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Conservative Bin-to-Bin Fractional Collisions

Robert Martin

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Spacecraft Propulsion Branch
Air Force Research Laboratory
Edwards Air Force Base, CA USA

30th International Symposium on Rarefied Gas Dynamics
Distribution A: Approved for Public Release; Distribution Unlimited; PA #16326

Outline

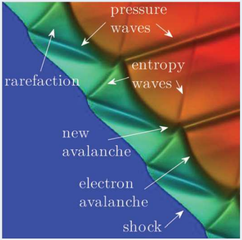
Important Collisions in Spacecraft Propulsion:

- Discharge and Breakdown in FRC
- Collisional Radiative Cooling/Ionization
- Combustion Chemistry

Common Features in Spacecraft Collisions:

- Relevant Densities Spanning Many Orders of Magnitude — 6+
- Transitions from Collisional to Collisionless
- Tiny Early e^- or Radical Populations Critical to Induction Delay
- Many types of Inelastic Collisions with Unknown Effects on Distribution Shapes

Shock Ionization



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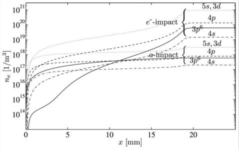

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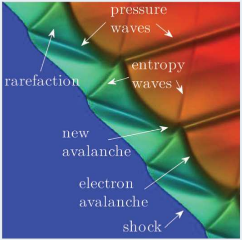
Need Low Noise & High Dynamic Range Collision Algorithms

Shock Ionization



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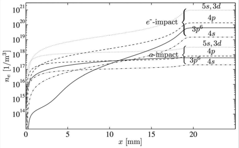

Standard Collision Models

Previous Collision Methods:

- Monte Carlo Collisions (MCC)
  - Particles Collide with Background "Fluid"
  - Often Used in Plasma/PIC Simulation
  - Ion-e^- Collisions Assume Stationary Ions
  - No Conservation/Detailed Balance
- Direct Simulation Monte Carlo Collisions (DSMC)
  - Most Modern Versions use No-Time Counter (NTC) Method
  - Conservative/Reversible Collision
  - Satisfies Detailed Balance
  - Subset of Possible Collisions Sampled
  - Random Selection vs Z_{ij} for All/Nothing Collision

All Random Flip vs Number of Collisions: $Z_{ij} = \frac{n_i n_j}{2} \langle \sigma v \rangle$ dt

Variable Weights for Dynamic Range

Continuum to Discrete Representation:

- Many Particles $\overset{\sim}{\rightarrow}$ Continuous Distribution

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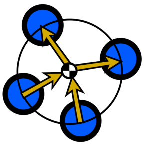
Variable Weight "All-or-Nothing" Collisions?



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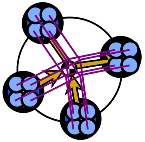

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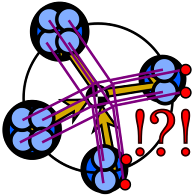
Variable Weight "All-or-Nothing" Collisions?
Physically Inconsistent!
(Mixing Violates Momentum/Energy Conservation)



NTC Collisions:

- (Collision Rate Volume):(Cell Volume)

$$Z_{ij} = \frac{n_i n_j}{2} \left\langle \sigma v \right\rangle_{ij} \mathrm{dt} = \frac{w_i w_j}{2V_{cell}^2} \left\langle \sigma v \right\rangle_{ij} \mathrm{dt}$$

Fractional-NTC Collisions:

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$$P_{ij} = w \langle \sigma v \rangle_{ij} \mathrm{dt}/V_{cell}$$
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$$N_{select} = \frac{N_p^2}{2} F_n \langle \sigma v \rangle_{ij}^{max} \mathrm{dt}/V_{cell}$$

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Collide if:

$$\mathtt{Rand(1)} < \frac{N_{collide}}{N_{select}} = \frac{P_{ij}}{P_{max}} = \frac{\langle \sigma v \rangle_{ij}}{\langle \sigma v \rangle_{ij}^{max}}$$

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- Select f by Cost/Accuracy Tradeoff

$$N_{select} = f N_p$$

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Fractional-NTC Collisions:

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Fractional-NTC Collisions:

- Select f by Cost/Accuracy Tradeoff
- Collision Δw Scaled for Skipped
- Add Particles & Original Reduced

$$N_{select} = f\,N_p$$

$$\Delta w_{ij} = \frac{N_p^2/2}{N_{select}} Z_{ij}$$

$$w_i = w_i - \Delta w_{ij} \;\&\; w_j = w_j - \Delta w_{ij}$$

$$w_{(N_p+1)} = \Delta w_{ij} \;\&\; w_{(N_p+2)} = \Delta w_{ij}$$

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- +2 Particles/Collision! → Must Merge

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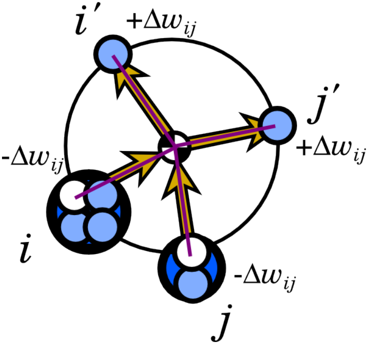

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Stochastic Weighted Particle Method:

- Developed by Rjasanow & Wagner

Attempted Collisions/Cell:

$$\nu = f(2\bar{w} - w_{min}) N_p (N_p - 1) \langle \sigma v \rangle^{max} dt$$

Select Pair (i,j) if:

$$\texttt{Rand} < \frac{w_i + w_j - w_{min}}{N_p(N_p - 1)(2\bar{w} - w_{min})}$$

-or-

$$\texttt{Rand} < \frac{w_i + w_j - w_{min}}{(2w_{max} - w_{min})}$$

Collide If:

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Perform Standard VHS Collisions

Generate/Modify Particles with:

$$\pm \Delta w / f = \pm \texttt{min}(w_i, w_j)/f$$

Update $\langle \sigma v \rangle^{max}$

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- Still Requires Merge $w_i \neq \texttt{const}$

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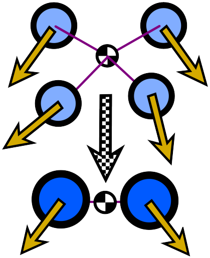
Update $\langle \sigma v \rangle^{max}$

Merge to Pair → DOF for Conservation:

- (n+2):2 yields Exact Mass, Momentum, and Kinetic Energy Conservation
- Applied Spatially also Shown to Conserve Electrostatic Energy
- Though Energy Conserving, Still Thermalizes VDF



$$w_{cell} = \sum_i^{(n+2)} w_i$$
$$\overline{\vec{v}} = \frac{1}{w_{cell}} \sum_i^{(n+2)} w_i \vec{v}_i$$
$$\overline{V^2} = \frac{1}{w_{cell}} \sum_i^{(n+2)} w_i \left(\vec{v}_i - \overline{\vec{v}}\right)^2$$

$$w_{(a/b)} = w_m/2$$
$$\vec{v}_{(a/b)} = \overline{\vec{v}} \pm \hat{\mathcal{R}} \sqrt{\overline{V^2}}$$

Similarly: $\vec{x}_{(a/b)} = \overline{\vec{x}} \pm \hat{\mathcal{R}} \sqrt{\overline{X^2}}$

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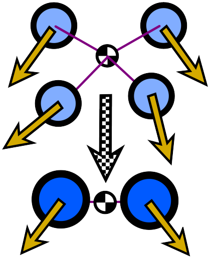
Selection of Near Neighbors in VDF Limits Thermalization

(≈ Near Neighbor Pairs in 2:1 Merges that Limit Numerical Cooling)



$$w_{cell} = \sum_i^{(n+2)} w_i$$

$$\bar{\vec{v}} = \frac{1}{w_{cell}} \sum_i^{(n+2)} w_i \vec{v}_i$$

$$\overline{V^2} = \frac{1}{w_{cell}} \sum_i^{(n+2)} w_i \left(\vec{v}_i - \bar{\vec{v}}\right)^2$$

$$w_{(a/b)} = w_m/2$$

$$\vec{v}_{(a/b)} = \bar{\vec{v}} \pm \hat{\mathcal{R}} \sqrt{\overline{V^2}}$$

Similarly: $\vec{x}_{(a/b)} = \bar{\vec{x}} \pm \hat{\mathcal{R}} \sqrt{\overline{X^2}}$

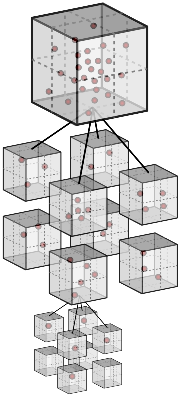
Merge to Pair → DOF for Conservation:

- (n+2):2 yields Exact Mass, Momentum, and Kinetic Energy Conservation
- Applied Spatially also Shown to Conserve Electrostatic Energy
- Though Energy Conserving, Still Thermalizes VDF

Selection of Near Neighbors in VDF Limits Thermalization

(≈ Near Neighbor Pairs in 2:1 Merges that Limit Numerical Cooling)

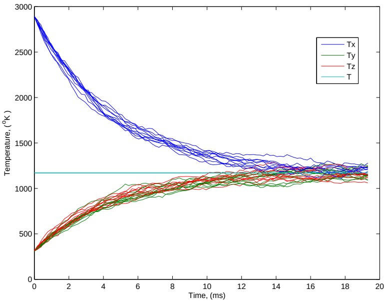
Octree Velocity Bins



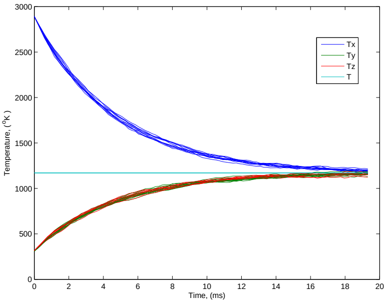
Efficient Neighbor Selection

Bi-Maxwellian Thermalization Results

Original-NTC



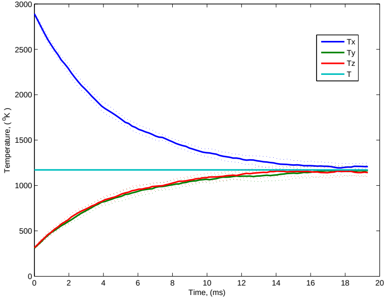
Fractional-NTC



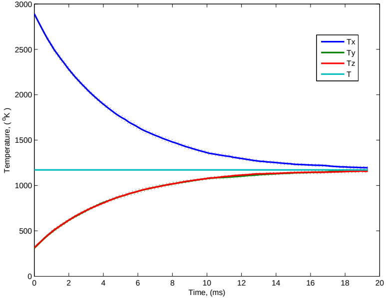
Comparison of 10x Runs from Same Initial Distribution

Bi-Maxwellian Thermalization Results

Original-NTC



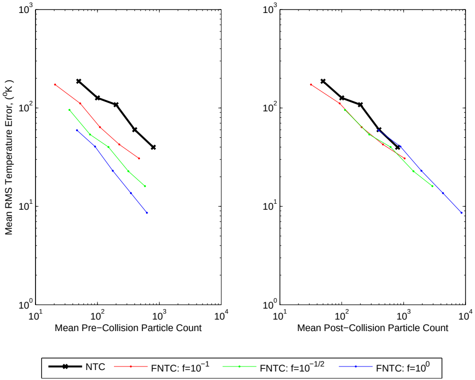
Fractional-NTC



Mean and RMS Fluctuation of Sample Runs

Fluctuations Level Tuneable with f Independent of Particles Count

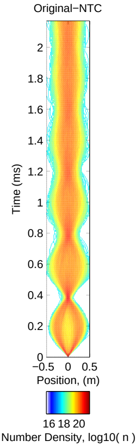
Bi-Maxwellian Thermalization Results



Fluctuations Level Tuneable with f Independent of Particles Count

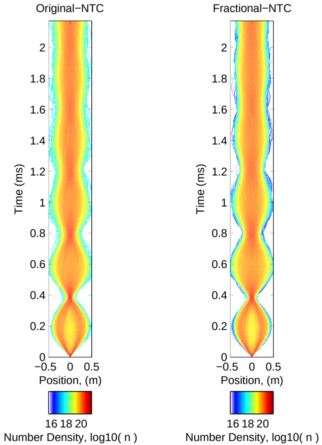
- Initial Bi-Maxwellian Distribution in Potential Well

- Initial Bi-Maxwellian Distribution in Potential Well
- NTC Collisions Results in Beam Thermalization

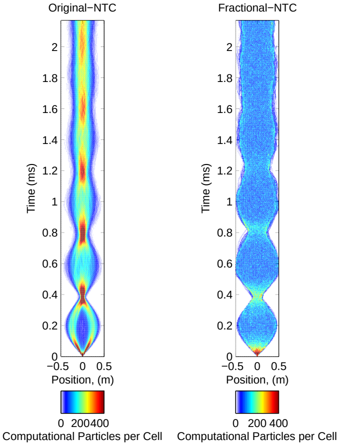


Collisional Beams in Potential Well

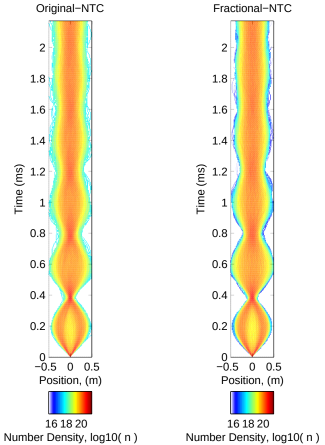
- Initial Bi-Maxwellian Distribution in Potential Well
- NTC Collisions Results in Beam Thermalization
- Fractional-NTC Collisions Produce Same Behavior



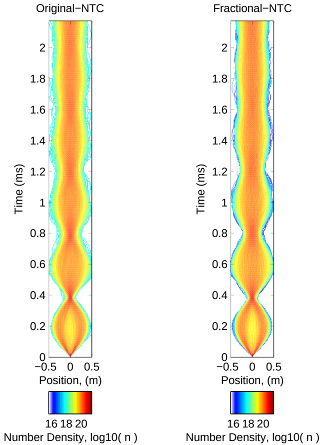
- Initial Bi-Maxwellian Distribution in Potential Well
- NTC Collisions Results in Beam Thermalization
- Fractional-NTC Collisions Produce Same Behavior
- Particles/Cell Dramatically Different



- Initial Bi-Maxwellian Distribution in Potential Well
- NTC Collisions Results in Beam Thermalization
- Fractional-NTC Collisions Produce Same Behavior
- Particles/Cell Dramatically Different
- Fringe Extends to Lower Densities with Variable Weights

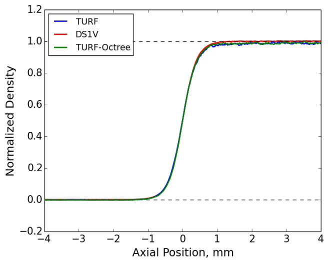


- Initial Bi-Maxwellian Distribution in Potential Well
- NTC Collisions Results in Beam Thermalization
- Fractional-NTC Collisions Produce Same Behavior
- Particles/Cell Dramatically Different
- Fringe Extends to Lower Densities with Variable Weights
- Relative 'Error' Unknown without Analytical Solution or High Fidelity Simulation



1D Normal Argon Shock Test

- Simple Verification vs. DS1V
- Initial Conditions:
  $T_0 = 293\text{K}$, $n_0 = 1\text{E}22/\text{m}^3$, $v_0 = 637.4(\text{m/s})$
- Initial Jump to Post-Shock at 1cm
- VHS Collisions:
  T_{ref}=273K, d_{ref}=4.17Å, ω_{VHS}=0.81

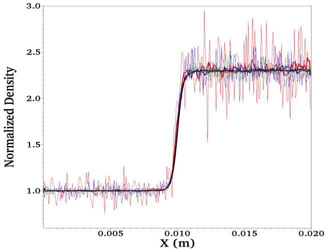


Mach 2 Argon Shock

1D Normal Argon Shock Test

- Simple Verification vs. DS1V
- Initial Conditions:
  $T_0 = 293\text{K}$, $n_0 = 1\text{E}22/\text{m}^3$, $v_0 = 637.4(\text{m/s})$
- Initial Jump to Post-Shock at 1cm
- VHS Collisions:
  T_{ref}=273K, d_{ref}=4.17Å, ω_{VHS}=0.81
- Time Average:
  $\bar{n}$ from t$\in [80, 100)\mu$s

TURF - SWPM+Octree

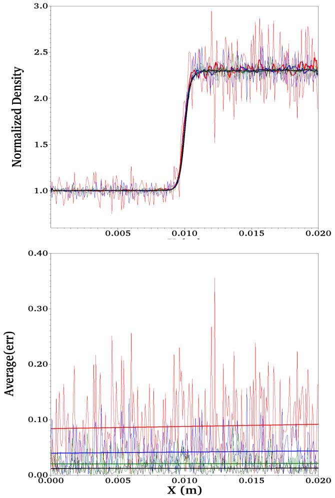


Target N/Cell Quadrupled per Line

1D Normal Argon Shock Test

- Simple Verification vs. DS1V
- Initial Conditions:
  $T_0 = 293\text{K}$, $n_0 = 1\text{E}22/\text{m}^3$, $v_0 = 637.4(\text{m/s})$
- Initial Jump to Post-Shock at 1cm
- VHS Collisions:
  T_{ref}=273K, d_{ref}=4.17Å, ω_{VHS}=0.81
- Time Average:
  $\bar{n}$ from t$\in [80, 100)\mu$s
- Error (Normalized L_1):
  $err = |n - \bar{n}|/\bar{n}$
- Error Controlled: $err \propto \sqrt{N/cell}$

TURF - SWPM+Octree

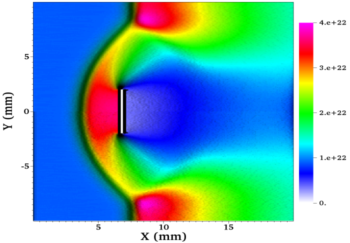


Target N/Cell Quadrupled per Line

2D Argon Shock Test

- Initial Conditions like M=2 Except: $v_0 = 2550\text{m/s}$
- Specular: x=5 – 5.04mm with y=±2mm
- Half Domain Modeled: $80\mu\text{m} \times 80\mu\text{m}$ Cells

TURF: n - Standard DSMC

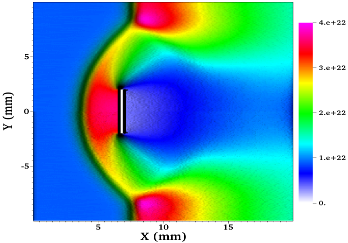


TURF: n - SWPM+Octree

2D Argon Shock Test

- Initial Conditions like M=2 Except: $v_0 = 2550$m/s
- Specular: x=5 − 5.04mm with y=±2mm
- Half Domain Modeled: 80μm × 80μm Cells
- Time Average: $\bar{n}$ from t∈ [80, 100)μs
- SWPM Similar to Standard DSMC

TURF: n - Standard DSMC

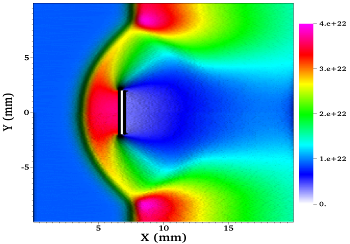


TURF: n - SWPM+Octree

2D Argon Shock Test

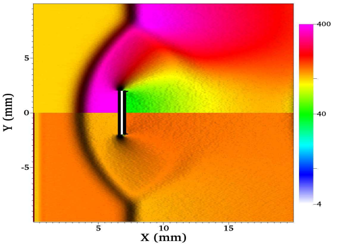
- Initial Conditions like M=2 Except:
  $v_0 = 2550\text{m/s}$
- Specular: x=5 − 5.04mm with y=±2mm
- Half Domain Modeled:
  $80\mu\text{m} \times 80\mu\text{m}$ Cells
- Time Average:
  $\bar{n}$ from t$\in [80, 100)\mu$s
- SWPM Similar to Standard DSMC
- Despite Different $Np/Cell$

TURF: n - Standard DSMC



TURF: n - SWPM+Octree

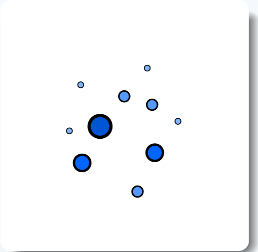
TURF Np/Cell - Standard DSMC



TURF Np/Cell - SWPM+Octree

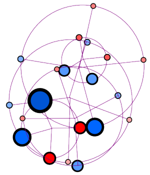
Issue with Collide then Merge

- Larger N_{select} → Better Approx. of Collision Integral

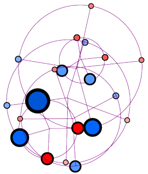


Issue with Collide then Merge

- Larger $N_{select} \rightarrow$ Better Approx. of Collision Integral
- f-NTC Produces 2x-Particles per $N_{select} = f\ N_p$
- Particle Memory Requires $\propto\ N_{max} \rightarrow (1+2f)N_{max}$

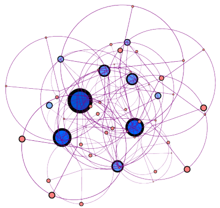


- Larger $N_{select} \to$ Better Approx. of Collision Integral
- f-NTC Produces 2x-Particles per $N_{select} = f\ N_p$
- Particle Memory Requires $\propto\ N_{max} \to (1+2f)N_{max}$
- For DSMC-like Results, $f \approx O(1)$

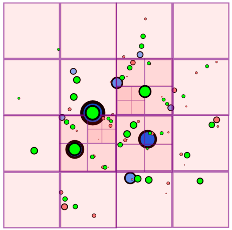


Issue with Collide then Merge

- Larger $N_{select} \rightarrow$ Better Approx. of Collision Integral
- f-NTC Produces 2x-Particles per $N_{select} = f\ N_p$
- Particle Memory Requires $\propto\ N_{max} \rightarrow (1+2f)N_{max}$
- For DSMC-like Results, $f \approx O(1)$
- Time Accurate or Dense Simulations, $f \approx O(10)+?$

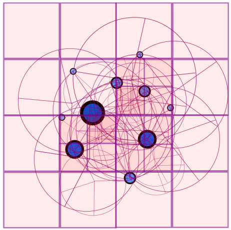


- Larger N_{select} $\rightarrow$ Better Approx. of Collision Integral
- f-NTC Produces 2x-Particles per $N_{select} = f\ N_p$
- Particle Memory Requires $\propto\ N_{max} \rightarrow (1+2f)N_{max}$
- For DSMC-like Results, $f \approx O(1)$
- Time Accurate or Dense Simulations, $f \approx O(10)+?$
- Merge Contracts back to $O(N_{max})$ Particles
- Merge Immediately after Collide per Spatial Cell?..
- Sort for Merge still $\propto (1+2f)\ log(1+2f)$?



Issue with Collide then Merge

- Larger $N_{select} \rightarrow$ Better Approx. of Collision Integral
- f-NTC Produces 2x-Particles per $N_{select} = f\ N_p$
- Particle Memory Requires $\propto\ N_{max} \rightarrow (1+2f)N_{max}$
- For DSMC-like Results, $f \approx O(1)$
- Time Accurate or Dense Simulations, $f \approx O(10)+?$
- Merge Contracts back to $O(N_{max})$ Particles
- Merge Immediately after Collide per Spatial Cell?..
- Sort for Merge still $\propto (1+2f)\ log(1+2f)$?
- Combine Collision and Merge in Single Step?



- Fractional Collision as Rate Equation

$$\begin{bmatrix} \vdots \\ \dot{w}_i \\ \vdots \\ \dot{w}_j \\ \vdots \\ \dot{w}_{i'} \\ \vdots \\ \dot{w}_{j'} \\ \vdots \end{bmatrix} = \sum_{k=1}^{N_{select}} \begin{bmatrix} \vdots \\ -w_i \left\langle \sigma v \right\rangle_{ij}^{k} w_j \\ \vdots \\ -w_i \left\langle \sigma v \right\rangle_{ij}^{k} w_j \\ \vdots \\ w_i \left\langle \sigma v \right\rangle_{ij}^{k} w_j \\ \vdots \\ w_i \left\langle \sigma v \right\rangle_{ij}^{k} w_j \\ \vdots \end{bmatrix}$$

- Fractional Collision as Rate Equation
- Bin Moments needed for Particle Pairs

$$\begin{bmatrix} \dot{w}_i \\ \dot{w}_j \\ \dot{w}_{i'} \\ \dot{w}_{j'} \\ - \\ \dot{(wv)}_i \\ \dot{(wv)}_j \\ \dot{(wv)}_{i'} \\ \dot{(wv)}_{j'} \\ - \\ \dot{(wv^2)}_i \\ \dot{(wv^2)}_j \\ \dot{(wv^2)}_{i'} \\ \dot{(wv^2)}_{j'} \end{bmatrix} = \sum_{k=1}^{N_{select}} \begin{bmatrix} -\Delta w_{ij} \\ -\Delta w_{ij} \\ \Delta w_{ij} \\ \Delta w_{ij} \\ - \\ -\Delta w_{ij} v_i \\ -\Delta w_{ij} v_j \\ \Delta w_{ij} v_{i'} \\ \Delta w_{ij} v_{j'} \\ - \\ -\Delta w_{ij} v_i^2 \\ -\Delta w_{ij} v_j^2 \\ \Delta w_{ij} v_{i'}^2 \\ \Delta w_{ij} v_{j'}^2 \end{bmatrix}$$

- Fractional Collision as Rate Equation
- Bin Moments needed for Particle Pairs
- Particle Pairs (i,j) Picked Randomly
- DSMC-like Collision (VHS,VSS,etc.)
  Random $\chi, \theta \rightarrow (v_{i'}, v_{j'})$

$$\begin{bmatrix} \dot{w}_i \\ \dot{w}_j \\ \dot{w}_{i'} \\ \dot{w}_{j'} \\ - \\ \dot{(wv)}_i \\ \dot{(wv)}_j \\ \dot{(wv)}_{i'} \\ \dot{(wv)}_{j'} \\ - \\ \dot{(wv^2)}_i \\ \dot{(wv^2)}_j \\ \dot{(wv^2)}_{i'} \\ \dot{(wv^2)}_{j'} \end{bmatrix} = \sum_{k=1}^{N_{select}} \begin{bmatrix} -\Delta w_{ij} \\ -\Delta w_{ij} \\ \Delta w_{ij} \\ \Delta w_{ij} \\ - \\ -\Delta w_{ij} v_i \\ -\Delta w_{ij} v_j \\ \Delta w_{ij} v_{i'} \\ \Delta w_{ij} v_{j'} \\ - \\ -\Delta w_{ij} v_i^2 \\ -\Delta w_{ij} v_j^2 \\ \Delta w_{ij} v_{i'}^2 \\ \Delta w_{ij} v_{j'}^2 \end{bmatrix}$$

Collide to Bins

- Fractional Collision as Rate Equation
- Bin Moments needed for Particle Pairs
- Particle Pairs (i,j) Picked Randomly
- DSMC-like Collision (VHS,VSS,etc.)
  Random $\chi, \theta \rightarrow (v_{i'}, v_{j'})$
- Octree to Find i' and j' Bins
  $8^L \rightarrow$ Few Levels to Search

$$\begin{bmatrix} \dot{w}_i \\ \dot{w}_j \\ \dot{w}_{i'} \\ \dot{w}_{j'} \\ - \\ \dot{(wv)}_i \\ \dot{(wv)}_j \\ \dot{(wv)}_{i'} \\ \dot{(wv)}_{j'} \\ - \\ \dot{(wv^2)}_i \\ \dot{(wv^2)}_j \\ \dot{(wv^2)}_{i'} \\ \dot{(wv^2)}_{j'} \end{bmatrix} = \sum_{k=1}^{N_{select}} \begin{bmatrix} -\Delta w_{ij} \\ -\Delta w_{ij} \\ \Delta w_{ij} \\ \Delta w_{ij} \\ - \\ -\Delta w_{ij} v_i \\ -\Delta w_{ij} v_j \\ \Delta w_{ij} v_{i'} \\ \Delta w_{ij} v_{j'} \\ - \\ -\Delta w_{ij} v_i^2 \\ -\Delta w_{ij} v_j^2 \\ \Delta w_{ij} v_{i'}^2 \\ \Delta w_{ij} v_{j'}^2 \end{bmatrix}$$

- Fractional Collision as Rate Equation
- Bin Moments needed for Particle Pairs
- Particle Pairs (i,j) Picked Randomly
- DSMC-like Collision (VHS,VSS,etc.)
  Random $\chi, \theta \rightarrow (v_{i'}, v_{j'})$
- Octree to Find i' and j' Bins
  $8^L \rightarrow$ Few Levels to Search

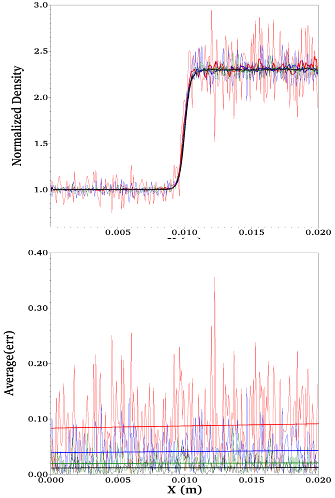
Conserve Mass, Momentum, and Energy
Memory Constant Independent of N^{select}

$$\begin{bmatrix} \dot{w}_i \\ \dot{w}_j \\ \dot{w}_{i'} \\ \dot{w}_{j'} \\ - \\ \dot{(wv)}_i \\ \dot{(wv)}_j \\ \dot{(wv)}_{i'} \\ \dot{(wv)}_{j'} \\ - \\ \dot{(wv^2)}_i \\ \dot{(wv^2)}_j \\ \dot{(wv^2)}_{i'} \\ \dot{(wv^2)}_{j'} \end{bmatrix} = \sum_{k=1}^{N_{select}} \begin{bmatrix} -\Delta w_{ij} \\ -\Delta w_{ij} \\ \Delta w_{ij} \\ \Delta w_{ij} \\ - \\ -\Delta w_{ij} v_i \\ -\Delta w_{ij} v_j \\ \Delta w_{ij} v_{i'} \\ \Delta w_{ij} v_{j'} \\ - \\ -\Delta w_{ij} v_i^2 \\ -\Delta w_{ij} v_j^2 \\ \Delta w_{ij} v_{i'}^2 \\ \Delta w_{ij} v_{j'}^2 \end{bmatrix}$$

1D Normal Argon Shock Test

- Mach 2 Case Repeated

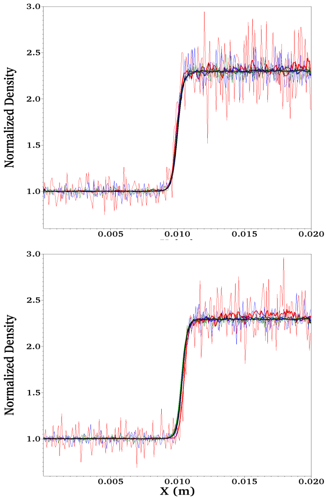
TURF - SWPM+Octree



1D Normal Argon Shock Test

- Mach 2 Case Repeated
- Bin-to-Bin Collsions Results Similar

TURF - SWPM+Octree

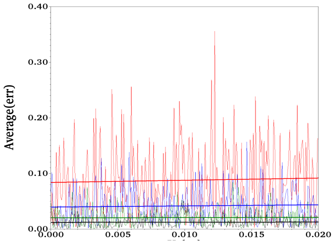


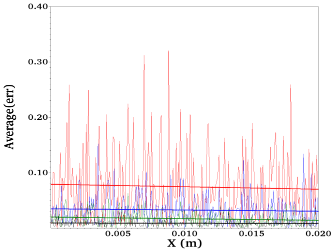
TURF - Bin to Bin

1D Normal Argon Shock Test

- Mach 2 Case Repeated
- Bin-to-Bin Collsions Results Similar
- Target *Np*/*Cell* Still Error Control
  (Target N/Cell Quadrupled per Line)

TURF - Octree



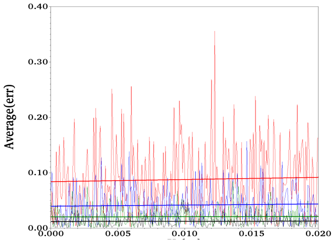


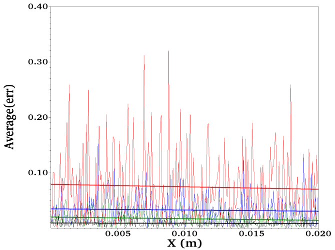
TURF - Bin to Bin

1D Normal Argon Shock Test

- Mach 2 Case Repeated
- Bin-to-Bin Collsions Results Similar
- Target *Np*/*Cell* Still Error Control (Target N/Cell Quadrupled per Line)
- Collision Core ≈3x Slower
- Non-Ideal: Dynamic Range Low

TURF - Octree



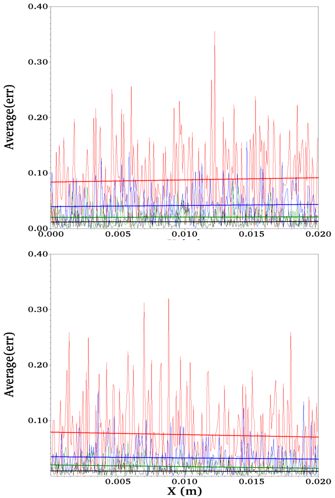


TURF - Bin to Bin

1D Normal Argon Shock Test

- Mach 2 Case Repeated
- Bin-to-Bin Collsions Results Similar
- Target $Np/Cell$ Still Error Control (Target N/Cell Quadrupled per Line)
- Collision Core ≈3x Slower
- Non-Ideal: Dynamic Range Low
- Proof-of-Concept with Real X-Section
- Expansion/Plume will be Better Case

TURF - Octree

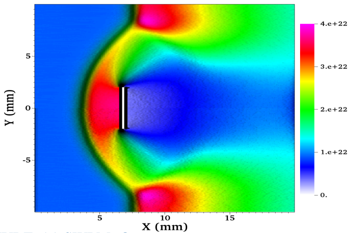


TURF - Bin to Bin

2D Argon Shock Test

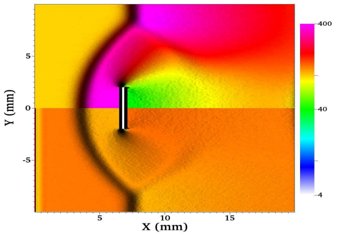
- Mach 8 Case Also Repeated

TURF: n - Standard DSMC



TURF: (n) SWPM+Octree

TURF: Np/Cell - Standard DSMC

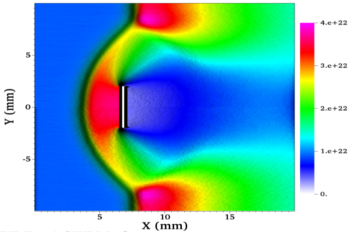


TURF: (Np/Cell) SWPM+Octree

2D Argon Shock Test

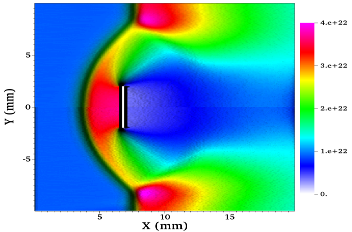
- Mach 8 Case Also Repeated
- Bin-to-Bin Collsions Results Similar

TURF: n - Standard DSMC



TURF: (n) SWPM+Octree

TURF: n - Standard DSMC



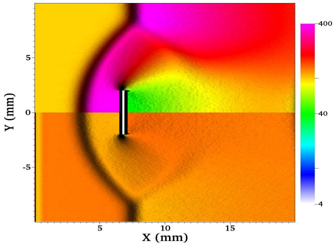
TURF: (n) Bin-to-Bin

Mach 8 Argon Bow Shock

2D Argon Shock Test

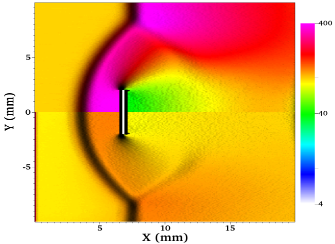
- Mach 8 Case Also Repeated
- Bin-to-Bin Collsions Results Similar
- Target *Np*/*Cell* Still Error Control

TURF: Np/Cell - Standard DSMC



TURF: (Np/Cell) SWPM+Octree

TURF: Np/Cell - Standard DSMC



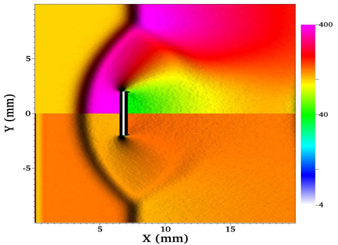
TURF: (Np/Cell) Bin-to-Bin

2D Argon Shock Test

- Mach 8 Case Also Repeated
- Bin-to-Bin Collsions Results Similar
- Target $Np/Cell$ Still Error Control
- B2B Run with f=4x Collisions
  (Note: SWPM+Octree f=1x)
-

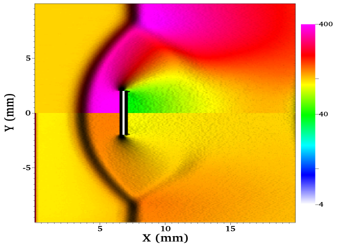
Standard - Collisions	548.9s	1x
Standard - Total Run	7945.3s	100%
SWPM+Octree - Collisions	2719.6s	4.95x
SWPM+Octree - Total Run	9542.4s	120%
Bin-to-Bin - Collisions	13163.6s	24.0x
Bin-to-Bin - Total Run	18860.5s	237%

TURF: Np/Cell - Standard DSMC



TURF: (Np/Cell) SWPM+Octree

TURF: Np/Cell - Standard DSMC



TURF: (Np/Cell) Bin-to-Bin

Mach 8 Argon Bow Shock

2D Argon Shock Test

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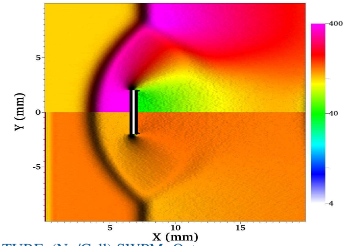
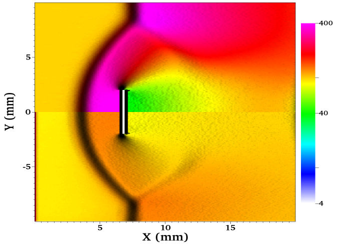
Bin-to-Bin - Collisions	13163.6s	24.0x
Bin-to-Bin - Total Run	18860.5s	237%
- Some Cost Compensated by Lower *Np*
- Too much Fill for Better Wake
- Significant Optimizations Still Needed
  (i.e. Data Structures, Sort->Sums, v-Bounds, Morton curve)

TURF: Np/Cell - Standard DSMC



TURF: (Np/Cell) SWPM+Octree
TURF: Np/Cell - Standard DSMC



TURF: (Np/Cell) Bin-to-Bin

CONCLUSION

- Standard Collision Incompatible with Variable Weight
- SWPM+Octree Option for Variable Weight Collision
- Bin-To-Bin Potentially Alleviates Memory Constraints
- Initial Verification vs. Standard Shock Cases Positive
- Limited Utility in Standard Shock Cases
- Performance with Strong Expansion/Plume Needed
- SWPM/Bin-to-Bin more Useful for Trace Species?

Thank You

Questions?