Sensitivity of Particle Size in Discrete Element Method to Particle Gas Method (DEM_PGM) Coupling in Underbody Blast Simulations

Venkatesh Babu, Kumar Kulkarni, Sanjay Kankanalapalli, Ravi Thyagarajan

U.S. Army, Research Development & Engineering Command, (RDECOM), Tank Automotive Research Development & Engineering Center (TARDEC),
Warren MI 48397

Abstract

In this paper, the capability of two methods of modelling detonation of high explosives (HE) buried in soil viz., (1) coupled discrete element & particle gas methods (DEM-PGM) and (2) Arbitrary Lagrangian-Eulerian (ALE), are investigated. The ALE method of modeling the effects of buried charges in soil is well known and widely used in blast simulations today [6]. Due to high computational costs, inconsistent robustness and long run times, alternate modeling methods such as Smoothed Particle Hydrodynamics (SPH) [7] and DEM are gaining more traction. In all these methods, accuracy of the analysis relies not only on the fidelity of the soil and high explosive models but also on the robustness of fluid-structure interaction. These high-fidelity models are also useful in generating fast running models (FRM) useful for rapid generation of blast simulation results of acceptable accuracy [8-14]. In this paper, the effect of sensitivity of particle size in the performance of the DEM_PGM blast simulation is compared to that of the ALE blast simulation method. The main focus of this study is to understand the strengths of DEM_PGM and identify the limitations/strengths compared to the ALE method. Discrete Element Method (DEM) can model individual particle directly, and displace independently which is based on Cundall & Strack [1]

A 2m x 2m x 2m volume is filled with ALE elements of 10mm side length. Three sets of ALE-equivalent DEM models are created using 3mm, 4mm and 5mm radius spheres. High explosive TNT is buried 50 mm deep in these three DEM soils and modeled as particles using PARTICLE_BLAST. Each of the 3 sets of DEM are analyzed for 100k, 250k, 500k and 750k TNT particles to understand the sensitivity of the DEM_PGM coupling and how the soil impulse, kinetic energy and translational energy are affected. This analysis has been extended to evaluate the TARDEC generic hull (GH) structural performance and compared to ALE method. Results show that DEM_PGM method reduces the computational time significantly when compared to the ALE method, and soil, a granular material by nature, can be well represented by fine particles in its discontinuous form.

Generic DEM_PGM model

Figure 1 below shows the generic DEM_PGM model set-up. Soil is filled with discrete element spheres referred to as DEM particles in this paper in a 2m x 2m x 2m volume. High explosive (HE) TNT material is buried inside this soil volume 50 mm from the ground surface (*aka* Depth of Burial, or DOB) and represented as blast particles (PARTICLE_BLAST). The blast simulation by coupling these two methods is referred to as DEM_PGM coupling in this paper.

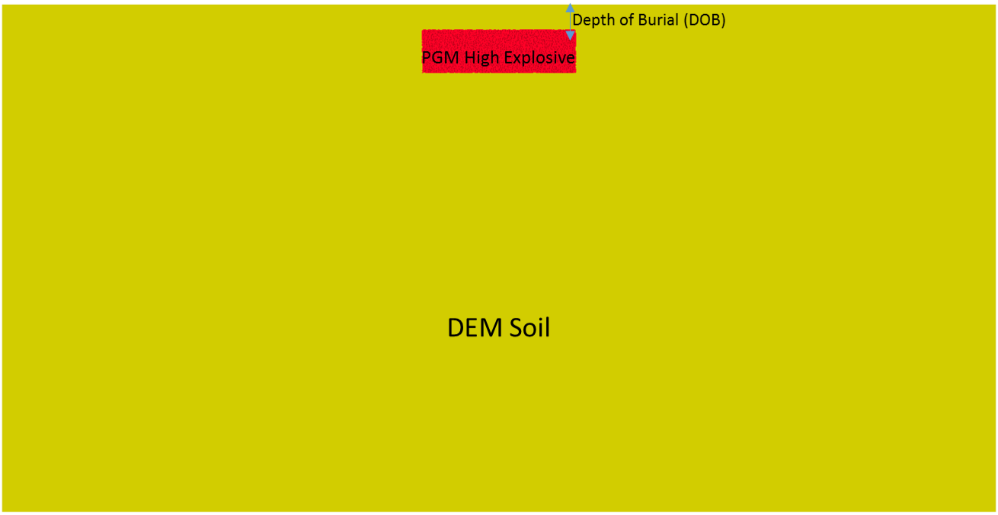
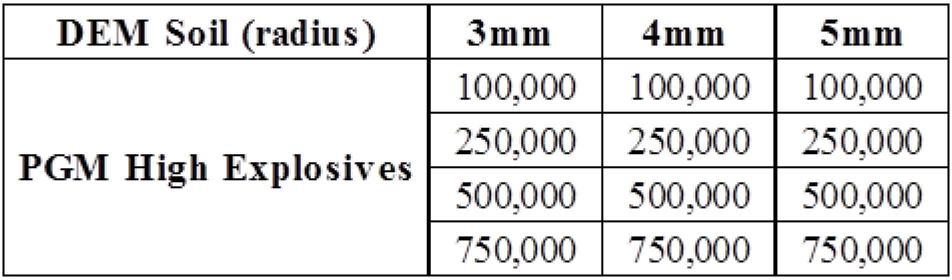


Figure 1. DEM_PGM model.

Initial set of analysis was performed using the matrix show below in Table 1. This way it gives an overall idea of how small the soil particles will capture the effect of blast response. Soil volume was filled with discrete element sphere particles [2] in LS-PREPOST. To fill the particles beyond 5 million elements was challenging due to preprocessor and hardware limitations. Table 1 below shows the DEM soil particle radius and HE particle analysis matrix

Table 1. DEM soil radius and PGM HE matrix



DEM Soil (radius)	3mm	4mm	5mm
PGM High Explosives	100,000	100,000	100,000
	250,000	250,000	250,000
	500,000	500,000	500,000
	750,000	750,000	750,000

DEM soil models created with three different radius 3mm, 4mm and 5mm. Inside these soils, HE particles were filled according to the table above ranging from 100,000 particles to 750,000 particles. TNT was used as high explosive material. Particle blast and HE geometry definition cards are shown below.

```
*PARTICLE_BLAST
$#      ssid    sstype      spid    sptype      hpid     htype
           0         0     10101         0         5         2
$#      nphe     npair     iunit
      750000         0         1
$#   ihetype   density    energy     gamma     covol     detov
           3    1590.0   6.20e+9       1.4       0.3    6741.0
$#      detx      dety      detz      tdet     btend
         0.0       0.0    -0.041       0.0       0.0
$#      bcxo      bcx1      bcy0      bcy1      bcz0      bcz1
         0.0       0.0       0.0       0.0       0.0       0.0
```

```
*DEFINE_PBLAST_GEOMETRY
$#       gid    gtype1
           5         3
$#        xa        ya        za        xb        yb        zb
         1.0       0.0       0.0       0.0       1.0       0.0
$#        xc        yc        zc
         0.0       0.0    -0.041
$#        g1        g2        g3
       0.122     0.082       0.0
```

Using these parameters DEM_PGM was evaluated using LS-DYNA MPP R8 version. Main focus of this analysis was to understand the soil pulse, estimate computation time and compare to equivalent ALE method. Figure 2 shows the soil impulse vs number HE particles where the soil impulse is directly proportional to the HE particle count. Figure 3 shows that the impulse is directly proportional to the soil particle radius. Figure 4 and 5 compares the computational time vs number HE particles and soil particle radius respectively. Figure 4 indicates that higher the HE particle count higher the impulse.

From Figure 4 & 5 we can infer that larger soil particle radius with lower HE particles reduces the computation time. Conversely, smaller soil particle radius with larger HE particle count increases computation time

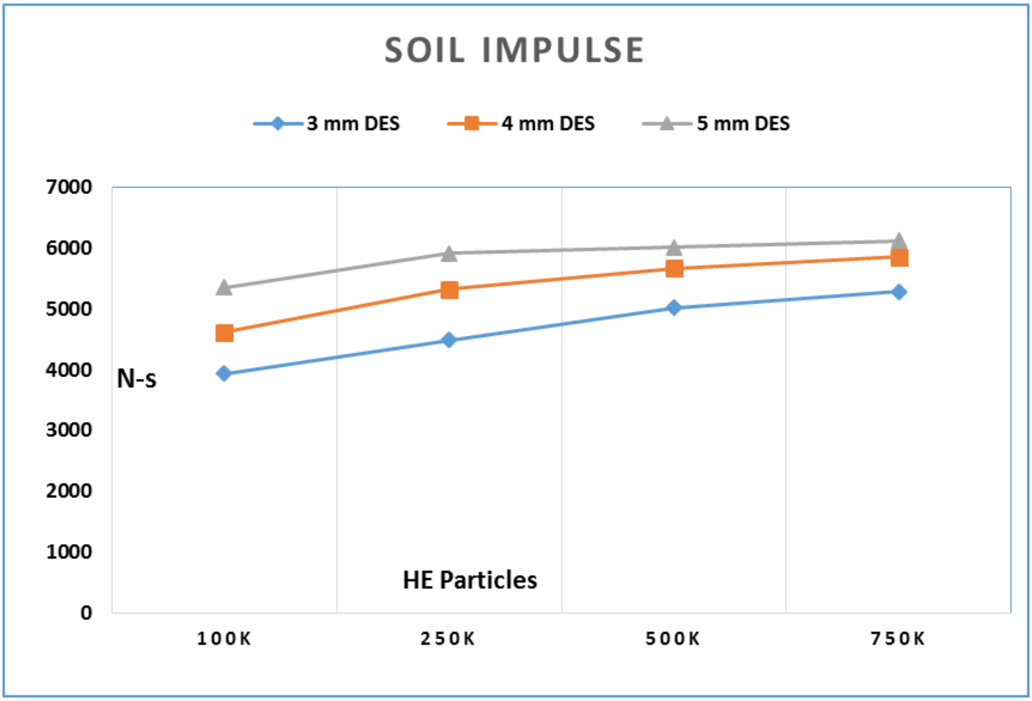


Figure 2. Soil impulse Vs. HE particles

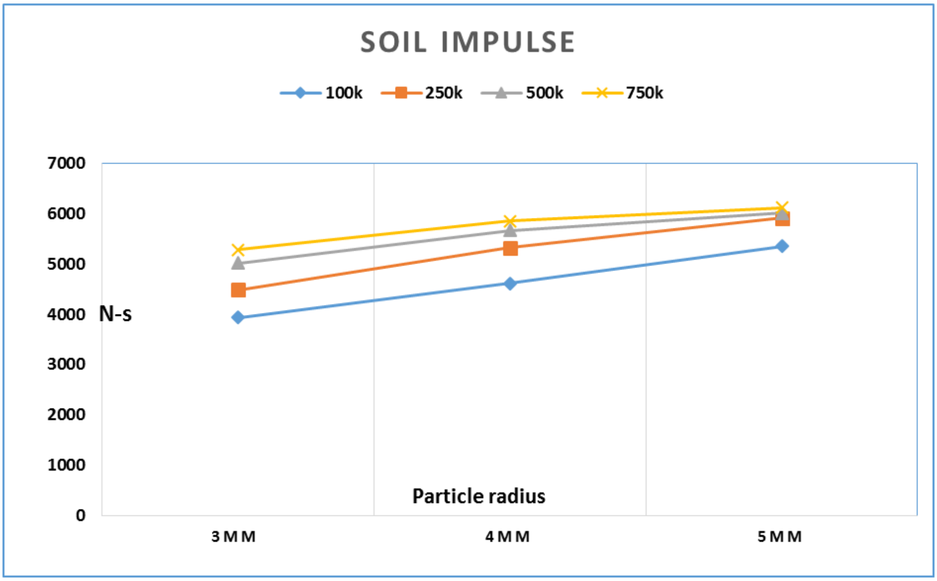


Figure 3. Soil impulse Vs. Soil Particle Radius

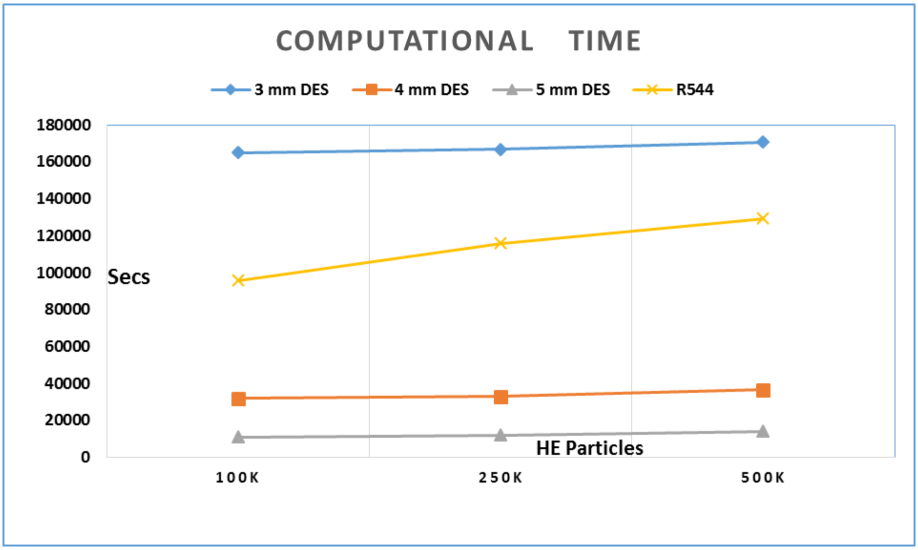


Figure 4. Computation time Vs. HE particles

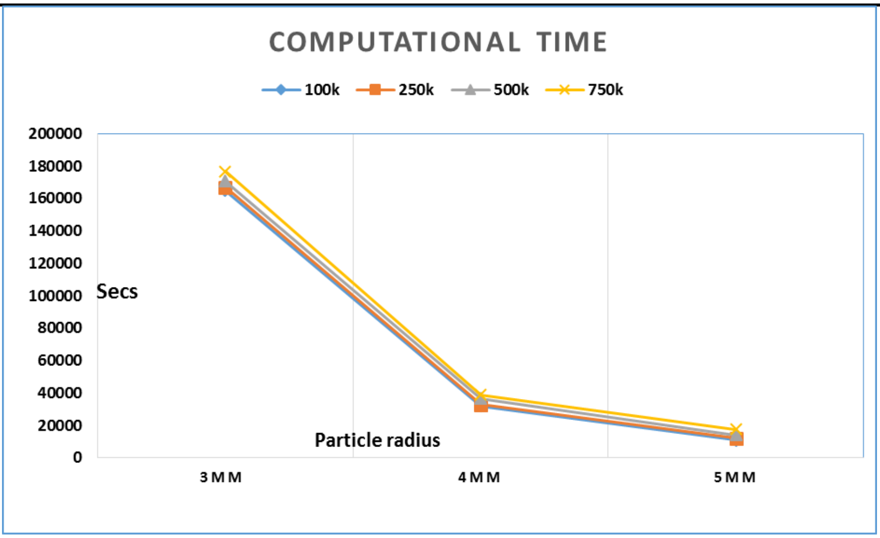


Figure 5. Computation time Vs. Soil particle radius

Figure 6 shows the comparison of soil kinetic energies between ALE and DEM_PGM. Since ALE is a continuum approach and DEM_PGM is based on discrete elements, it does not have internal energy component; instead, all the energies will be stored as kinetic energies and translational energies. Kinetic energies of ALE soil and DEM_PGM soils compares very well from Figure 6.

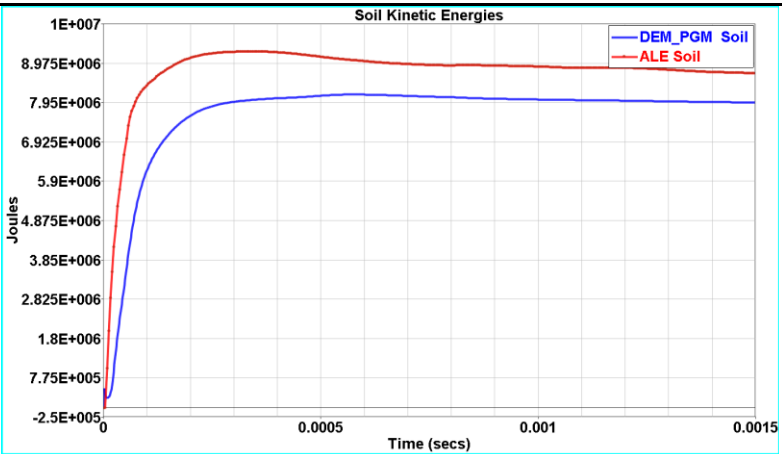


Figure 6. Soil kinetic energies

Table 2. Summarizes all the results of 3mm, 4mm and 5mm soil particle radius and different HE particles count. Last column shows the values for equivalent ALE method. Initial HE energy is almost unchanged in all the analysis including ALE. Soil kinetic energies tend to increase with increased number of HE particles for a given soil particle radius. Same set of analysis performed with the new developer version 105123, computational time is reduced by over 40% on average. Close observation of the table below reveals that at 5mm soil radius with 500,000 HE particles, computational time (9,481 seconds) is about half of equivalent ALE (19,888) method. ALE impulse is significantly higher than that of DEM_PGM due to soil mass differences between ALE and DEM_PGM. Due to voids DEM will not fill the soil volume completely and maximum packing density is observed at 56.3%. This motivated us to explore further evaluation of this DEM_PGM using a generic hull structure.

Table 2. Summary of DEM_PGM and ALE analysis results

Soil DES	**3mm**				**4mm**				**5mm**				**ALE**
HE Particles	100k	250k	500k	750k	100k	250k	500k	750k	100k	250k	500k	750k	10mm
Soil KE	7.92E+06	7.93E+06	8.17E+06	8.24E+06	7.90E+06	8.22E+06	8.32E+06	8.33E+06	7.98E+06	8.13E+06	8.22E+06	8.25E+06	7.98E+06
Impulse	3,940	4,490	5,023	5,285	4,615	5,328	5,673	5,857	5,357	5,918	6,019	6,123	13,864
HE ke	2.32E+07	2.32E+07	2.34E+07	2.36E+07	2.30E+07	2.33E+07	2.34E+07	2.35E+07	2.31E+07	2.33E+07	2.34E+07	2.34E+07	2.35E+07
HE TE	1.43E+07	1.48E+07	1.47E+07	1.49E+07	1.44E+07	1.46E+07	1.47E+07	1.47E+07	1.44E+07	1.46E+07	1.47E+07	1.47E+07	1.67E+06
Total Energy	2.84E+07	2.76E+07	2.52E+07	2.35E+07	2.82E+07	2.46E+07	2.26E+07	2.18E+07	2.56E+07	2.26E+08	2.09E+07	2.03E+07	2.35E+07
LS-DYNA R8 (secs)	165,025	166,809	170,721	176,857	32,055	32,945	36,653	38,918	11,005	11,992	14,131	17,511	19,888
LS-DYNA V105123 (secs)	105,616	115,855	129,357	134,190	16,062	17,653	20,473	23,220	5,661	6,879	9,421	12,241	19,888

Generic Hull Model

In order to evaluate DEM_PGM further TARDEC- developed Generic Hull (GH) model was used [7]. The TARDEC Generic Hull is used as the Cab and is comprised of 780,842 solid elements shown in Figure 7 and weighs 16,000lbs. Many unclassified studies from past researchers have utilized fictitious vehicle geometry due to the unavailability of realistic information. Due to the sensitive nature of the work performed by the Department of Defense, data generated from testing military vehicles is usually classified, making it difficult to share data in the public domain.

In order to increase the operational relevance of studies performed by the wider scientific community, the US Army Tank Automotive Research, Development and Engineering Center (RDECOM-TARDEC) recently fabricated a generic vehicle hull, aka TARDEC Generic Hull, shown in Figure 7, with the intent to:

- Subject it to an underbody mine blast test
- Share the data publicly
- Evaluate blast mitigation technologies

Generic Hull is modeled as a Lagrangian foreground mesh in this project and coupled to the DEM_PGM using PARTICLE_BLAST and DEFINE_DE_TO_SURFACE_COUPLING as a slave parts to discrete HE and Soil particles.

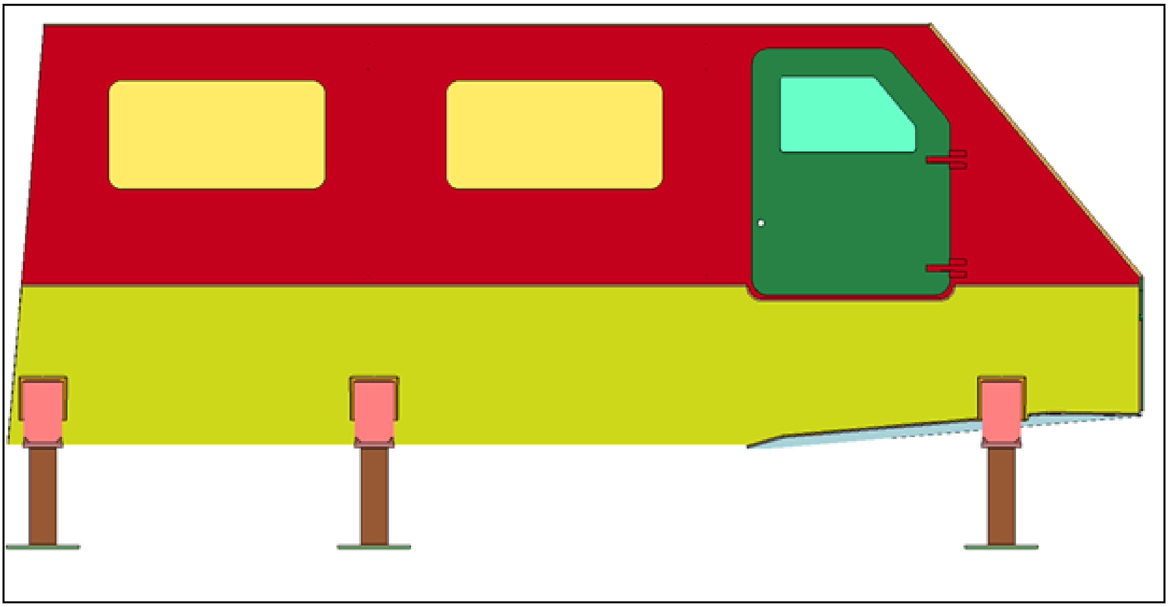


Figure 7. TARDEC Generic Hull Structure model

First, GH model was evaluated with ALE to establish the vehicle impulse, hull displacements and computation time. An equivalent DEM_PGM model was created and evaluated for similar responses with the new LS-DYNA developer version 101523. Total number of ALE Soil =2.46e+6, DEM Particles =4.06E+6. Figure 8 shows the ALE model with GH structure and equivalent DEM_PGM model with GH structure.

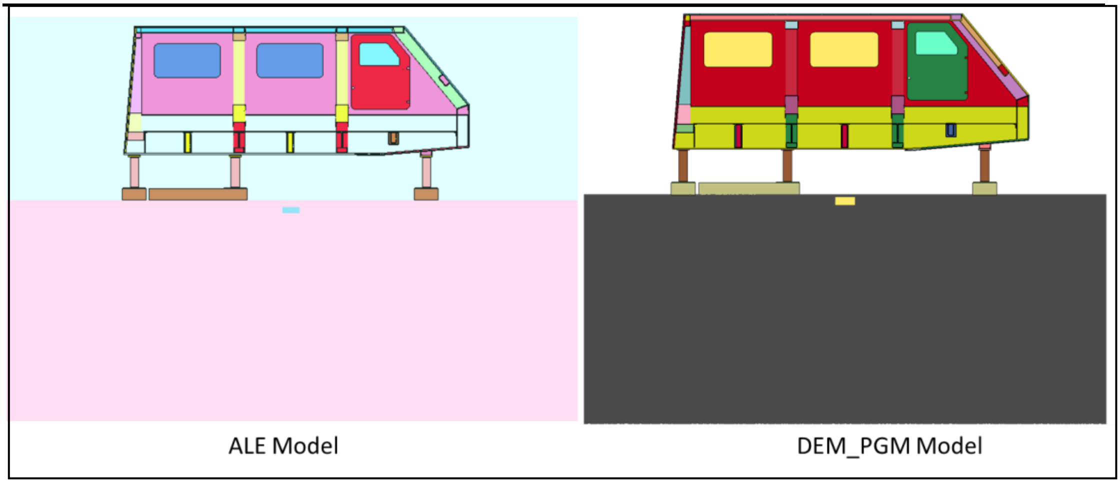


Figure 8. GH model setup in ALE and DEM_PGM

Overall system positions at 16 milliseconds are show in Figure 8. Soil crater dimensions between ALE and DEM_PGM are comparable within 90%. The DEM_PGM results shown are based on the initial set of values in *CONTROL_DISCRETE_ELEMENT card. ALE shows smoother transition between HE gases and soil, compared to DEM_PGM's soil particle to gas particle interaction.

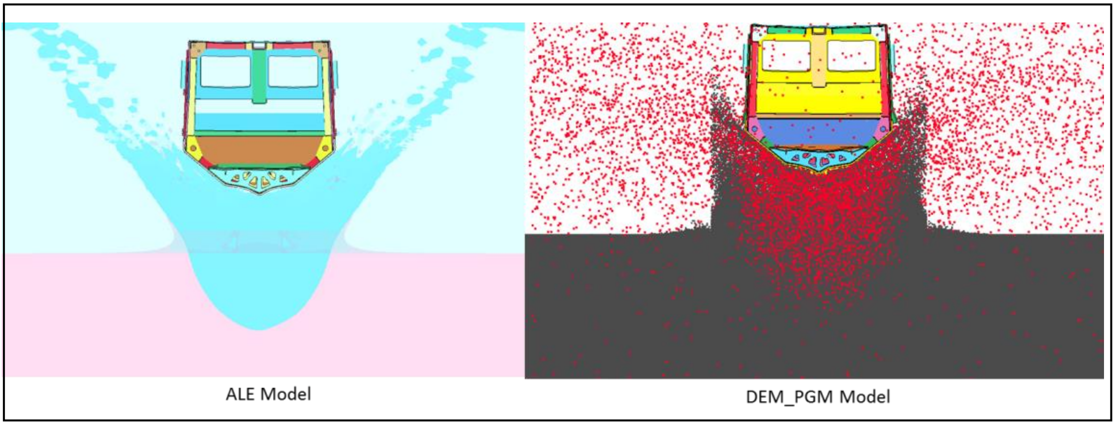
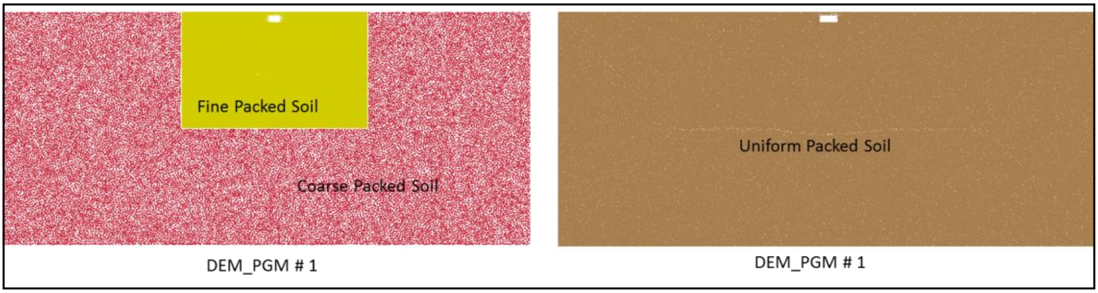


Figure 9. Animated snapshot at 16 milliseconds

DEM_PGM model

Initial comparison of ALE to DEM_PGM highlighted the need for further parameter evaluation to improve the soil to HE gas interaction. Two sets of DEM_PGM models were created. In the first model, DEM soils are created with two volumes, finely packed soil near the HE and coarsely packed soil away from the HE. This model has 4.06 million soil particles. In the second model, entire soil volume is created with 10 mm constant radius containing 19.3 million soil particles. Figure 10 shows the two DEM_PGM models.



DEM_PGM # 1 DEM_PGM # 1

Figure 10. DEM_PGM soil models

1. CONTROL_DISCRETE_ELEMENT

```
$------------1-------------2-------------3-------------4-------------5-----------6---------7---------8
$     Ndamp       tdamp           fric          fricr       normk       sheark      cap   mxnsc
  1.000E-00  0.200E+00 0.110E+00   0.110E+00   0.90E+00  0.285E+0          1
$     Gamma       capvol           ang
   0.728e-01         0.66          5.00
```

This card sets the global control parameters for discrete element spheres. *NDAMP* and *NORMK* are the normal damping and stiffness coefficients and values have bigger influence on the output responses compared to the other variables. *TDAMP* tangential damping coefficients and *FRC*, *FRICR* friction coefficients. Values of liquid surface tension (*Gamma*), *Capvol* and *Ang*les are associated with capillary forces. The parameters that define the behavior of wet material can be found from the equations developed by Rabinovich et al. [3, 4]

2. PARTICLE_BLAST

```
$-----------1------------2------------3------------4------------5------------6---------7--------8
$#      ssid    sstype      spid    sptype      hpid     htype
           0         0     10101         0         5         2
$#      nphe     npair     iunit
      750000         0         1
$#   ihetype   density    energy     gamma     covol     detov
           3    1590.0   6.20e+9       1.4       0.3    6741.0
$#      detx      dety      detz      tdet     btend
         0.0       0.0    -0.041       0.0       0.0
$#      bcxo      bcx1      bcy0      bcy1      bcz0      bcz1
         0.0       0.0       0.0       0.0       0.0       0.0
```

Variable *NPHE* (number of HE particles) has significance influence in terms of computation time and to accuracy. Higher the value of *NPHE*, higher the computation time and vice versa. Influence of *NPAIR* number of air particles is not evaluated in this study. Material properties of high explosives (HE) are defined in line 3 of PARTICLE_BLAST card

3. DEFINE_DE_TO_SURFACE_COUPLING

```
$-----------1------------2------------3------------4------------5------------6---------7--------8
$#     slave    master     stype     mtype
       10101      1102         0         0
$#     FricS     FricD      damp     bsort    LCVx    LCVy    LCVz   WEARC
         0.3       0.3       0.9         5
```

Variables *DAMP* and *BSORT* influences the computation time and leakage control of particles (spheres). Most of the analysis was performed without any air particles. One case was analyzed with air particles to understand whether air influences the results or not. Results showed that air particles has minimal effect on the overall results, decided to pursue the rest of the analysis without air as per Table 3 below. Soil particles represented as spheres will result in void and requires a very fine packing to minimize the void. Because of this, soil mass in DEM may not match the soil mass in ALE which is represented accurately for a given volume. To address this issue first, 7 DEM_PGM analysis was performed with soil density of 2,736 kg/m*3 compared to the actual density of 2,034 kg/m*3 to account for the void and also not to add too much mass into the system. One of the key observation from packing the soil using LS-PREPOST is that, packing density remained constant at 56.3% irrespective of the particle diameter which is significantly less than the theoretical Gaussian highest average density. In this project particle diameters varied from 3 mm to 10 mm. This could be due to packing algorithm in LS-PREPOST needs further investigation.

In geometry, close-packing of equal spheres is a dense arrangement of congruent spheres in an infinite, regular arrangement (or lattice). Carl Friedrich Gauss proved that the highest average density – that is, the greatest fraction of space occupied by spheres – that can be achieved by a lattice packing is [15]

$$\frac{\pi}{3\sqrt{2}} \simeq 0.74048.$$

Table 3. Analysis matrix

		Number of Soil Elements	**Soil Mass (kgs)**
	ALE	2.46E+06	2.99E+05
DEM_PGM	**Soil Density=2736 kg/m*3**		
	HE=75K	4.06E+06	2.26E+05
	HE=125k	4.06E+06	2.26E+05
	HE=250k	4.06E+06	2.26E+05
	HE=75K with;		
	NDAMP=1.0	4.06E+06	2.26E+05
	NDAMP=1.0, NORMK=1.0	4.06E+06	2.26E+05
	DEM=10mm constant	1.93E+07	1.76E+05
	Soil Density = 3,756 kg/m*3	4.06E+06	3.00E+05
	Soil Density = 2,034 kg/m*3	4.06E+06	1.68E+05
	Optimum DEM Parameters	4.06E+06	1.68E+05

DEM_PGM Results

Figure 11 below shows animated results between the ALE and DEM_PGM at 1, 6 and 16 milliseconds. DEM_PGM results compares to the ALE very well. By increasing the *NDAMP* and *NORMK* values, the DEM soil to HE interaction looks continuous without any leakage. Similar responses are observed on all the DEM_PGM combinations shown in Table 3 above. Figure 12 shows the ALE and DEM_PGM overlay

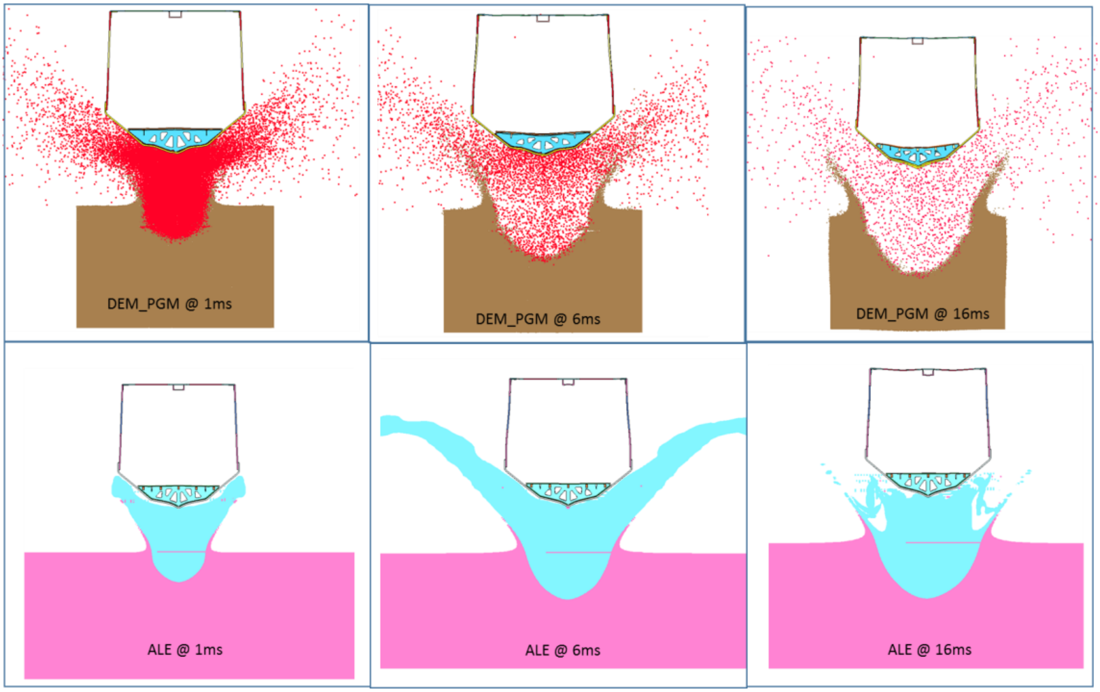


Figure 11. Snapshot results from ALE and DEM at different times

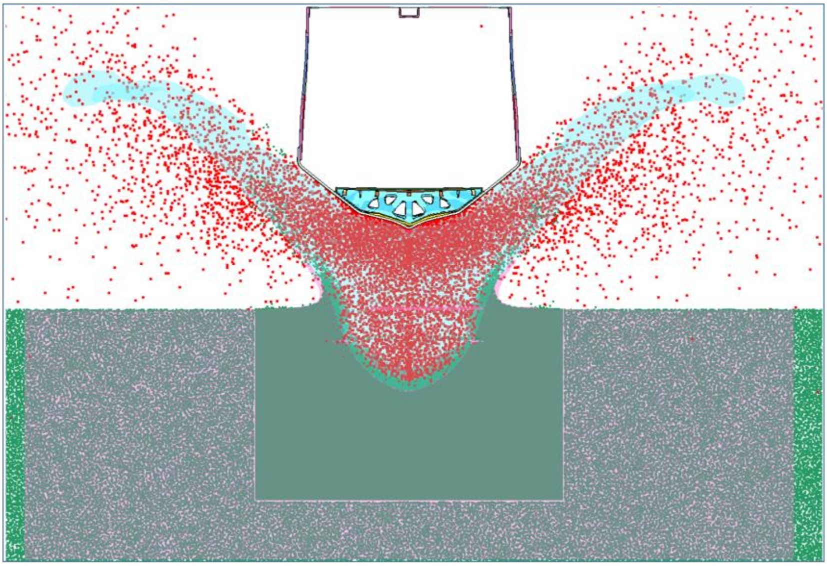


Figure 12. ALE and DEM_PGM overlay

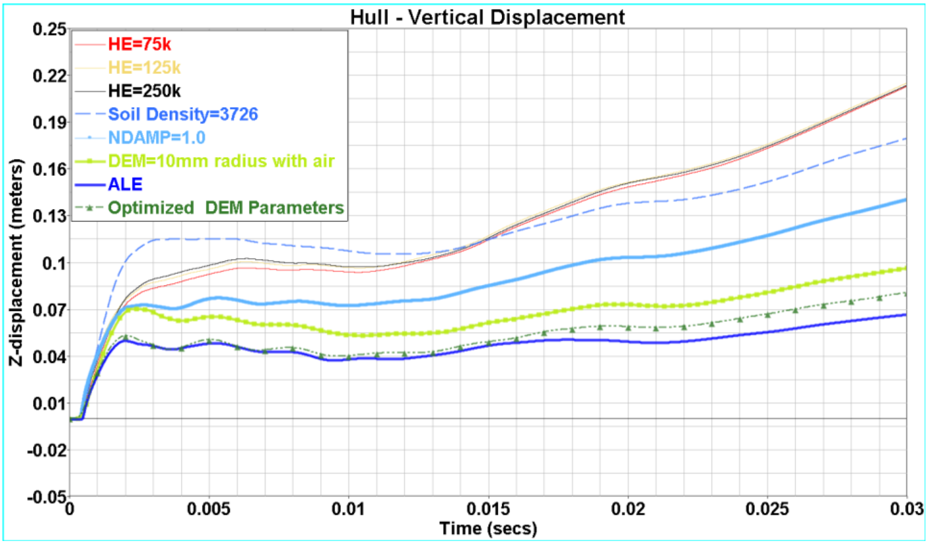


Figure 13. Hull vertical displacement

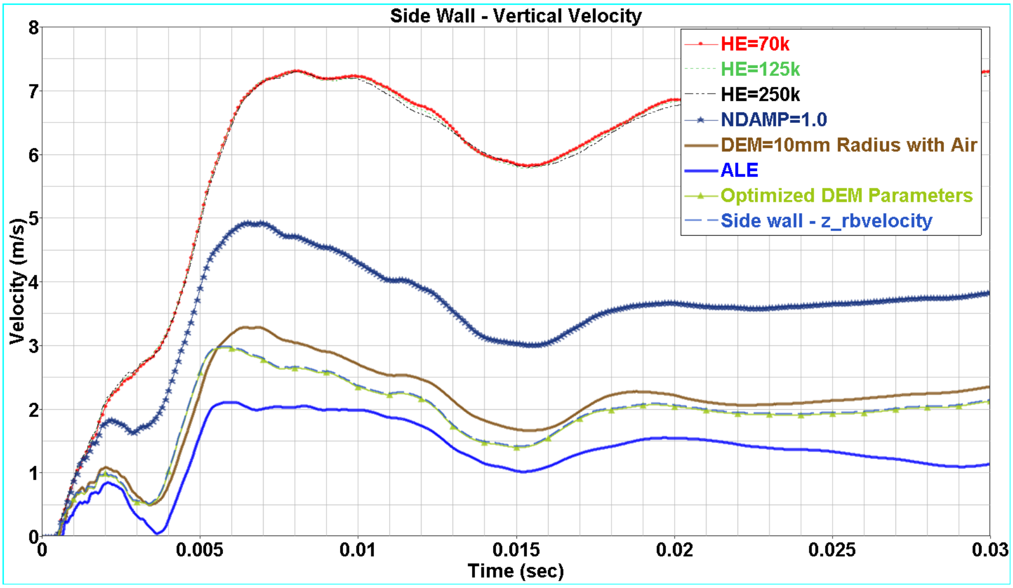


Figure 14. Side wall vertical velocities

Hull vertical displacement and side wall vertical velocities are shown in Figures 13 and 14 respectively. ALE simulation shows 50 mm hull displacement and 2.1 m/s side wall vertical velocities. Main objective of this DEM_PGM analysis was to compare the results to ALE results. By using LS-DYNA default parameters in CONTROL_DISCRETE_ELEMENT and DEFINE_DE_TO_SURFACE_COUPLING with the three different HE particles count studied and with soil particles assigned density of 2736 kg/m*3, the vertical hull displacement is 100 mm which is twice that of the ALE simulation. Side wall vertical velocities are at 7.2 m/s three and a half times that of the ALE. Increasing the soil density to 3726 kg/m*3 to match the soil mass results in much higher hull vertical displacements. Increasing the number of HE particles did not influence either the hull displacements or the side wall velocities. Moreover computational time increases by increasing the number of HE particles. 75,000 HE particles was chosen for the remainder of the analysis.

By locking HE particles at 75,000, variables *NDAMP* and *NORMK* values were changed from default values to 1.0. By increasing these values hull vertical displacements reduced from 100mm to 70 mm closer to ALE value of 50 mm. Please note that all these changes were made with soil density at 2736 kg/m*3 which is 34% higher than the actual soil density of 2034 kg/m*3 to account for the loss of mass due to void in DEM_PGM.

In the next analysis, HE=75,000 NDAMP=1.0, NORMK=1.0 and soil density changed from 2,736 kg/m*3 to 2,034 kg/m*3. As mentioned earlier, density of soil increased to 2,736 kg/m*3 to accommodate the void. This change resulted in hull vertical displacement of 52 mm and side wall vertical velocity of 2.7 m/s significantly improved performance and much closer to ALE results of 50 mm and 2.1 m/s. Vehicle impulse follows the similar trend and is shown in Figure 15 for all the combinations.

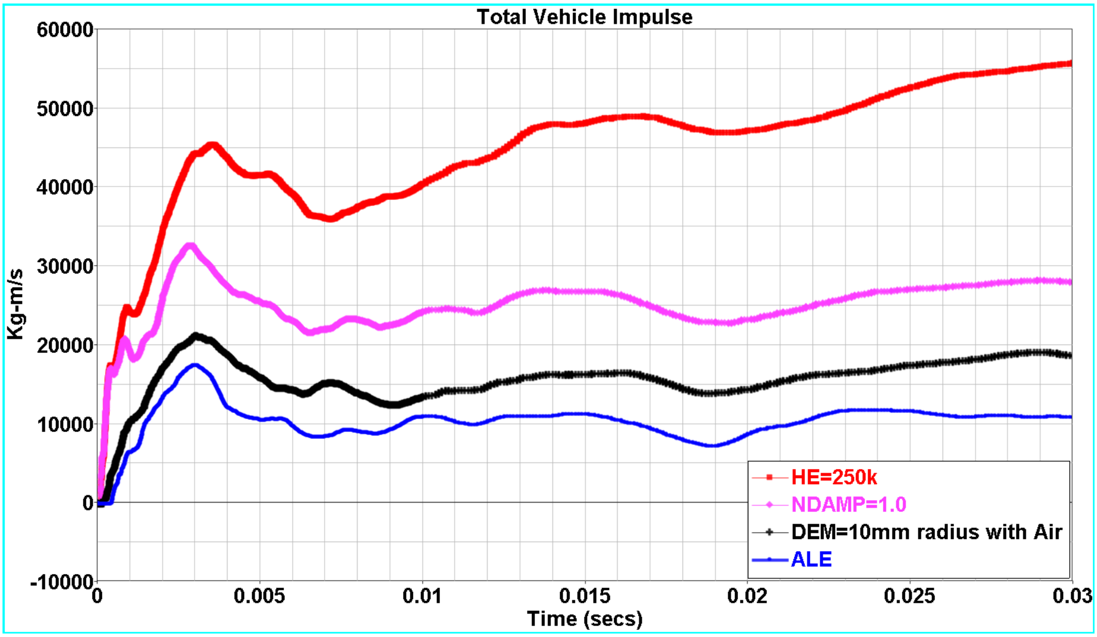


Figure 15. Vehicle impulse

Table 5 summarizes the results from all the ALE and DEM_PGM analysis. Key observation from the summary is that ALE takes 563,518 secs or 156 hours and optimized DEM_PGM results takes 29.5 hours significantly less for similar output response. Moreover DEM_PGM has 4 million soil particles compared to 2.4 million ALE elements. One case was analyzed with 19 million soil particles to see whether close packing of soil has any benefit in improving the responses. The discrete elements generated within this volume will use a constant user-defined radius [5]. This resulted in increased computation time similar to ALE but no added benefit.

Table 5. Summary of ALE and DEM_PGM results

		Hull Vertical Displacement (mm)	Side Wall Vertical Velocity (m/s)	Vehicle Impulse (kg-m/s)	Computation Time (secs)	Number of Soil Elements	Soil Mass (kgs)
	ALE	**50**	**2.11**	**17,449**	**563,518**	2.46E+06	2.99E+05
DEM_PGM	**Soil Density=2736 kg/m*3**						
	HE=75K	97	7.29	45,476	105,929	4.06E+06	2.26E+05
	HE=125k	100	7.29	45,476	182,793	4.06E+06	2.26E+05
	HE=250k	102	7.29	45,476	191,642	4.06E+06	2.26E+05
	HE=75K with					4.06E+06	2.26E+05
	NDAMP=1.0	77	4.93	32,659	106,200	4.06E+06	2.26E+05
	NDAMP=1.0, NORMK=1.0	70	4.8	32,113	106,200	4.06E+06	2.26E+05
	DEM=10mm constant	64	3.28	21,115	506,474	1.93E+07	1.76E+05
	Soil Density = 3756 kg/m*3	114	5.77	37,388	106,200	4.06E+06	3.00E+05
	Soil Density = 2034 kg/m*3	53	2.95	17,995	106,200	4.06E+06	1.68E+05
	Optimum DEM Parameters	**52**	**2.9**	**17,892**	**106,200**	4.06E+06	1.68E+05

Conclusions

In an effort to understand the sensitivity of the DEM_PGM variables, a detailed DEM_PGM model and ALE model created. As a first step, DEM_PGM model was analyzed without any structure to establish the initial corridor for the variables. Once identified the initial critical DEM_PGM variables, analysis has been extended to include TARDEC-developed Generic Hull structure. Using this model several other variables associated with DEM_PGM model such as normal and tangential damping coefficients, normal and shear stiffness coefficients, effect of liquid surface tension, liquid bridge volume were analyzed. Effect of friction, and damping coefficients were assessed by using DEFINE_DE_TO_SURFACE_COUPLING card. Some of the key findings from the analyses are

- Density of soil affects the overall responses
- Higher the number of HE particles higher the computation time, with minimal change in output responses
- Larger the number of soil particles, higher the computation cost with moderate change in output responses.
- *NDAMP* and *NORMK* in CONTROL_DISCRETE_ELEMENT value have a significant influence on soil particles and their cohesiveness and better response.
- *DAMP* and *BSORT* values in CONTROL_DE_TO_SURFACE_COUPLING reduces the leakage between DEM soil and the Lagrange structure. Lower *BSORT* values with higher *DAMP* value worked well in this analysis.
- Lastly, DEM_PGM reduces the computation time significantly compared to equivalent ALE method. ALE method computation time for 60 milliseconds was 156 hours and 29 hours for DEM_PGM method

DEM_PGM method is promising as an alternative to the ALE method particularly in underbody blast simulations, given the large deformation and high computation cost normally involved with the latter. The DEM_PGM analysis method will also be extended to different areas involving large deformation and high strains in addition to blast simulations.

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