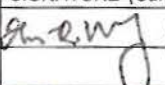
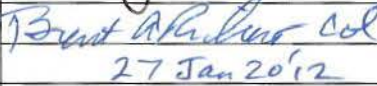


STAFF SUMMARY SHEET

	TO	ACTION	SIGNATURE (Surname), GRADE AND DATE		TO	ACTION	SIGNATURE (Surname), GRADE AND DATE
1	DFR	sig	 MAREY AD-25 2012	6			
2	DFER	approve	 27 Jan 2012	7			
3	DFR	action	(Author /Originator)	8			
4				9			
5				10			

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				USAFA-DF-PA-24

SUMMARY

1. PURPOSE. To provide security and policy review on the document at Tab 1 prior to release to the public.

2. BACKGROUND.

Authors: Alex Halloran, Colton Huntsman, Chad Demers, and Michael Courtney

Title: More Inaccurate Specifications of Ballistic Coefficients

Circle one: Abstract Tech Report Journal Article Speech Paper Presentation Poster
Thesis/Dissertation Book Other: _____

Check all that apply (For Communications Purposes):

☒ CRADA (Cooperative Research and Development Agreement) exists

☐ Photo/ Video Opportunities ☐ STEM-outreach Related ☐ New Invention/ Discovery/ Patent

Description: Scientific paper.


Release Information: DoD Technical Report (DTIC) and submission to Varmint Hunter Magazine

Previous Clearance information: (If applicable) Not Applicable.

Recommended Distribution Statement: Distribution A: approved for public release, distribution unlimited

3. DISCUSSION. Not applicable.

4. RECOMMENDATION. Sign coord block above indicating document is suitable for public release. Suitability is based solely on the document being unclassified, not jeopardizing DoD interests, and accurately portraying official policy.


(signature)
Michael Courtney, PhD AD-25
Director, Quantitative Reasoning Center
Assistant Professor of Mathematical Sciences

1 Tab

1. More Inaccurate Specifications of Ballistic Coefficients

More Inaccurate Specifications of Ballistic Coefficients

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Abstract: A ballistic coefficient (BC) can be determined by using two chronographs a measured distance away from each other and shooting a bullet so the velocity is measured by the two devices. Ballistic modeling software requires accurate measurement of ballistic coefficients to accurately predict downrange trajectories, wind drift, and retained energy. This article presents new measurements for 21 different bullets and shows that BCs can differ significantly from the claims of the manufacturer. These differences can cause significant differences in the predictions of ballistic modeling software.

Introduction: A ballistic coefficient is the measure of how well a bullet retains its velocity as it travels through the air. Assuming all other factors are equal, a high BC means the bullet will travel faster, farther, and more accurately than a bullet with a lower BC. As Brian Litz explains in his book, "Applied Ballistics for Long Range Shooting," there are different drag models used to describe the aerodynamic drag of supersonic projectiles. Most bullet manufacturers specify the BCs of their bullets relative to the G1 drag model; however, the G7 drag model is more appropriate for boat tail bullets. (Litz 2009a) The mathematical details are more complicated, but the G1 BC can be thought of as the fraction of 1000 yards it takes for a projectile to lose half of its energy. For example, a projectile with a G1 BC of .267 should lose approximately half of its energy by 267 yards in a standard atmosphere.

Much conversation among target shooters and hunters regarding bullet selection includes justification for choosing higher BC bullets. This might have the effect of creating temptation among manufacturers to publish overly optimistic BC specifications. However, a careful reading of the Litz book (Litz 2009a) as well as prior published measurements of ballistic coefficients by independent parties (Courtney and Courtney 2009) suggests that many of the BC numbers published by bullet manufacturers fail to be confirmed by independent measurements, and most published BC specifications are overly optimistic when compared with independent measurements.

Even though prior publications are in agreement in failing to confirm the high BC claims of manufacturers for many bullets, there has been some disagreement in cases where different parties have measured BCs for the same make and model of bullet (Litz 2009b). At the time of these earlier publications, it seems that in some cases, disagreement could be attributed to lot-to-lot variations, and in one specific case of the 115 grain Berger VLD in 0.257, dimensional variations were identified. There also seemed to be systematically lower BCs (8% or so) measured by Courtney and Courtney (2009) compared with the measurements of Litz (2009b). The dual chronograph method of Courtney and Courtney gives close agreement (< 3%) with simultaneous use of an acoustic method (Courtney and Courtney 2007). The Courtneys' error estimate for their dual chronograph measurement was around 3% for the bullets in question, and Brian Litz estimates his uncertainty at near 1%. Consequently, it was hard to attribute the discrepancies to measurement uncertainties. After a series of email exchanges and phone calls considering possible sources of the measurement differences, Brian Litz published the hypothesis that perhaps the thin sporter barrels used by Courtney and Courtney (2009) were

causing a larger peak yaw angle (near 11 degrees) than the heavier custom Palma barrel used by Litz (Litz 2009b).

The idea of different rifle barrels giving BC differences as large as 8% is certainly interesting and perplexing with lots of practical implications. Fortunately, the Litz hypothesis of increased yaw being the source of BC differences has several consequences that are experimentally testable. First of all, since the peak yaw is predicted to damp out from 11 degrees near the barrel to 2 degrees at 100 yards, BC measurements over a longer distance (say 200 yards) should give closer agreement with the Litz measurements. Secondly, if the variation is due to yaw and not lot-to-lot variations in the bullets or a different cause related to being shot from a different rifle, then it should not be present when the bullets are shot from a heavier barrel. Third, there is the possibility of directly observing the yaw with a high-speed video camera.

Method: A near chronograph (Oehler Model 35) was placed 15 to 30 feet from the muzzle, and a second chronograph (CED Millenium) was placed 300 feet further downrange. The separation between the two chronographs was carefully measured with a tape measure and was accurate to 4 inches or better. The ambient pressure, temperature, and humidity were determined with a Kestrel 4500 weather meter. Four to six shots were fired for each type of bullet and the near and far velocities were recorded for each shot. G1 and G7 BCs were determined with the JBM Ballistic Coefficient Calculator (<http://www.jbmballistics.com/cgi-bin/jbmbscv-5.1.cgi>) by entering the near and far velocities for each shot, along with the ambient conditions and desired drag model. Three bullet designs were tested with a chronograph separation of 190 yards. We had wanted to test with a 200 yard chronograph separation, but a dip in the ground (apparently remaining from construction of the backstop) required the far chronograph to be placed 190 yards downrange instead. Putting a chronograph further than 100 yards gets risky, depending on the accuracy of the loads and rifles and how much the wind is blowing. We've destroyed a few chronographs in other experiments, but managed to preserve the far chronograph in this one.

Given the care in measuring the chronograph separation distance and the specified accuracy of the chronographs (0.3%) combined with confirming the expected reading on the chronographs when they are placed back-to-back (2 feet apart), the uncertainty in the BC determination is dominated by shot to shot variations in most cases. The standard error from the mean is determined as the standard deviation of the samples divided by the square root of the number of shots for a given bullet.

The rifles used in the study were all Remington 700s with factory barrels. The 223 Remington, 308 Winchester, and 30-06 were all ADLs with sporter barrels. The 25-06 was a Sendero with a varmint weight barrel. The 300 Winchester Magnum had the 5-R milspec barrel, which is a bull barrel similar to that used in the U.S. Army M24 Sniper Weapon System.

Results: Table 1 shows our BC measurements along with the published specifications of the bullet companies. There are several manufacturers and many different bullets shown in the table. From the BC measurements presented, it seems that many bullets have their BCs significantly overestimated in the manufacturer specifications. G7 ballistic coefficients are also reported for boat tail bullets, because these are generally regarded as more accurate for predicting long-range trajectories and are not generally available from manufacturers other than Berger Bullets.

Nosler's overly optimistic estimate (by 44.8%) of the BC of the 150 grain E-Tip is particularly notable. The claimed G1 BC for this bullet is 0.469, but we measure 0.324 with an uncertainty of 0.007. Upon initial inspection, the BC claim of 0.469 seemed unrealistic. Very few 150 grain .308 bullets have BCs this high. For example, Hornady advertises a BC of 0.415 for the 150 grain GMX as well as for the 150 grain SST. Barnes advertises a BC of 0.420 for their tipped TSXBT. The highest BC bullets in this weight class have much longer boat tails (0.15 to 0.16") than the E-Tip which measures under 0.1". Probably even more detrimental to the E-Tip's BC is the pronounced shoulder between the plastic tip and the copper portion. The base of the plastic tip measures 0.138" but the shoulder quickly reaches a diameter of 0.165" before the transition to the ogive is really complete. This is the first rifle bullet design we've seen with a distinct shoulder between tip and metal portion, as the other designs we've inspected (TTSX, Ballistic Tip, Accubond, AMAX, VMAX, etc.) all have a near seamless transition between the plastic tip and metal portion of the bullet.

Company	Caliber	Style	Weight	Published	Measured	Measured	Near Vel	Distance	Cartridge	Overestimate
	inches		grains	G1 BC	G1 BC	G7 BC	fps	yards		%
Barnes	0.257	XBT	100	0.42	0.355(6)	0.174(3)	3271	100	25-06	18.3
Speer	0.257	SpBT	100	0.393	0.387(10)	0.191(5)	3119	100	25-06	1.6
Berger	0.257	VLD	115	0.479	0.476(11)	0.236(6)	3051	100	25-06	0.6
Barnes	0.257	TTSXBT	100	0.357	0.399(4)	0.197(2)	3170	100	25-06	-10.5
Nosler	0.257	NABBT	110	0.418	0.414(20)	0.206(10)	3079	100	25-06	1.0
Nosler	0.308	CTBST	168	0.49	0.419(8)	0.211(4)	2594	190	308 Win	16.9
Nosler	0.308	NBT	125	0.366	0.313(2)	0.157(2)	2832	190	308 Win	16.9
Nosler	0.308	NBT	125	0.366	0.283(4)	0.141(2)	3008	100	300 Win Mag	29.3
Berger	0.308	FBBT	155.5	0.464	0.409(22)	0.204(11)	3089	100	300 Win Mag	13.4
Berger	0.308	FBBT	155.5	0.464	0.406(6)	0.205(3)	2620	100	308 Win	14.3
Berger	0.308	FBBT	155.5	0.464	0.430(10)	0.217(5)	2620	190	308 Win	7.9
Berger	0.308	VLD	168	0.473	0.421(7)	0.212(3)	2650	100	308 Win	12.4
Barnes	0.308	TTSXBT	168	0.47	0.388(11)	0.195(6)	2716	100	30-06	21.1
Nosler	0.308	NET	150	0.469	0.324(7)	0.163(4)	2671	100	308 Win	44.8
LC M855	0.224	FMJBT	62	0.304	0.273(9)	0.134(4)	3270	100	223 Rem	11.4
LC XM193	0.224	FMJBT	55	0.243	0.233(14)	0.112(7)	3470	100	223 Rem	4.3
Berger	0.224	FBHP	62	0.291	0.245(2)	NA	2950	100	223 Rem	18.8
Berger	0.224	FBHP	62	0.291	0.245(6)	NA	2230	100	223 Rem	18.8
Nosler	0.224	NBT	40	0.221	0.186(2)	0.091(1)	3394	100	223 Rem	18.8
Nosler	0.224	NBT	40	0.221	0.181(1)	0.091(1)	2630	100	223 Rem	22.1
Nosler	0.224	NBT	55	0.267	0.243(3)	0.119(1)	3230	100	223 Rem	9.9
Nosler	0.224	NBT	55	0.267	0.229(2)	0.114(1)	2344	100	223 Rem	16.6
Nosler	0.224	NCC	69	0.305	0.285(4)	0.142(2)	3018	100	223 Rem	7.0
Federal AE	0.224	JHP	50	0.208	0.169(4)	NA	3354	100	223 Rem	23.1
Hornady	0.224	VMAX	60	0.265	0.246(1)	NA	3208	100	223 Rem	7.7
Nosler	0.224	NBTLF	40	0.221	0.205(3)	NA	3347	100	223 Rem	7.8
Hornady	0.224	VMAX	40	0.2	0.200(2)	NA	2685	100	223 Rem	0

Table 1: Measured G1 and G7 ballistic coefficients along with the BC advertised by the bullet manufacturers. The overestimate is the percent difference between the manufacturer's claim and the present study.

Discussion: When comparing the manufacturer's specifications and our measurements, we found that in many cases the advertised numbers are a significant overestimate. How much does this affect the usability? For the case of the 150 grain Nosler E-Tip, the advertised BC of 0.469, together with a muzzle velocity of 2738 fps (given by a maximum load of Varget in the test rifle with a 22" barrel) yields a prediction of retained velocity of 1810 fps, a drop of 48 inches, and a wind drift of 22 inches at 500 yards, using a 200 yard zero and ambient conditions

of 0% relative humidity, 29.92 in Hg of pressure, and 30° F. Using the measured BC of 0.324 gives 59 inches of drop, 35 inches of wind drift (in a 10 mph cross wind), and a retained velocity of only 1472 fps at 500 yards under the same conditions. Since Nosler's recommended minimum impact velocity for reliable expansion is 1800 fps, the usable range of this bullet is shortened to 350 yards, since beyond that the bullet will be impacting with less than 1800 fps. If one prefers an impact velocity of 2200 fps to ensure greater expansion, then the usable range of this bullet is reduced to 200 yards. Figure 1 shows a graph of retained velocity out to 1000 yards for the advertised and measured BCs.

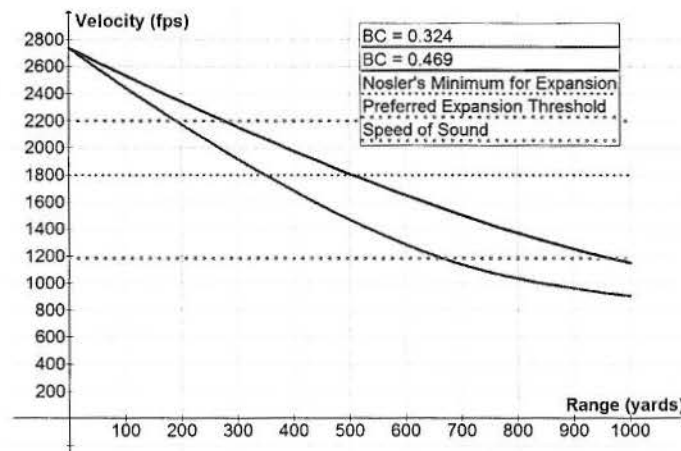


Figure 1: Predicted velocities for the 150 grain Nosler E-Tip out to 1000 yards for advertised BC of 0.469 and measured BC of 0.324.

Considering the results in Table 1 and reviewing the overestimates of the various bullets shows that although overestimates as large as 40% are relatively uncommon, a lot of BCs are overestimated by 15-25%. How much does this affect trajectory predictions? Consider the Berger 62 grain flat base varmint bullet. This bullet demonstrates excellent accuracy (0.5 MOA) in the Remington 700 test rifle out to 300 yards and is a candidate for both match and varmint shooting out to 600 yards. Berger's advertised BC of 0.291 implies a drop of 98 inches, a wind drift of 57 inches, and a retained velocity of 1289 fps at 600 yards under the same atmospheric conditions as before. The measured BC of 0.245 implies a drop of 117 inches, a wind drift of 73 inches and a retained velocity of 1110 fps at 600 yards. This retained velocity is probably too close to the sonic transition for the comfort level of most shooters given the transonic drag rise and possible degradation of accuracy if the bullet velocity falls through the speed of sound en route to the target. Table 1 also shows that this bullet maintains its BC of 0.245 even at the reduced velocity of 2230 fps showing that the G1 is a fairly good drag model for this bullet and long range trajectories based on this BC are likely to be reasonably accurate, at least until the bullet approaches the sonic transition.

These experimental results also allow us to close the loop on several issues discussed by Brian Litz in his article "Accurate Specifications of Ballistic Coefficients" (Litz 2009b). The 115 grain VLD in .257 had been advertised with a BC of 0.523, but independent tests (Courtney and Courtney 2009) had measured 0.419. After that article, using Brian Litz's input, Berger reassessed and published a BC of 0.479. Berger also sent one of the authors (MC) a couple of boxes of new bullets as the remaining BC difference was determined to be most likely attributable to dimensional variations of an old lot of bullets made with a worn out forming die.

These bullets were tested in the present study yielding a BC of 0.476 ± 0.011 which is within the uncertainty of both Berger's original revised estimate of 0.479 and their later published number of 0.466.

New data also allows evaluation of the Litz hypothesis that increased yaw due to thin sporter barrels was the primary causal factor of the BC differences measured by Litz using his acoustic method and medium weight Palma barrel and BC measurements of Courtney and Courtney (2009) and the present study using dual chronographs and thin sporter barrels. Table 1 shows three BC measurements with a chronograph separation of 190 yards. The Litz hypothesis implies that BC measurements using thin sporter barrels over a longer range should be much closer to the Litz measurements with a thicker barrel because the hypothetical yaw damps out quickly and will not continue to increase drag after damping out. The earlier paper (Courtney and Courtney 2009) reported a BC of 0.421 ± 0.004 for the 168 grain CTBST over 97 yards. Table 1 reports 0.419 ± 0.008 for the same bullet over 190 yards. The fact that the BC measurement did not increase over the longer range suggests that yaw is not a factor. The earlier paper (Courtney and Courtney 2009) reported a BC of 0.308 ± 0.010 for the 125 grain Nosler Ballistic Tip from the Remington 700 ADL in .308 Winchester over 97 yards. The present study reports a BC of 0.313 ± 0.002 for the same bullet over 190 yards. This is not significantly closer to the Litz measurement of 0.345. Of the three bullets with BC measurements available at both 97/100 yard and 190 yard chronograph separation, the only case where the 190 yard BC measurement was significantly closer to the Litz measurement was the 155.5 grain Fullbore Palma. Litz reports a BC of 0.464 for this bullet. We measured a BC of 0.406 ± 0.006 over 100 yards and a BC of 0.430 ± 0.010 over 190 yards. The numbers and the uncertainties involved are not compelling either way.

Another approach to evaluate the Litz hypothesis of barrel whip causing yaw leading to increased drag and lower BCs is to shoot the same bullets (same make and model, same lot number, same box of bullets) from a rifle with a heavier barrel. We obtained a Remington 700 in 300 Winchester Magnum with the heavy 5-R milspec barrel for this purpose. If barrel whip from the skinny barrel on the 308 Winchester were causing increased drag, then the same bullets should have higher BCs over 100 yards when fired from the heavier barreled rifle. In fact, we measured the 125 grain Nosler Ballistic Tip to have a BC of 0.283 ± 0.004 and the 155.5 grain Berger Fullbore Palma to have a BC of 0.409 ± 0.022 from the thicker barrel. Since the thicker barrel would eliminate barrel whip, yet the BC was still well below other measurements, the hypothesis that barrel whip was the cause of reduced BC measurements was not supported.

A final approach to evaluating the Litz barrel whip hypothesis suggesting that peak yaw angles are in the neighborhood of 11° is to determine the yaw directly via high-speed video. Numerous shots and many frames were analyzed for the 168 CTBST, the 125 NBT, the 155.5 FBBT, and the 150 E-Tip, all bullets with BC measurements significantly below either the Litz measurements or the manufacturer's claims. The maximum yaw observed in any frame was 4° . Based on the totality of the evidence failing to support the Litz hypothesis, it seems that the Litz hypothesis of yaw-induced drag increase is an unlikely explanation of lower BC in most cases. Further work will be necessary to determine the cause of BC variations between measurements conducted by Litz and other independent measurements. We suspect some other causal factor related to lot-to-lot bullet variations or something related to firing from different rifles. There is also the possibility of a larger than estimated error in the acoustic method or the dual chronograph method.

It seems that different lot numbers and different rifles can routinely produce as much as 20% differences in BC when firing the same make and model of bullet. This suggests that predictions of ballistics programs are no more than a starting point for long-range shooting, and that predictions might be much more accurate if a BC is determined for a particular bullet in a particular rifle. One of the authors (MC) achieved his longest kill on a deer (550 yards) using the older 115 grain VLD with a BC significantly below the one advertised. Success in this case resulted from measuring the actual drop at 550 yards as well as experimentally determining the scope adjustment (in MOA) required to compensate at that range. Predictions of modeling software are no substitute for experience and practice. Brian Litz has a great Yogi Berra quote at his website (www.appliedballisticsllc.com):

In theory there is no difference between theory and practice. But in practice, there is.

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