REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
25-08-2015	Final Report	17-Aug-2012 - 16-Dec-2015

4. TITLE AND SUBTITLE

Final Report: L1 Approximating Splines with Locally Computed Coefficients for Discrete Data

5a. CONTRACT NUMBER

5b. GRANT NUMBER

W911NF-04-D-0003

5c. PROGRAM ELEMENT NUMBER

611102

6. AUTHORS

John Lavery, S.C. Fang

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES

North Carolina State University
2701 Sullivan Drive
Suite 240
Raleigh, NC 27695 -7514

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)

U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)

ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

62145-MA-SR.13

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

The objective of the proposed research is to shift from the global structure previously used for L1 approximating splines to the local structure that has been so successful for L1 interpolating splines. Larger effects of this univariate research include: 1) It will lead to computationally cheap, shape-preserving nonparametric and parametric bi- and multivariate L1 spline fits for modeling irregular, multiscale 3D point clouds for reconstruction and texturing of urban terrain and of irregular geometrical objects. 2) It may lead to a new class of bi-level optimization algorithms.

15. SUBJECT TERMS

approximation, interpolation, L1 Major Component Detection and Analysis, L1 splines, spline fit

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			John Lavery
UU	UU	UU	UU		19b. TELEPHONE NUMBER 000-000-0000

Report Title

Final Report: L1 Approximating Splines with Locally Computed Coefficients for Discrete Data

ABSTRACT

The objective of the proposed research is to shift from the global structure previously used for L1 approximating splines to the local structure that has been so successful for L1 interpolating splines. Larger effects of this univariate research include: 1) It will lead to computationally cheap, shape-preserving nonparametric and parametric bi- and multivariate L1 spline fits for modeling irregular, multiscale 3D point clouds for reconstruction and texturing of urban terrain and of irregular geometrical objects. 2) It may lead to a new class of bi-level optimization algorithms.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
08/19/2013	1.00	John Lavery. Univariate Lp and lp Averaging, 0 < p < 1, in Polynomial Time by Utilization of Statistical Structure, Algorithms, (10 2012): 0. doi: 10.3390/a5040421
08/25/2015	8.00	Shu-Cherng Fang, Ye Tian, Qinwei Jin, John E. Lavery. l1 Major Component Detection and Analysis (l1 MCDA):Foundations in Two Dimensions, Algorithms, (12 2013): 12. doi:
08/25/2015	7.00	John E. Lavery, Shu-Cherng Fang, Lu Yu, Zhibin Deng, Dimitri Bulatov. Comparison of an l1-regression-based and a RANSAC-based Planar Segmentation Procedure forUrban Terrain Data with Many Outliers, Proceeding SPI 8892, (12 2013): 1. doi:
08/25/2015	5.00	John E. Lavery. Univariate Lp and lp Averaging, 0 < p < 1, in Polynomial Time by Utilization of Statistical Structure, , (2012): 421. doi:
08/25/2015	9.00	Shu-Cherng Fang, Ziteng Wang, John E. Lavery. Approximation of Irregular Geometric Data by LocallyCalculated Univariate Cubic L1 Spline Fits, Annals of Data Science, (12 2014): 5. doi:
08/25/2015	10.00	John E. Lavery, Shu-Cherng Fang, Zhibin Deng, Jian Luo. l1 Major Component Detection and Analysis (l1 MCDA) inThree and Higher Dimensional Spaces, Algorithms, (12 2014): 429. doi:
08/25/2015	11.00	John E. Lavery. Toward Lp and ┐p minimization, 0 < p < 1, in polynomial time:Lp averaging for hyperspherically symmetric distributions, Journal of Mathematical Analysis and Applications, (12 2015): 0. doi:
08/25/2015	12.00	John E. Lavery, Shu-Cherng Fang, Ziteng Wang. On Shape-preserving Capability of Cubic L1 Spline Fits, Computer Aided Geometric Design, (12 2016): 0. doi:
TOTAL:	**8**	

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received		Paper
08/19/2013	3.00	Ye Tian, Qingwei Jin, John E. Lavery, Shu-Cherng Fang. ?1 Major Component Detection and Analysis (?1 MCDA):Foundations in Two Dimensions, , (01 2013): 0. doi:
08/19/2013	4.00	Ziteng Wang, John Lavery, Shu-Cherng Fang. Approximation of Irregular Geometric Data byLocally Calculated Univariate Cubic L1 Spline Fits, , (09 2013): 0. doi:
08/25/2015	6.00	John E. Lavery, Shu-Cherng Fang, Lu Yu, Qingwei Jin. Univariate cubic L1 interpolating splines basedon the first derivative and on 5-point windows: analysis,algorithm and shape-preserving properties, Computational Optimization and Applications, (12 2012): 575. doi:
TOTAL:		**3**

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper
TOTAL:	

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

TOTAL:

Number of Manuscripts:

Books

Received Book

TOTAL:

Received	Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Ye Tian	1.00	
Zhibin Deng	1.00	
Jian Luo	1.00	
Ziteng Wang	1.00	
Tiantian Nie	1.00	
FTE Equivalent:	**5.00**	
Total Number:	**5**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	0.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:.....	0.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:.....	0.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):.....	0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:.....	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:	0.00

Names of Personnel receiving masters degrees

NAME	
Ye Tian	
Zhibin Deng	
Jian Luo	
Ziteng Wang	
Tiantian Nie	
Total Number:	**5**

Names of personnel receiving PHDs

NAME	
Ye Tian	
Zhibin Deng	
Jian Luo	
Ziteng Wang	
Total Number:	**4**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Through extensive mathematical analysis, the project has successfully shown the feasibility of constructing computationally efficient shape-preserving L1 approximating splines using the concept of locally computed coefficients. The properties, performance and other key findings have been documented in Pub. 2 and Pub 5. The work has been implemented and tested for modeling irregular, multiscale 3D point clouds for reconstruction and texturing of urban terrain and of irregular geometrical objects. A good comparison with the popular RANSAC-based planar segmentation procedure for urban terrain data with many outliers has been reported in Pub 3.

The research results have opened a door for exploring new approaches to identifying multiple major components embedded in irregular multiscale data clouds. The basic models of constructing ?1-based multi-component detection technology have been proposed in Pub 4 and Pub 6. Unlike the commonly used “principal component analysis,” the new technology has the capability of detecting non-orthogonal major components with heavy-tailed distributions.

This project has also led to the study of the fundamentals of shape-preservation capability of spline functions. An analytical measure of shape-preservation has been proposed in Pub 8 for the first time. Additional research works on exploring the new frontiers for adopting “sub-1 Lp and lp splines” (i.e., $0 < p < 1$) have been reported in Pub 1 and Pub 7.

Technology Transfer

STAFF RESEARCH PROPOSAL

L1 APPROXIMATING SPLINES WITH LOCALLY COMPUTED COEFFICIENTS

Project Objective:

The objective of the proposed research is to shift from the global structure previously used for L1 approximating splines to the local structure that has been so successful for L1 interpolating splines. Larger effects of this univariate research include: 1) It will lead to computationally cheap, shape-preserving nonparametric and parametric bi- and multivariate L1 spline fits for modeling irregular, multiscale 3D point clouds for reconstruction and texturing of urban terrain and of irregular geometrical objects. 2) It may lead to a new class of bi-level optimization algorithms.

Research Achievements:

Through extensive mathematical analysis, the project has successfully shown the feasibility of constructing computationally efficient shape-preserving L1 approximating splines using the concept of locally computed coefficients. The properties, performance and other key findings have been documented in Pub. 2 and Pub 5. The work has been implemented and tested for modeling irregular, multiscale 3D point clouds for reconstruction and texturing of urban terrain and of irregular geometrical objects. A good comparison with the popular RANSAC-based planar segmentation procedure for urban terrain data with many outliers has been reported in Pub 3.

The research results have opened a door for exploring new approaches to identifying multiple major components embedded in irregular multiscale data clouds. The basic models of constructing ℓ1-based multi-component detection technology have been proposed in Pub 4 and Pub 6. Unlike the commonly used “principal component analysis,” the new technology has the capability of detecting non-orthogonal major components with heavy-tailed distributions.

This project has also led to the study of the fundamentals of shape-preservation capability of spline functions. An analytical measure of shape-preservation has been proposed in Pub 8 for the first time. Additional research works on exploring the new frontiers for adopting “sub-1 Lp and lp splines” (i.e., $0 < p < 1$) have been reported in Pub 1 and Pub 7.

Publications:

1. Lavery, J.E., Univariate Lp and lp Averaging, $0 < p < 1$, in Polynomial Time by Utilization of Statistical Structure, Algorithms, 5(2012), 421-432.

2. Q. Jin, Yu, L., Lavery, J.E., Fang, S.-C., Univariate cubic L 1 interpolating splines based on the first derivative and on 5-point windows: analysis, algorithm and shape-preserving properties, Computational Optimization and Applications, 51(20012), 575-600.

3. Luo, J., Deng, Z., Bulatov, D., Lavery, J.E., Fang, S.-C., Comparison of an ℓ1-MCDA-based and a RANSAC-based Planar Segmentation Procedure for Urban Terrain Data with Many Outliers, Proc. SPIE 8892, Image and Signal Processing for Remote Sensing XIX, 889209 (October 17, 2013); doi:10.1117/12.2028627

4. Tian Y., Jin Q., Lavery, J.E., Fang, S.-C., ℓ1 Major Component Detection and Analysis (ℓ1 MCDA): Foundations in Two Dimensions, Algorithms, 6(2013), 12-28.

5. Wang, Z., Lavery, J.E., Fang, S.-C., Approximation of Irregular Geometric Data by Locally Calculated Univariate Cubic L1 Spline Fits, Annals of Data Science, 1 (2014), 5-14.

6. Deng, Z., Lavery, J.E., Fang, S.-C., Luo, J., ℓ1 Major Component Detection and Analysis (ℓ1 MCDA) in Three and Higher Dimensional Spaces, Algorithms, 7(2014), 429-443.

7. Lavery, J.E., Toward Lp and lp Minimization, $0 < p < 1$, in Polynomial Time: Lp Averaging for Hyperspherically Symmetric Distributions, Journal of Mathematical Analysis and Applications, accepted for publication, 2015.

8. Wang, Z., Lavery, J.E., Fang, S.-C., On Shape-preserving Capability of Cubic L1 Spline Fits, with Z. Wang and J E Lavery, to appear in Computer Aided Geometric Design, 2015.

Research Collaboration:

In addition to working with faculty and students at the North Carolina State University, collaborative research has been extended to include Professor Olivier Gibaru and Professor Eric Nyiri (Department of Applied Mathematics, Paris Tech, France) and Dr. Dimitri Bulatov (Fraunhofer Institute of Optronics, System Technologies and Image Exploitation (IOSB), Ettlingen, Germany).

Education Support:

This project has provided an opportunity to motivate and recruit young researchers to engage research in the field. A total of five PhD students of NC State and one Fulbright PhD student from France have been involved.

PhD Student Supervision at NC State University:

Mr. Ye TIAN (PhD Received in December 2012),

Mr. Zhibin DENG (PhD Received in December 2013)

Mr. Jian LUO (PhD Received in December 2014)

Mr. Ziteng WANG (PhD Received in May 2015)

Ms. Tiantian Nie (PhD student to graduate in 2016)

Fulbright PhD Student Supervision:

Mr. Laurent GAJNY, École Nationale Supérieure des Arts et Métiers, Lille, France, September 2013 – February 2014. (PhD Received in May 2015)