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Flight test campaigns benefit best from computational-based decision tools when these can operate in near real-time. This is because, for example, the actual flight test conditions often differ from the planned ones. For this reason, aeroelastic reduced-order models (ROMs), which seek the simplest models that capture the dominant dynamics of an aeroelastic system, are often advocated for supporting flight test. However, aeroelastic ROMs can be exploited in near real-time only after they have been precomputed for specific aircraft configurations and flight conditions. Furthermore, constructing an aeroelastic ROM is typically as computationally intensive as a high-fidelity nonlinear aeroelastic simulation. Therefore, an unplanned flight configuration and/or condition cannot be addressed in near real-time by the standard ROM technology. Updating aeroelastic ROMs to reflect flight test operations can solve this critical problem but is a significant challenge. Hence, the main objective of this research proposal is to develop reliable computational technologies for accelerating the computation of aeroelastic ROMs, and updating real-time computational points.

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Final Report

Parametrized Aeorelastic Reduced-Order Modeling of Fighters

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For Performance Period:

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Parameterized Aeroelastic Reduced-Order Modeling of Fighters

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ABSTRACT

Accurate predictions of aeroelastic phenomena such as flutter, limit cycle oscillations, and buffeting can play a significant role in improving the efficiency and safety of flight testing in general, and envelope expansion flight testing in particular. Several high-fidelity, nonlinear, aeroelastic computational models have already demonstrated a great potential for practical accuracy. Some of them have also been successfully validated for fighter aircraft in clean wing configurations, subsonic, transonic, and supersonic airstreams, but at low angles of attack. However, these high-fidelity, nonlinear mathematical models are computationally intensive. For example, using a 128-processor computing platform, they require on the order of one day to predict the aeroelastic parameters of a complete F-16 configuration at five different Mach numbers in the transonic regime. On the other hand, flight test campaigns benefit best from computational-based decision tools when these can operate in near real-time. This is because, for example, the actual flight test conditions often differ from the planned ones. For this reason, aeroelastic reduced-order models (ROMs), which seek the simplest models that capture the dominant dynamics of an aeroelastic system, are often advocated for supporting flight test. However, aeroelastic ROMs can be exploited in near real-time only after they have been precomputed for specific aircraft configurations and flight conditions. Furthermore, constructing an aeroelastic ROM is typically as computationally intensive as a high-fidelity nonlinear aeroelastic simulation. Therefore, an unplanned flight configuration and/or condition cannot be addressed in near real-time by the standard ROM technology either. Updating aeroelastic ROMs to reflect flight test operations can solve this critical problem but is a significant challenge. Hence, the main objective of this research proposal is to develop reliable computational technologies for accelerating the computation of aeroelastic ROMs, and updating them in near real-time to different operational points. The proposed computational technologies combine reconstruction algorithms based on higher-order sensitivities and Padé approximants, effective parameterizations for mass distribution, structural stiffness, altitude, Mach number and angles of attack, and fast algebraic solvers. The anticipated outcome of this research effort is a set of validated, state-of-the-art, aeroelastic ROM computational technologies which are expected to reduce the number or ROMs to be generated and imported in the control room, and increase both flight test efficiency and safety.

1 OBJECTIVES

The objectives of this research proposal are: (a) to accelerate the construction of Proper Orthogonal Decomposition (POD)-based aeroelastic reduced-order models (ROMs) for specified aircraft configuration and

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flight conditions, (b) to develop accurate and robust computational methodologies for updating precomputed aeroelastic ROMs to reflect flight test operations, and (c) if time permits, to collaborate with the Flight Test Center at the Edwards Air Force Base on developing and testing a path for integrating these computational methodologies into a manned flight simulator.

To this effect, the following research goals and corresponding statement of work are formulated.

1.1 RESEARCH GOALS

1) *Fast generation of high-fidelity computational snapshots.* It is well-known that the CPU time required for constructing a POD-based aeroelastic ROM at a specified operational point is dominated by the CPU time required for generating the snapshot solutions of the underlying fluid dynamic equations of equilibrium. Hence, the first research goal is to accelerate this process by an order of magnitude in order to make aeroelastic ROMs intrinsically more appealing for supporting flight test.

2) *Parameterization of operational mass and structural stiffness.* Fuel and other payloads such as launchers, missiles and stores are the most important sources of mass and stiffness variations in flight test operations. The second research goal is to identify, design, and implement effective parameterizations of these quantities that are most suitable for computing higher-order sensitivities of the snapshots to operational mass and to structural stiffness.

3) *Fast adaptation of aeroelastic ROMs.* Changes in one or several operating aircraft configurations and/or flight test conditions such as mass distribution, structural stiffness, Mach number, or angle of attack often necessitate the reconstruction of an aeroelastic ROM in order to maintain accuracy. Unfortunately, this destroys the sought-after near real-time operation. Straightforward approaches for ROM adaptation, including global projection methods and direct interpolation of reduced-basis vectors, have been attempted in the past; however, these approaches have also been shown to produce inaccurate results, particularly in the transonic flight regime. Hence, the third research goal is to develop new approaches for updating aeroelastic ROMs in near real-time to different operational points.

4) *Integration of parameterized aeroelastic ROMs into a manned flight simulator.* Finally, if time permits, the fourth research goal is to investigate, in collaboration with the Flight Test Center at the Edwards Air Force Base, a path for integrating the parameterized aeroelastic ROM technologies into a manned flight simulator.

1.2 STATEMENT OF WORK

Computational methods and corresponding numerical algorithms will be developed for accelerating the construction of Proper Orthogonal Decomposition (POD)-based aeroelastic reduced-order models (ROMs) at given operational points, and adapting them in near real-time to different operational conditions. Therefore, the expected outcome of this research effort is the development of parameterized computational aeroelastic ROMs that can be effectively used by flight test engineers as computational-based decision tools in order to increase both flight test efficiency and safety.

The generation of POD snapshots for training a fluid ROM is the most computationally intensive step of the construction of an aeroelastic ROM. When performed in the frequency domain, this step implies the solution of multiple linearized systems of equations, each associated with a different frequency. To accelerate this training process, an alternative computational methodology will be developed as follows. First, the repeated derivatives with respect to frequency of the snapshot solutions will be characterized as the solutions of a sequence of linearized (algebraic) problems that share the *same* operator (left-hand side), but differ in the excitation terms (right-hand side). Next, a fast algorithm will be designed to solve this sequence of algebraic problems at essentially the computational cost of solving a system of algebraic equations associated with a single frequency. Then, using these derivatives, Padé approximants or Wynn's algorithm will be developed to reconstruct the snapshot solutions at the other frequencies.

Two computational strategies for updating in near real-time aeroelastic ROMs to different operational points will also be developed. The first one will address mass distribution and structural stiffness parameters, and the second one will treat aerodynamic parameters. In the first strategy, mass distribution parameters will focus on the fuel system and on the external stores, the wing tip launchers and the sidewinders, as these are the most important mass parameters for the Flight Test Center at the Edwards Air Force Base. The structural stiffness parameters will mainly address the wing tip launchers and the sidewinders for a similar reason, and because these secondary systems can significantly affect the torsional stiffness of the wings. Fast and robust semi-analytical numerical algorithms will be developed for evaluating the higher-order sensitivities of the snapshots of the aeroelastic solutions computed at nominal values of these mass and structural stiffness parameters. Again, reconstruction algorithms based on Padé approximants will also be designed for exploiting in near real-time these precomputed sensitivities to update an aeroelastic ROM to different operational points. The second computational strategy will address variations in the altitude, Mach number, and angles of attack. For these parameters, the subspace angle approach originated by the proposers and a collaborator will be pursued and combined with reconstruction techniques similar to the ones outlined above to update aeroelastic ROMs in near real-time to variations in these parameters.

Finally, if time permits, a path for integrating the computational aeroelastic ROM technologies described above into a manned flight simulator will be developed and demonstrated in collaboration with the Flight Test Center at the Edwards Air Force Base.

2 TECHNICAL PROPOSAL

2.1 RESEARCH EFFORT

2.1.1 Introduction

2.1.1.1 Computational-Based Aeroelastic Decision Tools for Flutter Flight Test

Traditionally, computational aeroelasticity for loads and flutter prediction combines a linear finite element formulation for the structure with linear aerodynamic methods. At the same time, the prediction of aerodynamic performance and control surface effectiveness accounts for the effects of the structural elastic deformations on the external aerodynamics by means of correction factors applied to the results obtained when the aircraft is assumed to be rigid. Both practices are well-established in aircraft design and give accurate, reliable, and rather inexpensive predictions for static and dynamic effects at subsonic and supersonic speeds. In the transonic flight regime and for rapid maneuvering conditions where nonlinear aerodynamic effects may become non negligible, the aircraft design, development, and certification processes rely today on expensive flutter and aerodynamic experimental models and on extensive flight testing. Adoption of innovative, unconventional designs, and aerodynamically unstable configurations in modern aircraft exacerbates the presence and impact of aerodynamic nonlinearities. A possible consequence of the inaccurate prediction of aerodynamic loads involving nonlinear phenomena — such as shocks, vortices and separated flows — flutter, limit cycle oscillations (LCO), and other adverse aeroelastic effects remain unveiled until the flight tests. Unfortunately, flight tests are expensive, can be dangerous, and have already cost many pilot lives.

Perhaps for these reasons, a leading aeroelastician at Boeing's Phantom Works wrote in 2001 *"The results of a finite number of* [nonlinear] CFD [Computational Fluid Dynamics] *solutions could be used as a replacement for wind tunnel testing, assuming a validated code was available"* [1], and *"Even at present, existing CFD codes should be able to obtain five flutter solutions in one year"* [1]. Indeed, state-of-the-art CFD-based nonlinear aeroelastic simulation technologies [2–5] exemplified by the AERO code [2, 3] have recently become a superior choice over linear computational methods [6, 7], and a viable complement or alternative to scaled wind-tunnel testing for many types of aeroelastic analyses such as flutter prediction.

Flight testing can also benefit from aeroelastic numerical simulators in many ways; for example, in planning and reducing the number of sorties, anticipating critical points, and expanding flutter envelopes. However, for several practical reasons including the fact that actual flight test conditions often differ from planned ones, advanced aeroelastic analysis tools need not only be validated, but must also operate in near real-time to be accepted by flight test engineers as computational-based decision tools. Unfortunately, while some CFD-based aeroelastic simulators such as AERO have demonstrated a significant potential for predicting accurately transonic flutter, LCO, and other nonlinear aeroelastic phenomena, none of them can operate today in near real-time on high-fidelity, full-order, nonlinear computational models. For example, the AERO code developed by the proposers and their collaborators was reported in [2, 3] to accurately predict the aeroelastic parameters of a complete F-16 configuration at five different Mach numbers in the transonic regime. However, even when executed on a 128-processor system, this code requires about half-a-day to deliver the aforementioned numerical results.

2.1.1.2 Aeroelastic Reduced-Order Modeling

The major computational cost incurred by CFD-based nonlinear aeroelastic simulations is attributable to the need for high-fidelity fluid models in order to resolve the complex flow patterns that are present in the transonic regime. Because of this computational cost, the potential of CFD-based nonlinear aeroelastic codes is currently limited to the analysis of a few, carefully chosen configurations, rather than routine analysis. It is possible however to address this limitation by constructing CFD-based aeroelastic reduced-order models (ROMs) that are simple enough to *operate* in near real-time, and yet sufficiently accurate to capture the dominant aeroelastic behavior of a given system. For example, it was recently shown in [8–19] that fluid and/or aeroelastic ROMs constructed by a variety of methods, including the popular Proper Orthogonal Decomposition (POD) method [20, 21], can produce numerical results that compare well with those generated by full-order nonlinear counterparts. In particular, the POD method has been successfully applied to simple airfoils [9–12], panels [22], wings [13, 15, 16], turbine blades [17, 18], and most recently to complete aircraft configurations [23]. However, the *construction* of a CFD-based ROM is not a task that can be accomplished today in near real-time. For example, it was shown in [33] that for an F-16 aircraft, constructing a POD-based aeroelastic ROM that is suitable for flutter predictions in the transonic regime is as computationally intensive as simulating the first half-a-second of the transient response of this aircraft to specified initial conditions using a full-order, nonlinear, CFD-based computational model. Furthermore, a ROM constructed by POD or any other similar technique is usually not robust with respect to change in a model parameter [10, 24]. Hence, even amortizing the cost of a precomputed, CFD-based, aeroelastic ROM is technically challenging. Most importantly, for flight test operations, a method for the near real-time adaptation of precomputed ROMs to changing flight conditions is needed in order to avoid the unaffordable overhead cost associated with ROM reconstruction. Some progress in this area has been recently reported for the case of structural parameters [15]. However, with few exceptions [10, 16, 23, 25], little has been reported for changes in the free-stream Mach number and angles of attack. In any case, the topic of near real-time adaptation of aeroelastic ROMs is still in its infancy. Further research in this area is needed before CFD-based aeroelastic ROMs can address the technical challenges faced by flight test centers.

2.1.1.3 Unique and Proven Aeroelastic High-Fidelity and ROM Capabilities

The Principal Investigator (PI) is the lead developer of the AERO simulation platform [2, 3] which is considered by many to be the state-of-the-art of coupled, nonlinear aeroelastic simulation tools. This code is currently used at the Edwards Air Force Base, Lockheed-Martin Aeronautics, the Naval Research Laboratory, the Sandia National Laboratories, and several other institutions for applications ranging from the parametric identification of modern fighters to the design of dynamic data-driven systems for submarine applications. AERO consists of three main modules, namely, AERO-F, AERO-S, and MATCHER, and a suite of drivers for optimization and vehicle performance analysis. AERO-F is a domain-decomposition-based, massively parallel, three-dimensional, arbitrary Lagrangian-Eulerian (ALE), implicit, Navier-Stokes compressible flow solver. It features a combination of second-order finite volume and finite element discretization and sixth-order numerical dissipation methods on unstructured tetrahedral meshes. It performs turbulence modeling by solving either the one-equation Spalart-Allmaras model or the two-equation k-ε model, and can couple either of them with a wall function and Spalding's wall boundary condition for the eddy viscosity. It performs large eddy simulations using computationally efficient, VMS-LES and dynamic VMS-LES methods on unstructured and dynamic grids [26, 27]. It is also capable of Detached Eddy Simulations (DES) on moving grids. In the low-speed limit, it resorts to preconditioning to overcome the usual numerical difficulties encountered in this case by compressible flow solvers. For time-integration, AERO-F is equipped with an ALE version of the three-point backward-difference implicit scheme that satisfies its discrete geometric conservation law, and is proven to be stable and second-order time-accurate on moving grids. It supports two robust structure-analogy methods for constructing dynamic meshes. The first one is based on time-dependent torsional springs, and the second on the total Lagrangian approach for solving a fictitious nonlinear elasticity problem. For applications such as maneuvering where the structure can undergo large displacements and rotations, AERO-F relies on a corotational scheme for accelerating the update of the mesh motion [28].

AERO-F also embeds AERO-FL, a module for computing linearized flow perturbations around a specified equilibrium solution, predicting linearized aeroelastic responses assuming a modalized structure, generating snapshots for constructing a POD basis, generating an aeroelastic ROM in the frequency domain, and computing aeroelastic ROM solutions in the time-domain assuming a modalized structure.

AERO-S is a massively parallel structural Lagrangian code capable of linear as well as geometrically nonlinear static, sensitivity, eigenvalue, and transient finite element analyses of restrained and unrestrained homogeneous and composite structures. Control surfaces, sensors, actuators and simple propulsion systems can be easily modeled in AERO-S in order to simulate realistic maneuvers.

AERO-F and AERO-S are loosely coupled by a state-of-the-art staggered solution procedure that was recently proved to be formally second-order time-accurate and numerically demonstrated to be stable [29]. In this procedure, AERO-F and AERO-S communicate via run-time software channels. They exchange aerodynamic and elastodynamic data across non-matching fluid and structure mesh interfaces using data structures generated by the preprocessor MATCHER [30, 31]. Such exchanges are governed by a conservative algorithm for discretizing the transmission conditions at the fluid-structure interface.

AERO was initially validated with the flutter analysis of the AGARD Wing 445.6 [32]. Next, it was validated with the parametric identification of a complete F-16 Block 40 configuration in various free streams and at various angles of attack [2, 3], and the prediction of the limit-cycle-oscillation of a complete F-18 configuration. More recently, the aeroelastic ROM capability of AERO was also validated using the same complete F-16 configuration. In all cases, AERO produced aeroelastic simulation results that correlate reasonably well with flight test data. Sample validation results for the F-16 aircraft are summarized in Fig. 1, where the information displayed between parentheses corresponds to the number of unknowns associated with the adopted computational model.

2.1.2 Research Plan

2.1.2.1 Scope and Approach

The computational methodologies to be developed and validated under the proposed research effort will focus mainly on the transonic flutter problem.

Variations in the mass distribution of a given aircraft will be assumed to be either induced by variations in the amount of on-board fuel, or by changes or downloads of external stores, and/or wing tip launchers,

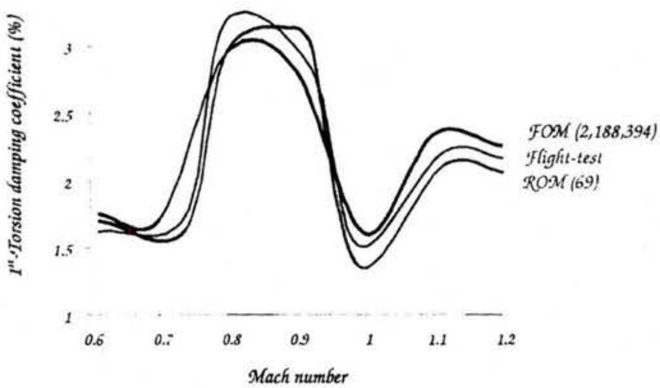

Figure 1: Variation with the free-stream Mach number of the damping ratio coefficient associated with the first torsional mode of an F-16 Block 40 aircraft: comparison of the results generated by AERO's aeroelastic ROM and full-order nonlinear computational model (FOM), with the counterparts obtained from flight test data

and/or sidewinders. Similarly, changes in the structural stiffness of a given aircraft will be assumed to be due to changes or downloads of wing tip launchers and/or sidewinders. These scenarios are chosen because of their relevance to the activities of the Flight Test Center at the Edwards Air Force Base.

Throughout the proposed research project, a given nonlinear aeroelastic system will be represented by the three-field Arbitrary Lagrangian-Eulerian (ALE) formulation [34]. After semi-discretization by a finite element (FE) or finite volume method, this formulation gives rise to three coupled ordinary differential equations

$$\left(\mathbf{A}(\mathbf{x})\mathbf{w}\right)_{,t} + \mathbf{F}(\mathbf{w},\mathbf{x},\dot{\mathbf{x}}) = 0 \tag{1}$$

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{f}^{int}(\mathbf{u},\dot{\mathbf{u}}) = \mathbf{f}^{ext}(\mathbf{u},\mathbf{w}) \tag{2}$$

$$\tilde{\mathbf{K}}\mathbf{x} = \tilde{\mathbf{K}}_c\mathbf{u} \tag{3}$$

Here, equation (1) represents a finite volume discretization of the ALE conservation form of the fluid equations: $\mathbf{A}$ denotes the diagonal matrix of cell volumes, $\mathbf{F}$ the nonlinear numerical flux function, and $\mathbf{w}$ the conservative state vector of the fluid subsystem. Equation (2) is a FE discretization of the structural equations of dynamic equilibrium: $\mathbf{M}$ denotes the FE mass matrix, $\mathbf{f}^{int}$ the vector of internal forces, $\mathbf{f}^{ext}$ the vector of external forces applied on the structure, and $\mathbf{u}$ the structural displacement vector. Both of the $,t$ and dot notations represent a partial derivative with respect to time. The various Dirichlet and Neumann boundary conditions associated with the fluid and structural subproblems are embedded in the above system and, for simplicity, are not explicitly stated. In many aeroelastic applications, the boundary of the fluid domain is required to deform according to the motion of the wet surface of the structure. This is represented here by equation (3) which models the fluid mesh as a pseudo-structure with a piece-wise static behavior [37]: $\mathbf{x}$ denotes the fluid mesh motion, $\tilde{\mathbf{K}}$ a fictitious stiffness matrix, and $\tilde{\mathbf{K}}_c$ a transfer matrix describing the effect of structural motions on the fluid mesh at the fluid-structure interface [31].

For the purpose of constructing a ROM, the above formulation of an aeroelastic problem is simplified as follows.

First, both the fluid and structural equations are linearized around an equilibrium point designated by the subscript o and satisfying $\dot{\mathbf{w}}_o = \dot{\mathbf{x}}_o = 0$. The fluid equation is also adimensionalized. The purpose of the adimensionalization is to remove from the resulting ROM any dependence on the dimensional free-stream pressure and density. Thus, the steady state flow solution will only have a dependence on the free-stream Mach number, M_∞, and angles of attack.

The linearization of the fluid subsystem adopted for the proposed research effort follows the development described first in references [35, 38], where the system in equation (1) is perturbed about an equilibrium configuration $(\mathbf{w}_o, \dot{\mathbf{w}}_o, \mathbf{x}, \dot{\mathbf{x}}_o)$ so that

$$\begin{aligned} \mathbf{w}(M_\infty, \alpha) &= \mathbf{w}_o(M_\infty, \alpha) + \delta\mathbf{w}, & \dot{\mathbf{w}}(M_\infty, \alpha) &= \dot{\mathbf{w}}_o(M_\infty, \alpha) + \delta\dot{\mathbf{w}} \\ \mathbf{x}(M_\infty, \alpha) &= \mathbf{x}_o(M_\infty, \alpha) + \delta\mathbf{x}, & \dot{\mathbf{x}}(M_\infty, \alpha) &= \dot{\mathbf{x}}_o(M_\infty, \alpha) + \delta\dot{\mathbf{x}} \end{aligned}$$

Then, the resulting linearized system is adimensionalized [23], to make it dependent only on the free-stream Mach number and angles of attack. Equation (1) is thus transformed into

$$\bar{\mathbf{A}}_o(\delta\bar{\mathbf{w}})_{,\tau} + \bar{\mathbf{H}}_o\delta\bar{\mathbf{w}} + (\bar{\mathbf{E}}_o + \bar{\mathbf{C}}_o)\delta\dot{\bar{\mathbf{x}}} + \bar{\mathbf{G}}_o\delta\bar{\mathbf{x}} = 0 \tag{4}$$

where

$$\begin{aligned} \bar{\mathbf{A}}_o &= \bar{\mathbf{A}}(\bar{\mathbf{x}}_o), \quad \bar{\mathbf{H}}_o = \frac{\partial\bar{\mathbf{F}}}{\partial\bar{\mathbf{w}}}(\bar{\mathbf{w}}_o, \bar{\mathbf{x}}_o, \dot{\bar{\mathbf{x}}}_o) \\ \bar{\mathbf{E}}_o = \frac{\partial\bar{\mathbf{A}}}{\partial\bar{\mathbf{x}}}(\mathbf{x}_o)\bar{\mathbf{w}}_o, \quad \bar{\mathbf{C}}_o &= \frac{\partial\bar{\mathbf{F}}}{\partial\dot{\bar{\mathbf{x}}}}(\bar{\mathbf{w}}_o, \bar{\mathbf{x}}_o, \dot{\bar{\mathbf{x}}}_o), \quad \bar{\mathbf{G}}_o = \frac{\partial\bar{\mathbf{F}}}{\partial\bar{\mathbf{x}}}(\bar{\mathbf{w}}_o, \bar{\mathbf{x}}_o, \dot{\bar{\mathbf{x}}}_o) \end{aligned}$$

The matrices $\bar{\mathbf{H}}_o$, $\bar{\mathbf{G}}_o$, $\bar{\mathbf{E}}_o$, and $\bar{\mathbf{C}}_o$ are the first-order terms of a Taylor expansion of the adimensionalized numerical flux function around the adimensionalized operating point ($\bar{\mathbf{w}}_o$, $\bar{\mathbf{x}}_o$, $\dot{\bar{\mathbf{x}}}_o$). The bar notation indicates that a quantity is an adimensionalized one. The matrix $\bar{\mathbf{H}}_o$ is the gradient of the adimensionalized numerical flux function with respect to the adimensionalized fluid state vector and thus has in general a rank equal to the number of fluid degrees of freedom (dofs). The coupling matrices $\bar{\mathbf{G}}_o$ and $\bar{\mathbf{C}}_o$ are gradients of the adimensionalized flux function with respect to the adimensionalized fluid mesh motion. The coupling matrix $\bar{\mathbf{E}}_o$ is the gradient of the adimensionalized cell volumes with respect to the adimensionalized fluid mesh motion. Finally, the $,\tau$ notation denotes a partial derivative with respect to the adimensionalized time. Likewise, a dot notation in the context of a "bar" quantity also indicates a partial derivative with respect to the adimensionalized time.

The linearization of the structural subsystem is accomplished similarly by perturbing the system in equation (2) around an equilibrium state, which leads to

$$\mathbf{M}\delta\ddot{\mathbf{u}} + \mathbf{D}_o\delta\dot{\mathbf{u}} + \mathbf{K}_s\delta\mathbf{u} = \mathbf{P}_o\delta\bar{\mathbf{w}} \tag{5}$$

where

$$\begin{aligned} \mathbf{K}_o &= \frac{\partial\mathbf{f}^{int}}{\partial\mathbf{u}}(\mathbf{u}_o, \dot{\mathbf{u}}_o), \quad \mathbf{K}_s = \mathbf{K}_o - \frac{\partial\mathbf{f}^{ext}}{\partial\mathbf{u}}(\mathbf{w}_o, \mathbf{u}_o) \\ \mathbf{D}_o &= \frac{\partial\mathbf{f}^{int}}{\partial\dot{\mathbf{u}}}(\mathbf{u}_o, \dot{\mathbf{u}}_o), \quad \mathbf{P}_o = \frac{\partial\mathbf{f}^{ext}}{\partial\bar{\mathbf{w}}}(\mathbf{w}_o, \mathbf{u}_o) \end{aligned}$$

The matrix $\mathbf{K}_o$ denotes the structural stiffness matrix at the operating point $\mathbf{u}_o$, $\mathbf{K}_s$ an adjusted structural stiffness matrix at the same equilibrium point, and $\mathbf{P}_o$ arises from the linearization of the external loading with respect to the adimensionalized fluid state vector.

To keep the notation as compact as possible, the subscript o and the prefix δ are dropped in the remainder of this proposal. The same variables $\bar{\mathbf{w}}$, $\bar{\mathbf{x}}$, and $\mathbf{u}$ are used to denote the perturbations of the fluid state, mesh motion, and structural motion vectors, respectively, around the chosen equilibrium point.

The fluid mesh position and velocity variables $\bar{\mathbf{x}}$ and $\dot{\bar{\mathbf{x}}}$ are eliminated from the coupled system of linearized equations by introducing

$$\bar{\mathbf{K}} = \tilde{\mathbf{K}}^{-1}\tilde{\mathbf{K}}_c \tag{6}$$

so that

$$\bar{\mathbf{x}} = \bar{\mathbf{K}}\bar{\mathbf{u}}, \quad \dot{\bar{\mathbf{x}}} = \bar{\mathbf{K}}\dot{\bar{\mathbf{u}}} \tag{7}$$

where $\bar{\mathbf{u}}$ is the adimensionalized displacement of the structure measured with respect to its equilibrium configuration. The above algebraic manipulations allow re-writing equation (4) as

$$\bar{\mathbf{A}}\bar{\mathbf{w}}_{,\tau} + \bar{\mathbf{H}}\bar{\mathbf{w}} + (\bar{\mathbf{E}} + \bar{\mathbf{C}})\bar{\mathbf{K}}\dot{\bar{\mathbf{u}}} + \bar{\mathbf{G}}\bar{\mathbf{K}}\bar{\mathbf{u}} = 0 \tag{8}$$

Next, neglecting the effect of $\dfrac{\partial \mathbf{f}^{ext}}{\partial \mathbf{u}}|_o$ and $\mathbf{D}_o$ — that is, assuming that $\mathbf{K}_s \approx \mathbf{K}_o$ and zero damping, equation (5) is projected on a basis of dry, natural, structural modes and therefore is transformed into

$$\mathbf{I}\ddot{\mathbf{u}}_m + \mathbf{\Omega}^2\mathbf{u}_m = \mathbf{P}_m\bar{\mathbf{w}} \tag{9}$$

where $\mathbf{u}_m$ is the generalized (modal) displacement coordinates of the structure, $\mathbf{\Omega}$ is the diagonal matrix of the squared natural pulsations of the structure, $\mathbf{P}_m$ is the generalized external force matrix, and $\mathbf{I}$ is the identity matrix. The use of a modal basis to represent the structure accomplishes two important goals. First, it reduces the number of dofs for the structure, thus contributing to a more compact aeroelastic ROM. Second, it reduces the size of the coupling matrices in the linearized fluid equation (8).

Equation (8) and equation (9) define the linearized aeroelastic formulation that will be adopted in the proposed research effort.

The computational expense associated with the solution of the linearized coupled system of equations (8,9) can be expected to be less than that associated with the solution of its nonlinear counterpart; however, this expense can still be considerable because of the size of the discrete fluid subsystem. The aim of a POD-based ROM, such as the one overviewed below, is therefore to address the latter issue.

POD is a method that provides a basis for representing a given data set from which a lower-dimensional subspace can be identified. When the given data set is, in some way, representative of a physical system, the resulting reduced basis can be deemed a low-order model of the original full-order model representing that system. The theory and application of POD is covered in many publications [15, 39–41]. To keep this research proposal as self-contained as possible, the POD procedure that will be used within the aeroelastic computational framework outlined above is summarized below. (It is noted that this procedure is similar in spirit to the work presented in [11, 12]).

1. *Generate complex-valued snapshot solutions of equation* (8) *in the frequency domain for a varying reduced frequency* k

$$\bar{\mathbf{w}}_j(k) = (ik\bar{\mathbf{A}} + \bar{\mathbf{H}})^{-1}\left(ik(\bar{\mathbf{E}} + \bar{\mathbf{C}}) + \bar{\mathbf{G}}\right)\bar{\mathbf{K}}\bar{\mathbf{u}}_j \tag{10}$$

Equation (10) above is obtained by assuming a periodic solution of the form $\bar{\mathbf{w}} = \bar{\mathbf{w}}_j e^{ik\tau}$ and a periodic excitation of the form $\bar{\mathbf{u}} = \bar{\mathbf{u}}_j e^{ik\tau}$, where $\bar{\mathbf{u}}_j$ is a prescribed structural displacement field and $i = \sqrt{-1}$ is the imaginary number. For each specified value of $\bar{\mathbf{u}}_j$, a sweep is performed on the reduced frequency k and several snapshots $\bar{\mathbf{w}}_j(k)$ are generated. The reduced frequency k is defined here as the product of the angular frequency, ω, and the ratio of a reference length, L_r, and reference velocity, v_r, as stated below

$$k = \frac{L_r}{v_r}\omega \tag{11}$$

Typically, $\bar{\mathbf{u}}_j$ is chosen as a dry natural mode of the structure and therefore the total number of generated snapshots is equal to the product of the number of excitation modes and the number of considered reduced frequencies.

2. *Form the real-valued correlation matrix*

$$\mathbf{R} = \mathbf{S}\mathbf{S}^T \tag{12}$$

where the superscript T designates the transpose,

$$\mathbf{S} = \left[Re(\bar{\mathbf{W}}) \quad Im(\bar{\mathbf{W}})\right] \tag{13}$$

and each column of $\bar{\mathbf{W}}$ contains a complex-valued snapshot of the form given in (10).

3. *Compute the eigenvalues and eigenvectors of the correlation matrix* $\mathbf{R}$. However, since all non-zero eigenvalues of the matrix $\mathbf{S}^T\mathbf{S}$ are also eigenvalues of the correlation matrix $\mathbf{R}$, and the size of $\mathbf{S}^T\mathbf{S}$ is significantly smaller than that of $\mathbf{R}$, it is more attractive to replace this step by the solution of the alternative generalized eigenvalue problem $\mathbf{S}^T\mathbf{S}\mathbf{\Psi} = \mathbf{\Psi}\Lambda$ [36]. Then, form the following POD basis

$$\mathbf{\Phi} = \mathbf{S}\mathbf{\Psi}\Lambda^{-\frac{1}{2}} \tag{14}$$

which satisfies

$$\mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{I} \tag{15}$$

4. *Form a truncated POD-basis, $\mathbf{\Phi}_r$, by reducing the size of the matrix $\mathbf{\Phi}$ to a few r columns.* Usually, the magnitude of the eigenvalue value associated with a column of $\mathbf{\Psi}$ (and therefore $\mathbf{\Phi}$) is used as a criterion for deciding which POD vector to retain and which to discard.

5. *Project the snapshots on the truncated POD basis*

$$\bar{\mathbf{w}} \approx \mathbf{\Phi}_r\bar{\mathbf{w}}_r \leftarrow \bar{\mathbf{w}}_r = \mathbf{\Phi}_r^T\bar{\mathbf{w}} \tag{16}$$

6. *Project the governing fluid equation* (8) *onto the POD basis.* This step leads to

$$(\bar{\mathbf{w}}_r)_{,\tau} = -\mathbf{\Phi}_r^T\bar{\mathbf{A}}^{-1}\bar{\mathbf{H}}\mathbf{\Phi}_r\bar{\mathbf{w}}_r - \mathbf{\Phi}_r^T\bar{\mathbf{A}}^{-1}\left((\bar{\mathbf{E}}+\bar{\mathbf{C}})\bar{\mathbf{K}}\dot{\bar{\mathbf{u}}} + \bar{\mathbf{G}}\bar{\mathbf{K}}\bar{\mathbf{u}}\right) \tag{17}$$

which is re-written here as

$$(\bar{\mathbf{w}}_r)_{,\tau} = \bar{\mathbf{H}}_r\bar{\mathbf{w}} - \bar{\mathbf{B}}_r\bar{\mathbf{y}}_r \tag{18}$$

where

$$\begin{aligned}
\bar{\mathbf{H}}_r &= -\mathbf{\Phi}_r^T(\bar{\mathbf{A}}^{-1}\bar{\mathbf{H}})\mathbf{\Phi}_r \\
\bar{\mathbf{B}}_r &= \mathbf{\Phi}_r^T\bar{\mathbf{A}}^{-1}\left[(\bar{\mathbf{E}}+\bar{\mathbf{C}})\bar{\mathbf{K}} \quad \bar{\mathbf{G}}\bar{\mathbf{K}}\right] \\
\bar{\mathbf{y}}_r &= \begin{bmatrix}\dot{\bar{\mathbf{u}}} \\ \bar{\mathbf{u}}\end{bmatrix}
\end{aligned}$$

Depending on the size of the truncated POD basis defined in step 4, the adimensionalized reduced-order fluid state vector, $\bar{\mathbf{w}}_r$, the fluid subsystem matrix $\bar{\mathbf{H}}_r$, and the coupling matrix $\bar{\mathbf{B}}_r$ can be significantly smaller than their full-order counterparts.

In summary, the POD process outlined above leads to a reduced basis that can be used for constructing a fluid ROM for a specified free-stream Mach number and specified angles of attack. The corresponding aeroelastic ROM is obtained by coupling equation (18) with equation (9). The fluid ROM, and therefore the aeroelastic ROM, may be used for computing flows at the specified free-stream Mach number, but for variable free-stream pressure and density and therefore for a *variable altitude*.

The overall approach outlined above for constructing a POD-based aeroelastic ROM is available in the AERO code used at the Flight Test Center at the Edwards Air Force Base. Therefore, the outcome of the proposed research on ROM acceleration and ROM adaptation will be implemented in AERO.

2.1.2.2 Fast Generation of High-Fidelity POD Snapshots

Research Issues. The CPU cost associated with Step 1 of the POD method outlined in Section 2.1.2.1 of this research proposal dominates the overall CPU cost of this procedure when applied to construct a ROM for the fluid system. For example, consider the case of an F-16 Block 40 aircraft where the CFD mesh generated for the FOM contains 403,919 vertices, the structure is represented by its first nine natural modes, five values of the reduced frequency k are used in Step 1, and therefore 90 snapshots are generated in this first POD step (45 real-valued and 45 imaginary-valued vector components). In this case, it was reported in [33] that using a Linux cluster with 32 Pentium-4 processors, Step 1 where the snapshots are generated consumes 3.04 hours of CPU time, whereas all six steps of the POD procedure combined consume 3.43 hours of CPU time. Hence, accelerating the construction of a POD-based ROM in view of enabling the practical usage of this computational technology in a flight test center requires speeding up the generation of the underlying computational snapshots.

Related Research. For each excitation displacement $\bar{\mathbf{u}}_j$, equation (10) can be rewritten as

$$\text{Solve } (ik\bar{\mathbf{A}} + \bar{\mathbf{H}})\bar{\mathbf{w}}_j(k) = \big(ik(\bar{\mathbf{E}} + \bar{\mathbf{C}}) + \bar{\mathbf{G}}\big)\bar{\mathbf{K}}\bar{\mathbf{u}}_j, \quad k = k_1,\ k_2,\ \dots \tag{19}$$

This shows that the sought-after computational snapshots are obtained by solving repeated systems of equations where the left-hand sides differ because of changes in the reduced frequency k, and the right-hand sides differ because of changes in both the reduced frequency k and the excitation displacement $\bar{\mathbf{u}}_j$. Similar systems of equations also arise in the solution of acoustic problems in frequency bands [46], where the PI and his research group have recently contributed a fast solution method that demonstrated — for realistic problems — more than an order of magnitude speedup over conventional approaches for solving repeated problems of the form given in equation (19). Essentially, the solution approach proposed in [46] consists of transforming the above problem into an equivalent one where the systems of equations differ only by their right-hand sides, rather than by both their left- and right-hand sides. Such linear systems of equations can then be solved at essentially the same CPU cost as one system of linear equations, particularly when a direct method can be employed for this purpose. A similar approach will be adopted here for accelerating by an order of magnitude the generation of the computational snapshots needed for constructing a POD-based fluid ROM.

Approach. It is proposed to expedite the snapshot solution process outlined in equation (19) by focusing on a single reduced frequency (or sampling a small range of reduced frequencies) for the snapshot solution(s) and its (their) respective derivatives with respect to reduced frequency, then using Padé approximants to reconstruct the snapshots at the other desired frequencies. (One can also think of a Taylor's series, however such an approach is known to be numerically unstable (for example, see [46])). Hence, the key aspect of this computational approach is to obtain accurate sensitivities of the snapshot solution with respect to the reduced frequency, k. This can be achieved by repeatedly differentiating equation (19) to obtain

$$(ik\bar{\mathbf{A}} + \bar{\mathbf{H}})\frac{\partial \mathbf{w}_j(k)}{\partial k} = i\big((\bar{\mathbf{E}} + \bar{\mathbf{C}})\bar{\mathbf{K}}\bar{\mathbf{u}}_j - \bar{\mathbf{A}}\bar{\mathbf{w}}_j(k)\big) \tag{20}$$

and

$$(ik\bar{\mathbf{A}} + \bar{\mathbf{H}})\frac{\partial^n \mathbf{w}_j(k)}{\partial k^n} = -ni\bar{\mathbf{A}}\frac{\partial^{n-1} \mathbf{w}_j(k)}{\partial k^{n-1}} \qquad n > 1 \tag{21}$$

which shows that the snapshot solution at a given frequency k and all its derivatives with respect to k can be obtained by solving a single problem with repeated right-hand sides. If a sparse direct method is chosen for solving equations (19–21), the matrix $(ik\bar{\mathbf{A}} + \bar{\mathbf{H}})$ is factored once, then the snapshot solution $\bar{\mathbf{w}}_j(k)$ and all its derivatives $\dfrac{\partial \mathbf{w}_j(k)}{\partial k}$ are obtained by computationally inexpensive forward and backward substitutions.

Then, for any other reduced frequency $k' = k + \Delta k$ that is reasonably close to k, the snapshot solution can be reconstructed in real-time by using Padé approximants computed from the knowledge of $\bar{\mathbf{w}}_j(k)$ and $\frac{\partial \mathbf{w}_j(k)}{\partial k}$, $n \geq 1$. This summarizes the crux of the approach that will be adopted for accelerating the generation of high-fidelity POD fluid snapshots.

Unfortunately, for CFD applications, the size of the FOM may be such that the matrix $(ik\bar{\mathbf{A}} + \bar{\mathbf{H}})$ is too large to be stored even in a sparse format, in which case the solution of problems (19–21) by a direct method may be ruled out. On the other hand, iterative solvers are in general ill-suited for the solution of problems with repeated right-hand sides because they often must restart from scratch the iteration process for every different right-hand side. However, Krylov-type iterative solvers can be tailored to efficiently solve problems with repeated right-hand sides (for example, see [47, 48]), particularly when preconditioned by domain decomposition-based operators (for example, see [49]) as in the AERO code. The challenge will be to extend such proven approaches to complex, three-dimensional, CFD problems.

2.1.2.3 Parameterization of the Fuel System, External Stores, Launchers and Sidewinders

The fuel system can be assumed to be rigid. Its mass can be assumed to be uniformly or piece-wise uniformly distributed. Hence, it will be addressed by a few parameters describing the fuel level at one or more locations inside the aircraft. The external stores, launchers, and sidewinders will be represented as flexible beams. Each such secondary system will be parameterized by its length, sectional geometry, Young modulus, moments of inertia, and discrete mass. The attachments of these systems to the main wings will be represented by springs and parameterized by the corresponding stiffness coefficients. Hence, variations in the mass and structural properties of these subsystems will be formulated in terms of these parameters. Variations due to a download of one (or more) of these subsystems will be accounted for by precomputing one (or more) aeroelastic ROM(s) for the resulting wing configuration(s).

2.1.2.4 Near Real-Time Adaptation of Aeroelastic ROMs

Changes in Mass Distribution and Structural Stiffness

Research Issues. The straightforward approach for adapting a POD basis to change in a structural parameter consists of generating new snapshots corresponding to the new mode shapes of the structure, and thus repeating the POD procedure. However, this cannot be performed in near-real time, and therefore is not realistic for supporting flight test operations. Further research is needed to develop a more economical alternative.

Related Research. Little has been published about adapting aeroelastic ROMs to changes in structural mass and/or stiffness properties. When the stiffness and mass of a structure are altered, the natural modes of the structure are affected, and therefore all the POD snapshots must be recomputed in principle. However, it was numerically shown in [15] that in such cases, it suffices to augment the original POD snapshots with those new ones corresponding only to both end points of the frequency range of interest. The works described in [42] and [43] on using ROMs for aerodynamic shape optimization, even though restricted to two-dimensional airfoils, also address related issues.

Approach. The sensitivity of the snapshot solution to a parameter s can be obtained from the differentiation of equation (19) with respect to s, which gives

$$(ik\bar{\mathbf{A}} + \bar{\mathbf{H}})\frac{\partial \bar{\mathbf{w}}_j(k)}{\partial s} = \left(ik(\frac{\partial \bar{\mathbf{E}}}{\partial s} + \frac{\partial \bar{\mathbf{C}}}{\partial s}) + \frac{\partial \bar{\mathbf{G}}}{\partial s}\right)\bar{\mathbf{K}}\bar{\mathbf{u}}_j + \left(ik(\bar{\mathbf{E}} + \bar{\mathbf{C}}) + \bar{\mathbf{G}}\right)\frac{\partial \bar{\mathbf{K}}}{\partial s}\bar{\mathbf{u}}_j \tag{22}$$

Using the chain rule, the various sensitivities appearing in the right-hand side of the above equation can be computed as

$$\frac{\partial \bar{\mathbf{E}}}{\partial s} = \frac{\partial \bar{\mathbf{E}}}{\partial \mathbf{X}_j}\frac{\partial \mathbf{X}_j}{\partial s}, \quad \frac{\partial \bar{\mathbf{C}}}{\partial s} = \frac{\partial \bar{\mathbf{C}}}{\partial \mathbf{X}_j}\frac{\partial \mathbf{X}_j}{\partial s}, \quad \frac{\partial \bar{\mathbf{G}}}{\partial s} = \frac{\partial \bar{\mathbf{G}}}{\partial \mathbf{X}_j}\frac{\partial \mathbf{X}_j}{\partial s} \tag{23}$$

where $\mathbf{X}_j$ is the j-th mode shape of the structure around which $\bar{\mathbf{E}}$, $\bar{\mathbf{C}}$, and $\bar{\mathbf{G}}$ are computed (the reviewer is reminded here that $\bar{\mathbf{u}}_j$ is typically chosen as a dry natural mode of the structure (see equation (10) and Section 2.1.2.1), in which case $\bar{\mathbf{E}}$, $\bar{\mathbf{C}}$, and $\bar{\mathbf{G}}$ explicitly depend on $\mathbf{X}_j$). Similarly, the higher-order sensitivies of the snapshot solution $\bar{\mathbf{w}}_j(k)$ with respect to the parameter s can be obtained from the recursive differentiation of equation (19) with respect to s, which leads to

$$(ik\bar{\mathbf{A}} + \bar{\mathbf{H}})\frac{\partial^n \bar{\mathbf{w}}_j(k)}{\partial s^n} = \sum_{r=0}^{n} \frac{n!}{r!(n-r)!}\left(\left(ik(\frac{\partial^{n-r}\bar{\mathbf{E}}}{\partial s^{n-r}} + \frac{\partial^{n-r}\bar{\mathbf{C}}}{\partial s^{n-r}}) + \frac{\partial^{n-r}\bar{\mathbf{G}}}{\partial s^{n-r}}\right)\frac{\partial^r \bar{\mathbf{K}}}{\partial s^r}\bar{\mathbf{u}}_j\right) \tag{24}$$

From equation (23), it follows that computing the higher-order derivatives $\frac{\partial^{n-r}\bar{\mathbf{E}}}{\partial s^{n-r}}$, $\frac{\partial^{n-r}\bar{\mathbf{C}}}{\partial s^{n-r}}$, and $\frac{\partial^{n-r}\bar{\mathbf{G}}}{\partial s^{n-r}}$ requires evaluating the sensitivity of each considered mode shape, $\mathbf{X}_j$, to the varied structural parameter, s. To this effect, the following is noted. The mode shapes are solution of the generalized eigenvalue problem

$$\mathbf{K}\mathbf{X}_j = \lambda_j \mathbf{M}\mathbf{X}_j \tag{25}$$

where λ_j denotes the j-th eigenvalue associated with the mode shape $\mathbf{X}_j$, and therefore form a $\mathbf{K}$- and $\mathbf{M}$-orthogonal basis. Hence, the sensitivity of a mode shape with respect to the varied parameter can be written as

$$\frac{\partial \mathbf{X}_j}{\partial s} = \sum_{r=1}^{N} a_{jr}\mathbf{X}_r \tag{26}$$

Assuming that the basis of mode shapes has been mass-normalized, substituting equation (26) into equation (25) after differentiating the latter with respect to s, and using the orthogonality properties of the mode shapes leads to

$$\begin{aligned} a_{jr} &= \frac{1}{\lambda_r - \lambda_j}\mathbf{X}_r^T(\lambda_j \frac{\partial \mathbf{M}}{\partial s} - \frac{\partial \mathbf{K}}{\partial s})\mathbf{X}_j \qquad j \neq r \\ a_{jj} &= -\frac{1}{2}\mathbf{X}_j^T(\frac{\partial \mathbf{M}}{\partial s})\mathbf{X}_j \end{aligned} \tag{27}$$

The above analysis suggests that if all the higher-order sensitivities identified above are precomputed, given a variation Δs in a parameter s, any previously computed snapshot can be adapted in near real-time to Δs using Padé approximants constructed from the knowledge of the unperturbed snapshot and its derivatives with respect to s. This is the proposed approach for adapting in near real-time a precomputed aeroelastic ROM to changes in mass distribution and/or structural stiffness.

Changes in Altitude, Mach Number and Angle of Attack

Research Issues. Here, the research issues are similar to those identified in the previous section on changes in mass distribution and structural stiffness.

Related Research. Several approaches for adapting fluid ROMs to changes in the Mach number, altitude, or angle of attack have been described in the literature. In [10], it was shown numerically that for airfoils in the subsonic regime, a fluid ROM is relatively insensitive to small changes in the Mach number. This property was subsequently confirmed in [16]. However, the proposers have recently established that this property does not hold in the transonic and supersonic regimes [33]. Attempts to adapt fluid ROMs to changes in the aforementioned aerodynamic parameters include the direct interpolation of the ROM basis vectors [16], the global POD (GPOD) method [44], the multi-POD method [45], and the subspace angle interpolation method [16,23,25]. The GPOD method agglomerates snapshots from varying flow parameters such as the Mach number or angle of attack. It was demonstrated in [44] for a micro-air vehicle at very low Mach numbers (0.04 - 0.05) and moderately large angles of attack (0° - 20°). The multi-POD technique expands the applicable parameter space of a ROM by selecting the most appropriate pre-computed basis during a simulation. It was illustrated in [45] for an airfoil subjected to a variable angle of attack (0° - 2°). In [16,23], the direct interpolation and GPOD methods were shown numerically to be ineffective in the transonic regime. In [23,25], the subspace angle interpolation method was proposed as a robust alternative for complete aircraft configurations in the transonic regime. In [33], this method was successfully applied to adapting POD-based aeroelastic ROMs for large variations of the Mach number (0.6 - 1.2) and moderate changes of the angle of attack [33].

Approach. In principle, the aeroelastic ROM adaptation proposed above for addressing changes in mass distribution and structural stiffness is also suitable for treating the case of changes in the Mach number and angle of attack. (As mentioned in Section 2.1.1.2, changes in altitude are automatically handled by the decoupled fluid/structure aeroelastic formulation and the adimensionalization of the fluid state variables.) Indeed, equations (22–24) are valid for any parameter s. In this case however, equations (26,27) are inappropriate as the mode shapes $\mathbf{X}_j$ do not depend on the aerodynamic parameters. The sensitivities $\frac{\partial \bar{\mathbf{E}}}{\partial s}$, $\frac{\partial \bar{\mathbf{C}}}{\partial s}$, and $\frac{\partial \bar{\mathbf{G}}}{\partial s}$ can rather be evaluated by automatic differentiation with respect to s of the concerned discrete matrices. Hence, the reconstruction approach outlined in Section 2.1.2.4 will also be considered in this case.

Fig. 2 (left) displays the variation with the Mach number, for a fixed angle of attack, of a sample component of a POD-generated aeroelastic ROM basis vector for a complete configuration of an F-16 Block 40 aircraft. This figure, which reveals a strong nonlinear variation of the aforementioned quantity, explains why the direct *linear* interpolation of two aeroelastic ROMs cannot successfully adapt them to a third operational Mach number. On the other hand, Fig. 2 (right) reveals that the subspace angles between two POD-generated aeroelastic ROM basis vectors — which were introduced in [24] — particularly for the first few ROM basis vectors, vary almost linearly with the Mach number. Again, this explains why the subspace angle interpolation method was successfully used to adapt two aeroelastic ROMs precomputed at two different Mach numbers to a third operational Mach number [33]. Similar success was reported in [33] when using the subspace angle interpolation method to adapt an aeroelastic ROM to change in the angle of attack. Hence, it is also proposed to pursue this approach and extend it to: (a) higher-order interpolations, and (b) combined variations of the Mach number and angle of attack in order to address trimmed flight.

2.1.2.5 Integration of Aeroelastic ROMs into a Flight Simulator

Finally, if time permits, a path for integrating the computational aeroelastic ROM technologies developed under the research tasks outlined in the previous sections will be investigated. A preliminary vision for this path is summarized in Fig. 3. Essentially, a *Data Handler* will be developed to exchange data such as flight conditions and post-processed aeroelastic results such as time-response curves or damping coefficients between the *Flight Simulator* and the *Model/Solution Composer*. Based on the data communicated by the *Flight Simulator*, the *Model/Solution Composer* will identify and select the most suitable aeroelastic ROM that is stored in the *Basis Solutions Database*, or decide to compose a new aeroelastic ROM by adapting precomputed ones to the specified flight conditions. The *Basis Solutions Database* can be continuously populated by constructing ROMs in the *ROM* module using snapshots obtained from high-fidelity models stored in the *Hi-Fi Model* module.

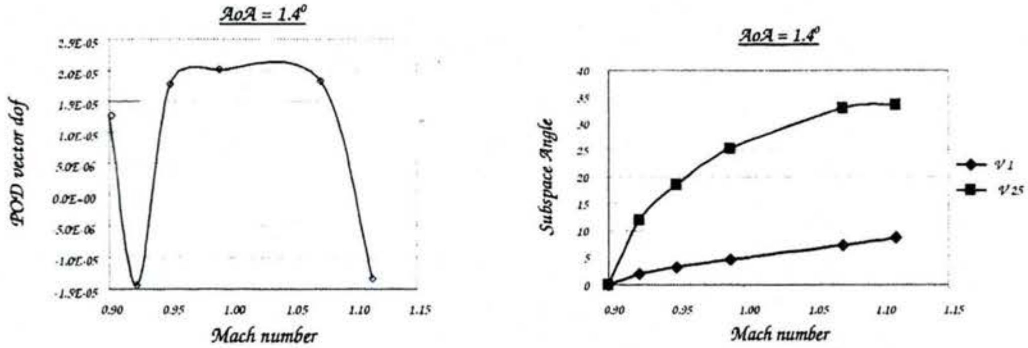

Figure 2: F-16 Block 40: variation with the Mach number, for a fixed angle of attack, of a sample component of a POD-generated aeroelastic ROM basis vector (left) — variation with the Mach number, for a fixed angle of attack, of the subspace angles between the POD-generated aeroelastic ROM basis vectors (right)

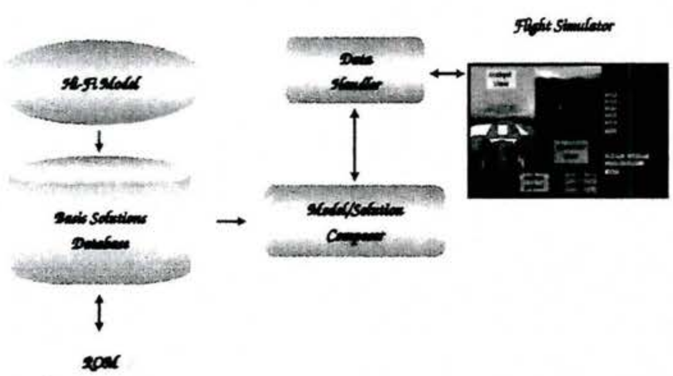

Figure 3: Path for integrating parameterized aeroelastic ROMs into a manned flight simulator

2.1.3 Collaboration with the Flight Test Center at the Edwards Air Force Base

For many years, the proposing research team has collaborated with personnel at the Flight Test Center (FTC) at the Edwards Air Force Base. Therefore, this team has a good understanding of the objectives and technical challenges faced by this Test and Evaluation Center, particularly in the area of aeroelastic simulations. To maximize the significance and potential impact of the outcome of this proposed research project, the proposing team plans on continuing this technical collaboration so that academic efforts lead to usable tools for the flight test engineer.

2.1.4 Project Schedule, Milestones and Deliverables

An upgraded version of the AERO-F code implementing the results of the research task focusing on the fast generation of high-fidelity POD snapshots will be delivered to the Flight Test Center at the Edwards Air Force Base at the end of the first year of funding. An upgraded version of the same code implementing also the results of the research task focusing on the near real-time adaptation of aeroelastic ROMs for changes in aerodynamic parameters (mass distribution and structural stiffness) will be delivered at the end of the second (third) year of funding.

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2.2 PRINCIPAL INVESTIGATOR TIME

2.2.1 Time Committment to this Research Project

The PI of this proposed research project is Professor Charbel Farhat. He will dedicate at least 1% of his academic time and 16.66% of his summer time to the proposed research effort. Professor Farhat will also supervise one full-time graduate student and one part-time post-doctoral assistant who will contribute to this proposed research project.

2.2.2 Current and Pending Support

Professor Farhat is currently the PI of the following research grants which extend beyond February 1st, 2006:

- Grant: Acoustic Signatures of Mines Located Near the Ocean Bottom. Agency: High Performance Technologies Inc. Commitment: 5% AY.
- Grant: A Dynamic Data-Driven System for Structural Health Monitoring and Critical Event Prediction. Agency: National Science Foundation. Commitment: 5% AY and 0.5 month summer.
- Grant: Aerodynamic/Aeroelastic Effects on a Class of High-Speed Vehicles. Agency: Toyota Motor Corporation. Commitment: 5% AY.
- Grant: High-Resolution Methods for the Solution of Direct and Inverse Acoustic Scattering Problems. Agency: Office of Naval Research. Commitment: 10% AY and 0.5 month summer.
- Grant: A Collaborative for Naval Computational Mechanics. Agency: Office of Naval Research. Commitment: 10% AY and 0.5 month summer.
- Grant: High Performance Computing Modernization Program – Programming Environment and Training (PET). Agency: High Performance Technologies Inc. Commitment: 5% AY.
- Grant: Scalable Substructuring Methods for Linear and Nonlinear Dynamics Problems. Agency: Sandia National Laboratories. Commitment: 10% AY and 0.5 month summer.
- Grant: Hybrid Unsteady Simulation for Helicopters. Agency: Defense Advanced Research Projects Agency. Commitment: 5% AY and 1 week summer.

2.3 FACILITIES

Professor Farhat operates at Stanford University a High-Performance Computing and Visualization Laboratory that can serve as a development and application platform for the proposed research. The laboratory is equipped with a Linux Cluster system with 160 Intel Xeon 3.056 GHz processors and 320 GBytes of memory. This parallel processor is connected to a Panasas Storage Cluster with direct node-to-disk access and to several front-end and visualization systems.

2.4 KEY PERSONNEL

The key personnel for this proposed research project includes Professor Charbel Farhat, Dr. Thuan Lieu, and a graduate student.

2.4.1 Charbel Farhat

Biographical Sketch

Charbel Farhat is Professor of Mechanical Engineering, Professor, by courtesy, of Aeronautics and Astronautics, and Professor in the Institute for Computational and Mathematical Engineering, all at Stanford University. Previously, he held the positions of Professor and Chair of Aerospace Engineering Sciences and Director of the Center for Aerospace Structures at the University of Colorado at Boulder. He holds a Ph.D. in Civil Engineering from the University of California at Berkeley (1987). He is the recipient of several prestigious awards including the Institute of Electrical and Electronics Engineers (IEEE) Computer Society Gordon Bell Award (2002), the International Association of Computational Mechanics (IACM) Computational Mechanics Award (2002), the Department of Defense Modeling and Simulation Award (2001), the US Association of Computational Mechanics (USACM) Medal of Computational and Applied Sciences (2001), the IACM Award in Computational Mechanics for Young Investigators (1998), the USACM R. H. Gallagher Special Achievement Award for Young Investigators (1997), the IEEE Computer Society Sidney Fernbach Award (1997), the American Society of Mechanical Engineers (ASME) Aerospace Structures and Materials Award (1994), and the United States Presidential Young Investigator Award (1989).

Professor Farhat is currently Vice Chair of the Society for Industrial and Applied Mathematics' Activity Group on Supercomputing (2003-2006), and Associate Editor of the International Journal for Numerical Methods in Engineering. He also serves on the editorial board of eleven other international scientific journals, and on the technical assessment board of several national research councils and foundations. He is a Fellow of the American Society of Mechanical Engineers (2003), Fellow of the International Association of Computational Mechanics (2002), Fellow of the World Innovation Foundation (2001), Fellow of the US Association of Computational Mechanics (2001), and Fellow of the American Institute of Aeronautics and Astronautics (1999). He has been an AGARD lecturer on aeroelasticity and computational mechanics at several distinguished European institutions, and a keynote speaker at numerous international scientific meetings. He is the author of over 200 refereed publications on fluid/structure interaction, computational fluid dynamics on moving grids, computational structural mechanics, computational acoustics, supercomputing, and parallel processing. His research program has been and is currently funded by several government and private agencies including the National Science Foundation, the Air Force Office of Scientific Research, the NASA Langley Research Center, the NASA Ames Research Center, the NASA Lewis Research Center, the Naval Research Laboratory, the Office of Naval Research, the Department of Energy, the Sandia National Laboratories, the Defense Advanced Research Projects Agency, TRW, the FMC Corporation, the Lockheed-Martin Corporation, High Performance Technologies, and the Toyota Motor Corporation.

Selected Publications

1 *"Reduced-Order Fluid/Structure Modeling of a Complete Aircraft Configuration,"* (with T. Lieu and M. Lesoinne), Computer Methods in Applied Mechanics and Engineering, (in press)

2 *"Adaptation of POD-based Aeroelastic ROMs for Varying Mach Number and Angle of Attack: Application to a Complete F-16 Configuration,"* (with T. Lieu), AIAA Paper 2005-7666, U.S. Air Force T&E Days, Nashville, Tennessee, December 6-8 (2005)

3 *"POD-based Aeroelastic Analysis of a Complete F-16 Configuration: ROM Adaptation and Demonstration,"* (with T. Lieu and M. Lesoinne), AIAA Paper 2005-2295, 46th Structural Dynamics & Materials Conference, Austin, Texas, April 18-21 (2005)

4 *"Expanding a Flutter Envelope Using Accelerated Flight Data: Application to an F-16 Fighter Configuration,"* (with C. Harris and D. Rixen), AIAA Paper 2000-1702, 41st Structural Dynamics & Materials Conference, Atlanta, GA, April 3-6 (2000)

5 *"A Linearized Method For the Frequency Analysis of Three-Dimensional Fluid/Structure Interaction Problems in all Flow Regimes,"* (with M. Lesoinne, M. Sarkis and U. Hetmaniuk), Computer Methods in Applied Mechanics and Engineering, Vol. 190, pp. 3121-3146 (2001)

6 *"A CFD Based Method for Solving Aeroelastic Eigenproblems in the Subsonic, Transonic, and Supersonic Regimes,"* (with M. Lesoinne), AIAA Journal of Aircraft, Vol. 38, pp. 628-635 (2001)

7 *"Application of a Three-Field Nonlinear Fluid-Structure Formulation to the Prediction of the Aeroelastic Parameters of an F-16 Fighter,"* (with P. Geuzaine and G. Brown), Computers and Fluids, Vol. 32, pp. 3-29 (2003)

8 *"Aeroelastic Dynamic Analysis of a Full F-16 Configuration for Various Flight Conditions,"* (with P. Geuzaine, G. Brown and C. Harris), AIAA Journal, Vol. 41, pp. 363-371 (2003)

2.4.2 Thuan Lieu

Biographical Sketch

Dr. Thuan Lieu is a part-time post-doctoral research assistant at Stanford University. He holds a Ph.D. in Aerospace Engineering Sciences from the University of Colorado at Boulder (2004). In his thesis entitled *Parameter Adaptation of Reduced-Order Models for Aeroelasticity*, he made important contributions to the field of aeroelastic reduced-order modeling that have been incorporated in the AERO simulator deployed at the Edwards Air Force Base. Dr. Lieu is the recipient of several awards, including a National Science Foundation Graduate University Fellowship (2001).

Selected Publications

1 *"Reduced-Order Fluid/Structure Modeling of a Complete Aircraft Configuration,"* (with C. Farhat and M. Lesoinne), Computer Methods in Applied Mechanics and Engineering, (in press)

2 *"Adaptation of POD-based Aeroelastic ROMs for Varying Mach Number and Angle of Attack: Application to a Complete F-16 Configuration,"* (with C. Farhat), AIAA Paper 2005-7666, U.S. Air Force T&E Days, Nashville, Tennessee, December 6-8 (2005)

3 *"POD-based Aeroelastic Analysis of a Complete F-16 Configuration: ROM Adaptation and Demonstration,"* (with C. Farhat and M. Lesoinne), AIAA Paper 2005-2295, 46th Structural Dynamics & Materials Conference, Austin, Texas, April 18-21 (2005)

4 *"Parameter Adaptation of Reduced-Order Models for Three-Dimensional Flutter Analysis,"* (with M. Lesoinne), 42nd AIAA Aerospace Sciences Meeting & Exhibit, Reno, Nevada, January 5-8 (2004)

2.5 COST PROPOSAL

The budget includes yearly support for: one full-time graduate student; 10.75% of the time (only during the first two years) of one post-doctoral research assistant with expertise in aeroelastic reduced-order modeling and experience with the technical needs of the Flight Test Center at the Edwards Airforce Base in the area of computational aeroelasticity; 1% of the academic time and two weeks of the summer time of the PI to supervise and contribute to this research project; and travel to attend the annual AFOSR Grantees Review Meeting.