



**AFRL-AFOSR-VA-TR-2016-0283**

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CHASE: CONTROL OF HETEROGENEOUS AUTONOMOUS  
SENSORS FOR SITUATIONAL AWARENESS

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**08/03/2016**  
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<b>1. REPORT DATE (DD-MM-YYYY)</b> 31-07-2016	<b>2. REPORT TYPE</b> Final Project Report	<b>3. DATES COVERED (From - To)</b> August 2010 - April 2016
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<b>4. TITLE AND SUBTITLE</b> CHASE: CONTROL OF HETEROGENEOUS AUTONOMOUS SENSORS FOR SITUATIONAL AWARENESS	<b>5a. CONTRACT NUMBER</b>
	<b>5b. GRANT NUMBER</b> FA9550-10-1-0567
	<b>5c. PROGRAM ELEMENT NUMBER</b>

<b>6. AUTHOR(S)</b> D. E. Koditschek, PI	<b>5d. PROJECT NUMBER</b>
	<b>5e. TASK NUMBER</b>
	<b>5f. WORK UNIT NUMBER</b>

<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> University of Pennsylvania	<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>
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<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> Air Force Office of Scientific Research 875 North Randolph Street Suite 325, Room 3112 Arlington VA, 22203	<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b> AFOSR
	<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b>

**12. DISTRIBUTION/AVAILABILITY STATEMENT**  
DISTRIBUTION A: Distribution approved for public release.

**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**  
The project aimed to forge a rigorous new perspective on the joint control of multiple information sources of disparate types to simultaneously achieve quantified information and physical objectives. The overarching goal throughout the six years of the project's existence remained the discovery and analysis of new foundational methodology for information collection and fusion that exercises rigorous feedback control over information collection assets, simultaneously managing information and physical aspects of their states. The project generated 169 peer-reviewed papers acknowledging this award. Their contents and significance are summarized in this report.

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Final Report: AFOSR MURI FA9550-10-1-0567  
CHASE: CONTROL OF HETEROGENEOUS AUTONOMOUS  
SENSORS FOR SITUATIONAL AWARENESS

D. E. Koditschek, PI

July 31, 2016

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# 1 Introduction

The Penn led AFOSR MURI on Control Science for Next Generation Sensing (originally titled “CHASE: Control of Heterogeneous Autonomous SENSors for Situational Awareness”) ran from August, 2010, through April 2016 and brought together a group of ten prominent university researchers along with with roughly two dozen PhD students and postdoctoral fellows supported across four participating universities on a total budget of  $\sim$  \$7.5M over the six years of the project. There have now appeared in the peer-reviewed literature nearly 170 papers acknowledging support from this award, including 39 publications in top archival engineering and mathematics journals.<sup>1</sup>

Over the course of the project period and its immediate aftermath, our faculty were recognized by a number of highly prestigious honors and awards including

- Vijay Kumar’s 2013 election to the US National Academy of Engineering
- G. B. Giannakis’s 2015 winning the inaugural IEEE Fourier Award, the Technical Field Award in Signal Processing
- D. E. Koditschek’s 2016 winning the IEEE Robotics and Automation Society Pioneer Award
- the 2016 award to both D. E. Koditschek and Ali Jadbababaie of the US Office of the Secretary of Defense Vannevar Bush (National Security Science and Engineering) Faculty Fellowship
- Claire Tomlin’s 2017 winning the IEEE Transportation Technologies Award

The team included experts in control theory, signal and information theory, computational sciences, optimization and robotics. Our final report is organized largely according to the accounts of the work conducted by the ten research groups led by each of the project PIs. This introductory section continues with an overview of how these efforts fit together along with some highlights of the intellectual and DoD legacy.

## 1.1 Project Overview

The project aimed to forge a rigorous new perspective on the joint control of multiple information sources of disparate types to simultaneously achieve quantified information and physical objectives. New foundational methodology for information collection and fusion that exercises rigorous feedback control over information collection assets, simultaneously managing information and

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<sup>1</sup> As part of this scholarly dissemination of project activities, we organized and ran a full day workshop titled “Opportunities and Challenges of Joint Inference and Control in Mobile Robotics” <http://kodlab.seas.upenn.edu/Main/WorkshopICRA2014> at the 2014 IEEE ICRA (International Conference on Robotics and Automation) devoted to the MURI and related research topics.

physical aspects of their states, remained the overarching goal throughout the six years of the project's existence. A rough guide to the main intellectual currents running through our project (and the report itself) can be organized along the following broad lines of PI inquiry.

### **1.1.1 Representation for Networked Signal Processing, Estimation, Localization and Control**

Work led by S. Sastry, summarized in Section 3 explored pricing mechanisms as a means to achieve coordinated behavior in group settings. Work led by C. Tomlin, summarized in Section 4 pursued reachability analysis for control in networked settings. Work led by G. Giannakis summarized in Section 6 focused significant effort in general signal processing, reducing communications costs by considering various methods of compression. Work led by S. Roumeliotis summarized in Section 7 focused in substantial measure on extracting better control of uncertainty and consistency using vestibular (e.g. inertial motion units) and exteroceptive (e.g. cameras or laser systems) sensory systems in centralized and distributed settings as well. Work led by V. Kumar, summarized in Section 10 explored the use of finite set statistics and control for estimation in networked mobile sensors.

### **1.1.2 Networked Coordination of Joint Physical-Information State**

Work led by K. Ramchandran summarized in Section 2 focused on managing information and memory structures for networked agents. Work led by A. Jadbabaie summarized in Section 8 concerned networked estimation and social learning. Work led by A. Ribeiro summarized in Section 11 investigated metric representations of network data and the connection to multi-agent systems with incomplete information.

### **1.1.3 Information-Actuation Linkage**

Work led by Y. Baryshnikov summarized in Section 5 addressed the configuration space of multiple particles as exhibiting a joint physical-information interpretation: potentially a literal system of multiple physically located agents; or an abstract dataset of multiply recorded observations; or some conjunction of the two. Work led by D. Koditschek summarized in Section 9 focused on the foundations of clustering-based and other configuration-space approaches to the coordination of sensorimotor agents.

## **1.2 Intellectual Legacy**

### **1.2.1 New Applications of Mathematics**

The project was particularly focused on integrating into the team's mix of work new ideas from nontraditional intellectual sources within the pure mathematical fields of topology and geometry. Here, we were well led by the notable



insights and guidance of PI Y. Baryshnikov, whose contributions included a new result bearing on the millennial “sphere packing” problem (see item 1 in Section 5), as well as the proposal for a new concept of problem complexity in reactive motion planning and related controls settings (see item 5 Section 5). Baryshnikov’s intellectual influence within the project is particularly visible in the clustering-based navigation work reported in Section 9.1.2 wherein his suggestion to explore the relation between the topology of hierarchy in the continuous space of particle configurations and in the discrete space of trees led to a completely new abstraction for flexible but precise motion planning. The echo of his ideas can also be seen in the new clustering-based work on classification reported in Section 9.1.3 as well as the axiomatic approach to metric analysis of asymmetric networks in Section 11.2.1.

In the general area of novel stochastic systems analysis it seems appropriate to mention the pioneering work on non-Bayesian distributed learning reported in Section 8.1. We believe the work reported in Section 10.1 represents the very first time that finite set statistics have been applied to realtime physical settings.

### **1.2.2 DoD Transition**

Our team was very fortunate to be well received and supported within the Sensor Directorate, Section RYAT, of the Air Force Research Laboratory at Wright-Patterson Air Force Base. We were particularly lucky to be assigned a participant from amongst that group, the mathematician J. Culbertson, whose collaborative efforts within the team are partly represented by the work reported in Section 9.1.3. In part because of his insights, guidance, and translational efforts, and in part due to the intellectual boldness of the RYAT leadership, a set of three new follow-on projects stemming from the work originating with this MURI has been initiated within the AFRL RYAT division in collaboration with some of the team. We are hopeful that this new effort will lead to a direct transition of the fundamental MURI research into immediate application within RYAT.

### **1.2.3 Acknowledgment**

In this final communication, on behalf of the entire MURI team, it seems fitting to end with a grateful acknowledgment and thanks to our AFOSR Program Manager, Dr. Tristan Nguyen. Rarely will a group of researchers encounter a PM who has the background, training, and taste to so thoroughly understand the team’s intellectual aspirations as to continually cheer them on — and often to seem as much an eager participant as a diligent caretaker of resources in their pursuit of knowledge. We have been extraordinarily lucky to find ourselves in the charge of this selfless, deeply committed, and wise steward. We owe Dr. Nguyen a debt of gratitude that we can only hope has been partly repaid by the accomplishments of the team whose freedom and encouragement to pursue new horizons is in no small measure a testament to his insights and guidance.

## 2 University of California Berkeley — Kannan Ramchandran

### 2.1 Update Report 2015 CHASE

We have made progress on four parallel thrusts related to this project during 2012-2015:

- Sparse Signal Processing and Machine Learning
- Graph Signal Processing
- Secure and Reliable Distributed Storage

#### 2.1.1 Sparse Signal Processing and Machine Learning

In this research thrust, we wish to usher the science of sparse signal processing into the next generation, and explore their connections to some of the important machine learning problems. Compressed sensing has recently emerged as a powerful paradigm for understanding the fundamental limits of sparse signal processing and high dimensional learning problems. While this has sparked excitement in efficient randomized approaches to signal acquisition and recovery, the current generation of algorithms relies predominantly on convex relaxations that, being in the problem dimension, are difficult to scale computationally. This forms the key intellectual motivation for this proposal: how to address the challenge of scale in the theory and design of sparse recovery algorithms. The goal is to achieve real-time or near-real-time processing for massive datasets featuring sparsity, which are relevant to a multitude of practical applications.

We exploit a novel and unexplored interdisciplinary toolkit from "modern coding theory", "graph theory", "number theory", and "statistical signal processing". This allows for the devising of new and powerful computational primitives similar in spirit to state-of-the-art codes like LDPC (Low Density Parity Check) codes and fountain codes that have revolutionized modern communication systems. We were able establish new theoretical foundations and computationally efficient algorithms for many data-intensive applications, as detailed below.

**Discrete Fourier Transform (DFT) and Walsh-Hadamard Transform (WHT)** In [1], we addressed the problem of computing an  $n$ -length DFT of signals that have  $k$ -sparse Fourier transform, where  $k \ll n$ . We proposed a novel FFAST algorithm that cleverly exploits filterless subsampling operation to induce aliasing artifacts, similar to parity-check constraints of good erasure-correcting sparse-graph codes, on the spectral coefficients. Then, we formally connected the problem of computing sparse DFT to that of decoding of appropriate sparse-graph codes. This connection was further exploited to design a sub-linear complexity FFAST peeling-style back-end decoder. Further, we analyzed the performance of the FFAST algorithm, using well known density

evolution techniques from coding theory, to show that our proposed algorithm computes the  $k$ -sparse  $n$ -length DFT using only  $O(k)$  samples in  $O(k \log k)$  arithmetic operations, with high probability. The constants in the big Oh notation for both sample and computational cost are small. In particular, the sample cost is less than  $4k$ . We also provide simulation results, that are in tight agreement with our theoretical findings. This work further extends to the noisy situation in [2] and [3], with applications in finite rate-of-innovation sampling [4] and image processing applications based on the 2-dimensional case in [5].

By exploiting the idea of aliasing and sparse-graph codes similar to that in the FFAST algorithm, we proposed a novel SPRIGHT framework in [6] and [7] to compute an  $n$ -length WHT of signals that have  $k$  WHT coefficients ( $k \ll n$ ) in the presence of noise. The framework comes in two options for noisy WHT computations, where the NSO-SPRIGHT algorithm uses  $O(k \log^2 n)$  samples and  $O(k \log^3 n)$  operations while the SO-SPRIGHT algorithm maintains the optimal sample scaling  $O(k \log n)$  and complexity  $O(k \log^2 n)$  as that of the noiseless case. Our approach is based on strategic subsampling of the input noisy samples using a small set of randomly shifted patterns that are carefully designed, which achieves a vanishing failure probability.

**Sparse Recovery** One of the most influential sparse recovery problems is compressed sensing. In [7] and [8], we consider the problem of recovering the support of an arbitrary  $k$ -sparse  $n$ -length vector in the presence of noise. A new family of sparse measurement matrices is introduced with low-complexity recovery algorithms, which achieves a near-optimal measurement cost  $O(k \log^{4/3} n)$  and sub-linear computational complexity  $O(k \log^{4/3} n)$ . The proposed method also admits the option of using  $O(k \log n)$  measurements to recover the sparse signal with near-linear time  $O(n \log n)$ . Our measurement system is designed to capture observations of the signal through sparse-graph codes, and to recover the signal by using a simple peeling decoder. We formally connect general sparse recovery problems with sparse-graph decoding in packet-communication systems, and showcase our design in terms of the measurement cost, computational complexity and recovery performance.

Another problem we studied is the estimation of an  $n$ -by- $n$  sparse covariance matrices with  $k$  sparse off-diagonal covariance entries. In [9], we consider the problem of recovering a sparse covariance matrix from quadratic measurements. In particular, we introduce two low complexity algorithms, the first a message-passing algorithm and the second a forward algorithm, that are based on a sparse-graph coding framework. We show that under some simplifying assumptions, the message passing algorithm can recover an arbitrarily-large fraction of the  $k$  non-zero components with  $ck$  measurements, where  $c$  is a small constant that can be precisely characterized. We further show that the forward algorithm can recover all the  $K$  non-zero entries with high probability with  $m = O(k)$  measurements and  $O(k \log k)$  decoding complexity. However, the forward algorithm suffers from significantly larger constants in terms of the number of required measurements, and is indeed less practical despite providing

stronger theoretical guarantees. We then consider the noisy setting, and show that both proposed algorithms can be robustified to noise with  $m = O(k \log^2 n)$  measurements. Finally, we provide extensive simulation results that support our theoretical claims.

In the work [10], we further consider a sparse recovery problem of group testing. In this work, we have proposed SAFFRON (Sparse-grAph codes Framework For gROup testiNg), which recovers an arbitrarily-close-to-one  $(1 - a)$ -fraction of  $k$  defective items with high probability with  $6c(a)k \log^2 n$  tests, where  $c(a)$  is a relatively small constant that depends only on  $a$ . Also, the computational complexity of the decoding algorithm of SAFFRON is order-optimal. We have described the design and analysis of SAFFRON based on the powerful modern coding-theoretic tools of sparse-graph coding and density evolution. We have also proposed a variant of SAFFRON, Singleton-Only-SAFFRON, which recovers all defective items with  $2e(1 + b)k \log k \log^2 n$  tests, with probability  $1 - O(1/k^b)$ . Further, we robustify SAFFRON and Singleton-Only-SAFFRON by using modern error-correcting codes so that they can recover the set of defective items with noisy test results. To support our theoretical results, we have provided extensive simulation results that validate the theoretical efficacy and the practical potential of SAFFRON.

**Machine Learning and Big Data** In the work [11], we consider the important problem in machine learning of learning a sparse boolean polynomial, which consists of  $2^n$  monomials of  $n$  boolean variables, in which only  $s \ll 2^n$  coefficients are non-zero. The goal is to learn the polynomial by querying the values of the polynomial. We introduce an active learning framework that is associated with a low query cost and computational runtime. The significant savings are enabled by leveraging sampling strategies based on modern coding theory, specifically, the design and analysis of sparse-graph codes, which represent the state-of-the-art of modern packet communications. More significantly, we show how this design perspective leads to exciting, and to the best of our knowledge, largely unexplored intellectual connections between learning and coding.

The key is to relax the worst-case assumption with an ensemble-average setting, where the polynomial is assumed to be drawn uniformly at random from the ensemble of all polynomials (of a given size  $n$  and sparsity  $s$ ). Our framework succeeds with high probability with respect to the polynomial ensemble, where the polynomial can be exactly learned using  $O(ns)$  queries in time  $O(ns \log s)$ , even if the queries are perturbed by Gaussian noise. We further apply the proposed framework to graph sketching, which is the problem of inferring sparse graphs by querying graph cuts. By writing the cut function as a polynomial and exploiting the graph structure, we propose a sketching algorithm to learn the an arbitrary  $n$ -node unknown graph using only few cut queries, which scales almost linearly in the number of edges and sub-linearly in the graph size  $n$ . Experiments on real datasets show significant reductions in the runtime and query complexity compared with competitive schemes.

Reducing latency in distributed computing and data storage systems is gain-

ing increasing importance. Several empirical works have reported on the efficacy of scheduling redundant requests in such systems. That is, one may reduce job latency by 1) scheduling the same job at more than one server and 2) waiting only until the fastest of them responds. Several theoretical models have been proposed to explain the power of using redundant requests, and all of the existing results rely heavily on a common assumption: all redundant requests of a job can be immediately cancelled as soon as one of them is completed. In [12], we study how one should schedule redundant requests when such assumption does not hold. This is of great importance in practice since cancellation of running jobs typically incurs non-negligible delays. In order to bridge the gap between the existing models and practice, we propose a new queueing model that captures such cancellation delays. We then find how one can schedule redundant requests to achieve the optimal average job latency under the new model. Our results show that even with a small cancellation overhead, the actual optimal scheduling policy differs significantly from the optimal scheduling policy when the overhead is zero. Further, we study optimal dynamic scheduling policies, which appropriately schedule redundant requests based on the number of jobs in the system. Our analysis reveals that for the two-server case, the optimal dynamic scheduler can achieve 7% to 16% lower average job latency, compared with the optimal static scheduler.

### 2.1.2 Graph Signal Processing

Graph-structured data is present in numerous modern applications, such as social media services (e.g. Facebook and Twitter), wireless sensors (e.g. temperature measurements), power networks, computer graphics, and finite-element meshes. Most of these graphs have attributes associated with the nodes or edges. For example, sensor nodes have measurement values associated with each node, and social media graphics have attributes like the name, age, gender, or number of ad clicks associated with each node in the graph. Given such data, problems of interest include finding patterns, predicting unobserved data, or obtaining multiscale representations of the graph and the associated data for efficient processing.

We explore fundamental signal processing operations on circulant graphs to a substantive depth. In particular, we analyze the properties of the Graph Fourier Transform (GFT) as defined in the literature for circulant graphs. We define basic operations like shifting, sampling and graph reconnection strategies for circulant graphs and analyze the corresponding properties in the spectral domain. Fundamental sampling theorems and uncertainty principles are derived. These form the basis for filter design and multi-resolution analysis.

We design three classes of two-channel filter bank structures for signals on circulant graphs. These are shown to satisfy different desirable properties of multi resolution filter banks. Further they provide wavelet bases at different levels which in turn result in a multi scale representation of the given graphical data.

For analyzing signals on general graphs, we provide a decomposition of an

arbitrary graph into circulant graphs [13, 14]. In particular, we show that the adjacency matrix of a given graph can be written as a suitable linear combination of the adjacency matrices of individual circulant graphs. Fundamental operations such as sampling and filter design are extended to general graphs through this decomposition .

Two-channel filter bank structures are designed for general graphs that help obtain a multi scale representation of any given data on a general graph. These filter bank structures are shown to satisfy desirable properties like critical-sampling and perfect-reconstruction [15, 16].

We discuss a specific application related to a class of semi-supervised ML algorithms, known as graph semi supervised learning (GSSL) algorithms. Some of the existing GSSL algorithms can be viewed as appropriate filter designs in the graph domain. A wavelet based semi-supervised algorithm is proposed and the performance is evaluated on different datasets in comparison to existing techniques [15, 16].

### 2.1.3 Secure and Reliable Distributed Storage [17, 18, 19]

We design erasure codes and storage algorithms that are efficient in terms of resource usage such as storage and network bandwidth, but also have strong (and optimal) security guarantees. In particular, we consider the notion of information-theoretic security where unlike cryptographic security, the adversary cannot obtain any information about the data even if it has unlimited computational power. We design algorithms for protection from various kinds of security threats, including those of (a) preserving privacy of the data in the presence of adversaries who may be able to read the data, (b) securing the data from malicious corruption when an adversary can modify the data, (c) securely transmitting data across a network, and (d) preserving privacy of any request made to a database by the user. Our algorithm are practical from a computational complexity and implementation standpoint, and are also proven to be theoretically optimal in terms of the various resources at hand.

## 2.2 Update Report 2012

We have made progress on two parallel research thrusts related to the project:

- Collaborative signal processing theory and algorithms for high-accuracy localization of CHASE agents;
- Characterizing the fundamental information-theoretic bounds as well as the design and construction of information-theoretically optimal network codes for reliable and secure distributed storage of information in the CHASE network.
- Fast and robust algorithms for compressed sensing.

A brief summary of each thrust follows.

### 2.2.1 High-accuracy localization

We have developed a robust architecture as well as the accompanying theory and algorithms for enabling high-accuracy localization of mobile CHASE agents even in harsh 3-D indoor multipath environments. A key novelty of our localization system is its use of inexpensive infrastructure comprising a redundant collection of and easy-to-deploy RFID-supertags in the CHASE environment. These RFID-based supertags form “virtual antenna arrays,” thereby resulting in the seamless upgrade of simple inexpensive single-antenna narrowband radios into sophisticated virtual multi-antenna radio systems, with the accompanying benefits that MIMO systems bring. Specifically, we exploit the redundancy afforded by the high density of these RFID tags, as well as their ability to create a virtual MIMO antenna array that can be used to distinguish multipath reflections, together with novel collaborative signal processing algorithms to overcome the “multipath barrier” that plagues high-precision localization. Our algorithms allow for simultaneous localization of the mobile CHASE agents and calibration of the RFID-supertags in real-time. Our results will be presented (1) in IEEE PLANS 2012.

We have also studied some of the key theoretical aspects of non-line-of-sight (NLOS) localization. We cast the problem of localization in an optimization framework and invoke results from semi-definite programming and matrix low-rank + sparse decomposition literature to provide theoretical guarantees for NLOS localization. Our results have been submitted (2) to IEEE SSP12, and will be available on arxiv (3) shortly.

### 2.2.2 Secure and Reliable Distributed Storage

We have addressed this problem recently using an information-theoretic setting. Particularly interesting are our results in the area of secure distributed storage of state information across CHASE agents under active adversarial settings. Specifically, in a distributed system architecture, it is desirable for the local state information as well as the global state information to be reliably stored in the network of CHASE agents. However, it is important to safeguard the system from some of the CHASE agents being potentially compromised or destroyed by adversarial attacks. In such a case, it is important to understand the fundamental information-theoretic bounds on the minimum communication cost needed to reliably and securely recover this information from the survivors in the network. Further, it is critical for the adversary to not be able to gain any knowledge about the state information stored in the network by compromising only a subset of the agents. We have studied this in an information-theoretic setting, and have developed network codes that are provably information theoretically optimal in achieving the optimal tradeoff between communication and storage cost while providing information-theoretic (i.e. computationally unbounded) security.

### 2.2.3 Compressed sensing

We use a hybrid mix of the Discrete Fourier Transform (DFT), an old workhorse in digital signal processing, and Low Density Parity Check (LDPC) codes, a recent workhorse in coding theory, to generate a linear measurement lens through which to perform compressive sensing (CS) of sparse high-dimensional signals. This novel hybrid DFT-LDPC framework represents a new family of sparse measurement matrices, and induces a fast algorithm (dubbed the Short-and-Wide Iterative Fast Transform based or SWIFT algorithm) for robustly recovering a high-dimensional  $k$ -sparse signal  $\mathbf{x}$ , in  $\mathbb{C}^n$ , from a near-optimal number of (upto a small constant multiple of  $k$ ) linear observations, with a decoding complexity of  $k$  steps, under high SNR.

### References

- [1] Sameer Pawar and Kannan Ramchandran. “Computing a  $k$ -sparse  $n$ -length discrete fourier transform using at most  $4k$  samples and  $\mathcal{O}(k \log k)$  complexity”. In: *IEEE International Symposium on Information Theory Proceedings (ISIT) 2013* (2013), pp. 464–468.
- [2] Sameer Pawar and Kannan Ramchandran. “A robust sub-linear time R-FFAST algorithm for computing a sparse DFT”. In: *arXiv preprint arXiv:1501.00320* (2015).
- [3] Sameer Pawar and Kannan Ramchandran. “A robust R-FFAST framework for computing a  $k$ -sparse  $n$ -length DFT in  $\mathcal{O}(k \log n)$  sample complexity using sparse-graph codes”. In: *IEEE International Symposium on Information Theory (ISIT) 2014* (2014), pp. 1852–1856.
- [4] Sameer Pawar, Venkatesan Ekambaram, and Kannan Ramchandran. “Computationally-efficient blind sub-Nyquist sampling for sparse spectra”. In: *Global Conference on Signal and Information Processing (GlobalSIP), 2013 IEEE* (2013), pp. 1065–1068.
- [5] Frank Ong, Sameer Pawar, and Kannan Ramchandran. “Fast and Efficient Sparse 2D Discrete Fourier Transform using Sparse-Graph Codes”. In: *arXiv preprint arXiv:1509.05849* (2015).
- [6] Xiao Li, Joseph K Bradley, Sameer Pawar, and Kannan Ramchandran. “SPRIGHT: A Fast and Robust Framework for Sparse Walsh-Hadamard Transform”. In: *arXiv preprint arXiv:1508.06336* (2015).
- [7] Xiao Li, Joseph Kurata Bradley, Sameer Pawar, and Kannan Ramchandran. “The SPRIGHT algorithm for robust sparse Hadamard Transforms”. In: *IEEE International Symposium on Information Theory (ISIT) 2014* (2014), pp. 1857–1861.
- [8] Xiao Li, Sameer Pawar, and Kannan Ramchandran. “Sub-linear time support recovery for compressed sensing using sparse-graph codes”. In: *arXiv preprint arXiv:1412.7646* (2014).



- [9] Xiao Li and Kannan Ramchandran. “Sparse polynomial learning and graph sketching”. In: *accepted by Advances in Neural Information Processing Systems (NIPS)* (2015).
- [10] Kangwook Lee, Ramtin Pedarsani, and Kannan Ramchandran. “SAFFRON: A Fast, Efficient, and Robust Framework for Group Testing based on Sparse-Graph Codes”. In: *arXiv preprint arXiv:1508.04485* (2015).
- [11] Ramtin Pedarsani, Kangwook Lee, and Kannan Ramchandran. “Sparse Covariance Estimation Based on Sparse-Graph Codes”. In: *53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)* (2015).
- [12] Kangwook Lee, Ramtin Pedarsani, and Kannan Ramchandran. “On Scheduling Redundant Requests with Cancellation Overheads”. In: *53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)* (2015).
- [13] Venkatesan N Ekambaram, Giulia C Fanti, Babak Ayazifar, and Kannan Ramchandran. “Multiresolution graph signal processing via circulant structures”. In: *IEEE Digital Signal Processing and Signal Processing Education Meeting (DSP/SPE) 2013* (2013), pp. 112–117.
- [14] Venkatesan N Ekambaram, Giulia C Fanti, Babak Ayazifar, and Kannan Ramchandran. “Circulant structures and graph signal processing”. In: *20th IEEE International Conference on Image Processing (ICIP) 2013* (2013), pp. 834–838.
- [15] Venkatesan N Ekambaram, Giulia Fanti, Babak Ayazifar, and Kannan Ramchandran. “Critically-sampled perfect-reconstruction spline-wavelet filterbanks for graph signals”. In: *IEEE Global Conference on Signal and Information Processing (GlobalSIP)* (2013), pp. 475–478.
- [16] V. Ekambaram, G. Fanti, B. Ayazifar, and K. Ramchandran. “Spline-like Wavelet Filter Banks for Multiresolution Analysis of Graph-structured Data”. In: *IEEE Transactions on Signal and Information Processing over Networks* (2015).
- [17] NB Shah, KV Rashmi, and Kannan Ramchandran. “One extra bit of download ensures perfectly private information retrieval”. In: *IEEE International Symposium on Information Theory (ISIT) 2014* (2014), pp. 856–860.
- [18] Nihar B Shah, KV Rashmi, and Kannan Ramchandran. “Secure network coding for distributed secret sharing with low communication cost”. In: *IEEE International Symposium on Information Theory Proceedings (ISIT) 2013* (2013), pp. 2404–2408.
- [19] Nihar B Shah, KV Rashmi, Kannan Ramchandran, and P Vijay Kumar. “Information-theoretically Secure Erasure Codes for Distributed Storage”. In: *arXiv preprint arXiv:1508.03787* (2015).

## 3 University of California Berkeley — S. Shankar Sastry

### 3.1 November 2015 report

In general, we are interested in interactions between competitive agents in dynamic settings and how to induce cooperation among them. We study this cooperation in multiple different contexts including coordination in multi-pursuer pursuit-evasion games, pricing design for optimal cooperation in linear-quadratic dynamic games, pricing design in nonlinear open-loop differential games, and more recently optimize macroscopic traffic behavior on highways.

#### 3.1.1 Pricing Design for Linear-Quadratic Dynamic Games [1, 2]

We investigate the use of pricing mechanisms as a means to achieve a desired feedback control strategy among selfish agents. We study a hierarchical linear-quadratic game with many dynamically coupled Nash followers and an uncoupled leader. The leader influences the game by choosing the quadratic dependence on control actions for each followers cost function. We show that determining whether the leader can establish the desired feedback control as a Nash equilibrium among the followers is a convex feasibility problem. We also discuss methods for the leader to ensure that the prices are budget balanced as well as robust to perturbations in the problem parameters and the followers' strategies. We apply the proposed method to the problem of ensuring the security of a multi-network, controlled diffusion in a general network, and efficient energy resource allocation in buildings.

#### 3.1.2 Pricing for Nonlinear Open-Loop Dynamic Games [3]

We extend our results on pricing design in linear quadratic games to general open-loop nonlinear dynamic games. We show that a leader or social planner can design the quadratic dependence on the followers control actions to make a set of desired control signals a local Nash equilibria of the nonlinear dynamic game. In addition, we can ensure that the induced equilibria is isolated and stable with respect to gradient play updates of the followers' control strategies. We apply these techniques to the problem of incentivizing members of a multi-network to invest in cyber-security.

#### 3.1.3 Toll Pricing for Optimal Traffic Flow on Highways [4]

We develop a new macroscopic model of traffic flow that incorporates competitive lane-changing behavior in order to study optimal toll pricing design for high occupancy toll (HOT) lanes. We use our model to design optimal tolls that maximize a social planner's objective while take into account drivers' selfish behavior.

### 3.1.4 Decentralized Control in Pursuit Evasion Games [5]

We study a multi-player pursuit evasion game and seek to understand how collaboration effects the outcome. We consider a scenario where we have  $N$  pursuers and  $N$  evaders, all the pursuers are faster than all the evaders, and the environment is unbounded. We seek to assign each pursuer to capture one evader and to minimize the sum of all the capture times. A naive approach to this problem is to phrase it as an optimal bipartite matching problem where the weights are the time for pursuer  $i$  to capture evader  $j$ . We study the difference between this naive approach and strategies that take advantage of coordination among pursuers.

## References

- [1] L. Ratliff, S. Coogan, D. Calderone, and S. Shankar Sastry. “Pricing in linear-quadratic dynamic games”. In: *Proc. Fiftieth Annual Allerton Conference on Communication, Control, and Computing* (2012).
- [2] D. Calderone, L. Ratliff, and S. Shankar Sastry. “Pricing Design for Robustness in Linear Quadratic Games”. In: *Decision and Control (CDC), 2013 IEEE 52th Annual Conference on*. Dec. 2013.
- [3] D. Calderone, L. Ratliff, and S. Shankar Sastry. “Pricing for Coordination in Open-Loop Differential Games”. In: *Proceedings of the 19th IFAC World Congress*. 2014.
- [4] D. Calderone, L. Ratliff, and S. Shankar Sastry. “Lane Pricing via Decision-Theoretic Lane Changing Model of Driver Behavior”. In: *Decision and Control (CDC), 2015 IEEE 54th Annual Conference on (to appear)*. Dec. 2015.
- [5] S. Coogan, L. Ratliff, D. Calderone, C. Tomlin, and S. Shankar Sastry. “Energy Management via Pricing in LQ Dynamic Games”. In: *Proc. American Control Conference (ACC)* (2013).

## 4 University of California Berkeley — Claire Tomlin

### 4.1 November 2015 AFOSR CHASE MURI Update

Our research on CHASE has continued in the area of reachability theory as well as applications in multi-agent systems. In the theoretical front, we have proposed new Hamilton-Jacobi formulations to the reachability problem in order to overcome the curse of dimensionality. In the applications front, we have investigated how to utilize reachability analysis in multi-vehicle systems to provide safety and liveness guarantees.

#### 4.1.1 Theoretical advances in addressing the curse of dimensionality in reachability analysis

In [1], we proposed a Hamilton-Jacobi (HJ) variational inequality that allows the computation of reachable sets in the presence of time-varying dynamics, moving target sets, and moving obstacles. When addressing problems with time invariant dynamics, moving target sets, and moving constraint sets, previous methods required augmentation of the state space with time as a new variable. Since HJ partial differential equations and variational inequalities are typically solved numerically on a grid, the computation complexity scales exponentially with the number of state space dimensions, and state augmentation is therefore expensive. Our new HJ variational inequality is able to bypass state augmentation and solve time-varying problems with moving target sets and constraint sets in the state space of the original system. Future work will involve extending our approach to hybrid systems.

In [2], we presented a special formulation of the Hamilton-Jacobi partial differential equation for decoupled systems. Previously, solutions to some high dimensional reachability problems can be approximated by methods involving techniques such as projection or sampling. To address the curse of dimensionality in another way, we have proposed an efficient way to solve reachability problems involving decoupled systems. Our decoupled HJ formulation takes advantage of the problem structure to compute the solution to a high dimensional reachability problem by performing simple computations in the lower dimensional decoupled components. Unlike many previous methods, our computed solution is exact.

#### 4.1.2 Practical applications in multi-vehicle systems

Applications of reachability analysis are limited largely by imagination. We have utilized our recent theoretical advances to provide different ways of guaranteeing safety and liveness of multi-vehicle systems.

In [3], we couple reachability analysis with basic graph theory to analyze a multiplayer reach-avoid game, in which a team of  $N$  attackers aim to reach some target set while avoiding capture from a team of  $N$  defenders. A direct

application of reachability analysis to this problem is intractable; however, by considering pairwise outcomes between attackers and defenders, constructing a graph representing the pairwise outcomes, and finding the maximum matching of the graph, we were able to find an upper bound on the number of attackers that can reach the target.

In [4], we present the sequential path planning of multi-vehicle systems, which efficiently guarantees liveness and safety through priority assignment. In the sequential path planning problem, multiple vehicles aim to arrive at each of their destinations before a scheduled time of arrival. Such a problem could be addressed with a high dimensional reachability approach; however, the exponential scaling of computation complexity with the number of vehicles makes this traditional approach intractable. By assigning priorities to each vehicle and treating higher priority vehicles as moving obstacles, we are able to efficiently solve the multi-vehicle path planning problem. Future work will involve incorporating uncertainty in the vehicle dynamics and trajectories.

In [5], we present the platooning of unmanned aerial vehicles, which combines the areas of reachability and hybrid systems to put the unmanned air space into a safe and intuitively structured environment. With the growing interest in using unmanned aerial vehicles (UAVs) for civil purposes such as package delivery, thousands of UAVs could be flying simultaneously in the airspace in the near future. Due to the curse of dimensionality, the safety and liveness of the joint system of thousands of vehicles cannot be guaranteed without imposing additional structure. In our platooning approach, we propose methods for air highway placement, model vehicles as hybrid systems, and guarantee the safety and liveness of all vehicles via reachability analysis, taking advantage of the decoupled HJ formulation for real time computations.

## 4.2 April 2012 report

Our research on CHASE has focused on the computation and use of reachable sets for aiding in human situational awareness for interacting and controlling CHASE agents.

We first recall that a reachable set describes a subset of the state space from which a specification may be reached. As we are interested in computing these sets using control inputs, in the face of disturbances, we consider two model problems:

- Given a desired target set, the backwards reachable set describes all initial conditions from which the system can reach the target set, within a specified time horizon, despite the worst possible disturbance;
- Given an unsafe set (to be avoided), the backwards reachable set describes all initial conditions from which, despite the best possible control action, the system can reach the unsafe set within a specified time horizon.

We have shown in earlier work that these reachable sets may be characterized as the sub-zero level sets of an appropriate Hamilton-Jacobi-Isaacs equation, and

we have designed a level set toolbox to solve these equations and compute the sets. The methods are exponential in the dimension of the continuous state, and hence computationally intensive for large state spaces, which is typical for large numbers of agents.

Our CHASE research has two directions:

- Fast methods for computation of sensible (not too conservative) approximations of these reachable sets.
- The use of these reachable sets as a user interface between a human controller and a set of CHASE agents.

#### 4.2.1 Fast computation methods

We have developed two new methods for reachable set computation: (a) open loop method, and (b) open loop iterative method. In the open loop method, one player (the control) assumes that other players (disturbances) follow their worst possible action over the entire time horizon of the game. Thus one strategy is fixed, and the game problem transforms into an optimal control problem which is causal, meaning the value function solution of the Hamilton-Jacobi equation increases monotonically along the characteristic curves (or, equivalently time-to-reach-target should decrease monotonically along optimal trajectories.) This allows a particular ordering of the grid nodes, thereby yielding a single pass algorithm: we have used a Fast Marching Method to solve the problem. In the HJI case, causality does not hold. Mathematically, this is due to the fact that the Hamiltonian of the HJI equation is non-convex. In general, this open loop game is more conservative for the control than the closed loop game, yet we have found in practice that it is not too conservative. We are characterizing the speed up and it appears that it is of several orders of magnitude over the level set methods that we use for general HJI equations.

In addition, we are now developing an “iterative open loop” solution, which recomputes the open loop solution at each time step, using the actual state as the updated initial state. This still allows us to use Fast Marching, though we reduce conservatism of the reachable sets using the iteration. We are currently characterizing the level of approximation of both in the application described below.

#### 4.2.2 Use of reachable sets in situational awareness

We have formed a testbed with human interaction with our quadrotors: reachable sets are computed for a game played in real time, and displayed to human players on smart phones. These sets give winning and losing regions of the game, and also regions of uncertainty which guide the human users on where to direct the quadrotors to search for opposing players (regions to avoid) or goal regions (regions to go to).

## References

- [1] J. Fisac, M. Chen, C. J. Tomlin, and S. S. Sastry. “Reach-Avoid Problems with Time-Varying Dynamics, Targets and Constraints”. In: *Proceedings of the 18th International Conference on Hybrid Systems: Computation and Control*. Seattle, WA, Apr. 2015.
- [2] M. Chen and C. J. Tomlin. “Exact and Efficient Hamilton-Jacobi Reachability for Decoupled Systems”. In: *Proceedings of the 54th IEEE Conference on Decision and Control*. Osaka, Japan, Dec. 2015.
- [3] M. Chen, Z. Zhou, and C. J. Tomlin. “Multiplayer Reach-Avoid Games via Pairwise Outcomes”. In: *IEEE Transactions on Automatic Control* Submitted (2015).
- [4] M. Chen, J. Fisac, S. S. Sastry, and C. J. Tomlin. “Safe Sequential Path Planning of Multi-Vehicle Systems via Double-Obstacle Hamilton-Jacobi-Isaacs Variational Inequality”. In: *Proceedings of the 14th Annual European Control Conference*. Linz, Austria, July 2015.
- [5] M. Chen, Q. Hu, C. Mackin, J. Fisac, and C. J. Tomlin. “Safe Platooning of Unmanned Aerial Vehicles via Reachability”. In: *Proceedings of the 54th IEEE Conference on Decision and Control*. Osaka, Japan, Dec. 2015.

## 5 University of Illinois at Urbana-Champaign — Yuliy Baryshnikov

Highlights:

1. Configuration spaces of hard disks investigated using Morse theory [1].
2. Shape formation by decentralized mechanisms studied in several context: from variational Plateau-like problems emulating lipid films formation to the symmetry breaking in rapidly-growing random trees [2].
3. Novel choreographies in multi-agent systems: a large number of results deal with the existence and stability of circular choreographies (in which the shape of the formation remains the same) - we, in a contrast, produce a novel type of choreographies, depending on the underlying topological properties of the formation [3, 4, 5].
4. Novel algorithms for robotic exploration, leading to provable uniform coverage of a domain [6].
5. Introduction of “topological perplexity” concept as a measure of mismatch between topology of an environment and a goal set within it [7].

### References

- [1] M. Kahle Y. Baryshnikov P. Bubenik. “On Configuration Spaces of Hard Spheres”. In: *Intl. Math. Research Notes* (to appear) ().
- [2] S. Lavalley M. Arnold Y. Baryshnikov. “Convex Hull Asymptotic Shape Evolution”. In: *proceedings WAFR’12* ().
- [3] Yuliy Baryshnikov and Andrey Sarychev. “Cyclic Vectors of Associative Matrix Algebras and Reachability Criteria for Linear and Nonlinear Control Systems”. In: *Proc. CDC* (2015).
- [4] Maxim Arnold, Yuliy Baryshnikov, and Daniel Liberzon. “Cyclic pursuit without coordinates: convergence to regular polygon formations”. In: *Proc. CDC, 2014* ().
- [5] Yuliy Baryshnikov and Cheng Chen. “Shapes of Cyclic Pursuit and Their Evolution”. In: *Proc. ACC, 2016* ().
- [6] Han Wang, Cheng Chen, and Yuliy Baryshnikov. “A Topological Perspective on Cycling Robots for Full Tree Coverage”. In: *Proc. WAFR* (2014).
- [7] B. Shapiro Y. Baryshnikov. “How to Run a Roach: a Topological Perspective”. In: *Geometric Control Theory and Sub-Riemannian Geometry*. Springer, 2014, pp. 37–51.



## 6 University of Minnesota — Georgios Giannakis

### 6.1 December 2015, AFOSR MURI-CHASE Final Report

#### 6.1.1 Summary of main contributions

**Leveraging sparsity and low rank for decentralized and online inference** [1, 2, 3, 4, 5] Inference from Big Data is rendered feasible by inherent regularities in the data that are often contaminated by presence of anomalies and misses. We have unveiled that most data matrices can be decomposed into a low rank plus sparse matrix, respectively capturing the nominal regularity in the data, and anomalies that are inherently sparse. Leveraging these properties, we have developed sparsity- and rank-regularized estimators for prediction, imputation of missing data, and anomaly identification, with rigorously derived recovery guarantees. In order to facilitate large-scale and real-time analytics, we have developed online and decentralized algorithms for network data traffic and anomaly prediction, robust subspace tracking, and tensor completion and extrapolation. This contribution is relevant to the approximate robust inference algorithms (Section 2.3.2.3 in the proposal), and also to robust sensing, cooperative localization, mapping and tracking (Section 2.3.1.1 in the proposal).

#### **Nonparametric basis pursuit via sparse kernel-based learning** [6, 7]

Reproducing kernel Hilbert spaces (RKHSs) constitute a powerful framework for nonparametric estimation and learning. We have endowed this framework with contemporary advances in sparsity-aware modeling and processing, to develop nonparametric basis pursuit tools for sparse linear regression, nuclear norm regularization, and dictionary learning. Our sparse kernel-based learning framework is general enough to incorporate new possibilities such as multi-kernel selection and matrix smoothing, with test cases from wireless cognitive radio sensing, microarray data imputation, and network traffic prediction. This contribution is relevant to robust sensing, cooperative localization, mapping and tracking (Section 2.3.1.1 in the proposal).

#### **Notable Recognitions**

- The conference version of [3] won the Best Student Paper Award in SPAWC-2012 [8].
- G. Mateos (a PhD graduate supported by this MURI-AFOSR-CHASE grant) joined the University of Rochester as an Assistant Professor of the Dept. of ECE.
- Prof. G. B. Giannakis was the inaugural recipient of the IEEE-level Technical Field Award (IEEE Fourier Award) “For contributions to the theory and practice of statistical signal processing with applications to wireless communications.” The selection criteria published include: Impact on the

field of signal processing technology, including innovation; leadership; and seminal contributions as evidenced by publications or patents or transition to practice.

### 6.1.2 Opportunities and challenges for future funding

Sparsity and low rank are attributes that naturally arise in most complex networks (e.g., social nets, power grids, and brain nets). For example, networks often exhibit edge sparsity, while nodes cluster into ‘communities’ admitting low rank matrix representations. Coupling our results with emerging trends in network science will open up opportunities for prediction of evolving network behavior, or identification of anomalies in social nets due to e.g., security threats, or cyber criminals. Furthermore, diffusion processes over complex networks are driven by the underlying network topology through (possibly) nonlinear and dynamical models. Identification of such models for prediction tasks over networks naturally lends itself to our nonparametric basis pursuit framework.

## 6.2 August 2012 report

Highlights of our research for the CHASE project until August 31, 2012 include:

- Incorporation of censoring theory, and its fundamental performance limits in complementing or replacing sensor selection and quantization schemes for data reduction in sensing, estimation and tracking using wireless sensor networks (WSNs).
- Establishment of a neat link between the emerging theories of compressive sampling and sparsity-aware signal processing with a basic aspect of statistical inference, namely that of universal robustness against outliers, even when the signals involved are not sparse.
- Joint exploitation of the low intrinsic-dimensionality of origin-destination flows and the sparse nature of anomalies, to formulate a convex program capable of unveiling traffic anomalies across flows and time due to denial of service attacks or jamming.

### 6.2.1 Distributed Data Censoring for Estimation and Tracking via WSNs [9].

Using interval censoring as a data-reduction tool, we have developed a distributed approach to select judiciously a subset of sensors, each deciding separately whether to censor its acquired measurements, so as to save communication resources while minimally impacting performance of the inference task at hand. Quantization of the uncensored measurements offers an additional degree of freedom in the rate resource conservation versus estimator error reduction trade-off, which we have rigorously delineated through the development of benchmark bounds assessing the performance of joint censoring-quantization

for WSN-based estimation. This line of work is relevant to the resource-aware nonlinear cooperative localization, mapping, and tracking tasks involved in layered sensing (Section 2.3.1.1 in the proposal), as well as to those required for sensory awareness (Section 2.3.2.1 in the proposal). Furthermore, it directly addresses the rate-performance tradeoffs involved by the underlying network affordances (Section 2.4.2.2 in the proposal).

### **6.2.2 Sparsity Control for Universally Robust Inference [10, 11, 12, 13, 14].**

The recent upsurge of research toward compressive sampling and parsimonious signal representations hinges on signals being sparse, either naturally, or, after projecting them on a proper basis. Interestingly, we have unveiled that by controlling sparsity of model residuals enables computationally affordable statistical inference that is universally robust to outliers (even when the underlying signals are not sparse), thus establishing a surprising link between sparsity and robust statistics. Universality pertains to criteria (loss functions and regularizing complexity controlling terms), signal and outlier models, as well as WSN-based inference tasks including robust parameter batch, nonparametric [10] and recursive estimation [11], distributed tracking, principal component analysis [14], classification, clustering, and multidimensional scaling [15, 13]. This contribution is relevant to the approximate robust inference algorithms (Section 2.3.2.3 in the proposal), and it is also instrumental for robust sensing, cooperative localization, mapping and tracking (Section 2.3.1.1 in the proposal).

### **6.2.3 Network-Compressive Coding for WSNs with Correlated Data [16].**

In large-scale WSN deployments, relaying information over several hops becomes increasingly energy inefficient. On the other hand, observations from nearby sensors may be highly correlated; for instance, in chemical contaminant monitoring or intrusion detection systems. For such settings, we jointly exploited the network graph along with the underlying communication graph to leverage spatial correlation for in-network data compression, thus effecting significant energy savings and prolonged network lifetime. Capturing statistical dependencies via factor graphs, we demonstrated how inference can be achieved at sink nodes based on compressed data, and using the method of types we derived error exponents for cyclic and acyclic factor graphs. Interestingly, the latter revealed that sensory observations can be recovered with arbitrarily low probability of error as the network scale grows. The resultant network-compressive coding framework addresses the fundamental resource-performance tradeoffs depending on network affordances (Section 2.4.2.1 in the proposal), along with the required sensory coordination efforts (Section 2.4.2.1 in the proposal), as well as the compressive sensing tasks for distributed inference (Section 2.2.2.1 in the proposal).

### 6.2.4 Unveiling Anomalies in Large-Scale Networks via Sparsity and Low Rank [17, 18, 4, 19, 8, 20, 21].

In the backbone of large-scale networks, traffic flows experience abrupt unusual changes which can result in congestion, and limit the extent to which end-user quality of service requirements are met. Diagnosing such traffic volume anomalies is crucial towards engineering the network traffic. This is challenging however, since the available data are the superposition of unobservable origin-to-destination (OD) flows per link. Leveraging the low intrinsic-dimensionality of OD flows and the sparse nature of anomalies, a convex program is formulated to unveil anomalies across flows and time. A centralized solver is put forth using the proximal gradient algorithm, which offers provable iteration complexity guarantees. An equivalent nonconvex but separable criterion enables in-network processing of link-load measurements, when optimized via the alternating-direction method of multipliers. The novel distributed iterations entail reduced-complexity local tasks, and affordable message passing between neighboring nodes. Interestingly, under mild conditions the distributed algorithm approaches its centralized counterpart.

### References

- [1] Morteza Mardani and Georgios B Giannakis. “Estimating Traffic and Anomaly Maps via Network Tomography”. In: (2015).
- [2] M. Mardani, G. Mateos, and G. B. Giannakis. “Subspace Learning and Imputation for Streaming Big Data Matrices and Tensors”. In: *IEEE Trans. Sig. Proc.* 63.10 (May 2015), pp. 2663–2677.
- [3] M. Mardani, G. Mateos, and G. B. Giannakis. “Decentralized Sparsity-Regularized Rank Minimization: Algorithms and Applications”. In: *IEEE Trans. Sig. Proc.* 61.21 (Nov. 2013), pp. 5374–5388.
- [4] M. Mardani, G. Mateos, and G. B. Giannakis. “Dynamic Anomalography: Tracking Network Anomalies via Sparsity and Low Rank”. In: *IEEE J. Sel. Topics Sig. Proc.* 07.1 (Feb. 2013), pp. 50–66.
- [5] P. A. Forero, V. Kekatos, and G. B. Giannakis. “Robust Clustering Using Outlier-Sparsity Regularization”. In: *IEEE Trans. Sig. Proc.* 60.8 (Aug. 2012), pp. 4163–4177.
- [6] J. A. Bazerque, G. Mateos, and G. B. Giannakis. “Rank Regularization and Bayesian Inference for Tensor Completion and Extrapolation”. In: *IEEE Trans. Sig. Proc.* 61.22 (Nov. 2013), pp. 5689–5703.
- [7] J. A. Bazerque and G. B. Giannakis. “Nonparametric Basis Pursuit via Sparse Kernel-Based Learning”. In: *IEEE Sig. Proc. Mag.* 30.4 (July 2013), pp. 112–125.

- [8] M. Mardani, G. Mateos, and G. B. Giannakis. “Distributed Nuclear Norm Minimization for Matrix Completion”. In: *Proc. of 13th Wrkshp. on Signal Processing Advances in Wireless Communications*. Cesme, Turkey, June 2012.
- [9] E. Msechu and G. B. Giannakis. “Sensor-Centric Data Reduction for Estimation With WSNs via Censoring and Quantization”. In: *IEEE Trans. Sig. Proc.* 60.1 (Jan. 2012), pp. 400–414.
- [10] G. Mateos and G. B. Giannakis. “Robust Nonparametric Regression via Sparsity Control With Application to Load Curve Data Cleansing”. In: *IEEE Trans. Sig. Proc.* 60.6 (Apr. 2012), pp. 1571–1584.
- [11] S. Farahmand and G. B. Giannakis. “Robust RLS in the Presence of Correlated Noise Using Outlier Sparsity”. In: *IEEE Trans. Sig. Proc.* 60.6 (June 2012), pp. 3308–3313.
- [12] G. Mateos and G. B. Giannakis. “Distributed Recursive Least-Squares: Stability and Performance Analysis”. In: *IEEE Trans. Sig. Proc.* 60.7 (July 2012), pp. 3740–3754.
- [13] P. Forero and G. B. Giannakis. “Sparsity-Exploiting Robust Multidimensional Scaling”. In: *IEEE Trans. Sig. Proc.* 60.8 (Aug. 2012), pp. 4418–4134.
- [14] G. Mateos and G. B. Giannakis. “Robust PCA as Bilinear Decomposition with Outlier-Sparsity Regularization”. In: *IEEE Trans. Sig. Proc.* 60.10 (Oct. 2012), pp. 5176–5190.
- [15] P. Forero and G. B. Giannakis. “Robust Multi-Dimensional Scaling via Outlier-Sparsity Control”. In: *Proc. of 45th Asilomar Conf. on Signals, Systems, and Computers*. Pacific Grove, CA, Nov. 2011.
- [16] K. Rajawat, A. Cano, and G. B. Giannakis. “Network-Compressive Coding for Wireless Sensors with Correlated Data”. In: *IEEE Trans. Wireless Commun.* 11.12 (Dec. 2012), pp. 4264–4274.
- [17] M. Mardani, G. Mateos, and G. B. Giannakis. “In-network Rank Minimization and Sparsity Regularization: Algorithms and Applications”. In: *IEEE Trans. on Sig. Proc.*, submitted Jan. 2013 ().
- [18] M. Mardani, G. Mateos, and G. B. Giannakis. “Recovery of Low-Rank Plus Compressed Sparse Matrices with Application to Unveiling Traffic Anomalies”. In: *IEEE Trans. on Info. Theory*, submitted Apr. 2012; revised Dec. 2012 ().
- [19] M. Mardani, G. Mateos, and G. B. Giannakis. “Unveiling Anomalies in Large-scale Networks via Sparsity and Low Rank”. In: *Proc. of 45th Asilomar Conf. on Signals, Systems, and Computers*. Pacific Grove, CA, Nov. 2011.
- [20] J. A. Bazerque, G. Mateos, and G. B. Giannakis. “Nonparametric Low-Rank Tensor Imputation”. In: *Proc. of IEEE Wrkshp. on Statistical Signal Processing*. Ann Arbor, MI, Aug. 2012.

- [21] M. Mardani, G. Mateos, and G. B. Giannakis. “Exact Recovery of Low-Rank Plus Compressed Sparse Matrices”. In: *Proc. of IEEE Wrkshp. on Statistical Signal Processing*. Ann Arbor, MI, Aug. 2012.

## 7 University of Minnesota — Stergios Roumeliotis

### 7.1 November 2015, FINAL REPORT

#### 7.1.1 Active Sensing for cooperative localization and target tracking

A fundamental problem we have address during this project is that of using mobility for maximizing the information acquired by a team of robots/sensors. To this end, we have studied the problem of determining the location that a mobile sensor should move to in order to acquire the most informative measurements (distance and/or bearing) for estimating the position of a moving target, while considering motion constraints, imposed by the kinematics of the sensor and/or obstacles in the environment. Despite the fact that this is a non-convex optimization problem, we were able to determine nonlinear transformations of the optimization variables that allowed us to compute the global minimum analytically. Also, we extended this approach to the case of teams of mobile sensors collaborating to track a target where we have introduced appropriate relaxations that have only linear (instead of exponential) in the number of sensors cost, while at the same time achieving performance indistinguishable from that of exhaustive-search-based approaches [1]. Lastly, we have extended this approach to determine the minimal deviation of robot teams motion from a desired formation that maximizes the information gain from inter-robot measurements while considering motion constraints [2].

#### 7.1.2 Sparsity-aware information compression

One of the main limitations of cooperative localization (i.e., the problem of jointly estimating the poses of  $N$  robots from proprioceptive and inter-robot observations) is that MMSE estimators have computational cost up to  $O(N^4)$ . Existing alternatives invoke, often ad hoc, approximations whose impact on performance cannot be quantified. To address this problem we introduced lossless information compression at the sensor fusion center in the form of a sparsity-aware QR algorithm that reduces the processing cost by an order of magnitude [i.e., to  $O(N^3)$ ] [3].

#### 7.1.3 Hybrid (analog/quantized) estimators

Previously, we had introduced MMSE and MAP estimators that are able to operate under stringent communication bandwidth constraints where each measurement is represented and broadcasted using one or few bits. We had also shown that using 4-5 bits per (scalar) measurement suffices for achieving performance very close to that of the corresponding estimator processing analog observations. These approaches, however, suffer from a fundamental limitation: In order to ensure consistency between all estimators in a sensor network (as required), they discard the analog measurements locally available to each sensor/robot; instead they only consider their quantized versions. This

requirement results in information loss, which, if properly addressed, can lead to significant performance improvement even when each measurement is quantized using a single bit. We achieved this objective by introducing ‘hybrid’ estimators that process both locally-available analog observations, as well as remotely-communicated quantized measurements. This was possible by designing new quantization rules that explicitly consider information metrics for characterizing the estimators’ performance [4].

#### 7.1.4 Analytically-guided-sampling-based particle filtering

A long-standing problem in particle filtering is that of particle depletion. Further, it is well-known that the optimal (in the sense that it maximizes the variance of the particles weights and hence reduces the probability of particle depletion) proposal distribution for drawing samples from is the posterior probability density function (pdf). The posterior pdf, however, is, in general, not available or easy to compute. Previous efforts to approximate the posterior pdf have often used uni-modal distributions and seldom considered the structure of the measurement equations, or their impact on the posterior pdf. In our work, we addressed the most challenging case where non-linear measurement functions cause the posterior pdf to be multi-modal and introduced an efficient method for determining all its modes analytically. This allowed us to draw samples from a significantly more accurate parametric approximation of the posterior pdf (using mixtures of Gaussians) and thus reduce the impact of particle depletion [5, 6].

#### 7.1.5 On the observability, consistency, and erroneous information acquisition of linearized systems

Previously, we had investigated the inconsistency for mobile robots navigating in 2D and determined that the main cause of this problem is that the linearized system and measurement models, employed by a linearized estimator [e.g., the extended Kalman filter (EKF)] often (and erroneously, since their Jacobians can only be evaluated at the current state estimate instead at the true state) have fewer unobservable directions than the corresponding nonlinear system. This mismatch in the observability properties of the two systems leads to erroneous information acquisition along unobservable directions, which causes the estimator to become overly confident, loose accuracy, and eventually diverge. To address this problem, we had introduced a systematic approach that, given the unobservable modes of the system, can efficiently (i.e., at negligible processing cost) modify the systems Jacobians so as to guarantee that the observability properties of the nonlinear system are preserved in the linearized one. For the purposes of this project, we extended this work to the significantly more challenging case of 3D navigation using visual (e.g., camera or RGBD) and inertial sensors, such as an inertial measurement unit (IMU). In particular, we introduced a systematic methodology for analytically determining the observable and unobservable modes of any nonlinear system [7] (note: to the best of



our knowledge, no such methodology existed before and, as a result, proving that a system is unobservable and finding its unobservable modes, was possible for only relatively simple systems). Moreover, we employed and applied this methodology to the problems of monocular-camera-based navigation [8], IMU-RGBD [9] and IMU-camera and based navigation using point [10, 11, 12, 7], or line features [13, 14], and considered special cases of motion, such as restricted on a plane [15]. Furthermore, we extended our analysis of linearized estimator inconsistency to address the case of nonlinear systems that although they are observable, they exhibit the same erroneous information acquisition when each measurement provides information for only part of the estimated state (e.g., two radars each measuring its distance to a moving target) [16]. Lastly, and with the aim to extend our investigation to more complex systems that provide sensor information over a wider spectrum, we addressed and analytically solved the problem of extrinsic calibration of a 3D laser scanner with respect to an omnidirectional-camera [17].

#### 7.1.6 High-efficiency vision-aided inertial navigation systems (VINS)

In addition to consistency (see 2.5), in this project, we focused on improving the efficiency and robustness of estimators used for 3D localization and mapping. In particular, we introduced a new form of estimator, the square-root inverse sliding window filter (SR-ISWF) [18], that employs the measurements available during a sliding time window to optimally determine the 3D pose of an IMU-camera system at half the time as compared to the current state of the art. This was possible by meticulously investigating and exploiting the particular structure of the problem to realize efficiencies in the numerical parts of the estimation algorithm, but also by wisely selecting and processing the most informative measurements [19]. Additionally, we considered the case of 3D mapping and introduced a new methodology for designing approximate estimators that create maps using a sparse, “island-like” representation of the space, thus realizing significant savings and allowing for the first time ever, real-time consistent mapping on mobile devices [20, 21]. Furthermore, and in order to increase the robustness and accuracy of the aforementioned estimators, we introduced methods for explicitly modeling and compensating for the lack of time synchronization between the systems sensors [22], the effect of the camera rolling shutter [22], and conditions where the motion becomes unobservable (hovering over the same scene) [23]. Lastly, we extended our work on VINS for the case of multiple aerial vehicles flying over the same scene [24].

#### 7.1.7 Autonomous quadrotor navigation through image-defined paths

We addressed the problem of autonomous quadrotor navigation within GPS-denied, and in particular indoor spaces. Specifically, we introduced an algorithm for constructing (offline) a visual map of the area, represented as a graph of linked images, based on visual and inertial data collected beforehand. This topological/appearance-based representation of the space is then used for spec-

ifying visual paths for the quadrotor to follow (e.g., during a patrol task). The path-relevant information, compressed to a binary representation of the features that will be seen along the quadrotors path, is then provided to the quadrotor which employs a geometric/probabilistic approach to determine the type of motion it needs to follow (e.g., moving on an arc versus rotating in place) in order to visually servo between successive reference images towards its goal. Moreover, and in order to improve the robustness of the system, we empowered the quadrotor with place recognition capabilities which employ a highly efficient data structure for querying and retrieving matches between the current image and previously mapped images. Lastly, we introduced an adaptive optical-flow algorithm that can accurately estimate the quadrotors horizontal velocity under adverse conditions (e.g., when flying over dark, texture-less areas) by progressively using additional information from the available images [25].

## References

- [1] K. Zhou and S.I. Roumeliotis. “Multi-robot Active Target Tracking with Combinations of Relative Observations”. In: *IEEE Transactions on Robotics* 27.4 (Aug. 2011), pp. 678–695.
- [2] X.S. Zhou, K.X. Zhou, and S.I. Roumeliotis. “Optimized Motion Strategies for Formation Localization”. In: *In preparation* (2016).
- [3] K.X. Zhou and S.I. Roumeliotis. “A Sparsity-aware QR Decomposition Algorithm for Efficient Cooperative Localization”. In: *Proc. IEEE International Conference on Robotics and Automation (ICRA’12)*. Saint Paul, MN, May 2012, pp. 799–806.
- [4] E.D. Nerurkar and S.I. Roumeliotis. “Hybrid Maximum a Posteriori Estimation under Communication Constraints”. In: *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP’13)*. Vancouver, Canada, May 2013.
- [5] G.P. Huang and S.I. Roumeliotis. “An Analytically-guided Sampling-based Particle Filter Applied to Range-only Target Tracking”. In: *Proc. IEEE International Conference on Robotics and Automation (ICRA’13)*. Karlsruhe, Germany, May 2013.
- [6] G.P. Huang, K.X. Zhou, N. Trawny, and S.I. Roumeliotis. “A Bank of Maximum A Posteriori (MAP) Estimators for Target Tracking”. In: *IEEE Transactions on Robotics* 31.1 (Feb. 2015), pp. 85–103.
- [7] J.A. Hesch, D.G. Kottas, S.L. Bowman, and S.I. Roumeliotis. “Camera-IMU-based Localization: Observability Analysis and Consistency Improvement”. In: *International Journal of Robotics Research* 33.1 (Jan. 2014), pp. 182–201.
- [8] J.A. Hesch and S.I. Roumeliotis. “Consistency Analysis and Improvement for Single-camera Localization”. In: *Proc. of the 2nd IEEE Workshop on Egocentric (First-Person) Vision, Conference on Computer Vision and Pattern Recognition (CVPR’12)*. Providence, RI, June 2012.

- [9] C.X. Guo and S.I. Roumeliotis. “IMU-RGBD Camera Extrinsic Calibration: Observability Analysis and Consistency Improvement”. In: *Proc. IEEE International Conference on Robotics and Automation (ICRA ’13)*. Karlsruhe, Germany, May 2013.
- [10] J.A. Hesch, D.G. Kottas, S.L. Bowman, and S.I. Roumeliotis. “Towards Consistent Vision-aided Inertial Navigation”. In: *Proc. 10th International Workshop on the Algorithmic Foundations of Robotics (WAFR’12)*. Boston, MA, June 2012.
- [11] D.G. Kottas, J.A. Hesch, S.L. Bowman, and S.I. Roumeliotis. “On the consistency of Vision-aided Inertial Navigation”. In: *Proc. 13th International Symposium on Experimental Robotics (ISER’12)*. Quebec City, Canada, June 2012.
- [12] J.A. Hesch, D.G. Kottas, S.L. Bowman, and S.I. Roumeliotis. “Consistency Analysis and Improvement of Vision-aided Inertial Navigation”. In: *IEEE Transactions on Robotics* 30.1 (Feb. 2014), pp. 158–176.
- [13] D.G. Kottas and S.I. Roumeliotis. “Efficient and Consistent Vision-aided Inertial Navigation using Line Observations”. In: *Proc. IEEE International Conference on Robotics and Automation (ICRA ’13)*. Karlsruhe, Germany, May 2013.
- [14] D.G. Kottas and S.I. Roumeliotis. “Exploiting Urban Scenes for Vision-aided Inertial Navigation”. In: *Proc. Robotics: Science and Systems (RSS’13)*. Berlin, Germany, June 2013.
- [15] G. Panahandeh, C.X. Guo, M. Jansson, and S.I. Roumeliotis. “Observability Analysis of a Vision-aided Inertial Navigation System Using Planar Features on the Ground”. In: *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS’13)*. Tokyo, Japan, Nov. 2013.
- [16] G.P. Huang and S.I. Roumeliotis. “On Filter Consistency of Discrete-time Nonlinear Systems with Partial-state Measurements”. In: *Proc. IEEE American Control Conference (ACC’13)*. Washington, DC, June 2013.
- [17] F.M. Mirzaei, D.G. Kottas, and S.I. Roumeliotis. “3D Lidar-Camera Intrinsic and Extrinsic Calibration: Observability Analysis and Analytical Least Squares-based Initialization”. In: *International Journal of Robotics Research , Special Issue on Robot Vision* 31.4 (Apr. 2012), pp. 452–467.
- [18] K. Wu, A. Ahmed, G. Georgiou, and S.I. Roumeliotis. “A Square Root Inverse Filter for Efficient Vision-aided Inertial Navigation on Mobile Devices”. In: *Robotics: Science and Systems (RSS’15)*. Rome, Italy, July 2015.
- [19] D.G. Kottas, R.C. DuToit, A. Ahmed, C.X. Guo, G. Georgiou, R. Li, and S.I. Roumeliotis. “A Resource-aware Vision-aided Inertial Navigation System for Wearable and Portable Computers”. In: *Workshop: Long Term Autonomy, IEEE International Conference on Robotics and Automation (ICRA ’14)*. Hong Kong, China, May 2014.

- [20] E.D. Nerurkar, K.J. Wu, and S.I. Roumeliotis. “C-KLAM: Constrained Keyframe-Based Localization and Mapping”. In: *Workshop: Multi-View Geometry in Robotics, Robotics: Science and Systems (RSS’13)*. Berlin, Germany, June 2013.
- [21] E.D. Nerurkar, K.J. Wu, and S.I. Roumeliotis. “C-KLAM: Constrained Keyframe-Based Localization and Mapping”. In: *Proc. IEEE International Conference on Robotics and Automation (ICRA’14)*. Hong Kong, China, May 2014.
- [22] C.X. Guo, D.G. Kottas, R.C. DuToit, A. Ahmed, R. Li, and S.I. Roumeliotis. “Efficient Visual-Inertial Navigation using a Rolling-Shutter Camera with Inaccurate Timestamps”. In: *Robotics: Science and Systems (RSS’14)*. Berkeley, California, July 2014.
- [23] D.G. Kottas, K.J. Wu, and S.I. Roumeliotis. “Detecting and Dealing with Hovering Maneuvers in Vision-aided Inertial Navigation Systems”. In: *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS’13)*. Tokyo, Japan, Nov. 2013.
- [24] I.V. Melnyk, J.A. Hesch, and S.I. Roumeliotis. “Cooperative Vision-aided Inertial Navigation using Overlapping Views”. In: *Proc. IEEE International Conference on Robotics and Automation (ICRA’12)*. Saint Paul, MN, pp. 936–943.
- [25] T. Do, L.C. Carrillo-Arce, and S.I. Roumeliotis. “Autonomous Flights through Image-denied Paths”. In: *Proc. International Symposium of Robotics Research (ISRR)*. Sestri Levante, Italy, Sept. 2015.

## 8 University of Pennsylvania — Ali Jadbabaie

### 8.1 October 2015 report

Our Research on distributed estimation and social learning has continued in the following directions:

We have shown that a variant of the social learning model presented at the last review and discussed in [1] has a very interesting interpretation in terms of a new techniques for distributed estimation. In other words, the problem of finding a true state of the world in a network of agents receiving a sequence of signals that are partially informative about the true state can be formulated as a distributed optimization problem called distributed dual averaging [2].

In [3, 4], we have characterized the rates of convergence for the learning model presented in [1]. We have shown that the naive updating used lets agents learn the unknown state exponentially fast—as if they are sophisticated Bayesian agents. We also have characterized the exponent in terms of the information contained in agents’ observations and their centralities in the network. As we argued in [3], for the learning process to be efficient, agents with the most informative observations (as measured by Kullback-Leibler divergence) need to also be the ones most centrally located in the network (as measured by eigenvector centrality).

We have extended the Bayesian learning framework discussed in Section 5 of [1] to the case where agents play a game with other agents in the network. We have argued that when agents have a coordination motive, in addition to the estimation motive, even if they play according to the selfishly and myopically optimal strategies, they will eventually reach consensus on the (socially) optimal action. We also have shown that when agents signals are Gaussian and their utility functions are quadratic, the belief update can be carried out in a tractable fashion similar to a Kalman filter.

In [5] we analyze a social learning model in which agents follow Bayes rule yet they do not recall their history of past observations and cannot reason about how other agents’ beliefs are formed. They do so by making rational inferences about their observations which include a sequence of independent and identically distributed private signals as well as the beliefs of their neighboring agents at each time. Successive applications of Bayes rule to the entire history of past observations leads to forebodingly complex inferences due to lack of knowledge about the global network structure that causes those observations. To address these complexities, in [5] we consider a Bayesian without Recall (BWR) model of inference, which in addition to providing a tractable framework for analyzing the behavior of rational agents in social networks, can also provide a behavioral foundation for the variety of non-Bayesian update rules in the literature.

In [6] we study the problem of non-Bayesian learning over social networks by taking an axiomatic approach. As our main behavioral assumption, we postulate that agents follow social learning rules that satisfy imperfect recall, according to which they treat the current beliefs of their neighbors as sufficient statistics for all the information available to them. We establish that as long as

imperfect recall represents the only point of departure from Bayesian rationality, agents social learning rules take a log-linear form. Our approach also enables us to provide a taxonomy of behavioral assumptions that underpin various non-Bayesian models of learning, including the canonical model of DeGroot. We then show that for a fairly large class of learning rules, the form of bounded rationality represented by imperfect recall is not an impediment to asymptotic learning, as long as agents assign weights of equal orders of magnitude to every independent piece of information. Finally, we show how the dispersion of information among different individuals in the social network determines the rate of learning.

We have developed a family of distributed algorithms for transshipment problems that rely on local information and exhibit quadratic convergence rates. More recently, we have shown in [7] that this approach can also be used in network flow problems where the demand vector is uncertain

In [8], we have extended our work on single time scale distributed parameter estimation to the case where the dynamics of the parameter is unstable. We have characterized bounds that relate the mixing properties of the network to instability of the dynamics parameter to be estimated, leading to the notion of network tracking capacity. A journal version of this result plus optimality bounds is submitted for publication in [8].

In [9] we consider the problems of optimal scheduling and estimation for measurements that are subject to latency of data acquisition. The latency in data acquisition refers to the time-delay between measurement and processing and it is inherent to dynamic and decentralized measurement architectures, where the processing is contingent upon acquisition of several data points each of which is spaced in time. Our aim in [9] is to investigate a design framework under which the deteriorating effect of data acquisition latency on the estimation performance is minimized. To this end, we consider a measurement scenario where the designer aims to estimate the unknown state of a discrete-time linear time-invariant (LTI) system but she is constrained to measure only one of the 'm' scalar outputs at every point in time. Subsequently, the designer would have a choice to make of which output to measure each time. We use an appropriate adaption of the Kalman filter and proffer efficient semi-definite programs to design the output measurement schedules that optimize the estimation performance. We consider both periodic and random schedules and in each case offer numerical examples to test the performance of optimally designed schedules.

## References

- [1] Pooya Molavi, Ali Jadbabaie, Kamiar Rahnema Rad, and Alireza Tahbaz-Salehi. "Reaching Consensus with Increasing Information". In: *IEEE Journal of Selected Topics in Signal Processing (accepted for publication)* (2013).
- [2] Shahin Shahrapour and Ali Jadbabaie. "Exponentially Fast Parameter Estimation in Networks Using Distributed Dual Averaging". In: *Proceedings of the 52nd IEEE Conference on Decision and Control*. Dec. 2013, pp. 6196–6201.

- [3] Ceyhun Eksin, Pooya Molavi, Alejandro Ribeiro, and Ali Jadbabaie. “Bayesian quadratic network game filters”. In: *Proceedings of International Conference on Acoustics, Speech, and Signal Processing (submitted)* (May 2013).
- [4] Ceyhun Eksin, Pooya Molavi, Alejandro Ribeiro, and Ali Jadbabaie. “Bayesian quadratic network game filters”. In: *IEEE Transactions on Signal Processing (submitted)* (Dec. 2012).
- [5] M. A. Rahimian, P. Molavi, and Ali Jadbabaie. “(Non-)Bayesian Learning without Recall”. In: *Proceedings of the 53rd IEEE Conference on Decision and Control*. Dec. 2014, pp. 5730–5735.
- [6] Pooya Molavi, Alireza Tahbaz-Salehi, and Ali Jadbabaie. “Foundations of Non-Bayesian Social Learning”. In: (2015). Columbia Business School Research Paper.
- [7] Michael Zargham, Alejandro Ribeiro, and Ali Jadbabaie. “Network Optimization Under Uncertainty”. In: *Proceedings of the 51st IEEE Conference on Decision and Control*. Dec. 2012.
- [8] Ceyhun Eksin, Pooya Molavi, Alejandro Ribeiro, and Ali Jadbabaie. “Learning in network games with incomplete information”. In: *IEEE Signal Processing Magazine* (May 2013).
- [9] M. A. Rahimian, S. A. Aleem, F. Koufogiannis, A. Jadbabaie, and G. J. Pappas. “Estimation and Scheduling with Data Acquisition Latency”. In: *Proceedings of the American Control Conference (submitted)*. July 2016.

## 9 University of Pennsylvania — Daniel E. Koditschek

### 9.1 October 2015 KodLab AFOSR CHASE MURI update

#### 9.1.1 Learning and navigation in uncertain spaces (Reverdy, Ilhan and Koditschek)

Motivated by the notions from ecology that the fitness of an embodied autonomous system is a function of 1) the body, 2) the environment, and 3) the task(s) which the body must perform, [1], we have focused on developing formal notions of environments. We seek to develop tractable representations of the task-relevant and learnable features of environments that would afford embodied autonomous systems the ability to improve their performance over time.

In [2], we consider a navigation task along the lines of [3] take the viewpoint that the environment is fixed but cluttered with obstacles of unknown shape and location. We develop a stochastic control policy that permits a natural mapping of the obstacles into two categories depending on their geometry, which carries direct implications into the degree of difficulty of avoiding them with the control policy.

In [4], we consider the problem of understanding the spatial distribution of objects of interest in a domain. We adopt a statistical model of object distribution based on the Poisson point process from spatial point process theory [5], for which the single parameter represents the mean density of objects in the domain. We then consider the estimation problem for this parameter and develop a method to produce reliable parameter estimates using a mobile robot as the platform for a sensor that can detect and register the objects.

**Acknowledgements** Items [2] and [4] represent original work funded in part by AFOSR CHASE MURI.

#### 9.1.2 Clustering-Based Robot Navigation and Control (Arslan, Guralnik and Koditschek)

Clustering is traditionally an unsupervised learning method aimed at discovering coherent groups (clusters) in a given unlabeled dataset to model its unknown global organizational structure and/or to determine a local neighborhood of every data point [6].

Inspired by its use for modelling global organizational structure, we introduce a novel application of clustering to the problem of coordinated robot navigation [7]. The notion of hierarchical clustering offers a natural abstraction for ensemble task encoding and control in terms of precise yet flexible organizational specifications at different resolutions, by relating the continuous space of configurations to the combinatorial space of trees. Based on this new abstraction, we propose a provably correct, computationally effective generic hierarchical navigation framework for collision-free motion design towards any given destination via a sequence of hierarchy preserving controllers. More precisely, as intrinsically suggested by our hierarchical abstraction, we introduce a two-level



navigation strategy for coordinated motion design: (i) at the low-level perform finer adjustments on configurations using hierarchy preserving vector fields [8], (ii) and at the high-level resolve structural conflicts between configurations using a discrete transition policy in tree space [9]; and the connection between these two levels is established by an optimal selection of a portal configuration supporting two adjacent hierarchies [10, 11]. Work now in progress targets a distributed implementation of our navigation framework based on our results on anytime hierarchical clustering [12, 13].

Inspired by the use of clustering for locality identification, we introduce a new application of generalized Voronoi diagrams to identify collision free (multi)robot configurations [14, 15]. In [14] we consider distributed mobile sensing applications of heterogeneous agents and propose a provable correct, collision free coverage and congestion control algorithm for heterogeneous disk-shaped robots. We are currently exploring another extension of Voronoi-based coverage control for hierarchical settings, based on nested partitions of convex environments. What is more, we also reconsider the problem of reactive navigation in sphere worlds and propose a convex optimization framework whose continuous evaluation is used to solve the collision free robot navigation problem with a unique attractor at a designated goal location [15]. Work now in progress targets navigation among convex obstacles using separating hyperplanes of convex bodies and robot navigation using a fixed radius sensory footprint [16].

**Acknowledgements** Items [7, 8, 9, 10, 11, 12, 13, 14, 15, 16] represent original work funded in part by AFOSR CHASE MURI.

### 9.1.3 Classification and Representation

**Functorial Hierarchical Overlapping Clustering (Guralnik, Culbertson and Stiller)** Much of the practice of clustering is based on geometric ideas regarding the role ought to be played by proximity, distribution, shape and relative position among members of a set of data points (usually “feature vectors”) in a Euclidean space [6]. More recently (though some methods are with us from the 1960s [17]), the emergent need for unsupervised clustering in so-called “big data” applications has put an emphasis on the need for a principled approach to clustering in settings where, instead of embedding the data set in some normed space (thereby exposing the data to unknown, poorly understood biases), a practitioner prefers to express their domain expertise in weighting pairs, triples etc. of data points with quantities representing an empirically assigned “degree of dissimilarity” / “degree of proximity”, and derive the latent classification (hard/soft/hierarchical...) directly from that structure while “letting the data speak for itself”. A great variety of methods of the latter kind have been engineered over the years, starting from classical agglomerative neighbor-joining methods [17, 18, 19] to the more statistically motivated ones [20, 21], just to name a few. Overall, however, the practice of clustering – especially that of dissimilarity clustering – has become more of an art of fitting a method to a problem, often through much trial and error, rather than a science

assigning appropriate methods to satisfy a set of specifications. This difficulty has brought into sharp focus the need for a notion of consistency for clustering methods, producing a string of “impossibility theorems” for constructing various axiomatic frameworks for dissimilarity-based partitioning (and the evaluation of results thereof) one might find intuitively plausible [22, 23]. This has given rise to one of the two notable exceptions to the state of affairs just described: work by Carlsson and Mémoli [24, 25, 26] convincingly establishes the category-theoretic notion of functoriality as a first decent candidate for a notion of consistency of a clustering method, though they limit themselves to hard (flat and hierarchical) clustering. The second of these exceptions is the notion of metric clustering pioneered by Bunemann [27, 28] and developed into a full-blown field (see [29, 30] for a review) of applied mathematical research following the seminal work of Bandelt and Dress on split decompositions [31, 32], studying hard clustering with overlaps, with roots in Isbell’s enquiries [33] into the geometry and topology of injective metric spaces from the point of view of studying the category of metric spaces with non-expansive maps.

Our work [34, 35, 36] represents an attempt to (1) unify these two schools of thought (functorial clustering) while (2) leveraging the deep connections between injectivity in metric spaces, non-positive curvature and cut decompositions of metric spaces [34, 37] in order to (3) study and construct functorial clustering methods which allow overlaps, thus potentially overcoming the limitations of the “chaining phenomenon” which turns most practitioners away from single linkage hierarchical clustering. Extending the results of [26] from partitions to the class of “flag covers”, we establish that all functorial (with respect to non-expansive maps) distance-based flat clustering methods are refined by the Rips complex [35, 37]. Furthermore, passing to a persistent version of flag covers called ‘sieves’, we provide a framework generalizing the observations of Kleinberg [22] and Carlsson-Mémoli [24] to a characterization of the possible ‘ranges’ of distance-based functorial clustering maps [36]. Unfortunately, one outcome of this generalization is the conclusion that the Bandelt-Dress approach to clustering is irreconcilable with a functorial approach within the framework of the category of metric spaces with non-expansive maps.

**Acknowledgements** Items [34, 35, 36] represent original work funded by in part by AFOSR CHASE MURI.

**Universal Memory Architectures (Guralnik and Koditschek)** A major obstacle on the way to producing general autonomous agents is the problem of maintaining a scalable internal representation of the agent’s experiences and predictions. Settled, accepted approaches to this problem in AI [38, 39, 40, 41, 42, 43, 44] are challenged [45, 46, 47] by: (1) the necessity to provide a-priori limited models of the interactions between the agent and its environment; (2) high-dimensional data structures incurring prohibitive maintenance costs, and/or (3) mathematical opacity of the representation denying the designer any ability to provide ‘formal’ performance guarantees for the agent being constructed. The

direction of research by Guralnik and Koditschek into UMAs, emerging from the CHASE MURI and of interest to AFRL/RV in this context [48], is focused on an agnostic, modality-independent approach to the extraction and aggregation of semantics from multiple heterogeneous binary data streams while (a) avoiding bias from prior modeling assumptions, (b) keeping the maintenance costs of the internal representation in check and, at the same time, (c) optimizing the internal representation for provable greedy planning [49, 50]. Moreover, the mathematical theory underlying UMA agents holds much promise in the way of its potential to support the "formal" characterization of application domains where autonomous agents of this kind could bring a task to successful completion despite being provided with only vague a-priori knowledge of the possible obstacles.

Directions for ongoing and future research are: (1) the development of a principled, decision-theoretic approach to the control of learning architecture parameters in UMA agents; (2) the characterization of sufficient descriptor sets for specific complex application domains; and (3) the introduction of a capability for self-enrichment through autonomous learning of meaningful binary descriptors.

**Acknowledgements** Items [49, 50] represent original work funded by in part by AFOSR CHASE MURI and in part by National Science Foundation grant CDI-II-1028237.

**Statistical Estimation and Information Theory of Hierarchical Clustering (Guralnik, Moran and collaborators)** Distance-based hierarchical clustering (DHC) methods are widely used in unsupervised data analysis [6]. Essentially all applications of distance-based clustering occur in highly uncertain domains: This uncertainty can be an inherent 'noise' in the measurement process but in many applications might just be a technique for modeling the unknowns in the weight assignment process. Conventional approaches to statistical estimation of partitions and hierarchies view the objects to be clustered as random samples of certain distributions over a prescribed geometry (e.g. Gaussian mixture model estimation using expectation-maximization in Euclidean spaces). Instead, we propose to directly attribute uncertainty to the process of obtaining values for the pairwise distances rather than distort the data by mapping it into one's 'favorite space'. To the best of our knowledge, very little work has been done in this vein, e.g. [51, 52]. We incorporate a statistical model of the uncertainty through corruption or noise in the pairwise distances and investigate the problem of estimating the DHC as unknown parameters from measurements. With work by Carlsson and Mémoli [24, 25, 26] establishing the primacy of Single Linkage Hierarchical Clustering as the only natural candidate for DHC, we focus on single linkage hierarchical clustering (SLHC). Statistical estimation of SLHC is of particular importance due to evidence that the negative effects of the so-called 'chaining phenomenon' which tends to turn users away from SLHC (which otherwise would be a preferred tool in their toolkit) may be mitigated

by the introduction of noise (‘dithering’) [53]. In [54], we study the geometry of SLHC point-pre-images and uncover the inherent super-exponential complexity of maximum likelihood estimation (MLE) of SLHC. We find conditions on the distance measurement noise guaranteeing that SLHC is equivalent to maximum partial profile likelihood estimation (MPPLE) ignoring certain measured distances. At the same time, we show that direct evaluation of SLHC on noisy data yields a consistent estimator. Consequently, a full maximum likelihood estimation (MLE) is expected to perform better than SLHC in getting the correct HC results for the ground truth metric – a claim supported by our work on numerical approximation of the MLE estimator of SLHC [55, 56].

A statistical outlook on HC motivates additional questions. It must be noted that various measures of separation and commutation (e.g. medians) have been considered for hierarchies/dendrograms for a long while [57, 58, 59], mostly on “ad-hoc” combinatorial and geometric grounds having little to do with the statistical nature of the applications for which they are needed. A fundamental problem in this field is then: *Is there a principled way to extend the notions of Shannon entropy and relative entropy, already employed in non-hierarchical clustering, from the domain of partitions to the domain of hierarchies (dendrograms) while retaining their qualities as statistically meaningful notions of diversity and divergence?* The usefulness of this kind of approach has already been verified in the non-hierarchical setting [60] and managed to extend it to a fully information-theoretic soft clustering tool in [20]. Using a well-known equivalence between dendrograms and ultra-metrics, we extend the Statistical Mechanics approach to defining the entropy of a partition to ultra-metrics, showing how to view the (weighted) collection of minimum spanning trees of an ultra-metric as a measure of diversity. Computing the weights has turned out to be the crux of the project: direct methods are intractable due to a very tight relationship between this problem and the long open problem of enumerating the extremal directions of the metric cone [61]. Still, progress has been made towards a solution using the theory of Multivariate Splines [62]. The current focus of the project is on computing what appears to be a new integration kernel that will allow one to bypass the combinatorial complexity of the boundary of the metric cone, and enable at least the verification of the standard axiomatic properties of entropy for the diversity measure we have defined, and perhaps even the computation of useful approximations thereof [63].

**Acknowledgements** Items [54, 55, 56, 63] represent original work funded in part by AFOSR CHASE MURI, by AFOSR grant FA2386-13-1-4080 and in part by the China Scholarship Council.

## 9.2 April 2012 KodLab AFOSR CHASE MURI Update

### 9.2.1 Project RCA.1(i) Control and Estimation of Hierarchy (with Baryshnikov)

**On the Homotopy Type of Point Cluster Hierarchies** In [64] we address the homotopy problem for point cluster hierarchies. Namely, we introduce the configuration space of  $n$  distinct points in  $\mathbb{R}^d$  and consider the homotopy type of a ‘stratum’ - a subset of configurations whose clusters give rise to the same hierarchy. Here, the term ‘hierarchy’ denotes a tree whose edges represent nested subsets of the index set  $J := 1, \dots, n$ . Namely, these subsets arise as the lattice of successively refined partitions of  $J$ , whose ‘top’ partition is the singleton of all indices, whose ‘bottom’ partition is the set of singleton indices, and whose intermediate partitions are defined by the manner in which subsets of ‘closest neighbors’ form and coalesce as the geometric scale of ‘close’ is gradually increased from 0 to the diameter of the entire point cloud.

There are many different ways to define a subset of ‘closest neighbors’ and we consider two distinct mechanisms. First, we examine a bottom up mechanism called ‘single linkage’ clustering whose lattice of partitions is defined by connected components formed from unions of particle-centered disks of increasing radii. Alternatively, we consider a top down mechanism called ‘barycentric’ clustering whose partition lattice arises from nested subgroupings that are maximally separated. Here, ‘maximal separation’ is interpreted to mean that the disjoint subsets comprising the sub-partition within a given cluster yield centroids that are as mutually far separated as possible given any other possible sub-partition.

The central result of this analysis is that the non-degenerate strata have the homotopy type of a product of spheres - essentially the epicycles of classical cosmology. This is established formally for the barycentric hierarchical clusterings. In contrast, while sufficiently ‘scale separated’ single linkage hierarchical clusters enjoy the same property, it is not clear (and seems unlikely) that this homotopy type persists over an entire stratum. Thus, whereas single linkage clustering gives rise to dendrograms (well studied objects accompanied by a formalism for endowing the set of combinatorial trees with the structure of a continuous metric space) and lends itself to strongly decentralized applications, barycentric clustering, which has a more centralized construction, seems to reveal its topological structure a bit more readily.

**Hierarchical Formation Control** We leverage and apply to the problem of relaxed formation control in [8] the results of [64] as follows. We pose the problem of navigating a swarm of fully actuated, non-intersecting point robots in  $\mathbb{R}^d$  to a specified configuration. We design a centralized (two-stage) hybrid controller whose basin of attraction is the entire barycentric-cluster hierarchy stratum of the specified goal as follows.

A simple  $\mathbb{R}^{nd}$ -Euclidean distance-gradient vector field centered on the goal configuration induces a basin around that attractor whose intersection with

the stratum leaves positive invariant a conical neighborhood from the origin spreading out to include that goal along the way toward including its increasingly scale-separated projections. This gradient-like field represents the second stage of the hybrid controller. The first stage targets a scale-separated projection of the goal configuration guaranteed to lie within the basin of the second. This first stage controller is systematically designed to rotate and narrow each successively refined cluster of a hierarchy in the goal’s stratum toward the corresponding rigid formation of the goal’s corresponding cluster in its the scale separated projection. In essence, this first stage controller imposes orbits along the epicycles prescribed by the goal stratum’s homotopy type, known according to [64]. It can be shown that this first stage controller leaves the stratum positive invariant and attracts all but a zero measure set of initial configurations to the scale-separated projection of the goal, whereupon the second controller can be engaged via a sequential composition [65].

### 9.2.2 Project RCA.3(iii) Sensor and Actuator Planning (with Moran & Howard)

**Localization over Partitions** We have introduced a new approach to highly uncertain motion estimation and control based upon histogram filters over cellular decompositions of a state space. In the very simplest problem we consider the task of a commanding a one degree of freedom ‘robot’ to approach and grab a one degree of freedom ‘object’ and bring it to a specified ‘nest’ configuration. As a deterministic problem this is the simplest example we know of a (non-classical) non-holonomically constrained control system, a class of problems for which smooth stabilization is known to be impossible, but which in this elementary setting admits of an obvious hybrid stabilizing controller [66].

The simplest partition-based stochastic version of this problem, introduced in [67], assumes that the robot has a perfect sense of its own position but only a noisy view of the object which improves with relative proximity. For example, the simple measurement model we are presently working with assumes an unbiased Gaussian distribution whose variance is affine in the relative distance. The novelty of this work arises from our estimator’s design as a histogram filter over a simple partition of the event space. For example, because the initial measurement model is unimodal, it seems clear that we should be able to work with a three cell approximation to the posterior distribution, throwing away any other detail about past measurements. Extensive simulations with this model suggest that such filters will converge unless the variance of the measurement model is ‘outlandishly’ pessimistic. However, developing a proof that this is correct has been challenging exactly because the measurement model’s sensitivity to relative position implies that the succession of measurements will be neither identically distributed nor independent. In fact, we know of no existing result in the localization estimation literature where convergence has been proven absent an iid assumption on the measurements. We expect that a formal result in even this very simple case will suggest a general pattern of how to conceptualize the quality of piecewise constant approximations to the posterior required

to achieve convergence in localization problems. This would, in turn, begin to inform our broader thinking about how to select and parametrize the class of cellular decompositions over which these piecewise constant approximations are to be developed.

**Optimal Action Policies for Localization over Partitions** A variant of the robot-grab-object task just described focuses attention on the longstanding question of direct vs. indirect control. The localization approach, above, presumes a pre-existing deterministic controller respecting which estimates of whatever kind available are applied to that controller, yielding a methodology known as “certainty equivalence” in the controls literature. The phrase “whatever kind available” bespeaks the proliferation of disparate loss functions relative to which the estimator might be optimal.

In contrast, a direct controls approach would instead impose a loss function over the allowed control actions at any state, and seek to develop sensory reaction policies that are optimal with respect to that grading of outcomes. Now, rather than discretizing the space of sensory inputs, the problem is to quantize effectively the space of control actions that are allowable at any state. We are presently exploring numerically the sort of robot localization behavior obtained in this simple robot-grab-object task by partitioning the action set with a simple three cell decomposition on which we impose a reasonable loss function whose appropriately discounted iterates are to be minimized. The resulting ‘direct’ optimal control policy appears to perform at least as well as the previously discussed indirect scheme.

An immediate next open question under investigation in this branch of the problem include, of course, the prospects for provable convergence properties as mediated by the choice of partition. Longer term, this problem domain presents an appealingly simple setting within which to explore the tradeoffs between direct and indirect control - now with a component of information-action linkage. We suspect that complicated controls problems (e.g, the complete robot-grab-object-and-bring-to-nest problem which is a non-classical non-holonomically constrained problem and, hence, does not admit any smooth point-stabilizing feedback controller even in the deterministic setting [66]) may be harder to solve with direct than with indirect approaches.

**Acknowledgements** References [64, 8, 67] represent original work funded in part by the CHASE MURI.

## References

- [1] Stuart Kauffman and Simon Levin. “Towards a general theory of adaptive walks on rugged landscapes”. In: *Journal of theoretical Biology* 128.1 (1987), pp. 11–45.

- [2] P. Reverdy, B. D. Ilhan, and D. E. Koditschek. “A drift-diffusion model for robotic obstacle avoidance”. In: *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. Sept. 2015.
- [3] D.E. Koditschek and E. Rimon. “Robot Navigation Functions on Manifolds with Boundary”. In: *Advances in Applied Mathematics* 11.4 (1990), pp. 412–442.
- [4] P. Reverdy and D. E. Koditschek. “Mobile robots as remote sensors for spatial point process models”. In: *(submitted to IEEE ICRA 2016)*. 2016.
- [5] Sung Nok Chiu, Dietrich Stoyan, Wilfrid S. Kendall, and Joseph Mecke. *Stochastic Geometry and Its Applications*. en. John Wiley & Sons, June 2013. ISBN: 9781118658253.
- [6] Anil K Jain. “Data clustering: 50 years beyond K-means”. In: *Pattern recognition letters* 31.8 (2010), pp. 651–666.
- [7] O. Arslan, D.P. Guralnik, and D.E. Koditschek. “Coordinated Robot Navigation via Hierarchical Clustering”. In: *submitted to IEEE Transactions of Robotics* (2015).
- [8] O. Arslan, Yu. Baryshnikov, D.P. Guralnik, and D.E. Koditschek. “Hierarchically Clustered Navigation of Distinct Euclidean Particles”. In: *Communication, Control, and Computing (Allerton), 2012 50th Annual Allerton Conference on*. 2012, pp. 946–953.
- [9] O. Arslan, D.P. Guralnik, and D.E. Koditschek. “Discriminative Measures for Comparison of Phylogenetic Trees”. In: *submitted to Discrete Applied Mathematics* (2015).
- [10] O. Arslan, D.P. Guralnik, and D.E. Koditschek. “Navigation of Distinct Euclidean Particles via Hierarchical Clustering”. In: *Algorithmic Foundations of Robotics XI, Springer Tracts in Advanced Robotics* 107 (2015), pp. 19–36.
- [11] O. Arslan and D.E. Koditschek. “On the Optimality of Napoleon Triangles”. In: *submitted to Journal of Optimization Theory and Applications* (2015).
- [12] O. Arslan and D.E. Koditschek. “Anytime Hierarchical Clustering”. In: *arXiv preprint arXiv:1404.3439* (2014).
- [13] O. Arslan and D.E. Koditschek. “A Recursive, Distributed Minimum Spanning Tree Algorithm for Mobile Ad Hoc Networks”. In: *Poster presented at RSS2014 Workshop on Communication-aware Robotics: New Tools for Multi-Robot Networks, Autonomous Vehicles, and Localization* (2014).
- [14] O. Arslan and D.E. Koditschek. “Voronoi-Based Coverage Control of Heterogeneous Disk-Shaped Robots”. In: *(submitted to) the 2016 IEEE International Conference on Robotics and Automation (ICRA)*. 2016.



- [15] O. Arslan and D.E. Koditschek. “Exact Robot Navigation Using Power Diagrams”. In: *(submitted to) the 2016 IEEE International Conference on Robotics and Automation (ICRA)*. 2016.
- [16] O. Arslan and D.E. Koditschek. “Exact Robot Navigation Using Separating Hyperplanes of Convex Bodies”. In: *in preparation* (2015).
- [17] Joe H Ward Jr. “Hierarchical grouping to optimize an objective function”. In: *Journal of the American statistical association* 58.301 (1963), pp. 236–244.
- [18] Nicholas Jardine and Robin Sibson. “Mathematical taxonomy”. In: *London etc.: John Wiley* (1971).
- [19] Dan Levy, Ruriko Yoshida, and Lior Pachter. “Beyond pairwise distances: neighbor-joining with phylogenetic diversity estimates”. In: *Molecular Biology and Evolution* 23.3 (2006), pp. 491–498.
- [20] Noam Slonim, Gurinder Singh Atwal, Gašper Tkačik, and William Bialek. “Information-based clustering”. In: *Proceedings of the National Academy of Sciences of the United States of America* 102.51 (2005), pp. 18297–18302.
- [21] David M Blei and Peter I Frazier. “Distance dependent Chinese restaurant processes”. In: *The Journal of Machine Learning Research* 12 (2011), pp. 2461–2488.
- [22] Jon Kleinberg. “An impossibility theorem for clustering”. In: *Advances in neural information processing systems* (2003), pp. 463–470.
- [23] Marina Meilua. “Comparing clusterings: an axiomatic view”. In: *Proceedings of the 22nd international conference on Machine learning*. ACM, 2005, pp. 577–584.
- [24] Gunnar Carlsson and Facundo Mémoli. “Persistent clustering and a theorem of J. Kleinberg”. In: *arXiv preprint arXiv:0808.2241* (2008).
- [25] Gunnar Carlsson and Facundo Mémoli. “Characterization, Stability and Convergence of Hierarchical Clustering Methods”. In: *J. Mach. Learn. Res.* 11 (Aug. 2010), pp. 1425–1470.
- [26] Gunnar Carlsson and Facundo Mémoli. “Classifying clustering schemes”. In: *arXiv preprint arXiv:1011.5270* (2010).
- [27] Bunemann P. “The recovery of trees from measures of dissimilarity”. In: *Mathematics in the Archaeological and Historical Sciences*. Edinburgh University Press, Edinburgh, 1971, pp. 387–395.
- [28] Vincent Moulton and Mike Steel. “Retractions of finite distance functions onto tree metrics”. In: *Discrete Applied Mathematics* 91.1 (1999), pp. 215–233.
- [29] A. Dress, K. T. Huber, and V. Moulton. “Metric spaces in pure and applied mathematics”. In: *Proceedings of the Conference on Quadratic Forms and Related Topics (Baton Rouge, LA, 2001)*. Extra Vol. 2001, pp. 121–139.

- [30] Bernd Sturmfels. “Can biology lead to new theorems”. In: *Annual report of the Clay Mathematics Institute* (2005), pp. 13–26.
- [31] Andreas W. M. Dress. “Trees, tight extensions of metric spaces, and the cohomological dimension of certain groups: a note on combinatorial properties of metric spaces”. In: *Adv. in Math.* 53.3 (1984), pp. 321–402. ISSN: 0001-8708. DOI: [10.1016/0001-8708\(84\)90029-X](https://doi.org/10.1016/0001-8708(84)90029-X).
- [32] Hans-Jürgen Bandelt and Andreas W. M. Dress. “A canonical decomposition theory for metrics on a finite set”. In: *Adv. Math.* 92.1 (1992), pp. 47–105. ISSN: 0001-8708. DOI: [10.1016/0001-8708\(92\)90061-0](https://doi.org/10.1016/0001-8708(92)90061-0).
- [33] J. R. Isbell. “Six theorems about injective metric spaces”. In: *Comment. Math. Helv.* 39 (1964), pp. 65–76. ISSN: 0010-2571.
- [34] Jared Culbertson, Dan P. Guralnik, and Peter F. Stiller. “Injective metrizable and the duality theory of cubings”. In: *(submitted to Pacific Journal of Mathematics)* (2015).
- [35] Jared Culbertson, Dan P. Guralnik, Jakob Hansen, and Peter F. Stiller. “Consistency Constraints for Overlapping Data Clustering”. In: *in preparation* (2015).
- [36] Jared Culbertson, Dan P. Guralnik, and Peter F. Stiller. “Functorial hierarchical clustering with overlaps”. In: *in preparation* (2015).
- [37] Urs Lang. “Injective hulls of certain discrete metric spaces and groups”. In: *J. Topol. Anal.* 5.3 (2013), pp. 297–331. ISSN: 1793-5253. DOI: [10.1142/S1793525313500118](https://doi.org/10.1142/S1793525313500118).
- [38] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. Vol. 1. 1. MIT press Cambridge, 1998.
- [39] Andrew G Barto and Sridhar Mahadevan. “Recent advances in hierarchical reinforcement learning”. In: *Discrete Event Dynamical Systems* 13.4 (2003), pp. 341–379.
- [40] Pat Langley, John E Laird, and Seth Rogers. “Cognitive architectures: Research issues and challenges”. In: *Cognitive Systems Research* 10.2 (2009), pp. 141–160.
- [41] John Laird. *The Soar cognitive architecture*. MIT Press, 2012.
- [42] Michael L Littman, Richard S Sutton, and Satinder P Singh. “Predictive representations of state”. In: *NIPS*. Vol. 14. 2001, pp. 1555–1561.
- [43] Honglak Lee, Roger Grosse, Rajesh Ranganath, and Andrew Y Ng. “Convolutional deep belief networks for scalable unsupervised learning of hierarchical representations”. In: *Proceedings of the 26th Annual International Conference on Machine Learning*. ACM. 2009, pp. 609–616.
- [44] Geoffrey E Hinton and Ruslan R Salakhutdinov. “Reducing the dimensionality of data with neural networks”. In: *Science* 313.5786 (2006), pp. 504–507.

- [45] John R Anderson and Christian Lebiere. “The Newell test for a theory of cognition”. In: *Behavioral and Brain Sciences* 26.05 (2003), pp. 587–601.
- [46] Bas R Steunebrink, Jan Koutník, Kristinn R Thórisson, Eric Nivel, and Jürgen Schmidhuber. “Resource-bounded machines are motivated to be effective, efficient, and curious”. In: *Artificial General Intelligence*. Springer, 2013, pp. 119–129.
- [47] Helgi Páll Helgason. “General Attention Mechanism for Artificial Intelligence Systems”. PhD thesis. Reykjavik University, 2013. URL: <http://skemman.is/en/item/view/1946/16163>.
- [48] Christopher Curtis, Matthew Lenzo, Matthew McClure, and Bruce Preiss. “The layered sensing operations center: a modeling and simulation approach to developing complex ISR networks”. In: *SPIE Defense, Security, and Sensing*. International Society for Optics and Photonics. 2010, pp. 769415–769415.
- [49] Dan P Guralnik and Daniel E Koditschek. “Toward a memory model for autonomous topological mapping and navigation: The case of binary sensors and discrete actions”. In: *Communication, Control, and Computing (Allerton), 2012 50th Annual Allerton Conference on*. IEEE. 2012, pp. 936–945.
- [50] Dan P Guralnik and Daniel E Koditschek. “Universal Memory Architectures for Autonomous Machines”. In: *arXiv preprint arXiv:1502.06132* (2015).
- [51] Yuriy Mileyko, Sayan Mukherjee, and John Harer. “Probability measures on the space of persistence diagrams”. In: *Inverse Problems* 27.12 (2011), p. 124007.
- [52] Mark Coates, Rui Castro, Robert Nowak, Manik Gadhiok, Ryan King, and Yolanda Tsang. “Maximum likelihood network topology identification from edge-based unicast measurements”. In: *ACM SIGMETRICS Performance Evaluation Review*. Vol. 30. 1. ACM. 2002, pp. 11–20.
- [53] Fernando Gama, Santiago Segarra, and Alejandro Ribeiro. “Overlapping clustering of network data using cut metrics”. In: *Acoustics, Speech and Signal Processing (ICASSP), 2015 IEEE International Conference on*. IEEE. 2015.
- [54] Dekang Zhu, Dan Guralnik, Xuezhi Wang, Xiang Li, and Bill Moran. “Statistical Properties of the Single Linkage Hierarchical Clustering Estimator”. In: *submitted to Journal of Computational and Graphical Statistics* (2015).
- [55] Dekang Zhu, Dan Guralnik, Xuezhi Wang, Xiang Li, and Bill Moran. “Statistical estimation for Single Linkage Hierarchical Clustering”. In: *Cyber Technology in Automation, Control, and Intelligent Systems (CYBER), 2015 IEEE International Conference on*. IEEE. 2015, pp. 745–750.

- [56] Dekang Zhu, Dan Guralnik, Xuezhi Wang, Xiang Li, and Bill Moran. “Maximum Likelihood Estimation for Single Linkage Hierarchical Clustering”. In: *submitted to Mathematical Problems in Engineering* (2015).
- [57] Louis J Billera, Susan P Holmes, and Karen Vogtmann. “Geometry of the space of phylogenetic trees”. In: *Advances in Applied Mathematics* 27.4 (2001), pp. 733–767.
- [58] Megan Owen and J Scott Provan. “A fast algorithm for computing geodesic distances in tree space”. In: *IEEE/ACM Transactions on Computational Biology and Bioinformatics (TCBB)* 8.1 (2011), pp. 2–13.
- [59] Bo Lin, Bernd Sturmfels, Xiaoxian Tang, and Ruriko Yoshida. “Convexity in Tree Spaces”. In: *arXiv:1510.08797v1 [math.MG]* (2015).
- [60] Susanne Still and William Bialek. “How many clusters? an information-theoretic perspective”. In: *Neural computation* 16.12 (2004), pp. 2483–2506.
- [61] Antoine Deza, Komei Fukuda, Dmitrii Pasechnik, and Masanori Sato. *On the skeleton of the metric polytope*. Springer, 2001.
- [62] Corrado De Concini and Claudio Procesi. *Topics in hyperplane arrangements, polytopes and box-splines*. Springer Science & Business Media, 2010.
- [63] Dan Guralnik, Stephen Howard, and Bill Moran. “Information Theory of Hierarchical Clustering”. In: *In Preparation* (2015).
- [64] Yu. Baryshnikov and D.P. Guralnik. “Hierarchical Clustering and Configuration Spaces”. In: *preprint* (2012).
- [65] Robert R Burridge, Alfred A Rizzi, and Daniel E Koditschek. “Sequential composition of dynamically dexterous robot behaviors”. In: *The International Journal of Robotics Research* 18.6 (1999), pp. 534–555.
- [66] Daniel E Koditschek. “An approach to autonomous robot assembly”. In: *Robotica* 12.02 (1994), pp. 137–155.
- [67] Avik De, Alejandro Ribeiro, William Moran, and Daniel E Koditschek. “Convergence of Bayesian histogram filters for location estimation”. In: *52nd IEEE Conference on Decision and Control*. IEEE. 2013, pp. 7047–7053.

## 10 University of Pennsylvania — Vijay Kumar

### 10.1 October 2015 report

We have focused on developing novel active information gathering strategies for teams of robots, focusing on the tasks of exploring unknown environments and detecting, localizing, and tracking multiple targets. Such tasks arise in a variety of real-world situations, including security and surveillance, search and rescue, infrastructure inspection, environmental monitoring, mapping, and first responder. In particular, first responder scenarios require robots to be able to successfully navigate through unknown environments while searching for objects of interest, combining the tasks of exploration and target tracking.

In [1] we proposed an entropy-based information gathering strategy for multi-robot exploration and coverage in unknown, indoor environments. The underlying mathematical principle involved was to model the Shannon entropy of the occupancy probability (a measure of uncertainty) as a Riemannian metric, and thus perform an optimal coverage and exploration in this abstract Riemannian manifold by minimizing a generalized coverage functional. The formal mathematical backbone was formalized, and convergence and stability of the algorithm were analyzed in [2]. This also let us develop algorithms for collaborative exploration and coverage of unknown environments by heterogeneous teams of humans and robots, as demonstrated in [2].

We also explored the application of topological reasoning in partially known environments for effective deployment of multi-agent teams for the task of information gathering. In [3] we showed how a group of autonomous agents can split into subgroups, based on the knowledge of existing topological classes of trajectories in a partially-known environment, for effective collaborative information gathering and exploration. Upon exhausting all available topological classes of trajectories to explore, a subgroup would rejoin a different subgroup in order to maximize exploration. We applied this fundamental principle of topological exploration and information gathering in [4] for the purpose of human-robot collaborative topological exploration and information gathering in context of search and rescue missions. In this research robots would identify topological classes of trajectories that are complementary to the classes that are being followed by human team members in order to maximize information gain.

In [5, 6] we use an information-based control law to explore unknown environments and to create high-quality 3D maps. The robots plan paths over a long time horizon, considering the effects of taking multiple measurements rather than adopting a myopic strategy. We use Cauchy-Schwartz Quadratic Mutual Information (CSQMI) to greatly increase the speed of the control computations. In [6] we also use a gradient-based optimization technique to locally refine the trajectories. This leads the robots to complete the exploration task significantly faster, on par with human performance. The control law in [5, 6] takes into account the mobility and sensing constraints of the robot to select useful locations to visit and to determine when a robot will not be able to make more progress. This also allows the same control law to be used on a variety of

robots, including on a team of aerial and ground robots.

We have also used the same type of information-based control law to detect, localize, and track targets. In [7, 8, 9] we use a team of robots to detect and localize a small, but unknown, number of targets in an environment. The robots are equipped with binary sensors that provide only a single bit of information: if there is any object of interest within the sensor footprint or not. The sensor is also susceptible to false positive and false negative detections. Despite this, the team is able to successfully determine both the number of targets and their locations using a hierarchical decomposition of the environment. In [10, 11, 12] we extend our prior work to be able to localize an unknown and arbitrarily large number of target and to use arbitrary sensors. In [12] we provide experimental results of a team of ground robots exploring an office environment in search of reflective markers using bearing-only sensors. Again, the robots are able to successfully determine the number of targets and their locations. In [13] we extend this work to tracking an unknown number of moving targets, with simulation results of a team of 2-4 fixed-wing aircraft tracking upwards of 80 taxis in a city. We also have work on tracking a single moving target using a team of robots equipped with range-only sensors in [14, 15, 16]. In all of this target tracking work we develop realistic sensor models, and the robots use these to both the estimate the target positions and to make control decisions. Other than the sensor and robot motion models, the estimation and control algorithms are platform-independent. This flexibility is one of the strongest features of our estimation and control framework.

## 10.2 April 2012 report

In [7] we propose a decentralized algorithm for driving a team of resource-constrained robots to localize an unknown number of targets in an environment, while simultaneously avoiding failures due to unknown hazards. Robots are equipped with noisy, binary sensors which could describe, for example, whether a radio receives a signal. Each robot then uses the measurements from its own sensor, and those shared by neighboring robots, to maintain an estimate of target locations using a recursive Bayesian filter over an adaptive discretization of the environment, where areas that are likely to contain targets are given higher resolution. Robots then follow an approximation to the analytic gradient of mutual information between the sensor readings and target locations, which is based on the fact that real sensors have a finite field of view in the environment and the intuition that robots which see the same region of the environment should coordinate their actions while those that are sufficiently far away may act as independent agents. This provides a methodology for selecting which robots to coordinate in order to fit a given computational budget. Finally we present results from numerical simulations, showing that our approximation method performs favorably compared to other existing methods in terms of accuracy and speed, and that a team of robots running this algorithm is able to successfully localize an unknown number of targets.

This algorithm is based on finite set statistics, which provides a rigorous

probabilistic framework for multi-target tracking. This was first developed for use in the radar-based tracking community but has since been used in simultaneous localization and mapping with mobile robots. Then in [8] we extend our previous work to include more complex sensors, such as cameras, for estimation, which have non-isotropic fields of view, can return multiple detections, and provide position information. A binary approximation of the sensor is used for the mutual information gradient calculations for numerical tractability. We present numerical simulations showing the successful localization of an unknown number of targets in several example environments, and are currently working to collect experimental results.

#### Acknowledgements:

Items [2, 5, 6, 8, 9, 14, 15, 16, 7, 8] represent original work funded in part by AFOSR CHASE MURI.

#### References

- [1] Subhrajit Bhattacharya, Nathan Michael, and Vijay Kumar. “Distributed Coverage and Exploration in Unknown Non-Convex Environments”. In: *Proceedings of 10th International Symposium on Distributed Autonomous Robotics Systems*. Springer, Jan. 2010.
- [2] Subhrajit Bhattacharya, Robert Ghrist, and Vijay Kumar. “Multi-robot Coverage and Exploration on Riemannian Manifolds with Boundary”. In: *International Journal of Robotics Research* 33.1 (Jan. 2014). DOI: 10.1177/0278364913507324, pp. 113–137.
- [3] Soonkyum Kim, Subhrajit Bhattacharya, Robert Ghrist, and Vijay Kumar. “Topological Exploration of Unknown and Partially Known Environments”. In: *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. [DOI: 10.1109/IROS.2013.6696907]. Tokyo, Japan, Nov. 2013.
- [4] Vijay Govindarajan, Subhrajit Bhattacharya, and Vijay Kumar. “Human-Robot Collaborative Topological Exploration for Search and Rescue Applications”. In: *International Symposium on Distributed Autonomous Robotic Systems (DARS)*. 2014.
- [5] Benjamin Charrow, Sikang Liu, Vijay Kumar, and Nathan Michael. “Information-Theoretic Mapping Using Cauchy-Schwarz Quadratic Mutual Information”. In: *IEEE International Conference on Robotics and Automation (ICRA)*. 2015.
- [6] Benjamin Charrow, Gregory Kahn, Sachin Patil, Sikang Liu, Ken Goldberg, Pieter Abbeel, Nathan Michael, and Vijay Kumar. “Information-Theoretic Planning with Trajectory Optimization for Dense 3D Mapping”. In: *Robotics: Science and Systems (RSS)*. 2015.

- [7] P. Dames, M. Schwager, V. Kumar, and D. Rus. “A Decentralized Control Policy for Adaptive Information Gathering in Hazardous Environments”. In: *IEEE Conference on Decision and Control (CDC)*. Dec. 2012.
- [8] P. Dames, D. Thakur, M. Schwager, and V. Kumar. “Adaptive Information Gathering Using Visual Sensors”. In: *IEEE International Conference on Robotics and Automation (ICRA)* (2012).
- [9] Philip Dames, Dinesh Thakur, Mac Schwager, and Vijay Kumar. “Playing Fetch with Your Robot: The Ability of Robots to Locate and Interact with Objects”. In: *IEEE Robotics and Automation Magazine* 21.2 (2013), pp. 46–52. DOI: [10.1109/MRA.2013.2295947](https://doi.org/10.1109/MRA.2013.2295947).
- [10] P. Dames and V. Kumar. “Cooperative Multi-Target Localization with Noisy Sensors”. In: *IEEE International Conference on Robotics and Automation (ICRA)*. 2013.
- [11] Philip Dames and Vijay Kumar. “Automated Detection, Localization, and Registration of Smart Devices With Multiple Robots”. In: *Proceedings of the IEEE International Conference on Automation Science and Engineering*. Gothenburg, Sweden, 2015. DOI: <http://dx.doi.org/10.1109/CoASE.2015.7294139>.
- [12] Philip M Dames and Vijay Kumar. “Autonomous Localization of an Unknown Number of Targets without Data Association Using Teams of Mobile Sensors”. In: *IEEE Transactions on Automation Science and Engineering* 12.3 (2015), pp. 850–864. DOI: [10.1109/TASE.2015.2425212](https://doi.org/10.1109/TASE.2015.2425212).
- [13] Philip Dames, Pratap Tokekar, and Vijay Kumar. “Detecting, Localizing, and Tracking an Unknown Number of Moving Targets Using a Team of Mobile Robots”. In: *Proceedings of the International Symposium on Robotics Research*. Sestri Levante, Italy, 2015.
- [14] B. Charrow, N. Michael, and V. Kumar. “Cooperative Multi-Robot Estimation and Control for Radio Source Localization”. In: *International Symposium on Experimental Robotics (ISER)* (2012).
- [15] B. Charrow, N. Michael, and V. Kumar. “Cooperative Multi-Robot Estimation and Control for Radio Source Localization”. In: *International Journal of Robotics Research* 33.4 (Apr. 2014). DOI: [10.1177/0278364913500542](https://doi.org/10.1177/0278364913500542), pp. 569–580.
- [16] B. Charrow, V. Kumar, and N. Michael. “Approximate representations for multi-robot control policies that maximize mutual information”. In: *Autonomous Robots* 37.4 (Dec. 2014), pp. 383–400.



## 11 University of Pennsylvania — Alejandro Ribeiro

### 11.1 October 2015 report

#### 11.1.1 Metric Representations of Network Data

Networks are data structures that encode relationships between elements and can be thought of as signals that, instead of having values associated with elements, have values associated with pairs of elements. As such, they play an important role in our current scientific understanding of problems in which relationships between elements are important. These problems include interactions between proteins or organisms in biology, individuals or institutions in sociology, and neurons or regions in the brain.

Despite their pervasive presence, tools to analyze networks and algorithms that exploit network data are not as well developed as tools and algorithms for processing of conventional signals. Although some of this lag can be attributed to different developmental stages, there is also the matter of the complexity of network data. To understand this latter point it is instructive to observe that particular cases of signals that encode relationships between elements are well understood and pose little challenge for analysis and algorithm design. E.g., a correlation matrix is a representation of the proximity between components of a random signal and a finite metric space defines distances between elements of a space. Both can be considered as particular cases of networks and the understanding of both is on par with the understanding of signals. Since correlation matrices and metric spaces have been studied for longer, the relative lag of network analysis can be again attributed to different developmental stages. However, the available evidence suggests otherwise.

Indeed, consider a problem of proximity search in which we are given a network and an element whose dissimilarity to different nodes of the network can be determined. We are asked to find the element that is least dissimilar. In an arbitrary network finding the least dissimilar node requires comparison against all nodes and incurs a complexity that is linear in the size of the network. In a metric space, however, the triangle inequality encodes a transitive notion of proximity. If two points are close to each other in a metric space and one of them is close to a third point, then the other one is also close to this third point. This characteristic can be exploited to design efficient search methods using metric trees whose complexity is logarithmic in the number of nodes [1, 2]. Likewise, many hard combinatorial problems on graphs are known to be approximable in metric spaces but not approximable in generic networks. The traveling salesman problem, for instance, is not approximable in generic graphs but it is approximable in polynomial time to within a factor of  $3/2$  in metric spaces [3]. In either case, the advantage of the metric space is that the triangle inequality endows it with a structure that an arbitrary network lacks. It is this structure that makes network analysis and algorithm design tractable.

If metric spaces are easier to handle than arbitrary networks, a possible route for network analysis is to design projection operators to map arbitrary networks

into the subset of networks that represent metric spaces. The design of the aforementioned projection operators is the subject of current work. The main efforts are being focused on how to utilize these projections to efficiently search a network as well as their utility in generating approximations for combinatorial optimization problems on graphs.

Current results are reported in [4].

### 11.1.2 Other Types of Network Metrics

The purpose of this research is to develop network discrimination tools that can be applied to network comparison problems that appear in neuroscience, biology, and the social sciences. As a prototypical example consider neurodegenerative diseases for which ultimate causes remain unknown but for which proximate causes are alterations in the pattern of brain connectivity. Memory, cognitive, coordination, and behavioral changes associated with Parkinsons, Alzheimers, and Huntingtons diseases have all been related with patterns of brain activity that have distinct markers when compared with the activity patterns of healthy individuals [5]. Knowing these alterations in brain connectivity is not only useful to foster our understanding of these disorders but also as a diagnostic tool. The outcome of these research effort is a network discrimination tool that can solve this diagnostic question and other similar questions such as discerning collaboration mores of research communities [6] and predicting the mortality of an emergent virus by studying the shape of its evolutionary tree [7].

Irrespectively of the application, the challenge in making network comparisons is the difficulty of computing proper distances between networks. In neurodegenerative disorders the changes in brain activity tend not to be specific to a region of the brain but more about global properties of the network. Alzheimers disease is not characterized by, say, a decreased connectivity between the frontal and parietal lobes, but by decreases in the richness of the connectivity between areas of the brain that differ from patient to patient. This means that networks have to be compared as unlabeled entities so that a decrease in connectivity is identified as a marker of the disease irrespectively of whether it occurs in the frontal lobe or the occipital lobe. In this specific example the alteration can be identified by the average node degree the average number of connections between brain regions , but the question remains of how to identify more subtle changes and of what improvement can be gleaned from a more thorough comparison.

Our technical approach to resolving this conundrum is to define and estimate distances between unlabeled networks. This is an improvement upon the current practice of comparing heuristic network features such as node degrees, centrality measures, motifs, and cuts. Network features that are relevant to a specific discrimination problem are likely irrelevant for others and features can yield conflicting comparative judgements, like two participants being close to a third person but far from each other because the triangle inequality is not necessarily valid. A proper distance between networks would overcome these drawbacks. Distances are universal and avoid conflicting judgements because the triangle

inequality is valid.

We have defined two families of valid metrics in the space of networks by associating the ideas of the Gromov-Hausdorff distance between point sets and searching mappings among nodes between networks[6, 8]. As for the current work, Instead of searching mappings between nodes, network discrimination problems can also be solved by considering that each network represents a space. Under such setup, network distances can be evaluated as the difference between the first network and the embedding of the second network onto the space defined by the first network. We have proved that this formulation yields valid network metric. Our current work focuses on the efficient implementation of the algorithms.

### 11.1.3 Multi-agent systems with incomplete information

In many multi-agent systems a team of autonomous agents want to complete a task but each agent has different and incomplete information about the task. In these settings, the systems can be modeled by an underlying environment, knowledge about the state of the environment that the agents acquire, and a state dependent global objective that agents affect through their individual actions. The optimal action profile maximizes this global objective for the realized environment's state with the optimal action of an agent given by the corresponding action in the profile. The problem we address in this work is the determination of suitable actions when the probability distributions that agents have on the state of the environment are possibly different. These not entirely congruous beliefs result in mismatches between the action profiles that different agents deem to be optimal. As a consequence, when a given agent chooses an action to execute, it is important for it to reason about what the beliefs of other agents may be and what are the consequent actions that other agents may take. In this work, we pose this problem as an incomplete information network game [Ch. 6, [9]] with aligned interests [10].

For further intuition consider the target covering problem where a team of robots wants to cover the entrances to an office floor. The environmental information gives the position of the doors as well as the positions of the robots. The goal of the robots is to cover all the entrances while minimizing the total work - which is proportional to the sum over all robots of the path integrals of the norms of the robots accelerations - required to do so. If there is perfect environmental information available, the robots can solve the global work minimization problem locally. Since there is nothing random on this problem formulation this is a relatively straightforward assignment and path planning problem. When there is uncertainty about the environment but the robots have sufficient time to coordinate, they can share all of their environmental observations. Once this is done all agents have access to the same information and can proceed to minimize the expected work. Since all base their solutions in the same information, their trajectories are compatible and the robots just proceed to move according to the computed plans. The problem arises when the environment's information is not perfect and the coordination delay is undesirable.

In that scenario robots make an estimate of the path they expect other agents to choose and minimize the expected total work based on these expected paths. Even though the interests of the members of the autonomous team are aligned, they have to resort to strategic reasoning and end up playing a game against uncertainty.

The solutions that we propose to the problem above are variations of the fictitious play algorithm that take into account the distributed nature of the multi-agent system and the fact that the state of the environment is not perfectly known [11]. In conventional fictitious play, agents build beliefs on others' future behavior by computing histograms of past actions and best respond to their expected payoffs integrated with respect to these histograms [12]. In a game of incomplete information, expected payoff computation in traditional fictitious play consists of integrating the payoff with respect to both the local belief on the state of the environment and the local beliefs on the behavior of other agents. However, in a networked setting only local information can be available and agents need to reason about the behavior of non-neighboring agents based on past observations of its neighbors only. In the variations developed here histograms are built using knowledge of actions taken by nearby agents and best responses are further integrated with respect to the local beliefs on the state of the environment. This algorithmic behavior is shown to be asymptotically optimal in the sense that if agents move towards a common belief, the actions they select are optimal with respect to the corresponding expected utility.

These results are reported in [11].

### **Acknowledgements:**

Items [4, 6, 8, 11] represent original work funded in part by AFOSR CHASE MURI.

## **11.2 April 2012 report**

### **11.2.1 Hierarchical Clustering in Asymmetric Networks**

Miranda trusts Billy who trusts Ariel who trusts Miranda, but there has not been enough interactions in the opposite direction to establish trust. When these three people meet, shall they trust each other? I.e., are they part of a circle of trust? The objective of this project is to develop an axiomatic theory to provide an answer to this question. In general, we start with a network where nodes represent individuals and directed edges represent a trust dissimilarity from the originating node to the end node. Small values of this dissimilarity signify large amounts of trust of the edge's source node on the edge's destination. Our goal is the study of the formation of trust groups in the network. I.e., the determination of the level of trust at which two individuals are integrated in a trust cluster given not only their direct interactions but their indirect interactions through other members of the network. It may make sense for Miranda, Billy, and Ariel to trust each other, because they all either trust each other directly, or have trust on someone that trusts the person they don't know.

Once the problem is written in this language it is clear that determining circles of trust is akin to finding clusters in an asymmetric network for a given resolution level. The determination of a family of clusterings indexed by this resolution parameter is a problem known as hierarchical clustering. Simple as this sounds, the problem is that clustering in general and clustering using asymmetric data in particular is a poorly understood problem. There are plethora of methods that can be chosen to perform clustering, but these methods are based on heuristic intuition, not fundamental principles. Beyond purist concerns, lack of theoretical understanding is also a practical problem for clustering of asymmetric data because the intuition backing clustering methods is drawn from geometric point clouds. This intuition does not carry when the given data is not metric as in the case of asymmetric trust dissimilarities.

Even though asymmetric clustering intuition is difficult in general, there are some particular specks of intuition that we can exploit to gain insight into the general problem. These intuitive statements can be postulated as axioms that restrict the space of allowable asymmetric hierarchical clustering methods. To the extent that the axioms are true, the properties of this reduced space of methods are fundamental properties of asymmetric hierarchical clustering and by extension fundamental properties of the formation of circles of trust. In our investigations we have postulated three axioms that we call the axioms of value, influence, and transformation. These axioms are stated formally in [13] but they correspond to the following intuitions:

\*(A1) Axiom of Value. For a network with two nodes the nodes are clustered together at the resolution at which both trust each other, namely, the maximum of the two trust dissimilarities between them.

\*(A2) Axiom of Influence. There cannot be any circles of trust formed at resolutions that do not allow formation of bidirectional, possibly indirect, trust relationships.

\*(A3) Axiom of Transformation. If we consider a network and reduce all pairwise trust dissimilarities, the level at which two nodes become part of the same circle of trust is not larger than the level at which they were clustered together in the original network.

Despite their apparent weakness, axioms (A1)-(A3) are a source of strong structure. Our first result is the derivation of two asymmetric hierarchical clustering methods that abide to these axioms. The first method insists that trust propagate only through arcs in which there is bidirectional trust and is therefore termed reciprocal clustering. The second method allows trust to propagate unidirectionally and is thus termed nonreciprocal clustering. That these methods comply with (A1)-(A3) is not particularly surprising. However, we have proved that any clustering method that satisfies axioms (A1)-(A3) lies between reciprocal and nonreciprocal clustering in a well defined sense. Specifically, any clustering method that satisfies axioms (A1)-(A3) forms circles of trust at resolutions larger than the resolutions at which they are formed with nonreciprocal clustering, and smaller than the resolutions at which they are formed with reciprocal clustering. These preliminary result endows reciprocal and nonreciprocal clustering with special meaning. For a given resolution level, nodes that do not

cluster together with nonreciprocal clustering cannot be part of a circle of trust. Nodes that do cluster together with reciprocal clustering are definitely part of a circle of trust. In between, the answer depends on the extent to which reciprocal trust propagation is required or nonreciprocal trust propagation is acceptable.

This work has been reported in [13].

### 11.2.2 Distributed Network Optimization with Heuristic Rational Agents

Network optimization problems entail a group of agents with certain underlying connectivity that strive to minimize a global cost through appropriate selection of local variables. Optimal determination of local variables requires, in principle, global coordination of all agents. In distributed network optimization, agent coordination is further restricted to neighboring nodes. The optimization of the global objective is then achieved through iterative application of local optimization rules that update local variables based on information about the state of neighboring agents. Distributed network optimization is a common solution method for estimation and detection problems in CHASE networks.

Beyond its use in engineered systems, distributed network optimization is also used to model the emergence of global behavior in biological and social networks. In this context, the optimization cost models global network behavior that emerges through the application of the local optimization rules. In biological systems, network optimization models that mimic natural phenomena like bird flocking or animal swarming have been introduced. Bird flocking models posit that individual birds try to optimize total drag by adjusting their individual positions and velocities based on the observed behavior of neighboring birds within their field of vision. Similarly, the foraging behavior of animal herds and fish schools can be explained as the optimization of an objective that includes terms to account for the value of food, the value of cohesion and the cost of excessive proximity. As in the case of bird flocks, members of the herd or school adjust their positions with respect to the observed positions of nearby peers. Notice how these models exhibit the three hallmarks of distributed network optimization. They start from a global objective that the network agents want to optimize - like total drag for bird flocks - through the selection of local variables - birds' positions and velocities - while restricting interactions to neighboring agents - positions and velocities are updated relative to the closest neighboring birds on the field of vision. Consensus formation and opinion propagation in social networks can also be understood in terms of distributed network optimization. In this case network nodes represent social agents having differing opinions that they update over time based on the observed opinions of neighboring nodes. Agents determine these updates by minimizing a local measure of disagreement with their neighbors. As a result, the network as a whole is minimizing a global measure of disagreement. The difference between consensus and opinion propagation models is that in the former all nodes attempt to increase harmony, while in the latter some stubborn agents do not change their opinions.

The goal of this project is to propose and study more realistic models whereby agents execute actions that are optimal in an average sense only. We name these rules and the agents that use them as heuristic rational, since we think of them as the application of a heuristic rule that is intent on being optimal even though it may not be so. We show that models commonly used to study propagation of opinions in social networks foraging of animal herds and quantization and communication issues in field estimation using WSNs can be cast in the language of heuristic rational optimization. We also study the behavior of networks composed of heuristic rational agents and show that: (i) The global network behavior visits a neighborhood of optimality infinitely often. (ii) The probability of straying away from this neighborhood by more than a given amount is exponentially bounded. These results can be interpreted as an explanation for the emergence [cf. (i)] and sustenance [cf. (ii)] of global network behavior that is close to optimal despite imperfect decision making of individual agents in natural and social systems.

This work has been reported in [14, 15, 16].

### 11.2.3 Distributed Maximum a Posteriori Probability Estimation of Dynamic Systems

In this project we consider the problem of estimating a time-varying signal with a distributed sensor network which collects noisy observations of the signal of interest. Our goal is to implement a distributed and adaptive estimation algorithm to track this dynamical system relying on local observations and communication with neighboring nodes. To meet this goal we utilize maximum a posteriori probability (MAP) estimates and design a mechanism to incorporate global information into local estimates. We want sensors to compute estimates at the current time estimating the state of the system at the same time while coming close to the optimal centralized MAP that could be computed if all the observations were available at a central location. This algorithm is instrumental to the implementation of distributed data aggregation algorithms for CHASE systems.

The first idea proposed to mediate the incorporation of global information within local estimates is the consensus algorithm in which sensors update their estimates through iterative averaging of neighboring values. Consensus algorithms are well studied for static estimation problems and have also been adapted for dynamic estimation. An alternative approach to mediate the incorporation of global information is through the introduction of Lagrange multipliers, effectively setting a price on disagreement which sensors try to minimize; a feat which can be accomplished in a distributed manner using dual subgradient descent techniques.

Most work on distributed estimation for time-varying parameters assumes that communications occur in a time scale separate from the timeline of the dynamic system. This assumption is necessary because the algorithms are iterative. Thus, their implementation in a dynamic setting requires the assumption that an infinite number of communication steps occur between subsequent

states of the dynamic system. We have generalized price mediation algorithms to nonlinear dynamic estimation problems while using a common time scale for communications and the evolution of the process. When using a single time scale, each iteration of the price update algorithm brings the sensors closer to agreement on the MAP estimate, while at the same time the process, and thus the MAP estimate, drifts to a new value. Our technical contribution is to characterize this tradeoff by showing that local estimates approach the centralized MAP estimator with a small error which we characterize in terms of problem-specific constants.

This work has been reported in [17] and [18].

### Acknowledgements:

Items [13, 14, 15, 16, 17, 18] represent original work funded in part by AFOSR CHASE MURI.

### References

- [1] Peter N Yianilos. “Data structures and algorithms for nearest neighbor search in general metric spaces”. In: *SODA*. Vol. 93. 194. 1993, pp. 311–321.
- [2] Jeffrey K Uhlmann. “Satisfying general proximity/similarity queries with metric trees”. In: *Information processing letters* 40.4 (1991), pp. 175–179.
- [3] Nicos Christofides. *Worst-case analysis of a new heuristic for the travelling salesman problem*. Tech. rep. DTIC Document, 1976.
- [4] S. Segarra, G. Carlsson, F. Memoli, and A. Ribeiro. “Metric Representations of Network Data”. In: *IEEE Trans. Signal Process.* (submitted) (2015).
- [5] Maja AA Binnewijzend, Joost PA Kuijjer, Wiesje M van der Flier, Marije R Benedictus, Christiane M Möller, Yolande AL Pijnenburg, Afina W Lemstra, Niels D Prins, Mike P Wattjes, Bart NM van Berckel, et al. “Distinct perfusion patterns in Alzheimer’s disease, frontotemporal dementia and dementia with Lewy bodies”. In: *European radiology* 24.9 (2014), pp. 2326–2333.
- [6] W. Huang and A. Ribeiro. “Metrics in the space of high order networks”. In: *IEEE Trans. Signal Process.* (to appear) (Sept. 2015).
- [7] R Burke Squires, Brett E Pickett, Sajal Das, and Richard H Scheuermann. “Toward a method for tracking virus evolutionary trajectory applied to the pandemic H1N1 2009 influenza virus”. In: *Infection, Genetics and Evolution* 28 (2014), pp. 351–357.
- [8] W. Huang and A. Ribeiro. “Persistent Homology Lower Bounds on High Order Network Distances”. In: *IEEE Trans. Signal Process.* (submitted) (July 2015).



- [9] Drew Fudenberg and Jean Tirole. “Game Theory”. In: *MIT press* 393 (1991).
- [10] Dov Monderer and Lloyd S Shapley. “Potential games”. In: *Games and economic behavior* 14.1 (1996), pp. 124–143.
- [11] C. Eksin and A. Ribeiro. “Distributed Fictitious Play for Optimal Behavior of Multi-Agent Systems with Incomplete Information”. In: *IEEE Trans. Automatic Control* (submitted) (July 2015).
- [12] Dov Monderer and Lloyd S Shapley. “Fictitious play property for games with identical interests”. In: *Journal of economic theory* 68.1 (1996), pp. 258–265.
- [13] G. Carlsson, F. Memoli, A. Ribeiro, and S. Segarra. “Axiomatic construction of hierarchical clustering in asymmetric networks”. In: *Proc. Int. Conf. Acoustics Speech Signal Process.* May 2013, pp. 5219–5223.
- [14] C. Eksin and A. Ribeiro. “Distributed network optimization with heuristic rational agents”. In: *IEEE Trans. Signal Process* 60.10 (Oct. 2012).
- [15] Ceyhun Eksin and Alejandro Ribeiro. “Heuristic rational models in social networks”. In: *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE. 2012, pp. 3077–3080.
- [16] Ceyhun Eksin and Alejandro Ribeiro. “Network optimization with heuristic rational agents”. In: *2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*. IEEE. 2011, pp. 53–57.
- [17] F. Jakubiec and A. Ribeiro. “Distributed maximum a posteriori probability estimation of dynamic systems with wireless sensor networks”. In: *Proc. Int. Conf. Acoustics Speech Signal Process.* (to appear) (Mar. 2012).
- [18] F. Jakubiec and A. Ribeiro. “Distributed maximum a posteriori probability estimation of dynamic systems”. In: *IEEE Trans. Signal Process.* (to appear).8 (Aug. 2012).

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215-898-9506

## Organization / Institution name

University of Pennsylvania

## Grant/Contract Title

The full title of the funded effort.

(MURI-10) CHASE: CONTROL OF HETEROGENEOUS AUTONOMOUS SENSORS FOR SITUATIONAL AWARENESS; Control Science for Next Generation Sensing; Control of Information: Collection and Fusion

## Grant/Contract Number

AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".

FA9550-10-1-0567

## Principal Investigator Name

The full name of the principal investigator on the grant or contract.

Daniel E. Koditschek

## Program Manager

The AFOSR Program Manager currently assigned to the award

Tristan Nguyen

## Reporting Period Start Date

08/16/2010

## Reporting Period End Date

04/16/2016

## Abstract

The project aimed to forge a rigorous new perspective on the joint control of multiple information sources of disparate types to simultaneously achieve quantified information and physical objectives. The overarching goal throughout the six years of the project's existence remained the discovery and analysis of new foundational methodology for information collection and fusion that exercises rigorous feedback control over information collection assets, simultaneously managing information and physical aspects of their states. The project generated 169 peer-reviewed papers acknowledging this award. Their contents and significance are summarized in this report.

## Distribution Statement

This is block 12 on the SF298 form.

Distribution A - Approved for Public Release

## Explanation for Distribution Statement

If this is not approved for public release, please provide a short explanation. E.g., contains proprietary information.

DISTRIBUTION A: Distribution approved for public release.

**SF298 Form**

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**Archival Publications (published) during reporting period:**

Listed and Detailed in the report

**2. New discoveries, inventions, or patent disclosures:**

**Do you have any discoveries, inventions, or patent disclosures to report for this period?**

No

**Please describe and include any notable dates**

**Do you plan to pursue a claim for personal or organizational intellectual property?**

**Changes in research objectives (if any):**

None

**Change in AFOSR Program Manager, if any:**

None

**Extensions granted or milestones slipped, if any:**

None

**AFOSR LRIR Number**

**LRIR Title**

**Reporting Period**

**Laboratory Task Manager**

**Program Officer**

**Research Objectives**

**Technical Summary**

**Funding Summary by Cost Category (by FY, \$K)**

	Starting FY	FY+1	FY+2
Salary			
Equipment/Facilities			
Supplies			
Total			

**Report Document**

**Report Document - Text Analysis**

**Report Document - Text Analysis**

**Appendix Documents**

**2. Thank You**

**E-mail user**