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TITLE

UNDERWATER ACOUSTIC RADIATION OF AN ELASTIC RECTANGULAR PLATE COVERED BY A DECOUPLING COATING

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Underwater Acoustic Radiation of an Elastic Rectangular Plate Covered by a Decoupling Coating

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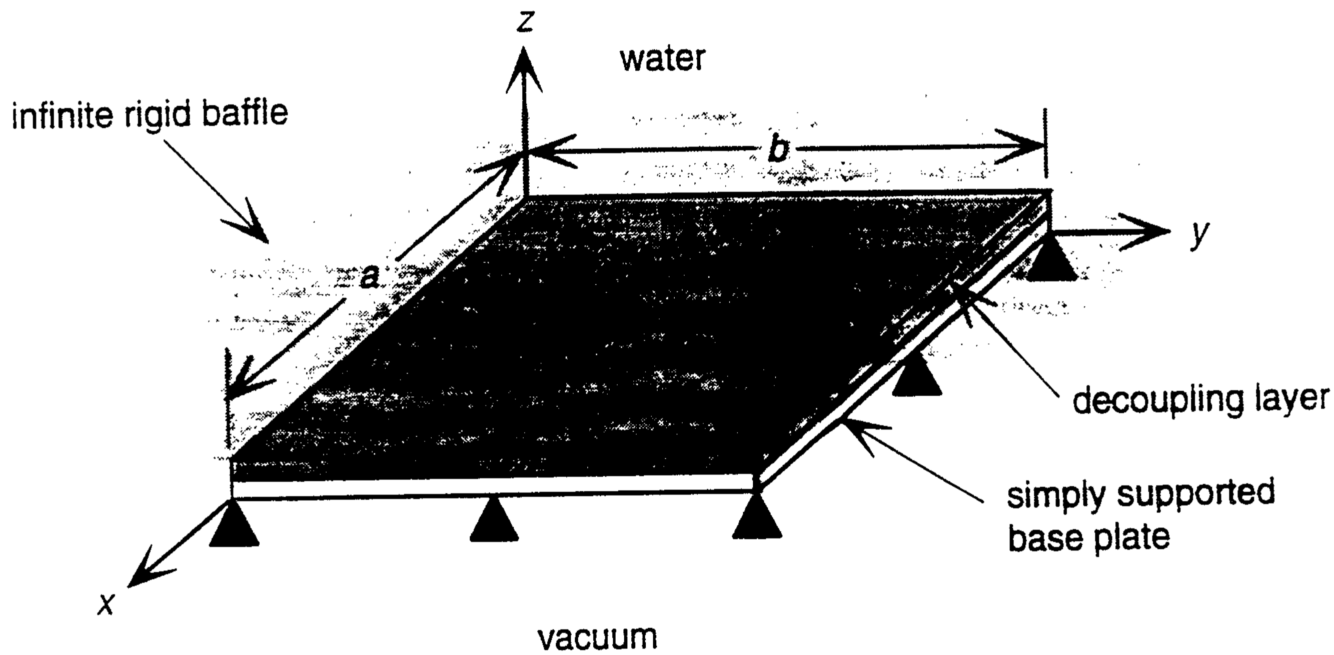
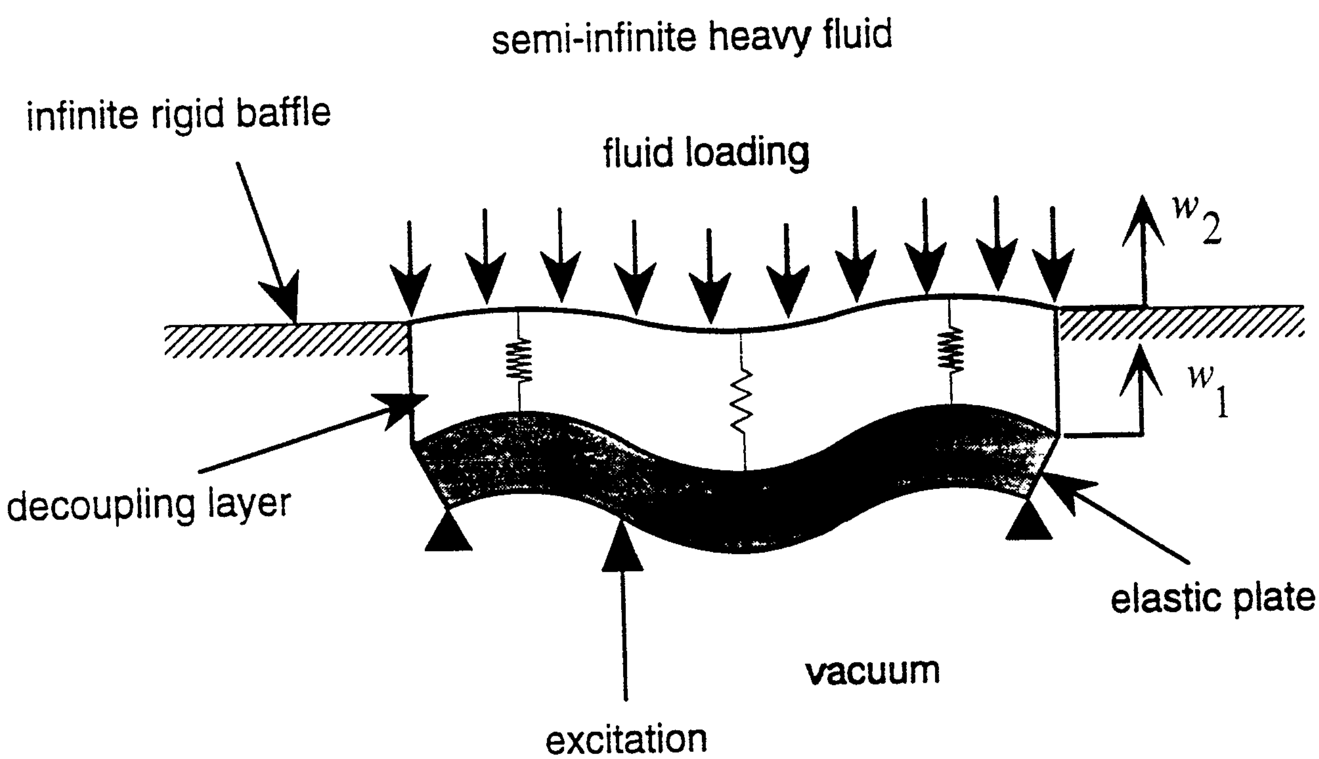
ABSTRACT

The reduction of the sound radiated by a finite structure immersed in water on one side is of concern in this paper. The purpose of this study is to investigate the decoupling effect of adding a compliant layer between a base structure and the fluid. The vibration response and the acoustic radiation of a baffled, elastic plate covered by a decoupling layer immersed in water is analyzed. The theoretical model uses the Love-Kirchhoff theory to describe the motion of the base plate whereas the locally reacting model is used for the behavior of the coating. A parametric study is performed in order to understand the decoupling mechanism and the noise reduction effect of such structures.

The locally reacting model assumes that the decoupling layer behaves like evenly distributed springs in the transverse direction. The unknowns of the problem (i.e. transverse displacement of the base plate and acoustic pressure in the fluid) are expanded with trigonometric functions that satisfy the simply supported boundary condition. The radiation impedances of the plate-fluid system are calculated using a numerical algorithm.

SCOPE


z
water
infinite rigid baffle
b
y
a
decoupling layer
simply supported
base plate
x
vacuum


semi-infinite heavy fluid
infinite rigid baffle
fluid loading
w_2
w_1
decoupling layer
elastic plate
vacuum
excitation

MOTIVATIONS AND OBJECTIVES

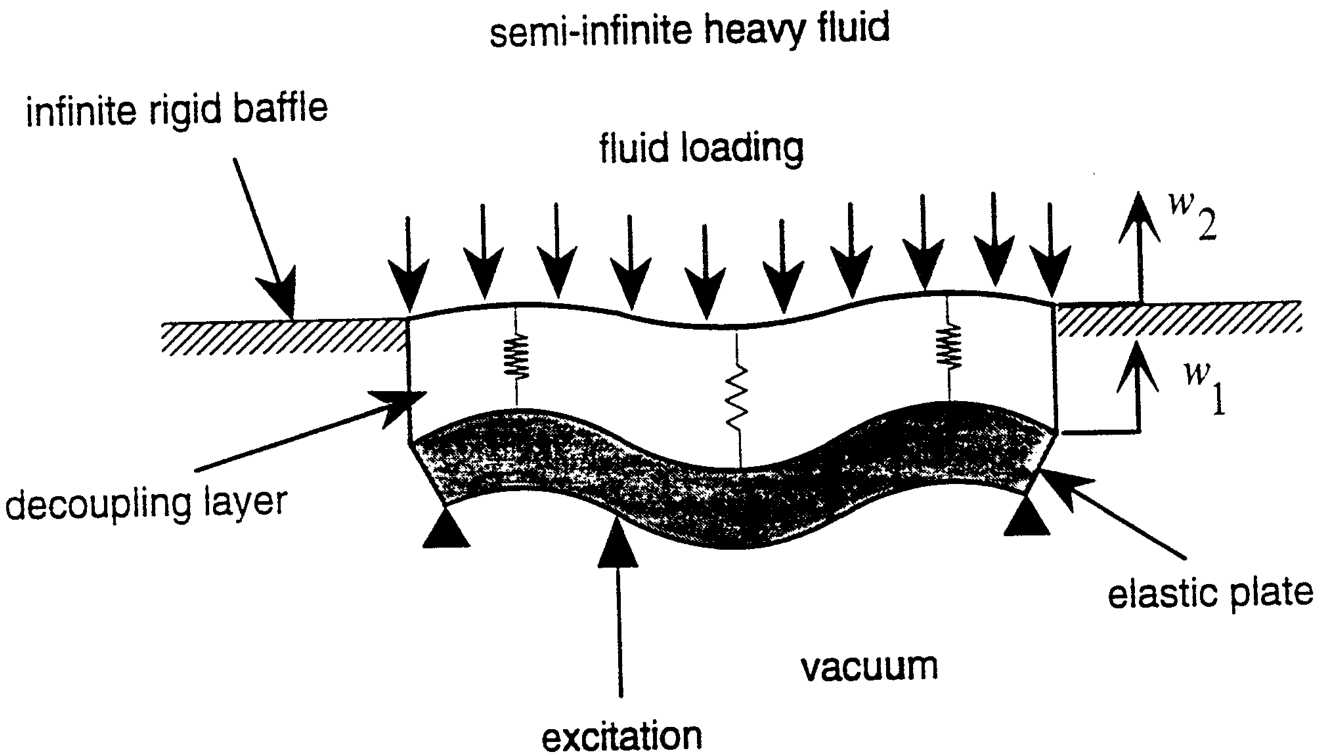
❑ Motivations

- Amount of acoustic attenuation provided by decoupling treatment on ship structure in water ?
- How to relate small scale laboratory experiments to full scale case ?
- What « figure of merit » for acoustic characterization / ranking of decoupling materials ?

❑ Objectives

- Develop an exact model of Vibration / Sound Radiation into water from finite plate with decoupling layer
- Provide DREA with simple prediction tool to analyze decoupling treatments
- Suggest a global vibratory indicator to characterize acoustic performance of decoupling treatment, independently of substrate size

THEORY : STRUCTURAL MOTION



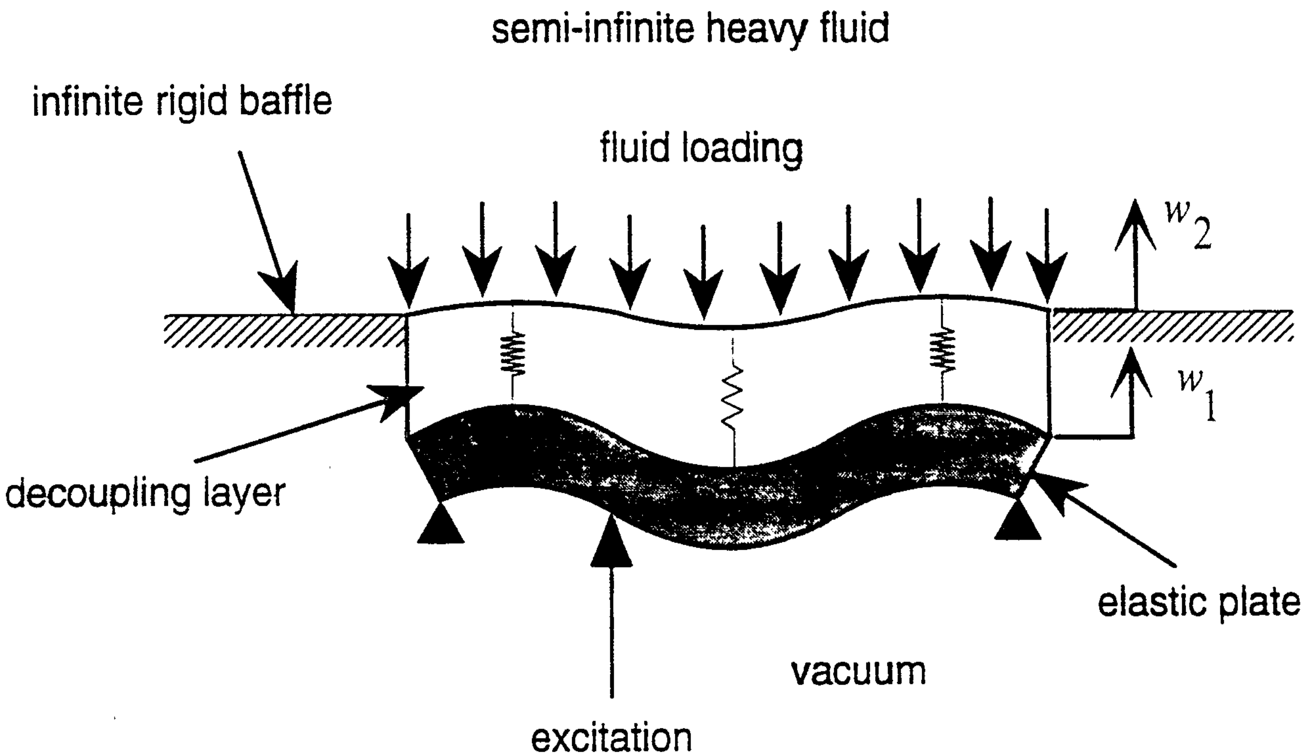
❑ *Assumptions*

- Substrate : Pure bending (Love-Kirchhoff plate)
- Decoupling layer : Locally reacting, massless material (deformation in the transverse direction)
- Fluid loading of the structure

❑ *Equation of motion of the base plate*

$$\widetilde{D}\nabla^4 \widetilde{w}_1(Q) - \rho h \omega^2 \widetilde{w}_1(Q) = \widetilde{f}(Q) - \widetilde{P}(Q)$$

THEORY :: STRUCTURAL MOTION (...)



❑ *Equation of motion of the decoupling layer*

$$\widetilde{P}(Q) = Z_c\left(\widetilde{w}_2(Q) - \widetilde{w}_1(Q)\right)$$

Z_c : Impedance of the decoupling material :

$$Z_c = \frac{\widetilde{B}_c}{h_c}$$

$\widetilde{B}_c$: Complex bulk modulus of the decoupling layer : $\widetilde{B}_c = B_c(1 + j\eta_c)$

h_c : Thickness of the decoupling layer

THEORY :: FLUID-STRUCTURE COUPLING

❑ ***Helmholtz equation***

$$\Delta\widetilde{P}(M)+k_0^2\widetilde{P}(M)=0$$

❑ ***Continuity of the structural and acoustic normal accelerations on the outer surface of the decoupling material***

$$\frac{\partial\widetilde{P}(Q)}{\partial z}=\rho_0\omega^2\widetilde{w}_2(Q)$$

❑ ***Acoustic pressure in the fluid : Rayleigh integral***

$$\widetilde{P}(Q)=-\omega^2\rho_0\iint_S\widetilde{w}_2(M)G(M,Q)ds_M$$

Green's function $G(M,Q)$:

$$G(M,Q)=\frac{e^{-jk_0R}}{2\pi R}$$

R : distance between point M and Q

THEORY : COUPLED ELASTOACOUSTIC SYSTEM

❑ *Elastoacoustic unknowns*

$\tilde{w}_1$: Transverse displacement of the base plate

$\tilde{w}_2$: Transverse displacement of the outer surface of the decoupling layer

$\tilde{P}$: Surface acoustic pressure in the fluid

❑ *Eliminating $\tilde{w}_2$ using equation of motion of decoupling layer*

$$\tilde{P}(Q) = Z_c\left(\tilde{w}_2(Q) - \tilde{w}_1(Q)\right)$$

❑ *Expanding* $\tilde{w}_1(Q)$ and $\tilde{P}(Q)$ *over in-vacuo modes of the substrate plate*

$$\begin{cases} \tilde{w}_1(Q) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{a}_{mn} w_{mn}(Q) \\ \tilde{P}(Q) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{b}_{mn} w_{mn}(Q) \end{cases}$$

where $w_{mn}(Q) = \sin\left(m\pi\frac{x}{a}\right)\sin\left(n\pi\frac{y}{b}\right)$

Elastoacoustic system in matrix form

$$\begin{bmatrix}[A] & [B]\\ [C] & [D]\end{bmatrix}\begin{Bmatrix}\tilde{a}_{mn}\\ \tilde{b}_{mn}\end{Bmatrix}=\begin{Bmatrix}\tilde{f}_{mn}\\ 0\end{Bmatrix}$$

$$\begin{cases}[A]=\begin{bmatrix}\ddots & & \\ & \rho\frac{ab}{4}\left(\omega_{mn}^2(1+j\eta)-\omega^2\right) & \\ & & \ddots\end{bmatrix} \text{ (diagonal matrix)},\\[2ex] [B]=\begin{bmatrix}\ddots & & \\ & \frac{ab}{4} & \\ & & \ddots\end{bmatrix} \text{ (diagonal matrix)},\\[2ex] [C]=\begin{bmatrix} & -j\omega Z_{mnpq} & \end{bmatrix} \text{ (full, symmetric matrix)},\\[2ex] [D]=\begin{bmatrix} & -j\omega\frac{1}{Z_c}Z_{mnpq} & \end{bmatrix}+\begin{bmatrix}\ddots & & \\ & \frac{ab}{4} & \\ & & \ddots\end{bmatrix} \text{ (full, symmetric matrix)}.\end{cases}$$

Intermodal radiation impedance coefficients

$$Z_{mnpq}=j\rho_0\omega\iint_S\iint_S w_{pq}(Q)G(Q,M)w_{mn}(M)ds_Q ds_M$$

Numerical calculation of the radiation impedance coefficients : Sandman method

THEORY :: VIBRO-ACOUSTIC INDICATORS

❑ ***Mean square velocity of the base plate***

$$\langle V_1\rangle^2 = \frac{\omega^2}{2s}\iint_S \tilde{w}_1(Q)\tilde{w}_1^*(Q)ds_Q = \frac{\omega^2}{8}\sum_{m=1}^{N}\sum_{n=1}^{N}\left|\tilde{a}_{mn}\right|^2$$

❑ ***Mean square velocity of the outer surface of the decoupling layer***

$$\langle V_2\rangle^2 = \frac{\omega^2}{2s}\iint_S \tilde{w}_2(Q)\tilde{w}_2^*(Q)ds_Q = \frac{\omega^2}{8}\sum_{m=1}^{N}\sum_{n=1}^{N}\left|\frac{\tilde{b}_{mn}}{Z_c}+\tilde{a}_{mn}\right|^2$$

❑ ***Radiated sound power in the heavy fluid***

$$W = \frac{1}{2}\mathrm{Re}\left[\iint_S \tilde{P}(Q)(-j\omega)\tilde{w}_2^*(Q)ds_Q\right]$$

❑ ***Radiation efficiency in the heavy fluid***

$$\sigma = \frac{W}{\rho_0 c_0 s\langle V_2\rangle^2}$$

SIMPLIFIED THEORY FOR LARGE DECOUPLING

Assumptions

- Fluid loading is neglected in the equation of motion of the base plate
- The displacement of the outer surface of the decouling layer can be neglected in comparison to the motion of the base plate :

$$\left|\tilde{w}_2(Q)\right| << \left|\tilde{w}_1(Q)\right|$$

$$\begin{cases} \tilde{w}_1(Q) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{a}_{mn} w_{mn}(Q) \\ \tilde{w}_2(Q) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{c}_{mn} w_{mn}(Q) \end{cases}$$

$$\begin{bmatrix} [A] & [0] \\ [E] & [D] \end{bmatrix} \begin{Bmatrix} \tilde{a}_{mn} \\ \tilde{c}_{mn} \end{Bmatrix} = \begin{Bmatrix} \tilde{f}_{mn} \\ 0 \end{Bmatrix}$$

$$\begin{cases} [A] = \begin{bmatrix} \ddots & & \\ & \rho \frac{ab}{4}\left(\omega_{mn}^2(1+j\eta) - \omega^2\right) & \\ & & \ddots \end{bmatrix} \text{ (diagonal matrix)} \\ [E] = \begin{bmatrix} -\frac{ab}{4} Z_c \end{bmatrix} \text{ (diagonal matrix),} \\ [F] = \begin{bmatrix} -j\omega Z_{mnpq} \end{bmatrix} \text{ (full, symmetric matrix)} \end{cases}$$

NUMERICAL RESULTS : LOW DECOUPLING

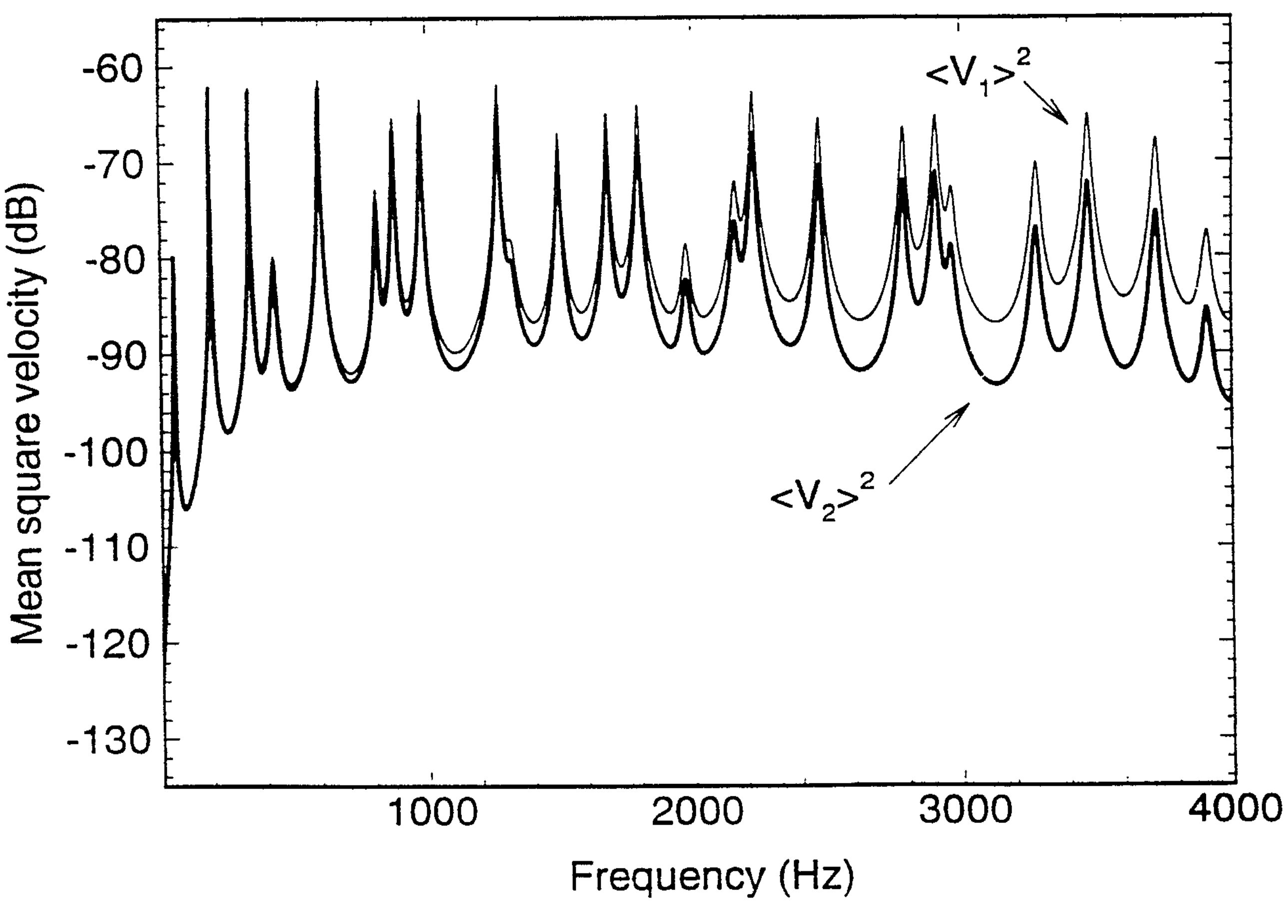
Bulk modulus of the decoupling layer : $B_c = 10^8$ Pa

❑ *Characteristics*

	Length (m)	Width (m)	Thickness (mm)	Density (kg/m^3)	Young/bulk modulus (Pa)	Loss factor	Poisson ratio
Base plate	0.6	0.6	9	7850	2.1×10^{11}	0.005	0.3
Decpl. layer	0.6	0.6	10	-	10^8	0	-

Excitation : Point force applied at $x_0 = y_0 = 0.06$ m from a corner of the plate

❑ *Mean square velocity*



NUMERICAL RESULTS : LARGE DECOUPLING

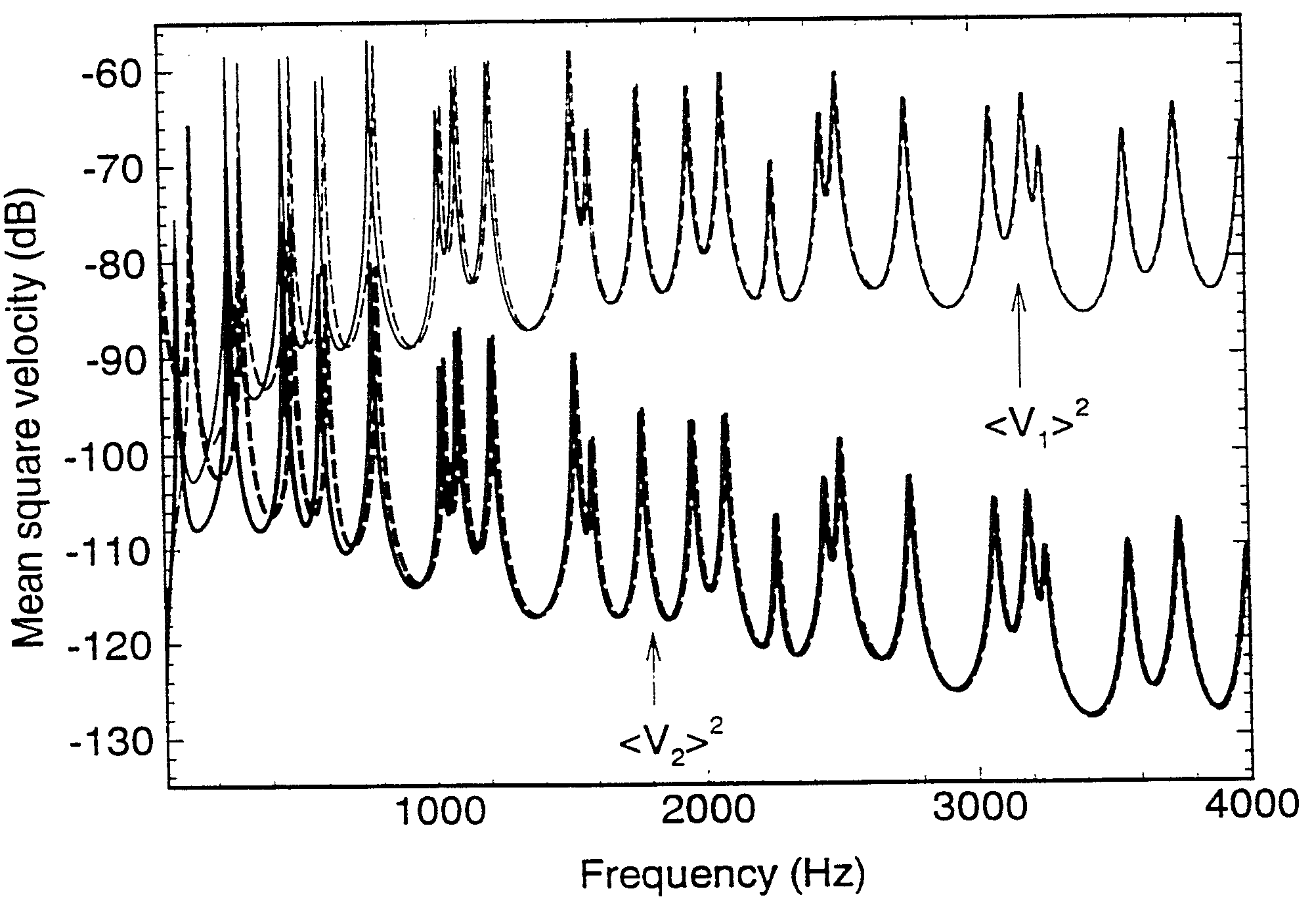
Bulk modulus of the decoupling layer : $B_c = 10^6$ Pa

❑ *Characteristics*

	Length (m)	Width (m)	Thickness (mm)	Density (kg/m^3)	Young/bulk modulus (Pa)	Loss factor	Poisson ratio
Base plate	0.6	0.6	9	7850	2.1×10^{11}	0.005	0.3
Decpl. layer	0.6	0.6	10	-	10^6	0	-

Excitation : Point force applied at $x_0 = y_0 = 0.06$ m from a corner of the plate

❑ *Mean square velocity*



NUMERICAL RESULTS : COMPARISON WITH SANDMAN

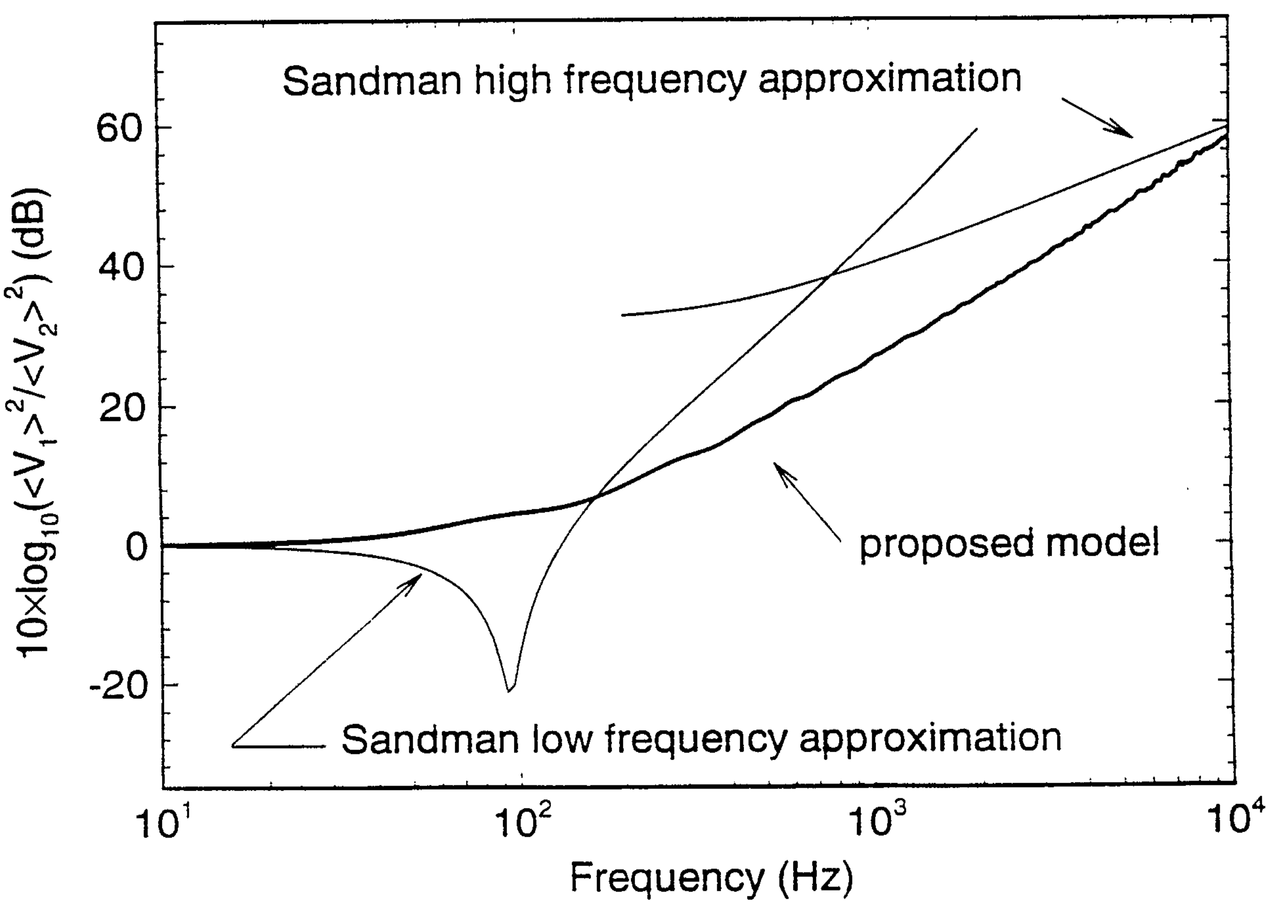
Bulk modulus of the decoupling layer : $B_c = 10^6$ Pa

❑ *Characteristics*

	Length (m)	Width (m)	Thickness (mm)	Density (kg/m^3)	Young/bulk modulus (Pa)	Loss factor	Poisson ratio
Base plate	0.6	0.6	9	7850	2.1×10^{11}	0.005	0.3
Decpl. layer	0.6	0.6	10	-	10^6	0	-

Excitation : Point force applied at $x_0 = y_0 = 0.06$ m from a corner of the plate

❑ *Mean square velocity ratio :* $10\log(\langle V_1\rangle^2/\langle V_2\rangle^2)$



NUMERICAL RESULTS : RADIATION EFFICIENCY

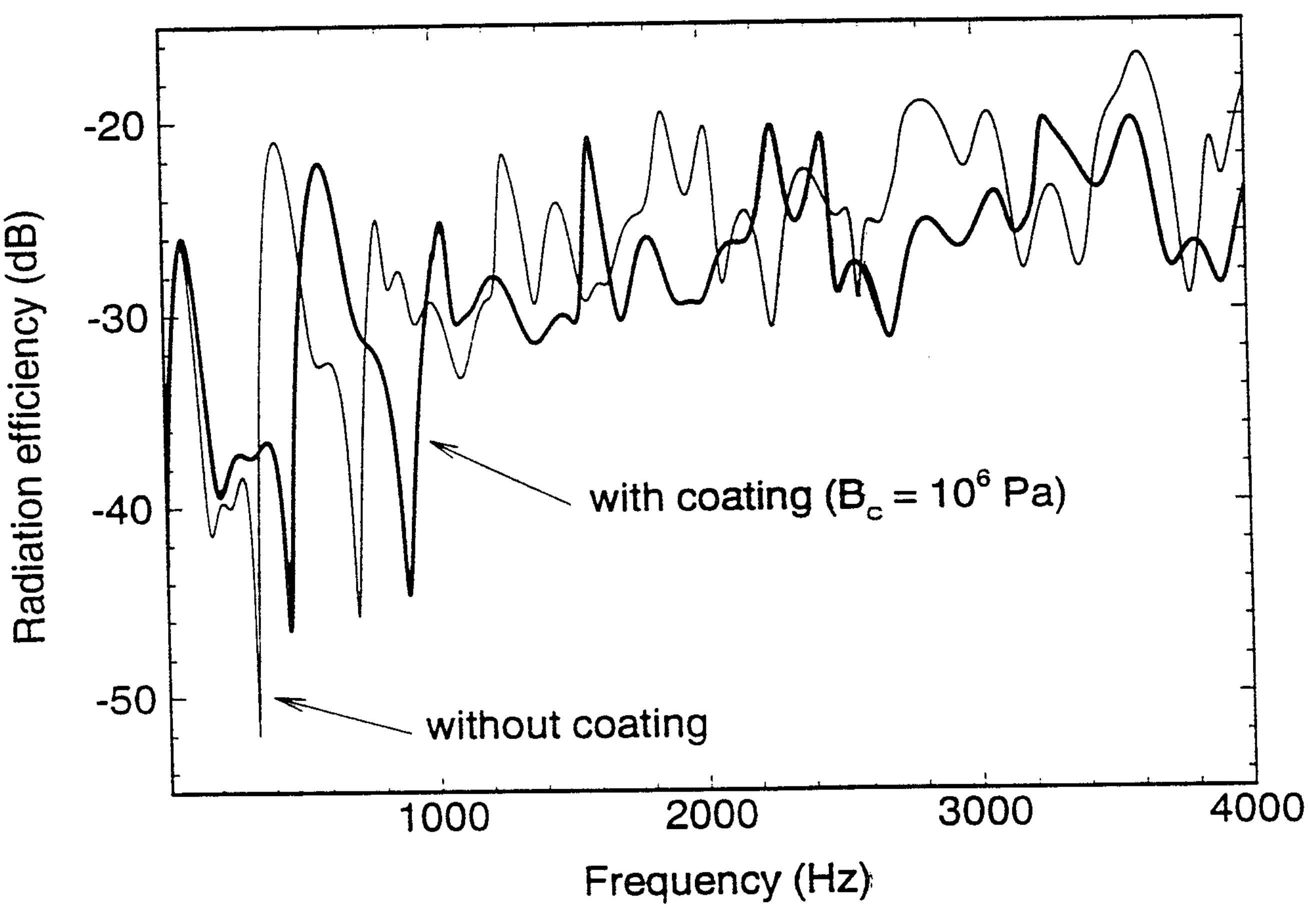
Bulk modulus of the decoupling layer : $B_c = 10^6$ Pa

❑ *Characteristics*

	Length (m)	Width (m)	Thickness (mm)	Density (kg/m^3)	Young/bulk modulus (Pa)	Loss factor	Poisson ratio
Base plate	0.6	0.6	9	7850	2.1×10^{11}	0.005	0.3
Decpl. layer	0.6	0.6	10	-	10^6	0	-

Excitation : Point force applied at $x_0 = y_0 = 0.06$ m from a corner of the plate

❑ *Radiation efficiency*



NUMERICAL RESULTS : RADIATED SOUND POWER

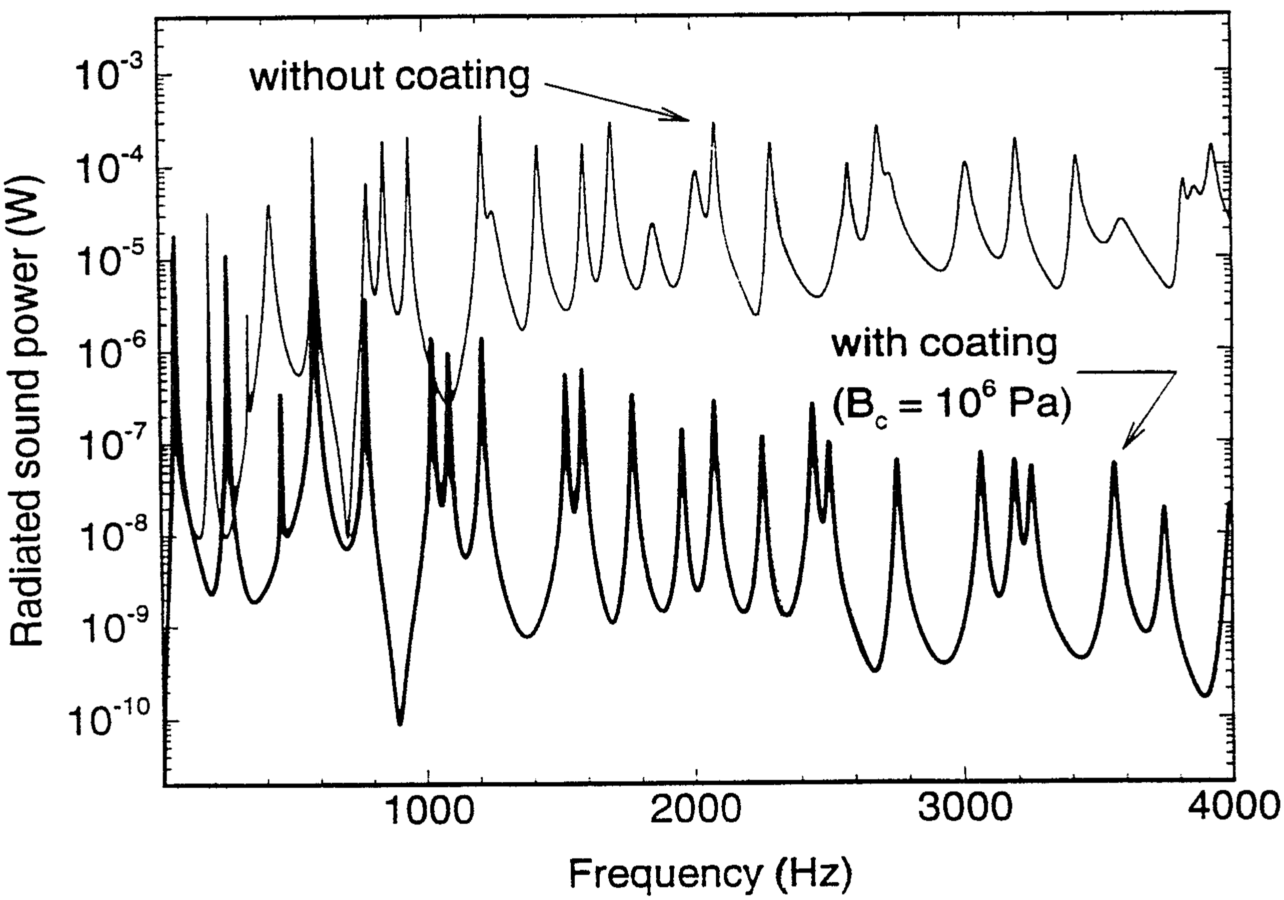
Acoustic efficiency of a decoupling layer

❑ *Characteristics*

	Length (m)	Width (m)	Thickness (mm)	Density (kg/m^3)	Young/bulk modulus (Pa)	Loss factor	Poisson ratio
Base plate	0.6	0.6	9	7850	2.1×10^{11}	0.005	0.3
Decpl. layer	0.6	0.6	10	-	10^6	0	-

Excitation : Point force applied at $x_0 = y_0 = 0.06$ m from a corner of the plate

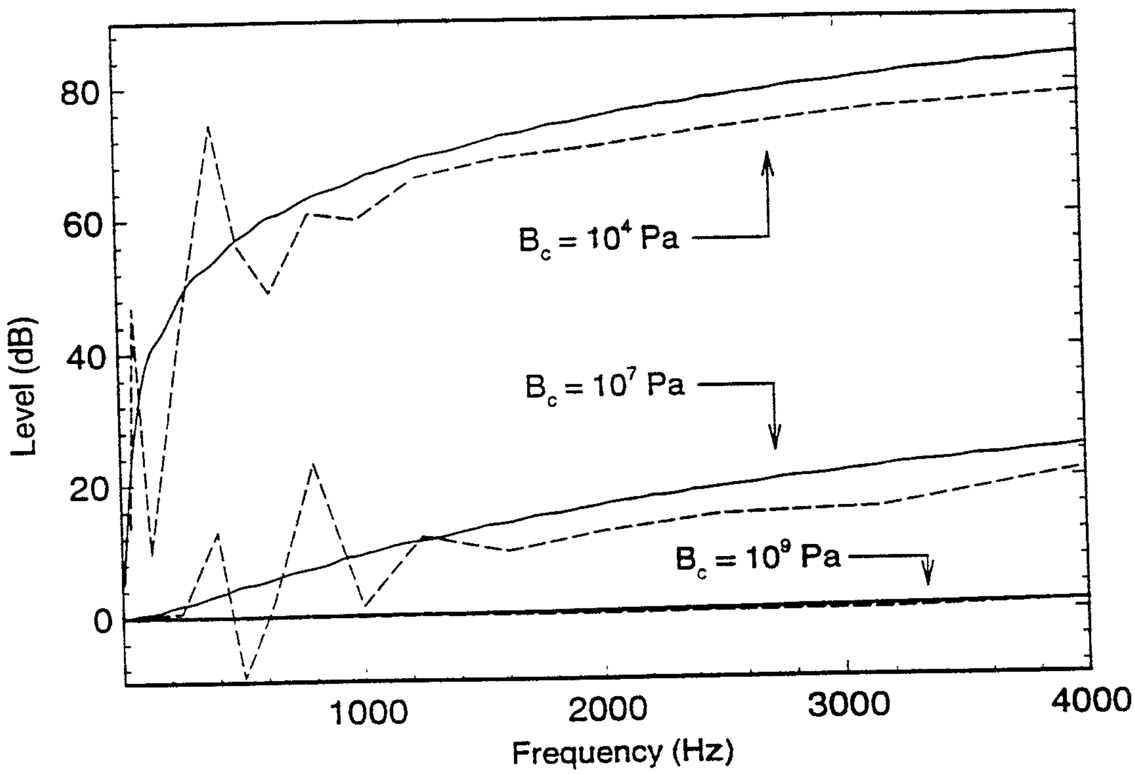
❑ *Radiated sound power*



NUMERICAL RESULTS : VELOCITY RATIO VS. ACOUSTIC INSERTION LOSS

- *Looking for a simple vibratory indicator representative of acoustic performance of decoupling treatments*

- *Comparison of the mean square velocity ratio* $10\log(<V_1>^2/<V_2>^2)$ *and the acoustic insertion loss*



The mean square velocity ratio $10\log(<V_1>^2/<V_2>^2)$ *is an appropriate measure of the acoustic performance of a decoupling treatment*

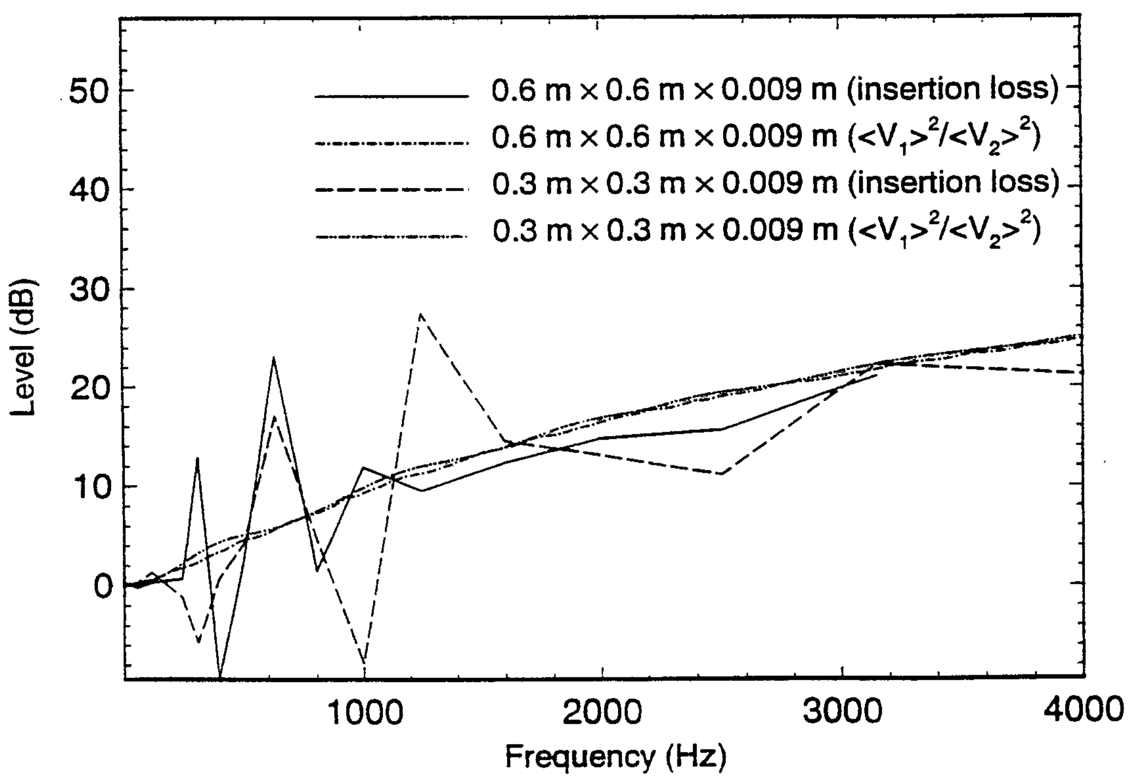
NUMERICAL RESULTS : EFFECT OF SUBSTRATE SIZE

- ***Is the proposed decoupling indicator dependent on substrate size ? (cf. small-scale laboratory characterization)***

- ***Comparing the mean square velocity ratio and acoustic insertion loss for 2 plate x- and y-dimensions***

- ***Characteristics***

	Thickness (mm)	Density (kg/m^3)	Young/bulk modulus (Pa)	Loss factor	Poisson ratio
Base plate	9	7850	2.1×10^{11}	0.005	0.3
Decpl. layer	10	-	10^7	0	-



Conclusions

- ❑ *A model for Vibration / Sound Radiation into water from baffled, rectangular, S-S Love-Kirchhoff plate covered by compliant, locally-reacting, massless decoupling layer*

- ❑ *Simplified theory when decoupling effect is large (low impedance / high frequency)*

- ❑ *The ratio* $10\log(\langle V_1\rangle^2/\langle V_2\rangle^2)$ of *average vibration velocity of substrate to average vibration velocity at layer's free surface :*

  - *is a smooth, non-modal indicator approximately independent of substrate dimensions*

  - *is representative of the acoustic insertion loss provided by the decoupling treatment*

- ❑ *Perspectives :*

  - *theory of elasticity to account for flexural, shear and extensional strain of decoupling layer*

  - *Experimental validations*