

ABSTRACT

Title of Dissertation: DETERMINISTIC MULTI-OBJECTIVE
ROBUST OPTIMIZATION FOR SINGLE
PRODUCT AND PRODUCT LINE
ENGINEERING WITH DESIGN AND
MARKETING CONSIDERATIONS

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Nearly all engineering design problems have multiple objectives with parameters that have uncontrollable variations due to noise or uncertainty. Such variations can significantly degrade performance of design solutions or can even make them infeasible. The variations can also adversely affect customer's preferences for a product design alternative and its success in the market.

This dissertation presents two multi-objective optimization approaches for obtaining robustly optimal design solutions. The two approaches use the same method to obtain a feasibly robust solution: one that does not violate any constraint due to uncontrollable variations. However, each approach uses a different method to obtain multi-objectively robust solutions. Approach 1 obtains a multi-objectively robust solution in which, with respect to a target point and under uncontrollable variations, the distance between worst case and target design points and the distance between worst and best case design points are minimized. Approach 2 obtains a multi-objectively robust solution

which is optimal for nominal values of parameters and at the same time maintains an acceptable range of variability with respect to individual objective functions. Approach 2 is used within an integrated design and marketing framework to facilitate the generation of a robustly optimal set of single product design alternatives and a robustly optimal product line design alternative. By way of this framework, in the design domain, Approach 2 evaluates performance and robustness of design alternatives. While in the marketing domain, it considers designs that are robust with respect to customer preferences for variations propagated from the design domain as well as inherent variations due to the fit of a preference model to sampled marketing data.

The applicability and differences of the two robust optimization approaches are demonstrated and explored with a numerical and an engineering example. In particular, since Approach 2 is more flexible and less conservative than Approach 1, it has been applied and demonstrated with a real-world case study in single product and product line engineering of a power tool with both design and marketing considerations.

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2006

DEDICATION

To my parents

And

To my wife, Leila

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NOMENCLATURE

AIC	Akaike's information criterion
C_{aH}	cost of assembling the components into the final product H
C_{mH}	maintenance cost of product H
C_{r^tH}	unit cost of the r^{th} component of type t
C_{sH}	salvage cost
$cons_s$	constant term representing no-choice utility for consumers in s^{th} segment
ED	expected utility dominance
$F_r^{t(q)}$	shared fitness for product alternative q in subset r
FC	fixed cost for a product line
f_i	i^{th} objective function
g_j	j^{th} constraint function
H_r	r^{th} product alternative
I	total number of objective functions
J	total number of constraint functions
K	total number of choice sets
L	total number of parameters
LL	overall log-likelihood of choice sets
M	total number of products in each choice set
MPV	measurement of performance variation
MS_l	market share for l^{th} product in product line l
MS_{upper_bound}	upper bound of market share for product
MS_{lower_bound}	lower bound of market share for product
N	total number of design variables
N_B	number of function calls to obtain design best case scenario
N_F	number of function calls to verify design feasibility robustness
N_m	market size
N_v	number of variants in a product line
\bar{N}_v	maximum number of variants in a product line
N_{v_shrt}	number of products sharing the t^{th} type of component r in a line
N_W	number of function calls to obtain design worst case scenario
n_q	niche count
n	number of function calls
n_D	number of designs passed from top level
n_F	number of non feasibly robust designs passed from top level
P	price
P_l	price for variant l in a product line

\Pr_{cmks}	choice probability for consumer c of profile m from choice set k conditional on consumer c belonging to s^{th} segment
\Pr_{cmk}	unconditional probability of choice for consumer c of profile m from choice set k
P_{mk}	price of profile m in k^{th} choice set
P_{mmk}	price of profile mm in k^{th} choice set
\mathbf{p}	vector of design parameters
\mathbf{p}_0	vector of nominal values of design parameters
\mathbf{p}_L	vector of upper bounds of design parameters
\mathbf{p}_U	vector of lower bounds of design parameters
p_v	v^{th} design parameter value
q	number of parameters to be estimated
R	total number of competitive products
Rev	product manufacturer's revenue
S	total number of market segments
SS	sample size
SS_s	size of s^{th} segment
$U_{\text{lower_bound},s}$	lower bound of conjoint utility for product in s^{th} segment
$U_{\text{upper_bound},s}$	upper bound of conjoint utility in s^{th} segment
$U_{cp_r,\text{lower_bound},s}$	lower bound of conjoint utility for r^{th} competitive product in s^{th} segment
$U_{cp_r,\text{upper_bound},s}$	upper bound of conjoint utility for r^{th} competitive product in s^{th} segment
u_{cmks}	utility of an individual c for profile m in choice set k , given that this individual belongs to segment s
u_U	upper bound of attribute utility estimate
u_L	lower bound of attribute utility estimate
VC_H	variable cost of H^{th} variant in a product line
W	production capacity
\mathbf{x}	vector of design variables
x_g	design variable for gear reduction set
x_m	design variable for motor
x_r	design variable for gear ratio
\mathbf{y}_{mk}	vector of product attributes of profile m in k^{th} choice set
\mathbf{y}_{mmk}	vector of product attributes of profile mm in k^{th} choice set
z	standard error of the point estimate of the conjoint part-worth
z_1	standard error of the point estimate of the conjoint part-worth at price level P_1
z_2	standard error of the point estimate of the conjoint part-worth at price level P_2
z_{12}	covariance of the two conjoint part-worth estimates of price levels
α	feasibility robustness penalty coefficient
β	objective robustness penalty coefficient
β_{sy}	vector of parameter coefficients weighting product attribute level
β_{sp}	vector of parameter coefficients for prices

Δf_i	variation of the i^{th} objective function
Δf_i^W	maximum observed variation of i^{th} objective function from nominal value
Δf_i^D	acceptable (known) variation of i^{th} objective function from nominal value
ε_{mk}	random component of the utility for profile m in k^{th} choice set
λ_{rt}	discount factor
ρ_{rt}	commonality significance factor
π	profit

CHAPTER 1

INTRODUCTION

1.1. MOTIVATION AND OBJECTIVE

The existing research has shown that an effective integration of engineering design and marketing domains can have a positive impact in the success and performance of a product design (e.g., [Griffin and Hauser, 1992]). It is no surprise that the specifics of such an integration have been the focus of research in the last decade or so, e.g., the quality function deployment approach [Griffin and Hauser, 1993], decision-based design [Hazelrigg, 1998], and integration of customer requirements into product design [Bailetti and Litva, 1995].

The key characteristic in the integration of engineering design and marketing domains is that it provides the means for consideration of a large number of factors, some of which are specific and unique to one domain and some are common across both domains. Examples of such factors are ambient temperature, power source voltage, brand, and price. Often, some of these factors are interrelated and affect the decisions that fall under either of design or marketing domains. An effective and efficient method for considering and integrating these factors is critical for reducing the time and cost in product design.

An important step in the design domain is the generation of a set of product design alternatives. A product design alternative is generated by identifying its factors and features in both design and marketing domains. For instance, a corded power tool design can be generated partly by a set of engineering design factors (later referred to as

design variables), e.g., choice of motor type, gear type, gear ratio, switch type. There are also other factors that more fully define the product design from a marketing perspective, e.g., choice of brand, price. A modification of such factors in either domain can result in a different product design alternative. In this regard, several researchers have developed methods such as combinatorial permutation of design characteristics and multi-objective design optimization methods for generation of design alternatives (e.g., [Fuhita and Ishii, 1997] [Shi and Schmidt, 2003] [Deb and Jain, 2003]). However, these and other similar design generation methods are developed based on engineering design aspects of a product and neglect the marketing aspects of the problem.

More importantly, a product design process has to accommodate uncontrollable variations (or uncertainties) that occur due to different operation conditions and usage situations of a product that affect its performance (e.g., [McAllister and Simpson, 2003]). For a power tool such as a grinder, the uncontrollable variations in power source voltage, current, and ambient temperature are examples of uncontrollable parameter variations in operation conditions. The changes in the load bias (i.e., the force that user imposes on a tool), different application material (e.g., wood or concrete) are examples of uncontrollable parameters in usage situations. All of these are just a few examples of parameters with uncontrollable variations that can play a major role in engineering design performance of a product design. Note that parameters that are the source of uncontrollable variations in design performance can also cause variations in consumer preferences and thus affect marketing success of a product. For instance, power tool products that are very sensitive to voltage variations, or those that malfunction shortly after they are subject to a user's load bias cannot sustain in the marketplace. As a result, it

is quite important to account for the variations in both engineering design attributes (e.g., maximum no-load motor speed, motor temperature) as well as the marketing attributes (e.g., retail price, life of the product). In other words, any new product that is designed should be “robustly” optimum to these variations. A robust optimum design alternative is one that has (i) the best possible (engineering and market) performance under the worst case variations, and (ii) the least possible (or acceptable) variability in its performance under uncontrollable variations of parameters.

Figure 1.1 encapsulates the main elements of the problem that will be considered in this dissertation. In this problem, the product design process is focused on two domains: engineering design and marketing. In the engineering design domain, the focus is on the performance, feasibility, and robustness of a product design alternative. In this regard, a simulation tool is used to obtain and measure the performance and feasibility of a product design alternative. In the marketing domain, the focus is on capturing the customers’ needs and preferences with respect to the performance and features of a product design alternative. The competition has also been accounted for since a product cannot be successful in the market if the characteristics in the competitive products are not taken into account. There is also variability or uncertainty involved in parameters in both design and marketing domain. The robust product design optimization presented in this dissertation is the pivotal element that provides a link between the design and marketing domains and generates a solution for this integrated design and marketing problem. The solution can be in the form of a single product or a family of products hereafter referred to as a product line.

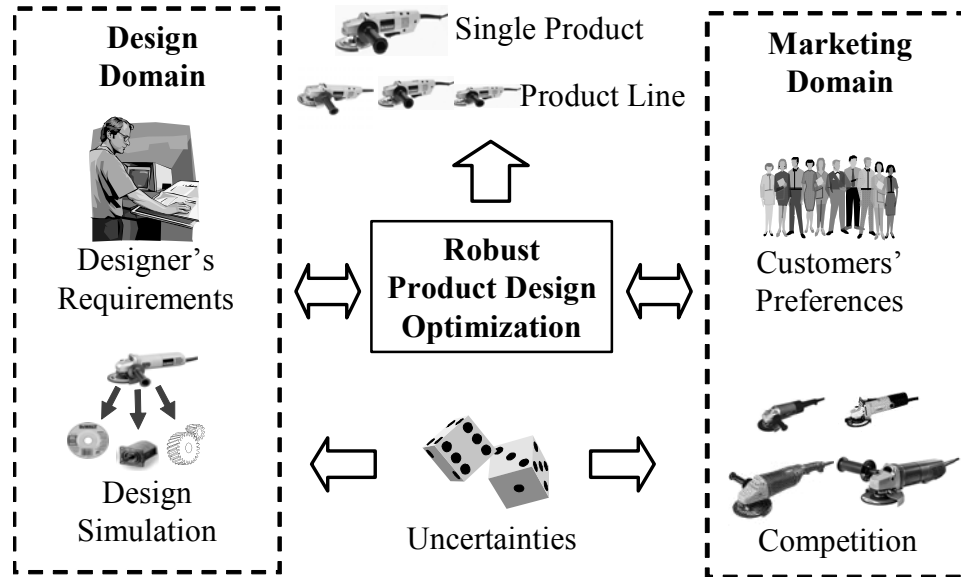


Figure 1.1: Problem Definition: Robust Product Design Optimization

Although the effect of uncontrollable variations of design parameters in performance attributes have been the subject of research investigations for more than a decade (e.g., [Taguchi et al, 1989] [Parkinson et al 1993]), these investigations neglect to consider the effect of variations of design parameters in customer preferences. All of the above-mentioned discussions have direct implications on a single product design problem, where the assumption is that the manufacturer launches only one product to the market. However, the issue of variability and its effects on both design and marketing domains can be extended to include a product line (i.e., a set of products or variants that share the same attributes).

The overall objective of this dissertation is to develop an approach that takes into account variability in a number of design and marketing parameters, and obtain a set of “best” possible single product design or product line design alternatives for launching into the market.

1.2. RESEARCH THRUSTS: PROBLEM DEFINITIONS AND OBJECTIVES

To achieve the above mentioned overall objective, three research thrusts have been identified and pursued in this dissertation. These are: (i) Research Thrust 1: Multi-Objective Robust Design Optimization, (ii) Research Thrust 2: Single Product Robust Optimization, and (iii) Research Thrust 3: Product Line Robust Optimization.

A brief description of the motivation and objective behind each research thrust is given in the next three Sections: Sections 1.2.1 to 1.2.3.

1.2.1. Research Thrust 1: Multi-Objective Robust Optimization

A multi-objective optimization problem is one that has several design objectives that are at least partly conflicting, and has constraints. For such a problem, due to variations in parameters that are not under a designer's control (i.e., uncontrollable parameters), there may exist unacceptable variations in design objectives and/or constraints. This research thrust is aimed at obtaining solutions to a multi-objective optimization problem that are not only feasible and optimal but also their objective and constraint functions are allowed to have acceptable (or minimal) variations caused by uncontrollable parameters.

The objective of this research thrust is to develop some measures that will help assess the robustness of a design alternatives, and to incorporate these measures in a multi-objective design optimization methodology.

1.2.2. Research Thrust 2: Single Product Robust Optimization

A successful product design alternative needs to satisfy the requirements of both engineering design and marketing domains. The engineering design domain deals with factors of a product design which are crucial to its performance and feasibility. The

marketing domain is concerned with customers' inputs and their perceptions with respect to the product. The requirements of both design and marketing domains need to be concurrently accounted for. The advantage of using the marketing information during (and not after) the design is to ensure that potentially desirable design alternatives in the market are not eliminated during the design stage.

The objective of this research thrust is to develop an approach that accounts for variations in design domain, marketing attributes, and customer preferences to generate a set of robust optimum product alternatives that not only satisfy the requirements in the design domain but also show a good performance in the marketing domain where several competitive products are present.

1.2.3. Research Thrust 3: Product Line Robust Optimization

Since customer needs across a market is often diverse, in many cases, introducing a single product may not satisfy the requirements of all segments in the market. In particular, when there are competitive products in the market, customers might switch to the competition resulting in a low market share for a producer. To address this problem, product manufacturers launch more than one variant of a product (or a product line) to the market in order to gain a higher market share. However, a product line comes at a cost and in order to reduce the cost, the variants in the product line are made to share common components. The third research thrust is concerned with a framework that generates a set of robust and optimum product line designs and then select a product line design from this set while taking into account customers input from all market segments and competitive products.

The objective of this research thrust is to develop an approach for robust optimization of product line alternatives that preserves robustness of variants in a set of product line designs, as much as possible, while taking into account customers preferences and market competition.

1.3. ORGANIZATION OF DISSERTATION

The organization of the rest of the dissertation is as follows. Chapter 2 provides definitions and terminology used throughout the dissertation, as well as a review of related work in the literature. In Chapter 3, the robustness measures and two approaches for multi-objective robust design optimization are developed. Next, in Chapter 4, an approach to integrate the engineering design and marketing aspects of a product design within a multi-objective robust optimization scheme is presented. Chapter 5 is devoted to an extension of the approach in Chapter 4 to a product line robust optimization. To demonstrate the applications of the proposed method, an engineering design example as well as a numerical example is provided in Chapter 3. The case study presented in Chapter 4 and Chapter 5 is based on a real-world design simulation and marketing data. Finally, the dissertation is concluded with some remarks as well as discussions on the contributions of the dissertation along with suggestions for future research directions.

CHAPTER 2

DEFINITIONS AND PREVIOUS WORK

2.1. INTRODUCTION

The purpose of this chapter is to provide definitions and terminologies that are used throughout this dissertation. In addition, a review of previous work in robust optimization and those in integration of engineering design with marketing for both single product and product line design is presented.

In Section 2.2, the related definitions and terminology are presented. Next, Section 2.3 is devoted to a literature review for the three research thrusts. In particular, Section 2.3.1 covers the previous work in robust optimization techniques, Section 2.3.2 discusses the literature on single product robust optimization methods, and Section 2.3.3 is devoted to the literature review of robust product line optimization methods. Finally, Section 2.4 provides a summary of the chapter.

2.2. DEFINITIONS AND TERMINOLOGY

The formulation of a multi-objective optimization problem can be written as shown in Eq. (1).

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && f_i(\mathbf{x}, \mathbf{p}) && i = 1, \dots, I \\ & \text{subject to :} && g_j(\mathbf{x}, \mathbf{p}) \leq 0 && j = 1, \dots, J \end{aligned} \tag{2.1}$$

where f_i is the i^{th} objective function, g_j is the j^{th} inequality constraint function, and $\mathbf{x} = (x_1, \dots, x_N)$ is the vector of design variables, $\mathbf{p} = (p_1, \dots, p_V)$ is the vector of design parameters. It is assumed that design variables, \mathbf{x} , can be changed by the optimizer, while

design parameters, \mathbf{p} , are fixed to their nominal value, to generate different design alternatives. Note that in Eq.(2.1), it is generally assumed that some of the \mathbf{p} components have uncontrollable variations with their range of variation presumed to be known. Note also that some of the \mathbf{x} components can have uncontrollable variations too, in which case the set \mathbf{p} also includes these \mathbf{x} components. Some researchers prefer to differentiate between variations in design variables and variations in design parameters, the so-called type-1 and type-2 variations [Chen et al., 1996] and [Kalsi et al., 2001]. For simplicity, in this dissertation, that distinction is not made.

The following are a few definitions of the concepts and terminology that are used throughout the dissertation.

Design Variable Space: The N -dimensional space whose coordinates are the components of the design variable vector \mathbf{x} . Every point in this space represents a design alternative. In this space, design alternatives can be generated by manipulating design variables. For instance, a particular choice of a motor, gearbox, housing, etc., forms a power tool design alternative.

Design Objective Space: The I -dimensional space whose coordinates are design objective functions (i.e., f_1, \dots, f_I). The performance attributes of a design alternative are evaluated in this space.

Design Parameter Space: The V -dimensional space whose coordinates are the elements of the design parameter vector \mathbf{p} . Corresponding to every design candidate (i.e., a point in the design variable space), there exists a point in the design parameter space. Design parameters affect the design performance attributes (i.e., objective functions and

constraints). In general, the designer does not have control over variations in design parameters.

Design Constraint Space: The J -dimensional space formed by the inequality constraints is referred to as design constraint space.

Design Attributes: The outputs of a design simulation model represent a set of performance attributes that are also called: design attributes. It should be noted that the outputs of the design simulation can be used to form design objective and constraint functions.

Marketing Attributes: The set of product design attributes that are specific to a marketing study is called marketing attributes. It should be noted that purely marketing attributes do not play a role in engineering design performance of a product.

Common Attributes: These are a set of attributes that are common in both engineering design and marketing domains. Weight of a product is an example of a common attribute.

Design alternatives: The collection of design variables (x_i s), when fixed to a certain value or level, forms a design alternative. In other words, each design alternative can be represented by a vector of design variables.

Product Alternatives: Each design alternative can be enumerated over marketing attribute levels, and generate several product alternatives. For instance, a single design alternative of a corded power tool can be offered at several different price (which is a marketing attribute) levels, each representing a product alternative. Note that, there is a difference between a product alternative and a design alternative.

Product Line: A product line (or product family) refers to a set of product alternatives that have the same basic function, but each alternative has a different combination of attribute levels. Each alternative within a product line is referred to as a variant.

Multi-objective Dominance: A product design alternative multi-objectively dominates another design alternative, if and only if it is strictly dominant (i.e., is better) in terms of at least one objective function and at the same time not inferior (i.e., is not worse) in terms of remaining objective functions. Figure 2.1 depicts a two-dimensional objective space in which both objectives are minimized. Design alternative A dominates design alternative B (i.e., A is better than B in terms of both objective functions). Also, design A dominates design C (i.e., A is better in terms of f_1 , and is not worse in term of f_2). However, design A and design D are non-dominated with respect to one another. The shaded region shows the region where all designs in that region are dominated by design A.

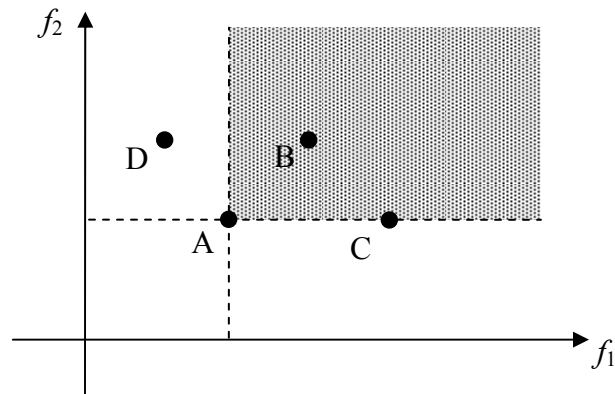


Figure 2.1: Multi-objective dominance

Tradeoff Set: A set of design alternatives (i.e., points in objective space) forms a tradeoff set if all of the points are non-dominated with respect to each other. For instance, in Figure 2.1 designs B and C form a tradeoff set.

Pareto Set, Pareto Frontier: The design alternatives that are not dominated (or are non-dominated) by any other design point in the feasible region (i.e., set of all feasible design alternatives) form a set which is called a Pareto set. The plot of Pareto set in the objective space is referred to as Pareto frontier. The feasible region of a two dimensional minimization problem along with the corresponding Pareto frontier is shown in Figure 2.2.

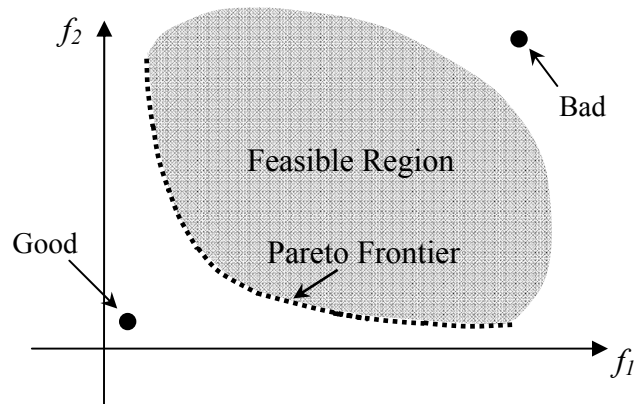


Figure 2.2: The design objective space

Dominance Number: The dominance number for each design alternative is defined as the number of design alternatives that dominates it in the objective space. The lower the magnitude of the dominance number, the better the design is. As an example, the dominance number of Pareto design alternatives is zero.

Good and Bad Reference Points: A designer can provide a good (i.e., target) value as well as a bad value for each objective function. These good and bad points are an estimate of the ideal and nadir points, respectively [Miettinen, 1999]. These values are used to normalize the objective and constraint functions so that they have the same order of magnitude. They are also used as a reference in the proposed robust optimization method.

An example of these reference points, i.e., the good and bad points, in a two-objective minimization problem is shown in Figure 2.2.

Normalization: The design objectives are normalized to have the same order of magnitude. To do the normalization, it is assumed that for each objective the designer provides a target value f_i^g (an estimate of a desired target or good design) and a bad value f_i^b (an estimate of an undesired design). The normalization is shown in Eq. (2.2) with the quantity f_i^N representing the normalized function value.

$$f_i^N = \frac{f_i - f_i^b}{f_i^g - f_i^b}, \quad i = 1, \dots, I \quad (2.2)$$

Objective Robustness: The objective robustness is a property of a design alternative whose objective functions (i.e., performance attributes) are “insensitive” to variations caused by uncontrollable design parameters. In other words, the objective function values for a design that is objectively robust show minimum (or limited) change in their value under uncontrollable parameters’ variations.

Feasibility Robustness: The feasibility robustness is a property of a design alternative whose inequality constraints are always satisfied regardless of the variations in uncontrollable design parameters.

Robust Design: It refers to a design alternative that has both objective robustness and feasibility robustness.

Nominal Pareto Set: It is the Pareto set of a multi-objective optimization problem where the design parameters are fixed at their nominal values.

Robust Pareto Set: A trade-off set whose elements are non-dominated in the objective space and also possess both objective robustness and feasibility robustness.

2.3. OVERVIEW OF PREVIOUS WORK

Robust design optimization methods have become very popular and many researchers have investigated the effect of changes in uncontrollable design parameters in the context of engineering design (e.g., [Taguchi et al., 1989] [Parkinson et al., 1993] [Badhrinath and Rao, 1994] [Chen and Yuan, 1999] [Gunawan and Azarm, 2005]). The performance and robustness of a product design plays an important role in its success in the marketplace too. The success requirement of a product in the market has also triggered the need for incorporation of customers' inputs into the design process. In that regard, a number of design-marketing integration schemes have been introduced in the literature (e.g., [Urban and Hauser, 1980] [Li and Azarm, 2000] [Michalek et al., 2005]). Such approaches are introduced for both single product design (e.g., [Li and Azarm, 2000] [Chen and Yuan, 1999]) and product line design (e.g., [Li and Azarm, 2002] [Simpson, 2003]) problems. In particular, the approach introduced by Li and Azarm [Li and Azarm, 2000] addresses the design performance and marketing performance of a product in two separate and sequential stages, and that the effect of uncontrollable design parameters to the marketing performance of a product has not been addressed. Other approaches (e.g., [Michalek et al., 2005] [Simpson, 2003]) have not addressed the effect of uncontrollable design parameters (i.e., engineering design domain) in the overall performance of a product design, and particularly ignored the marketing performance (i.e., marketing domain) of a product.

In the next three subsections, a detailed review of literature for robust optimization techniques, single product robust optimization, and product line robust optimization are provided.

2.3.1. Literature Review of Robust Optimization Techniques

Most the existing robust optimization techniques are exclusively focused on single objective design optimization problems (e.g., [Taguchi et al, 1989] [Sundaresan et al, 1992] [Parkinson et al, 1993] [Badhrinath and Rao 1994] [Chen and Yuan, 1999]). The work by Taguchi [Taguchi, 1978] is perhaps one of the earliest publications that address the robustness issue. Taguchi [Taguchi et al., 1989] defined robustness as: “The state where the technology, product, or process performance is minimally sensitive to factors causing variability (either in manufacturing or in the user’s environment) and aging at the lowest manufacturing cost”. In an attempt to categorize robustness concepts, the paper by [Parkinson et al, 1993] has classified robustness for a design alternative to two classes: 1) “feasibility robustness” that implies maintaining constraint satisfaction under the uncontrollable parameter variations, and 2) “objective robustness” that implies maintaining least objective function sensitivity under the uncontrollable parameter variations. The majority of the robust optimization methods in the literature are either probabilistic (e.g., [Chen and Yuan, 1999] [DeLaurentis and Mavris, 2000]) or deterministic (e.g., [Su and Renaud, 1997] [Roy and Parmee, 1996] [Zhu and Ting, 2001]). Some of the deterministic methods obtain a robust optimum design by computing its sensitivity using first-order derivative of design attributes and then incorporate these measures when optimizing the design (e.g., [Sundaresan et al., 1992] [Badhrinath and Rao, 1994]). Probabilistic methods, on the other hand, use statistical concepts and techniques to estimate performance (i.e., objective functions) and/or feasibility sensitivity [Parkinson et al., 1993] [Du et al., 2004] of a design and then try to obtain an optimized solution that has the least amount of sensitivity with respect to the variations. One of the assumptions in probabilistic methods is that the probability distribution of uncontrollable

parameters is known or can be obtained upfront (e.g., [Chen and Yuan, 1999] [Choi and Youn, 2002] [Jung and Lee, 2002] [Teich, 2001] [Hughes, 2001]). The reliability-based design optimization (RBDO) methods are part of probabilistic methods that focus on the feasibility of a design under variation (e.g., [Youn et al., 2004]). Overall, the probabilistic methods such as RBDO tend to be more effective when the probability distribution function for uncontrollable parameters is available. In the absence of the probability distributions, possibility-based design optimization (e.g., [Choi et al., 2004]) or deterministic methods can be used. Unfortunately, a major difficulty in applying probabilistic methods is that the probability distribution for parameter variations may not be available or if available, it may not be valid [Haimes, 1998]. Furthermore, probabilistic methods for robust optimization often use the expected value of the objective functions or constraints which could lead to misleading results and interpretations [Haimes, 1998]. On the other hand, the main shortcoming of current deterministic methods is that the approximations used in these methods (e.g., [Yu and Ishii, 1989]) are generally valid only for a small range around the nominal or they are not applicable when the objective/constraint functions have discontinuity with respect to design parameters. One of the deterministic methods that did not have the limitations of approximation of functions within a small region of the nominal was recently developed [Gunawan and Azarm, 2005]. However, the approach by [Gunawan and Azarm, 2005] was based on the assumption that the objective and/or constraint functions are continuous but not necessarily differentiable with respect to uncontrollable parameters. The deterministic approach by [Li et al., 2005] does not have some of the limitations (e.g., continuity of objective functions / constraints with respect to design parameters) of the

approach by [Gunawan and Azarm, 2005]. However, both approaches: [Gunawan and Azarm, 2005] and [Li et al., 2005], require a presumed acceptable range of variations for objective functions for which solution existence cannot be guaranteed. Among the other deterministic approaches, a few methods take design performance at the worst case scenario of design parameters into consideration (e.g., [Shimizu et al, 1997] [Kouvelis and Yu, 1997]). In particular, Kouvelis and Yu have used a min-max approach to address robustness for single objective problems. Kouvelis and Yu's single objective robust optimization approach has been extended to problems with multiple objectives using a min-max approach [Shimizu et al, 1997]. While a min-max approach seems to address multi-objective problems, relying only on a min-max approach can yield very conservative results

Two deterministic robust optimization approaches will be proposed and implemented in this dissertation. The first approach (Approach 1) was inspired by the method of Kouvelis and Yu [Kouvelis and Yu, 1997]. However, the first approach is applicable to problems with multiple objectives. Furthermore, the first approach is guaranteed to obtain robust solutions that have the best multi-objective performance under the worst case scenario and at the same time show minimum variability in their objective function values. The proposed second robust optimization approach (Approach 2) is tailored for problems where the variability of some objective functions needs to be limited (and not minimized). Both approaches, as developed in this dissertation, do not need the probability distribution of the uncontrollable parameters, and can be applied to problems in which the range of uncontrollable parameter variations is large.

2.3.2. Literature Review of Single Product Robust Optimization

There are several published methodologies in the engineering design literature for generation of design alternatives. Among those, the design concept generation methods based on grammar rules (e.g., [Schmidt and Cagan, 1997] [Hsu and Woon, 1998] [Schmidt et al., 2005] [Jin et al., 2005]) or functions (e.g., [Pahl and Beitz, 1984] [Hirtz et al., 2002]) are widely used in the literature. Another class of design generation methods, particularly applicable at the detailed design stage, are based on design optimization with multiple objectives (e.g., [Narayanan and Azarm, 1999] [Deb and Jain, 2003] [McAllister et al., 2005]), and are those that use permutations over multiple levels of attributes (e.g., [Li and Azarm, 2000]). Majority of the design optimization methods for product alternative generation address the optimality and feasibility of the generated alternatives. An important drawback of the majority of the above-mentioned approaches is that they are developed based on design factors or “design attributes” and may overlook other major factors such as “marketing attributes” that impact customer preferences and eventual success of a product.

Extant research in marketing and management science literature has shown that an effective integration of engineering design and marketing factors can have positive impact on product development cycle time [Griffin, 1997] [Sherman et al., 2000] [Urban et al., 1997], project performance [Griffin and Hauser, 1992] [Olson et al., 2001], and overall company and market performance [Gemser and leenders, 2001] [Griffin and Hauser, 1996] [Tatikonda and Montoya-Weiss, 2001]. The specifics of integrated design-marketing approaches have been the focus of research in the last decade: e.g., quality function deployment approach [Griffin, 1992] [Griffin and Hauser, 1993] [Hauser and Clausing, 1988], lead user analysis [Urban and Von Hippel, 1988], and integrating

customer requirements into product designs [Bailetti and Litva, 1995] [Urban et al., 1997] have received particular attention. Nevertheless, most of the above-mentioned approaches in the literature are focused on the effect of market characteristics [Urban and Hauser, 1980] or customer-oriented attributes [Wassenaar et al., 2005] of the products.

In summary, the previous methods are focused on either engineering design factors or marketing aspects of a product, and the interaction of design factors with marketing factors in the presence of uncontrollable parameters in both domains has not been addressed fully in the literature. There are a few reported integrated design-marketing approaches that have limitations. For instance, the approach developed by [Li and Azarm, 2000] handles the design and marketing domains in two separate and sequential stages. On the other hand among other works on the integrated design and marketing optimization (e.g., [Michalek et al., 2005]) the issue of the design performance variability (design robustness) and its effect in the marketing performance of the product has not been explored.

2.3.3. Literature Review of Product Line Robust Optimization

The problem of designing successful product lines has received particular attention in the engineering, marketing and management science literature for the last two decades (e.g., [Green and Kreiger, 1985], [Chen and Hausman, 2000], [Simpson et al, 2001], [Balakrishnan and Gupta, 2004]). Manufacturers often launch variants of a product (or a product line) to meet customer requirements in different segments of a market. As [Pine, 1993] writes, “The customers can no longer be lumped together into a huge homogenous market, but are individuals whose individual wants and needs can be

ascertained and fulfilled”. Identifying the needs of customers in different segments of the market is vital to the success of a product line in the market.

In the marketing and management science literature, the focus of the research has been to obtain a product line that not only satisfies heterogeneous customers’ preferences, but also achieves an economy of scale (i.e., reduction per unit cost by mass production) (e.g., [Green and Kreiger, 1985] [McBride and Zufryden, 1988] [Ramdas and Sawhney, 2001]). Also several methods in the literature have discussed the issue of market segmentation. For instance, [Desai et al, 2001] have characterized the market by two segments (high and low valuation) and assumed the manufacturer produces two products each aimed at each segment. Assuming that the market is structured (e.g., [Kannan and Wright, 1991]), the product manufacturers can use the platform-based designs to create the product lines that have sufficient variety for different segments, while maintaining lower cost within their manufacturing processes. For instance, Black and Decker have built a line of products around a scalable motor platform [Meyer and Lehnerd, 1997]. Furthermore, these designs share components that reduces the overall manufacturing costs across the product line ([Ramdas and Sawhney, 2001], [Fisher et al, 1999], [Gupta and Krishnan, 1999]). Most of the research in this area has been focused on the revenue maximization aspect of the product line design. The cost aspect of the product line design has been simplified to fixed cost only (e.g. [Dobson and Kalish, 1988] and [McBride and Zufryden, 1988]). One exception is the work of Ramdas and Sawhney. [Ramdas and Sawhney, 2001]. In particular Ramadas and Sawhney [Ramadas and Sawhney, 2001] have developed a model to account for the fact that firms can save money on development costs when components can be shared by variants in a product line. There

are many factors that are involved in determining the overall production cost of a product design, some of which have been explored by researchers in both engineering design and management areas (e.g., [Taylor, G.D., 1997] [Park and Simpson, 2003]). The commonality among the variants in a product line and its effect on the overall production cost of the product family has also been investigated by several researchers (e.g., [Morgan et al., 2001], [Park and Simpson, 2005]). A comparison between several commonality measures can be found in [Thevenot and Simpson, 2004].

Another major focus of the marketing and management science literature has been on the optimization-based approaches to obtain the optimal product line. In particular, these approaches are based on two classes of problem formulations. There are either maximization of producer's profit (i.e., sellers return) or customer utility (i.e., buyer's welfare) (e.g., [Green and Kreiger, 1985], [McBride and Zufryden, 1988]).

In the engineering design literature on product line design, the focus has been on cost reduction due to commonality among the variants in a product line and platform management (e.g., [Morgan et al., 2001], [Park and Simpson, 2005]). In that regards many measures for degree of commonality for product families (or lines) have been developed (e.g., [Martin and Ishii, 1997] [Kota et al., 2000]), and Thevenot et al. have compared many of these measures [Thevenot et al., 2004]. Recently, many researchers in engineering design disciplines have investigated this problem from a profit maximizing perspective (e.g., [Li and Azarm, 2002] [Michalek et al., 2005]).

The researchers in both engineering design and marketing and management science disciplines have used a wide variety of optimization methods to obtain an optimal product line. For instance, the method of integer programming [McBride and Zufryden,

1988], and genetic algorithms (e.g., [Alexouda and Paparrizos, 2001] [D'Souza and Simpson, 2003]) are among the frequently used approaches to find an optimal product line. Due to the complexity of product line optimization problems, several researchers have developed heuristics to obtain near optimal solutions. Among these, the dynamic programming heuristic approach [Kohli and Sukumar, 1990], beam heuristic search approach (e.g., [Nair et al, 1995]), [Thakur et al., 2000]) are used to obtain a product line that maximizes either the seller's return or buyers' welfare. However, the heuristic approaches may not converge to true optimum solution.

Finally a number of researchers have taken steps to develop integrated approaches where the criteria for both engineering design and marketing disciplines are taken into account. Among them, the approach developed by [Li and Azarm, 2002] performs a product line design generation (taking only engineering design objectives into account) and evaluation (taking marketing and business goals into account). As a result of the separation of engineering design objectives and marketing attributes, some of the promising product line candidates (from the marketing perspective) may be eliminated during the design optimization. In another integrated approach [D'Souza and Simpson, 2003], the design generation and evaluation stages are performed simultaneously using a genetic algorithm technique to obtain the final product. However, the criterion used for the fitness assignment is entirely based on the manufacturing cost. In addition, the approach by [Morgan, et al., 2001] has some simplifications such as hypothesizing a market with only one competitive product.

One of the important issues that has not been addressed in the both engineering design and marketing and management science literature is the effect of variations in

uncontrollable design parameters to both engineering performance attributes of the variants in the product line as well as the customer utilities. Furthermore, in most of the engineering-based approaches the relationship of the market segmentation to the product line alternative generation has not been addressed. Here, an integrated design-marketing approach will be presented that takes the issues of variations in both design and marketing domains. The approach is divided into two stages to ease the computational complexity of the problem. However, unlike some of the previous work (e.g., [Li and Azarm, 2002]) any elimination scheme in the approach in this dissertation is geared to remove product designs that are unacceptable for either of the two engineering design and marketing disciplines. The details of the proposed approach are presented in Chapter 5 of this dissertation.

2.4. SUMMARY

In this chapter, the definitions and terminology used throughout this dissertation are presented. A literature review for each of the research thrusts is presented. The shortcomings of the previous works in each of the proposed research thrusts are summarized as in the following.

- The majority of methods of robust optimization in the literature handle single objective optimization problems. Those that account for multiple objectives either require the probability distribution of uncontrollable design parameters or majority of them (i.e., deterministic approaches) are based on the assumption that the objective functions and/or constraints are continuous and differentiable with respect to uncontrollable design parameters. Unlike some of the existing methods in the

literature, both approaches in this dissertation do not have the above-mentioned limitations. In particular, Approach 1 is guaranteed to obtain robust optimal solutions that show best performance under worst case scenario, with minimum variability in objective function values, and Approach 2 works for problems where the designer requires an acceptable range of variation for each objective function, and obtains a robust Pareto set. However, there is a computational complexity issue that is associated with each approach.

- The majority of the approaches for integration of engineering design and marketing in the literature are focused on engineering design aspects of a product design and the marketing aspects are either not considered or simplified. The methods in the marketing literature do not address the problem beyond the marketing performance of a product and the possible implications of design domain and the interactions between design and marketing domains have not been accounted for. On the other hand, the extant literature on integration of engineering design optimization and marketing (e.g., [Michalek et al., 2005]) has not yet addressed the issue of design robustness and its impact on the customers' choice of a product design. The approach in this dissertation integrates the engineering design requirements (including design robustness) and marketing implications of each within a framework that generates an optimal and robust set of product design alternatives.
- One of the important issues that has not been addressed in the literature is the effect of variations in uncontrollable design parameters to engineering performance attributes of the variants in the product line as well as the customer utilities. In the engineering-based approaches, the relationship of the market segmentation to the

product line alternative generation has not been addressed. To overcome these shortcomings, a robust product line optimization approach is developed that generates an optimal product line that is desirable in the engineering design as well as the marketing domains and accounts for variability in both domains.

In the next chapter, the first research thrust of this dissertation, Multi-Objective Robust Optimization is presented.

CHAPTER 3

MULTI-OBJECTIVE ROBUST OPTIMIZATION

3.1. INTRODUCTION

Engineering design optimization problems in general have parameters with variations (due to noise or uncertainty) that a designer cannot control. As a result of such variations, the performance (the value of objective functions) of an optimized design solution might degrade significantly and/or its feasibility might be violated. Uncontrollable design parameter variations can occur, for instance, in material properties such as density and modulus of elasticity, in part dimensions due to manufacturing errors, and in usage conditions such as ambient temperature and humidity.

The purpose of this chapter is to introduce two deterministic approaches for robust multi-objective optimization based on a sensitivity estimation of a design alternative in the objective and/or constraint space. The reason to introduce two robust optimization approaches in this chapter is that there are different types of design optimization problems where a unique approach may not be as effective to obtain robustly optimal solutions. As will be shown in this chapter, both approaches can handle problems in which the objective and/or constraint functions are discontinuous with respect to uncontrollable design parameters and/or variables. Moreover, both of the approaches do not require a presumed probability distribution for uncontrollable variations in parameters and are applicable when parameter variations are large. As will be shown, based on a feasibility robustness measure, the approach can also obtain design solutions that are guaranteed to be feasible when uncontrollable variations in parameters

occur. Furthermore, based on objective robustness measures in either of the approaches, the obtained solutions have limited (or minimal) performance variability in the objective space.

The organization of this chapter is as follows. An overview of the concepts used in the first robust optimization approach is provided in Section 3.2. Then in Section 3.3 the details of the robust measures, namely feasibility robustness and multi-objective robustness are given. Next, the first robust optimization method is presented. Section 3.4 is devoted to an alternative robust optimization method. Depending upon the problem and the designer's requirements, either of the two methods can be used. Both of the approaches are demonstrated by an application of both methods to a numerical and an engineering design example in Section 3.5. A comparison between the two robust optimization methods is given in Section 3.6. Finally, this chapter is concluded with a summary in Section 3.7.

3.2. ROBUSTNESS MEASURES

In this section, two alternative robustness measures are introduced that can be used to capture either the sensitivity (i.e., variability) in performance or the worst case scenario performance for a design alternative. These measures are later used in the first robustness assessment approach for evaluation of every design alternative during robust optimization. The reason for developing two different methods for objective robustness assessment is that each of these methods is suitable for a different problem setting and designer's inputs. In particular, one group of problems may require minimum variability for the objective function values (e.g., precision devices with minimum tolerance) and at

the same time the worst case performance may have to be taken into account. In such a case, it is assumed that a designer can provide a target and a bad value for each objective function. The robust optimization approach 1 can handle these problems. In another group of design problems the variability of some of the objective functions need to be limited (and not minimized), and a designer can provide the specific limits for the variability of each objective function. The robust optimization approach 2 is suitable for this class of design problems. The details of these methods and their differences are given later in this chapter.

But first it is necessary to describe how the sensitivity of each design alternative in terms of the objective functions and constraints can be obtained, as discussed next.

3.2.1. Sensitivity Region

The performance attributes (i.e., objective and constraint functions) of design alternatives, as discussed before in Eq. (2.1), are generally sensitive to variations due to uncontrollable design parameters (i.e., \mathbf{p}). In order to capture the sensitivity it is necessary to map every point in the design parameter space, as the uncontrollable parameters change for a design alternative, to a corresponding point in the design objective and/or constraint space. In other words, an objective sensitivity region for each design alternative can be obtained by a mapping from the parameter space, p_2 vs. p_1 space, onto the design objective space, f_2 vs. f_1 space, as shown in Figure 3.1. Similarly, a constraint sensitivity region can be obtained by a mapping from the parameter space to constraint space. The mappings can be performed using design simulation software. Figure 3.1 shows an example of such a mapping for a design alternative A. The solid point in the parameter space (left side of Figure 3.1) corresponds to the nominal values of

each uncontrollable design parameter. The solid point in the objective space (right side of Figure 3.1) corresponds to the design alternative's objective values at the nominal levels for uncontrollable design parameters. The box (or the hyper-box in more than 3 dimensions) in the design parameter space represents the known ranges of variation for design parameters. Every point in this box is mapped onto a corresponding point in the objective space. As shown in the right side of Figure 3.1, the region that is formed by these mappings is called a sensitivity region

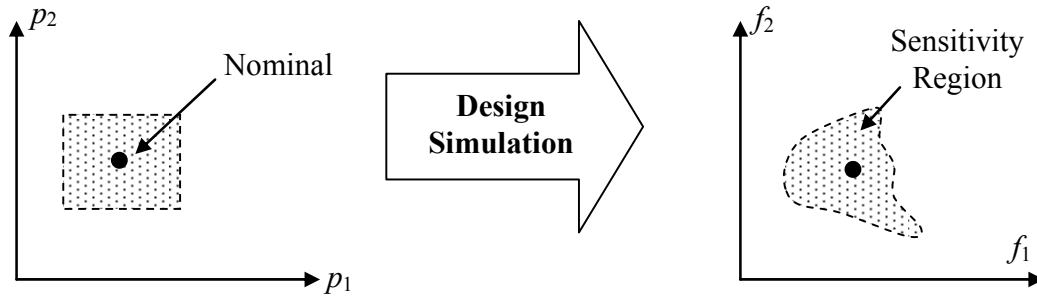


Figure 3.1: Sensitivity region

Again, it should be noted that the region shown on the right side of Figure 3.1 only represents the sensitivity of a design alternative in the objective space. However, as it will be shown in Section 3.2.3, the feasibility of each design alternative is also sensitive to the variations in uncontrollable design parameters. Hence, in a similar fashion, the sensitivity region for a design can be obtained in the design constraint space where the constraint functions are used to perform the mappings from design parameter space.

Briefly, three robustness measures are derived in the following subsections for approach 1. The first two measures are for multi-objective robustness (i.e., “worst case scenario distance from target” and “multi-objective variability”). The third measure is for feasibility robustness, which is used in both approach 1 and approach 2.

3.2.2. Objective Robustness Measure for Approach 1- Worst Case Scenario Distance from Target

One of the important issues in robustness evaluation of a design alternative is to examine how a design performs under a worst case scenario of design parameters. In single objective design optimization problems obtaining the worst case scenario is straightforward. For instance in a problem where the objective is to minimize the stress in a component, the worst case scenario for a design alternative can be obtained for a set of uncontrollable parameters (e.g., loading condition, ambient temperature, manufacturing tolerance, etc) so that it gives the highest level of stress. In multi-objective optimization, however, the worst case scenario must be obtained considering all objective functions. One possible method to determine the worst case, as proposed here, is based on a distance metric. In this method, the designer selects two levels for each objective function; one desirable or good level and one undesirable or bad level. The corresponding point in the objective space to the good level of each objective function is called the target point. In a similar fashion the corresponding level for the bad level for each objective function in the objective space is called the bad point. The target and bad points are shown in Figure 3.2(a).

The proposed first metric is intended to capture the worst case scenario performance of each design based on the distance of the farthest point of its sensitivity region to a target point. In other words, the Worst Case Scenario Distance (*WCSD*) from target is obtained based on how far a design point is from a target in the objective space under worst case values of parameters, see Figure 3.2. Eq. (3.1) calculates, based on an L_q norm (see [Miettinen, 1999] for a definition of this norm). The worst case parameter value is shown by \mathbf{p}^W (See Figure 3.2), for which a design \mathbf{x} is farthest from a target:

$$WCSD = \max_{\mathbf{p}} \left[\sum_{i=1}^M \left| \frac{f_i(\mathbf{x}, \mathbf{p}) - f_i^g}{f_i^b - f_i^g} \right|^q \right]^{1/q} \quad (3.1)$$

where $\mathbf{p} \in [\mathbf{p}_L, \mathbf{p}_U]$, and \mathbf{p}_L and \mathbf{p}_U are the vectors of known lower and upper bounds of uncontrollable design parameters. An appropriate global optimization method could be used to find the *WCSD* from the target for each design point.

3.2.3. Objective Robustness Measure for Approach 1- Multi-Objective Variability

To obtain a measure of multi-objective variability, first the *WCSD* value is obtained by Eq. (3.1) and then the maximization in Eq. (3.1) is converted to a minimization form to find the Best Case Scenario Distance (*BCSD*) (closest point to the target, as shown in Figure 3.2). Multi-objective variability is defined as the distance between the *WCSD* and *BCSD* points, as shown by the dashed line in Figure 3.2. The measure for variability has several components as follows. Firstly, the metric must account for the wideness of the sensitivity region. A distance metric seems to fit this quite naturally. Secondly, the variability metric must account for the location of the target (and the bad point for the scaling purpose) so that the variability could represent the change in performance in terms of all objective function values. Finally, the metric must account for all objective function values at once in order to reduce the computational cost of the method.

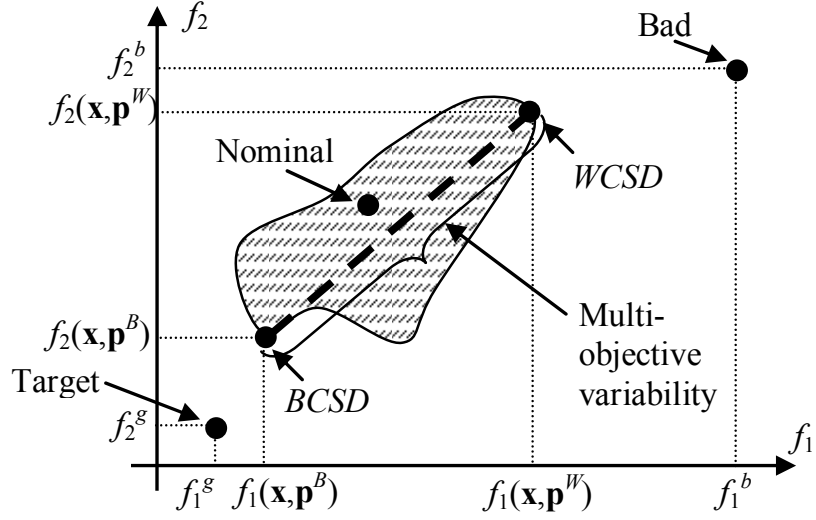


Figure 3.2: Multi-objective variability using L_2 norm

The mathematical formulation for the multi-objective variability measure is given in Eq. (3.2)

$$Variability = \left[\sum_{i=1}^M \left| \frac{f_i(\mathbf{x}, \mathbf{p}^W) - f_i(\mathbf{x}, \mathbf{p}^B)}{f_i^b - f_i^g} \right|^q \right]^{1/q} \quad (3.2)$$

where \mathbf{p}^W and \mathbf{p}^B represent the worst case scenario and the best case scenario parameter sets, respectively. Also, $f_i(\mathbf{x}, \mathbf{p}^W)$ and $f_i(\mathbf{x}, \mathbf{p}^B)$ are for the worst case and best case scenarios (or values) of the i^{th} objective function, respectively.

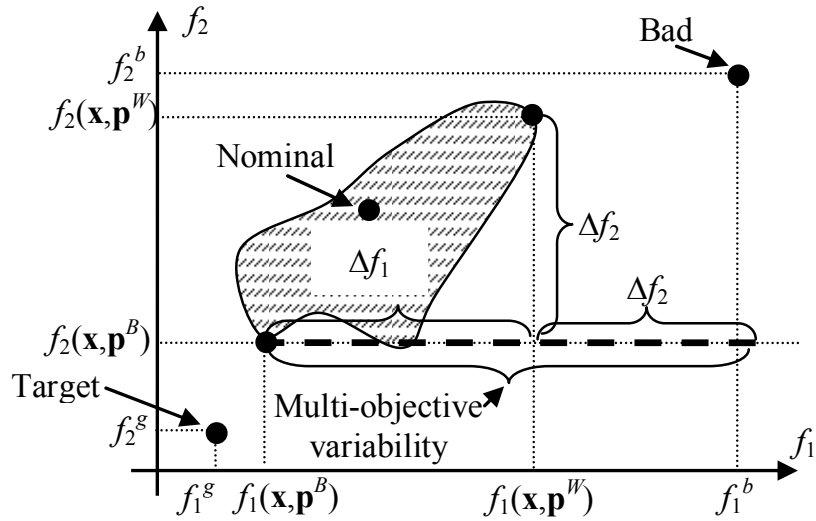


Figure 3.3: Multi-objective variability using L_1 norm

The multi-objective variability can also be obtained using an L_1 norm (i.e., $q = 1$ in Eq. 3.2). It should be noted that the location of the worst case and best case points can be altered using different norms. Figure 3.3 shows the multi-objective variability when the L_1 norm metric is used in Eq. (3.2). Furthermore, it is also shown that the multi-objective variability when an L_∞ norm is used instead, see Figure 3.4.

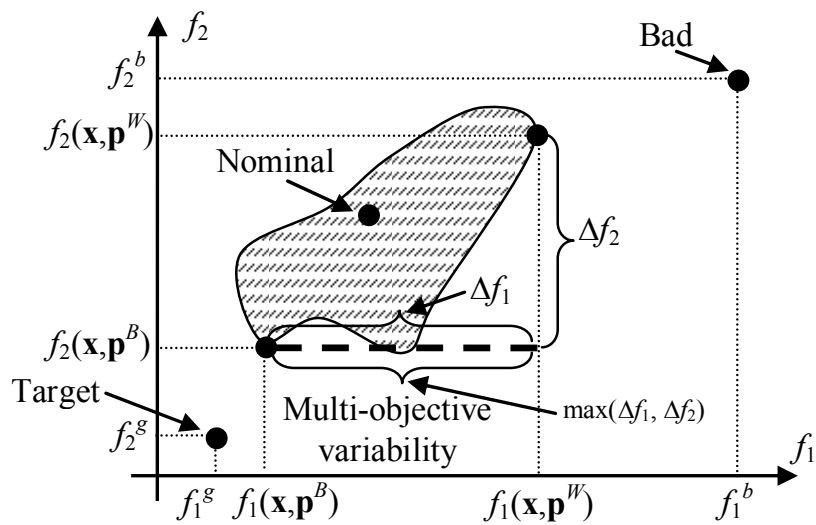


Figure 3.4: Multi-objective variability using L_∞ norm

3.2.4. Feasibility Robustness Measure for Approaches 1 and 2

The variations in uncontrollable design parameters not only affects the objective function values which represent the performance of a design point, but also have a direct impact on the constraint values which ensure feasibility of each design. The main purpose of feasibility robustness is to check if a design maintains its feasibility under parameter variation. To determine if a design is feasibly robust, it is examined whether the inequality constraints are violated under the worst case scenario of parameters, i.e.,

$$\left(\sum_{j=1}^J \left(\max_{\mathbf{p}} [g_j(\mathbf{x}, \mathbf{p}), 0] \right)^q \right)^{\frac{1}{q}} \leq 0 \quad (3.3)$$

subject to : $\mathbf{p}_L \leq \mathbf{p} \leq \mathbf{p}_U$

where g_j is the j^{th} constraint. Figure 3.5 shows the constraint space with two constraint functions g_1 and g_2 . It should be noted again that the above inequality has to be examined by a global optimization method. Two design alternatives and their corresponding constraint sensitivity region are shown. The light shaded area represents the feasible region (generated for nominal uncontrollable parameter values) where both constraints are less than or equal to zero. Any point from the sensitivity region that goes beyond the feasible region makes the design feasibly non-robust.

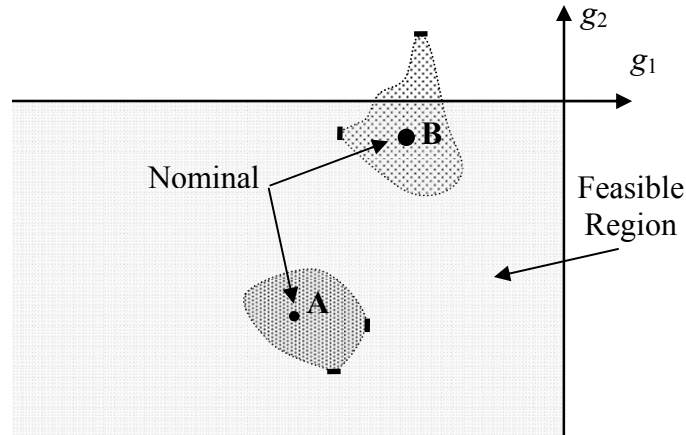


Figure 3.5: Feasibility robustness comparison for two design candidates

Clearly both designs shown in Figure 3.5 do not violate constraints for nominal parameter values (i.e., the solid points) and therefore are (nominally) feasible. However, the sensitivity region of design alternatives **B** is extended outside the feasible region and therefore **B** is not feasiably robust. On the other hand, design **A** is feasiably robust.

3.3. MULTI-OBJECTIVE ROBUST OPTIMIZATION: APPROACH 1

Figure 3.6 encapsulates the approach 1. Beginning from the top block, a design point \mathbf{x} is passed on to the middle level block to check for its feasibility robustness. If it does not pass the feasibility robustness (as verified by the middle block), then it will be eliminated and the next design \mathbf{x} is chosen by an optimizer in the top block and passed on to the middle level block. The selection of the next design is done by the upper level (i.e., main) optimization scheme. If the design \mathbf{x} satisfies the feasibility robustness, it will be passed on to the bottom level block in which the *WCSD* and *Variability* measures are calculated (using Eqs. 3.1 and 3.2) implemented within a global optimization technique

(with respect to \mathbf{p}) and their values for the current design are returned to the top level block. The top level block has the main multi-objective optimization model that creates the set of points that have minimum $WCSD$ and minimum multi-objective variability. Therefore it is expected that the solutions that are obtained using this method perform better under the worst case scenario of uncontrollable design parameters, and at the same time their objective function values are not “sensitive”, based on the measure defined, to the parameter variations

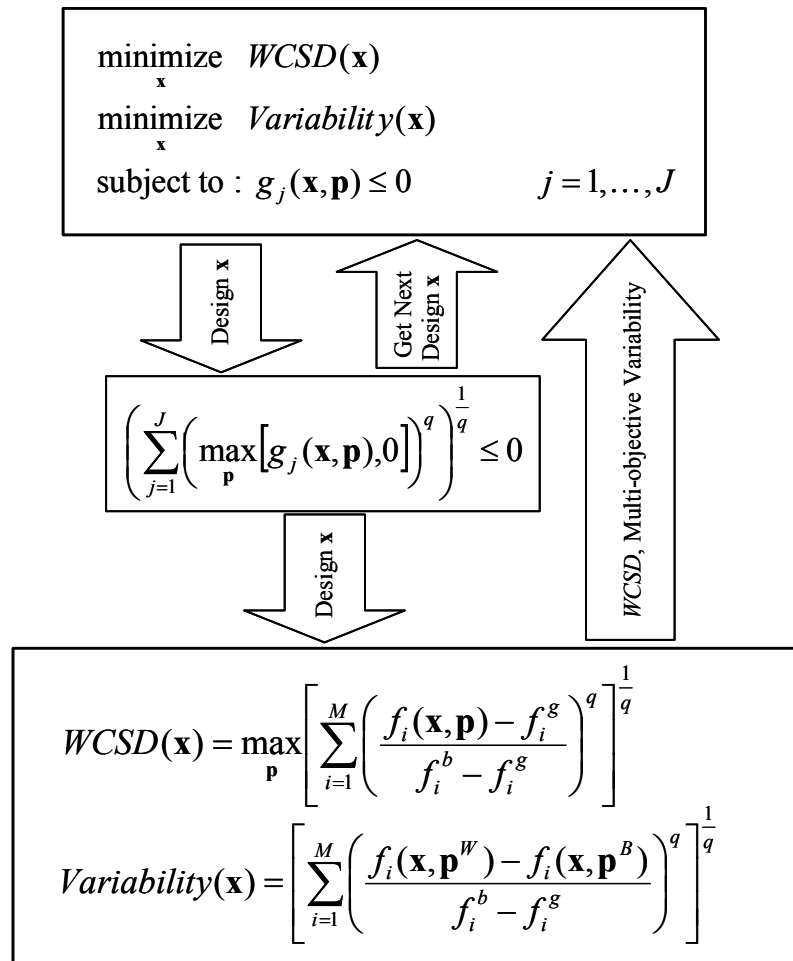


Figure 3.6: Robust multi-objective optimization approach 1

3.3.1. Steps in Robust Optimization for Approach 1

A step by step procedure of the approach described on section 3.3 is shown in Figure 3.7, as described as the following steps.

Step 1: Select initial values for design variables \mathbf{x} . Choose nominal values and ranges of uncontrollable parameters. Select target and bad points in the design objective space.

Step 2: Pass the initial (or current) design variables as well as the values of uncontrollable parameters to the design simulation to compute the objective and constraint functions. The search for the worst case constraint values as well as parameters corresponding to minimum and maximum distance from the target in design objective space is performed (see Eqs. (3.1) and (3.2)).

Step 3: Based on the worst case values of constraints, feasibility robustness for design \mathbf{x} can be examined (see Eq. (3.3)). The procedure eliminates any design \mathbf{x} that violates the feasibility robustness and goes to Step 6. The procedure continues, with any design \mathbf{x} that passes the feasibility robustness, to Step 4.

Step 4: Each feasibly robust design point \mathbf{x} will be evaluated based on $WCSD$ and multi-objective variability robustness measures. Depending on the optimization technique, an improvement of the design is performed.

Step 5: If the optimizer is converged, the procedure will end; otherwise, it goes to Step 6.

Step 6: Select the values for the next set of design variables \mathbf{x} , via the optimizer, and go to Step 2.

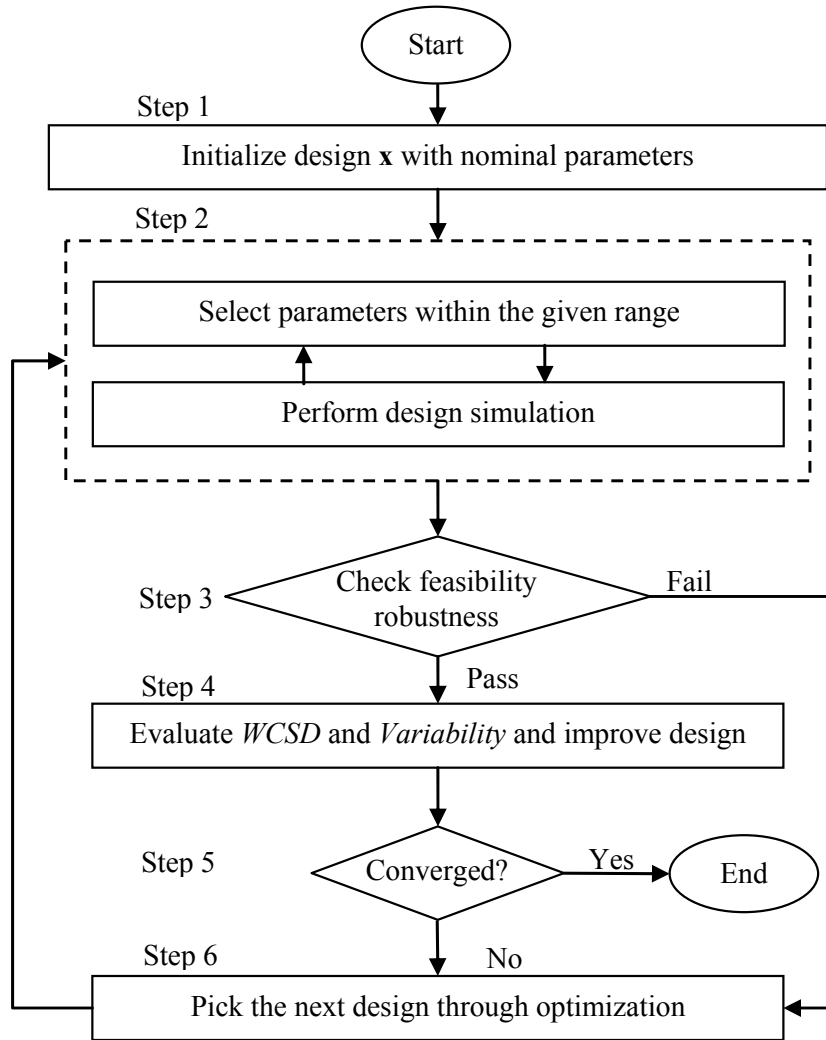


Figure 3.7: The flowchart of robust optimization approach 1

3.4. MEASURES AND APPROACH 2 FOR MULTI-OBJECTIVE ROBUST OPTIMIZATION

While there are many advantages in using the approach 1 with worst case scenario performance and variability, for some engineering problems, the results can become unnecessarily too conservative. Furthermore, in many problems, a designer is interested to limit the amount of variations for certain objectives without necessarily minimizing

them. In other words, approach 1 can potentially converge to designs that have extremely low variability and poor performance in a nominal case of design parameters (see Section 3.6.2.1, for an example). In order to allow for the designer to have control over limiting the variations, approach 2 has been developed to assess the multi-objective robustness of design alternatives. The description of the approach 2 and its details are given in Sections 3.4.1 and 3.4.2. A computational cost comparison of these two robust optimization approaches are given in Section 3.5, followed by two examples in Section 3.6.

3.4.1. Multi-objective Robustness for Approach 2

Approach 2 allows the designer to control the sensitivity of each objective function with respect to uncontrollable design parameters separately. A design is defined to be multi-objectively robust if the variation in each of its objective function values is bounded within a pre-specified acceptable range. In order to formulate the multi-objective robustness, a measure for multi-objective variability needs to be introduced. Here, the multi-objective variability for each design alternative is based on the maximum variation of the objective function values from the nominal values, Δf_i^W :

$$\Delta f_i^W = \max_{\mathbf{p}} |f_i(\mathbf{x}, \mathbf{p}) - f_i(\mathbf{x}, \mathbf{p}_0)| \quad i=1, \dots, I \quad (3.4)$$

where \mathbf{p}_0 is the vector of nominal parameter values for design \mathbf{x} , and \mathbf{p} is between \mathbf{p}_L and \mathbf{p}_U , the known lower and upper bounds on design parameters, respectively. An appropriate optimization technique could be used to find Δf_i^W for each design alternative \mathbf{x} .

To assess the multi-objective robustness of each design, first the designer needs to provide the maximum acceptable range from nominal for each objective function. The maximum acceptable variation for the i^{th} objective function value is shown as Δf_i^D in Figure 3.8. One needs to solve for the Δf_i^W using Eq. (3.4) to obtain the maximum observed variation of an objective function from its nominal value as shown in Figure 3.8. Based on this approach, the observed maximum variations of every objective function from its nominal value (e.g., Δf_i^W) should be smaller than an acceptable range (e.g., Δf_i^D) provided by the designer. It should be noted that both hyper-boxes are generated symmetrically with respect to the nominal point. Schematically, as shown in Figure 3.8, a design has multi-objective robustness, if its sensitivity region does not go beyond the dashed box (created by ranges provided by the designer). The above-mentioned statement can also be verified by ensuring that the dotted box (which is obtained by solving for the maximum variation from the nominal) remains inside the dashed box.

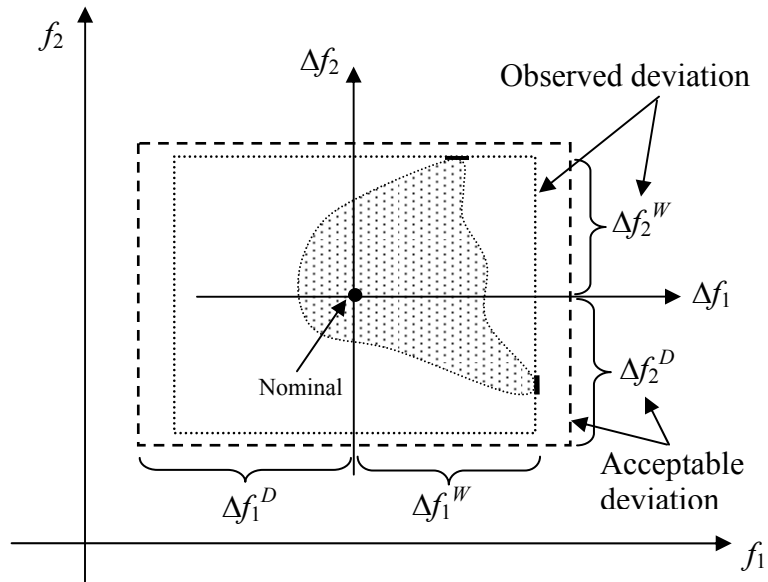


Figure 3.8: Multi-objective robustness

Figure 3.9 displays an example with two design alternatives **A** and **B** with their nominal objective values. The sensitivity region for each design alternative is also displayed. It can be observed that while the nominal design **A** outperforms nominal design **B** (when both objectives are being minimized), design **B** maintains multi-objective robustness according to the acceptable ranges (i.e., Δf_i^D). On the other hand, design **A** violates the acceptable range, particularly with respect to objective function f_2 . The approach eliminates any design such as design **A** that does not satisfy the acceptable range for robustness.

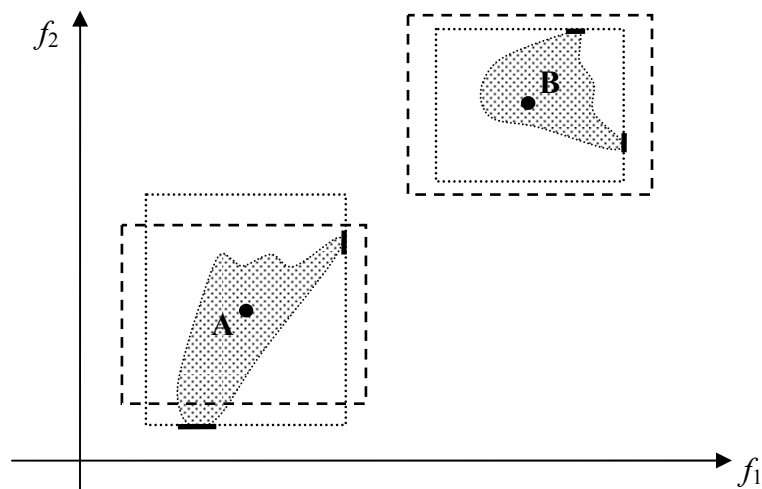


Figure 3.9: Multi-objective robustness comparison of two design candidates

3.4.2. Objective Robustness Measure for Approach 2- Robust Design Optimization

The robust design optimization for approach 2 is shown in Figure 3.10. It should be mentioned that the feasibility robustness assessment in the approach 2 is the same as that in approach 1. The main multi-objective optimization is performed in the upper block of the flowchart. In the upper block, the original constraints are revised to ensure the

feasibility robustness. To evaluate the second constraint (i.e., multi-objective robustness), each design point \mathbf{x} is passed on to the lower block of the flowchart, where the maximum variation from the nominal value for each objective function is calculated and returned to the upper block. This procedure continues until the optimization stopping criteria are met.

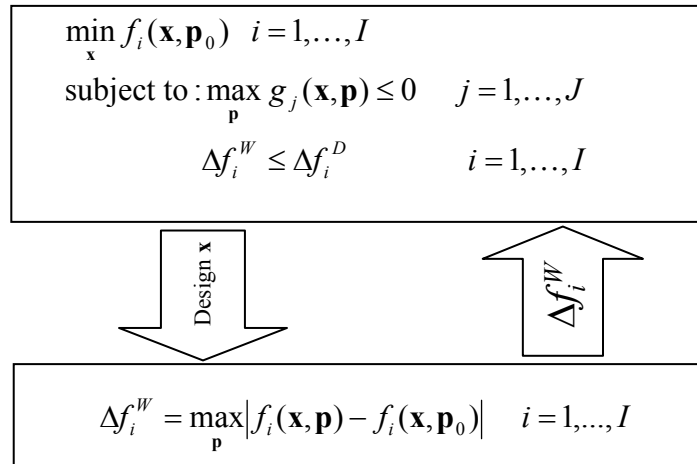


Figure 3.10: Robust optimization approach

3.4.3. Steps in Robust Optimization for Approach 2

Similarly to approach 1, a step by step procedure for the approach 2 has been developed, Figure 3.11, as described in the following steps.

- Step 1: Select initial values for design variables \mathbf{x} , and choose the nominal values of uncontrollable design parameters.
- Step 2: Pass the initial design variables as well as uncontrollable design parameters (within the given range) to design simulation to compute the objective and constraint functions. The search for worst case constraint values as well as maximum deviation from the nominal objective function values in the design objective space is performed in this step (see Eq. (3.4)).

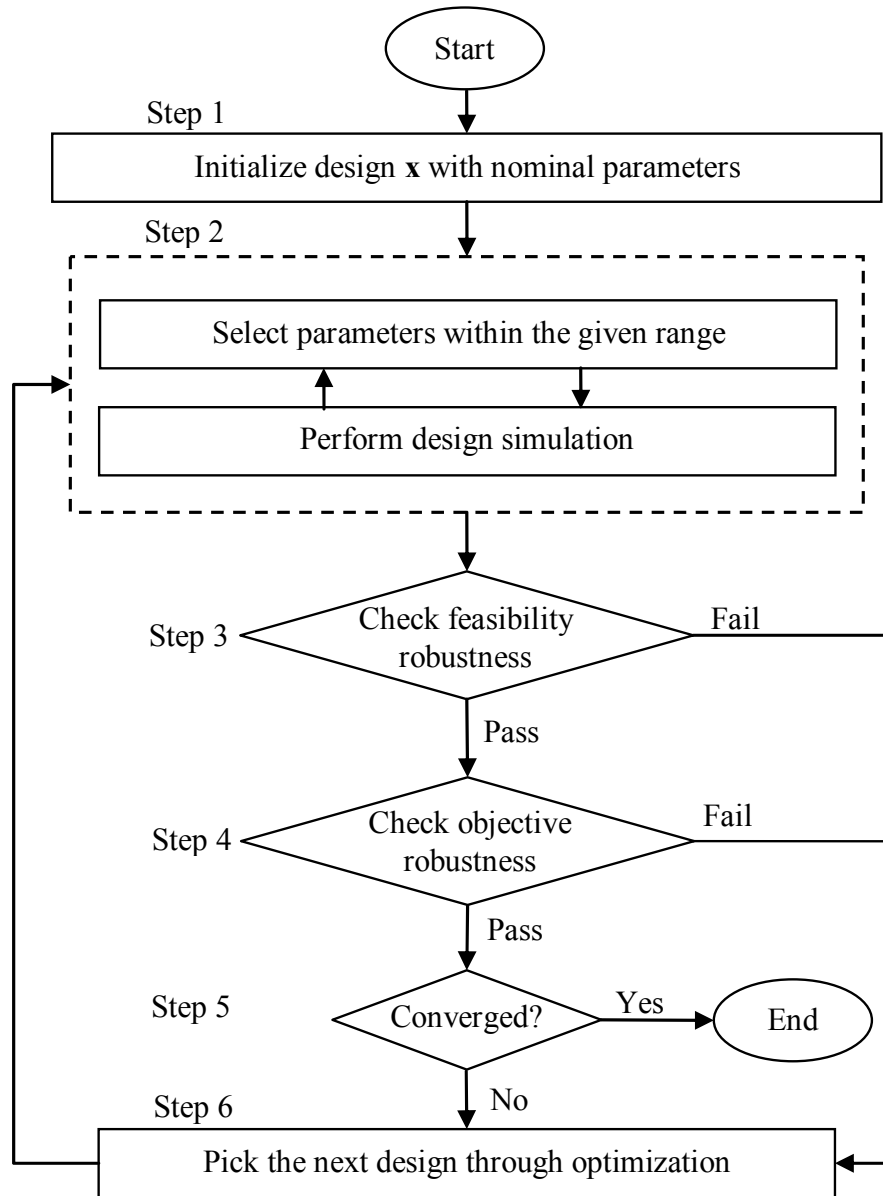


Figure 3.11: The flowchart of robust optimization approach 2

Step 3: Based on the worst case values of constraints, feasibility robustness for design x can be examined (see Eq. (3.3)). The procedure eliminates any design x that violates the feasibility robustness and goes to Step 6. The procedure continues, with any design x that passes the feasibility robustness, to Step 4.

Step 4: Each feasibly robust design point \mathbf{x} will be evaluated to identify whether or not its maximum deviation of objective function values is within the designer's acceptable range. Any design that violates the objective robustness criteria is eliminated in this step and the procedure continues to Step 6. In this step, depending on the optimization technique, the feasibly and multi-objectively robust designs are improved.

Step 5: If the optimizer is converged, the procedure will end; otherwise, it goes to Step 6.

Step 6: Select the values for the next set of design variables \mathbf{x} , via the optimizer, and go to Step 2.

3.5. COMPUTATIONAL COST

Both of the robust optimization approaches, given in Sections 3.3 and 3.4, are composed of a top level optimization problem and a number of sub-level optimizations. Both approaches require one sub-level optimization for feasibility robustness assessment. Approach 1 requires two sub-level optimizations for calculating multi-objective robustness measures. However, approach 2 requires I sub-level optimizations, where I is the number of objective functions.

An estimate the computational cost for both approaches is given next.

3.5.1. Computational cost of the approach 1

Assume that the top level optimization problem requires n_D number of function calls (i.e., for n_D number of design alternatives) to converge. Each of the n_D design alternatives that is passed from the top level optimization model needs to be evaluated for

feasibility robustness. Among the n_D designs, suppose n_F design alternatives do not meet the requirement for feasibility robustness and the rest, $n_D - n_F$, are evaluated for multi-objective robustness. The number of function calls required for verifying feasibility robustness of each design is assumed to be N_F , and the number of function calls to obtain the worst case and the best case scenario points for each design is N_W and N_B , respectively. Hence, Eq. (3.5) can be used to compute the total number of function calls n required to run the robust optimization approach.

$$n = n_D \cdot N_F + (n_D - n_F) \cdot (N_W + N_B) \quad (3.5)$$

The maximum (or worst case) number of function calls occurs when all of the design alternatives that are evaluated in the top block of Figure 3.7 (i.e., n_D) satisfy the feasibility robustness, in that case the number of function calls will be: $n = n_D \cdot (N_F + N_W + N_B)$.

3.5.2. Computational cost of the approach 2

Similar to the approach 1, it can be assumed that the top level optimization problem requires n_D number of function calls to converge. Since the feasibility robustness method for both approaches is the same, similar to the first approach suppose each of the n_D design alternatives that is passed from the top level optimization can be divided into n_F design alternatives that do not meet the requirement for feasibility robustness and the rest, $n_D - n_F$, are evaluated for multi-objective robustness. Furthermore, the number of function calls required for verifying feasibility robustness of each design is assumed to be N_F . In this case however the number of function calls to examine the feasibility robustness with respect to the i^{th} objective function for each design is assumed to be N_I .

Hence, Eq. (3.6) can be used to compute the total number of function calls n required to run the robust optimization approach.

$$n = n_D \cdot N_F + (n_D - n_F) \cdot \sum_{i=1}^H (N_I) \quad (3.6)$$

where, H represents the number of objective functions that designer wishes to consider for multi-objective robustness. One of the strength of the second approach is that it is not necessary to have limits on variations for all objective functions.

Similarly to the approach 1, the maximum (or worst case) number of function calls occurs when all of the design alternatives that are evaluated in the top block of Figure 3.11 (i.e., n_D) satisfy the feasibility robustness, and at the same time all of the objective functions need to be considered for objective robustness. In that case the

number of function calls will be: $n = n_D \cdot (N_F + \sum_{i=1}^I (N_I))$.

A quantitative comparison of the Eqs (3.5) and (3.6) reveals that the objective robustness in the first approach is computationally more efficient than that of the second approach. The first approach requires only two sub-level optimizations (i.e., one to obtain the *WCSD* and one to obtain the *BCSD* and therefore *Variability*). However, the number of sub-level optimizations for the second approach can be as high as the total number of objective functions.

3.6. DEMONSTRATION EXAMPLES

As a demonstration, both approaches are applied to a numerical example and an engineering design problem. Both examples have two objective functions and have parameter variations. The numerical example is an optimization problem which has a discrete parameter among its uncontrollable design parameters, while the engineering

design example has a discrete variable among its design variables. The purpose of the numerical example is to provide a step-by-step description for each of the two approaches. The engineering design example is an application to a real world design problem.

Both approaches are implemented with a Multi-Objective Genetic Algorithm (MOGA) [Fonseca and Fleming, 1993] optimizer with Kurapati et al.'s constraint handling technique [Kurapati et al., 2002]. The corresponding MOGA parameter values are provided in Table 3.1.

Parameter	Value
Length of Chromosomes	10
Population Size	100
Population Replacement	10
Crossover Probability	90%
Mutation Probability	5%
Selection Type	Stochastic Universal Selection
Number of Generations	300

Table 3.1: MOGA parameters

3.6.1. Numerical Example

This is a numerical test example which was originally formulated and solved by Poloni et al. [Poloni et al., 2000] and others [Deb, 2001] to demonstrate approaches 1 and 2. However, two uncontrollable design parameters (one continuous and the other discrete) are added to the original formulation. As defined originally [Poloni et al., 2000] there are two continuous design variables x_1 and x_2 that are bounded in the range: $[-\pi, \pi]$, and two objective functions. Here, the optimization problem is revised as in Eq. (3.7).

minimize $f_1(x_1, x_2, p_1, p_2) = 1 + (A_1 - B_1)^2 + (A_2 - B_2)^2$
 minimize $f_2(x_1, x_2, p_1, p_2) = (x_1 + p_1 + 3)^2 + (x_2 + p_2 + 1)^2$
 where:

$$\begin{aligned}
 A_1 &= 0.5 \sin 1 - 2 \cos 1 + \sin 2 - 1.5 \cos 2 \\
 A_2 &= 1.5 \sin 1 - \cos 1 + 2 \sin 2 - 0.5 \cos 2 \\
 B_1 &= 0.5 \sin(x_1 + p_1) - 2 \cos(x_1 + p_1) + \sin(x_2 + p_2) - 1.5 \cos(x_2 + p_2) \\
 B_2 &= 1.5 \sin(x_1 + p_1) - \cos(x_1 + p_1) + 2 \sin(x_2 + p_2) - 0.5 \cos(x_2 + p_2) \\
 -0.2 &\leq p_1 \leq 0.2 \\
 p_2 &\in \{-0.2, -0.1, 0.0, 0.1, 0.2\} \\
 -\pi &\leq x_1 + p_1 \leq \pi \\
 -\pi &\leq x_2 + p_2 \leq \pi
 \end{aligned}$$

(3.7)

Figure 3.12 shows the feasible domain in the design objective space (showed in gray), and the Pareto frontier (shown in black) which is disconnected. The feasible domain in Figure 3.12 is generated exhaustively to clarify the shape of the objective space.

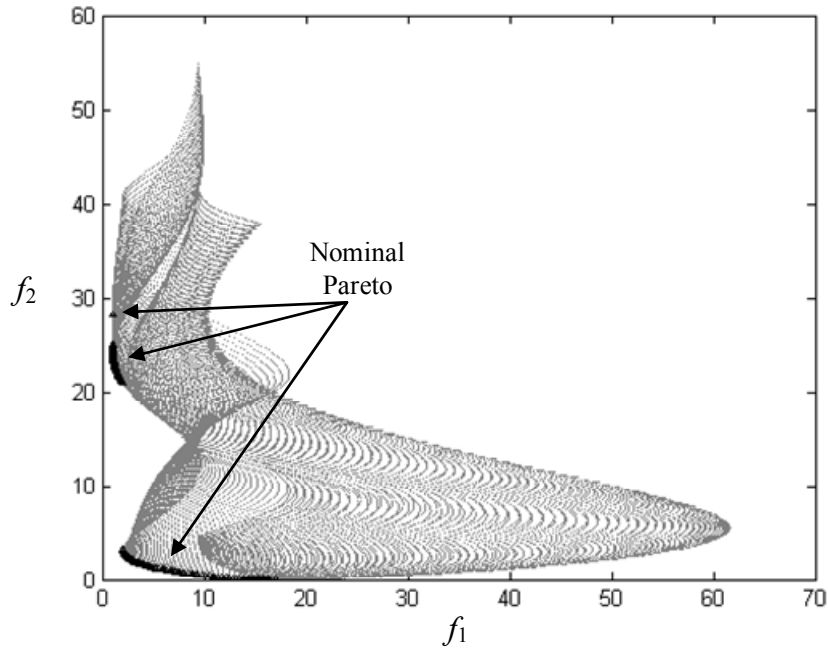


Figure 3.12: The nominal Pareto set in the objective space

It is assumed that p_1 and p_2 are the two uncontrollable design parameters. The parameter p_1 is a continuous parameter and its variation range is within $[-0.2, 0.2]$, and p_2 is a discrete parameter with values taken from $\{-0.2, -0.1, 0, 0.1, 0.2\}$. In addition, the nominal values for both p_1 and p_2 are assumed to be zero.

The feasibility robustness of every point in the design objective space is examined and Figure 3.13 is generated. Before proceeding with either of the robust optimization approach, and in particular the objective robustness issues, from Figure 3.13 it can be observed that the effect of parameter variations causes a portion (i.e., the dark points) of the design variable space to become infeasible. In this example, to ensure the feasibility robustness, as shown in Eq. (3.7) both x_1+p_1 and x_2+p_2 should stay in the range of $[-\pi, \pi]$. Figure 3.13 also shows the points that do not satisfy the feasibility robustness requirement. The dark region is formed by the points that violate the feasibility robustness criterion and are eliminated during the process of the robust multi-objective optimization approach 1.

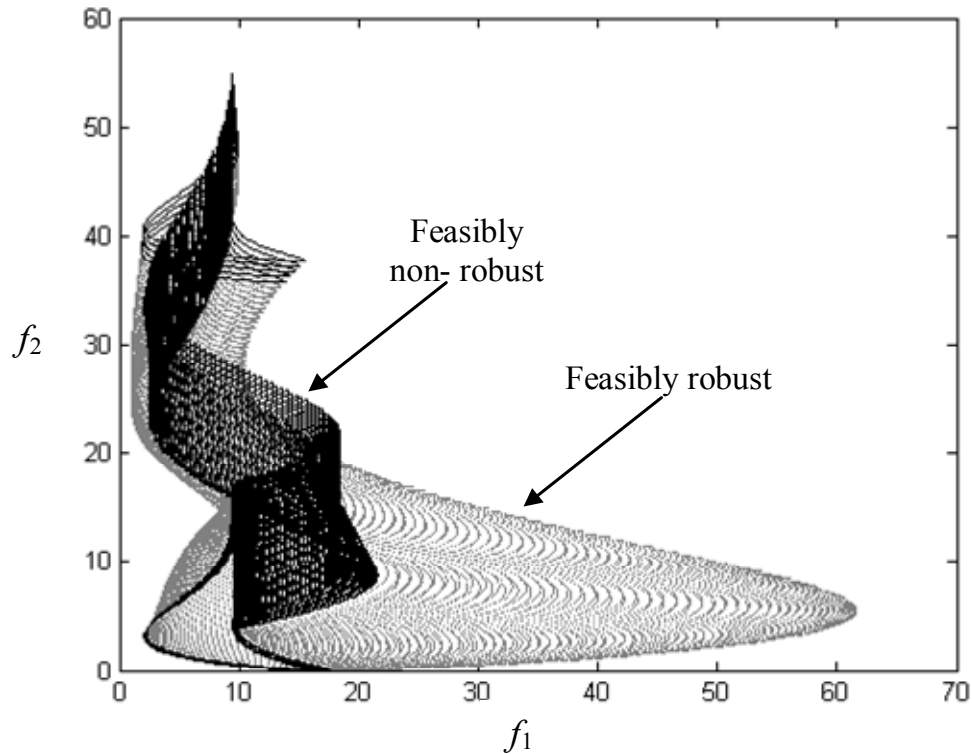


Figure 3.13: The portion of the objective space that is feasibly non-robust

3.6.1.1. The Solution Using Robust Optimization in Approach 1:

To obtain the *WCSD* and multi-objective variability measures, first the location of target and bad points needs to be identified: $(f_1, f_2) = (0, 0)$ and $(f_1, f_2) = (70, 60)$, respectively. For every point (f_1, f_2) in the objective space, a corresponding (*WCSD* and multi-objective variability) point can be obtained. Initially, the L_2 norm (i.e., $q = 2$ in Eqs. (3.3) and (3.4)) is used to obtain *WCSD* and multi-objective variability for each design. The region created by these points is shown in Figure 3.14. The region created already accounts for feasibly robustness. Finally, the solutions to the problem of minimizing *WCSD* and minimizing multi-objective variability, as illustrated in Figure 3.6, form a set

of robustly (feasibly and multi-objectively) non-dominated points, as identified in two clusters of A and B in Figure 3.14.

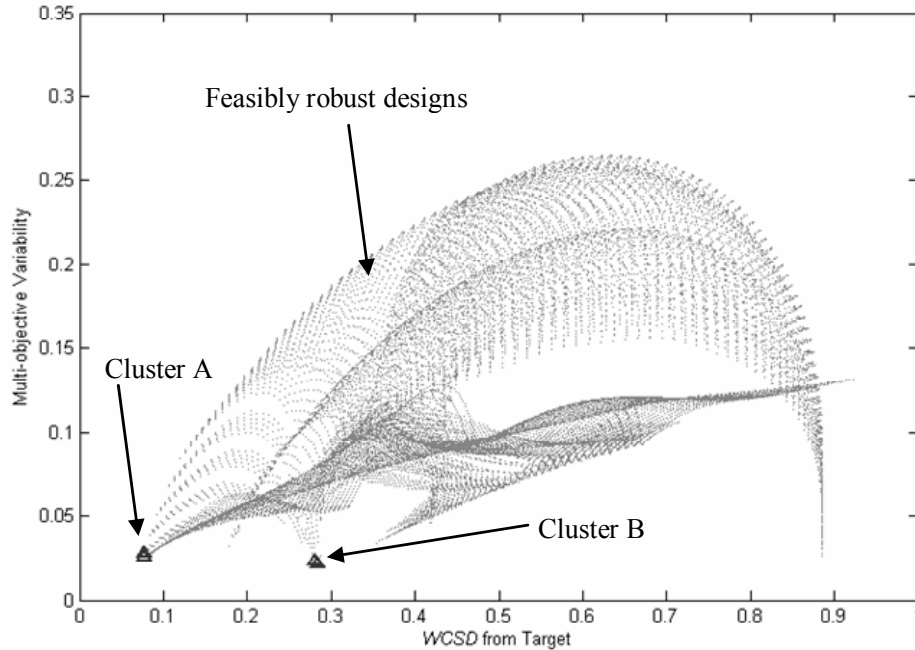


Figure 3.14: Robust optimal non-dominated points

Clusters of A and B solution points are also shown in the design objective space in Figure 3.15. It can be observed that the points in cluster A are very close to the set of nominal Pareto optimal points which were shown in Figure 3.12. However, the points in cluster B are marginally better in terms of the multi-objective variability (see Figure 3.14) but are farther away from the target (see Figure 3.15). To examine the feasibility robustness of the obtained solutions, a snapshot of Figure 3.13 is shown in Figure 3.15. It can be observed that the designs on the bottom left boundary of the design objective space are not feasibly robust. However, all of the designs in cluster A are objectively robust and satisfy the feasibility robustness criteria. It is difficult to visually examine the feasibility robustness of designs in cluster B since these designs are located behind the

feasibly non-robust points given in Figure 3.13 (or its snapshot in Figure 3.15). Indeed, the design alternatives corresponding to both clusters satisfy the feasibility robustness criteria.

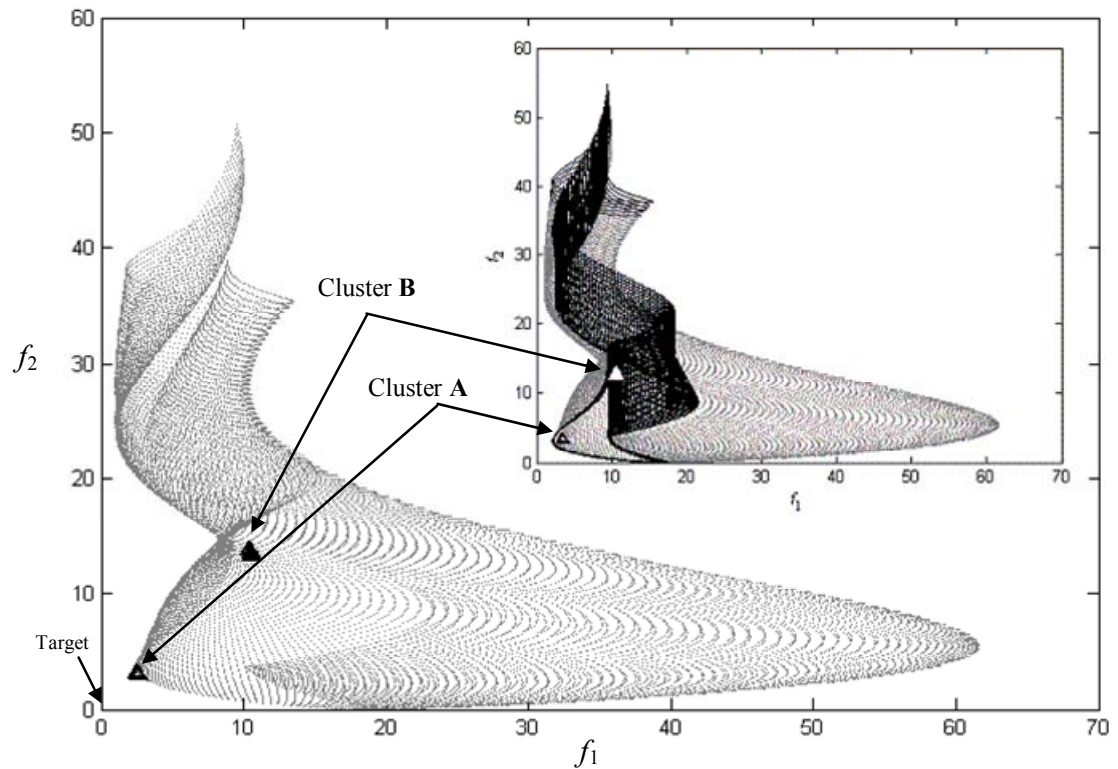


Figure 3.15: Robustly non-dominated optimal points in the objective space

To investigate the effect of using different norms in approach 1, the numerical example is also solved with L_1 and L_∞ norms. Figure 3.16 shows the solution obtained by different norms in the design objective space.

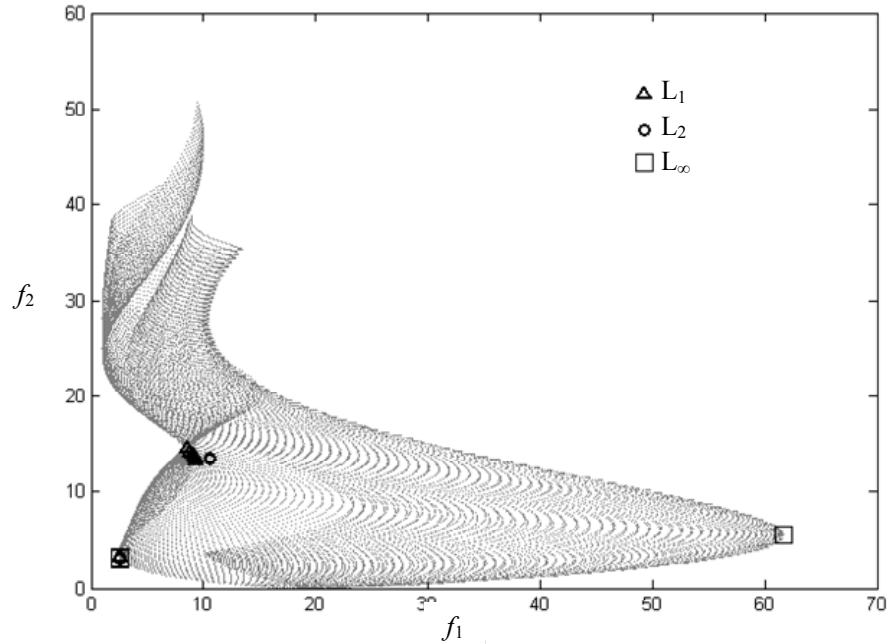


Figure 3.16: Solutions obtained by different norms

As shown in Figure 3.16, all of the norms generate solutions at the bottom corner (like cluster A in Figure 3.15). However, the L_1 norm generates more solutions around cluster A. Furthermore, L_1 or L_2 norms generate somewhat similar results. However the L_∞ norm also generates a solution that is to right of the design objective space, as marked in Figure 3.16.

3.6.1.2. The Solution Using the Robust Optimization in Approach 2:

The approach 2 described in Section 3.4 is used here to obtain solutions to this numerical problem. The feasibility robustness criteria for both approaches are identical. Therefore the non-feasibly robust points in both Figures 3.11 and 3.13 are being eliminated in the approach 2 as well. However, for the second approach, one needs to specify the maximum acceptable variability of each objective function. It is assumed that

the $\Delta f_1^D = 3$ and $\Delta f_2^D = 2$. It should be noted that by increasing the maximum acceptable variation values, a larger number of designs pass the objective robustness criteria for the approach 2. The variation of the objective functions for feasibly robust designs are shown in Figure 3.17. The variability of solutions in Figure 3.17 is within the acceptable ranges specified above. The dark region shows designs that basically do not violate the objective robustness criteria using the approach 2.

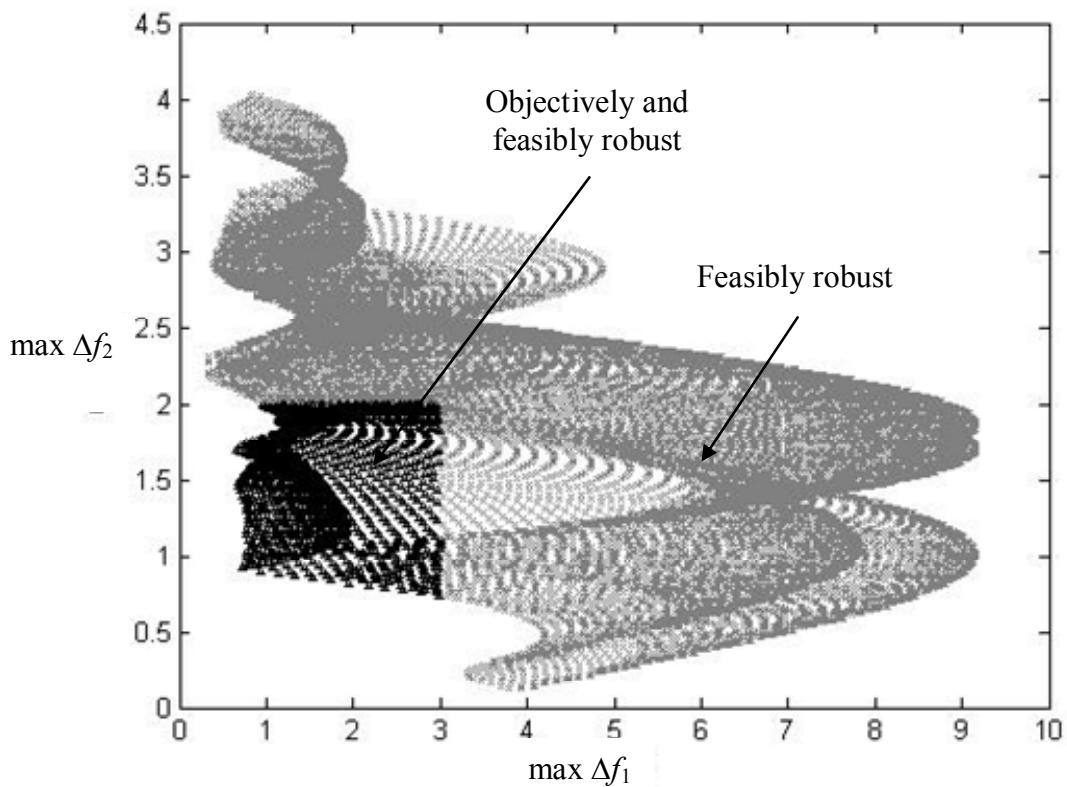


Figure 3.17: The feasibly robust vs. feasibly/objectively robust points

Every gray point in Figure 3.17 represents the maximum observed variability in terms of both design objective functions. After setting the criteria for maximum acceptable variability, only a subset of the feasibly robust solution satisfied those criteria.

The objectively and feasibly robust points are identified as dark points in Figure 3.17. In order to visualize and compare the dark points from Figure 3.17 in the design objective space, every point was mapped back to the objective space and the result is shown in Figure 3.18.

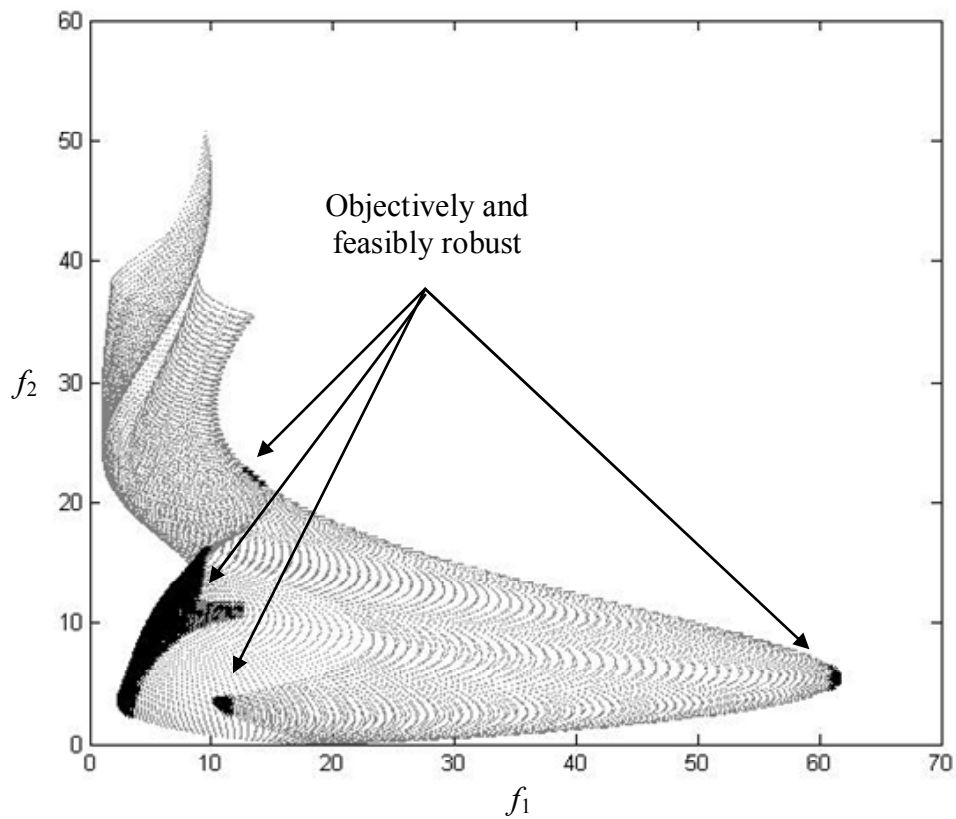


Figure 3.18: The feasibly robust vs. feasibly/objectively robust points in objective space

Now, the Pareto points among the feasibly and objectively robust designs shown in Figure 3.18 are obtained. The final results of the robust optimization approach 2 are given in Figure 3.19.

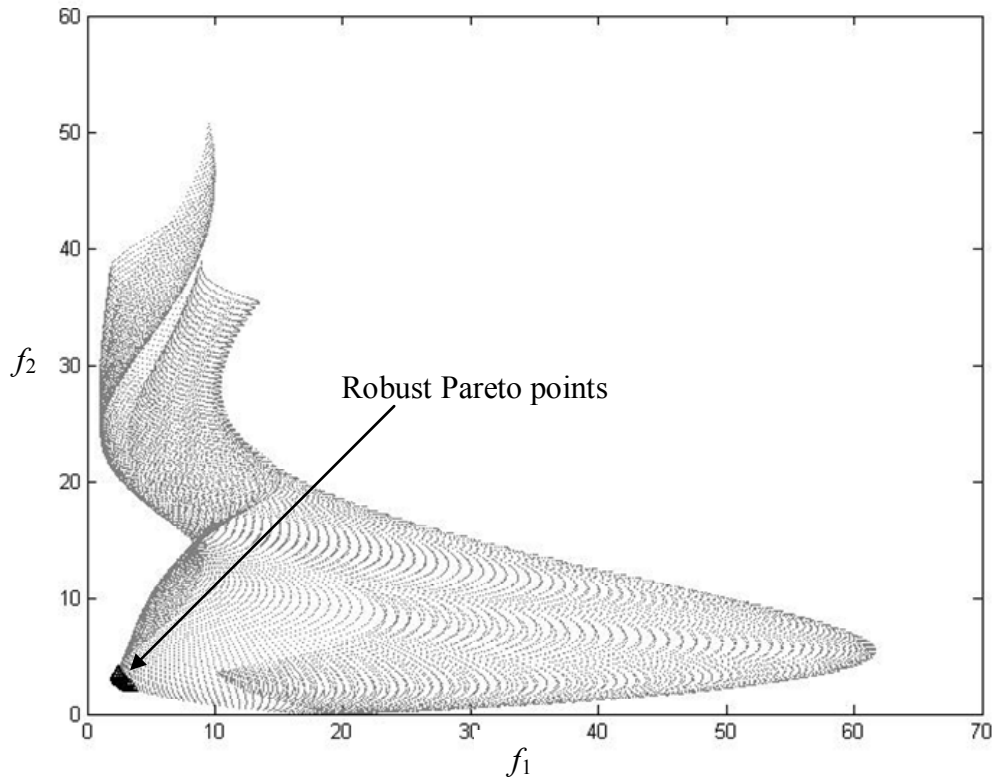


Figure 3.19: The robust Pareto set of designs

3.5.1.3. Comparison Study:

To investigate the results obtained by each of the two approaches, and make a comparison, six points are selected and identified in the design objective space. Point 1 is selected from the obtained robust results of both approaches (i.e., common between both of the approaches). Point 2 is selected from the robustly non-dominated optimal set using either L_1 or L_2 norms (i.e., Figure 3.15) from the approach 1. Points 3 and 4 are selected from the nominal Pareto set (i.e., Figure 3.12), point 5 is selected arbitrarily from the objective space, and finally point 6 is selected from the robustly non-dominated optimal set using L_∞ norm. These points are selected to quantitatively compare the performance

and robustness of nominal vs. robust optimum solutions. The sensitivity regions for these six points are created and shown (dark shaded) in Figure 3.20.

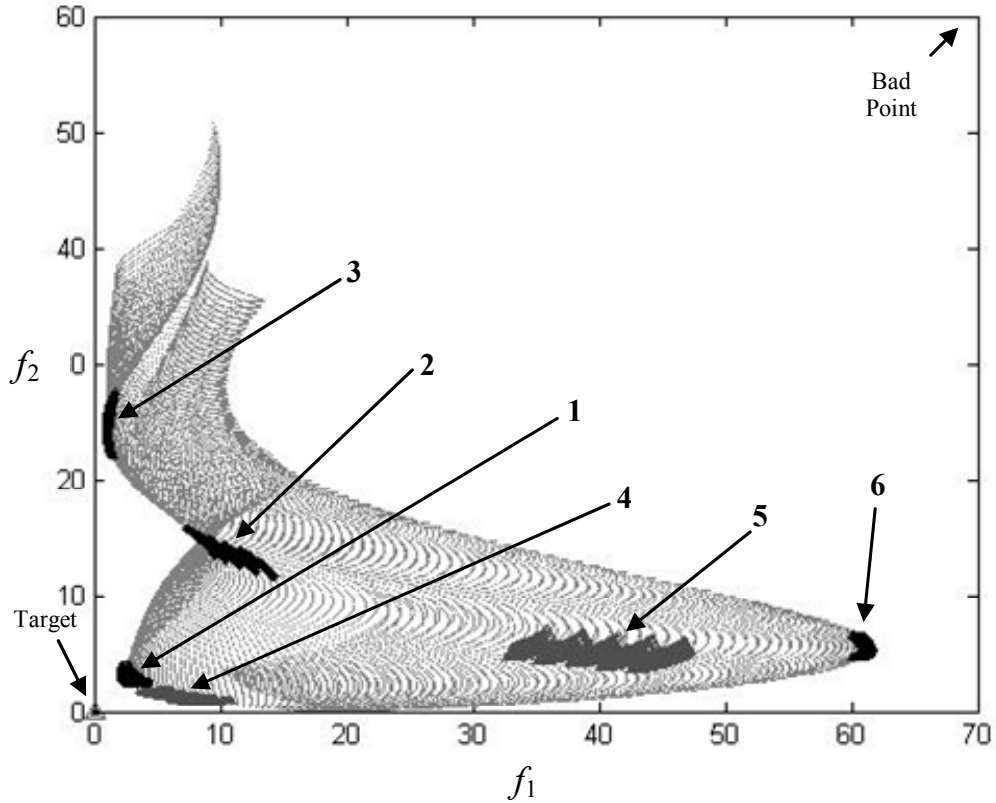


Figure 3.20: Sensitivity region for selected points

Points 1 and 2 are selected from the obtained robust solutions using approach 1. By examining the results shown in Table 3.2, it can be observed that both points 1 and 2 have a small *WCSD* and/or that their sensitivity regions along the direction of target are narrower than the other three points. This is a very satisfactory result for approach 1. However, in particular point 2 which is not a result of the approach 2 shows a relatively large variation in terms of the objective function f_1 . Point 1 is among the results of both approaches and has a low (i.e., acceptable) variation in terms of both objectives. Point 3

and 4 that belong to the nominal Pareto set show a larger amount of variability compared to points 1 and 2. Point 5 that does not belong to any Pareto set happens to have the highest $WCSD$ and variability among the rest of the points. Finally point 6 which is the result of the approach 1 using L_∞ norm, shows relatively low variability in terms of both objective functions. However, design 6 has a very large value of objective function f_1 . It is interesting to observe that design 6 is acceptable for the approach 2 (see Figure 3.18). However, due to poor performance in the nominal case it is eliminated by approach 2. A quantitative comparison of these six points is shown in Table 3.2.

Point	x_1	x_2	f_1	f_2	Δf_1^W	Δf_2^W	$WCSD$	$Variability$
1	-2.90	0.78	2.520	3.172	1.75	0.83	0.077	0.025
2	-0.18	1.38	10.562	13.600	3.66	2.16	0.284	0.024
3	1.13	1.76	1.058	24.674	0.60	2.83	0.459	0.092
4	-2.93	0.12	6.558	1.258	4.57	0.56	0.160	0.110
5	-1.07	-2.2	40.257	5.165	7.15	1.72	0.672	0.194
6	-0.66	-1.06	61.63	5.47	1.60	1.04	0.8804	0.0229

Table 3.2: Optimality and robustness comparison of selected points

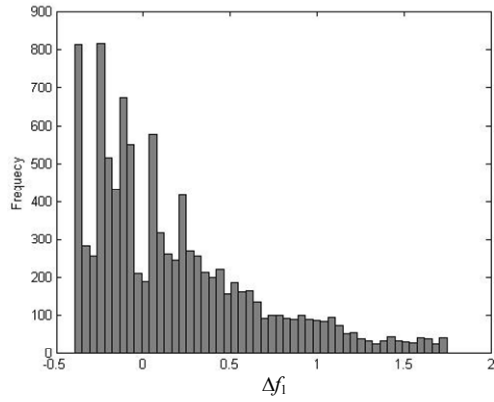
The total number of function calls using approach 1 is determined as follows. The top level problem requires about 1000 function calls, and the feasibility robustness, $WCSD$, and $BCSD$ calculations, require 100, 200, and 200, respectively. In the worst case (see Section 3.5.1), the number of function calls for this example using approach 1 is estimated to be about 500,000 using a genetic algorithm based technique for the optimizer. The total number of function calls for this example using approach 2 can be determined as follows. Similar to approach 1, the number of function calls for the top level optimization is about 1000, the feasibility robustness, objective robustness for f_1 and

objective robustness for f_2 require 100, 300, and 300 respectively. Therefore, in the worst case, the number of function calls for approach 2 implemented with a genetic algorithm based technique for this example is about 700,000.

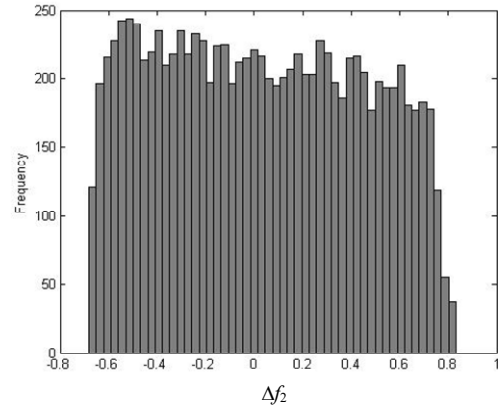
For this particular example, since there are only two objective functions, approach 2 is not dramatically less efficient than approach 1. However, for the cases that the number of objective functions is large, approach 1 is computationally more efficient (i.e., only two sub-optimization problems to assess the objective robustness).

3.6.1.3. Verification:

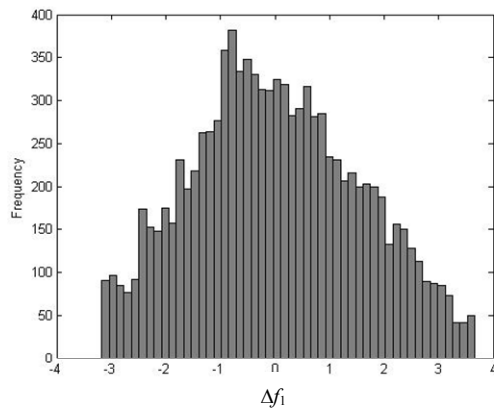
To verify the robustness of results obtained by each of the approaches, the design parameter, p_1 , was randomly perturbed 10000 times around its nominal values within the given range between -0.2 and 0.2 Also, it is randomly chosen any of the possible discrete combination given in the problem description for design parameter p_2 , 10000 times. Then the new objective function values are calculated. In other words, a Monte Carlo simulation using a uniform distribution is performed to examine the robustness of each of the above-mentioned five design points. It is also assumed that the parameters p_1 and p_2 are statistically independent. The histograms of the output of the simulation for each design are provided in Figure 3.21 (a) – (l).



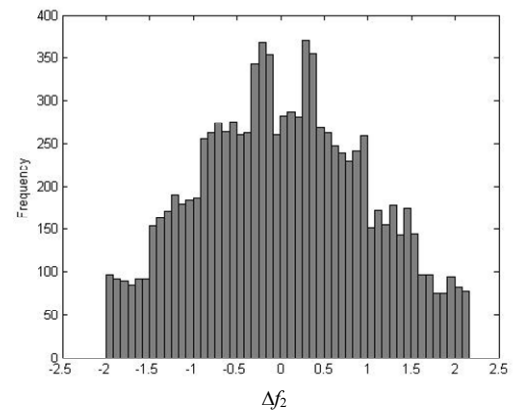
(a) Design 1 – Δf_1



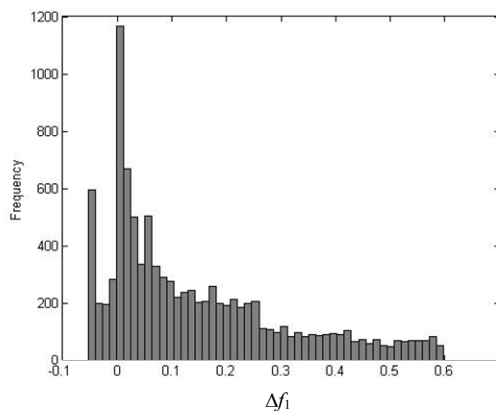
(b) Design 1 – Δf_2



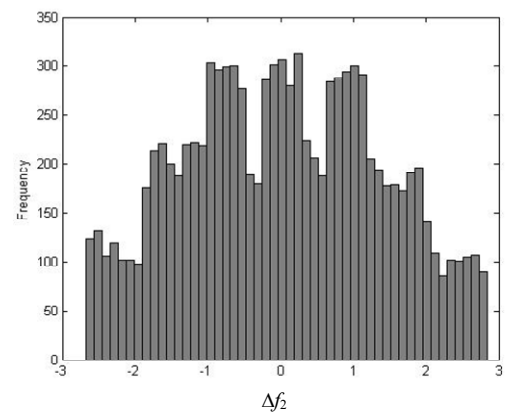
(c) Design 2 – Δf_1



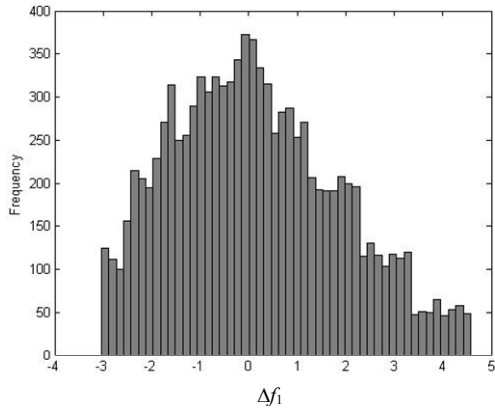
(d) Design 2 – Δf_2



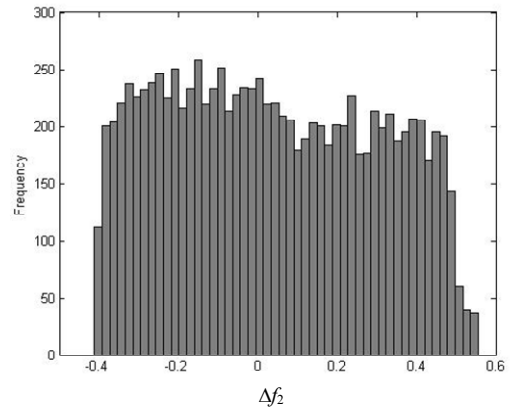
(e) Design 3 – Δf_1



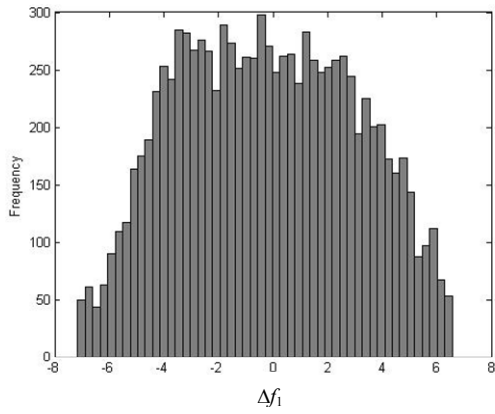
(f) Design 3 – Δf_2



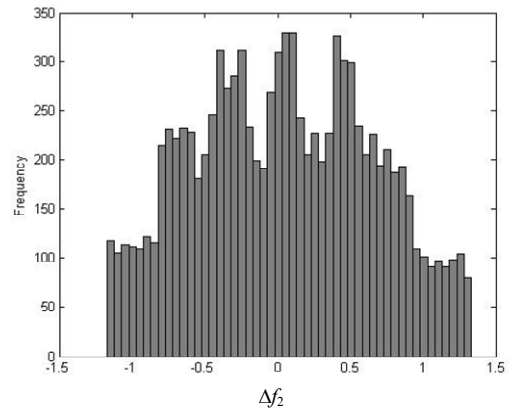
(g) Design 4 – Δf_1



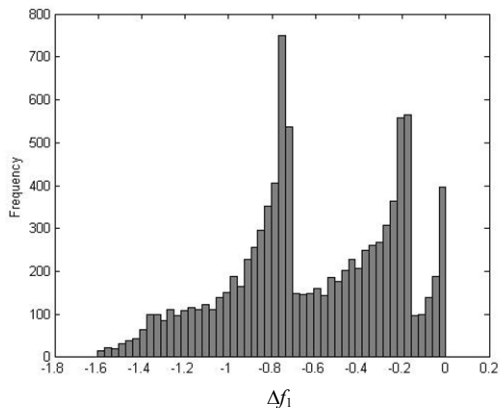
(h) Design 4 – Δf_2



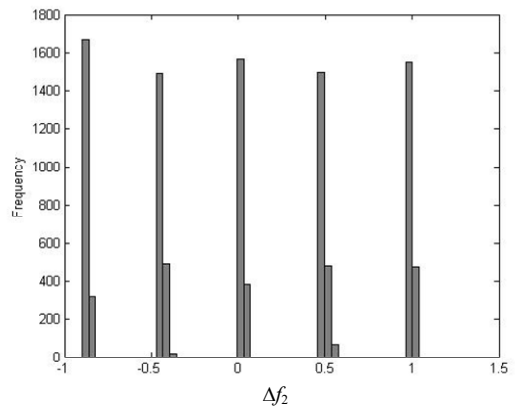
(i) Design 5 – Δf_1



(j) Design 5 – Δf_2



(k) Design 6 – Δf_1



(l) Design 6 – Δf_2

Figure 3.21: Histogram of simulation output for selected designs

For each of the histograms, the maximum absolute deviation from nominal (i.e., Δf^W) for both objective functions are obtained and the results are shown in Table 3.3.

Point	Δf_1^W	Δf_2^W
1	1.7469	0.8319
2	3.6641	2.1600
3	0.5983	2.8354
4	4.5720	0.5557
5	7.1536	1.7190
6	1.6022	1.0400

Table 3.3: Comparison of individual objective function variation for selected designs

Recall that the acceptable range of variations from nominal performance was defined as $\Delta f_1^D = 3$ and $\Delta f_2^D = 2$. All of the points in Table 3.3 except point 1 and point 6 exceed these acceptable limits, and hence are not multi-objectively robust (using approach 2). Point 6 is an exception among the results obtained by approach 1. Point 6 is obtained using L_∞ norm in approach 1 and it satisfy the objective robustness criteria of the approach 2.

3.6.2. Engineering Example: Design of a Vibrating Platform

This problem was originally modeled by [Messac, 1996] and later reformulated as a multi-objective design optimization problem by [Narayanan and Azarm, 1999]. The objective functions are to maximize the fundamental frequency of the system and to minimize the material cost. The platform consists of five layers from three materials as

shown in Figure 3.22. The material properties are given in Table 3.4. The outer layer, two inner layers and the center layer are assumed to be made of different materials.

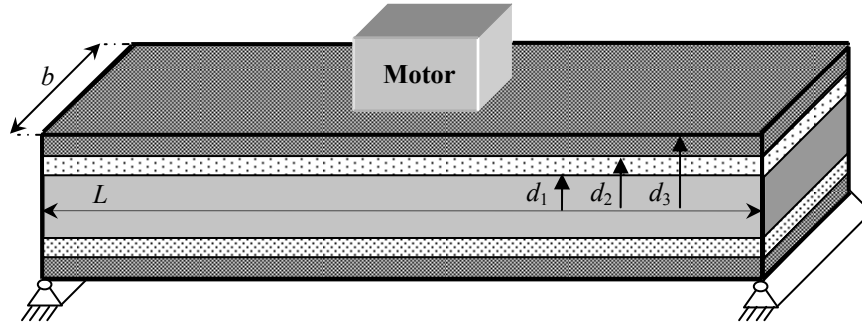


Figure 3.22: Vibrating platform

There are five design variables, L , b , d_1 , d_2 , and d_3 , as shown in Figure 3.22. Also, there are six possible combinations for assigning three materials to the three layers since adjacent layers are not allowed to have the same material.

	Material A	Material B	Material C
ρ (Kg/m ³)	100	2770	7780
E (GPa)	1.6	70	200
c (\$/m ³)	500	1500	800

Table 3.4: Material properties (nominal values)

The optimization formulation for this problem is given in Eq. (3.8).

$$\begin{aligned}
&\text{maximize} && f_n = \left(\frac{\pi}{2L^2} \right) \left(\frac{EI}{\mu} \right)^{\frac{1}{2}} \\
&\text{minimize} && \text{Cost} = 2b[c_1 d_1 + c_2 (d_2 - d_1) + c_3 (d_3 - d_2)] \\
&\text{subject to :} && g_1 = \mu L - 2800 \leq 0 && 0.05 \leq d_1 \leq 0.5 && (3.8) \\
&&& g_2 = d_1 - d_2 \leq 0 && 0.2 \leq d_2 \leq 0.5 \\
&&& g_3 = d_2 - d_1 - 0.15 \leq 0 && 0.2 \leq d_3 \leq 0.6 \\
&&& g_4 = d_2 - d_3 \leq 0 && 0.35 \leq b \leq 0.5 \\
&&& g_5 = d_3 - d_2 - 0.01 \leq 0 && 3.0 \leq L \leq 6.0
\end{aligned}$$

where,

$$\begin{aligned}
EI &= \left(\frac{2b}{3} \right) [E_1 d_1^3 + E_2 (d_2^3 - d_1^3) + E_3 (d_3^3 - d_2^3)] \\
\mu &= 2b[\rho_1 d_1 + \rho_2 (d_2 - d_1) + \rho_3 (d_3 - d_2)]
\end{aligned}$$

There are several uncontrollable design parameters in this problem that can be subject to variations. The density and cost per m³ of material A (ρ_A and c_A , respectively) are chosen to be varying within 5% of the nominal values given in Table 3.4. Therefore the density of material A is varied in the range: [95, 105] Kg/m³, and its cost per volume in the range: [\$475, \$525]. It should be noted that since none of the constraints are a function of density or cost of material A, then the feasibility robustness of every design solution is guaranteed.

3.6.2.1. The Solution Using Robust Optimization Approach 1:

An implementation of MOGA [Fonseca and Fleming, 1993] is used in conjunction with the approach 1 to obtain the results. It is assumed that the objective function values of the target point are 500Hz and \$20 respectively. The coordinates of the bad point are 0Hz and \$1000 respectively. The results are shown in Figure 3.23.

As shown in Figure 3.23, the robustly non-dominated optimal solutions are inferior to the nominal Pareto solutions. This implies that sometimes, to achieve robustness, the optimality may have to be sacrificed. It can be observed that several points in robust optimal non-dominated set are very close to the nominal Pareto points, and one can choose either of those designs to preserve the performance. However, these designs show more variability in performance than the ones that are farther from the nominal Pareto set.

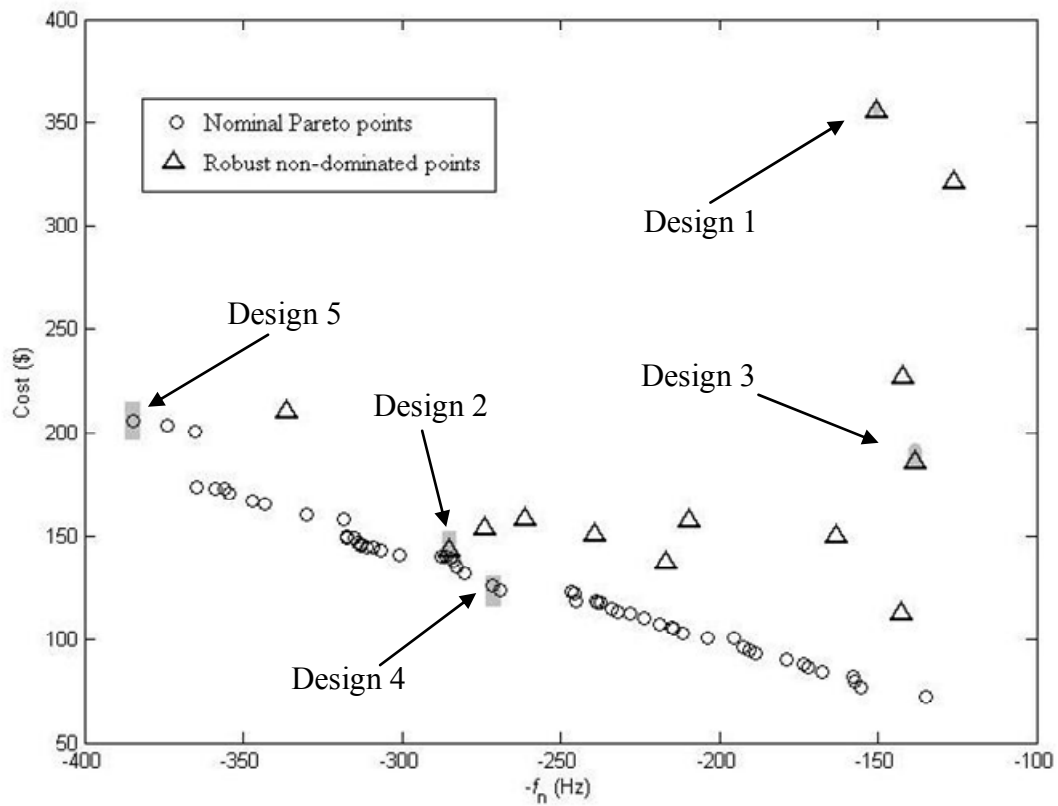


Figure 3.23. Nominal Pareto and robustly non-dominated optimal set for vibrating platform example

To demonstrate the robustness of solutions, five design alternatives are selected, as shown in Figure 3.23 and their sensitivity regions are obtained. The *WCSD* from the target and variability for each alternative is calculated. The purpose of selection of these points is entirely for demonstration purposes, and in many cases where the objective space is not two dimensional, the visual comparison may not be possible. Three of the designs are from robust optimal non-dominated set and design 4 and 5 are from the nominal Pareto set. The results are shown in Table 3.5 and Figure 3.23 (gray sensitivity regions).

Design	f_1	f_2	WCSD	Variability
1	150.710	355.782	0.779	0.001
2	284.988	143.156	0.455	0.007
3	138.484	186.044	0.744	0.002
4	271.462	125.895	0.487	0.011
5	384.392	205.861	0.315	0.015

Table 3.5: Optimality and robustness comparison of selected points

Again, to investigate the effect of the choice of different norms on the obtained results, the problem is solved using L_1 and L_∞ norms as well. The comparison is given in Figure 3.24. It can be observed that the obtained results using L_2 norm, for this particular example, include more solutions that are close to the nominal Pareto. In general, the results obtained from the approach 1 depend upon the choice of the norm used and are problem-dependent.

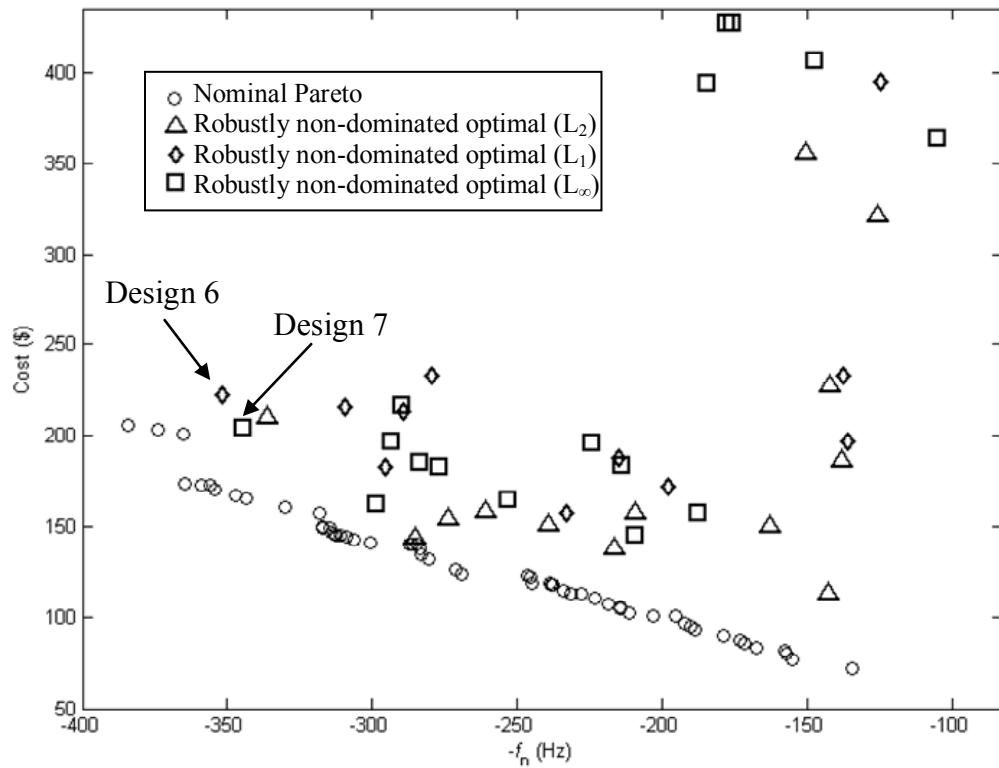


Figure 3.24: Comparison of results obtained by different norms

The design 6 and design 7 are selected from the robustly non-dominated points obtained by approach 1. In particular design 6 is obtained using L_1 norm and design 7 is obtained using L_2 norm within approach 1.

3.6.2.2. The Solution Using Robust Optimization Approach 2:

The approach 2 described in Section 3.4 is used to obtain the solutions to the vibrating platform problem. As mentioned before, the feasibility robustness criteria in this example is always guaranteed. Therefore multi-objective robustness assessment should be carried out. First it is necessary to specify the maximum acceptable variability

of each objective function. It is decided to let the maximum $\Delta f_n^D = 5$ Hz and maximum $\Delta Cost^D = \$5$. The robust Pareto optimal results are shown in Figure 3.25. In addition to five previously selected designs, two designs from the robust Pareto set (obtained using approach 2) are selected.

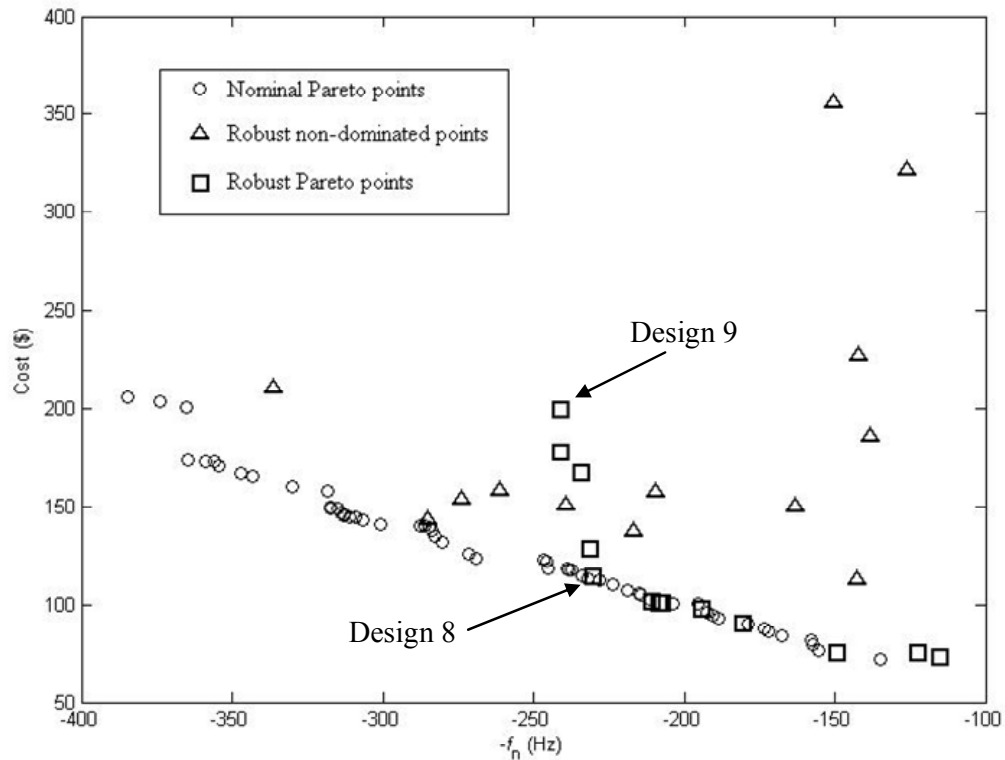


Figure 3.25: Nominal Pareto, robust non-dominated set, and robust Pareto points

As shown in Figure 3.25, the robust Pareto points obtained by approach 2 are *generally* closer to the nominal Pareto (i.e., have better performance) comparing to the robustly non-dominated optimal set obtained by approach 1 (except a few points that are obtained using other norms). The corresponding design variables for all nine designated designs are given in Table 3.6.

Design	d_1	d_2	d_3	b	L	m_1	m_2	m_3
1	0.305	0.365	0.367	0.365	3.10	B	A	C
2	0.313	0.354	0.359	0.387	3.01	A	C	B
3	0.148	0.269	0.277	0.360	3.53	A	B	C
4	0.327	0.339	0.340	0.353	3.00	A	C	B
5	0.401	0.487	0.492	0.372	3.00	A	C	B
6	0.426	0.482	0.485	0.385	3.08	A	B	C
7	0.394	0.455	0.462	0.411	3.09	A	C	B
8	0.255	0.293	0.295	0.369	3.03	A	C	B
9	0.239	0.327	0.336	0.394	3.07	A	B	C

Table 3.6: Design variable values of selected designs

As depicted in Figure 3.23, design 2 that is very close to the nominal Pareto frontier shows relatively larger variability (particularly in terms of cost) compared to the design 1 and 3 that are farther from the nominal Pareto set. For instance, design 1 has the lowest variability among all of the points shown in Figure 3.23. However, design 1 is the only design that has material B in the center layer and this has contributed to a significant degradation in its objective function values. This can be intuitively justified since material B is the most expensive material and the variations come from the material A's properties. It is quite interesting to compare design 2 from the robustly non-dominated optimal set (using L_2 norm) to design 4 from the nominal Pareto set. Although both designs show little difference in their performance (i.e., objective function values), design 2 has smaller variability and better $WCSD$ from target. Again, this can be quickly identified in design variable space because both designs use material A in the center layer but the center layer of design 2 is thinner than design 4. This makes the resulting variability relatively smaller in design 2 compared to design 4. Similar analogy can be applied to design 5 which uses material A in its center layer and has the highest thickness for the center layer among all of the five selected designs. Design 6 and 7 are obtained by

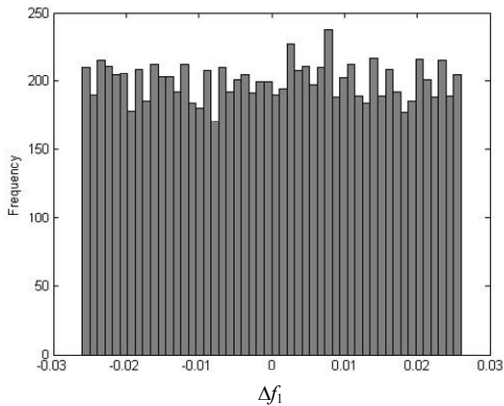
approach 1 but using L_1 and L_∞ respectively. Both of them are relatively close to nominal Pareto. However, both of them, similar to design 4 have thicker center layer of material A, resulting in higher variability particularly in terms of their cost. Design 8 and design 9 are generated using the approach 2. In particular, design 8, is almost a part of the nominal Pareto in terms of the nominal performance. If one compares designs 2,4, and, 8, while they are almost non-dominated with respect to each other in terms of nominal performance, design 8 has much lower variability (from nominal) in terms of both objective functions (see Table 3.7). Also, design 9 which is not close to the nominal Pareto, performs better than design 1 (and is non-dominated with respect to design 3). Nevertheless, the purpose of obtaining an optimal robust design is not to minimize the variability but to limit the adverse effects of variability in performance to a limited amount. Hence, the results of approach 2 while satisfy this criterion, are performing better in nominal cases. In order to have a quantitative comparison and to verify the robustness of the obtained solutions, a verification study on the selected design points is performed in the following sub-section.

Again, it is estimated that the total number of function calls required for this example using both of the robust optimization approaches. The top level problem in both approaches requires about 3000, and for the feasibility robustness for both approaches need to have 500 function calls. The approach 1 needs about 1000, and 1000 function calls for *WCSD* and *BCSD* respectively. The approach 2 requires around 1000 and 1000 calls for obtaining maximum deviation from nominal values for first and second objective functions. In the worst case, for this example, the number of function calls using both approaches is estimated to be equal to 7,500,000 using a genetic algorithm based

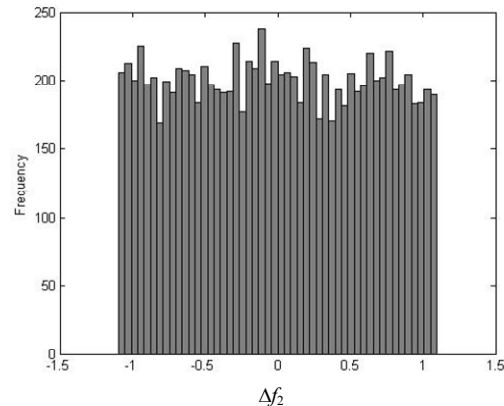
technique for the optimizer. Again, since the number of objective functions for this example is only two, both approaches have approximately same computational efficiency.

3.6.2.3. Verification Study:

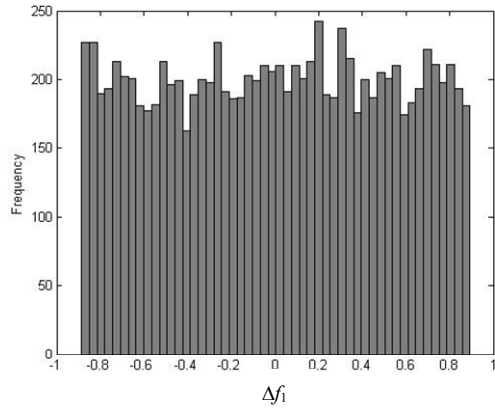
To verify the robustness of the results obtained by each of the approaches, design parameters p_1 and p_2 are randomly perturbed for 10000 times around their nominal values within the given ranges of variation. Then the new objective function values are calculated (i.e., a Monte Carlo simulation using uniform distribution similar to that of numerical example). The histograms of the output of the simulation for each of the selected designs are provided in Figure 3.26 (a) – (n). The solutions that their variability is within the ranges specified above, are also identified.



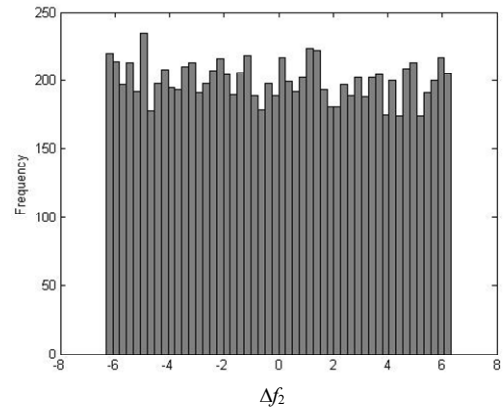
(a) Design 1 – Δf_1



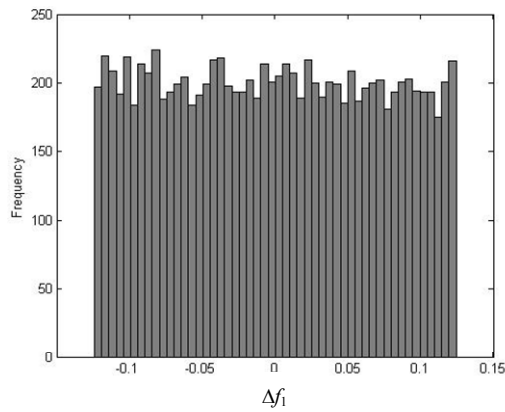
(b) Design 1 – Δf_2



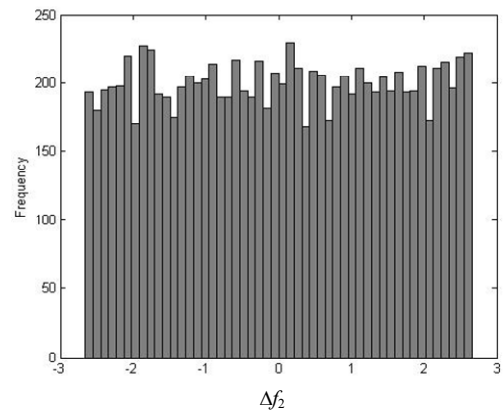
(c) Design 2 – Δf_1



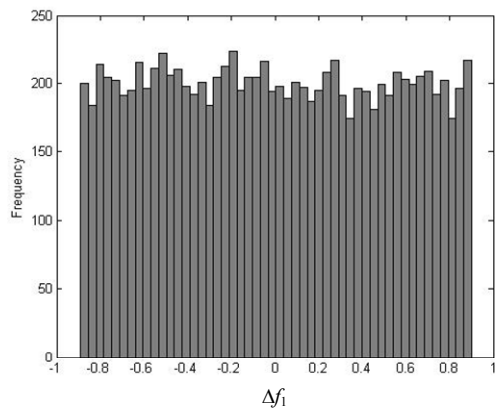
(d) Design 2 – Δf_2



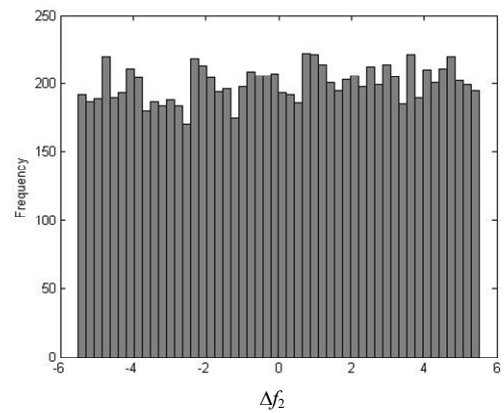
(e) Design 3 – Δf_1



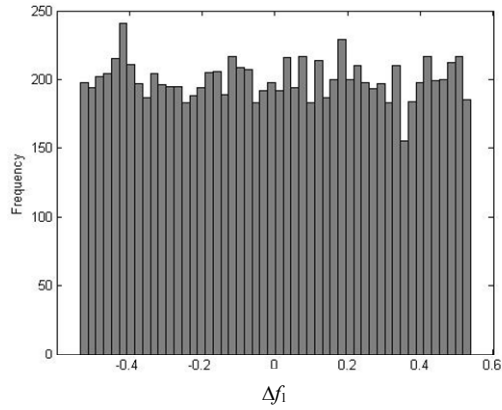
(f) Design 3 – Δf_2



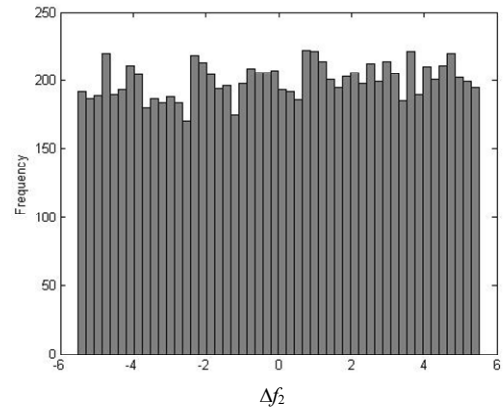
(g) Design 4 – Δf_1



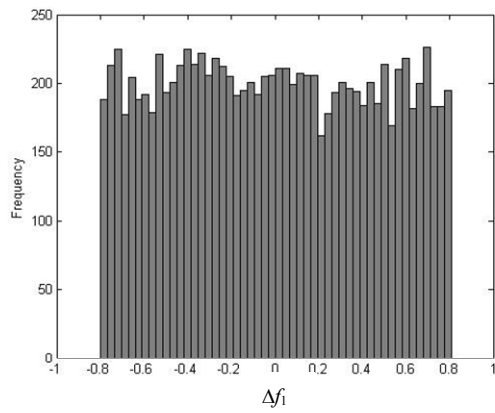
(h) Design 4 – Δf_2



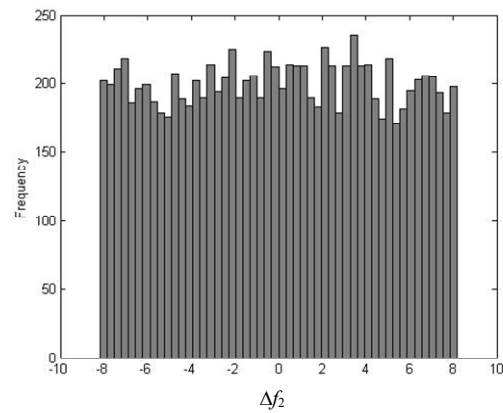
(i) Design 5 – Δf_1



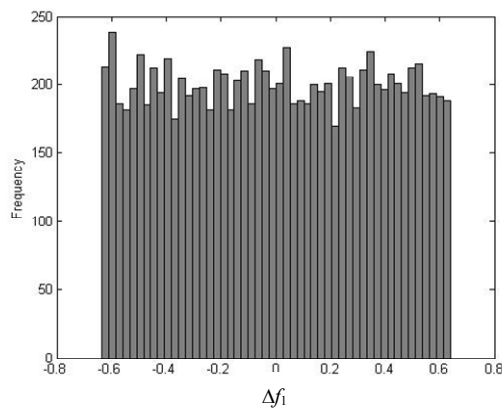
(j) Design 5 – Δf_2



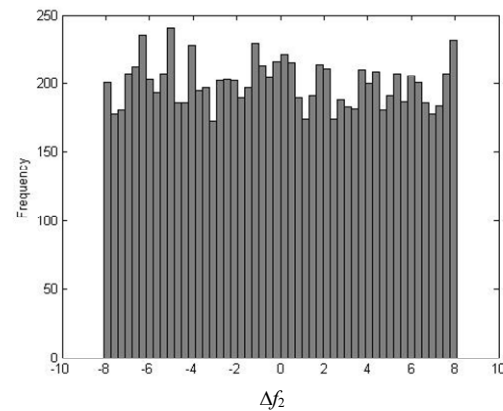
(k) Design 6 – Δf_1



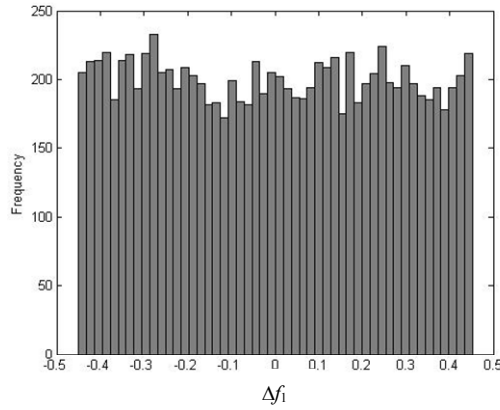
(l) Design 6 – Δf_2



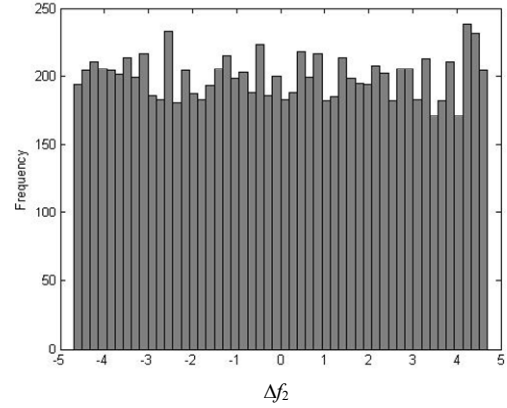
(m) Design 7 – Δf_1



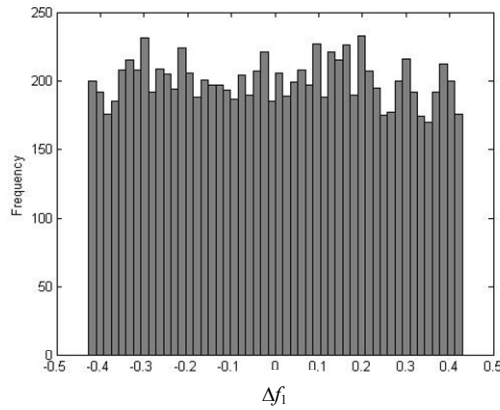
(n) Design 7 – Δf_2



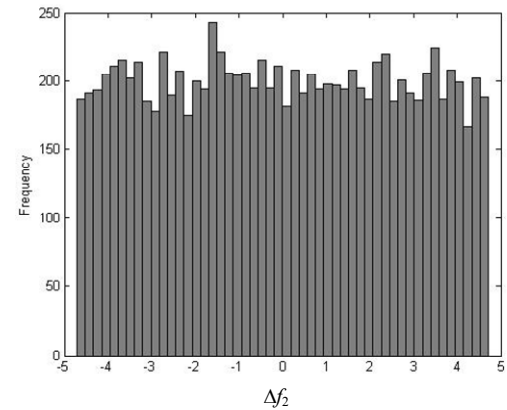
(o) Design 8 – Δf_1



(p) Design 8 – Δf_2



(q) Design 9 – Δf_1



(r) Design 9 – Δf_2

Figure 3.26: Histogram of the output for selected vibrating platform designs

Similar to that of the numerical example, the maximum absolute deviation from nominal (i.e., Δf^H) for f_1 and f_2 are obtained and the results are shown in Table 3.7. Also, in order to be able to make a quantitative performance comparison among the selected designs the nominal objective function values are provided in the first two columns of Table 3.7.

Point	f_1	f_2	Δf_1^W	Δf_2^W
1	150.7108	355.7819	0.0260	1.0960
2	284.9879	143.1559	0.8940	6.3346
3	138.4839	186.0439	0.1245	2.6597
4	271.4622	125.8953	0.9005	5.5117
5	384.3916	205.8603	0.5380	7.4621
6	351.6329	222.0769	0.6388	8.1077
7	344.4149	204.2766	0.8065	8.1914
8	229.9232	114.1587	0.4520	4.7013
9	240.7242	199.2154	0.4288	4.7100

Table 3.7: Comparison of objective function variation from nominal values

As expected the selected designs from robust Pareto (i.e., designs 8 and 9) exhibit variability within the designer’s requirements (i.e., $\Delta f_1 \leq 5$ and $\Delta f_2 \leq 5$) while their multi-objective performance is often better than the solutions obtained by the approach 1 (e.g., compare design 8 with design 1 or design 3). In particular, the designs 6 and 7 which are obtained by the approach 1 using L_1 and L_∞ norms respectively, show much higher (and unacceptable) variations with respect to the second objective function.

3.7. COMPARISON OF THE TWO ROBUST OPTIMIZATION APPROACHES

Two different approaches for multi-objective robust optimization were developed in Sections 3.3 and 3.4 and demonstrated with two examples in Section 3.6. While both approaches use the same measure for feasibility robustness assessment, the procedure for multi-objective robustness assessment in each approach is different. In the following, the differences are reviewed and the advantages and shortcomings of each approach are discussed.

The approach 1 needs two reference (i.e., target and bad) points in the design objective space and uses a distance metric (L_q norm) to locate multi-objective worst case scenario for each design. Another similar distance metric is used to calculate the variability of a design along the direction for target (i.e. distance between worst case and best case points in the objective space with respect to target).

There are a few advantages in using the approach 1. As discussed in Section 3.5, the approach 1 is computationally more efficient especially for problems where the number of objective functions is large. For any multi-objective robust optimization problem with I objective functions, approach 1 needs to solve only two optimization sub-problems in order to calculate the objective robustness measures. Moreover, since the approach 1 takes the worst case performance as one of the objectives into account, the acceptable performance of the solutions at the worst case scenario is guaranteed. These facts can be observed by the examples. The solutions obtained by approach 1 have a lower variability and $WCSD$ measure compared to the nominal Pareto solutions or those solutions obtained by approach 2 (e.g., see designs 1 and 6 in the first example or designs 2, 6, and, 7 in the second example).

The approach 1 despite its strengths has a number of shortcomings. Optimizing the worst case value of the objective function(s) can be very conservative, and most of the obtained solutions are likely to perform poorly in the nominal case of design parameters. Furthermore, a variability measure in the direction of a target point can overlook a large variability of one objective function which is not in the direction of target (for instance, see design 2 in Figure 3.20). Also, approach 1 is not applicable to cases where a designer is interested to limit the variability of some (not all) of the

objective functions. Finally, the obtained solutions can be sensitive to the location of the reference points (i.e., target and bad points).

The approach 2 (discussed in Section 3.4) does not use a distance metric to assess the multi-objective robustness of a design alternative. It also does not require any reference point in the objective space. The solutions obtained by approach 2 are much less conservative compared to those obtained by approach 1, because only those designs that exhibit unacceptable variability are eliminated. Since the variability is calculated for any individual objective function separately (and not in any particular direction) it does not overlook non-robust designs. Also, the designer has the flexibility to limit the variability of all or only a selected number of objective functions. The main drawback of the approach 2 is its computational complexity. As discussed in Section 3.5, it requires I sub-optimization problems to assess the objective robustness of any solution where there are I -objective functions (assuming the designer has limits on variability for all objective functions).

Overall, each presented robust approach can be used with any robust multi-objective design optimization problem, and depending on the problem a designer can choose either of the discussed approaches to obtain robust solutions. Due to its flexibility, it is decided to use robust optimization approach 2 to address the robust single product and robust product line design optimization problems that are discussed in Chapters 4 and 5 respectively.

3.8. SUMMARY

This chapter has presented two different deterministic approaches for robust multi-objective design optimization problems. First, the idea of design sensitivity in more

than one dimension is explored using the sensitivity region concept. Both approaches utilized this concept for robustness assessment. The approach 1 used an L_q norm distance metric to obtain two measures, namely, the worst case scenario distance from target and the variability of each design alternative. The approach 2 examined whether or not the variability in the objective function values for each design alternative is within an acceptable range. For both approaches, in order to examine the feasibility of each design under variation of uncontrollable design parameters, the worst case value of each design constraint was obtained and checked for feasibility. If any constraint was violated under variation of design parameters, then the corresponding design alternative was identified as one that does not have a feasibility robustness property. The robust multi-objective design optimization approach 1 was formulated based on the above-mentioned measures and the feasibility robustness criterion (see Figure 3.6). In order to avoid some of the limitations of the approach 1, approach 2 to robust multi-objective design optimization was introduced. The approach 2 utilized the same feasibility robustness assessment module, while the objective robustness was used as additional constraints to the original problem. While it was less conservative and more flexible, in reality it is computationally more expensive than the approach 1.

To demonstrate each of the robust optimization approaches, each approach is applied to a numerical and an engineering design optimization problem (i.e., design of a vibrating platform). The results were compared and the advantages and the shortcomings of each method were illustrated by comparing a few selected solutions. To perform a verification study, a sensitivity analysis was carried out using Monte Carlo simulation to

verify and compare the robustness of selected solutions. Finally, a detailed comparison between both of the robust multi-objective design optimization approaches is provided.

In the next chapter a bi-disciplinary (i.e., engineering design-marketing) framework is developed that applies robust optimization approach 2 to design of a single product (a power tool). The results of the implementation of this framework are designs that are robustly optimum in both design and marketing domains.

CHAPTER 4

SINGLE PRODUCT ROBUST OPTIMIZATION

4.1. INTRODUCTION

In product design, both design and marketing related attributes are likely to have variability. The source of this variability is parameters that the designer does not have control over. Such variations can cause unwanted changes in product performance that in turn may affect customers' preferences for a product. For instance, in a corded power tool, design attributes that might have variability include engineering specifications of the tool such as armature temperature and output torque at a specified motor speed. The marketing attribute of the power tool such as life may also vary due to changes in parameters in the design domain. Variability in marketing attributes can also arise due to variances inherent in marketing parameters when marketing researchers estimate customer preferences for such attributes [McFadden, 1986].

From an engineering design perspective, a design alternative should maintain its feasibility under variations from uncontrollable parameters, have variations in its performance that are within an acceptable range, and most importantly, also exhibit the best possible performance.

From a marketing perspective, one should also consider the uncertainties in the preference estimation due to the fact that there is never a perfect fit between the preference elicitation model and the collected marketing survey data. In the proposed approach in this chapter, the "preference robustness" is considered as a criterion that accounts for: 1) the impact of variations in the design domain on the values of marketing

attributes; and 2) the variations inherent in the marketing (conjoint) model parameter estimates.

The purpose of this chapter is to present an integrated design-marketing approach that takes all of the above mentioned issues in both design and marketing domains. The single product design solutions obtained by the proposed approach are optimum and robust in both domains.

The organization of this chapter is as follows. Section 4.2 gives a general overview of the approach. Section 4.3 presents details of the robustness assessment modules in both design and marketing discipline. First, a description of the design robustness model followed by the marketing preference robustness model will be provided. Details of the overall integrated approach are presented in Section 4.4 followed by an example in Section 4.5. Finally, the chapter is concluded in Section 4.6 with a summary.

The marketing conjoint model and the associated data analysis presented in Section 4 are borrowed from papers co-authored with marketing colleagues (i.e., [Besharati et al., 2004] [Luo et al., 2005]).

4.2. THE OVERALL APPROACH

Figure 4.1 gives a flowchart of the overall approach. It is assumed that initial exploratory studies have already been conducted by a product development team (consisting of marketing and design experts) in identifying general dimensions along which the product is expected to do better compared to existing products in the market. These dimensions form the basis for the selection of design and marketing attributes considered in the approach.

The approach has two main components: Engineering design model (left column of Figure 4.1) and marketing model (right column of Figure 4.1). In the design model, first, the nominal values and the range of variations for uncontrollable design parameters are identified. For example, for a corded power tool, such as a grinder, these design parameters could be ambient temperature, source voltage and current, for which the ranges of variations are specified. Next, in the design model, a set of design inputs (i.e., design variables, nominal values and ranges of variations of design parameters) is selected. Design inputs are fed into design simulation software that calculates an estimate of design attributes (or performance) for each design alternative under consideration. Some design attributes are expected to show little or no variation while others may exceed beyond an acceptable range. Depending upon how performance and/or feasibility of a design responds to such variations, two measures for design robustness, namely multi-objective robustness and feasibility robustness, are developed and used to measure “engineering robustness”, as shown in the flowchart. (Details of the design robustness measures are given in Chapter 3.)

Some product attributes not only reflect engineering design performance of a product but also are key elements to a customer’s purchase decision (e.g., product life, maximum output power, in the case of a power tool). In the proposed approach, this type of product attribute is designated as a “common attribute”. Such an attribute is common in both marketing and engineering design models. Some common attributes, such as number of operations per battery charge for a cordless power tool, are derived from the design simulation and can be used directly in the marketing module. Others are mapped (or converted) to marketing attributes for capturing the preferences of consumers. For

example, in the case study in Section 4.6, the design attribute “maximum output power” of a product is mapped to the marketing attribute “amp rating”, an attribute that power tool customers usually recognize¹. The notion of common attributes is introduced to ensure that design alternatives that have potential appeal in the market are not eliminated during a robust design optimization process. On the other hand, there may be product attributes that are not common to both marketing and engineering models. For example, in a corded power tool, attributes like brand, switch type, and girth size, which do not affect product design performance, are quite important to the market performance of a product and hence appear purely as marketing attributes. The proposed approach takes into account the variability in customers’ preferences (or utilities) for common attributes (such as life and amp rating) that come from both design and marketing domains. It should be noted that only common attributes are relevant to measuring simultaneous robustness in design and marketing.

In the marketing model (initiated and developed by the colleagues in marketing [Luo et al., 2005] [Besharati et al., 2004]) (right column of Figure 4.1), the most important customer needs are first identified based on an a priori exploratory market study. Important customer needs can be marketing attributes such as retail price, brand name, power (e.g., amp rating), and product life. Once these attributes and their possible levels are identified, a marketing technique known as “finite mixture conjoint analysis” is used (see Section 4.4) to estimate customer utilities for different levels of attributes. In a typical conjoint experiment, consumer’s preferences are estimated through their evaluations of a set of hypothetical product profiles (or alternatives) specified in terms of

¹ This is based on the discussions with the industrial partner. In working with industrial users of their grinders they have found “amp rating” being used as an indicator for power by the users.

a combination of levels of different product attributes. Estimated attribute-level utilities are then used to calculate potential market shares of the proposed product alternatives against existing competitors' products. The proposed approach also accounts for variability in the market share estimates. Hence, the output of the marketing model includes estimates of market share and its variation which can be used to measure preference robustness. The preference robustness measures together with the engineering design robustness criteria are used in the optimizer to generate a set robust product design alternatives, each of which not only performs well from both engineering design and marketing performance points of view but also exhibits low variation with respect to its performance levels under uncontrollable parameter variations. The ranking rules used in the proposed bi-disciplinary optimizer are given later on in Section 4.5. The optimizer continues to iterate until a stopping criterion is reached. The output of the optimization includes a set of product designs. These products are not only optimal in both design and marketing domains, but also their performance is ensured not to fluctuate beyond an acceptable range.

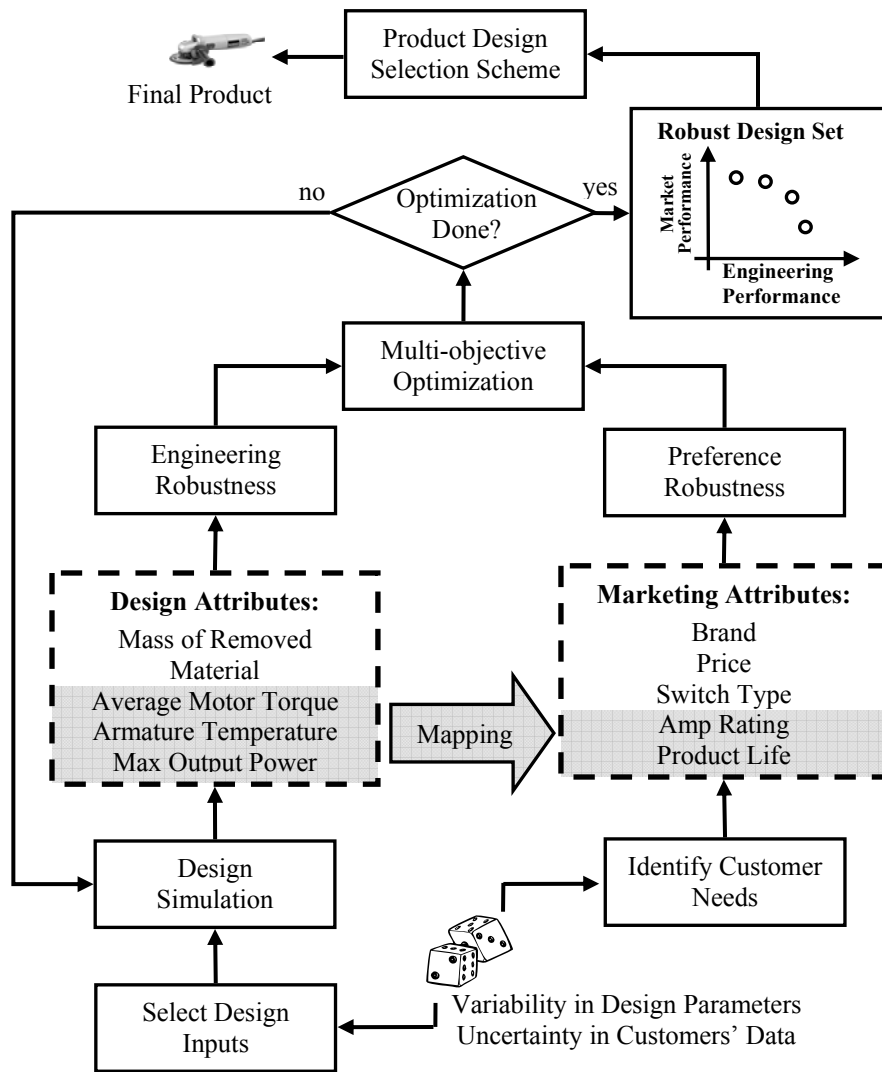


Figure 4.1: Overall approach

In the final stage of the proposed product design development process, one may need to make a selection among the generated robust product design alternatives (see, e.g., [Fuhita et al, 1997] [Li and Azarm, 2000] [Besharati et al., 2004]). For instance, the producer can develop these designs further into prototypes and conduct additional performance evaluation in the field to select the final product for mass-production. Also, the managers can make a selection decision based upon the market positions of competitive products to maximize the new product's competitive advantage. Finally, the optimal product can be chosen based on the long-term profit that it will provide. For

example, a technique such as the life-cycle product cost-benefit analysis [Ramdas and Sawhney, 2001] can be used to map out the most profitable product based on the combination of production cost and life-cycle operating cost incurred over the product's life cycle. The issue of product design selection is beyond the scope of this dissertation. However, a copy of a proposed approach for product design selection under uncertainty is provided in Appendix I [Besharati et al., 2005].

4.3. ROBUSTNESS ASSESSMENT

As mentioned in Section 4.2, the goal in robustness assessment is to determine if a product design satisfies the robustness requirements in both design and marketing domains. Two separate modules, namely design robustness and preference robustness, are used to examine the robustness of each product design. The design robustness module is built based upon the assumption that the simulation software is deterministic and that it receives a set of design variables and uncontrollable design parameters and computes a corresponding set of design attributes (e.g., maximum output power, weight). The design performance attributes can be used as objective functions and/or constraints in the robust design optimization approach. The details of the proposed robustness assessment module were already provided in Chapter 3 (see approach 2 in that chapter).

As highlighted earlier, there are several attributes that are specific to marketing domain and do not play a role in design performance (e.g., brand, price). However, the attributes that are common to both design and marketing domain (such as product life) do have a role in the design module. In this chapter, the marketing attributes (excluding the common attributes) are all discrete. Each design alternative can be enumerated over

marketing attribute levels, and thus generate several product alternatives (that are based on the same design alternative).

Since the marketing information is not considered in the robust design optimization methods of Chapter 3, the proposed robust design optimization approach (see Figure 3.8) may overlook design candidates that are good alternatives from a marketing performance viewpoint. Therefore, it is important to also take the marketing aspects of the product into account during the design process, as discussed next.

4.3.1. Marketing Model with Preference Variation

A successful product design should not only satisfy engineering design requirements but should also perform well in the market. In order to assess the marketing performance of a product, a marketing model is developed. Specifically, a finite mixture conjoint model is used to capture the customers' preferences and determine a product's impact on the market.

The details of such a model are given in Section 4.3.1.1. Section 4.3.1.2 covers the sources of variability in customers' preferences and the approach for modeling such variations. Finally, Section 4.3.1.3 is devoted to the proposed robust marketing optimization approach.

4.3.1.1. Finite Mixture Conjoint Model

An important goal in product design development is to respond optimally to customer needs in order to obtain higher profits and/or market shares for the manufacturer. There are many methods in the literature to model and measure customer preferences and utilities. Among these methods, conjoint analysis is very popular in the marketing literature [Green and Srinivasan, 1990].

In a typical conjoint-based method, customers' utilities for different levels of marketing attributes are estimated through customers' evaluations of a set of hypothetical product profiles (or alternatives). The simple premise in conjoint models is that customers evaluate the overall utility of a product by combining the separate utility value (i.e., part-worth) of specific levels of marketing attributes that define the product. Estimated part-worth utilities are then used to calculate the utility of each proposed product against alternative products and existing competitors' products. Since consumers generally have heterogeneous preferences towards the products, a finite mixture multinomial logit model is commonly used to capture the preferences of different market segments ([Kamakura and Russell, 1989] [Vriens et al., 1996]). In such a model, it is assumed that there are several segments in the market. Across different market segments, consumers have different preferences towards products. Within each market segment, the consumers are assumed to have identical preferences. Finite mixture model provides a way to segment the market based on consumers' responses to the conjoint experiment, and the number of segments is determined by the Akaike's Information Criterion [Akaike, 1973]. Akaike's Information Criterion is commonly used in the comparison of competing models in order to identify a model that best explains the observed data, penalized by the additional complexity of the model [Kamakura and Russell, 1989]. The details of the marketing model are presented as follows.

A choice model for a conjoint choice experiment starts with J individuals (consumers), each evaluating K different sets of product alternatives (called choice sets). Each of the K choice sets contains M product alternatives. Each product alternative is defined by the combination of different levels of marketing attributes. A customer

chooses a profile from each of the K choice sets based on his preference for the products. Assuming the existence of $s = 1, \dots, S$ market segments with segment size SS_s , the utility u of an individual c for product m in choice set k , given that this individual belongs to segment s , is defined as follows [McFadden, 1986]:

$$u_{cs}(\mathbf{y}_{mk}, P_{mk}) = (\mathbf{y}_{mk} \boldsymbol{\beta}_{sy} + P_{mk} \boldsymbol{\beta}_{sp}) + \varepsilon_{csmk} \quad (4.1)$$

In this case, all marketing attributes are coded as dummy variables. Therefore, \mathbf{y}_{mk} is an $\alpha \times 1$ vector of zeros and ones with ones representing the corresponding marketing attribute levels of product m . Because of the linear dependency nature of these dummy variables within each marketing attribute, in order for this model to be identified, one level for each attribute is omitted in the estimation. Furthermore, a value that is equal to the negative of the sum of the utility estimates of all other levels is used as the utility for the missing level. Assuming that the random component ε_{csmk} follows an independent identical double exponential distribution, the probability that product m is chosen from choice set k , subject to consumer c being a member of segment s , can be expressed as follows:

$$\Pr_{cmk s} = \frac{\exp(\mathbf{y}_{mk} \boldsymbol{\beta}_{sy} + P_{mk} \boldsymbol{\beta}_{sp})}{\sum_{mm=1}^M \exp(\mathbf{y}_{mmk} \boldsymbol{\beta}_{sy} + P_{mmk} \boldsymbol{\beta}_{sp}) + \exp(\text{cons}_s)} \quad (4.2)$$

Based on the conditional probability in Eq. (4.2), if θ_s represents the likelihood that a consumer is a member of market segment s , the unconditional probability of consumer c choosing product m from choice set k can be computed as ([Kamakura and Russell, 1989]):

$$\Pr_{cmk} = \sum_{s=1}^S \theta_s \Pr_{cmk s} \quad (4.3)$$

Based on Eq. (4.3), the log-likelihood of observing all the choices in all the choice sets for all the customers in the sample can be written as [Kamakura and Russell, 1989]:

$$LL = \sum_{c=1}^C \sum_{m=1}^M \sum_{k=1}^K \ln(\text{Pr}_{cmk}) \quad (4.4)$$

Using maximum likelihood estimation method on Eq. (4.4), a set of estimates of part-worth utilities for each level of marketing attributes can be identified that are most likely leading to the choices observed in this case.

The above estimation procedure is conducted for each model (scenario) as defined by the number of segments in the market (one segment through five segments). Finally, Akaike's information criterion (*AIC*) is used to determine the optimal number of segments in the market. The model (scenario) with the smallest *AIC* value is the one that best explains the observed choices without overfitting the data (see [Vriens,1996] [Akaike, 1973]). Akaike Information Criterion is defined as in Eq. (4.5) where *LL* is the log-likelihood value from Eq. (4.4), *q* is the number of part-worth utilities estimated in the model (scenario) and *SS* is the sample size (number of customers times the number of choice sets).

$$AIC = -\frac{2(LL - q)}{SS} \quad (4.5)$$

The estimation procedure, thus, provides the estimates of the number of segments in the market, part-worth utilities for each level of the marketing attributes for each segment, and the asymptotic variance-covariance matrix of part-worth utilities [Luo et al., 2005] [Carrol and Green, 1995]. In addition, the posterior probability that a customer is a member of a particular segment *s* can be estimated by updating in a Bayesian fashion the

prior probability of membership θ_s , using the observed choices of the consumer as a conditioning event.

4.3.1.2. Construction of Preference Variation

Based on the outputs of the estimation procedure outlined in Section 4.3.1.1, one is able to obtain not only the point estimates of the part-worth utilities of each level of marketing attributes but also a variance and co-variance matrix of the part-worth utility estimates. In the context of preference robustness, the proposed methodology integrates the following types of variations in consumer preference. First, variation from the engineering domain in attributes that are common between the engineering design module and the marketing module (e.g., the actual amp rating of the product may vary when used in different seasons of the year, say between 6 and 7 amps). Second, variations inherent in the conjoint part-worth estimation because of the imperfect model-data fit. According to [Ben-Akiva and Lerman, 1985], choice-based conjoint part-worth utility estimates can be considered as asymptotically normal when the sample size is sufficiently large [Ben-Akiva and Lerman, 1985]. Therefore, the method described in the following paragraphs can be used to construct the interval estimates of the part-worth utilities for various design alternatives considered in the design process.

The following paragraphs explain the procedure to calculate the interval estimate of the conjoint part-worths at a segment level. For continuous product attributes (such as price and amp rating), the standard procedure of pair-wise linear interpolation is used to calculate the point estimates in between specified conjoint levels. Next, the segment-level interval conjoint estimates are used to construct the interval estimate of market share of each hypothesized product design given a set of competitors.

For discrete product attributes (such as brand, switch type, and girth type in the case of a power tool design), the interval estimate (95% two-sided confidence level) of the conjoint part-worth utilities can be constructed as follows²:

$$[u_L, u_U] = [u - 1.96z, u + 1.96z] \quad (4.6)$$

where u_L represents the lower bound of the utility estimate and u_U represents the upper bound of the utility estimate; u represents the point estimate of the conjoint part-worth utility; and z represents the standard error of the point estimate of the conjoint part-worth utility, which is basically the square-root of the variance measure associated with the specific part-worth utility in the asymptotic variance-covariance matrix.

For continuous and non-common product attributes (such as price), the standard procedure of pair-wise linear interpolation (See Sawtooth Choice-Based Conjoint User Manual, 2001) is used to calculate the point estimate and the lower and upper bounds of the 95% simultaneous confidence levels for utilities of price that are in between specified levels. For example, for a price (P) that is in between two specified price levels (P_1 and P_2) in the conjoint study, the point estimate of the conjoint part-worth utility can be calculated as follows:

$$u(P) = \frac{(P_2 - P)}{(P_2 - P_1)} u(P_1) + \frac{(P - P_1)}{(P_2 - P_1)} u(P_2) \quad (4.7)$$

where $u(P_1)$ represents the point estimate of the conjoint part-worth utility at price level P_1 and $u(P_2)$ represents the point estimate of the conjoint part-worth utility at price level P_2 . And the interval estimate of the conjoint part-worth utility for this price can be calculated as:

² 95% confidence level is used here because this is the most commonly criterion in statistics literature [Greene, 2000]. This percentage can be adjusted based on the product manager's preference and in this regard the essence of the proposed approach will not be affected.

$$[u_L, u_U] = [u - 1.96(\text{var}(u))^{1/2}, u + 1.96(\text{var}(u))^{1/2}] \quad (4.8)$$

where, $\text{var}(u)$, the variance of the utility, can be obtained using Eq. (4.9) [Greene, 2000]:

$$\text{var}(u) = \left(\frac{P_2 - P}{P_2 - P_1} \right)^2 z_1^2 + \left(\frac{P - P_1}{P_2 - P_1} \right)^2 z_2^2 + 2z_{12} \frac{(P_2 - P)(P - P_1)}{(P_2 - P_1)^2} \quad (4.9)$$

where z_1 represents the standard error of the point estimate of the conjoint part-worth utility at price level P_1 ; z_2 represents the standard error of the point estimate of the conjoint part-worth utility at price level P_2 ; and z_{12} represents the covariance of the two conjoint part-worth utility estimates.

In equations (4.6) – (4.9), only one component of the preference robustness is addressed that accounts for the uncertainties in customer choices in the preference ranking process. The second component of the preference robustness in the marketing model comes from the variation in the performance of the product in the engineering domain. For example, when the tool is used in different usage situations and under different conditions, the actual amp rating of the power tool may vary ± 0.5 amps from the nominal value. This variation will also have impact on consumer's preferences for the tool. In the proposed model, the impact of such variation on the consumer's preference is accounted for. First, the ranges of utility variation are calculated for the lower and the upper bounds of the power amps variation using Eq. (4.7) – (4.9). Next, the lower and the upper bounds of the conjoint utility for one nominal value of power rating are constructed by considering both components of the preference robustness. Figure 4.2 plots the lower and the upper bounds of conjoint utilities when amp rating changes. For each point that is in-between levels, Eq. (4.7) is used to calculate the point utility estimate and Eqs. (4.8) – (4.9) are used to calculate the upper and lower bounds. For simplification, only the upper

and lower bounds of the utility estimates are highlighted and the point utility estimates are not shown in Figure 4.2.

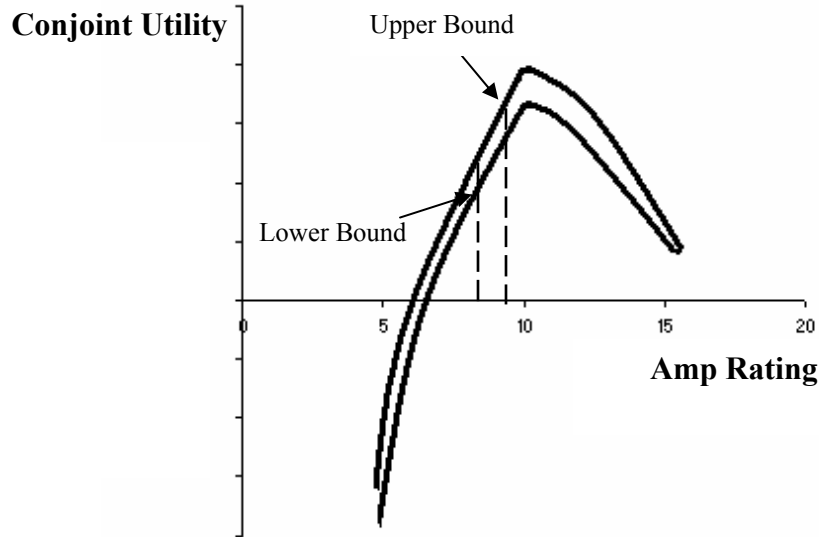


Figure 4.2: Variability calculation for common attributes

Once the interval estimates of conjoint part-worth utilities for each level of the marketing attributes are obtained at the segment level, the upper and lower bounds of the conjoint utility for each product alternative (at the segment level) can be calculated by summing up the lower and upper bounds of conjoint part-worth utility estimates for each marketing attribute. When calculating the market shares, the impact of variation needs to be considered not only on the product being developed (hereafter called the “own” product) but also on the competing products. In other words, the interval estimates of the conjoint part-worths are used in the utility calculation of the competing products too. The Eq. (4.6) is used to obtain the interval conjoint part-worth utility estimates for discrete marketing attributes. The formulae in Eqs. (4.7) – (4.9) are used to calculate the interval conjoint estimate for continuous and non-common attributes. With regard to the common attributes, the variation information from the engineering lab is obtained. For example,

after testing the competing products in different usage situations and under different conditions, it is found that the actual amp rating of the competing product 1 varies from 8.3 to 9.5 amps while its nominal value of the amp rating is 9 amps. Such information is used to calculate the interval conjoint part-worth utility estimates of the common attributes for the competing products. The calculation procedure is the same as the one described in Figure 4.2.

Thus, when calculating the upper and lower bounds of market shares for product alternatives being designed, one should consider not only the worst and best possible market performance of the alternative being designed but also those of the competing products.

The lower bound of conjoint utility for the own product in s^{th} segment is denoted as $U_{\text{lower_bound},s}$, the upper bound of conjoint utility for own product in s^{th} segment is denoted as $U_{\text{upper_bound},s}$. For competing products (cp_1, \dots, cp_R), the lower bound of conjoint utility for r^{th} product in s^{th} segment is denoted as $U_{cp_r,\text{lower_bound},s}$, the upper bound of conjoint utility for r^{th} product in s^{th} segment is denoted as $U_{cp_r,\text{upper_bound},s}$. A measure of Market Share Variation (MSV) is defined as follows:

$$MSV = MS_{\text{upper_bound}} - MS_{\text{lower_bound}} \quad (4.10)$$

where $MS_{\text{upper_bound}}$ and $MS_{\text{lower_bound}}$ are upper bound and lower bound of market share for a product respectively and defined as:

$$MS_{\text{lower_bound}} = \sum_{s=1}^S \theta_s \frac{\exp(U_{\text{lower_bound},s})}{\exp(U_{\text{lower_bound},s}) + \sum_{r=1}^R \exp(U_{cp_r,\text{upper_bound},s}) + \exp(\text{cons}_s)}$$

$$MS_{upper_bound} = \sum_{s=1}^S \theta_s \frac{\exp(U_{upper_bound,s})}{\exp(U_{upper_bound,s}) + \sum_{r=1}^R \exp(U_{cp_r,lower_bound,s}) + \exp(cons_s)} \quad (4.11)$$

4.3.1.3. Robust Marketing Optimization

Figure 4.3 provides the flowchart of the proposed marketing approach which is used to obtain a set of robust product design alternatives. The approach has two starting points. From the marketing end, market researchers first conduct focus group study to decide the most important product attributes for the end users and the set of competitors in the marketplace. Based on this, a conjoint study is designed and conducted in the field. Next, the finite mixture conjoint model as outlined in Section 4.3.1.1 is used to calculate the conjoint part-worth utilities and variance-covariance matrix of these estimates. Another starting point of Figure 4.3 is from the engineering end. It starts with a design alternative whose design attributes are calculated by a design simulation. The common attributes are either derived directly (e.g., total weight of a product) or through appropriate mapping functions (e.g., product amp rating as a function of the maximum motor output power). For each design alternative, several different product alternatives can be created by enumeration over the marketing attributes (e.g., price, switch type). Given the specification of these product design alternatives and their variations, the market share and its variation (*MSV*) for every product alternative are calculated within the preference robustness procedure as described above. The optimizer obtains the set of products that not only has a high market share value but also has small variations in market share. The optimum set is obtained among all design alternatives provided by design simulation at their optimum levels of marketing attributes.

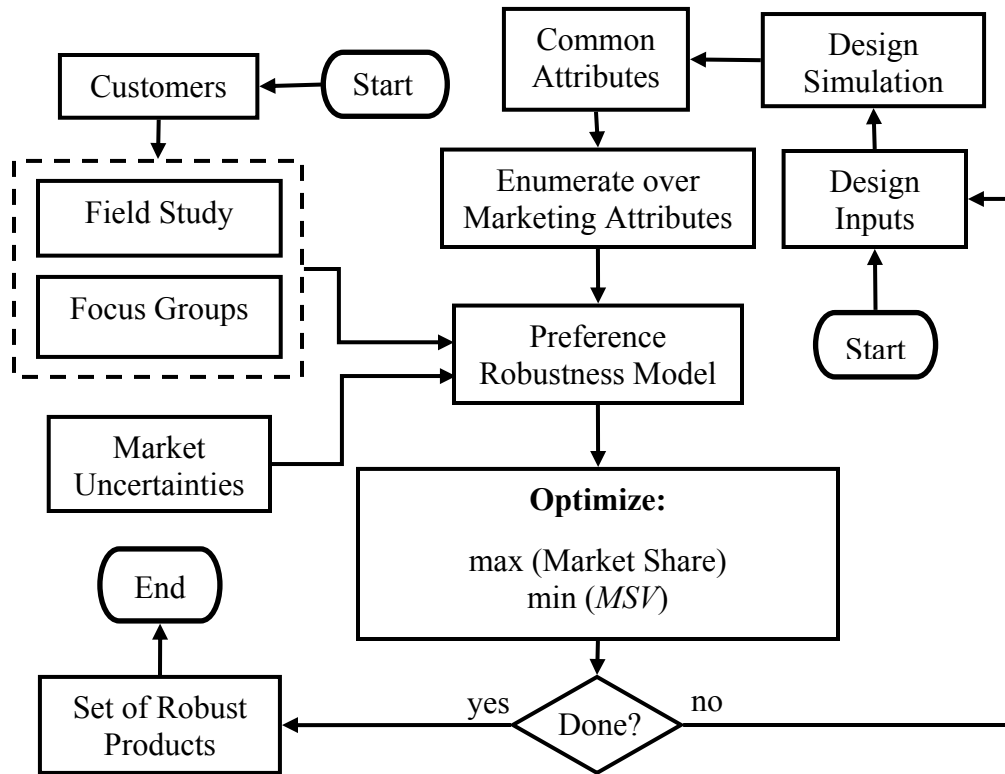


Figure 4.3: Robust marketing approach

4.4. INTEGRATED DESIGN-MARKETING APPROACH

The integrated design-marketing approach is shown in Figure 4.4. The approach has two starting points, one for the design model and the other for the marketing model. In the design model, the approach starts with a design alternative x . The design x is passed on to the design simulation to obtain the design attribute values (i.e., engineering performance objectives and constraints). Next, the design is evaluated for feasibility and multi-objective robustness. If it is not satisfying the feasibility robustness criterion or does not meet designer's requirements for multi-objective robustness, its objective function values are penalized (e.g., a positive penalty is added to each objective function for the case of minimization). Then, the design objective function values are analyzed

within the optimizer. An optimization technique can be used within the framework to perform the search. Each design is enumerated over the marketing attributes (excluding brand) to produce corresponding product alternatives that can be evaluated from the marketing point of view. In the marketing model, after estimating customer utilities from the conjoint analysis model, the overall customer utility and market share for that design is computed and passed on to the optimizer. The marketing objectives are to maximize the market share and to minimize its variability. Even though past research mainly focuses on market share maximization, one may argue that it is also important for product designers to weight between market share and its variability. When two product alternatives have comparable market shares, the product with smaller market share variability should be favored because there is a less amount of uncertainty associated with how this product will perform in the marketplace. The details of the proposed ranking algorithm are given in Section 4.4.1. This procedure continues until the stopping criterion, such as a maximum number of iterations, is reached. The approach ends after identifying a set of robust design alternatives.

4.4.1. Design-Marketing Evaluation of Product Alternatives

The optimizer used in the proposed approach (Figure 4.4) should evaluate and compare products based on their engineering design as well as market performance. The performance measures in both domains (disciplines) were defined in previous sections. Here the product evaluation is performed at the domain level (i.e., marketing or engineering design domain), and the optimizer obtains the product designs that show superior performance in both domains.

In the engineering design domain, robust products are considered (from both feasibility and multi-objective robustness points of view) and their performance and feasibility is evaluated using the design objective and constraint functions. In the marketing domain, both market share and its variations (*MSV*) are considered to assess the marketing performance and robustness of the products (maximize market share and minimize variation, given a set of competitive products). The rank ordering rule, which is used in the optimizer, is as follows: product **X** dominates product **Y** if it dominates (i.e., has a better performance) in at least one of the domains (i.e., design or marketing) while not dominated in the other. Alternatively, product **X** is dominated by **Y** if it does not dominate **Y** in any domain while being dominated by **Y** in at least one domain. If neither of these conditions holds, then products **X** and **Y** are non-dominated.

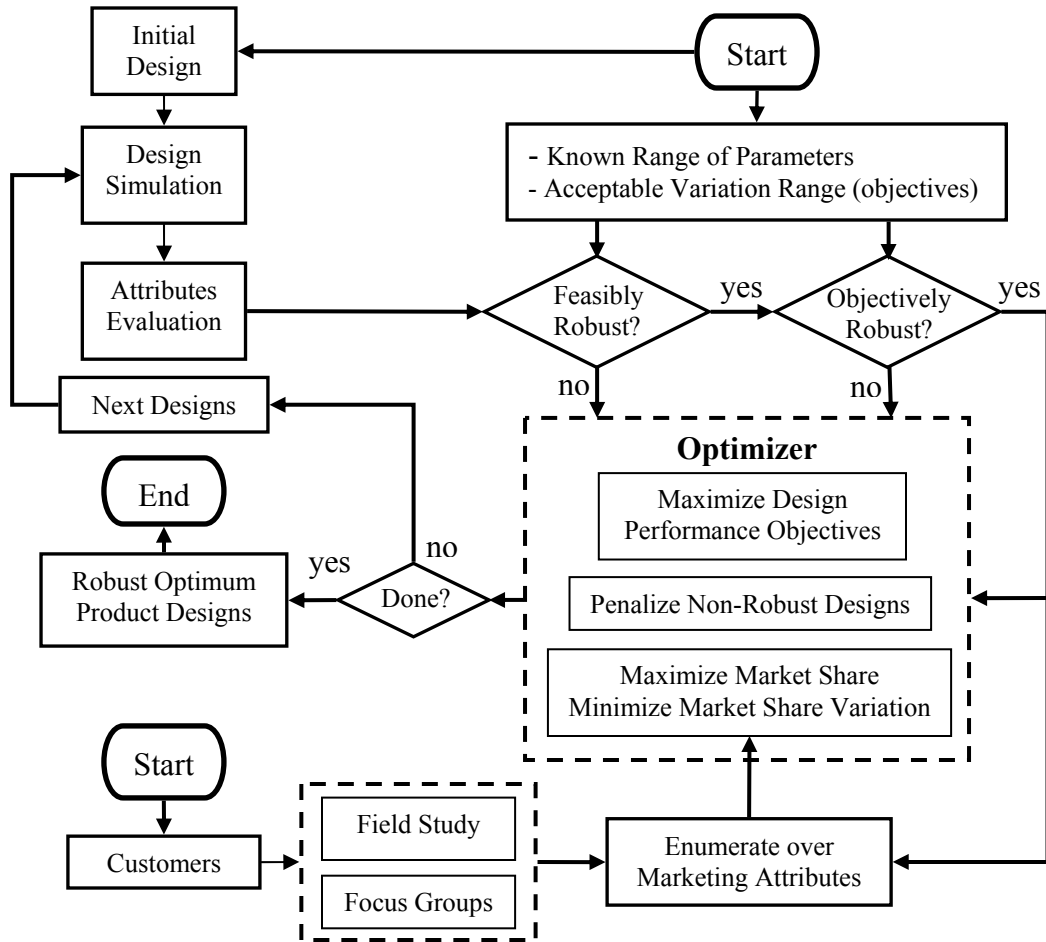


Figure 4.4: Integrated design-marketing approach

Figure 4.5 shows an example in which three product designs **A**, **B**, and **C** are being rank ordered. The left hand figure depicts the engineering design domain in which output speed is minimized while mass removed is maximized. The right hand side figure shows the marketing domain in which the market share of the product is maximized while the variation in market share estimates is minimized. In order to rank order the products, it is necessary to compare each pair separately. **B** dominates **A** in both design and marketing domains, and therefore, overall **B** dominates **A**. However, **A** dominates **C** in the design domain, but is dominated by **C** in the marketing domain. Such a conflict leads to declaring both **A** and **C** to be non-dominated products. Furthermore, between **B**

and C, **B** dominates C in the design domain. However in the marketing domain, **B** and C are non-dominated. Therefore, based on the above-mentioned ranking rule, **B** dominates C. In short, considering both design and marketing domains, **B** gets the highest (i.e., first) rank (no product dominates it), while both **A** and **C** are non-dominated with respect to each other, and are ranked second.

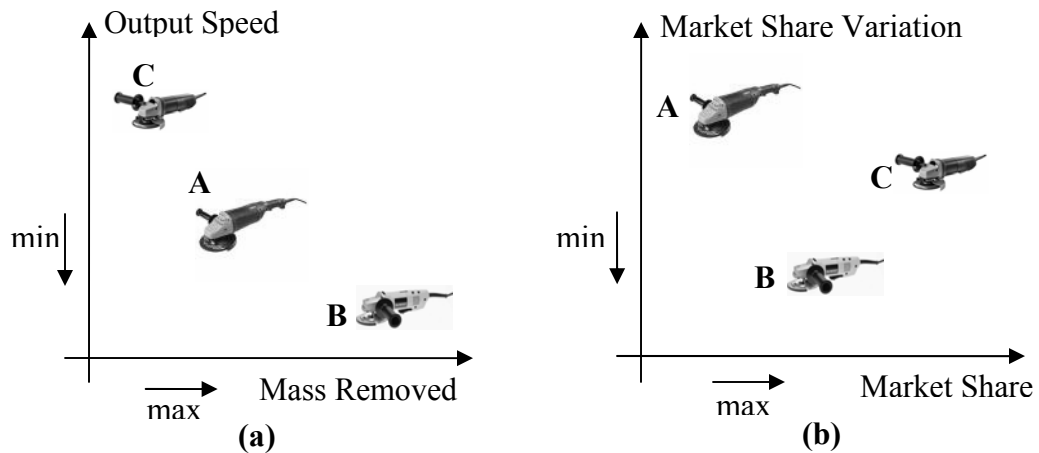


Figure 4.5: Rank ordering under uncertainty:
(a) design domain, and (b) marketing domain

4.5. EXAMPLE

This section demonstrates the proposed approach with an example: design of a corded power tool; a small angle grinder. The data and definitions (or preliminaries) for the example are given in Section 4.5 .1 followed by the robust design model in Section 4.5.2 and robust marketing model in Section 4.5.3. The set of robust product design alternatives is presented in Section 4.5.4.

4.5.1. Preliminaries

To begin with, it is necessary to survey the market for corded power tools to identify key attributes of the product that are important to customers and then establish a

set of common attributes between engineering design and marketing disciplines. Working as a team with an industrial partner, several focus group studies are conducted to first identify a set of attributes that are considered as the most critical by the end users. Six marketing attributes have been identified for this product: brand, price, amp rating, switch type, life, and girth size. The engineering design attributes (i.e., output from the design simulation) are maximum output power, output speed, armature temperature, and brush temperature. Among these attributes, amp rating and life of the product are common attributes between the design and marketing domains. Amp rating is obtained using maximum motor output power, and an estimate of product life can also be obtained by a heuristic that takes motor output speed and armature temperature. The application (i.e., type of material and the duration of use) is assumed to be the same for all design alternatives. However, depending on the motor used, the average application current is different for each design alternative. The set of design variables are: choice of motor (x_m) which is a discrete variable between 1 to 10, choice of speed reduction unit or gearbox (x_g), a discrete variable between 1 and 6, the gear ratio (x_r) which is a continuous variable between 3.5 and 5.0. There are 5 design parameters that affect the performance of each design alternative. The design parameters' with their uncontrollable variability information is given at Table 4.1. The values in Table 4.1 are obtained by examining the experimental (or historical) values for each design parameter. In some cases an expert or designer can provide these values.

Design Parameter	Nominal	Lower bound	Upper bound
Source Voltage (V)	110	95	125
Ambient Temperature (C)	25	-10	50
User Load Bias (lb)	6	3	9
Fan CFM Degradation (%)	20	0	80
Application Torque Adjustment (%)	0	-20	20

Table 4.1: Design parameters' information

The variability in the marketing is discussed in Section 4.5.3. It is assumed that in the market for this power tool, there are 3 competitive products. Their specifications in terms of marketing attributes (including the common attributes) are given in Table 4.2 below.

Competitive Product	Price	Amp rating	Switch type	Life	Girth size
Brand 1	\$99	9	Side Slider	120hrs	Large
Brand 2	\$129	12	Paddle	150hrs	Small
Brand 3	\$79	6	Paddle	80hrs	Small

Table 4.2: Competitive products specifications

The set of robust design alternatives considering only engineering design robustness aspects are discussed in the next Section.

4.5.2. Robust Designs using Engineering Design Robustness

To ensure performance and efficiency of the product and reduce the effects of vibration to the user, the engineering design objectives are defined as follows. The product's output motor speed is minimized while the amount (i.e., mass) of material removed is maximized. To guarantee that the product does not fail (i.e., burn out) under demanding application conditions, a design constraint is imposed to keep the motor temperature (which is the larger of armature temperature and field temperature) less than

220°C [Medinger, 2005]. Given these two objectives and constraint, without considering the effects of parameter variations on them, a Multi-Objective Genetic Algorithm (MOGA) [Narayanan and Azarm, 1999] with Kurapati et al.'s constraint handling technique [Kurapati et al., 2002] was used as an optimizer to obtain the set of (nominal) Pareto designs. The reason for choosing an optimizer based on Genetic Algorithm is that this case study involves both discrete and continuous variables. Figure 4.6 shows the results. Nominal Pareto design points are highlighted by diamond symbols in Figure 4.6. There are gaps among the clusters of design alternatives as depicted in Figure 4.6. The primary reason for these gaps is due to dramatic changes in performance based on the choice of available components in the database. The parameters used for MOGA are the same of those values given Table 3.1.

At this point, for expositional purposes, the marketing module is not considered.

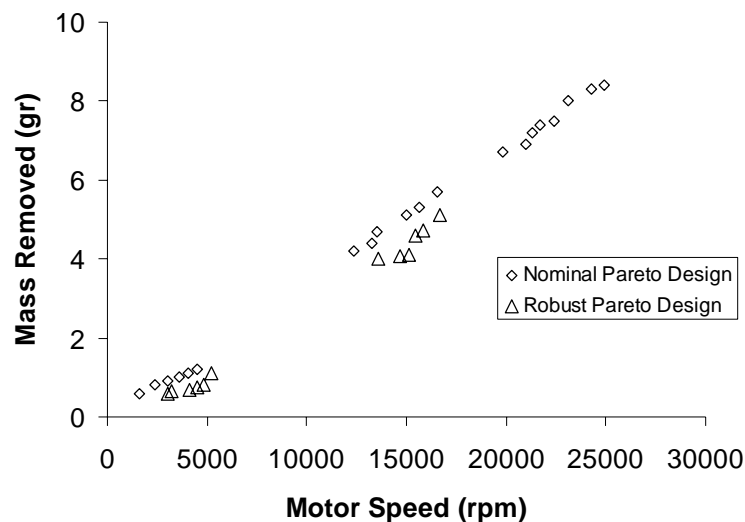


Figure 4.6: Set of nominal and robust Pareto design alternatives

Using the model provided in Section 3.4, with a genetic algorithm as the optimizer, the maximum variation from nominal values of motor speed and the mass of

removed material are calculated for every design alternative. In this example, the variation from nominal value for motor speed must be less than 8,000 rpm. In addition, the variation in the mass of removed material in one application (of the tool on a steel plate) must be less than 5 grams. The robust designs are those that satisfy these requirements as well as the feasibility robustness requirement. Likewise, the model of Section 3.4.2 is used to identify feasibly robust design alternatives. For a power tool design, to operate for long and intensive applications, the motor temperature should not exceed a certain level. There are several parameters that can influence motor temperature in a power tool. Among those, the ambient temperature, user load bias, and power supply voltage and current can have considerable effects on the motor temperature. The design alternatives that are not feasibly robust are eliminated during the optimization.

The robust Pareto design alternatives are obtained following the framework given in Figure 3.8. Again, MOGA with parameters in Table 4.3 is used as the optimizer. The robust Pareto points for this example are also shown in Figure 4.6 along with the nominal Pareto points. It can be observed that in this example almost all of the robust Pareto points are inferior (in terms of performance) to the nominal Pareto points. However, it was verified that none of the robust designs show unacceptable variation in performance which can lead to failure of the product.

In the next section, the effect of customer preferences is studied (without using engineering design objectives and constraints mentioned above) in the generation of product design alternatives.

4.5.3. Robust Design using Preference Robustness

Based on some exploratory research, four different brands (one of which is the producer's own brand) are chosen along with three levels of price, three levels of amp ratings, four types of switch, three levels for product life, and two levels for girth size. Respondents for this study include metal workers and construction workers (who make up 80% of the user base for the tool) recruited from job sites and construction sites. The interviews were conducted with 249 respondents. Each respondent was given 18 choice scenarios (16 were used for conjoint estimations and 2 for verification). Each choice scenario included two product design alternatives and a no-choice option with verbal descriptions indicating the levels of marketing attributes (brand, price, amp rating, switch type, life, and girth size). Respondents were asked to consider different usage situations when making their choices. The data was collected, coded and analyzed using Sawtooth Software. The finite mixture module of the Sawtooth Software was used to obtain the part-worth estimates, standard errors, and the variances and co-variances associated with each attribute level at the segment level. The scenarios of 1, 2, 3, 4, and 5 market segments are examined. The number of market segments is determined by choosing the segment with the minimal *AIC* value (see Eq. (4.5)), which turned out to be four segments. Therefore, four market segments are formed. For each segment, its segment size estimate, conjoint part-worth utility estimates, and the variance-covariance matrix of the conjoint part-worth utility estimates are known. Using the approach outlined in the marketing model, 95% Simultaneous Confidence Interval (SCI) for the utility estimate of each attribute level are calculated. Table 4.3 below provides the part-worth utility estimates associated with each attribute level and the utility estimate for "no-choice" in each market segment. In this table, the values of segment sizes are also provided. In

addition, a 14×14 variance and co-variance matrix of the conjoint estimates for each market segment is obtained. The diagonal elements of the matrices are all positive numbers and they represent the variances of the conjoint estimates. The off-diagonal elements describe the co-variances of the conjoint estimates.

The information provided in Table 4.3 can be used to illustrate how the utility of a product is calculated. For a product with own brand, \$79 retail price, amp rating of 9, 110 hours of product life, top slider switch, and small girth, its utility for consumers in segment 1 is 1.3 (i.e. $(-.54)+(-.11)+.13+1.33+(-1.01)+1.5=1.3$). For consumers in segment 2, its utility is -1.47 (i.e. $.45+(-0.09)+(-1.42)+(-.47)+(-.65)+.71=-1.47$). Similarly, this product's utility for consumers in segment 3 can be calculated as -5.79 and for consumers in segment 4 as -.99. Similar approach can be used to calculate the lower and upper bound utility for each product alternative and the competitor products.

To assess the face validity of the model estimates, the estimated market shares for the existing products with actual market share data obtained from Power Tool Institute (PTI) are compared. PTI is an organization that provides its member companies with market level data such as the market shares of different power tool products. It is found that the discrepancies between the estimated market shares from the conjoint experiment and the actual shares are within 5-7%. As a result, it can be implied that the proposed model estimates are reasonably in line with the actual market share values.

	Segment 1	Segment 2	Segment 3	Segment 4
Segment Size	0.378	0.248	0.121	0.253
	Part-worth	Part-worth	Part-worth	Part-worth
Brand 0 (own)	-0.54	0.45	2.21	-0.16
Brand 1	0.18	1.06	-2.37	-0.2
Brand 2	0.83	0.11	-1.5	1.15
Brand 3	-0.46	-1.61	1.66	-0.79
Price \$79	-0.11	-0.09	0	-0.01
Price \$99	-0.89	-1.15	1.91	-0.24
Price \$129	1	1.23	-1.91	0.25
Amp 6	1.25	0.45	-1.48	-0.45
Amp 9	0.13	-1.42	-0.65	-2.38
Amp 12	-1.38	0.97	2.13	2.82
Life 80	-0.86	-0.12	-4.71	0.8
Life 110	1.33	-0.47	-5.82	0.74
Life 150	-0.47	0.6	10.53	-1.54
Paddle	0.42	0.29	-3.29	-0.65
Top Slider	-1.01	-0.65	-3.04	0.41
Side Slider	2.39	-0.07	2.46	0.56
Trigger	-1.8	0.42	3.87	-0.31
SmallGirth	1.5	0.71	1.51	0.41
LargeGirth	-1.5	-0.71	-1.51	-0.41
No-Choice	-0.02	-0.02	-0.02	-0.02

Table 4.3: Conjoint part-worth estimates

Based on the outputs from the conjoint estimation and the procedure described in Section 4.3, the interval utility estimates for all the product alternatives are calculated. These interval utility estimates were used to calculate an upper and lower bound of market share for each design alternative, based on Eqs. (4.6) – (4.11). The variation

between the upper and lower bound of the market share is used as a measure of preference robustness.

Next, using the approach described in Section 4.3, design alternatives are generated through passing numerous combinations of design variables using the optimizer to the corded power tool simulation. The output generated by the design simulation is used to obtain (directly or via mappings) the common attributes. As mentioned before, two common attributes are mapped from design simulation output, namely, amp rating and life of the product. Next, there are three non-common marketing attributes that contribute towards the generation of the set of product alternatives. These attributes are price, switch type and girth size. Due to the fact that brand name is generally fixed for any particular manufacturer, the brand name is fixed to “own brand” for all the product alternatives in the MOGA. By enumerating each design alternative attributes over these non-common attributes, numerous product alternatives can be generated. For each generated product alternative, using the information provided in Table 4.3 and Eqs. (4.7) – (4.11) the market share and its variation can be estimated. MOGA is used with similar parameters as those in Section 4.5.2 to obtain the set of robust product alternatives that have maximum market share and minimum variation in market share. In the initial population, the market share variability ranges from 5% to 20%. The optimization results are shown in Figure 4.7. Every product design point in Figure 4.7 is obtained by mapping from design simulation results to marketing related attributes. For instance, for product design A, the common attribute values are 5.71 Amps and 119 hours for amp rating and life, respectively, under continuous application of the tool on a steel plate. The maximum deviation from the nominal design objectives values

are 3.7 grams and 7,654 rpms which both satisfy the designer's acceptable ranges. Since the nominal market share value and its variation for product A (i.e., 22.8% and 3.68%) is among the best possible values, product A appears in the marketing Pareto products set. Similar to the results in Figure 4.6, there are gaps and clusters in the obtained solutions because of dramatic changes in performance of a product design due to discrete choice of components such as a motor.

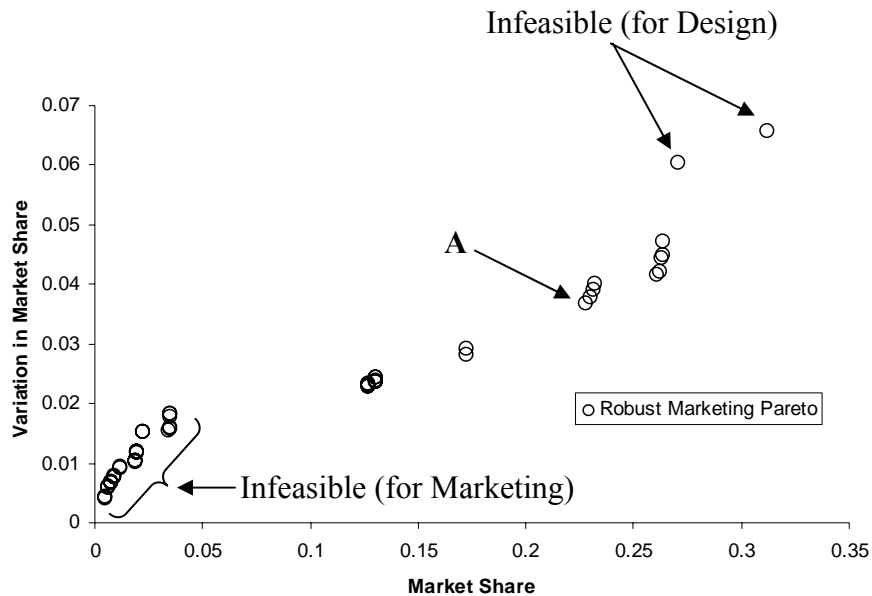


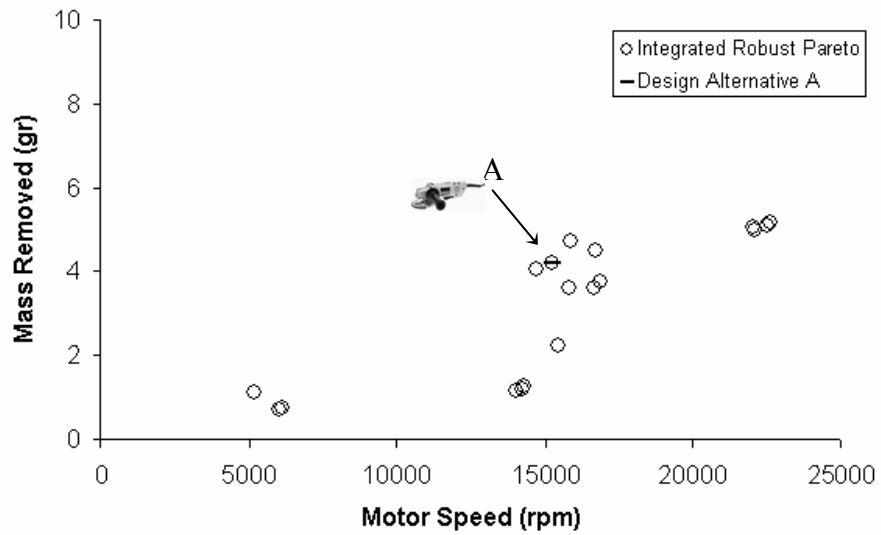
Figure 4.7: Set of robust marketing Pareto product alternatives

As shown in Figure 4.7, 53 product alternatives form the robust marketing Pareto products, all of which have less than 7% of market share variations. It should be noted that in calculating of market share and its variation only pure marketing attributes such as price and switch type along with common attributes (life and amp rating) are considered. Therefore, several other critical engineering aspects of the designs are not accounted for. For instance, two products in the upper right side of the Figure 4.7 are indicated as infeasible from engineering design aspects. Both of the products violate the constraint on motor temperature and therefore are not good candidates for the prototyping stage.

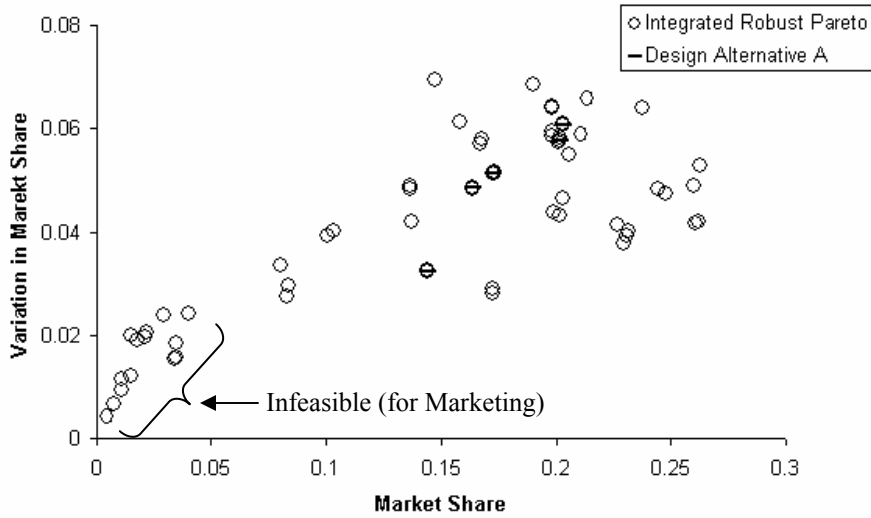
Similarly, the product alternatives in the bottom left corner of Figure 4.7 are infeasible from the marketing perspective. Even though the market performance of these product alternatives does not vary much, the market shares of these products are all less than 5%. Such low market shares are considered as infeasible because these product alternatives cannot generate requisite revenue to recover the fixed costs needed for the development of these products.

4.5.4. Robust Design using Integrated Design and Marketing Approach

The integrated robust design and marketing approach given in Section 4.4 is now applied to the example. Similar to the discussion for Figure 4.4, each corded power tool design alternative is evaluated for performance and robustness. After obtaining the product alternatives, the market share and its variation are calculated for each product alternative. The evaluation is performed at a discipline level according to the rules given in Section 4.4.1. The final set of robust products based on the proposed integrated approach is shown in Figure 4.8.



(a)



(b)

**Figure 4.8: Final set of robust design and product alternatives:
(a) engineering design domain, and (b) marketing domain**

There are 18 design alternatives in the design objective space that are identified as robust in design objective space (Figure 4.8a). As mentioned before, every design alternative is enumerated over non-common marketing attributes to produce several product alternatives. In this example, there are three non-common attributes, namely, switch type, price and girth size that overall create 24 possible product alternatives for

each design. It should be noted that not all of the possible generated combinations for each design have optimum performance in both design and marketing domain. In this example, corresponding to the 18 designs in the design domain (Figure 4.8a), there exist 62 product alternatives in the marketing domain (Figure 4.8b). For example, design alternative A in Figure 4.8a corresponds to the five optimum products in Figure 4.8b. Table 4.4 tabulates the properties of these products.

Product	Motor no.	Gear no.	Gear ratio	Price	Switch type	Girth size	Market share	Var. of market share
1	6	4	4.9	\$129	Top Slider	Small	0.144	0.032
2	6	4	4.9	\$129	Trigger	Small	0.164	0.049
3	6	4	4.9	\$129	Top Slider	Large	0.173	0.051
4	6	4	4.9	\$129	Trigger	Large	0.203	0.060
5	6	4	4.9	\$129	Side Slider	Small	0.201	0.058

Table 4.4: List of product alternatives corresponding to design A

Furthermore, among the 18 design alternatives in design objective space (i.e., Figure 4.8a), only 4 of them also belong to the robust set obtained through robust design-only approach (i.e., Figure 4.6). On the other hand, the comparison of marketing domain in final robust products with marketing-only approach (i.e., Figure 4.7) reveals that only 14 products are common between two sets. Such comparisons could help the designer in making a selection among the generated set of product alternatives.

Overall, the integrated approach in this chapter obtains solutions (as shown in Figure 4.8) that are superior in terms of design performance, marketing performance or both. The next step in the product development process is to make a selection among the

products and then the selected products can be carried forward for the prototyping stage. Using Figure 4.8 and locating products in both domains, it would allow a product design manager to evaluate each product from both design performance (and robustness) as well as its market performance. Since it may not be feasible to carry forward 62 products to the prototyping stage, the design and marketing teams may decide to reduce the number of the final products. First, some of these alternatives can be eliminated through a more stringent criteria for robustness (for example, by reducing the acceptable range of variability in the design and/or marketing dimensions), which can reduce the number in the Pareto set. Second, as mentioned before, the marketing team may decide to eliminate solutions that have a low level of predicted market share (e.g., below 5%). This will reduce the number of robust products to 48. Third, the marketing team may prefer to target at a particular price point for the new product after accounting for retailers' existing assortments and their preferences. As a result, the only product alternatives with this price point will be considered. Finally, a similar procedure can be carried out in design domain and the design team can eliminate the designs that have higher production costs (when offered at the same price) to increase the projected profit. While there are many techniques to aid in making a selection among the final product alternatives, the discussion of such techniques is beyond the scope of this dissertation. The main focus here is to present an integrated design-marketing robust optimization approach to identify a robust optimal set of product alternatives for the new product development team facing a complex design problem with interdisciplinary objectives and infinite number of design alternatives.

4.6. SUMMARY

In this chapter an integrated approach for a single product robust optimization is presented. Engineering design and marketing are two major domains (or disciplines) that are covered in the approach. The performance and robustness of a single product alternative in each domain were also evaluated, and the relation between the design and marketing attributes were established by identifying a set of common attributes. The approach 2 to robust design optimization described in Section 3.4 was used along with a preference robustness approach based on a finite mixture conjoint model. The essentials of the preference robustness measures and their relation with variability due to uncontrollable design parameters were provided in details.

A bi-disciplinary (i.e., marketing-design) optimization criterion was used to generate and rank order a set of product design alternatives, which could then be taken to the prototype development stage. This assured that the prototypes being tested are robust not only from a design perspective but also from a customer preference perspective. In this regard, it is important to note that the integrated approach was not a sequential elimination scheme. Instead every product was evaluated in both design and marketing domains. Only those products that may become infeasible or have inferior performance in at least one domain were eliminated in the process.

To demonstrate the integrated design and marketing robust optimization framework, the presented approach is applied to a design problem, design of a small angle grinder. The results are obtained in three different cases; first when only design robustness is taken into account, second when only preference robustness is taken into

account and third, when both design robustness and preference robustness are considered simultaneously.

In the next chapter the proposed approach for single product robust optimization is extended to product line design where instead of just one product a set of variants are to be selected to target different segments of a market.

CHAPTER 5

PRODUCT LINE ROBUST OPTIMIZATION

5.1. INTRODUCTION

In Chapter 4, a method for robust optimization in single product design was presented. There, the main assumption was that the product manufacturer is able to achieve a reasonable market share and profit by launching only a single product to the market. This chapter is focused on product line design. A product line refers to a collection of single products that essentially have the same function but with different attribute levels. Each individual product in a product line is called a variant. Product manufacturers often want to develop a product line in order to meet the needs of different market segments and thus obtain a broad market for their products.

The purpose of this chapter is to develop an approach for robust product line optimization. From Engineering design point of view, the variants in a product line must be robust (See Chapter 3), and from a marketing point of view, the variants should be robust and collectively produce maximum possible profit for the product manufacturer.

The organization of this chapter is as follows. Section 5.2 provides an overview of the robust product line design optimization problem. Section 5.3 gives a description for a two-stage approach for obtaining a robustly optimum product line design. Next, Section 5.4 illustrates the approach with an example for robust optimal product line design of a corded power tool. Finally the chapter is concluded by a summary in Section 5.5.

5.2. OVERVIEW OF ROBUST PRODUCT LINE DESIGN PROBLEM

The product line design optimization problem here is to obtain a set of N product alternatives (or variants) from a large number of product candidates and then select K single products out of the obtained finite set of N single product candidates to form a profit maximizing product line. Usually there is a maximum number of variants in a product line (i.e., $K \leq K_{\max}$). This set of N product alternatives is either pre-determined by the product designer (e.g. [Chen and Hausman, 2000] [Ramdas and Sawhney, 2001]) or obtained by permutation of all possible combinations of attribute levels in a conjoint study, e.g., [Nair et al., 1995]. From a product manufacturer's perspective, however, the set of technologically and economically feasible product alternatives, from design perspective, can be very large. In fact, the size of this set of feasible product design alternatives can theoretically be infinite when some attributes are continuous (e.g., weight and product price). A subjective selection of some "good" product alternatives from such a large number of feasible product alternatives generally results in a suboptimal product line [Nair et al., 1995]. Furthermore, for most products, the product design space has a dimension which goes beyond that of the space defined for marketing attributes. Hence, similarly to the single product design method, as discussed in Chapter 4, the process for product line design also relies on the integration of engineering design and marketing domains.

The optimal product line design is carried out in a sequential two-stage approach, as shown in Figure 5.1: Stage I is for robustly optimal single product alternative generation, and Stage II is for product line design optimization.

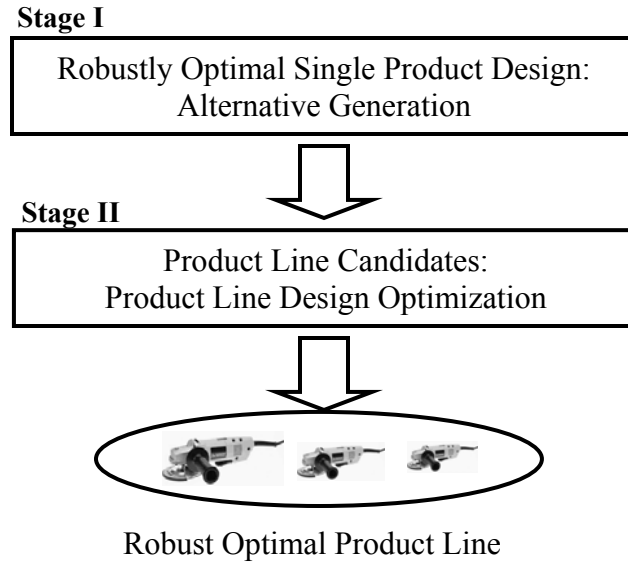


Figure 5.1: Overview of product line robust optimization approach

In Stage I, the focus is on generating a set of single product design alternatives that are individually robust. In contrast to the approach in Chapter 4, as well as previous research in single product optimization (e.g. [Balakrishnan and Jacob 1996] [Besharati et al., 2004] [Luo et al., 2005] and [Michalek et al., 2005]), the main focus in Stage I is to just eliminate “undesirable” product alternatives. The undesirable alternatives refer to those that do not satisfy the design robustness requirements (see Chapter 3). The approach in Stage I is geared to reduce the number of product design alternatives from very large to a finite number. This set of generated robust product design alternatives from Stage I is then used as variants for the creation of product line alternatives in Stage II.

In Stage II, a set of product line design alternatives are generated, from the robust optimal single product designs produced in Stage I. In Stage II, a Genetic Algorithm (GA) based combinatorial optimization approach (e.g., [Deb, 2001]) is used to obtain an

optimal product line. The details of the optimization problem for Stage II are provided in Section 5.3. Next, based upon the previous work (e.g., [Ramdas and Sawhney, 2001]) a cost model for platform-based products is introduced with the assumption that the product manufacturer buys components off the shelf from outside vendors and conducts the assembly in house.

In the following sections it is attempted to bridge the gap between the marketing and the engineering literature by developing a model that accounts for different aspects of product line profit maximization problem such as competitive products offerings and the cost savings associated with component sharing among variants.

5.3. APPROACH FOR PRODUCT LINE DESIGN PROBLEM

The robust product line design problem is viewed as an optimization problem with two stages; Stage I: robustly optimal single product alternative generation (Figure 5.2), and Stage II: product line design optimization (Figure 5.5). The details of the approach in Stage I and Stage II are provided in Sections 5.3.1 and 5.3.2 respectively.

5.3.1. Stage I: Robustly Optimal Single Product Design - Alternative Generation

As shown in Figure 5.2, Stage I considers both engineering design and marketing domains. The purpose of the Stage I approach is to select a set of robust and optimal product design alternatives in each market segment. Starting with a large number of product candidates, a GA based approach is used to reduce the number of product alternatives. In the engineering design domain (bottom block, left column of Figure 5.2), a set of design inputs is identified.

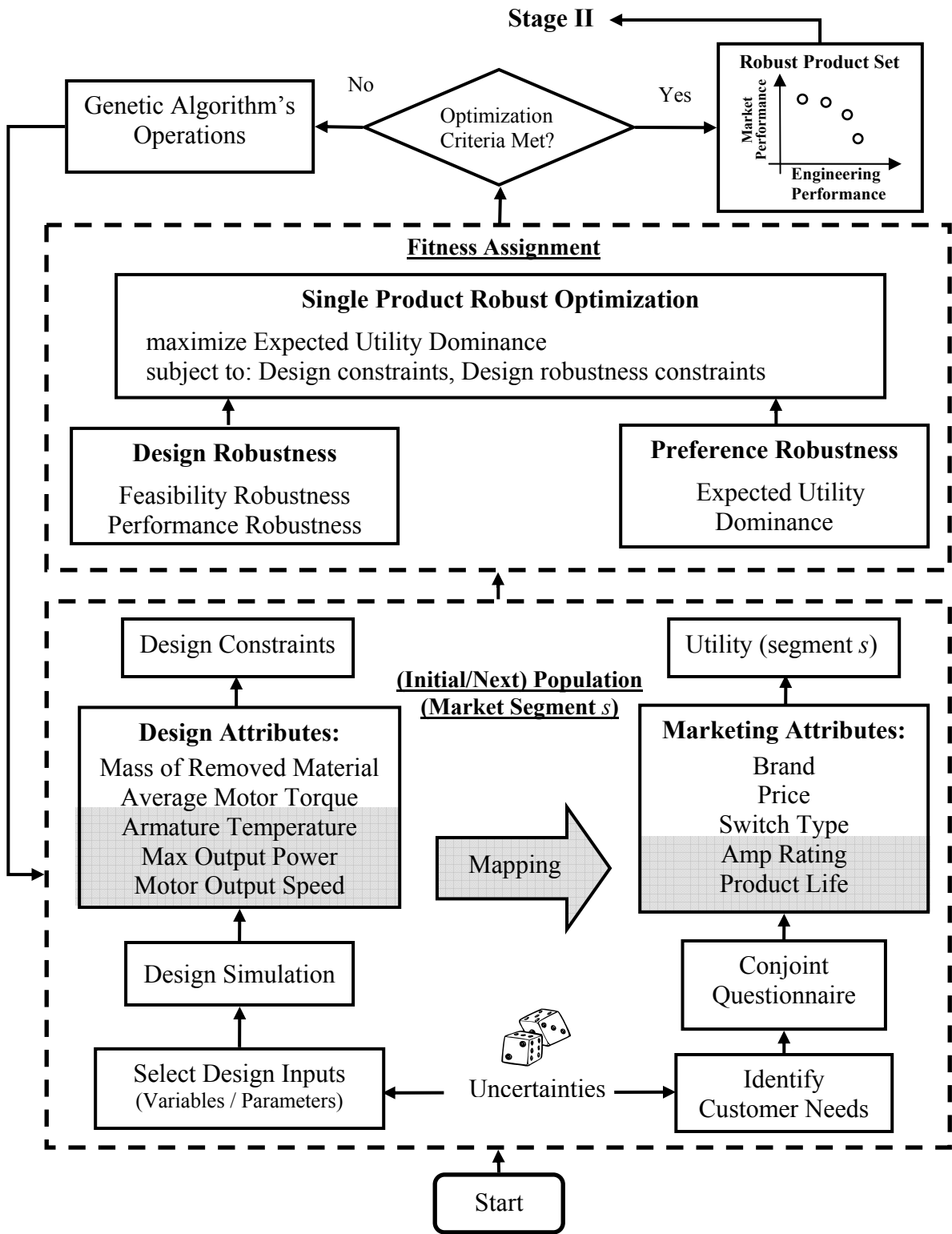


Figure 5.2: The approach for Stage I of product line optimization

Design inputs include both design variables and uncontrollable design parameters. The set of design variables define a design alternative. Similar to the approach in Chapter 4, a deterministic design simulation software tool is used to receive the value of design inputs and obtain corresponding design attribute values (e.g., armature temperature and maximum output power).

From the marketing domain (bottom block, right column of Figure 5.2), the most important attributes for consumers are identified through an exploratory marketing research study. Similarly to the approach for single product robust optimization of Chapter 4, some of the common attributes such as amp rating and product life are obtained by a mapping function from the engineering design domain. Next, the levels of each attribute are decided. Once the attribute levels are known, a choice-based conjoint questionnaire (see, e.g., [Kamakura and Russell, 1989]) is developed for consumer preference elicitation. A finite mixture conjoint model (e.g., [Vriens et al., 1996]) is used to address consumer heterogeneity. This model can be used with Akaike Information Criterion (AIC) [Akaike, 1973] to obtain the optimal number of market segments. The finite mixture conjoint model is then applied to calculate the utility of the attribute levels for each market segment. The utility of a product alternative in each segment is also calculated in this block (recall Section 4.3.1.1).

In the top block of Figure 5.2, the effects of uncertainties in both engineering and marketing domains are considered. In particular, similar to that in Chapter 4, the design robustness box determines whether or not each single product design candidate satisfies the requirements for feasibility robustness and objective robustness.

To generate the initial product population in GA, the design inputs and the marketing attributes (excluding brand) are used. It should be noted that design input form a design alternative and combination of design inputs and marketing attributes form the product alternatives. Initially a set of product alternatives is randomly generated for each market segment. Then in the top block each product alternative is evaluated, and a fitness value is assigned to each product in the population. The product alternatives that do not satisfy design robustness or design constraints are penalized, and those that are not penalized, are evaluated based on an expected utility dominance measure for consumer utilities. The definition and details of the measure are given in Section 5.3.1.1.

The GA procedures including population generation, fitness assignment, and genetic operations are repeated in several iterations till the stopping criterion is satisfied. The stopping criterion is defined as follows. When there is no significant change in the expected utility dominance value of a certain portion of the best individuals in the population, the procedure stops. The result of the GA optimization in Stage I is a set of robust products for each market segment. In the next few sections the specifics of the expected utility dominance measure, the optimization problem, and, the fitness assignment procedure are discussed.

5.3.1.1. Stochastic dominance

Stochastic dominance is used to compare two alternatives under uncertainty (i.e., when the distributions are known) [Mislevy et al., 1992] [Clemen and Reilly, 2001]. Figure 5.3 gives an example where the cumulative distribution function (CDF) of the utility for three product alternatives is given. Product H_3 stochastically dominates product

H_1 because the CDF for H_3 is entirely on the right side of CDF for H_1 . However, product H_2 neither dominates nor is dominated by the other two alternatives.

By definition, a product alternative H_1 stochastically dominates product H_2 , if for any given utility value, U , product H_1 gives a higher probability than does product H_2 . Let H_1 and H_2 denote two single product design alternatives. Suppose the utility distribution of product H_1 in segment s is U_{H_1} and that of product H_2 is U_{H_2} . In a comparison of the conjoint utilities of these two products:

$$\Pr(H_1 \succ H_2 | s) = \Pr[(U_{H_1} - U_{H_2}) > 0] \quad (5.1)$$

where the symbol ' \succ ' is used for stochastic domination of an alternative over the other.

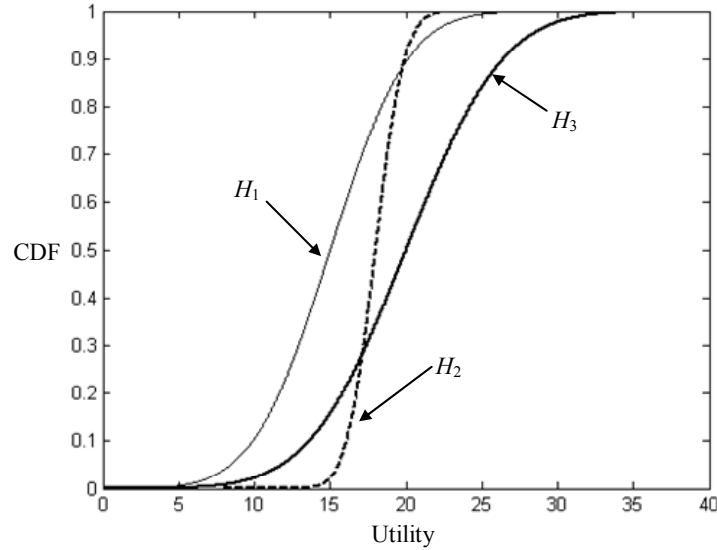


Figure 5.3: Stochastic dominance comparison of three single products

For the product design alternative H_r in a population of size R , the expected dominance value in market segment s is defined:

$$ED(H_r | s) = \sum_{r, r'=1}^{R-1} \Pr(H_r \succ H_{r'} | s) \quad \text{where, } r \neq r' \quad (r = 1, \dots, R) \quad (5.2)$$

This measure is used in the GA approach in Stage I as an objective which needs to be maximized. In the following subsection, a proposed integrated fitness assignment approach is given in detail.

5.3.1.2. Stage I Optimization Model

The optimization formulation for Stage I is shown in Figure 5.4.

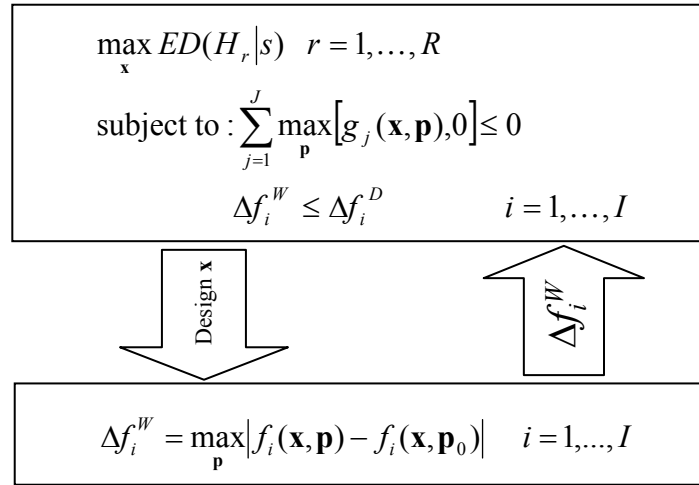


Figure 5.4: The robust optimization approach for stage I

As shown in Figure 5.4, vector H_r represents the r^{th} product alternative, and its components are composed of design attributes and marketing attributes. The symbol s represents the s^{th} market segment. The expected dominance or ED function is calculated using the obtained utility distributions for a set of R product alternatives. The f_i represents the i^{th} performance attribute whose variability must be limited. Δf_i^W and Δf_i^D are the maximum deviation and acceptable deviation from nominal value of i^{th} performance attribute respectively. Also g_j represents the j^{th} design constraint which must remain feasible. It should be noted that there is a difference between the flowchart in Figure 5.4 and Figure 3.8. Here the objective function is the expected utility dominance (i.e., ED) of

each product and the performance attributes, f_i s are not being treated as objective functions and therefore are not being optimized. The details of the expected dominance function are provided in Section 5.3.1.1.

The optimization in the Stage I is carried out at the segment level. Note that in this stage, optimization is not conducted at the entire market-level, consisting of all segments at the same time. This is because a market-level analysis generally multiplies the segment size by the segment-level conjoint utility to obtain the weighted average market-level utility. As a result, such an analysis will always favor product alternatives appealing to the largest segment. This approach is against the basic principle of product line design in providing a variety of products to satisfy consumers in different market segments. In contrast, the proposed segment-level based approach is more appropriate because it can guarantee that product design alternatives that might be appealing to smaller market segments are not eliminated. The outcomes of the Stage I optimization is a set of robust products with high conjoint utilities in each market segment under the uncertainties. To evaluate each product design alternative during the GA optimization, a fitness value needs to be assigned to each alternative. The details of fitness assignment procedure are provided in the following section.

5.3.1.3. Integrated Fitness Assignment Approach and Implementation

The motivation behind the proposed integrated fitness assignment is (1) to ensure that products that are not generated from robust design alternatives are penalized; and (2) to generate a set of single product alternatives that have higher ranks in terms of expected conjoint utility dominance among the rest of the individuals in a GA population.

The fitness assignment approach used in Non-Dominated Sorting Genetic Algorithm (NSGA) [Deb, 2001] is tailored to address the problem here. Three criteria are examined to assign a fitness value to each product alternative in a population. First each product is examined to identify whether it corresponds to a feasibly robust design. If it does not come from a design that satisfies the feasibility robustness defined in Eq.(3.3), a negative value, F_{FR} is added to its fitness value. In a similar fashion, a negative value, F_{PR} is added to any product that corresponds to a design that does not satisfy the objective robustness criteria define in Figure 3.8. The following procedure can be used to assign the fitness to each product in a population:

Step 1. Choose a sharing parameter, σ_{share} , and a small positive number, ε , and let $F_{min} = N + \varepsilon$. Here, N is the number of products in the initial population and F_{min} is the initial fitness assigned to every individual. There are certain methods to obtain a value for σ_{share} (e.g., Press, et al., 1988). In this case the following is used:

$$\sigma_{share} = \frac{1}{N-1} (ED^H - ED^L) \quad (5.3)$$

where the ED^H and ED^L are the highest and lowest values for expected dominance number for the current population. Also, set counter $r = 1$.

Step 2. Rank order population based on the expected dominance value as described in Section 5.3.1.1. For instance a population size of N can be sorted as (P_1, P_2, \dots, P_s) , where, P_1 represents a subset of the population with highest expected dominance number, and P_s is the subset of the population with the lowest

dominance number. It should be noted that each subset can comprise of as little as only one product alternative. Also, each product within the population is checked for feasibility robustness and objective robustness. Any product, q , that violates feasibility robustness is set to: $F_{FR}^{(q)} = -\alpha N$; otherwise it is set to: $F_{FR}^{(q)} = 0$. Likewise, for any product q that violates the objective robustness: $F_{PR}^{(q)} = -\beta N$; otherwise: $F_{PR}^{(q)} = 0$. The quantity α and β are the robustness penalty coefficients that designer can choose depending upon the importance of each feasibility and objective robustness criteria.

Step 3. For each product alternative q in subset P_r , perform the following procedure:

Step 3a: Assign fitness as $F_r^{(q)} = F_{min} - \varepsilon$.

Step 3b: Calculate niche count (a measure for diversity) among the products in subset P_r as:

$$nc_q = \sum_{i=1}^N Sh(d_{qi}) \quad (5.4)$$

$$\text{where, } Sh(d_{qi}) = \begin{cases} 1 - \frac{d_{qi}}{\sigma_{share}} & d_{qi} \leq \sigma_{share} \\ 0 & otherwise \end{cases} \quad (5.5)$$

where d_{qi} is the difference between the expected dominance value of product alternative q from the i^{th} product alternative in subset P_r .

Step 3c: Calculate the shared fitness as

$$F_r^{(q)} = F_r^{(q)} / nc_q. \quad (5.6)$$

Step 4. Set $F_{\min} = \min\left(F_r^{(q)} : q \in P_r\right)$, then set the overall shared fitness of product

alternative q as:

$$F_r^{(q)} = F_r^{(q)} + F_{FR}^{(q)} + F_{PR}^{(q)} \quad (5.7)$$

and set counter $r = r + 1$.

Step 5. If $r \leq s$ go to Step 3. Otherwise, the fitness assignment for the current population is complete.

5.3.2. STAGE II OPTIMIZATION MODEL

The Stage I optimization produces a set of robust products in each market segment. The sets of robust single products across different market segments can be combined to form a pool of variants for candidate product line alternatives. In Stage II, the goal is to find an optimal product line that maximizes the product manufacturer's profit. Figure 5.5 depicts Stage II of the approach. Stage II starts with a set of robustly optimal single products obtained from Stage I. As shown in the middle block of Figure 5.5, in order to estimate the product manufacturer's profit, it is necessary to calculate the product line cost and the product manufacturer's revenue. The cost of each product line candidate has two main components; fixed cost and variable cost. The fixed cost is mainly determined by factory setup cost and equipment cost. The variable cost comprises component cost, assembly cost, maintenance cost, salvage cost. It is also important to consider the cost savings due to commonality of parts/features among variants in product line. Moreover, the product manufacturer's revenue is obtained considering the current competitive product offerings, heterogeneous customers' preferences, and the

composition of a product line. In the top block of Figure 5.5 the optimization problem is provided. The objective is to maximize the product line profit. There are also two constraints: an upper bound on the maximum production capacity and an upper bound on the number of variants in a product line.

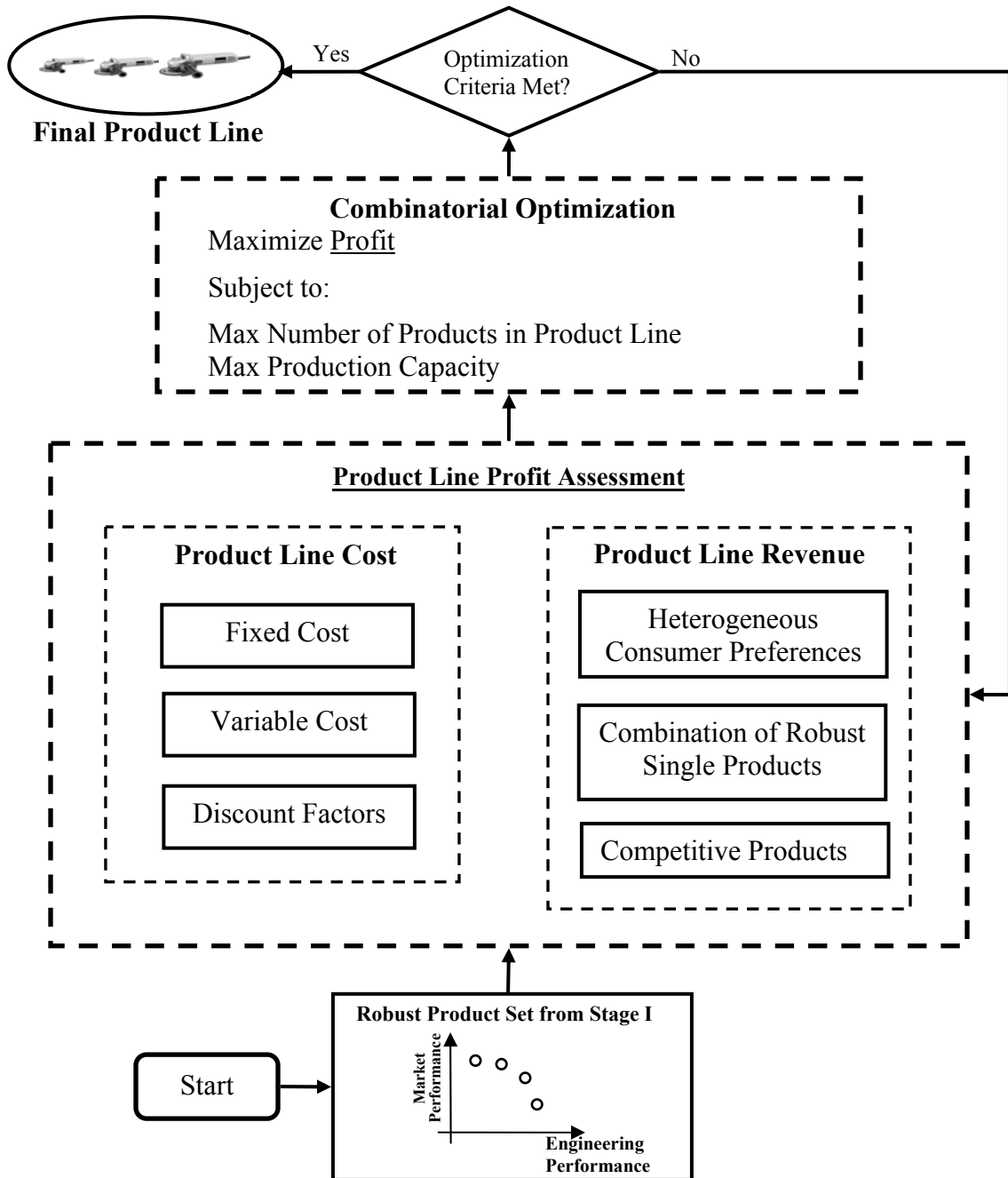


Figure 5.5: The approach for Stage II of product line optimization

It should be noted that the rest of the variants in the product line are treated as competitive products to account for cannibalization effect. Given the competitive products information the finite mixture conjoint is used to estimate the market share of each product alternative in the product line. The optimization model for the Stage II is provided in Eq. (5.8):

$$\begin{aligned} \max_{\{N_v=1,2,\dots,\bar{N}_v\}} \pi &= \sum_{H=1}^{N_v} MS_H \cdot (P_H - VC_H) \cdot N_m - FC \\ \text{subject to :} & \\ N_v &\leq \bar{N}_v \\ MS_H \cdot N_m &\leq W \quad H = 1, \dots, N_v \end{aligned} \tag{5.8}$$

The objective function π is to maximize the product line profit. The first constraint ensures that the number of products in the product line does not exceed a pre-specified upper limit. The second constraint is based on production volume for each product (e.g., [Bradley, 2004]). In equation (5.8), the index H denotes the H^{th} variant in a product line with N_v variants, MS_H , P_H , and VC_H stand for the market share, price, and variable cost of product H , N_m is the market size in units of potential purchase, FC represents the fixed cost of product line, \bar{N}_v is the maximum number of variants in a product line, and W denotes the production capacity constraint for each variant.

In the following, the details of the calculation for two components of product line profit; product line cost and revenue are provided.

5.3.2.1. Product Line Market Share and Revenue Calculation

The revenue calculation for a product line requires the market share values of each variant in the line, price of each variant, and the market size (N_m). The product line revenue ‘ rev ’ can be calculated by:

$$rev = \sum_{H=1}^{N_v} MS_H \cdot P_H \cdot N_m \quad (5.9)$$

In order to calculate the market share, one needs to identify a set of competitor product offerings, if exist. In the absence of competitive products, the variants within the product line compete with one another. Once the competitive set is determined, the market share of a product H is calculated as:

$$MS_H = \sum_{s=1}^S \theta_s \cdot MS_{Hs} \quad (5.10)$$

Also, similar to the approach in Chapter 4 (recall Section 4.3.1.2), the market share of each variant in a particular segment s is calculated as:

$$MS_{Hs} = \frac{\exp(U_{Hs})}{\left[\sum_{H=1}^{N_v} \exp(U_{Hs}) + \sum_{q=1}^Q \exp(U_{cp_q}) + \exp(cons_s) \right]} \quad (5.11)$$

In Eq. (5.11), the market share of product H is the weighted average of this product’s market shares in different market segments with θ_s representing the corresponding segment size. In particular, product H ’s market share in segment s (denoted as MS_{Hs}) can be calculated using the segment-level conjoint part-worth estimates from the finite mixture conjoint analysis. In the calculation of MS_{Hs} , the set of nominal product attribute values for both own and competitive products is used. These values are used for calculation of market shares because consumers generally base their

purchase decisions according to these directly accessible attribute values. In addition, for attributes that are continuous in nature (such as price and amp rating), a pair-wise linear interpolation procedure is used to calculate the conjoint utility associated with any attribute whose value is in between levels. Finally, it should be pointed out that for a variant H in a product line, the rest of the variants are considered as competitor products in order to account for cannibalization effects, as shown as the first component in the denominator of the expression for MS_{H_s} in Eq. (5.11). The competitive products outside the product line are indexed by q with $q=1,\dots,Q$, as the second component in the denominator of MS_{H_s} in Eq. (5.11). It can be shown that the proposed model is flexible enough to accommodate the cases in which one or more competitors have more than one product offerings in the market, as will be illustrated in the example section (see Section 5.4). Similarly to the approach for single product in Chapter 4, the option of “no-choice” is included as the last component in the denominator of MS_{H_s} . This component is included here to capture market expansion in that the share of “no-choice” can expand or shrink based on the overall attractiveness of the product offerings in the marketplace.

5.3.2.2. Product Line Cost Assessment

The product line cost assessment method consists of two parts; (i) calculation of variable cost, and (ii) obtaining the fixed cost. The approach for the variable cost of each variant in the product line is built upon the previous work of Ramdas and Sawhney [Ramadas and Sawhney, 2001] and Morgan et al. [Morgan et al., 2001]. The proposed approach here somewhat differs from that of Ramdas and Sawhney by the fact that the cost model here focuses on platform-based product categories in which the manufacturer

purchases the components (or building blocks) of the variant from outside vendors and assembles the components into a final product. Given the fact that more and more U.S. manufacturers are adopting this model [Meyer and Lehnerd, 1997], the proposed cost model provides a useful guide in the examination of the component sharing effect for such products. Here, a simplified cost model has been used that takes the cost savings corresponding to the commonality of components (across variants) and manufacturing (e.g., assembly) costs into account. The variable cost of product H in product line is calculated as follows:

$$VC_H = \sum_{r=1}^R (1 - \lambda_{rt}) \cdot C_{rtH} + C_{aH} + C_{mH} + C_{sH} \quad (5.12)$$

In equation (5.12), the variable cost VC_H is jointly determined by the unit cost of the r^{th} component C_{rtH} scaled down by a discount factor λ_{rt} . The quantity r is the index for component label (e.g., motor, switch), and t is the index for the type of the component (e.g. motor #1, paddle switch), the assembly cost C_{aH} , the maintenance cost C_{mH} , and the salvage cost C_{sH} . In other words, the main assumption behind the cost model is that the product (in this case consumer durable) manufacturer purchases the components from outside vendors, assembles the components into the final products, and sells the products to end users. The product manufacturer provides after-sale maintenance support such as replacement of malfunctioned products during the warranty period. Finally, at the end of product life cycle, the product manufacturer may be required to salvage the products. Many states have adopted regulations that require the product manufacturers to salvage their products at the end of their life cycle. While the salvage process can be costly to a product manufacturer, the salvage/disposal of a product does

not necessarily incur extra cost to a product manufacturer. In some cases reused parts or their refurbishment can actually make the salvage cost a benefit and therefore the incurred salvage cost to the product manufacturer can be negative.

Due to the fact that different products within a product line can share the same types of components, the cost associated with acquiring the shared components is commonly scaled down due to a high purchase volume and the economy of scale in working with fewer vendors. In the proposed cost model, a discount factor λ_{rt} is considered to account for this effect. In particular, this discount factor is defined as follows:

$$\lambda_{rt} = \begin{cases} 0 & \text{No commonality among the variants} \\ \frac{N_{vshrt}}{N_v} \rho_{rt} & \text{Otherwise} \end{cases} \quad (5.13)$$

When there is no sharing of component r among the variants in the product line, the discount factor λ_{rt} is set to be zero. When there is component sharing, ρ_{rt} (called “commonality significance factor”) represents the degree of cost benefit by sharing the t^{th} type of component r in the product line. As discussed by Morgan et al., [Morgan et al., 2001] and Ramdas and Sawhney [Ramadas and Sawhney, 2001], ρ_{rt} is generally evaluated from historical data on a case-by-case basis. When all the variants in the product line use the same component type, ρ_{rt} is equal to the discount factor λ_{rt} . Otherwise, the discount factor λ_{rt} is a proportion of ρ_{rt} depending on the degree of component sharing in the product line. The idea behind identifying a measure for component sharing is inspired by the prior literature on the effect of design commonality and its effect on a product family cost and performance (e.g., [Collier, 1981] [Martin and

Ishii, 1997] [Kota et al., 2000]). As shown in equation (5.13), proportion is defined as the number of products sharing the t^{th} type of component r (denoted as N_{vshrt}) divided by the number of variants in the line (i.e., $\frac{N_{vshrt}}{N_v}$). It should be emphasized that in many product development projects, the proportion of the cost saving due to component sharing may differ from the above-mentioned proportion. In the proposed approach, it is assumed that the unit cost of assembling the components into the final product H (denoted as C_{aH}) is determined by the specific selection of the component type (such as housing type) and/or the specific combination of product components. Since certain equipment is used to assemble similar components, there is a cost saving associated with it.

The maintenance cost of product H in the approach (denoted as C_{mH}) is negatively proportional to the product life under uncertainty. As discussed earlier in Chapters 3 and 4, the uncertainty from the uncontrollable design parameters (e.g., different usage situations and operations conditions) affects the life of a product. The WCS of product life represents the lower bound of the product life variation, which is estimated earlier in the first stage optimization. In particular, the maintenance cost in equation (5.14) is defined with $Cm_1 > Cm_2 > \dots > Cm_k$ and $L_1 < L_2 < \dots < L_k$. The values of Cm_1, \dots, Cm_k and the cutoff life estimate points of L_1, \dots, L_k can be estimated through an examination of the historical data on servicing the products after sales.

$$C_{mH} = \begin{cases} Cm_1 & Life(WCS) \leq L_1 \\ Cm_2 & L_1 < Life(WCS) \leq L_2 \\ \dots & \\ Cm_k & Life(WCS) > L_k \end{cases} \quad (5.14)$$

Finally, the salvage cost C_{sH} is obtained through a look-up table for a product H . Besides the variable cost, another important element of the product line cost is the fixed cost. The fixed cost is calculated as the sum of the equipment cost and the factory setup cost. As the breadth of the product line increases, the associated fixed cost of manufacturing the product line also increases. Based on the required equipment costs and factory setup costs, the firm can estimate the fixed costs of making one product, two products, till \bar{N}_v products.

In the next section, the proposed two-stage approach is applied to a product line design example.

5.4. EXAMPLE

In this section, the proposed two-stage approach with an example is demonstrated: design of a corded grinder product line. The data and definitions for the example are given in Section 5.4.1 followed by the Stage I analysis; robust product line design generation model in Section 5.4.2 and the Stage II; robust product line optimization model in Section 5.4.3. The set of robust product line alternatives is presented in Section 5.4.4.

5.4.1. Preliminaries

Several focus group studies were conducted to first identify a set of attributes that are considered as the most critical by the end users. Six marketing attributes have been identified for this product: brand, price, amp rating, switch type, life, and girth size. The engineering design attributes (i.e., output from the design simulation) are maximum output power, output speed, field armature and brush temperature values, and mass of

removed material. All of these attributes are shared among the variants within a product line. In this example, among the above-mentioned attributes, amp rating and life of the product are common attributes between the design and marketing domains. Amp rating is obtained using maximum motor output power, and an estimate of product life can also be obtained by a heuristic that takes motor output speed and motor temperature. The application (i.e., type of material and the duration of use) is assumed to be the same for all product design alternatives. However, depending on the motor used, the average application current is different for each design alternative. The set of design variables are: choice of motor (x_m) which is a discrete variable between 1 to 10, choice of speed reduction unit or gearbox (x_g), a discrete variable between 1 and 6, the gear ratio (x_r) which is a continuous variable between 3.5 and 5.0. There are 5 design parameters that affect the performance of each design alternative. The design parameters' with their uncontrollable variability information are the same as the single product robust optimization example given at Table 4.1 in Chapter 4.

It is assumed that in the market for this family of power tool product, there are 3 competitive brands and 5 competitive products. Their specifications in terms of marketing attributes (including the common attributes) are given in Table 5.1 below.

Competitive Product	Price	Amp rating	Switch type	Life	Girth size
Competitor 1, Product 1	\$99	9	Side Slider	110hrs	Large
Competitor 1, Product 2	\$59	5.5	Top Slider	90hrs	Small
Competitor 2, Product 1	\$129	12	Paddle	150hrs	Small
Competitor 2, Product 2	\$89	8.5	Side Slider	105hrs	Large
Competitor 3, Product 1	\$79	6	Paddle	80hrs	Small

Table 5.1: Competitive products specifications

The market study also concluded that there are three market segments for the intended product line, and there are already five competitive product offerings from three competitive brands available in the market. The corresponding utility values for every competitive product in each segment are provided in Table 5.2. The last row in Table 5.2 represents the “No-Choice” option.

Competitive Product	Segment 1	Segment 2	Segment 3
Competitor 1, Product 1	6.50	-0.17	0.18
Competitor 1, Product 2	-3.26	-0.09	0.92
Competitor 2, Product 1	7.35	1.10	1.60
Competitor 2, Product 2	6.26	-1.32	0.95
Competitor 3, Product 1	-0.73	-0.08	0.34
No-Choice	3.17	-0.08	0.70

Table 5.2: Competitive products utilities

The conjoint model and the associated data analysis are borrowed from the literature (i.e., [Luo et al., 2005] [Besharati et al., 2004]). Respondents for this study included users from different trades such as metal and concrete. The conjoint study was conducted with 740 respondents across the US market. Each respondent was given 18 choice scenarios (16 were used for conjoint estimations and 2 for validation). Sawtooth Software [Sawtooth Manual, 2001] was used to create a fractional factorial design with over 80% efficiency. Each choice scenario included two alternative designs and a no-choice option. Respondents were asked to consider different usage situations when making their choices. The finite mixture choice-based conjoint model provides an estimation of the number of market segments along with the segment sizes. In the Table 5.3 below, the part-worth utility estimates associated with each attribute level and the utility estimate for “no-choice” in each market segment are provided.

	Segment 1	Segment 2	Segment 3
Segment Size	0.16	0.22	0.62
	Part-worth	Part-worth	Part-worth
Brand 0 (own)	1.56	0.12	0.36
Brand 1	-3.89	-0.27	-0.22
Brand 2	1.99	-0.02	0.40
Brand 3	0.35	0.17	-0.54
Price \$79	-0.28	0.03	-0.08
Price \$99	-0.64	-0.65	0.32
Price \$129	0.93	0.62	-0.24
Amp 6	1.17	-0.49	-0.33
Amp 9	-1.89	0.52	-0.03
Amp 12	0.73	-0.03	0.36
Life 80	-5.01	0.26	-0.38
Life 110	1.71	-0.32	0.11
Life 150	3.30	0.06	0.27
Paddle	-1.96	0.26	-0.66
Top Slider	1.94	0.24	0.71
Side Slider	4.16	-0.55	0.38
Trigger	-4.14	0.05	-0.43
SmallGirth	-1.54	0.23	0.1
LargeGirth	1.54	-0.23	-0.1
No-Choice	3.17	-0.08	0.7

Table 5.3: Conjoint part-worth estimates

In the next Section the results from Stage I of the proposed approach are presented in which a set of robust single product candidates are generated.

5.4.2. Stage I: Robust Optimal Single Product Generation

As mentioned before, the objective of Stage I is to eliminate single product design alternatives that are not robust from the engineering design point of view and only keep those alternatives that yield higher expected dominance values of utilities in each market segment. To ensure a sustained acceptable performance of the power tool, two design attributes are considered for objective robustness; the output motor speed and the amount of mass removal. On the other hand an engineering design constraint is identified that ensures that the product does not fail to operate under different usage situations and conditions. The imposed constraint is defined as the motor temperature (which is the larger of armature temperature and field temperature) must be maintained at less than 200°C. Given these design attributes (i.e., objectives and constraints), the feasibility and objective robustness for each product design alternative can be evaluated using the approach 2 in Chapter 3, and in particular by examining the Equation in Figure 3.8.

A GA technique with a fitness assignment scheme as described in Section 5.3.1.2 is used to obtain the results for each of the three market segments. The reason for choosing an optimizer based on GA for this example is that problems of this nature involve both discrete and continuous variables and parameters. The parameters used for the GA are similar to those in Table 4.3 in Chapter 4. In particular, since the feasibility robustness aspect of the products is of more importance to the designer, the feasibility robustness penalty coefficients (i.e., α) is assigned to be 3 and the objective robustness penalty coefficient (i.e., β) is assigned to be 2 (recall Section 5.3.1.3).

The optimization is performed with respect to each individual market segment. For segments 1, 2, and 3, the Stage I approach has obtained 50, 53, and 48 robust product

candidate solutions, respectively. These robust product candidate solutions together form a set of 151 single product design alternatives that need to be evaluated in Stage II of the approach. The design information of a subset of these products is provided in Table 5.4.

Alt. No.	Motor	G. Ratio	Life	Switch	Girth	Price	Gear Type
1	2	4.2	121.81 hr	3	Large	\$129	Helical
10	3	3.8	130.68 hr	3	Large	\$79	Helical
49	3	4.9	127.26 hr	3	Large	\$79	Helical
16	3	3.9	130.58 hr	2	Large	\$99	Helical
51	2	4.2	121.81 hr	1	Small	\$129	Helical
62	2	4.5	121.33 hr	4	Small	\$129	Helical
82	3	4.0	129.86 hr	1	Small	\$129	Helical
95	3	4.6	128.78 hr	2	Small	\$129	Helical
110	2	4.8	120.82 hr	2	Large	\$99	Helical
127	3	4.2	129.97 hr	3	Small	\$99	Helical
135	3	4.5	129.10 hr	2	Large	\$99	Helical
151	3	5.0	127.26 hr	3	Small	\$99	Helical

Table 5.4: Specifications of a subset of single product results for stage I

Next, 12 single product alternatives are arbitrarily selected from the set of 151 robust optimal products obtained using Stage I approach and are shown in Table 5.4. In particular the first 4 products in Table 5.4 are selected from the obtained results for segment 1, and the next 4 products are selected from the optimal results corresponding to segment 2 and the last 4 products are optimal for the third market segment. Among the available motors, only motor 2 and 3 performed satisfactory under different usage situations and uncontrollable parameters, and products that have utilized motor 3 have a lightly higher life compared to those utilizing motor 2. In the next section the product line alternatives composed of combination of the obtained 151 single products are evaluated. Again, it should be noted that the above 12 selected points are just a snapshot of the overall 151 single product designs obtained in stage I and the optimization in stage II is actually performed on all 151 single product designs.

5.4.3. Stage II: Robust product line optimization

In Stage II, the product line alternatives that can be generated using the obtained solutions in Stage I are evaluated. The cost and commonality issue among the variants of a product line as well as the performance of product line candidates in the market (e.g., overall profit) is being used to select the robust optimal product line design. It should be noted that the cost information provided here is camouflaged to safe guard the proprietary information of the industrial partner.

The cost information related to this example is as follows. The fixed cost of manufacturing consists of the equipment cost and factory setup cost. Both elements are dependent on the production capacity. The maximum number of variants within a product line is assumed to be specified by a product manufacturer. For production of a single variant, two-variant, and three-variant product lines, the corresponding fixed cost is determined to be \$15M, \$18M, and \$25M.

The variable cost elements associated with a product line in this example are the cost associated with purchasing components (i.e., parts), cost of assembly, salvage cost, maintenance cost. The following tables provide the specifics of each variable cost element.

The single product results obtained in Stage I have either of the motor number 2 or 3. The corresponding cost and commonality significance factor of motors, switch types and housings are provided in Table 5.5.

Part / Feature	Unit Cost	Commonality significance factor
Motor 2	\$15	0.2
Motor 3	\$10	0.3
Switch 1	\$10	0.05
Switch 2	\$20	0.1
Switch 3	\$15	0.12
Switch 4	\$10	0.08
Housing A	\$10	0.1
Housing B	\$15	0.06

Table 5.5: Unit cost and commonality significance factor for parts/features

The salvage cost for each product is assumed to be \$3. The maintenance cost (see Eq. (5.13)) is assumed to be obtained from the following formula.

$$C_{m_i} = \begin{cases} 5 & Life(WCS) \geq 95 \\ 15 & 90 < Life(WCS) < 95 \\ 50 & Life(WCS) \leq 90 \end{cases} \quad (5.15)$$

Furthermore, the market size is assumed to be 1,800,000. The production capacity constraint is also determined as the following.

Number of Variants in Product Line	Production Capacity (# of products)
1	1,700,000
2	1,200,000
3	1,000,000

Table 5.6: Production capacity constraint

Now, using the information provided above along with the competitive products information and utilities given in Table 5.2 and 5.3., Stage II of the proposed approach is run to obtain the optimal product line solution.

The second stage optimization is carried out. As mentioned before the maximum number of variants within the product line (i.e., \bar{N}_v) is set to 3. Among the 151 variants to be considered for a product line, the following is obtained as the final optimum product line:

Variant	Motor	Gear ratio	Life	Switch	Girth	Price	Gear type
13	3	3.8	130.7 hr	3	Large	\$79	Helical
79	3	3.9	130.7 hr	2	Small	\$129	Helical

Table 5.7: The optimum product line solution

The corresponding profit for the solution is estimated as \$13,068,122.

In order to make some comparison, the solution is obtained using the following constraints; (i) only one variant in the product line, (ii) only three variants in the product line. The corresponding results are shown in Tables 5.9 and 5.10.

Variant	Motor	Gear ratio	Life	Switch	Girth	Price	Gear type
13	3	3.8	130.7 hr	3	Large	\$79	Helical

Table 5.8: The product line solution – single variant

The corresponding profit for the best product line with one variant in this example is calculated as \$9,396,698.

Variant	Motor	Amp rating	Life	Switch	Girth	Price	Gear type
13	3	11.46 A	130.7 hr	3	Large	\$79	Helical
79	3	11.46 A	130.7 hr	2	Small	\$129	Helical
81	3	11.46 A	127.3 hr	2	Small	\$99	Helical

Table 5.9: The product line solution – three variants

The corresponding profit for the best product line with three variants in this example is calculated as \$8,075,881.

Tables 5.7 - 5.9 provide the optimal solutions when there are 1, 2, or 3 variants in the product line. As mentioned at the end of the first stage, only two motors could satisfy objective/feasibility robustness when the product is used under varying usage situations.

Hence, all product alternatives obtained in Stage I have either of these two motors. As shown in Table 5.7, the optimal product line is composed of two differentiated products with a few common product components. However, motor number 3 costs less and at the same time the product motor (as one of the major parts of the product) has the highest commonality significance factor among the components (see Table 5.5). Therefore it is of no surprise that all variants in the optimal product line share motor 3. Using the same motor in variants in a product line causes somewhat similar life for each variant (i.e., in majority of cases the product life is over when the motor fails to operate).

It should be noted that the variants in the optimal product line have different switch types and girth sizes to satisfy the heterogeneous consumer preferences. Given the existing competitive products and the current cost structure, the product line shown in Table 5.7 yields the highest profit for the product manufacturer. In the following section, the effect of a few parameters on the final optimal product line solution is discussed.

5.4.4. Post-optimality analysis and verification

In order to examine the stability of the obtained solutions, two factors (i.e., fixed cost and production capacity) are identified. A stable solution would not change when these factors vary. These factors play a role in determination of the number of variants in a product line. A product line with more variants can potentially capture larger market share. However, in order to produce higher profit, one should consider the fixed cost as well as the feasibility of the production for product manufacturer. In the following, the effect of each of these factors to the final solutions is discussed.

5.4.4.1. Fixed cost

The fixed cost difference between the one variant and two variant lines is \$3M, and the difference for two-variant and three variant product lines is \$7M. By comparing the product line alternatives presented in Tables 5.7 – 5.9, the following observations can be made.

The optimal product line with two variants produces about \$3.6M more profit than that of the single variant line. Therefore, if the fixed cost difference (which is currently \$3M) becomes more than \$3.6M (i.e., the fixed cost for two variant line is more than \$21.6M) then the optimal product line will become the single variant alternative 13 shown in Table 5.8. In order for a three variant product line alternative to incur a higher profit than the one in Table 5.7, its corresponding fixed cost must become over \$23M, and the fixed cost of the single variant lines should become over \$16.4). In that case the alternative shown in Table 5.9 will become the optimal product line.

The composition of the optimal product line alternatives in Tables 5.7 – 5.9 does not change under fixed cost variations. The effect of another parameter (i.e., production capacity) on the result is discussed in the following.

5.4.4.2. Production capacity constraint

The production capacity W in Eq. (5.8) has a direct impact on the revenue and therefore the profit of the product manufacturer. When the number of variants increases, the volume of the manufactured products for each variant will decrease. The data for production capacity in this example is provided in Table 5.6. The difference in production for a single variant product line or a two-variant product line is 500,000, and

the difference between two-variant lines and three-variant lines is 200,000. The change in these values has an impact on the final product line result. To show the sensitivity of the profit values for each of the single-variant, two-variant, or three-variant product lines with respect to changes in the production capacity, each of the values (i.e., W) in Table 5.6 are varied within a range of $[W-500,000, W+500,000]$. The following figure depicts the change in the optimal product line profit values when the production capacity varies.

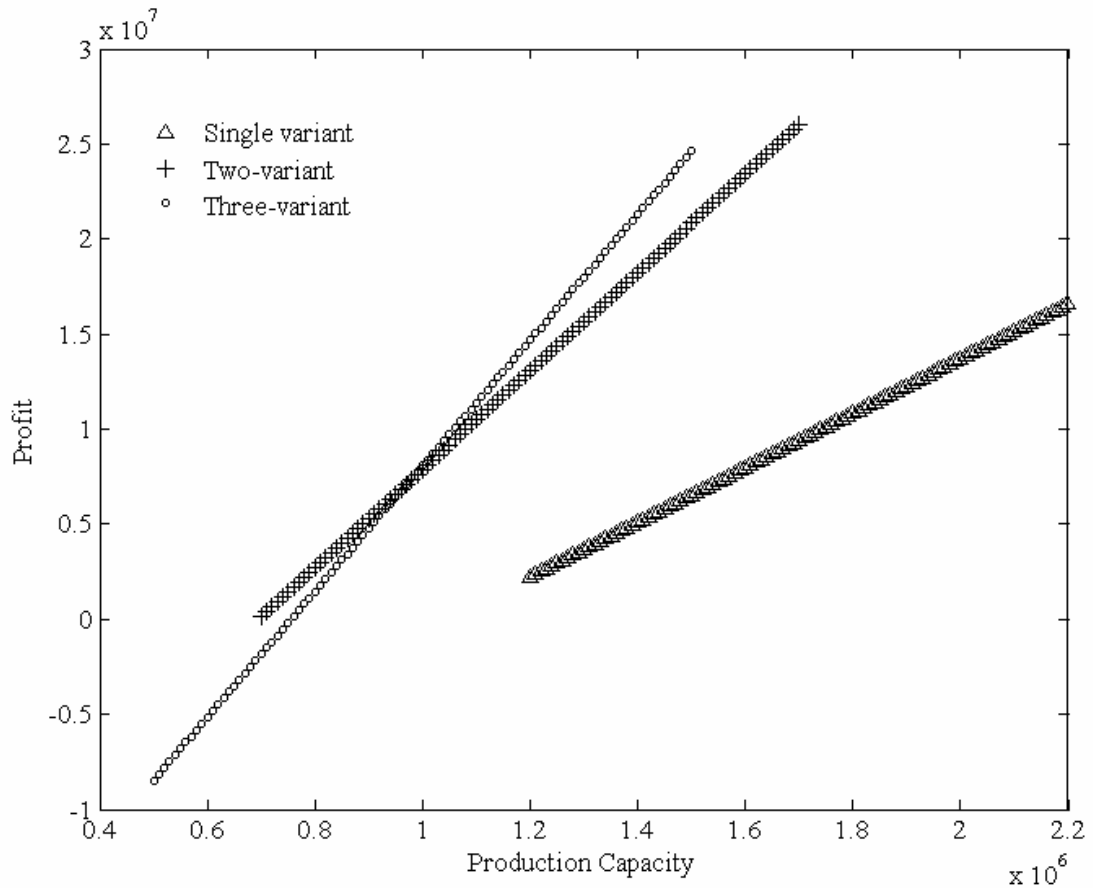


Figure 5.6: Effect of production capacity on overall profit

As shown in Figure 5.6 for the given ranges of product capacity, the incurred profit for two-variant and three-variant product lines is always higher than that of a single variant line. However, if the production capacity for the three-variant product line case

goes above 1,000,000 the optimal product line will have three variants as shown in Table 5.9. Again regardless of the changes in production capacity, the composition of the obtained optimal product lines does not change.

There are other parameters that can impact the composition of variants in the optimal product line. The competitor products information and the customers' responses to the questionnaires (and therefore the partworth utility values) are two examples of such parameters.

The results of Stage II of the example have been verified by an exhaustive search approach to ensure the accuracy of the solutions. The obtained solutions match those obtained by the GA shown in Tables 5.7 – 5.9.

5.5. SUMMARY

In this chapter an approach for robust product line design optimization is presented. As reported in the literature, a product line optimization problem is computationally expensive, and therefore, the robust optimization approach for a product line problem becomes computationally prohibitive. A two-stage approach to alleviate the computational burden is developed. Similar to the single product robust optimization in chapter 4, two domains are covered; engineering design and marketing. Unlike some of the previous two-stage approaches in this area, the presented method was tailored to eliminate only those product alternatives that are non-robust (i.e., perform unsatisfactorily under different usage situations and environments).

The second approach to robust design optimization in chapter 3 is used to determine whether or not each design alternative is robust. Stage I of the approach uses a

GA technique to obtain a promising set of robust product design alternatives. An integrated fitness assignment technique for the GA is adopted that considers engineering design robustness measures (e.g., feasibility and objective robustness) as well as a stochastic dominance measure for the conjoint utilities. Such fitness assignment technique provides a GA with the necessary means to obtain product alternatives that are not only robust in design domain, but have higher utility values under uncertainties. These products yield higher market share values, and are passed onto the second stage of the approach.

The second stage of the approach performs the combinatorial optimization on the results obtained by the first stage to obtain the optimal product line. The product offerings of the competitors, the obtained revenue, the overall incurred cost, the effects of component sharing on the overall variable cost of a product, production capacity are among the factors that are considered for the profit maximizing approach in the second stage.

To demonstrate the two- stage approach to robust product line optimization, a product line design problem is used. The marketing data for this example is obtained through an online survey of power tool users. The results for three cases are obtained; (i) only one variant in the product line, (ii) two variants in the product line, and, (iii) three variants in the product line. The optimum result was the case in which there were only two variants in the product line. In the next chapter, the concluding remarks for this dissertation are provided.

CHAPTER 6

CONCLUSION

6.1. INTRODUCTION

This dissertation has presented three research thrusts. In the first research thrust, which is for multi-objective robust optimization, two different approaches are developed to assess the robustness of a design alternative. In the second research thrust, which is for single product robust optimization, an integrated robust optimization approach is developed for single product engineering with design and marketing considerations. Finally, in the third research thrust, which is for product line robust optimization, the approach for single product robust optimization has been extended to product line robust optimization.

The balance of this chapter is as follows. In Sections 6.1.1 – 6.1.3, a discussion on each research thrust together with advantages and disadvantages of the proposed methods and models are provided. Section 6.2 highlights the contributions of this dissertation, and Section 6.3 provides some ideas for future research directions.

6.1.1. Discussion for Research Thrust 1: Multi-Objective Robust Optimization

The two approaches for multi-objective robust optimization can be applied to a wide variety of engineering design optimization problems. They have the following advantages and disadvantages.

6.1.1.1. Advantages

- Both approaches are deterministic and hence do not require the probability distribution for uncontrollable parameters. Also, both approaches are applicable beyond a small range where a linear approximation scheme is valid.
- Approach 1 can handle problems in which the objective and/or constraint functions are discontinuous with respect to uncontrollable parameters. Furthermore, Approach 1 guarantees the existence of robust solutions that have minimal variability and best performance under worst case scenario.
- Approach 2 provides a means to limit the acceptable range of variability for each objective function.

6.1.1.2. Disadvantages

- Approach 1 requires the location of target and bad points in the design objective space. The results obtained by Approach 1 can be sensitive to the location of these points.
- The robustness measures in Approach 1 are intended to estimate variability along the direction of target and bad points in the objective space, and ignore variability in terms of individual objective functions.
- Both approaches can be computationally expensive especially when the objective and/or constraint functions are expensive to compute.

6.1.2. Discussion for Research Thrust 2: Single Product Robust Optimization

The design robustness measures for Approach 2 are used to assess the robustness of a product design alternative. The single product robust optimization methodology from this research thrust accounts for the requirements from both design and marketing

domains. In this approach, the effects of variations in design performance are mapped to the marketing domain to evaluate variability in customer's preferences. The set of product alternatives generated by the methodology consists of products that not only are optimum and robust from engineering design point of view, but also yield higher market share for the manufacturer. The methodology has the following advantages and disadvantages.

6.1.2.1. Advantages

- The solutions obtained are a set of single product designs that show the best possible performance and maintain feasibility even if they are subject to applications and environments that are different from their standard laboratory conditions.
- The uncertainty in estimating customer utilities due to sampling errors, which is an important factor, is considered.
- The bi-disciplinary rules, discussed in Section 4.4, rank orders product alternatives based on their performance and robustness in both design and marketing discipline, and can be easily extended beyond these two disciplines.

6.1.2.2. Disadvantages

- The single product robust optimization approach can generate a large set of optimal robust product design alternatives. Making a selection from such a large set may not be an easy task. One possible remedy is to perform design robustness prior to the integration with the marketing model, i.e., in a sequential process. However, such a sequential process may eliminate potentially good design

alternatives. Another remedy is to tighten the acceptable range for each objective function. Finally, a selection approach can be used, such as that provided in Appendix I.

- The proposed approach is limited to design and marketing and ignores disciplines such as manufacturing and retail channel. In particular, the approach ignores to account for the concept of a “powerful” channel such as Home Depot and Wal-Mart.

6.1.3. Discussion for Research Thrust 3: Product Line Robust Optimization

The product line robust optimization approach is an integrated sequential two-stage design-marketing technique. This technique significantly reduces the computational cost. Since the robustness is a property of a single product, the same design robustness assessment technique used in Chapters 3 and 4, i.e., Approach 2, is used in the first stage. The customer utilities across market segments are used in Approach 2 to obtain product alternatives that are robust under design parameter variations and also their conjoint utility estimates dominate the rest of the alternatives. In the second stage, combinatorial optimization is used to obtain a product line that yields the maximum profit for the manufacturer. The advantages and disadvantages of the robust product line optimization approach are provided as follows.

6.1.3.1. Advantages

- The approach examines each product design alternative at the first stage in terms of engineering design robustness (i.e., feasibility and objective robustness) as well as its market performance (i.e., the customers’ utility) to identify the most promising product design candidates.

- The first stage of the approach reduces the size of potential product design candidates to a manageable size so that the second stage optimization can be performed.
- The optimization at the first stage is performed at the market segment level, rather than the entire market level. This tactic ensures that the product alternatives that are appealing to the smaller market segments are not eliminated during the first stage optimization.
- The second stage approach takes into account the overall manufacturer's profit. In particular, the cost saving due to components among variants in a product line has been addressed.

6.1.3.2. Disadvantages

- Although the first stage of the approach accounts for both design and marketing considerations, it is still possible that some promising single product candidates be eliminated during the first stage of the approach.
- In spite of the reduction in the computational cost, the two stages can still be computationally expensive. In particular, in the first stage during the fitness assignment step, every product design alternative needs to be evaluated in terms of design robustness and the stochastic utility dominance (within a generation). These evaluations can add a significant computational complexity to the proposed approach, as discussed in the next section.

6.2. Remarks on Computational Costs

The majority of methods and procedures discussed in this dissertation are based on the two robust optimization approaches discussed in Chapter 3. These two approaches

are structured as an outer-inner optimization problem. In the outer level, a multi-objective optimization is performed, and in the inner level, several single objective optimization problems are solved to help in robustness assessment of solution candidates in the upper level problem. This bi-level structure contributes significantly to the computational complexity of the approach especially when the inner level optimizer needs large number of function calls to converge to a solution. It should be noted that the inner level single objective optimizations have to be solved using a global optimization technique. If for any case the range of design parameter variations is not wide or if the objective functions or constraints are convex or monotonic and differentiable with respect to the design parameters, then a traditional optimization technique can be used, and that reduces the computational cost of the proposed approach significantly. Most of the traditional gradient based optimization techniques converge to a solution in about 10^2 order of magnitude function calls. However, since the approaches are intended to address a wide variety of real-world engineering design problems, a global optimization technique such as a genetic algorithm has been used.

As mentioned in Chapter 3, a few observations are made with respect to the number of function calls required for each approach:

- For Approach 1, the *WCSD* and variability measures need to be obtained in the bottom level block of Figure 3.4. Moreover, the feasibility robustness for each design alternative passed from the upper level optimization is determined in the middle level block of Figure 3.4. The number of function calls required in Approach 1 was provided in Eq. (3.5). An estimate of the actual number of function calls for both numerical and engineering examples were also provided in

Chapter 3. In particular, the numerical example required about 500,000 function calls to obtain robust optimal solutions, and the engineering example required about 7,500,000.

- Approach 2 (See Figure 3.8) needs to obtain the maximum variation from the nominal value for all objective functions to assess the objective robustness of each design alternative. This means that the number of inner optimization problems for objective robustness assessment is equal to the number of objective functions. The feasibility robustness in this method is the same as that in Approach 1. The total number of function calls required for both objective robustness and feasibility robustness using Approach 2 has also been provided in Eq. (3.6). Again, an estimate of the actual number of function calls for the numerical case examples using Approach 2 was about 700,000. Also, the total number of function calls for the engineering example using the approach 2 was estimated to be about 7,500,000.
- Although the actual number of function calls of both Approach 1 and Approach 2 was comparable for the two examples in Chapter 3, it should be noted that, generally speaking, Approach 2 is computationally more expensive than Approach 1. The main reason that the number of function calls was close or were about the same was that both examples had only two objective functions. If the number of objective functions increases, the computational cost for Approach 2 will be higher than that of Approach 1 (compare Eqs. 3.5 and 3.6).

In the next section we highlight the main contributions of this dissertation.

6.3. Contributions

The following gives a summary of the contributions for this dissertation:

- Developed two new approaches for multi-objective robust design optimization. Each method can obtain a set of design solutions that are optimally robust with respect to uncontrollable parameter variations. Unlike most of the reported robust optimization methods in the literature, neither of the approaches requires the probability distributions of uncontrollable parameters. Moreover, neither approach uses an approximation scheme which could limit applicability to problems in which the range of parameters variations is small. Also, both approaches can be used for design problems where the objective and constraint functions are non-differentiable or discontinues with respect to the uncontrollable design parameters.
- Developed two new measures for multi-objective robustness for Approach 1. These measures are based on the concepts of the sensitivity of a design in the objective space. The first measure, *WCSD*, is based upon how far the worst case scenario point in that sensitivity region is located from a target point specified by designer. The second measure, variability, is based upon how far the worse case scenario and best case scenario points are from each other.
- Developed a method to assess the feasibility robustness of design solutions. This method is used in both robust multi-objective optimization approaches. The feasibility robustness method ensures the design alternatives to remain feasible under uncontrollable variations of design parameter.

- Developed a new method to assess the robustness of multi-objective of design solutions for Approach 2. This method is based on the concept of sensitivity of the objective function due to uncontrollable parameters. Approach 2 can be applied to problems where the designer needs to limit variability of each individual objective function.
- Developed an integrated framework for single product optimization that combines the criteria from both engineering design and marketing domains. Unlike most of the approaches in the extant literature, the proposed framework takes into account the effect of the variations in design parameters from the design domain to the marketing performance of a product. The single product solutions obtained are not only optimum and robust from engineering design point of view, but also have high market share.
- Developed an integrated two-stage framework that efficiently reduces the size of the initial set of single product candidates to a manageable size in the first stage and then obtains an optimum and robust product line alternative based on a second stage approach. The novel aspect of the proposed product line robust optimization approach is that it goes beyond just a profit maximization technique and actually accounts for the effect of the uncontrollable parameter variations on design performance and feasibility as well as the customers' preferences.

Next section provides some suggestions that can be considered for future research.

6.4. Future Research Directions

This section briefly presents some general research directions.

- The proposed robust design optimization approaches are based upon some worst case scenarios. Therefore, the solutions obtained by these methods can be very conservative. If there is more information about parameter variations (e.g., probability distribution is available) then these approaches can be improved to produce less conservative solutions.
- Due to the outer-inner structure, both robust optimization methods are computationally expensive especially when the objective and/or constraint functions are obtained using a computationally expensive design simulation software tool. In this regard, the use of an approximation technique with the proposed robust optimization approaches should be explored and expected to alleviate the computational burden of both approaches significantly.
- The integrated robust single product optimization approach currently accounts for two disciplines, namely, engineering design and marketing. However, the approach can be extended to a multidisciplinary case where other disciplines such as manufacturing, finance, retail, etc. are to consider for product design development.
- The number of final product designs obtained by the single product robust optimization approach can be large, making the selection among a large number of product candidates a non-trivial task. One remedy to this problem is to perform the robust optimization for each discipline (i.e. design and marketing) in a separate and sequential manner. However, doing so can result in elimination of many good product candidates. An appropriate product design selection technique can be combined with the approach. An example for such a technique is presented

in Appendix I. However, the approach presented in Appendix I requires known probability distribution of design attributes.

- The proposed product line optimization approach is a two-stage technique and may eliminate good product candidates in each stage. Certainly, an integrated single stage approach should be able to obtain better solutions.
- The constraints defined for the multi-objective optimization problems in this dissertation must be satisfied at the same time. In other words, the constraints are expressions that are linked with a logical ‘AND’ operation. In some cases the ‘AND’ operation may not characterize the links between the constraints, and other Boolean operations (e.g., OR, XOR, etc) may need to be used. The constraint evaluation module in the implementation of the approach can be extended to allow for such Boolean operations.
- The ranges of uncontrollable parameters in this dissertation are defined based on an assumption that the upper and lower bounds of parameters variations are fixed (i.e., known and deterministic). However, in some cases, a designer may not be able to provide exact values for these upper and lower bounds of parameter variations. Therefore, allowing for ‘fuzzy’ and/or ‘uncertain’ values for these bounds can help characterize the cases when the bounds are imprecise and/or uncertain.

APPENDIX I.

This appendix provides an approach for a single product design selection under uncertainty. The main assumption behind the approach in this appendix is that the probability distribution of uncertain design attributes is known. Also it is assumed that design and marketing domains have the same set of attributes.

A DECISION SUPPORT SYSTEM FOR PRODUCT DESIGN SELECTION: A GENERALIZED PURCHASE MODELING APPROACH*

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Abstract

Selection of a final design for a new product that is to be introduced in the market is a very critical step in the new product development process. The selection needs to consider three factors of importance: anticipated market demand for the design, designer's preferences, and uncertainty in achieving predicted design attribute levels under different usage conditions and situations. We propose a generalized purchase modeling approach that considers all of the above factors and develop a customer based expected utility metric that forms the basis for a Decision Support System (DSS) for

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supporting the selection in product design. We illustrate the modeling approach and the use of DSS with the help of a case example that highlights the utility of the proposed DSS.

Keywords: Product design selection, selection under uncertainty, multi-attribute decision making

A-I.1. Introduction

The final decision to select a particular design for a given product is perhaps the most critical stage in product design development. Obviously, such a decision is influenced by many factors, the specifics of which are not known a priori during the design stage. As such, a quantitative basis for comparison and selection of the best design solution among a host of alternatives could greatly impact the eventual success or failure of a product in the market. The importance of this issue prompts for more sophisticated design selection criteria and methods to incorporate all important factors of interest into the selection of a single final design.

There are three main factors that influence a successful product design selection: 1) market demand based on customers' preferences; 2) designers' preferences based on his/her knowledge and experience with design issues and market issues; and 3) uncertainty in achieving the predicted design attribute levels (or performance). For example, a design alternative may fail to become a successful product if it does not gain and maintain enough market demand. On the other hand, considering the market demand by itself does not secure a successful product in the market. For instance, introducing a product at low price into the market might increase the initial product demand

significantly. However, it may not be possible to sustain such a demand in the longer run (repurchase of the product) due to the poor performance (and reliability) with respect to some of the product attributes. A designer's knowledge and experience can be very useful in predicting product performance if customers' evaluations are not known a priori. As such, the designer's preferences can be used to specify the product performance in terms of its attributes, which customers may not know or consider at the point of purchase. Also, a designer can incorporate the specifics of competitive products in his/her preference function so that the new design can be appropriately positioned in the market relative to competition [22]. Finally, because of the uncertainty in product design parameters (such as, manufacturing tolerances and variations in the product usage environment), the design attribute levels can deviate from their nominal values and affect the product performance. Such uncertainty can make or break a product in the market and it is, thus, important to consider these variations in selecting a final product design.

There are many individual and group decision making techniques in the literature that can be used for product design selection, among which Multi-Attribute Utility Theory (MAUT) [19], Analytical Hierarchy Process (AHP) [25], and Conjoint Analysis [16],[17] are used extensively. While many of these techniques address a subset of the above identified factors, none of them address all three factors simultaneously. For instance, MAUT employs the von Neumann and Morgenstern (vNM) utility theory to model an individual's (a designer's or customer's) preferences [28]. Many applications of MAUT that are used for modeling a decision maker's preferences for rank ordering and selection among a set of alternatives can be found in marketing and management science literature [4],[9],[11]. However, applications of MAUT for customer elicitation using a

lottery technique are mostly limited to highly educated respondents [18], which may be applicable to a small segment of the market. While AHP is relatively easier to implement for any customer group [15], its simplification and unwanted rank reversal may sacrifice its predictive validity especially when the design attributes are significantly correlated. Conjoint Analysis (CA) is another approach for multi-attribute decision making problems where the focus is preference elicitation at the individual customer level. Two types of conjoint models are discussed in the literature: compositional (self explicated) and decompositional. In the self explicated methodology, the customers are asked to give the importance weight for each attribute followed by the rank ordering of distinct levels for each attribute. There are several issues related to the self explicated approaches. First, the customers may not be able to provide the accurate information in terms of the weights (e.g., due to socially accepted values). Moreover, there is a low chance of detecting potential nonlinearity in partworth (e.g., utility) function. Conversely, in decompositional approaches the customers are only asked to express their preference or choice among the product profiles, and it is the researchers' responsibility to ensure that relevant attributes considered in the profile generation. In most cases, the preference model is presumed to be of the same general structure for all individuals in a population sample [17]. As such, even by allowing an error term, a CA model might not represent the precise behavior of all individuals in the sample. In other words, different segments (or even individuals) in the market may have different preference structures. Therefore, a reasonably accurate customer categorization (market segmentation) is perhaps the most critical step in the marketing study [24].

In addition, in order to overcome preference aggregation problems and account for choice uncertainty, a conjoint model can be based on a discrete choice model that utilizes an individual's selection behavior [20]. Among the discrete choice models, the probabilistic-based models such as multinomial logit [21], [5], probit [12], mixed logit-probit [7], and also deterministic models such as the first choice model [26] are based on the customers' utilities. All of the above-mentioned selection models are compensatory, and are likely to select the product with the highest customer utility. However one can argue that the purchase decision rules can be non-compensatory. In other words, many customers may not choose the product with the highest aggregated utility due to economic or other considerations (e.g., purchase reservation prices). This issue eventually makes the pure compensatory decision rules difficult to implement, especially in industrial markets where decision rules are not purely compensatory. Hence, the choice model needs to allow for the consumers' acceptable bounds on each attribute while taking into account their interactions. In many situations in industrial markets the manufacturers (i.e., product components consumers) set acceptable bounds on the product specifications (i.e., clearly explicated decision rules for selection of equipments). Our methodology takes into account both compensatory and non-compensatory decision rules through our generalized purchase modeling approach. The customers purchase decision rules is obtained using a self explicated technique. Moreover, our approach not only considers the customers purchase criteria and designer's preferences in the selection of the product design, but also it allows for the uncertainties in attaining the specified nominal attribute levels. A decision support system based on the three mentioned factors

will, therefore, be market focused and also take into account the realities of the design development process.

There are several market-based DSS methodologies reported in the literature to aid product selection [10], [22], single product design selection [2], [3], and product line design [1]. The selection criteria in these methods are mostly either based on maximization of the market share, the seller's return or minimization of job completion time. Nevertheless, there are uncertainties involved with each of the mentioned problems that can affect the results significantly. We propose a generalized purchase function, an extension of our approach [6] to model the customer purchase behavior to capture the impact of all the above mentioned three factors. The customers purchase criteria (captured using a self-explicated approach) can be given as an input to our newly developed DSS for final product design selection. The capability of the new DSS to handle sophisticated and realistic decision-making situations is demonstrated with an example in industrial market, i.e., product design selection of a power electronic module.

The organization of the rest of this paper is as follows. In Section A-I.2, a description of the Customer-based Expected Utility (*CEU*) metric is provided, along with a generalized DSS to model customer purchase decision. Section A-I.3 is devoted to an application of the proposed methodology to an example: product design selection for power electronic modules. Finally, the concluding remarks of the paper are provided in Section A-I.4.

A-I.2. Methodology

As discussed in the previous section, we account for three factors that impact product design selection significantly: 1) market demand; 2) uncertainty in achieving

nominal attribute levels; and 3) designer's preferences. The overall framework of our approach is shown in Figure A-I.1. A number of product alternatives are generated within the design process. The product attributes (both performance and market related) can be obtained using design simulation tools and marketing models. The main objective of this paper is to present a DSS that aggregates the above factors into a single-valued (scalar) metric, one that accounts for the utility function of the designer, the product's demand (based on customers' preferences), and the uncertainty in attaining a desired attribute level. Thus, our DSS can be used to identify the optimal product design from a large set of product design alternatives. In the following subsections, each of the factors used in our DSS is discussed separately.

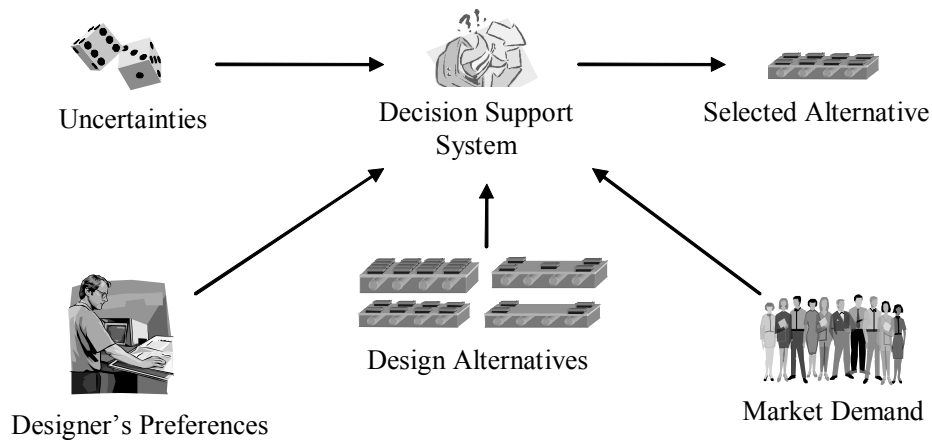


Figure A-I.1: Overall product design selection framework

A-I.2.1. Normalized Market Demand

We define the normalized market demand of a product as the percentage of customers in a market who decide to purchase a product with a given combination of attribute levels. One could predict whether or not a customer buys a certain product with a combination of attribute levels. By aggregating such a purchase decision over a

representative sample of customers, the demand of a product can then be estimated. Here, it is assumed that the customers have prior experiences with similar existing products in the market and therefore, they can evaluate the product and make a purchase decision based on the product attributes. The non-compensatory choice models do not make tradeoffs among the attributes directly such as compensatory models do (e.g., multi attribute utility function). The three most important noncompensatory approaches are the conjunctive, disjunctive and lexicographic [14]. In the following, we define a generalized purchase decision model, and relate it to noncompensatory choice models.

Generalized Purchase Decision: A customer's purchase decision function $D_p(\mathbf{x})$ is defined as follows:

$$D_p(\mathbf{x}) = \begin{cases} 1 & \text{If customer buys a product at } \mathbf{x} & (\mathbf{x} \text{ is within customer's acceptable ranges}) \\ 0 & \text{If customer does not buy a product at } \mathbf{x} & (\text{Otherwise}) \end{cases} \quad (\text{A-I.1})$$

where $\mathbf{x} = (x_1, \dots, x_n)$ is the vector of design attributes.

The above definition does not clearly address the relation between components (attributes) of vector \mathbf{x} . To address the issue of the interactions between attributes, we introduce the noncompensatory customer choice models:

Conjunctive: In a conjunctive choice model, the customer would purchase the product only if all of the attributes of the product are within the customer's acceptable ranges. If any attribute is deficient, the purchase decision function for that design alternative becomes zero.

(All attributes are within customer's acceptable range)

(Otherwise)

$$D_p(\mathbf{x}) = \begin{cases} 1 & \text{If customer buys a product at } \mathbf{x} \\ 0 & \text{If customer does not buy a product at } \mathbf{x} \end{cases} \quad (\text{A-I.2})$$

Disjunctive: In a disjunctive model it is sufficient that at least one attribute of the product satisfies the customer. For instance, under the conjunctive model, the customer may insist on purchasing a light weight *and* inexpensive product. However under the disjunctive model the customer would settle for a product with either low weight *or* low price.

$$D_p(\mathbf{x}) = \begin{cases} 1 & \text{If customer buys a product at } \mathbf{x} & \text{(At least one attribute is within} \\ & & \text{customer's acceptable range)} \\ 0 & \text{If customer does not buy a product at } \mathbf{x} & \text{(Otherwise)} \end{cases} \quad (\text{A-I.3})$$

Lexicographic: In a lexicographic model all attributes of the product are considered in a hierarchical manner from the most important to the customer all the way to the least important. In other words, the product is evaluated based on the most influential attribute first, and if there is a tie, the second most influential attribute is used and so on until there is no tie among the products under consideration.

A customer's preferences with respect to several attributes may be too complicated to be modeled simultaneously by one of the above-mentioned choice models. However a combination of noncompensatory models can capture the interactions among attributes more naturally. For example, the following customer's purchase scenario cannot be handled with a pure disjunctive or conjunctive choice model:

“The price should not be over \$100, and the weight needs to be no more than 3 lbs, but if the product is on sale for less than \$60, then I am willing to buy one that is up to 5 lbs in weight”.

By using Boolean expression, we can model the above customer’s purchase decision, as shown in Eq. (A-I.4). The binary decision diagram of such a customer is depicted in Figure A-I.2. The solid lines in Figure A-I.2 are used when the statement (or event) holds, while the dashed lines indicate that the event does not hold. For a thorough description of binary function representations, see [8].

$$D_p(\mathbf{x}) = \begin{cases} 1 & [(\text{cost} < 100) \wedge (\text{weight} < 3)] \vee [(\text{cost} < 60) \wedge (\text{weight} < 5)] \\ 0 & \text{Otherwise} \end{cases}$$

$$= \begin{cases} 1 & (A \wedge B) \vee (C \wedge D) \\ 0 & \text{Otherwise} \end{cases}$$

(A-I.4)

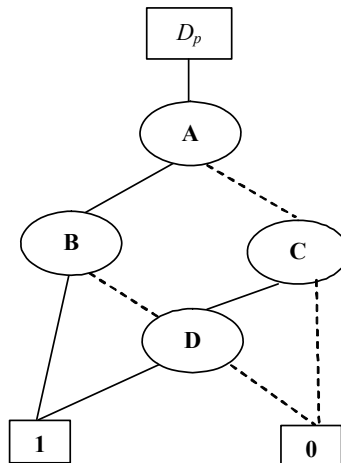


Figure A-I.2: Binary decision diagram representation of a customer choice model

The customer purchase decisions can be modeled by a combination of the aforementioned noncompensatory choice models. Basically it is possible to represent every Boolean expression using a Conjunctive Normal Form (CNF) or Disjunctive Normal Form (DNF) and either one can be converted to the other. The CNF can be constructed by the conjunction of disjunctive expressions. The general form of CNF is shown in Eq. (A-I.5).

$$\left(t_1^1 \vee t_2^1 \vee t_3^1 \vee \dots \vee t_{k_1}^1\right) \wedge \dots \wedge \left(t_1^l \vee t_2^l \vee t_3^l \vee \dots \vee t_{k_l}^l\right) \equiv \bigwedge_{j=1}^l \left(\bigvee_{i=1}^{k_j} t_i^j\right) \quad (\text{A-I.5})$$

Likewise, DNF can be shown as in Eq. (6).

$$\left(t_1^1 \wedge t_2^1 \wedge t_3^1 \wedge \dots \wedge t_{k_1}^1\right) \vee \dots \vee \left(t_1^l \wedge t_2^l \wedge t_3^l \wedge \dots \wedge t_{k_l}^l\right) \equiv \bigvee_{j=1}^l \left(\bigwedge_{i=1}^{k_j} t_i^j\right) \quad (\text{A-I.6})$$

The purchase decision of the customer shown in Figure 2 is defined as a DNF. However, it can be converted to CNF as shown in Eq. (A-I.7).

$$D_p(\mathbf{x}) = \begin{cases} 1 & (A \vee C) \wedge (B \vee C) \wedge (A \vee D) \wedge (B \vee D) \\ 0 & \text{Otherwise} \end{cases} \quad (\text{A-I.7})$$

For a given sample of customers, the normalized demand q of a product with a vector of attribute \mathbf{x} can be calculated by:

$$q(\mathbf{x}) = \frac{\sum_{i=1}^N D_{p_i}(\mathbf{x})}{N} \quad (\text{A-I.8})$$

where N stands for the total number of customers in the sample, and D_{p_i} refers to the purchase decision of the i -th customer in the sample.

In obtaining the normalized demand, the purchase decision rules for each customer are captured through a self-explicated approach. Every customer expresses his/her purchase decision criteria in one of the above mentioned normal forms. In this model, it is assumed that the sampling errors are insignificant (i.e., the sample resembles the whole population). In industrial markets, which are the focus of our study, it is quite common for sales team to interact with customers to understand client requirements and criteria better. In many cases, clients may explicitly provide their specific criteria and information on acceptable upper and lower bounds on attributes (arising from performance and quality considerations). However, tradeoff information between attributes is generally not provided. This information is obtained directly through self-explicated responses (as in a conjoint study) from the buyers/buyer segments.

A-I.2.2. Uncertainty in Achieving Nominal Attribute Levels

In a product design process, it is common to use design simulation tools. These tools can help to simulate the performance (design attributes) of a design to the variation in input parameters. The uncertainty in an attribute level is generally due to uncontrollable randomness in input design parameters (such as manufacturing tolerances, deviation in the source voltage and frequency, and changes in the environment temperature). As a result, the design attribute levels may deviate from the nominal values. When there is enough information (e.g., data) about the uncertainty in input parameters, an appropriate probability distribution can be constructed. The variations in the design attribute levels can be modeled by mapping from the input design parameters space to the design attribute space through the design simulation tools. Monte Carlo simulation is one of the common methods for modeling the uncertainties by constructing a design attribute

distribution, hereafter referred to as p_i . In Monte Carlo simulation, a sample of possible input design parameters (representing an appropriate distribution) is selected and mapped into the corresponding attribute levels, which in turn creates a probability distribution for the uncertainty in an attribute level. Such mappings can be performed by the functional form of the design performance attributes (if available) or by the design simulation software (e.g., numerical results of a finite element analysis, computational fluid dynamics, etc.). As an example, we can estimate the overall weight of a product (e.g., a single chip module) by adding the weight of the chip and the PCB board. Then, the variability in the total weight of the module can be estimated by sampling the weight of each component (Chip and PCB) and calculate the total weight by adding up the component weights.

It should be noted that many important aspects of a product can be simulated by using design simulation software. The performance and quality of a product is then directly assessed by examining the impact of product design attributes on product quality and performance based on the simulation output results. For example, in power electronic device development, the thermal performance of a device (e.g., junction temperature) and the development cost (e.g., planning, design, parts, assembly, etc) are two attributes that represent the quality and performance of that design alternative. Our methodology uses the design simulation software and subsequently takes the performance and quality aspects of each design alternative into account during the selection process.

The proposed design selection approach is able to allow for the uncertainties in the design performance attributes and capture the customers' purchase decisions along with the designer's preference for selecting the optimal product design.

A-I.2.3. Designer's Preference

The designer's preference is one of the key elements in product design and development. It reflects the designer's experience and expertise of the design and knowledge of the market. Moreover, it enables the consideration of potential design alternatives that are promising from the designer's (or producer's) point of view (for example, identifying designs that can have superior performance and reliability or designs that can offer better competition along several dimensions, which consumers may not have knowledge about). Thus, the designer's preference in our DSS is used to ensure the quality of the product that may not be explicitly known to ordinary customers. This issue becomes more demanding when we plan to launch a product that has desired performance in long term both in the market and field. We have used a MAUT approach [19] to capture the designer's preferences. As mentioned earlier, MAUT is an effective and powerful methodology for preference modeling especially where only a single respondent (i.e., the designer) is able to understand and respond to lottery technique questions. Although there are several forms of the utility function that can be used to model the designer's preferences, for our DSS we have chosen a multiplicative form that is able to handle the interaction among the attributes. Also, with respect to each individual attribute a quadratic form of the utility function is used. The general form of a multiplicative utility function is shown in Eq. (A-I.9). The details of capturing the designer's utility function (U) and the scaling constants k 's and K are beyond the scope of this paper.

$$\begin{aligned}
U(\mathbf{x}) = & \sum_{i=1}^n k_i u_i(x_i) + K \sum_{\substack{i=1 \\ j>i}}^n k_i k_j u_i(x_i) u_j(x_j) + K^2 \sum_{\substack{i=1 \\ j>i \\ l>j}}^n k_i k_j u_i(x_i) u_j(x_j) \\
& + \dots + K^{n-1} \prod_{i=1}^n k_i u_i(x_i)
\end{aligned}
\tag{A-I.9}$$

$$u_i(x_i) = a_i x_i^2 + b_i x_i + c_i$$

where K is the scaling constant calculated from Eq. (A-I.10).

$$1 + K = \prod_{i=1}^n (1 + K k_i) \tag{A-I.10}$$

A-I.2.4. Customer Based Expected Utility Metric

Suh [27] introduced a metric known as a *probability of success* in product design that combined the uncertainty in each attribute level with a customer's acceptable range. As shown in Figure A-I.3, Suh's metric is defined as the area under the probability density function (PDF) that falls within a customer acceptable range for that attribute (i.e., the overlap between the design and customer ranges). In essence, Suh's metric reflects the probability that the product attribute level will fall in the range that a customer deems desirable or acceptable.

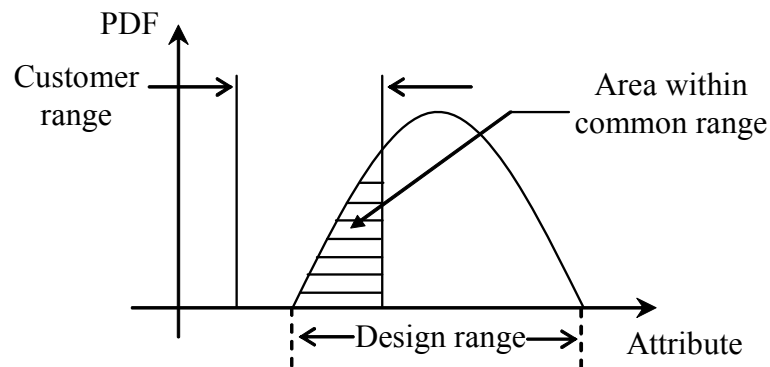


Figure A-I.3: Probability of success [27]

Using the terminology introduced in this paper, Suh's probability of success, P_s , can be reformulated as:

$$P_s = \int \int \cdots \int D_p(\mathbf{x}) p(\mathbf{x}) dx_1 dx_2 \dots dx_n \quad (\text{A-I.11})$$

where $D_p(\mathbf{x})$ is the purchase decision function, and $p(\mathbf{x})$ is the joint probability distribution for design attributes.

In formulating our metric, we extend Suh's probability of success measure by taking into account not only the uncertainty in the design but also customer's purchase decision and the designer's preference. We define the Customer-based Expected Utility (*CEU*) metric by weighting Suh's probability of success measure with the designer's utility over the ranges of attributes that are of interest to the customer. As shown in Eq. (A-I.12), *CEU* is an estimate of the expected designer's utility under the condition that the attribute level falls in the acceptable range of customer. If the designer and customer share the same acceptable range for the product attributes (i.e., complete overlap of the customer's range and designer's range), then the *CEU* metric turns into the designer's expected utility, and the customer input does not play a role in the utility calculation for that particular design. On the other hand, if there is no overlap between the designer's range and customer's range for a design alternative, then it implies that the design is not likely to succeed in the market (i.e., zero probability of success), yielding the lowest *CEU*, which is equal to zero. The reason for incorporating the designer's preferences in the *CEU* metric is to ensure the consideration of quality and performance of the product that may not be explicitly known to ordinary customers. In addition, the appropriate

product positioning in the market can also be considered using the designer's preferences. In the case that several design alternatives are within the customer acceptable ranges and exhibit acceptable technical performance, the designer can choose to give a higher utility to the alternative that is different from the current competitive products in the market. There are many real world industrial situations in which the designer's preference and his/her knowledge about the customer requirements play a key role in the success of the product development. The new design for Airbus A380 [22] is an example where the design project manager could decide on several technical challenges to satisfy conflicting customers' requirements in the presence of competition.

Basically, the most influential part of the *CEU* is decided by the customers in terms of their acceptable range for each attribute. The successful product design candidates are the ones that can accommodate the widest customer range, and among those (if there is a tie), the one with the highest designer's utility is selected by our DSS.

Next, the application of the *CEU* metric is discussed for different cases of single/multiple market segments.

Case 1: Market characterized by a single segment

In this case there is only one segment characterizing the market whose purchase decision for a product is captured by function D_p . The uncertainty at a single attribute level is given by a probability distribution function p . Figure A-I.4 demonstrates these functions along with the designer's utility U in the case of a single attribute x .

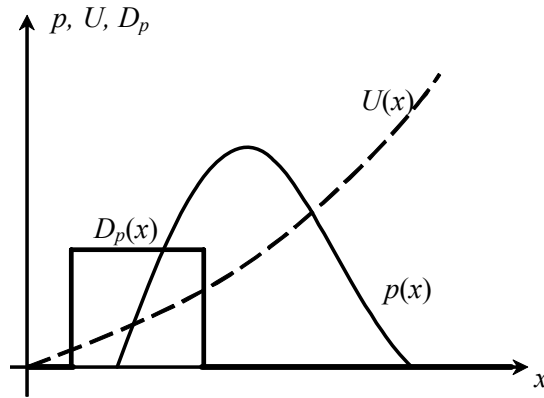


Figure A-I.4: The components of *CEU* for a single customer

The *CEU* of a design alternative can then be defined as follows:

$$CEU(\mathbf{x}) = \iint \dots \int D_p(\mathbf{x}) p(\mathbf{x}) U(\mathbf{x}) dx_1 dx_2 \dots dx_n \quad (\text{A-I.12})$$

where $D_p(\mathbf{x})$ is purchase decision function, $p(\mathbf{x})$ is the joint probability distribution for design attributes, and $U(\mathbf{x})$ is the designer's utility function. The *CEU* function reflects the expected value of the designer's utility while accounting for the market information (i.e., desired range and purchase decision of attributes). According to this metric, a design alternative with the set of attribute values that are not able to satisfy the market (i.e., not within the range of attributes as wanted by customers) will yield a zero *CEU* value. On the other hand, a design alternative for which the design and the market have the maximum common range and at the same time has the highest designer's utility yields the highest *CEU* value. Such an alternative is the one, among all alternatives under consideration, which is most likely to satisfy the customers while also being preferred by the designer.

However, real-world product design selection usually involves a market with numerous customers whose purchase decisions might be different or even conflicting with one another. The next subsection focuses on multi-segment market.

Case 2: Multiple segments

To account for multiple-segment preferences, the normalized demand of a product is used instead of a purchase decision function in formulating the *CEU* metric. In Figure A-I.5, the normalized demand of a product, $q(x)$, is shown as a function of the vector of attribute levels. As mentioned before, demand can easily be obtained by aggregating the purchase decisions D_p of each customer segment in the market.

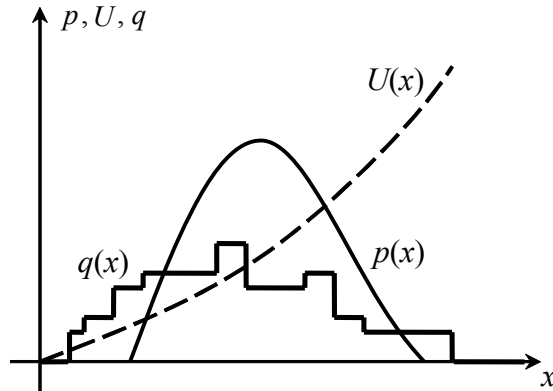


Figure A-I.5: The components of *CEU* for multiple segments (single attribute)

Therefore, the *CEU* of a design alternative can be obtained by replacing the individual's D_p in Eq. (A-I.12) with an estimated normalized demand $q(x)$ obtained from Eq. (A-I.8).

$$CEU(\mathbf{x}) = \int \int \cdots \int q(\mathbf{x}) p(\mathbf{x}) U(\mathbf{x}) dx_1 dx_2 \dots dx_n \quad (\text{A-I.13})$$

Figure A-I.6 shows an example of the most general case for product design selection

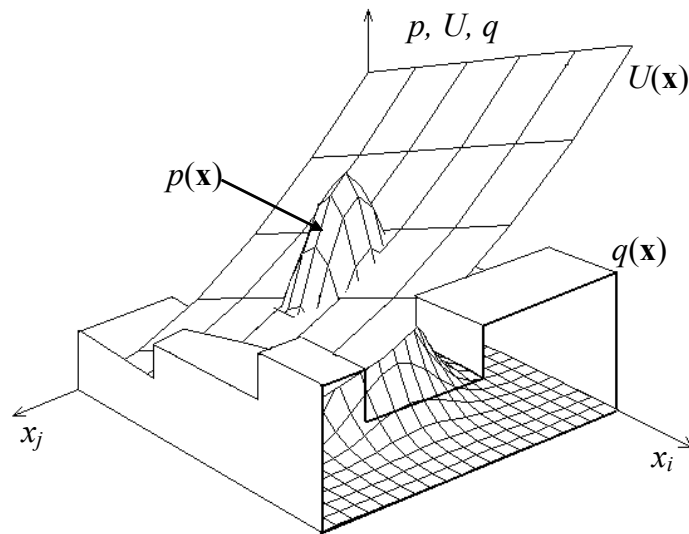


Figure A-I.6: The components of *CEU* for multiple segments (two attributes)

The next section describes the general selection (DSS) framework based upon the proposed *CEU* metric.

A-I.3. DSS for Design Selection

The DSS for the design selection process is shown in Figure A-I.7. It is assumed that the design input parameters are subject to a random variation (or noise) due to environmental and/or other conditions. The design simulation model receives the values of design parameters as input and returns the values of attribute levels as output. A Monte Carlo simulation is employed to sample uncertainties in the design parameters and

compute the PDF of attribute levels (i.e., $p(\mathbf{x})$). Next, the designer's utility and also the generalized purchase decisions for each market segment are obtained. The normalized demand of a design alternative is then estimated by aggregating the purchase decisions. Finally the *CEU* metric is calculated for a given design alternative. This procedure has to be performed over all design alternatives under consideration, and the output of the DSS is the alternative with the highest *CEU* value that meets both designer's preferences and market demand the best.

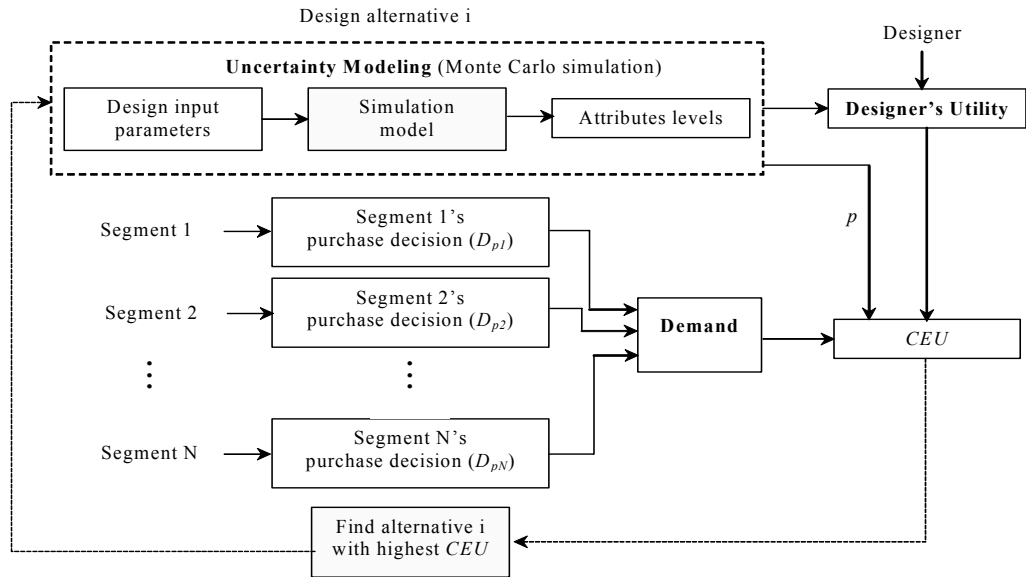


Figure A-I.7: DSS for design selection

A-I.3.1. Case Example

The proposed DSS is applied to the design and selection of a power electronic device with three performance attributes. The attributes are: manufacturing cost (x_c), junction temperature (x_T), and thermal cycles to failure (x_F). As a demonstration of our approach, we only consider ten design alternatives (that have tradeoffs with respect to one another) for their rank ordering.

Three design disciplines are involved to simulate the performance of each design alternative given the input design parameters (e.g., the geometry of power chips on the module, coolant flow rate, ambient temperature, market prices). A screenshot of the DSS user interface is shown in Figure A-I.8. Most of the engineering design simulators do not provide a closed functional form for the output responses as a function of inputs (e.g., finite element models, computational fluid dynamics simulators). An evolutionary algorithm, Multi-Objective Genetic Algorithm (MOGA), is used in our case study to help with searching the design space. (Details of MOGA is beyond the scope of this paper, however, for a review of MOGAs and other evolutionary algorithms refer to [13].) The solution from a multi-objective optimization problem as stated in this example is a set of design alternatives (called a Pareto set). The goal is to use our DSS for selection of the most promising design alternative from this set of Pareto alternatives.

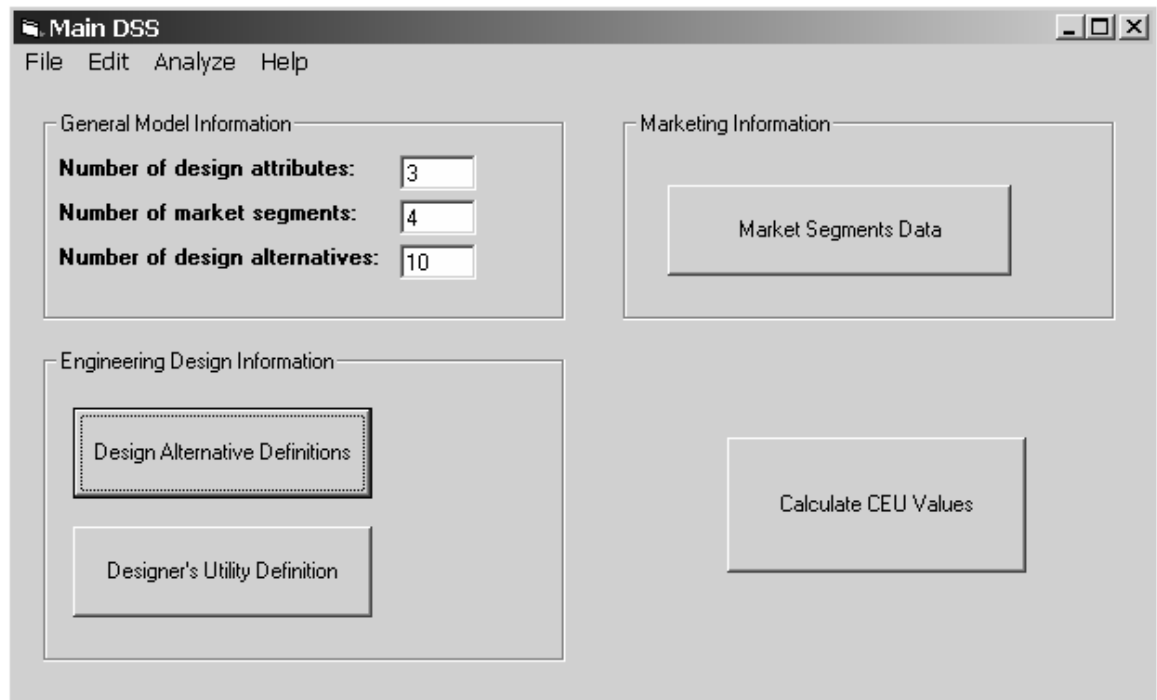


Figure A-I.8: DSS user interface - main window

Based on historical market data and design laboratory experiments, an appropriate distribution is fit to the data collected for input design parameters. Figure 9 shows the design alternative generation process. Using the distribution obtained for input design parameters, a Monte Carlo simulation is performed. With the Monte Carlo simulation, the design alternative generator (i.e., multi-objective genetic algorithm optimizer) is used for all sampled input parameters to obtain distributions of the output performance attribute levels. It is determined that the normal distribution is the best fit for all three attributes. For simplicity, it is also assumed that the probability distributions of the attributes are statistically uncorrelated. The nominal values of attribute levels for these alternatives are shown in Table A-I.1. The standard deviation for junction temperature is estimated as 3.5°C, for cycles to failure 100 cycles, and for cost \$1.67. It is assumed that there is a fixed profit margin of \$100 on each product (and it is the same for each design alternative.) To enter design attributes information, the user needs to click on Design Attribute Definitions and enter the appropriate values for each attribute as shown in Figure A-I.10. There are mainly three segments in the market for this power electronic device, namely, power vehicles, naval ships, power adaptors.

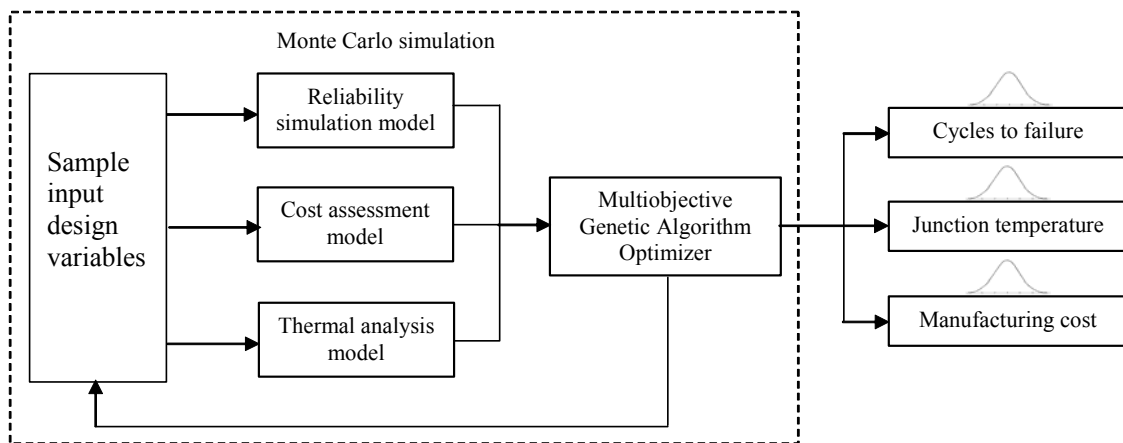


Figure A-I.9: Design alternative generation process (The distributions shown here are only schematic.)

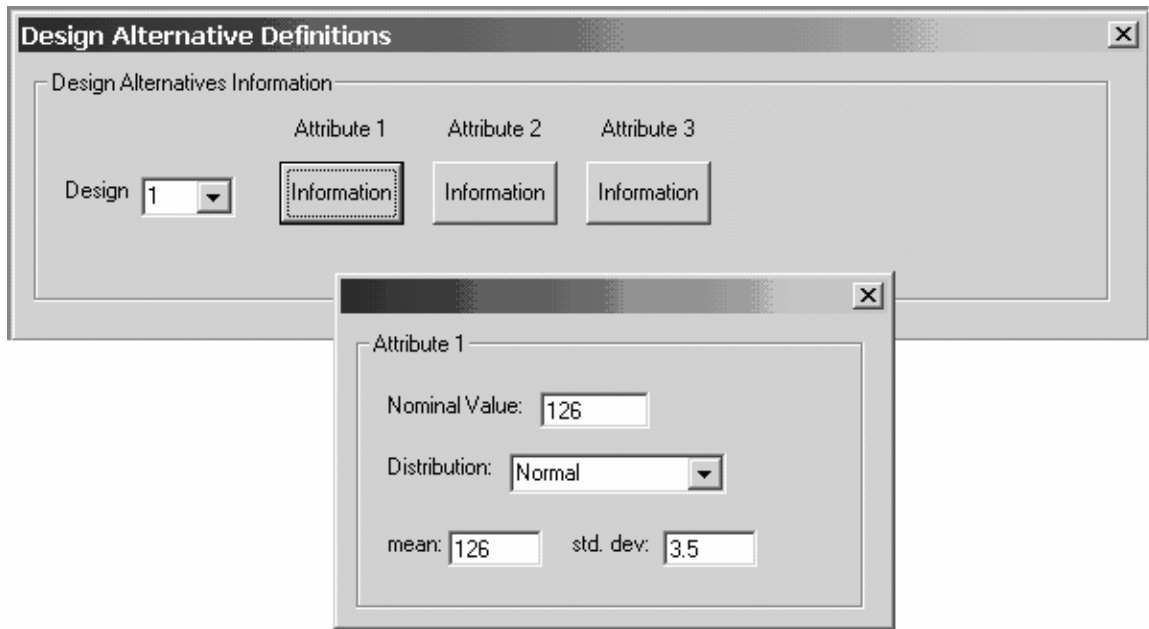


Figure A-I.10: Design definitions

<i>Design #</i>	<i>Junction temperature (°C)</i>	<i>Cycles to failure</i>	<i>Manufacturing cost (US \$)</i>
1	126	22,000	85
2	105	38,000	99
3	138	14,000	65
4	140	13,000	60
5	147	10,600	52
6	116	27,000	88
7	112	32,000	92
8	132	17,000	75
9	122	23,500	85
10	135	15,000	62

Table A-I.1: Description of design alternatives

Next, the utility of the designer over each attribute of the product is captured. In this example, we use a linearly additive utility function with utility-independent attributes. The designer is assumed to show a slight risk taking behavior towards the cost of the product, but his preference behavior towards the failure and also temperature of the product is assumed to be risk averse. On the other hand the designer considers that the cost of the product is more important than the cycles to failure which is more important than the junction temperature. Using the methodology introduced by Keeney and Raiffa [19], the scaling constants of the utility function are estimated as (Although it is integrated in the DSS, the details of computing the utilities are beyond the scope of this paper; see [19].): $k_c=0.60$, $k_F=0.25$, $k_T=0.15$ and obtain

$$\begin{aligned} u_c(x_c) &= 2 \times 10^{-4} x_c^2 - 0.05x_c + 2.88 \\ u_T(x_T) &= -2 \times 10^{-4} x_T^2 + 0.03x_T + 0.002 \\ u_F(x_F) &= -9 \times 10^{-10} x_F^2 + 7.6 \times 10^{-5} x_F - 0.61 \end{aligned}$$

$$U(\mathbf{x}) = k_c u_c(x_c) + k_T u_T(x_T) + k_F u_F(x_F) \quad (\text{A-I.14})$$

where, x_c , x_T , and x_F are the manufacturing cost, junction temperature and cycles to failure respectively. k_c , k_T , and k_F are their scaling constants, u_c , u_T , and u_F are the single attribute utilities, and U is the multi-attribute utility for design alternative \mathbf{x} .

User can enter the designer's utility function by clicking on the Designer's Utility Definition button on the main DSS window as depicted in Figure A-I.11. As we mentioned in section A-I.2.3., the individual elements of the multiplicative utility function (i.e., utility function with respect to each individual attribute) are assumed to be of quadratic form. However, one may argue that the quadratic form of individual utilities or the multiplicative model may not be able to address the designer's preferences for all

occasions. In that case, our DSS can take a custom utility function simulator. The custom utility simulator is an executable program that takes the attribute levels for each product from the main DSS software and writes the corresponding overall designer utility into a text file (named utility.txt). The schematic framework of this connection between the DSS and utility simulator is shown in Figure A-I.12.

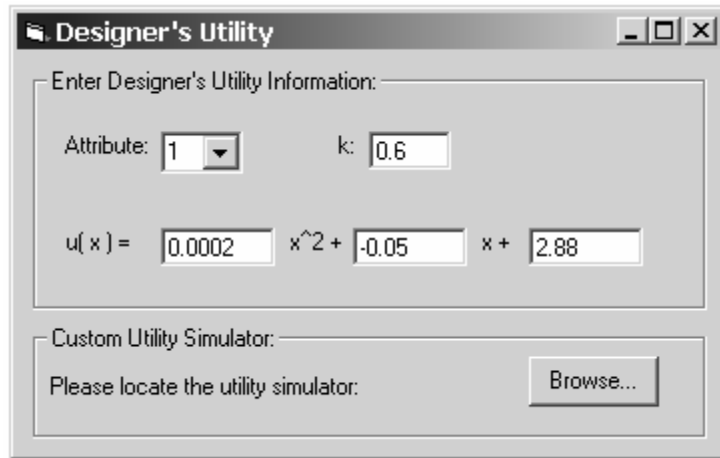


Figure A-I.11: Designer’s utility definition with respect to design attributes

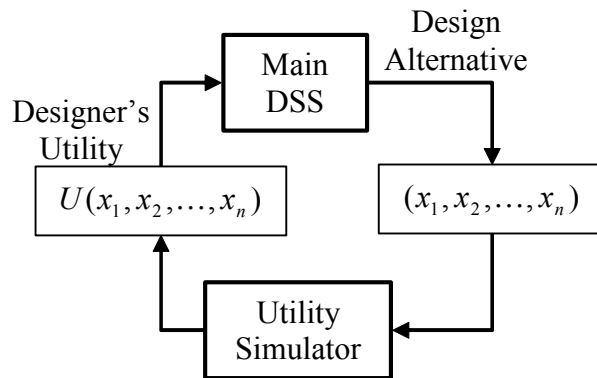


Figure A-I.12: Interaction between the DSS and the custom utility simulator

To demonstrate the application of *CEU* metric, several scenarios with no customer information and with different customer's purchase decisions are illustrated in the scenarios below.

Scenario 1 – No market information: In this scenario, only the designer's preferences are accounted for (i.e., via a utility function) while the customer's purchase decision is ignored as appropriate information is not available. The vNM expected utility (*EU*) of the designer can be calculated for each design attribute and then aggregated as shown below:

$$\begin{aligned}
 EU_c &= \int \frac{1}{\sqrt{2\pi}\sigma_c} u_c(x_c) e^{-\frac{(x_c-\mu_c)^2}{2\sigma_c^2}} dx_c \\
 EU_T &= \int \frac{1}{\sqrt{2\pi}\sigma_T} u_T(x_T) e^{-\frac{(x_T-\mu_T)^2}{2\sigma_T^2}} dx_T \\
 EU_F &= \int \frac{1}{\sqrt{2\pi}\sigma_F} u_F(x_F) e^{-\frac{(x_F-\mu_F)^2}{2\sigma_F^2}} dx_F \\
 EU &= k_c EU_c + k_T EU_T + k_F EU_F
 \end{aligned} \tag{A-I.15}$$

where μ_c , σ_c and μ_T , σ_T and μ_F , σ_F stand for the means and standard deviations of cost, junction temperature and cycles to failure, respectively, and $u_c(x_c)$, $u_T(x_T)$ and $u_F(x_F)$ and scaling constants k_c , k_T and k_F are given in Eq. (A-I.14). (It is assumed that the designer's utility is zero outside the design range.) The Expected Multi-Attribute Utility (*EU*) of each design alternative is then calculated from the above equations, as listed in Table A-I.2.

Alternative	EU_c	EU_T	EU_F	EU
1	0.25	0.61	0.64	0.40
2	0.09	0.95	0.99	0.45
3	0.61	0.33	0.29	0.49
4	0.73	0.28	0.23	0.54
5	0.93	0.09	0.10	0.60
6	0.21	0.79	0.81	0.45
7	0.16	0.85	0.93	0.46
8	0.41	0.48	0.43	0.43
9	0.25	0.69	0.69	0.43
10	0.68	0.41	0.34	0.55

Table A-I.2: Designer's expected utilities

The EU ranking of Table A-I.2 can be interpreted as follows: (i) cost is more important to the designer than cycles to failure than junction temperature, and (ii) design alternatives with lower costs are of more interest to the designer. Therefore, the design alternative 5 has the highest EU .

Scenario 2 - Single segment: In this case, the market information is also accounted for in the selection process. Suppose that a segment of the market seeks a device with the following specifications:

- The device has to endure at least 25,000 cycles, or its junction temperature must remain less than 130°C
- The customer is willing to purchase the device if the price is less than \$170 (i.e., manufacturing cost less than \$70), and it lasts at least 20,000 cycles.

This translates into the following purchase decision function (a disjunctive normal form):

$$D_p(\mathbf{x}) = \begin{cases} 1 & [(x_F \geq 25,000) \vee (x_T \leq 130)] \vee [(x_c \leq 70) \wedge (x_F \geq 20,000)] \\ 0 & \text{Otherwise} \end{cases}$$

(A-I.16)

We define: $A \equiv \{\mathbf{x} | x_F \geq 25,000\}$; $B \equiv \{\mathbf{x} | x_T \leq 130\}$; $C \equiv \{\mathbf{x} | x_c \leq 70\}$; and $D \equiv \{\mathbf{x} | x_F \geq 20,000\}$. The set corresponding to purchase decision of 1 can be written as:

$$S = A \cup B \cup (C \cap D)$$

The binary decision diagram of the above-mentioned customer's purchase decision function is depicted in Figure A-I.13.

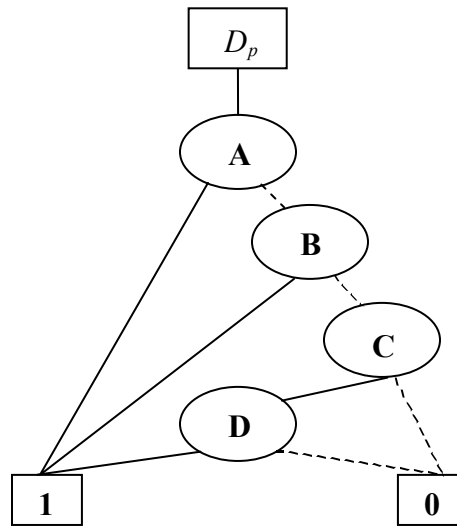


Figure A-I.13: Binary decision diagram for the customer's purchase decision function

We need to keep in mind that the above-mentioned sets are not mutually exclusive. In other words, it is necessary to account for the overlaps between the sets and subtract the intersections. For instance:

$$\int D_p(\mathbf{x})U(\mathbf{x})p(\mathbf{x})d\mathbf{x} = \int_A U(\mathbf{x})p(\mathbf{x})d\mathbf{x} + \int_B U(\mathbf{x})p(\mathbf{x})d\mathbf{x} + \int_{C \cap D} U(\mathbf{x})p(\mathbf{x})d\mathbf{x} - \int_{A \cap B} U(\mathbf{x})p(\mathbf{x})d\mathbf{x} - \int_{A \cap C \cap D} U(\mathbf{x})p(\mathbf{x})d\mathbf{x} - \int_{B \cap C \cap D} U(\mathbf{x})p(\mathbf{x})d\mathbf{x} + \int_{A \cap B \cap C \cap D} U(\mathbf{x})p(\mathbf{x})d\mathbf{x} \quad (\text{A-I.17})$$

Now, the *CEU* of each design alternative can be calculated using Eq. (A-I.12) and Eq. (A-I.16) for the two attributes:

$$CEU(\mathbf{x}) = \int \int \int D_p(\mathbf{x})U(\mathbf{x})p(\mathbf{x})dx_c dx_T dx_F \quad (\text{A-I.18})$$

The results are tabulated in Table A-I.3.

Alternative	<i>CEU</i>
1	0.24
2	0.22
3	0.05
4	0.00
5	0.00
6	0.33
7	0.34
8	0.10
9	0.31
10	0.04

Table A-I.3: *CEU* of design alternatives

According to Table A-I.3, the design alternatives that are less satisfactory to the market (i.e., outside the customer range) are ranked lower. Alternatives 4 and 5 are completely outside the customer range, yielding a zero *CEU* value. Alternatives 3, 8 and 10 have nominal attribute levels outside but close to the boundary of ranges defined by

the customer. In other words, they are likely to fall inside the customer ranges yielding negligible *CEU*. In contrast, alternatives 1, 2, 6, 7, and 9 are in the customer range with respect to all attributes. Among them, alternatives 6,7, and 9 have a wider acceptable customer range of attribute levels, and therefore have higher *CEU* values. Nevertheless, the designer's utility value of alternative 7 is higher than that of 6 and 9, and thus, alternative 7 is ranked the highest in the set.

Now, we can quantitatively compare the vNM expected utility metric (*EU*) with our metric (*CEU*) by evaluating the results in scenario 1 and scenario 2. In scenario 1 since the low cost is preferred by designer, alternative 5 with a big cost difference (than other alternatives) will be the output of the vNM expected utility method. However, in presence of the given customer requirements, design alternative 5 fails to satisfy the customer technical needs in terms of cycles to failure and is eliminated. Conversely, design alternative 7, which is the second most expensive alternative, has the highest *CEU* value because its attribute levels fall in the middle of the customer ranges, and also yields a high designer's utility.

Scenario 3 - Multiple segments: Assume in this case, there are four customer segments involved. The purchase decisions are defined as following:

- Segment 1: The device needs to tolerate at least 20,000 cycles. Its junction temperature should not exceed 130°C. The available budget for this purchase is no more than \$185 per product item.

- Segment 2: The desired device needs to have one of the following criteria: endurable more than 35,000 cycles, junction temperature less than 110°C, the price less than \$160.
- Segment 3: The budget does not exceed \$185 per product item and the eligible device needs to satisfy either one of the following criteria: lasting more than 2,000 cycles, junction temperature less than 130°C.
- Segment 4: The desired device should tolerate at least 3,000 cycles and its junction temperature should not exceed 110°C.

The user can enter the market segment preference information by clicking on Market Segment Data button on main DSS window as shown in Figure A-I.14.

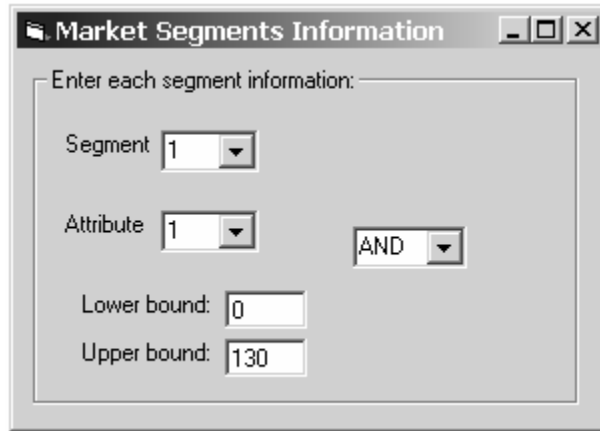
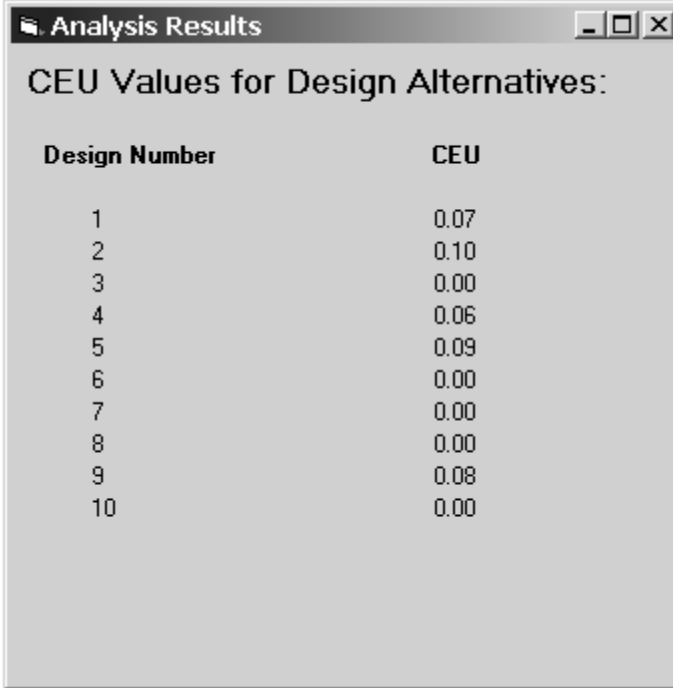


Figure A-I.14: Market segments preference information

The *CEU* of each design alternative can be calculated as follows:

$$CEU(\mathbf{x}) = \int \int \int q(\mathbf{x}) U(\mathbf{x}) p(\mathbf{x}) dx_c dx_T dx_F \quad (\text{A-I.19})$$

and the results of the DSS program are shown in Figure A-I.15.



The screenshot shows a window titled "Analysis Results" with a table of CEU values for design alternatives. The table has two columns: "Design Number" and "CEU". The data is as follows:

Design Number	CEU
1	0.07
2	0.10
3	0.00
4	0.06
5	0.09
6	0.00
7	0.00
8	0.00
9	0.08
10	0.00

Figure A-I.15: Final results

A closer look at the results shown in Figure A-I.15 reveals that those alternatives that are within the market acceptable ranges and at the same time yield the highest designer's utility are ranked higher by this metric. Alternatives 3, 6, 7, 8, and 10 are outside all customers' ranges yielding zero *CEU*. Moreover, alternatives 4 and 5 are acceptable for only one customer resulting in relatively low *CEU*. The remaining alternatives are acceptable to two customers, and among them alternative 2 gets the highest *CEU* because of its higher designer's utility value.

A-I.4. Concluding Remarks

The customer-based expected utility metric presented in this paper accounts for uncertainties associated with design attribute levels as well as the success of the product in the market and its desirability to the designer. As demonstrated in the examples, the

approach guides the designer to determine which of the alternatives could possibly satisfy more customers and thus gain a higher potential demand. The generalized definition of the purchase decision function can model the customers' choice patterns more suitably than a pure conjunctive choice model. Such generalized model allows for the interaction among attributes from customers' point of view. It is shown that those alternatives that fall outside the customer range have a lower chance of success (i.e., lower *CEU* value) than those within the range. Although the proposed approach is unique in the sense that it accounts for both customers' and designer's preferences as well as manufacturing uncertainties, it has some limitations. The designer and the customers share the same attributes. This may be a valid assumption for many cases; however, one could face a situation where the designer deals with technical attributes that are not of any interest to a customer (or are beyond customer's knowledge). One way to handle such situations is to consider no customer preference for those technical attributes, and proceed with Eq. (A-I.11) without any bounds for those specific attributes. As we mentioned, the *CEU* metric maps three important factors: product demand, uncertainties, and the designer's preferences, into a single scalar for design selection. However, one may argue that there are several other important factors that affect the product development and are not considered in the *CEU* metric. While the engineering design related factors such as performance and quality of the product can be modeled as product attributes, some of the market related issues such as pricing strategies and advertising may not be directly addressed by our metric.

Also, our purchase decision modeling is based on the buy/no buy decision of each customer. In other words, the approach does not address whether or not the customers

decide to buy which competitive product (i.e., market share estimate). However, we argue that the designer should have good knowledge of the market including the competitive products. In general, the designer looks for attributes or dimensions along which they can do better with respect to competitive products – this can be captured by giving higher weights to the designer’s preferences for attributes that make the new product different and better than the competition. In the validation stage, a choice based conjoint study could then be conducted to directly evaluate the impact of competitive products. Finally, the presented approach is only for introducing a single product in the market and the issues of product families and cannibalization effects are among the next steps of our future research.

Overall our approach (or a variation of it) will have value in both academic and industrial settings. In industrial design development teams, where there are multiple disciplines involved in the product design, there are always tradeoffs among several design alternatives with respect to each discipline. Similarly, in academic problems, when we deal with a multi-objective optimization case study, there are many design alternatives that are equally optimum and feasible with respect to the design objectives and constraints. In both situations, selecting a product design (or a family of products) from this set is not a trivial task. A decision support tool such as the one that we have developed for this research can help the managers and practitioners make such decisions.

A-I.5. Acknowledgment

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Appendix II.

AN INTEGRATED SOFTWARE FOR SINGLE PRODUCT ROBUST OPTIMIZATION

A-II.1. Overview of the Software

This appendix gives an overview of an integrated robust design/marketing software package designed and tailored specifically to be used with an executable engineering simulation tool and obtain the optimum and/or robust design solutions (i.e., from design point of view). Furthermore, the enumeration and cost assessment module along with marketing module provide the means of estimating the market share and profit of each solution. Ultimately, the final ranking module is used to perform the final selected product(s). The schematic relation between the software components and user inputs (as text interface files) are shown in Figure A-II.1.

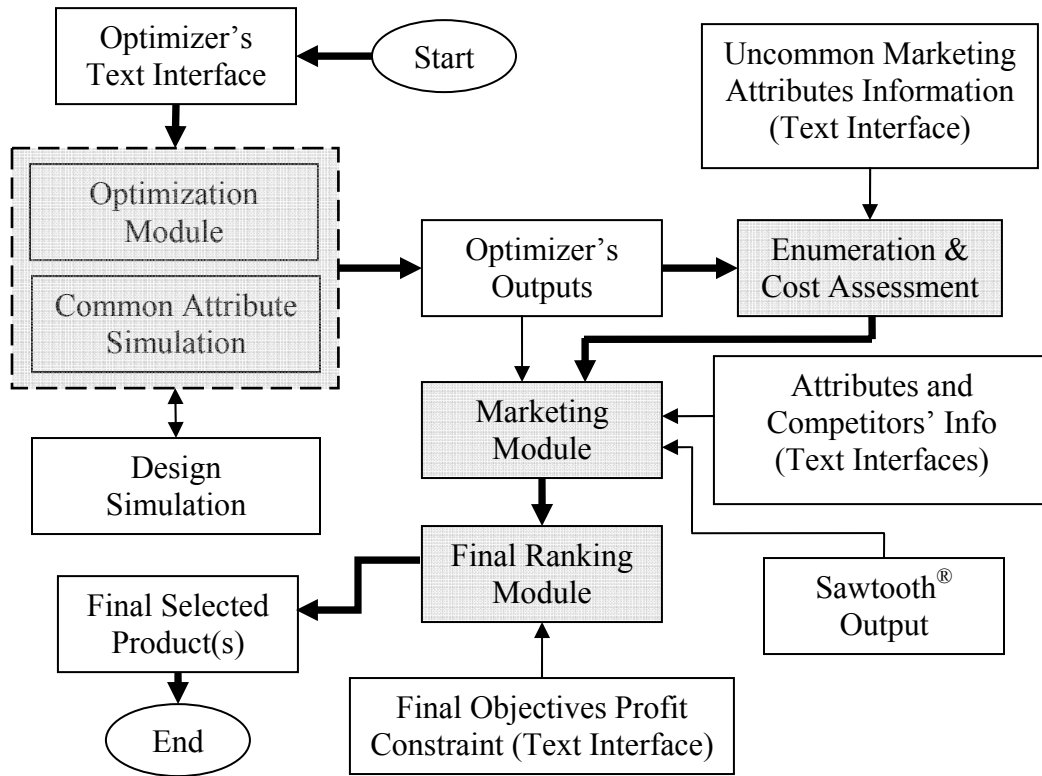


Figure A-II.1: Overall schematic view of the software components

The thick arrows show the order of execution of modules in the software. There are two ways to run the software. First, the user can run each individual module in the order shown in Figure A-II.1, sequentially. The other alternative is to run a loader program that calls each module sequentially in the right order. The details for running the integrated software are given in Section A-II.3.

A-II.2. Individual Components (Modules)

There are four main components (modules) that create the integrated robust design/marketing software. Each component is shown by a gray box in Figure A-II.1. In the following sections, the detailed description of inputs and outputs for each module is given.

A-II.2.1. Robust Optimization Module

The robust optimization module can be launched by double clicking the `optimizer.exe` file. In order for optimizer to proceed, the input files listed in section A-II.2.1.1 must be present in the same folder as the executable. The optimizer will generate results in several output files given in section A-II.2.1.2.

A-II.2.1.1. Required Input Files:

- **User.dat** This is a text user interface which contains the commands for the optimizer to perform its task. The first line of this file contains the name of the simulation tool which is used for engineering design simulation. The order and syntax format of each command are given at the bottom of the file. The optimizer does NOT run if this file is missing or invalid.
- **CAsim.exe** The common attribute simulator is an executable file which calculates the common design/marketing attributes using the output of simulation tool. This executable file is called after every call of the simulation, and generates a temporary file called “**commonAttribute.tmp**”. This temporary file contains the values of each common attribute. The number of such attributes is given in **User.dat** file. The user can write his/her own executable common attribute simulator which reads the design simulation output (e.g., **sim.dat**) and generates the above-mentioned temporary file. The optimizer does NOT run if “**CAsim.exe**” is missing or invalid.

- **Simulation.exe** This file is the simulation tool which is used during the optimization. As mentioned above, the name of this file should match the first line of “**User.dat**” file. The optimizer does NOT run if this file is missing or has a different name compared to the one defined in “**User.dat**” file.

A-II.2.1.2. Generated Output Files:

The optimizer generates the following files. The format of all generated output files is simple text. The **.xls** extension is only used to ease viewing the results in Microsoft® Excel®. The following are the generated output files:

Population.xls	All generated design alternatives
FeasiblePopulation.xls	All feasible design alternatives
NominalPareto.xls	Set of Pareto designs among the feasible population
BCSPopulation.xls	All generated alternatives at Best Case Scenario
WCSPopulation.xls	All generated alternatives at Worst Case Scenario
BCSFeasiblyRobust.xls	All feasibly robust designs at Best Case Scenario
WCSFeasiblyRobust.xls	All feasibly robust designs at Worst Case Scenario
WCSPareto.xls	Pareto set among the WCS Feasibly Robust designs
RobustPareto.xls	Pareto set in terms of WCS objectives & variability
NominalAndWCSPareto.xls	Intersection of Nominal Pareto and WCS Pareto
NominalAndRobustPareto.xls	Intersection of Nominal Pareto and Robust Pareto designs

A-II.2.1.3. Flowchart of Robust Optimization Module:

The overall flowchart of the robust optimization module is given in Figure A-II.2. The obtained results (output files) are used in the next components to calculate several marketing aspects of the product such as overall cost, market share, profit, etc.

It should be noted that the robust optimizer can be used by itself for engineering design or robust design optimization purposes. However, to incorporate the other aspects of the design (e.g., marketing) in the selection process, the following results are passed onto the next components: Feasible population, Feasibly robust alternatives at their best case scenario (design aspect) and Feasibly robust alternatives at their worst case scenario (design aspect).

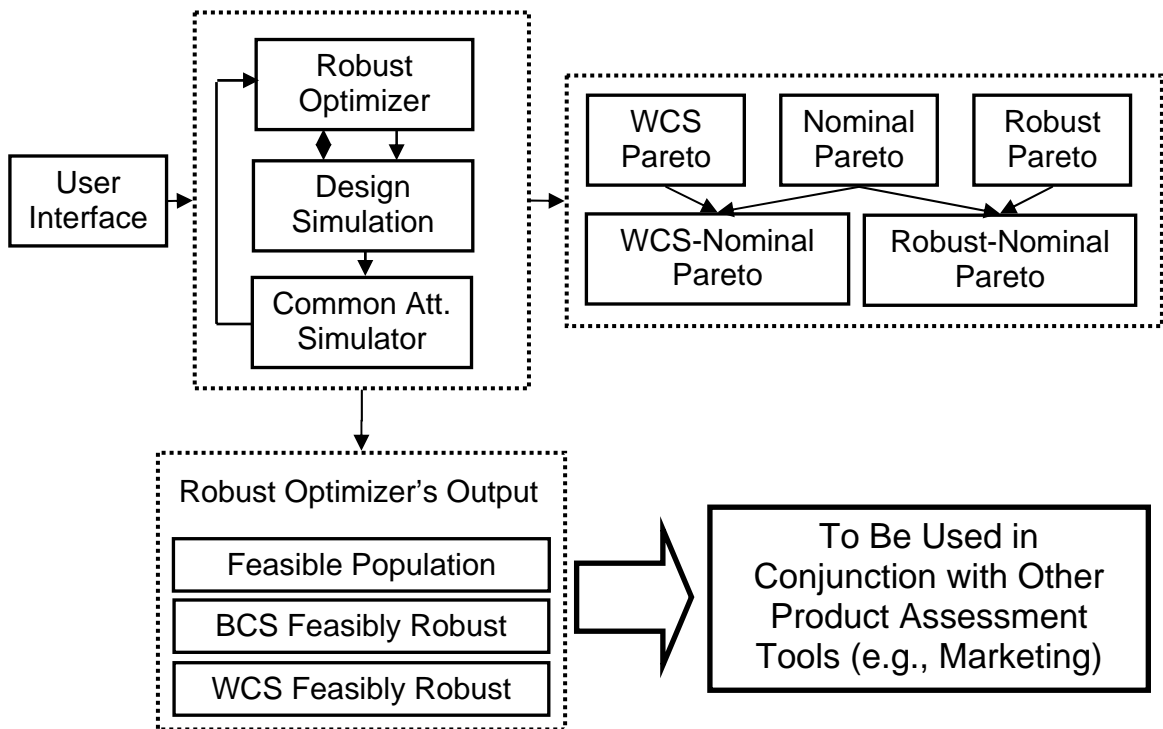


Figure A-II.2: Flowchart of robust optimization module

A-II.2.2. Enumeration and Cost Assessment Module

There are several aspects of the product that are beyond the scope of a design simulation. Issues such as the pricing of a product, cost assessment of different parts, and ergonomic aspects of a product (e.g., switch type, girth size) can have a significant impact in the marketing performance of any product. The enumeration and cost assessment module is used to perform two main tasks: (i) enumerate each design alternative generated by robust optimizer over the marketing non-common attributes such as price, switch type, girth size, etc, (2) calculate the total cost of each product by adding up the platform cost (obtained from design simulation) and the cost associated with each level of marketing non-common attributes. For example each switch type has an associated cost with it. The input and output files used for enumeration and cost assessment module, “**InterSim.exe**” are given in the following subsections.

A-II.2.2.1. Required Input Files:

- **MarketInput.dat** This is a text user interface which contains the required marketing non-common attribute information as well as the cost associated with each level of those attributes. The syntax and format of each is given on the bottom of the file. The enumeration and cost assessment module does NOT work if this file is not present or is invalid.
- **User.dat** This is a text user interface that was used for the optimizer. The number of common attributes and their corresponding labels are read from this file. The enumeration and cost assessment module does NOT run if this file is missing or invalid.

- **FeasiblePopulation.xls** This file is generated by the robust optimizer and contains all feasible design alternatives. Each of these design alternatives is enumerated over marketing non-common attributes and creates a product alternative to be assessed in marketing module in the next step. The enumeration and cost assessment module does NOT run if this file is missing or invalid.
- **BCSFeasiblyRobust.xls** This file contains all feasibly robust designs at Best Case Scenario attribute values. It is used to estimate the BCS values of performance attributes and also for variability calculations. The enumeration and cost assessment module does NOT run if this file is missing or invalid.
- **WCSFeasiblyRobust.xls** This file contains all feasibly robust designs at Worst Case Scenario attribute values. It is used to estimate the WCS values of performance attributes and also for variability calculations. The enumeration and cost assessment module does NOT run if this file is missing or invalid.

A-II.2.2.2. Generated Output Files:

- **FeasibleProducts.xls** All generated feasible products after enumeration over non-common attributes and overall cost assessment
- **FeasiblyRobustProducts.xls** All feasibly robust products after enumeration over non-common attributes and overall cost assessment

A-II.2.3. Marketing Assessment Module

The “**marketing_module.exe**” is the executable file that once is called performs the market share and profit calculation for each product alternative obtained from previous steps. It is important to make sure that the required input files (listed in section 2.3.1) are present in the same directory as the executable. The Output files (listed in section 2.3.2) are generated in the same directory as the executable. Steps that are followed if any of the input files are missing is explained within section 2.3.1.

A-II.2.3.1. Required Input Files:

- **Bdcbc_lc.lcs** This is the output generated by the Sawtooth. The order of attributes must be consistent with that in “**attribute.txt**” and “**competitor.txt**” files. It is advised that the user refers to this file while entering the attribute information in “**attribute.txt**” and “**competitor.txt**” files. The marketing module will display a message “Cannot Open Sawtooth File” and will terminate if this file is missing or invalid.
- **Attribute.txt** This is a text file that contains information about each attribute and levels within each attribute. This file is necessary to correlate “**Bdcbc_lc.lcs**”, “**FeasibleProducts.xls**”, and “**FeasiblyRobustProducts.xls**” files. The names of the attributes must be consistent with those in “**FeasibleProduct.xls**” and “**FeasiblyRobustProducts.xls**” files. The exact file description is present at the bottom of the file itself. If the number of attributes and/or the number of levels

within each attribute in “**Attribute.txt**” is different from the format in the Sawtooth output file “**Bdcbc_lc.lcs**”, the program will display an error message “Error in attribute.txt file. Program Terminated”. If missing, a computer interface will ask the user to enter necessary information.

- **Competitor.txt** A text document having information about each competitor. This file is also required to correlate “**Bdcbc_lc.lcs**”, “**FeasibleProducts.xls**”, and “**FeasiblyRobustProducts.xls**” files. The exact file description is present at the bottom of the file itself. The order of the attributes and attribute levels must be consistent with the format in “**Bdcbc_lc.lcs**” and “**Attribute.txt**”. This file has an option of including one or more own products in the “**competitor.txt**” file, for the case of line extension. Own brand is coded as ‘0’. Competitive brands are coded as ‘1’, ‘2’, and as on, with the same order as in “**Bdcbc_lc.lcs**” and “**Attribute.txt**” files. The values of other discrete product attributes start from ‘1’. The values of continuous and non-common product attributes should be the actual values. With regard to continuous and common attributes, we use two columns to represent the actual values of these attributes (the first column) and the percentage variation of the competitor product on this attribute under various usage situations and conditions (the second column). If missing, a computer interface will ask the user to enter necessary information.

- **FeasiblyRobustProducts.xls** File containing Feasibly Robust Product designs. Name of discrete attributes and Best Case and Worst Case values for continuous attributes must be present. The names of the attributes must be consistent with those in “**attribute.txt**” file. If missing: The software displays a message indicating that the file is missing and will terminate.
- **FeasibleProducts.xls** File containing all the feasible products. It is similar to FeasiblyRobustProducts.xls except that here we have only one value for both Discrete and Continuous attributes. If missing: The software displays a message indicating that the file is missing and will terminate.

A-II.2.3.2. Generated Output Files

- **Share.xls** This file is the same as “**FeasiblyRobustProducts.xls**” with additional columns for own market share (BCS), own market share (WCS), own profit (BCS), own profit (WCS), competitor 1 market share (BCS), competitor 1 market share (WCS), competitor 2 market share (BCS), competitor 2 market share (WCS), and so on. (Note: the calculation of own profit here is based on the assumption “market size = 1”).
- **Share1.xls** This file is the same as “**FeasibleProducts.xls**” with additional columns for own market share, own profit, competitor 1 market share, competitor 2 market share, and so on. (Note: the calculation of own profit here is based on the assumption “market size = 1”).

A-II.2.4. Final Ranking Module

After obtaining the set of feasible products and feasibly robust products, the last component of the software is used to rank order and select desired feasible/feasibly robust products. The “Final_Ranking.exe” is called and performs the ranking task. The required input files and generated output files are described in the following sub-sections.

A-II.2.4.1. Required Input Files:

- **Interface.dat** This is a text file that is used to provide the objective function information (excluding market share which is always maximized). It also provides the market size and minimum desired profit. It should be noted that the profit constraint is only applied to the feasible product alternatives. The final selection of the feasibly robust product alternatives is based upon the robustness measures defined in design and the interval estimates of market share. The order and syntax of the information is given at the bottom of the file. The final ranking module does NOT run if this file is missing or invalid.
- **Share.xls** This file contains the set of feasibly robust product alternatives. The robust ranking is performed on the contents of this file. The final ranking module does NOT run if this file is missing or invalid.

- **Share1.xls** This file contains the set of feasible product and the final ranking is performed on the contents of this file. The profit constraint is also taken into the account for the ranking and selection of the products from this set.
- **User.dat** This is the text user interface that was used for the optimizer. The number of common attributes is read from this file. The final ranking module does NOT run if this file is missing or invalid.
- **MarketInput.dat** This is the text user interface that was used for enumeration and cost assessment module. The number of simulation software arguments and the number of marketing non-common attributes are read from this file. The final ranking module does NOT run if this file is missing or invalid.
- **Competitor.txt** This is a text user interface that contains the competitor's information. The number of competitors is read from this file. The final ranking module does NOT run if this file is missing or invalid.

A-II.2.4.2. Generated Output Files:

- **NominalParetoProducts.xls** Set of Pareto products considering market share point estimate and objectives given in “**Interface.dat**” file. The market size and its effect on overall profit is taken into account here.

- **FeasibleParetoProducts.xls** Set of Pareto products, same as the “**NominalParetoProducts.xls**” only the constraint on profit is taken into account in selection in this file.
- **RobustParetoProducts.xls** Set of customer-based robust Pareto products, where the WCS distance from target, multi-objective variability and the market share (and its variation) are taken into account in rank ordering the products.

A-II.3. The Combined Software

In the above sections, each component of the integrated robust design/marketing software is presented. While a user can run each component one after the other (by double clicking the executable files or by calling them in command prompt), a loader is created to perform this task.

The “**Loader.exe**” file can be used to launch the modules of the software sequentially in the right order. However, the user should be very cautious about using the loader. If any module of the software fails to perform its task (and terminates), the loader however does not terminate and proceeds with calling the next module. It is advised that the loader be used when the input files are verified and are present in the path of the software. The loader does not have any user interface or parameter associated with it.

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