parameters are perturbed to evaluate the impact on the cost function, and the parameters are iteratively optimized to minimize the cost function.

Inverse problems are usually ill-posed because well-posed problems typically involve systems with known parameters. The considerations in evaluating the posedness of a problem are existence of a solution, uniqueness of a solution, and sensitivity of the solution to the model parameters. As such, part of this effort examines the forward problem to improve understanding of the physical mechanisms that impact EM propagation through the MABL. More specifically, examining the sensitivity of VTRPE to variations in environmental parameters, including evaluating which environmental parameters cause the largest variations in propagation patterns. If large variations in atmospheric parameters have little impact on the EM field, then it will not be possible to deduce the parameters from the data. Prior research has shown that differences in the refractivity profile do change the EM field in a significant way. Furthermore, prior research (e.g., see review by Karimian et al., 2011) examining use of inverse problem approaches to infer atmospheric refractivity profiles from measured sea clutter data have resulted in encouraging results. This sensitivity study implements a global sensitivity approach that considers how environmental parameters interact with each other to either enhance or reduce sensitivities of any individual parameter. In problems of this level of complexity neglecting such interactions can lead to false conclusions regarding the importance of various parameters. The results of this sensitivity study also enable the cost function of the inversion problem to be optimized. Specifically, it can be used to tailor the cost function to place more emphasis on certain parameters, e.g., the duct height.

Another factor that impacts the ability to deduce atmospheric parameters from an inverse problem is the number of degrees of freedom, i.e., the number of parameters that need to be optimized. Rigorous evaluation of different parametric formulations of the refractivity profile is undertaken to ensure that the parameters being recovered can sufficiently capture the aspects of the refractivity profile that influence the propagation. These simplified parametric refractivity models must sufficiently match measured atmospheric refractivity profiles. In addition, use of these models to generate refractivity profiles that are subsequently input to EM propagation models must generate propagation patterns that match those of propagation simulated using directly measured refraction profiles. It is not a given that a good match to the atmospheric profile will yield a good match to the propagation as these simplified models can introduce artifacts that can cause significant discrepancies in the propagation despite a reasonably good match with atmospheric measurements. Thus, this part of the approach must include evaluation of simplified refractivity models for both atmospheric data and propagation prediction.

This effort extends previous efforts in this area in a couple important ways. First, this research examines what environmental parameters have the largest impact on VTRPE propagation prediction, providing much needed insight into the sensitivity of EM propagation to various environmental factors including refractivity. Study of the forward problem in this context improves our understanding of the physical mechanisms impacting EM propagation thru the MABL. In order to solve the inverse problem, the number of parameters representing, for example, the refractivity profile needs to be reduced to make the numerical optimization feasible, while still generating sufficiently accurate representations of the refractivity for propagation prediction. A rigorous evaluation of simplified refractivity models

relative to both atmospheric measurements as well as propagation prediction has not been performed previously; it has generally been assumed that such models are sufficient.

Second, surrogate VTRPE-generated data is used in the inverse approach rather than measured sea clutter data. Use of simulated data permits the refractivity profiles recovered in the inversion process to be quantitatively evaluated because the exact solution is known. Furthermore, synthetic data permits interrogation of the EM field at any point in space instead of only examining the returned clutter signal. Operationally, this implies that separate transmit and receive antennas could be leveraged to improve the amount of information deduced from the inversion in comparison to use of only the returned sea surface clutter signal. When clutter data is used for the observational piece, other unknowns affect the inversion such as the sea surface reflectivity. The sea surface reflectivity, which impacts a clutter inversion, is not sufficiently understood and is generally empirical, especially for low grazing angles. Use of model data as a synthetic measurement decouples some of the uncertainty about the sea surface reflectivity in the inverse solution. Moreover, prior studies have, for the most part, not been extended into X- and K-bands.

The surrogate data that is generated as part of this effort is leveraged in other related efforts to extend M&S capabilities. In summary, this effort because it uses surrogate data allows investigation into the limitations of inverse approaches, as well as extensions to previous approaches, which have only used measured clutter data. Therefore, this research extends knowledge and capabilities in the use and development of inverse problem techniques to deduce atmospheric parameters.

WORK COMPLETED

The research completed under this grant includes three main components. First, the development, implementation, and execution of a global sensitivity study examining the sensitivity of radar propagation as computed using VTRPE to 16 different environmental parameters: 8 related to the sea surface and 8 related to the vertical structure of the refractivity profile. The study was carried out for three different frequencies spanning C- to K-band and two polarizations (HH and VV). The details and results of this study were published in Lentini and Hackett (2015).

Second, an evaluation of several simplified refractivity models was completed. This study included evaluation of the models relative to measured atmospheric refractivity profiles as well as comparisons of how well the simplified models predicted the true propagation when the true propagation is assumed to be that computed based on input of the measured atmospheric conditions. The details and results of this study were published in Saeger et al. (2015).

Lastly, the results of these two efforts were used to implement an inversion approach that recovers three parameters that can be used to define an evaporation duct profile. The results of the evaluation of several simplified refractivity models were used to create a new model. This model contains few enough parameters to allow the inverse approach to converge onto a unique solution for propagation over smooth sea surfaces, but represents the refractivity profile adequately for the purposes of propagation prediction. This two-layer model is:

$$\begin{aligned} M(z) &= M_0 + c_0 \left(z - z_d \ln \left(\frac{z+0.0015}{0.0015} \right) \right) & z \leq z_L \\ M(z) &= M_0 + m_1 z & z > z_L \end{aligned} \quad (3)$$

where, M_0 is the surface refractivity, c_0 is a parameter influencing the M-deficit and consequently the curvature surrounding the duct height, z_d is the duct height, and m_1 is the slope in the mixed layer. The top of the evaporation layer, z_L , is defined as twice the duct height. The parameters z_d , c_0 , and m_1 are the three parameters that the inversion approach determines in the optimization. This model is referred to as the “Stacked model.”

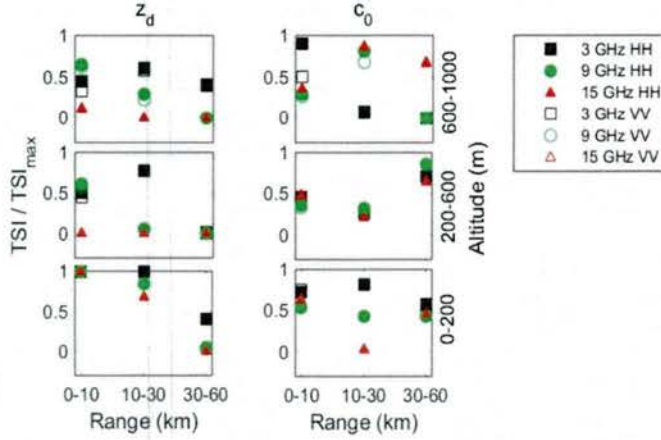


Figure 1: Total sensitivity (normalized by the maximum total sensitivity) of propagation to evaporation layer parameters for various altitudes and ranges for three different frequencies and two polarizations (see legend). A value of one on the vertical axis indicates that it is the most important parameter in that region.

layer parameters, z_d and c_0 , which shows the duct height to be the most influential parameter on propagation in the short-range low-altitude region for all frequencies and polarizations, and the curvature parameter, c_0 , to be most influential in the long range region particularly at the higher frequencies. Consequently, certain regions of the domain are weighted more heavily than others in the cost function. These weightings are illustrated in Figure 2.

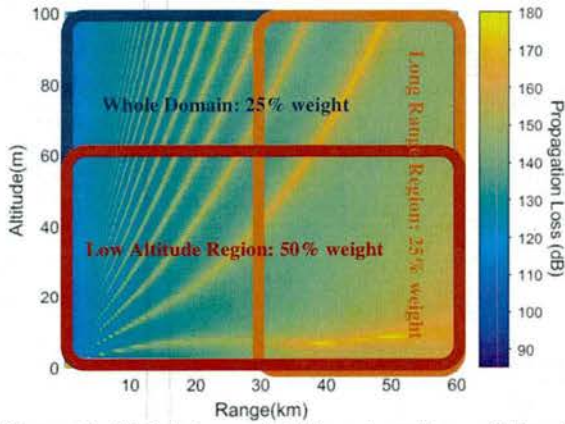


Figure 2: Weightings of different regions of the domain in the cost function of the inversion approach.

shows that the curvature and duct height parameters were repeatedly retrieved with very high accuracy (errors < 10%). The mixed layer slope parameter contained some instances of poor retrieval, but these were constrained to scenarios where the duct height was high; thus, the mixed layer represented a very

The results of the sensitivity study were used to optimize the cost function of the inversion approach in order to focus the inversion on accurate solutions of the parameters of most relevance to accurate propagation prediction, which are z_d and c_0 . These two parameters are considered more important to accurately recover in comparison to the mixed layer slope, m_1 , because the mixed layer slope is a parameter that can be more easily obtained with bulk environmental measurements or numerical weather prediction models. Although the inversion aims to get all three parameters with high accuracy, if one parameter must be retrieved at lower accuracy the cost function is setup to optimize accuracy of the other two parameters first. Figure 1 shows the results of the sensitivity study for the evaporation

The optimization process (i.e., inversion process) is executed using genetic algorithms (GA). The work completed under this grant also involved simple parametric studies of varying GA settings to configure the optimization algorithms. Figure 3 shows a flow chart of the algorithm design and Table 1 shows the GA settings for the inversion scheme. This inversion approach was tested for a set of refractivity profiles shown in Figure 4. A sample inversion result is shown in Figure 5, and the accuracy of the recovered parameters for the cases shown in Figure 4 are shown in Figure 6. Figure 6

small part of the domain, which diminishes sensitivity of the propagation pattern to it. In other words, the mixed layer slope is of little consequence to the propagation in these cases so the inaccurate mixed layer slope is irrelevant.

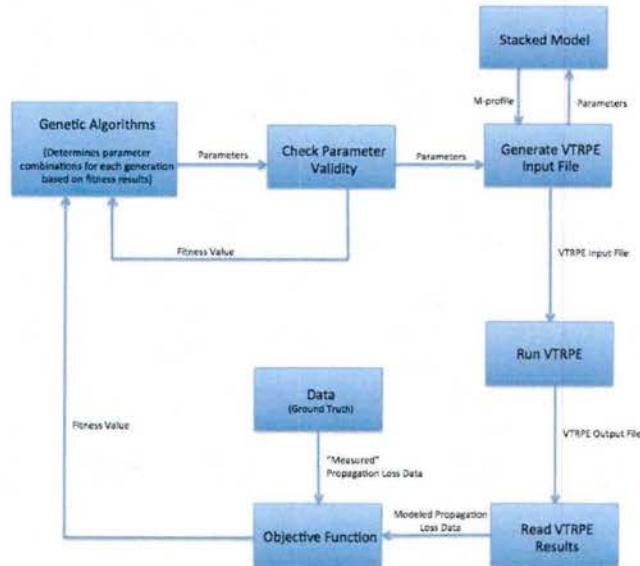


Figure 3: Flow chart of the algorithm design for the inversion approach.

RESULTS

Several key pieces of knowledge were gained in this research. First, it was found that log-linear simplified evaporation duct models are able to accurately (errors less than 1 M-unit on average) replicate refractivity profiles obtained under a variety of conditions in comparison to atmospheric measurements with a specific version of the model formulation (see Eqn. 3). Furthermore, they are able to generate accurate propagation patterns when propagation is computed using atmospheric conditions based on the simplified model in comparison to propagation computed using measured atmospheric conditions. Prior studies, which in some cases suggest that such a model is inadequate, do not allow for adjustment of

the curvature surrounding the duct and have a fixed slope in the mixed layer above the evaporation duct. Simple log-linear models that have a parameter to adjust the M-deficit and consequently the curvature surrounding the duct that does not change the duct height allows a much wider range of profiles to be accurately modeled. In addition, the slope in the mixed layer above the evaporation layer also needs to vary independently of the other parameters, which is not the case in the standard Paulus-Jeske formulation (Paulus, 1990). Thus, it is determined that a log-linear model formulation can be adequate for inversion problems seeking to determine evaporation duct refractivity models if there are at least three parameters: z_d , m_I , and c_0 (i.e., Eqn. 3). Such a model is only appropriate for inversion problems because there is no physical basis for the c_0 parameter; thus, it cannot be predicted in a forward sense. Linking this parameter to environmental conditions is an area of future research that might make such a model more viable for a forward prediction based on bulk environmental measurements.

The second key piece of information determined from this research is spatial maps of the sensitivity of EM propagation to 16 environmental parameters for three different frequency bands and two polarizations. These sensitivity estimates include how the 16 parameters can interact with each other to reinforce or diminish the importance of an individual parameter. The 16 environmental parameters include eight parameters related to the sea surface and eight parameters related to the vertical structure of refractivity. With these maps that cover 60 km in range and 1000 m in altitude, one can determine a part of the domain that is of interest (along with selection of frequency and polarization) and determine which environmental parameters influence the propagation most significantly in that region. This type of information is important for a number of potential reasons: (i) to design experiments that target the correct environmental measurements when attempting comparisons between propagation measurements and propagation predictions, (ii) focusing numerical weather prediction improvements

on the parameters that will most significantly improve propagation predictions, and (iii) knowing what are the most important parameters to target in inversion problem approaches to recovering environmental parameters.

Table 1: GA settings for the inversion approach.

Feature	Value
Population Size	25
Specified Initial Population	15
Elite Count	2
Fitness Function	Mean Square Error (50/25/25)
Fitness Limit	2 dB ²
Selection Function	Roulette
Fitness Scaling Function	Rank
Crossover Functions	Scattered
Crossover Fraction	20%
Mutation Function	Adaptive Feasible

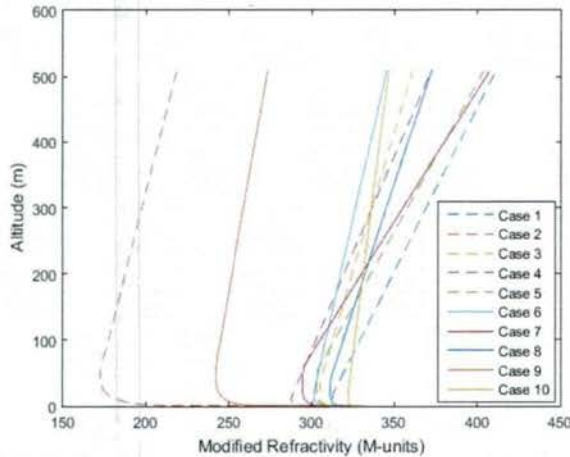


Figure 4: Set of refractivity profiles used to test the inversion algorithms. The inversion process seeks to recover these profiles. Because the solution is known, the accuracy of the inversion can be quantitatively evaluated.

sufficient to accurately model the refractivity profile can be repeatedly recovered via an inversion process with accuracy better than 90%. This canonical study demonstrates that inversion methods are clearly a viable approach under optimal conditions (i.e., unlimited data, smooth sea surface, and no instrumentation noise). Future research should now focus on determining how the accuracy degrades with incorporation of complicating factors such as limited data availability, measurement uncertainty, and rough ocean surface effects. It is important to understand what is the best possible accuracy of refractivity profiles recovered during an inversion process to determine the range of viable applications for this type of approach to determining refractivity.

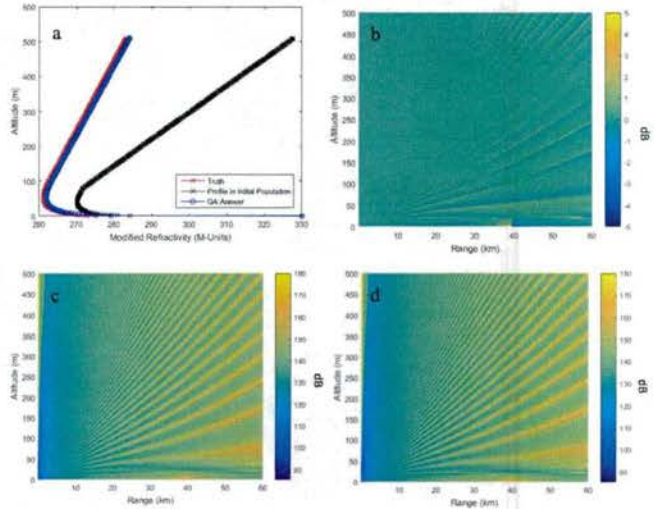


Figure 5: Sample inversion result. (a) Refractivity profile of the known solution (Truth), the inversion result (GA Answer), and a sample refractivity profile from the initialization of the GA. (b) The difference between the propagation pattern produced from the known solution refractivity profile and that produced from the inversion result. (c) The propagation pattern based on the known solution for the refractivity profile. (d) The propagation pattern based on the inversion solution refractivity profile.

Finally, through incorporation of the information gained in the aforementioned studies, an idealized inversion approach was designed, implemented, and tested with synthetic data over a smooth sea surface. The results of this testing demonstrated that parameters describing the vertical structure of the refractivity profile that were deemed

IMPACT/APPLICATIONS

The results of this study contribute towards further development of techniques and methods that can provide the fleet with an “EM forecast.” It is crucially important to understand the environment in which operations are occurring so that their affect on sensor performance can be taken into account in evaluating own system performance as well as susceptibility by adversary’s systems. The results of this study contribute toward improving an “EM forecast” capability by advancing methodologies associated with inversion approaches to determining the environment and by providing new insight into sensitivities of EM propagation to environmental parameters.

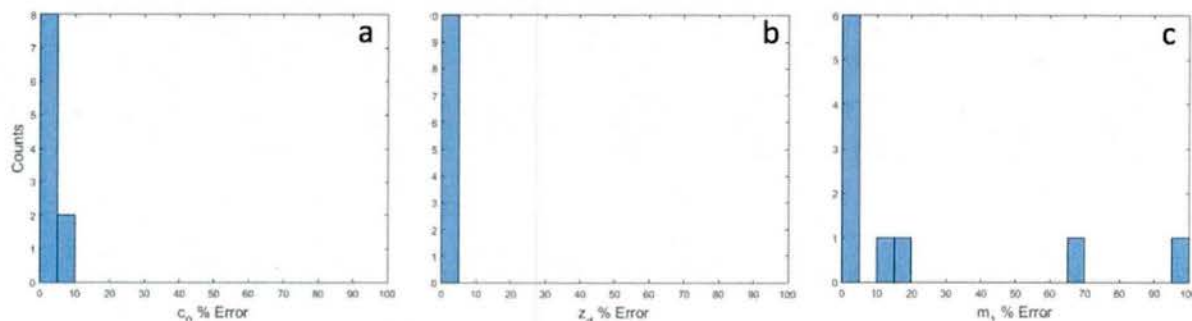


Figure 6: Histograms of percent error in refractivity parameters recovered from the inversion process for the cases presented in Figure 4. (a) Shows errors for duct “curvature” parameter, c_0 ; (b) shows errors for duct height, z_d ; and (c) shows errors for the slope in the mixed layer, m_1 . Errors for duct height are consistently below 5% and errors for the “curvature” parameter are consistently below 10%, which are the two parameters targeted most strongly in the inversion process. Large errors in the mixed layer slope occur for cases with a high duct height; in such cases, the mixed layer is a small percentage of the entire vertical domain (100 m) resulting in decreased sensitivity of the inversion process to the accuracy of the mixed layer slope.

RELATED PROJECTS

This research is closely linked to the ONR MURI called CASPER (<http://met.nps.edu/~qwang/casper/home/home.php>) (PI Qing Wang), which seeks to better understand environmental effects on EM propagation. PI Hackett has been involved with the CASPER MURI as a collaborator and has assisted with the execution of the CASPER East experiments.

The PI has submitted an ONR proposal to continue/extend the line of research discussed in this report, which was recently funded (N00014-16-1-2075). The focus of this research is to evaluate the impact of EM data quantity and location on the accuracy of the inversion solution as well as to evaluate the impact of the rough ocean surface on inversion solution accuracy. These studies will be carried out with synthetic data that enables quantification of the reduction in accuracy. The last part of this effort will utilize measured data from the CASPER experiments (and other similar experiments) to evaluate the inversion approach with measured EM data.

Research conducted by the Naval Research Laboratory (NRL) in Monterey, CA in the area of numerical weather prediction (NWP) is also closely related to this effort. In particular, numerical weather prediction can get environmental estimates in the correct ballpark but lacks the detail needed for EM propagation prediction in some applications. Linking together NWP and inversion methods may improve accuracy of both approaches; thus, the PI has been kept apprised of developments from

NWP studies (via meeting with NRL researchers at scientific conferences; Tracy Haack) in order to eventually leverage those capabilities into the inversion approach being developed.

Lastly, Chris Earls at Cornell University has a couple ONR-funded research efforts that seek to also inversely determine refractivity. Their study differs in that they are using a pure data-driven approach that does not utilize any EM propagation physics directly. EM propagation predictions produced from VTRPE simulations generated as part of the effort described in this report are also used to provide synthetic data to Chris Earls to support his data-driven inversion approach.

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Saeger, J.T., N.G. Grimes, H.E. Rickard, and E.E. Hackett, 2015, Evaluation of simplified evaporation duct refractivity models for inversion problems, *Radio Science*, 50, doi:10.1002/2014RS005642. [published, refereed]

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EDUCATION

One graduate student was educated under funding from this research that graduated in 2015. Several undergraduate researchers were also educated and supported.