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PRACTICAL NONLINEARITIES

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Final Report**

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14. ABSTRACT The Defense Advanced Research Projects Agency (DARPA) Dynamics-Enabled Frequency Sources (DEFYS) program is focused on the convergence of nonlinear dynamics and microelectromechanical systems (MEMS) to achieve breakthroughs in miniature oscillator performance for Department of Defense (DoD) applications. This project focused on working to demonstrate the advantages of nonlinear dynamics for high-performance oscillators, while confining the activities to a technology platform that has already been commercialized, and which can meet the challenging size and temperature coefficient objectives while maintaining a firm connection to manufacturability.					
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1.0 BACKGROUND

The Defense Advanced Research Projects Agency (DARPA) Dynamics-Enabled Frequency Sources (DEFYS) program is focused on the convergence of nonlinear dynamics and microelectromechanical systems (MEMS) to achieve breakthroughs in miniature oscillator performance for Department of Defense (DoD) applications. Early work in this program has shown that nonlinear dynamics can provide performance advantages. However, the pathway from initial results to manufacturable devices is far from certain for these ideas. Our project hopes to help improve the transition pathways for the best ideas in this program.

In our project, we worked to demonstrate the advantages of nonlinear dynamics for high-performance oscillators, while confining our activities to a technology platform that has already been commercialized, and which can meet the challenging size and temperature coefficient objectives while maintaining a firm connection to manufacturability.

The key elements of our project are:

Leveraging Nonlinearities for Performance: Our group has previously demonstrated the operation of MEMS oscillators far beyond the limits of linear dynamics, often described as the “Critical Bifurcation Limit” in publications. We’ve shown that operation in this regime can overcome the high impedances of micromechanical resonators while still leveraging the other advantages of MEMS. In this project, we will seek opportunities to leverage nonlinearities to achieve improvements in phase noise, in particular by **Trading Phase Noise for Amplitude Noise:** For highly-nonlinear oscillators, there are operating points which accentuate the amplitude noise, and can therefore be used to suppress phase noise. We’ve been able to access such states in MEMS resonators and we are prepared to exploit these states to meet DEFYS objectives in oscillators in this project.

Suppression of the Temperature Coefficient of Frequency (TCF): In this effort, we will demonstrate the use of doping for TCF suppression within our existing process, thereby meeting this important metric while staying within a manufacturable process.

Our focus has been on the use of near-degenerate doping of the silicon MEMS resonators, which has been shown to impact the temperature dependence of the modulus. Through this effort, we have discovered significant interdependency between doping and nonlinearity, and have begun new effort to characterize and understand this effect to enable predictive modeling of MEMS devices.

Manufacturability: We will carry out all design and fabrication within the constraints of our existing wafer-scale encapsulation process.

Supporting DEFYS Performers: In this project, we also offer to fabricate devices for all other DEFYS performers within our process, as well as for future collaborators with performers in the DEFYS program, and provide a pathway to insertion for all other ideas and technologies demonstrated within the DEFYS program.

Milestones for Year 3 of Stanford's Practical Nonlinearities

The project schedule is connected to a set of 3 core fabrication runs, one in each year. The milestones associated with these tasks are generally connected to these runs, through design, testing, and optimization of the various aspects of the resonators for this program. Deliverables are based on the execution of these runs.

Year 3

- Month 27 All Initial Designs for Final Fab Run
- Month 29 Masks and wafers prepared for Final Fab Run
- Month 30 Start of Final Fab Run
- Month 35 Completion of Final Fab Run
- Month 36 Delivery of devices based on designs from other DEFYS performers

Because of momentum from efforts prior to the start of this project, we were always significantly ahead of the schedule for the scheduled fabrication runs, leading to early delivery of parts to performers, and an opportunity to initiate a 4th run before the end of the project. This "Run #4" is still being completed with funds from the Positioning, Navigation and Timing (PNT) Program, and will give us and all other performers an important added opportunity to leverage the accumulated experience and knowledge from prior runs.

2.0 PROJECT RESULTS SUMMARY

As of the date of this report (June 2016), our project is completed, with most tasks completed ahead of schedule. For example, the 3rd Fab Run started 5 months early and was completed in spring 2015. A 4th Fab Run was initiated prior to the completion of this project, and is currently approaching completion. The activities in the last year of this project were dominated by our efforts to complete this fabrication run. Fab run activities do not generate significant content for technical progress reports, so the remaining results sections in this report are somewhat less lengthy, reflecting the effort being spent on the fabrication.

We began this project with considerable momentum due to effort supported by Bosch on encapsulation of pressure sensors and inertial sensors for automotive and consumer electronics applications. Additionally, DARPA funded effort in the Micro Positioning, Navigation, and Timing (uPNT) program has contributed to support for recent fabrication runs with preliminary DEFYs-Inspired devices. Some of the results from characterization of these devices led directly to improved designs for this effort. The ongoing work is co-funded by this uPNT program effort, allowing performers from Mesodynamic Architectures (MESO) and uPNT all to include devices in these runs. This cost-sharing was planned, and is necessary for success, as the funds allocated to the fabrication activity in the MESO and uPNT tasks are not enough to cover the cost and effort of these runs. Also helping reduce the cost of this effort, and allowing an expanded activity under these funds, we have a strong group of PhD students engaged in the project and prepared to support ongoing activities. Note that Ahadi was partly supported by fellowship, and that Gerrard and Heinz were fully supported by external fellowships (National Science Foundation (NSF) and National Defense Science and Engineering Graduate (NDSEG)), enabling significant student participation at reduced cost to DARPA.

The “3rd fabrication run” was completed in spring 2015. MESO performers from HRL, University of California, Santa Barbara (UCSB), Michigan State University (MSU), Rockwell Collins, and Yale contributed to this design opportunity, and functional devices were being prepared for delivery to these teams at the end of the period of this report. Our preliminary probing indicated that there was high yield on all basic test structures.

In August, 2015, we held an “Epi-Seal Symposium”, which sought to gather all the contributors to the most recent fab runs to discuss their results and make requests for future fabrication run opportunities. Presentations by UCSB, MSU, HRL and Yale were included in this meeting, and it is clear that MESO performers have benefitted from these runs. MSU has an ongoing collaboration with UCSB and Stanford, and has sought NSF funding and other DARPA funding to continue these efforts. HRL used devices from this run to support proposals to other DARPA programs. At the symposium, there was a decision to focus on a version of the run that enables large gaps, small gaps, release of large structures without release holes, and other features that are generally aligned with achieving higher Q and greater opportunities for exploring nonlinearities.

The first two runs under this project produced complete suites of devices, enabling extraction of the temperature dependence of the elastic modulus of doped silicon, pointing towards devices and designs that can have significantly reduced temperature dependence which should help all

performers approach the DEFYs temperature coefficient goals. A complete manuscript was published in JMEMS [1], and has already been available via Open Access on the JMEMS Website for more than 9 months. **As a result of this effort, it is now possible to carry out predictive modeling of the temperature dependence of doped MEMS resonators, which is an important tool for designers wishing to leverage this effect. This is one of the most important results from this project, and is currently being used in academia and industry for design of high-performance MEMS devices.**

We're also continuing to use these same suites of devices to identify the temperature and doping dependence of the nonlinear elasticity of these resonators. Results were presented at the 2015 IEEE MEMS Conference (in a contributed paper and in a plenary presentation), and in the 2015 Conference on Solid State Sensors, Actuators and Microsystems, and our studies are continuing. Researchers from MSU and UCSB have also produced publications based on these results. The initial results indicate that the nonlinear elasticity is highly dependent on many factors, such as doping, temperature, orientation, and mode shape. The combination of all these effects allows devices to exhibit a great variety of nonlinearities, opening a pathway towards tailored nonlinearities for oscillator performance enhancement. Our recent effort has focused on the use of "ringdown" characterization in addition to frequency sweeps to identify and isolate the effects from electrostatic nonlinearities from mechanical and material nonlinearities [2]. A more recent project, in collaboration with the UC Davis group, used these models and methods to explore the limits of nonlinear resonator operation [7].

An important realization from these studies is that the nonlinear elasticity for doped and undoped silicon is not well understood [3]. For certain simple devices and particular orientations, there have been empirical experiments to extract predictable fitted relationships.

However, we have seen that the general problem is extraordinarily complex, with significant variations in nonlinearity arising at the higher doping levels necessary for temperature compensation. We have shifted some of our focus to a more comprehensive study of these effects, with a goal of providing a capability for predictive modeling of the nonlinearity. This work is ongoing, and we are seeking funding from NSF and from other parts of DARPA to support ongoing effort.

During this period, we have continued working closely with Steve Shaw and Mark Dykman at MSU. Their particular expertise is in modeling the physics associated with nonlinearities in MEMS resonators, focusing on the impact of these effects on the noise in MEMS resonators and oscillators. We have had several meetings within the team at Stanford, at MSU, at conferences and at DARPA meetings, and many more phone calls. Through this collaboration, our team has been looking very closely at the waveform produced during ringdown from high-amplitude drive, and this has led to several interesting and important observations. We clearly see an amplitude dependent frequency shift, associated with the geometric and material nonlinearity, and we also see an amplitude dependent change in the damping coefficient perhaps the first observation of nonlinear damping in a MEMS device. Further, detailed examination of the phase of the resonator during ringdown enables us to identify and quantify distinct noise sources. This collaboration was only effective during the last phase of our project, and has already produced several important results [2, 8].

With the winding down of the MESO program, we have faced the unfortunate fact that only 1 year of the collaboration with MSU is supported in this program. We have invested significant time and effort in the past few months in search of funds through the PNT program to continue these activities, as we are just beginning to see substantial and important results. So far, there is no good news here, and we are continuing.

During this work, and in the parallel effort funded by the PNT Program, managed by Dr. Robert Lutwak, we have begun investigating the effect of nonlinearities within MEMS Gyroscopes. Technically, this work is not within the explicit goals of the MESO program, but it certainly draws inspiration from the things we're learning in MESO, and has led to some important contributions to the performance of MEMS gyroscopes. In particular, we have demonstrated for the first time that Parametric Amplification can improve the performance of a MEMS gyroscope [4] and we are exploring other ways that material nonlinearities can create beneficial nonlinear dynamical effects [5]. The MESO program team should view these results as a product of MESO!

3.0 PROJECT STATUS DETAILS

We conclude by reporting on new results obtained from existing devices from recent fabrication activities.

3.1 Encapsulated Bulk Acoustic Mode Resonators

As described in the last status report, we successfully built families of encapsulated resonators using a variant of the episeal process, which allowed devices with large released areas to be formed without etch release holes. By eliminating these etch release holes, we eliminate many stress concentrations that contribute to ThermoElastic Dissipation, and have enabled Q values to be increased by approximately 10X. Some of these results were reported at the Transducers 2015 Conference (June 2015, Anchorage, AK, see Figure 1, right) [6], and a more complete manuscript has recently appeared in the literature [9]. Devices made in this process were delivered to other performers at the end of the run. We hope to see research results from other performers after they receive and characterize their parts.

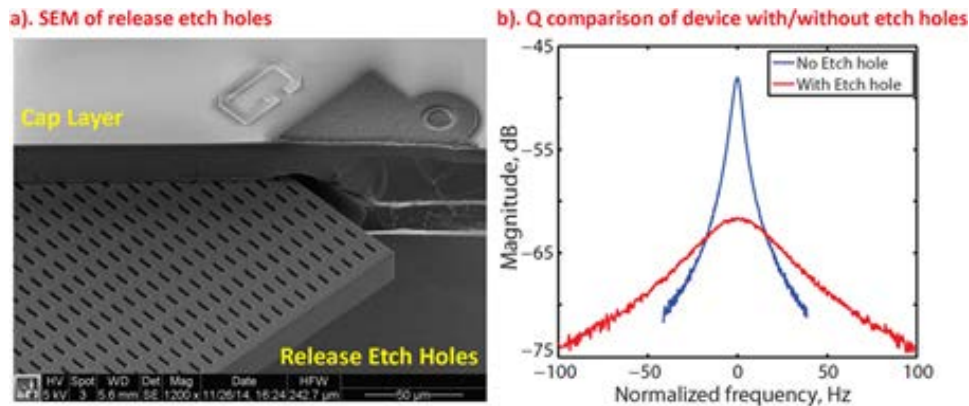


Figure 1: (a) SEM of a LE Resonator with Etch-holes in the Device Layer and (b) Comparison of Frequency Response of LE Resonators with/without Release Etch-hole

An important result from this recent publication [9] is the demonstration of extreme long-term stability for MEMS resonators built in this modified encapsulation process. The figures below show the temperature dependence of frequency and the long term stability of a temperature compensated Lamb mode resonator built in this process. We see a “Turnover Temperature” at 82 °C, which is ideal for operation of an ovenized clock. Further, we see open loop frequency stability approaching a few ppb over a month, showing unprecedented stability for a MEMS resonator.

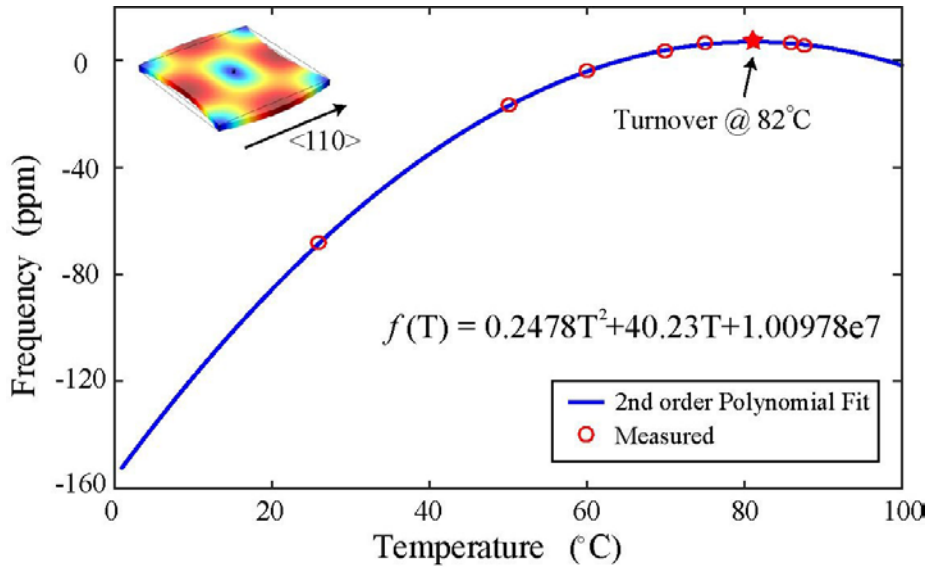


Figure 2: Frequency Temperature Measurement of a Lamé Mode Resonator
The turnover point can be found by fitting a 2nd order polynomial to the measured data, and is found to be around 82 °C.

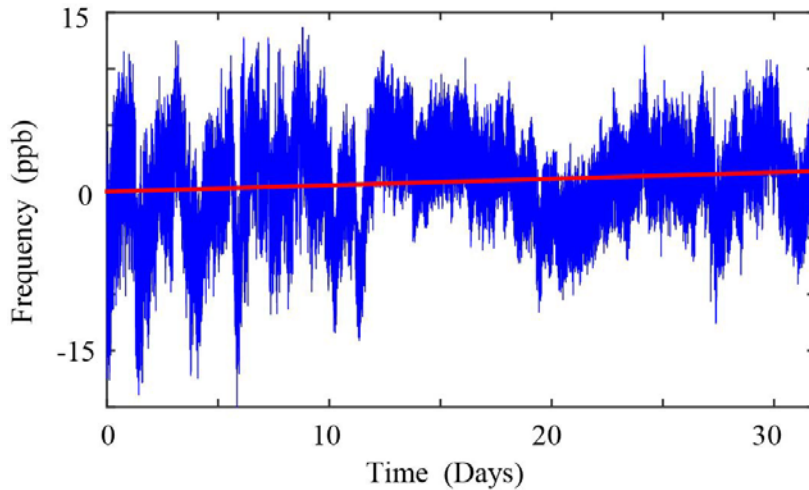


Figure 3: Frequency Stability of a Lamé Mode Resonator Operated at the Turnover Temperature of 82 °C

The red line is a linear fit to the data, which projects frequency drift over one year to be <30 ppb.

3.2 Temperature Compensation

As discussed in our proposal and in the first four progress reports, it has been shown that highly doped silicon resonators can exhibit reduced temperature dependence of frequency. The complete relationship between the elasticity, the doping, and the temperature is very complex, and requires determination of the three independent elastic constants as a function of doping and temperature. A journal publication has appeared with all these elastic constants, suitable for use by designers [1]. The data shown above leverages this result, showing a device designed to eliminate the linear temperature coefficient of frequency at 82 °C, enabling operation of an ultra-stable time reference. As a reminder all of this is demonstrated within a process that is

compatible with the existing SiTime manufacturing process, and can leverage all existing low cost, high volume production, circuitry, and packaging.

3.3 Nonlinear Elasticity

The elastic constants of silicon are known to have a mechanical stiffening nonlinearity, which can be represented as a nonlinear contribution to the elastic constants.

Inspired by initial conversations with our new colleagues at MSU, we began investigation of the relationship between doping and nonlinear elasticity during the period of the last report.

We observe that heavily doped silicon has some very strong and complicated intrinsic nonlinearities. We believe it is important to study this effect because heavily doped silicon is the best method for avoiding the temperature dependent frequency effects that are present in lightly doped silicon. Correcting this temperature dependence of frequency is a fundamental requirement of the MESO program, and lightly doped silicon has a temperature dependence that is at least 100X too strong. If performers are interested in heavily doped silicon to cure the temperature coefficient of frequency, and they are interested in nonlinear dynamics, they will run headlong into the very complex effects we're seeing in our first devices. Further, there is no model or capability to predict these effects. We have initiated a focused effort to measure and model the material dependent nonlinear stiffness observed in these devices. This work is ongoing, and will continue through the final period of this program. Reference 9 includes some preliminary efforts on characterizing of more nonlinear devices, and there is a more complete publication currently in review on the broader characterization of these effects.

We have initiated a study, similar to the one already completed to determine the temperature dependence of the elastic constants for doped silicon, in which we will try to determine the nonlinear elastic coefficients for these same doping levels. This is a very complex task, as we need to determine at least 27 independent parameters to extract the results we need, and we are not seeing a straightforward path to these results.

The main message from this part of this report is that we are using our fabrication process to develop large suites of MEMS resonators with controlled variations in design, doping, and other parameters. The ability to fabricate and yield these large numbers of devices provides an opportunity to understand complex phenomena and extract broadly useful data for designers. In our specific case, we have extracted a complete set of doping-dependent temperature dependence for the elastic constants of crystalline silicon, and we are working to extract the doping-dependent nonlinear elastic constants. In addition to these basic and fundamental results, we are beginning to have insight to some very subtle phenomena, such as nonlinear friction and nonlinear elasticity in these devices. While these are many steps away from the ultimate goals of the program, these fundamental contributions will be necessary to fully utilize the discoveries of this program.

3.4 Collaboration Activities with Michigan State University

These activities were discussed at great length in the last progress report and are continuing. A specific aspect of this effort has been the development of a method to extract nonlinear dynamical properties and information about the character of the damping and noise mechanisms from the details of a ringdown experiment. Of particular importance is the ability to separate the effects of conservative and non-conservative dissipation mechanisms. The non-conservative mechanisms are linked to noise phenomena through the Fluctuation-Dissipation Theorem, whereas the conservative mechanisms should be noise free. In this work, we capture the waveform of the ringdown by over sampling at more than 100x the resonant frequency (see Figure 4). We can then fit the data to exact forms for damped oscillator response, including the nonlinearities in the amplitude frequency relationship, as well as the amplitude dependent dissipation coefficients [2].

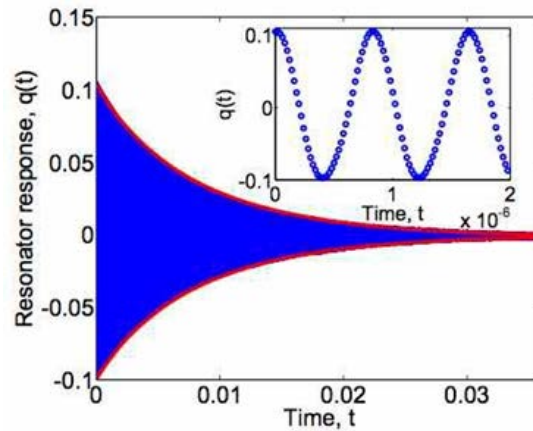


Figure 4: Illustration of the Ringdown Waveform Captured by Over Sampling at more than 100x the Resonant Frequency

Careful analysis of these fitted ringdown measurements shows that the frequency of the oscillation and the decay rate are both amplitude dependent. The data in Figure 5 shows the change in the period of the oscillation as a function of time as the oscillator decays from a high amplitude to a low amplitude. We are able to fit this data to a simple amplitude dependent stiffness coefficient for these devices. The data in Figure 6 shows the nonlinear decay of amplitude indicating nonlinear friction in these devices. All of this data is used to extract the strength of the conservative and non-conservative dissipation mechanisms, which we are currently analyzing for comparisons with noise measurements.

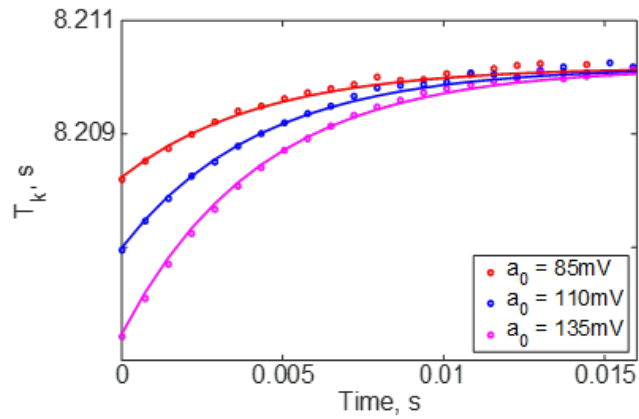


Figure 5: Change in the Period of the Oscillation as a Function of Time as the Oscillator Decays from a High Amplitude to a Low Amplitude

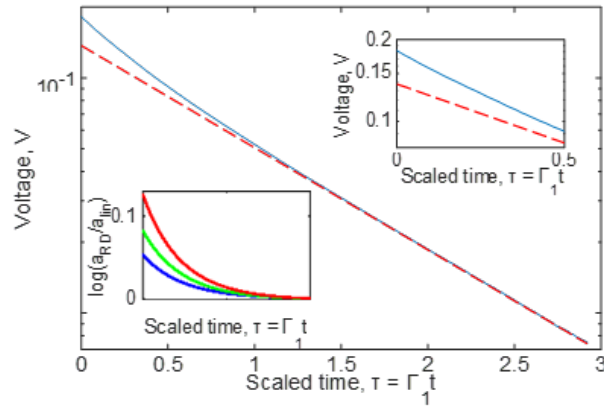


Figure 6: Nonlinear Decay of Amplitude Indicating Nonlinear Friction in the Devices

4.0 FINANCIAL STATUS

At the end of spring 2016, all funds for the project have been expended. Our internal systems are currently showing an overdraft of \$900, and this will be cleared as we reconcile some very minor accounting details. All invoices for reimbursement on this contract should have been sent by now, and all transactions between MSU and Stanford have similarly been settled.

We learned at the 2015 principal investigator meeting that there are no plans to fund the second year of the MSU subcontract so we completed the activities that we could within the available time and funds. We did request and receive a no cost extension of the Stanford contract to cover the end date of the MSU subcontract.

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LIST OF ACRONYMS, ABBREVIATIONS, AND SYMBOLS

ACRONYM	DESCRIPTION
DARPA	Defense Advanced Research Projects Agency
DEFYS	Dynamics-Enabled Frequency Sources
DoD	Department of Defense
MEMS	Microelectromechanical Systems
MESO	Mesodynamic Architectures
MSU	Michigan State University
NDSEG	National Defense Science and Engineering Graduate
NSF	National Science Foundation
PNT	Positioning, Navigation and Timing
TCF	Temperature Coefficient of Frequency
UCSB	University of California, Santa Barbara
uPNT	Micro Positioning, Navigation, and Timing