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Application of a Dynamic Programming Algorithm for Weapon Target Assignment

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ABSTRACT

Threat evaluation and weapon assignment procedures are an integral part of achieving successful outcomes in naval combat scenarios, both in terms of defending one's assets and destroying enemy targets. To assist with the ever more complex and rapidly changing combat environment, warfighters require access to real-time decision aids based on optimisation techniques to support the decision making process. This report documents the methodology used to identify, develop and assess a dynamic programming algorithm for Weapon Target Assignment which, after more rigorous testing, could be used as a concept demonstrator and as an auxiliary decision tool in real-time combat situations.

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Executive Summary

The requirement to make rapid real-time assessments of potential threats to one's naval assets and to assign weapons accordingly has become an increasingly more complex and sophisticated task for combat systems operators who are required to make split-second decisions based on incomplete information and often based purely on their training, experience and wits.

As the task of threat evaluation and weapon assignment (TEWA) becomes more complex, more rapidly changing and with larger volumes of data and information to be processed under time critical conditions, research into optimisation techniques that are capable of offering semi-automated, real-time decision support have become more imperative. The TEWA problem is typically split into its two components, TE and WA. This report is concerned solely with the issue of weapon assignment, or as preferred in conventional nomenclature, weapon-target assignment (WTA). The objective of a weapon target assignment is to determine the number of particular weapon types to be assigned to specific targets in order to minimise the overall target threat, given realistic limitations.

The aim of this work is to identify, develop and evaluate optimisation procedures for determining an efficient, real-time approach for optimising the allocation of weapon responses to target threats in a maritime scenario. Several echelons of the WTA problem, beginning with the simplest case of a static WTA environment and finishing with the staged shoot-look-shoot sequence of a deterministic dynamic WTA are presented. For this approach, a deterministic dynamic algorithm is developed in the MATLAB environment. A satisfactory but limited validation of the algorithm is accomplished through reproducing results, for example, problems previously worked through in Microsoft EXCEL. Supplementary work is required to rigorously validate this algorithm and to tailor its inputs and outputs in accordance with Australian maritime tactical procedures. Further, it is intended that a mature WTA algorithm be incorporated into a DST Group software test-bed for further testing and evaluation.

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Contents

Acronyms

C2	Command and Control
DDP	Deterministic Dynamic Programming
LAP	Linear Assignment Problem
MDP	Markov Decision Process
MMR	Maximum Marginal Return
NP	Non-deterministic Polynomial time
pdf	Probability density function
PT	Polynomial Time
SWTA	Static Weapon Target Assignment
TE	Threat Evaluation
TEWA	Threat Evaluation and Weapon Assignment
WA	Weapon Assignment
WTA	Weapon Target Assignment

Symbols

$c(x)$	Effective contribution of decision x to the objective function
F_{obj}	Objective function of system for entire WTA analysis
$f_j(s_j,x_j)$	Objective function of individual stage for dynamic WTA analysis
$f^*(s_j)$	Optimised objective function of system stage, s
i	Weapon index parameter
j	Target index parameter
N_m	Maximum number of weapons in any one salvo fired
N_w	Number of weapons available
N_T	Number of known targets
p_{kill}	Probability weapon destroys target (static WTA)
$p_{kill}(x)$	Probability x weapons destroy target (dynamic WTA)
s	State of system (e.g., number of weapons available)
V_j	Target value
x_{ij}	Decision to assign weapon i to a target j (static WTA)
x_j	Number of weapons allocated to target j (dynamic WTA)

1. Introduction

In a maritime combat scenario, the objective of minimising the threat to one's defended assets[1] requires rapid real-time assessment of potential threats, the prioritisation of the identified threats and the optimal assignment of weapons to the known targets [1]. To complicate matters, decisions must be made in a dynamic environment often using incomplete or uncertain information. Combat systems operators are required to make split-second decisions based on their training, experience, situational awareness and knowledge of the capabilities of their platform's combat systems. In addition, environmental factors (e.g., geography, sea-state, etc.), vessel manoeuvrability and one's own Fire Control Radar emissions that may be detectable by the enemy need to be considered once combat begins.

The work reported here aims to identify, develop and assess procedures for optimising the allocation of weapon responses to target threats in a dynamic maritime scenario. Furthermore, for such procedures to be effective, they need to be conducted in real-time. More specifically, this project seeks to address a requirement for the development of an algorithm for Weapon Target Assignment (WTA) for specific use in maritime combat scenarios. After successful testing, the WTA algorithm could potentially be used as a training aid, a concept demonstrator, or as a decision support tool in real-time combat operations.

2. Background

2.1 Threat Evaluation and Weapon Assignment

Threat evaluation and weapon assignment (TEWA) procedures have historically relied on the expertise of the combat systems operator. In more recent times, as combat scenarios inevitably become more complex and rapidly changing, research into optimisation techniques offering automated and semi-automated, real-time decision support have received greater focus, particularly as larger volumes of data and information must be processed under real-time conditions [2].

The TEWA problem is more easily comprehended by being split into its two components, Threat Evaluation (TE) and Weapon Assignment (WA). TE depends on the situational awareness information available to the decision makers, who can then prioritise targets (threats) according to a number of factors. Such factors include target type, target value and the imminent threat to the defended asset.

Threat prioritisation rules may differ from one scenario to the next, based on the asset's operational priorities. Once targets have been categorised into a prioritised engagement list, weapons must be optimally allocated to the targets to maximise the probability of

[1] In this document, we will henceforth refer to a single rather than multiple defended assets.

survival of the asset. The asset and target based views of tackling this problem are discussed further in Section 2.2.4.

TE analysis utilises the available tactical picture to provide situational awareness to combat systems operators to produce a prioritised list of potential threats. The list of prioritized threats is used as an input for the second stage of the TEWA process, i.e., WA.

TE is not discussed further in this report. The process of assigning weapons to targets, or weapon-target assignment (WTA) as it is more generally known, is the focus of this report. An excellent overview of WTA issues and the related resource allocation problem in the Command and Control (C2) environment are given by many authors, including Pierre Plamandon in his PhD thesis dissertation [3] and Fredrik Johansson [4].

Combat scenarios referred to in this report involve a single naval ship which has positively identified a number of targets as threats, whether they are an enemy ship (i.e., specifically, its weapon launcher systems) or a directed weapon, such as an anti-ship missile. The asset must allocate its limited weapon resources to specific targets to optimise its chances of survival. An asset will typically carry weapons of different types, so the objective of a WTA is to determine the number of each weapon type to be assigned to specific targets in order to minimise the overall target threat.

In this report, the focus is on WTA on a single maritime platform. WTA in a multi-platform environment is a problem to be addressed in the future.

2.2 Weapon Target Assignment Formulations

The WTA optimisation problem arises in all defence combat arenas, including air, land and sea. WTA is a specific resource allocation problem where the objective is to optimally assign a range of weapons to a finite list of known threats (i.e., targets) in order that the overall threat to one's assets can be minimised.

A number of different approaches have been applied to the WTA problem, with various forms of static and dynamic [5] [6] [7] formulations investigated. Heuristic rather than exhaustive (i.e., exact) approaches [8] [4] [3] tend to be the only practicable option for anything other than the simplest engagement scenarios.

When considering a WTA problem, a number of factors need to be considered and defined before any approach can be adopted. Some of these factors are discussed below.

2.2.1 Linear versus Nonlinear Assignment Problems

The generalised linear assignment problem (LAP) of allocating agents to tasks (in this case, weapons to targets) is a fundamental problem of combinatorial optimisation [9], where the latter is recognised as a branch of Operations Research mathematics.

In the simplest case, the number of weapons and targets are equal, with only one weapon being assigned to any one threat in an allocation. The requirement then is to optimise the assignment of weapons to targets by simply optimizing the total 'cost' of all assignments (in the WTA case the cost relates to the threat minimisation benefit).

LAPs can also be represented in graph theory as bipartite graphs as shown in Figure 2-1(a). In the LAP graph illustrated, agents cannot be assigned to more than one task, as in scientists to projects, bound by the constraints that only one scientist can be assigned to only one project, ensuring that all scientists and projects are assigned.

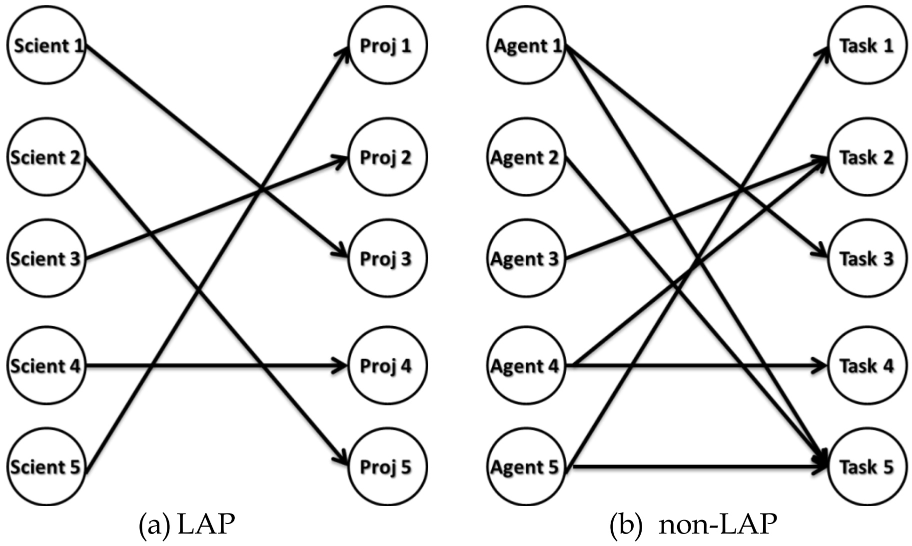

Figure 2-1 Example of a linear and a nonlinear bipartite graph, respectively

When agents are assigned to more than one task or tasks are assigned to more than one agent, the assignment problem becomes nonlinear [10] as illustrated by the bipartite graph in Figure 2-1(b). WTAs are generally viewed as nonlinear assignment problems (non-LAP) in all but the simplest cases; i.e., the optimal solution is nonlinear but is still constrained to integer values as in the LAP case[2].

For the WTA scenario, one must optimally assign N_w weapons to N_t targets with the latter having ascribed values V as taken from the viewpoint of the defended asset. The target values depend on the context of the combat scenario: target values can be assessed as either their relative value (e.g., in terms of a high value target) or in terms of a threat to the defended asset. The likelihood of a number of weapons successfully destroying a series of targets, or more specifically removing their threat capability, is described by a probability of kill matrix [p_{kill}]. The elements of this matrix denote the probability of a given weapon in the inventory successfully limiting the destructive capability of a known target. The degree to which these parameters can be established *a priori* will determine the effectiveness of the methodology.

[2] This nonlinear assignment problem is constrained to integer solutions, and is often referred to as integer linear programming.

2.2.2 Exact versus Heuristic Techniques

The necessity for WTA problems to have real-time solutions is fundamental and requires no explanation. However, WTA problems belong to a class of combinatorial optimization problems which are shown to be NP-complete, where NP refers to Nondeterministic Polynomial time[3] [11] [12]. Informally, this means that there is no robust known method for arriving at an optimal solution without conducting an exhaustive analysis of all options (i.e., every possible WTA optimum solution must be considered to identify the optimal solution[4]).

More formally, polynomial time (PT) algorithms are limited in size by a simple, deterministic, polynomial expression; i.e., a fast, feasible solution, which is a critical concept when considering optimal WTA solutions. The runtime for an algorithm can be expressed in terms of the order of the size of the input 'n', using $T(n) = O(n^x)$ where 'x' is a constant (e.g., if $x=1$, solution is linear, if $x=2$, solution is exponential, etc.). A 'fast' solution is considered as one in which the runtime of the algorithm is of the order of the size of the input 'n'; i.e., $T(n) = O(n^1)$. In the case of exhaustive enumeration WTA techniques, $T(N_w, N_t) = O(N_w e^{N_t})$; where N_w and N_t are number of weapons and targets, respectively.

Since no real time exact solutions to WTAs are available, either for static or dynamic analyses, alternative approximation methodologies must be considered, including heuristic techniques. That said, the procedures described in this report cannot be guaranteed as the optimal solution, merely as the optimal solution based on the technique used. As a precaution, such techniques are generally referred to as suboptimal.

As mentioned above, an exact solution can only be found through exhaustive or branch-and-bound techniques (the latter being a modified exhaustive search). If, for example, all agent-task combinations are expressed in a decision tree, then the exhaustive technique will weigh up the benefit of each single agent-task option as it moves through the branches of the decision tree. The only difference with a branch and bound algorithm is that upper and/or lower bounds are applied to the exhaustive enumeration trees, thus removing agent-task assignments (i.e., particular tree branches) that are clearly sub-optimal. In general, exhaustive techniques are not practicable since solutions must be achieved in real-time.

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[3] The class of problems for which a solution can quickly provide an answer in Polynomial Time (PT) is simply called class P. Because there are some problems where no such solutions exist, but a solution may exist that can be verified in PT, then this type of solution is referred to as Nondeterministic Polynomial (NP) complete [11]. So although a given solution to such a problem can be verified quickly, there is no known algorithm to identify a solution in the first place. Typically, NP-complete problems are addressed by using approximation algorithms.

[4] The author uses the term 'the optimal solution' to indicate the most favourable solution possible globally, whereas 'an optimum solution' is a local optimal solution, but not necessarily the global optimal solution.

2.2.3 Markov Decision Processes

As mentioned in the preceding section, heuristic methods are necessary for determining optimum WTA solutions. Markov Decision Processes (MDPs) have been used for this purpose since their introduction by Bellman in 1957 [13]. In their conventional formulation, they can be used to either minimise or maximise the related 'objective function', discussed in more detail in the following section. MDPs have been found to be suitable for modelling stochastic processes; i.e., processes that operate in an environment that is not fully defined or known and the outcomes of decisions are probabilistic in nature, as is typically the case for combat scenarios.

Although MDPs are widely used to model stochastic processes, they are still ultimately limited by the solution runtime since, as mentioned previously, as certain processes become more complex their solutions inevitably become more numerically expensive. As discussed in Section 2.2.2, as the number of parameters increase, the runtime grows exponentially, which is a limiting factor for the MDP approach.

2.2.4 Asset-Based versus Target-Based

A WTA problem can be viewed from either a target-based or an asset-based perspective [16] [14]. From the target-based perspective, values are assigned to each potential target (or threat) in relation to their perceived value and/or their potential ability to cause damage to the defended asset. The objective of the target-based WTA solution is to minimise the survivability of the incoming targets.

Conversely, in an asset based perspective values are assigned to the assets rather than the targets. This WTA problem is where weapons are assigned such that the combined value of assets is maximised. Furthermore, the asset-based approach requires information on which targets are approaching the defended asset(s), either as an inbound missile or as a weapons system about to launch a missile. Such an approach requires a high degree of situational awareness [14], whereas a target based approach is more appropriate when the situational awareness is less well known. The conventional approach, which is used here, is the target-based perspective.

2.2.5 Static versus Dynamic

In the case of a static analysis [5] [17], there is no sequence of events; rather, all weapon assignments are made in a time independent environment and are assumed to be fired simultaneously.

Dynamic environments [7] [15] [16] [17] are those where there may be a sequence of firing events with each subsequent firing dependent on the outcome of the previous one. This is often referred to as a shoot-look-shoot scenario [18], since observations of the outcome of previous firings are performed before future assignments are made.

2.2.6 Deterministic versus Stochastic

In naval and indeed most forms of combat scenarios, weapon-target assignments produce outcomes which are inherently stochastic in nature [7] [19]. In other words, the dynamic scenario's next state in time is not completely known because of a constantly changing environment and also because of the pseudo-random nature of the outcome of any firing event.

In simple, idealised formulations, it is acceptable to assume that the next state of the environment is fully determined by the current state. This may be true for particular parameters and a good approximation for others. In such cases, this environment is referred to as deterministic, even though decisions made at any stage are done on the basis of available probability of kill data. By adopting deterministic techniques, the warfighter is predicting a particular outcome for a WTA, the actual outcome of which is rapidly corroborated or otherwise. Either way, the next WTA decision will be based on the result of the preceding one.

Stochastic techniques are also available, particularly in the case where there is no ability to observe the outcome of each firing stage. In such cases, one must assume a probability density function (pdf) for the possible outcomes. That is, the combat environment is no longer treated as deterministic, but probabilistic. Such methodology rapidly becomes computationally challenging, depending on the complexity of the probability density function (pdf) used. In most combat engagements, the war fighters are able to observe the results of their firing which will influence the next firing decisions, making probabilistic techniques unnecessary. For the purposes of this report, stochastic WTAs are not considered.

3. Static Weapon Target Assignment

The solution described here is for the case of a single maritime asset with multiple hostile threats, more commonly referred to as targets. As mentioned in Section 2.2.4, the approach used is target-based. Our objective is to optimally assign N_w weapons to N_t targets in order to minimise the survivability of the targets, or in other words, to maximise the survivability of the assets.

The simplest methodology for the WTA problem is to employ a static MDP. Static MDPs optimally assign all weapons to all targets as a single assignment based on a formulation of perceived target threat, value and likelihood of kill. In effect, this means there is no assessment of damage between firings; firings are made purely on the perceived threat value of targets and the weapons available.

The formulation [20] [14] assumes an asset has $i = 1, 2, \ldots, N_w$ weapons available and there are $j=1,2,\ldots, N_t$, potentially hostile, independent targets. The decision to assign a particular weapon i to a target j is denoted, x_{ij}; where:

$$x_{ij} = \begin{cases} 1; & \text{when weapon } i \text{ is assigned to target } j \\ 0; & \text{otherwise} \end{cases}$$

Avoiding the issue of different levels of damage, the outcome of this weapon target assignment is either a hit or a miss, i.e., the target is either destroyed or still has a full threat capability. Given the stochastic nature of the outcome, each firing follows a Bernoulli distribution; i.e.:

Probability weapon *i* destroys target *j*: p_{kill}^{ij}
Probability weapon *i* doesn't destroy target *j*: $1 - p_{kill}^{ij}$

If each target has a value, V_j, as perceived from the perspective of the asset, then for a basic WTA, we wish to minimise the overall objective function, F_{obj}, which is the total expected value of the surviving targets after all allocated weapons have been released [21]:

$$F_{obj} = \sum_{j=1}^{N_t} V_j \prod_{i=1}^{N_w} (1 - p_{kill}^{ij})^{x_{ij}} \tag{1}$$

Subject to all weapons being assigned to any of the N_t targets:

$$\sum_{i=1}^{N_w} \sum_{j=1}^{N_t} x_{ij} = N_w \tag{2}$$

and where

$$\prod_{i=1}^{N_w} (1 - p_{kill}^{ij})^{x_{ij}} \tag{3}$$

represents the survivability of a target *j* against all the weapons assigned to it. F_{obj} is used as a relative benefit measure of a specific WTA. The objective function can be viewed as the average survival value of the collective threat. It is defined according to the analyst's requirements based on which parameters are deemed significant costs.

3.1 Static Weapon Target Assignment using Exhaustive Enumeration

As previously mentioned, since there is no exact solution to determining the optimal value of the objective function, it is necessary to use either a heuristic solution or in limited cases, a brute force approach, where the latter computes every possible value of F_{obj}. Clearly, such strategies can only work for scenarios with a limited number of weapons and targets.

However, in order to demonstrate alternative, heuristic techniques, it is useful, at least initially, to compare the results of a chosen approach with the results of an exhaustive enumeration. An exhaustive search algorithm was written in MATLAB code to generate the optimal solution for this particular scenario.

In the following static WTA example, a single defended asset has $N_w = 5$ identical weapons (i.e., each weapon has an identical value of p_{kill} for all targets) available to defend itself against $N_t = 3$ identified hostile targets, each of different perceived value (i.e., 0.30, 0.65

and 0.90 for targets 1, 2 and 3, respectively). The asset must optimise the use of its weapons. This analysis is relatively trivial, since the p_{kill} values are identical. In reality, the resultant WTAs depend on the defended asset's knowledge of the targets' positions and the known effectiveness of these weapons against the target type (encapsulated in p_{kill}). For this scenario, the MATLAB static exhaustive enumeration WTA algorithm generated the following optimal WTA:

- Assign 1 weapon to Target 1
- Assign 2 weapons to Target 2
- Assign 2 weapons to Target 3

This result will be compared to that of the Maximum Marginal Return (MMR) algorithm in the following section.

3.2 Static Weapon Target Assignment using MMR Algorithm

There are a few algorithms that are commonly used to find an *optimised* (i.e., local minimum), as opposed to the *optimal* (i.e., global minimum), solution to the Static WTA problem, such as the Maximum Marginal Return (MMR) [14] [22] and the Local Search algorithms [14] [22] [23], since the solution is known to be NP-hard (refer Section 2.2.2) and only heuristic formulations are practicable.

In the previous section, an exhaustive enumeration algorithm was used to generate the optimum solution to a simple WTA problem. Although the solution given is exact, since all possible WTAs are considered, this technique is of limited value except in only the simplest of scenarios. The computational cost of the exhaustive enumeration algorithm varies as $N_w e^{N_t}$.

A number of heuristic approaches have been developed for use in Static WTA problems. The MMR algorithm [14] [24] [25] [21], uses a greedy, heuristic approach, varying as $N_w N_t$, that is computationally more efficient than the exhaustive approach. The MMR algorithm exhaustively tests each weapon in sequence against a single target until the objective function for that target, otherwise referred to as the target's MMR, is a maximum. When the target's MMR is identified, the overall MMR is updated to reflect this assignment. It is assumed that the assignment of a weapon to a target will result in removal of that target. This procedure is carried out for all targets. Depending on the probability of kill, p_{kill},, and the target value, V_j, for a particular weapon-target combination, some targets may have multiple weapons assigned, whereas other targets may not have any weapon assigned.

The MATLAB MMR algorithm was run using the example scenario data from the previous section and generated the following results:

- Assign 1 weapon to Target 1
- Assign 2 weapons to Target 2
- Assign 2 weapons to Target 3

These assignments are in agreement with the results of the exhaustive enumeration (i.e., exact search) algorithm described in the previous section which provides some initial confidence in the approach.

3.3 Comparison of Exhaustive Enumeration and MMR Algorithms

To more thoroughly assess the capability of the MMR approach against the exhaustive enumeration algorithm, a considerable number of WTA scenarios would need to be tested with, for example, varying p_{kill} values for each weapon. It is not the intention of this study to conduct extensive analyses at this time, but to consider the static WTA analyses as a step in the development of dynamic WTA methods.

However, by way of example, a single analysis is presented here to highlight that the MMR algorithm may not always produce optimal results. By using the data from the previous example, but varying the p_{kill} values for each of the five weapons; e.g., p_{kill} = [0.44 , 0.35, 0.26, 0.56, 0.65], the results obtained in Table 3-1 are obtained.

Table 3-1 Comparison of WTA algorithm results from the exhaustive enumeration (exact) and MMR algorithms (N_t =3, N_w = 5)

Algorithm	Weapon #1	Weapon #2	Weapon #3	Weapon #4	Weapon #5	Obj. Fn. value
Exact	Target 3	Target 1	Target 2	Target 2	Target 3	0.5830
MMR	Target 3	Target 2	Target 3	Target 2	Target 3	0.6164

There are clear differences in the results as the exhaustive enumeration algorithm assigns weapons 2 and 3 to targets 1 and 2, respectively, whereas the MMR algorithm assigns weapons 2 and 3 to targets 2 and 3, respectively. Given that exhaustive enumeration delivers the optimal solution, the MMR's result is deemed sub-optimal. This is also reflected in the objective function values shown for each algorithmic approach.

4. Dynamic Weapon Target Assignment

A static approach to WTA is, in general, overly simplistic as WTA is typically a dynamic, time-dependent operation [26] [18]; with the option that at the end of each firing an assessment of damage is made before the next weapon assignments are decided. This suggests that the WTA problem can be divided into simple, discrete stages, which is only true to an extent. In reality, the arrival of hostile targets won't follow a simple, staged schedule. However, for the purposes of obtaining useable optimised WTA solutions, the dynamic assignment approach assumes regular, distinct stages where it is assumed that the outcome of the next firing is completely determined by the current state and the related decisions made, as discussed in Section 2.2.6. This is specifically referred to as a deterministic, dynamic weapon assignment.

Historically the wider problem of dynamic assignment is more generally referred to as dynamic programming – a term coined by Richard Bellman [13] in the 1950's and that came into use long before computer programming was possible [13] [12]. Programming in this sense refers to scheduling. Dynamic Programming can be considered as either deterministic or probabilistic. From the deterministic viewpoint, it is assumed the state at the next time stage is completely determined by the current state and the decisions made at this stage.

The case of probabilistic dynamic programming is more complex, since the 'deterministic' assumption is deemed to no longer hold. In this case, the state at the next stage is not completely known and the subsequent state is now described by a probability distribution. That said, the associated pdf is completely determined by the current state and the decisions made at this stage, as in the deterministic case. It is henceforth assumed that all dynamic WTA analyses referred to in this report are deterministic. Furthermore, the examples and discussion below assume that all weapons available are identical. For the static WTA discussed previously, the parameter x_{ij} represents the decision to assign Weapon *i* to Target *j*. In the case of dynamic WTA, the parameter x_j denotes the number of weapons assigned to Target *j*.

4.1 Network Flow Decision Diagrams

For the relatively less complex problems, network flow decision diagrams offer a simple, visual analysis tool for dynamic WTA problems; the solution being readily comparable to alternative solutions [7] [27]. A network flow diagram allows the analyst to track the options available and to gain a better intuitive understanding of potential solutions to a WTA problem.

4.1.1 Network Flow Example

Consider a dynamic WTA scenario where there is a defended asset with four weapons to confront five hostile targets. Figure 4-1 shows the network flow decision diagram for the scenario considered. The problem reads from the first to last target (left to right across the page), with the initial target ($j=1$) showing a state of $s_1 = 4$ (i.e., 4 weapons available). In this case the analysis is dynamic rather than static as each target is dealt with sequentially, i.e., in successive stages.

For ease of reading, Figure 4-1 only shows the decisions, x_j, relating to the fourth state, s_4, are labelled at each stage; with the label for decisions at other states following the same pattern. The successive target stages are listed horizontally across the page, whereas the possible weapon states are listed vertically down the page. A dynamic programming mathematical analysis is described for this example in Section 4.2.2.

As can be seen from Figure 4-1, a network flow diagram's usefulness naturally becomes more limited as the number of stages and possible states increase, leading to an exponential rise in complexity.

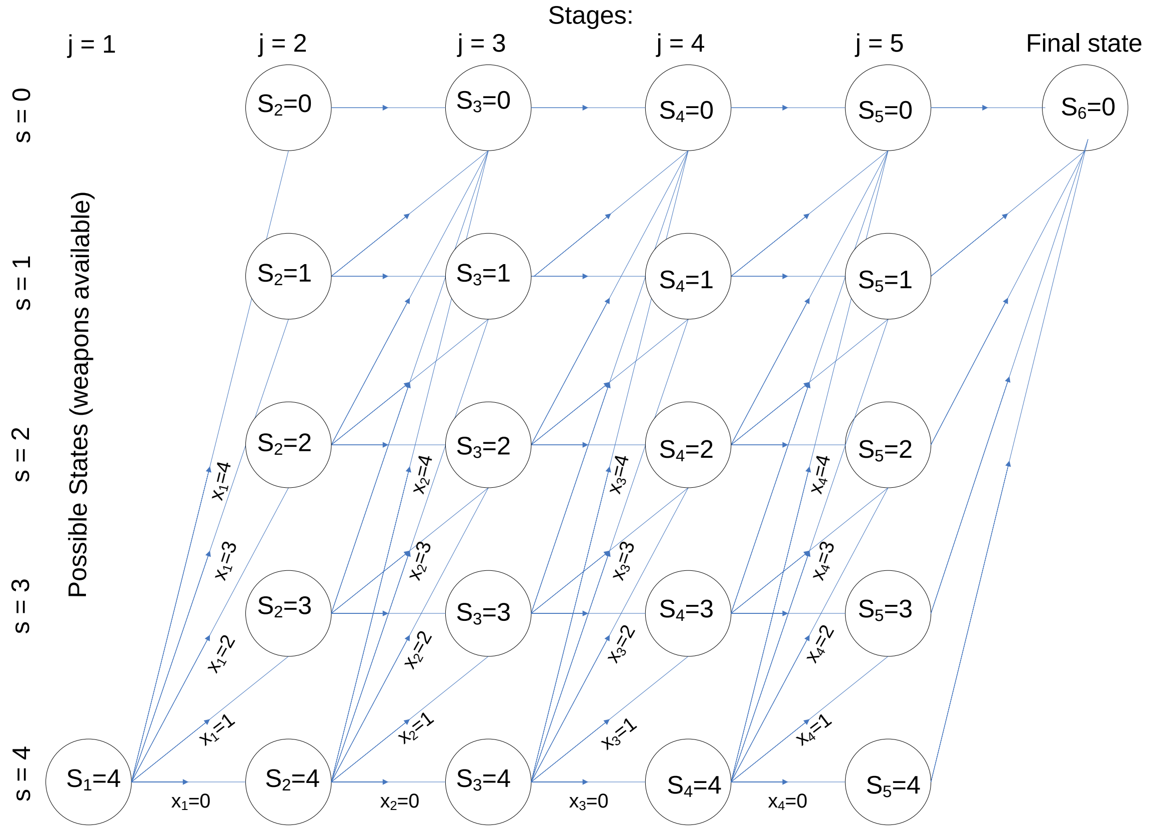

Figure 4-1 A decision tree diagram showing target stages (left to right) and possible weapon states (top to bottom) for a five target, four weapon dynamic programming scenario

4.2 Deterministic Dynamic Programming

4.2.1 Analysis Model Formulation

In an example maritime combat scenario, a single asset has access to N_w weapons and is confronted by N_t hostile targets. The number of remaining weapons in the weapon system at any stage j is given by the state, s_j. The allocation of weapons to a particular target at a specific stage is given by the decision variable, x_j.[5] The optimum assignment of weapons to targets can be determined by maximising the effective threat potential of the targets.

To find an optimal solution using Deterministic Dynamic Programming (DDP), a recursive relationship is used to identify the optimum decision for each stage j, having already determined the optimum decisions for stage $j+1$, $j+2$, etc. [7]. Furthermore, a recursive solution starts with the proposed optimum state at the final stage (i.e., typically having all weapons expended) and works backwards to the initial stage. It is in essence, a Markovian Decision Process (MDP), since all the information required to make an optimised decision

[5] The definition of x_j is different to that used in the theory presented for the static WTA. In the static case, $x_j \in \{0,1\}$ depending on whether the weapon was assigned or not, to a target j, respectively. Here x_j refers to the number of weapons assigned to a particular target, j.

is contained in the current state description at any stage. The objective function for each stage is given by:

$$f_j(s_j, x_j) = c(x_j) + f^*_{j+1}(s_{j+1}), \tag{4}$$

with optimum value:

$$f^*_{j+1}(s_{j+1}) = \max_{x_j = 0,1,\ldots,s_j} f_j(s_j, x_j) \tag{4a}$$

given:

$$s_{j+1} = s_j - x_j \tag{5}$$

and

$$0 \le x_j \le s_j, \tag{6}$$

where:

j: identity of the current target stage ($j = 1, 2, \ldots, N_t$)
s_j: available weapon state for target j
x_j: number of weapons allocated to target j (i.e., $x_j = 0, 1, 2, \ldots$)
$c(x_j)$: effective contribution of decision x_j to the objective function
$f_j(s_j, x_j)$: objective function for target j given weapon state s_j and weapon allocation x_j.
$f_j^*(s_j)$: optimal value of the objective function for target j given weapon state s_j

Equation 4 states that the objective function at stage j is given by the cost of decision x_j given state s_j in addition to the cost of all 'previous'[6] stage contributions, $f^*_{j+1}(s_{j+1})$. The final constraint states that there cannot be more weapons available in a later state, s_{j+1}, than in the previous one, s_j.

In DDP WTA analysis, the effective contribution $c(x_j)$ to target threat reduction of the decision x_j at the current stage j is given by the product:

$$c(x_j) = V_j \, p_{kill}(x_j) \tag{7}$$

where $p_{kill}(x_j)$ represents the probability that x_j weapons result in target j being killed. Assuming x_j^* to be an optimal decision, it produces a maximised objective function at stage j given by:

$$f^*_j(s_j) = \max_{x_j}\left[f_j(s_j, x_j)\right] = f_j(s_j, x^*_j) \tag{8}$$

In other words, the current value of the objective function is given by the cost of the current weapon allocation (current decision) plus the optimum (maximum) cost of all future stages. For WTA target-based analyses, the aim is to maximise the objective function at each stage:

$$f_j(s_j, x_j) = V_j \, p_{kill}(x_j) + f^*_{j+1}(s_{j+1}) \tag{9}$$

This relationship between successive stages is shown schematically in Figure 4-2.

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[6] Given that this is a recursive analysis the term 'previous' refers to the prior analyses of later states, not earlier states.

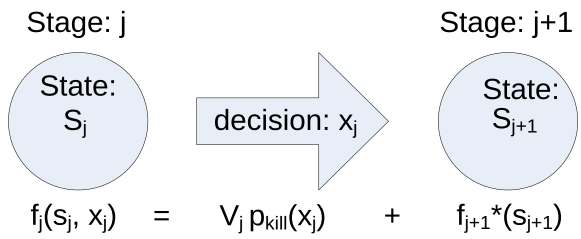

Figure 4-2 Schematic of the relationship between successive stages, j, j+1, in a DDP analysis

4.2.2 WTA DDP Example

Consider, for example, the hypothetical case (introduced in the previous section) of an asset with $N_w = 4$ identical weapons available and $N_t = 5$ distinct targets that have been deemed as threats. The asset has to quickly determine the best use of its weapons. These $p_{kill}(x_j)$ values used here differ from those used in the static WTA example since here all weapons are the same. Furthermore, the probability that a target is killed by multiple weapons fired in a single salvo (i.e., $x_j > 1$) is calculated using[7]:

$$p_{kill}(x_j) = 1 - (1 - p_{kill}(1))^{x_j} \tag{10}$$

An *a priori* knowledge of the weapons' capabilities against the targets is summarised in Table 4-1 with calculated kill probabilities for salvo firings provided in Table 4-2.

Table 4-1 Nominal target values, V_j, and kill probabilities, $p_{kill}(x_j)$, for a single weapon firing (i.e., $x_j = 1$)

Target #	Target Value	kill probability:
j	V_j	$p_{kill}(x_j)$
1	1.000	0.18
2	0.920	0.33
3	0.920	0.15
4	0.750	0.26
5	0.750	0.28

[7] Equation 10 uses the Bernoulli distribution, which is a special case of the binomial distribution with the number of experiments ' n' being unity (i.e., in this case, weapons).

Table 4-2 Target kill probabilities, $p_{kill}(x_j)$, calculated for x_j = {1, 2, 3, 4} weapons/salvo. The $p_{kill}(x_j)$ values were calculated using equation 10 for an example WTA scenario (N_t = 5, N_w = 4).

Target #	Calculated C probability: $p_{kill}(x_j)$			
j	$p_{kill}(1)$	$p_{kill}(2)$	$p_{kill}(3)$	$p_{kill}(4)$
1	0.18	0.33	0.45	0.80
2	0.33	0.55	0.70	0.96
3	0.15	0.28	0.39	0.73
4	0.26	0.45	0.59	0.91
5	0.28	0.48	0.63	0.93

The DDP analysis commences by considering the last target *j*=5. Using Equation 9, contributions to the objective function are calculated using:

$$f_5(s_5,x_5) = V_5 p_{kill}(x_5) + f_6^*(s_6) .$$

But since there is no 6th stage or target, $f_6^*(s_6)$ is defined as zero. Thus:

$$f_5(s_5,x_5) = V_5 p_{kill}(x_5) .$$

The objective function values are calculated for each individual target stage using the formulae listed in Table 4-3. Their associated stage contributions to these objective function values are listed in Table 4-4. The objective function values, $f_j(s_j,x_j)$, for each of the five target stages are listed in Table 4-5 to Table 4-9, respectively.

It can be seen from Table 4-9 that the maximum value of $f_1(s_1,x_1)$ is $f_1^*(x_1) = 0.912$ which corresponds to $x_1^* = 0$ weapons allocated. For the next stage involving Target 2, the weapon state (i.e., available weapons) is calculated using $s_{j+1} = s_j - x_j$, which gives $s_2 = 4 - 0 = 4$. For state $s_2 = 4$, Table 4-8 indicates that $f_2^*(x_2) = 0.912$, corresponding to $x_2^* = 2$ weapons allocated. Similarly, for Target 3, the state, $s_3 = 4 - 2 = 2$, yields $f_3^*(x_3) = 0.405$ from Table 4-7, corresponding to $x_3^* = 0$ weapons allocated. For Target 4, the state, $s_4 = 2 - 0 = 2$, yields $f_2^*(x_2) = 0.405$ from Table 4-6 corresponding to $x_4^* = 1$ weapons allocated. Finally, for Target 5, the state $s_5 = 2 - 1 = 1$, yields $f_5^*(x_5) = 0.210$ from Table 4-5 corresponding to $x_5^* = 1$ weapons allocated. As a check, $s_6 = 1 - 1 = 0$ confirms all weapons are used. In summary, the recursive DDP analysis above has yielded the following optimum policy decision based on the objective function given in equation (9):

- $x_1^* = 0$: no weapons assigned to Target 1
- $x_2^* = 2$: two weapons assigned to Target 2
- $x_3^* = 0$: no weapons assigned to Target 3
- $x_4^* = 1$: one weapon assigned to Target 4
- $x_5^* = 1$: one weapon assigned to Target 5

The cost of this decision is determined by calculating the value of the overall objective function, which in this example is 3.44. The author produced a series of random WTA policy decisions (not shown here) which were not determined using an optimisation scheme. Although this wasn't an exhaustive analysis, these policy decisions all produced costs (i.e., objective function values) higher than that provided by the WTA analysis above, giving some confidence in the result.

Table 4-3 Objective functions for each target stage and the associated table reference data

Stage	objective function	reference data
Target 5	$f_5(s_5,x_5) = V_5 p_{kill}(x_5)$	Table 4-5
Target 4	$f_4(s_4,x_4) = V_4 p_{kill}(x_4) + f_5^*(s_5)$	Table 4-6
Target 3	$f_3(s_3,x_3) = V_3 p_{kill}(x_3) + f_4^*(s_4)$	Table 4-7
Target 2	$f_2(s_2,x_2) = V_2 p_{kill}(x_2) + f_3^*(s_3)$	Table 4-8
Target 1	$f_1(s_1,x_1) = V_1 p_{kill}(x_1) + f_2^*(s_2)$	Table 4-9

Table 4-4 Contributions, $V_j p_{kill}(x_j)$, of 'x_j' weapons/salvo on each target to individual objective functions listed in Table 4-3 using $p_{kill}(x_j)$ data from Table 4-1

	$V_j p_{kill}(x_j)$				
target number	$x_j = 0$	$x_j = 1$	$x_j = 2$	$x_j = 3$	$x_j = 4$
$j = 1$	0.000	0.180	0.328	0.449	0.796
$j = 2$	0.000	0.304	0.507	0.643	0.883
$j = 3$	0.000	0.138	0.255	0.355	0.669
$j = 4$	0.000	0.195	0.339	0.446	0.683
$j = 5$	0.000	0.210	0.361	0.470	0.696

Table 4-5 Objective function $f_5(s_5,x_5)$ values for Target 5, denoted $f_5^*(s_5)$

state number	$x_5 = 0$	$x_5 = 1$	$x_5 = 2$	$x_5 = 3$	$x_5 = 4$	$f_5^*(x_5)$
$s_5 = 0$	0.000	x	x	x	x	0.000
$s_5 = 1$	x	0.210	x	x	x	0.210
$s_5 = 2$	x	x	0.361	x	x	0.372
$s_5 = 3$	x	x	x	0.470	x	0.470
$s_5 = 4$	x	x	x	x	0.696	0.696

Table 4-6 Objective function $f_4(s_4,x_4)$ values for Target 4, denoted $f_4^*(s_4)$

state number	$x_4 = 0$	$x_4 = 1$	$x_4 = 2$	$x_4 = 3$	$x_4 = 4$	$f_4^*(x_4)$
$s_4 = 0$	0.000	x	x	x	x	0.000
$s_4 = 1$	0.210	0.195	x	x	x	0.210
$s_4 = 2$	0.361	0.405	0.339	x	x	0.405
$s_4 = 3$	0.470	0.556	0.549	0.446	x	0.556
$s_4 = 4$	0.696	0.665	0.701	0.656	0.683	0.701

Table 4-7 Objective function $f_3(s_3,x_3)$ values for Target 3, denoted $f_3^*(s_3)$

state number	$x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$f_3^*(x_3)$
$s_3 = 0$	0.000	x	x	x	x	0.000
$s_3 = 1$	0.210	0.138	x	x	x	0.210
$s_3 = 2$	0.405	0.348	0.255	x	x	0.405
$s_3 = 3$	0.556	0.543	0.465	0.355	x	0.556
$s_3 = 4$	0.701	0.694	0.660	0.565	0.669	0.701

Table 4-8 Objective function $f_2(s_2,x_2)$ values for Target 2, denoted $f_2^*(s_2)$

state number	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$f_2^*(x_2)$
$s_2 = 0$	0.000	x	x	x	x	0.000
$s_2 = 1$	0.210	0.304	x	x	x	0.304
$s_2 = 2$	0.405	0.514	0.507	x	x	0.514
$s_2 = 3$	0.556	0.709	0.717	0.643	x	0.717
$s_2 = 4$	0.701	0.860	0.912	0.853	0.883	0.912

Table 4-9 Objective function $f_1(s_1,x_1)$ values for Target 1, denoted $f_1^*(s_1)$

state number	$x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$f_1^*(x_1)$
$s_1 = 4$	0.912	0.897	0.841	0.752	0.796	0.912

All things being equal, it would be reasonable to assume that weapons would be allocated one per target, with one target having no weapon assigned. However, it is notable that the analysis allocates two weapons to Target 1 and no weapons to Targets 3 or 4. This is because the objective function contributions, $c(x_j)$, calculated by the product, $V_j p_{kill}(x_j)$, of these weapon assignments is relatively lower than others, largely due to a lower kill probability value, $p_{kill}(x_j)$, than the target value, V_j.

This analysis can also be shown on a revised version of the previous network flow diagram shown in Figure 4-3. In this diagram, only the optimum decisions and corresponding states are shown (highlighted in red). To determine the solution's effectiveness, we can determine the total expected, average survivability value of the targets after the weapons have been fired, otherwise known as the overall objective function, F_{obj}. The value of the F_{obj} for the DDP solution is given by [28]:

$$F_{obj} = \sum_{j=1}^{Nt} V_j \prod_{i=1}^{Nw} (1 - p_{kill}^{ij})^{x_{ij}} \tag{11}$$

F_{obj} for the DDP solution is calculated using the data in Table 4-10 to be 3.44. This value can be compared to values for other WTA options and shown to be a minimum.

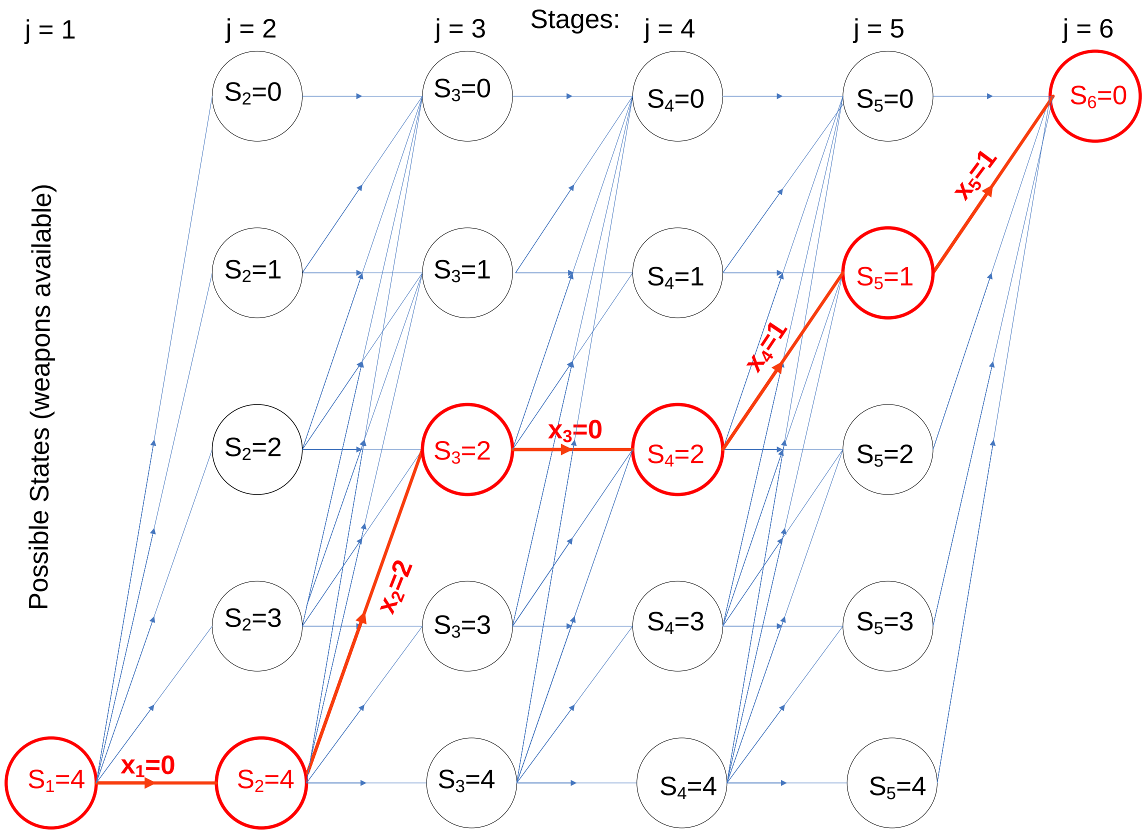

Figure 4-3 Optimal solution for five target, four weapon example problem

Table 4-10 Calculation of the objective function, F_{obj}

Target	$x_j = 0$	$x_j = 1$	$x_j = 2$	$x_j = 3$	$x_j = 4$	$\prod_{i=1}^{Nw}(1-P_k^{ij})^{x_{ij}}$
j = 1	1.000	1.000	1.000	1.000	1.000	1.000
j = 2	0.920	0.920	0.413	0.413	0.413	0.413
j = 3	0.920	0.920	0.920	0.920	0.920	0.920
j = 4	0.750	0.555	0.555	0.555	0.555	0.555
j = 5	0.750	0.540	0.540	0.540	0.540	0.540
			$F_{obj} = \sum_{j=1}^{Nt} V_j \prod_{i=1}^{Nw}(1-P_k^{ij})^{x_{ij}}$			3.43

4.3 WTA DDP Algorithm

An algorithm, which uses the DDP methodology described in the previous section, was written in MATLAB code. As in the static analysis case, the WTA_DDP MATLAB routine has a range of input parameters, including target number, N_t, weapon number, N_w, and kill probabilities, p_{kill}, for each weapon-target assignment. This routine also includes an additional parameter, N_m which is the number of weapons fired per salvo (i.e., the firing of multiple weapons in a firing) and p_{kill} values are generated using Equation 9. The user can also vary the individual target values, V_j, if required.

4.3.1 WTA DDP Algorithm Example

The example presented below uses the WTA_DDP algorithm, in which p_{kill} distributions were generated using fictitious $p_{kill}(1)$ values as shown in Figure 4-4. The scenario involves six targets of varying value, 11 weapons with different, target dependent p_{kill} values, and a limitation of three weapon firings per salvo for any single target. A run of the algorithm produced the results listed in Table 4-11and displayed in the WTA plot of Figure 4-5.

Table 4-11 Target values for the 6 targets, 11 weapons (limited to 3 weapons per salvo) and the resultant WTAs, x_j, generated using WTA_DDP MATLAB algorithm

–	Value	*kill probabilities* (N_m = 3)			#weapons
j	V_j	$p_{kill}(1)$	$p_{kill}(2)$	$p_{kill}(3)$	x_j
1	0.1	0.520	0.770	0.889	1
2	0.1	0.310	0.524	0.671	0
3	0.1	0.420	0.664	0.805	1
4	1.0	0.450	0.698	0.834	3
5	1.0	0.490	0.740	0.867	3
6	1.0	0.380	0.616	0.762	3

A separate EXCEL spreadsheet was used to validate the output of the WTA_DDP algorithm. The algorithm and spreadsheet produced identical results.

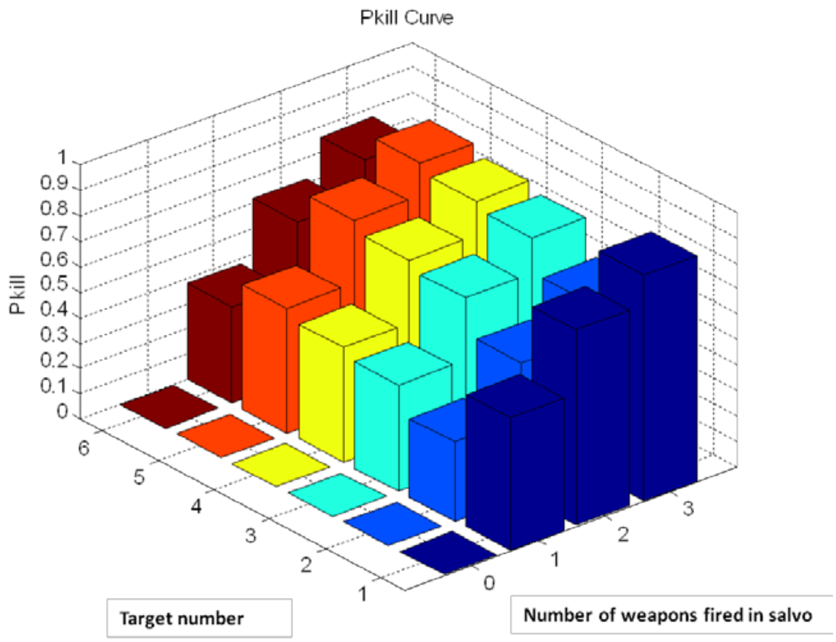

Figure 4-4 pkill distribution for the WTA DDP example with Nt = 6, Nw = 11 and Nm = 3

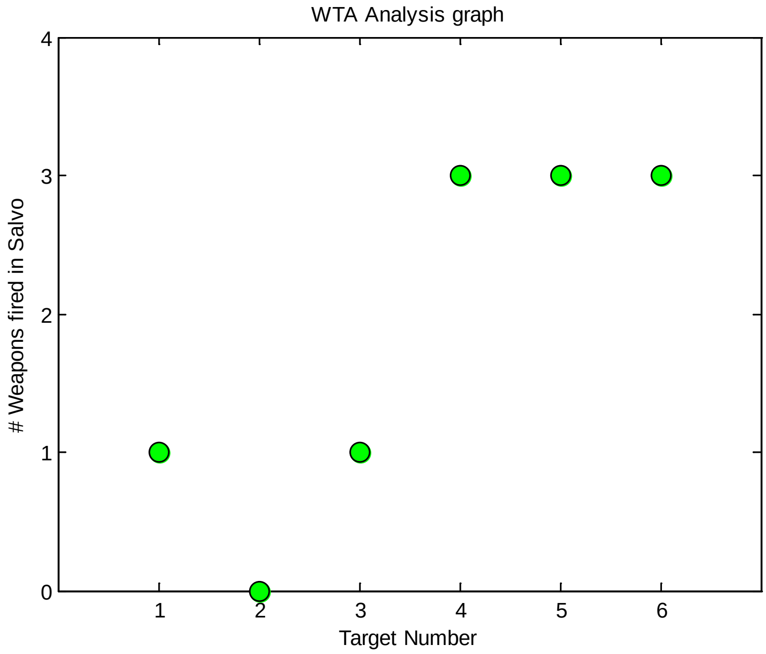

Figure 4-5 Optimised weapon-target allocation result from the WTA_DDP algorithm with N_t = *6,* N_w = *11 and* N_m = *3, using the* p_{kill} *data shown in Figure 4-4*

5. Discussion and Conclusions

The task of assigning weapons to targets in an optimal manner in a maritime combat scenario presents many challenges, particularly when many of the parameters that must be included in any analysis are not fully known. The work presented here outlines the static and dynamic WTA procedures, both of which have been well documented in earlier texts and articles. The static analysis makes the simplification that all assignments of weapons to targets are decided and are fired simultaneously. In contrast, the dynamic analysis assumes a sequence of firing events, with each subsequent event dependent on the observed outcome of the previous one. The dynamic WTA procedure produces a series of sequential optimal weapon allocation decisions for a finite number of known targets with a finite number of available weapons. However, due to the stochastic nature of the dynamic analysis, there is no certainty of success after each firing. If at any stage, the observed outcome of a weapon firing is unsuccessful, the WTA procedure must be rerun for the remaining weapons and targets. This is typically referred to as a shoot-look-shoot procedure. The methodology for both of these analysis types have been discussed in this report.

Dynamic WTAs are also based on a number of assumptions, as they are typically presented as a deterministic analysis, although it is well understood that in reality, naval and other combat scenarios have outcomes that are inherently stochastic. It has been demonstrated in this report that such uncertainty can be dealt with by the use of a deterministic dynamic programming (DDP) formulation for WTA.

The DDP WTA example analyses were initially described with the aid of decision tree diagrams and EXCEL worksheets, although neither of these methodologies are viable as real-time solutions. For this purpose, a MATLAB routine, WTA_DDP, was developed and applied to the examples presented in EXCEL worksheets for cross-validation purposes. Once validated, the WTA_DDP algorithm was applied to more numerically complex examples.

Further development of the algorithm and the associated analysis techniques reported here will potentially lead to a useful resource as a training aid and as a real-time decision-tool. Ultimately, the value of any WTA methodologies developed would be fully realised once combined with threat evaluation techniques into a fully integrated TEWA system.

The WTA work presented in this report is an initial step toward a deeper understanding and development of a coordinated Hard Kill / Soft Kill capability, with the ultimate aim of this work to integrate hard kill and soft kill maritime combat procedures. In the near term, the author has proposed to undertake further work to develop the WTA methodology and associated algorithm to model multiple assets, as opposed to single assets, under threat. Incorporation of this and future algorithms into a practical software test bed for test and evaluation purposes is also highly desirable.

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19. ABSTRACT

Threat evaluation and weapon assignment procedures are an integral part of achieving successful outcomes in naval combat scenarios, both in terms of defending one's assets and destroying enemy targets. To assist with the ever more complex and rapidly changing combat environment, warfighters require access to real-time decision aids based on optimisation techniques to support the decision making process. This report documents the methodology used to identify, develop and assess a dynamic programming algorithm for Weapon Target Assignment which, after more rigorous testing, could be used as a concept demonstrator and as an auxiliary decision tool in real-time combat situations.

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