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ENHANCED CONSTRAINED PREDICTIVE CONTROL FOR APPLICATIONS TO AUTONOMOUS VEHICLES AND MISSIONS

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Final Report

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14. ABSTRACT
This project report summarizes outcomes of a one year research project into enhancing nonlinear constrained model predictive control capabilities for autonomous vehicles and missions. For the problem of under actuated autonomous spacecraft control, we have demonstrated, analyzed and enhanced, in terms of computations, CNMPC solutions which are systematic, lead to stabilizing, discontinuous feedback laws with exponential convergence rates, and are suitable for on-board implementation. For the problem of the constrained control of the coupled rotational and translational relative motion spacecraft dynamics, a periodic reference governor strategy has been developed which permits the deputy spacecraft to dock with a chief spacecraft in orbit subject to control constraints and without collisions. For the problem of control of spacecraft formations, we have developed and demonstrated a predictive time shift governor control scheme which stabilizes several spacecraft to a given formation while adhering to control, communication, and collision constraints.

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1.0 SUMMARY

There is a growing need in developing Model Predictive Control (MPC) solutions and implementing related predictive control schemes (such as reference and extended command governors) for control of autonomous vehicles. These feedback control methods rely on minimizing an objective function subject to user defined constraints and re-computing the solution along the trajectory while the prediction horizon is receded.

Recent advances in computing hardware and algorithms have allowed for the implementation of simple, linear quadratic MPC solutions in real-time by solving a quadratic minimization problem. At the same time, there still exists many challenges to implementing more complex constrained nonlinear MPC (CNMPC) controllers. These challenges are more prevalent in spacecraft systems, where computational ability in orbit is limited. Thus, the focus of this research was to expand the capabilities of CNMPC and other predictive control schemes for spacecraft applications.

This one year research project addressed three thrust areas:

1. Computational enhancements to CNMPC.
2. Applications of CNMPC and constrained control to combined translational and rotational spacecraft relative motion control.
3. Developing MPC solutions for maneuvering autonomous networked spacecraft formations with debris avoidance.

The following articles have been written and published related to the subject matter of this report:

1. Petersen, C., Leve, F., and Kolmanovsky, I., "Model Predictive Control of an Underactuated Spacecraft with Two Reaction Wheels," AIAA *Journal of Guidance, Control, and Dynamics*, accepted for publication May 29, 2016, to appear, http://dx.doi.org/10.2514/1.G000320.2016

2. Petersen, C., and Kolmanovsky, I., "Coupled Translation and Rotation Dynamics for Precise Rendezvous and Docking with Periodic Reference Governor Control Scheme," *Proceedings of 26th AAS/AIAA Space Flight Mechanics Meeting*, Paper No. AAS 16-507, AAS/AIAA, Napa, CA, 2016.

3. Frey, G., Petersen, C., Leve, F., Garone, E., Kolmanovsky, I.V., Girard, A., "Time Shift Governor for Coordinated Control of Two Spacecraft Formations, *Proceedings of 2016 IFAC Symposium on Nonlinear Control System Design*, Monterey, California, August, 2016.

2.0 INTRODUCTION

Constraint handling is a required and challenging aspect of controlling autonomous vehicles. Constraints may include actuator magnitude and rate limits, safety limits and obstacle (e.g., debris in the spacecraft context) avoidance requirements. To ensure robust operation in the presence of uncertainties and disturbances, feedback rather than open-loop strategies are highly desirable in these applications. Applicable literature to this work are in references [1]-[10].

Effective feedback controllers for systems with constraints are typically nonlinear and are often, but not always, predictive. In particular, they can be based on Model Predictive Control (MPC) [8, 4] and chained invariant/contractive set techniques [1], [6], [11], [12]. References [11-13] describe the previous applications of these techniques to spacecraft relative motion and attitude control.

From the computational standpoint, MPC involves solving at consecutive time instants an optimal control problem subject to constraints, which is parametrically dependent on an estimate of the initial state. The main challenges in implementing MPC stem from the requirement for a solution to an optimal control problem to be generated (or updated) in real-time and due to obstacle avoidance constraints being nonconvex.

Constrained nonlinear MPC (CNMPC) [4] represents a general MPC framework distinguished by either the model used for prediction or the constraints being nonlinear, and/or the cost function being non-quadratic, and nonlinear constraints. Coupled translational/rotational relative motion control (involving a deputy and a chief spacecraft), and networked spacecraft formation control with debris avoidance involving multiple spacecraft represent important research frontiers for the autonomous spacecraft and for applications of CNMPC.

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

3.1 Computational Enhancements to CNMPC

The development and enhancement of CNMPC solutions has been considered in the context of real-time attitude control of an autonomous underactuated spacecraft with two Reaction Wheels (RWs) and zero angular momentum. This problem is of particular relevance given multiple instances of spacecraft RW failures (Kepler, FUSE, Haybusa, etc) that have led to the need to control the spacecraft in this configuration. Furthermore, future spacecraft missions may consider two RW configurations due to packaging and power constraints.

The control of underactuated autonomous spacecraft with two RWs is also representative of a broader class of problems with nonlinear dynamics and constraints, similar to nonholonomic systems, for which conventional control system design techniques do not apply while general and systematic control system design techniques are lacking.

Several previous publications have proposed non-MPC type, discontinuous control laws that successfully perform rest-to-rest attitude maneuvers for such an underactuated spacecraft. The motivation for exploiting CNMPC in this problem is not only dealing with constraints but also the ability of CNMPC to generate stabilizing feedback laws that are nonsmooth or even discontinuous as a function of the state which is necessary in the case of underactuated spacecraft. Thus by utilizing CNMPC, the obstruction to stabilizability can be overcome and attitude maneuvers can be performed while enforcing constraints.

3.1.1 Assumptions

For this research, a spacecraft configuration consisting of a spacecraft bus and two RWs is considered. The RWs are assumed to be thin and symmetric. The orientation of the spacecraft is characterized by 3-2-1 Euler Angles yaw (ψ), pitch (θ), and roll (ϕ), with the kinematics prescribed by

$$\dot{\Theta} = M(\Theta)\omega, \tag{1}$$

where Θ is a vector of Euler angles, ω is a vector of angular velocities, and $M(\Theta)$ is a matrix which is a function of Θ. Any three parameter parameterization of orientation has singularities. It is assumed that the spacecraft's mission does not require large attitude transients and never approaches singularity.

Let the total angular momentum of the entire spacecraft configuration be conserved and zero. Assuming that the uncontrollable axis of the spacecraft corresponds to the third component of angular velocity, and using the fact the maneuvers are small, the equations of motion are approximated as

$$\begin{aligned}
\dot{\phi} &= (\alpha_1 \nu_1 + \alpha_2 \nu_2) \\
\dot{\theta} &= (\beta_1 \nu_1 + \beta_2\, \nu_2), \\
\dot{\psi} &= (\beta_1 \nu_1 + \beta_2 \nu_2)\phi, \\
\dot{\nu}_1 &= u_1, \\
\dot{\nu}_2 &= u_2,
\end{aligned} \tag{2}$$

where ν_1, ν_2 are the RW speeds, u_1, u_2 are the control signals, and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants which are functions of the spacecraft and RW inertias. The use of a nonlinear prediction model rather than a linearized model is essential to be able to achieve stabilization with MPC. This is due to the fact that the linearized model is not controllable and thus a linear controller cannot be designed for this problem. In contrast, the simplified nonlinear model is locally controllable and stabilizable and an MPC controller designed using it will be shown to stabilize the desired attitude equilibrium of the full nonlinear model as well.

For the implementation of MPC, these continuous-time equations of motion are discretized. Assuming a sampling period of *T sec*, and that the control input is zero-order hold, the discrete-time dynamics can be given as

$$X_{k+1} = F(X_k, u_{1,k}, u_{2,k}), \tag{3}$$

where $X = [\phi \ \ \theta \ \ \psi \ \ \nu_1 \ \ \nu_2 \ ]^{\mathrm{T}}$, and $*_k = *\,(kT)$.

3.1.2 Method

The MPC optimization problem that needs to be solved for under-actuated spacecraft at each discrete-time step has the following form:

$$\min_{u_{1,k}, u_{2,k} \ k=0,1,\ldots,N-1} \sum_{k=0}^{N-1} X_k^{\mathrm{T}} Q X_k + r_1 u_{1,k}^2 + r_2 u_{2,k}^2 \,, \tag{4}$$

subject to

$$\begin{gathered} X_0 = X(t), \\ X_{k+1} = F\left(X_k, u_{1,k}, u_{2,k}\right), \quad k = 0,1,\ldots,N-1, \\ X_N = 0, \\ \max\left(\nu_{1,k}, \nu_{2,k}\right) \leq \nu_{max}, \ \ k = 0,1,\ldots,N, \\ \max\left(u_{1,k}, u_{2,k}\right) \leq u_{max}, \ \ k = 0,1,\ldots,N-1, \end{gathered} \tag{5}$$

where *N* is the optimization horizon, ν_{max} is the maximum RW speed, u_{max} is the maximum control allowable, $Q = Q^{\mathrm{T}} > 0$ is the state weighting matrix and $r_1, r_2 > 0$ are control input weights. Note that our formulation of CNMPC uses the terminal state constraint to ensure the closed-loop stability [4]. To demonstrate that CMPC generates a stabilizing discontinuous feedback law (in terms of state), the control signal $u_{1,0}$ is generated by solving the above optimization problem for a variety of spacecraft attitude initial conditions. The results are given in Figures 1 and 2 showing the required control law discontinuity.

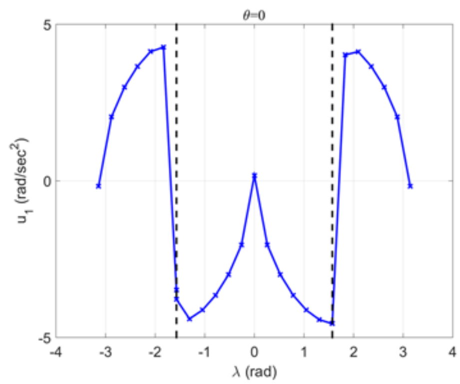


Figure 1. Feedback Control u_1Generated When $\phi = 0.5\, cos\, \lambda$, $\theta = 0$, $\psi = 0.5\, sin\, \lambda$

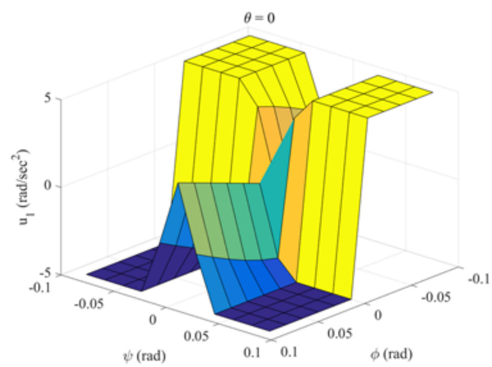


Figure 2. Feedback Control u_1 Generated When $\theta = 0$ and ϕ & ψ are Varied

3.2 Applications of CNMPC and Constrained Control to Combined Translational and Rotational Spacecraft Relative Motion Control

In this part of the research project, we started by deriving equations of motion which accounted for the coupling between the rotational and translational dynamics by modeling the motion between an arbitrary point on a rigid deputy spacecraft and an arbitrary point on a rigid chief spacecraft. The derived equations of motion facilitate the design of coordinated maneuvers when the chief satellite is rotating at a constant rate in the Hill's relative motion frame, and can be used for precise rendezvous operations in which the spacecraft parts to be docked are offset from their Center-of-Masses (COMs). To demonstrate such rendezvous operations with a rotating chief spacecraft we develop a Periodic Linear Quadratic Controller (PLQR) to locally stabilize the desired relative position and orientation and a periodic reference governor, which is a predictive control scheme that enforces constraints during rendezvous.

3.2.1 Assumptions

Consider a chief spacecraft and a deputy spacecraft in orbit about the Earth. Assume that both spacecraft are rigid and that gravity from the Earth is the only external force acting on both spacecraft. The motion of each spacecraft's COM is governed by the two-body problem equations. In addition, let the deputy spacecraft be equipped with an array of thrusters such that instantaneous control forces and torques can be applied in any direction. The configuration of the thrusters is such that the forces and moments for the orientation control and translational control can be independently generated. Hence the deputy spacecraft is instantaneously controllable. In contrast to the deputy spacecraft, we assume that the chief spacecraft is not actuated, but let the chief's COM follow a circular orbit, defined by mean motion *n*. The full coupled equations of motion may be found in [14].

Let the rotational and translational equations be linearized about an equilibrium corresponding to $\rho = \dot{\rho} = \bar{q} = \omega = 0$, where ρ is the distance of a point on the chief spacecraft to a point on the deputy spacecraft, $\bar{q}$ is the vector part of the quaternion giving the attitude of the deputy spacecraft relative to the chief spacecraft , and ω is the angular velocity vector of the deputy spacecraft relative to the chief spacecraft. This results in a set of time-varying, linear differential equations. If this linear model is discretized with a sampling period of T, the discrete-time equations of motion become

$$X_{k+1} = A_k X_k + B_k U_k, \tag{6}$$

$$Y_k = C_k X_k + D_k U_k, \tag{7}$$

where a subscript k designates the discrete time instant at which the vector or a matrix are sampled, i.e. $*_k = *\,(kT)$, X is the state vector consisting of positions, orientations, and velocities, U is the deputy spacecraft control vector consisting of control moments and forces, and Y is the output vector for set-point following. Note that the state, control, and output matrices are time-varying, though there are cases when these matrices are constant (an example being when the chief is not spinning, $\omega_c = 0$). Due to the assumption on deputy spacecraft actuators, the pair (A_k, B_k) satisfies the controllability rank condition for all k.

3.2.2 Methods

Since the linearized system is periodic and controllable, a p-periodic feedback matrix K_k can be constructed such that the control law

$$U_k = -K_k X_k, \tag{8}$$

uniformly asymptotically stabilizes the system to the origin as $k \to \infty$ while minimizing the cost function J^*,

$$J^* = \sum_{k=0}^{\infty} X_k^{\mathrm{T}} Q X_k + U_k^{\mathrm{T}} R U_k, \tag{9}$$

where $Q = Q^T > 0$ and $R = R^T > 0$ are state error and control weighting matrices. This linear quadratic (LQ) feedback approach is known as periodic linear quadratic regulator (PLQR). Now let the control be augmented such that

$$U_k = K_k X_k + \Gamma_k v_k, \tag{10}$$

where v_k is the set-point and Γ_k is a p-periodic matrix that guarantees, under suitable assumptions, for a constant reference $v_k = v$, that

$$Y_k \to \; v \; as \; k \to \infty. \tag{11}$$

In order to enforce constraints on the periodic linear system (resulting assuming that $\dot{\omega}_c = 0$), a reference governor approach is used. A reference governor is an add-on to an existing closed-loop system that modifies the desired reference r_k^* to v_k in order to enforce constraints. At each discrete time instant kT, v_k is determined by solving the optimization problem,

$$\min_{v} ||r_k^* - v||^2 \;, \tag{12}$$

subject to

$$(X_k, v) \in \; O_{\infty,k}, \tag{13}$$

where $O_{\infty,k}$ is an output constraint admissible set that is p-periodic, (i.e., $O_{\infty,k} = O_{\infty,k+p}$). The set $O_{\infty,k}$ comprises of all pairs (X_k, v) such that, under constant reference command, v, the closed-loop response satisfies constraints for all future time instants. Since the reference governor is based on this prediction of the closed-loop response, it is thus classified as a predictive control scheme. The two constraints considered in this research are a control input constraint and a collision avoidance constraint between the deputy spacecraft and the chief spacecraft during docking.

3.3 Developing MPC Solutions for Maneuvering Autonomous Networked Spacecraft Formations with Debris Avoidance

3.3.1 Assumptions

In this part of research, we have defined a predictive control scheme called a parameter governor in order to form and maintain a spacecraft formation. The objective is to place all spacecraft on the same closed, unforced natural motion trajectory in a relative motion frame with the correct phasing. To accomplish this, we model the relative motion dynamics for each spacecraft using the Hill equations. In discrete-time, the state $X^{(i)}$ of the ith spacecraft evolves according to the following linearized equations of motion,

$$X^{(i)}(t+1) = AX^{(i)}(t) + BU^{(i)}(t), \tag{14}$$

where t designates the discrete time instants, $X^{(i)}$ is the state vector of the ith spacecraft that includes the relative position coordinates and velocities of the spacecraft in Hill's frame, $U^{(i)}(t)$ are control inputs that correspond to instantaneous velocity change, and A and B are the discrete time dynamics and input matrices, respectively. The desired natural motion trajectory is expressed as

$$\bar{X}_d(t+1) = A\bar{X}_d(t), \tag{15}$$

with an appropriately selected initial condition, $\bar{X}_d(0)$. We assume that each spacecraft is controlled by a nominal static state feedback law of the form

$$U^{(i)}(t) = K(X^{(i)}(t) - X^{(di)}(t)), \tag{16}$$

where $X^{(di)}(t)$ is the target state for the ith spacecraft and K is a feedback gain matrix for which $A + BK$ is Schur. To achieve a prescribed position in formation, the target to each satellite is commanded as $X^{(di)}(t) = \bar{X}_d(t + \theta^{(di)})$ where $\theta^{(di)}$ specifies the desired phase shift along the trajectory. While this controller is capable of forming the desired formation, constraints on control or state variables may not be satisfied.

3.3.2 Methods

To enforce constraints, an add-on predictive parameter governor controller is developed. This parameter governor adjusts the target provided to each spacecraft by adjusting the time shift along the reference trajectory,

$$X^{(di)}(t) = \bar{X}_d(t + \theta^{(di)} + \tau_i(t)), \tag{17}$$

with the time-shift parameter $\tau_i(t)$. Therefore, this parameter governor is referred to as the Time Shift Governor (TSG). We limit the values the parameters may take to a finite, discrete set $p_i = \theta^{(di)} \in \Psi$. We consider pointwise-in-time constraints on the control vector and the state vector of the overall system, and define the cost functional as

$$J(t, p, X(t)) = W(t) + \Omega(t), \tag{18}$$

where $W(t)$ is a term depending only on the parameter such that $W(t) = 0$ when the desired formation is attained and $\Omega(t)$ contains penalties on the state error and control. The parameter governor updates $p(t)$ subject to the condition that with $p(t + k) = p(t)$ maintained constant over the prediction horizon, the constraints are enforced. Specifically, the following optimization problem is considered:

$$\min_p J\big(t, p, X(t)\big), \tag{19}$$

subject to

$$\begin{gathered} X(t+k|t) \in \mathbb{X}, U(t+k|t) \in \mathbb{U}, \\ p(t+k) = p, \\ p \in \Psi^n, \\ X(t|t) = X(t), \end{gathered} \tag{20}$$

where X, U are the combined state and control vectors of all spacecraft, and $\mathbb{X}, \mathbb{U}$ are the sets defined by the state and control constraints.

4.0 RESULTS AND DISCUSSION

4.1 Computational Enhancements to CNMPC

In this section, the CNMPC is applied to the actual nonlinear model of the underactuated spacecraft with two RWs. The spacecraft bus in these simulations is assumed to have principal moments of inertia equal to 430, 1210, and 1300 kg m^2, respectively. The reaction wheels are assumed symmetric, thin, and are mounted such that the COM of the spacecraft bus and total spacecraft assembly coincide. The inertias of the RWs about their spin axes are 0.043 kg m^2. The two RWs are aligned with the minor and intermediate principal axes of the spacecraft bus. The RWs are constrained such that $u_{max} = 5$ rad/s^2 and $\nu_{max} = 100$ rad/s. The sampling period is chosen to be T=10 s and the optimization horizon is N=30 steps. Figure 3 and 4 below demonstrate that the MPC formulation which uses an approximate model for prediction, is able to stabilize the attitude of the underactuated spacecraft to the desired pointing orientation while enforcing control constraints on the exact model of the spacecraft. Moreover, the convergence rates in both simulations appear to be exponential. As these results demonstrates, CNMPC framework enables a systematic solution to a difficult nonlinear control problem which is not solvable by most of the conventional systematic control system design techniques which generate smooth controllers.

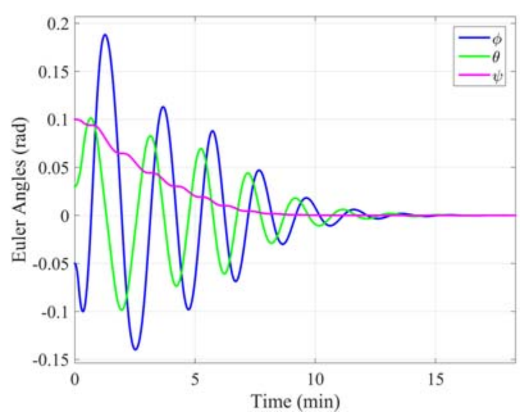
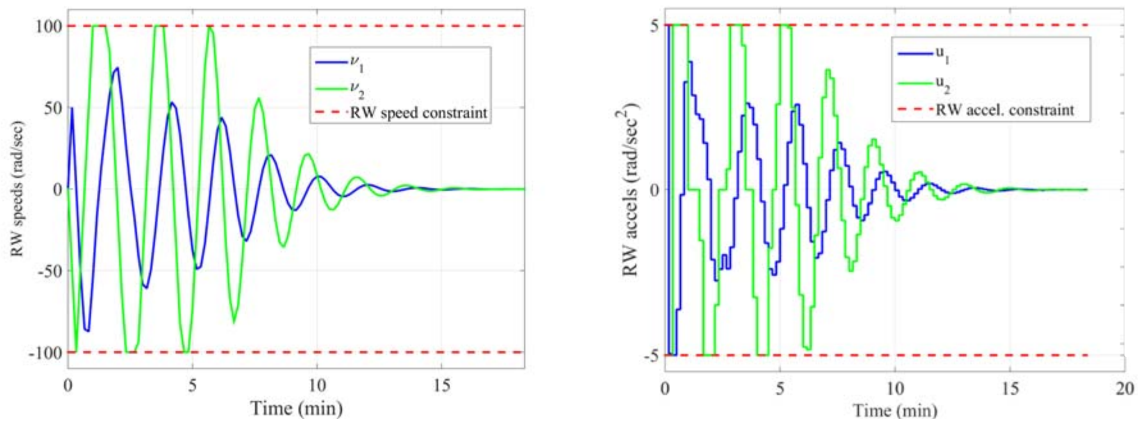


Figure 3. Euler Angles of an Underactuated Spacecraft Utilizing MPC



(a)

(b)

Figure 4. MPC Response of an Underactuated Spacecraft (a) RW Speeds, (b) RW Accelerations

To demonstrate the range of at-rest initial conditions the nonlinear MPC controller can stabilize, 1000 random test simulations were run with initial Euler angles belonging to the interval of [-180, 180] deg, initial zero angular velocity, and RW speeds initially at 0 rad/s. Figure 5 gives an approximation of the region of attraction based on if the controller was able to converge to a 0.01 rad (0.573 deg) Euler angle box and a 0.001 rad/s angular velocity box. As can be seen, the region of attraction is quite large, despite the small angle assumption being used in the controller design.

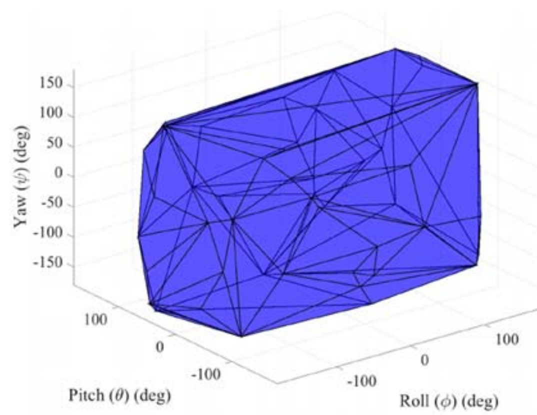


Figure 5. Approximation of Nonlinear MPC Region of Attraction for Rest-to-Rest Maneuvers

The MPC optimization problem this simulation was solved using an interior-point method with MATLAB's *fmincon* function. The average and worst case computation time needed to solve the optimization problem in Section 3.1.2 using a standard computer with 2.4 GHz clock speed were 1.2 s and 2.4 s, respectively. Both times are less than the sample time T in these simulations. Using custom solvers optimized for real-time implementation as a C code will clearly reduce computation time. For instance, techniques we developed in [15], which exploit symbolic computations and code optimization can drastically improve the computation time. Furthermore, the techniques we developed in [31] provide a speed-up by exploiting Newton-Kantorovich (chord) method which avoids the need to re-compute the Jacobians every time step and every iteration. In [31], we demonstrate the application of Newton-Kantorovich's method to fully actuated spacecraft.

Though common spaceflight hardware have processing power typically in the MHz range, the general trend has been towards growing computing power. In fact, there are now more powerful spaceflight processors available such as the 1 GHz PROTON-200k, the 1.5 GHz PROTON-400k-3X, and the 3 GHz PROTON-200k-3X [16]. Reconfigurable Field Programmable Gate Arrays (FPGAs) can also be used for spacecraft missions with large processing demands [17]. It should finally be noted that RWs are used for non-agile maneuvers [18]. Since the closed-loop bandwidth for actuation is in the range 0.01 Hz to 1 Hz [19], control solutions do not need to be computed as rapidly as for other real-time systems. Thus our application may not be dissimilar from other applications for which successful real-world implementations of nonlinear MPC have been reported [20-23]. For spacecraft with limited onboard computational ability, an explicit implementation may be used where the nonlinear MPC is precomputed offline and approximately function-fitted; the fitted function is then used online [24-27]. Such an implementation is still fundamentally based on computational optimization.

Note that the underlying optimization problem is nonlinear and non-convex. Thus there are no a priori guarantees other than offline testing by running multiple simulations that the solver used will converge to a solution. Continuation and warm starting strategies can mitigate the risk of the solver not converging [28-30]. For some implementations, convergence is not required, only feasibility and cost decrease.

4.2 Applications of CNMPC and Constrained Control to Combined Translational and Rotational Spacecraft Relative Motion Control

To demonstrate that the coupled translation and rotation equations can be used for precise rendezvous and docking in low-Earth orbit ($n = 0.0012$), we present a simulation that considers a chief spacecraft rotating at a constant rate with respect to the Hill frame. Both the chief and deputy spacecraft are modeled as constant density cuboids with dimensions 0.6 m × 0.6 m × 0.8 m for the deputy and 1.04 m × 1.15 m × 1.57 m for the chief. The masses of the deputy and chief spacecraft are 130 kg and 360 kg, respectively. The points on the deputy spacecraft and chief spacecraft to be docked together are chosen to lie on the surfaces of each cuboid, and are denoted by x's in Figure 6. The trajectory in Figure 6 shows that the deputy spacecraft successfully performs a rendezvous with the chief and avoids collision. The control plot in Figure 7 demonstrates that the control input never exceeds its limit. The satisfaction of constraints is reflected in the modified reference in Figure 8.

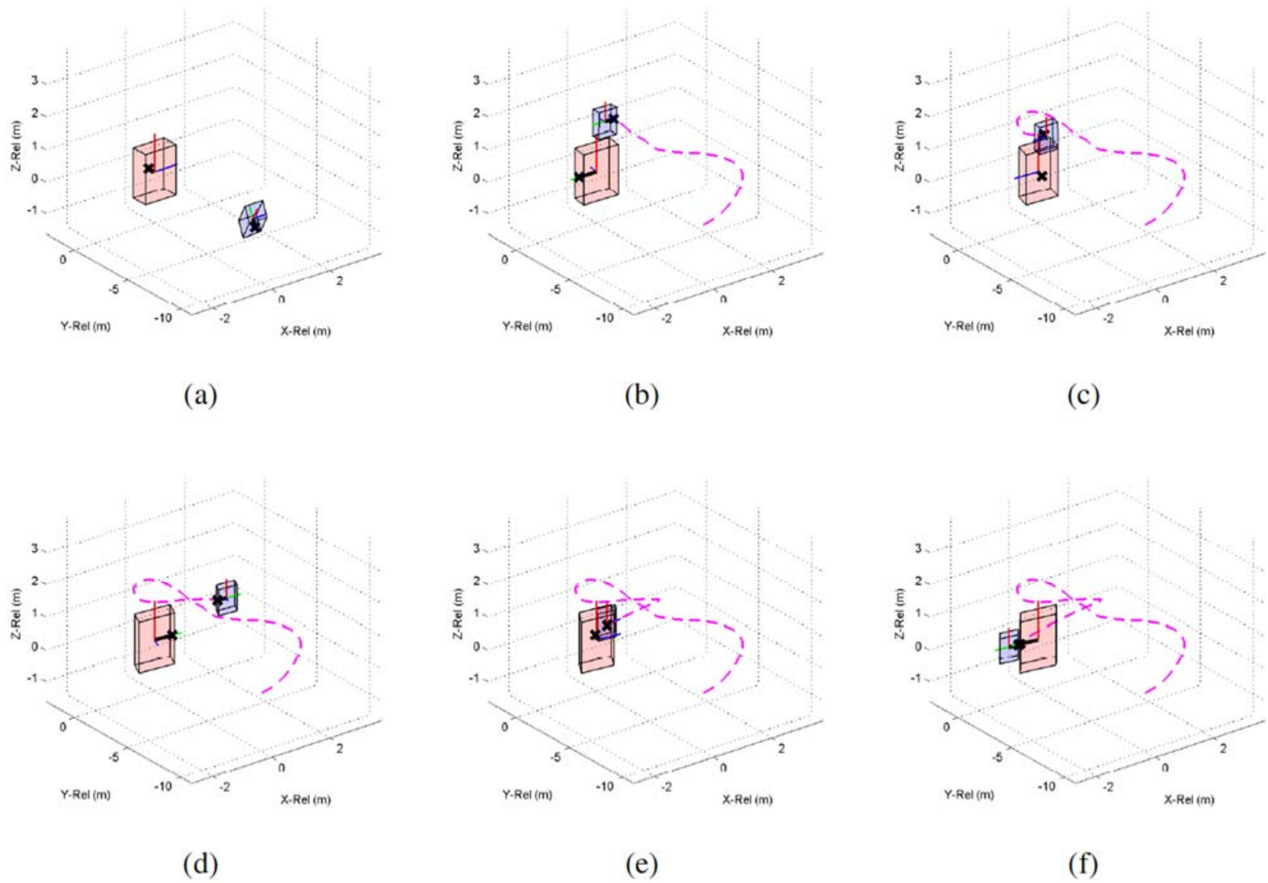


Figure 6. Spacecraft Rendezvous to a Rotating Chief Spacecraft After (a) 0 s, (b) 150 s, (c) 300 s, (d) 450 s, (e) 600 s, (f) 750 s

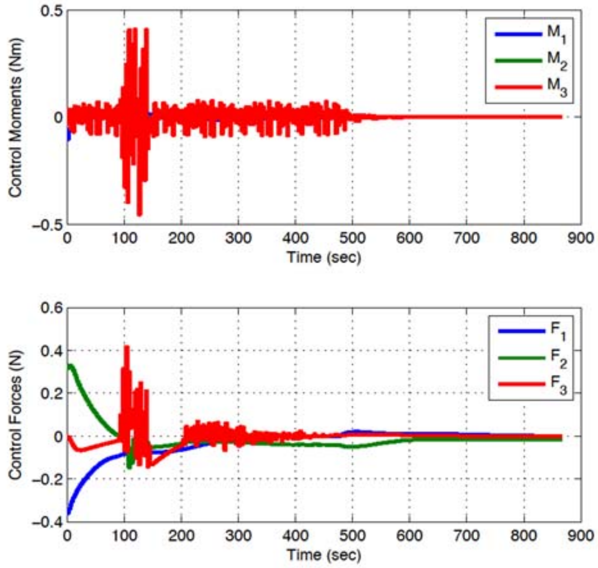


Figure 7. Control Inputs for a Spacecraft Rendezvous to a Rotating Chief Spacecraft

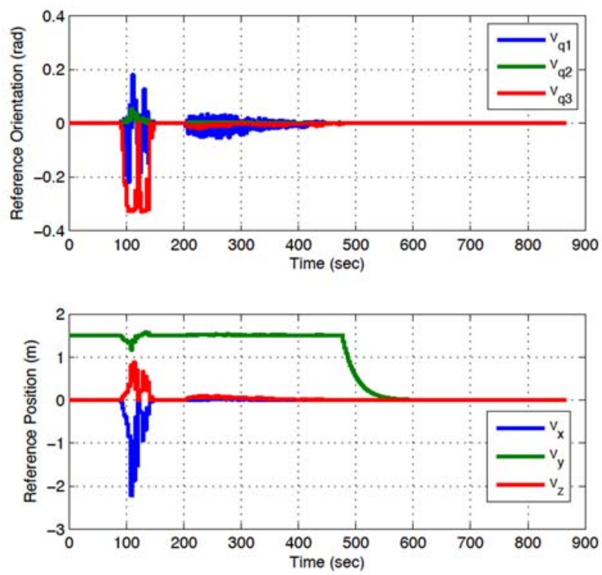


Figure 8. Orientation and Position Reference Commands v_k for a Spacecraft Rendezvous to a Rotating Chief Spacecraft

4.3 Developing MPC Solutions for Maneuvering Autonomous Networked Spacecraft Formations with Debris Avoidance

The simulations presented in this section utilize the TSG to form and maintain a formation of three spacecraft. The desired formation is one in which all three spacecraft travel along a $2x1$ natural motion ellipse in the $X - Y$ plane separated by approximately 120°. The parameters used for the simulation are shown in Table 1.

Table 1. Parameters used for TSG and SSG Simulations

Parameter	Units	Value
Parameter set for τ	-	$\tau = 0{:}1{:}49$
Nominal Circular orbit altitude	Km	$R_0 = 350$
Relative phase shift parameters	-	$\theta^{(d1)} = 16, \theta^{(d2)} = 0, \theta^{(d3)} = 33$
LQR State weighting matrix	-	$Q = diag(1, 1, 1, 10^{-3}, 10^{-3}, 10^{-3})$
LQR Control weighting matrix	-	$R = 10^8 \times diag(1,1,1)$
Maximum allowable ΔV	km/s	0.001
Minimum allowable separation distance between spacecraft	Km	1
Maximum allowable angle between relative orientations	Deg	60

Figure 9 shows a simulation where the TSG is used to form and maintain the desired formation. The top-left quadrant shows that with the TSG inactive constraints are violated while the top right quadrant shows that with the TSG active constraints are satisfied. The bottom left quadrant shows the parameter values and W(t) vs. time and the bottom right quadrant shows system trajectories.

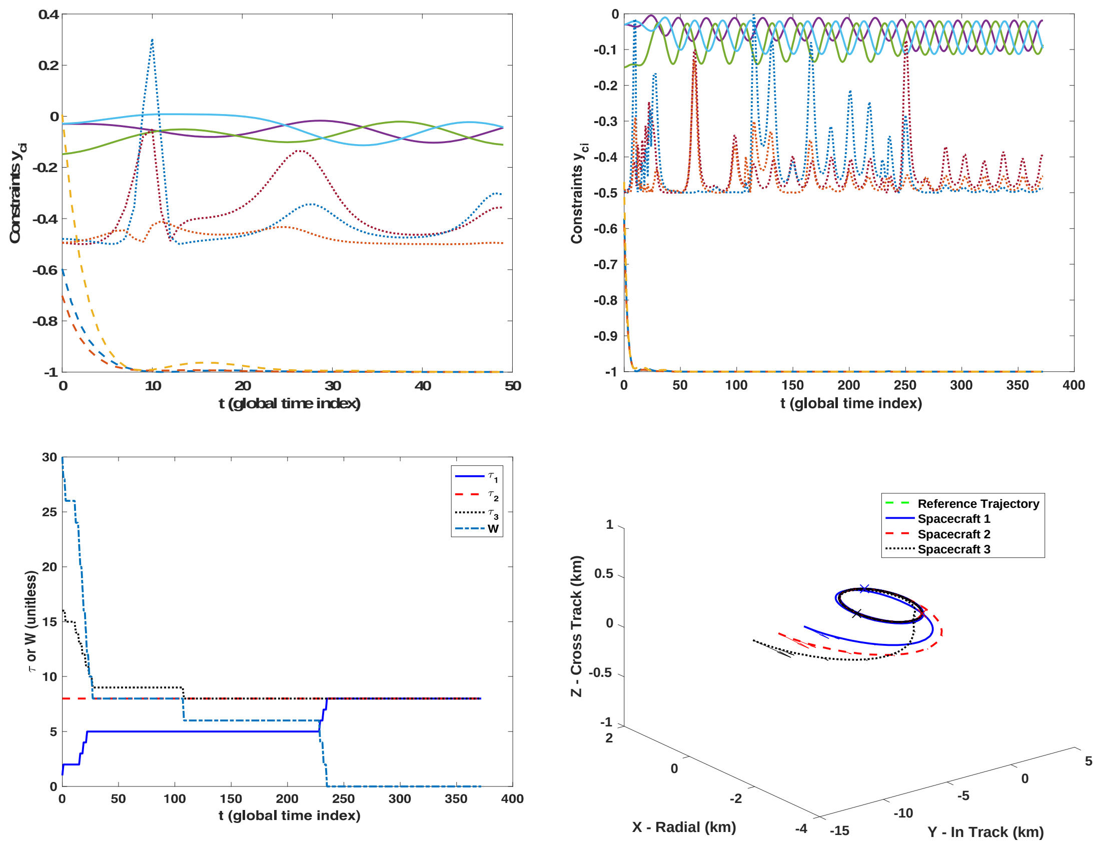


Figure 9. Demonstration of the TSG. Clockwise from top left: Constraints with TSG Inactive, Constraints with TSG Active, Parameters and W(t) vs. Time and System Trajectories

For constraints, solid lines denote separation distance constraints, dashed lines denote control constraints and dotted lines denote communication constraints.

5.0 CONCLUSIONS

Several outcomes have been achieved as the result of the one year research project into enhancing constrained predictive control for applications to autonomous vehicles and missions. For the challenging problem of controlling an under actuated autonomous spacecraft with two RWs, we have demonstrated, analyzed and enhanced, in terms of computations, the CNMPC solutions which are systematic, lead to stabilizing, discontinuous feedback laws with exponential convergence rates, and are suitable for on-board implementation. For the problem of the constrained control of the coupled rotational and translational relative motion dynamics, effective models have been established and a periodic reference governor strategy has been developed which permits the deputy spacecraft to dock with a chief spacecraft in orbit subject to control constraints and without collisions. For control of spacecraft formations, we have developed a predictive time shift governor control scheme which stabilizes several spacecraft to a given formation while adhering to control, communication, and collision constraints.

REFERENCES

[1] Blanchini, F., and Miani, S., *Set-Theoretic Methods in Control*, Birkhauser, 2008.

[2] Gilbert, E.G., and Kolmanovsky, I.V., "Nonlinear Tracking Control in the Presence of State and Control Constraints: A Generalized Reference Governor," *Automatica*, Vol. 38, No. 12, pp. 2063-2073, 2002.

[3] Ghaemi, R., Sun, J., and Kolmanovsky, I.V., "An Integrated Perturbation Analysis and Sequential Quadratic Programming Approach for Model Predictive Control," *Automatica*, Vol. 45, pp. 2412-2418, 2009.

[4] Grune, L., and Pannek, J., *Nonlinear Model Predictive Control*, Springer, 2011.

[5] Guibout, V.M., and Scheeres, D.J., "Spacecraft Formation Dynamics and Design," *Journal of Guidance, Control and Dynamics*, Vol. 29, No. 1, 2006.

[6] Kolmanovsky, I.V. and Gilbert, E.G., "Theory and Computation of Disturbance Invariant Sets for Discrete-Time Linear Systems," *Mathematical Problems in Engineering*, Vol. 4, No. 4, pp. 317–367, 1998.

[7] Kalabic, U., Gupta, R., Di Cairano, S., Bloch, A., and Kolmanovsky, I.V., "Constrained Spacecraft Attitude Control on SO(3) Using Reference Governors and Nonlinear Model Predictive Control," *Proceedings of 2014 American Control Conference*, Portland, OR, 2014.

[8] Mayne, D.Q., "Model Predictive Control: Recent Developments and Future Promise," *Automatica*, Vol. 50, No. 12, pp. 2967–2986, 2014.

[9] Petersen, C., Baldwin, M., and Kolmanovsky, I.V., "Model Predictive Control Guidance with Extended Command Governor Inner-Loop Flight Control for Hypersonic Vehicles," *Proceedings of AIAA Guidance, Navigation and Control Conference*, AIAA 2013, Boston, MA.

[10] Starek, J., and Kolmanovsky, I.V., "Nonlinear Model Predictive Control Strategy for Low Thrust Spacecraft Missions," *Optimal Control, Applications and Methods*, Vol. 35, pp. 1-20, 2014.

[11] Weiss, A., Petersen, C., Baldwin, M., Erwin, R.S., and Kolmanovsky, I.V., "Safe Positively Invariant Sets for Spacecraft Obstacle Avoidance," *Journal of Guidance, Control, and Dynamics*, Vol. 38, No. 4, pp. 720-732, 2015.

[12] Weiss, A., Leve, F., Baldwin, M., Forbes, J., and Kolmanovsky, I.V., "Spacecraft Constrained Attitude Control Using Positively Invariant Constraint Admissible Sets on SO(3)$\times R^3$," *Proceedings of 2014 American Control Conference*, Portland, OR, 2014.

[13] Weiss, A., Baldwin, M., Erwin, R.S., and Kolmanovsky, I.V., "Model Predictive Control for Spacecraft Rendezvous and Docking: Strategies for Handling Constraints and Case Studies," *IEEE Transactions on Control Systems Technology*, Vol. 24, No. 4, pp. 1638-1647, 2015.

[14] Petersen, C., and Kolmanovsky, I., "Coupled Translation and Rotation Dynamics for Precise Rendezvous and Docking with Periodic Reference Governor Control Scheme," *Proceedings of Space Flight Mechanics Meeting*, No. 16-507, AAS/AIAA, Napa, CA, 2016.

[15] Walker, K., Samadi, B., Huang, M., Gerhard, J., et al., "Design Environment for Nonlinear Model Predictive Control," SAE International, Technical Paper 2016-01-0627, 2016.

[16] N.A., "Aerospace Technology," Advanced Technology Institute LLC, 2016, http://www.ati-space.com/ATI.files/11_index.files/utyuu_aboute.htm, [retrieved 16-October-2014].

[17] Pingree, P.J., "Advancing NASA's On-Board Processing Capabilities with Reconfigurable FPGA Technologies," *Aerospace Technologies Advancements*, edited by T.T. Arif, INTECH, 2010, pp. 69–86.

[18] Leve, F.A., Hamilton, B.J., and Peck, M.A., "Momentum Device Technology Tradeoffs," *Spacecraft Momentum Control Systems*, Vol. 1010, Springer, 2015, pp. 49–55.

[19] Leve, F.A., Hamilton, B.J., and Peck, M.A., "Momentum Control Devices and Attitude Control Systems," *Spacecraft Momentum Control Systems*, Vol. 1010, Springer, 2015, pp. 5–10.

[20] N.A., "Boiler Optimization Systems," ffwd *Power Generation Special Issue*, 2011, http://www02.abb.com, [retrieved 10-March-2016].

[21] N.A., "Basell Optimizes Ethylene Plant with Honeywell MPC and RTO," *Honeywell Case Study*, Jan., 2007, https://www.honeywellprocess.com, [retrieved 10-March-2016].

[22] Diehl, M., Uslu, I., Findeisen, R., Schwarzkopf, S., Allgöwer, F., Bock, H.G., Bürner, T., Gilles, E.D., Kienle, A., Schlöder, J.P., and Stein, E., "Real-Time Optimization for Large Scale Processes: Nonlinear Model Predictive Control of a High Purity Distillation Column," in *Online Optimization of Large Scale Systems*, 2001, pp. 363–383.

[23] Diehl, M., Bock, H.G., Schlöder, J.P., Findeisen, R., Nagu, Z., Allgöwer, F., "Real-Time Optimization and Nonlinear Model Predictive Control of Processes Governed by Differential-Algebraic Equations," *Journal of Process Control*, Vol. 12, Issue 4, Elsevier, 2002, pp. 577–585.

[24] Bemporad, A., Morari, M., Dua, V., and Pistikopoulos, E., "The Explicit Linear Quadratic Regulator for Constrained Systems," *Automatica*, Vol. 28, Issue 1, Elsevier, 2002, pp. 3–20.

[25] Seron, M.M., Donà, J.A. and Goodwin, G.C., "Global Analytical Model Predictive Control with Input Constraints," *Proceedings of the 39th Conference on Decision and Control*, IEEE, Sydney, Australia, 2000.

[26] Tøndel, P., Johansen, T.A., and Bemporad, A., "An Algorithm for Multi-Parametric Quadratic Programming and Explicit MPC Solutions," *Automatica*, Vol. 39, Issue 3, Elsevier, 2003, pp. 489– 497.

[27] Grieder, P., Borrelli, F., Torrisi, F., and Morari, M., “Computation of the Constrained Infinite Time Linear Quadratic Regulator,” *Automatica*, Vol. 40, Issue 4, Elsevier, 2004, pp. 701–708.

[28] Dinh, Q.T., Savorgnan, C., and Diehl, M., “Adjoint-Based Predictor-Corrector Sequential Convex Programming for Parametric Nonlinear Optimization,” *SIAM Journal on Optimization*, Vol. 22, No. 4, 2012, pp. 1258–1284.

[29] Zavala, V.M., Laird, C.D., and Biegler, L.T., “A Fast Moving Horizon Estimation Algorithm Based on Nonlinear Programming Sensitivity,” *Journal of Process Control*, Vol. 18, Issue 9, Elsevier, 2008, pp. 876–884.

[30] Kungurtsev, V. and Diehl, M., “SQP Methods for Parametric Nonlinear Optimization,” Optimization Online, 2014, pp. 1–35.

[31] Butts, K., Dontchev, A., Huang, M., Kolmanovsky, I.V., “A Perturbed Chord (Newton-Kantorovich) Method for Constrained Nonlinear Model Predictive Control,” *Proceedings of 10th IFAC Symposium on Nonlinear Control Systems*, Monterey, California, 2016.

LIST OF SYMBOLS, ABBREVIATIONS, AND ACRONYMS

Acronyms:

COM	Center-of-mass
CNMPC	Constrained Nonlinear Model Predictive Control
FPGAs	Field Programmable Gate Arrays
MPC	Model Predictive Control
LQR	Linear Quadratic Regulator
PLQR	Periodic Linear Quadratic Regulator
RWs	Reaction Wheels
TSG	Time Shift Governor

Symbols:

A	Discrete time dynamics matrix
B	Discrete time input matrix
C	Discrete time output matrix
D	Discrete time direct feed-through matrix
ϕ	Roll angle
θ	Pitch angle
ψ	Yaw angle
Θ	Vector of Euler angles
$\mathrm{M}(\Theta)$	Kinematics matrix
ν_1, ν_2	RW speeds
u_1, u_2	Control signas
α_1, α_2	Constants
β_1, β_2	Constants

X	State vector
U	Control vector, input to the model
Y	Output vector from the model
$X^{(i)}$	State of the ith spacecraft in the formation
$U^{(i)}$	Control applied to the ith spacecraft in the formation
Q	State weighting matrix
R	Control weighting matrix
r_1, r_2	Control weights
N	Prediction horizon of MPC problem
$(\ )_{max}$	Maximum bound
λ	Auxiliary variable
ρ	Distance from a point on the chief spacecraft to the point on the deputy spacecraft
$\bar{q}$	Vector part of the quaternion
T	Sampling period
$(\quad)_k$	Designates time instant at which a vector or a matrix are sampled
ω_c	Angular velocity of the chief
O_∞	Output constraint admissible set
K	Control gain matrix
Γ	Design matrix in the control law
r^*	Actual reference command
v	Modified reference command by reference governor
$*\,(t)$	Sampled value at the discrete-time instant, t
$\Theta^{(di)}$	Desired phase shift along the trajectory
τ	Time shift parameter
p	Parameter driven by the parameter governors

$\mathbb{X}$ State constraint set

$\mathbb{U}$ Control constraint set

Ψ Set of feasible parameter values

W Parameter-dependent term in the cost used by the parameter governor

Ω State and control-dependent term in the cost used by the parameter governor

J Total cost used by the parameter governor

$diag$ Diagonal matrix

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