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14. ABSTRACT This report provides an overview of a one-year research project designed to strengthen the mathematical foundations of the social sciences. A major focus of this project has been to explore the way in which an axiomatic methodology can help integrate social theory and higher mathematical models as part of the larger research process.

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Report Title

Final Report: HOMOTOPY TYPES AND SOCIAL THEORY: THEORETICAL FOUNDATIONS OF STRATEGIC DYNAMICS

ABSTRACT

This report provides an overview of a one-year research project designed to strengthen the mathematical foundations of the social sciences. A major focus of this project has been to explore the way in which an axiomatic methodology can help integrate social theory and higher mathematical models as part of the larger research process.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

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TOTAL:

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(b) Papers published in non-peer-reviewed journals (N/A for none)

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Number of Papers published in non peer-reviewed journals:

(c) Presentations

Universal constraints of social order: A formal foundation. Presentation at the 2015 American Sociological Association.

Causal analysis and mechanism design: Reconciliation via type theory. Presentation at the 2015 International Network of Analytical Sociologists, Boston.

Sorokin cycles and axial transformations: A road forward. Presentation at the 2014 Conference of the International Society for Comparative Study of Civilizations.

Social theories and mechanisms: Meta-levels of social modeling. Prepared for Social Type Theory Tutorial.

Number of Presentations: 4.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

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Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

04/03/2016 1.00 David L. Sallach. Universal Constraints of Social Order: A Formal Foundation,
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TOTAL: 1

Number of Manuscripts:

Books

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Patents Submitted

Patents Awarded

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<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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Names of Post Doctorates

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Scientific Progress

INTRODUCTION

This Final Report to the Army Research Office provides an overview of a research project that draws upon higher mathematics to assist social researchers in modeling the complexities of cultural, social and economic processes and their interactions. The broader focus has been to establish a framework that helps move the social sciences toward a methodology that is both rigorous and expressive. Specifically, the project strategy was to: 1) specify a formal framework representing social interaction in all of its diverse forms, 2) assess and gauge the complexities of multiscale social interactions, and 3) establish a rigorous procedure for identifying and tracking major social processes, as well as the contextual factors that influence their direction and momentum.

The project made significant progress on each of the three priorities. From this foundation, new and more effective forms of social science methodology (and thus, social science insight), could be, and is being, developed.

AXIOMATIC METHODS

Praxeology was defined by Mises to be a “logic of action” (Hülsmann 2007:594), and the concept was elaborated in a number of his subsequent economic writings (1966; 1976; 1985; 2006). As Hoppe observed (2007:49), the last three of these books focus on establishing the logical and epistemological foundations of economics. He also noted that a cluster of cogent economic propositions was derived from Mises’ axiom of action.

Hoppe provides a concise overview of the resulting Austrian method. Work in Austrian economics provides the most fully developed example of axiomatic social science, one with the potential to be broadened beyond economics. Accordingly, it appears that this theoretical tradition provides a foundation for the task of determining how best to use axiomatic methods across social science disciplines, and relative to complex social settings.

However, the mathematics available for social modeling has advanced considerably since Mises’ times. Category theory (Lambek & Scott 1986; McLarty 1996; Baez 1998; Awodey 2010), algebraic topology (Sato 1999; Crole 1993:120-153) and, homotopy type theory (Voevodsky 2013) together define an integrated framework (cf., Rodin 2014a) that can support effective social models.

SOCIAL TRAJECTORIES

The variety of circumstances, priorities and strategies of operative social forces are vast, especially recognizing that social actors can be active at various scales, and their activities sometimes have cross-scale consequences. Accordingly, modeling the interaction of diverse social actors necessitates theorization at a high level of abstraction. Discussion in this section of the Report is designed to illustrate how such a general theory has been formulated and can be applied.

The initial formulation developed during the project was based on the copious interactions of three types of social relation: 1) advantage, 2) reciprocity and 3) dissolution. The first occurs when one or more social actors attempt to use a specific type of advantage to generate benefits at the expense of another (or others). The second occurs when two (or more) social actors give each other reciprocal benefits (or impose reciprocal costs). The third occurs when established interaction patterns weaken and, no longer commanding the priority they once did, fall into disuse. Dissolution includes a gradual reduction of resources from the focal relation, up to and including complete withdrawal.

There is no simple sequence among these relations. Rather, in any interaction, each social actor assesses whether to remain in the current relational pattern or to change a relation type. Further, each contemplated alteration will usually include an estimate of how other relevant social actors are likely to respond to a prospective change, this may influence the decision. In addition, as introduced below, it is necessary to explicitly take deceit into account. This goes beyond the initial concept, but makes working models more robust.

Advantage. When social actors have an advantage over others, they tend to persist in that relation. Advantage may be established and extended through strategy, industry and/or good fortune. Its preservation is typically a salient priority, even for actors without a strong strategic orientation.

The benefits of advantage may change over time. Based on shifting markets, they may become more valuable. Benefits may also become less valuable, perhaps due to ancillary costs, or other more attractive benefits may become available through another relation. The costs of retaining a benefit may increase, whether due to resistance of the subordinate actor, or to competition from other social actors who seek to replace the superordinate actor.

Reciprocity. The mutuality that arises in reciprocal relations tends to make them more stable than other types of relations. Each benefit (or cost) exchanged, elicits a corresponding response. Each actor in a reciprocal relation receives something quite distinct. For example, a parent and child may receive quite different benefits from their relationship and, further, these are likely to change over time. But, diverse forms notwithstanding, all such interactions are mutually reinforcing.

Dissolution. Relationships sometimes erode over time, or are rapidly disrupted. Benefits may decline and/or costs may increase. Actors may choose to withdraw from present relations; new actors may compete to participate in previously established relations. The deterioration of some relations may contribute to the establishment of new relations.

Deceit. Previous paragraphs imply the prevalence of open, straight-forward interactions, but deception may also play an important role. Actors may promise benefits that they cannot or will not provide. Similarly, they may make threats that they have no intention of carrying out. False promises and threats constitute a distinct layer that shapes the outcomes across all other relations.

SOCIAL AXIOMS

One of the most compelling advantages of the mathematical foundation described here is the ability to express the relevant relations axiomatically and, thus, be able to derive extended insights therefrom. Category and type theories have been used to advance axiomatic methods as well (cf., Rodin 2014a; 2014b). In particular, the identification and development of a constructive axiomatics that incorporates types (cf., Rodin 2014a), can be readily mapped to empirical patterns. Following are some examples of such axioms, as developed in the present project.

Axiomatic Advantage. Advantage is a relation in which one actor (or a group of actors) benefits more than the other(s) in the relation. While the ways in which either actor benefits may vary, each balances the costs and benefits of the relation.

Axiom 1a. The more intense the negative affect directed at their target, the greater the advantage sought by a social actor or cultural trend.

Axiom 1b. The greater the advantage sought by a social actor or cultural trend, the more intense the negative affect directed at their target.

Axiom 1 illustrates how diverse motives can be mutually reinforcing. Axiom 1a is punitive, seeking advantage from the actions that aroused the negative affect. Axiom 1b is self-aggrandizing, by interpreting (excessively or falsely) actions as requiring relative advantages. As long as their focus is aligned within a single actor, 1a and 1b can comfortably coexist.

Axiomatic Reciprocity. Reciprocity is a relation in which actors mutually benefit from (or are mutually damaged by) their repeated exchanges. Each benefit (or cost) reinforces the overall pattern.

Axiom 2. Within available resources, a change in benefits (costs) by one party evokes a comparable change by the other.

Because the resulting action patterns are mutually reinforcing, reciprocity is an open-ended process.

Axiomatic Dissolution. Dissolution is the process by which an established relation breaks down or is compelled to end. In general, this occurs when the reward or cost is either too small to matter, or too large to be fulfilled.

Axiom 3a. When the results of a recurrent interaction appear to be inconsequential, the motive to continue the relation dissipates.

Axiom 3b. When the requirements of a recurrent interaction become infeasible, the relation is terminated or radically restructured.

Axiomatic Extremes. Another way that axiomatic models contribute to the clarity of social science theory is through the identification of extreme axioms that appear rarely, but are powerful in their effects.

Several examples of extremal axioms include:

Axiom 4a. The more directly that the path of a social actor or cultural trend approaches an idealized conceptual pole, the higher is the n-category required to characterize its structure.

Axiom 4b. The more obliquely that a social actor or cultural trend approaches an idealized conceptual pole, the stronger the attraction that local and empirical forces will have on the emerging path trajectory.

Axiom 5. The more rapidly a social actor or cultural trend approaches an idealized conceptual pole, the more morphisms it manifests.

These axioms are not exhaustive, but they do illustrate some of the potential of axiomatic analysis. When coupled with other axioms of an (emerging) social type theory, and the parameters that characterize particular social and/or historical circumstances, extremal axioms can contribute to the characterization and anticipation of disruptive social trends and outcomes.

RESULTS AND PROSPECTS

The focus of this report is to summarize how the project employed axiomatic methods to integrate social theory and higher mathematical models and, thereby, make both more readily available to working analysts who are assessing potentially disruptive events and trends. This integration moves social analysis toward a higher and more cogent framework.

This framework has been shown to provide effective capabilities for analysis and outcome projection. It is also beneficial that it is applicable across a wide range of social science domains and policy issues. The project described in this report has defined the foundation for such a framework, and suggests the range of social issues to which that framework can be applied.

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