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A BROWNIAN BRIDGE MOVEMENT MODEL TO TRACK MOBILE TARGETS

by

Chun Chieh Cheng

September 2016

Thesis Advisor:	Dashi I. Singham
Second Reader:	Michael P. Atkinson

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This thesis focused on the use of a network of unmanned sensors as the means for target detection. The relationship between the sensors' attributes and the probability of detection is explored through a meta-experiment. The experiment utilizes a three-stage algorithm that generates heat maps, deploys sensors and randomizes intelligence inputs, and measures the probability of detection. A trade-off analysis was conducted and showed that to achieve a higher probability of detection, it is more effective to have sensors cover a wider area at fewer discrete points in time than to have a greater number of discrete looks using sensors covering smaller areas.

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A BROWNIAN BRIDGE MOVEMENT MODEL TO TRACK MOBILE TARGETS

Chun Chieh Cheng
Major, Singapore Armed Forces, Army
B.S., Nanyang Technological University, 2010

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Approved by: Dashi I. Singham
Thesis Advisor

Michael P. Atkinson
Second Reader

Patricia A. Jacobs
Chair, Department of Operations Research

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ABSTRACT

The Brownian bridge movement model (BBMM) models target movement between two known points as a Brownian bridge. This thesis extended the BBMM to account for multiple starting and ending points and to account for intelligence inputs midway through the target movement. The BBMM is applied to a military scenario where U.S. forces are conducting surveillance to monitor the breakout of Chinese forces in the South China Sea. Probability heat maps, depicting the probability of a target location at discrete times, are generated through simulations in MATLAB. Using the heat maps, this thesis developed an algorithm to automate the placement of sensors to detect the target.

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LIST OF ACRONYMS AND ABBREVIATIONS

A2AD	anti-access area denial
BBMM	Brownian bridge model
DPP	Democratic Progressive Party (Taiwan)
GOP-RDR	generalized orienteering problem with resource dependent rewards
LOCUST	low-cost unmanned aerial vehicles swarming technology
NOLH	nearly-orthogonal Latin hypercube
PLAN	People's Liberation Army Navy (China)
SAM	surface-to-air missile
SEA	system engineering analysis
UAV	unmanned aerial vehicle

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EXECUTIVE SUMMARY

The search and detection of a target is a classical problem that has been examined in numerous studies and in various applications, such as anti-submarine warfare, search and rescue, counter-piracy and counter-smuggling operations. There are various techniques in search and detection theory, like simulation, data analysis and probability models, to provide quantitative estimates of the probability of detection. This thesis uses a probability model, the Brownian bridge movement model (BBMM), to depict the probability density of the target's location at discrete times, and using that probability density, chooses the deployment of unmanned sensors to detect the target.

Horne et al. (2007) provide an in-depth introduction of the BBMM and used the BBMM to study movement and migration patterns of animals and birds. The BBMM is essentially a combination of Brownian bridges, which are Brownian motion tied at particular values at two points in time, usually the starting and ending point of the movement. The BBMM is largely dependent on the Brownian motion variance parameter, and this thesis describes a method to estimate this variance by using the probability that the target will travel outside the limits of a given distance. The BBMM is extended to account for multiple starting and ending points as well as to incorporate intelligence inputs of a target's location midway. The BBMM is applied to a military scenario in the South China Sea, where the U.S. forces are interested in monitoring and tracking high-value units, such as an aircraft carrier, to prevent the projection of forces by the Chinese. In particular, this thesis is interested in the employment of unmanned sensors, especially a future system where the sensors are connected in a network to provide data-link relay through line-of-sight communications and surveillance capabilities.

Through simulations of the BBMM, this thesis generates probability heat maps of the target's location at discrete times. With the heat maps, the thesis develops an algorithm to automate the placement of the sensors at the highest probability regions at discrete time points. The probability of detection by the sensor was also estimated in the algorithm based on attributes of the sensors that can be specified by the user.

A meta-experiment is also designed using the BBMM and a three-stage algorithm. The first stage involves generating the probability heat map using the BBMM. In the second stage, the areas with the highest likelihood at specified times are identified and the sensors are placed at those areas. In the last stage, the sensors are given randomized intelligence updates, and using the sensor's probability of detection, the total probability of detection by the sensors is estimated. Using this algorithm, the thesis explores the effects of the attributes of the sensors on the overall probability of detection.

A trade-off analysis was conducted between the number of sensor placements and the size of the sensor detection region. The results were benchmarked against the sensor referenced from the Lynx multi-mode radar system. From the analysis, it was evident that having a wider detection range offers a higher probability of detection. Having multiple discrete sensors could reduce the variance in the probability of detection and there is a minimal search width required, beyond which it is difficult to detect a target even with a high number of discrete looks. This could, however, be because the deployment algorithm only places the sensors at the regions of highest probability, which may be along the most likely central paths, thereby limiting them to picking up Brownian bridges that have already been detected by prior sensors.

The thesis recommends a possible configuration for the network of unmanned sensors, where multiple sensors at discrete locations should overlap one another to create wider detection regions. Given that the technology of unmanned systems is maturing, this thesis also recommends further research to develop autonomous algorithms to enable sensors to move and deploy themselves to strategic locations, while maximizing the probability of detection.

References

Horne, J. S., Garton, E. O., Krone, S. M., & Lewis, J. S. (2007). Analyzing animal movements using Brownian bridges. *Ecology*, 2354–2363.

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I. INTRODUCTION

The United States has publicly announced that the Asia-Pacific region is a "top-priority" of their security policy in 2011 (BBC, 2011) and has maintained a sizeable force in the region since then. One key area of concern is the South China Sea, where China has been assertive in its influence and territorial claims and where U.S. forces have conducted their freedom of navigation operations. There have been a number of incidents in the region between the militaries of the two powers (Tomlinson, 2016), and the chance of a potential confrontation is not negligible. China has also been steadily constructing and building up defense capabilities in the region, particularly in disputed areas such as the Spratly and Paracel Islands, which could bolster China's anti-access and area denial (A2AD) capabilities. This could potentially disrupt the combat abilities of the U.S. forces, especially in gathering target information and surveillance, which could in turn decrease the effectiveness of engagements with China and other adversaries.

The use of unmanned vehicles or drones for surveillance and targeting operations in a contested area has been discussed in the literature, such as in a project by the System Engineering Analysis Cohort 23 (2016), which proposed the use of unmanned systems to create a web of sensors to overcome the A2AD capabilities by the Chinese within the South China Sea while providing line-of-sight relay communications and target information for precision strikes. An advantage that these unmanned vehicles could offer is resilience to an enemy's jamming and detection, while extending the reach of a nation's own military forces (SEA Cohort 23, 2016).

Using the above scenario as the backdrop, the main motivation behind this thesis is to explore how mathematical methods can be used to place autonomous sensors. Next, we estimate the probability of detection associated with the placement of sensors in regions where the target is most likely to be at different points in time. Three main attributes of the sensors—detection range, sensor detection probability and the number of discrete sensor placements—are varied to explore the effects of the attributes on the probability of detection.

A. SEARCH AND DETECTION

Searching for and detecting a target is a classical problem that has been examined in numerous studies and in various applications. Nuun (1981) stated that some of the earliest developments in search theory were made during World War II by Bernard Koopman and his colleagues in the Anti-Submarine Warfare Operations Research Group. Since then, search theory has been used and applied extensively to other fields, such as search and rescue (Hunt, 2015), counter-piracy (Slootmaker, 2011) and counter-smuggling (Campos, 2014) operations. Besides these extensive studies, there are also many techniques used to solve search-related problems.

For example, Slootmaker (2011) examined the use of a probability model to combat piracy by estimating the probability of a pirate attack at various locations and times. In another work, Johnston (1995) applied search theory to estimate the probability of detection of maritime vessels by aircraft searchers for search areas of different sizes. He provided quantitative estimates through the use of simulations, taking into account the characteristics of the maritime vessels, the type of sensors, and the aircrafts used for detection in that area of operation (Johnston, 1995). With the development of unmanned aerial vehicles (UAVs), there has also been interest in the application of search theory to develop autonomous algorithms to conduct area searches more effectively. For example, Lau (2015) used two coverage algorithms in his work, a simple greedy algorithm that assigns search cells to UAVs based on closest distance and a fixed-lane algorithm that minimizes overlaps by pre-assigning search cells, to demonstrate that swarm UAVs are capable of coordinating among themselves to conduct autonomous area searches.

In another work, Campos (2014) applied the use of mathematical models to optimize search assets to detect drug smugglers. His work included the use of a probability model, developed by the U.S. Naval Research Laboratory, that generates the probability that a pirate will attack at a given time and location in the Horn of Africa (Hansen et al., 2011). Campos (2014) created a conversion algorithm that could convert the heat map output into an optimization model, developed by Pietz and Royset (2013), to generate a search and interdiction plan. The optimization model by Pietz and Royset (2013), termed the Smuggler Search Problem, is a “specific application to a more general

class of problems called the Generalized Orienteering Problem with Resource Dependent Rewards (GOP-RDR)" (p. 294). Both models require intelligence inputs consisting of waypoints, departure times, velocities, drug loads and environmental factors to estimate the target's location and optimize the search plan (Campos, 2014).

The approach in this thesis is to make use of a probability model that could depict the probability density of the target's location at specific times, which we would use to deploy sensors. Thereafter, using the model, we could estimate the probability of detection by the sensors and investigate the attributes of the sensors to be deployed.

B. OVERVIEW OF THE MODEL

In order for the placement of sensors to be modeled effectively, some prior information on the target would be required. Often, there could be some intelligence collected from other sources, such as satellite imagery, interception of enemy transmissions or human intelligence. Such information could give clues to the target's departure point, intended route, arrival point and likely time of departure. As such, there is a need to develop a model that could make use of information that could be gathered, to better represent the likelihood of a target's location over time and use that to deploy the sensors.

Therefore, the thesis will utilize the Brownian bridge movement model (BBMM), which requires essentially the starting point, ending point, and likely time of departure of a target, as well as a Brownian motion variance parameter. These are inputs to a mathematical model to estimate the location of a target over a time period (Horne, Garton, Krone, & Lewis, 2007). The basis of the BBMM is the Brownian bridge, which is a stochastic process that is Brownian motion with particular values tied at two points in time. The BBMM is relatively simple and has been extensively employed in animal studies, for example, in the study of movement and migration patterns of black bears (Horne et al., 2007), mule deer (Bunnefeld et al., 2011) and vultures (Fischer, Walter, & Avery, 2013).

The BBMM used in the animal studies does not account for the probability density at a specified time period, however. Instead, they are primarily used to generate the probability heat map of finding an animal within a region, independent of time.

The thesis will use the BBMM to model the movement of a target and generate the probability heat map of the target's location at discrete time steps. We will extend the BBMM to model the scenario in the South China Sea. Some key aspects of the extension would include incorporating multiple starting and ending points to depict a target with more than one possible intended ending location. We will also update the BBMM with discrete intelligence inputs pertaining to the target's location while the target is still moving. In addition, the thesis will discuss the method to estimate the probability of detection by a deployed sensor in the model.

With the probability heat maps generated at discrete time steps, the thesis will use them to automate the placement of sensors by selecting areas with the highest probability of detection. By adjusting the sensors' attributes and analyzing the probability of detection, the thesis will also conduct trade-off analysis between the number of discrete sensors and the sensors' detection range.

C. RESEARCH QUESTIONS

The intent of the thesis is to explore the use of the BBMM in a military scenario, extend the BBMM to generate probability heat maps and place sensors to estimate the probability of detecting a target with uncertain behavior. Results from the model are interpreted to analyze the effect of various sensor attributes. Some extensions of the model include accounting for multiple starting and ending points of a target's movement, way points along the target's path, updating the heat map with discrete intelligence inputs and estimating the probability of detection. This thesis will thus be guided by the following questions:

- How can we develop and enhance the BBMM to effectively model the inclusion of way points and multiple start and end points?
- How can intelligence inputs collected from sensors be used to update the heat maps and target's location?

- How can we generate probability heat maps using the BBMM that can be used to deploy sensors to increase the probability of detection?
- Which attributes of the sensors (detection range, detection probability, or number of discrete sensors) are most important in increasing the probability of detection?

D. BENEFITS OF THIS THESIS

This thesis is intended to develop a viable solution to estimate and present the likelihood of target movement based on multiple intelligence updates in the form of heat maps. The main focus of this research is to use these heat maps to place sensors and to understand the attributes of the sensors and their effects on the probability of detection. This research would also lay the foundation for future researchers to develop autonomous algorithms to dynamically deploy a web of sensors over a course of time to maximize the probability of detection. With the use of such autonomous unmanned systems, militaries can operate on a leaner manpower structure, while extending their operating reach beyond the enemies' horizon.

E. METHODOLOGY

This thesis begins with an in-depth review of the possible military scenario in the South China Sea and provides a detailed description of the BBMM. The BBMM is applied to the scenario and is enhanced to include way points and multiple starting and ending points for the movement of the target. Using the BBMM, probability heat maps of a target's location over time are generated and interpreted to automate the placement of discrete sensors in those regions with the highest probability of finding a target. Next, we will estimate the detection probability of the target based on the sensors' parameters. The thesis conducts a meta-experiment to examine the effects of the attributes of the sensors on the probability of detection, and a trade-off analysis between the number of discrete sensors and the detection range is conducted. This will help to determine which attribute is critical to improving the probability of detection.

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II. OVERVIEW OF SCENARIO

The South China Sea, covering about 3.5 million square km, is one of the world's most vital maritime areas. An article in Time magazine (Beech, 2016) highlighted "that more than $5 trillion in trade flows through the waters each year, about one-third of all global maritime commerce" (p. 44). The area also boasts a huge supply of seafood and untapped oil and natural gas deposits. Such importance has led a number of countries to lay competing claims to various parts of this waterway. China's increasing militarization of the islands and strong stance in exerting their claim would pave the way to possible conflicts within the region.

A. CONTEST IN THE SOUTH CHINA SEA

China, Vietnam, the Philippines, Taiwan, Malaysia and Brunei have competing claims in the South China Sea, as depicted in Figure 1. China's claim is the most expansive, based on the nine-dash line, a demarcation of maritime borders from historic rights.

Figure 1. Disputed Claims in the South China Sea.
Source: Rosenberg (n.d.).

China has been very active in establishing its claim and control, through the reclamation of islands and maintaining a "maritime militia" comprised of Chinese fishing fleets (Glaser, 2015). Their reclamation efforts have been astounding, as seen in the images of Fiery Cross Reef in Figure 2. In a brief span of two years running tracks, basketball courts and an aircraft runway have been introduced (Center for Strategic & International Studies, n.d.). These reclamation efforts have been seen by analysts as a means to militarize the area and have been dubbed the "Great Wall of Sand," by Admiral Harry Harris Jr., head of the U.S. Pacific Command (Browne, 2016). In addition, these islands would definitely boost China's anti-access, area denial capabilities through the deployment of a variety of weapons, equipment and platforms such as radar, surface-to-air missiles (SAM), anti-ship cruise missiles and electronic jamming equipment (Glaser, 2015). They could also possibly serve as military outposts during times of conflict.

Figure 2. Construction and Reclamation of Fiery Cross Reef. Adapted from the Center for Strategic & International Studies (n.d.).

While the U.S. does not have any claims to the islands in the South China Sea, President Barack Obama has presided over a pivotal shift in focus to Asia, including the deployment of more American troops to the region, as well as a projected 60% of the U.S. Navy fleet to the Pacific by 2020 (Marcus, 2012). See Figure 3 for the number of

forces stationed in Southeast Asia. U.S. forces continue to patrol the region in their attempt to ensure navigation freedom in the region.

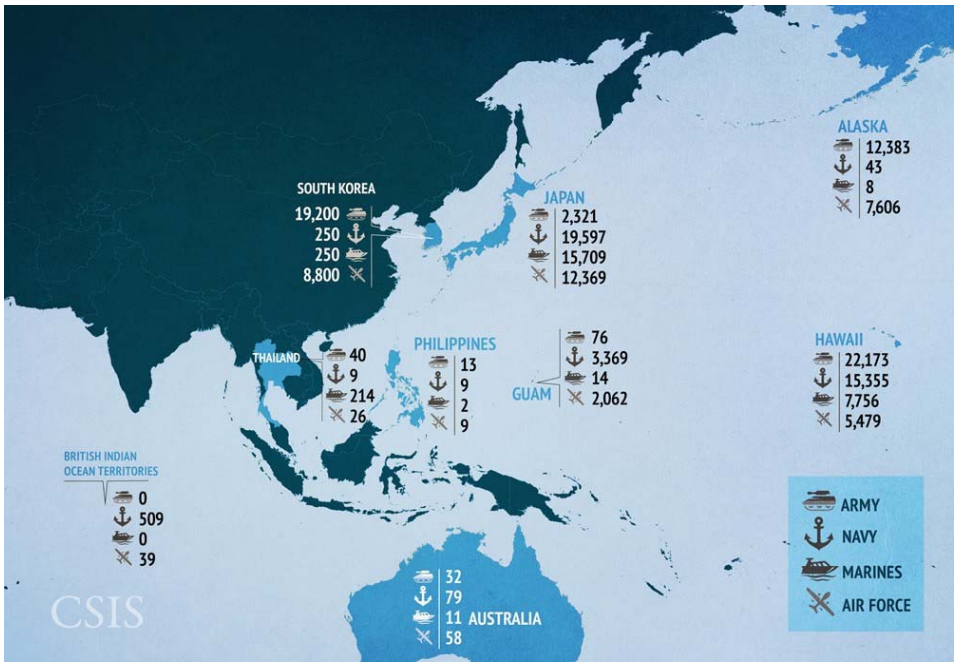

Figure 3. The Number of U.S. Forces Deployed in Southeast Asia. Source: Center for Strategic & International Studies (2014).

In July 2016, an international tribunal, the Permanent Court of Arbitration in The Hague, dismissed China's claims to the Spratly and Paracel Islands and Scarborough Shoal. The arbitration was initiated unilaterally by the Philippines to seek a peaceful resolution to the matter, but China has refused to participate and maintains that it will not acknowledge the court's decision (BBC, 2016). China continued to maintain its uncompromising stance on its claims in the South China Sea and has continued to bolster its military presence on the man-made islands, as well as closing off a part of the South China Sea for military exercises (Bodeen, 2016).

As U.S. forces continue to patrol and survey the waters in the South China Sea as part of their freedom of navigation operations, the number of incidents between Chinese forces and U.S. forces in the region has been increasing. One such incident occurred on June 7, 2016, when a RC-135 reconnaissance plane from the U.S. Air Force was

intercepted “unsafely” by a Chinese J-10 fighter jet, reportedly in international airspace (Starr, 2016). In the report, the unsafe manner was defined as the high speed with which the Chinese fighter closed in and that it was flying at the same altitude as the U.S. reconnaissance plane (Starr, 2016). This incident came on the heels of another incident the previous month, where another U.S. reconnaissance plane was intercepted by Chinese jets, which came within 50 feet of the U.S. plane at one point during the incident (Kube, 2016). Admiral Sun Jianguo, Deputy Chief of the Joint Staff Department of the Central Military Commission, stated that China opposes these freedom of navigation patrols and that such military freedom of navigation could play out in a disastrous way (Blanchard, 2016).

The contest for the South China Sea could potentially spark a dangerous confrontation between the two superpowers that would involve all countries within the region. It is this scenario that sets the stage for the research in this thesis.

B. CHINA’S CARRIER PROGRAM

In the lead-up to any military confrontation, Chinese forces would probably have to project a number of forces to the buffer islands to provide perimeter security for mainland China and to deny access to the South China Sea. In the 2015 Annual Report to Congress, it was mentioned that China has started to pursue an indigenous aircraft carrier program and could build multiple carriers in the next 15 years, which would give Chinese forces the capability to project large forces and extend their operating reach (Office of the Secretary of Defense, 2015). Currently, China has begun carrier training onboard Liaoning (shown in Figure 4), an ex-Soviet aircraft carrier that was bought from Ukraine, refurbished and commissioned in 2012. On December 15, the Chinese People’s Liberation Army Navy (PLAN) qualified a new batch of aviators flying the Shenyang J-15 Flying Shark onboard the carrier. It would seem that China is making remarkable progress in learning the science and art of carrier aviation and likely will have a powerful fleet in the next 15 years (Majumdar, 2015).

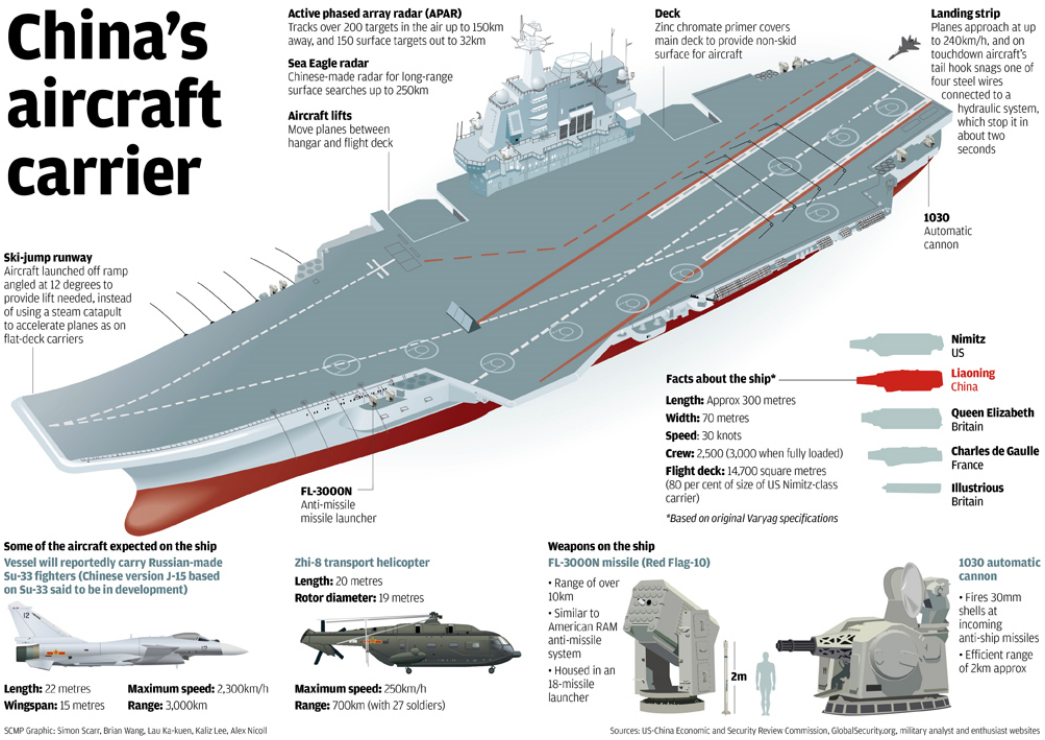

Figure 4. China's Aircraft Carrier, Liaoning. Source: Shrivastava (2013).

Besides providing protection and a buffer to the South China Sea, these aircraft carriers are also important assets that could extend the reach of China's military into the Pacific to deter enemies from coming close to the South China Sea. These aircraft carriers would thus be considered as high-value units that would require constant tracking and monitoring of their locations. With the A2AD capabilities of the Chinese, however, it could prove difficult to do so using conventional means, particularly during times before the start of a conflict when key assets have not yet been deployed. Thus, a system of unmanned drones autonomously moving and detecting targets would be an attractive solution.

C. ADVANCEMENT OF UNMANNED VEHICLES

Development of UAVs or drones has been increasing in recent years, with both commercial and military entities looking to harness their capabilities for various uses, such as surveillance, making deliveries, filming or aerial photography and many more (Top 12 Nonmilitary Uses, n.d.). For example, an Israeli startup company, Airobotics,

has developed a fully autonomous drone that is capable of taking off, conducting a surveillance flight and then landing to charge batteries and prepare for the next mission (Ackerman, 2016).

In particular, the development of cooperative and swarming drone technology would certainly shape the battlefield of the future. Many militaries have embarked on programs in this area, such as the U.S. Office of Naval Research, which started the Low-Cost UAV Swarming Technology (LOCUST) (Hambling, 2016) to look into firing many small drones as a means of ship attack, rather than using just one missile. The Navy also developed a swarming operation using unmanned boats that could swarm toward an enemy ship, to prevent it from getting close to the high-value boat (McGarry, 2014).

Besides developing swarming technology, the use of renewable energy sources to create long-endurance airplanes has also become a possibility. One such plane, the Solar Impulse 2, a zero-fuel aircraft, embarked on and completed a journey around the world without using a single drop of fuel. Its longest flight lasted five days and five nights, from Nagoya, Japan to Hawaii ("About Impulse," n.d.).

Thus, it would be very possible, in the near future, that militaries can field a large number of surveillance drones, with long endurance and cooperative knowledge, to monitor and communicate on the battlefield. One of the capstone projects by SEA-23, students discussed the use of unmanned systems in integrating cross-domain naval fire for the year 2025. They proposed a system that could create a web line-of-sight communication using unmanned systems comprising aerial, surface and sub-surface vehicles, which could relay target information back to a "mother ship" for precision fire (see Figure 5). These systems with line-of-sight relay communications could provide an ad-hoc network that would be resilient to the enemy's reach and jamming. With the need to maintain distance between the sensors, however, the sensors would need to move in cognizance of other sensors' position and not employ conventional search techniques. Therefore, it would be ideal if algorithms existed to achieve autonomous deployment while maximizing the probability of detection of targets.

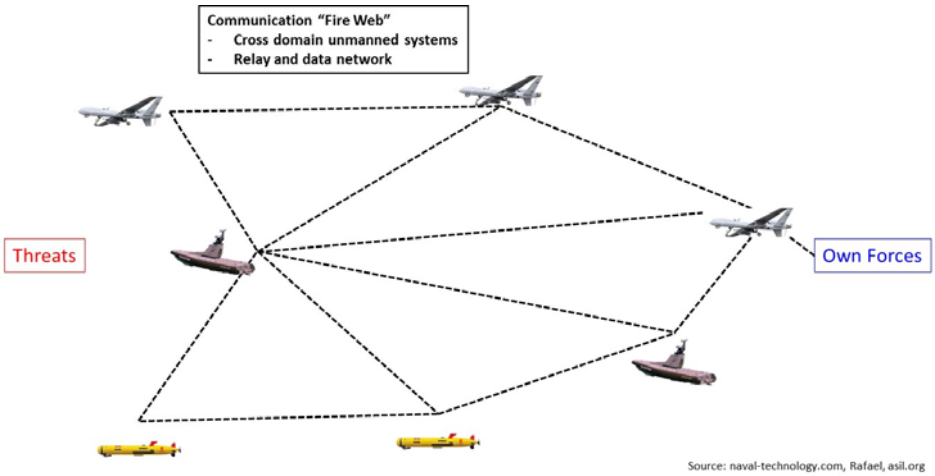

Figure 5. Capstone Project by SEA-23, a Web of Sensors. Adapted from SEA Cohort 23 (2016).

This thesis will build upon the use of such a communications web for the detection of targets in the South China Sea. Specifically, the thesis will utilize the BBMM to generate probability heat maps for a target location. These heat maps help determine where to employ sensors in a web. This thesis also will discuss the type of unmanned systems, the detection range, and the quantity and quality of sensors required in such a web.

D. SCENARIO TO BE USED IN THE MODEL

China's primary naval operating base is off Hainan, and China has been constructing important and vital defense assets in several areas in the Spratly and Paracel Islands. At the Shangri-La dialogue in Singapore in May 2015, Admiral Sun Jianguo, China's representative, mentioned that these islands will provide all-round and comprehensive services to meet civilian demands, but he also noted that they will be used to meet necessary defense needs (Glaser, 2015). Indeed, China has begun to deploy defensive capabilities such as its advanced fighter aircraft, J-11, and its SAMs to Woody Island, which can be seen as a move to militarize these islands (Minnick, 2015).

Separately, China has always maintained its stance on a "one-China" policy, which states that Taiwan is part of China. The delicate balance between China and Taiwan, however, has been upset with the election of the Democratic Progressive Party

(DPP) into Taiwanese government, a party that has strongly advocated the preservation of freedom and democracy of Taiwan. Taiwan's location in the South China Sea could also provide military significance for China to establish buffers in the region, especially against an invading force. As such, Taiwan could become a likely target of interest in the wake of military confrontations.

Thus, the thesis will use the scenario of tracking Chinese aircraft carriers through three passage lanes, from Hainan to the three possible locations of the Spratly Islands, Scarborough Shoal off the Philippines, and Taiwan, as shown in Figure 6. These passage lanes could be also used by Chinese forces to break out of the South China Sea region into the Pacific, to control buffer islands and to deter any establishment of a naval blockade that could affect freedom of navigation and trade in China. Therefore, it would be ideal if targets could be tracked and intercepted by the web of sensors before they could reach their intended location.

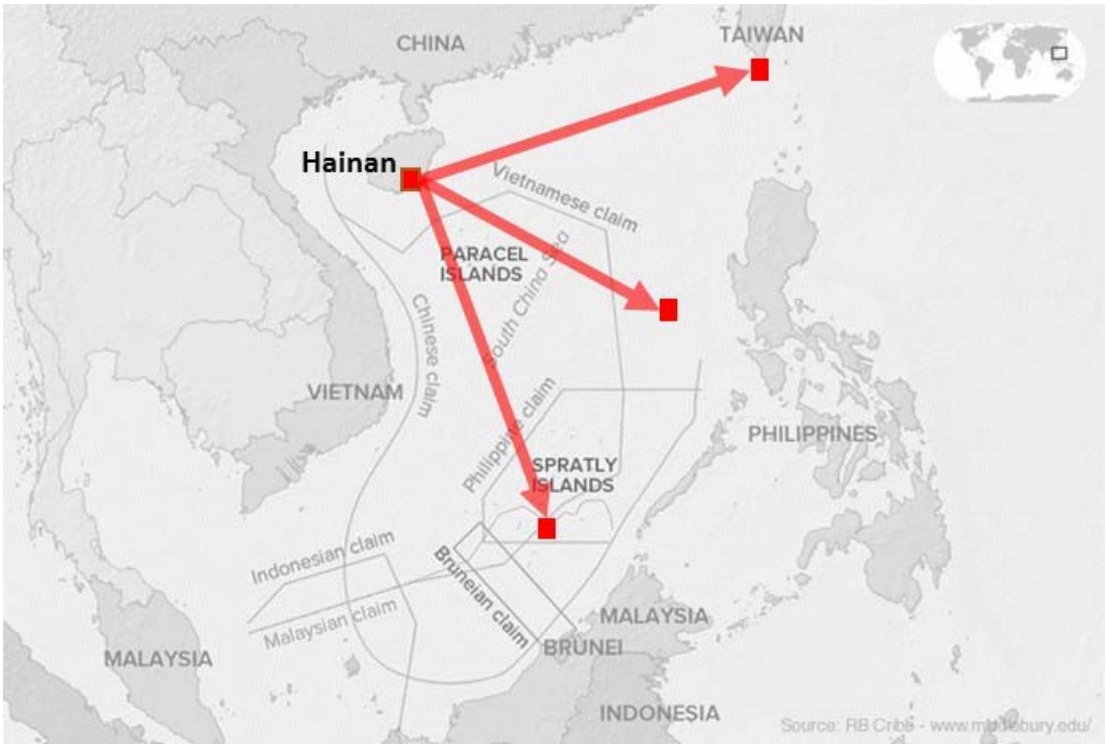

Figure 6. Possible Passage Lanes in the South China Sea during Conflicts. Adapted from Cribb (n.d.).

III. THE BROWNIAN BRIDGE MOVEMENT MODEL

Given the starting and ending points of a target trajectory, the target's location in between can be modeled as a Brownian motion, the parameters of which are controlled through the choice of variance parameters (Horne et al., 2007). This approach, termed the Brownian bridge movement model (BBMM), models uncertainty in a target's path and allows for path dependency, where a target's location is highly dependent on its previous locations. While a relatively simple model, the randomness can capture aggregate-level fluctuations due to environmental factors, tactics and timing.

This chapter provides an understanding of the BBMM and how it will be applied in this thesis. The thesis will also extend the BBMM in order to model a target using three possible paths to traverse the South China Sea. Multiple Brownian bridges of a target's possible paths will be simulated, and they will be compiled to generate probability heat maps of the target's location at discrete time steps.

A. THE BROWNIAN BRIDGE MOVEMENT MODEL

The Brownian bridge is a stochastic process in which Brownian motion is tied to particular values at two points in time. The standard Brownian bridge has a variance of 1 and exists over the time range [0,1], taking the value zero at times 0 and 1. It can be rescaled according to time, distance and variance (Ross, 1983). Bullard (1991) described a Brownian bridge model that uses two-dimensional Brownian bridges to estimate an animal's location and plot the probability density function as a contour map (see Figure 7). This thesis will utilize a similar approach, but instead of determining the distribution over an aggregate period of time, the model will look into the distribution at a particular instance or specific time *t*, and display the probabilities in a heat map.

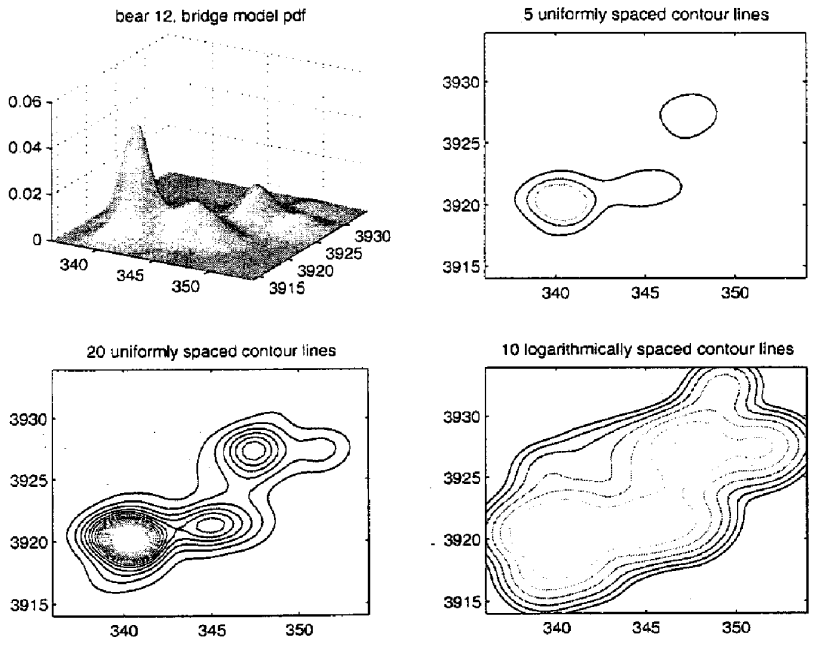

Figure 7. Contour Map of a Bear's Movement, Modeled Using Brownian Bridges. Source: Bullard (1991).

1. Two-Dimensional Brownian Bridges

In a two-dimensional Brownian bridge, Brownian movement is allowed in a two-dimensional space. While the movement of a target between two points cannot be exactly predicted, it could be approximated by the use of random walk or its continuous counterpart, Brownian motion (Turchin, 1998), which can be characterized by Gaussian probability distributions. By conditioning on the starting and ending points and applying Brownian motion in between, we have the stochastic process of a Brownian bridge (Ross, 1983).

With the knowledge of the starting and ending points of a target at given times, such that a target begins at a point (0, 0) at time $t = 0$ and will end at a point $\vec{b} = (b_1, b_2)$, at time T, the target's position between time 0 and T can be estimated using the two-dimensional Brownian bridge. The position (x, y) can be written as (Bullard, 1991):

$$\begin{bmatrix} x \\ y \end{bmatrix} = N(\vec{\mu}(t), \sigma^2(t) I_2),$$

where

$$\vec{\mu}(t) = \frac{t}{T}\vec{b}$$

and

$$\sigma^2(t) = \frac{t(T-t)}{T}{\sigma^2}_b.$$

The parameter ${\sigma^2}_b$ is the variance of the Brownian motion and I_2 is a *2 x 2* identity matrix. Using this model, the variance will be small when *t* is near times 0 and *T* and will be largest when *t = T/2* (Bullard, 1991). The probability that multi-dimensional Brownian bridges cross a boundary is studied in Atkinson and Singham (2016).

While it is relatively easy to obtain information on the possible starting and ending points of a target, it is, however, difficult to determine the value of ${\sigma^2}_b$. Horne et al. (2007) estimated this variance by using observed locations and assumed that the path connecting the two locations is a Brownian bridge. They went on to construct a likelihood function comprising the unknown estimate of Brownian motion variance. Its maximum likelihood estimate is obtained numerically by optimizing the likelihood function over values of ${\sigma^2}_b$, using the Golden Search routine (Horne et al., 2007).

This thesis will attempt to estimate the Brownian motion variance parameter without using tracking data of the target, since such data are not available for this research. The variance parameter represents the deviations in the path that the target will travel relative to when it travels in a straight line. A smaller variance implies a more straightforward path, while a higher variance would indicate a more indirect path, with a higher probability of it traveling backwards from its destination. Here, we used a method to estimate the variance parameter by using probabilities that the target will stray outside a given distance from the central direct path. Figure 8 shows this distance, *a*, for a Brownian bridge *B(t)* over time $t \in [0, T]$.

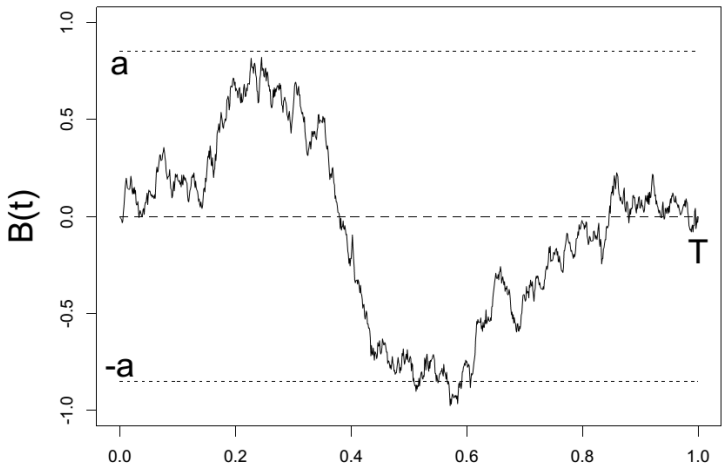

Figure 8. Brownian Bridge over Time [0, *T*].

If *P* is defined as the probability that a target will travel outside the limits of a given distance, bounded by (-*a*, *a*), then the variance can be found using the following equations (Atkinson & Singham, 2016):

$$P := Prob(|B(t)| > a) = e^{-2a^2/\sigma^2 T}$$

$$\Rightarrow \sigma^2 = -\frac{2a^2}{T \ \ln P}$$

The value of *P* is a probability that we assumed is estimated by a subject matter expert. Subsequently, the simulated paths can be checked to see if they align with the expected behavior or any available tracked data of the target, to determine if the estimated variance is satisfactory.

2. Uncertainty of Starting and Ending Points

In this thesis, the BBMM assumes uncertainty in the starting and ending points as well as the departure times of the target. It is often true that a target's starting location and departure timing are not known exactly, given that intelligence sources are not perfectly accurate. In the model, the thesis will introduce this uncertainty by randomizing the starting points, ending points and departure times of each Brownian bridge simulated, by assuming a uniform distribution over time and space over the ranges of uncertainty.

B. USE OF THE BBMM IN ANIMAL STUDIES

The use of such a model was first proposed by Bullard (1991), who applied the model in estimating the home ranges of animals. Since then, the BBMM has been used widely in understanding the movement patterns of animals (Horne et al., 2007). In the work by Horne et al. (2007), the use of the BBMM to estimate the home range of a male black bear was studied. The BBMM was used to generate an overall probability map for the black bear's location by aggregating many Brownian bridges based on tracked movement, collected through GPS collars. Horne et al. (2007) also compared the probability map with that generated using a fixed-kernel method, and found that the results were similar (see Figure 9), with a 77% overlap in the areas represented by both the BBMM and the fixed-kernel estimate. The model has also been expanded to other studies such as "stopover" habitats used by migrating mule deer (Bunnefeld et al., 2011) and home ranges of black vultures (Fischer et al., 2013).

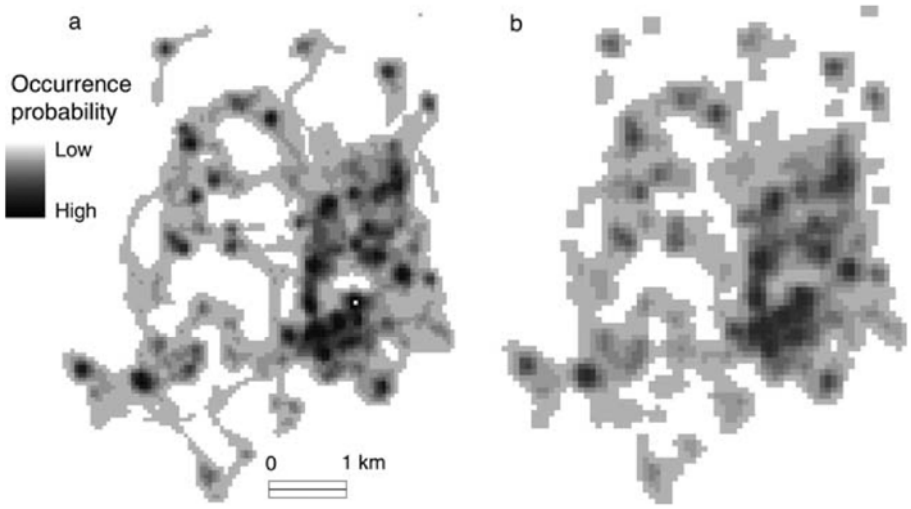

Figure 9. Estimated Home Range of a Male Black Bear using both the BBMM and Fixed-Kernel Methods. Source: Horne et al. (2007).

It is noted, however, that most animal studies created probability maps that only depict the probability distribution of an animal's location, independent of time. Whereas in this thesis, we developed a BBMM that would generate probability heat maps as time

changes, such that it could be updated with discrete intelligence inputs. The BBMM was first modeled in the defense context to examine the case of interdicting drug smugglers using naval assets, and to generate probability heat maps that are used to present the likelihood of the target's location in the present and future. Such heat maps would provide operators with a map-based visual view of uncertainty in target location, which would greatly aid in their decision in allocating detection and interdiction assets.

C. GENERATING PROBABILITY HEAT MAPS

This thesis follows an approach outlined by Horne et al. (2007) to determine the probability of finding a target in a specific area at a specific time, *t*. The starting and ending points of the target are known. We employ a two-dimensional Brownian bridge between the starting and ending points, which uses independent Brownian bridges to model movement in the *x* and *y* directions in $\mathbb{R}^2$. The functions, $f(x)$ and $f(y)$ are the probability density functions of the Brownian bridge in each direction at a given point in time *t*. Next, denote Z_t as a Brownian bridge at time *t*. The probability the target is in region *A* at time t is defined as

$$P(Z_t \in A) = \iint_A f(x)f(y)dxdy.$$

Instead of calculating these integrals, which can be numerically tedious especially when we incorporate intelligence updates, this thesis would employ the use of simulations in MATLAB to estimate the probability of the target's location at a specific time, *t*. To determine the probability of the target's location in an area, the region of interest is divided into multiple cells. The starting and ending points of the target are randomized according to some specified distribution and used to simulate multiple Brownian bridges. At a specific time period, *t*, the number of the Brownian bridges located within a cell A is recorded and divided by the total number of Brownian bridges to calculate the probability that the target is in that cell. This information is subsequently plotted in heat maps, which change over time, as shown in Figure 10.

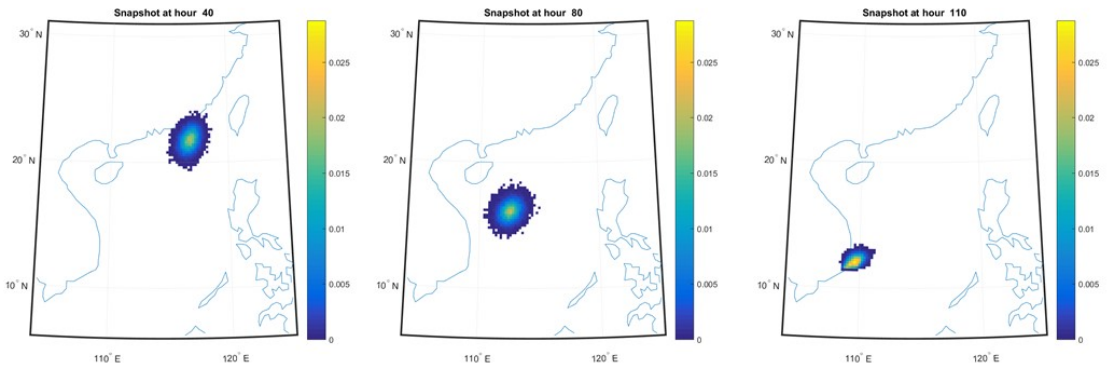

Figure 10. Probability Heat Map of a Target Moving across the South China Sea over a Time Period.

The simulation allows for the generation of the Brownian bridges through a few parameters, which include the variance of the Brownian motion, the velocity of the target and the expected time range of departure of the target.

D. EXTENSIONS TO THE BBMM

As discussed in the scenario in Chapter II, there is a likelihood that the target will end up at one of three different locations. Also, with the deployment of sensors, one would expect that there could be intelligence updates at snapshots of time, which could be used to update the BBMM. Therefore, this thesis will discuss three extensions of the BBMM in the simulations such that it could be applied to the scenario.

1. Incorporating Waypoints

If there is a known path of which the target will travel on, waypoints can be included in the BBMM to mimic that known path. The model does this by assuming that the paths connecting the waypoints would be independent Brownian bridges. This is depicted in Figure 11 by the grey lines between Point A and Point D. As the Brownian bridges are assumed to be independent, we also assumed that the heat map associated with the Brownian bridges connecting Point A to B is independent of the heat map connecting Point B to C, and so on.

Figure 11. Example of Three Brownian Bridges Connecting the Way Points between Points A, B, C and D.

2. Incorporating Multiple Starting and Ending Points

To model the three paths shown in the South China Sea scenario, there is a need to include multiple end points. This was achieved by generating multiple, independent Brownian bridges to the different end points and weighting them by probabilities that the target will travel towards that end point. Such probabilities could be deduced from other intelligence sources or scenario experts. In addition, the starting and ending points are simulated with uncertainty to generate the Brownian bridges. The combined weighted probability heat maps across the three paths are then plotted accordingly. Figure 12 shows the probability heat map of a target moving from the island of Hainan according to the three paths illustrated in the scenario. Notice that at t = 100hrs, the probability heat map of the target's location is a lot smaller compared to the earlier times. This is because the target has reached the possible ending locations. Since the variance of the Brownian motion is small at the starting and ending points, the probability heat map shown will have a narrower density at the three ending points.

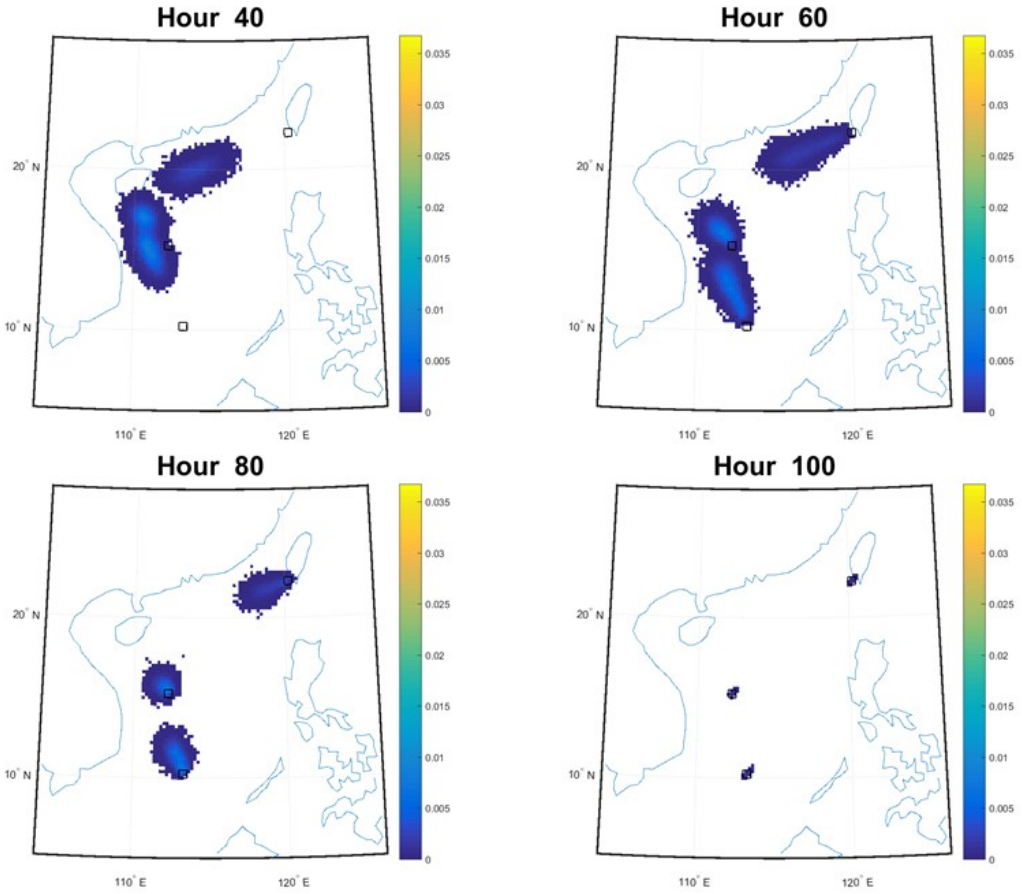

Figure 12. Example of Probability Heat Maps with Multiple Ending Points.

3. Incorporating Intelligence Inputs

As the target moves along the path, there could be real-time intelligence inputs, whether through detection by sensors or other means of intelligence, to update the location of the target. These inputs are modeled with a binary variable {0, 1} in the BBMM, to indicate if the target is detected within the sensor coverage area or not at that specific time. If the target is detected, it will be assigned a value of 1, and only the Brownian bridges that pass within the sensor coverage area are retained. If the target is not detected, the Brownian bridges within the sensor coverage area are filtered out instead. This is illustrated in Figure 13, where we have three red boxes denoting the presence of sensors at these boxes. At t = 30hrs, the first sensor is not active; therefore, we did not observe any changes to the probability heat map. At t = 50hrs, the heat map

indicates that the target is present in the sensor coverage area, and we observe that the heat map collapses within the box, indicating that only the Brownian bridges within the box are retained. At t = 60hrs, the intelligence input is that the target is not present in the new sensor coverage area, which could be due either to a failed detection or the fact that the target is outside the sensor detection range at that point of time. Here, we observe that the heat map in the rightmost red box is empty, indicating that the Brownian bridges within the box have been filtered. At t = 70hrs, we observe the target continuing to move to its likely ending point based on the remaining Brownian bridges left in the model.

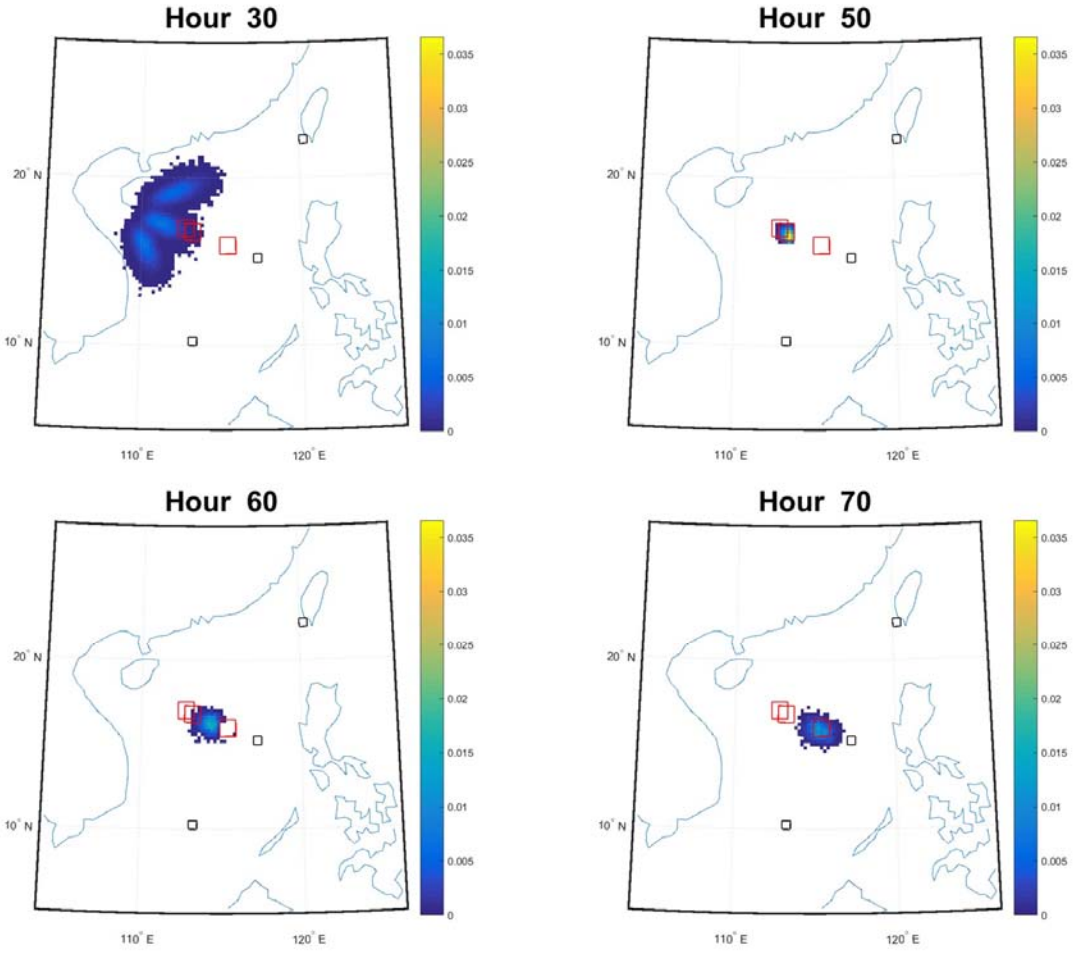

Figure 13. Probability Heat Maps Incorporating Intelligence Updates.

IV. ESTIMATING THE PROBABILITY OF DETECTION

With the probability heat maps generated to depict the probability of a target's location over time, the probability of detection of a target by a sensor at that snapshot of time (called a discrete look) can be estimated. This chapter will outline the various steps used to calculate and simulate the probability of detection of a target, which will be the main measure of effectiveness of a given sensor configuration.

A. THE PROBABILITY OF DETECTION BY A SENSOR

In signal detection theory, the goal is to estimate two key parameters, the first being the signal strength relative to the noise signals, and the second is the response on whether the target is present or not (Abdi, n.d.). Table 1 lists the four possible scenarios that could occur based on the response and the presence of signals. From these four scenarios, we are able to estimate the probability of a false alarm (positive reading when the target is not there) and the probability of a missed detection (negative reading when the target is there).

Table 1. List of Four Possible Scenarios in Signal Detection Theory.

	Target Present "Signal"	**Target not Present "Noise"**
Positive Sighting	Detection	False Alarm
Negative Sighting	Missed Detection	Correct Rejection

From the probability heat maps generated, we can obtain the probability that a target is at a certain location at a discrete point in time. As such, we can summarize the information we can estimate thus far in Table 2.

Table 2. Probabilities and Their Representations.

Probability	**Representations**
$P(T)$	Probability that the target is within an area at a discrete point in time, which is obtained from the probability heat maps.
$P(FA)$	Probability of a false alarm by the sensor, which is based on the sensor's signal to noise ratio.
$P(D\|T)$	Probability that the sensor gives a positive sighting given that the target is in the area, which is obtained usually through field tests of the sensor.
$P(D)$	Probability that the sensor gives a positive sighting, regardless of whether the target is present or not $= [P(D\|T) \times P(T)] + [P(FA) \times (1 - P(T))]$.

We can use Bayes' theorem to calculate $P(T|D)$, which is the probability that the target is in a given area for which the sensor gives a positive reading. $P(T|D)$ will henceforth be denoted as "probability of detection," which will be our main measure of effectiveness in later experiments. The following formula shows how we calculate the probability of detection:

$$P(T|D) = \frac{P(D|T) \times P(T)}{P(D)}.$$

B. USING THE BBMM TO ESTIMATE THE PROBABILITY OF DETECTION OF A SINGLE SENSOR

We first assumed that the sensor coverage area is rectangular in shape and that the sensors deployed will have a constant probability of detection and false alarm throughout one experiment run. To estimate the probability of detection, the sensor coverage area was first plotted onto the probability heat map. At that snapshot of time when the sensor is active, the number of Brownian bridges passing through the sensor coverage area is flagged. The probability of the target passing through the coverage area or areas was estimated by dividing that number of Brownian bridges flagged by the total number of Brownian bridges rendered in the simulation. This estimates the value for $P(T)$, which is the probability of the target passing through sensor coverage areas. In a scenario where

there are multiple end points, the number of Brownian bridges flagged from each path is weighted accordingly. An example is shown in Figure 14, where the red box in the figure depicts the sensor coverage area. Afterward, we can calculate the probability of detection, $P(T|D)$, with specified values of $P(D|T)$ and $P(FA)$, which are inputs to the model that define the capabilities of the sensors.

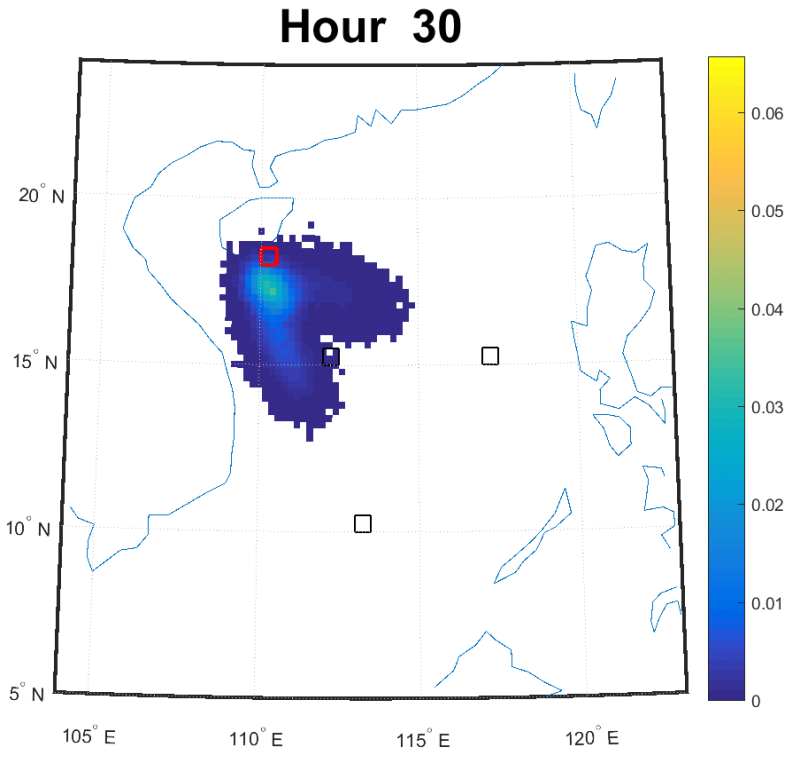

Figure 14. Estimating the Probability a Target is Present in a Discrete Sensor Using a Multiple Paths Scenario.

C. THE PROBABILITY OF DETECTION WITH MULTIPLE SENSORS AND INTELLIGENCE INPUTS

To determine the value of $P(T)$ when we have multiple sensors, we use the number of different Brownian bridges that have been flagged with a positive sighting at each sensor coverage area. As such, Brownian bridges that passed through a sensor coverage area with negative sighting, or a Brownian bridge that has already been detected before will not be counted. The value of $P(T)$ is then calculated by dividing the number

of Brownian bridges flagged by the sensors by the total number of Brownian bridges simulated. This is the value of the probability of locating the target at all sensor coverage areas with a positive sighting. With $P(T)$ found, we apply the approach in Section A to determine the probability of detection by the sensors based on the assumption that all sensors have the same detection capabilities in each experiment run.

An example is shown in Figure 15, where we have a target moving to three multiple end points and three discrete sensors are placed at t = 40hrs, 50hrs and 60hrs. It happens that all three sensors gave a positive sighting of the target, so we see that only the Brownian bridges within the sensor coverage areas (denoted by the red boxes) are depicted.

In this simulation run, we simulated 20,000 Brownian bridges for each of the three paths. The value of $P(T)$ is then obtained by the method described above, and was 0.0552. The sensors here have a detection probability of 0.8 with a detection range of 0.4 degrees (approximately 45km) and a false alarm rate of 0.05. Using Bayes' theorem, the probability of detection was found to be 0.48.

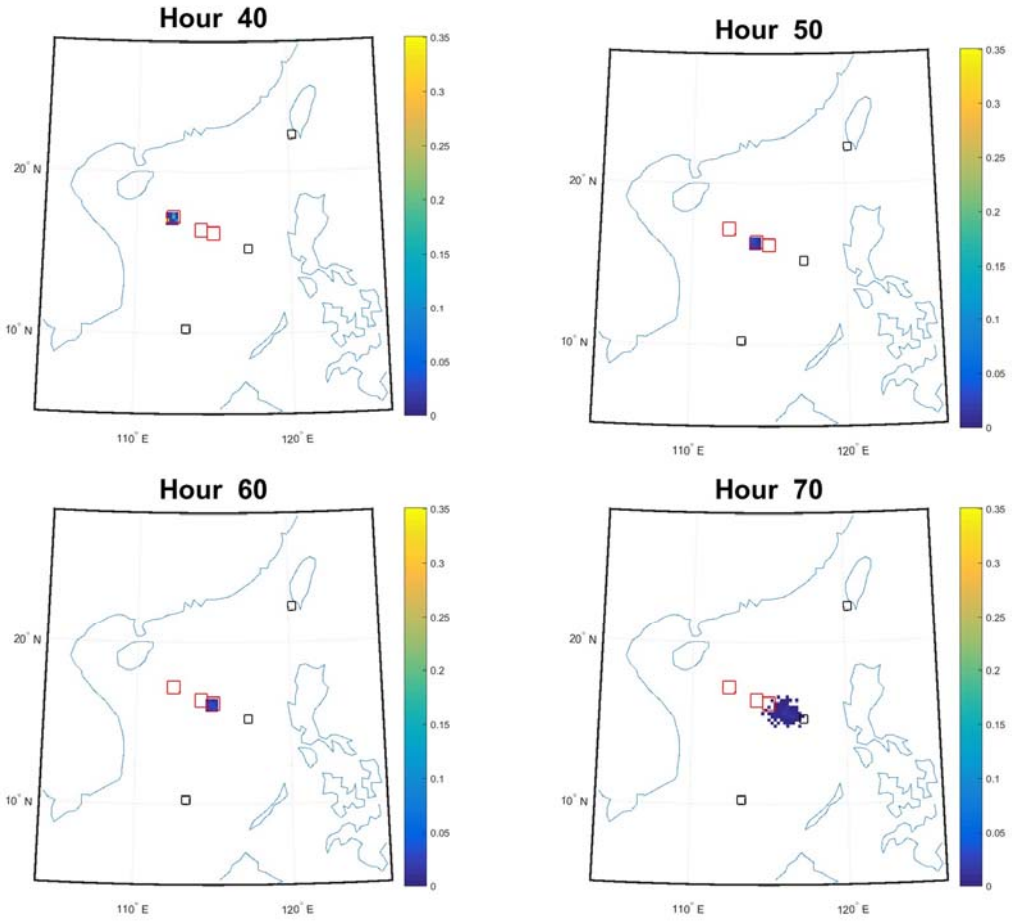

Figure 15. Probability Heat Maps Showing Discrete Sensors Detecting a Target.

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V. META-EXPERIMENT

In order to understand the different attributes of the sensors and how they affect the probability of detection, a meta-experiment was designed and carried out, using the BBMM as the underlying model. The experiment uses a three-stage algorithm to generate probability heat maps, deploy sensors at discrete times, and estimate the probability of detection based on intelligence inputs. Certain key parameters are varied to study and understand their effect on the probability of detection. The results are compared to provide an analysis on the relationship between the attributes of the sensors and the probability of detection.

A. A THREE-STAGE ALGORITHM FOR THE EXPERIMENT

In order to evaluate the different configurations of the sensors, an algorithm was designed to automate the placement of sensors and calculate the total probability of detection. The algorithm is broken down into three stages, as seen in Figure 16.

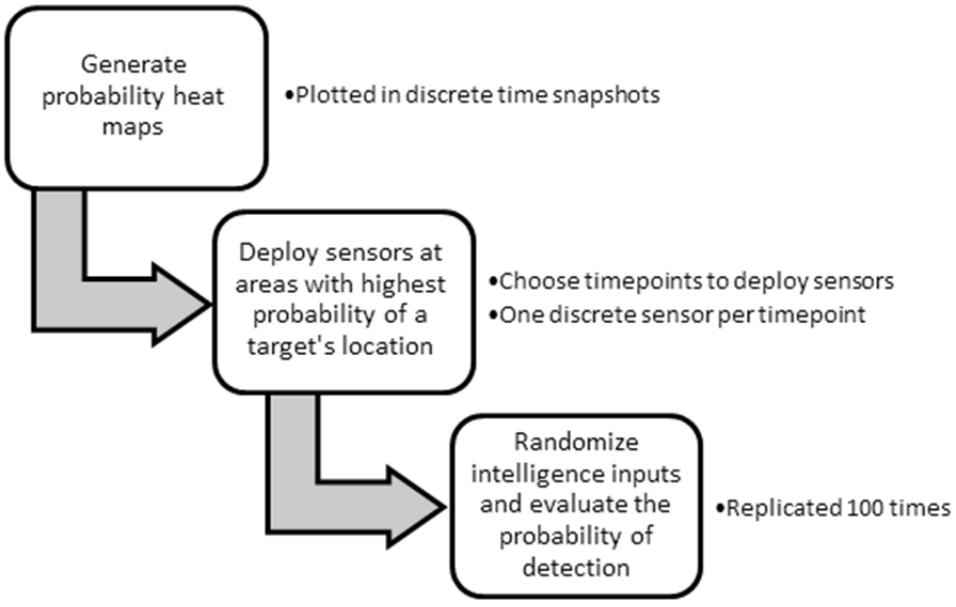

Figure 16. Three-Stage Algorithm for the Meta-Experiment.

The first stage involves generating the probability heat map using the BBMM based on inputs for the target information. In the second stage, the areas with the highest likelihood of a target's location at the specified time steps are identified and the sensors are deployed to those areas. The sensors deployed will observe the location at varying discrete times, but only one sensor will be active at each discrete time. Figure 17 illustrates the probability heat map of a target moving to three possible locations with one discrete sensor placed at the highest likelihood of the target's location at t = 40, 50, 60hrs.

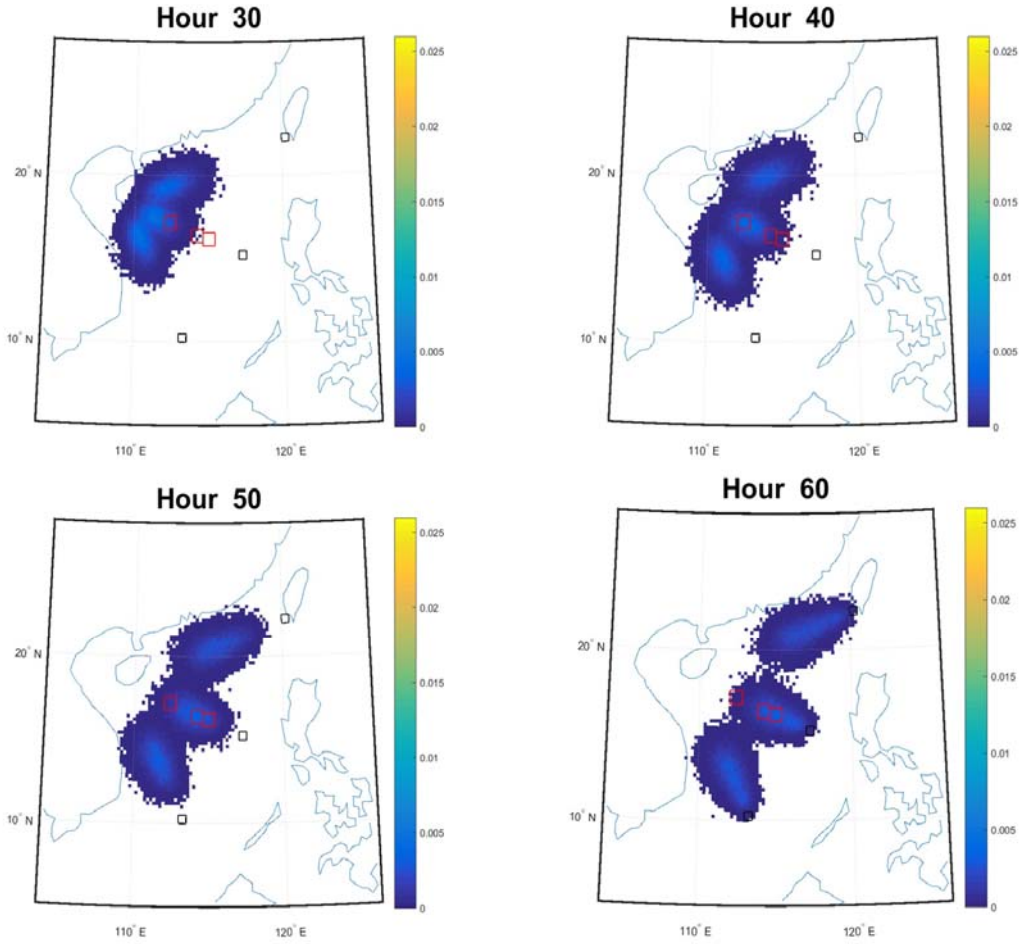

Figure 17. First and Second Stage of the Meta-Experiment.

In the last stage, the sensor sightings are randomized to either a positive or a negative detection using a Bernoulli distribution with $P(D)$, probability a sensor gives a positive sighting, assigned as the probability parameter. The sensor sightings are randomized in each replication and the heat maps are updated accordingly. The randomized sightings are based on the sensors' detection probability and a higher detection probability means a higher chance of a positive sighting. With the updated heat maps, the total probability of detection by the sensors is estimated for each replication and consolidated for statistical analysis. The results are illustrated in Figure 18, where we observed that each of the sensors was able to detect the target in one replication of the experiment. The probability of detection was estimated and recorded. The experiment was replicated 100 times with randomized intelligence updates and the probability of detection for each run was compiled for further analysis. See Figure 19 for the boxplot of the probability of detection in the 100 runs. The boxplot shows the 25% quantile, the median value, and the 75% quantile. The black dots represent the outliers from the dataset and the whiskers of the boxplot extends to the outermost data that falls within 1.5 times the interquartile range from the third and first quartile. The mean of the probability of detection here is 0.448, with a 95% CI of [0.4379, 0.4581]. From the data, we observed some outliers, which could be due to the varying intelligence inputs. If only one out of the three sensors registered a positive sighting, the overall probability of detection could be affected greatly, hence the low probability of detection. However, with the sensors' detection probability set at 0.8, this does not happen frequently and thus the low probability becomes an outlier in this dataset.

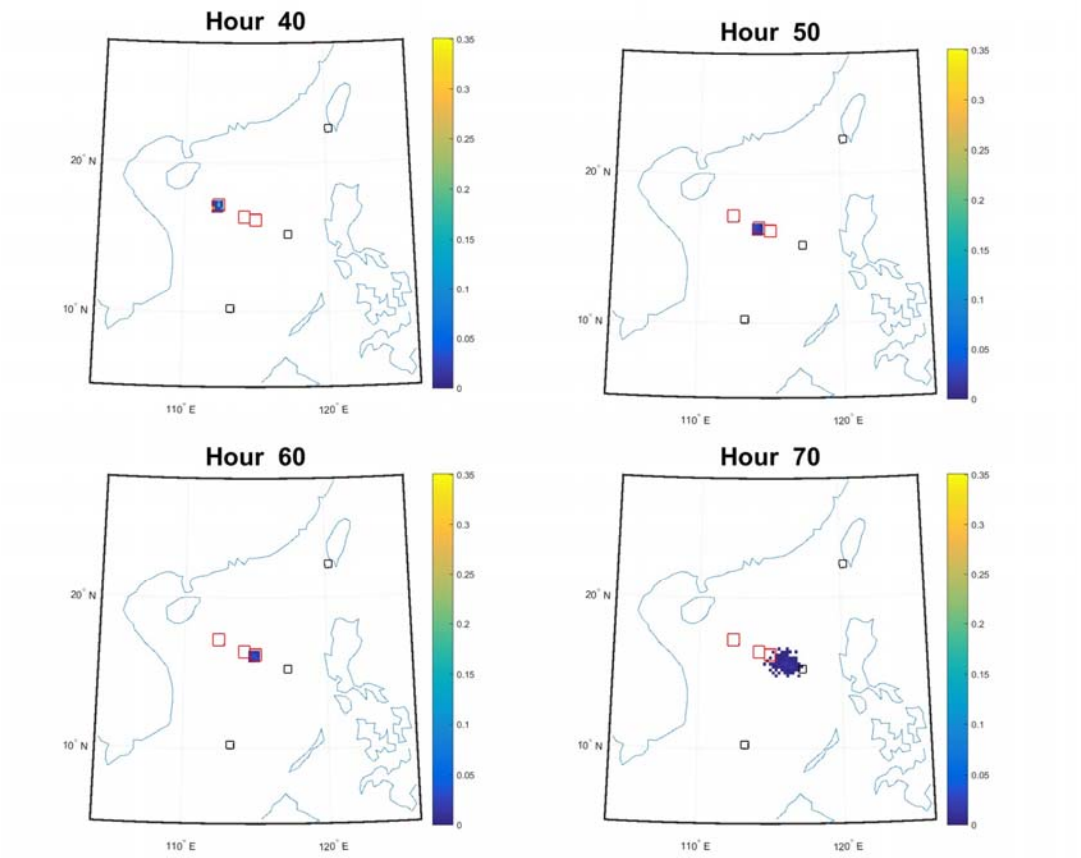

Figure 18. Third Stage of the Meta-Experiment.

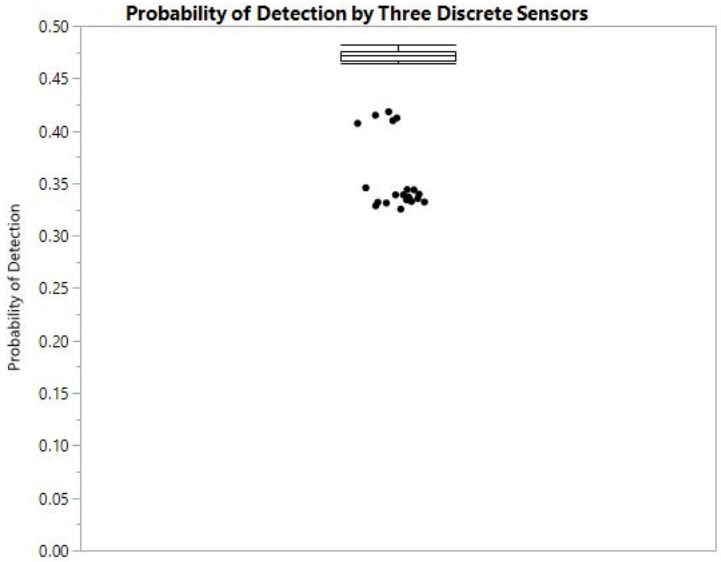

Figure 19. Boxplot of the Probability of Detection.

B. LIMITATIONS AND ASSUMPTIONS

There are some limitations in the model and algorithm, specifically in the deployment of sensors, as it is not possible to represent a web of sensors fully. The experiment currently is based on the number of discrete time sensor placements available during the target's movement, and has yet to represent a sensor deployed continuously. A continuous time sensor placement can, however, be approximated by dividing the time duration of deployment into a number of discrete looks, but it could be computationally expensive. Instead, the thesis will see each discrete time sensor placement as a different sensor that is stationary, and each sensor will only have one chance to detect a target moving across the South China Sea.

One limitation in the model is that the deployment algorithm for the sensors is based on the initial probability heat map generated without any intelligence input and the sensors are placed at the regions with the highest likelihood of locating a target. This is because the plotting of the probability heat map with intelligence inputs is computationally expensive, especially if a high number of sensors are placed. Therefore, the thesis took the simpler and faster approach to deploy the sensors without accounting for intelligence inputs. A dynamic deployment algorithm whereby sensors deployed will be based on the updated probability heat map conditional on information collected by the prior sensor could be investigated in future research.

One point about the model is that the regions with the highest probability of locating a target could be at the starting and ending points, given that there is less variability at those locations. It would therefore be intuitive to locate sensors at these points. In this model, however, we also assume that sensors should not be deployed near the starting and ending points. This is because the starting point could be under the enemy's influence and defense systems, which could take down the sensors easily. Inputs from sensors deployed near the ending points would not provide any tactical advantage since the enemy has already reached their intended location.

Another assumption that the model makes is that we are not able to cover the entire area of interest with unmanned sensors such that no areas are unmonitored at any discrete point of time. We believe that the number of sensors to achieve such a feat would be logistically challenging and such a full coverage network could be susceptible to environmental factors or enemy jamming and interference.

C. PARAMETERS FOR THE EXPERIMENT

In the design of the algorithm, the data inputs and parameters are kept separated from the model, in another file, such that a user can adjust and calibrate the model accordingly. The data parameters that can be varied are categorized into two sections; the first section contains the information on the target and the second section contains the attributes of the sensors. The key information required for the target includes the starting and ending points, velocity of travel and probability weightings on each path. Parameters that will affect the Brownian bridges are the variance of the Brownian motion and the estimated departure times of the target. These parameters will affect the variance of the Brownian bridges in two dimensions and thus affect the range of the probability heat map distribution. The data parameters required for the target are listed in Table 3. Here, we will be using parameters of a Chinese aircraft carrier, as described in Chapter II.

Table 3. Data Parameters Required for the Target.

Parameters	**Representation**
Starting and ending points	There will be uncertainty in the starting and ending points of the target.
Variance	Variance parameter of the Brownian bridges.
Weighting	Estimated probability that the target will traverse along the path. This probability could be gathered from subject matter experts or prior intelligence.
Velocity	Estimated speed of the target.
Time interval	Time range of the departure of the target from the starting point.

The data parameters for the sensors are listed in Table 4. These parameters will affect the placement of the sensors, as well as the calculation of the probability of detection.

Table 4. Data Parameters Required for the Sensors.

Parameters	**Representation**
Discrete sensor placement	Number of discrete sensors available for placement to detect the target.
Time interval	Time range for the detection by sensors. In the experiment, this is used to control the distances away from the starting and ending points.
Sensor size	The sensor detection area half-width. A high-quality sensor will have a higher range of detection capability. The units used here will be in degrees.
Detection probability	The probability of detection of the sensor. A high-quality sensor will have a higher probability of detection.
Rate of false alarm	False alarm rate of the sensor. A high-quality sensor will have a lower rate of false alarm.

To determine the parameters used for the sensor configuration, some references were made based on current unmanned aerial systems. For example, Lynx multi-mode radar provides high-resolution imagery and has a range of about 80km with an endurance of close to 48 hours (General Atomics Aeronautical, n.d.). It is utilized on the newer U.S. Army Gray Eagle unmanned aircraft system, which has a unit cost that is close to USD $7 million, and 36 of the improved versions had been purchased by the U.S. Army (Drew, 2015). Given the high costs, it would not be possible to field a large number of these unmanned sensors to collect intelligence.

In contrast, if an autonomous network systems of sensors is developed, one could expect that a large number of sensors would be required. Given the need to field a large number, the cost of each sensor would have to be cheaper, meaning that its capabilities could be less effective.

Therefore, the thesis will seek to conduct a trade-off analysis between the number of sensor placements and the size of the sensor detection region. This would allow us to determine the quantity of less effective sensors required to achieve a comparable probability of detection by the higher quality sensors, like those found in the Gray Eagle.

D. MEASURE OF EFFECTIVENESS

The measure of effectiveness used in the experiment is the probability of detection, which is calculated based on the method outlined in Chapter IV. In the following experiments, single factors are varied one at a time, to provide an initial analysis of the model. A total of 100 replications are carried out within each experiment and the mean and 95% confidence interval are recorded. Version 12 of JMP (2016) was used as the statistical tool to analyze the results.

E. VARYING THE NUMBER OF DISCRETE TIME SENSORS

The number of discrete time sensors is varied in the first experiment, with a detection range of 0.4 degrees and a detection probability of 0.8. From Figure 20, we see that the median of the probability of detection increases as the number of discrete sensors increases. We observe an increase of approximately 0.03 in the mean probability of detection between 3 and 10 sensors, and an increase of 0.04 in the probability of detection between 10 and 20 sensors, as listed in Table 5. The table includes the values of the mean, 95% confidence interval, standard deviation and standard error of the mean. While it is logical that as the number of sensors increases, the likelihood of detection would increase, from the results, the increase was not as significant as we would expect it to be. Presumably, this could be due to the placement of the sensors, which focused on the high-probability detection points, which meant that additional sensors might not detect the target that traversed on the lower probability paths. We also observed some outliers in the dataset, particularly more so when the number of discrete sensors is small. The small probability occurs when all, or almost all of the sensors report a negative sighting and this is more likely to happen when the number of discrete sensors is small. When the number of discrete sensors is high, the probability the majority of the sensors register a negative sighting is small, hence fewer outliers.

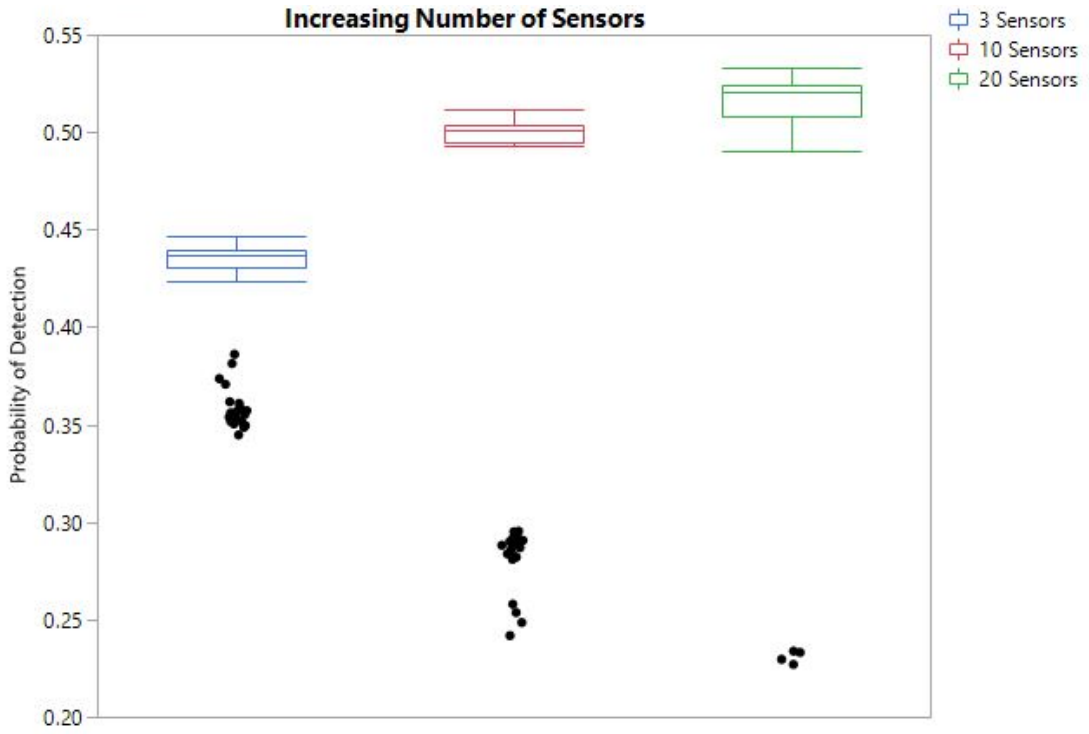

Figure 20. Boxplot of the Probability of Detection by Varying the Number of Discrete Time Sensors.

Table 5. Probability of Detection with Increasing Discrete Time Sensors.

Number of Discrete Sensors	Mean	Lower 95% CI	Upper 95% CI	Std Dev	Std Error Mean
3	0.4214	0.4148	0.4279	0.03312	0.00331
10	0.4578	0.4400	0.4755	0.08947	0.00895
20	0.5054	0.4941	0.5168	0.05737	0.00574

F. VARYING THE SENSOR DETECTION PROBABILITY

Next, the thesis varied the sensor's probability of detection using 10 discrete sensors with a size of 0.4. From Figure 21, we observe an increase in the probability of detection as the detection probability increases. Also, the variability of the results is higher when the detection probability is low, which can be observed from the size of the boxplot and the standard deviations reported in Table 6. It would seem that the sensor detection probability plays a significant role in the probability of detection, given that

there was a difference of approximately 0.15 when the sensor's detection probability is reduced from 0.9 to 0.6. This could be due to the fact that a high detection probability would mean higher chances of locating a target so it is more likely that many of the sensors deployed would indicate a positive sighting, thereby reducing the variability of the results. Additional sightings of the same target do not add to the probability of detection.

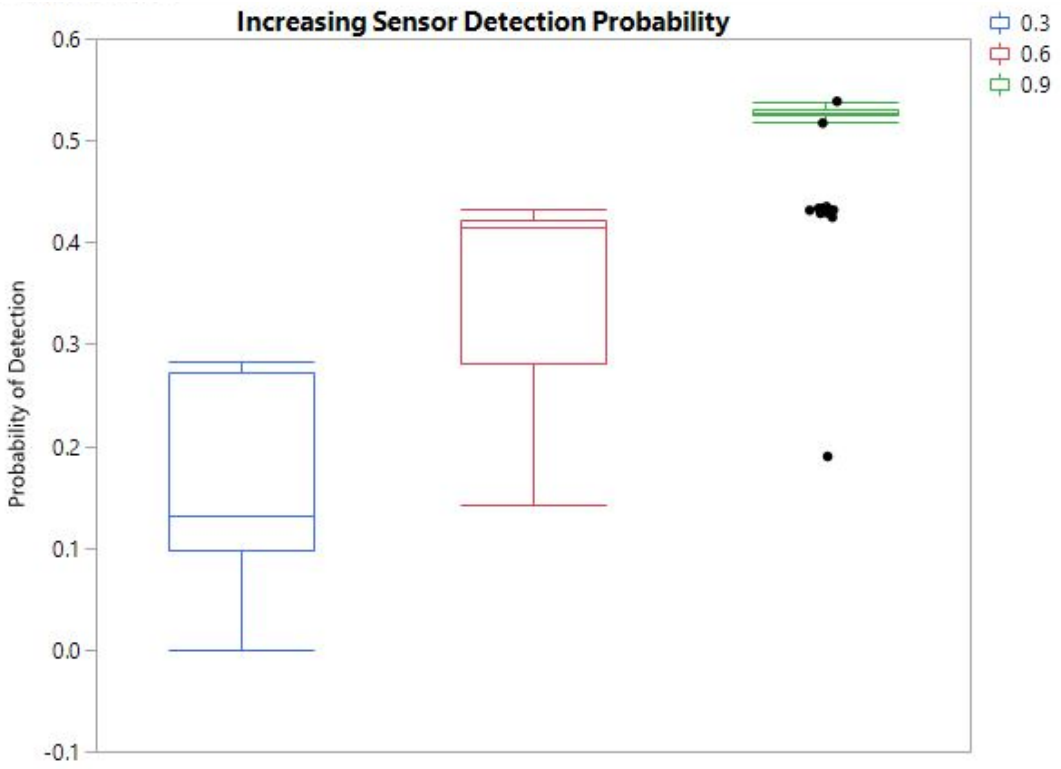

Figure 21. Boxplot of the Probability of Detection by Varying Sensor the Detection Probability.

Table 6. Probability of Detection by Increasing the Sensor Detection Probability.

Sensor Detection Probability	Mean	Lower 95% CI	Upper 95% CI	Std Dev	Std Error Mean
0.3	0.1652	0.1486	0.1818	0.08368	0.00837
0.6	0.3687	0.3529	0.3845	0.07963	0.00796
0.9	0.5137	0.5043	0.5225	0.04579	0.00458

G. VARYING THE SENSOR DETECTION REGION

In the last experiment, we varied the sensor's detection region, using 10 discrete sensors each with a detection probability of 0.9. The boxplot in Figure 22 shows an increase in the probability of detection as the detection region increases. There are a few outliers in the dataset for each case. In Table 7, we see that 10 sensors with a detection region of 0.8 (in degrees), similar to those for the Lynx multi-mode radar, can offer a probability of detection close to 0.8. If that region is reduced by half to 0.4, however, the probability of detection decreases by about 0.3. Thus, it would seem that a wider detection range can capture a higher number of Brownian bridges per sensor, thereby increasing the likelihood of locating the target.

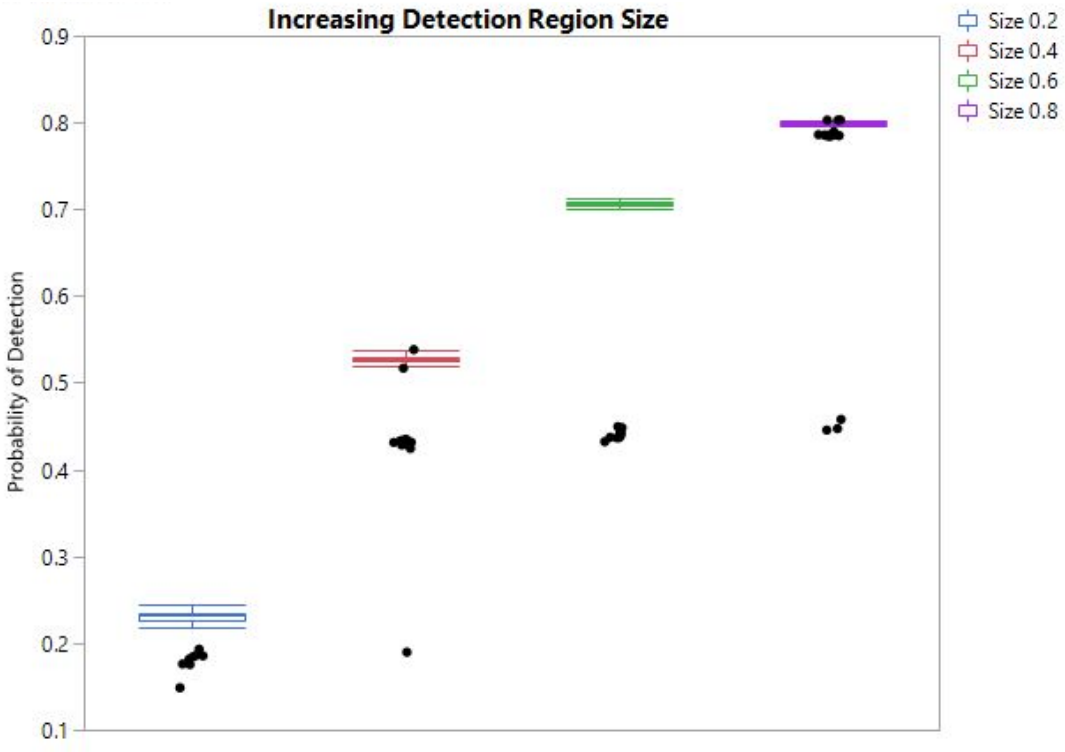

Figure 22. Boxplot of the Probability of Detection by Varying Detection Region Size.

Table 7. Probability of Detection by Increasing Detection Region Size.

Sensor Region Half Width	Mean	Lower 95% CI	Upper 95% CI	Std Dev	Std Error Mean
0.2	0.2272	0.2241	0.2303	0.0158	0.00158
0.4	0.5137	0.5043	0.5225	0.0458	0.00458
0.6	0.6832	0.6680	0.6984	0.0767	0.00767
0.8	0.7876	0.7757	0.7995	0.0598	0.00598

H. TRADE-OFF ANALYSIS

A trade-off analysis was conducted between the number of discrete sensors and the detection range, to investigate whether having a higher number of short-range unmanned sensors, like a swarm of small-sized drones, could replace the more expensive UAVs such as the Gray Eagle. The thesis varied the configuration of the parameters, as shown in Table 8, replicated the experiment 100 times with each configuration, and compared the results to those of a sensor having capabilities referenced from the Lynx multi-mode radar system. The sensors' detection probability in the configurations was kept constant at 0.8. The summary of results is shown in Table 8. In the first configuration, the mean of the probability of detection was the highest at 0.68, but it also has the highest variability, with its standard deviation at 0.168. The second configuration produces a probability of detection of 0.6, which is within one standard deviation of the first configuration. Both the third and fourth configurations have the lowest values for the probability of detection at about 0.2. From the boxplot in Figure 23, we observed a noticeable decrease in the probability of detection as the sensor width decreases, even though the number of discrete sensors is increasing.

Table 8. Probability of Detection for the Various Configurations.

Configurations	Mean	Lower 95% CI	Upper 95% CI	Std Dev	Std Error Mean
Benchmark, based on Lynx multi-mode radar and 10 discrete sensors	0.7876	0.7757	0.7995	0.0598	0.00598
Configuration 1: 15 discrete sensors, sensor width of 0.7	0.6821	0.6487	0.7155	0.1682	0.0168
Configuration 2: 20 discrete sensors, sensor width of 0.5	0.5962	0.5843	0.6080	0.0598	0.00598
Configuration 3: 30 discrete sensors, sensor width of 0.2	0.2031	0.1953	0.2109	0.0392	0.00393
Configuration 4: 125 discrete sensors, sensor width of 0.2	0.1987	0.1890	0.2084	0.0489	0.00489

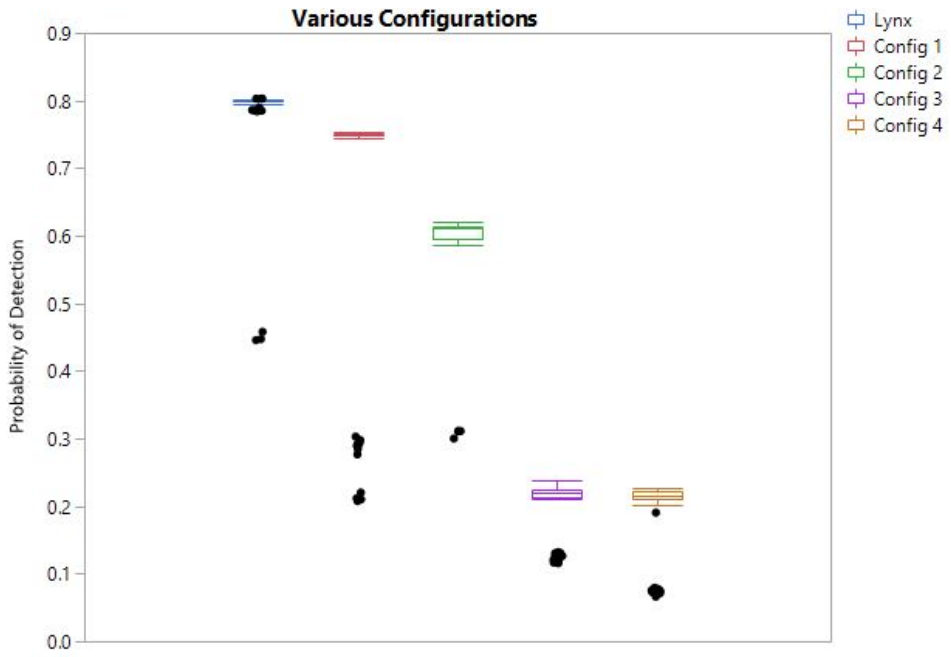

Figure 23. Boxplot of the Probability of Detection by Various Configurations.

From the results, it seems that a high-quality sensor, like that of the Lynx, is still effective in detecting a target even if the number of discrete sensors is small. As the sensor size decreases, intuitively, it is expected that a higher number of discrete sensors would be required. Looking at the third and fourth configurations, it would seem that a minimal sensor width is required, beyond which there is only a very low chance of detecting a target. This could, however, be due to the method used to estimate the probability of detection with intelligence inputs. At each sensor coverage area, only Brownian bridges that are not yet flagged will be flagged and added up. Coupled with the small sensor size and the fact that we deploy the sensors at the highest probability regions, the sensors would likely be concentrated along the most likely paths of the target. As such, these sensors would keep picking up Brownian bridges that have already been flagged, which could explain the low probability. One possible idea to avoid this issue would be to regenerate the Brownian bridges based on the intelligence inputs at each discrete time step by treating the sensor coverage area as a waypoint. Future work will develop more complex algorithms for deploying sensors that will avoid this overlap.

It is recommended that more research be done in exploring the relationship of the sensors' attributes and the probability of detection through other forms of deployment algorithms, such as random deployment or a fixed deployment scheme. It could also be followed up with a nearly-orthogonal Latin hypercube (NOLH) design of experiment to fully investigate the relationship between the attributes and the probability of detection.

I. ANALYSIS OF RESULTS

In summary, the probability of detection of a target is largely dependent on the sensor width and the number of discrete sensors. The sensors' detection probability has an impact on the variability of the probability of detection. From the trade-off analysis, it would seem that a Lynx multi-mode radar or equivalent is still effective at detecting targets even when just deploying a small number of discrete sensors.

Given the limitations of size and electrical power of an unmanned system, it could be difficult to boost the range of a sensor any further. As such, the thesis would recommend that if the sensor's detection range is small, it is perhaps cost effective to

overlap the sensors to achieve a wider detection range at a discrete time, as shown in Figure 24. This could be programmed through cooperative algorithms that are described in Lau's (2015) work. If not, it would probably be ideal to continue using unmanned vehicles with capabilities close to those of the Lynx multi-mode radar sensor for the deployment of the networked sensors, instead of using cheaper but less effective alternatives.

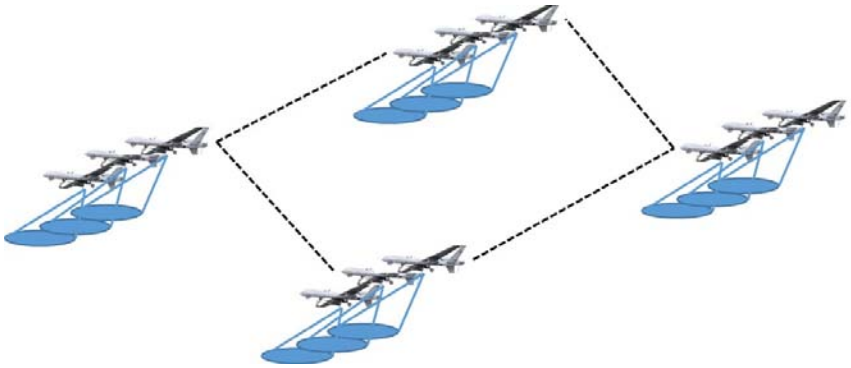

Figure 24. Overlapping Multiple Sensors to Achieve Wider Search Areas in a Networked System.

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VI. CONCLUSION AND RECOMMENDATIONS

Given that the technology of unmanned systems is advancing and maturing, we believe that it is a good opportunity to research cooperative and autonomous algorithms to deploy such sensors. This research developed one such model that we could employ to automate the placement of sensors, as part of the work to achieve a fully autonomous algorithm in a "field and forget" concept. In Chapter V, we also demonstrated the use of the model to provide an analysis of the attributes of the sensors to be deployed to improve the probability of detection.

A. DEVELOPING THE BBMM

The thesis focused on developing the BBMM to represent a possible military scenario and utilized it as a model to estimate the probability of detection. We describe modifications and extensions to the BBMM to make it useful in generating heat maps for target location given uncertain intelligence. We were able to use the BBMM to determine the locations with highest probability of detection at discrete points in time and deploy the sensors to those locations. Using the BBMM, the thesis demonstrated one approach to automate the placement of sensors, laying the foundation for future research in autonomous algorithms involving a network of sensors.

B. EMPLOYING THE BBMM TO STUDY ATTRIBUTES OF SENSORS

Using the BBMM, the thesis also went on to investigate the attributes of the sensors and their effect on the probability of detection of the target. The thesis also did a simple trade-off analysis between the number of discrete sensors and the size of the detection region, using parameters referenced from current unmanned sensors. The analysis provided an initial recommendation that such a network of sensors should exploit a wider sensor detection region at discrete points of time. The thesis also suggested a possible configuration for the deployment of such a network, which is to have wider search areas by overlapping multiple sensors.

C. FUTURE WORK

There is room for more follow-on research in two areas. The first is to continue work to extend the BBMM, such that it could account for continuous looks using sensors that are either stationary or moving. One possible extension of the model would be to look at the deployment algorithm, using a random deployment or deploying the next sensor based on the intelligence inputs from the previous sensor. This would allow us to understand the different possible types of deployment configurations and how it could affect the probability of detection as well.

The second area is to continue to investigate the effects of the attributes of the sensors on the probability of detection. This could be achieved through the design of a NOLH experiment, to ensure a more comprehensive understanding of the effects and their interactions.

With more time and research, we foresee that we can truly exploit a network of fully autonomous systems to make better decisions in the face of uncertainty in intelligence collection.

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