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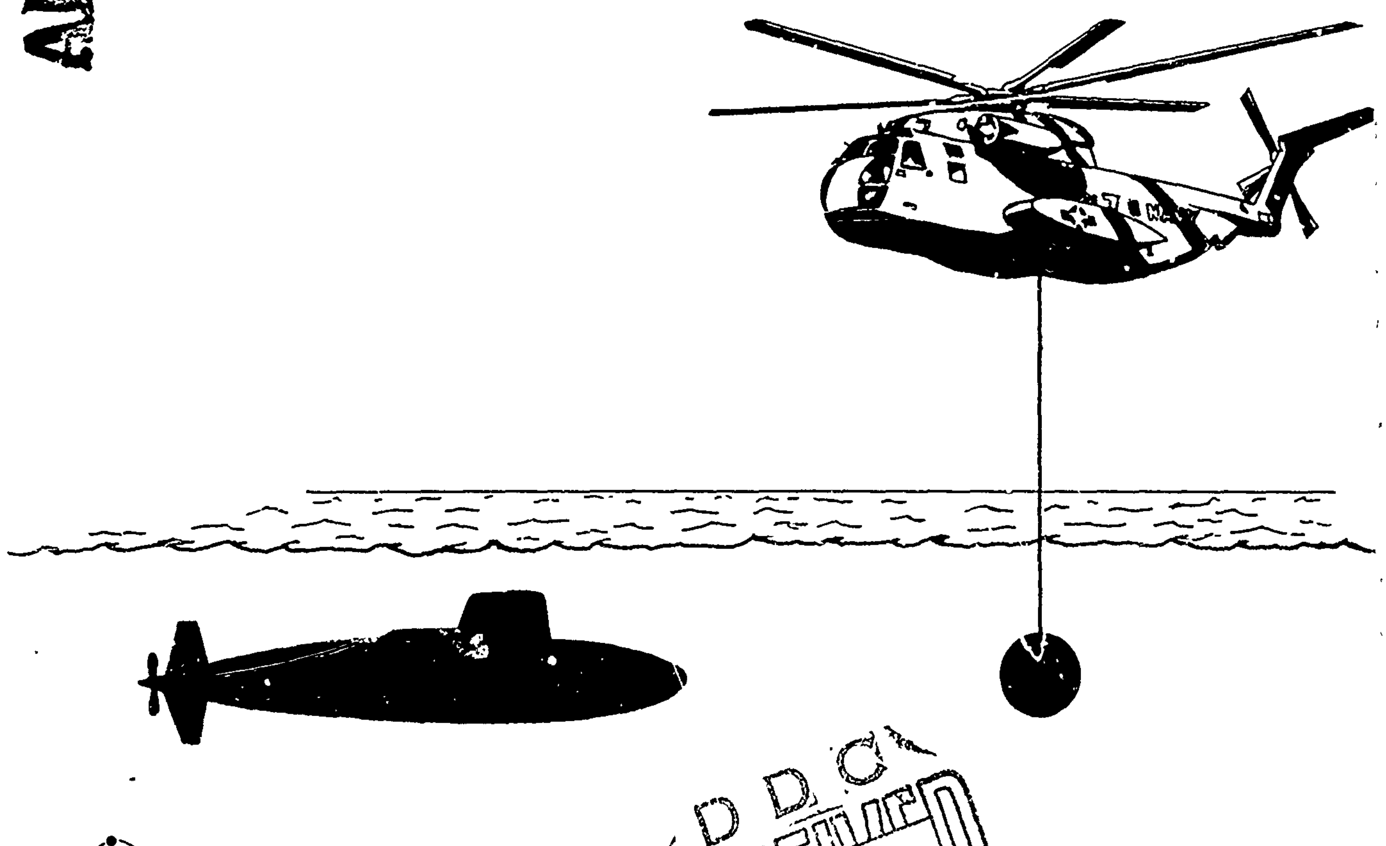
A STUDY OF MEASURES OF EFFECTIVENESS USED IN NAVAL ANALYSIS STUDIES



ONR CONTRACT:
N00014-71-C-0247
FINAL REPORT
31 OCTOBER 1972

ULTRASYSTEMS, INC.
500 NEWPORT CENTER DRIVE
NEWPORT BEACH, CALIFORNIA

VOLUME 2
STUDY REVIEW
SUMMARIES, PART I

A STUDY OF MEASURES OF EFFECTIVENESS USED IN NAVAL ANALYSIS STUDIES

Volume 2

Study Review Summaries, Part I

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ONR Contract: N00014-71-C-0247

Final Report

31 October 1972

Prepared for

Office of Naval Research
Naval Analysis Programs

Prepared by

ULTRASYSTEMS, INC.
500 Newport Center Drive
Newport Beach, California 92660

LIST OF STUDY REVIEW SUMMARIES, PART I

(1)

AIRBORNE ASW

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STUDY REVIEW SUMMARY NO (1)-1

A. STUDY DESCRIPTION

1) Originating Activity: Systems Analysis Office, ASW Systems Project Office, White Oak, Maryland
2) Report Title: Single Helicopter Tactical Effectiveness Study
3) Authors: R. W. Bryant and R. H. Dickman
4) Report Number: SAG Technical Memorandum 67-3 (AD-385 182)
5) Date: August 1967
6) Classification: Confidential
7) Abstract: The objective of this report was to determine the relative importance of incremental gains resulting from certain proposed changes in the sensor and weapon configuration of an ASW helicopter. The analysis is based on an analytical model and uses results of OPTEVFOR investigations. The approach taken in this analysis is to determine the comparative effectiveness of given ASW helicopter systems in detecting, localizing and killing both conventional and nuclear submarines.
8) Descriptors: Antisubmarine warfare, detection probability, dipping sonar, helicopter, kill probability, localization probability, MAD, Markov process, search, submarine, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Submarine search
   3.1) Definition: A helicopter searches for a submarine using a dipped sonar. Upon detection, localization is attempted using either the dipped sonar or MAD. The helicopter then flies to datum and launches a torpedo (snake search or circle search).

3.2) Criterion For Success: Detection, localization and kill of submarine

3.3) MOE Selected: Probability of submarine detection, localization and kill

3.4) Functional Form Of MOE:

<u>Case 1</u> - Dipped sonar used for both detection and localization and circle search torpedo is deployed

$$MOE = f_1(x_1, x_2, x_3)$$

where

x_1 = a vector whose i^{th} component represents the probability of detection on the i^{th} sonar dip in the datum area

$= g_1(x_4, \ldots, x_9)$

x_2 = probability of submarine localization within the acquisition range of the circle search torpedo

$= g_2(x_{10}, x_{11})$

x_3 = kill probability of the circle search torpedo

x_4 = dipped sonar range

x_5 = initial datum size (radius of uncertainty)

x_6 = submarine speed

$$= \begin{cases} x_{12} & \text{if alerted} \\ x_{13} & \text{if unalerted} \end{cases}$$

x_7 = helicopter time late at datum

x_8 = number of dips

x_9 = time to recover transducer, accelerate, fly to new dip station, decelerate, and transmit six pings

x_{10} = circle search torpedo acquisition radius

x_{11} = time to raise transducer fly, redip. and obtain six pings at the mean detection range

x_{12} = alerted submarine speed

x_{13} = unalerted submarine speed

Case 2 - Dipped sonar used for detection, MAD for localization and circle search torpedo is deployed

$$MOE = f_2(x_1, x_3, x_{14})$$

where

x_{14} = probability of submarine localization with MAD
$= g_3(x_{15}, x_{16})$

x_{15} = mean area swept by MAD
$= h_1(x_{17}, \ldots, x_{20})$

x_{16} = mean area of uncertainty of submarine position
$= h_2(x_{12}, x_{19}, x_{20})$

x_{17} = helicopter speed

x_{18} = MAD sweep width

x_{19} = time at which MAD search ends
$= i_1(x_{12}, x_{21}, \ldots, x_{24})$

x_{20} = time at which MAD search begins
$= i_2(x_{25}, x_{26}, x_{27})$

x_{21} = mean detection range

x_{22} = time to decelerate

x_{23} = time to lower ball

x_{24} = time to receive a signal

x_{25} = time to raise ball

x_{26} = time to accelerate

x_{27} = time to fly to datum

Case 3 - Dipped sonar used for both detection and localization and snake search torpedo is deployed

$$MOE = f_3(x_1, x_{28}, x_{29})$$

where

x_{28} = probability of submarine localization within the acquisition range of the snake search torpedo
$= g_4(x_9, x_{12}, x_{30})$

x_{29} = kill probability of the snake search torpedo

x_{30} = optimum launch range for snake search torpedo

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) The helicopter operates independently in obtaining a detection and prosecuting an attack.

(b) Since the helicopter is using active sonar, the submarine is alerted by the first ping and immediately evades at twice its unalerted speed in a random direction.

(c) No degradation of performance is considered for operator or equipment failures.

(d) The dipped sonar is used in a circle search mode.

(e) The submarine is randomly distributed within the datum area on a random course, and the helicopter dips in such a way that dipping sonar search area is always within the datum area. If detection is not obtained on a given dip, the helicopter moves a distance equal to twice the sonar range and redips.

(f) The detection phase is limited to seven dips since by this time the datum area has become so large that detection is unlikely.

(g) In defining the probability of localizing a target with MAD, the helicopter breaks dip when sonar detection is obtained, flies to datum (on the proper bearing and at a distance equal to the mean detection range). To insure that sonar contact can be regained, the MAD search must cease before the submarine is beyond sonar detection range.

(h) Kill probabilities used are averages over all launch conditions and submarine aspects and include the probability of hit and the probability of kill given hit for various modes of operation.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

   2.1) Qualitative Factor:

      (a) conventional/nuclear

   2.2) Quantitative Factors:

      (a) unalerted submarine speed

      (b) alerted submarine speed

3) Friendly Force Composition: Helicopter (SH-3A)

   3.1) Quantitative Factors:

      (a) helicopter time late at datum

      (b) helicopter speed

      (c) time to decelerate

      (d) time to accelerate

      (e) time to fly to datum

   3.2) Sensors: Dipped sonar and MAD

      3.2.1) Type: Dipped sonar

         3.2.1.1) Quantitative Factors:

            (a) dipped sonar range

            (b) mean detection range

      3.2.2) Type: MAD

         3.2.2.1) Quantitative Factors:

            (a) MAD sweep width

            (b) time to receive a signal

            (c) time to lower ball

            (d) time to raise ball

   3.3) Armament: Circle search and snake search torpedoes

      3.3.1) Type: Circle search torpedo

         3.3.1.1) Quantitative Factor:

            (a) circle search torpedo acquisition range

      3.3.2) Type: Snake search torpedo

         3.3.2.1) Quantitative Factor:

            (a) optimum launch range for snake search torpedo

3.4) Tactics:

3.4.1) Qualitative Factor:

(a) conduct dipped sonar search within the datum area

3.4.2) Quantitative Factors:

(a) time to recover transducer, accelerate, fly to new dip station, decelerate, and transmit six pings

(b) number of dips

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Helicopter - Submarine

4.1.1) Quantitative Factors:

(a) initial datum size (radius of uncertainty)

(b) time to raise transducer, fly, redip, and obtain six pings at the mean detection range

4.2) Armament - Platform:

4.2.1) Type: Circle search torpedo - Submarine

4.2.1.1) Quantitative Factor:

(a) kill probability of the circle search torpedo

4.2.2) Type: Snake search torpedo - Submarine

4.2.2.1) Quantitative Factor:

(a) kill probability of the snake search torpedo

STUDY REVIEW SUMMARY NO.(1)·2

A. STUDY DESCRIPTION

1) Originating Activity: University of Wisconsin, Madison, Wisconsin
2) Paper Title: "A Helicopter Versus Submarine Search Game"
3) Author: J. M. Danskin
4) Source: Operations Research, Vol. 16, No. 3, May-June 1963, pp. 509-517
5) Classification: Unclassified
6) Abstract: The problem for solution was to find an optimal search strategy for helicopters using their dipping sonar to search for a submarine, which had been sighted a short time before, now submerged and attempting to escape. By some approximations to reality and a shift in the point of view, the problem is brought to a simple two-person zero-sum game in which one side varies areas and the other probability distributions. The solutions are exhibited and proved to be solutions by direct application of the definition of optimal strategies for a game.
7) Descriptors: Antisubmarine warfare, contact investigation, detection probability, dipping sonar, game theory, helicopter, search, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Contact investigation
   - 3.1) Definition. Sonar-carrying helicopters are dispatched to search for submerged submarine that had been sighted earlier.
   - 3.2) Criteron For Success: Detection of submarine
   - 3.3) MOE Selected: Probability of submarine detection resulting from optimal strategies of helicopters and submarine

3.4) Functional Form Of MOE:

$$MOE = \min_{x_2} \max_{x_1} f(x_1, x_2)$$

where

$f(x_1,x_2)$ = is the probability of detection resulting from a helicopter strategy x_1 and a submarine mixed strategy x_2.

x_1 = helicopter strategy
= $g_1(x_3, x_4)$

x_2 = submarine mixed strategy
= $g_2(x_4, x_5)$

x_3 = total area of speed space scanned by helicopter
= $h_1(x_6, x_7, x_8)$

x_4 = area of speed circle
= $h_2(x_9)$

x_5 = submarine speed

x_6 = total number of helicopters

x_7 = detection radius of sonar dip

x_8 = a vector whose i^{th} component represents the time of the i^{th} sonar dip
= $i(x_{10}, x_{11}, x_{12})$

x_9 = maximum possible speed of submarine without sound detection

x_{10} = late time of helicopter

x_{11} = time between sonar dips

x_{12} = total number of dips

4) MOE Usage In Study: The problem was formulated in the setting of a two-person zero-sum game. An optimal strategy was found using the techniques of game theory. This strategy was used to prepare helicopter search patterns for the U.S.N.

5) Special Study Assumptions:

(a) The point where the submarine was detected is precisely known.

(b) The submarine will not exceed a speed beyond which the aircraft listening devices will hear them.

(c) The submarine's speed is constant from the moment he is detected.

(d) The helicopters arrive at some time late, and thereafter make active sonar dips at fixed time intervals determined by the time required to rotate the sonar heads and to fly to "fresh water".

(e) Each helicopter makes a number of dips determined by his time available on-station after the flight from the carrier.

(f) The submarine obtains no useful information from the sounds he hears from the helicopters.

(g) The helicopter sonars illuminate perfectly on each dip a constant circular area and this is done essentially instantly.

(h) The submarine strategy consists of a choice of direction and speed. Both of these are held throughout the search.

(i) There is no overlapping of sonar dips.

(j) All sonar dips are inside the speed circle.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factor:

(a) maximum speed of submarine possible without sound detection

2.2) Deployment:

2.2.1) Qualitative Factor:

(a) submerged

2.3) Tactics:

2 3.1) Qualitative Factor:

(a) avoid detection

2.3.2) Quantitative Factor:

(a) submarine speed

3) Friendly Force Composition: Helicopters

3.1) Qualitative Factor:

(a) homogeneous units

3.2) Quantitative Factors:

(a) total number of helicopters

(b) late time of helicopter

3.3) Sensor: Dipping sonar

3.3.1) Quantitative Factors:

(a) detection radius of sonar

(b) time between sonar dips

(c) total number of dips

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STUDY REVIEW SUMMARY NO. (1)-3

A. STUDY DESCRIPTION

1) Originating Activity: Bell Helicopter Company, Fort Worth, Texas
2) Report Title: Redetecting An Inexactly Located Submarine
3) Author: F.R.S. Dressler
4) Report Number: Operations Analysis Report 299-197-007
5) Date: 15 October 1965
6) Classification: Unclassified
7) Abstract: This document presents the developement, methodology, and results of an analysis on the use of a manned helicopter working with a destroyer escort to redetect an inexactly located submarine. Important system parameters are identified, and their influence on system capability is shown in simple, graphical form.
8) Descriptors: Antisubmarine warfare, destroyer escort, detection probability, helicopter, sonobuoy, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Contact investigation/prosecution
   3.1) Definition: An enemy submarine is first detected by an escort's long-range sonar. A helicopter flies from the escort to redetect, localize, classify and, if necessary, kill the target.
   3.2) Criterion For Success: Detection of submarine
   3.3) MOE Selected: Probability of detection
      3.3.1) Rationale For Selection: The critical phase of the helicopter's mission is redetection. Without an adequate redetection capability, the escort/helicopter system cannot successfully accomplish its mission. Techniques for classification and weapon delivery have been relatively well studied.

3.4) Functional Form Of MOE:

Case 1 – Escort can supply timely data concerning the submarine's position after the helicopter has left

$$MOE = f_1(x_1, \ldots, x_8)$$

where

x_1 = detection area

x_2 = standard deviation of submarine location error, range

x_3 = standard deviation of submarine location error, lateral

x_4 = mean range position of the sonobuoy relative to the submarine

x_5 = mean lateral position of the sonobuoy relative to the submarine

x_6 = standard deviation of sonobuoy position, range

x_7 = standard deviation of sonobuoy position, lateral

x_8 = number of sonobuoys employed

Case 2 – Escort loses contact and is unable to predict the submarine's future mean position

$$MOE = f_2(x_1, x_9)$$

where

x_9 = area of submarine position uncertainty

Case 3 – Escort has sufficient time (contacts) to predict the future course of the submarine before losing contact

$$MOE = f_3(x_1, \ldots, x_{11})$$

where

x_{10} = time from first to last contact

x_{11} = time after last contact

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) In Case 1,

(a.1) The escort can estimate submarine mean position.

(b) In Case 2,

(b.1) Both the submarine and sensor positions are distributed randomly and uniformly in the area of uncertainty.

(b.2) Edge effects on detection are ignored.

(c) Sensor has circular symmetry in its detection capability.

(d) Sensor detection capability can be described by a diffused (exponential) detection function. Rationale for this assumption is that at short ranges detection is virtually certain, whereas at long ranges little or no detection capability exists.

(e) Detection of the submarine is independent of submarine depth.

(f) Submarine/sensor location errors are described by a circular normal probability function.

(g) All sonobuoys are dropped with a common aim point.

(h) Sonobuoy locations are statistically independent.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Deployment:

2.1.1) Qualitative Factor:

(a) submerged

3) Friendly Force Composition: Destroyer Escort and helicopter

3.1) Platform Type: Destroyer Escort

3.1.1) Sensor: Sonar

3.2) Platform Type: Helicopter

3.2.1) Sensor: Sonobuoys

3.2.1.1) Quantitative Factors:

(a) number of sonobuoys employed

(b) detection area

(c) standard deviation of submarine location error, range

(d) standard deviation of submarine location error, lateral

(e) standard deviation of sonobuoy position, range

(f) standard deviation of sonobuoy position, lateral

3.?.1.3) Deployment:

3.2.1.2.1) Qualitative Factor:

(a) on surface

4) Friendly Force - Threat Interaction:

4.1) Quantitative Factors:

(a) time from first to last contact

(b) time after last contact

4.2) Sensor - Platform:

4.2.1) Type: Sonar - Submarine

4.2.1.1) Quantitative Factor:

(a) area of submarine position uncertainty

4.2.2) Type: Sonobuoy - Submarine

4.2.2.1) Quantitative Factors:

(a) mean range position of the sonobuoy relative to the submarine

(b) mean lateral position of the sonobuoy relative to the submarine

STUDY REVIEW SUMMARY NO.(1)-4

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: A Study of Airborne ASW
3) Author: L. G. Hunt
4) Report Number: Naval Warfare Analysis Group Research Contribution No. 121 (AD-508 075)
5) Date: October 1968
6) Classification: Secret
7) Contract: N00014-68-A-0091 (Office of Naval Research)
8) Abstract: The encounter between an ASW aircraft and a submarine can be described as a Markov process. Such a characterization means that overall effectiveness, as measured by kill probability, can be accumulated simultaneously for all sensors over all attack sequences, provided that transition probabilities between sensors can be obtained. These transition probabilities may be calculated or may be derived from fleet exercise data. Such a model is developed and used to study various airborne ASW missions. Barriers, convoy screens, and area search operations with conventional weapons are considered.
9) Descriptors: Aircraft, antisubmarine warfare, barrier, Codar, detection, Jezebel, kill probability, Lofar, MAD, Markov process, radar, sonobuoy, submarine, tracking, visual

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Sonobuoy barrier patrol
   3.1) Definition: An aircraft patrols a specified area listening for submarines on passive sonobuoys. Once detected, a submarine is localized using Codar buoys and final fix is obtained by MAD. The submarine is then attacked by torpedoes.

3.2) Criterion For Success: Suppression of submarine activity

3.3) MOE Selected: Probability of killing an enemy submarine

3.3.1) Rationale For Selection: While kill probability is only one of many measures of effectiveness of airborne ASW, it is often useful as a point of departure, since its calculation requires a thorough examination of the aircraft-submarine duel. Other measures, such as reducing the mobility of the enemy submarine force or denying it the surface, may have greater strategic consequence than an unqualified kill probability, but they are usually peripheral in an analytical sense.

3.4) Functional Form Of MOE:

$$MOE = f(x_1)$$

where

x_1 = a 2-dimensional array whose $(i,j)^{th}$ entry is the one-step transitional probability from state i to state j for $i,j = 1,2,\ldots,10$.

The system states are defined as follows:

State	Description
1	Opportunity
2	Lofar detection
3	Radar detection
4	Visual detection
5	Codar fix
6	Entrapment
7	Tracking
8	Acquisition
9	Kill
10	Submarine escape

3.5) Additional MOE's Identified:

(a) Reduction of the enemy submarine force mobility

(b) Denial of the surface to the enemy submarine force

4) MOE Usage In Study: Model was formulated and used to analyze fleet exercise data from various airborne ASW missions (barriers, convoy screens, and area search operations).

5) Special Study Assumptions:

(a) The encounter between an ASW aircraft and a submarine is described by a stationary Markov process. Such a characterization means that overall effectiveness can be accumulated simultaneously for all sensors over all attack sequences.

(b) The events of the encounter occur in discrete time increments. The length of these time increments is chosen so that each is long enough for any transition. They are not necessarily of equal duration.

(c) The time during which the submarine is submerged does not exist as far as detection is concerned.

(d) Once alerted, the submarine changes tactics to minimize snorkeling or exposure.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

3) Friendly Force Composition: Aircraft (P-3A)

3.1) Sensors: Directional Lofar sonobuoys, Codar sonobuoys, and periscope detection radar

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Aircraft (P-3A) - Submarine

4.1.1) Quantitative Factors:

(a) the one step transition probability from state 1 to state 1

(b) the one step transition probability from state 1 to state 4

(c) the one step transition probability from state 2 to state 1

(d) the one step transition probability from state 2 to state 4

(e) the one step transition probability from state 3 to state 1

(f) the one step transition probability from state 3 to state 4

(g) the one step transition probability from state 3 to state 6

(h) the one step transition probability from state 3 to state 7

(i) the one step transition probability from state 4 to state 1

(j) the one step transition probability from state 4 to state 6

(k) the one step transition probability from state 4 to state 7

(l) the one step transition probability from state 4 to state 8

(m) the one step transition probability from state 5 to state 1

(n) the one step transition probability from state 5 to state 4

(o) the one step transition probability from state 5 to state 6

(p) the one step transition probability from state 6 to state 7

(q) the one step transition probability from state 6 to state 10

(r) the one step transition probability from state 7 to state 6

(s) the one step transition probability from state 7 to state 8

(t) the one step transition probability from state 7 to state 10

(u) the one step transition probability from state 8 to state 7

(v) the one step transition probability from state 8 to state 9

(w) the one step transition probability from state 8 to state 10

(x) the one step transition probability from state 9 to state 10

(y) the one step transition probability from state 10 to state 10

4.2) Sensor - Platform:

4.2.1) Type: Lofar - Submarine

4.2.1.1) Quantitative Factors:

(a) the one step transition probability from state 1 to state 2

(b) the one step transition probability from state 3 to state 2

(c) the one step transition probability from state 4 to state 2

4.2.2) Type: Radar - Submarine

4.2.2.1) Quantitative Factors:

(a) the one step transition probability from state 1 to state 3

(b) the one step transition probability from state 2 to state 3

(c) the one step transition probability from state 5 to state 3

4.2.3) Type: Codar - Submarine

4.2.3.1) Quantitative Factor:

(a) the one step transition probability from state 2 to state 5

STUDY REVIEW SUMMARY NO.(1)-5

A. STUDY DESCRIPTION

1) Originating Activity: Systems Analysis Office, ASW Systems Project Office, White Oak, Maryland
2) Report Title: A Comparative Analysis of VP Lofar Tactics Against a Nuclear Target
3) Author: J. B. Parkerson
4) Report Number: SAO Technical Memorandum 69-12 (AD-510 283)
5) Date: December 1969
6) Classification: Confidential
7) Abstract: The objective of this study was to compare the effectiveness of proposed new VP Lofar tactics against nuclear targets to that of tactics currently in fleet use. Specifically, the study addresses the question: Given a buoy spacing based upon a known environment and a correctly classified target, which buoy pattern is more effective in detecting the target? Study results, based upon a limited number of scenarios and parameter variations, indicate that further study is needed to determine the effect of datum uncertainty upon buoy pattern optimization.
8) Descriptors: Aircraft, antisubmarine warfare, barrier, detection probability, Jezebel, Lofar, sonobuoy, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Barrier placement/patrol
   - 3.1) Definition: An ASW aircraft places Jezebel buoys in either a circular (containing barrier) or straight line pattern to redetect a previously contacted submarine.
   - 3.2) Criterion For Success: Detection of submarine
   - 3.3) MOE Selected: Probability of submarine detection

3.4) Functional Form Of MOE:

$$\text{MOE} = \begin{cases} f_1(x_1, \ldots, x_5) & \text{for a containing barrier} \\ f_2(x_6, x_7) & \text{for a straight line barrier} \end{cases}$$

where

x_1 = mean detection range
$= g_1(x_8, x_9, x_{10})$

x_2 = sonobuoy monitoring time

x_3 = radius of submarine initial position uncertainty

x_4 = time late to datum

x_5 = sonobuoy spacing factor

x_6 = probability of an opportunity
$= g_2(x_{11}, x_{12})$

x_7 = probability of detection given an opportunity
$= g_3(x_1, x_2, x_5)$

x_8 = mean source level

x_9 = mean ambient noise

x_{10} = mean recognition differential

x_{11} = length of sonobuoy pattern (barrier segment) maintained by one aircraft
$= h(x_{13}, x_{14})$

x_{12} = total length of line across which submarines pass with a uniform probability distribution

x_{13} = number of sonobuoys

x_{14} = spacing between sonobuoys

4) MOE Usage In Study: Three combinations of aircraft and Jezebel equipment were used to compute the effectiveness of two sonobuoy placement tactics.

5) Special Study Assumptions:

(a) In the straight line barrier case,

(a.1) The initial submarine position was chosen uniformly along a line parallel to the barrier, but displaced at a constant distance.

(a.2) Target course, held constant throughout each pass through the geometry, was picked at the beginning of each pass from a uniform distribution.

(a.3) The length of the line of initial positions was chosen such that the target would pass through the barrier during each engagement.

(b) In the containing barrier case,

(b.1) The submarine initial position was chosen from a bivariate normal distribution centered at the nominal datum point and truncated at 3 sigma.

(b.2) The target course was uniformly distributed throughout 360° and held constant throughout each pass through the geometry.

(b.3) The first buoy to be laid was placed on a line between the datum and the initial aircraft position.

(c) The aircraft was considered to be within RF range of all the buoys.

(d) The sonobuoys were operable from the time they were laid to the end of the pass through the geometry.

(e) Navigation errors were not allowed.

(f) Sonobuoys were laid precisely according to pattern requirements.

(g) Detection of a nuclear target was accomplished when one line was observed on any buoy.

(h) The equipment was always operable.

(i) There were no surface ship contacts.

(j) Submarine speed and depth are constant.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) mean ambient noise

2) Threat Composition: Submarine

2.1) Qualitative Factor:

(a) quiet transiting and nuclear

2.2) Quantitative Factor:

(a) mean source level

2.3) Deployment:

2.3.1) Qualitative Factor:

(a) submerged

2.3.2) Quantitative Factors:

(a) total length of line across which submarines pass with a uniform probability distribution

(b) radius of submarine initial position uncertainty

3) Friendly Force Composition: Aircraft (P-2 and P-3A/B)

3.1) Quantitative Factor:

(a) time late to datum

3.2) Sensor: Sonobuoys

3.2.1) Quantitative Factors:

(a) sonobuoy monitoring time

(b) sonobuoy spacing factor

(c) mean recognition differential

(d) number of sonobuoys

(e) spacing between sonobuoys

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STUDY REVIEW SUMMARY NO.(1)-6

A. STUDY DESCRIPTION

1) Originating Activity: Raff Analytic Study Associates, Inc., Silver Spring, Maryland
2) Report Title: Mathematical Model for Cost Effectiveness Analysis of Small Acoustic Sensors
3) Authors: S. J. Raff and J. E. Roth
4) Report Number: Final Report 66-21 (AD-379 966)
5) Date: 23 January 1967
6) Classification: Confidential
7) Contract: N00014-66-C-0131 (Office of Naval Research)
8) Abstract: A mathematical model for the cost-effectiveness of small inexpensive sonobuoys has been developed. This model is flexible and its accuracy is adequate for determination of cost-effectiveness of classes of devices which are in the research stage. Factors which enter into the model are detection range, variance in detection range, failure rate, false alarm rate, volume and cost of the sonobuoy devices, ratio of submarine to search plane speeds and a few tactical alternatives. The outputs are overall cost of sonobuoys and implantation, cost of wasted weapons, and probability of sufficiently adequate submarine localization for kill. The model itself is presented in graphical form on the accompanying series of large charts and nomograms. The computational procedure required to exercise the model is the straightforward use of these charts and nomograms in accordance with the explanatory notes which are on the charts themselves. This report contains a description of the charts and an explanation of their derivation.
9) Descriptors: Aircraft, antisubmarine warfare, barrier, contact investigation, cost effectiveness, localization, nomograph, reliability, sonobuoy, submarine, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Contact investigation
   3.1) Definition: An aircraft starts its search from a datum and lays out a pattern of sonobuoys, called a coarse pattern. When one of the sonobuoys in this pattern indicates the presence of a submarine, the aircraft will then lay a pattern of sonobuoys, called a fine pattern, in the immediate vicinity of the indicated contact.
   3.2) Criterion For Success: Localization of submarine
   3.3) MOE Selected: Total cost of the exercise for a specified probability of localization
   3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2, x_3)$$

where

x_1 = cost of flight time
$= g_1(x_4, x_5, x_6)$

x_2 = expect cost of wasted weapons
$= g_2(x_7, x_8)$

x_3 = total cost of sensors
$= g_3(x_9, x_{10})$

x_4 = number of aircraft required
$= h_1(x_{10}, x_{11}, x_{12})$

x_5 = total flight time

x_6 = cost per hour per aircraft

x_7 = probability of wasted weapon

$$= \begin{cases} h_2(x_{13},x_{14},x_{15}) & \text{for single criterion (i.e., one fine pattern signal sufficient to drop torpedo)} \\ h_3(x_{13},\ldots,x_{18}) & \text{for double criterion (i.e., a torpedo is launched only if signal is received from at least two of } x_{16} \text{ adjacent sonobuoys)} \end{cases}$$

x_8 = cost of each weapon

x_9 = cost of each sensor

x_{10} = total number of sensors in exercise
= $h_4(x_{19},x_{20})$

x_{11} = stowage volume of aircraft

x_{12} = volume of each sensor

x_{13} = probability of detection - coarse pattern

x_{14} = probability of false alarm - coarse pattern
= $i_1(x_{17},x_{18},x_{19})$

x_{15} = probability of false alarm - fine pattern
= $i_2(x_{17},x_{18},x_{20})$

x_{16} = number of adjacent sensors considered in establishing the double criterion

x_{17} = total mission time

x_{18} = false alarm rate

x_{19} = total number of sensors in coarse pattern
= $j_1(x_{23},x_{24})$

x_{20} = total number of sensors in fine patterns
= $j_2(x_{13},x_{21},x_{22})$

x_{21} = number of false alarms in coarse pattern
= $k_1(x_{17},x_{18},x_{19})$

x_{22} = number of sensors used in each fine pattern
= $k_2(x_{23},x_{24})$

x_{23} = physical length of barrier

$= \begin{cases} l_1(x_{25},x_{26}) \text{ for coarse pattern} \\ l_2(x_{25},x_{27}) \text{ for fine pattern} \end{cases}$

x_{24} = sensor spacing

x_{25} = length of barrier, normalized to the datum accuracy, corrected for the travel of the target during the time required for laying down the barrier

$= m(x_{28},\ldots,x_{31})$

x_{26} = datum accuracy for coarse pattern

x_{27} = datum accuracy for fine pattern

x_{28} = effective barrier length, normalized to the datum accuracy

$= n(x_{32})$

x_{29} = search aircraft speed

x_{30} = target submarine speed

x_{31} = number of effective lengths

x_{32} = ideal probability of detection

$= \begin{cases} o_1(x_{13},x_{33}) \\ o_2(x_{33},x_{34}) \end{cases}$

x_{33} = degradation factor

$= p(x_{35},x_{36})$

x_{34} = probability of detection - fine pattern

x_{35} = range variability factor

$= q_1(x_{24},x_{37},x_{38})$

x_{36} = reliability factor

$= q_2(x_{17},x_{18})$

x_{37} = sonobuoy detection range standard deviation

x_{38} = mean detection range of sensor

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

   (a) Sensors are uniformly spaced.

   (b) Sensors are assumed to detect omnidirectionally at their mean detection range and to be placed as to completely cover the barrier with no overlap.

   (c) Sensor detection range is lognormally distributed.

   (d) The double criterion probability of two wasted weapons is negligible.

   (e) If two torpedoes are launched in error, the mission itself is considered to be wasted.

   (f) A fine pattern will be laid down to cover each signal from the coarse pattern.

   (g) The cost of flight time for the first plane is not charged to the exercise as it would be needed at the same cost for any search/localization techniques.

   (h) The gap in the line of sensors due to sensor failure is proportional to that part of the total number which have failed. This assumption will generally be satisfactory for coarse localization, but in fine localization one is likely to find appreciable overlap in the ranges of adjacent sensors.

   (i) Sonobuoy range is less than the torpedo acquisition range

   (j) Sensor spacing is greater than the sensor mean detection range.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

   2.1) Quantitative Factor:

      (a) target submarine speed

3) Friendly Force Composition: Aircraft

  3.1) Quantitative Factors:

    (a) stowage volume of aircraft

    (b) search aircraft speed

    (c) cost per hour per aircraft

  3.2) Sensor: Sonobuoys

    3.2.1) Quantitative Factors:

      (a) cost of each sensor

      (b) volume of each sensor

      (c) false alarm rate

      (d) sonobuoy detection range standard deviation

      (e) mean detection range of sensor

    3.2.2) Deployment:

      3.2.2.1) Quantitative Factor:

        (a) sensor spacing

    3.2.3) Tactics:

      3.2.3.1) Quantitative Factor:

        (a) number of adjacent sensors considered in establishing the double criterion

  3.3) Armament: Torpedoes

    3.3.1) Quantitative Factor:

      (a) cost of each weapon

4) Friendly Force - Threat Interaction:

  4.1) Platform - Platform: Aircraft - Submarine

    4.1.1) Quantitative Factors:

      (a) total flight time

      (b) total mission time

      (c) number of effective lengths

  4.2) Sensor - Platform: Sonobuoy - Submarine

    4.2.1) Quantitative Factors:

      (a) probability of detection - coarse pattern

      (b) probability of detection - fine pattern

      (c) datum accuracy for coarse pattern

      (d) datum accuracy for fine pattern

STUDY REVIEW SUMMARY NO. (1)-7

A. STUDY DESCRIPTION

1) Originating Activity: Systems Analysis Office, ASW Systems Project Office, White Oak, Maryland
2) Report Title: Proposed ASW Measure of Effectiveness, MOE-7
3) Author: C. W. Kissinger
4) Report Number: ASW-1412 memo to File
5) Date: 17 January 1969
6) Classification: Secret
7) Abstract: This memo presents a measure of effectiveness for air ASW, and a preliminary sensitivity analysis of this MOE to various input parameters.
8) Descriptors: Aircraft, antisubmarine warfare, barrier, carrier based aircraft, kill probability, sonobuoy, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Sonobuoy barrier patrol
   - 3.1) Definition: Aircraft (VS and VP) attempt to detect, localize, and kill submarines which pass through a sonobuoy field.
   - 3.2) Criterion For Success: Suppression of submarine activity
   - 3.3) MOE Selected: Ratio of the difference of a reference level of damage sustained minus the potential damage sustained to the total damage capability
     - 3.3.1) Rationale For Selection: This MOE is proportional to both the quantity and quality of own forces. It also reflects both the enemy force's ability to penetrate a screen and its damage capability. This MOE has a meaningful range from +1 to -1.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2, x_3)$$

where

x_1 = reference level of damage sustained by own forces

x_2 = potential damage sustained by own forces

$= g_1(x_4, \ldots, x_{10})$

x_3 = total damage capability of enemy submarines

$= g_2(x_4, x_5)$

x_4 = a vector whose $i^{\underline{th}}$ component represents the damage capability for the enemy submarine of type i

x_5 = a vector whose $i^{\underline{th}}$ component represents the number of enemy submarines of type i

x_6 = total desired barrier length

x_7 = a vector whose $i^{\underline{th}}$ component represents the number of aircraft of type i

x_8 = a vector whose $i^{\underline{th}}$ component represents the base loading factor of type i aircraft

x_9 = a vector whose $i^{\underline{th}}$ component represents the length of barrier maintained by one type i aircraft

x_{10} = a matrix whose $(i, j)^{\underline{th}}$ entry represents the kill probability of a type i aircraft against a type j submarine

3.5) Additional MOE Identified:

(a) Ratio of damage averted by own forces to total damage capability of enemy submarines

4) MOE Usage In Study: The MOE was formulated and used to investigate the sensitivity of various input parameters.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component represents the damage capability for the enemy submarine of type i

(b) a vector whose $i^{\underline{th}}$ component represents the number of enemy submarines of type i

3) Friendly Force Composition: Aircraft (VP and VS)

3.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component represents the number of aircraft of type i

(b) a vector whose $i^{\underline{th}}$ component represents the base loading factor of type i aircraft

3.2) Deployment:

3.2.1) Qualitative Factor:

(a) patrol sonobuoy barrier

3.2.2) Quantitative Factors:

(a) total desired barrier length

(b) a vector whose $i^{\underline{th}}$ component represents the length of barrier maintained by one type i aircraft

3.3) Tactics:

3.3.1) Quantitative Factor:

(a) reference level of damage sustained by own forces

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Aircraft - Submarine

4.1.1) Quantitative Factor:

(a) a matrix whose $(i, j)^{\underline{th}}$ entry represents the kill probability of a type i aircraft against a type j submarine

STUDY REVIEW SUMMARY NO.(1)-8

A. STUDY DESCRIPTION

1) Originating Activity: Veda Incorporated, Ann Arbor, Michigan
2) Report Title: Cost Effectiveness Models for Airborne ASW Search and Detection Systems
3) Report Number: V-0513C/2.503 (AD-507 666)
4) Date: 15 September 1967
5) Classification: Confidential
6) Contract: N62269-67-C0405 (U.S. Naval Air Development Center)
7) Abstract: This report contains the general cost models used in determining the cost effectiveness for an airborne ASW mission. The models include both recurring and non-recurring costs for the aircraft, crew and sensor. The effectiveness models postulated in a previous study are exercised for both a radar and an acoustic barrier mission, and the resulting effectiveness numbers are used to find example cost-effectiveness numbers. No attempt has been made to compare the effectiveness or cost-effectiveness of the radar and acoustic barriers since only the search and detection portions of each mission are considered in this study.
8) Descriptors: Aircraft, antisubmarine warfare, barrier, cost, cost effectiveness, detection probability, Lofar, radar, search, sonobuoy, submarine, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Barrier placement/patrol

3.1) Definition: An aircraft patrols a barrier according to a prescribed path using a sensor, either radar or sonar. If the presence of a submarine is detected, the patrolling aircraft performs a contact investigation or localization procedure.

3.2) Criterion For Success: Detection of submarine

3.3) MOE Selected: Cost-effectiveness, defined by the ratio of total mission cost to the probability of detecting a submarine at least once as it passes through the barrier

3.3.1) Rationale For Selection: Cost-effectiveness was chosen because it provides a method of assessing the mission costs for successfully accomplishing the mission at a given effectiveness level.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2, x_3)$$

where

x_1 = total mission time

x_2 = total system costs in dollars per mission hour

$= g_1(x_1, x_4, \ldots, x_{12})$

x_3 = probability of detecting a submarine at least once as it passes through the barrier (called search and detection effectiveness)

$$= \begin{cases} g_2(x_{13}, x_{14}, x_{15}) & \text{for radar sensor} \\ g_3(x_{13}, x_{14}, x_{16}) & \text{for sonar sensor} \end{cases}$$

x_4 = aircraft non-recurring costs in dollars per aircraft flight hour

x_5 = aircraft recurring costs in dollars per aircraft flight hour

x_6 = number of aircraft flight hours performing the mission

x_7 = non-recurring costs per crew flight hour

x_8 = recurring costs per crew flight hour

x_9 = number of crew use hours

x_{10} = non-recurring costs per sensor flight hour

x_{11} = recurring costs per sensor flight hour

x_{12} = number of sensor use hours

x_{13} = maximum number of encounters possible

x_{14} = sensor cumulative probability of detection given an encounter

x_{15} = probability distribution of the number of encounters when intermittent operation and contact investigations are included

$= h_1(x_{13}, x_{17}, x_{18})$

x_{16} = probability distribution of the number of encounters

$= h_2(x_{19}, \ldots, x_{26})$

x_{17} = ratio of the area covered with intermittent operation and contact investigation to area covered without intermittent operation and contact investigation

x_{18} = probability distribution of the number of encounters when intermittent operation and contact investigation are not included

x_{19} = sonobuoy monitor time

x_{20} = revisit time per sonobuoy

$= i(x_{24}, x_{27}, \ldots, x_{32})$

x_{21} = submarine snorkel time

x_{22} = writeout time per buoy

x_{23} = sonobuoy effective range

x_{24} = flight perimeter

x_{25} = number of writers on aircraft

x_{26} = number of Lofar radio channels

x_{27} = aircraft sonobuoy carrying capacity

x_{28} = aircraft speed

x_{29} = on-station time

x_{30} = aircraft endurance

x_{31} = barrier width

$= j_1(x_{23}, x_{27}, x_{28}, x_{29}, x_{33}, x_{34})$

x_{32} = barrier length

$= j_2(x_{23}, x_{27}, x_{28}, x_{29}, x_{33}, x_{34})$

x_{33} = barrier distance from home base

x_{34} = maximum distance submarine can remain submerged without snorkeling

3.5) Additional MOE's Identified:

(a) Joint probability of at least one detection and initial localization to within the performance capability of the final localization technique

(b) Search and detection effectiveness = x_3

4) MOE Usage In Study: Illustrative development of cost-effectiveness models to estimate the value of adding a sensor to planned airborne ASW systems. For the case of the sonobuoy barrier, no claim is made that this model approach adequately represents acoustical detection performance. It is recognized that better models exist which include many effects omitted here and that better model the effects which have been included. The only purpose of the examples presented was to demonstrate the cost-effectiveness models and the effects of variations in detection performance on the models.

5) Special Study Assumptions:

(a) For the radar barrier the patrol aircraft flies a rectangular racetrack type patrol path.

(b) For the sonobuoy barrier a staggered array of sonobuoys is used rather than a rectangular array, because it makes more efficient use of sonobuoy performance capabilities and is less likely to allow the undetected passage of a submarine because of a slight miscalculation in detection range.

(c) The barrier is assumed to exist for the full time required for a submarine to transit the entire barrier width.

(d) For the acoustic barrier two different aircraft patrol patterns are considered, a simple racetrack and an alternating row type of flight path.

(e) For the sonobuoy barrier the flight altitude of the patrol aircraft is governed by the requirement that enough buoys be within the radio horizon to utilize all the writers' time available, yet not so high as to have two buoys that broadcast on the same channel within radio range simultaneously.

(f) Submarine is unaware of the existence of the barrier.

(g) Submarine carries a fully operable and alerted ECM system and will dive within a specified time period after being alerted to electronic searches. Any such dive will not cause any deviation from the submarine's penetration path, but the submarine is assumed to remain submerged for a predetermined length of time.

(h) Sensor probability of detection is constant over the entire area of surveillance.

(i) For the radar barrier the patrol aircraft flies at such an altitude that the radar horizon range is equal to the radar maximum detection range. This results in the best radar performance (least clutter) and the least amount of radar energy transmitted into the region beyond maximum detection range.

(j) The radar is operated intermittently, that is, radiating for a fixed period of time, then placed in standby, then repeating this cycle. (Note: The radar off-period was chosen to minimize the lost area for a specified radar range).

(k) A 360 degree scan radar is assumed.

(l) Acoustic detection performance is based upon consideration of the primary zone acoustic range only, that is, convergence zone effects are omitted. This assumption was necessary to keep the geometry of detections within manageable bounds for hand calculations and was not done without a recognition of its limitations.

(m) The outer limit of primary detection zone was held constant and independent of both time and location. This was done for simplicity of manipulations.

(n) Equal buoy writeout times were assumed.

(o) Longer writeout times were not used around the ends of the barrier, even when the flight path allowed it.

(p) For the sonobuoy barrier case the aircraft returns to the point on the patrol path at which the aircraft originally left to prosecute the potential contacts.

(q) A single aircraft is used to monitor the acoustic barrier.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

   2.1) Qualitative Factor:

      (a) conventionally powered (diesel-electric)

   2.2) Quantitative Factor:

      (a) maximum distance submarine can remain submerged without snorkeling

   2.3) Tactics:

      2.3.1) Qualitative Factor:

         (a) attempt to transit through the barrier using a snorkel-submerged cycle run according to some regular time cycle

2.3.2) Quantitative Factor:

(a) submarine snorkel time

3) Friendly Force Composition: Aircraft (S2, P3, VSX-type)

3.1) Quantitative Factors:

(a) aircraft non-recurring costs in dollars per aircraft flight hour

(b) aircraft recurring costs in dollars per aircraft flight hour

(c) number of aircraft flight hours performing the mission

(d) non-recurring costs per crew flight hour

(e) recurring costs per crew flight hour

(f) non-recurring costs per sensor flight hour

(g) recurring costs per sensor flight hour

(h) number of sensor use hours

(i) number of writers on aircraft

(j) number of Lofar radio channels

(k) aircraft sonobuoy carrying capacity

(l) aircraft speed

(m) on-station time

(n) aircraft endurance

(o) number of crew use hours

3.2) Sensors: Radar and sonobuoys

3.2.1) Type: Radar

3.2.1.1) Quantitative Factor:

(a) ratio of the area covered with intermittent operation and contact investigation to area covered without intermittent operation and contact investigation

3.2.1.2) Tactics:

3.2.1.2.1) Qualitative Factor:

(a) intermittent operation

3.2.2) Type: Sonobuoy

3.2.2.1) Quantitative Factors:

(a) sonobuoy monitor time

(b) writeout time per sonobuoy

(c) sonobuoy effective range

3.2.2.2) Deployment:

3.2.2.2.1) Qualitative Factor:

(a) floating low in the water

3.2.2.2.2) Quantitative Factor:

(a) flight perimeter

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Aircraft - Submarine

4.1.1) Quantitative Factors:

(a) total mission time

(b) maximum number of encounters possible

(c) barrier distance from home base

4.2) Sensor - Platform:

4.2.1) Type: Radar - Submarine

4.2.1.1) Quantitative Factors:

(a) sensor cumulative probability of detection given an encounter

(b) probability distribution of the number of encounters when intermittent operation and contact investigation are not included

4.2.2) Type: Sonobuoy - Submarine

4.2.2.1) Quantitative Factor:

(a) sensor cumulative probability of detection given an encounter

A. STUDY DESCRIPTION

1) Originating Activity: Systems Analysis Office, ASW Systems Project Office, Silver Spring, Maryland
2) Report Title: Air ASW MOE
3) Report Number: ASW 14 memo Ser. 69-003 to CNO Op-953G
4) Date: 29 January 1969
5) Classification: Secret
6) Abstract: A description and preliminary analysis of the air ASW measure of effectiveness proposed by Op-953 is presented.
7) Descriptors: Aircraft, antisubmarine warfare, barrier, carrier, carrier based aircraft, detection, kill probability, localization, sonobuoy, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Sonobuoy barrier patrol
   - 3.1) Definition: Aircraft attempt to detect, localize, and kill submarines which pass through a sonobuoy field.
   - 3.2) Criterion For Success: Detection, localization and kill of submarine
   - 3.3) MOE Selected: Average effective length of air ASW (sonobuoy) barrier that can be maintained per enemy submarine
   - 3.4) Functional Form Of MOE:

$$\text{MOE} = \begin{cases} f_1(x_1, \ldots, x_5), & \text{if numbers of rows and spacing are not selectable as a function of the environmental conditions encountered} \\ f_2(x_1, x_2, x_5, x_{13}, x_{14}, x_{18}), & \text{if number of rows and spacing are selectable as a function of the environmental conditions encountered} \end{cases}$$

where

x_1 = a vector whose i^{th} component represents the number of ASW aircraft of type i

x_2 = a vector whose i^{th} component represents the base loading factor for aircraft type i

$$= \begin{cases} g_1(x_6, x_7, x_8) & \text{for VP aircraft} \\ g_2(x_6, \ldots, x_9) & \text{for VS aircraft} \end{cases}$$

x_3 = a vector whose i^{th} component represents the length of barrier that can be monitored by one aircraft of type i

$= g_3(x_{10}, x_{11}, x_{12})$

x_4 = a vector whose i^{th} component represents the kill probability for an aircraft of type i monitoring a barrier of the length of the i^{th} component of x_3, averaged over all enemy submarine types

$= g_4(x_5, x_{13}, x_{14}, x_{15})$

x_5 = total number of enemy submarines of all types

x_6 = a vector whose i^{th} component represents the monthly utilization of aircraft type i

x_7 = a vector whose i^{th} component represents the average mission duration of aircraft type i

x_8 = a vector whose i^{th} component represents the time on station at nominal distance from base for aircraft type i

$= h(x_7, x_{16}, x_{17})$

x_9 = a vector whose i^{th} component represents the number of CVS necessary to keep one aircraft of type i on station

x_{10} = a vector whose i^{th} component represents the number of sonobuoy channels that can be monitored by aircraft of type i

x_{11} = number of rows of sonobuoys in the barrier

x_{12} = sonobuoy spacing

x_{13} = a vector of matrices whose i^{th} component is a matrix having the property that its $(j, k)^{th}$ entry represents the probability of kill for an aircraft of type i monitoring a barrier of the length of the i^{th} component of x_3 against a submarine of type j in environmental conditions of type k

x_{14} = a vector whose i^{th} component represents the number of enemy submarines of type i

x_{15} = a vector whose i^{th} component represents the probability of occurrence of environment type i

x_{16} = transit distance from base to station

x_{17} = a vector whose i^{th} component represents the transit speed from base to station for the i^{th} aircraft

x_{18} = a matrix whose $(i, j)^{th}$ entry represents the length of barrier that can be monitored by one aircraft of type i in environmental condition j

$= g_5(x_{19}, x_{20}, x_{21})$

x_{19} = a matrix whose $(i, j)^{th}$ entry represents the number of sonobuoy channels that can be monitored by aircraft of type i in environmental condition j

x_{20} = a vector whose i^{th} component represents the number of rows of sonobuoys in the barrier in environmental condition i

x_{21} = a vector whose i^{th} component represents the spacing of the sonobuoys in environmental condition i

4) MOE Usage in Study: Formulation only

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) a vector whose i^{th} component represents the probability of occurrence of environment type i

2) Threat Composition: Submarines

2.1) Quantitative Factors:

(a) total number of enemy submarines of all types

(b) a vector whose i^{th} component represents the number of enemy submarines of type i

3) Friendly Force Composition: Aircraft (VP and VS) and carrier (CVS)

3.1) Platform Type: Aircraft (VP and VS)

3.1.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component represents the number of ASW aircraft of type i

(b) a vector whose $i^{\underline{th}}$ component represents the transit speed from base to station for the $i^{\underline{th}}$ aircraft

3.1.2) Sensors: Sonobuoys

3.1.2.1) Quantitative Factor:

(a) a vector whose $i^{\underline{th}}$ component represents the number of sonobuoy channels that can be monitored by aircraft of type i

3.1.2.2) Deployment:

3.1.2.2.1) Quantitative Factors:

(a) number of rows of sonobuoys in the barrier

(b) sonobuoy spacing

3.1.3) Deployment:

3.1.3.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component represents the monthly utilization of aircraft type i

(b) a vector whose $i^{\underline{th}}$ component represents the average mission duration of aircraft type i

3.2) Platform Type: Carrier (CVS)

3.2.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component represents the number of CVS necessary to keep one aircraft of type i on station

(b) transit distance from base to station

4) Friendly Force - Physical Environment Interaction:

4.1) Sensor - Environment: Sonobuoy

4.1.1) Quantitative Factors:

(a) a matrix whose $(i, j)^{\underline{th}}$ entry represents the number of sonobuoy channels that can be monitored by aircraft of type i in environmental condition j

(b) a vector whose $i^{\underline{th}}$ component represents the number of rows of sonobuoys in the barrier in environmental condition i

(c) a vector whose $i^{\underline{th}}$ component represents the spacing of the sonobuoys in environmental condition i

5) Friendly Force - Threat - Physical Environment Interaction:

5.1) Platform - Platform - Environment: Aircraft - Submarine

5.1.1) Quantitative Factor:

(a) a vector of matrices whose $i^{\underline{th}}$ component is a matrix having the property that its $(j, k)^{\underline{th}}$ entry represents the probability of kill for an aircraft of type i monitoring a barrier of the length of the $i^{\underline{th}}$ component of x_3 against a submarine of type j in environmental conditions of type k

STUDY REVIEW SUMMARY NO. (1)-10

A. STUDY DESCRIPTION

1) Originating Activity: Systems Analysis Office, ASW Systems Project Office, White Oak, Maryland
2) Report Title: Some Results of a Preliminary Study of Measures of Effectiveness for Air ASW
3) Report Number: ASW-1412 memo Ser. 68-0064 to Files
4) Date: 11 December 1968
5) Classification: Secret
6) Abstract: This memo provides representative measures of effectiveness for Air ASW and a preliminary sensitivity analysis of these MOE's to projected changes in our own and enemy force structure.
7) Descriptors: Aircraft, antisubmarine warfare, barrier, carrier based aircraft, detection, kill probability, localization, sonobuoy, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Sonobuoy barrier patrol
   - 3.1) Definition: Aircraft (VS and VP) attempt to detect, localize, and kill submarines which pass through a sonobuoy field.
   - 3.2) Criterion For Success: Suppression of submarine activity
   - 3.3) MOE's Selected:

     $(MOE)_1$ = Ratio of average effective barrier length to total number of enemy submarines

     $(MOE)_2$ = Ratio of average effective barrier length to total damage capability of enemy submarines

     $(MOE)_3$ = Ratio of fraction of submarines killed to damage sustained by own forces

     $(MOE)_4$ = Ratio of damage averted by own force to total damage capability of enemy submarines

$(MOE)_5$ = Reciprocal of damage sustained by own forces

$(MOE)_6$ = Fraction of submarines killed

$(MOE)_7$ = Ratio of the product of damage averted by own forces and the total damage sustained by own forces to total damage capability of enemy submarines

3.3.1) Rationale For Selection: $(MOE)_1$ reflects the quality of own forces in performing a barrier operation. $(MOE)_2$ reflects the ability of enemy submarines to inflict damage on own forces. $(MOE)_3$ reflects the fact that quantities of own forces can be insufficient to require that all enemy submarines must pass through a barrier. $(MOE)_4$ reflects own force capability of averting damage from enemy submarines, but does not reflect damage sustained. $(MOE)_5$ reflects enemy submarines ability to inflict damage on own forces. It is not as sensitive to enemy losses as is $(MOE)_3$, which is directly proportional to the fraction of submarines killed. $(MOE)_6$ reflects the ability of own forces to kill enemy submarines. $(MOE)_7$ reflects the damage sustained due to enemy submarines not killed.

3.4) Functional Form Of MOE's:

$(MOE)_1 = f_1(x_1, x_2)$

$(MOE)_2 = f_2(x_1, x_3)$

$(MOE)_3 = f_3(x_4, x_5)$

$(MOE)_4 = f_4(x_3, x_6)$

$(MOE)_5 = f_5(x_5)$

$(MOE)_6 = f_6(x_4)$

$(MOE)_7 = f_7(x_3, x_5, x_6)$

where

x_1 = average effective barrier length

$= g_1(x_2, x_7, x_8)$

x_2 = total number of enemy submarines

= $g_2(x_8)$

x_3 = total damage capability [illegible] enemy submarines

= $g_3(x_8, x_9)$

x_4 = fraction [illegible]arines killed

= g_4[illegible]$_7, x_8, x_{10})$

x_5 = [illegible] damage sustained by own forces

= [illegible]$_7, \ldots, x_{10})$

x_6 [illegible]mage averted by own force

= $g_6(x_7, \ldots x_{10})$

[illegible] a vector whose i^{th} component represents the effective length of barrier for an enemy submarine of type i

= $h(x_{11}, \ldots, x_{14})$

[illegible] = a vector whose i^{th} component represents the number of enemy submarines of type i

x_9 = a vector whose i^{th} component represents the damage factor for the enemy submarine of type i, i.e., the relative penalty of failing to stop a type i submarine

x_{10} = total desired barrier length

x_{11} = a vector whose i^{th} component represents the number of aircraft of type i

x_{12} = a vector whose i^{th} component represents the base loading factor of type i aircraft

= $i(x_{15}, \ldots, x_{18})$

x_{13} = a vector whose i^{th} component represents the length of barrier maintained by one type i aircraft

x_{14} = a matrix whose $(i, j)^{th}$ entry represents the kill probability of a type i aircraft against type j submarine

x_{15} = a vector whose i^{th} component represents the utilization (hours/month) of type i aircraft

x_{16} = a vector whose $i^{\underline{th}}$ component represents the average mission duration for type i aircraft

x_{17} = transit speed of aircraft from base to station

x_{18} = transit distance from base to station

4) MOE Usage In Study: MOE's were formulated and used to study the sensitivity of the various input parameters.

5) Special Study Assumptions:

(a) Only one submarine pass through the field is explicitly considered, i.e., effects of survival for multiple patrols is not explicitly included.

(b) The role of other ASW forces is not explicitly considered.

(c) Cost of either side's forces is not considered.

(d) Those MOE's that are a function of the total desired barrier length, x_{10}, implicitly assume that the average effective length of barrier, x_1, is a physically realizable barrier length of kill probability equal to one. This assumption implies that the $(i, j)^{\underline{th}}$ entry in x_{14} is inversely proportional to the $i^{\underline{th}}$ component of x_{13}, which changes with buoy spacing and/or number of rows of buoys. This is approximately true only if the buoys detect independently, and if for each i and j the kill probability of a type i aircraft against a type j submarine is small compared to unity. The effect is not considered serious for a preliminary sensitivity analysis of MOE's, but should be corrected if it is desired to obtain accurate absolute values for the MOE's.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component represents the number of enemy submarines of type i

(b) a vector whose i^{th} component represents the damage factor for the enemy submarine of type i, i.e., the relative penalty of failing to stop a type i submarine

3) Friendly Force Composition: Aircraft (VP and VS)

3.1) Quantitative Factors:

(a) a vector whose i^{th} component represents the number of aircraft of type i

(b) transit speed of aircraft from base to station

3.2) Deployment:

3.2.1) Qualitative Factor:

(a) patrol of sonobuoy barrier

3.2.2) Quantitative Factors:

(a) [illegible] desired barrier length

(b) a vector whose i^{th} component represents the utilization (hours/month) of type i aircraft

(c) a vector whose i^{th} component represents the average mission duration for type i aircraft

(d) transit distance from base to station

(e) a vector whose i^{th} component represents the length of barrier maintained by one type i aircraft

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Aircraft - Submarine

4.1.1) Quantitative Factor:

(a) a matrix whose $(i, j)^{th}$ entry represents the kill probability of a type i aircraft against type j submarine

STUDY REVIEW SUMMARY NO. (1)-11

A. STUDY DESCRIPTION

1) Originating Activity: Cornell Aeronautical Laboratory, Inc., Buffalo, New York
2) Report Title: Cost-Effectiveness Comparison Between ASW Air/Sea Craft and Conventional ASW Aircraft
3) Author: B. B. Levitt
4) Report Number: GM-1968-G-2 (AD-507 541)
5) Date: 30 January 1967
6) Classification: Secret
7) Contract: Nonr 4545(00) (Office of Naval Research)
8) Abstract: The objective of this study is to conduct a cost-effectiveness comparison between the most promising air/sea craft systems and conventional ASW aircraft in the performance of the ASW barrier mission. An analysis is made of the performance characteristics of the P-3A (ANEW) and VSX in conducting the mission. Performance characteristics of the primary ASW search sensors are developed for the P-3A/VSX, using retrievable sonar buoys. The results are used to determine optimum sensor emplacement patterns, operational modes for the various vehicles and force level requirements for the mission. Cost factors are derived for all aircraft and air/sea craft and are applied to the force level requirements to permit a comparison of aircraft cost-effectiveness as a function of contact investigation capability.
9) Descriptors: Aircraft, antisubmarine warfare, barrier, carrier, carrier based aircraft, Cass, contact investigation, cost, cost-effectiveness, detection, localization, Lofar, sonobuoy, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Barrier placement/patrol

3.1) Definition: A submarine attempts to penetrate a sonar buoy barrier, established by aircraft deployed sonar buoys.

3.2) Criterion For Success: Achieve maximum contact investigation capability in contacts per day at least cost

3.3) MOE's Selected:

$(MOE)_1$ = Total lifetime cost to achieve maximum contact investigation capability

$(MOE)_2$ = Total mission cost to achieve maximum contact investigation capability

3.4) Functional Form Of MOE:

$$(MOE)_1 = f_1(x_1, \ldots, x_4)$$

$$(MOE)_2 = f_2(x_5, \ldots, x_8)$$

where

x_1 = total number of aircraft required for a specified mission length
$= g_1(x_9, x_{10})$

x_2 = initial aircraft investment cost
$= g_2(x_{11}, \ldots, x_{16})$

x_3 = annual aircraft operating cost
$= g_3(x_{17}, \ldots, x_{21})$

x_4 = number of years of operation

x_5 = single sortie cost
$= g_4(x_3, x_{22})$

x_6 = number of sorties that must be flown during a specified mission length
$= g_5(x_{10}, x_{23}, x_{24}, x_{25})$

x_7 = carrier basing cost
$= g_6(x_1, x_{26}, x_{27}, x_{28})$

x_8 = cost of buoy loss

$$= \begin{cases} g_7(x_{29}, \ldots, x_{33}) & \text{for retrievable buoys} \\ g_8(x_6, x_9, x_{34}, x_{35}, x_{36}) & \text{for non-retrievable buoys} \end{cases}$$

x_9 = number of echelons required

$= h_1(x_{24}, x_{25}, x_{37}, x_{38}, x_{39})$

x_{10} = number of aircraft per echelon

x_{11} = airframe and engine investment cost

x_{12} = fixed payload (permanently installed avionics and sensor equipment) investment cost

x_{13} = airframe spares investment cost

x_{14} = engine spares investment cost

x_{15} = avionics spares investment cost

x_{16} = special support equipment investment cost

x_{17} = system utilization cost

x_{18} = fuel costs

x_{19} = overhaul and maintenance costs

x_{20} = personnel costs

x_{21} = base costs

x_{22} = number of sorties flown per year

x_{23} = period of time the barrier must be maintained

x_{24} = total on-station time available

x_{25} = time required to emplace and retrieve barrier sonobuoys

x_{26} = fraction of the carrier deck space occupied by aircraft

x_{27} = annual operating cost of the carrier

x_{28} = fraction of the year the carrier is used for the mission

x_{29} = retrievable buoy loss rate

x_{30} = retrievable buoy reliability

x_{31} = number of retrievable buoys emplaced

x_{32} = weight of retrievable buoys in pounds

x_{33} = cost per pound of retrievable buoys

x_{34} = non-retrievable buoy reliability

x_{35} = cost of non-retrievable buoy

x_{36} = number of sonobuoys in the water at all times

$= h_2(x_{40}, \ldots, x_{44})$

x_{37} = base turnaround time

x_{38} = transit distance between base and barrier station

x_{39} = transit cruise speed
x_{40} = sonobuoy spacing
x_{41} = sea state
x_{42} = barrier width
x_{43} = barrier length
x_{44} = minimum effective width

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Enemy submarine speed is held constant.

(b) Sufficient range information is available, and bearing accuracy is such that a barrier may be constructed of a single row of buoys.

(c) Each aircraft monitors its buoys from the center of its assigned section.

(d) Sonobuoy spacing is based on a specified probability of detection and a rough localization criterion.

(e) A minimum of two aircraft or air/sea craft must be maintained on station at all times.

(f) Aircraft and air/sea craft operate from a phantom shore base or an aircraft carrier that remains a constant distance from all points on the barrier equivalent to the mission radius of the respective aircraft.

(g) There is no range limitation on communications capability between aircraft and sensors.

(h) Sonar buoy emplacement and retrieval times are constant.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) sea state

2) Threat Composition: Submarine

2.1) Qualitative Factor:

(a) nuclear

2.2) Deployment.

2.2.1) Qualitative Factor:

(a) submerged

2.3) Tactics:

2.3.1) Qualitative Factor:

(a) non-cavitating

3) Friendly Force Composition: Aircraft (P-3A/VSX and air/sea craft) and carrier

3.1) Platform Type: Aircraft

3.1.1) Quantitative Factors:

(a) airframe and engine investment cost
(b) fixed payload (permanently installed avionics and sensor equipment) investment cost
(c) airframe spares investment cost
(d) engine spares investment cost
(e) avionics spares investment cost
(f) special support equipment investment cost
(g) system utilization cost
(h) fuel costs
(i) overhaul and maintenance costs
(j) personnel costs
(k) base costs
(l) time required to emplace and retrieve barrier sonobuoys
(m) base turnaround time
(n) transit cruise speed

3.1.2) Sensors: Retrievable and non-retrievable sonar buoys

3.1.2.1) Type: Retrievable sonar buoys

3.1.2.1.1) Quantitative Factors:

(a) retrievable buoy loss rate
(b) retrievable buoy reliability
(c) number of retrievable buoys emplaced
(d) weight of retrievable buoys in pounds
(e) cost per pound of retrievable buoys

3.1.2.2) Type: Non-retrievable sonar buoys

3.1.2.2.1) Quantitative Factors:

(a) non-retrievable buoy reliability

(b) cost of non-retrievable buoy

(c) sonobuoy spacing

3.1.3) Deployment:

3.1.3.1) Quantitative Factors:

(a) number of years of operation

(b) number of aircraft per echelon

(c) number of sorties flown per year

(d) total on-station time available

(e) transit distance between base and barrier station

3.1.4) Tactics:

3.1.4.1) Quantitative Factor:

(a) period of time the barrier must be maintained

3.2) Platform Type: Carrier

3.2.1) Quantitative Factors:

(a) fraction of the carrier deck space occupied by aircraft

(b) annual operating cost of the carrier

3.2.2) Deployment:

3.2.2.1) Quantitative Factor:

(a) fraction of the year the carrier is used for the mission

4) Friendly Force - Threat Interaction:

4.1) Sensor - Platform: Sonar buoys - Submarine

4.1.1) Quantitative Factors:

(a) barrier width

(b) barrier length

(c) minimum effective width

STUDY REVIEW SUMMARY NO. (1)-12

A. STUDY DESCRIPTION

1) Originating Activity: Cornell Aeronautical Laboratory, Inc. Buffalo, New York
2) Report Title: Improved Air ASW Effectiveness by the Employment of Acoustic Countermeasures in Task Force Operations
3) Authors: B. B. Levitt and M. W. Zumwalt
4) Report Number: CAL Report No. GM-2268-G-4 (AD-506 225)
5) Date: 30 September 1969
6) Classification: Secret (NOFORN)
7) Contract: N00014-66-C-0232 (Office of Naval Research)
8) Abstract: This research effort is a further extension of previous studies conducted in the area of acoustic countermeasures (ACM) and tactical deception techniques. The present study effort is intended to examine the manner in which ACM devices improve the effectiveness of ASW support forces assigned to provide protection to a carrier task force.
9) Descriptors: Acoustic decoy, aircraft, antisubmarine warfare, barrier, carrier, contact investigation, countermeasure, detection probability, localization, normal density function, sonobuoy, SOSUS, submarine, surveillance, task force

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Contact investigation
   3.1) Definition: A carrier task force (CTF) transits through an area in which it is likely that enemy submarines may be encountered. ASW aircraft are being used to provide support against any contacts obtained in the vicinity of the CTF or along its projected track. Initial contact is made by a

remote surveillance system and then aircraft respond by planting a pattern of sonobuoys in the contact area in order to detect and localize the position of the submarine.

3.2) Criterion For Success: Detection of submarine

3.3) MOE Selected: Percent of a specified area in which the probability of submarine detection by the ASW support forces is equal to or greater than a stated level

3.4) Functional Form Of MOE:

$MOE = f(x_1, x_2)$

where

x_1 = SSN action range
$= g_1(x_3)$

x_2 = cumulative probability of SSN detection by the ASW support aircraft
$= g_2(x_4, x_5)$

x_3 = initial detection range of the carrier task force by the SSN

x_4 = time available for the ASW support aircraft to detect the SSN target
$= h_1(x_6, x_7, x_8)$

x_5 = cumulative probability of detection vector in which the $i^{\underline{th}}$ component represents the cumulative probability of detection of the SSN by the $i^{\underline{th}}$ look epoch
$= h_2(x_9, \ldots, x_{14})$

x_6 = effective ASW time
$= i_1(x_{15}, x_{16})$

x_7 = time required for target localization

x_8 = time required for target attack

x_9 = total number of buoys in the pattern

x_{10} = sonobuoy monitor matrix
$= i_2(x_9, x_{17})$

x_{11} = number of grid points in the SSN uncertainty area

= $i_3(x_{18})$

x_{12} = a vector whose $i^{\underline{th}}$ coordinate represents the matrix of possible submarine positions at the time of the $i^{\underline{th}}$ look epoch

= $i_4(x_{11}, x_{19}, \ldots, x_{22})$

x_{13} = matrix of sonobuoy position coordinates with respect to the uncertainty area

x_{14} = sonobuoy detection probability function

= $i_5(x_{23}, x_{24})$

x_{15} = total time available to the carrier task force and its ASW support aircraft in which ASW action must be prosecuted

= $h_1(x_1, x_{25}, x_{26})$

x_{16} = ASW support aircraft time to respond

= $h_2(x_{27}, x_{28}, x_{29})$

x_{17} = number of look time entries in the buoy look schedule

x_{18} = size of uncertainty area

x_{19} = orientation, relative to the SSN course, of the major axis of the target uncertainty area

x_{20} = CTF track

x_{21} = SSN intercept start bearing

x_{22} = SSN speed

= $j_1(x_1, x_{25}, x_{32})$

x_{23} = signal excess

= $j_2(x_{30}, x_{31})$

x_{24} = standard deviation of signal excess

x_{25} = distance matrix of submarine position relative to CTF track as a function of time

= $i_6(x_{12}, x_{20})$

x_{26} = speed of relative motion, or the rate at which the range between the carrier task force and the SSN is being closed

= $i_7(x_1, x_{25}, x_{32})$

x_{27} = expected surveillance system delay

x_{28} = time allotted for aircraft ground operations

x_{29} = aircraft time to fly out to the SSN advanced position

x_{30} = sonobuoy figure of merit

$= k(x_{33}, x_{34}, x_{35})$

x_{31} = propagation loss

x_{32} = CTF speed

x_{33} = target radiated noise level

$= l_1(x_{22})$

x_{34} = background noise level

x_{35} = recognition differential

$= l_2(x_{36}, x_{37}, x_{38})$

x_{36} = sonobuoy bandwidth

x_{37} = target signal frequency

x_{38} = signal integration time

4) MOE Usage In Study: A model was formulated and developed to determine the effectiveness of the ASW support aircraft and the change in this effectiveness due to the employment of ACM devices.

5) Special Study Assumptions:

(a) A single CVA simulator, located at a fixed distance and bearing from the CTF, is used as the basic ACM device.

(b) The ASW support aircraft are utilized in an on-call mode of operation and are dispatched to the scene when a threatening contact is obtained.

(c) Initial detection of the threatening submarine results in an uncertainty area of specified size that is assumed to be elliptical in shape. Target location within the uncertainty area is described by a bivariate normal distribution. The semi-major and semi-minor axes of the ellipse are equal in length to the standard deviation of target location in that direction.

(d) The probability distribution of the target in the uncertainty area is discretely approximated by a rectangular grid and an array of grid points representing the center of a grid cell and having associated with it the probability that the target is in the grid cell. Because of this discrete approximation, the location probabilities are normalized so that they collectively sum to 1.0.

(e) The target uncertainty area does not grow or become distorted during the time interval between initial SOSUS detection and commencement of rough localization operations by the ASW aircraft. This uncertainty area does, however, translate in the direction of the SSN intercept course and at the SSN intercept speed (or speed regime in an ACM environment).

(f) The SSN selects the minimum speed required to intercept the CTF. The rationale for selection of the minimum speed intercept solution is as follows: (1) the minimum speed intercept solution represents a unique solution to the intercept problem, one of but two such unique solutions - the other being the maximum speed solution, and (2) the minimum speed solution results in the SSN presenting the most difficult target to the passive detection devices due to the low level of radiated noise associated with the minimum speed.

(g) The surveillance system is capable of detecting, locating and tracking the SSN at its minimum speed and radiated noise level.

(h) In an ACM-free environment, the SSN intercept speed remains constant. In an ACM environment, however, the speed of the submarine changes during the course of the intercept as it attempts to distinguish between the CTF and the simulator.

(i) ASW support aircraft use directional sonobuoys for initial detection and rough localization operations.

(j) Each sonobuoy becomes operationally active within a fixed period after it is planted, i.e., is considered to be receiving and transmitting data after this initial activation interval.

(k) Propagation loss is considered to be range-independent and to consist of spherical spreading.

(l) The recognition differential is defined to be the signal to noise level required for 50% probability of detection.

(m) SSN probability of detection by a sonobuoy at a given range is based upon use of the normal probability density function.

(n) The number of buoys in the total pattern exceeds the monitoring capability of the ASW support aircraft.

(o) The buoy monitor/look schedule is such that no section of the buoy field is ignored for a period sufficient for the target SSN to slip through the buoy field.

(p) Sonobuoy reliability is 100% during the emplacement of the pattern.

(q) When sonobuoy life expires, they are replaced without loss of coverage.

(r) The ACM device does not affect the SSN approach on the CTF until it has closed to at least a specified distance. Consequently, the SSN selects the minimum intercept speed until it reaches this distance and thereafter follows the speed regime established for an ACM environment intercept.

(s) Speed/bearing histories of the SSN relative to the CTF track are assumed to be symmetrical on either side.

(t) The CTF commander receives early warning of the impending SSN attack.

(u) SSN action range is defined as the range at which detection of the CTF has occurred (with at least a specified probability), the SSN has properly classified its target, correctly solved the intercept problem and initiated action to close the CTF to within weapon range.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Qualitative Factor:

(a) geographic area

1.2) Quantitative Factor:

(a) background noise level

2) Threat Composition: Submarine (SSN)

2.1) Qualitative Factor:

(a) nuclear

2.2) Quantitative Factor:

(a) target signal frequency

2.3) Tactics:

2.3.1) Quantitative Factor:

(a) SSN intercept start bearing

3) Friendly Force Composition: SOSUS, aircraft and carrier task force

3.1) Platform Type: SOSUS

3.1.1) Quantitative Factor:

(a) expected surveillance system delay

3.2) Platform Type: Aircraft

3.2.1) Quantitative Factor:

(a) time allotted for aircraft ground operations

3.2.2) Sensor: Sonobuoys

3.2.2.1) Quantitative Factors:

(a) sonobuoy bandwidth

(b) signal integration time

3.2.2.2) Deployment:

3.2.2.2.1) Quantitative Factor:

(a) number of look time entries in the buoy look schedule

3.2.2.3) Tactics:

3.2.2.3.1) Quantitative Factor:

(a) number of look time entries in the buoy look schedule

3.2.3) Deployment:

3.2.3.1) Quantitative Factor:

(a) aircraft time to fly out to the SSN advanced position

3.3) Platform Type: Carrier task force

3.3.1) Quantitative Factor:

(a) CTF speed

3.3.2) Deployment:

3.3.2.1) Quantitative Factor:

(a) CTF track

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform:

4.1.1) Type: Carrier task force - Submarine

4.1.1.1) Quantitative Factor:

(a) initial detection range of the carrier task force by the SSN

4.1.2) Type: Aircraft - Submarine

4.1.2.1) Quantitative Factors:

(a) time required for target localization

(b) time required for target attack

4.1.3) Type: SOSUS - Submarine

4.1.3.1) Quantitative Factors:

(a) size of uncertainty area

(b) orientation, relative to the SSN course, or the major axis of the target uncertainty area

4.2) Sensor - Platform: Sonobuoy - Submarine

4.2.1) Quantitative Factors:

(a) matrix of sonobuoy position coordinates with respect to the uncertainty area

(b) standard deviation of signal excess

5) Friendly Force - Environment Interaction:

5.1) Sensor - Environment:

5.1.1) Quantitative Factor:

(a) propagation loss

STUDY REVIEW SUMMARY NO. (1)-13

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Model and Computer Program for Calculating the Kill Probabilities for Certain ASW Tactics
3) Authors: S.H. Howe, P.W. McCree, Jr. and R.R. Adams
4) Report Number: OEG Research Contribution No. 47 (AD-424 761)
5) Date: 25 October 1963
6) Classification: Unclassified
7) Abstract: This Research Contribution describes a model and computer program designed to compute kill probabilities for certain firing tactics waged against an evading submarine. The model was developed assuming that the attack is imminent and that the weapon will be directed at the point where the submarine was last contacted. The model's design is centered around a determination of the set of points which represent the locus of the evading submarine. The locus is determined by variation of a simple evasion tactic. The aimpoint is considered to be circularly, normally distributed about the true position of the submarine at the time of last contact.
8) Descriptors: Aircraft, antisubmarine warfare, bomb, contact prosecution, helicopter, kill probability, normal density function, sonar, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Contact prosecution
   3.1) Definition: A helicopter, assisting a weapon delivery aircraft in an attack on an evading submarine, has a firmly established sonar contact with the submarine. The weapon delivery aircraft must await communications and direction from the assisting helicopter and then delay for at least some minimum time before

maneuvering to the predicted position and dropping the weapon.

3.2) Criterion For Success: Destruction of submarine

3.3) MOE Selected: Average kill probability

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_{10})$$

where

x_1 = maximum turn angle, i.e., the maximum angle through which the submarine can execute an evasive turn

x_2 = vector of possible submarine turn angles

x_3 = blind time, defined to be the time between the bomb burst and the time when sonar contact is severed

x_4 = vector of possible evasive turn times

x_5 = radius of the submarine's evasive turn

x_6 = submarine speed

x_7 = damage radius, i.e., the distance from the point of the bomb burst within which a 100 percent probability of kill will be satisfied

x_8 = sonar azimuth error

x_9 = distance at which the target was last sighted

x_{10} = sonar range error

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) If it is found that the assisting helicopter's distance from the submarine is within a minimum safe distance, certain tasks must be performed before the assisting helicopter can retire, under emergency power, to the minimum safe distance.

(b) If it is found that the assisting helicopter is outside the minimum safe distance from the submarine's position, the assisting helicopter tracks the submarine and continuously vectors the delivery aircraft until the weapon is dropped.

(c) During a multiple of a fraction of the blind time, the submarine cruises at a specified speed. After this period of time, the submarine executes a turn through a multiple of a fraction of the turn angle at a specified average speed. If there is any remaining blind time, the submarine cruises along its new course at its original speed until all the blind time is consumed.

(d) The weapon aimpoint is considered to be circularly, normally distributed about the true position of the submarine at the time of last contact.

(e) The turn time is uniformly distributed throughout the blind time and the turn angle is also uniformly distributed through all possible values of that angle.

(f) The blind time is expected to be of short duration. The blind time, thus, does limit the design of a more sophisticated evasion which one could expect a submarine to accomplish under imminent attack.

(g) The instantaneous speed of a submarine during a turn is a continuous, linear function of the turn angle, and the speed decreases to 55% of its initial value for a 90 degree turn angle.

(h) The point at which the target was last observed is assumed to be circularly, normally distributed about the true position of the submarine at the beginning of the blind time.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) maximum turn angle, i.e., the maximum angle through which the submarine can execute an evasive turn

(b) submarine speed

2.2) Tactics:

2.2.1) Quantitative Factors:

(a) vector of possible submarine turn angles

(b) vector of possible evasive turn times

(c) radius of the submarine's evasive turn

3) Friendly Force Composition: Helicopter and aircraft

3.1) Platform Type: Helicopter

3.1.1) Sensor: Sonar

(3.1.1.1) Quantitative Factors:

(a) sonar azimuth error

(b) distance at which the target was last sighted

(c) sonar range error

3.2) Platform Type: Aircraft

3.2.1) Armament: Bomb

3.2.1.1) Quantitative Factor:

(a) damage radius, i.e., the distance from the point of the bomb burst within which a 100 percent probability of kill will be satisfied

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Aircraft - Submarine

4.1.1) Quantitative Factor:

(a) blind time, defined to be the time between the bomb burst and the time when sonar contact is severed

STUDY REVIEW SUMMARY NO. (1)-14

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Ordnance Laboratory, White Oak, Maryland

2) Report Title: The Effect of Adding Passive Sensors to the SH-3D Helicopter for Barrier Screening and Datum Investigation Missions

3) Author: R.E. Muir

4) Report Number: NOLTR 67-4 (AD-386 190)

5) Date: 27 January 1967

6) Classification: Secret

7) Project Number: RUDC 3B-000/212-1/F001-10-02

8) Abstract: This report provides a preliminary estimate of the operational impact caused by the addition of passive sensors to the SH-3D armament. Two operational situations, barrier screening and datum investigation, were studied under the assumption of both quiet and noisy targets.

9) Descriptors: Antisubmarine warfare, barrier, contact investigation, detection probability, Difar, dipping sonar, helicopter, Jezebel, Julie, localization probability, Lcfar, normal density function, sonar, sonobuoy, submarine, task force

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System

2) Function: Airborne ASW

3) Missions:

3.1) Mission Type: Barrier placement/patrol

3.1.1) Definition: A helicopter, using a passive sonar system, maintains a barrier a specified distance from a task force or convoy. Upon receipt of a passive contact, the helicopter attempts to convert to an active sonar contact.

3.1.2) Criterion For Success: Detection of submarine

3.1.3) MOE's Selected:

$(MOE)_1$ = Maximum width of barrier that can be maintained and still ensure a 50% probability of initial detection

$(MOE)_2$ = Number of detection opportunities converted to active contacts

3.1.3.1) Rationale For Selection: $(MOE)_1$ provides a relative measure of search phase effectiveness which is degraded to account for helicopter limitations in endurance and availability. $(MOE)_2$ provides an overall measure of the barrier screening mission effectiveness. For this mission there exist operational situations where detection without localization would be worthwhile, hence two measures are introduced.

3.1.4) Functional Form Of MOE's:

$$(MOE)_1 = f_1(x_1, x_2)$$

$$(MOE)_2 = f_2(x_1, \ldots, x_4)$$

where

x_1 = width of barrier that can be patrolled to ensure a 50% probability of initial detection

$$= \begin{cases} g_1(x_5, x_6) & \text{for passive omnidirectional sonobuoy system} \\ g_2(x_{13}, x_{14}) & \text{for passive directional sonobuoy system} \\ g_3(x_8, x_{12}, x_{19}) & \text{for passive towed line array system deployed in a back-and-forth patrol, } (x_8 > x_{12}) \\ g_4(x_{20}, x_{22}) & \text{for passive towed line array system in a dipping mode deployed in a crossover pattern and towing during integration time } (x_8 < x_{23}) \\ g_5(x_{27}, x_{28}) & \text{for passive vertical line array system deployed in a dipping crossover pattern} \end{cases}$$

$g_6(x_{33}, x_{34})$ for passive/active dipping sonar in passive mode deployed in a dipping crossover pattern

x_2 = percentage of time a patrol could be maintained from the operating base of the helicopter

= $g_7(x_{41}, x_{42})$

x_3 = target density (in attempted target crossings per unit of patrol width)

x_4 = probability of conversion from an initial passive detection to an active contact

=

- $g_8(x_{56}, x_{57})$ for entrapment ring about a hyperbolic fix tactic using passive omnidirectional sonobuoy and active dipping sonar
- $g_9(x_{12}, x_{44}, \ldots, x_{49})$ for line of bearing tactic using passive directional sonobuoy and active dipping sonar (single passive contact)
- $g_{10}(x_8, x_{15}, x_{49}, x_{53})$ for entrapment ring using passive directional sonobuoys and active dipping sonar (two passive contacts)
- $g_{11}(x_{12}, x_{44}, \ldots, x_{48}, x_{50})$ for line of bearing tactic using passive towed line array sonar and active dipping sonar
- $g_{12}(x_8, x_{20}, x_{50}, x_{53})$ for entrapment ring tactic using passive towed line array sonar and active dipping sonar
- $g_{13}(x_{12}, x_{44}, \ldots, x_{48}, x_{51})$ for line of bearing tactic using passive vertical line array sonar and active dipping sonar

$$\begin{cases} g_{14}(x_8, x_{27}, x_{51}, x_{53}) & \text{for entrapment ring tactic using passive vertical line array sonar and active dipping sonar} \\ g_{15}(x_{12}, x_{44}, \ldots, x_{48}, x_{52}) & \text{for line of bearing tactic using passive/active dipping sonar} \\ g_{16}(x_8, x_{33}, x_{52}, x_{53}) & \text{for entrapment ring tactic using passive/active dipping sonar} \end{cases}$$

x_5 = number of passive omnidirectional sonobuoys that can be patrolled by the helicopter

x_6 = passive omnidirectional sonobuoy spacing in barrier

$$= \begin{cases} h_1(x_7, x_8, x_9) & \text{for no blind time} \\ h_2(x_5, x_7, \ldots, x_{12}) & \text{for a specified blind time} \end{cases}$$

x_7 = detection range of passive omnidirectional sonobuoy

$= h_3(x_{61})$

x_8 = target speed

x_9 = passive omnidirectional sonobuoy system integration time

x_{10} = blind time between periods of monitoring passive omnidirectional sonobuoys

x_{11} = maximum horizontal range at which effective monitoring of passive omnidirectional sonobuoys can be achieved

x_{12} = available flying speed of helicopter

x_{13} = number of passive directional sonobuoys that can be patrolled by the helicopter

x_{14} = passive directional sonobuoy spacing in barrier

$$= \begin{cases} h_4(x_8, x_{15}, x_{16}) & \text{for no blind time} \\ h_5(x_8, x_{12}, x_{13}, x_{15}, \ldots, x_{18}) & \text{for a specified blind time} \end{cases}$$

x_{15} = detection range of passive directional sonobuoy

$= h_6(x_{66})$

x_{16} = passive directional sonobuoy system integration time

x_{17} = blind time between periods of monitoring passive directional sonobuoys

x_{18} = maximum horizontal range at which effective monitoring of passive directional sonobuoys can be achieved

x_{19} = maximum distance from the helicopter the target will pass in transiting the patrol line

$= h_6(x_8, x_{12}, x_{20}, x_{21})$

x_{20} = detection range of passive towed line arrayed sonar

$= i_1(x_{69})$

x_{21} = passive towed line arrayed sonar integration time

x_{22} = length of side leg of crossover pattern using passive towed line arrayed sonar in a dipping mode

$= h_7(x_8, x_{12}, x_{20}, x_{21}, x_{23}, x_{24}, x_{25})$

x_{23} = effective speed of advance of helicopter using passive towed line arrayed sonar in a dipping mode

$= i_2(x_8, x_{12}, x_{20}, x_{26})$

x_{24} = time required to lower passive towed line arrayed sonar

x_{25} = time required to raise passive towed line arrayed sonar

x_{26} = total time spent in hover at each dip station using passive towed line arrayed sonar in a dipping mode

x_{27} = detection range of passive vertical line arrayed sonar

$= h_8(x_{72})$

x_{28} = length of side leg of crossover pattern using passive vertical line arrayed sonar

$= h_9(x_8, x_{12}, x_{27}, x_{29}, x_{30}, x_{31}, x_{39})$

x_{29} = effective speed of advance of helicopter using passive vertical line arrayed sonar

$= i_3(x_8, x_{12}, x_{27}, x_{32})$

x_{30} = time required to lower passive vertical line arrayed sonar

x_{31} = time required to raise passive vertical line arrayed sonar

x_{32} = total time spent in hover at each dip station using passive vertical line arrayed sonar

x_{33} = detection range of passive/active dipping sonar in passive mode

= $h_{10}(x_{75})$

x_{34} = length of side leg of crossover pattern using passive/active dipping sonar in passive mode

= $h_{11}(x_8, x_{12}, x_{33}, x_{35}, x_{36}, x_{37}, x_{40})$

x_{35} = effective speed of advance of helicopter using passive/active dipping sonar in passive mode

= $i_4(x_8, x_{12}, x_{33}, x_{38})$

x_{36} = time required to lower passive/active dipping sonar

x_{37} = time required to raise passive/active dipping sonar

x_{38} = total time spent in hover at each dip station using passive/active dipping sonar

x_{39} = passive vertical line arrayed sonar integration time

x_{40} = passive/active sonar integration time in passive mode

x_{41} = force level of helicopters equipped with passive sensors

x_{42} = number of helicopters required in the complement to keep one helicopter on station at distance x_{43} from the operating base

= $h_{12}(x_{43})$

x_{43} = distance of barrier from operating base

x_{44} = length of barrier in line of bearing tactic

x_{45} = spacing between dips in line of bearing tactics

= $h_{13}(x_{46})$

x_{46} = detection range of active dipping sonar

= $i_5(x_{78})$

x_{47} = detected target range from datum

x_{48} = total time spent in hover at each dip station using active dipping sonar

x_{49} = bearing error of passive directional sonobuoys

x_{50} = bearing error of passive towed line arrayed sonar

x_{51} = bearing error of vertical line arrayed sonar

x_{52} = bearing error of passive/active sonar in passive mode

x_{53} = entrapment ring radius

= $h_{14}($[illegible]$, x_{46}, x_{54}, x_{55})$

x_{54} = number of active sonar dips in entrapment tactics

x_{55} = time between active sonar dips

= [illegible]$_{6}(x_{12}, x_{46}, x_{48})$

x_{56} probability of obtaining a hyperbolic fix using omnidirectional sonobuoys given an initial detection

= $h_{15}(x_7, x_{58})$

x_{57} = probability of entrapping target using active sonar

= $h_{16}(x_8, x_{53}, x_{59}, x_{60})$

x_{58} = radius of passive omnidirectional sonobuoy from hyperbolic fix center

x_{59} = helicopter time late to datum in entrapment tactic after a hyperbolic fix

x_{60} = passive omnidirectional sonobuoy range error

x_{61} = transmission loss for passive omnidirectional sonobuoy

= $i_7(x_{62}, \ldots, x_{65})$

x_{62} = target radiated noise

x_{63} = recognition differential for passive omnidirectional sonobuoy

x_{64} = array gain for passive omnidirectional sonobuoy

x_{65} = background noise level

x_{66} = transmission loss for passive directional sonobuoy

= $i_8(x_{62}, x_{65}, x_{67}, x_{68})$

x_{67} = recognition differential for passive directional sonobuoy

x_{68} = array gain for passive directional sonobuoy

x_{69} = transmission loss for passive towed line arrayed sonar

= $j(x_{62}, x_{65}, x_{70}, x_{71})$

x_{70} = recognition differential for passive towed line arrayed sonar

x_{71} = array gain for passive towed line arrayed sonar

x_{72} = transmission loss for passive vertical line arrayed sonar

= $i_9(x_{62}, x_{65}, x_{73}, x_{74})$

x_{73} = recognition differential for passive vertical line arrayed sonar

x_{74} = array gain for passive vertical line arrayed sonar

x_{75} = transmission loss for passive/active dipping sonar in passive mode

= $i_{10}(x_{62}, x_{65}, x_{76}, x_{77})$

x_{76} = recognition differential for passive/active dipping sonar in passive mode

x_{77} = array gain for passive/active dipping sonar in passive mode

x_{78} = allowable one way transmission loss for active dipping sonar

= $i_{11}(x_{62}, x_{79}, x_{80})$

x_{79} = minimum detectable signal for a 50 percent probability of detection for active dipping sonar

x_{80} = target signal strength

3.2) Mission Type: Contact investigation

3.2.1) Definition: A helicopter flies to a datum (obtained as an initial contact by some platform within the task force or convoy) and attempts to reacquire it passively. If a passive redetection can be achieved, the helicopter will then attempt to convert to an active detection.

3.2.2) Criterion For Success: Detection and localization of submarine

3.2.3) MOE Selected: Cumulative probability of reacquiring and converting the target to an active contact asystems

3.2.4) Functional Form Of MOE:

$$(MOE)_3 = f_3(x_4, x_{81})$$

where

x_{81} = probability of reacquiring target

$$= \begin{cases} g_{17}(x_7, x_8, x_{60}, x_{82}) & \text{for passive omnidirectional sonobuoy} \\ g_{18}(x_8, x_{15}, x_{82}, x_{83}) & \text{for passive directional sonobuoy} \\ g_{19}(x_8, x_{20}, x_{82}, x_{84}) & \text{for passive towed line arrayed sonar} \\ g_{20}(x_8, x_{27}, x_{82}, x_{85}) & \text{for passive vertical line arrayed sonar} \\ g_{21}(x_8, x_{33}, x_{82}, x_{86}) & \text{for passive/active sonar in passive mode} \end{cases}$$

x_{82} = helicopter time late to contact datum

x_{83} = passive directional sonobuoy range error

x_{84} = passive towed line arrayed sonar range error

x_{85} = passive vertical line arrayed sonar range error

x_{86} = passive/active sonar (in passive mode) range error

4) MOE Usage In Study: The MOE's were formulated and used as the basis for comparing the increased mission effectiveness resulting from adding passive sensors to the SH-3D helicopter.

5) Special Study Assumptions:

(a) Both missions are conducted in areas where system performance would not be degraded by task force radiated noise.

(b) Transmission loss in the ocean is assumed to obey a square law spreading loss function.

(c) The signal-to-noise ratio at any range of the target from the passive plant is assumed normally distributed.

(d) The speed of the target submarine is known.

(e) In the case of the barrier placement/patrol mission,

(e.1) The most probable course of the target submarine can be estimated.

(e.2) There is no previous knowledge of the target prior to initial detection on the passive sonar.

(e.3) The targets are assumed to be uniformly distributed in the area prior to detection.

(e.4) A helicopter sets up a barrier type patrol which assumes that all targets of a given speed transiting normal to the patrol line and within the lateral limits of its effective width are within 50 percent detection range of the detection system for at least the minimum integration time.

(e.5) For the dipping systems, a dipping crossover pattern designed to ensure a given probability of initial detection is used.

(e.6) Continuously towed systems are employed in a back-and-forth patrol.

(e.7) If an omnidirectional sonobuoy is employed, hyperbolic fixing is assumed.

(f) In the case of the contact investigation mission,

(f.1) The acquisition probability is calculated assuming a single passive plant is made on the datum and assuming a circular normal target distribution around the datum.

(f.2) The conversion probability is calculated using line-of-bearing, entrapment and hyperbolic fixing models assuming the distribution of detected targets from the datum is circular normal distributed.

(f.3) A definite range law of detection is assumed.

(g) The line-of-bearing tactic used in the prosecution phase consists of active sonar plants (constant spacing) along a bearing line defined by the initial passive directional plant. A Gaussian distribution of targets normal to the bearing line is assumed, and this distribution is dependent on the passive directional plant bearing error.

(h) The entrapment tactic used in the prosecution phase consists of two directional plants in obtaining the estimated position of the target submarine for subsequent entrapment using active dipping sonar.

(i) The hyperbolic fixing tactic used in the prosecution phase consists of a fix using three omnidirectional buoys subsequent to an entrapment tactic using active dipping sonar.

(j) In calculating the probability of a hyperbolic fix,

(j.1) The distribution of detected targets from the initial detection point is uniform.

(j.2) Detection at the second and third passive plants is independent of the initial detection.

(j.3) Detection at the third plant is independent of the second plant.

(j.4) The probability of detection at the second and third plants is a function of the signal-to-noise ratio.

(j.5) The signal to noise ratio is normally distributed.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) background noise level

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) target speed

(b) target radiated noise

(c) target signal strength

2.2) Tactics:

2.2.1) Quantitative Factor:

(a) target density (in attempted target crossings per unit of patrol width)

3) Friendly Force Composition: Helicopter

3.1) Quantitative Factors:

(a) available flying speed of helicopter

(b) force level of helicopters equipped with passive sensors

3.2) Sensors: Passive omnidirectional sonobuoy, passive directional sonobuoy, passive towed line arrayed sonar, passive vertical line arrayed sonar, passive/active dipping sonar and active dipping sonar

3.2.1) Type: Passive omnidirectional sonobuoy

3.2.1.1) Quantitative Factors:

(a) number of passive omnidirectional sonobuoys that can be patrolled by the helicopter

(b) passive omnidirectional sonobuoy system integration time

(c) blind time between periods of monitoring passive omnidirectional sonobuoys

(d) maximum horizontal range at which effective monitoring of passive omnidirectional sonobuoys can be achieved

(e) passive omnidirectional sonobuoy range error

(f) recognition differential for passive omnidirectional sonobuoy

(g) array gain for passive omnidirectional sonobuoy

(h) radius of passive omnidirectional sonobuoy from hyperbolic fix center

3.2.2) Type: Passive directional sonobuoy

3.2.2.1) Quantitative Factors:

(a) number of passive directional sonobuoys that can be patrolled by the helicopter

(b) passive directional sonobuoy system integration time

(c) blind time between periods of monitoring passive directional sonobuoys

(d) maximum horizontal range at which effective monitoring of passive directional sonobuoys can be achieved

(e) bearing error of passive directional sonobuoys

(f) recognition differential for passive directional sonobuoy

(g) array gain for passive directional sonobuoy

(h) passive directional sonobuoy range error

3.2.3) Type: Passive towed line arrayed sonar

3.2.3.1) Quantitative Factors:

(a) passive towed line arrayed sonar integration time

(b) time required to lower passive towed line arrayed sonar

(c) time required to raise passive towed line arrayed sonar

(d) total time spent in hover at each dip station using passive towed line arrayed sonar in a dipping mode

(e) bearing error of passive towed line arrayed sonar

(f) recognition differential for passive towed line arrayed sonar

(g) array gain for passive towed line arrayed sonar

(h) passive towed line arrayed sonar range error

3.2.4) Type: Passive vertical line arrayed sonar

3.2.4.1) Quantitative Factors:

(a) time required to lower passive vertical line arrayed sonar

(b) time required to raise passive vertical line arrayed sonar

(c) total time spent in hover at each dip station using passive vertical line arrayed sonar

(d) passive vertical line arrayed sonar integration time

(e) bearing error of vertical line arrayed sonar

(f) recognition differential for passive vertical line arrayed sonar

(g) array gain for passive vertical line arrayed sonar

(h) passive vertical line arrayed sonar range error

3.2.5) Type: Passive/active dipping sonar

3.2.5.1) Quantitative Factors:

(a) time required to lower passive/active dipping sonar

(b) time required to raise passive/active dipping sonar

(c) total time spent in hover at each dip station using passive/active dipping sonar

(d) passive/active sonar integration time in passive mode

(e) bearing error of passive/active sonar in passive mode

(f) recognition differential for passive/active dipping sonar in passive mode

(g) array gain for passive/active dipping sonar in passive mode

(h) passive/active sonar (in passive mode) range error

3.2.6) Type: Active dipping sonar

3.2.6.1) Quantitative Factors:

(a) total time spent in hover at each dip station using active dipping sonar

(b) minimum detectable signal for a 50 percent probability of detection for active dipping sonar

3.3) Tactics:

3.3.1) Quantitative Factors:

(a) distance of barrier from operating base

(b) length of barrier in line of bearing tactic

(c) number of active sonar dips in entrapment tactics

(d) helicopter time late to datum in entrapment tactic after a hyperbolic fix

(e) helicopter time late to contact datum

4) Friendly Force - Threat Interaction:

4.1) Sensor - Platform: Passive/active dipping sonar - Submarine

4.1.1) Quantitative Factor:

(a) detected target range from datum

STUDY REVIEW SUMMARY NO. (1)-15

A. STUDY DESCRIPTION

1) Originating Activity: Systems Analysis Office, ASW Systems Project Office, White Oak, Maryland
2) Report Title: An Analytical Procedure for Optimizing Buoy Patterns
3) Author: J.B. Parkerson
4) Report Number: SAOTM 70-1 (AD-517 859)
5) Date: December 1970
6) Classification: Confidential
7) Abstract: A probabilistic model for optimizing the effectiveness of the Lofar containing barrier tactic against nuclear targets is developed using estimates of area of uncertainty associated with the datum, target speed, and median detection range as inputs. Plots of field parameters yielding the most effective pattern are presented.
8) Descriptors: Aircraft, antisubmarine warfare, barrier, contact investigation, detection probability, Lofar, sonobuoy, submarine, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Barrier placement/patrol
   3.1) Definition: Lofar buoys are deployed according to a containing barrier tactic in the search for transiting submarines.
   3.2) Criterion For Success: Detection of submarine
   3.3) MOE Selected: Probability of detection
   3.4) Functional Form Of MOE:
   $MOE = f(x_1, \ldots, x_4)$

where

x_1 = probability of detection given the target is initially inside the circle of buoys

$= g_1(x_5, x_6, x_7)$

x_2 = probability the target is initially inside the circle of buoys

$$= \begin{cases} g_2(x_{26}) & \text{for normally distributed targets} \\ g_3(x_{26}, x_{27}) & \text{for uniformly distributed targets} \end{cases}$$

x_3 = probability of detection given the target is initially outside and will pass through the circle of buoys

$$= \begin{cases} g_4(x_{26}, x_{27}) & \text{for normally distributed targets} \\ g_5(x_{26}, x_{27}) & \text{for uniformly distributed targets} \end{cases}$$

x_4 = probability the target is initially outside and will pass through the circle of buoys

$= g_6(x_1)$

x_5 = probability of a buoy detection given the buoy is monitored

x_6 = probability of monitoring a buoy given there is an opportunity for detection

$= h_1(x_8, x_9, x_{10})$

x_7 = probability of an opportunity for detection given the target is initially inside the circle of buoys

$$= \begin{cases} h_2(x_{13}, x_{24}, x_{25}) & \text{for } x_{16} \text{ not "small"} \\ h_3(x_{13}, x_{17}, x_{25}) & \text{for } x_{16} \text{ "small"} \end{cases}$$

x_8 = time required to monitor all buoy sets

$= i_1(x_9, x_{11}, x_{12})$

x_9 = time each buoy set is monitored before switching to another set

x_{10} = duration of target travel time within the definite range of detection circle

$$= \begin{cases} i_2(x_{13}, x_{14}) & \text{for no integration time} \\ i_3(x_{13}, \ldots, x_{16}) & \text{for integration time required to detect a weak Lofar signal} \\ i_4(x_{14}, x_{17}) & \text{for } x_{16} \text{ "small"} \end{cases}$$

x_{11} = total number of buoys to be monitored

x_{12} = number of r.f. channels to be monitored simultaneously

x_{13} = buoy median detection range

x_{14} = submarine speed

x_{15} = integration time

x_{16} = reduced detection range due to integration time

$= j_1(x_{13}, x_{14}, x_{15})$

x_{17} = target to buoy range within which the signal received will have been enhanced by at least the processing gain achievable as a result of the estimated integration time

$= j_2(x_{13}, x_{18})$

x_{18} = processing gain during integration time

$= k(x_{19}, \ldots, x_{23})$

x_{19} = signal excess

x_{20} = source level

x_{21} = propagation loss

x_{22} = ambient noise

x_{23} = recognition differential

x_{24} = buoy effective range of detection

x_{25} = spacing factor

x_{26} = radius of the circles of buoys

$= h_4(x_{11}, x_{13}, x_{25})$

x_{27} = radius beyond which there are no targets initially

4) MOE Usage In Study: The MOE was formulated and used to find the optimal barrier design.

5) Special Study Assumptions:

(a) A circularly shaped buoy pattern is used. The rational for a circular ring of buoys about the datum coordinates is based on the assumptions that the target will eventually pass through the barrier and that proper buoy spacing along the circumference will yield high detection probabilities.

(b) Target course information is not available.

(c) Distribution of targets in the area of uncertainty is assumed to be either normally or uniformly distributed.

(d) The probability of detection given an intersection is the same regardless of the direction of target motion with respect to the buoy circle.

(e) Each buoy is considered equally likely to detect targets which pass within some specific range of the buoy.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factors:

(a) propagation loss

(b) ambient noise

2) Target Composition: Submarine

2.1) Quantitative Factors:

(a) submarine speed

(b) source level

3) Friendly Force Composition: VP Aircraft

3.1) Sensor: Sonobuoys

3.1.1) Quantitative Factors:

(a) total number of buoys to be monitored

(b) number of r.f. channels to be monitored simultaneously

(c) buoy median detection range

(d) integration time

(e) recognition differential

(f) buoy effective range of detection

3.1.2) Deployment:

3.1.2.1) Quantitative Factor:

(a) spacing factor

3.1.3) Tactics:

3.1.3.1) Quantitative Factor:

(a) time each buoy set is monitored before switching to another set

4) Friendly Force - Target Interaction:

4.1) Sensor - Platform: Sonobuoy - Submarine

4.1.1) Quantitative Factors:

(a) probability of a buoy detection given the buoy is monitored

(b) signal excess

(c) radius beyond which there are no targets initially

Itasystems

STUDY REVIEW SUMMARY NO. (1)-16

A. STUDY DESCRIPTION

1) Originating Activity: Bureau of Naval Weapons, Washington, D.C.
2) Report Title: Cost Effectiveness of Carrier Based ASW Aircraft
3) Author: C.M. Boorer
4) Report Number: R-5-64-20 (AD-355 704)
5) Date: September 1964
6) Classification: Confidential
7) Abstract: This study investigates the cost effectiveness of several carrier based ASW aircraft. For the purposes of the study new aircraft designs were selected which would accommodate the A-NEW avionics system and which would generally meet the primary mission requirements for carrier based ASW aircraft.
8) Descriptors: Aircraft, antisubmarine warfare, availability, carrier, carrier based aircraft, cost, search, surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Submarine search
   - 3.1) Definition: A carrier based ASW aircraft searches for enemy submarines.
   - 3.2) Criterion For Success: Maintain on-station search capability
   - 3.3) MOE's Selected:

     $(MOE)_1$ = Aircraft operating cost per on-station hour

     $(MOE)_2$ = Aircraft operating cost per search mile
   - 3.4) Functional Form Of MOE's:

     $(MOE)_1 = f_1(x_1, x_2)$

     $(MOE)_2 = f_2(x_1, x_3)$

where

x_1 = aircraft operating cost per hour (this includes capital cost amortized over an eight year period, crew, fuel and maintenance and spare parts costs)

x_2 = maximum number of hours per day that each aircraft could be on-station

$= g_1(x_4, \ldots, x_7)$

x_3 = maximum number of search miles per day for each aircraft

$= g_2(x_4, \ldots, x_8)$

x_4 = aircraft availability

x_5 = mission time including time on-station plus transit time

x_6 = time on-station for each aircraft

x_7 = turnaround time between missions

x_8 = true airspeed at which the mission was flown

3.5) Additional MOE's Identified:

(a) Search hours per day per carrier deckload for a specified mission radius

(b) Search miles per day per carrier deckload for a specified mission radius

4) MOE Usage In Study: Formulation and numerical examples

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarines

3) Friendly Force Composition: Carrier and aircraft (S-2E)

3.1) Platform Type: Carrier

3.1.1) Quantitative Factor:

(a) turnaround time between missions

3.2) Platform Type: Aircraft (S-2E)

3.2.1) Quantitative Factors:

(a) aircraft operating cost per hour (this includes capital cost amortized over an eight year period, crew, fuel and maintenance and spare parts costs)

(b) aircraft availability

(c) true airspeed at which the mission was flown

3.2.2) Deployment:

3.2.2.1) Quantitative Factors:

(a) mission time including time on-station plus transit time

(b) time on-station for each aircraft

STUDY REVIEW SUMMARY NO. (1)-17

A. STUDY DESCRIPTION

1) Originating Activity: Vitro Laboratories, Silver Spring, Maryland
2) Report Title: Concept Formulation Study for Independent ASW Localization and Attack System for Surface Ships, Vol. 8 Cost Effectiveness
3) Report Number: TR 1993-10-2 (AD-377 818)
4) Date: 14 November 1966
5) Classification: Confidential
6) Contract: NOW 66-0513c (Naval Air Systems Command)
7) Abstract: This report presents the results of a study to establish concepts for exploiting the convergence zone sonar detection capability from Destroyer Escorts. The study involved an analysis of the requirements and capabilities available in the areas of vehicles, ships, ASW sensors, processing and display and airborne electronics. It included an analysis and definition of requirements in reliability, maintainability, logistics, personnel, training and system costs. Competing systems were analyzed and compared to the candidate concepts as part of an overall system and cost effectiveness study. This volume of the concept formulation study determines the estimated cost of systems required to meet specified operational requirements and of the variation of the overall cost with different configurations and their performance capabilities.
8) Descriptors: Aircraft, antisubmarine warfare, barrier, Cass, classification, contact investigation, contact prosecution, cost, cost effectiveness, destroyer, detection, Difar, false target, helicopter, localization, Lofar, MAD, maintainability, reliability, smoke bomb, sonar, sonobuoy, submarine, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne ASW
3) Mission: Contact investigation/prosecution
   - 3.1) Definition: A destroyer-based ASW helicopter places a sonobuoy barrier in an attempt to redetect, localize, classify and attack a previously detected submarine.
   - 3.2) Criterion For Success: Performance of mission requirements at least cost
   - 3.3) MOE Selected: Total system cost for specified level of wartime and peacetime utilization
   - 3.4) Functional Form Of MOE:

     $MOE = f(x_1, x_2, x_3)$

where

x_1 = total program operating and maintenance cost
$= g_1(x_4, \ldots, x_{12})$

x_2 = total program investment cost
$= g_2(x_{37}, \ldots, x_{42})$

x_3 = total payload cost
$= g_3(x_{57}, x_{58}, x_{59})$

x_4 = operating and maintenance cost for flight operations
$= h_1(x_{13}, \ldots, x_{17})$

x_5 = operating and maintenance cost for vehicle rework or overhaul
$= h_2(x_{15}, \ldots, x_{19})$

x_6 = operating and maintenance cost for engine overhaul
$= h_3(x_{14}, \ldots, x_{17}, x_{20})$

x_7 = operating and maintenance cost for PAMN spare parts
$= h_4(x_{14}, \ldots, x_{17}, x_{21})$

x_8 = operating and maintenance cost for flight personnel
= $h_5(x_{15}, x_{22},\ldots, x_{26})$

x_9 = operating and maintenance cost for maintenance personnel
= $h_6(x_{15}, x_{27},\ldots, x_{30})$

x_{10} = operating and maintenance cost for general support
= $h_7(x_{14},\ldots, x_{16}, x_{31})$

x_{11} = operating and maintenance cost for support operations
= $h_8(x_{15}, x_{16}, x_{17}, x_{32})$

x_{12} = operating and maintenance cost for maintenance training
= $h_9(x_{33},\ldots, x_{36})$

x_{13} = LAAV operating cost per flight hour

x_{14} = number of hours one vehicle is flown per month for LAAV and miscellaneous missions

x_{15} = number of years of the program

x_{16} = average number of operating aircraft (base requirement for LAAV program)

x_{17} = percentage of base procurement quantity needed for training requirements

x_{18} = annual proportion of operating vehicles requiring service

x_{19} = unit cost of rework

x_{20} = engine overhaul cost per flight hour

x_{21} = spare parts cost per flight hour

x_{22} = average annual pay and allowances per officer

x_{23} = average annual special duty flight pay per officer

x_{24} = number of pilots

x_{25} = number of TACO's which are officers

x_{26} = number of officers other than pilots and TACO's

x_{27} = average annual pay for enlisted personnel

x_{28} = average annual special duty flight pay per enlisted man

x_{29} = number of enlisted men other than TACO's

x_{30} = number of TACO's which are enlisted men

x_{31} = general support cost per flight hour

x_{32} = operations support cost per annum per operating vehicle

x_{33} = trainer's cost

x_{34} = training parts cost

x_{35} = factory training cost

x_{36} = contractor's technical services cost

x_{37} = initial vehicle procurement and initial spares cost

$= h_{10}(x_{16}, x_{44}, x_{45})$

x_{38} = pool and pipeline overbuy costs

$= h_{11}(x_{44}, \ldots, x_{47})$

x_{39} = attrition overbuy costs

$= h_{12}(x_{44}, x_{45}, x_{47}, x_{48})$

x_{40} = training costs

$= h_{13}(x_{16}, x_{17}, x_{24}, x_{25}, x_{44}, x_{45}, x_{47}, x_{48})$

x_{41} = research and test vehicle cost

$= h_{14}(x_{44}, x_{45}, x_{51})$

x_{42} = special support costs

$= h_{15}(x_{44}, x_{52}, \ldots, x_{55})$

x_{43} = publications cost

$= h_{16}(x_{44}, x_{52}, x_{56})$

x_{44} = vehicle unit flyaway cost

x_{45} = percentage of vehicle flyaway cost applicable to spare parts

x_{46} = percentage of basic procurement and training vehicles for meeting pool and pipeline requirements

x_{47} = number of operating and training aircraft

$= i_1(x_{13}, x_{16})$

x_{48} = percentage of basic procurement and training vehicles necessary for expected attrition during the program

x_{49} = cost of training one pilot

x_{50} = cost of training one TACO

x_{51} = number of vehicles needed for research and testing

x_{52} = total vehicle buy

$= i_2(x_{16}, x_{17}, x_{45}, x_{47}, x_{50})$

x_{53} = percentage of flyaway cost spent on airframe support equipment

x_{54} = percentage of flyaway cost spent on power plant support equipment

x_{55} = percentage of flyaway cost spent on avionics support equipment

x_{56} = percentage of flyaway cost spent on publications

x_{57} = payload plus avionics and ASW electronics RDT&E cost

x_{58} = cost for non-expendables

$= h_{17}(x_{16}, x_{17}, x_{46}, x_{48}, x_{51})$

x_{59} = cost for expendables

$= h_{18}(x_{60}, x_{61}, x_{62})$

x_{60} = cost of sonobuoys

$= i_3(x_{63}, x_{64})$

x_{61} = cost of torpedoes

$= i_4(x_{65}, x_{66})$

x_{62} = cost of smoke bombs

$= i_5(x_{67}, x_{68})$

x_{63} = expected number of sonobuoys expended during program

$= j_1(x_{69}, x_{70})$

x_{64} = unit cost of sonobuoy

x_{65} = expected number of torpedoes expended during program

$= j_2(x_{71}, x_{72})$

x_{66} = unit cost of torpedo

x_{67} = expected number of smoke bombs expended during program

$= j_3(x_{14}, x_{72}, x_{73}, x_{74}, x_{76})$

x_{68} = unit cost of smoke bomb

x_{69} = expected number of sonobuoys expended on true missions during program

$= k_1(x_{14}, x_{72}, \ldots, x_{75}, x_{77}, x_{78})$

x_{70} = expected number of sonobuoys expended on false missions during program

$= k_2(x_{14}, x_{72}, \ldots, x_{75}, x_{79}, x_{80})$

x_{71} = expected number of torpedoes expended on true missions during the program

$= k_3(x_{14}, x_{72}, \ldots, x_{75}, x_{81})$

x_{72} = percent of time destroyer spends on-station

x_{73} = number of destroyers available for the program

x_{74} = number of ships that the destroyer contacts per two week period

x_{75} = percentage of contacts which are true

x_{76} = number of smoke bombs expended per mission

x_{77} = number of type-one sonobuoys dropped per true mission

x_{78} = number of type-two sonobuoys dropped per true mission

x_{79} = number of type-one sonobuoys dropped per false mission

x_{80} = number of type-two sonobuoys dropped per false mission

x_{81} = number of torpedoes expended per true mission

4) MOE Usage In Study: This MOE was formulated and used in conjunction with an effectiveness measure developed in another volume of this study to determine the optimal cost-effective LAAV system configuration.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarines

3) Friendly Force Composition: Destroyer, helicopter, personnel

   3.1) Platform Type: Destroyer

      3.1.1) Quantitative Factor:

         (a) number of destroyers available for the program

      3.1.2) Deployment:

         3.1.2.1) Quantitative Factor:

            (a) percent of time destroyer spends on-station

   3.2) Platform Type: Helicopter

      3.2.1) Quantitative Factors:

         (a) LAAV operating cost per flight hour
         (b) average number of operating aircraft (base requirement for LAAV program)
         (c) percentage of base procurement quantity needed for training requirements
         (d) annual proportion of operating vehicles requiring service
         (e) unit cost of rework
         (f) engine overhaul cost per flight hour
         (g) spare parts cost per flight hour
         (h) general support cost per flight hour
         (i) operations support cost per annum per operating vehicle
         (j) trainer's cost
         (k) training parts cost
         (l) factory training cost
         (m) contractor's technical services cost
         (n) vehicle unit flyaway cost

(o) percentage of vehicle flyaway cost applicable to spare parts

(p) percentage of basic procurement and training vehicles for meeting pool and pipeline requirements

(q) percentage of basic procurement and training vehicles necessary for expected attrition during the program

(r) number of vehicles needed for research and testing

(s) percentage of flyaway cost spent on airframe support equipment

(t) percentage of flyaway cost spent on power plant support equipment

(u) percentage of flyaway cost spent on avionics support equipment

(v) percentage of flyaway cost spent on publications

(w) payload plus avionics and ASW electronics RDT&E cost

3.2.2) Sensors: Sonobuoys and smoke bombs

3.2.2.1) Type: Sonobuoy

3.2.2.1.1) Quantitative Factor:

(a) unit cost of sonobuoy

3.2.2.1.2) Tactics:

3.2.2.1.2.1) Quantitative Factors:

(a) number of type-one sonobuoys dropped per true mission

(b) number of type-two sonobuoys dropped per true mission

(c) number of type-one sonobuoys dropped per false mission

(d) number of type-two sonobuoys dropped per false mission

3.2.2.2) Type: Smoke bomb

3.2.2.2.1) Quantitative Factor:

(a) unit cost of smoke bomb

3.2.2.2.2) Tactics:

3.2.2.2.2.1) Quantitative Factor:

(a) number of smoke bombs expended per mission

3.2.3) Armament: Torpedo

3.2.3.1) Quantitative Factor:

(a) unit cost of torpedo

3.2.3.2) Tactics:

3.2.3.2.1) Quantitative Factor:

(a) number of torpedoes expended per true mission

3.2.4) Deployment:

3.2.4.1) Quantitative Factors:

(a) number of hours one vehicle is flown per month for LAAV and miscellaneous missions

(b) number of years of the program

3.3) Platform Type: Personnel

3.3.1) Quantitative Factors:

(a) average annual pay and allowances per officer

(b) average annual special duty flight pay per officer

(c) number of pilots

(d) number of TACO's which are officers

(e) number of officers other than pilots and TACO's

(f) average annual pay for enlisted personnel

(g) average annual special duty flight pay per enlisted man

(h) number of enlisted men other than TACO's

(i) number of TACO's which are enlisted men

(j) cost of training one pilot

(k) cost of training one TACO

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Destroyer - Submarine

4.1.1) Quantitative Factors:

(a) number of ships that the destroyer contacts per two week period

(b) percentage of contacts which are true

(2)

AIRBORNE AAW

STUDY REVIEW SUMMARY NO.(2)-1

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Weapon Systems Analysis Office, Marine Corps Air Station, Quantico, Virginia
2) Report Title: SPARROW III Effectiveness and Cost Comparison
3) Authors: J. J. Bellaschi and R. J. Lange
4) Report Number: WSAO-R-63-1 (AD-369 114)
5) Date: 1 September 1965
6) Classification: Secret
7) Abstract: This study provides a cost effectiveness analysis between SPARROW III-6A and 6B; including shelf-life, attendant maintenance and rework costs; approved F-4 aircraft program and planned research and development programs. Study includes a cost effectiveness comparison of AIM-7D, 7E and later 7F missiles, to determine procurement alternatives, modification feasibilities, and overall effectiveness (including launcher alternatives, environmental considerations, target mixes, lethality).
8) Descriptors: Aircraft, air superiority, air-to-air missile, antiair warfare, bomber defense, combat air patrol, kill probability, reliability, survivability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne AAW
3) Missions:
   3.1) Mission Type: Defense against bomber attack
      3.1.1) Definition: Aircraft equipped with air-to-air missiles are engaged in anti-air warfare in a standard CAP operation to defend against bomber aircraft having no self-defense capability.

3.1.2) Criterion For Success: Destruction of bombers

3.1.3) MOE's Selected:

$(MOE)_1$ = Probability that friendly aircraft will destroy a bomber

$(MOE)_2$ = Expected number of kills a friendly aircraft will achieve if it is directed against two bombers per sortie

3.1.4) Functional Form Of MOE's:

Case 1 - One missile fired head-on

$(MOE)_1 = f_1(x_1,x_2,x_3)$

where

x_1 = probability of detection and conversion of bomber by aircraft in head-on attack

x_2 = missile reliability

x_3 = probability of a lethal hit on bomber in head-on attack

Case 2 - Two missiles ripple fired during a head-on attack

$(MOE)_1 = f_2(x_1,x_2,x_3)$

Case 3 - Three missiles launched, two during a head-on attack and one during a subsequent tail attack

$(MOE)_1 = f_3(x_1,\ldots,x_5)$

where

x_4 = probability of detection and conversion of bomber by aircraft in tail attack

x_5 = probability of a lethal hit on bomber in tail attack

Case 4 - Four missiles launched, two during a head-on attack and two during a subsequent tail attack

$(MOE)_1 = f_4(x_1, x_2, x_3, x_6)$

where

x_6 = probability of killing a bomber with two missiles ripple fired during a tail attack following a head-on attack

= $g(x_2, x_4, x_5)$

Case 5 - Two head-on attacks with two missiles ripple fired at each target

$(MOE)_2 = f_5(x_1, x_2, x_3)$

3.2) Mission Type: Air superiority

3.2.1) Definition: Aircraft equipped with air-to-air missiles are engaged in anti-air warfare in a standard CAP operation to defend against fighter aircraft (escorting a bombing force) which are also equipped with air-to-air missiles.

3.2.2) Criterion For Success: Survival of friendly aircraft and destruction of enemy interceptor

3.2.3) MOE's Selected:

$(MOE)_3$ = Probability that aircraft will survive the enemy interceptor

$(MOE)_4$ = Probability that aircraft will survive the engagement with enemy interceptor

3.2.4) Functional Form Of MOE's:

Case 1 - Friendly aircraft launches two missiles first

$(MOE)_3 = f_6(x_2, x_7, x_8)$

$(MOE)_4 = f_7(x_2, x_7, \ldots, x_{11})$

where

x_7 = probability of detection and conversion of enemy interceptor by aircraft in head-on attack

x_8 = probability of lethal hit on enemy interceptor in head-on attack

x_9 = probability of detection and conversion of friendly aircraft by enemy interceptor in head-on attack

x_{10} = probability of lethal hit on friendly aircraft in head-on attack

x_{11} = enemy interceptor missile reliability

Case 2' - Friendly aircraft launches two missiles last

$(MOE)_3 = f_8(x_2, x_7, \ldots, x_{11})$

$(MOE)_4 = f_9(x_9, x_{10}, x_{11})$

4) MOE Usage In Study: The MOE's were formulated and used to compare various air-to-air missile systems using operational data.

5) Special Study Assumptions:

(a) All operations are in a clear environment, i.e., electronic countermeasures are not employed by the enemy aircraft.

(b) Friendly aircraft carries 4 air-to-air missiles.

(c) In the bomber defense mission,

(c.1) The attacker is detected by either a ship-based or airborne warning and control system which then vectors the friendly aircraft on a collision course toward the target.

(c.2) After detection of the target by radar, the friendly aircraft converts into launch position.

(c.3) One or two missiles are launched as soon as the interceptor is within both the maximum missile aerodynamic and seeker ranges and before minimum missile-launch rate is reached.

(c.4) If a tail attack follows the head-on attack, the friendly aircraft makes a 180^{o} turn to arrive at a rear hemisphere launch position.

(d) In the bomber defense role against two targets in trail, friendly aircraft first converts to a head-on attack and ripple fires two missiles against the first bomber. After the first attack, the friendly aircraft is revectored to another head-on attack against a second bomber and two missiles are launched against it.

(e) In the air superiority mission,

(e.1) Each fighter aircraft ripple fire two missiles as soon as the target has been detected and is being tracked and both aerodynamic and seeker range limits are reached.

(e.2) No reattacks are considered after the initial attack.

(e.3) A period of approximately two seconds will elapse between the first and second launches in a ripple fire. Because this time is so short, the probability that an opponent will fire a missile during this particular period is ignored.

(e.4) Fighter aircraft are non-maneuvering.

(f) The numerical evaluations for each mission are based on missile reliability data developed during Production Monitoring Tests without taking into account degradation that might occur as a result of missile aging.

(g) Factors such as probability of detection by warning and control systems, aircraft reliability, abort or attrition rates, and human factors which might normally be considered as part of true probability of kill equations, have been intentionally omitted since they would be the same for all missiles being evaluated and are not pertinent to a relative comparison.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Bombers (Badger and Blinder) and fighter aircraft (Fitter)

   2.1) Platform Type: Fighter aircraft

      2.1.1) Armament: Air-to-air missiles (AA-4 and AA-5)

         2.1.1.1) Quantitative Factor:

            (a) enemy interceptor missile reliability

3) Friendly Force Composition: Fighter aircraft (F-4)

   3.1) Armament: Air-to-air missiles (AIM-7D,-7E and -7F)

      3.1.1) Quantitative Factor:

         (a) missile reliability

4) Friendly Force - Threat Interaction:

   4.1) Platform - Platform:

      4.1.1) Type: Fighter aircraft - Bomber

         4.1.1.1) Quantitative Factors:

            (a) probability of detection and conversion of bomber by aircraft in head-on attack

            (b) probability of detection and conversion of bomber by aircraft in tail attack

      4.1.2) Type: Fighter aircraft - Fighter aircraft

         4.1.2.1) Quantitative Factors:

            (a) probability of detection and conversion of enemy interceptor by aircraft in head-on attack

            (b) probability of detection and conversion of friendly aircraft by enemy interceptor in head-on attack

   4.2) Platform - Armament: Fighter aircraft - Air-to-air missile

4.2.1) Quantitative Factor:

(a) probability of lethal hit on friendly aircraft in head-on attack

4.3) Armament - Platform:

4.3.1) Type: Air-to-air missile - Bomber

4.3.1.1) Quantitative Factors:

(a) probability of lethal hit on bomber in head-on attack

(b) probability of lethal hit on bomber in tail attack

4.3.2) Type: Air-to-air missile - Fighter aircraft

4.3.2.1) Quantitative Factor:

(a) probability of lethal hit on enemy interceptor in head-on attack

STUDY REVIEW SUMMARY NO.(2)-2

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Missile Center, Point Mugu, California
2) Report Title: Mission Success
3) Authors: E. Q. Smith, Jr. and E. F. King
4) Report Number: NMC-MP-65-12
5) Date: 10 February 1966
6) Classification: Unclassified
7) Abstract: The performance of a military weapon system is defined in such a way as to lead to a quantitative assessment of mission success. This can be employed equally well to compute effectiveness per dollar or to measure the ratio of improvement to added cost in existing or prototype systems. Further, a method of locating the more promising (subsystem) areas of improvement is considered.
8) Descriptors: Aircraft, antiair warfare, cost, countermeasure, kill probability, missile, reliability, survivability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne AAW
3) Mission: Air superiority
   - 3.1) Definition: Fighter aircraft attack airborne targets.
   - 3.2) Criterion For Success: Destruction of target
   - 3.3) MOE Selected: Ratio of the incremental improvement in accomplishing the mission to the incremental monetary cost of such an improvement
     - 3.3.1) Rationale For Selection: The terms in the MOE formulation may represent positive changes from an existing situation or a total change from "zero". In the first case, improvement of a system is evaluated; the second case considers the worth of the system per se (presumably in comparison with alternative ways of accomplishing the desired results).

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2)$$

where

x_1 = incremental improvement in accomplishing the mission
$= g_1(x_3, \ldots, x_7)$

x_2 = increased monetary cost of such an improvement
$= g_2(x_8, x_9)$

x_3 = reliability of the system
$= h_1(x_{16}, x_{17})$

x_4 = availability of the system
$= h_2(x_{16}, x_{18})$

x_5 = single-shot kill probability of each missile
$= h_3(x_{19}, \ldots, x_{24})$

x_6 = number of missiles per aircraft
x_7 = number of aircraft assigned to a mission
x_8 = initial system cost
x_9 = support cost
$= h_4(x_{10}, \ldots, x_{16})$

x_{10} = total number of systems
x_{11} = average operating time per system per month
x_{12} = average repair cost per unit
x_{13} = system useful life
x_{14} = cost of spare systems
x_{15} = time spent in the supply line
x_{16} = system mean time between failures
x_{17} = mission time
x_{18} = system mean time to repair
x_{19} = personnel performance factor
x_{20} = weather performance factor
x_{21} = performance factor in ECM environment

x_{22} = target-weapon lethality
x_{23} = weapon launch range
x_{24} = weapon launch angle

4) MOE Usage In Study: Formulation only
5) Special Study Assumptions:
   (a) Each aircraft fires all of its missiles at the target.
   (b) All assigned aircraft are used on the mission.
   (c) All aircraft survive the mission.

C. EFFECTIVENESS FACTORS

1) Physical Environment:
   1.1) Quantitative Factor:
      (a) weather performance factor
2) Target Composition: Not stated
3) Friendly Force Composition: Fighter aircraft
   3.1) Quantitative Factors:
      (a) system mean time between failures
      (b) system mean time to repair
      (c) number of aircraft assigned to a mission
      (d) initial system cost
      (e) total number of systems
      (f) average operating time per system per month
      (g) average repair cost per unit
      (h) system useful life
      (i) cost of spare systems
      (j) time spent in the supply line
      (k) mission time
      (l) personnel performance factor
      (m) performance factor in ECM environment
   3.2) Armament: Missiles

3.2.1) Quantitative Factors:

(a) number of missiles per aircraft

(b) weapon launch range

(c) weapon launch angle

4) Friendly Force - Target Interaction:

4.1) Armament - Platform: Missiles - Target

4.1.1) Quantitative Factor:

(a) target - weapon lethality

STUDY REVIEW SUMMARY NO. (2)-3

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Weapons Center, China Lake, California
2) Report Title: Efficient Use of Combat Air Patrol Against Cruise Missiles
3) Report Number: TN 127-17
4) Date: January 1970
5) Classification: Secret
6) Abstract: This report provides a geometric interpretation of potential combat air patrol (CAP) operations against cruise missiles radially closing a surface ship target. A methodology to examine the use of CAP for the special purpose of cruise missile interdiction is derived in this study. This methodology allows the investigator of CAP defense and the task force defense planner to characterize CAP defense by a CAP envelope much as a missile is characterized by a missile envelope.
7) Descriptors: Aircraft, antiair warfare, carrier, combat air patrol, cruise missile, detection, ship defense

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne AAW
3) Mission: Surface ship defense
   - 3.1) Definition: CAP aircraft, on-station at a designated point relative to a CVA, are used to intercept cruise missiles directed at a surface ship formation.
   - 3.2) Criterion For Success: Detection of cruise missile raid at a range which allows for missile intercept at useful ranges
   - 3.3) MOE Selected: Detection range of raid relative to vital area center (CVA) for a given intercept range

3.4) Functional Form Of MOE:

$MOE = f(x_1, \ldots, x_7)$

where

x_1 = CAP on-station distance from CVA

x_2 = angle between the line-of-sight to the CAP station and the approach vector of the incoming missile raid

x_3 = average velocity of the missile raid

x_4 = CAP aircraft speed

x_5 = length of path of CAP aircraft's flight to intercept
$= g_1(x_1, x_2, x_8)$

x_6 = straight line distance traveled by CAP aircraft in accelerating from an initial velocity x_9 to a velocity x_4
$= g_2(x_4, x_9, x_{10})$

x_7 = effective reaction time
$= g_3(x_4, x_6, x_{10}, x_{11}, x_{12})$

x_8 = specified distance of intercept from the CVA

x_9 = CAP aircraft initial velocity

x_{10} = acceleration function of the CAP aircraft

x_{11} = reaction time from detection until CAP aircraft leaves to intercept

x_{12} = time required for the interceptor to accelerate from an initial velocity x_9 to a velocity x_4
$= h(x_4, x_9, x_{10})$

4) MOE Usage In Study: CAP detection envelopes were prepared for the current F-4B aircraft operating against various missile threats.

5) Special Study Assumptions:

(a) The CAP aircraft is assumed to fly a straight line path from its station to the intercept point. With straight paths, calculations can be made using simple trigonometry and complicated equations can be avoided. Furthermore, this assumption is felt to introduce only small errors.

(b) The raid is assumed to consist of nonmaneuvering, radially closing cruise missiles that proceed in a straight line toward the CVA.

(c) Both missile speed and CAP aircraft speed are held constant.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Cruise missiles

  2.1) Quantitative Factor:

    (a) average velocity of the missile raid

  2.2) Tactics:

    2.2.1) Qualitative Factors:

      (a) nonmaneuvering

      (b) radially closing

3) Friendly Force Composition: CAP aircraft and CVA

  3.1) Platform Type: CAP aircraft

    3.1.1) Quantitative Factors:

      (a) CAP aircraft speed

      (b) CAP aircraft initial velocity

      (c) acceleration function of the CAP aircraft

    3.1.2) Deployment:

      3.1.2.1) Quantitative Factor:

        (a) CAP on-station distance from CVA

  3.2) Platform Type: CVA

    3.2.1) Tactics:

      3.2.1.1) Quantitative Factor:

        (a) reaction time from detection until CAP aircraft leaves to intercept

4) Friendly Force - Threat Interaction:

  4.1) Platform - Platform: CVA - Cruise missiles

    4.1.1) Quantitative Factors:

      (a) angle between the line-of-sight to the CAP station and the approach vector of the incoming missile raid

      (b) specified distance of intercept from the CVA

Intasystems

STUDY REVIEW SUMMARY NO. (2)-4

A. STUDY DESCRIPTION

1) Originating Activity: ARINC Research Corporation, Washington, D.C.
2) Report Title: A Cost Effectiveness Study of the F4B Airborne Missile Control System
3) Authors: P.E. Oyerly, E. Rappaport, J.F. Thiel, D. Lott and D. Frick
4) Report Number: Publication No. 285-01-2-399 (AD-356 908)
5) Date: 1 October 1963
6) Classification: Confidential
7) Contract: N123 (61756) 32994A (PMR) (U.S. Naval Missile Center)
8) Abstract: The purpose of this study was to develop a cost-effectiveness model for an airborne missile control system (AMCS). The primary purpose of the model is to provide a basis for determining how available funds can best be used to increase the effectiveness of the AMCS, or, conversely, where a cut in funds would have least influence on system effectiveness. It is believed that this study represents the first comprehensive measurement of all the pertinent system attributes of the system, subsystem, and unit level for a complex electronic system during actual operational usage.
9) Descriptors: Aircraft, air-to-air missile, availability, cost, cost effectiveness, fire control, maintainability, missile, radar, reliability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Subsystem
2) Function: Airborne AAW
3) Tactical Situation: Performance of mission mix
   3.1) Definition: A squadron of F-4 aircraft, equipped with an airborne missile control system, perform a variety of mission types such as: Act as Bogey, Combat Air Patrol, Deck-Launched Intercept, Escort Fighter, Maintenance Test, Mirror Landing Practice, Pilot Familiar-

ization, Carrier Qualification, Sparrow III Launch Attempt, Sidewinder Launch Attempt, Cross Country and In-Flight Refueling.

3.2) Criterion For Success: Performance of mission functions when utilized

3.3) MOE Selected: Total system operating costs over a specified time for a specified system utilization

3.3.1) Rationale For Selection: This MOE is a measure of cost-effectiveness for an airborne missile control system. In developing the cost-effectiveness measure, it was recognized that cost alone is not the sole criterion for measuring system worth. Accordingly, the measure was developed in such a manner that other important system attributes, such as reliability, would be presented simultaneously. This measure thus provides a broad basis for intelligent management and technical decisions.

3.4) Functional Form Of MOE:

$MOE = f(x_1, \ldots, x_4)$

where

x_1 = system initial cost amortized over specified time period

$= g_1(x_5, x_6)$

x_2 = system special test and bench equipment cost amortized over specified time period

$= g_2(x_5, x_7)$

x_3 = system operating cost for Radar Intercept Officer over specified time period

$= g_3(x_8, x_9)$

x_4 = system maintenance cost over specified time period

$= g_4(x_{11}, x_{12}, x_{13}, x_{18})$

x_5 = specified calendar time over which costs are computed

x_6 = system initial cost

x_7 = system special test and bench equipment cost

x_8 = total system operating hours required for specified time period

$= h_1(x_5, x_{10})$

x_9 = hourly cost for Radar Intercept Officer

x_{10} = system utilization factor

x_{11} = number of subsystems of airborne missile control system

x_{12} = a vector whose $i^{\underline{th}}$ component represents the average cost to maintain the $i^{\underline{th}}$ subsystem when it requires maintenance

$= h_2(x_{14}, x_{15}, x_{16})$

x_{13} = a vector whose $i^{\underline{th}}$ component represents the repair rate for the $i^{\underline{th}}$ subsystem

x_{14} = number of levels of maintenance

x_{15} = a matrix whose $(i,j)^{\underline{th}}$ entry represents the probability of the $i^{\underline{th}}$ subsystem being repaired at the $j^{\underline{th}}$ level of maintenance

x_{16} = a matrix whose $(i,j)^{\underline{th}}$ entry represents the average cost incurred when the $i^{\underline{th}}$ item is repaired at the $j^{\underline{th}}$ maintenance level

$= i(x_{17}, x_{18}, x_{19})$

x_{17} = a matrix whose $(i,j)^{\underline{th}}$ entry represents the average repair time (man-hours) of the $i^{\underline{th}}$ subsystem in the $j^{\underline{th}}$ level of maintenance

x_{18} = a matrix whose $(i,j)^{\underline{th}}$ entry represents the hourly rate for maintenance men when repairing the $i^{\underline{th}}$ subsystem at the $j^{\underline{th}}$ level of maintenance

x_{19} = a matrix whose $(i,j)^{\underline{th}}$ entry represents the average cost of parts to maintain the $i^{\underline{th}}$ subsystem at the $j^{\underline{th}}$ level of maintenance

4) MOE Usage In Study: The MOE was formulated and used to evaluate in detail the cost-effectiveness of the AERO 1A airborne missile control system over several time periods.

5) Special Study Assumptions:

   (a) The airborne missile control system and its associated test equipment are considered to have useful lives of a specified duration and depreciated linearly.

   (b) The only operating costs that are considered are the charges of the Radar Intercept Officer.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Friendly Force Composition: Airborne missile control system, Radar Intercept Officer, and maintenance shop

   2.1) Platform Type: Airborne missile control system

      2.1.1) Quantitative Factors:

         (a) system initial cost

         (b) number of subsystems of airborne missile control system

         (c) a matrix whose $(i,j)^{th}$ entry represents the average cost of parts to maintain the i^{th} subsystem at the j^{th} level of maintenance

      2.1.2) Deployment:

         2.1.2.1) Quantitative Factors:

            (a) specified calendar time over which costs are computed

            (b) system utilization factor

   2.2) Platform Type: Radar Intercept Officer

      2.2.1) Quantitative Factor:

         (a) hourly cost for Radar Intercept Officer

   2.3) Platform Type: Maintenance shop

2.3.1) Quantitative Factors:

(a) system special test and bench equipment cost

(b) a vector whose $i^{\underline{th}}$ component represents the repair rate for the $i^{\underline{th}}$ subsystem

(c) number of levels of maintenance

(d) a matrix whose $(i,j)^{\underline{th}}$ entry represents the probability of the $i^{\underline{th}}$ subsystem being repaired at the $j^{\underline{th}}$ level of maintenance

(e) a matrix whose $(i,j)^{\underline{th}}$ entry represents the average repair time (man-hours) of the $i^{\underline{th}}$ subsystem in the $j^{\underline{th}}$ level of maintenance

(f) a matrix whose $(i,j)^{\underline{th}}$ entry represents the hourly rate for maintenance men when repairing the $i^{\underline{th}}$ subsystem at the $j^{\underline{th}}$ level of maintenance

(3)

AIRBORNE ATTACK

STUDY REVIEW SUMMARY NO.(3)-1

A. STUDY DESCRIPTION

1) Originating Activity: North American Rockwell Corporation, Columbus, Ohio
2) Paper Title: "Cost-Effectiveness Evaluation for Mixes of Naval Air Weapons Systems"
3) Author: B. S. Albert
4) Source: Operations Research, Vol. 11, No. 2, Mar.-Apr. 1963, pp.173-193
5) Classification: Unclassified
6) Contract: NOa(s)59-6074c (Bureau of Naval Weapons)
7) Abstract: The problem for solution was to develop a method for determining cost and a means for employing cost together with effectiveness in the determination of the next generation attack air weapon system for the Navy. A number of factors in this problem are different from the usual cost-effectiveness problem; some aircraft being procured today will still be in use, Naval aircraft have multiple mission capability (both strategic and tactical), the ships being built now or already commissioned are those from which next generation aircraft will operate and aircraft carriers have a fixed area for accomodating aircraft and relatively fixed personnel accomodations. In addition, when we consider a decision that must be made in the relatively near future, we must allow that not only is the total Naval budget limited, but that each kind of major procurement appropriation (for aircraft, missiles, ships, etc.) is also limited. These considerations led to the formulation of a weapon system costing methodology that was tailored to Naval operating characteristics, and the development of a cost-effectiveness evaluation methodology by way of linear programming to include the various restrictions cited.
8) Descriptors: Airborne attack, aircraft, carrier, carrier based aircraft, cost effectiveness, linear programming, missile, surface ship, target mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Airborne Attack
3) Mission: Air strike

3.1) Definition: A mix of attack aircraft, defense aircraft, and missiles attack a variety of targets in a mix of war types (nuclear all-out, nuclear limited, and conventional limited).

3.2) Criterion For Success: Destruction of target

3.3) MOE Selected: Weighted maximum effectiveness for a mix of conflicts, tactical profiles (mission profile and tactics) and war importance factors

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_5)$$

where

x_1 = vector of importance factors for each type of war

x_2 = vector of specified achievement levels for each type of war

x_3 = vector of vehicle carrier survival probabilities for a specified type of conflict and tactical profile

x_4 = vector of time proportions for which a particular tactical profile is employed during a given conflict type

x_5 = vector of maximum total system effectiveness for a given tactical profile and conflict type

$= g(x_6, x_9, x_{10}, \ldots, x_{32})$

x_6 = vector of weighted effectiveness per vehicle for each target type, given a tactical profile and conflict type

$= h(x_7, x_8)$

x_7 = vector of effectiveness of a particular vehicle attacking a particular type of target

x_8 = vector of relative target values

x_9 = vector of attack aircraft costs

x_{10} = attack aircraft cost budget

x_{11} = vector of defense aircraft costs

x_{12} = defense aircraft cost budget

x_{13} = vector of attack missile costs

x_{14} = attack missile cost budget

x_{15} = vector of defense missile costs

x_{16} = defense missile cost budget

x_{17} = vector of ship cost-basic aircraft or missile pro-rata share of its basing ship's cost

x_{18} = budget for ship cost-basic aircraft or missile pro-rata share of its basing ship's cost

x_{19} = vector of installation costs

x_{20} = installation cost budget

x_{21} = vector of minimum production levels for each vehicle

x_{22} = total minimum production level for all vehicles

x_{23} = vector of maximum production levels for each vehicle

x_{24} = total maximum production level for all vehicles

x_{25} = vector of target distribution for a given vehicle type, tactical profile and conflict type

x_{26} = total number of targets that can be attacked by a given vehicle for given tactical profile and conflict type

x_{27} = vector of available deck space per attack aircraft

x_{28} = total available deck space for attack aircraft

x_{29} = vector of available deck space per defense aircraft

x_{30} = total available deck space for defense aircraft

x_{31} = vector of personnel required per air vehicle

x_{32} = total personnel available for air vehicles

3.5) Additional MOE's Identified:

(a) Weighted effectiveness per aircraft (or missile) = (Effectiveness of any vehicle in destroying a target class per aircraft (or missile)) x (Relative value of target)

(b) Effectiveness of attack vehicles = targets destroyed per aircraft over a given time period

(c) Effectiveness of defense vehicles = number of bombers attacked against a point raid for a given time period

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) Each target in a class is assumed to have the same kill probability when attacked by any one particular type of air vehicle.

(b) Objective function and constraints are linear so that linear programming can be employed.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Not stated

3) Target Composition: Not stated

3.1) Quantitative Factor:

(a) vector of relative target values

4) Friendly Force Composition: Attack aircraft, defense aircraft, carrier, and factory

4.1) Platform Type: Attack aircraft

4.1.1) Quantitative Factors:

(a) vector of attack aircraft costs

(b) attack aircraft cost budget

4.1.2) Armament: Attack missiles

4.1.2.1) Quantitative Factors:

(a) vector of attack missile costs

(b) attack missile cost budget

4.2) Platform Type: Defense aircraft

4.2.1) Quantitative Factors:

(a) vector of defense aircraft costs

(b) defense aircraft cost budget

4.2.2) Armament: Defense missiles

4.2.2.1) Quantitative Factors:

(a) vector of defense missile costs

(b) defense missile cost budget

4.3) Platform Type: Carrier

4.3.1) Quantitative Factors:

(a) vector of ship cost-basic aircraft or missile pro-rata share of its basing ship's cost

(b) budget for ship cost-basic aircraft or missile pro-rata share of its basing ship's cost

(c) vector of installation costs

(d) installation cost budget

(e) vector of available deck space per attack aircraft

(f) total available deck space for attack aircraft

(g) vector of available deck space per defense aircraft

(h) total available deck space for defense aircraft

(i) vector of personnel required per air vehicle

(j) total personnel available for air vehicles

4.4) Platform Type: Factory

4.4.1) Quantitative Factors:

(a) vector of minimum production levels for each vehicle

(b) total minimum production level for all vehicles

(c) vector of maximum production levels for each vehicle

(d) total maximum production level for all vehicles

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform: Carrier - Threat

5.1.1) Quantitative Factor:

(a) vector of vehicle carrier survival probabilities for a specified type of conflict and tactical profile

6) Friendly Force - Target Interaction:

6.1) Platform - Platform: Attack aircraft/defense aircraft - Target

6.1.1) Quantitative Factors:

(a) vector of effectiveness of a particular vehicle attacking a particular type of target

(b) vector of target distribution for a given vehicle type, tactical profile and conflict type

(c) total number of targets that can be attacked by a given vehicle for given tactical profile and conflict type

(d) vector of time proportions for which a particular tactical profile is employed during a given conflict type

(e) vector of importance factors for each type of war

(f) vector of specified achievement levels for each type of war

STUDY REVIEW SUMMARY NO. (3)-2

A. STUDY DESCRIPTION

1) Originating Activity: North Americal Rockwell Corporation, Columbus, Ohio
2) Paper Title: Addendum to "Cost-Effectiveness Evaluation for Mixes on Naval Air Weapons Systems"
3) Author: G. P. Jones
4) Source: Operations Research, Vol. 11, No. 2, Mar.-Apr. 1963, pp. 189-193
5) Classification: Unclassified
6) Contract: NOa(s) 59-6074c (Bureau of Naval Weapons)
7) Abstract: The method describ . in the paper to which this article is an addendum forms the basis of the selection procedure used in the Navy Attack Study (contract number NOa(s) 59-6074c). An effectiveness measure is formulated for aid in the evaluation of the Carrier Force.
8) Descriptors: Airborne attack, carrier, carrier based aircraft, cost effectiveness, linear programming, survivability, target mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Airborne Attack
3) Mission: Air strike
   - 3.1) Definition: Aircraft launched from a carrier penetrate area and local defenses to attack a mix of targets.
   - 3.2) Criterion For Success: Destruction of target
   - 3.3) MOE Selected: Expected number of targets destroyed in a given period of time
   - 3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_{10})$$

where

x_1 = probability of reliable operation of the air attack system

x_2 = probability of correct navigation to and identification of the target

x_3 = probability of damaging the target

x_4 = number of flights in an engagement

x_5 = number of aircraft participating in each flight

x_6 = probability that the aircraft survives the area defenses on the in-going leg of each flight

x_7 = probability that the base vessel survives until the launching of the flight

x_8 = number of local defenses encountered in the flight

x_9 = number of target opportunities per aircraft within each local defense area

x_{10} = probability that the aircraft survives a local defense to or from the target (or targets) on the flight

4) MOE Usage In Study: Formulation only

C. <u>EFFECTIVENESS FACTORS</u>

1) Physical Environment: Not stated
2) Threat Composition: Not stated
   2.1) Quantitative Factor:
      (a) number of local defenses encountered in the flight
3) Target Composition: Not stated
4) Friendly Force Composition: Carrier and aircraft
   4.1) Platform: Aircraft
      4.1.1) Quantitative Factors:
         (a) probability of reliable operation of the air attack system
         (b) probability of correct navigation to and identification of the target
         (c) number of flights in an engagement
         (d) number of aircraft participating in each flight
5) Friendly Force - Threat Interaction:
   5.1) Platform - Platform:
      5.1.1) Type: Aircraft - Threat

5.1.1.1) Quantitative Factors:

(a) probability that the aircraft survives the area defenses on the in-going leg of each flight

(b) probability that the aircraft survives a local defense to or from the target (or targets) on the flight

5.1.2) Type: Carrier - Threat

5.1.2.1) Quantitative Factor:

(a) probability that the base vessel survives until the launching of the flight

6) Friendly Force - Target Interaction:

6.1) Platform - Platform: Aircraft - Target

6.1.1) Quantitative Factors:

(a) probability of damaging the target

(b) number of target opportunities per aircraft within each local defense area

STUDY REVIEW SUMMARY NO.(3)-3

A. STUDY DESCRIPTION

1) Originating Activity: Vought Aeronautics Division, LTV Aerospace Corporation, Dallas, Texas
2) Report Title: Research Investigations in Naval Attack Aircraft, Including Armament, Volumes 1-3
3) Author: B. G. Kohr
4) Report Numbers: 2-55100/9R-2656, -2657, -2658 (AD-507 443, AD-507 444, AD-507 445)
5) Date: December 1969
6) Classification: Confidential
7) Contract: N00019-68-C-0326 (Naval Air Systems Command)
8) Abstract: This report evaluates and compares four attack aircraft. One subsonic close air support mission aircraft and three deep strike aircraft with speed capabilities nominally supersonic, subsonic, and transonic. The effects of variation of mission design parameters were investigated within the framework of the baseline design; variations caused by changes in ordnance load, dash radius, total mission radius, and time-on-station were included along with alternate mission capabilities. Extensive operational analyses evaluate performance effectiveness, payload potential, sortie rate sensitivity, survival against enemy air defenses, and target kill potential for the close air support and deep strike designs.
9) Descriptors: Airborne attack, aircraft, cost, close air support, design, exponential density function, Poisson density function, survivability, target mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System

2) Function: Airborne Attack

3) Mission: Close air support

3.1) Definition: Aircraft attack hostile ground targets which are close to friendly forces.

3.2) Criterion For Success: Successful attack capability

3.3) MOE's Selected:

$(MOE)_1$ = Payload potential

$(MOE)_2$ = Number of sorties performed within a specified operational period

$(MOE)_3$ = Force size requirement for fulfilling 90% of close air support requests

3.3.1) Rationale For Selection: The payload potential is a scale of measure for the evaluation of the load-carrying performance of close air support aircraft. The number of sorties performed within a specified operational period is a measure of the aircraft sortie rate capability. The number of close air support aircraft required to be on-station in order to satisfy 90% of all requests is a measure of total force requirements for close air support.

3.4) Functional Form Of MOE's:

$$(MOE)_1 = f_1(x_1, x_2, x_3)$$

$$(MOE)_2 = f_2(x_4, \ldots, x_8)$$

$$(MOE)_3 = f_3(x_{16}, x_{17})$$

where

x_1 = payload weight

x_2 = radius of action

x_3 = takeoff gross weight

x_4 = aircraft availability factor

$= g_1(x_9, \ldots, x_{15})$

x_5 = mission cycle time

x_6 = time on-station

x_7 = turn around time

x_8 = length of flying day

x_9 = mean time between failure for airframe system

x_{10} = mean time between failure for avionic system

x_{11} = mean time to repair airframe system

x_{12} = mean time to repair avionic system

x_{13} = probability of being a deck dud due to failure in airframe system

x_{14} = probability of being a deck dud due to failure in avionic system

x_{15} = probability of survival from previous sortie

x_{16} = aircraft request rate

x_{17} = number of aircraft required to support one aircraft on-station

$= g_2(x_6, x_8, x_{18})$

x_{18} = number of sorties per flying day for one aircraft

$= h(x_4, \ldots, x_8)$

4) MOE Usage In Study: MOE's were formulated and used to evaluate the relative effectiveness of close air support and various deep strike aircraft designs.

5) Special Study Assumptions:

(a) In the case of payload potential, full internal fuel and 5-minute military rated thrust at sea level was assumed.

(b) In the case of sortie rate capabilities,

(b.1) The aircraft is in the target area at the start of the first flying day.

(b.2) The aircraft continues cycling until no time is left to attack targets that day.

(b.3) Exponential failure laws for each of two complementary subsystems is assumed.

(b.4) The aircraft is fully checked out after each cycle, and the ready force at the beginning of any cycle becomes a homogeneous group of zero flight time aircraft.

(c) In the case of force size requirement,

(c.1) The aircraft loiters near the battle area until directed to a target by requests arriving randomly from a forward air controller, or until the fuel available for loiter is exhausted.

(c.2) Aircraft requests are randomly distributed in time with equal probability of occurrence in any given interval during the flight day (i.e., the expected occurrence in any time interval follows a Poisson distribution).

(c.3) Ordnance is loaded at a constant specified rate.

(c.4) Re-arming and other re-cycling tasks requiring 0.5 hours are performed concurrently during the first half hour of the turn-around cycle.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) length of flying day

2) Threat Composition: Antiaircraft artillery and surface-to-air missiles

3) Target Composition: Medium tank and rocket launcher/truck

4) Friendly Force Composition: Aircraft

4.1) Quantitative Factors:

(a) payload weight

(b) radius of action

(c) takeoff gross weight

(d) turn around time

(e) mean time between failure for airframe system

(f) mean time between failure for avionic system

(g) mean time to repair airframe system

(h) mean time to repair avionic system

(i) probability of being a deck dud due to failure in airframe system

(j) probability of being a deck dud due to failure in avionic system

4.2) Deployment:

4.2.1) Quantitative Factors:

(a) mission cycle time

(b) time on-station

(c) aircraft request rate

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform: Aircraft - AAA and SAM's

5.1.1) Quantitative Factor:

(a) probability of survival from previous sortie

STUDY REVIEW SUMMARY NO. (3)-4

A. STUDY DESCRIPTION

1) Originating Activity: Lockheed California Company, Burbank, California
2) Report Title: Final Report Navy Close Support Aircraft Study
3) Report Number: LR 21063, LAC 618582
4) Date: 18 December 1967
5) Classification: Secret
6) Contract: N00019-67-C-0290 (Naval Air Systems Command)
7) Abstract: The primary objective in this study was to generate data contributing to a proposed technical approach for future U.S. Navy requirements for carrier based close support aircraft. In this context, a baseline concept was established and the design was parametrically varied and evaluated from the standpoint of vehicle performance, armor, armament, and avionics consideration pertinent to mission considerations in order to provide information necessary for judgments on a cost-effectiveness basis.
8) Descriptors: Airborne attack, aircraft, antiaircraft gunnery, close air support, cost effectiveness, detection, gun, kill probability, radar, surface target, target mix, visual, weapon mix, survivability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne Attack
3) Mission: Close air support
   3.1) Definition: Aircraft attack hostile targets which are close to friendly forces.
   3.2) Criterion For Success: Destruction of target
   3.3) MOE's Selected:
   $(MOE)_1$ = Expected number of targets killed per day
   $(MOE)_2$ = Expected number of targets killed during the system's lifetime

3.3.1) Rationale For Selection: $(MOE)_1$, called the user's measure, reflects the daily operational capability of the system and is a function of the in-commission rate of the Close Air Support system. $(MOE)_2$, called the buyer's measure, is based on system lifetime capability rather than daily capability.

3.4) Functional Form Of MOE's:

$$(MOE)_1 = f_1(x_1, \ldots, x_7)$$

$$(MOE)_2 = f_2(x_1, \ldots, x_4, x_6)$$

where

x_1 = in-commission probability

$= g_1(x_8, x_9, x_{10})$

x_2 = target encounter probability

$$= \begin{cases} 1.0 & \text{if no loiter is allowed} \\ g_2(x_{11}, x_{12}) & \text{if loiter is allowed} \end{cases}$$

x_3 = target acquisition probability

$= g_3(x_{13}, \ldots, x_{20})$

x_4 = survival to target probability

$= g_4(x_{21}, \ldots, x_{35})$

x_5 = target kill probability

x_6 = overall survival probability

$= g_5(x_{21}, \ldots, x_{35})$

x_7 = number of sorties per day

$= g_6(x_{11}, x_{36}, \ldots, x_{45})$

x_8 = frequency of subsystem failures

x_9 = repair time distribution

x_{10} = fault isolation time

x_{11} = loiter time

x_{12} = mean time between target occurrences

x_{13} = aircraft speed

x_{14} = aircraft altitude

x_{15} = target size

x_{16} = target/background contrast

x_{17} = search time

x_{18} = search area

x_{19} = range to target

x_{20} = number of sensor resolution elements

x_{21} = number of guns

x_{22} = number of gun locations

x_{23} = firing time duration

x_{24} = probability of visual detection of the aircraft

x_{25} = probability of radar detection of the aircraft

x_{26} = rate of fire

x_{27} = number of rounds available

x_{28} = gun effective range

x_{29} = gun slewing rate

x_{30} = gun elevation angle limit

x_{31} = gun depression angle limit

x_{32} = ready-to-fire time
$= h_1(x_{46}, \ldots., x_{51})$

x_{33} = probability of the projectile hitting the target
$= h_2(x_{52}, \ldots., x_{55})$

x_{34} = aircraft vulnerable area

x_{35} = probability of kill given hit by a particular projectile type

x_{36} = command and control time

x_{37} = time to take-off and rendezvous at the base

x_{38} = time for transit from the base to the forward edge of the battle area (FEBA)

x_{39} = time to transit from the FEBA to the target area

x_{40} = time to detect the target

x_{41} = time to convert on the target

x_{42} = time to deliver ordnance

x_{43} = time to return to FEBA

x_{44} = time to transit from FEBA to the base

x_{45} = time to perform all functions necessary to enable the aircraft to go on another close support sortie

x_{46} = time to detect aircraft

x_{47} = time of first detection

x_{48} = sound alarm time

x_{49} = gun battery reaction time

x_{50} = visual tracking time

x_{51} = radar tracking time

x_{52} = distance from gun site to intercept

x_{53} = projectile miss distance (standard deviation)

x_{54} = relative azimuth approach angle of the projectile

x_{55} = relative elevation approach angle of the projectile

3.5) Additional MOE Identified:

(a) Total system cost for a prescribed level of effectiveness

4) MOE Usage In Study: $(MOE)_1$ and $(MOE)_2$ were computed for seven targets and four aircraft candidate configurations. In addition, an overall MOE was computed as follows: Overall $(MOE)_i = .6(MOE)_i(\text{with loiter})_i + .4(MOE)_i(\text{no loiter})$ for $i = 1,2$.

5) Special Study Assumptions:

(a) When an aircraft is sent on a mission, it is loaded only with the best weapon for the target it is going to attack.

(b) A mix of the same type of aircraft, each carrying a different type weapon, was assumed to be on loiter status if more than one target type was to be attacked.

(c) Each aircraft dropped or launched its entire weapon load on the target.

(d) External store payload is invulnerable to the anti-air defenses.

(e) Target acquisition probability is dependent upon target type, but independent of the type of aircraft.

(f) In-commission probability is constant and the same for all aircraft configurations.

(g) All sources of projectile dispersion are regarded as random effects and are independent from shot to shot.

(h) Projectile trajectories are based on nominal trajectory data.

(i) Line-of-sight for determining range from gun site to aircraft is based upon measurements from a stored grid of terrain elevations.

(j) Loiter occurs on the friendly side of the FEBA.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Anti-aircraft artillery

2.1) Quantitative Factors:

(a) number of guns

(b) number of gun locations

(c) firing time duration

(d) rate of fire

(e) gun effective range

(f) gun slewing rate

(g) gun elevation angle limit

(h) gun depression angle limit

(i) sound alarm time

(j) gun battery reaction time

2.2) Sensors: Visual and radar

2.2.1) Type: Visual

2.2.1.1) Quantitative Factors:

(a) probability of visual detection of the aircraft

(b) visual tracking time

2.2.2) Type: Radar

2.2.2.1) Quantitative Factors:

(a) probability of radar detection of the aircraft

(b) radar tracking time

2.3) Armament: Projectiles

2.3.1) Quantitative Factors:

(a) number of rounds available

(b) probability of kill given a hit by a particular projectile type

(c) projectile miss distance (standard deviation)

(d) relative azimuth approach angle of the projectile

(e) relative elevation approach angle of the projectile

3) Target Composition: Personnel (100 men), light artillery mortar (or AAA emplacement), vehicle, armored personnel carrier (APC), medium tank and bridge

3.1) Quantitative Factors:

(a) target size

(b) target/background contrast

(c) mean time between target occurrences

4) Friendly Force Composition: Aircraft

4.1) Quantitative Factors:

(a) frequency of subsystem failures

(b) repair time distribution

(c) fault isolation time

(d) aircraft speed

(e) aircraft altitude

(f) aircraft vulnerable area

4.2) Sensor: Not stated

4.2.1) Quantitative Factor:

(a) number of sensor resolution elements

4.3) Armament: General purpose bombs, penetration bombs, guided bombs, cluster bombs, fire bombs, dispensers, rocket pods, gun pods, mines, air-to-ground missiles, and anti-radiation missiles

4.4) Tactics:

4.4.1) Quantitative Factors:

(a) command and control time

(b) time to take-off and rendezvous at the base

(c) time for transit from the base to the forward edge of the battle area (FEBA)

(d) time to transit from the FEBA to the target area

(e) time to detect the target

(f) time to convert on the target

(g) time to deliver ordnance

(h) time to return to FEBA

(i) time to transit from FEBA to the base

(j) time to perform all functions necessary to enable the aircraft to go on another close support sortie

(k) loiter time

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform: Aircraft - Anti-aircraft artillery

5.1.1) Quantitative Factors:

(a) time to detect aircraft

(b) time of first detection

(c) distance from gun site to intercept

6) Friendly Force - Target Interaction:

6.1) Platform - Platform: Aircraft - Target

6.1.1) Quantitative Factors:

(a) search time

(b) target kill probability

(c) search area

(d) range to target

Ultrasystems

STUDY REVIEW SUMMARY NO. (3)-5

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Weapons Center, China Lake, California
2) Report Title: Tactical Air Armament Study Phase 1B, Vol. I Summary Report, and Vol II analyses of Specific Subjects, Chapter 4 - Utility and Cost Effectiveness of Data Link Controlled Electro Optical Guide Weapons
3) Report Number: NWC Document 12-803
4) Date: May 1970
5) Classification: Secret
6) Abstract: The intent of this investigation is to determine whether the change in tactics allowed through the use of data link control increases the utility and effectiveness of the system. Comparison of the Data-Link Walleye II with the Walleye II gives the difference in cost created by the use of data link. Since the cost to achieve target kill criteria is indicative of the utility and effectiveness of the system, a measure of the utility and effectiveness of a data link is established. The investigation takes the form of a simple cost effectiveness comparison of the Walleye II system with and without data link control.
7) Descriptors: Airborne attack, aircraft, antiaircraft defense, antiaircraft gunnery, attrition, bomb, bridge, data link, interdiction, kill probability, reliability, surface-to-air missile

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne Attack
3) Mission: Interdiction
   - 3.1) Definition: Aircraft launched from an offshore CVA penetrates through an area defended by AAA and SAM sites to attack a bridge and a power plant.
   - 3.2) Criterion For Success: Destruction of target

3.3) MOE Selected: Probable destroyed value of the target = (probability of achieving the target kill criteria) X (target's assigned military value)

3.4) Functional Form Of MOE:

$$MOE = \begin{cases} f(x_1, \ldots, x_7, x_9, x_{11}, x_{12}, x_{13}, x_{15}, x_{17}, x_{18}, x_{19}), & \text{for bridge target} \\ f(x_1, \ldots, x_6, x_8, x_{10}, x_{11}, x_{12}, x_{14}, x_{16}, x_{17}, x_{18}, x_{20}), & \text{for power plant target} \end{cases}$$

where

x_1 = aircraft weapon load

x_2 = salvo size

x_3 = maximum raid size

x_4 = raid size increment (step increase allowable to achieve the selected target kill probability)

x_5 = weapon launch range

x_6 = avionics reliability

x_7 = weapon reliability against bridge target

x_8 = weapon reliability against power plant target

x_9 = distance from carrier to bridge target

x_{10} = distance from carrier to power plant target

x_{11} = area defense radius

x_{12} = local defense radius

x_{13} = value of bridge target

x_{14} = value of power plant target

x_{15} = target planning kill level for bridge

x_{16} = target planning kill level for power plant

x_{17} = area defense attrition probability (losses per unit distance flown)

x_{18} = local defense attrition probability (losses per unit distance flown)

x_{19} = probability of dropping one span of the bridge

x_{20} = probability of achieving a fractional kill of 50% or better against the power plant (i.e., the power output is reduced 50% or more)

3.5) Additional MOE Identified:

(a) Cost Effectiveness, defined as the ratio of the probable destroyed value to total costs incurred while achieving the desired effectiveness (NOTE: Total costs include weapon procurement cost, weapon RDT&E cost (amortized over the weapon buy level), aircraft operating costs, and replacement cost for aircraft lost due to attrition by enemy defensive systems.)

4) MOE Usage In Study: Basis for comparison of the Walleye II system with and without data link control

5) Special Study Assumptions:

(a) Only one weapon is dropped (per section) on each pass at the target.

(b) Each aircraft attacks only one target on a given mission.

(c) The number of weapons launched against each target is the smallest integral number for which the probability of inflicting the required target kill criteria is equal to or greater that the desired target kill level.

(d) Sufficient target information is available to allow the pilot to accurately launch the weapon at the target.

(e) Probability of detection and acquisition is 1.0, i.e., reconnaissance information is sufficient to allow the attack pilot to locate the target.

(f) Strikes are only made in clear weather (visual acuity and radar resolution are excellent).

(g) The bridge and thermal power plant are assigned the same military value.

(h) The probability the system will not acquire and lock on is contingent on the system avionics performance and is reflected in the reliability values of avionics and the weapons.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Qualitative Factor:

(a) clear weather

2) Threat Composition: Anti-aircraft artillery and surface-to-air missiles

2.1) Platform Type: Anti-aircraft artillery

2.1.1) Quantitative Factor:

(a) local defense radius

2.1.2) Deployment:

2.1.2.1) Qualitative Factor:

(a) local defense at targets

2.2) Platform Type: Surface-to-air missiles (SA-2 and SA-3)

2.2.1) Quantitative Factor:

(a) area defense radius

2.2.2) Deployment:

2.2.2.1) Qualitative Factor:

(a) area defense surrounding targets

3) Target Composition: Bridge and power plant

3.1) Platform Type: Bridge

3.1.1) Quantitative Factors:

(a) value of bridge target

(b) target planning kill level for bridge

3.2) Platform Type: Power plant

3.2.1) Quantitative Factors:

(a) value of power plant target

(b) target planning kill level for power plant

4) Friendly Force Composition: Aircraft and carrier (CVA)

4.1) Platform Type: Aircraft

4.1.1) Quantitative Factors:

(a) aircraft weapon load

(b) maximum raid size

(c) raid size increment

(d) avionics reliability

4.1.2) Armament: Walleye II bombs:

4.1.2.1) Quantitative Factors:

(a) salvo size

(b) weapon launch range

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform:

5.1.1) Type: Aircraft - Anti-aircraft artillery

5.1.1.1) Quantitative Factor:

(a) local defense attrition probability

5.1.2) Type: Aircraft - Surface-to-air missiles

5.1.2.1) Quantitative Factor:

(a) area defense attrition probability

6) Friendly Force - Target Interaction:

6.1) Platform - Platform:

6.1.1) Type: Carrier - Bridge

6.1.1.1) Quantitative Factor:

(a) distance from carrier to bridge target

6.1.2) Type: Carrier - Power plant

6.1.2.1) Quantitative Factor:

(a) distance from carrier to power plant target

6.2) Armament - Platform:

6.2.1) Type: Walleye II bombs - Bridge

6.2.1.1) Quantitative Factors:

(a) probability of dropping one span of the bridge

(b) weapon reliability against bridge target

6.2.2) Type: Walleye II bombs - Power plant

6.2.2.1) Quantitative Factors:

(a) probability of achieving a fractional kill of 50% or better against the power plant

(b) weapon reliability against power plant target

Litasystems

STUDY REVIEW SUMMARY NO. (3)-6

A. STUDY DESCRIPTION

1) Originating Activity: Naval Air Systems Command, Bureau of Naval Weapons, Washington, D.C.
2) Report Title: Close Support Effectiveness of the VAX and Other Aircraft
3) Authors: S.L. Taffel, M.R. Perry and C.M. Boorer
4) Report Number: R-581-62-1/2 (AD-375 708)
5) Date: December 1962
6) Classification: Confidential
7) Abstract: A model is developed which analytically describes a particular type of close support operation. In this operation aircraft occupying a specified deck space are used to maintain weapons on-station near the area of ground operations so that air support will be readily available when needed by ground forces. The average number of weapons on-station is determined after consideration of aircraft spotting, performance characteristics, maintenance and loading requirements, weapons characteristics, and the effect of weather restrictions. The results are presented in a form which permits separate effectiveness comparisons for each weapon type studied. It is concluded that the VAX provides a significantly higher average number of weapons on-station than do competing interim aircraft types.
8) Descriptors: Airborne attack, bomb, carrier, carrier based aircraft, close air support, missile, rocket, weapon mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne Attack
3) Mission: Close air support

3.1) Definition: Carrier based aircraft attempt to remain on-station near the vicinity of a ground force operating area.

3.2) Criterion For Success: Maintenance of sufficient weapons on station to provide rapid response to close support requests

3.3) MOE Selected: Average number of weapons on-station

3.3.1) Rationale For Selection: This measure takes into account the ordnance load carried by an aircraft and the portion of time during which this ordnance is available on-station.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2, x_3)$$

where

x_1 = number of aircraft spotted in the available carrier deck space

x_2 = a vector whose $i^{\underline{th}}$ component represents the number of type i weapons carried per aircraft

x_3 = probability that aircraft will be on-station at any time

= $g_1(x_4, x_5, x_6)$

x_4 = probability of operable weather for aircraft

x_5 = aircraft cycle time (i.e., the time between consecutive take-offs)

= $h(x_6, \ldots, x_9)$

x_6 = aircraft on-station time

= $i_1(x_2, x_{12})$

x_7 = time required for aircraft to fly to and return from station

= $i_2(x_2, x_{12})$

x_8 = aircraft average repair or not ready time

= $i_3(x_6, x_7, x_{10})$

x_9 = aircraft loading time

= $i_4(x_2, x_{11})$

x_{10} = not ready hours per flight hour

x_{11} = a vector whose i^{th} component is the unit weight of type i weapon

x_{12} = on-station radius

4) MOE Usage In Study: The MOE was formulated and used to compare the effectiveness of various aircraft weapon systems operating at a particular radius.

5) Special Study Assumptions:

(a) The aircraft takes off from the CVA and flies out, under best cruise conditions, to its station radius. Upon arrival on-station the aircraft descends to a specified altitude and remains there as long as fuel permits. The aircraft then returns to the CVA. On arrival at the carrier each aircraft is placed in a not ready state.

(b) Reliability is not included in the formulation of the measure of effectiveness due to the nonexistence of usable reliability data.

(c) The affect of weather is treated in an oversimplified manner because weather information of the type required is not readily available.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) probability of operable weather for aircraft

2) Friendly Force Composition: Carrier (CVA) and aircraft

2.1) Platform Type: Aircraft

2.1.1) Quantitative Factors:

(a) number of aircraft spotted in the available carrier deck space

(b) not ready hours per flight hour

2.1.2) Armament:

2.1.2.1) Quantitative Factor:

(a) a vector whose i^{th} component represents the number of type i weapons carried per aircraft

(b) a vector whose i^{th} component is the unit weight of a type i weapon

2.1.3) Deployment:

2.1.3.1) Quantitative Factor:

(a) on-station radius

A. STUDY DESCRIPTION

1) Originating Activity: Vought Aeronautics Division, LTV Aerospace Corporation, Dallas, Texas
2) Report Title: Advanced Carrier Based V/STOL Close Air Support Aircraft Requirements Study and Appendices
3) Author: H. L. Brautigam
4) Report Number: 2-55200/9R-50576 (AD-506 069 and AD-506 065)
5) Date: 28 May 1969
6) Classification: Secret (NOFORN) and Appendices - Confidential
7) Contract: N00019-68-C-0539 (Naval Air Systems Command)
8) Abstract: Study objectives placed primary emphasis on definition of design and operational requirements most appropriate for V/STOL and CTOL close air support systems envisioned for the 1975-1985 time period. The study has been structured and conducted so as to provide a detailed evaluation and comparison of the operational worth of V/STOL and CTOL performance parameters as they might be employed in the battle area. A comprehensive configuration/tactics evaluation model was employed extensively in the study to provide these results. The model permitted optimization of weapon delivery tactics for each type of V/STOL or CTOL aircraft, so that the resultant cost effectiveness comparisons (in terms of cost per target killed) would represent the best employment of each close air support system concept evaluated. Weapon delivery tactics were optimized to minimize overall cost per target destroyed.
9) Descriptors: Airborne attack, attrition, bomb, carrier based aircraft, cost, cost effectiveness, gun, kill probability, radar, surface-to-air missile, target mix, vulnerability, weapon mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System

2) Function: Airborne Attack

3) Mission: Close air support

3.1) Definition: Aircraft, under the direction of a forward controller, provide air support to ground forces in attacking a variety of ground targets.

3.2) Criterion For Success: Destruction of target

3.3) MOE Selected: Cost effectiveness, defined as cost per target killed

3.3.1) Rationale For Selection: This MOE provides a flexible tool for evaluating aircraft, in comparing aircraft capabilities to desired tactics, and in determining desirable configuration changes.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_4)$$

where

x_1 = cost of aircraft lost due to enemy action while conducting sufficient passes to obtain an expected kill on the target
$= g_1(x_5, x_6, x_7)$

x_2 = cost of aircraft lost due to normal operations in conducting a sufficient number of missions to obtain expected kill on the target
$= g_2(x_5, x_8, x_9)$

x_3 = cost of normal operations, operations cost, incurred in conducting a sufficient number of missions to achieve target kill
$= g_3(x_8, x_{10}, x_{11})$

x_4 = cost of ordnance required to achieve target kill
$= g_4(x_7, x_{12})$

x_5 = aircraft flyaway cost
$= h_1(x_{13}, \ldots, x_{16})$

x_6 = single pass aircraft attrition
$= h_2(x_{17}, x_{18})$

x_7 = number of passes required to achieve target kill

= $h_3(x_{19})$

x_8 = missions flown per target killed

= $h_4(x_{10}, x_{20}, x_{21})$

x_9 = aircraft operational loss rate per mission

x_{10} = aircraft operating cost per hour

= $h_5(x_{31}, x_{32})$

x_{11} = mission time in hours

x_{12} = cost of ordnance expended per pass

x_{13} = cost of avionics

x_{14} = cost of armor

x_{15} = cost of airframe

x_{16} = cost of propulsion system

x_{17} = probability that aircraft survives guns during one pass

= $i_1(x_{22}, x_{23})$

x_{18} = probability that aircraft survives Redeye-type missile during one pass

= $i_2(x_{24}, x_{25})$

x_{19} = probability that the target is destroyed on a single pass by the given ordnance, tactics, aircraft, avionics combination

= $i_3(x_{43}, x_{44})$

x_{20} = weight of unit ordnance

x_{21} = aircraft payload

x_{22} = probability that the aircraft is killed on a single pass by antiaircraft guns

= $j_1(x_{26}, \ldots, x_{31})$

x_{23} = expected number of guns available to effectively fire upon aircraft

= $j_2(x_{41}, x_{42})$

x_{24} = probability that the aircraft is killed on a single pass by Redeye missile

x_{25} = expected number of Redeye weapons fired per pass

x_{26} = aircraft bottom vulnerable area

x_{27} = aircraft side vulnerable area

x_{28} = aircraft front vulnerable area

x_{29} = expected hits per square foot of vulnerable area on bottom

x_{30} = expected hits per square foot of vulnerable area on side

x_{31} = expected hits per square foot of vulnerable area on front

x_{32} = total annual aircraft operating cost

$= i_4(x_{34}, \ldots, x_{40})$

x_{33} = number of flight hours per month

x_{34} = annual personnel cost

x_{35} = annual operating consumables cost

x_{36} = annual parts and rework cost

x_{37} = annual support materials cost

x_{38} = annual civil engineering cost

x_{39} = annual service wide operations cost

x_{40} = annual weapons and facilities cost

x_{41} = AA weapon density

x_{42} = AA weapon effective area

x_{43} = probability of hitting the target

$= j_3(x_{45}, x_{46})$

x_{44} = probability of killing the target given a hit

x_{45} = effective target radius

$= k(x_{48}, x_{49}, x_{50})$

x_{46} = circular error probable (bombs), or error in mils (for guns)

x_{47} = actual target radius

x_{48} = weapon damage radius

x_{49} = percent coverage required to achieve a kill

4) MOE Usage In Study: The MOE was formulated and used to provide a basis for a detailed evaluation and comparison of the operational worth of V/STOL and CTOL in the close air support role.

5) Special Study Assumptions:
   - (a) No fractional ordnance loads are employed.
   - (b) One bomb is expended per pass against the tank target.
   - (c) Against the soft company, two conventional bombs or one Rockeye is expended per pass.
   - (d) Passes are made until all ordnance is expended.
   - (e) The error sources for weapon delivery are assumed to be statistically independent.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: AA guns and Redeye-type missiles
   - 2.1) Platform Type: AA guns
     - 2.1.1) Quantitative Factor:
       - (a) AA weapon effective area
     - 2.1.2) Deployment:
       - 2.1.2.1) Quantitative Factor:
         - (a) AA weapon density

3) Target Composition: Tanks and personnel
   - 3.1) Quantitative Factor:
     - (a) actual target radius

4) Friendly Force Composition: Aircraft
   - 4.1) Quantitative Factors:
     - (a) cost of avionics
     - (b) cost of armor
     - (c) cost of airframe
     - (d) cost of propulsion system
     - (e) aircraft payload
     - (f) aircraft bottom vulnerable area
     - (g) aircraft side vulnerable area

(h) aircraft front vulnerable area

(i) annual personnel cost

(j) annual operating consumables cost

(k) annual parts and rework cost

(l) annual support materials cost

(m) annual civil engineering cost

(n) annual service wide operations cost

(o) annual weapons and facilities cost

4.2) Armament: Bombs and guns

4.2.1) Quantitative Factors:

(a) cost of ordnance expended per pass

(b) weight of unit ordnance

(c) circular error probable (bombs), or error in mils (for guns)

(d) weapon damage radius

4.3) Deployment:

4.3.1) Quantitative Factors:

(a) mission time in hours

(b) number of flight hours per month

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform:

5.1.1) Type: Aircraft - Redeye-type missiles

5.1.1.1) Quantitative Factors:

(a) probability that the aircraft is killed on a single pass by Redeye missile

(b) expected number of Redeye weapons fired per pass

5.1.2) Type: Aircraft - AA guns and Redeye-type missiles

5.1.2.1) Quantitative Factors:

(a) aircraft operational loss rate per mission

(b) expected hits per square foot of vulnerable area on bottom

(c) expected hits per square foot of vulnerable area on side

(d) expected hits per square foot of vulnerable area on front

6) Friendly Force - Target Interaction:

6.1) Armament - Platform: Bombs and guns - Tanks and personnel

6.1.1) Quantitative Factors:

(a) probability of killing the target given a hit

(b) percent coverage required to achieve a kill

STUDY REVIEW SUMMARY NO. (3)-8

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Postgraduate School, Monterey, California
2) Report Title: A Formulation of the Allocation of Attack Aircraft to Fixed Location Targets
3) Authors: P.A. Banks and K. Russell
4) Report Identification: Thesis for the Master of Science in Operations Research, (AD-475 306)
5) Date: May 1965
6) Classification: Unclassified
7) Abstract: This paper formulates a stochastic non-linear model for assigning a force of attack aircraft on a single sortie against fixed location targets. The number of aircraft alive at weapon release on any pass of a series against a given target is treated as a random variable. The total value of damage to all targets is taken as the measure of effectiveness and a particular form of the objective function derived. The parameters of the model and the form of the constraint equations are also discussed.
8) Descriptors: Airborne attack, aircraft, antiaircraft defense, attrition, force allocation, kill probability, nonlinear integer programming, survivability, target mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne Attack
3) Mission: Air strike
   3.1) Definition: A mix of attack aircraft are allocated to attack fixed location targets.
   3.2) Criterion For Success: Destruction of target

3.3) MOE Selected: Maximum total value of damage inflicted upon all targets

3.4) Functional Form Of MOE:

$$MOE = \max_{x_1} f(x_1, \ldots, x_{12})$$

where

$f(x_1, \ldots, x_{12})$ = total expected value of damage to all targets if attack aircraft are allocated by strategy matrix x_1

x_1 = a matrix whose $(i,j)^{\underline{th}}$ entry represents the number of $j^{\underline{th}}$ base-aircraft type assigned to the $i^{\underline{th}}$ target

x_2 = a vector whose $i^{\underline{th}}$ component represents the random variable which represents the total number of passes against the $i^{\underline{th}}$ target

= $g_1(x_1, x_{13})$

x_3 = a vector whose $i^{\underline{th}}$ component represents the probability distribution of the $i^{\underline{th}}$ component of x_2

= $g_2(x_2, x_{15})$

x_4 = a vector whose $i^{\underline{th}}$ component represents the pre-assigned value of the $i^{\underline{th}}$ target

x_5 = a vector whose $i^{\underline{th}}$ component represents the probability that the $i^{\underline{th}}$ target is killed by exactly one pass given that a specified number of preferred weapons are delivered on that pass

x_6 = a vector whose $i^{\underline{th}}$ component represents the total number of $i^{\underline{th}}$ base-aircraft type

x_7 = a matrix whose $(i,j)^{\underline{th}}$ entry represents the fuel required for the $j^{\underline{th}}$ base-aircraft type to strike the $i^{\underline{th}}$ target

x_8 = a vector whose $i^{\underline{th}}$ component represents the total fuel available for the $i^{\underline{th}}$ base-aircraft type

x_9 = a matrix whose $(i,j)^{\underline{th}}$ entry represents the number of preferred weapons the $j^{\underline{th}}$ base-aircraft type carries to the $i^{\underline{th}}$ target

x_{10} = a vector whose i^{th} component represents the number of weapons available for the i^{th} base-aircraft type

x_{11} = a matrix whose $(i,j)^{th}$ entry represents the expected percentage attrition of the j^{th} base-aircraft assigned the i^{th} target

x_{12} = a vector whose i^{th} component represents the maximum acceptable number of i^{th} base-aircraft losses

x_{13} = a matrix whose $(i,j)^{th}$ entry represents the number of passes per aircraft planned by the j^{th} base-aircraft type assigned the i^{th} target

x_{14} = a matrix whose $(i,j,k)^{th}$ entry represents the random variable denoting the number of the j^{th} base-aircraft type alive at release on the k^{th} pass against the i^{th} target

$= h_1(x_1)$

x_{15} = a matrix whose $(i,j,k)^{th}$ entry represents the probability distribution of the $(i,j,k)^{th}$ entry of x_{14}

$= h_2(x_{14}, x_{16}, \ldots, x_{20})$

x_{16} = a matrix whose $(i,j,k)^{th}$ entry represents the probability that a raid of size k is detected en route from the j^{th} base-aircraft location to the i^{th} target

x_{17} = a matrix whose $(i,j,k,l)^{th}$ entry represents the probability that a raid of size k is engaged en route from the j^{th} base-aircraft location to the i^{th} target given that the raid is detected and a total of l aircraft are employed in the strike operation

x_{18} = probability that any aircraft in a raid is killed en route given that the raid is engaged

x_{19} = a matrix whose $(i,j,k)^{th}$ entry represents the probability that a raid from the j^{th} base-aircraft location finds the i^{th} target given that k aircraft survive en route

x_{20} = a matrix whose $(i,j,k,l)^{\underline{th}}$ entry represents the probability that any aircraft in a raid from the $j^{\underline{th}}$ base-aircraft location survives until the $k^{\underline{th}}$ release against the $i^{\underline{th}}$ target given that l aircraft are alive commencing the first pass (k=1), or given that l aircraft are alive at the $(k-1)^{\underline{st}}$ release for the second and subsequent passes $(k \geq 2)$

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) The damage inflicted on a target is nonlinear with respect to the number of aircraft passes made against it.

(b) The number of aircraft alive on a particular pass against a given target is treated as a random variable.

(c) Each aircraft assigned to a target delivers the same number of preferred weapons on each pass against that target. A preferred weapon implies that for a specific target there exists a weapon which is most effective in destroying that target.

(d) On any pass against a target assume that the target is either killed or not killed.

(e) Aircraft make passes until all weapons are expended or until the aircraft is killed.

(f) An individual aircraft is assigned only one target per sortie.

(g) A raid is composed of one base-aircraft type, but more than one raid can be assigned to a target.

(h) Enemy fighters, if scrambled against a raid, are sent in numbers sufficient to engage each aircraft in that raid.

(i) The number of a given base-aircraft type alive at release on any pass over a given target is statistically independent of any other base-aircraft types alive over the same target on any pass.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Aircraft

3) Target Composition: Fixed location targets

   3.1) Quantitative Factor:

   (a) a vector whose i^{th} component represents the pre-assigned value of the i^{th} target

4) Friendly Force Composition: Aircraft

   4.1) Quantitative Factors:

   (a) a vector whose i^{th} component represents the total number of i^{th} base-aircraft type

   (b) a vector whose i^{th} component represents the total fuel available for the i^{th} base-aircraft type

   4.2) Armament: Weapons

   4.2.1) Quantitative Factor:

   (a) a vector whose i^{th} component represents the number of weapons available for the i^{th} base-aircraft type

   4.3) Tactics:

   4.3.1) Quantitative Factor:

   (a) a vector whose i^{th} component represents the maximum acceptable number of i^{th} base-aircraft losses

5) Friendly Force - Threat Interaction:

   5.1) Platform - Platform: Aircraft - Aircraft

   5.1.1) Quantitative Factor:

   (a) probability that any aircraft in a raid is killed en route given that the raid is engaged

6) Friendly Force - Target Interaction:

   6.1) Platform - Platform: Aircraft - Fixed location targets

   6.1.1) Quantitative Factors:

   (a) a vector whose i^{th} component represents the probability that the i^{th} target is killed by exactly one pass given that a specified number of preferred weapons are delivered on that pass

(b) a matrix whose $(i,j)^{th}$ entry represents the fuel required for the j^{th} base-aircraft type to strike the i^{th} target

(c) a matrix whose $(i,j)^{th}$ entry represents the expected percentage attrition of the j^{th} base-aircraft assigned the i^{th} target

(d) a matrix whose $(i,j)^{th}$ entry represents the number of passes per aircraft planned by the j^{th} base-aircraft type assigned the i^{th} target

6.2) Armament - Platform: Weapons - Fixed location targets

6.2.1) Quantitative Factor:

(a) a matrix whose $(i,j)^{th}$ entry represents the number of preferred weapons the j^{th} base-aircraft type carries to the i^{th} target

7) Friendly Force - Threat - Target Interaction:

7.1) Platform - Platform - Platform: Aircraft - Aircraft - Fixed location targets

7.1.1) Quantitative Factors:

(a) a matrix whose $(i,j,k)^{th}$ entry represents the probability that a raid of size k is detected en route from the j^{th} base-aircraft location to the i^{th} target

(b) a matrix whose $(i,j,k,l)^{th}$ entry represents the probability that a raid of size k is engaged en route from the j^{th} base-aircraft location to the i^{th} target given that the raid is detected and given that a total of l aircraft are employed in the strike operation

(c) a matrix whose $(i,j,k)^{th}$ entry represents the probability that a raid from the j^{th} base-aircraft location finds the i^{th} target given that k aircraft survive en route

(d) a matrix whose $(i,j,k,l)^{\underline{th}}$ entry represents the probability that any aircraft in a raid from the $j^{\underline{th}}$ base-aircraft location survives until the $k^{\underline{th}}$ release against the $i^{\underline{th}}$ target given that l aircraft are alive commencing the first pass (k=1), or given that l aircraft are alive at the $(k-1)^{\underline{st}}$ release for the second and subsequent passes $(k \geq 2)$

Intrasystems

A. STUDY DESCRIPTION

1) Originating Activity: Analytic Services Inc., Falls Church, Virginia

2) Report Title: Air Interdiction: Analysis of Self-Contained Operations Against Mobile Targets

3) Author: G.J. Miller

4) Report Number: Tactical Division Note 71-5 (AD-519 465)

5) Date: December 1971

6) Classification: Confidential

7) Contract: F44620-69-C-0014 (Directorate of Operational Requirements and Development Plans, DCS/R&D Hq. USAF)

8) Abstract: This Tactical Division Note describes in detail an analytic model developed to evaluate the effectiveness and costs associated with an air interdiction campaign against mobile targets located in a network. The aircraft used in this campaign are assumed to be self-contained search and attack systems (aircraft which operate independently, both detecting and attacking targets). Assumptions, limitations, data requirements, and available output are discussed. Illustrative results indicate the model's use in examining operational alternatives, sensitivity to a single parameter, and system trade-offs. Appendices develop the model's equations and document a computer version. Conclusions are that the model's limitations should not prevent its use as an aid in (1) determining effective munitions allocation strategies, (2) evaluating alternative systems for performing the self-contained search and attack mission, and (3) estimating the impact of a self-contained search and attack campaign on the overall supply interdiction mission.

9) Descriptors: Airborne attack, aircraft, antiaircraft defense, attrition, bomb, cost, detection probability, false target, interdiction, kill probability, Poisson density function, surface target, survivability, target mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne Attack
3) Mission: Interdiction
   3.1) Definition: A self-contained search and attack aircraft conducts an air interdiction campaign against mobile targets (such as trucks, tanks or railroad trains) located in a lines-of-communication (LOC) network (such as a system of roads or tracks).
   3.2) Criterion For Success: Destruction of target
   3.3) MOE's Selected:
   $(MOE)_1$ = Expected number of targets (or target elements) destroyed per sortie
   $(MOE)_2$ = Expected cost per target destroyed
   $(MOE)_3$ = Expected aircraft lost per target destroyed
   3.3.1) Rationale For Selection: The correct decision among alternative choices concerning this type of mission depends on the measure of effectiveness used in making the choice. Under different circumstances, different measures are appropriate. In an engagement in which an interdicting force must have the highest possible impact in a short time, the number of sorties which can be flown is likely to be the limiting factor, and the force commander might well want to make decisions aimed at maximizing the expected number of targets destroyed per sortie. In a longer less intense campaign

in which the number of aircraft available to the interdicting force is likely to be the limiting factor (possibly because of high attrition and a long production cycle), the force commander might prefer to minimize the expected number of aircraft lost per target destroyed. A development planner on the other hand, might have the task of allocating scarce dollars among competing programs and thus might want to allocate his resources so as to provide for killing the largest possible number of targets for the dollar invested. Thus, he would seek to minimize the expected cost per target killed, although his decision must be based on knowledge of the fact that a commander in the field may be making decisions according to some other criterion.

3.4) Functional Form Of MOE's:

$(MOE)_1 = f_1(x_1, \ldots, x_5)$

$(MOE)_2 = f_2(x_1, \ldots, x_5, x_{10}, \ldots, x_{30})$

$(MOE)_3 = f_3(x_1, \ldots, x_5, x_{10}, \ldots, x_{27})$

where

x_1 = maximum number of attacks possible on each sortie $= g_1(x_6, x_7)$

x_2 = a vector whose i^{th} component represents the expected number of target elements destroyed in the i^{th} target attack, given it is a real target $= g_2(x_7, x_8, x_9)$

x_3 = probability that a given target under attack is real $= g_3(x_{14}, \ldots, x_{20}, x_{23})$

x_4 = a vector whose i^{th} component represents the probability that the amount of search distance required until completion of the i^{th} attack (including search distance forfeited because of fuel consumed while attacking) is less than or equal to the total search distance available to the aircraft per sortie $= g_4(x_{10})$

x_5 = a vector whose $i^{\underline{th}}$ component represents the probability that the aircraft survives until the $i^{\underline{th}}$ attack given that the amount of search distance required until completion of the $i^{\underline{th}}$ attack is less than or equal to the total search distance available to the aircraft per sortie

$= g_5(x_3, x_8, x_{11}, x_{12}, x_{13})$

x_6 = number of units of ordnance available per sortie

x_7 = a vector whose $i^{\underline{th}}$ component represents the number of units of ordnance released on the $i^{\underline{th}}$ target attack

x_8 = a vector whose $i^{\underline{th}}$ component represents the probability that a real target under attack is of type i

$= h_1(x_{16}, x_{17})$

x_9 = a matrix whose $(i,j)^{\underline{th}}$ entry represents the expected number of target elements destroyed when a target of type i is attacked with j units of ordnance

x_{10} = a vector whose $i^{\underline{th}}$ component represents the Poisson parameter associated with the $i^{\underline{th}}$ attack

$= h_2(x_{14}, \ldots, x_{23})$

x_{11} = probability that the aircraft is destroyed traveling from its base to the search area

x_{12} = a vector whose $i^{\underline{th}}$ component represents the probability that the aircraft is destroyed while making an attack on a target of type i

x_{13} = probability that the aircraft is destroyed while making an attack on a false target

x_{14} = expected number of targets traveling in the LOC network

x_{15} = length of LOC network

x_{16} = a vector whose $i^{\underline{th}}$ component represents the probability that a target of type i is detected and attacked, given that it is flown over

x_{17} = a vector whose $i^{\underline{th}}$ component represents the probability that a target which is flown over is of type i

x_{18} = rate of occurrence of false alarms

x_{19} = probability that a false alarm is detected

x_{20} = probability that a detected false alarm is incorrectly identified as a true target and that an attack occurs

x_{21} = maximum search distance available to the aircraft per sortie

x_{22} = search distance lost per attack as a result of fuel consumed during attack

x_{23} = total number of target types

x_{24} = probability that the aircraft is destroyed during the return trip from the search area to the base

x_{25} = a vector whose i^{th} component represents the average density of ground defenses of type i

x_{26} = a vector whose i^{th} component represents the probability that the aircraft is destroyed while flying over a ground defense type i

x_{27} = maximum number of different types of ground defenses

x_{28} = operating cost per sortie

x_{29} = cost of replacing aircraft

x_{30} = munitions cost per unit of ordnance

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) The self-contained search and attack aircraft flies from its base to a search area, searches along the LOC network for a target, attacks a target when it is found, and then resumes the search. This search-and-attack process continues until one of three things happens: (1) the aircraft runs out of ordnance; (2) the aircraft expends all fuel which has been allowed for the search; (3) the aircraft is destroyed by ground fire. Following either of the first two occurrences, the aircraft returns to its base.

(b) The number of targets of a given type (real or false) which the aircraft flies over for a given search distance is a random variable obeying a Poisson probability law. This assumption has been adopted for the following reasons: It has been found to be an exact expression of or a close approximation to many processes of this general type; its mathematical expression is relatively easy to manipulate; it is a conservative assumption in that the nature of the distribution precludes planning a sortie by predicting at what point during the search an attack will occur (an encounter is equally likely at any point in the search regardless of when the previous encounter occurred).

(c) The Poisson assumption would obviously not be valid if applied to individual vehicles traveling in convoys. The model therefore requires that the assumption apply to the convoy as a whole (that is, the convoy is considered to be the target, each vehicle in it being a "target element").

(d) A single attack of each target is all that is allowed the aircraft, i.e., no reattacks. The aircraft never follows an attack with a return to assess damage as the basis for a decision as to whether to reattack. This assumption is probably reasonable in the case of fleeting targets (such as trucks traveling on a road) which can leave the road and hide in foliage upon being warned of the aircraft's presence and so are not reattackable or in the case of heavily defended targets which are designated in advance as causing unacceptable attrition on reattacks.

(e) Each attack requires the same amount of fuel.

(f) The allocation of ordnance is assumed to be made in advance of the mission.

(g) The threat is assumed to pose a constant probability of loss to the aircraft while it flies from its base to the search area. Similarly, aircraft which survive until the end of the search portion are assumed to be subject to a constant probability of loss on the return flight to base. A constant loss probability is assumed each time an attack occurs. (This probability may vary with target type. It also can be different (probably lower) if the attack is on a false target.)

(h) Losses occurring during the attack are assumed not to affect the success of that attack.

(i) Losses occurring while the aircraft is searching for targets are assumed to occur according to a Poisson process, under the assumption that ground defenses which are not centered around the targets are randomly located along the aircraft's search path.

(j) Two types of false signals are assumed to contribute to the false alarm rate, namely: (1) random faults within the sensor mechanism, and (2) the presence of actual signal-generating objects along the aircraft's flight path, such as civilian vehicles or signal producing decoys placed there by an enemy. These effects are reflected in the value of the conversion probability associated with false alarms.

(k) A single target type is assumed to travel in the network.

(l) Target density is assumed to remain constant throughout the interdiction campaign.

(m) Weapons released simultaneously are assumed to have equal and independent effect on the target at which they are aimed.

(n) Costs are treated as though variable costs are the only costs accrued by the interdicting force (a reasonable approximation of the situation wherein development and basing construction costs are sunk costs).

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factors:

(a) length of LOC network

(b) rate of occurrence of false alarms

2) Threat Composition: Ground defenses

2.1) Quantitative Factor:

(a) maximum number of different types of ground defenses

2.2) Deployment:

2.2.1) Quantitative Factor:

(a) a vector whose $i^{\underline{th}}$ component represents the average density of ground defenses of type i

3) Target Composition: Mobile targets (trucks, tanks or railroad trains)

3.1) Quantitative Factor:

(a) total number of target types

3.2) Deployment:

3.2.1) Quantitative Factor:

(a) expected number of targets traveling in t[illegible] LOC network

4) Friendly Force Composition: Aircraft

4.1) Quantitative Factors:

(a) maximum search distance available to the aircraft per sortie

(b) search distance lost per attack as a result of fuel consumed during attack

(c) operating cost per sortie

(d) cost of replacing aircraft

4.2) Armament: Bombs

4.2.1) Quantitative Factors:

(a) munitions cost per unit of ordnance

(b) number of units of ordnance available per sortie

4.3) Tactics:

4.3.1) Quantitative Factor:

(a) a vector whose $i^{\underline{th}}$ component represents the number of units of ordnance released on the $i^{\underline{th}}$ target attack

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform: Aircraft - Ground defenses

5.1.1) Quantitative Factors:

(a) probability that the aircraft is destroyed traveling from its base to the search area

(b) probability that the aircraft is destroyed during the return trip from the search area to the base

(c) a vector whose $i^{\underline{th}}$ component represents the probability that the aircraft is destroyed while flying over a ground defense type i

6) Friendly Force - Target Interaction:

6.1) Platform - Platform: Aircraft - Mobile targets

6.1.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component represents the probability that a target of type i is detected and attacked, given that it is flown over

(b) a vector whose $i^{\underline{th}}$ component represents the probability that a target which is flown over is of type i

6.2) Armament - Platform: Bombs - Mobile targets

6.2.1) Quantitative Factor:

(a) a matrix whose $(i,j)^{\underline{th}}$ entry represents the expected number of target elements destroyed when a target of type i is attacked with j units of ordnance

7) Friendly Force - Physical Environment Interaction:

7.1) Platform - Environment: Aircraft

7.1.1) Quantitative Factors:

(a) probability that a false alarm is detected

(b) probability that a detected false alarm is incorrectly identified as a true target and that an attack occurs

8) Friendly Force - Threat - Target Interaction:

8.1) Platform - Platform - Platform: Aircraft - Ground defenses - Mobile targets

8.1.1) Quantitative Factor:

(a) a vector whose $i^{\underline{th}}$ component represents the probability that the aircraft is destroyed while making an attack on a target of type i

9) Friendly Force - Threat - Physical Environment Interaction:

9.1) Platform - Platform - Environment: Aircraft - Ground defenses

9.1.1) Quantitative Factor:

(a) probability that the aircraft is destroyed while making an attack on a false target

STUDY REVIEW SUMMARY NO. (3)-10

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Tactical Air Warfare Study II, Volume I - Summary Report and Volume III - Effectiveness Analysis
3) Author: R.E. Beatty, Jr.
4) Report Number: NWAG Study No. 42
5) Date: 16 November 1965
6) Classification: Secret
7) Contract: NONR 3732 (00) (Office of Naval Research)
8) Abstract: The relative effectiveness of equal-cost tactical air mixes is measured in a major, 30-day conventional combat situation. Each of the mixes has a different proportion of land-based and sea-based tactical air forces. Analysis of costs leads to an approximate equal trade-off in numbers of aircraft between land and sea-based forces. Substitution of sea-based for land-based tactical air is shown to result in increased kills, sorties and ordnance delivered in large scale non-nuclear wars. A sensitivity analysis is made to demonstrate the range of validity of the findings.
9) Descriptors: Airborne attack, aircraft, air superiority, attrition, availability, carrier, close air support, cost, force allocation, interdiction, kill, survivability, target mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne Attack
3) Mission: Air strike
   3.1) Definition: A mix of attack aircraft are allocated to attack fixed location targets.

3.2) Criterion For Success: Destruction of target

3.3) MOE Selected: Total target kill potential

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_{11})$$

where

x_1 = duration of war in days

x_2 = number of aircraft types

x_3 = number of target types

x_4 = replacement schedule for attrited aircraft

x_5 = attrition per sortie as a function of war duration

x_6 = a vector whose i^{th} component represents the initial number of aircraft type i

x_7 = a matrix whose $(i,j)^{th}$ entry represents the fraction of available aircraft type i allocated to target type j each day of the war

x_8 = a matrix whose $(i,j)^{th}$ entry represents the number of kills per sortie of aircraft type i against target type j

x_9 = a matrix whose $(i,j)^{th}$ entry represents the number of augmented aircraft type i previously used for air defenses released on day j of the war for attack missions

x_{10} = a matrix whose $(i,j)^{th}$ entry represents the fraction of bases on-line for aircraft type i on day j of the war

x_{11} = a vector whose i^{th} component represents the sortie rate per day of aircraft type i

4) MOE Usage In Study: The MOE was used to compare the relative effectiveness of equal-cost mixes of land-based and sea-based tactical air forces.

5) Special Study Assumptions:

(a) Kills per sortie were calculated using the same target spectrum for all aircraft.

(b) All strike aircraft, sea-based or land-based, were assumed subject to the same attrition rate per sortie. 50 percent of the attrition is assumed to occur prior to reaching the target and 50 percent after completion of target runs.

(c) Losses due to enemy attacks on bases were not considered.

(d) Sufficient POL and ordnance are available at all times for land-based aircraft.

(e) Reliability effects are ignored.

(f) Sea-based and land-based tactical air elements are maintained at equal combat readiness.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Target Composition: Target mix

2.1) Quantitative Factor:

(a) number of target types

3) Threat Composition: Threat mix

4) Friendly Force Composition: Aircraft and bases (sea or land)

4.1) Platform Type: Aircraft

4.1.1) Quantitative Factors:

(a) number of aircraft types

(b) a vector whose i^{th} component represents the initial number of aircraft type i

4.1.2) Deployment:

4.1.2.1) Quantitative Factor:

(a) duration of war in days

4.1.3) Tactics:

4.1.3.1) Quantitative Factors:

(a) replacement schedule for attrited aircraft

(b) a matrix whose $(i,j)^{th}$ entry represents the number of augmented aircraft type i previously used for air defenses released on day j of the war for attack missions

(c) a vector whose i^{th} component represents the sortie rate per day of aircraft type i

4.2) Platform Type: Bases (sea or land)

4.2.1) Quantitative Factor:

(a) a matrix whose (i,j)th entry represents the fraction of bases on-line for aircraft type i on day j of the war

5) Friendly Force - Target Interaction:

5.1) Platform - Platform: Aircraft - Target mix

5.1.1) Quantitative Factors:

(a) a matrix whose (i,j)th entry represents the fraction of available aircraft type i allocated to target type j each day of the war

(b) a matrix whose (i,j)th entry represents the number of kills per sortie of aircraft type i against target type j

6) Friendly Force - Threat Interaction:

6.1) Platform - Platform: Aircraft - Threat mix

6.1.1) Quantitative Factor:

(a) attrition per sortie as a function of war duration

STUDY REVIEW SUMMARY NO. (3)-11

A. STUDY DESCRIPTION

1) Originating Activity: Analytic Services Inc., Falls Church, Virginia
2) Report Title: Air Interdiction: Models for Armed Reconnaissance in a Permissive Environment
3) Author: D.J. Van Arman
4) Report Number: Tactical Division Note TDN 71-4
5) Date: September 1971
6) Classification: Unclassified
7) Contract: F44620-69-C-0014 (Directorate of Operational Requirements and Development Plans, DCS/R&D, Hq. USAF)
8) Abstract: This Tactical Division Note analyzes a munitions and time-limited hunt for targets of opportunity, e.g., the hunt for truck convoys by a self-contained adverse-weather / night attack (SCANA) aircraft. Stochastic models are formulated for various situations in which one or more strikes are made on targets encountered. Among factors considered in these models are target density, target ability to hide, target size, hunter ability to detect targets, false targets, munitions carried, munitions effectiveness, time per strike, and hunt time. Attrition is not considered. Various munitions allocation strategies are defined and examined with respect to their effects on expected damage per hunt. In addition to defining reasonable munitions allocation strategies for a particular hunter, the models permit comparison of the effectiveness of different hunters (e.g., different types of SCANA aircraft).
9) Descriptors: Airborne attack, aircraft, detection probability, false target, interdiction, kill probability, reconnaissance, search, target mix, weapon mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System

2) Function: Airborne Attack

3) Mission: Interdiction

3.1) Definition: An attack aircraft (called the hunter) attempts to inflict damage on targets of opportunity that he meets and attacks in a time-limited hunt in a permissive environment.

3.2) Criterion For Success: Destruction of target

3.3) MOE Selected: Total damage expected in a hunt of specified duration

3.4) Functional Form Of MOE:

Case 1 - Fixed allocation of munitions with one strike per target

$MOE = f_1(x_1, \ldots, x_4)$

where

x_1 = probability that an arbitrary strike is against a true target

x_2 = probability that a true target that is attacked is damaged
$= g_1(x_4, x_5)$

x_3 = probability density function of the number of attacks per hunt
$= g_2(x_6, x_7, x_8)$

x_4 = number of units of munitions allocated to each target attacked
$= g_3(x_6, \ldots, x_9)$

x_5 = probability of a unit of munitions killing a target

x_6 = total hunt time

x_7 = expected duration of attack on one target

x_8 = average number of targets attacked per unit of search time
$= h_1(x_{10}, x_{11}, x_{12})$

x_9 = total units of munitions allocated per aircraft per hunt

x_{10} = average number of targets flown over per unit of search time

x_{11} = probability that a target that is flown over is detected

x_{12} = probability that a detected target is attackable

Case 2 - Sequential allocation of munitions with one strike per target

MOE = $f_2(x_1, x_3, x_{13}, x_{14})$

where

x_{13} = a vector whose i^{th} component represents the probability that a true target that is attacked on the i^{th} attack is killed

= $g_4(x_5, x_{14})$

x_{14} = a vector whose i^{th} component represents the number of units of munitions allocated to the i^{th} target attacked

= $h_2(x_6, x_7, x_9, x_{15}, x_{16})$

x_{15} = a vector whose i^{th} component represents the search time for the i^{th} target

x_{16} = a vector whose i^{th} component represents the attack time for the i^{th} target

Case 3 - Fixed allocation of munitions with the number of strikes determined by the nature of the target

MOE = $f_3(x_1, x_3, x_{17}, x_{18})$

where

x_{17} = probability that a true target that is attacked is damaged

= $g_5(x_5, x_{18})$

x_{18} = number of units of munitions allocated to each target attacked

= $g_6(x_5, x_6, x_9, x_{19}, \ldots, x_{22})$

x_{19} = number of target types

x_{20} = a vector whose i^{th} component represents the average number of targets type i attacked per unit of search time

x_{21} = expected number of strikes per attack

= $h_3(x_{19}, x_{20})$

x_{22} = expected time for an attack

= $h_4(x_{19}, x_{20}, x_{23})$

x_{23} = a vector whose $i^{\underline{th}}$ component represents the expected time for attack on targets type i

Case 4 - Sequential allocation of munitions with the number of strikes determined by the nature of the target

MOE = $f_4(x_1, x_3, x_{24}, x_{25})$

where

x_{24} = a vector whose $i^{\underline{th}}$ component represents the probability that a true target that is attacked on the $i^{\underline{th}}$ attack is killed

= $g_7(x_5, x_{25})$

x_{25} = a vector whose $i^{\underline{th}}$ component represents the number of units of munitions allocated to the $i^{\underline{th}}$ attack

= $h_5(x_6, x_7, x_9, x_{15}, x_{26})$

x_{26} = a vector whose $i^{\underline{th}}$ component represents number of strikes made on the $i^{\underline{th}}$ attack

Case 5 - Fixed allocation of munitions per strike with restrike possible

MOE = $f_5(x_1, x_6, \ldots, x_9, x_{26}, \ldots, x_{29})$

where

x_{27} = probability that a true target that is attacked is damaged

= $g_8(x_5, x_{28})$

x_{28} = units of munitions preplanned for each strike

x_{29} = number of strikes for which munitions are available

= $h_6(x_9, x_{27})$

x_{30} = probability of a target hiding after an unsuccessful strike

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) The number of targets met in total search time is a Poisson random variable.

(b) The search time for a target is an exponentially distributed random variable.

(c) The attack times are independent random variables with the same distribution.

(d) The probability of damage to a target as a function of the number of units of munitions used on the target is assumed to be concave monotonic increasing.

(e) In cases 3 and 4,

(e.1) Targets are classified according to the number of strikes the hunter will make on them, so that targets of type i will have i or more elements of which i will be struck once.

(e.2) The number of targets type i found in the total search time is a Poisson random variable and the occurrence of different types of targets is independent.

(f) In case 5,

(f.1) The number of units of munitions allocated per strike is preplanned.

(f.2) A hunter strikes a simple target repeatedly until one of the following occurs: (1) the target is destroyed, (2) the target has hidden, (3) the hunter is out of time.

(g) Attrition suffered by the hunter is not considered.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Target Composition: Target mix

3) Friendly Force: Aircraft

3.1) Armament: Munitions

3.1.1) Quantitative Factor:

(a) total units of munitions allocated per aircraft per hunt

3.2) Tactics:

3.2.1) Quantitative Factors:

(a) total hunt time

(b) units of munitions preplanned for each strike

4) Friendly Force - Target Interaction:

4.1) Platform - Platform: Aircraft - Target mix

4.1.1) Quantitative Factors:

(a) probability that an arbitrary strike is against a true target

(b) expected duration of attack on one target

(c) average number of targets flown over per unit of search time

(d) probability that a target that is flown over is detected

(e) probability that a detected target is attackable

(f) a vector whose i^{th} component represents the search time for the i^{th} target

(g) a vector whose i^{th} component represents the attack time for the i^{th} target

(h) number of target types

(i) a vector whose i^{th} component represents the average number of targets type i attacked per unit of search time

(j) a vector whose i^{th} component represents the expected time for attack on targets type i

(k) a vector whose i^{th} component represents number of strikes made on the i^{th} attack

(l) probability of a target hiding after an unsuccessful strike

4.2) Armament - Platform: Munitions - Target mix

4.2.1) Quantitative Factor:

(a) probability of a unit of munitions killing a target

STUDY REVIEW SUMMARY NO. (3)-12

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Missile Center, Point Mugu, California
2) Report Title: STANDARD ARM (Mod 0) Weapon System Performance Analysis
3) Authors: R. Lerner, R. Nauman, D.O. Crozier, L.K. Dunbar and M.A. Garcia
4) Report Number: TM-68-29 (AD-389 997)
5) Date: 10 May 1968
6) Classification: Secret (NOFORN)
7) Project Number: Local Project L-2519 (Naval Air Systems Command)
8) Abstract: This report presents a performance analysis of the STANDARD ARM (Mod 0) weapon system in tactical environments. In the analysis, the weapon system performance was determined in four radar environments of varying density, and the effects of these environments on component operation were examined. The environments were defined in terms of the number of radars in the area surrounding the strike targets. Modes of operation were related to human operator options and decisions.
9) Descriptors: Airborne attack, aircraft, air-to-surface missile, detection probability, fire control, missile, missile seeker, Poisson density function, radar, surface-to-air missile

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Subsystem
2) Function: Airborne Attack
3) Mission: Air strike
   3.1) Definition: An aircraft armed with an antiradiation air-to-surface missile system attacks surface-to-air missile fire-control radars in a multiradar environment.

3.2) Criterion For Success:

Case 1 - Evaluation of the Radar Homing and Warning subsystem

Criterion For Success: Detection of target radar and visual identification of its direction and signal intensity

Case 2 - Evaluation of the missile seeker subsystem

Criterion For Success: Detection and acquisition of target

3.3) MOE's Selected:

Case 1

$(MOE)_1$ = Probability of specific target radar detection in a multiradar environment and visual identification of its direction and signal intensity

Case 2

$(MOE)_2$ = Probability that the missile seeker will detect a specified target radar in a multiradar environment and that the missile will then acquire this radar as a target

3.4) Functional Form Of MOE's:

$(MOE)_1 = f_1(x_1, x_2)$

$(MOE)_2 = f_2(x_1, x_5, x_6, x_7)$

where

x_1 = number of radars in the environment contributing acceptable pulses

x_2 = expected number of pulses from the target radar during the dead period

$= g(x_3, x_4)$

x_3 = duration of dead period

x_4 = expected pulse repetition frequency

x_5 = missile seeker sweep period

x_6 = probability density function for the time it takes the missile seeker to first detect the signal from the target radar

x_7 = a vector whose i^{th} component represents the probability distribution function for the time it takes the missile seeker to first detect the signal from the i^{th} non-target radar

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) For the Radar Homing and Warning subsystem,

(a.1) The pulse from the target radar exceeds the intensity threshold of the subsystem and is included in the time sequence of acceptable pulses.

(a.2) The time sequence of acceptable pulses synthesized from all radars other than the target radar is random with an expected pulse repetition frequency (PRF) that is constant. Considered by itself, the time sequence of pulses from the target radar is not random in time.

(a.3) The period between pulses from the target radar is large in comparison to the dead period. Therefore, the target radar can have at most one pulse in a dead period, and there are many pulses from other radars between adjacent pulses of the target radar.

(a.4) The number of acceptable pulses within a dead period is a Poisson distributed random variable.

(a.5) The maximum number of pulses that can possibly occur in a dead period is at least equal to the number of radars contributing pulses to the time sequence of acceptable pulses.

(b) For the missile seeker,

(b.1) The signal from the target radar is contained within the composite signal received as an input to the missile seeker, and has an acceptable PRF.

(b.2) The individual radar signals that are synthesized into a composite input to the missile seeker are independent and of random phase in time.

(b.3) The sweep intensity threshold is initiated randomly in time.

(b.4) The time required to acquire a radar of an acceptable PRF once its signal is detected above the sweep intensity threshold is small in comparison to the sweep period.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Target Composition: SAM fire-control radar

   2.1) Quantitative Factors:

   (a) number of radars in the environment contributing acceptable pulses

   (b) expected pulse repetition frequency

3) Friendly Force Composition: Antiradiation ASM system

   3.1) Sensors: Radar Homing and warning subsystem and missile seeker

   3.1.1) Type: Radar Homing and Warning subsystem

   3.1.1.1) Quantitative Factor:

   (a) duration of dead period

   3.1.2) Type: Missile seeker

   3.1.2.1) Quantitative Factor:

   (a) missile seeker sweep period

4) Friendly Force - Target Interaction:

   4.1) Sensor - Platform: Missile seeker - SAM fire-control radar

   4.1.1) Quantitative Factors:

   (a) probability density function for the time it takes the missile seeker to first detect the signal from the target radar

   (b) a vector whose i^{th} component represents the probability distribution function for the time it takes the missile seeker to first detect the signal from the i^{th} non-target radar

STUDY REVIEW SUMMARY NO. (3)-13

A. STUDY DESCRIPTION

1) Originating Activity: Bureau of Naval Weapons, Washington, D.C.
2) Report Title: Cost-Effectiveness of CONDOR
3) Authors: S.L. Taffel, T.J. Schmitt and E.A. Thibault
4) Report Number: R-5-66-1 (AD-369 169)
5) Date: January 1966
6) Classification: Secret
7) Abstract: The effectiveness and associated cost incurred by aircraft employing CONDOR to destroy specified defended targets are compared to the effectiveness and cost associated with aircraft employing anti-radiation-missiles to attack the defenses and WALLEYE's to destroy the primary target. Bridge, POL dumps, SAM sites and aircraft type primary targets are considered. The analytical model developed to obtain the results takes into account such factors as raid size, attack aircraft loading, system reliability, weapon vulnerability, weapon lethality and defense characteristics. Three levels of enemy defense are investigated. The model also permits limited consideration of the use of countermeasures by either attackers or the defenders. Results are presented in terms of targets killed per unit cost and relative cost expenditure required to achieve specified target damage levels on a single strike operation. In addition the relative number of targets killed by each system during a single day's operation is determined.
9) Descriptors: Airborne attack, aircraft, air-to-surface missile, antiaircraft defense, antiaircraft gunnery, availability, bomb, carrier based aircraft, cost, cost-effectiveness, countermeasure, electronic warfare, gun, kill probability, radar, reliability, surface-to-air missile, survivability, target acquisition, target mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force

2) Function: Airborne Attack

3) Mission: Air strike

3.1) Definition: Aircraft launched from a carrier penetrate area and local defenses to attack a mix of targets.

3.2) Criterion For Success: Destruction of target

3.3) MOE's Selected:

$(MOE)_1$ = Expected number of primary targets killed per unit cost

$(MOE)_2$ = Expected number of primary targets killed per day for a given force level

3.4) Functional Form Of MOE's:

$$(MOE)_1 = f_1(x_1, x_2)$$

$$(MOE)_2 = f_2(x_1, x_{54}, x_{55}, x_{59}, x_{60})$$

where

x_1 = expected number of primary targets killed during one raid $= g_1(x_3, x_4)$

x_2 = cost of one raid $= g_2(x_{50}, \ldots, x_{57})$

x_3 = probability of killing a primary target $= h_1(x_4, \ldots, x_7)$

x_4 = number of primary targets being attacked

x_5 = average unit kill probability per weapon attacking the primary targets

x_6 = maximum number of passes per aircraft possible against the primary targets $= i_1(x_8, \ldots, x_{11})$

x_7 = a vector whose i^{th} component represents the expected number of weapons surviving through the gun defenses after being launched on the i^{th} pass against the primary targets

= $i_2(x_{12}, \ldots, x_{15})$

x_8 = number of weapons carried by each aircraft assigned to primary targets only

x_9 = number of primary target weapons carried by each aircraft assigned to both defense and primary target

x_{10} = number of primary target weapons launched per aircraft per pass by aircraft assigned to primary targets only

x_{11} = number of primary targets weapons launched per aircraft per pass by aircraft assigned to both defense and primary targets

x_{12} = target acquisition probability associated with primary targets

x_{13} = a vector whose i^{th} component represents the expected number of weapons surviving through the missile defense after being launched on the i^{th} pass against primary targets

= $j_1(x_{16}, \ldots, x_{26})$

x_{14} = average kill probability per pass achieved by a gun unit against a primary target weapon

x_{15} = number of enemy defensive gun units which can fire against attackers

x_{16} = a vector whose i^{th} component represents the expected number of aircraft assigned to primary targets surviving to launch on their i^{th} pass against the primary targets

= $k_1(x_{21}, \ldots, x_{24}, x_{26}, x_{27})$

x_{17} = a vector whose $i^{\underline{th}}$ component represents the expected number of aircraft assigned to both defense and primary targets surviving to launch on their $i^{\underline{th}}$ pass against the primary targets

= $k_2(x_{21}, \ldots, x_{24}, x_{26}, x_{28})$

x_{18} = a vector whose $i^{\underline{th}}$ component represents the number of primary weapons launched per aircraft on the $i^{\underline{th}}$ pass by aircraft assigned to primary targets only

= $k_3(x_8, x_{10})$

x_{19} = a vector whose $i^{\underline{th}}$ component represents the number of primary weapons launched per aircraft on the $i^{\underline{th}}$ pass by aircraft assigned to both defense and primary targets

= $k_4(x_9, x_{11})$

x_{20} = reliability of all aircraft systems needed for weapon launch

x_{21} = proportion of enemy defense capability retained after degradation due to friendly force use of ECM

x_{22} = probability of defense survival through all attacks against it

= $k_5(x_{29}, x_{33})$

x_{23} = reliability of enemy missile site excluding the missile and it launcher

x_{24} = a vector whose $i^{\underline{th}}$ component represents the number of enemy missile salvos intercepting primary target weapons launched on the $i^{\underline{th}}$ pass against primary targets

x_{25} = average missile salvo kill probability against weapons attacking the primary targets

x_{26} = average missile salvo kill probability against aircraft attacking the primary targets

x_{27} = expected number of aircraft assigned to primary targets only, surviving en route to defense complex

= $j_2(x_{34}, x_{35}, x_{36})$

x_{28} = expected number of aircraft assigned to both defense and primary targets that survive to attack primary targets

= $j_3(x_{21}, x_{23}, x_{29}, x_{37}, x_{38}, x_{39})$

x_{29} = a vector whose i^{th} component represents the probability of defense site surviving the i^{th} pass

= $j_4(x_{30},\ldots, x_{32})$

x_{30} = average unit kill probability per weapon attacking the missile site

x_{31} = a vector whose i^{th} component represents the expected number of anti-defense weapons surviving the gun defense on the i^{th} pass

= $k_6(x_{15}, x_{40}, x_{41}, x_{42})$

x_{32} = number of enemy defensive missile units which are attacked

x_{33} = maximum number of passes against the enemy defense

= $j_5(x_{48}, x_{49})$

x_{34} = probability of successful navigation to the target area

x_{35} = probability of aircraft survival en route to defense complex

x_{36} = number of aircraft in raid assigned to hit primary targets only

x_{37} = expected number of aircraft assigned to both defense and primary targets surviving en route to defense complex

= $k_7(x_{34}, x_{35}, x_{43})$

x_{38} = average missile salvo kill probability against aircraft attacking the defense

x_{39} = a vector whose $i^{\underline{th}}$ component represents the number of enemy missile salvos intercepting aircraft during the interval between the $(i-1)^{\underline{st}}$ and $i^{\underline{th}}$ pass against the defense

x_{40} = target acquisition probability for anti-defense weapon

x_{41} = a vector whose $i^{\underline{th}}$ component represents the expected number of anti-defense weapons surviving the missile defense on the $i^{\underline{th}}$ pass

$= l(x_{20}, x_{21}, x_{23}, x_{29}, x_{37}, x_{44}, \ldots, x_{47})$

x_{42} = average kill probability per pass achieved by a gun unit against an anti-defense weapon

x_{43} = number of aircraft in raid assigned to both defense and primary targets

x_{44} = a vector whose $i^{\underline{th}}$ component represents the number of anti-defense weapons launched per aircraft on the $i^{\underline{th}}$ pass

$= m(x_{33}, x_{48}, x_{49})$

x_{45} = probability of weapon guidance, if required after launch of anti-defense weapons

x_{46} = proportion of anti-defense weapons the survive enemy use of countermeasures to defeat incoming homing weapons

x_{47} = average missile salvo kill probability against weapons attacking the defense

x_{48} = number of aint-defense weapons carried by each of the aircraft assigned both defense and primary targets

x_{49} = number of antidefense weapons launched per pass

x_{50} = number of primary target weapons expended on one raid

$= h_2(x_8, x_9, x_{34}, x_{36}, x_{43})$

x_{51} = number of anti-defense weapons expended on one raid

$= h_3(x_{34}, x_{43}, x_{48})$

x_{52} = cost per primary target type weapon

x_{53} = cost per anti-defense type weapon

x_{54} = total number of attack aircraft per raid

= $h_4(x_{36}, x_{43})$

x_{55} = expected number of aircraft returning to the CVA from one raid

= $h_5(x_6, x_{16}, x_{17}, x_{34}, x_{54}, x_{58})$

x_{56} = replacement cost per aircraft

x_{57} = operating cost per aircraft per sortie

x_{58} = probability of aircraft survival on way out from defense complex

x_{59} = number of aircraft that can be spotted aboard the specified deck space

x_{60} = number of raids possible per operating day

= $g_3(x_{60}, x_{61})$

x_{61} = number of raids limited by force size

= $h_6(x_{54}, x_{59})$

x_{62} = number of raids limited by time

= $h_7(x_{63}, x_{64}, x_{65})$

x_{63} = aircraft daily operating time

x_{64} = aircraft flight time

x_{65} = aircraft cycle time

3.5) Additional MOE Identified:

(a) Cost expenditure required to achieve specified target damage levels on a single strike operation.

4) MOE Usage In Study: The MOE's were formulated and used as the basis for comparing alternative weapon system configurations.

5) Special Study Assumptions:

(a) Enemy opposition is limited to those SAM sites and gun batteries which are near the target complex being attacked.

(b) The operation against a given defense complex is subdivided into a number of cycles. The first phase consists of attacks against the defense for the purpose of reducing their capability. The second phase consists of attacks against the primary targets by aircraft assigned only to attack the primary targets and by the survivors of the anti-defense attack phase.

(c) Airborne launched radar homing weapons that are reliable and survive defensive fire from missiles and guns are assumed to be equally distributed over all SAM sites in the target complex.

(d) If all weapons cannot be expended on a single pass, additional passes are carried out until all the ordnance has been expended.

(e) Simultaneous missile launches are assumed from each aircraft in the raid. This should be possible because of the multiplicity of guidance channels available.

(f) Each SAM site has a fixed number of missiles which it can expend.

(g) Enemy missile salvos are equally distributed over all incoming aircraft targets (where possible) until these aircraft release their weapons; at this point, enemy fire is then directed against these incoming weapons and is equally distributed over them.

(h) Aircraft that do not engage the defenses wait just outside of the defended zone until the attacks against the defense have been completed.

(i) All aircraft which survive are assumed to be able to return to the carrier, regardless of whether or not they were able to find the target.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: SAM sites and gun batteries

   2.1) Platform Type: Gun battery

2.1.1) Quantitative Factor:

(a) number of enemy defensive gun units which can fire against attackers

2.2) Platform Type: SAM site

2.2.1) Quantitative Factors:

(a) reliability of enemy missile site excluding the missile and it launcher

(b) number of enemy defensive missile units which are attacked

3) Target Composition: Target mix (Simple truss bridge, SAM site radar van, POL revetted, and parked aircraft)

4) Friendly Force Composition: Aircraft

4.1) Quantitative Factors:

(a) reliability of all aircraft systems needed for weapon launch

(b) probability of successful navigation to the target area

(c) probability of aircraft survival en route to defense complex

(d) replacement cost per aircraft

(e) operating cost per aircraft per sortie

(f) probability of aircraft survival on way out from defense complex

(g) number of aircraft that can be spotted aboard the specified desk space

4.2) Armament: ASM

4.2.1) Quantitative Factors:

(a) number of weapons carried by each aircraft assigned to primary targets only

(b) number of primary target weapons carried by each aircraft assigned to both defense and primary target

(c) probability of weapon guidance, if required after launch of anti-defense weapons

(d) number of anti-defense weapons carried by each of the aircraft assigned both defense and primary targets

(e) cost per primary target type weapon

(f) cost per anti-defense type weapon

4.3) Deployment:

4.3.1) Quantitative Factors:

(a) number of aircraft in raid assigned to both defense and primary targets

(b) aircraft daily operating time

(c) aircraft flight time

(d) aircraft cycle time

4.4) Tactics:

4.4.1) Quantitative Factors:

(a) number of primary targets being attacked

(b) number of primary target weapons launched per aircraft per pass by aircraft assigned to primary targets only

(c) number of primary targets weapons launched per aircraft per pass by aircraft assigned to both defense and primary targets

(d) number of aircraft in raid assigned to hit primary targets only

(e) number of anti-defense weapons launched per pass

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform:

5.1.1) Type: Aircraft - SAM sites

5.1.1.1) Quantitative Factors:

(a) proportion of enemy defense capability retained after degradation due to friendly force use of ECM

(b) average missile salvo kill probability against aircraft attacking the primary targets

(c) average missile salvo kill probability against aircraft attacking the defense

(d) a vector whose i^{th} component represents the number of enemy missile salvos intercepting aircraft during the interval between the $(i-1)^{st}$ and i^{th} pass against the defense

5.2) Armament - Platform:

5.2.1) Type: ASM - Gun batteries

5.2.1.1) Quantitative Factors:

(a) average kill probability per pass achieved by a gun unit against a primary target weapon

(b) average kill probability per pass achieved by a gun unit against an anti-defense weapon

5.2.2) Type: ASM - SAM site

5.2.2.1) Quantitative Factors:

(a) a vector whose i^{th} component represents the number of enemy missile salvos intercepting primary target weapons launched on the i^{th} pass against primary targets

(b) average missile salvo kill probability against weapons attacking the primary targets

(c) average unit kill probability per weapon attacking the missile site

(d) target acquisition probability for anti-defense weapon

(e) proportion of anti-defense weapons the survive enemy use of countermeasures to defeat incoming homing weapons

(f) average missile salvo kill probability against weapons attacking the defense

6) Friendly Force - Target Interaction:

6.1) Armament - Platform: ASM - Target

6.1.1) Quantitative Factors:

(a) average unit kill probability per weapon attacking the primary targets

(b) target acquisition probability associated with primary targets

Ultrasystems

STUDY REVIEW SUMMARY NO. (3)-14

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Passive Defense Aspects of Dispersed Formation Operation Under EMCON
3) Author: J. Ozols
4) Report Number: OEG Research Contribution 184 (AD-516 566)
5) Date: May 1971
6) Classification: Secret
7) Contract: N00014-68-A-0091 (Office of Naval Research)
8) Abstract: The tactic of aircraft carrier task force defense using dispersed formations and electronic emission control is examined. Data from fleet exercises are reported and theoretical parametric models derived. The effect of this tactic on carrier strike operations is discussed. Active AAW under these conditions is not treated.
9) Descriptors: Airborne attack, aircraft, antiair warfare, carrier, classification probability, detection, emission control, radar, search, task force, visual

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Airborne Attack
3) Mission: Aircraft attack on task force
   3.1) Definition: A carrier task force conducts air strike operations with ships dispersed over a large area and in "random" stations to disguise its appearance. An enemy aircraft searches

for the task force in order to locate and identify (either correctly or incorrectly) the aircraft carrier within it.

3.2) Criterion For Success: Detection and identification of aircraft carrier

3.3) MOE Selected: Probability that a search aircraft will locate the task force, and find and correctly identify the aircraft carrier within it

3.4) Functional Form Of MOE:

MOE = $f(x_1, x_2)$

where

x_1 = probability that the search aircraft will find the operating area of the task force

$$= \begin{cases} g_1(x_3, x_4, x_{11}) & \text{for visual search of a moderately-dispersed task force or radar search of a widely-dispersed task force} \\ g_2(x_3, x_4) & \text{for radar search of a moderately-dispersed formation} \\ g_3(x_7, x_8, x_{10}) & \text{for radar search of a task force in compact formation} \end{cases}$$

x_2 = probability that the search aircraft will correctly identify the aircraft carrier within the task force given that the task force has been located

= $g_4(x_{11}, x_{13})$

x_3 = probability of the search aircraft passing through the square containing the task force

= $h_1(x_5, x_6)$

x_4 = probability of detecting one ship in the square if the square is searched

$$= \begin{cases} h_2(x_7, x_8, x_{10}) & \text{for visual search of a moderately-dispersed task force or radar search of a widely-dispersed task force} \\ h_3(x_{11}) & \text{for radar search of a moderately-dispersed formation} \end{cases}$$

x_5 = number of squares contained in the area of uncertainty

$$= \begin{cases} i_1(x_9, x_{10}) & \text{for visual search of a moderately-dispersed task force or radar search of a widely-dispersed task force} \\ i_2(x_9, x_{10}) & \text{for radar search of a moderately-dispersed formation} \end{cases}$$

x_6 = number of squares search aircraft will actually pass through during search

$$= \begin{cases} i_3(x_{10}, x_{12}) & \text{for visual search of a moderately-dispersed task force or radar search of a widely-dispersed task force} \\ i_4(x_{10}, x_{12}) & \text{for radar search of a moderately-dispersed formation} \end{cases}$$

x_7 = length of search aircraft's path within region actually occupied by the task force

$= i_5(x_{10})$

x_8 = sweep width of search aircraft

x_9 = area of the region in which the task force is located

x_{10} = area of the region actually occupied by the task force

x_{11} = number of units in the task force

x_{12} = length of path flown by the search aircraft within the region in which the task force is located

x_{13} = probability that the airborne search radar operator will incorrectly classify another ship as the aircraft carrier

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Area actually occupied by the task force is assumed to be much smaller than the area of the region in which the task force is located.

(b) Once one ship of the task force has been detected, it is possible for the aircraft to localize (although not necessarily identify) the other ships at will using its airborne search radar.

(c) In the visual search and radar search case,

(c.1) The operating area of the task force can be approximated by a square that is $\sqrt{x_{10}}$ on a side. This assumption is convenient for derivation of the equations, but is not critical to the result. Ships are stationed randomly within the square.

(c.2) The aircraft search path can be divided into disjoint segments such that the detection events along any segment are independent random events.

(c.3) The ship's position is uniformly distributed in the region actually occupied by the task force.

(c.4) The aircraft's path is random in the region actually occupied by the task force.

(c.5) On any portion of the path which is small relative to the total length of the path, but decidedly larger than the range of possible detection, the aircraft always detects the ship within the lateral range $x_8/2$ on either side of the path but never beyond.

(d) During the enemy's search phase, the task force observes complete electronic silence. After the operating area has been discovered, units are allowed to use nonidentifying emitters and deception devices.

(e) The task force is assumed to be dispersed over an area so as to appear random to an airborne observer. To assure this apparent randomness, each unit moves randomly around its assigned station but stays within a specified radius of the station.

(f) The search aircraft examines each target in turn, until one is found that is classified, correctly or incorrectly, as the aircraft carrier, at which point the search terminates.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) area of the region in which the task force is located

2) Threat Composition: Aircraft

2.1) Sensor: Radar

2.1.1) Quantitative Factor:

(a) sweep width of search aircraft

3) Friendly Force Composition: Task force

3.1) Quantitative Factor:

(a) number of units in the task force

3.2) Tactics:

3.2.1) Qualitative Factor:

(a) task force may assume widely-dispersed, moderately-dispersed, or compact formation

3.2.2) Quantitative Factor:

(a) area of region actually occupied by the task force

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Task force - Aircraft

4.1.1) Quantitative Factor:

(a) length of path flown by the search aircraft within the region in which the task force is located

4.2) Platform - Sensor: Task force - Radar

4.2.1) Quantitative Factor:

(a) probability that the airborne search radar operator will incorrectly classify another ship as the aircraft carrier

(4)

ENVIRONMENTAL SYSTEMS

STUDY REVIEW SUMMARY NO.(4)-1

A. STUDY DESCRIPTION

1) Originating Activity: Arthur D. Little, Inc., Cambridge, Massachusetts
2) Report Title: Cost Effectiveness - Mechanical BT vs. Expendable BT
3) Report Number: (AD-367 030)
4) Date: July 1965
5) Classification: Confidential
6) Contract: NObsr-93055 (Department of the Navy, Bureau of Ships)
7) Abstract: In this report are compared the performance characteristics and costs of the mechanical bathythermograph (MBT) with those of the expendable bathythermograph (XBT). The XBT allows a ship to take a thermal profile in heavy seas without slowing down, changing course, or breaking formation, and this reduces the ship exposure to enemy attack. Because of having to slow down to employ an MBT and then catch-up, there results a difference in fuel expenditure. This difference is chosen as the basis for comparison.
8) Descriptors: Bathythermograph, environmental system, surface ship, task force

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Subsystem
2) Function: Environmental Systems
3) Mission: Bathythermograph maneuver
    3.1) Definition: A mechanical bathythermograph (MBT) ship traveling with a task force slows down, launches an MBT at low speed, and then catches up with the task force.
    3.2) Criterion For Success: Low cost measurement of the vertical ocean temperature profile
    3.3) MOE Selected: Difference in fuel consumption due to the bathythermograph maneuver

3.3.1) Rationale For Selection: This MOE gives a basis for comparison between the mechanical bathythermograph and the expendable bathythermograph, and since launching the expendable bathythermograph from a surface ship does not require any change in the operating schedule, there is no difference in fuel consumption.

3.4) Functional Form Of MOE:

Case 1 - With fuel expenditure during slowdown to launch bathythermograph

$$MOE = f_1(x_1,\ldots,x_8)$$

where

x_1 = catch-up speed
x_2 = task force speed
x_3 = MBT launching speed
x_4 = catch-up speed fuel demand
x_5 = task force speed fuel demand
x_6 = MBT launching speed fuel demand
x_7 = time to slow down
x_8 = time to launch MBT

Case 2 - No fuel expenditure during slowdown to launch bathythermograph

$$MOE = f_2(x_1,\ldots,x_{13})$$

where

x_9 = mass of ship
x_{10} = drag force
x_{11} = drag constant
x_{12} = power required to maintain catch-up speed
x_{13} = power required to increase speed from MBT launching speed to catch-up speed

4) MOE Usage In Study: The MOE was formulated and used as one basis for comparison between the mechanical bathythermograph and the expendable bathythermograph

5) Special Study Assumptions:

(a) In Case 1,

(a.1) The MBT ship, originally cruising with the task force, slows down to a speed at which it can launch an MBT. After the MBT launch the MBT ship speeds up to catch the task force, most of the time being spent at the catch up speed, very little of the time being spent in accelerating to the catch-up speed.

(a.2) During the MBT maneuver the task force and the MBT ship move along the same straight line, the task force maintaining its original speed.

(a.3) The average velocity during slow-down is one-half the sum of the task force speed and the MBT launch speed.

(b) In Case 2,

(b.1) Until the start of MBT maneuver, task force and the MBT ship are together traveling at task force speed. At the start of the maneuver the MBT ship completely cuts off power to coast to MBT launch speed. When the MBT ship reaches launch speed, it turns on enough power to maintain this speed.

(b.2) When the MBT ship finishes the MBT launch, it turns up power to start catching up with task force. When the MBT ship reaches catch-up speed, it reduces power to maintain this speed.

(b.3) At some time before regaining task force, the MBT ship completely cuts off power to coast to task force speed. When the MBT ship speed reaches task force speed, MBT ship regains task force, and turns on enough power to maintain task force speed.

(b.4) The MBT ship and task force travel in a straight line, the task force maintaining constant speed during the entire maneuver. Experience shows this assumption to be reasonable, and it is necessary to confine this investigation within practical limits.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Friendly Force Composition: Mechanical bathythermograph (MBT) ship and task force

   2.1) Platform Type: MBT ship

      2.1.1) Quantitative Factors:

         (a) catch-up speed
         (b) MBT launching speed
         (c) catch-up speed fuel demand
         (d) task force speed fuel demand
         (e) MBT launching speed fuel demand
         (f) time to slow down
         (g) time to launch MBT
         (h) mass of ship
         (i) power required to maintain catch-up speed
         (j) power required to increase speed from MBT launching speed to catch-up speed

      2.1.2) Sensor: MBT

         2.1.2.1) Quantitative Factors:

            (a) drag force
            (b) drag constant

   2.2) Platform Type: Task force

      2.2.1) Quantitative Factor:

         (a) task force speed

(5)

MINING

STUDY REVIEW SUMMARY NO.(5)-1

A. STUDY DESCRIPTION

1) Originating Activity: Operations Research Incorporated, Silver Spring, Maryland
2) Report Title: Operational Analysis of Aerial Minelaying Systems 1970-1975, Vol. II The Theory of Aerial Minelaying
3) Authors: L. Gilford and F. S. Zusman
4) Report Number: TR-364 (AD-377 015)
5) Date: 23 March 1966
6) Classification: Confidential
7) Contract: NONR-4955(00) (Office of Naval Research)
8) Abstract: This report presents a mathematical theory developed to describe the essential characteristics of an aerial-minelaying operation. The theory developed is used to evaluate the effectiveness of alternative minelaying systems. The theory relates specific well-defined input parameters to mission effectiveness and costs.
9) Descriptors: Aircraft, antiair warfare, attrition, binomial density function, cost, mining, Poisson density function, survivability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Mining
3) Mission: Aerial minelaying
   3.1) Definition: A wave of minelaying aircraft flies a specified number of sorties, in which a sortie consists of planting a series of mines and returning to a staging area. During any segment of the sortie, the aircraft may come under attack from airborne and/or surface anti-aircraft weapons.

3.2) Criterion For Success: Survival of aircraft and planting of mines

3.3) MOE Selected: Joint probability that a specified number of aircraft are killed and a specified number of mines are unplanted

3.3.1) Rationale For Selection: To evaluate the role of minelaying by aircraft and the comparative effectiveness of alternate aerial minelaying systems, it is necessary to develop a measure that reflects the stochastic nature of various possible outcomes, displaying the effectiveness (in terms of mines planted) as well as the costs (in terms of aircraft attrition).

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_{10})$$

where

x_1 = number of aircraft in the initial wave sortie

x_2 = number of sorties to be flown

x_3 = mine capacity of a single aircraft

x_4 = a vector whose i^{th} component is the number of "guns" (or launchers) operating in the i^{th} segment

x_5 = a matrix whose $(i,j)^{th}$ entry is the number of "shots" fired (or missiles launched) by the j^{th} gun in the i^{th} segment

x_6 = a matrix whose $(i,j)^{th}$ entry is the single-shot kill probability of the j^{th} gun in the i^{th} segment

x_7 = a matrix whose $(i,j)^{th}$ entry represents the firepower in terms of long-term average number of shots per time unit that the j^{th} gun is capable of sustaining in the i^{th} segment

x_8 = a vector whose i^{th} component represents the time duration of the i^{th} segment for all aircraft surviving the $(i-1)^{st}$ segment

x_9 = a matrix whose $(i,j)^{\underline{th}}$ entry represents the number of aircraft killed by the $j^{\underline{th}}$ gun in the $i^{\underline{th}}$ segment

x_{10} = a vector whose $i^{\underline{th}}$ component represents the number of aircraft surviving the first i segments

4) MOE Usage In Study: The MOE is formulated and applications of the MOE are presented.

5) Special Study Assumptions:

(a) Each aircraft has the same mine capacity.

(b) Each gun in each segment fires at a firepower rate in such a manner that the probability of getting off a shot in any small time interval is the same as any other equal sized, non-overlapping time interval, so that the number of shots fired follows a Poisson distribution.

(c) The conditional probability distribution of the number of aircraft killed by a specified gun in a specified segment is Binomial.

(d) There is no progressive damage to an aircraft, i.e., an aircraft is either killed or survives.

(e) The enemy strategy is optimal, i.e., if the number of guns is less than or equal to the number of surviving aircraft in a segment, the guns are in groups so that composite group kill probabilities are as close to equal as possible.

(f) No shots are wasted, i.e., there is no overkill. When the first shot from a group of two guns kills an aircraft, the second is used against another target.

(g) The enemy ammunition is effectively limitless.

(h) Each shot is independent of every other shot.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Anti-aircraft weapons

2.1) Quantitative Factors:

(a) a vector whose i^{th} component is the number of "guns" (or launchers) operating in the i^{th} segment

(b) a matrix whose $(i,j)^{th}$ entry is the number of "shots" fired (or missiles launched) by the j^{th} gun in the i^{th} segment

(c) a matrix whose $(i,j)^{th}$ entry represents the firepower in terms of long-term average number of shots per unit time that the j^{th} gun is capable of sustaining in the i^{th} segment

3) Friendly Force Composition: Aircraft minelayer

3.1) Quantitative Factor:

(a) mine capacity of a single aircraft

3.2) Tactics:

3.2.1) Qualitative Factor:

(a) fly a specified number of sorties, each of which consists of planting mines and returning to the staging area

3.2.2) Quantitative Factors:

(a) number of aircraft in the initial wave sortie

(b) number of sorties to be flown

(c) a vector whose i^{th} component represents the time duration of the i^{th} segment for all aircraft surviving the $(i-1)^{st}$ segment

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Aircraft minelayer - Antiaircraft weapons

4.1.1) Quantitative Factors:

(a) a matrix whose $(i,j)^{th}$ entry is the single-shot kill probability of the j^{th} gun in the i^{th} segment

(b) a matrix whose $(i,j)^{th}$ entry represents the number of aircraft killed by the j^{th} gun in the i^{th} segment

(c) a vector whose i^{th} component represents the number of aircraft surviving the first i segments

Hasystems

STUDY REVIEW SUMMARY NO. (5)-2

A. STUDY DESCRIPTION

1) Originating Activity: Operations Research Incorporated, Silver Spring, Maryland
2) Report Title: Operational Analysis of Aerial Minelaying Systems 1970-1975, Vol. I The Analysis of Aerial-Minelaying Systems
3) Author: S.E. Starley
4) Report Number: TR-364 (AD-377 014)
5) Date: 22 July 1966
6) Classification: Secret
7) Contract: NONR-4955(00) (Office of Naval Research)
8) Abstract: This report presents the analyses and results of the operational effectiveness of various aircraft performing an aerial-minelaying mission. The analysis considers a broad class of factors and parameters related to mining areas, minefield design, characteristics, and operation of minelaying aircraft, and characteristics and operation of enemy air defense systems.
9) Descriptors: Aircraft, air-to-air missile, antiaircraft defense, antiaircraft gunnery, antiair warfare, attrition, detection, detection probability, gun director, interception probability, kill probability, mining, projectile, radar, surface-to-air missile, survivability, vulnerability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Mining
3) Mission: Aerial minelaying
   3.1) Definition: A wave of minelaying aircraft flies a specified number of sorties, in which a sortie consists of planting a series of mines and returning to a staging area. During any segment of the sortie, the aircraft may come under attack from airborne and/or surface anti-aircraft weapons.

3.2) Criterion For Success: Survival of aircraft

3.3) MOE Selected: Total threat delivered to penetrating aircraft

3.3.1) Rationale For Selection: This measure can be used to determine the expected number of aircraft lost in accomplishing the mission and thus assess the impact on mission cost.

3.4) Functional Form Of MOE:

Case 1 - Manned interceptors used for defense

$$MOE = f_1(x_1, x_2)$$

where

x_1 = average single-shot kill probability of the air-to-air weapon delivered by the interceptor
$= g_1(x_3, x_4, x_5)$

x_2 = number of passes made by an interceptor
$= g_2(x_6, \ldots, x_{14})$

x_3 = probability of achieving an intercept

x_4 = probability of detection and conversion by the interceptor against the raid, given that the intercept is possible

x_5 = kill probability of the weapon, given that detection and conversion are possible for the intercept

x_6 = radius of local defense zone

x_7 = separation distance between the interceptor air base and the center of the mining target

x_8 = range from the center of the target area at which the minelayers are first detected

x_9 = flight speed of the inbound minelaying aircraft

x_{10} = reaction time for interceptors, which is defined as the time that elapses from when the detection was made to when the interceptors are ready to take-off

x_{11} = time required for interceptors to become airborne and climb to the proper pursuit altitude

x_{12} = time required for a reattack pass

x_{13} = interceptor speed at the pursuit altitude

x_{14} = exit or dash-out speed of the minelaying aircraft

<u>Case 2</u> - Antiaircraft artillery is used for defense

$$MOE = f_2(x_{15}, x_{16}, x_{17})$$

where

x_{15} = sustained rate of fire of the AAA weapon

x_{16} = average single-shot kill probability of the AAA weapon $= g_3(x_{18}, x_{19}, x_{20})$

x_{17} = average time under fire $= g_4(x_9, x_{21}, \ldots, x_{28})$

x_{18} = explosive shell fuze reliability

x_{19} = total aircraft vulnerable area

x_{20} = standard deviation of projectile radial error

$$= \begin{cases} g_5(x_9, x_{27}, x_{28}) & \text{for computer-directed AA guns with radar ranging} \\ g_6(x_9) & \text{for shoulder-fired weapons or optically directed weapons on AA mounts} \end{cases}$$

x_{21} = number of projectile rounds available

x_{22} = total AAA site delay time $= h_1(x_{29}, \ldots, x_{32})$

x_{23} = maximum effective slant range of AA battery

x_{24} = minelaying aircraft altitude

x_{25} = slant range at which minelaying aircraft can first be observed $= h_2(x_{25}, x_{26}, x_{33})$

x_{26} = vector of minelaying aircraft penetration path offset distances

x_{27} = projectile flight time function

x_{28} = projectile ballistic coefficients

x_{29} = time required to evaluate the observation

x_{30} = time to train the weapon

x_{31} = time to acquire and track the target

x_{32} = time to commence firing

x_{33} = fire-control radar mask angle

Case 3 - Surface-to-air missiles used for defense

$MOE = f_3(x_{34}, x_{35})$

where

x_{34} = average single-shot kill probability of the missile

x_{35} = average number of missiles launched

$= g_7(x_9, x_{24}, x_{26}, x_{36}, \ldots, x_{42})$

x_{36} = maximum slant range at which the missile site radar can acquire the minelaying aircraft

x_{37} = missile site radar mask angle

x_{38} = acquisition-to-lock-on time

x_{39} = lock-on-to-fire time

x_{40} = missile flight time function

x_{41} = time between salvos in a ripple

x_{42} = number of missiles available

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) In Case 1,

(a.1) The local defense zone is a circle with center at the center of the mining target.

(a.2) The minelaying aircraft are vulnerable to interceptor attacks from the time they are detected until the time they pass out of the interceptor operating range.

(b) In Case 2, the vulnerability of an aircraft to AAA is defined as the probability of a single random hit causing an attrition kill (A-kill), i.e., the level of damage sufficient to cause the aircraft to fall out of control within 5 minutes.

(c) In Case 3, the missile single-shot kill probability is assumed to be independent of range and of target characteristics, and includes all probabilities associated with the system operation.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Interceptors, AAA site and SAM site

2.1) Platform Type: Interceptors

2.1.1) Quantitative Factors:

(a) reaction time for interceptors, which is defined as the time that elapses from when the detection was made to when the interceptors are ready to take-off

(b) time required for interceptors to become airborne and climb to the proper pursuit altitude

2.1.2) Armament: Air-to-air missile

2.1.3) Deployment:

2.1.3.1) Quantitative Factors:

(a) radius of local defense zone

(b) separation distance between the interceptor air base and the center of the mining target

2.1.4) Tactics:

2.1.4.1) Quantitative Factors:

(a) time required for a reattack pass

(b) interceptor speed at the pursuit altitude

2.2) Platform Type: AAA site

2.2.1) Quantitative Factors:

(a) time required to evaluate the observation

(b) time to train the weapon

(c) time to acquire and track the target

(d) time to commence firing

2.2.2) Sensor: Radar

2.2.2.1) Quantitative Factor:

(a) fire-control radar mask angle

2.2.3) Armament: Projectile

2.2.3.1) Quantitative Factors:

(a) sustained rate of fire of the AAA weapon

(b) explosive shell fuze reliability

(c) number of projectile rounds available

(d) maximum effective slant range of AA battery

(e) projectile flight time function

(f) projectile ballistic coefficients

2.3) Platform Type: SAM site

2.3.1) Quantitative Factors:

(a) acquisition-to-lock-on time

(b) lock-on-to-fire time

2.3.2) Sensor: Radar

2.3.2.1) Quantitative Factors:

(a) maximum slant range at which the missile site radar can acquire the minelaying aircraft

(b) missile site radar mask angle

2.3.3) Armament: Surface-to-air missile

2.3.3.1) Quantitative Factors:

(a) average single-shot kill probability of the missile

(b) missile flight time function

(c) number of missiles available

2.3.3.2) Tactics:

2.3.3.2.1) Quantitative Factor:

(a) time between salvos in a ripple

3) Target Composition: Mining target

4) Friendly Force Composition: Aircraft

4.1) Quantitative Factor:

(a) flight speed of the inbound minelaying aircraft

4.2) Deployment:

4.2.1) Quantitative Factors:

(a) minelaying aircraft altitude

(b) vector of minelaying aircraft penetration path offset distances

4.3) Tactics:

4.3.1) Quantitative Factor:

(a) exit or dash-out speed of the minelaying aircraft

5) Friendly Force - Threat Interaction:

5.1) Platform - Platform: Aircraft - Interceptor

5.1.1) Quantitative Factors:

(a) probability of achieving an intercept

(b) probability of detection and conversion by the interceptor against the raid, given that the intercept is possible

5.2) Platform - Armament:

5.2.1) Type: Aircraft - Air-to-air missile

5.2.1.1) Quantitative Factor:

(a) kill probability of the weapon, given that detection and conversion are possible for the intercept

5.2.2) Type: Aircraft - Projectile

5.2.2.1) Quantitative Factor:

(a) total aircraft vulnerable area

6) Friendly Force - Target Interaction:

6.1) Platform - Platform: Aircraft - Mining target

6.1.1) Quantitative Factor:

(a) range from the center of the target area at which the minelayers are first detected

(6)

MINE COUNTERMEASURES

STUDY REVIEW SUMMARY NO.(6)-1

A. STUDY DESCRIPTION

1) Originating Activity: Minesweeping Branch, Bureau of Ships, Washington, D.C.
2) Report Title: Integration of Minesweeping and Minehunting in Assault Operations
3) Author: R. K. Reber
4) Report Number: Minesweeping Branch Technical Memorandum No. 174 (AD-512 912)
5) Date: 1 July 1964
6) Classification: Confidential
7) Abstract: This study extends previous analysis on assault mine countermeasures. The game theoretic approach is extended to the case of imperfect information of pertinent parameters. Rationale for division of effort between sweeping and searching is given. Once an appropriate division is specified, sweeping and searching procedures can be specified even with imperfect information. Casualties to countermeasures vessels are considered and rules are given to carry out operations in each part of a channel or area.
8) Descriptors: Assault ship, game theory, mine, mine countermeasure, minehunter, minesweeper

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Mine Countermeasures
3) Mission: Mine clearance
   3.1) Definition: A combination of minesweepers and minehunters search for mines in an area to be traveled by assault ships.

Diagonal sweeping is employed for wide areas and central channel sweeping is employed for narrow channels.

3.2) Criterion For Success: Clearance of minefield

3.3) MOE Selected: Risk to ships in the assault operation.
For the wide area case the risk is defined to be the fraction of mines initially in the area or channel which are expected to be exploded by the ships. For the narrow channel case the risk is defined to be the ratio of the expected number of mines exploded by ships to the expected number of mines in a channel of width six times the standard deviation of the navigational error for assault ships.

3.4) Functional Form Of MOE:

Case 1 - Sweepable type mines are laid in a wide area

$$MOE = f_1(x_1, \ldots, x_5)$$

where

x_1 = percentage clearance which is obtained in the minehunting operation
$= g_1(x_6, \ldots, x_{10})$

x_2 = equivalent number of runs through full length of channel made by the sweeps actually used
$= g_2(x_{11}, x_{12})$

x_3 = number of transits of the channel by the assault ships (not including countermeasures vessels)

x_4 = aggregate actuation width of sweeps

x_5 = average aggregate actuation width of assault ships for sweepable mines

x_6 = fraction of all mines which are undetectable

x_7 = efficiency parameter for minehunting

x_8 = equivalent number of runs through full length of channel made by minehunters actually used
$= h_1(x_{11}, x_{13})$

x_9 = aggregate detection width of search gear

= $h_2(x_{14}, x_{15})$

x_{10} = width of area or channel

x_{11} = fraction of the total countermeasures effort which is in minesweeping

x_{12} = equivalent number of runs through the full length of channel made by sweeps assuming all countermeasures vessels are used as sweepers

= $h_3(x_{16}, x_{17})$

x_{13} = equivalent number of runs through full length of channel made by minehunters assuming all countermeasures vessels are used as hunters

= $i(x_{16}, x_{18})$

x_{14} = characteristic detection width of search gear

x_{15} = characteristic detection probability of search gear

x_{16} = total number of minehunters and sweepers (sweeper available for regular influence sweeping)

x_{17} = equivalent number of runs through full length of channel made by each sweep

x_{18} = equivalent number of runs through full length channel made by each minehunter

Case 2 - One type of sweepable and one type of unsweepable mine are laid in a wide area

MOE = $f_2(x_1, \ldots, x_5, x_{19}, x_{20})$

where

x_{19} = fraction of all mines which are sweepable

x_{20} = a parameter for unsweepable mines in wide area

= $g_3(x_3, x_{10}, x_{21})$

x_{21} = average aggregate actuation width of assault ships for unsweepable mines

Case 3 - Sweepable type mines are laid in a narrow channel

MOE = $f_3(x_1, \ldots, x_5)$

Case 4 - One type of sweepable and one type of unsweepable mine are laid in a narrow channel

MOE = $f_4(x_1, \ldots, x_5, x_{19}, x_{22})$

where

x_{22} = a parameter for unsweepable mines in a narrow channel

= $g_4(x_3, x_{21}, x_{23})$

x_{23} = standard deviation of navigational error for assault ships

3.5) Additional MOE's Identified:

(a) Expected number of casualties

(b) Expected fraction of mines not found in minehunting operation

(c) Expected percentage clearance obtained in the minehunting operation

4) MOE Usage In Study: The MOE's were formulated and used to find the approximate optimal division of effort between minesweeping and minehunting for mine neutralization operations when the exact values of the parameters are not known.

5) Special Study Assumptions:

(a) Each mine exploded by a ship sinks or disables the ship.

(b) A limited and definite time interval is available for carrying out countermeasures by a countermeasure force whose size is fixed.

(c) All divisions of effort between sweeping and hunting are possible, implying that all countermeasures vessels can be used as sweepers and that all can be used as hunters.

(d) The ship paths are distributed uniformly over the width of the channel or area.

(e) In wide area case,

(e.1) Diagonal sweeping and uniform searching methods are employed.

(e.2) When part of the sweeping or hunting effort consists of runs through only part of the channel, the fraction of the total operating time spent on turns is not significantly greater than it would be if all runs were of full length. (An approximate method of correcting for the greater time on turns for short runs is presented).

(f) In the narrow channel case,

(f.1) The ship paths are assumed to have a Gaussian across-channel distribution.

(f.2) A combination of uniform searching in a nominal channel of suitable width and an appropriate central channel sweeping operation is employed.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) width of area or channel

2) Threat Composition: Sweepable and unsweepable mines

2.1) Quantitative Factor:

(a) fraction of all mines which are sweepable

3) Friendly Force Composition: Minesweepers, minehunters and assault ships

3.1) Platform Type: Minesweeper

3.1.1) Quantitative Factors:

(a) aggregate actuation width of sweep

(b) characteristic detection width of search gear

(c) characteristic detection probability of search gear

(d) total number of sweepers

- 3.1.2) Tactics:
  - 3.1.2.1) Qualitative Factor:
    - (a) diagonal sweeping mode in wide area case and central channel sweeping mode in the narrow channel case
  - 3.1.2.2) Quantitative Factors:
    - (a) fraction of the total counter-measures effort which is [illegible] mine-sweeping
    - (b) equivalent number of runs through full length of channel made by each sweep
- 3.2) Platform Type: Minehunter
  - 3.2.1) Quantitative Factors:
    - (a) characteristic detection width of search gear
    - (b) characteristic detection probability of search gear
    - (c) total number of minehunters
  - 3.2.2) Tactics:
    - 3.2.2.1) Qualitative Factor:
      - (a) uniform searching
    - 3.2.2.2) Quantitative Factors:
      - (a) equivalent number of runs through full length of channel made by each minehunter
      - (b) efficiency parameter for minehunting
- 3.3) Platform Type: Assault ship
  - 3.3.1) Quantitative Factor:
    - (a) standard deviation of navigational error for assault ships

3.3.2) Deployment:

3.3.2.1) Qualitative Factor:

(a) ship paths distributed uniformly over the width of the channel or area

3.3.2.2) Quantitative Factor:

(a) number of transits of the channel by the assault ships

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform:

4.1.1) Type: Minehunter - Sweepable and unsweepable mines

4.1.1.1) Quantitative Factor:

(a) fraction of all mines which are undetectable

4.1.2) Type: Assault ship - Sweepable mines

4.1.2.1) Quantitative Factor:

(a) average aggregate actuation width of assault ships for sweepable mines

4.1.3) Type: Assault ship - Unsweepable mines

4.1.3.1) Quantitative Factor:

(a) average aggregate actuation width of assault ships for unsweepable mines

STUDY REVIEW SUMMARY NO.(6)-2

A. STUDY DESCRIPTION

1) Originating Activity: Operations Research Incorporated, Silver Spring, Maryland
2) Report Title: Cost-Effectiveness Analysis of Oceangoing Fully Supported Small-Displacement Mine-Clearance Ships
3) Authors: S. E. Starley, S. E. Gottlieb, J. M. Sheehan and F. P. Falci, Jr.
4) Report Number: TR-329 (AD-366 893)
5) Date: 25 June 1965
6) Classification: Secret
7) Contract: N600(61331)61097 (U.S. Navy Mine Defense Laboratory)
8) Abstract: This report examines the operation of small-displacement mine-clearance ships. Comparisons are made for alternative ship characteristics based on their operational effectiveness in clearing an enemy minefield. The mine-clearance operations examined in this analysis include those associated with offensive and defensive naval missions. These operations include minesweeping of both moored mines and bottom influence mines, and minehunting and neutralization of bottom mines.
9) Descriptors: Mine, mine countermeasures, minehunting, minesweeping, Poisson density function

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Mine Countermeasures
3) Mission: Mine clearance
    3.1) Definition: Mine-clearance ships operate in support of offensive amphibious assault operations and/or in support

of defensive operations, such as defense of harbors and over-the-beach logistic supply sites. This effort consists of minesweeping of moored mines and bottom influence mines, and minehunting and neutralization of bottom mines.

3.2) Criterion For Success: Clearance of minefield

3.3) MOE Selected: Total force level required to clear a given area in a given time

3.4) Functional Form Of MOE:

<u>Case 1</u> - Minesweeping force

$$MOE = f_1(x_1, x_2)$$

where

x_1 = number of ships required in the minesweeping force to clear a given area in a given time

$= g_1(x_3, \ldots, x_9)$

x_2 = expected number of minesweeping ships lost due to attrition in the minefield

x_3 = area to be cleared by minesweeping force

x_4 = turning time per turn for minesweeping ship

x_5 = speed of advance of minesweeping ship through minefield while sweeping

x_6 = track separation distance between two succeeding tracks of minesweeping vehicles

x_7 = time available for continual minesweeping operations

x_8 = sweep path distance along minesweeping track before a turn is required

x_9 = expected sweep time per day per minesweeping ship

$$= \begin{cases} h_1(x_{10}, x_{11}, x_{12}) & \text{for moored minesweeping} \\ h_2(x_{13}, x_{14}) & \text{for influence minesweeping} \end{cases}$$

x_{10} = maximum number of moored sweep gear failures possible in a day for a single ship

x_{11} = a random variable representing the minimum sweep time for a given number of moored sweep gear failures for a single ship with a given number of spare gears

$= i_1(x_{13}, \ldots, x_{18})$

x_{12} = probability density function for x_{11}

$= j_1(x_{11}, x_{19})$

x_{13} = daily time limit for minesweeping vehicle operation (limited by daylight or crew fatigue)

x_{14} = time for minesweeping ship to make a round-trip transit between the support area and the minefield plus the initial streaming and final retrieval of sweep gear

$= j_2(x_{17}, x_{18}, x_{25}, x_{26}, x_{27})$

x_{15} = number of moored sweep gear failures

x_{16} = number of moored sweep gear spares

x_{17} = clearance gear-streaming time for minesweeping ship

x_{18} = clearance gear-retrieval time for minesweeping ship

x_{19} = rate of encountering mines that will destroy moored sweep gear

$= k(x_5, x_6, x_{20})$

x_{20} = density of moored minefield devices (moored mines or moored obstruction mines) that will destroy moored sweep gear

$= i(x_{21}, \ldots, x_{27})$

x_{21} = probability that a moored minefield device is an obstructor mine

x_{22} = probability that a moored obstructor mine will destroy moored sweep gear

x_{23} = probability that a moored mine (non-obstructor) will destroy sweep gear

x_{24} = density of moored mines

x_{25} = free-route (no gear in tow) sustained speed for minesweeping ship

x_{26} = transit distance between support area and minefield for minesweeping ship

x_{27} = average minesweeping vehicle speed while streaming or retrieving gear

Case 2 - Minehunting force

$MOE = f_2(x_{28}, x_{29})$

where

x_{28} = number of ships required in the minehunting force to clear a given area in a given time

$= g_2(x_{30}, \ldots, x_{40})$

x_{29} = expected number of minehunting ships lost due to attrition in the minefield

x_{30} = time available for continual minehunting operations

x_{31} = daily time limit for minehunting vehicle operation (limited by daylight or crew fatigue)

x_{32} = time for minehunting ship to make a round-trip transit between the support area and the minefield plus initial streaming and final retrieval of clearance gear

$= h_3(x_{41}, \ldots, x_{45})$

x_{33} = number of coverages over the area

x_{34} = track separation distance between two succeeding tracks of a minehunting vehicle during detection operation

x_{35} = speed of advance of minehunting ship through minefield while detecting

x_{36} = turning time per turn for minehunting ship

x_{37} = sweep path distance along minehunting track before a turn is required

x_{38} = area to be cleared by minehunting force

x_{39} = expected value of the time out for classification and neutralization of mines

$= h_4(x_{33}, x_{46}, \ldots, x_{50})$

x_{40} = expected value of the time out for classification and neutralization of false targets

$= h_5(x_{33}, x_{47}, x_{48}, x_{51}, x_{52}, x_{53})$

x_{41} = free-route (no gear in tow) sustained speed of minehunting ship

x_{42} = transit distance between support area and minefield for minehunting ship

x_{43} = average minehunting vehicle speed while streaming or retrieving gear

x_{44} = clearance gear-streaming time for minehunting ship

x_{45} = clearance gear-retrieval time for minehunting ship

x_{46} = number of mines in area to be cleared by minehunting ship

$= i_2(x_{38}, x_{54})$

x_{47} = time to classify a detected mine or false target

x_{48} = time to conduct one neutralization attempt

x_{49} = time to classify a detected mine as a mine

x_{50} = probability of detecting a single mine in a single coverage

x_{51} = number of false targets in area to be cleared by minehunting ship

$= i_3(x_{38}, x_{55})$

x_{52} = probability of classifying a detected false target as a mine

x_{53} = probability of detecting a single false target in a single coverage

x_{54} = density of bottom mines

x_{55} = density of false targets

3.5) Additional MOE's Identified:

(a) Expected number of mines neutralized in minehunting operation

(b) Expected number of neutralization units required per day in minehunting operation

4) MOE Usage In Study: The MOE was formulated and used to compare the operational effectiveness of alternative small-displacement mine-clearance ships.

5) Special Study Assumptions:

(a) In the minesweeping case,

(a.1) The mines are uniformly distributed in space (latitude and longitude).

(a.2) For influence minesweeping no gear failures occur.

(a.3) The rate (in destroyers per sweep hour) of encountering mines that will destroy moored sweep gear is constant, and the probability density function of the number of mines that will destroy moored sweep gear is Poisson.

(b) In the minehunting case,

(b.1) All coverages are independent with respect to detection and classification, i.e., the probability of detecting a mine as a mine-like object and classifying it as a mine is the same for all coverages.

(b.2) For each detection of a mine, a classification operation is performed.

(b.3) A neutralization operation will occur only once for each detection classified as a mine. Contacts that have been subjected to a neutralization operation will be marked and ignored on succeeding coverages if they continue to be detected as mine-like.

(c) No consideration is given to the probability that the mine-clearance vessels are able to refuel successfully when necessary.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) density of false targets

2) Threat Composition: Moored mines and bottom influence mines

2.1) Platform Type: Moored mine

2.1.1) Deployment:

2.1.1.1) Qualitative Factor:

(a) uniformly distributed in area

2.1.1.2) Quantitative Factors:

(a) probability that a moored minefield device is an obstructor mine

(b) density of moored mines

2.2) Platform Type: Bottom influence mine

2.2.1) Deployment:

2.2.1.1) Quantitative Factor:

(a) density of bottom mines

3) Friendly Force Composition: Minesweepers and minehunters

3.1) Platform Type: Minesweeper

3.1.1) Quantitative Factors:

(a) turning time per turn for minesweeping ship

(b) speed of advance of minesweeping ship through minefield while sweeping

(c) time available for continual minesweeping operations

(d) maximum number of moored sweep gear failures possible in a day for a single ship

(e) daily time limit for minesweeping vehicle operation (limited by daylight or crew fatigue)

(f) number of moored sweep gear spares

(g) clearance gear-streaming time for minesweeping ship

(h) clearance gear-retrieval time for minesweeping ship

(i) free-route (no gear in tow) sustained speed for minesweeping ship

(j) average minesweeping vehicle speed while streaming or retrieving gear

(k) number of moored sweep gear failures

3.1.2) Tactics:

3.1.2.1) Qualitative Factors:

(a) for moored minesweeping either protected echelon or nonprotected formations are used

(b) for influence-sweeping the formation consists of single-coverage grid sweeping with overlapping of the sweep-gear characteristic actuation width

3.1.2.2) Quantitative Factors:

(a) track separation distance between two succeeding tracks of minesweeping vehicles

(b) sweep path distance along minesweeping track before a turn is required

3.2) Platform Type: Minehunter

3.2.1) Quantitative Factors:

(a) time available for continual minehunting operations

(b) daily time limit for minehunting vehicle operation (limited by daylight or crew fatigue)

(c) speed of advance of minehunting ship through minefield while detecting

(d) turning time per turn for minehunting ship

(e) free-route (no gear in tow) sustained speed of minehunting ship

(f) average minehunting vehicle speed while streaming or retrieving gear

(g) clearance gear-streaming time for minehunting ship

(h) clearance gear-retrieval time for minehunting ship

3.2.2) Tactics:

3.2.2.1) Qualitative Factor:

(a) minehunting tactics are based on a continuing series of single mine-clearance operations

3.2.2.2) Quantitative Factors:

(a) number of coverages over the area

(b) track separation distance between two succeeding tracks of a minehunting vehicle during detection operation

(c) sweep path distance along minehunting track before a turn is required

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform:

4.1.1) Type: Minesweeper - Moored mines/ bottom influence mines

4.1.1.1) Quantitative Factors:

(a) expected number of minesweeping ships lost due to attrition in the minefield

(b) area to be cleared by minesweeping force

(c) transit distance between support area and minefield for minesweeping ship

4.1.2) Type: Minesweeper - Moored mines

4.1.2.1) Quantitative Factors:

(a) probability that a moored obstructor mine will destroy moored sweep gear

(b) probability that a moored mine (non-obstructor) will destroy sweep gear

4.1.3) Type: Minehunter - Bottom Influence mines

4.1.3.1) Quantitative Factors:

(a) expected number of minehunting ships lost due to attrition in the minefield

(b) area to be cleared by minehunting force

(c) transit distance between support area and minefield for minehunting ship

(d) time to classify a detected mine or false target

(e) time to conduct one neutralization attempt

(f) time to classify a detected mine as a mine

(g) probability of detecting a single mine in a single coverage

(h) probability of classifying a detected false target as a mine

(i) probability of detecting a single false target in a single coverage

STUDY REVIEW SUMMARY NO. (6)-3

A. STUDY DESCRIPTION

1) Originating Activity: Minesweeping Branch, Bureau of Ships, Washington, D.C.
2) Report Title: Risk to Mine Countermeasures Vessels in Assault Operations
3) Author: R.K. Reber
4) Report Number: Minesweeping Branch Technical Note No. 36 (AD-517 452)
5) Date: 29 July 1964
6) Classification: Confidential
7) Abstract: The present paper considers risk to the mine countermeasures vessels as it relates to the problem of integrating minesweeping and minehunting in assault operations. In particular, consideration is given to the effect of risk to countermeasures vessels on the appropriate choice of a sequence of mine countermeasures operations and on the appropriate choice of a division of effort between sweeping and hunting. Also considered is the question of what can be accomplished by precursor sweeping in assault operations. Although the study is concerned mainly with assault operations, the simpler but related problem of countermeasures risk in clearance operations is considered briefly.
8) Descriptors: Assault ship, game theory, mine, mine countermeasure, minehunter, minesweeper, ship counter device

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Mine Countermeasures
3) Mission: Mine clearance
   3.1) Definition: A combination of minesweepers and minehunters search for mines in an area to be traveled by assault ships.

3.2) Criterion For Success: Clearance of minefield

3.3) MOE Selected: Risk to the countermeasures vessels, which is defined as the expected value of the ratio of the number of mines exploded within the damage radius of the countermeasures vessels to the number of mines initially in the channel or area in which countermeasures are carried out

3.4) Functional Form Of MOE:

Case 1 - Clearance sweeping

MOE = $f_1(x_1, x_2)$

where

x_1 = risk to minehunters in first minehunting operation in area clearance

= $g_1(x_3, x_4, x_5, x_8)$

x_2 = risk to minesweepers in sweeping operation in area clearance

= $g_2(x_9, x_{10}, x_{11})$

x_3 = characteristic detection width of search gear

x_4 = characteristic detection probability of search gear

x_5 = aggregate danger width of minehunter

= $h_1(x_6, x_7)$

x_6 = danger front of minehunter

x_7 = probability of actuating while within the danger area a mine with random athwartship position within the danger front

x_8 = specified clearance in fractional units of detectable 1-count mines during the minehunting searching operation taking into consideration both the mines which are detected and neutralized and the mines which are exploded by the field of the hunter

x_9 = aggregate actuation width of sweeps

x_{10} = aggregate danger width of sweeper

x_{11} = fraction of mines neutralized or removed during the hunting operation

$= h_2(x_8, x_{12}, x_{13}, x_{14})$

x_{12} = specified minehunting clearance of detectable mines in fractional units

x_{13} = fraction of mines in area that are undetectable

x_{14} = maximum ship-count setting

Case 2 - Non-clearance sweeping (in preparation for an assault operation

$MOE = f_2(x_{13}, x_{15}, x_{16})$

where

x_{15} = risk to countermeasures vessels from detectable mines

$= g_3(x_{17}, \ldots, x_{20})$

x_{16} = risk to countermeasures vessels from undetectable mines

$= g_4(x_{21}, \ldots, x_{24})$

x_{17} = risk to minesweepers from detectable mines in first minesweeping operation

$= h_3(x_9, x_{10}, x_{25}, x_{26})$

x_{18} = risk to minesweepers from detectable mines in second minesweeping operation

$= h_4(x_9, x_{10}, x_{25}, \ldots, x_{29})$

x_{19} = risk to minehunters from detectable mines in first minehunting operation

$= h_5(x_3, x_4, x_5, x_{25}, x_{26}, x_{29})$

x_{20} = risk to minehunters from detectable mines in second minehunting operation

$= h_6(x_3, x_4, x_5, x_{25}, x_{26}, x_{27}, x_{29}, x_{30})$

x_{21} = risk to minesweepers from undetectable mines in first minesweeping operation

$= h_7(x_9, x_{10}, x_{25}, x_{26})$

x_{22} = risk to minesweepers from undetectable mines in second minesweeping operation

$= h_8(x_9, x_{10}, x_{25}, \ldots, x_{33})$

x_{23} = risk to minehunters from undetectable mines in first minehunting operation

$= h_9(x_{25}, x_{29}, x_{32})$

x_{24} = risk to minehunters from undetectable mines in second minehunting operation

$= h_{10}(x_{25}, x_{27}, x_{30}, x_{33})$

x_{25} = average search density due to sweeps for the first sweeping operation

x_{26} = ship-count setting

x_{27} = average search density due to sweeps for the second sweeping operation

x_{28} = specified minehunting clearance of detectable mines in fractional units in the first searching operation

x_{29} = specified percentage clearance in fractional units of detectable 1-count mines during the first minehunting operation

x_{30} = specified percentage clearance in fractional units of detectable 1-count mines during the first and second minehunting operation

x_{31} = specified minehunting clearance of detectable mines in fractional units in the first and second searching operations

x_{32} = average sweeping density due to field of minehunters for first searching operation

x_{33} = average sweeping density due to field of minehunters for second searching operation

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) Mines which are detected on any given run are detected a safe distance ahead or abeam and they are then neutralized without risk to the hunter; this is called ahead neutralization.

(b) The major risk to the countermeasures vessels comes from acoustic mines.

(c) Results are obtained only for the case of wide channels or areas for which diagonal sweeping is appropriate.

(d) Mines which present a risk to the countermeasures vessels are sweepable by the sweep gear.

(e) Hunters do not tow acoustic sweeps since it appears impractical to do so in ahead neutralization.

(f) In clearance sweeping,

(f.1) The area may contain both sweepable and unsweepable mines and both detectable and undetectable mines.

(f.2) There is sufficient sweeping and hunting to obtain a very high percentage of clearance of all sweepable and all detectable mines.

(f.3) Hunting is carried out first with a searching level sufficient to obtain a specified value of x_{12}.

(f.4) The field of the hunter can not actuate mines outside the damage radius of the hunter.

(f.5) Risk from detectable mines with count setting higher than one will be assumed to be negligible.

(f.6) Ship count settings are uniformly distributed.

(f.7) There is no risk to the minehunters in the second hunting operation since after the first hunting operation all remaining sweepable mines are removed by a sweeping operation.

(f.8) Sweeping is by the skip track method.

(g) In non-clearance sweeping,

(g.1) Assault sweeping is carried out in two stages separated by a minehunting stage. The final sweeping stage is followed by a second minehunting stage.

(g.2) All sweeps are perfect.

(g.3) Sweeping in each stage is by the diagonal method in order to give maximum protection to the minehunters as well as to the assault ships.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Mines

   2.1) Qualitative Factor:

      (a) mix of sweepable and unsweepable

   2.2) Quantitative Factors:

      (a) fraction of mines in area that are undetectable

      (b) maximum ship-count setting

      (c) ship-count setting

3) Friendly Force Composition: Minesweepers and minehunters

   3.1) Platform: Minehunter

      3.1.1) Quantitative Factors:

         (a) characteristic detection width of search gear

         (b) characteristic detection probability of search gear

         (c) danger front of minehunter

   3.2) Platform: Minesweeper

      3.2.1) Quantitative Factor:

         (a) aggregate danger width of sweeper

   3.3) Tactics:

      3.3.1) Quantitative Factors:

         (a) average search density due to sweeps for the first sweeping operation

         (b) average search density due to sweeps for the second sweeping operation

         (c) average sweeping density due to field of minehunters for first searching operation

         (d) average sweeping density due to field of minehunters for second searching operation

4) Friendly Force - Threat Interaction:

   4.1) Platform - Platform:

      4.1.1) Type: Minehunter- Mines

4.1.1.1) Quantitative Factors:

(a) probability of actuating while within the danger area a mine with random athwartship position within the danger front

(b) specified clearance in fractional units of detectable 1-count mines during the minehunting searching operation taking into consideration both the mines which are detected and neutralized and the mines which are exploded by the field of the hunter

(c) specified minehunting clearance of detectable mines in fractional units

(d) specified minehunting clearance of detectable mines in fractional units in the first searching operation

(e) specified percentage clearance in fractional units of detectable 1-count mines during the first minehunting operation

(f) specified percentage clearance in fractional units of detectable 1-count mines during the first and second minehunting operation

(g) specified minehunting clearance of detectable mines in fractional units in the first and second searching operations

4.1.2) Type: Minesweeper - Mines

4.1.2.1) Quantitative Factor:

(a) aggregate actuation width of sweeps

(7)

OCEAN SURVEILLANCE

STUDY REVIEW SUMMARY NO.(7)-1

A. STUDY DESCRIPTION

1) Originating Activity: Planning Research Corporation, Los Angeles, California
2) Report Title: Cost and Effectiveness of Selected Ocean-Area Surveillance Systems
3) Authors: A. W. Corry and J. M. Chester
4) Report Number: PRC R-452 (AD-349 418)
5) Date: 31 December 1963
6) Classification: Secret
7) Abstract: This study deals with methods for accomplishing ocean-area surveillance. Consideration is given to the cost of satellite systems and the cost and effectiveness of aircraft systems for performing surveillance. Other techniques are discussed and comparisons are made relative to the practicality and applicability of such systems to the overall task of performing ocean-area surveillance.
8) Descriptors: Aircraft, cost, optical detection, optical sensor, optical tracking, satellite, surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Ocean Surveillance
3) Mission: Surveillance of ocean area
   - 3.1) Definition: A satellite using an optical sensor scans the ocean-area in search of ships.
   - 3.2) Criterion For Success: Surveillance and establishment of the track of ships at sea

3.3) MOE Selected: Probability of successful tracking of a vessel for a voyage of specified duration

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2, x_3)$$

where

x_1 = voyage duration

x_2 = one-half the maximum time duration between vessel sightings

x_3 = probability of tracking failure during a period of x_2 days

$= g(x_2, x_4, \ldots, x_7)$

x_4 = number of scans of a vessel in daylight hours

x_5 = number of times per day a vessel is scanned

$x_6 = h_1(x_5, x_8)$

$x_7 = h_2(x_5, x_9)$

x_8 = average duration (in days) of cloudy weather

x_9 = average duration (in days) of clear weather

3.5) Additional MOE Identified:

(a) Number of satellites required to provide a specified level of surveillance

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Photographs taken of individual ships are of sufficient quality that recognition can be accomplished.

(b) The vessel to be tracked must be seen at least once in consecutive fixed periods of a specified length during the voyage.

(c) The persistence of weather is taken into account for each period, but is ignored from one period to the next.

(d) Vessel sightings are independent from period to period.

C. EFFECTIVENESS FACTORS

1) Physical Environment:
   - 1.1) Quantitative Factors:
     - (a) average duration (in days) of cloudy weather
     - (b) average duration (in days) of clear weather

2) Target Composition: Vessel
   - 2.1) Deployment:
     - 2.1.1) Qualitative Factor:
       - (a) on ocean surface

3) Friendly Force Composition: Satellite
   - 3.1) Deployment:
     - 3.1.1) Qualitative Factor:
       - (a) orbiting the earth
     - 3.1.2) Quantitative Factors:
       - (a) voyage duration
       - (b) one-half the maximum time duration between vessel sightings

4) Friendly Force - Target Interaction:
   - 4.1) Platform - Platform: Satellite - Vessel
     - 4.1.1) Quantitative Factors:
       - (a) number of scans of a vessel in daylight hours
       - (b) number of times per day a vessel is scanned

STUDY REVIEW SUMMARY NO. (7)-2

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Postgraduate School, Monterey, California
2) Report Title: Optimal Allocation of Pacific Fleet Patrol Aircraft among Selected Deployment Sites
3) Author: S. S. Massey, Jr.
4) Report Identification: Thesis for the Masters of Science in Operations Research, (AD-704 083)
5) Date: October 1969
6) Classification: Unclassified
7) Abstract: A methodology is developed which determines the optimal allocation of patrol forces among selected deployment sites. The procedure uses a linear programming algorithm which minimizes a linear cost function, subject to restraining equations representing the total hours available, the relationship between on-station and transit hours, and base loading. The methodology can be utilized to determine the allocation of forces among selected bases, reallocation of forces when a base, or bases, must be removed from consideration, and the effect of utilizing additional bases.
8) Descriptors: Aircraft, cost, force allocation, linear programming, surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Ocean Surveillance
3) Mission: Surveillance of ocean area
   3.1) Definition: Patrol aircraft provide surveillance coverage of specific coastal or ocean areas.
   3.2) Criterion For Success: Provide required patrol coverage at least cost

3.3) MOE Selected: Minimum cost of providing the required on-station hours

3.4) Functional Form Of MOE:

$$MOE = \min_{x_1} f(x_1, \ldots, x_{12})$$

where

$f(x_1, \ldots, x_{12})$ = total cost of providing the required on-station hours given allocation strategy x_1

x_1 = a matrix whose (i,j,k)th entry represents the number of on-station hours per month allocated to area (i,j) from base k

x_2 = a matrix whose (i,j,k)th entry represents the cost per on-station hour in area (i,j) when flown from base k
= $g_1(x_8, x_9, x_{13})$

x_3 = a matrix whose (i,j)th entry represents the on-station hours per month required in area (i,j)

x_4 = a matrix whose (i,j,k)th entry represents the number of transit hours per month to area (i,j) from base k

x_5 = total hours available per month for training and miscellaneous flying at all bases

x_6 = a vector whose ith component represents the flight time in hours per month available from base i

x_7 = total flight hours available

x_8 = a vector whose ith component represents the average sortie length in hours from base i

x_9 = a matrix whose (i,j,k)th entry represents the distance in nautical miles from base k to operating area (i,j)

x_{10} = number of bases

x_{11} = number of vertical strips in the rectangular grid

x_{12} = number of horizontal strips in the rectangular grid

x_{13} = a vector whose ith component represents the cost per flight hour when flown from base i

x_{14} = length of each rectangular subarea

x_{15} = width of each rectangular subarea

x_{16} = a vector whose i^{th} component represents the location of base i

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) All aircraft used have identical characteristics.

(b) An area A exists into which it is desired to allocate a specific amount of patrol effort. To facilitate the development, a rectangular grid is superimposed upon A and its supporting bases. This grid is of sufficient size that all of area A and its supporting bases are enclosed within the borders of the rectangle. Moreover, the grid will subdivide area A into a number of subareas of equal size.

(c) Any flight designated to operate in a specific area will proceed to the center of that area prior to beginning its on-station period.

(d) The system under consideration, that of patrol aircraft and bases, has been in the operating forces for many years; hence any costs associated with any Research and Development, or Investment phase is not considered. The annual operating costs, those recurring outlays which are needed to operate and maintain activities in service, are the only costs considered.

(e) The cost is assumed to be a linear function of the on-station hours.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factors:

(a) number of vertical strips in the rectangular grid

(b) number of horizontal strips in the rectangular grid

(c) length of each rectangular subarea

(d) width of each rectangular subarea

2) Friendly Force Composition: Bases and Aircraft (P-3)

2.1) Platform Type: Base

2.1.1) Quantitative Factors:

(a) total hours available per month for training and miscellaneous flying at all bases

(b) a vector whose $i^{\underline{th}}$ component represents the flight time in hours per month available from base i

(c) a vector whose $i^{\underline{th}}$ component represents the average sortie length in hours from base i

(d) number of bases

(e) a vector whose $i^{\underline{th}}$ component represents the cost per flight hour when flown from base i

2.1.2) Deployment:

2.1.2.1) Quantitative Factor:

(a) a vector whose $i^{\underline{th}}$ component represents the location of base i

(b) a matrix whose $(i,j,k)^{\underline{th}}$ entry represents the distance in nautical miles from base k to operating area (i,j)

2.2) Platform Type: Aircraft

2.2.1) Deployment:

2.2.1.1) Quantitative Factor:

(a) total flight hours available

2.2.2) Tactics:

2.2.2.1) Quantitative Factors:

(a) a matrix whose $(i,j)^{\underline{th}}$ entry represents the on-station hours per month required in area (i,j)

(b) a matrix whose $(i,j,k)^{\underline{th}}$ entry represents the number of transit hours per month to area (i,j) from base k fasystems

STUDY REVIEW SUMMARY NO. (7)-3

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Electronics Laboratory Center, San Diego, California
2) Report Title: A General Localization Probability Model for EM Emitters in a DF Network
3) Report Author: D. M. Heller
4) Report Number: TN-1781
5) Date: 1 December 1970
6) Classification: Unclassified
7) Abstract: This report presents a methodology for establishing an analytical expression for the probability of localization as a function of bearing accuracy of direction finders (DF), the geometry of the communication situation, and the probability of signal detection for a defined network of direction finding sites.
8) Descriptors: Communications, detection, direction finding, localization, normal density function, submarine, surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Ocean Surveillance
3) Mission: Surveillance of ocean area
   3.1) Definition: A network of direction finding sites is distributed so as to provide surveillance over a large ocean area in which a patrolling submarine may, on occasion, come to the surface and transmit a brief radio message. This electromagnetic emission, when detected at one or more DF sites, initiates a submarine localization effort.
   3.2) Criterion For Success: Successful determination of bearing to transmitting submarine

3.3) MOE Selected: Probability that at least one pair of direction finding sites successfully determines bearings and the localization area to a specified size

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2, x_3)$$

where

x_1 = number of direction finding sites

x_2 = a vector whose i^{th} component is the probability that site i detects the communication and establishes a bearing on the submarine

x_3 = a matrix whose $(i,j)^{th}$ entry is the probability that the true location of the submarine is within area x_4 given that bearings from sites i and j are used to determine the center of x_4

$= g(x_1, x_4, \ldots, x_9)$

x_4 = required localization area

x_5 = a vector whose i^{th} component represents the bearing error for site i

x_6 = variance of the bearing error for each direction finding site

x_7 = a matrix whose $(i,j)^{th}$ entry is the distance between sites i and j

x_8 = horizontal coordinate of the true submarine location at time of communication

x_9 = vertical coordinate of the true submarine location at time of communication

3.5) MOE Usage In Study: Formulation and illustrative numerical results

3.6) Special Study Assumptions:

(a) The submarine electromagnetic emissions must meet specified direction finding site detection critera.

(b) True submarine location does not lie on a line connecting any two DF sites.

(c) The bearing error for each DF site is normally distributed with mean zero and constant variance (x_6), the same for each site.

(d) In the formulation of the MOE the events are treated as independent in the sense that the detection of the submarine communication by one site is independent of the detection of the submarine communication by another site. It is felt that, even though not correct, it is a good approximation.

(e) First order approximations are used in determining submarine position errors.

(f) The network of DF sites is stationary.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Platform: Submarine

2.1.1) Deployment:

2.1.1.1) Qualitative Factor:

(a) patrol of ocean area

2.1.1.2) Quantitative Factors:

(a) horizontal coordinate of the true submarine location at time of communication

(b) vertical coordinate of the true submarine location at time of communication

3) Friendly Force Composition: Network of direction finding (DF) sites

3.1) Platform: DF sites

3.1.1) Quantitative Factors:

(a) number of direction finding sites

(b) a vector whose i^{th} component represents the bearing error for site i

(c) variance of the bearing error for each direction finding site

3.1.2) Deployment:

3.1.2.1) Qualitative Factor:

(a) stationary array

3.1.2.2) Quantitative Factor:

(a) a matrix whose $(i,j)^{\underline{th}}$ entry is the distance between sites i and j

3.1.3) Tactics:

3.1.3.1) Quantitative Factors:

(a) required localization area

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: DF sites - Submarine

4.1.1) Quantitative Factors:

(a) a vector whose $i^{\underline{th}}$ component is the probability that site i detects the communication and extablishes a bearing on the submarine

(8)

SUBMARINE ASW

STUDY REVIEW SUMMARY NO.(8)-1

A. STUDY DESCRIPTION

1) Originating Activity: Daniel H. Wagner, Associates, Paoli, Pennsylvania
2) Report Title: Submarine - Versus - Submarine Secure Sweep Width Manual
3) Report Number: DHWA Log No. 13-504
4) Date: 17 December 1964
5) Classification: Confidential
6) Contract: Nonr-4192(00)(X) and Nobs-92146 (Ship Silencing Branch, Bureau of Ships)
7) Abstract: A method is presented in manual form for predicting a specific submarine's secure sweep width or secure sweep rate against a specific type of submarine target. These measures of effectiveness exclude detections previously counterdetected by the target. Either adversary may be nuclear or diesel. The methods are illustrated by examples and are applied to a variety of problems in choice of optimal speeds and other tactical parameters.
8) Descriptors: Antisubmarine warfare, detection, normal density function, sonar, submarine, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situations:
   3.1) Tactical Situation Type: SSK versus Transitor

3.1.1) Definition: A submarine covers a frontage against which enemy submarines attempt to penetrate or to transit past.

3.1.2) Criterion For Success: Obtain secure detection of submarine

3.1.3) MOE Selected: Secure sweep width, which is defined as the product of the width of frontage over which target crossings are equally likely at all points times the expected fraction of targets on which own ship makes secure detection

3.1.3.1) Rationale For Selection: Secure sweep width gauges the ability of own ship to cover frontage, and is particularly relevant to SSK versus transitor. This MOE combines a variety of acoustic effects, kinematic effects, and probabilistic effects on detection into a single number--this number may be used to compare one ship against another or one tactic against another, as well as to estimate force requirements for detection purposes. This MOE may also be used in planning and analyzing exercises. By restricting the MOE to secure detections, account is taken automatically of counterdetection possibilities.

3.1.4) Functional Form Of MOE:

Case 1 - SSK and transitor are both nuclear submarines

$(MOE)_1 = f_1(x_1, x_2)$

where

x_1 = tentative secure sweep width (nuclear vs. nuclear)

$= g_1(x_3, x_4)$

x_2 = kinematic correction factor for secure sweep width (nuclear vs. nuclear)

$$= \begin{cases} 1 & \text{if own ship hovers} \\ g_2(x_5,x_6) & \text{if own ship and target ship courses are perpendicular} \\ g_3(x_5,x_6) & \text{if own ship and target ship courses are random} \end{cases}$$

x_3 = insecure sweep width

$$= \begin{cases} h_1(x_7,\ldots,x_{11}) & \text{for environments where bottom bounce and surface duct are possible} \\ h_2(x_7,\ldots,x_{11}) & \text{for no bottom bounce propagation} \\ h_3(x_7,\ldots,x_{11}) & \text{for Arctic (under ice)} \\ h_4(x_7,\ldots,x_{11}) & \text{for shallow water} \\ h_5(x_7,\ldots,x_{11}) & \text{for sound channel or isovelocity} \end{cases}$$

x_4 = secure correction factor (nuclear vs. nuclear)

$$= \begin{cases} h_6(x_7,\ldots,x_{10},x_{12}) & \text{for environments where bottom bounce and surface duct are possible} \\ h_7(x_7,\ldots,x_{10},x_{12}) & \text{for no bottom bounce propagation} \\ h_8(x_7,\ldots,x_{10},x_{12}) & \text{for Arctic (under ice)} \\ h_9(x_7,\ldots,x_{10},x_{12}) & \text{for shallow water} \\ h_{10}(x_7,\ldots,x_{10},x_{12}) & \text{for sound channel or isovelocity} \end{cases}$$

x_5 = own ship speed

x_6 = target ship speed

x_7 = own ship depth

x_8 = target ship depth

x_9 = ocean layer depth

x_{10} = ocean bottom depth

x_{11} = figure of merit for own ship detecting target ship

$= i_1(x_{13},\ldots,x_{16})$

x_{12} = acoustic advantage

$= i_2(x_{11},x_{17})$

x_{13} = target ship radiated noise

x_{14} = own ship background noise (self noise and ambient noise)

x_{15} = directivity index of own ship sonar

x_{16} = own ship sonar recognition differential

x_{17} = figure of merit for target ship detecting own ship

$= j_1(x_{18},\ldots,x_{21})$

x_{18} = own ship radiated noise

x_{19} = target ship background noise (self noise and ambient noise)

x_{20} = directivity index of target ship sonar

x_{21} = target ship sonar recognition differential

Case 2 - SSK is a nuclear submarine and transitor is a diesel submarine

$(MOE)_i = f_2(x_{22},x_{23})$

where

x_{22} = tentative secure sweep width (nuclear vs. diesel)

$= g_4(x_3,x_{24})$

x_{23} = kinematic correction factor for secure sweep width (nuclear vs. diesel)

$$= \begin{cases} 1 & \text{if own ship hovers} \\ g_5(x_5,x_{25}) & \text{if own ship and target ship courses are perpendicular} \\ g_6(x_5,x_{25}) & \text{if own ship and target ship courses are random} \end{cases}$$

x_{24} = secure correction factor (nuclear vs. diesel)

$= h_{11}(x_{26},\ldots,x_{29})$

x_{25} = target ship speed of advance

x_{26} = target ship quiet mode relative distance

$= i_3(x_{30},x_{31})$

x_{27} = target ship noisy mode relative distance

$= i_4(x_{32},x_{33})$

x_{28} = target ship quiet mode correction factor

$$= \begin{cases} i_5(x_7,\ldots,x_{10},x_{12},x_{34}) & \text{for environments where bottom bounce and surface duct are possible} \\ i_6(x_7,\ldots,x_{10},x_{12},x_{34}) & \text{for no bottom bounce propagation} \\ i_7(x_7,\ldots,x_{10},x_{12},x_{34}) & \text{for Arctic (under ice)} \\ i_8(x_7,\ldots,x_{10},x_{12},x_{34}) & \text{for shallow water} \\ i_9(x_7,\ldots,x_{10},x_{12},x_{34}) & \text{for sound channel or isovelocity} \end{cases}$$

x_{29} = target ship noisy mode correction factor

$$= \begin{cases} i_{10}(x_7,\ldots,x_{10},x_{12},x_{35}) & \text{for environments where bottom bounce and surface duct are possible} \\ i_{11}(x_7,\ldots,x_{10},x_{12},x_{35}) & \text{for no bottom bounce propagation} \\ i_{12}(x_7,\ldots,x_{10},x_{12},x_{35}) & \text{for Arctic (under ice)} \\ i_{13}(x_7,\ldots,x_{10},x_{12},x_{35}) & \text{for shallow water} \\ i_{14}(x_7,\ldots,x_{10},x_{12},x_{35}) & \text{for sound channel or isovelocity} \end{cases}$$

x_{30} = time target ship spends quiet

$$x_{31} = \text{target ship quiet mode relative speed}$$

$$= \begin{cases} x_5 & \text{if own ship hovers} \\ x_{36} & \text{if target ship hovers} \\ j_2(x_5, x_{36}) & \text{if own ship and target ship courses are perpendicular} \\ j_3(x_5, x_{36}) & \text{if own ship and target ship courses are random} \end{cases}$$

$$x_{32} = \text{time target ship spends noisy}$$

$$x_{33} = \text{target ship noisy mode relative speed}$$

$$= \begin{cases} x_5 & \text{if own ship hovers} \\ x_{32} & \text{if target ship hovers} \\ j_4(x_5, x_{37}) & \text{if own ship and target ship courses are perpendicular} \\ j_5(x_5, x_{37}) & \text{if own ship and target ship courses are random} \end{cases}$$

$$x_{34} = \text{target ship quiet mode correction}$$

$$= \begin{cases} j_6(x_7, \ldots, x_{10}, x_{17}, x_{26}) & \text{for environments where bottom bounce and surface duct are possible} \\ j_7(x_7, \ldots, x_{10}, x_{17}, x_{26}) & \text{for no bottom bounce propagation} \\ j_8(x_7, \ldots, x_{10}, x_{17}, x_{26}) & \text{for Arctic (under ice)} \\ j_9(x_7, \ldots, x_{10}, x_{17}, x_{26}) & \text{for shallow water} \\ j_{10}(x_7, \ldots, x_{10}, x_{17}, x_{26}) & \text{for sound channel or isovelocity} \end{cases}$$

$$x_{35} = \text{target ship noisy mode correction}$$

$$= \begin{cases} j_{11}(x_7, \ldots, x_{11}, x_{27}) & \text{for environments where bottom bounce and surface duct are possible} \\ j_{12}(x_7, \ldots, x_{11}, x_{27}) & \text{for no bottom bounce propagation} \\ j_{13}(x_7, \ldots, x_{11}, x_{27}) & \text{for Arctic (under ice)} \\ j_{14}(x_7, \ldots, x_{11}, x_{27}) & \text{for shallow water} \\ j_{15}(x_7, \ldots, x_{11}, x_{27}) & \text{for sound channel or isovelocity} \end{cases}$$

x_{36} = target ship speed while quiet

x_{37} = target ship speed while noisy

Case 3 - SSK is a diesel submarine and transitor is a nuclear submarine

$(MOE)_1 = f_3(x_{38}, x_{39})$

where

x_{38} = tentative secure sweep width (diesel vs. nuclear)

$= g_7(x_3, x_{40})$

x_{39} = kinematic correction factor for secure sweep width (diesel vs. nuclear)

$$= \begin{cases} 1 & \text{if own ship hovers} \\ g_8(x_6, x_{41}) & \text{if own ship and target ship courses are perpendicular} \\ g_9(x_6, x_{41}) & \text{if own ship and target ship courses are random} \end{cases}$$

x_{40} = secure correction factor (diesel vs. nuclear

$= h_{12}(x_{42}, \ldots, x_{45})$

x_{41} = own ship speed of advance

x_{42} = own ship quiet mode relative distance

$= i_{15}(x_{46}, x_{47})$

x_{43} = own ship noisy mode relative distance

$= i_{16}(x_{48}, x_{49})$

x_{44} = own ship quiet mode correction factor

$$= \begin{cases} i_{17}(x_7, \ldots, x_{12}, x_{50}) & \text{for environments where bottom bounce and surface duct are possible} \\ i_{18}(x_7, \ldots, x_{12}, x_{50}) & \text{for no bottom bounce propagation} \\ i_{19}(x_7, \ldots, x_{12}, x_{50}) & \text{for Arctic (under ice)} \\ i_{20}(x_7, \ldots, x_{12}, x_{50}) & \text{for shallow water} \\ i_{21}(x_7, \ldots, x_{12}, x_{50}) & \text{for sound channel or isovelocity} \end{cases}$$

x_{45} = own ship noisy mode correction factor

$$
= \begin{cases}
i_{22}(x_7,\ldots,x_{12},x_{51}) & \text{for environments where bottom bounce and surface duct are possible} \\
i_{23}(x_7,\ldots,x_{12},x_{51}) & \text{for no bottom bounce propagation} \\
i_{24}(x_7,\ldots,x_{12},x_{51}) & \text{for Arctic (under ice)} \\
i_{25}(x_7,\ldots,x_{12},x_{51}) & \text{for shallow water} \\
i_{26}(x_7,\ldots,x_{12},x_{51}) & \text{for sound channel or isovelocity}
\end{cases}
$$

x_{46} = time own ship spends quiet

x_{47} = own ship quiet mode relative speed

$$
= \begin{cases}
x_{52} & \text{if own ship hovers} \\
x_6 & \text{if target ship hovers} \\
j_{16}(x_6,x_{52}) & \text{if own ship and target ship courses are perpendicular} \\
j_{17}(x_6,x_{52}) & \text{if own ship and target ship courses are random}
\end{cases}
$$

x_{48} = time own ship spends noisy

x_{49} = own ship noisy mode relative speed

$$
= \begin{cases}
x_{53} & \text{if own ship hovers} \\
x_6 & \text{if target ship hovers} \\
j_{18}(x_6,x_{53}) & \text{if own ship and target ship courses are perpendicular} \\
j_{19}(x_6,x_{53}) & \text{if own ship and target ship courses are random}
\end{cases}
$$

x_{50} = own ship quiet mode correction

$$= \begin{cases} j_{20}(x_7,\ldots,x_{11},x_{42}) & \text{for environments where bottom bounce and surface duct are possible} \\ j_{21}(x_7,\ldots,x_{11},x_{42}) & \text{for no bottom bounce propagation} \\ j_{22}(x_7,\ldots,x_{11},x_{42}) & \text{for Arctic (under ice)} \\ j_{23}(x_7,\ldots,x_{11},x_{42}) & \text{for shallow water} \\ j_{24}(x_7,\ldots,x_{11},x_{42}) & \text{for sound channel or isovelocity} \end{cases}$$

x_{51} = own ship noisy mode correction

$$= \begin{cases} j_{25}(x_7,\ldots,x_{10},x_{17},x_{43}) & \text{for environments where bottom bounce and surface duct are possible} \\ j_{26}(x_7,\ldots,x_{10},x_{17},x_{43}) & \text{for no bottom bounce propagation} \\ j_{27}(x_7,\ldots,x_{10},x_{17},x_{43}) & \text{for Arctic (under ice)} \\ j_{28}(x_7,\ldots,x_{10},x_{17},x_{43}) & \text{for shallow water} \\ j_{29}(x_7,\ldots,x_{10},x_{17},x_{43}) & \text{for sound channel or isovelocity} \end{cases}$$

x_{52} = own ship speed while quiet

x_{53} = own ship speed while noisy

Case 4 - SSK and transitor are both diesel submarines

$(MOE)_1 = f_4(x_{54},x_{55})$

where

x_{54} = tentative secure sweep width (diesel vs. diesel)

$= g_{10}(x_3,x_{56})$

x_{55} = kinematic correction factor for secure sweep width (diesel vs. diesel)

$$= \begin{cases} 1 & \text{if own ship hovers} \\ g_{11}(x_{25}, x_{41}) & \text{if own ship and target ship courses are perpendicular} \\ g_{12}(x_{25}, x_{41}) & \text{if own ship and target ship courses are random} \end{cases}$$

x_{56} = secure correction factor (diesel vs. diesel)

$= h_{13}(x_7, \ldots, x_{12}, x_{17}, x_{26}, x_{27}, x_{42}, x_{43})$

3.1.5) Additional MOE Identified:

(a) Expected number of secure detections the SSK will make on transitors

3.2) Tactical Situation Type: Submarine search by an intruder for a fleet ballistic missile (FBM) submarine (or an SSK on station)

3.2.1) Definition: A submarine searches an area for submarine targets which are presumed to be hiding at some unknown point in the area. (Alternatively, the intruder could be infiltrating a barrier, attempting to find and attack the SSK's in the barrier).

3.2.2) Criterion For Success: Obtain secure detection of submarine

3.2.3) MOE Selected: Secure sweep rate, which is defined as the product of the area of region in which target is equally likely at all points times the expected fraction of targets on which own ship makes secure detection divided by the searching time

3.2.3.1) Rationale For Selection: Secure sweep rate gauges own ship's ability to search an area, and applies particularly to an intruder searching for an FBM or an SSK on-station. This MOE combines a variety of acoustic effects, kinematic effects, and probabilistic effects on detection into a single number--this number may be used to compare on ship against another or one tactic against another, as well as to estimate force requirements for detection purposes. This MOE may also be used in planning and analyzing exercises. By restricting the MOE to secure detections, account is taken automatically of counter-detection possibilities.

3.2.4) Functional Form Of MOE:

Case 1 - Intruder and target are both nuclear submarines

$$(MOE)_2 = f_5(x_1, x_5, x_{57})$$

where

x_{57} = kinematic correction factor for secure sweep rate (nuclear vs. nuclear)

$$= \begin{cases} 1 & \text{if target ship hovers} \\ g_2(x_5, x_6) & \text{if own ship and target ship courses are perpendicular} \\ g_3(x_5, x_6) & \text{if own ship and target ship courses are random} \end{cases}$$

Case 2 - Intruder is a nuclear submarine and target is a diesel submarine

$$(MOE)_2 = f_6(x_5, x_{22}, x_{58})$$

where

x_{58} = kinematic correction factor for secure sweep rate (nuclear vs. diesel)

$$= \begin{cases} 1 & \text{if target ship hovers} \\ g_5(x_5, x_{25}) & \text{if own ship and target ship courses are perpendicular} \\ g_6(x_5, x_{25}) & \text{if own ship and target ship courses are random} \end{cases}$$

Case 3 - Intruder is a diesel submarine and target is nuclear submarine

$$(MOE)_2 = f_7(x_{38}, x_{41}, x_{59})$$

where

x_{59} = kinematic correction factor for secure sweep rate (diesel vs. nuclear)

$$= \begin{cases} 1 & \text{if target ship hovers} \\ g_8(x_6, x_{41}) & \text{if own ship and target ship courses are perpendicular} \\ g_9(x_6, x_{41}) & \text{if own ship and target ship courses are random} \end{cases}$$

Case 4 - Intruder and target are both diesel submarines

$(MOE)_2 = f_8(x_{41}, x_{54}, x_{60})$

where

x_{60} = kinematic correction factor for secure sweep rate (diesel vs. diesel)

$$= \begin{cases} 1 & \text{if target ship hovers} \\ g_{11}(x_{25}, x_{41}) & \text{if own ship and target ship courses are perpendicular} \\ g_{12}(x_{25}, x_{41}) & \text{if own ship and target ship courses are random} \end{cases}$$

4) MOE Usage In Study: The MOE's are formulated and numerical examples are presented. In addition, applications to various problems in choice of optimal speed, optimal snorkel-battery cycle, and optimal depth are discussed.

5) Special Study Assumptions:

(a) The target can usually prevent own ship from achieving its mission (at least to the extent of evading) by being the first of the two to detect the other.

(b) A secure detection is a detection in which the detector has not been previously counterdetected by its target during the encounter in question.

(c) The noisy mode of a diesel submarine is the operation of its diesel engines and is synonymous with snorkeling.

(d) The quiet mode of a diesel submarine is its non-cavitating battery operation.

(e) It is assumed as an approximation that a diesel submarine in the quiet mode is undetectable, and that in the noisy mode it is unable to make detections; neither of these assumptions is strictly correct, but they avoid much mathematical complications.

(f) The methods for computing the MOE's apply to the following types of motion: (1) own ship and target course perpendicular, (2) target course random with respect to own course, and (3) either ship is hovering. (Here "random" means that as far as own information is concerned, the target is just as likely to be on one course as any other course and a ship is said to be hovering if, during an encounter, it moves a distance which is small compared to the detection ranges involved; e.g., making minimum turns, or circling on-station).

(g) In the computation of the insecure sweep width it is assumed that the target takes no evasive action if it makes the first detection, and if target is diesel, then the target's radiated noise is continually that of its snorkeling condition (a similar assumption holds if own ship is diesel).

(h) In a given encounter, the acoustic parameters remain fixed throughout (although statistical variations from encounter to encounter are recognized).

(i) The sonar figure of merit minus propagation loss is normally distributed.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Qualitative Factor:

(a) possible propagation environments are bottom bounce and surface duct, no bottom bounce, Arctic (under ice), shallow water, sound channel or isovelocity

1.2) Quantitative Factors:

(a) ocean layer depth

(b) ocean bottom depth

2) Threat Composition: Submarines

2.1) Quantitative Factors:

(a) target ship speed

(b) target ship radiated noise

(c) target ship background noise (self noise and ambient noise)

(d) target ship speed of advance

2.2) Sensors: Sonar

2.2.1) Quantitative Factors:

(a) directivity index of target ship sonar

(b) target ship sonar recognition differential

2.3) Tactics:

2.3.1) Quantitative Factors:

(a) time target ship spends quiet

(b) time target ship spends noisy

(c) target ship depth

(d) target ship speed while quiet

(e) target ship speed while noisy

3) Friendly Force Composition: Submarine

3.1) Quantitative Factors:

(a) own ship speed

(b) own ship radiated noise

(c) own ship background noise (self noise and ambient noise)

(d) own ship speed of advance

3.2) Sensors: Sonar

3.2.1) Quantitative Factors:

(a) directivity index of own ship sonar

(b) own ship sonar recognition differential

3.3) Tactics:

3.3.1) Quantitative Factors:

(a) time own ship spends quiet

(b) time own ship spends noisy

(c) own ship depth

(d) own ship speed while quiet

(e) own ship speed while noisy

STUDY REVIEW SUMMARY NO.(8)-2

A. STUDY DESCRIPTION

1) Originating Activity: Office of the Chief of Naval Operations, Washington, D. C.
2) Paper Title: "The Development of Submarine Tactics for Antisubmarine Warfare"
3) Authors: S. Francis, W. H. Pugh and F. A. Andrews
4) Source: U.S. Navy Journal of Underwater Acoustics, Vol. 20, No. 3 (Supplement), July 1970, pp. 205-224 (AD-512 804)
5) Classification: Secret (NOFORN)
6) Abstract: A program for the development of antisubmarine submarine (SSK) tactics was started shortly after WW II. Methods for tactical interaction with enemy submarines have been defined, and a model for describing weapon system effectiveness now exists. The Joint SUBPAC/SUBLANT Program in tactical development is described and the role of Commander Submarine Development Group TWO as program coordinator is discussed. A review of the current and proposed elements of a submarine weapon system is made with a statement of operational capabilities and deficiencies which have been observed at sea. Finally, significant accomplishments in SSK tactical development over the past years are discussed and a listing is made of future tactical problems. This latter tactical discussion is based largely on the tactical summary resulting from over 4 years of fleet operational evaluations of the SSN 594 class submarine in antisubmarine warfare.
7) Descriptors: Antisubmarine warfare, classification, detection, kill, submarine, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situation: SSK versus Transitor

3.1) Definition: An SSK is deployed as a single-unit in an operational area through which enemy submarines must transit in order to arrive at their own patrol stations.

3.2) Criterion For Success: Detection and destruction of submarine

3.3) MOE Selected: Number of kills per engagement opportunity

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_5)$$

where

x_1 = probability that the SSK detects a transiting submarine without first being successfully counterattacked, given a detection opportunity existed

x_2 = probability that the SSK correctly classified a transiting submarine without being successfully counterattacked between time of detection and classification, given that the transitor has been detected

x_3 = probability that the SSK makes an attack against a transiting submarine without being successfully counterattacked between time of classification and attack, given that the transitor has been correctly classified

x_4 = probability that the SSK conducts an accurate attack against a transiting submarine without being successfully counterattacked between the time of attack and the time the launched weapon no longer requires control by the firing ship for successful culmination of the attack, given that an attack is made

x_5 = probability that a transiting submarine is destroyed, given that an accurate attack is made

4) MOE Usage In Study: MOE computations are presented based on at-sea evaluations of submarine performance against various type transitors. The data used for computations were collected over several years and, therefore, are averaged over many conditions of sea state, various sound velocity profiles, various crews, and material conditions of equipment.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

3) Friendly Force Composition: Submarine

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) probability that the SSK detects a transiting submarine without first being successfully counter-attacked, given a detection opportunity existed

(b) probability that the SSK correctly classified a transiting submarine without being successfully counterattacked between time of detection and classification, given that the transitor has been detected

(c) probability that the SSK makes an attack against a transiting submarine without being successfully counterattacked between time of classification and attack, given that the transitor has been correctly classified

(d) probability that the SSK conducts an accurate attack against a transiting submarine without being successfully counterattacked between the time of attack and the time the launched weapon no longer requires control by the firing ship for successful culmination of the attack, given that an attack is made

(e) probability that a transiting submarine is destroyed, given that an accurate attack is made

ultrasystems

STUDY REVIEW SUMMARY NO.(8)-3

A. STUDY DESCRIPTION

1) Originating Activity: Mystic Oceanographic Company, Mystic, Connecticut
2) Report Title: An Evaluative Model for SSN Active Sonar Missions
3) Author: R. B. Giddings and A. T. Mollegen Jr.
4) Report Number: D-103-70 (AD-511 719)
5) Date: 17 August 1970
6) Classification: Secret
7) Contract: N00024-69-C-5330 (Naval Ship Engineering Center)
8) Abstract: This report describes an event-based evaluative model for analyzing engagements involving active sonar. It is recommended that effectiveness be characterized (as is conventional) by event occurrences, but also, that valid failures to reach necessary events be attributed to failures to perform the necessary functions. With this approach, problem areas are recognizable and explainable in terms of function performance, as well as tactical stages.
9) Descriptors: Antisubmarine warfare, detection probability, fire control, kill probability, search, sonar, submarine, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Mission: Search and destroy
   3.1) Definition: A submarine searches for hostile submarines and attacks all those that it detects and for which it has an opportunity for attack.
   3.2) Criterion For Success: Destruction of submarine
   3.3) MOE Selected: Expected value of target killed

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_7)$$

where

x_1 = probability of a detection opportunity given search

x_2 = probability of detection given an opportunity

x_3 = probability an approach is initiated given a detection

x_4 = probability of weapon launch given an approach is initiated

x_5 = probability of an accurate fire control solution given weapon launch

x_6 = probability of target kill given an accurate fire control solution

x_7 = target value

3.5) Additional MOE Identified:

(a) Probability of target kill

4) MOE Usage In Study: Formulation only

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factor:

(a) target value

3) Friendly Force Composition: Submarine

3.1) Sensor: Sonar

3.2) Armament: Torpedo

4) Friendly Force - Threat Interaction

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) probability of a detection opportunity given search

(b) probability an approach is initiated given a detection

(c) probability of weapon launch given an approach is initiated

4.2) Sensor - Platform: Sonar - Submarine

4.2.1) Quantitative Factor:

(a) probability of detection given an opportunity

4.3) Armament - Platform: Torpedo - Submarine

4.3.1) Quantitative Factors:

(a) probability of an accurate fire control solution given weapon launch

(b) probability of target kill given an accurate fire control solution

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STUDY REVIEW SUMMARY NO.(8)-4

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Ordnance Laboratory, White Oak, Maryland
2) Report Title: The Application of Operations Analysis to Weapon Systems Development
3) Author: J. C. Hetzler, Jr.
4) Report Number: NOLTR 69-154 (AD-699 138)
5) Date: 5 August 1969
6) Classification: Unclassified
7) Abstract: This report contains a review of the methodology of operations analysis (OA). The basic steps required to formulate and solve an OA problem have been listed and discussed in detail. These procedures have been applied to a typical tactical situation--the submarine barrier patrol. An effectiveness model for a submarine using a hypothetical mix of weapons has been generated. Kill probabilities and cost-effectiveness comparisons have been made for a variety of weapon mix possibilities.
8) Descriptors: Antisubmarine warfare, barrier, cost effectiveness, detection, kill probability, submarine, torpedo, transitor, underwater-to-underwater missile, weapon mix

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situation: SSK versus Transitor
   - 3.1) Definition: An SSK is deployed as a single-unit in an operational area through which enemy submarines must transit in order to arrive at their own patrol stations.
   - 3.2) Criterion For Success: Detection and destruction of submarine

3.3) MOE Selected: Cost-effectiveness, defined to be average cost per kill

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2, x_3)$$

where

x_1 = kill probability of SSK
$= g_1(x_4, \ldots, x_7)$

x_2 = average cost to SSK per attack
$= g_2(x_8, x_9)$

x_3 = risk to SSK during an attack
$= g_3(x_9, \ldots, x_{12}, x_{25}, x_{26})$

x_4 = probability of detecting transitor as it tries to cross the barrier
$= h_1(x_{13}, \ldots, x_{16})$

x_5 = conditional probability of SSK achieving a tactical position that will permit an attack on a transitor, given that the transitor is detected
$= h_2(x_{17}, x_{18})$

x_6 = conditional probability that the SSK weapon system destroys the transitor given that it functions reliably
$= h_3(x_{19}, x_{20}, x_{21})$

x_7 = probability that the SSK weapon system is reliable
$= h_4(x_{22}, x_{23}, x_{24})$

x_8 = average cost of operating SSK on 30-day barrier patrol

x_9 = total cost of weapon equipped SSK
$= h_5(x_{25}, \ldots, x_{29})$

x_{10} = probability that the SSK commits itself to an attack
$= h_6(x_4, x_5)$

x_{11} = probability that SSK attack fails
$= h_7(x_6, x_7)$

x_{12} = probability that transitor destroys SSK in a counter-attack

x_{13} = SSK detection sweep width

x_{14} = barrier width

x_{15} = transitor speed

x_{16} = SSK speed

x_{17} = conditional probability of SSK achieving a tactical position that will permit an attack using torpedoes type I or II, given that the transitor is detected

$= i_1(x_{15}, x_{16})$

x_{18} = conditional probability of SSK achieving a tactical position that will permit an attack using sub-to-sub missiles, given that the transitor is detected

x_{19} = probability that an SSK attack with torpedoes of type I destroys the transitor

x_{20} = probability that an SSK attack with torpedoes of type II destroys the transitor

x_{21} = probability that an SSK attack with sub-to-sub missiles destroys the transitor

x_{22} = probability that a torpedo of type I is reliable

x_{23} = probability that a torpedo of type II is reliable

x_{24} = probability that the sub-to-sub missile is reliable

x_{25} = total cost of SSK equipped only with torpedoes

x_{26} = total cost of SSK equipped with both torpedoes and sub-to-sub missiles

x_{27} = total cost of torpedo type I system

x_{28} = total cost of torpedo type II system

x_{29} = total cost of sub-to-sub missile system

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) The sweep width of SSK's detection unit is constant.

(b) The transitor is assumed to try to cross the barrier on a path perpendicular to the barrier line.

(c) The transitor's unknown location is to be uniformly random with respect to both time and crossing point on barrier line.

(d) The SSK is equipped with two types of torpedoes and perhaps a sub-to-sub missile.

(e) If the SSK is using torpedoes, it cannot obtain an attack position unless the initial detection occurs before transitor crosses the barrier line.

(f) Using a sub-to-sub missile, the SSK can attack no matter where transitor is detected.

(g) The costs include all development, training, maintenance, and exercise costs on a pro rata basis.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine (transitor)

  2.1) Quantitative Factor:

    (a) transitor speed

3) Friendly Force Composition: Submarine (SSK)

  3.1) Quantitative Factors:

    (a) average cost of operating SSK on 30-day barrier patrol

    (b) SSK speed

    (c) total cost of SSK equipped only with torpedoes

    (d) total cost of SSK equipped with both torpedoes and sub-to-sub missiles

  3.2) Sensor: Sonar

    3.2.1) Quantitative Factor:

      (a) SSK detection sweep width

  3.3) Armament: Torpedoes (type I and II) and sub-to-sub missiles

    3.3.1) Type: Torpedo Type I

      3.3.1.1) Quantitative Factors:

        (a) total cost of torpedo type I system

(b) probability that a torpedo of type I is reliable

3.3.2) Type: Torpedo Type II

3.3.2.1) Quantitative Factors:

(a) total cost of torpedo type II system

(b) probability that a torpedo of type II is reliable

3.3.3) Type: Sub-to-sub missile

3.3.3.1) Quantitative Factors:

(a) total cost of sub-to-sub missile system

(b) probability that the sub-to-sub missile is reliable

3.4) Deployment:

3.4.1) Qualitative Factor:

(a) barrier

3.4.2) Quantitative Factor:

(a) barrier width

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) probability that transitor destroys SSK in a counterattack

(b) conditional probability of SSK achieving a tactical position that will permit an attack using sub-to-sub missiles, given that the transitor is detected

4.2) Armament - Platform:

4.2.1) Type: Torpedo Type I - Submarine

4.2.1.1) Quantitative Factor:

(a) probability that an SSK attack with torpedoes type I destroys the transitor

4.2.2) Type: Torpedo Type II - Submarine

4.2.2.1) Quantitative Factor:

(a) probability that an SSK attack with torpedoes type II destroys the transitor

4.2.3) Type: Sub-to-sub missile - Submarine

4.2.3.1) Quantitative Factor:

(a) probability that an SSK attack with sub-to-sub missiles destroys the transitor

STUDY REVIEW SUMMARY NO.(8)-5

A. STUDY DESCRIPTION

1) Originating Activity: Operations Research Incorporated, Silver Spring, Maryland
2) Report Title: Analysis of Design Goals for ASW Submarine Torpedoes
3) Authors: S. H. Howe and J. H. Horden
4) Report Number: NRC:CUW:0303 (ORI, TR-241) (AD-356 454)
5) Date: 1 December 1963
6) Classification: Secret
7) Contract Number: Nonr 2300(08) (Office of Naval Research/National Academy of Sciences)
8) Abstract: This report estimates and compares the operational value of different configurations of antisubmarine torpedoes that might be used by our attack nuclear submarines. The comparison mainly is drawn between an existing torpedo and a very advanced weapon which is being developed. Additional torpedo configurations are considered; first, in order to assess separately the value of increasing torpedo endurance and/or speed; second, to consider the operational value of an interim improvement over the existing torpedo that could, if necessary, be effected before the advanced torpedo can become operational. The analysis determines the fraction of targets detected that can be attacked by particular submarine weapon system.
9) Descriptors: Antisubmarine warfare, kill, normal density function, submarine, torpedo, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW

3) Tactical Situation: SSK versus Transitor

3.1) Definition: An attacking submarine tracks a transiting submarine with passive sonar. At some point in time, the attacker launches a wire-guided torpedo which is directed in the bearing-rider mode. In this mode, the torpedo is continuously redirected so as to position it on a continuously updated bearing from the firing ship to the target. When the torpedo has run out a certain distance, it's sonar is enabled, and it begins to search actively. At this moment the target is alerted and turns away from the torpedo.

3.2) Criterion For Success: Destruction of submarine

3.3) MOE Selected: Firing-range limit; defined as the maximum range-to-target, for a particular target aspect, at which a torpedo can be fired to achieve a specified probability (in this study 90 percent probability was used) of acquiring the target with sufficient endurance remaining for overtaking an alerted submarine that evades by running directly away at maximum speed

3.3.1) Rationale For Selection: This MOE is selected in order to assess how sensitive the performance of the torpedoes under study is to factors such as target capabilities, fire-control errors and torpedo-operating parameters.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2)$$

where

x_1 = probability of torpedo acquiring an unalerted target

$= g_1(x_3, x_4, x_5)$

x_2 = conditional probability that the torpedo overtakes the target given that the target was acquired while unalerted and is now alerted

$= g_2(x_{15}, x_{16})$

x_3 = angle subtended by the sonar beamwidth of the attack submarine

x_4 = runout range of torpedo intercept course to point of acquisition

= $h_1(x_6, \ldots, x_9)$

x_5 = standard deviation of errors in torpedo's lead angle

= $h_2(x_6, x_{10}, \ldots, x_{16})$

x_6 = torpedo lead angle

x_7 = target speed before acquisition

x_8 = torpedo speed

x_9 = range to target at launch of torpedo

x_{10} = sonar bearing error

x_{11} = torpedo gyro set error

x_{12} = torpedo gyro drift error

x_{13} = duration of run on the bearing-rider trajectory to point of acquisition

= $i_1(x_8, x_{14})$

x_{14} = runout range of the torpedo bearing-rider trajectory to point of acquisition

x_{15} = maximum alerting range from point of enable

= $h_3(x_8, x_{17}, x_{18}, x_{22})$

x_{16} = standard deviation of the range error at enabling time

x_{17} = torpedo runout to intercept

= $i_2(x_4, x_{19}, x_{20})$

x_{18} = time to overtake target (at maximum alerting range) after acquisition

= $i_3(x_7, x_8, x_{17}, x_{22}, \ldots, x_{25})$

x_{19} = endurance left for chase

= $j_1(x_{22}, x_{23})$

x_{20} = distance that target (at maximum alerting range) travels before being overtaken after acquisition

x_{21} = torpedo runout range to acoustic enable
x_{22} = total endurance of torpedo
x_{23} = maximum target velocity
x_{24} = target blind time
x_{25} = time for target to accelerate to maximum velocity

3.5) Additional MOE's Identified:

(a) Attack range; defined as the maximum lateral range (less than or equal to the sonar detection range) at which a transiting hostile submarine can be successfully attacked while passing a barrier submarine

(b) Fraction of targets detected that can be attacked by a particular submarine weapon system

(c) Number of hostile submarines sunk during a war of specified duration

4) MOE Usage In Study: A model is formulated and the results are used to evaluate the effectiveness of various types of torpedoes.

5) Special Study Assumptions:

(a) The target has already been detected, so that limitations of sonar and environment are ignored.

(b) The analysis is concerned only with torpedo-limited performance.

(c) The analysis is considered only for the condition of a first-pass interception of the alerted target.

(d) When torpedo sonar is enabled, target is alerted.

(e) For the calculation of the acquisition probability the errors in the target position are ignored and only the errors in sending the torpedo to a known point are considered. The target's actual position is assumed to be normally distributed about the aim point.

(f) The errors in bearing and gyro-set errors are included in the initial fire control solution.

(g) The gyro-drift errors accumulate during the course of the run, and the time used in calculating the contribution of drift errors is measured along the actual curved path rather than along the straight line intercept path.

(h) The gyro-set errors occurring each time commands are given are ignored.

(i) The specific shape of the beam pattern and the effect of having the target track intersect the pattern axis in a direction other than normal are made negligible by requiring the torpedo to reach the immediate vicinity of the target rather than just coming within acquisition range.

(j) The longitudinal contribution of the torpedo velocity error is ignored.

(k) The lateral drift of the torpedo is proportional to the fractional velocity and is related to the torpedo turning rate during the run.

(l) The lateral drift error arising from the torpedo velocity error varies with the shape of the trajectory.

(m) The actual curved path trajectory is 10 percent longer than the straight line intercept path except for bow and stern shots where the two paths are assumed equal.

(n) The torpedo sonar enabling point is chosen such that the torpedo has maximum probability of being enabled before reaching the target.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) target speed before acquisition

(b) maximum target velocity

(c) time for target to accelerate to maximum velocity

(d) target blind time

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2.2) Deployment:

2.2.1) Qualitative Factor:

(a) submerged

2.3) Tactics:

2.3.1) Qualitative Factor:

(a) after torpedo sonar enables, the target is alerted and evades by turning away from the torpedo and accelerating to maximum speed

3) Friendly Force Composition: Submarine

3.1) Sensor: Sonar

3.1.1) Quantitative Factors:

(a) angle subtended by the sonar beamwidth of the attack submarine

(b) sonar bearing error

3.2) Armament: Torpedoes

3.2.1) Qualitative Factor:

(a) wire-guided

3.2.2) Quantitative Factors:

(a) torpedo lead angle

(b) torpedo speed

(c) torpedo gyro-set error

(d) torpedo gyro-drift error

(e) standard deviation of the range error at enabling time

(f) torpedo runout range to acoustic enable

(g) total endurance of torpedo

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factor:

(a) range to target at launch of torpedo

4.2) Armament - Platform: Torpedo - Submarine

4.2.1) Quantitative Factors:

(a) runout range of the torpedo bearing-rider trajectory to point of acquisition

(b) distance that target (at maximum alerting range) travels before being overtaken after acquisition

STUDY REVIEW SUMMARY NO.(8)-6

A. STUDY DESCRIPTION

1) Originating Activity: Daniel H. Wagner, Associates, Paoli, Pennsylvania
2) Report Title: Barrier Effectiveness
3) Author: E. P. Loane
4) Report Identification: Memorandum Serial No. 508/116, DHWA Log No. 44-1448
5) Date: 23 December 1968
6) Classification: Confidential
7) Abstract: This memorandum presents a simplified attrition analysis of a barrier campaign and derives a computationally convenient measure of effectiveness for a barrier force. This measure uses Weapon System Effectiveness (WSE), and all of the prominent factors which should influence such a campaign, to predict the effect of an SSK barrier on suppressing enemy submarine activity in the open ocean. The simplifying assumptions required weaken the measure for purposes of producing absolute estimates of effectiveness; however, it is felt that in terms of relative comparisons and particularly for weighing the tradeoffs between WSE and Weapons System Vulnerability (WSV), the model is useful.
8) Descriptors: Antisubmarine warfare, barrier, Erlang density function, exponential density function, Poisson density function, submarine, transitor, vulnerability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situation: SSK versus Transitor
   - 3.1) Definition: SSK's are deployed as a barrier through which enemy submarines attempt to transit.
   - 3.2) Criterion For Success: Suppression of submarine activity
   - 3.3) MOE Selected: Expected enemy submarine activity
   - 3.4) Functional Form Of MOE:

     $MOE = f(x_1, x_2, x_3)$

where

x_1 = initial number of enemy forces

x_2 = expected duration (in months) of the barrier campaign

x_3 = function with values between 0 and 1

$= g_1(x_4, x_5)$.

$x_4 = h_1(x_2, x_5, \ldots, x_{11})$

$x_5 = h_2(x_1, x_2, x_7, x_8, x_{10}, x_{11}, x_{12})$

x_6 = initial number of SSK's

x_7 = availability of an SSK (i.e., the fraction of time the SSK spends on-station)

x_8 = rate at which enemy submarines transit in both directions (transits per month)

x_9 = weapons system effectiveness (WSE), estimated for a specified patrol area width

x_{10} = total width of barrier

x_{11} = patrol area width

x_{12} = weapons system vulnerability (WSV), estimated for a specified patrol area width

3.5) Additional MOE Identified:

(a) Expected enemy submarine activity if there is no attrition of enemy forces

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) A fixed fraction of the activity takes place in the open ocean.

(b) The MOE implicitly assumes some deployment of the enemy forces at the start of the campaign.

(c) The quality of the enemy and U.S. forces does not change with time and attrition (i.e., WSE and WSV do not change).

(d) The probability of an opportunity for an interaction between a particular transitor and the barrier submarines is inversely proportional to the number of U.S. ships (the constant of proportionality is taken to be the ratio of the patrol area width to the total width of the barrier).

(e) Enemy transits occur as a Poisson process in time.

(f) The duration of the campaign is a random variable with Erlang distribution. In particular, the campaign is divided into two segments, each of which has duration which is exponentially distributed with mean equal to one-half of the expected campaign duration.

(g) Probability that both an SSK and transitor are killed in an encounter is assumed negligible.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

  1.1) Qualitative Factor:

    (a) open ocean

2) Threat Composition: Submarines

  2.1) Quantitative Factor:

    (a) initial number of enemy forces

  2.2) Tactics:

    2.2.1) Qualitative Factor:

      (a) transit of open ocean barrier

    2.2.2) Quantitative Factor:

      (a) rate at which enemy submarines transit in both directions (transits per month)

3) Friendly Force Composition: Submarines (SSK)

  3.1) Quantitative Factors:

    (a) initial number of SSK's

    (b) availability of an SSK

  3.2) Deployment:

    3.2.1) Qualitative Factor:

      (a) barrier

    3.2.2) Quantitative Factors:

      (a) total width of barrier

      (b) patrol area width

      (c) expected duration (in months) of the barrier campaign

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) weapons system effectiveness

(b) weapons system vulnerability

STUDY REVIEW SUMMARY NO. (8)-7

A. STUDY DESCRIPTION

1) Originating Activity: Daniel H. Wagner, Associates, Paoli, Pennsylvania
2) Report Title: Barrier Measure of Effectiveness
3) Author: E.P. Loane
4) Report Identification: DHWA Log No. 64-1531
5) Date: 15 April 1969
6) Classification: Confidential
7) Abstract: This memorandum demonstrates that even with an impregnable barrier there will be some enemy submarine activity in the open ocean resulting from those ships already deployed at the start of the campaign. A new measure of barrier effectiveness, the expected number of successful enemy submarine transits over a campaign of random duration is developed.
8) Descriptors: Antisubmarine warfare, barrier, exponential density function, kill probability, Poisson density function, submarine, transitor, vulnerability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situation: SSK versus Transitor
   - 3.1) Definition: SSK's are deployed as a barrier through which enemy submarines attempt to transit.
   - 3.2) Criterion For Success: Suppression of submarine activity
   - 3.3) MOE Selected: Expected number of successful enemy transits
   - 3.4) Functional Form Of MOE:

     MOE = $f(x_1, \ldots, x_4)$

where

x_1 = initial number of enemy forces

x_2 = expected duration (in months) of the barrier campaign

x_3 = function with values between 0 and 1

$= g_1(x_4, x_5)$

x_4 = rate at which enemy submarines transit in both directions (transits per month)

$x_5 = h_1(x_1, x_2, x_7, x_8, x_{10}, x_{11}, x_{12})$

x_6 = initial number of SSK's

x_7 = availability of an SSK (i.e., the fraction of time the SSK spends on-station)

$x_8 = h_2(x_2, x_6, \ldots, x_{11})$

x_9 = weapons system effectiveness (WSE), estimated for a specified patrol area width

x_{10} = total width of barrier

x_{11} = patrol area width

x_{12} = weapons system vulnerability (WSV), estimated for a specified patrol area width

3.5) Additional MOE Identified:

(a) Probability that the transitor is killed in an encounter

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) Duration of the campaign is a random variable with an exponential distribution.

(b). Probability that a transitor is killed is linearly increasing with the number of SSK's available.

(c) Probability that the SSK is killed is linearly increasing with the number of SSK's available.

(d) A fixed fraction of the activity takes place in the open ocean.

(e) The MOE implicitly assumes some deployment of the enemy forces at the start of the campaign.

(f) The quality of the enemy and U.S. forces does not change with time and attrition (i.e., WSE and WSV do not change).

(g) The probability of an opportunity for an interaction between a particular transitor and the barrier submarines is inversely proportional to the number of U.S. ships.

(h) Enemy transits occur as a Poisson process in time.

(i) Probability that both an SSK and transitor are killed in an encounter is assumed negligible.

C. EFFECTIVENESS FACTORS

1) Physical Environment:
   - 1:1) Qualitative Factor:
     - (a) open ocean

2) Threat Composition: Submarines
   - 2.1) Quantitative Factor:
     - (a) initial number of enemy forces
   - 2.2) Tactics:
     - 2.2.1) Qualitative Factor:
       - (a) transit of an open ocean barrier
     - 2.2.2) Quantitative Factor:
       - (a) rate at which enemy submarines transit in both directions (transits per month)

3) Friendly Force Composition: Submarines (SSK)
   - 3.1) Quantitative Factors:
     - (a) initial number of SSK's
     - (b) availability of an SSK
   - 3.2) Deployment:
     - 3.2.1) Qualitative Factor:
       - (a) barrier
     - 3.2.2) Quantitative Factors:
       - (a) expected duration (in months) of the barrier campaign
       - (b) total width of barrier
       - (c) patrol area width

4) Friendly Force - Threat Interaction:
   - 4.1) Platform - Platform: Submarine - Submarine
     - 4.1.1) Quantitative Factors:
       - (a) weapons system effectiveness
       - (b) weapons system vulnerability

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STUDY REVIEW SUMMARY NO.(8)-8

A. STUDY DESCRIPTION

1) Originating Activity: Daniel H. Wagner, Associates, Paoli, Pennsylvania
2) Report Title: Two Pairs of Measures of Submarine Barrier Performance
3) Author: M. L. Yoseloff
4) Report Identification: DHWA Log No. 70-1557
5) Date: 3 June 1969
6) Classification: Confidential
7) Contract: N60921-69-C-0207
8) Abstract: This memorandum considers the problem of measuring the performance of a submarine barrier in suppressing the activity of enemy transitors. Two pairs of measures will be developed. The first pair will give the expected total number of submarine months of enemy activity, and the expected fractional decrease in total enemy activity for a campaign of fixed duration. The second pair will give the expected values of these for the entire campaign under the hypothesis that the duration of the campaign is exponentially distributed with specified mean. These measures treat each fleet as a homogeneous body with each ship having the average characteristics of the fleet as a whole.
9) Descriptors: Antisubmarine warfare, barrier, exponential density function, kill probability, Poisson density function, submarine, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situation: SSK versus Transitor
    3.1) Definition: SSK's are deployed as a barrier through which enemy submarines attempt to transit.
    3.2) Criterion For Success: Suppression of submarine activity
    3.3) MOE's Selected:

    Case 1 - Campaign has fixed duration

    $(MOE)_1$ = Expected total number of enemy submarine months of activity from the start of the campaign to time t

$(MOE)_2$ = Expected fractional portion of possible activity lost by the enemy because of the barrier

Case 2 - Campaign has uncertain duration

$(MOE)_3$ = Expected total enemy submarine activity for the entire campaign

$(MOE)_4$ = Expected cumulative fractional loss of possible activity by the enemy

3.4) Functional Form Of MOE's

$(MOE)_1 = f_1(x_1 \ldots, x_7)$

$(MOE)_2 = f_2(x_1 \ldots, x_7)$

$(MOE)_3 = f_3(x_1 \ldots, x_{10})$

$(MOE)_4 = f_4(x_1 \ldots, x_{10})$

where

x_1 = total number of SSK's at the start of the campaign

x_2 = total enemy fleet size under consideration at the start of the campaign

x_3 = fraction of enemy ships transiting the barrier per unit time

x_4 = fraction of SSK's on-station throughout the campaign

x_5 = proportionality constant related to the probability of an encounter

x_6 = probability that an SSK kills a transitor given an encounter between the two

x_7 = probability that a transitor counterkills an SSK given an encounter between the two

x_8 = probability that an SSK kills a transitor given a kill $= g_1(x_6, x_7)$

x_9 = probability that a transitor kills an SSK given a kill $= g_2(x_6, x_7)$

x_{10} = mean duration of the campaign

3.5) Additional MOE Identified:

(a) Expected number of enemy submarine months lost to the SSK's because of the barrier

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Mutual kill is impossible.

(b) Encounters and kills occur as a Poisson process with variable intensity proportional to the product of the number of SSK's on station and the number of enemy submarines transiting.

(c) In Case 2, the length of the campaign is assumed to be exponentially distributed.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarines

2.1) Quantitative Factor:

(a) total enemy fleet size under consideration at the start of the campaign

2.2) Tactics:

2.2.1) Qualitative Factor:

(a) transit of barrier

2.2.2) Quantitative Factor:

(a) fraction of enemy ships transiting the barrier per unit time

3) Friendly Force Composition: Submarines (SSK)

3.1) Quantitative Factor:

(a) total number of SSK's at the start of the campaign

3.2) Deployment:

3.2.1) Qualitative Factor:

(a) barrier

3.2.2) Quantitative Factors:

(a) fraction of SSK's on station throughout the campaign

(b) mean duration of the campaign

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) proportionality constant related to the probability of an encounter

(b) probability that an SSK kills a transitor given an encounter between the two

(c) probability that a transitor counterkills an SSK given an encounter between the two

STUDY REVIEW SUMMARY NO. (8)-9

A. STUDY DESCRIPTION

1) Originating Activity: Daniel H. Wagner, Associates, Paoli, Pennsylvania
2) Report Title: Submarines as ASW Escorts for Attack Carriers
3) Author: H. R. Richardson
4) Report Number: DHWA Log No. 15-950 (AD-393 465)
5) Date: 17 October 1966
6) Classification: Secret
7) Contract: N60921-7267 (Naval Ordnance Laboratory)
8) Abstract: This report presents an operations research analysis of the use of submarines as ASW escorts for attack carriers during the principal phases of a typical mission. The objectives of the analysis have been directed to the development of a rationale for specifying required sonar performance and for a preliminary assessment of the potential value of submarines when utilized in this mission. In the analysis emphasis is devoted to the determination of the influence of the detection range of the escort submarine on the expected number of enemy torpedo hits on the carriers.
9) Descriptors: Antisubmarine warfare, carrier, classification, escort submarine, exponential density function, hit probability, kill probability, reliability, submarine, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Mission: Carrier escort
   3.1) Definition: Attack submarines are used as ASW escorts (SSE's) for a carrier task force passing through an area known to contain hostile submarines.

3.2) Criterion For Success: Survival of carriers

3.3) MOE Selected: Expected number of enemy torpedo hits on a carrier for given detection range of the SSE active sonar

3.3.1) Rationale For Selection: For a given detection range of the SSE active sonar, this measure can be used to determine the percentage of the threat which must be countered by additional forces in order to prevent the expected number of hits against a single carrier from exceeding certain fixed levels.

3.4) Functional Form Of MOE:

Case 1 - Carriers and SSE's employ evasive tactics upon detection of enemy penetrators

$\therefore$ MOE = $f_1(x_1,\ldots,x_4)$

where

x_1 = fraction of penetrators which can attain firing range at or before a specified delay time (measured from detection) when the SSE sonar detection range is x_4
= $g_1(x_4,x_5,x_6)$

x_2 = density function of the total time between detection and carrier evasion (i.e., the delay time)
= $g_2(x_7,x_8)$

x_3 = expected number of hits on a carrier given a successful penetration

x_4 = SSE sonar detection range

x_5 = penetrator speed

x_6 = task force speed of advance

x_7 = mean classification time by SSE

x_8 = time required for the SSE to communicate with the task force and for the carriers to commence evasion

Case 2 - Carriers do not evade and SSE's attack penetrators upon detection

MOE = $f_2(x_3,x_9)$

where

x_9 = probability that the SSE fails to disable the penetrator given the SSE sonar detection range is x_4

$= g_3(x_7, x_{10}, x_{11}, x_{12})$

x_{10} = time available after detection to fire the torpedo

$= h_1(x_{13}, x_{14})$

x_{11} = time lag between classification and torpedo launch

x_{12} = single torpedo kill probability

$= h_2(x_{15}, x_{16}, x_{17})$

x_{13} = time available to the SSE to fire a torpedo when the SSE sonar detection range exceeds the critical range

$= i_1(x_4, x_5, x_6, x_{18})$

x_{14} = time required for the torpedo to intercept a point on the 3-hit contour

$= i_2(x_{18}, x_{19})$

x_{15} = torpedo reliability

x_{16} = warhead effectiveness

x_{17} = torpedo hit probability

x_{18} = distance from the SSE to the edge of the 3-hit contour

x_{19} = relative velocity of the torpedo with respect to the moving task force

3.5) Additional MOE Identified:

(a) Probability that the penetrator will attack before the task force has an opportunity to classify and react

4) MOE Usage In Study: The MOE is formulated for each case and numerical examples are presented based upon a postulated future enemy threat.

5) Special Study Assumptions:

(a) During the in-transit period of the mission,

(a.1) Two carriers are assumed to steam a specified distance apart flanked by two SSE's.

(a.2) Two constant task force speeds of advance are allowed. At the lower number, the SSE's move on a zig-zag track which enables one or the other to clear baffles and look astern periodically. In the higher speed of advance case, the SSE's simply parallel the carriers, since attack from astern is not considered likely.

(a.3) Both the nuclear and diesel penetrators are assumed to approach from ahead.

(b) During the on-station period of the mission,

(b.1) The carriers' line of advance is parallel to a nearby coastline.

(b.2) Upon commencement of this period, the carriers separate and operate independently, a single SSE being assigned to each.

(b.3) Diesel penetrators approach anti-parallel (i.e., parallel, but opposed) at a constant speed.

(b.4) Nuclear penetrators approach parallel to the direction of advance at constant speed.

(c) In Case 2 (aggressive escort),

(c.1) The carriers maintain a steady course throughout the engagement.

(c.2) Task force speed is constant and twice that of the penetrator.

(c.3) SSE patrol cycle is based on remaining at each of a series of stations a fixed amount of time and parallel to the course of the carriers.

(c.4) Twenty tracks (ten for the bow and orthogonal approaches, respectively) are considered to be representative of the penetrator distribution. The computations are performed for each track separately and then averaged uniformly.

(c.5) The torpedo employed is a straight-runner. This assumption is felt to be reasonable in the situation considered, since the incoming penetrators are intent upon sinking the carriers and will be maintaining a fairly straight course.

(c.6) Upon detection and classification of a penetrator, the escort submarine with the better chance of success fires a single torpedo.

(d) The escort submarines operate independently of any other ASW forces.

(e) Deterministic SSE sonar detection ranges are used in a cookie-cutter manner. The circular detection range has a 120° sector (called the baffled sector) removed astern of the SSE, and a specified sector on either side of the carrier is blanked, regardless of the carrier's bearing from the SSE.

(f) Enemy penetrators fire their torpedo weapon at a range which provides an expectation of 3 hits out of a salvo of 6 torpedoes.

(g) The fraction of penetrators which may attack within a given amount of time after detection is based upon averaging over all possible bearings.

(h) Upon detection of a penetrator, there is a delay time comprising the classification time required by the SSE and the reaction time of the carriers.

(i) The classification time is taken to be a random variable described by an exponential probability density function.

(j) The carrier reaction time is a specified constant.

(k) The penetrators are deployed in a uniform field, discretized and represented by a finite set of points.

(l) Deloading by penetrators of their torpedo tubes is ignored. This is because, in most cases, if the first salvo is unsuccessful, then the carriers will be able to evade at high speed before a second salvo can be loaded and fired.

(m) The SSE's are responsible for the entire ASW protection of the task force. In a real combat environment additional forces will always be available so that the stated detection range will be conservative to this extent.

(n) Penetrators make a submerged approach.

(o) Cumulative attrition is not accounted for, that is, SSE's and carriers are assumed to survive in both phases of the mission.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarines

 2.1) Qualitative Factor:

  (a) diesel/nuclear

 2.2) Quantitative Factor:

  (a) penetrator speed

 2.3) Tactics:

  2.3.1) Qualitative Factors:

   (a) penetrate totally submerged

   (b) fire 6 torpedoes at a range which will yield 3 hits on the average.

3) Friendly Force Composition: Carriers and submarines

 3.1) Platform: Carrier

  3.1.1) Quantitative Factors:

   (a) task force speed of advance

   (b) time required for the SSE to communicate with the task force and for the carriers to commence evasion

 3.2) Platform: Submarine (SSE)

  3.2.1) Quantitative Factors:

   (a) mean classification time by SSE

   (b) time lag between classification and torpedo launch

  3.2.2) Sensors: Sonar

   3.2.2.1) Quantitative Factor:

    (a) SSE sonar detection range

  3.2.3) Armament: Torpedo

   3.2.3.1) Quantitative Factors:

    (a) torpedo reliability

    (b) warhead effectiveness

    (c) torpedo hit probability

3.2.3.2) Deployment:

3.2.3.2.1) Quantitative Factor:

(a) relative velocity of the torpedo with respect to the moving task force

3.2.4) Deployment:

3.2.4.1) Quantitative Factor:

(a) distance from the SSE to the edge of the 3-hit contour

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Carrier - Submarine

4.1.1) Quantitative Factor:

(a) expected number of hits on a carrier given a successful penetration

STUDY REVIEW SUMMARY NO.(8)-10

A. STUDY DESCRIPTION

1) Originating Activity: Commander Submarine Force, U.S. Atlantic Fleet and Commander Submarine Force, U.S. Pacific Fleet
2) Report Title: Measure of Effectiveness Model for a Submarine in the Intruder Role
3) Report Number: Joint Letter - ComSubLant #0764 & ComSubPac #0710
4) Date: June 1968
5) Classification: Confidential
6) Abstract: This model treats the ability of the submarine in the Intruder role to search out and destroy an enemy submarine in the enemy's own patrol area. A measure of this ability is also indirectly a measure of the ability of the Intruder to inhibit SSK effectiveness since the magnitude of the Intruder's effect on SSK operations is reflected in the threat the Intruder poses. The Intruder may physically move to search his assigned area, or may use little movement in the expectation that the target will come to him.
7) Descriptors: Antisubmarine warfare, classification probability, detection probability, kill probability, search, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Mission: Search and destroy
   - 3.1) Definition: A submarine in the role of an intruder is to seek out and destroy an enemy submarine in the enemy submarine's own patrol area.
   - 3.2) Criterion For Success: Detection and destruction of submarine

3.3) MOE's Selected:

$(MOE)_1$ = Probability that the intruder will detect a target present in the patrol area in a specified time

$(MOE)_2$ = Probability that the intruder will kill the target given that he has detected the target

$(MOE)_3$ = Kill rate, which is defined as the rate at which enemy targets are killed as a function of intruder area size

$(MOE)_4$ = Exchange ratio, which is defined as the expected number of targets killed for each intruder killed

3.3.1) Rationale For Selection: Because of the diversity of intruder tactical concepts, it is difficult to describe a single measure of effectiveness for the intruder mission. It is recognized, however, that for some applications, i.e., ranking of tactics and/or systems, the effectiveness of an intruder must be represented by a single value. The measure "kill rate" is considered to be the most appropriate single value to describe intruder system effectiveness.

3.4) Functional Form Of MOE's:

$$(MOE)_1 = \begin{cases} f_1(x_1,x_2,x_3;t) & \text{when each exercise is the same} \\ f_2(x_4,x_5;t) & \text{otherwise} \end{cases}$$

$$(MOE)_2 = f_3(x_{11},\ldots,x_{17})$$

$$(MOE)_3 = f_4(x_4,x_{11},\ldots,x_{17})$$

$$(MOE)_4 = f_5(x_{18},x_{19})$$

where

t = time from beginning of search

x_1 = a vector whose i^{th} component represents the length of an i^{th} time interval of target exposure ordered so that the i^{th} component is greater than or equal to the $(i-1)^{st}$ component

x_2 = a vector whose $i^{\underline{th}}$ component represents the number of targets detected at or before an exposure of time equal to the $i^{\underline{th}}$ component of x_1

x_3 = a vector whose $i^{\underline{th}}$ component represents the number of targets having an exposure time of the $i^{\underline{th}}$ component of x_1 that have not been detected prior to that time

x_4 = intruder search rate
= $g(x_6,\ldots,x_{10})$

x_5 = size of intruder patrol area

x_6 = total number of detections made on targets that enter the intruder patrol area

x_7 = a vector whose $i^{\underline{th}}$ component represents the size of the $i^{\underline{th}}$ intruder's assigned area

x_8 = a vector whose $i^{\underline{th}}$ component represents the total number of targets present in the $i^{\underline{th}}$ intruder area during the period of the intruder's patrol in the area

x_9 = total number of intruder patrol areas

x_{10} = a matrix whose $(i,j)^{\underline{th}}$ entry represents the total search time for the $i^{\underline{th}}$ target in the $j^{\underline{th}}$ area

x_{11} = number of targets which enter the intruder patrol area and are detected by the intruder either before or after these targets enter the intruder patrol area

x_{12} = number of detected targets which are correctly classified by the intruder

x_{13} = number of targets which are correctly classified by the intruder that are valid approach opportunities

x_{14} = number of correctly classified targets which are attacked by the intruder

x_{15} = number of targets which are attacked by the intruder that are valid for evaluating attack accuracy

x_{16} = number of attacked targets which are accurately attacked by the intruder

x_{17} = probability that the type weapon fired will impact (kill) the target, given that an accurate attack is made

x_{18} = number of enemy targets killed

x_{19} = number of intruders killed

3.5) Additional MOE's Identified:

(a) Effective sonar sweep width of intruder

(b) Probability of kill by intruder as a function of time

(c) False attack ratio for intruder

(d) Probability that the intruder will detect a specified number of targets, given a specified number of targets present in the patrol area, as a function of time

(e) Probability that the intruder will detect at least a specified number of targets, given a specified number of targets present in the patrol area, as a function of time

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Initially the target's location is unknown to the intruder.

(b) The target is not required to travel appreciable distances in order to carry out his mission.

(c) The intruder is operating independently, i.e., not closely coordinated with other friendly forces.

(d) A target is said to be exposed to detection when it and the intruder are both within the intruder's patrol area. An interval of target exposure begins when either: (1) The target enters the intruder's patrol area and the intruder is already there, or (2) the intruder enters its patrol area and the target is already there, or (3) the target and intruder enter the patrol area simultaneously, or (4) the intruder reinitiates search after the target and intruder disengage following a detection (and possible attack) by the intruder.

(e) All intervals of target exposure resulting from comparable exercises are aggregated and then ordered by increasing length.

(f) Each interval of target exposure ends with a detection or a non-detection.

(g) For the second functional form, the probability of detection is assumed to be exponentially distributed.

(h) Detections do not occur unless both the target and Intruder are in the intruder area.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

3) Friendly Force Composition: Submarine

  3.1) Armament: Torpedoes

    3.1.1) Quantitative Factor:

      (a) probability that the type weapon fired will impact (kill) the target, given that an accurate attack is made

  3.2) Deployment:

    3.2.1) Quantitative Factors:

      (a) size of intruder patrol area

      (b) a vector whose $i^{\underline{th}}$ component represents the size of the $i^{\underline{th}}$ intruder's assigned area

      (c) total number of intruder patrol areas

  3.3) Tactics:

    3.3.1) Quantitative Factor:

      (a) a matrix whose $(i,j)^{\underline{th}}$ entry represents the total search time for the $i^{\underline{th}}$ target in the $j^{\underline{th}}$ area

4) Friendly Force - Threat Interaction:

  4.1) Platform - Platform: Submarine - Submarine

    4.1.1) Quantitative Factors:

      (a) a vector whose $i^{\underline{th}}$ component represents the length of an $i^{\underline{th}}$ time interval of target exposure ordered so that the $i^{\underline{th}}$ component is greater than or equal to the $(i-1)^{\underline{st}}$ component

(b) a vector whose i^{th} component represents the number of targets detected at or before an exposure of time equal to the i^{th} component of x_1

(c) a vector whose i^{th} component represents the number of targets having an exposure time of the i^{th} component of x_1 that have not been detected prior to that time

(d) total number of detections made on targets that enter the intruder patrol area

(e) a vector whose i^{th} component represents the total number of targets present in the i^{th} intruder area during the period of the intruder's patrol in the area

(f) number of targets which enter the intruder patrol area and are detected by the intruder either before or after these targets enter the intruder patrol area

(g) number of detected targets which are correctly classified by the intruder

(h) number of targets which are correctly classified by the intruder that are valid approach opportunities

(i) number of correctly classified targets which are attacked by the intruder

(j) number of targets which are attacked by the intruder that are valid for evaluating attack accuracy

(k) number of attacked targets which are accurately attacked by the intruder

(l) number of enemy targets killed

(m) number of intruders killed

STUDY REVIEW SUMMARY NO. (8)-11

A. STUDY DESCRIPTION

1) Originating Activity: Systems Analysis Office, ASW Systems Project Office, White Oak, Maryland
2) Report Title: Submarine Measures of Effectiveness
3) Report Number: ASW-1436 memo Ser. 69-0041 to ASW 14
4) Date: 4 November 1969
5) Classification: Secret
6) Abstract: Various measures of effectiveness for submarine ASW barrier missions are presented.
7) Descriptors: Antisubmarine warfare, availability, barrier, Erlang density function, kill probability, submarine, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situation: SSK versus Transitor
   - 3.1) Definition: SSK's are deployed as a barrier through which enemy submarines attempt to transit.
   - 3.2) Criterion For Success: Destruction of submarine
   - 3.3) MOE Selected: Expected percentage of enemy submarines killed attempting to penetrate barrier
     - 3.3.1) Rationale For Selection: This is a measure of mission success, i.e., prevention of enemy transits through a barrier.
   - 3.4) Functional Form Of MOE:

     MOE = $f(x_1)$

where

x_1 = a function with values between 0 and 1

$= g_1(x_2, x_3, x_4)$

x_2 = parameter of Erlang distribution governing length of campaign

x_3 = SSK kill rate

$= g_2(x_5, \ldots, x_{10})$

x_4 = enemy submarine counterkill rate

$= g_3(x_6, x_7, x_8, x_{10}, x_{11}, x_{12})$

x_5 = initial number of SSK's

x_6 = availability of SSK

x_7 = single enemy submarine transit rate (transits/month)

x_8 = expected duration of the campaign (months)

x_9 = one-on-one kill probability of barrier submarines

x_{10} = total required barrier width in n.m.

x_{11} = initial number of enemy submarines

x_{12} = one-on-one probability of counterkill

3.5) Additional MOE Identified:

(a) Expected number of enemy submarines killed attempting to penetrate barrier

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) The SSK-transitor encounters are one-on-one independent encounters.

(b) No replenishment of attrition losses in barrier.

(c) No change of tactics during campaign, i.e., x_9 and x_{11} remain fixed.

(d) On a given transit, at most one submarine is lost, i.e., mutual kill cannot occur.

(e) On a given transit, the probability that a barrier submarine kills a transitor, or vice-versa, is proportional to the number of barrier submarines remaining.

C. EFFECTIVENESS FACTORS

1) Physical Environment:
   - 1.1) Qualitative Factor:
     - (a) open ocean
2) Threat Composition: Submarine
   - 2.1) Quantitative Factor:
     - (a) initial number of enemy submarines
   - 2.2) Tactics:
     - 2.2.1) Qualitative Factor:
       - (a) transit of open ocean
     - 2.2.2) Quantitative Factor:
       - (a) single enemy submarine transit rate (transits/month)
3) Friendly Force Composition: Submarine (SSK)
   - 3.1) Quantitative Factors:
     - (a) initial number of SSKs
     - (b) availability of SSK
   - 3.2) Deployment:
     - 3.2.1) Qualitative Factor:
       - (a) in barrier
     - 3.2.2) Quantitative Factor:
       - (a) total required barrier width in n.m.
4) Friendly Force - Threat Interaction:
   - 4.1) Platform - Platform: Submarine - Submarine
     - 4.1.1) Quantitative Factors:
       - (a) parameter of Erlang distribution governing length of campaign
       - (b) expected duration of the campaign
       - (c) one-on-one kill probability of barrier submarines
       - (d) one-on-one probability of counterkill

STUDY REVIEW SUMMARY NO. (8)-12

A. STUDY DESCRIPTION

1) Originating Activity: Commander Submarine Force, U.S. Atlantic Fleet and Commander Submarine Force, U.S. Pacific Fleet
2) Report Title: Measure of Effectiveness Model for the SSK Versus Transitor Mission
3) Report Number: #3510, Ser. N352/0663
4) Date: 11 July 1969
5) Classification: Confidential
6) Abstract: An evaluative model is presented for use in measuring the effectiveness of the SSK versus Transitor mission.
7) Descriptors: Antisubmarine warfare, classification probability, detection probability, kill probability, submarine, transitor, vulnerability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Tactical Situation: SSK versus Transitor
   3.1) Definition: An SSK is to detect, attack and kill any enemy submarine which transits the SSK patrol area.
   3.2) Criterion For Success: Detection and destruction of submarine
   3.3) MCE's Selected:

$(MOE)_1$ = SSK/Transitor Effectiveness, which is defined as the probability of the SSK killing a transiting enemy submarine given a detection opportunity

$(MOE)_2$ = SSK/Transitor Vulnerability, which is defined as the probability of accurate counterattack by the SST, given a detection opportunity for the SSK

$(MOE)_3$ = Exchange ratio

3.3.1) Rationale For Selection: $(MOE)_1$ includes vulnerability implicitly in its definition, since, if the Transitor "kills" the SSK, the SSK's action for that transit is effectively terminated. Realizing this implicit reflection of vulnerability in $(MOE)_1$, it is useful to display vulnerability explicitly. This is done by a second related measure, $(MOE)_2$. A third measure, $(MOE)_3$, is related to the first two and is given by the ratio of $(MOE)_1$ to $(MOE)_2$. These three measures provide a complete measure of the effectiveness of the submarine in its role as an SSK.

3.4) Functional Form Of MOE's:

$$(MOE)_1 = f_1(x_1, \ldots, x_5)$$

$$(MOE)_2 = f_2(x_6, \ldots, x_{11})$$

$$(MOE)_3 = f_3(x_1, \ldots, x_{11})$$

where

x_1 = probability that the SSK detects a transiting submarine without first being successfully counterattacked, given a detection opportunity existed

x_2 = probability that the SSK correctly classifies a transiting submarine without being successfully counterattacked between time of detection and classification, given that the Transitor has been detected

x_3 = probability that the SSK makes an attack against a transiting submarine without being successfully counterattacked between time of classification and attack, given that the Transitor has been correctly classified

x_4 = probability that the SSK conducts an accurate attack against a transiting submarine without being successfully counterattacked between the time of attack and the time the launched weapon no longer requires control by the firing ship for successful culmination of the attack, given that an attack is made

x_5 = probability that a transiting submarine is destroyed, given that an accurate attack is made

x_6 = number of detection opportunities (i.e., the number of times a transiting submarine enters the SSK patrol area)

x_7 = number of SSK's accurately counterattacked by the Transitor before the SSK detects the Transitor

x_8 = number of SSK's accurately counterattacked by the Transitor after detection, but before classification by the SSK

x_9 = number of SSK's accurately counterattacked by the Transitor after classification, but before attack by the SSK

x_{10} = number of SSK's accurately counterattacked by the Transitor after attack, but before culmination of an accurate attack

x_{11} = number of SSK's accurately counterattacked by the Transitor after a successfully culminated attack by the SSK

4) MOE Usage In Study: Formulation only

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated
2) Threat Composition: Submarine
3) Friendly Force Composition: Submarine
4) Friendly Force - Threat Interaction:
   4.1) Platform - Platform: Submarine - Submarine
      4.1.1) Quantitative Factors:
         (a) probability that the SSK detects a transiting submarine without first being successfully counterattacked, given a detection opportunity existed

(b) probability that the SSK correctly classified a transiting submarine without being successfully counterattacked between time of detection and classification, given that the Transitor has been detected

(c) probability that the SSK makes an attack against a transiting submarine without being successfully counterattacked between time of classification and attack, given that the Transitor has been correctly classified

(d) probability that the SSK conducts an accurate attack against a transiting submarine without being successfully counterattacked between the time of attack and the time the launched weapon no longer requires control by the firing ship for successful culmination of the attack, given that an attack is made

(e) probability that a transiting submarine is destroyed, given that an accurate attack is made

(f) number of detection opportunities (i.e., the number of times a transiting submarine enters the SSK patrol area)

(g) number of SSK's accurately counterattacked by the Transitor before the SSK detects the Transitor

(h) number of SSK's accurately counterattacked by the Transitor after detection, but before classification by the SSK

(i) number of SSK's accurately counterattacked by the Transitor after classification, but before attack by the SSK

(j) number of SSK's accurately counterattacked by the Transitor after attack, but before culmination of an accurate attack

(k) number of SSK's accurately counterattacked by the Transitor after a successfully culminated attack by the SSK

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STUDY REVIEW SUMMARY NO. (8)-13

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Research Laboratory, Washington, D.C.
2) Report Title: Monte Carlo Simulations of Submarine Barrier Operations
3) Authors: I. Widman and J. S. Lee
4) Report Number: NRL Problem NO. 78801-10
5) Date: Not stated
6) Classification: Unclassified
7) Project Number: RR003-02-41-6153
8) Abstract: A Monte Carlo simulation, to study the effectiveness of the submarine barrier operation, is examined in this report. In the submarine barrier operation, the submarines are stationed in patrol areas to prevent the transit of enemy submarines into the open ocean. This study shows that enemy submarines can reduce their risk of interception by transiting in groups with proper spacing between transitors. It also shows that some of the conventional analytical techniques used in analyzing submarine detection and search operations would lead to erroneous results.
9) Descriptors: Antisubmarine warfare, barrier, detection probability, exponential density function, Monte Carlo method, sonar, submarine, torpedo, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Mission: Barrier placement/patrol
   - 3.1) Definition: To prevent an enemy submarine's transit into the open ocean, submarine barriers are used in the forward area controlled by enemy forces.
   - 3.2) Criterion For Success: Suppression of submarine activity

3.3) MOE's Selected:

In the case of a single enemy transit,

$(MOE)_1$ = Probability that the transiting submarine will be intercepted

In the case of group transits

$(MOE)_2$ = Probability of detection per transitor; i.e., expected number of detections divided by the number of transitors

3.4) Functional Form Of MOE's:

$(MOE)_1 = f_1(x_1, x_2)$

$(MOE)_2 = f_2(x_3, \ldots, x_{10}, x_{19}, x_{20}, x_{22})$

where

x_1 = probability that a single transiting submarine is detected in the barrier

$= g_1(x_3, \ldots, x_{10})$

x_2 = probability that single transiting submarine is intercepted given that it is detected in the barrier

$= g_2(x_{16})$

x_3 = transitor's initial point of entry into the patrol area

x_4 = barrier submarine's initial position when transitor enters patrol area

x_5 = length of the patrol area

x_6 = width of the patrol area

x_7 = width of safety zone

x_8 = time interval between observations

x_9 = search pattern path

$$= \begin{cases} h_1(x_5, x_7, x_{11}, x_{12}) & \text{for linear search} \\ h_2(x_6, x_7, x_{11}, \ldots, x_{14}) & \text{for crossover search} \end{cases}$$

x_{10} = detection probability law (probability of detecting a target as a function of the distance from the detector)

$$= \begin{cases} h_3(x_{15}) & \text{for exponential detection law} \\ h_4(x_{13}) & \text{for definite range law} \end{cases}$$

x_{11} = detection range

x_{12} = barrier submarine's initial position at beginning of search pattern

x_{13} = barrier submarine's speed

x_{14} = transitor's speed

x_{15} = a constant which represents the sensitivity of the detector and the condition of the environment

x_{16} = approach region

$= h_5(x_5, x_6, x_{13}, x_{14}, x_{17}, x_{18})$

x_{17} = firing range of submarine-launched torpedoes

x_{18} = barrier submarine's position at time it detects transitor

$= i(x_3, \ldots, x_{10})$

x_{19} = number of transitors in group transit attempt

x_{20} = factor of detection range increase

$= g_3(x_{11}, x_{19}, x_{21})$

x_{21} = spacing between transiting submarines

x_{22} = spacing between patrol areas

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) In these operations, the ocean area concerned is patrolled by a number of barrier submarines, each stationed in a rectangular area.

(b) Inside each rectangle a specified strip width is established as a safety zone.

(c) Barrier submarines maintain communication silence to avoid being detected by transiting submarines. For the same reason, passive sonar is the only detection device used in this operation.

(d) Transitors are assumed to be traveling at constant speed in a straight line perpendicular to the barrier line either in groups or singly. Both the barrier submarine and transiting submarines are traveling at a speed that achieves highest probability of secure detection.

(e) The barrier submarine must remain in the patrol area. It may fire into, but not enter, a safety zone.

(f) In the linear search case, the barrier submarine patrols along a fixed level parallel to the width of the patrol area, moving back and forth in the barrier region. The submarine changes direction when either it reaches the safety zone or its detection range reaches the boundary.

(g) The crossover search is used when the speed of the barrier submarine is greater that the speed of the transiting submarine.

(h) Discrete time steps are taken for the convenience of digital simulation. At each time step, a sonar contact is simulated to determine whether the barrier submarine detects the transitor. If the barrier submarine detects the transitor, the barrier submarine takes an intercept path to approach within firing range of the transitor. If the barrier submarine does not detect the transitor, the search and transit process proceed to the next time step.

(i) The region from which the barrier submarine can intercept a transitor when the transitor is detected at a specified location is called the approach region. If the barrier submarine is inside the approach region at the time of detection, the barrier submarine is sure to intercept the transitor. Conversely, the transitor will escape if the barrier submarine is outside the approach region at the time of detection.

(j) In the case of group transits,

(j.1) Transitors do not communicate since they wish to reduce the probability of being detected.

(j.2) Neighboring barrier submarines can not be called upon to assist in attacking transitors.

(j.3) As the spacing between transitors approach zero, the noise from various transitors combines to increase the effective detection range. As the spacing increases, it is assumed that the factor decreases exponentially to 1.

(j.4) If two or more detections are made at any observation, the barrier submarine will proceed in a manner that will permit two interceptions if possible, and, if not, to intercept the transitor that requires less chasing.

(j.5) Transitors are unaware of the boundaries of the patrol area, and so one submarine group may fall into two or more patrol areas.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated
2) Threat Composition: Submarine
 2.1) Quantitative Factor:
  (a) transitor's speed
 2.2) Tactics:
  2.2.1) Quantitative Factors:
   (a) number of transitors in group transit attempt
   (b) spacing between transiting submarines
3) Friendly Force Composition: Submarine
 3.1) Quantitative Factor:
  (a) barrier submarine's speed
 3.2) Sensors: Passive sonar
  3.2.1) Quantitative Factor:
   (a) detection range
 3.3) Armament: Torpedoes
  3.3.1) Quantitative Factor:
   (a) firing range of submarine-launched torpedoes
 3.4) Deployment:
  3.4.1) Quantitative Factors:
   (a) length of the patrol area
   (b) width of the patrol area
   (c) width of safety zone
   (d) spacing between patrol areas

3.5) Tactics:

3.5.1) Qualitative Factors:

(a) linear search pattern

(b) crossover search pattern

3.5.2) Quantitative Factors:

(a) time interval between observations

(b) barrier submarine's initial position at beginning of search pattern

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) transitor's initial point of entry into the patrol area

(b) barrier submarine's initial position when transitor enters patrol area

5) Friendly Force - Environment Interaction:

5.1) Sensor - Environment: Passive sonar

5.1.1) Quantitative Factor:

(a) a constant which represents the sensitivity of the detector and the condition of the environment

STUDY REVIEW SUMMARY NO. (8)-14

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Minimizing the Approach Time of an SSK to its Target
3) Author: J. Bram
4) Report Number: OEG Interim Research Memorandum No. 21 (AD-284 796)
5) Date: 13 August 1962
6) Classification: Unclassified
7) Abstract: The problem of finding the path which minimizes the approach time when a killer submarine wishes to overtake an enemy submarine without being counterdetected is formulated and solved.
8) Descriptors: Antisubmarine warfare, contact prosecution, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Mission: Contact prosecution
   3.1) Definition: A killer submarine detects an enemy submarine and attempts to place himself a specified distance directly ahead of the enemy submarine as quickly as possible.
   3.2) Criterion For Success: Preparation for attack in the least possible time without being counterdetected
   3.3) MOE Selected: Minimum approach time
   3.4) Functional Form Of MOE:

$$MOE = \min_{x_1} f(x_1, \ldots, x_5)$$

where

$f(x_1, \ldots x_5)$ = killer submarine approach time if killer submarine travels along approach path x_1

x_1 = killer submarine approach path

x_2 = enemy submarine velocity

x_3 = a function of the range between the killer submarine and the enemy submarine which represents the maximum speed that the killer submarine can travel and keep the probability of being counterdetected at a fixed level

x_4 = killer submarine position relative to the enemy submarine at time of enemy submarine detection

x_5 = distance in front of the enemy submarine that the killer submarine requires for his attack

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) Enemy submarine speed is constant.

(b) Killer submarine travels at maximum speed.

(c) Killer submarine knows how fast he can travel and keep the probability of counterdetection at a fixed level.

C. EFFECTIVENESS MEASUREMENT

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factor:

(a) enemy submarine velocity

3) Friendly Force Composition: (Killer) submarine

3.1) Tactics:

3.1.1) Quantitative Factor:

(a) distance in front of the enemy submarine that the killer submarine requires for his attack

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) a function of the range between the killer submarine and the enemy submarine which represents the maximum speed that the killer submarine can travel and keep the probability of being counterdetected at a fixed level

(b) killer submarine position relative to the enemy submarine at time of enemy submarine detection

STUDY REVIEW SUMMARY NO. (8)-15

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Analysis of the Effectiveness of an SSK Barrier
3) Report Number: OEG Study No. 460
4) Date: 10 October 1951
5) Classification: Confidential
6) Abstract: This study offers a method for estimating the potential effectiveness of the SSK, or antisubmarine submarine. For the sake of specificity, the SSK is taken to be guarding a channel in a specified ocean area, and its effectiveness is taken to be equivalent to the percentage of enemy submarine traffic in the channel which it can destroy. The model set up to describe this situation allows for variations in such factors as the sweep width of the SSK's detection gear, the SSK's submerged speed-endurance, the range of its torpedoes, and the enemy's choice of tactics in passing through the channel. The model therefore makes it possible to estimate the effectiveness not merely of one specific type of SSK, but of any submarine performing an SSK mission. Furthermore, the model can easily be extended to cover SSK operations other than the specific one chosen as an example, and this study considers, in addition to the case of a single SSK in a specific channel, the use of numbers of SSK's in offensive barriers of various shapes. Also discussed are the merits of a defensive SSK barrier, the value of the information on enemy submarine movements that an SSK barrier might provide, and the value of the antisubmarine contributions that an SSK barrier might make by delaying enemy submarines in transit.
7) Descriptors: Antisubmarine warfare, attrition, barrier, detection, kill, sonar, submarine, tactics, torpedo

ultrasystems

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System

2) Function: Submarine ASW

3) Tactical Situation: SSK versus Transitor

3.1) Definition: SSK's are deployed as a barrier through which enemy submarines attempt to transit.

3.2) Criterion For Success: Suppression of submarine activity

3.3) MOE Selected: Expected proportion of enemy submarine traffic destroyed by the SSK's

3.3.1) Rationale For Selection: This MOE is a direct measure of the SSK's effectiveness in performing its primary objective, namely, destruction of enemy submarines. Although the enemy can reduce the effectiveness of an SSK barrier, so far as the destruction of his submarines is concerned, by adoption of more extensive submergence tactics, he cannot do so without suffering the disadvantage that all of his submarines are slowed down.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2)$$

where

x_1 = number of SSK's operating in the channel

x_2 = expected proportion of enemy traffic destroyed by a single SSK

$= g_1(x_3, x_4, x_5)$

x_3 = contact factor (in contacts per SSK per transiting enemy), which is defined as the proportion of transiting submarines that is detected by a single SSK

$= h_1(x_6, x_7, x_8)$

x_4 = attack factor (in attacks per contact), which is defined as the proportion of transiting submarines that is attacked by a single SSK

$= h_2(x_7, x_9, x_{10})$

x_5 = kill factor (in kills per attack), which is defined as the proportion of attacked enemy submarines that is killed by a single SSK

x_6 = width of channel available to enemy submarines

x_7 = sweep width which an SSK can achieve against enemy submarines which are detectable

x_8 = enemy tactics factor, which is defined as the probability that enemy submarines will be detectable (i.e., snorkeling) when passing the SSK

$$= \begin{cases} i_1(x_7, x_{11}, x_{12}) & \text{if targets travel while charging batteries} \\ i_2(x_7, x_{14}, x_{15}) & \text{if targets lie to while charging batteries} \end{cases}$$

x_9 = approach distance, which is defined as the maximum distance to the target track from which an SSK can intercept a target

$= i_3(x_{16})$

x_{10} = torpedo forerun, which is defined as the maximum forerun of the torpedoes available to the SSK

x_{11} = distance enemy submarine travels during one complete snorkel cycle

$= j_1(x_{12}, x_{13})$

x_{12} = distance enemy submarine travels during that portion of one complete snorkel cycle when it is continuously snorkeling

$= k_1(x_{16}, x_{17})$

x_{13} = distance enemy submarine travels during that portion of one complete snorkel cycle when it is running submerged

$= k_2(x_{18}, x_{19})$

x_{14} = number of battery charges required to transit channel if targets lie to while charging batteries

$= j_2(x_{15}, x_{18})$

x_{15} = length of channel

x_{16} = enemy submarine speed while snorkeling

x_{17} = snorkeling time during one complete snorkel cycle
x_{18} = enemy submarine speed while running submerged
x_{19} = submerged time during one complete snorkel cycle

3.5) Additional MOE's Identified:

(a) Number of enemy submarines sunk in a given interval of time, if a specified number of SSK's are maintained on-station continuously during the same period

(b) Number of enemy submarines sunk in a given interval of time by a specified number of submarines available for use as SSK's

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) The SSK's operate in a rectangular channel that is long enough so that the enemy submarines will have to expose themselves somewhere in transiting the area. This assumption is made for mathematical convenience.

(b) The SSK's always use the means of detection that achieves the maximum sweep width against enemy submarines transiting the channel.

(c) The SSK's will remain completely submerged and attempt to hover as much as possible in the patrol area, and therefore these tactics will make the enemy's problem of locating the SSK's as difficult as possible; they will help the SSK's to achieve their maximum sweep width when relying upon listening equipment; they will help to maintain the average state of charge of the SSK batteries at a high level, thus maintaining their submerged endurance; and they will minimize contacts between SSK's.

(d) The enemy submarine may adopt any one of a wide range of tactics in transiting the channel. Therefore, an enemy tactic factor, which measures the fraction of the length of the channel in which the targets expose themselves to detection by SSK's, is introduced.

(e) The SSK's detection equipment obeys a definite range law, i.e., detecting all detectable targets which pass within the sweep width and none outside.

(f) An enemy submarine, when it is not snorkeling, is running at a depth and speed at which it does not cavitate and therefore cannot be detected.

(g) The enemy submarines are equally likely to transit the channel at any point of its width.

(h) An SSK does not start its approach until it is directly abeam of the target.

(i) An SSK is assumed to start the approach with a fully-charged battery. This is because the SSK is initially hovering (and conserving electrical power), so as to maximize its sweep width.

(j) The SSK is assumed to start its approach with perfect knowledge of target course and speed. This assumption is partially justifiable by the consideration that whenever the target is detected before the SSK finds itself abeam of the target, then some tracking information will be available before the start of the approach.

(k) The SSK is assumed to maintain contact on its target during the approach. This implies that the enemy does not take evasive action during the approach, and does not complete the snorkel cycle before the SSK completes its approach and attack.

(l) The SSK's do not communicate intelligence to each other which aids them in sinking their primary target, that is, the model will in general not be valid for any SSK wolfpack operations.

(m) The number of SSK's employed in the channel does not exceed x_6/x_7x_8.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factors:

(a) width of channel available to enemy submarines

(b) length of channel

2) Threat Composition: Submarines

2.1) Tactics:

2.1.1) Quantitative Factors:

(a) enemy submarine speed while snorkeling

(b) snorkeling time during one complete snorkel cycle

(c) enemy submarine speed while running submerged

(d) submerged time during one complete snorkel cycle

3) Friendly Force Compositions: SSK's

3.1) Sensors: Sonar

3.1.1) Quantitative Factor:

(a) sweep width which an SSK can achieve against enemy submarines which are detectable

3.2) Armament: Torpedoes

3.2.1) Quantitative Factor:

(a) torpedo forerun, which is defined as the maximum forerun of the torpedoes available to the SSK

3.3) Deployment:

3.3.1) Quantitative Factor:

(a) number of SSK's operating in the channel

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: SSK - Submarine

4.1.1) Quantitative Factor:

(a) kill factor (in kills per attack), which is defined as the proportion of attacked enemy submarines that is killed by a single SSK

STUDY REVIEW SUMMARY NO. (8)-16

A. STUDY DESCRIPTION

1) Originating Activity: Submarine Development Group Two, Groton, Connecticut
2) Report Title: A Measure of Detection Performance
3) Author: B.J. McCabe
4) Report Number: Research Technical Contribution 4-72 (AD-520 030)
5) Date: April 1972
6) Classification: Confidential
7) Abstract: The purpose of this report is to introduce a measure of submarine detection performance which can be estimated from exercise data, and which is proposed as a method for summarizing exercise or mission detection data. The principal goal was to develop a statistic which meets the following requirements: (1) it makes use of the information in the failures to detect as well as the detections; (2) it has a simple enough structure so that its probability distribution can be analyzed.
8) Descriptors: Antisubmarine warfare, detection, normal density function, sonar, statistics, submarine, transitor

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine ASW
3) Mission: Barrier placement/patrol
   3.1) Definition: An SSK, patrolling a barrier, attempts with passive sonar to detect enemy submarines transiting through this barrier.
   3.2) Criterion For Success: Detection of submarine

3.3) MOE Selected: Conceptual detection range, which is defined as the range at which the probability that the closest point of approach (CPA) does not exceed this range is equal to the total probability of detection

3.3.1) Rationale For Selection: The principal goal was to develop a measure of submarine detection performance which can be estimated from exercise data and which meets the following requirements: (1) It makes use of the information in the failures to detect as well as the detections; (2) it has a simple enough structure so that its probability distribution can be analyzed. Mean detection range is not a good measure because it fails to use any of the information in the runs on which detections did not occur. Area under the cumulative probability of detection (CPD) curve fails on the second count - its probability distribution is too intricate to be susceptible to analysis. There are two other measures which satisfy (1) and (2) but they suffer from other defects. The ratio of detections to opportunities may not be a good summary of performance because a biased CPA distribution may result in a misleading ratio of detections to opportunities. (Too many short CPA's inflate the measure, too many long CPA's deflate it.) Another measure - area under an estimate of the lateral range curve -is usually not usable due to the large data base required to estimate a lateral range curve validly. Conceptual detection range (CDR) satisfies both requirements (1) and (2), can remove the effects of a biased CPA distribution, and does not require an enormous data base (25 runs or greater is generally adequate). The statistic, CDR, does not use all the information in the detection, that is, the information in the fact that a detection has occurred is used, but not the detection range itself. The latter information is used in computing the CPD curve. However, this kind of criticism can be made of virtually every non-parametric statistic. The neglect of information in data is justified if some other end is served.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2)$$

where

x_1 = empirical distribution function for the closest point of approach
$= g_1(x_3, x_5)$

x_2 = empirical probability of detection
$= g_2(x_3, x_4)$

x_3 = number of transits of enemy submarine

x_4 = number of enemy submarines detected

x_5 = a vector whose i^{th} component represents the closest point of approach for the i^{th} transit

3.5) Additional MOE's Identified:

(a) Mean detection range

(b) Area under the cumulative probability of detection curve

(c) Ratio of detections to opportunities

(d) Area under the lateral range curve

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Each attempted passage through the barrier (called a run) produces either a detection range or a range of the closest point of approach (CPA).

(b) The SSK and transitors behave in such a way that successive runs produce CPA's to the SSK which are independent and governed by an unknown probability distribution function.

(c) The lateral range function (probability of detection for specified CPA) is unknown.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

   2.1) Tactics:

      2.1.1) Quantitative Factor:

         (a) number of transits of enemy submarine

3) Friendly Force Composition: Submarine

   3.1) Sensor: Passive sonar

4) Friendly Force - Threat Interaction:

   4.1) Platform - Platform: Submarine - Submarine

      4.1.1) Quantitative Factor:

         (a) a vector whose i^{th} component represents the closest point of approach for the i^{th} transit

   4.2) Sensor - Platform: Passive sonar - Submarine

      4.2.1) Quantitative Factor:

         (a) number of enemy submarines detected

STUDY REVIEW SUMMARY NO. (8)-17

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Postgraduate School, Monterey, California

2) Report Title: The Evaluation of Submarine Weapon Systems Effectiveness: An Analytical Approach

3) Authors: N.E. Prosser, W.E. Smith, Jr. and C.H. Van Landingham, Jr.

4) Report Identification: Thesis for the Master of Science in Operations Research, (AD-375 327)

5) Date: 1964

6) Classification: Confidential

7) Abstract: This thesis offers to those concerned with modern submarine warfare a methodology for deriving quantitative submarine ASW capability information. This methodology encompasses the following: First, it establishes a submarine ASW measure of effectiveness based on observed performance data. Second, it demonstrates the applications of the measure of effectiveness to operational situations. Third, it provides criteria for assigning varying degrees of realism to observed data, thus, insuring a valid data base necessary for the computation of a true measure of effectiveness. Fourth, the techniques of automatic data processing are applied to the collection, storage, recall and query of the required performance data. The methodology presented thus provides for the distillation and assemblage of a great volume of data into usable form which, in turn, can be employed as a basis for decision processes as applied to today's submarine forces.

8) Descriptors: Antisubmarine warfare, classification probability, contact prosecution, detection, detection probability, hit probability, kill probability, normal density function, Poisson density function, search, submarine, submarine attack, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force and System
2) Function: Submarine ASW
3) Tactical Situations:
   - 3.1) Tactical Situation Type: Submarine versus Submarine
     - 3.1.1) Definition: A friendly submarine engages an enemy submarine in a one-on-one situation.
     - 3.1.2) Criterion For Success: Destruction of enemy submarine and survival of friendly submarine
     - 3.1.3) MOE Selected: Weapon system effectiveness, which is defined as the conditional probability that friendly submarine obtains a hit on enemy submarine, given that friendly submarine survives the engagement and has an initial detection opportunity
       - 3.1.3.1) Rationale For Selection: This measure of effectiveness reflects a submarine's overall system capability and can be constructed using performance data.
     - 3.1.4) Functional Form Of MOE:

$$(MOE)_1 = f_1(x_1, x_2)$$

where

x_1 = weapon system capability, which is defined as the conditional probability that friendly submarine places a weapon within the average weapon acquisition range for a homing weapon, or at estimated center of target if non-homing weapon, given that friendly submarine survives the engagement and has an initial detection opportunity

$$= g_1(x_3, \ldots, x_6)$$

x_2 = weapon performance factor, which is defined as the conditional probability that weapon hits enemy submarine, given that it is placed within the average weapon acquisition range for a homing weapon, or at estimated middle of target for non-homing weapon

x_3 = conditional probability that friendly submarine detects enemy submarine, given that it survives the detection phase of the engagement and has an initial detection opportunity

x_4 = conditional probability that friendly submarine correctly classifies the enemy submarine, given that it survives the classification phase of the engagement and has detected the target

x_5 = conditional probability that friendly submarine attacks the enemy submarine, given that it survives the attack phase of the engagement and has classified the target

x_6 = conditional probability that friendly submarine places a weapon within the average weapon acquisition range for a homing weapon, or at estimated center of target if non-homing weapon, given that it survives the hit phase of engagement and has attacked the target

3.2) Tactical Situation Type: Submarine force versus Submarine force

3.2.1) Definition: A friendly submarine force engages an enemy submarine force that is operating in an ocean area.

3.2.2) Criterion For Success: Destruction of submarine

3.2.3) MOE Selected: Expected number of enemy submarines killed in a specified period of time

3.2.4) Functional Form Of MOE:

$$(MOE)_2 = f_2(x_7, x_8)$$

where

x_7 = fractional enemy force destroyed by friendly force in time x_{19}
$= g_2(x_9, x_{10}, x_{11}, x_{19})$

x_8 = total number of enemy submarine in search area
$= g_3(x_{12}, x_{13})$

x_9 = a vector whose $i^{\underline{th}}$ component represents the expected number of enemy submarines killed by type i friendly submarine
$= h_1(x_8, x_{12}, x_{14})$

x_{10} = a vector whose $i^{\underline{th}}$ component represents the coverage factor for the type i friendly submarine
$= h_2(x_{15}, \ldots, x_{19})$

x_{11} = number of types of friendly submarines in search area

x_{12} = a vector whose $i^{\underline{th}}$ component represents the number of enemy submarines of type i in search area

x_{13} = number of types of enemy submarines in search area

x_{14} = a matrix whose $(i,j)^{\underline{th}}$ entry represents the weapon system effectiveness of type i friendly submarines against type j enemy submarines

x_{15} = search area in square miles

x_{16} = a vector whose $i^{\underline{th}}$ component represents the number of friendly submarines of type i in search area

x_{17} = a vector whose $i^{\underline{th}}$ component represents the optimal search speed of friendly submarine of type i

x_{18} = a vector whose $i^{\underline{th}}$ component represents the range for friendly submarine type i at which 90% of all initial detections occur at ranges less than or equal to this range

x_{19} = time period in hours

4) MOE Usage In Study: The MOE's were formulated and numerical examples were presented. In particular, fleet data were used to determine values of $(MOE)_1$ which comprised the entries in the matrix x_{14}. In addition, $(MOE)_2$ was used to obtain force level requirements for a desired level of effectiveness in an area search ASW operation.

5) Special Study Assumptions:

(a) Initial detection ranges are normally distributed.

(b) A detection opportunity is defined as existing whenever a target closes to the 90% detection range.

(c) In the force level submarine versus submarine encounter,

(c.1) The number of each type of enemy submarine in the search area is known.

(c.2) The enemy submarines are assumed to be uniformly distributed throughout the search area.

(c.3) Each friendly submarine is assigned a particular subarea to search such that no two submarines cover the same area.

(c.4) Each friendly submarine searches its subarea in a random fashion.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) search area in square miles

2) Threat Composition: Submarine

2.1) Deployment:

2.1.1) Quantitative Factors:

(a) a vector whose i^{th} component represents the number of enemy submarines of type i in search area

(b) number of types of enemy submarines in search area

3) Friendly Force Composition: Submarine

3.1) Quantitative Factor:

(a) a vector whose i^{th} component represents the range for friendly submarine type i at which 90% of all initial detections occur at ranges less than or equal to this range

3.2) Armament: Torpedo

3.3) Deployment:

3.3.1) Quantitative Factors:

(a) number of types of friendly submarine in search area

(b) a vector whose i^{th} component represents the number of friendly submarines of type i in search area

(c) time period in hours

3.4) Tactics:

3.4.1) Quantitative Factor:

(a) a vector whose i^{th} component represents the optimal search speed of friendly submarine of type i

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Submarine

4.1.1) Quantitative Factors:

(a) conditional probability that friendly submarine detects enemy submarine, given that it survives the detection phase of the engagement and has an initial detection opportunity

(b) conditional probability that friendly submarine correctly classifies the enemy submarine, given that it survives the classification phase of the engagement and has detected the target

(c) conditional probability that friendly submarine attacks the enemy submarine, given that it survives the attack phase of the engagement and has classified the target

(d) conditional probability that friendly submarine places a weapon within the average weapon acquisition range for a homing weapon, or at estimated center of target if non-homing weapon, given that it survives the hit phase of the engagement and has attacked the target

(e) a matrix whose $(i,j)^{\underline{th}}$ entry represents the weapon system effectiveness of type i friendly submarines against type j enemy submarines

4.2) Armament - Platform: Torpedo - Submarine

4.2.1) Quantitative Factor:

(a) weapon performance factor, which is defined as the conditional probability that weapon hits enemy submarine, given that it is placed within the average weapon acquisition range for a homing weapon, or at estimated middle of target for non-homing weapon

STUDY REVIEW SUMMARY NO. (8)-18

A. STUDY DESCRIPTION

1) Originating Activity: Operations Research, Incorporated, Silver Spring, Maryland

2) Report Title: The Effect of Multiple Contacts on Passive Sonar Classification - An Analytic Approach

3) Author: P.M. Tullier

4) Report Number: TR 713

5) Date: June 1972

6) Classification: Confidential

7) Contract: N00014-71-C-0408 (Office of Naval Research)

8) Abstract: The time-dependent aspects of passive sonar classification are analyzed in this report. An analysis of sonar functions, such as providing input for fire control solutions, demonstrated the need for overlapping observations of several contacts. Thus the time to return to a contact depends on the load (number of contacts under observation). In the future, improved sonars with greater ranges will increase the system load whether or not ship densities change. As an attempt to understand sonar operations under heavy system loads, a model of contact classification, given a system load, is developed for a single sonar and contact of a single priority class. The result is a probability law for classifying a contact on a look starting at time t after the last look, given k contacts in the system when the last look began. Extensions that include multiple sonars and contacts of different priority classes are discussed. Data requirements also are discussed, giving sources, collection difficulties, and expected biases.

9) Descriptors: Antisubmarine warfare, classification, classification probability, exponential density function, Poisson density function, queuing, sonar, submarine, surveillance, undersea surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Subsystem
2) Function: Submarine ASW
3) Mission: Ocean Surveillance
   3.1) Definition: Submarine passive sonar system is used to classify all contacts received.
   3.2) Criterion For Success: Classification of contact
   3.3) MOE Selected: Probability of classifying a contact on a look starting at a specified time after the last look, given a specified number of contacts in the system when the last look began
   3.4) Functional Form Of MOE:

$$MOE = f(x_1)$$

where

x_1 = probability density function for classifying a contact on a look starting at time x_9 after the last look, given x_{10} contacts in the system when the last look began

$= g(x_2, x_3, x_4, x_9, x_{10})$

x_2 = probability of return to contact at time x_9, given x_{10} initial contacts in the system

$= h_1(x_5, x_6, x_7, x_9, x_{10})$

x_3 = probability density function of the look duration

$= h_2(x_8)$

x_4 = probability density function of classification of a contact in the look interval (x_9, x_9 + look time), given time of previous look x_{13}

$= h_3(x_9, x_{12}, x_{13}, x_{14})$

x_5 = a vector whose i^{th} component represents the convolution of i service time distributions

$= i_1(x_8)$

x_6 = a vector whose $i^{\underline{th}}$ component represents the probability of i contacts being encountered before time x_9, given x_{10} initial contacts in the system

= $i_2(x_9, x_{10}, x_{11})$

x_7 = sweep circle sector size

x_8 = mean look rate

x_9 = time since last look at reference contact

x_{10} = number of contacts in the system when the last look began

x_{11} = contact arrival rate

x_{12} = joint probability density function for a classification window in terms of its beginning and duration

= $i_3(x_{15})$

x_{13} = time last look ended

x_{14} = minimum time required in order to classify

x_{15} = signal to noise ratio

= $j(x_{16}, \ldots, x_{19})$

x_{16} = source level

x_{17} = propagation loss

x_{18} = directivity index

x_{19} = background noise

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) One passive sonar is present.

(b) The classification process relies on cumulative information available. The contact is available for classification during time intervals of random start and duration. These intervals are called classification windows.

(c) With regard to priority, all contacts are treated the same.

(d) The random arrivals of new contacts are assumed to be distributed according to the Poisson distribution law. Cases concerning bulk arrivals or fixed number of contacts are not treated explicitly in the analysis.

(e) The time spent by the sonar operator on each look at a contact is assumed to be exponentially distributed.

(f) The sonar operator's look must overlap the classification window for a minimum time on order to classify.

(g) Arrivals are uniform over all directions, including the baffled area. This assumption does not reflect the real world, but was used to facilitate the mathematical analysis.

(h) The sonar is assumed to sweep in a circular pattern. When a contact is encountered, it is observed and then the sweep proceeds in the same direction. This simple search pattern is used in order to reduce the mathematical complexity.

(i) Contacts are allowed to move in and out of detection range. However, the contact of reference (the one to be classified) is assumed to be on about the same bearing when the next observation is made. This means that the sonar will sweep out approximately 360 degrees between looks at a contact. This assumption is reasonable if the time between looks is short or the contact moves very slowly.

(j) It is assumed that if the sonar is trained on bearing θ_1 and there are no existing contacts between θ_1 and θ_2, then no contacts will be picked up during the sweep from θ_1 to θ_2. This assumption is reasonable because sonar sweeps are made very quickly. If the sweep was slow, arrivals could enter near θ_2 while the sonar transversed from θ_1 to θ_2, therefore the assumption would not be valid.

(k) The sonar sweep circle is divided into sectors large enough to hold only one contact.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factors:

(a) propagation loss

(b) background noise

2) Target Composition: Contacts

2.1) Quantitative Factor:

(a) source level

3) Friendly Force Composition: Submarine

3.1) Sensor: Passive sonar

3.1.1) Quantitative Factors:

(a) minimum time required in order to classify

(b) directivity index

3.2) Tactics:

3.2.1) Quantitative Factors:

(a) sweep circle sector size

(b) mean look rate

4) Friendly Force - Target Interaction:

4.1) Sensor - Platform: Passive sonar - Contacts

4.1.1) Quantitative Factors:

(a) time since last look at reference contact

(b) number of contacts in the system when the last look began

(c) contact arrival rate

(d) time last look ended

(9)

SUBMARINE ATTACK

STUDY REVIEW SUMMARY NO.(9)-1

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Postgraduate School, Monterey, California
2) Report Title: The Factors Affecting Antisubmarine Warfare Inside the Screen
3) Author: D. G. Clark
4) Report Identification: Thesis for the Master of Science In Operations Research, (AD-509 085)
5) Date: December 1968
6) Classification: Confidential
7) Abstract: This paper explores the situation where an attacking submarine has penetrated the escorting screen and is operating in the vicinity of the main body. Some recent studies from other sources are constructively criticized and ideas and models are postulated for future analysis to determine justification for the name "submarine haven" which has been given to this area in the acoustic shadow of the main body.
8) Descriptors: Convoy defense, detection, hit probability, submarine, submarine attack, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine Attack
3) Mission: Submarine attack on convoy
   3.1) Definition: A merchant convoy forms the target for an attacking submarine. The convoy is protected by destroyers in a circular area patrol screen. The submarine attempts to penetrate the screen in order to fire torpedoes at the convoy ships.
   3.2) Criterion For Success: Destruction of ships
   3.3) MOE Selected: Expected number of ships hit
   3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_6)$$

where

x_1 = probability the submarine penetrates the screen

$= g_1(x_7, x_8, x_9)$

x_2 = probability the submarine is detected while attacking

x_3 = probability the submarine chooses to evade after attack

x_4 = probability the submarine reattacks after attack

x_5 = probability of at least one hit in a salvo

$= g_2(x_{10}, x_{11})$

x_6 = number of torpedoes carried by the submarine

x_7 = number of screening ships

x_8 = sweep width of a screening ship

x_9 = radius of the screen circle

x_{10} = hit probability of a torpedo

x_{11} = number of torpedoes fired per salvo

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Torpedo firings are independent.

(b) All salvos are of three torpedos.

(c) Each destroyer in the screen is assigned a sector of the circle in relative degrees, and a minimum and maximum range from formation center. Within her sector, each patrols in a random manner at maximum effective sonar speed.

(d) Torpedoes detonate upon impact.

(e) The convoy is in a square formation with a specified constant distance between adjacent stations in both columns and rows.

(f) Torpedoes are fired from outside the screen.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Merchant ships and destroyers

2.1) Platform Type: Merchant ships

2.1.1) Deployment:

2.1.2.1) Qualitative Factor:

(a) in a square formation

2.2) Platform Type: Destroyer

2.2.1) Quantitative Factors:

(a) number of screening ships (destroyers)

(b) sweep width of a screening ship

2.2.2) Deployment:

2.2.2.1) Qualitative Factor:

(a) in a circular area patrol screen

2.2.2.2) Quantitative Factor:

(a) radius of screen circle

2.2.3) Tactics:

2.2.3.1) Qualitative Factor:

(a) random patrol within the assigned sector

3) Friendly Force Composition: Submarine

3.1) Tactics:

3.1.1) Qualitative Factor:

(a) penetrate screen and attack convoy ships

3.1.2) Quantitative Factors:

(a) probability the submarine chooses to evade after attack

(b) probability the submarine reattacks after attack

3.2) Armament: Antishipping torpedoes

3.2.1) Quantitative Factors:

(a) number of torpedoes carried by the submarine

(b) number of torpedoes fired per salvo

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Submarine - Destroyer

4.1.1) Quantitative Factor:

(a) probability the submarine is detected while attacking

4.2) Armament - Platform: Torpedo - Merchant ships

4.2.1) Quantitative Factor:

(a) hit probability of a torpedo

STUDY REVIEW SUMMARY NO. (9)-2

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Measures of Effectiveness in Submarine Warfare and their Relation to an Integrated Research Program
3) Author: W.J. Horvath
4) Report Number: OEG Report 52
5) Date: 20 June 1946
6) Classification: Confidential
7) Abstract: This report deals with the problem of determining quantitative measures for the effectiveness of submarine operations and applying the results of such studies to discover means for improving these operations. The techniques used for these studies apply generally to all types of warfare, but the present report is concerned only with offensive submarine warfare against enemy merchant shipping.
8) Descriptors: Detection, hit probability, kill probability, merchant ship, submarine, submarine attack, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine Attack
3) Mission: Submarine attack on convoy
   - 3.1) Definition: Submarines attack individual merchant ships and merchant ship convoys.
   - 3.2) Criterion For Success: Destruction of ships
   - 3.3) MOE Selected: Number of ships sunk per unit time spent in area
   - 3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_4)$$

where

x_1 = number of ships sighted per unit time spent in area

x_2 = conditional probability of attacking a ship given that it is sighted

x_3 = conditional probability of hitting a ship given that it is attacked

x_4 = conditional probability of sinking a ship given that it is hit

4) MOE Usage In Study: The MOE is formulated and used to discuss methods of increasing the overall effectiveness of submarine attacks on merchant ships by increasing the effectiveness in each of the four phases (sighting, approach, firing, and sinking) of the attack operation.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated
2) Target Composition: Merchant ships
3) Friendly Force Composition: Submarines
   3.1) Armament: Torpedoes
4) Friendly Force - Target Interaction:
   4.1) Platform - Platform: Submarine - Merchant ship
      4.1.1) Quantitative Factors:
         (a) number of ships sighted per unit time spent in area
         (b) conditional probability of attacking a ship given that it is sighted
         (c) conditional probability of hitting a ship given that it is attacked
   4.2) Armament - Platform: Torpedo - Merchant ship
      4.2.1) Quantitative Factor:
         (a) conditional probability of sinking a ship given that it is hit

Intasystems

STUDY REVIEW SUMMARY NO. (9)-3

A. STUDY DESCRIPTION

1) Originating Activity: Stanford Research Institute, Menlo Park, California
2) Report Title: Simulation Models of Search in the Presence of Decoys
3) Author: E.L. Wong
4) Report Number: Technical Note NWRC-TN-37
5) Date: April 1972
6) Classification: Unclassified
7) Contract: N00014-71-C-0119 (Office of Naval Research)
8) Abstract: A simulation model that represents a submarine's search for a high value target within a specified operating area is described in this report. This model was developed as an adjunct to the formulation and implementation of a computationally more efficient analytical model. The simulation model served two purposes. First, the simulation model provided a validation of the statistical inputs used for the analytical model. Specifically, the simulation studies validated the applicability of the analytical model for determining rate of encounter between submarine and targets. Second, results obtained through exercise of the simulation model provided a convenient check of the reasonableness of analytical model results.
9) Descriptors: Acoustic decoy, countermeasure, decoy, detection, detection probability, Monte Carlo method, search, submarine, tactics

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine Attack
3) Mission: Target search
   3.1) Definition: A submarine searches for a high value target (HVT) in a specified area.

3.2) Criterion For Success: Detection of target

3.3) MOE Selected: Elapsed time to target detection

3.3.1) Rationale For Selection: Time to (first) detection is equivalent to time to first encounter, where encounter occurs whenever the range between the submarine and the HVT is less than some predetermined value.

3.4) Functional Form Of MOE:

Case 1 - Continuous search, without false targets

MOE = $f_1(x_1, \ldots, x_7)$

where

x_1 = HVT velocity
x_2 = length of HVT track segments
x_3 = radius of objective area
x_4 = time increment between direction changes by HVT and submarine
x_5 = radius of detection of HVT by submarine
x_6 = submarine search velocity
x_7 = length of submarine track segments

Case 2 - Sprint/drift search, without false targets

MOE = $f_2(x_1, \ldots, x_4, x_8, \ldots, x_{13})$

where

x_8 = submarine sprint velocity
x_9 = submarine drift velocity
x_{10} = submarine sprint period
x_{11} = submarine drift period
x_{12} = radius of detection of HVT by submarine during sprint
x_{13} = radius of detection of HVT by submarine during drift

Case 3 - Continuous search, with false target field

MOE = $f_3(x_1, \ldots, x_7, x_{14}, \ldots, x_{19})$

where

x_{14} = length of decoy track segments

x_{15} = decoy velocity

x_{16} = decoy classification time

x_{17} = decoy "turned-off" time

x_{18} = radius of detection of decoy by submarine

x_{19} = number of decoys

4) MOE Usage in Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Objective area is considered to be circular in shape.

(b) Initial positions of the HVT and decoys (if included) are randomly distributed within the area.

(c) Initial HVT movement is radially away from the center of the area.

(d) The submarine searcher moves initially toward the center of the area.

(e) Direction changes by the HVT and submarine are made at the same time.

(f) Decoys, if not stationary, move initially in a manner similar to that exhibited by the HVT.

(g) Speeds of the HVT, decoys (if included) and submarine are held constant.

(h) The submarine possesses perfect information concerning the size and location of the search area.

(i) The detection capability of the submarine against either the decoys or the HVT is described by a definite range probability law or "cookie cutter".

(j) Courses traveled by the HVT, decoys (if included) and submarine are straight line.

(k) In Case 2, the submarine employs a tactic of first executing a high speed sprint and then a slow speed drift to attempt detection of the HVT.

(l) In Case 3,

(l.1) During the period the submarine is classifying a decoy, the submarine is precluded from making new detections on either the HVT or other decoys.

(1.2) After decoy classification, the decoy is turned off for x_{17} time units.

(1.3) When the submarine is within detection range of two or more decoys at any given instant, the submarine investigates and classifies the nearest decoy, ignoring the others.

(1.4) Decoy classification by the submarine is perfect.

(1.5) When the HVT is within range, it is detected and classified by the submarine without regard for whatever decoys may also be present.

C. EFFECTIVENESS FACTORS

1) Physical Environment:
   - 1.1) Quantitative Factor:
     - (a) radius of objective area
2) Target: High value target (HVT) and decoys
   - 2.1) Platform Type: High value target
     - 2.1.1) Quantitative Factor:
       - (a) HVT velocity
     - 2.1.2) Tactics:
       - 2.1.2.1) Quantitative Factor:
         - (a) length of HVT track segments
   - 2.2) Platform Type: Decoy
     - 2.2.1) Quantitative Factors:
       - (a) decoy velocity
       - (b) number of decoys
     - 2.2.2) Tactics:
       - 2.2.2.1) Quantitative Factor:
         - (a) length of decoy track segments
3) Friendly Force: Submarine
   - 3.1) Tactics:

3.1.1) Quantitative Factors:

(a) submarine search velocity

(b) length of submarine track segments

(c) submarine sprint velocity

(d) submarine drift velocity

(e) submarine sprint period

(f) submarine drift period

4) Friendly Force - Target Interaction:

4.1) Platform - Platform:

4.1.1) Type: Submarine - High value target

4.1.1.1) Quantitative Factor:

(a) time increment between direction changes by HVT and submarine

4.1.2) Type: Submarine - Decoy

4.1.2.1) Quantitative Factors:

(a) decoy classification time

(b) decoy "turned-off" time

5) Friendly Force - Target - Physical Environment Interaction:

5.1) Platform - Platform - Environment:

5.1.1) Type: Submarine - High value target

5.1.1.1) Quantitative Factors:

(a) radius of detection of HVT by submarine

(b) radius of detection of HVT by submarine during sprint

(c) radius of detection of HVT by submarine during drift

5.1.2) Type: Submarine - Decoy

5.1.2.1) Quantitative Factor:

(a) radius of detection of decoy by submarine

STUDY REVIEW SUMMARY NO. (9)-4

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Distribution of Losses in an Idealized Antishipping Campaign
3) Author: J. Hall
4) Report Number: CNA Research Contribution No. 120 (AD-857 966)
5) Date: 9 July 1969
6) Classification: Unclassified
7) Contract: N00014-68-A-0091 (Office of Naval Research)
8) Abstract: This paper describes an idealized, steady state antishipping campaign carried out by submarines whose operations are mutually independent. The probability distribution of the number of successful patrols per submarine is derived and the probability distribution of the total shipping losses (total number of ships hit) is approximated.
9) Descriptors: Antisubmarine warfare, attrition, barrier, binomial density function, convoy defense, normal density function, screen, submarine, submarine attack, surface ship, survivability

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Submarine Attack
3) Mission: Submarine attack on convoy
   3.1) Definition: Submarines cycle between a base and an operating area in which they attack surface ships defended by barriers and ASW screens.
   3.2) Criterion For Success: Survival of submarines and destruction of ships

3.3) MOE's Selected:

$(MOE)_1$ = Probability distribution of the number of successful patrols per submarine

$(MOE)_2$ = Probability distribution of total shipping losses (total number of surface ships hit)

Case 1 - Unlimited number of patrols per submarine

$(MOE)_1 = f_1(x_1)$

$(MOE)_2 = f_2(x_2, x_3)$

where

x_1 = probability that submarine survives transit out and first half of operating period, given it started from base
$= g_1(x_4, x_5)$

x_2 = average number of ships hit by submarine
$= g_2(x_6, x_7, x_8)$

x_3 = standard deviation of ships hit by submarines
$= g_3(x_6, x_7, x_9)$

x_4 = a vector whose i^{th} component represents the probability that a submarine survives the i^{th} barrier it transits from the base to the operating area

x_5 = probability the submarine survives an ASW screen given an encounter

x_6 = total number of submarines

x_7 = number of ships hit per successful patrol per submarine

x_8 = average number of successful patrols per submarine
$= h_1(x_1)$

x_9 = standard deviation of the number of successful patrols per submarine
$= h_2(x_1)$

Case 2 - Limited number of patrols per submarine

$(MOE)_1 = f_3(x_1, x_{10})$

$(MOE)_2 = f_4(x_{11}, x_{12})$

where

x_{10} = maximum number of patrols per submarine

x_{11} = average number of ships hit by submarines $= g_4(x_6, x_7, x_{13})$

x_{12} = standard deviation of ships hit by submarines $= g_5(x_6, x_7, x_{14})$

x_{13} = average number of successful patrols per submarine $= h_3(x_1, x_{10})$

x_{14} = standard deviation of the number of successful patrols per submarine $= h_4(x_1, x_{10})$

3.5) Additional MOE Identified: In Case 2,

(a) Probability distribution of the total number of submarines surviving after completion of as many patrols as possible

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) Submarine operations are mutually independent.

(b) The submarines cycle between base and an operating area in which they find a constant flux of targets. Each submarine sees the same flux.

(c) A patrol is "successful" if the submarine survives the transit out and the first half of the operating period.

(d) The number of submarines is assumed to be sufficiently large such that the distribution of the number of ships hit is approximately normal.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated
2) Threat Composition: Barriers and ASW screens
3) Target Composition: Surface ships
4) Friendly Force Composition: Submarines
   - 4.1) Quantitative Factor:
     - (a) total number of submarines
   - 4.2) Deployment:
     - 4.2.1) Quantitative Factor:
       - (a) maximum number of patrols per submarine
5) Friendly Force - Threat Interaction:
   - 5.1) Platform - Platform:
     - 5.1.1) Type: Submarine - Barrier
       - 5.1.1.1) Quantitative Factor:
         - (a) a vector whose $i^{\underline{th}}$ component represents the probability that a submarine survives the $i^{\underline{th}}$ barrier it transits from the base to the operating area
     - 5.1.2) Type: Submarine - ASW screen
       - 5.1.2.1) Quantitative Factor:
         - (a) probability the submarine survives an ASW screen given an encounter
6) Friendly Force - Target Interaction:
   - 6.1) Platform - Platform: Submarine - Surface ships
     - 6.1.1) Quantitative Factor:
       - (a) number of ships hit per successful patrol per submarine

(10)

SURFACE ASW

STUDY REVIEW SUMMARY NO. (10)-1

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Effectiveness of Acoustic Simulators
3) Authors: A. Hershaft and W. B. Buchanan
4) Report Number: OEG Study No. 704 (AD-384 385)
5) Date: 6 September 1967
6) Classification: Confidential
7) Contract: NONR 3732(00) (Office of Naval Research)
8) Abstract: This study examines the effect of sonic decoys on the search time required by a single submarine to locate a carrier operating within a fixed operating area. To frustrate the search procedures used by the submarine, the carrier distributes sonic decoys randomly in the operations area. The effectiveness of these decoys is then measured by the increase in the time for the submarine to locate the carrier. Parameters investigated are submarine speed, number of decoys, acoustic range of the decoys, carrier speed, carrier detectability and the range closure, i e., the range to which the submarine must close a target to classify it as ship or decoy.
9) Descriptors: Acoustic decoy, antisubmarine warfare, carrier, carrier based aircraft, escort ship, Monte Carlo method, submarine, task force

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Surface ASW
3) Tactical Situation: Carrier task group versus Submarine

3.1) Definition: A carrier task group, in the vicinity of an enemy coast, launches conventional strikes against inland targets. The carrier deploys carrier simulators and follows a constant speed evasive pattern of movement consistent with aircraft launch requirements, while the escort ships patrol their AAW stations. Opposing the carrier operations in the area is a single submarine, using passive sonar.

3.2) Criterion For Success: Prevention of detection and classification of the carrier

3.3) MOE Selected: Median time to closure

3.3.1) Rationale For Selection: The cumulative plot of closure times in most cases can be approximated by the exponential distribution, in which case the median time to closure would specify uniquely the entire distribution. Also, with exponential closure times, the median time to closure is equivalent to the "half-life" of the search periods.

3.4) Functional Form Of MOE:

MOE = $f(x_1,\ldots,x_{14})$

where

x_1 = radius of uncertainty area

x_2 = radius of operations area

x_3 = range of detection of simulator

x_4 = range of detection of escort ship

x_5 = range of detection of carrier

x_6 = speed of carrier

x_7 = speed of submarine

x_8 = number of simulators

x_9 = number of escorts

x_{10} = interval between possible carrier course changes

x_{11} = probability of carrier course change

x_{12} = range of closure (i.e., the range to which the submarine must close the target (ship or simulator) to obtain a correct target classification)

x_{13} = number of investigated simulators capable of being stored in submarine memory

x_{14} = duration of encounter

3.5) Additional MOE Identified:

(a) Percentage increase in closure probability at the end of a specified period of time attributable to the simulators

4) MOE Usage In Study: Monte Carlo simulation performed for a range of parameter values to determine MOE values.

5) Special Study Assumptions:

(a) Simulator operating life is ignored.

(b) No consideration is given to the logistic problem of refueling or replacing the buoys to maintain a field of simulators.

(c) The operations area is circular.

(d) The carrier proceeds on a random zig-zag course within the operations area.

(e) Escort ships patrol their AAW stations and act in effect as acoustic decoys.

(f) The submarine is not detected.

(g) Simulators are always detected within their detection range but never outside.

(h) The submarine attempts to classify every sound source by closing range.

(i) Whenever there is a choice, the submarine prosecutes the loudest signal source.

(j) The submarine may head toward a previously investigated buoy only after investigating a specified number of additional sound sources.

(k) The submarine is not able to distinguish buoy simulators from ships until the closure range is reached, i.e., the zero speed of the simulator does not give away the deception at ranges in excess of the closure range.

(l) The carrier is placed at random within the operations area, whereas the escort ships are distributed at random in one-half of the operations area, as if to counter the air threat.

(m) The simulators are deployed at random in the operations area.

(n) The operations area is situated at random within a larger uncertainty area created by the submarine's incomplete knowledge of the location of the operations area.

(o) The submarine is placed at random on the perimeter of the uncertainty area.

(p) The escort ships and the simulators are held stationary.

(q) The submarine stays on a straight course until he alters course to close a new sound source or until he would leave the uncertainty area.

(r) New course selection by the carrier is constrained by the fact that the carrier cannot leave the operations area.

(s) Upon closing a sound source to within the specified range of closure, the submarine resumes search or investigates the next strongest sound source (if one is available), eliminating the signal just investigated from consideration until a previously specified number of other signals have been investigated in turn. Whenever an investigated sound source turns out to be the carrier, the elapsed time is recorded as closure time.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

   2.1) Quantitative Factors:

      (a) speed of submarine

      (b) number of investigated simulators capable of being stored in submarine memory

   2.2) Deployment:

      2.2.1) Qualitative Factor:

         (a) random placement on the perimeter of the uncertainty area

2.3) Tactics:

2.3.1) Qualitative Factors:

(a) close on all sound sources to within the specified range of closure

(b) prosecutes the loudest sound source first

(c) heads toward a previously investigated buoy only after investigating a specified number of additional sound sources

(d) stays on a straight course until course is altered to close a new sound source or until he would leave the uncertainty area

2.3.2) Quantitative Factor:

(a) range of closure

3) Friendly Force Composition: Carrier, escort ships and buoy simulators

3.1) Platform Type: Carrier

3.1.1) Quantitative Factors:

(a) range of detection of carrier

(b) speed of carrier

3.1.2) Deployment:

3.1.2.1) Qualitative Factor:

(a) random placement within the operations area

3.1.3) Tactics:

3.1.3.1) Qualitative Factor:

(a) proceed on random zig-zag course within the operations area

3.1.3.2) Quantitative Factors:

(a) interval between possible carrier course changes

(b) probability of carrier course change

3.2) Platform Type: Escort ships

3.2.1) Quantitative Factors:

(a) range of detection of escort ship

(b) number of escorts

3.2.2) Deployment:

3.2.2.1) Qualitative Factor:

(a) randomly distributed in one-half the operations area

3.2.3) Tactics:

3.2.3.1) Qualitative Factor:

(a) held stationary

3.3) Platform Type: Buoy simulators

3.3.1) Quantitative Factors:

(a) range of detection of simulator

(b) number of simulators

3.3.2) Deployment:

3.3.2.1) Qualitative Factor:

(a) randomly distributed in the operations area

3.3.3) Tactics:

3.3.3.1) Qualitative Factor:

(a) held stationary

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Carrier - Submarine

4.1.1) Quantitative Factors:

(a) radius of uncertainty area

(b) radius of operations area

(c) duration of encounter

STUDY REVIEW SUMMARY NO. (10)-2

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Comparative Tactical Effectiveness of Advanced ASW Fire Control Computers
3) Author: R. G. Brown
4) Report Number: OEG Study 419 (revised), (AD-505 549)
5) Date: 6 December 1950
6) Classification: Confidential
7) Abstract: Four types of automatic shipboard ASW fire control computers are compared for tactical effectiveness. Two are existing types of computers, using linear prediction, one with least squares smoothing and one with exponential smoothing; the other two represent possible future trends in development. The probability that a weapon, which lands at the target's future position as determined by the computer, will hit the target is compared for a range of conditions representing most operating conditions.
8) Descriptors: Antisubmarine warfare, contact prosecution, computer, fire control, hit probability, normal density function, prediction, submarine, surface ship, tracking

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Subsystem
2) Function: Surface ASW
3) Mission: Contact prosecution
   3.1) Definition: An ASW fire control computer recieves target information from a sonar and then transmits aiming orders to a weapon. The weapon is then fired at a submerged target.
   3.2) Criterion For Success: Destruction of submarine
   3.3) MOE Selected: Maximum probability of a hit
      3.3.1) Rationale For Selection: The purpose of a fire control computer is to aim a weapon so that it will hit the target, and the probability of accomplishing that purpose is a

proper measure of tactical effectiveness. The tracking time is chosen so as to maximize the probability of a hit. The resulting maximum value is chosen as the MOE.

3.4) Functional Form Of MOE:

$$MOE = \max_{x_1} f(x_1, \ldots, x_{13})$$

where

$f(x_1, \ldots, x_{13})$ = hit probability

x_1 = length of tracking interval

x_2 = frequency of observations (data rate)

x_3 = standard deviation of the error distribution

x_4 = blind time

$= g(x_{14}, x_{15}, x_{16})$

x_5 = actuation radius

x_6 = ballistic dispersion

x_7 = smoothing function

x_8 = prediction function

x_9 = target depth

x_{10} = target path

x_{11} = target speed

x_{12} = radius of turn

x_{13} = size of pressure hull

x_{14} = dead time

x_{15} = time of flight

x_{16} = sinking time

4) MOE Usage In Study: Parametric variations of hit probability are performed as a function of the input parameters for each type of computer. It is pointed out, however, that the probability of hit is not the final criterion for comparative evaluation of computers intended for fleet use. Other factors must be considered such as the need for a higher probability of a hit, the cost of obtaining it, and the mechanical realization of the mathematical models which are used. Also mentioned is the fact that math-

ematical models were used because: (1) no reliable operational data on the accuracy of computers were available, (2) the method of analysis also is meant to be applicable during the design stages for new computers, and (3) only the relative tactical effectiveness of different methods of prediction under identical conditions is sought.

5) Special Study Assumptions:

(a) The error in each observation of each coordinate of the target position is statistically independent of the errors in all other observations, and are assumed to be independent of range.

(b) The distribution of errors for all observations is assumed to be Gaussian with zero mean.

(c) The standard deviation of the errors is the same in both coordinate directions and for all observations.

(d) The only errors in prediction are a random error due to the random errors in the observations and a systematic error introduced in the computer by the fact that the prediction function used is only an approximation to the equation which represents the target's actual path. Errors neglected were the errors in transmission of information, in own ship's course and speed, and inaccuracies in determining blind time.

(e) In the operation of computers, the internal mechanical errors are small enough to be neglected.

(f) For computers that use either the least squares or fixed memory point method of smoothing, the time to complete the solution after the last observation is negligible.

(g) Target speed and depth are constant.

(h) The target maintains the same path from the beginning of the tracking time until the end of the blind time. (NOTE: A method is presented to show how this assumption may be relaxed.)

(i) No "spot" corrections are made for a change in target course, and the only allowance to be made for a change in course during the tracking time is to start over again, using only data from the new path.

(j) A hit is said to have occurred if the center of the shot, or pattern, passes the target depth within the lethal radius of the shot, or pattern of shots, measured from the pressure hull of the submarine and in the horizontal plane of the target.

(k) The center of the weapon projectile, or pattern of projectiles, reaches target depth precisely at the point predicted by the computer as the future position of the target.

(l) The distribution of weapon impact points is circular normal, centered at the point of aim, and independent of the attacking ship's orientation.

(m) For the case of Hedgehog projectiles launched as a circular pattern, it is assumed that the projectiles hit the surface of the water simultaneously and sink so that the center of the pattern reaches depth at the point at which it was aimed.

(n) Ballistic dispersion of Hedgehog projectiles is sufficiently small that it may be neglected in computing the probability of a hit.

(o) Only one pattern of Hedgehog is launched in an attack.

(p) There are no errors aiming the weapon launcher at the point predicted by the computer as the future position of the target.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) target speed

(b) size of pressure hull

2.2) Deployment:

2.2.1) Qualitative Factor:

(a) submerged

2.2.2) Quantitative Factor:

(a) target depth

2.3) Tactics:

2.3.1) Qualitative Factors:

(a) straight run

(b) turn at fixed rate

2.3.2) Quantitative Factors:

(a) target path

(b) radius of turn

3) Friendly Force Composition: Surface ship

3.1) Sensor: Sonar

3.1.1) Quantitative Factors:

(a) frequency of observations (data rate)

(b) standard deviation of the error distribution

3.2) Computer: Fire control computer

3.2.1) Quantitative Factors:

(a) smoothing function

(b) prediction function

3.3) Armament: Hedgehog and weapon A

3.3.1) Quantitative Factors:

(a) actuation radius

(b) ballistic dispersion

(c) dead time

(d) time of flight

(e) sinking time

STUDY REVIEW SUMMARY NO.(10)-3

A. STUDY DESCRIPTION

1) Originating Activity: Arthur D. Little, Inc., Cambridge, Massachusetts
2) Paper Title: "Surveillance of a Region by Detection and Tracking Operations"
3) Author: J. M. Dobbie
4) Source: Operations Research, Vol. 12, No. 5, May-June 1964, pp. 379-394
5) Classification: Unclassified
6) Abstract: A study is made of the capabilities of a surveillance system to detect and track submarines that are in a region that is under surveillance. The operation consists of barrier searches for submarines entering the region, area searches for submarines that have entered the region undetected, tracking procedures to hold contact on detected submarines, and special searches to regain contact when contact has been lost. The capabilities of the surveillance system are found for a general distribution of submarine on-station times, under the assumption that the recontact rate decreases with increasing time after loss of contact.
7) Descriptors: Antisubmarine warfare, barrier, detection, search, submarine, surveillance, tracking

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Surface ASW
3) Mission: Ocean surveillance
   3.1) Definition: A region of the ocean is kept under surveillance to determine the existence of enemy submarines in the region and their locations. If a submarine is detected, either as it enters the region or after it is in the region, it will be tracked. If tracking contact is lost, a procedure to regain contact will be used. If contact is regained, the submarine again will be tracked.

3.2) Criterion For Success: Detection and tracking of submarine

3.3) MOE's Selected:

Case 1 - Contact can be transferred from a detecting unit to a tracking unit with certainty and in negligible time

Define the submarine states as follows:

State	Description
1	Submarine being tracked
2	Submarine not being tracked because contact has been lost
3	Submarine not detected

$(MOE)_1$ = Expected number of submarines in the region that are in state 1 at time t

$(MOE)_2$ = Expected number of submarines in the region that are in state 2 at time t

$(MOE)_3$ = Expected number of submarines in the region that are in state 3 at time t

Case 2 - Transfer interval is large and there is a non-negligible probability that a submarine is in a transfer state

Define the submarine states as follows:

State	Description
1'	Submarine being tracked by a mobile unit in the vicinity of the submarine
2'	Submarine previously tracked, contact recently lost, local search being made to regain tracking contact
3'	Submarine previously tracked, search to regain contact discontinued, new detection recently made by area search,

tracking unit(s) now en route to area or searching in an effort to obtain tracking contact

4'	Submarine previously tracked, search to regain contact discontinued, no new detection
5'	Submarine not previously tracked, recently detected by area search, tracking unit(s) now en route to area or searching in an effort to obtain tracking contact
6'	Submarine detected by the barrier as it enters the region, tracking unit(s) now en route to area or searching in an effort to obtain tracking contact
7'	Submarine not previously tracked and no previous detection, if any, is being used in an effort to obtain tracking contact

$(MOE)_4$ = Expected number of submarines in the region that are in state 1' at time t

$(MOE)_5$ = Expected number of submarines in the region that are in state 2' at time t

$(MOE)_6$ = Expected number of submarines in the region that are in state 3' at time t

$(MOE)_7$ = Expected number of submarines in the region that are in state 4' at time t

$(MOE)_8$ = Expected number of submarines in the region that are in state 5' at time t

$(MOE)_9$ = Expected number of submarines in the region that are in state 6' at time t

$(MOE)_{10}$ = Expected number of submarines in the region that are in state 7' at time t

3.4) Functional Form Of MOE:

Case 1

$(MOE)_1 = f_1(x_1, \ldots, x_7)$

$(MOE)_2 = f_2(x_1, x_2, x_5, x_6, x_7, x_{12}, x_{13})$

$(MOE)_3 = f_3(x_1, x_2, x_5, x_6, x_7, x_{14})$

where

x_1 = number of undetected submarines in the region at the start of the surveillance operation

x_2 = upper distribution function of remaining time on-station for submarines that are in the region at the start of the surveillance operation

= $g_1(x_6, x_8)$

x_3 = conditional probability that a submarine is in state 1 at time t after entry, given that it was in state 1 at entry and stays in the region during the interval

= $g_2(x_9, x_{10})$

x_4 = conditional probability that a submarine is in state 1 at time t after entry, given that it was in state 3 at entry and remains in the region during the interval

= $g_3(x_9, x_{10}, x_{11})$

x_5 = rate at which submarines enter the region at time t

x_6 = upper distribution function of submarine time on-station

x_7 = probability of detection by the barrier search

x_8 = average time on station

x_9 = probability of not regaining contact by time t after loss of contact

= $h_1(x_{15}, x_{16})$

x_{10} = rate of losing contact while tracking

x_{11} = rate of detection by the area search on a submarine not previously detected

x_{12} = conditional probability that a submarine is in state 2 at time t after entry, given that it was in state 1 at entry and stays in the region during the interval

$= g_4(x_3)$

x_{13} = conditional probability that a submarine is in state 2 at time t after entry, given that it was in state 3 at entry and remains in the region during the interval

$= g_5(x_4, x_{11})$

x_{14} = conditional probability that a submarine is in state 3 at time t after entry, given that it was in state 3 at entry and remains in the region during the interval

$= g_6(x_{11})$

x_{15} = rate of reduction by the area search

x_{16} = rate of regaining contact by the special search at time t after loss of contact

Case 2

$$(MOE)_4 = f_4(x_1, x_2, x_5, x_6, x_7, x_{17}, x_{24})$$

$$(MOE)_5 = f_5(x_1, x_2, x_5, x_6, x_7, x_{18}, x_{25})$$

$$(MOE)_6 = f_6(x_1, x_2, x_5, x_6, x_7, x_{19}, x_{26})$$

$$(MOE)_7 = f_7(x_1, x_2, x_5, x_6, x_7, x_{20}, x_{27})$$

$$(MOE)_8 = f_8(x_1, x_2, x_5, x_6, x_7, x_{21}, x_{28})$$

$$(MOE)_9 = f_9(x_1, x_2, x_5, x_6, x_7, x_{22})$$

$$(MOE)_{10} = f_{10}(x_1, x_2, x_5, x_6, x_7, x_{23}, x_{29})$$

x_{17} = conditional probability that a submarine is in state $1'$ at time t, given that it was in state $6'$ at time t=0 and stays in the region during the interval

= $g_7(x_{18}, \ldots, x_{23})$

x_{18} = conditional probability that a submarine is in state $2'$ at time t, given that it was in state $6'$ at time t=0 and stays in the region during the interval

= $g_8(x_{10}, x_{11}, x_{15}, x_{30}, x_{31}, x_{34}, x_{35})$

x_{19} = conditional probability that a submarine is in state $3'$ at time t, given that it was in state $6'$ at time t=0 and stays in the region during the interval

= $g_9(x_{10}, x_{11}, x_{15}, x_{30}, x_{31}, x_{34}, x_{35})$

x_{20} = conditional probability that a submarine is in state $4'$ at time t, given that it was in state $6'$ at time t=0 and stays in the region during the interval

= $g_{10}(x_{10}, x_{11}, x_{15}, x_{30}, x_{31}, x_{34}, x_{35})$

x_{21} = conditional probability that a submarine is in state $5'$ at time t, given that it was in state $6'$ at time t=0 and stays in the region during the interval

= $g_{11}(x_{11}, x_{33}, x_{34}, x_{37}, x_{38})$

x_{22} = conditional probability that a submarine is in state $6'$ at time t, given that it was in state $6'$ at time t=0 and stays in the region during the interval

= $g_{12}(x_{34})$

x_{23} = conditional probability that a submarine is in state $7'$ at time t, given that it was in state

$6'$ at time t=0 and stays in the region during the interval

$= g_{13}(x_{11}, x_{33}, x_{34}, x_{37}, x_{38})$

x_{24} = conditional probability that a submarine is in state $1'$ at time t, given that it was in state $7'$ at time t=0 and stays in the region during the interval

$= g_{14}(x_{18}, \ldots, x_{23})$

x_{25} = conditional probability that a submarine is in state $2'$ at time t, given that it was in state $7'$ at time t=0 and stays in the region during the interval

$= g_{15}(x_{10}, x_{11}, x_{15}, x_{30}, x_{31}, x_{34}, x_{35})$

x_{26} = conditional probability that a submarine is in state $3'$ at time t, given that it was in state $7'$ at time t=0 and stays in the region during the interval

$= g_{16}(x_{10}, x_{11}, x_{15}, x_{30}, x_{31}, x_{34}, x_{35})$

x_{27} = conditional probability that a submarine is in state $4'$ at time t, given that it was in state $7'$ at time t=0 and stays in the region during the interval

$= g_{17}(x_{10}, x_{11}, x_{15}, x_{30}, x_{31}, x_{34}, x_{35})$

x_{28} = conditional probability that a submarine is in state $5'$ at time t, given that it was in state $7'$ at time t=0 and stays in the region during the interval

$= g_{18}(x_{11}, x_{32}, x_{33}, x_{34}, x_{37}, x_{38})$

x_{29} = conditional probability that a submarine is in state $7'$ at time t, given that it was in state $7'$ at time t=0 and stays in the region during the interval

$= g_{19}(x_{11}, x_{33}, x_{34}, x_{37}, x_{38})$

x_{30} = probability that conversion has not been made and conversion attempts are continuing at time t after the submarine entered state 2'

= $h_2(x_{34}, x_{38})$

x_{31} = probability that conversion has not been made and conversion attempts are continuing at time t after the submarine entered state 3'

= $h_3(x_{35}, x_{39})$

x_{32} = probability that conversion has not been made and conversion attempts are continuing at time t after the submarine entered state 5'

= $h_4(x_{36}, x_{40})$

x_{33} = probability that conversion has not been made and conversion attempts are continuing at time t after the submarine entered state 6'

= $h_5(x_{37}, x_{41})$

x_{34} = rate of breaking off conversion attempts if submarine is in state 2'

x_{35} = rate of breaking off conversion attempts if submarine is in state 3'

x_{36} = rate of breaking off conversion attempts if submarine is in state 5'

x_{37} = rate of breaking off conversion attempts if submarine is in state 6'

x_{38} = rate of obtaining tracking contact at time t after the submarine enters state 2', provided the attempts to convert are continuing

x_{39} = rate of obtaining tracking contact at time t after the submarine enters state 3', provided the attempts to convert are continuing

x_{40} = rate of obtaining tracking contact at time t after the submarine enters state 5', provided the attempts to convert are continuing

x_{41} = rate of obtaining tracking contact at time t after the submarine enters state 6', provided the attempts to convert are continuing

3.5) Additional MOE's Identified:

(a) In case 1,

(a.1) Probability that a submarine is in state 1

(a.2) Probability that a submarine is in state 2

(a.3) Probability that a submarine is in state 3

(a.4) Expected number of submarines in the region at time t

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) Submarines enter the surveillance region at a known rate and remain on-station for an interval of time before leaving. The entry rate need not be known, if it is constant.

(b) The time on-station is a random variable from a known distribution.

(c) The detection capabilities of the barrier can be described adequately by a single probability of detection, the same for all submarines.

(d) The capabilities of the area search can be described adequately by two search rates, one for all submarines that had not been detected previously and the other for all submarines that had been detected previously, on a particular patrol. That is, a previous detection might increase the rate of detection by decreasing the recognition differential needed for detection. This gain in detection rate does not carry over from one patrol to a later patrol.

(e) The detection probabilities on two submarines are independent.

(f) After detection, contact is transferred from the detecting unit to a tracking unit. Two assumptions are considered here. In the main part of the paper, it is assumed that the time required for transfer is zero, and that the probability of transfer is one. In an extension, it is assumed that transfer is not certain, that the transfer time is not negligible, and that the transfer rate varies with time after detection, as well as with the type of detecting unit.

(g) After contact has been made by a tracking unit, contact will be maintained as long as possible while the submarine remains in the region. The tracking capability can be described adequately by a single rate of losing contact.

(h) If contact is lost, a special search to regain contact will be made by the tracking unit and, perhaps, by other units of the same type. The rate of regaining contact with this search is a known function of time after loss of contact. If contact is regained, it is assumed that tracking will be done by the detecting unit, or that contact can be transferred in negligible time.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarines

 2.1) Qualitative Factor:

  (a) homogeneous units

 2.2) Deployment:

  2.2.1) Quantitative Factor:

   (a) number of undetected submarines in the region at the start of the surveillance operation

 2.3) Tactics:

  2.3.1) Qualitative Factor:

   (a) randomly enter surveillance region

  2.3.2) Quantitative Factors:

   (a) rate at which submarines enter region

   (b) upper distribution function of submarine time on-station

   (c) average time on-station

3) Friendly Force Composition: Mobile units

3.1) Sensor: Passive sensor field

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Mobile units - Submarines

4.1.1) Quantitative Factors:

(a) rate of losing contact while tracking

(b) rate of detection by the area search on a submarine not previously detected

(c) rate of reduction by the area search

(d) rate of regaining contact by the special search after loss of contact

(e) rate of breaking off conversion attempt if submarine is in state 2'

(f) rate of breaking off conversion attempt if submarine is in state 3'

(g) rate of breaking off conversion attempt if submarine is in state 5'

(h) rate of breaking off conversion attempt if submarine is in state 6'

(i) rate of obtaining tracking contact after the submarine enters state 2'

(j) rate of obtaining tracking contact after the submarine enters state 3'

(k) rate of obtaining tracking contact after the submarine enters state 5'

(l) rate of obtaining tracking contact after the submarine enters state 6'

4.2) Sensor - Platform: Passive sensor field - Submarines

4.2.1) Quantitative Factor:

(a) probability of detection by the barrier search

STUDY REVIEW SUMMARY NO. (10)-4

A. STUDY DESCRIPTION

1) Originating Activity: Arthur D. Little, Inc., Cambridge, Massachusetts
2) Paper Title: "Transfer of Detection Contacts to Tracking Contacts in Surveillance"
3) Author: J. M. Dobbie
4) Source: Operations Research, Vol. 14, No. 5, Sept.-Oct. 1966, pp. 791-800
5) Classification: Unclassified
6) Abstract: Surveillance of a region is conducted by using a search system to detect targets and a tracking system or force to follow them. After the search system detects a target, the tracking force will attempt to gain contact, usually by a local search. Before the tracking force detects the target, and thereby completes transfer, contact may be lost by the search system. If recontact is made by the search system, the new contact will be used by the tracking force to localize its search for the target. As the local search by the tracking force proceeds, contact by the search system may be lost and regained a number of times. The probability of transfer is obtained under the assumption that the transfer rate decreases as the time since last contact by the search system increases. The particular problem considered is that of sea surveillance for submarines, in which the search system is a fixed field of sensors and the tracking force consists of mobile units, such as ships, aircraft, and helicopters. However, the model can be adopted to other transfer problems, such as the transfer of a contact by a search radar to contact by a tracking radar.
7) Descriptors: Antisubmarine warfare, detection, radar, search, submarine, surveillance, tracking

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW

3) Mission: Contact investigation

3.1) Definition: Submarine contact has been made by a sensor field and a tracker has been directed to the area to conduct a search for the suspected submarine.

3.2) Criterion For Success: Detection of submarine

3.3) MOE Selected: Probability that a submarine has been detected by the tracker

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_5)$$

where

x_1 = rate at which the sensor field loses contact

x_2 = rate at which the sensor field regains contact

x_3 = expected time for the tracker to reach the vicinity of the submarine

x_4 = rate of detection when contact is held by the sensor field

x_5 = rate of detection as a function of time after last contact is not held by the sensor field

3.5) Additional MOE Identified:

(a) Expected time to find the submarine after the tracker reaches the area

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) Rate at which the sensor field loses contact is constant.

(b) Rate at which the sensor field regains contact is constant.

(c) Rate of detection by the tracker remains constant as long as contact is held by the sensor field, and decreases monotonically after contact is lost.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Deployment:

2.1.1) Qualitative Factor:

(a) submerged

3) Friendly Force Composition: Tracking force

3.1) Sensor: Sensor field

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Tracking force - Submarine

4.1.1) Quantitative Factor:

(a) expected time for the tracker to reach the vicinity of the submarine

4.2) Sensor - Platform: Sensor field - Submarine

4.2.1) Quantitative Factors:

(a) rate at which the sensor field loses contact

(b) rate at which the sensor field regains contact

(c) rate of detection when contact is held by the sensor field

(d) rate of detection as a function of time after last contact when contact is not held by the sensor field

STUDY REVIEW SUMMARY NO.(10)-5

A. STUDY DESCRIPTION

1) Originating Activity: Arthur D. Little, Inc., Cambridge, Massachusetts
2) Report Title: The Feasibility of Surface Effect Vehicles in ASW Missions
3) Report Number: ADL C-71425 (AD-505 504)
4) Authors: R. A. Gallant, B. O. Koopman, F. Marbury and G. E. Miller
5) Date: June 1969
6) Classification: Secret
7) Contract Number: DAHC15-69-C-0257 (Advanced Research Projects Agency)
8) Abstract: The purpose of the study was to determine the feasibility of using Surface Effect Vehicles (SEV) in anti-submarine warfare in the 1980 timeframe. The analysis focused on the identification of: ASW missions that could be performed better with SEV's than with competing systems; tactics that could be used for each mission; sensor, weapon and SEV performance requirements for ASW missions; and advances in technology required to bring about a viable SEV-ASW system.
9) Descriptors: Amphibious operation, antisubmarine warfare, barrier, contact investigation, contact prosecution, convoy defense, screen, search, sonobuoy, SOSUS, submarine, surface effect vehicle, surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Missions:

3.1) Definition:

(a) Barrier placement/patrol: SEV's are either placed in the path of a detected submarine, on a known transit track, to shield a convoy or amphibious landing, or to guard relatively narrow portions of the sea.

(b) Escort/screen: SEV's are used as escorts or part of a protective screen to protect merchant convoys, task forces and amphibious landing craft formations from submarine attack.

(c) Contact investigation/prosecution: SEV's investigate submarine probable area obtained by SOSUS contact and fix to obtain more precise localization and then attack with torpedoes.

(d) Ocean surveillance: SEV's monitor restricted areas to accumulate observations concerning gathering places of potentially hostile submarines.

3.2) Criterion For Success: Detection of submarine

3.3) MOE's Selected:

$(MOE)_1$ = Kinetic search rate, which is defined to be the expected number of targets detected per unit time in a kinetic search procedure from a uniform distribution of targets spread with unit density over the area

$(MOE)_2$ = Static search rate, which is defined to be the expected number of targets detected per unit time in a static search procedure from a uniform distribution of targets spread with unit density over the area

3.4) Functional Form Of MOE's:

$(MOE)_1 = f_1(x_1, \ldots, x_4)$

$(MOE)_2 = f_2(x_1, x_5)$

where

x_1 = area of region to be searched
x_2 = time to lower sonar, scan and raise sonar
x_3 = maximum SEV speed
x_4 = range of passive sonar detection
$= g_1(x_6, x_7, x_8, x_9)$
x_5 = submarine speed
x_6 = source strength
x_7 = number of hydrophones
x_8 = background noise
x_9 = signal-to-noise ratio

4) MOE Usage In Study: The MOE's were formulated and used in a qualitative analysis of the effectiveness of the SEV's to perform the various ASW missions.

5) Special Study Assumptions:

(a) A definite range law is used in the search; that is, any target in a specified region is detected, while a target not in the region is not detected.

(b) In the formulation for the kinetic search rate, the target is either stationary or slowly moving.

C. EFFECTIVENESS FACTORS

1) Physical Environment:
   1.1) Quantitative Factor:
      (a) background noise
2) Threat Composition: Submarine
   2.1) Quantitative Factors:
      (a) submarine speed
      (b) source strength
3) Friendly Force Composition: Surface Effect Vehicles

3.1) Quantitative Factor:

(a) maximum SEV speed

3.2) Sensor: Passive sonar

3.2.1) Quantitative Factors:

(a) time to lower, scan, and raise sonar

(b) number of hydrophones

3.3) Tactics:

3.3.1) Qualitative Factors:

(a) search for submarine using acoustic sensor

(b) in the formulation for the kinetic search rate, the SEV carries out a sequence of cycles in which, starting in a still buoyancy mode, it lowers its acoustic sensor, carries out a full 360° scan with data processing, raises its sensor, and then it moves at maximum speed to the nearest minimally overlapping position, where it repeats the process.

(c) in the formulation of the static search rate, the SEV remains in the still buoyancy mode, constantly listening, and stationed in the presumed path of the target.

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Surface Effect Vehicles - Submarine

4.1.1) Quantitative Factor:

(a) area of region to be searched

4.2) Sensor - Platform: Passive sonar - Submarine

4.2.1) Quantitative Factor:

(a) signal-to-noise ratio

Ultrasystems

STUDY REVIEW SUMMARY NO.(10)-6

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington Virginia
2) Report Title: Advanced Surface Effect Vehicles for Anti-Submarine Warfare Missions
3) Author: R. D. Linnel
4) Report Number: Systems Evaluation Group Study 7 (AD-507 574)
5) Date: February 1970
6) Classification: Secret
7) Contract Number: N00014-68-A-0091 (Advanced Research Projects Agency)
8) Abstract: The use of potential Surface Effect Vehicles (SEV) in Anti-Submarine Warfare (ASW) missions is considered. Simple analytical parametric models for SEV are developed. Two models are presented: one for sea-mobile (only) vehicles (SES) and one for amphibious vehicles (SEA). Also, two types of ASW missions were considered in this investigation, the ASW area search mission and the ASW barrier mission.
9) Descriptors: Antisubmarine warfare, barrier, contact prosecution, detection probability, kill probability, localization probability, MAD, search, sonobuoy, submarine, surface effect vehicle, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Missions:
   3.1) Mission Type: Submarine search/contact prosecution
      3.1.1) Definition: An ASW vehicle proceeds to a search area and sweeps a designated area with its sensors, classifies and localizes all detections

as needed, and, in a shooting scenario, attacks all localizations classified as real targets.

3.1.2) Criterion For Success: Detection and destruction of submarine

3.1.3) MOE Selected: Effective cost ratio, which is defined as the ratio of the 10-year system cost for area search, to the product of the overall kill probability and the area swept

3.1.4) Functional Form Of MOE:

$$(MOE)_1 = f_1(x_1, x_2, x_3)$$

where

x_1 = 10-year system cost for area search
$= g_1(x_4, \ldots, x_7)$

x_2 = probability of overall submarine kill
$= g_2(x_8, \ldots, x_{12})$

x_3 = area swept
$= g_3(x_5, x_{13}, x_{14}, x_{15})$

x_4 = 10-year investment and operating cost for the vehicle
$= h_1(x_{16}, x_{17})$

x_5 = number of mission ready area search vehicles

x_6 = number of expendable reliable acoustic path (ERAP) sonobuoys per ready vehicle
$= h_2(x_8, x_{13}, x_{14}, x_{15}, x_{18})$

x_7 = number of localization sonobuoys per ready area-search vehicle

x_8 = coverage factor
$= h_3(x_3, x_{18})$

x_9 = detection probability within opportunity range

x_{10} = probability that submarine is classified as a submarine

x_{11} = probability that localization is obtained

= $h_4(x_{19}, \ldots, x_{24})$

x_{12} = probability of kill by 4-torpedo attack

x_{13} = hours on-station per day

= $h_5(x_{25}, x_{26}, x_{27})$

x_{14} = number of days of area search

x_{15} = sweep rate per vehicle in ASW area search

= $h_6(x_8, \ldots, x_{11}, x_{18}, x_{28}, \ldots, x_{35})$

x_{16} = average investment cost for SEV

= $i_1(x_{36}, \ldots, x_{42})$

x_{17} = annual operating cost for SEV

= $i_2(x_{55})$

x_{18} = opportunity range (i.e., maximum expected range of sonobuoy)

x_{19} = probability that MAD classifies after RAP

x_{20} = probability that passive sonar classifies after RAP

x_{21} = probability that MAD classifies after passive sonar

x_{22} = probability that active sonar classifies after RAP

x_{23} = probability that passive sonar classifies after active sonar

x_{24} = probability that MAD classifies after active sonar

x_{25} = fractional utilization of ASW vehicles

x_{26} = time on-station for the ASW vehicle

x_{27} = time per sortie

x_{28} = ASW area search mission speed

x_{29} = path factor for distance between primary sonobuoys

x_{30} = density of submarines

x_{31} = time required for initial classification
= $i_3(x_9, x_{18}, x_{30}, x_{34}, x_{43}, x_{44})$

x_{32} = time for localization/classification
= $i_4(x_{18}, x_{45}, x_{46}, x_{47})$

x_{33} = time required for 4-torpedo attack

x_{34} = density of false targets

x_{35} = probability of classification of noise as a submarine

x_{36} = cost for structures labor
= $j_1(x_{48}, x_{49})$

x_{37} = cost for structures material
= $j_2(x_{49}, x_{50})$

x_{38} = cost for equipment labor
= $j_3(x_{48}, x_{51})$

x_{39} = cost for equipment material
= $j_4(x_{50}, x_{51})$

x_{40} = cost for fan-propulsion labor
= $j_5(x_{48}, x_{52}, x_{53})$

x_{41} = cost for fan-propulsion material
= $j_6(x_{50}, x_{52}, x_{53})$

x_{42} = cost of payload installed
= $j_7(x_{54})$

x_{43} = total initial classification time per sortie

x_{44} = number of primary search sonobuoys

x_{45} = constant for localization distance

x_{46} = factor for localization distance

x_{47} = speed used for classification/localization

x_{48} = labor learning factor
= $k_1(x_5)$

x_{49} = weight of structure
= $k_2(x_{55}, x_{56})$

x_{50} = material learning factor
= $k_3(x_5)$

x_{51} = weight of equipment
= $k_4(x_{55}, x_{56}, x_{57})$

x_{52} = power required for fan
= $k_5(x_{55}, x_{58})$

x_{53} = power required for propulsion
= $k_6(x_{55}, x_{56}, x_{58})$

x_{54} = cost of dockside payload

x_{55} = displacement weight
= $m(x_{49}, x_{51}, \ldots, x_{59}, x_{60}, x_{61})$

x_{56} = speed for design mission

x_{57} = range for design mission

x_{58} = height of wave

x_{59} = weight of payload installed
= $n_1(x_{62}, x_{63})$

x_{60} = weight of fan-propulsion system
= $n_2(x_{55}, x_{64}, x_{65})$

x_{61} = weight of fuel carried
= $n_3(x_{66})$

x_{62} = installed payload weight factor

x_{63} = weight of dockside payload

x_{64} = factor for fan power

x_{65} = factor for propulsion power

x_{66} = weight of fuel for range
= $p(x_{55}, x_{56}, x_{57}, x_{64}, \ldots, x_{68})$

x_{67} = factor for operational degradation of fuel consumption

x_{68} = fuel specific consumption

x_{69} = factor for wave drag power

3.2) Mission Type: Barrier placement/patrol

3.2.1) Definition: Lines of sensors are positioned in a stationary strip that must be crossed by the threat submarines in carrying out their mission. Classification and localization are done for each detection by each sensor in the barrier, and, in the shooting scenario, attacks are made for all detections classified as real targets.

3.2.2) Criterion For Success: Detection and destruction of submarine

3.2.3) MOE Selected: Effective cost ratio, which is defined as the ratio of the 10-year system cost for ASW barriers to the product of overall kill probability and the length of the barrier

3.2.4) Functional Form Of MOE:

$$(MOE)_2 = f_2(x_2, x_{70}, x_{71})$$

where

x_{70} = ten-year system cost for barriers
$= g_4(x_4, x_{72}, x_{73})$

x_{71} = length of ASW barrier
$= g_5(x_5, x_{13}, x_{74}, \ldots, x_{77})$

x_{72} = number of mission ready barrier search vehicles

x_{73} = number of localization sonobuoys per ready barrier vehicle
$= h_7(x_5, x_8, x_9, x_{10}, x_{13}, x_{25}, x_{26}, x_{27}, x_{30}, x_{34}, x_{35}, x_{74}, \ldots, x_{78})$

x_{74} = factor for ASW barrier minimum width
$= i_5(x_{18}, x_{79}, x_{80})$

x_{75} = factor for ASW barrier swath width

= $i_6(x_{18}, x_{79}, x_{81})$

x_{76} = coverage rate for ASW barrier mission

= $i_7(x_8, \ldots, x_{11}, x_{18}, x_{30}, \ldots, x_{35}, x_{75}, x_{82})$

x_{77} = submarine speed in crossing an ASW barrier

x_{78} = number of days of barrier search

x_{79} = number of lines of sonobuoys in an ASW barrier

x_{80} = minimum width of an ASW barrier

x_{81} = swath width of an ASW barrier

x_{82} = ASW barrier mission speed

4) MOE Usage in Study: Formulation and numerical examples

C. EFFECTIVENESS FACTORS

1) Physical Environment:
   1.1) Quantitative Factors:
      (a) height of wave
      (b) density of false targets
2) Threat Composition: Submarine
   2.1) Quantitative Factors:
      (a) submarine speed in crossing an ASW barrier
      (b) density of submarines
3) Friendly Force Composition: Surface Effect Vehicles
   3.1) Quantitative Factors:
      (a) factor for fan power
      (b) factor for operational degradation of fuel consumption
      (c) factor for propulsion power

(d) factor for wave drag power
(e) fuel specific consumption
(f) number of mission ready area search vehicles
(g) range for design mission (area search/barrier search)
(h) speed for design mission (area search/barrier search)
(i) weight of dockside payload
(j) weight of payload installed
(k) fractional utilization of ASW vehicles
(l) time per sortie
(m) speed used for classification/localization
(n) ASW area search mission speed
(o) cost of dockside payload
(p) ASW barrier mission speed
(q) number of mission ready barrier search vehicles

3.2) Sensors: RAP (reliable acoustic path) sonobuoys

3.2.1) Quantitative Factors:

(a) path factor for distance between primary sonobuoys
(b) number of lines of sonobuoy in an ASW barrier
(c) number of localization sonobuoys per ready search vehicle
(d) number of primary search sonobuoys
(e) opportunity range of sonobuoy
(f) swath width of an ASW barrier
(g) probability of classification of noise as a submarine

3.3) Armament: Torpedoes

3.3.1) Quantitative Factor:

(a) time required for 4-torpedo attack

3.4) Deployment:

3.4.1) Quantitative Factors:

(a) time on-station for the ASW vehicle
(b) minimum width of ASW barrier
(c) number of days of area search
(d) total initial classification time per sortie
(e) number of days of barrier search

4) Friendly Force - Threat Interaction:

4.1) Sensor - Platform: RAP Sonobuoys - Submarine

4.1.1) Quantitative Factors:

(a) factor for localization distance
(b) constant for localization distance
(c) probability that active sonar classifies after RAP
(d) detection probability within opportunity range
(e) probability that MAD classifies after active sonar
(f) probability that MAD classifies after passive sonar
(g) probability that MAD classifies after RAP
(h) probability that submarine is classified as submarine
(i) probability that passive sonar classifies after RAP
(j) probability that passive sonar classifies after active sonar

4.2) Armament - Platform: Torpedoes - Submarine

4.2.1) Quantitative Factor:

(a) probability of kill by 4-torpedo attack

STUDY REVIEW SUMMARY NO. (10)-7

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Postgraduate School, Monterey, California
2) Report Title: Evaluating the Effectiveness of a Surface Ship ASW Screen
3) Author: P. S. Marsden
4) Report Identification: Thesis for the Master of Science in Operations Research, (AD-510 527)
5) Date: April 1970
6) Classification: Secret
7) Abstract: This thesis proposes a measure of effectiveness that reflects the basic defensive role of the ASW screen and which is evaluated by using data from current naval fleet exercises. Possible methods to improve the screen's effectiveness are proposed and evaluated in a Markov chain model.
8) Descriptors: Antisubmarine warfare. Markov process, screen, submarine, surface ship

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Mission: Escort/screen
   3.1) Definition: A surface ship ASW screen encounters a hostile submarine.
   3.2) Criterion For Success: Insurance of the safe passage of convoys, strike groups, and amphibious forces in the presence of hostile submarines
   3.3) MOE Selected: Probability that the submarine fails to attack the main body by direct or indirect action of the screen units
      3.3.1) Rationale For Selection: This measure reflects the defensive nature of the surface ship screen.
   3.4) Functional Form Of MOE:

$$MOE = f(x_1)$$

where

x_1 = a 2-dimensional array whose $(i,j)^{\underline{th}}$ entry is the one-step transition probability from state i to state j for i,j=1,2,...,8.

The system states are defined as follows:

State	Description
1	The submarine is outside the screen in a position to attack the screen units or main body of the surface formation.
2	The submarine is inside the screen and in a position to attack.
3	The presence of the submarine is detected by the surface units outside the screen prior to the submarine's attack on the main body.
4	The presence of the submarine is detected inside the screen prior to the submarine's attack.
5	The submarine attacks the main body of the formation.
6	The submarine fails to attack the main body through some direct or indirect action of the screen units.
7	The submarine interrupts its attack on the main body through some direct or indirect action of the screen.
8	The submarine successfully completes its attack on the main body.

4) MOE Usage In Study: Operational fleet exercise data, including environmental factors, were reviewed to obtain data for evaluation of the MOE.

5) Special Study Assumptions:

(a) The phenomenon could be described by a stationary Markov process.

(b) Initially, the tactical situation is described by State 1.

C. EFFECTIVENESS FACTORS:

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Tactics:

2.1.1) Qualitative Factor:

(a) penetrate screen and attack convoy

2.1.2) Quantitative Factors:

(a) the one-step transition probability from state 1 to state 2

(b) the one-step transition probability from state 1 to state 5

(c) the one-step transition probability from state 2 to state 5

(d) the one-step transition probability from state 5 to state 8

3) Friendly Force Composition: Surface ships

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Surface ships - Submarine

4.1.1) Quantitative Factors:

(a) the one-step transition probability from state 1 to state 3

(b) the one-step transition probability from state 1 to state 6

(c) the one-step transition probability from state 2 to state 4

(d) the one-step transition probability from state 2 to state 6

(e) the one-step transition probability from state 3 to state 2

(f) the one-step transition probability from state 3 to state 4

(g) the one-step transition probability from state 3 to state 5

(h) the one-step transition probability from state 3 to state 6

(i) the one-step transition probability from state 4 to state 5

(j) the one-step transition probability from state 5 to state 6

(k) the one-step transition probability from state 5 to state 7

STUDY REVIEW SUMMARY NO.(10)-8

A. STUDY DESCRIPTION

1) Originating Activity: U.S. Naval Postgraduate School, Monterey, California
2) Report Title: Application of Cost Effectiveness Techniques to Selection of Preferred Warship Characteristics
3) Author: L. K. McMillen, Jr.
4) Report Identification: Thesis for the Master of Science In Operations Research, (AD-481 402)
5) Date: 1966
6) Classification: Unclassified
7) Abstract: This paper discusses the applicability of cost effectiveness methods to the problem of determining preferred design characteristics of surface, anti-submarine warships. A short introduction to the concept of cost effectiveness as applied to military weapons systems is followed by a description of the methodology applicable to adapting cost effectiveness techniques to selection of preferred warship design characteristics. The surface anti-submarine vessel is used as a vehicle for adapting the cost effectiveness methodology; explanations as to how the cost effectiveness model may be expanded to include other types of surface ships are included.
8) Descriptors: Antisubmarine warfare, convoy escort, cost, cost effectiveness, screen, search, sonar, submarine, surface ship, surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force
2) Function: Surface ASW
3) Missions:
   3.1) Definition:
      (a) Ocean surveillance - The ASW patrol vessels are to cover a specified area by sonar surveillance within a

specified period of time on a continuing basis.

(b) Escort/screen - The ASW escort vessels are to provide complete sonar coverage across the front of a convoy of specified width.

3.2) Criterion For Success: Detection of submarine

3.3) MOE Selected: Total weapons system cost, over a specified period of time, to produce a specified degree of effectiveness

3.3.1) Rationale For Selection: The fixed effectiveness study is generally more applicable to weapons system selection problems than is a fixed cost study. It is usually easier to arrive at broad decisions with regard to further military needs in terms of specific capabilities than it is to decide initially to allot a fixed percentage of the national budget toward developing a specific military capability.

3.4) Functional Form Of MOE:

$$MOE = f(x_1, \ldots, x_6)$$

where

x_1 = cost of individual ship procurement

$= g_1(x_7, \ldots, x_{13})$

x_2 = annual cost of operating one ship while performing the specified mission

$= g_2(x_{14}, \ldots, x_{17})$

x_3 = initial cost of support facilities required to support one ship

x_4 = annual operating cost of the support facilities required to support one ship

x_5 = number of ships required to produce the fixed degree of effectiveness

$$= \begin{cases} g_3(x_{18}, \ldots, x_{24}) & \text{for the ASW area search mission} \\ g_4(x_{19}, x_{20}, x_{22}, \ldots, x_{26}) & \text{for the ASW escort/screen mission} \end{cases}$$

x_6 = time period over which the system is to be used

x_7 = procurement cost of hull
= $h_1(x_{27})$

x_8 = procurement cost of propulsion
= $h_2(x_{27}, x_{28})$

x_9 = procurement cost of anti-aircraft armament
= $h_3(x_{29}, x_{30})$

x_{10} = procurement cost of anti-submarine armament
= $h_4(x_{31}, x_{32})$

x_{11} = procurement cost of communications equipment

x_{12} = procurement cost of underwater search equipment
= $h_5(x_{19}, x_{33})$

x_{13} = procurement cost of above water search equipment

x_{14} = annual cost of fuel per ship
= $h_6(x_{20}, x_{28}, x_{34})$

x_{15} = annual cost of maintenance per ship
= $h_7(x_1, x_{20})$

x_{16} = annual cost of personnel per ship
= $h_8(x_{27}, x_{28})$

x_{17} = annual cost of consummable supplies per ship

x_{18} = area to be patrolled

x_{19} = sonar range for which probability of target detection is 50 per cent
= $i(x_{35}, \ldots, x_{43})$

x_{20} = cruising speed of ASW vehicle

x_{21} = revisit time of ASW patrol

x_{22} = endurance of the ASW vehicle

x_{23} = time required for replenishment and repair between patrols of ASW vehicle

x_{24} = average distance from support base to patrol area

x_{25} = width of convoy

x_{26} = convoy speed

x_{27} = length of ASW vehicle

x_{28} = maximum design speed of ASW vehicle

x_{29} = number of anti-aircraft weapons installed

x_{30} = fixed unit price of an anti-aircraft weapon

x_{31} = number of anti-submarine weapons installed

x_{32} = fixed unit price of an anti-submarine weapon

x_{33} = fixed electronic (for sonar) installation cost

x_{34} = total shaft horsepower

x_{35} = sonar sound attenuation coefficient

x_{36} = sonar frequency

x_{37} = attenuation anomaly (sound pressure level loss due to unexplained properties of the medium of propagation)

x_{38} = power output of sonar

x_{39} = target signal strength

x_{40} = sonar receiving directivity index $= j(x_{36}, x_{44}, x_{45})$

x_{41} = sonar ambient noise spectrum level

x_{42} = sonar recognition differential

x_{43} = sonar critical band width

x_{44} = vertical dimensions of sonar transducer

x_{45} = velocity of sound in water

3.5) Additional MOE's Identified:

(a) For the ocean surveillance mission,

(a.1) Number of ships required in the area at all times to accomplish the assigned task

(a.2) Number of ships that must be contained in the system to maintain one ship on-station continuously

(b) In the ASW escort/screen mission,

(b.1) Number of ships necessary to meet the specified escort requirements (b.2) Number of ships required to maintain one ship on continuous escort duty

4) MOE Usage In Study: Formulation only

5) Special Study Assumptions:

(a) The fixed effectiveness approach is used in developing this model, i.e., it is assumed that a prior decision that a certain ASW capability is required, and that this capability is best delivered by surface, ASW vessels. Since the fixed effectiveness approach is being considered here, the weapon system represented by the total cost equation must be capable of delivering the previously fixed degree of effectiveness.

(b) The measure of effectiveness used and the degree of effectiveness are initially specified. The number of ships required is then chosen as a function of the ship's efficiency in terms of the specified effectiveness.

(c) No attempt has been made to reduce either of the major cost divisions concerning support facilities to component cost elements. This is due primarily to the unavailability of sufficient data to determine the cost of installing and operating ship support facilities as a function of the physical characteristics of the weapon system.

(d) The geometrical shape of the hulls under consideration are similar, consequently the displacement of a ship can be represented as a linear function of its length.

(e) All the vessels considered in a study have a common type of propulsion system, consequently the cost of propulsion machinery can be determined as a linear function of shaft horsepower.

(f) The propulsive efficiency (i.e., the ratio between shaft horsepower and effective horsepower) is constant, an assumption which is justified because of the range of cost accuracy required for the study.

(g) The cost of any particular weapon with its associated fire control systems are considered fixed parameters. This assumption is reasonable in view of the fact that a great deal of time and effort is normally spent on individual weapon optimization, and weapons are often developed first with the ship being built around the latest weapon design.

(h) No attempt is made to determine the cost of shipboard communications equipment as a function of system characteristics. There is little data available in this field. Also, it is difficult to determine what physical characteristics of the weapon system that should be related to the cost of communications equipment.

(i) Only the costs for underwater search equipment are analyzed.

(j) The annual cost of scheduled overhaul is considered to be a fixed fraction of the total procurement cost of a ship.

(k) The costs of non-regularly scheduled maintenance vary principally as a function of cruising speed.

(l) The operating cost due to personnel required to man the electronic search equipment both above water and under water, is assumed to be a linear function of the detection range of the equipment.

(m) The operating costs attributed to personnel required to operate the ship's propulsion and associated auxiliary machinery varies linearly with shaft horsepower, whereas costs attributed to general shipkeeping personnel vary linearly with the displacement of the vessel.

(n) The operating cost due to personnel required to man the installed communications is considered constant.

(o) Since the installed armament is considered to be a parametrized value, the technically trained personnel required for the maintenance of the ordnance equipment is considered a parameterized cost.

(p) Cost of equipage is considered a constant here since any significant variance in usage rates will be covered under restricted availability.

(q) The definite range of detection law is used to compute sonar sweep widths.

(r) In the ASW convoy escort mission there are no support facilities between the initial and terminal points of the convoy, and no provisions for replenishment at sea.

(s) The transmitting directivity index of sonar is equal to the receiving directivity index of the sonar.

(t) No provisions were made for the effect of learning curves on the procurement costs of system units.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

  1.1) Quantitative Factor:

    (a) velocity of sound in water

2) Threat Composition: Submarine

  2.1) Quantitative Factor:

    (a) target signal strength

3) Friendly Force Composition:

  3.1) Platform type: Surface ASW vehicle

    3.1.1) Quantitative Factors:

      (a) initial cost of support facilities required to support one ship

      (b) annual operating cost of the support facilities required to support one ship

      (c) time period over which the system is to be used

      (d) procurement cost of communications equipment

      (e) annual cost of consummable supplies per ship

(f) cruising speed of ASW vehicle

(g) endurance of the ASW vehicle

(h) time required for replenishment and repair between patrols of ASW vehicle

(i) length of ASW vehicle

(j) maximum design speed of ASW vehicle

(k) total shaft horsepower

3.1.2) Sensors: Underwater and above water search equipment

3.1.2.1) Type: Underwater search equipment

3.1.2.1.1) Quantitative Factors:

(a) fixed electronic (for sonar) installation cost

(b) sonar sound attenuation coefficient

(c) sonar frequency

(d) attenuation anomaly (sound pressure level loss due to unexplained properties of the medium of propagation)

(e) power output of sonar

(f) sonar ambient noise spectrum level

(g) sonar recognition differential

(h) sonar critical band width

(i) vertical dimensions of sonar transducer

3.1.2.2) Type: Above water search equipment

3.1.2.2.1) Quantitative Factor:

(a) procurement cost of above water search equipment

3.1.3) Armament: Anti-aircraft and anti-submarine weapons

3.1.3.1) Type: Anti-aircraft weapons

3.1.3.1.1) Quantitative Factors:

(a) number of anti-aircraft weapons installed

(b) fixed unit price of an anti-aircraft weapon

3.1.3.2) Type: Anti-submarine weapons

3.1.3.2.1) Quantitative Factors:

(a) number of anti-submarine weapons installed

(b) fixed unit price of an anti-submarine weapon

3.1.4) Deployment:

3.1.4.1) Quantitative Factors:

(a) area to be patrolled

(b) revisit time of ASW patrol

(c) average distance from support base to patrol area

3.2) Platform type: Convoy

3.2.1) Quantitative Factors:

(a) width of convoy

(b) convoy speed

LXW

STUDY REVIEW SUMMARY NO.(10)-9

A. STUDY DESCRIPTION

1) Originating Activity: Boeing Airplane Company, Seattle, Washington
2) Report Title: A Technique for Analysis of Intermittent Search Operations Applicable to ASW
3) Authors: R. E. Nichols and W. M. Whisler
4) Report Number: D2-10868 (AD-868 707)
5) Date: 11 June 1961
6) Classification: Unclassified
7) Abstract: Determination of the probability of detection for an escort screen with an intermittent search capability as a function of all the variables involved would be difficult at best by analytical computations. An approximation can be obtained by the Monte Carlo technique. However, the number of combinations of parameters that must be evaluated is sizable. A procedure is given to reduce the task to the development of only one set of data by Monte Carlo. The method is applicable to the detection phase of similar barrier tasks that utilize intermittent search. Some specific applications are presented.
8) Descriptors: Antisubmarine warfare, convoy escort, detection probability, dipping sonar, Monte Carlo method, screen, search, submarine, surface effect vehicle

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Mission: Escort/screen
   3.1) Definition: Hydrofoil craft form an ASW surface escort force. They conduct a search ahead of the force for its entire width as well as maintaining stations at the side and rear of the force to detect any submarine attempting to intercept the force. Two methods of search are used. They are referred to as the zig-zag and double-line (or straight ahead) methods. The search patterns

are formed by a series of sonar dip cycles that consist of foiling to a fixed location and then conducting a sonar search.

3.2) Criterion For Success: Detection of submarine

3.3) MOE Selected: Probability of submarine detection

3.4) Functional Form Of MOE:

$$MOE = f(x_1, x_2)$$

where

x_1 = relative movement between sensor and target
$= g_1(x_3, \ldots, x_7)$

x_2 = spacing between adjacent sensor units
$= g_2(x_3, \ldots, x_6, x_8)$

x_3 = sonar detection range

x_4 = time to set up and search (dip time)
$= h(x_8, x_9)$

x_5 = average transit velocity

x_6 = speed of advance

x_7 = speed of target

x_8 = total time per search cycle

x_9 = time to transit to next search station

4) MOE Usage In Study: The probability of detection was determined by Monte Carlo simulation on a digital computer and parametric analyses were performed.

5) Special Study Assumptions:

(a) Sonar search time is directly proportional to the number of pings.

(b) Three-ping detection criterion was used.

(c) Target course is opposite that of the escort screen.

(d) Capability for detection only exists when the hydrofoil is stationary.

(e) Hydrofoil craft is "blind" when in transit on foils.

(f) All speeds are constant.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated
2) Threat Composition: Submarine
   2.1) Quantitative Factor:
      (a) speed of target
3) Friendly Force Composition: Hydrofoil craft
   3.1) Quantitative Factors:
      (a) speed of advance
      (b) time to transit to next search station
      (c) average transit velocity
      (d) total time per search cycle
   3.2) Sensor: Dipping sonar
      3.2.1) Quantitative Factor:
         (a) sonar detection range
   3.3) Tactics:
      3.3.1) Qualitative Factors:
         (a) foil to a fixed location and conduct a sonar search
         (b) zig-zag search pattern
         (c) double-line search plan

STUDY REVIEW SUMMARY NO.(10)-10

A. STUDY DESCRIPTION

1) Originating Activity: Daniel H. Wagner, Associates, Paoli, Pennsylvania
2) Report Title: The Influence of Destroyer Silencing on Mission Effectiveness
3) Author: H. R. Richardson
4) Report Number: DHWA Log. No. 21-982
5) Date: 31 December 1966
6) Classification: Secret
7) Contract: NObs-92146 (Naval Ship Systems Command)
8) Abstract: The influence of destroyer noise silencing on mission effectiveness is analyzed in the three tactical settings of screening, datum search, and open ocean search. In each situation, silencing is specified parametrically by reducing the self and radiated noise of a reference destroyer. The influence of silencing is reflected in measures of effectiveness appropriate to the respective tactical settings. The problem of estimating the cost associated with silencing is not considered. Of separate interest, optimal destroyer tactics in open ocean search and optimal submarine approach tactics are developed.
9) Descriptors: Antisubmarine warfare, barrier, convoy escort, destroyer, merchant ship, missile, normal density function, Poisson density function, search, submarine, surveillance, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force and System
2) Function: Surface ASW

3) Missions:

3.1) Mission Type: Escort/screen

3.1.1) Definition: Merchant vessels are escorted by convoys that are protected by destroyers. Enemy submarines attempt to penetrate the screen.

3.1.2) Criterion For Success: Prevention of submarine penetration of convoy screen

3.1.3) MOE's Selected:

$(MOE)_1$ = Expected number of merchant vessels sunk during a single attack by a diesel submarine

$(MOE)_2$ = Probability that a diesel submarine is sunk at some point during a single attack on a convoy

$(MOE)_3$ = Probability that a destroyer is sunk during a single attack on a convoy by a diesel submarine

$(MOE)_4$ = Expected number of merchant vessels sunk by diesel submarines during one month

$(MOE)_5$ = Expected number of diesel submarines sunk during one month

3.1.4) Functional Form Of MOE's:

$$(MOE)_1 = f_1(x_1,\ldots,x_4,x_6,x_7,x_9)$$

$$(MOE)_2 = f_2(x_1,\ldots,x_4,x_8)$$

$$(MOE)_3 = f_3(x_2,x_5)$$

$$(MOE)_4 = f_4(x_{17},\ldots,x_{24})$$

$$(MOE)_5 = f_5(x_{17},x_{19},\ldots,x_{29})$$

where

x_1 = probability that the submarine is detected in the advanced detection zone

x_2 = probability that the submarine is detected in the forward screen

x_3 = probability that the submarine survives an attack in the advanced detection zone

x_4 = probability that the submarine survives the duel in the forward screen

$= g_1(x_{10}, x_{12})$

x_5 = probability that the destroyer survives the duel in the forward screen

$= g_2(x_{10}, x_{11})$

x_6 = number of tubes in the submarine

x_7 = number of torpedoes expended while dueling with the destroyer

x_8 = probability that the submarine is sunk by a surface attack unit while leaving the convoy

x_9 = probability that a merchant ship is sunk by a single torpedo

$= g_3(x_{17}, x_{18}, x_{19})$

x_{10} = probability that both ships survive the forward screen duel

$= h_1(x_{13}, \ldots, x_{16})$

x_{11} = probability that the destroyer survives and the submarine is sunk in the forward screen duel

$= h_2(x_{13}, \ldots, x_{16})$

x_{12} = probability that the submarine survives and the destroyer is sunk in the forward screen duel

$= h_3(x_{13}, x_{14}, x_{15})$

x_{13} = probability that submarine is sunk by initial two-torpedo attack from destroyer

x_{14} = probability that destroyer sinks submarine after initial attack

x_{15} = probability that submarine sinks destroyer after initial attack

x_{16} = number of attacks by the destroyer in the forward screen duel

x_{17} = submarine torpedo kill probability, given a hit

x_{18} = probability that an aimed shot hits the target (merchant ship)

x_{19} = probability that the aimed shot misses but hits another merchant ship

$= h_4(x_{20}, x_{49})$

x_{20} = separation between merchant vessels

x_{21} = number of submarines which exist at the beginning of the month

x_{22} = expected number of merchant vessels sunk during a single submarine cycle

$= g_4(x_{30}, x_{31}, x_{32})$

x_{23} = probability of submarine attacking a convoy in Region A

$= g_5(x_{33}, \ldots, x_{36})$

x_{24} = probability of submarine attacking a convoy in Region B

$= g_6(x_{34}, x_{37})$

x_{25} = probability of submarine attacking a convoy in Region C

$= g_7(x_{35}, x_{36}, x_{37})$

x_{26} = cycle time of submarine assigned to Region A (specified geographical area)

x_{27} = cycle time of submarine assigned to Region B (specified geographical area)

x_{28} = cycle time of submarine assigned to Region C (specified geographical area)

x_{29} = probability of submarine being sunk in the course of a cycle

= $g_8(x_{31},x_{32})$

x_{30} = expected number of merchant vessels sunk during a single attack by a diesel submarine

= $f_1(x_1,\ldots,x_4,x_6,x_7,x_8)$

x_{31} = probability that a diesel submarine is sunk at some point during a single attack on a convoy

= $f_2(x_1,\ldots,x_4,x_8)$

x_{32} = probability that submarine is sunk in the barrier in a one-way transit

x_{33} = fraction of submarines assigned to Region A (specified geographical area)

x_{34} = fraction of submarines assigned to Region B (specified goegraphical area)

x_{35} = fraction of submarines assigned to Region C (specified geographical area)

x_{36} = probability of diversion in Region A

= $h_5(x_{38})$

x_{37} = probability of diversion in Region B

= $h_5(x_{38})$

x_{38} = probability that a convoy which passes a given submarine will be detected at some point from which the submarine can carry out an approach on the convoy

= $i(x_{39},\ldots,x_{43})$

x_{39} = width of uncertainty as to convoy route

x_{40} = detection and approach sweep width by the submarine on the convoy

= $j(x_{42},x_{44},\ldots,x_{49})$

x_{41} = submarine's back-and-forth patrol speed while on-station

x_{42} = convoy speed of advance

x_{43} = length of one leg of submarine's back-and-forth patrol

x_{44} = submarine detection range

x_{45} = submarine approach speed

x_{46} = submarine torpedo speed

x_{47} = convoy half-width

x_{48} = convoy half-length

x_{49} = torpedo range

3.1.5) Additional MOE's Identified:

(a) Expected number of merchant vessels sunk during a six-month period

(b) Expected number of merchant vessels sunk during a single attack by a nuclear submarine

(c) Probability that a nuclear submarine is sunk at some point during a single attack on a convoy

(d) Probability that a destroyer is sunk during a single attack on a convoy by a nuclear submarine

(e) Expected number of merchant vessels sunk by a nuclear submarine during one month

(f) Expected number of nuclear submarines sunk during one month

(g) Expected number of destroyers sunk by diesel submarines during one month

(h) Expected number of destroyers sunk by nuclear submarines during one month

3.2) Mission Type: Submarine search

3.2.1) Definition: A single destroyer attempts to detect enemy submarine operating in a specified search area.

3.2.2) Criterion For Success: Detection of submarine

3.2.3) MOE Selected: Effective sweep rate

3.2.3.1) Rationale For Selection: This MOE is selected so that the result of multiplying it by the destroyer's search time and the target density gives the expected number of targets detected during the period of search.

3.2.4) Functional Form Of MOE:

$$(MOE)_6 = f_6(x_{50},\ldots,x_{54})$$

where

x_{50} = destroyer active time

x_{51} = destroyer quiet (no pinging) time

x_{52} = destroyer detection range

x_{53} = distance destroyer travels during a complete cycle (active and quiet)

$= g_9(x_{50},x_{51},x_{55},x_{56})$

x_{54} = cumulative detection probability of the destroyer (the probability of at least one detection by the destroyer along the path)

$= g_{10}(x_{57},x_{58},x_{59})$

x_{55} = destroyer active speed

x_{56} = destroyer quiet speed

x_{57} = a vector whose i^{th} component represents the destroyer's detection probability for the i^{th} ping

$= h_7(x_{59},x_{60})$

x_{58} = a vector whose i^{th} component represents the probability of a jump in the Poisson process in the time period between the $(i-1)^{st}$ and i^{th} ping

$= h_8(x_{60})$

x_{59} = the median active single-ping destroyer detection range

x_{60} = total number of pings

4) MOE Usage In Study: The MOE's were formulated and used to determine the influence of destroyer silencing on mission effectiveness based on a postulated enemy threat and specified operational area of the world.

5) Special Study Assumptions:

(a) In the ASW escort/screen mission,

(a.1) A submarine always makes a cautious approach to the convoy (i.e., it does not initiate an engagement with a destroyer).

(a.2) Detection and identification of a screening destroyer by its radiated noise will enable the submarine to plan an evasive course during the penetration. An optimal tactic (within certain speed and course constraints) is employed for this purpose.

(a.3) In a given screen penetration, a diesel submarine will fire only the torpedoes in its tubes, and no opportunity is available for shots from a reload.

(a.4) When the submarines make their approach on a convoy, there will be occasions when they are detected and engage in a duel with the escort destroyers. It is assumed that in these circumstances they will fire acoustic homing torpedoes in salvos of two.

(a.5) There are three zones where the submarines may interact with the convoy's defenses: The advanced detection zone (ADZ), the forward screen, and the stern detection zone (SDZ). Whenever detection is made in the ADZ, a long range weapon will be fired. Because of the relatively few destroyers available per convoy, it is assumed that detection in the SDZ is not possible.

(a.6) A submarine which penetrates the forward screen successfully fires the torpedoes remaining in its tubes at the merchant vessels. After firing its torpedoes, the submarine attempts to exit from the convoy, at which time it is possible that it may be attacked by a surface attack unit.

(a.7) A submarine will duel with at most one destroyer in the forward screen.

(a.8) In the event a submarine is detected at the forward screen, the destroyer will commence an attack with two torpedoes. If the submarine survives this initial attack, then 3 minutes later both adversaries exchange weapons in salvos of 2. If both ships survive, then the submarine concentrates upon penetrating the protected body and does not return fire any further. The destroyer, however, continues to fire at a rate of 1 salvo of 2 torpedoes every 3 minutes until the submarine is sunk or attains the protected body. At this time the duel is discontinued, since if the submarine runs shallow, then the destroyer's homing torpedoes will pose a greater threat to the noisy merchant vessels than to the submarine.

(a.9) A specified percentage of submarines are being overhauled at any time.

(a.10) The rate of flows of convoys are uniformly spaced in time.

(b) In the ASW area search mission,

(b.1) The submarine tactic is to avoid detection, and in so doing traces out a relative path which is divided into three parts: The part which is traveled before the submarine is alerted to the presence of the destroyer; the part which is traveled from the time the submarine is alerted until the submarine is abeam of the destroyer; and the part which is traveled after the submarine passes abeam of the destroyer.

(b.2) Before the initial alert, the submarine is patrolling at top speed, in a random direction with respect to the destroyer's path.

(b.3) Upon being alerted to the presence of the destroyer, the submarine follows the best straight-line course for evasion.

(b.4) As the destroyer changes mode from active to quiet, (or quiet to active), the submarine's relative motion track changes angle and the relative speed changes.

(b.5) Once the submarine is abeam of the destroyer, the submarine tactic is to run directly away.

(b.6) The destroyer's detection range is treated as a stochastic process.

(b.7) The destroyer detection range is a lognormal distribution, which is equivalent to assuming that propagation loss follows a spreading law and signal excess has a normal distribution.

(b.8) The fluctuations in signal excess about the mean are described by a step process. The realization of such a process remains fixed until a jump occurs according to a Poisson distribution.

(b.9) The probability of detection is approximated by assuming that all relative paths have unimodal single-ping detection probabilities.

(b.10) The destroyer's detection range is greater than the maximum of the range at which the submarine may intercept the destroyer's pings, and the range at which the submarine may detect the destroyer (in active mode) by radiated noise.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated
2) Threat Composition: Diesel submarines

2.1) Quantitative Factors:

(a) number of submarines which exist at the beginning of the month

2.2) Sensor: Sonar

2.2.1) Quantitative Factor:

(a) submarine detection range

2.3) Armament: Torpedoes

2.3.1) Quantitative Factors:

(a) number of tubes in the submarine

(b) submarine torpedo kill probability, given a hit

(c) torpedo range

(d) submarine torpedo speed

2.4) Deployment:

2.4.1) Quantitative Factors:

(a) cycle time of submarine assigned to Region A (specified geographical area)

(b) cycle time of submarine assigned to Region B (specified geographical area)

(c) cycle time of submarine assigned to Region C (specified geographical area)

2.5) Tactics:

2.5.1) Qualitative Factor:

(a) See assumptions (a.1), (a.6), (a.7), (a.8), (b.1), (b.3), and (b.5)

2.5.2) Quantitative Factors:

(a) fraction of submarines assigned to Region A (specified geographical area)

(b) fraction of submarines assigned to Region B (specified geographical area)

(c) fraction of submarines assigned to Region C (specified geographical area)

(d) length of one leg of submarine's back-and-forth patrol

(e) submarine approach speed

(f) submarine back-and-forth patrol speed while on-station

3) Friendly Force Composition: Destroyers, merchant vessels and ASW barrier

3.1) Platform Type: Destroyer

3.1.1) Quantitative Factors:

(a) destroyer active speed

(b) destroyer quiet speed

3.1.2) Sensor: Sonar

3.1.2.1) Quantitative Factors:

(a) destroyer detection range

(b) the median active single-ping destroyer detection range

3.1.3) Armament: Torpedoes

3.1.4) Tactics:

3.1.4.1) Qualitative Factor:

(a) in the open search mission, search on a constant course using active sonar intermittently or continuously, which ever provides the best results

3.1.4.2) Quantitative Factors:

(a) destroyer active time

(b) destroyer quiet time

(c) total number of pings

3.2) Platform Type: Merchant vessels

3.2.1) Quantitative Factor:

(a) convoy speed of advance

3.2.2) Deployment:

3.2.2.1) Quantitative Factors:

(a) convoy half-width

(b) convoy half-length

(c) separation between merchant vessels

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform:

4.1.1) Type: Destroyer - Submarine

4.1.1.1) Quantitative Factors:

(a) number of torpedoes expended while dueling with the destroyer

(b) probability that submarine is sunk by initial two-torpedo attack from destroyer

(c) probability that destroyer sinks submarine after initial attack

(d) probability that submarine sinks destroyer after initial attack

(e) number of attacks by the destroyer in the forward screen duel

(f) probability that the submarine is sunk by a surface attack unit while leaving the convoy

4.1.2) Type: Merchant vessel - Submarine

4.1.2.1) Quantitative Factors:

(a) probability that an aimed shot hits the target (merchant ship)

(b) width of uncertainty as to convoy route

4.1.3) Type: ASW barrier - Submarine

4.1.3.1) Quantitative Factor:

(a) probability that submarine is sunk in the barrier in a one-way transit

4.2) Sensor - Platform: Sonar - Submarine

4.2.1) Quantitative Factors:

(a) probability that the submarine is detected in the advanced detection zone

(b) probability that the submarine is detected in the forward screen

4.3) Armament - Platform: Torpedo - Submarine

4.3.1) Quantitative Factor:

(a) probability that the submarine survives an attack in the advanced detection zone

STUDY REVIEW SUMMARY NO. (10)-11

A. STUDY DESCRIPTION

1) Originating Activity: Center for Naval Analyses, Arlington, Virginia
2) Report Title: Design of Antisubmarine Attack Models
3) Author: A. Herschaft
4) Report Number: OEG Study No. 690 (AD-363 555)
5) Date: 6 July 1965
6) Classification: Confidential
7) Contract: NONR 3732(00) (Office of Naval Research)
8) Abstract: A general discussion of the problems underlying the construction of antisubmarine attack models is presented. A broad spectrum of component errors and other factors which bear on the fire control problem are considered, and means for evaluating their contribution to the final probability of damage are provided. A fairly general attack model is developed for computing by Monte Carlo simulation the over-all probability of placing a weapon within a specified distance from the target. A description of the actual computer program and a comparison of several solutions of a typical problem are appended.
9) Descriptors: Antisubmarine warfare, contact prosecution, fire control, kill probability, Monte Carlo method, normal density function, sonar, submarine, tactics, tracking

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Mission: Contact prosecution
   - 3.1) Definition: An attack unit attacks an enemy submarine which has been detected and correctly classified.
   - 3.2) Criterion For Success: Destruction of submarine
   - 3.3) MOE Selected: Probability that submarine is damaged

3.4) Functional Form Of MOE:

$MOE = f(x_1, x_2, x_3)$

where

x_1 = probability density function of weapon impact point about the true target position

$= g_1(x_4, \ldots, x_{30})$

x_2 = conditional probability density function of submarine damage, given weapon impact point and true target position

x_3 = operational reliability of weapon system

x_4 = radial distance from attack unit to submarine's true position at the time attack is initiated

x_5 = bias error of radial distance from attack unit to submarine's true position at the time attack is initiated

x_6 = standard deviation of random error of radial distance from attack unit to submarine's true position at the time attack is initiated

x_7 = radius of turn which submarine executes at the beginning of its evasion maneuver

x_8 = bearing from attack unit to submarine's true position at the time attack is initiated

x_9 = bias error of bearing from attack unit to submarine's true position at the time attack is initiated

x_{10} = standard deviation of random error of bearing from attack unit to submarine's true position at the time attack is initiated

x_{11} = distribution of angle of turn which submarine executes at the beginning of its evasion maneuver

x_{12} = bias error of submarine's course (before its evasion maneuver)

x_{13} = standard deviation of random error of submarine's course (before its evasion maneuver)

x_{14} = true speed of the submarine between the time attack is initiated and time the submarine begins its evasion maneuver

x_{15} = true speed of the submarine during its evasion maneuver

x_{16} = true speed of the submarine between the time it ends its evasion maneuver and the time the weapon is activated

x_{17} = bias error of the true speed of the submarine between the time attack is initiated and time the submarine begins its evasion maneuver

x_{18} = standard deviation of random error of the true speed of the submarine between the time attack is initiated and time the submarine begins its evasion maneuver

x_{19} = distribution of time at which submarine starts evasion maneuver

x_{20} = estimated time of activation of the weapon

x_{21} = weapon aimpoint offset angle

x_{22} = weapon aimpoint offset distance

x_{23} = weapon placement bias range error

x_{24} = standard deviation of weapon placement random range error

x_{25} = weapon placement bias lateral displacement error

x_{26} = standard deviation of weapon placement random lateral displacement error

x_{27} = weapon placement bias bearing error

x_{28} = standard deviation of weapon placement random bearing error

x_{29} = distribution of depth of submarine when weapon is activated

x_{30} = velocity of the weapon

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) At time attack is initiated the submarine is heading on a straight course at speed x_{14}. After a time lapse the submarine executes a turn at speed x_{15} and continues on a tangential course at speed x_{16} until weapon is activated. Up to the beginning of the evasion maneuver, the target course and speed remain constant.

(b) The angle of turn which the submarine executes at the beginning of its evasion maneuver is assumed to be either normally distributed, uniformly distributed or a fixed value.

(c) The time at which the submarine begins its evasion maneuver is assumed to be either normally distributed, uniformly distributed or a fixed value.

(d) The depth of the submarine at the time the weapon is activated is assumed to be either normally distributed, uniformly distributed or a fixed value.

(e) All errors in weapon delivery and placement are random and distributed either normally or uniformly.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Tactics:

2.1.1) Quantitative Factors:

(a) radius of turn which submarine executes at the beginning of its evasion maneuver

(b) distribution of angle of turn which submarine executes at the beginning of its evasion maneuver

(c) bias error of submarine's course (before its evasion maneuver)

(d) standard deviation of random error of submarine's course (before its evasion maneuver)

(e) true speed of the submarine between the time attack is initiated and time the submarine begins its evasion maneuver

(f) true speed of the submarine during its evasion maneuver

(g) true speed of the submarine between the time it ends its evasion maneuver and the time the weapon is activated

(h) bias error of the true speed of the submarine between the time attack is initiated and time the submarine begins its evasion maneuver

(i) standard deviation of random error of the true speed of the submarine between the time attack is initiated and time the submarine begins its evasion maneuver

(j) distribution of time at which submarine starts evasion maneuver

3) Friendly Force Composition: Attack unit

3.1) Sensor: Sonar

3.2) Armament: Antisubmarine weapon

3.2.1) Quantitative Factors:

(a) operational reliability of weapon system

(b) estimated time of activation of the weapon

(c) weapon aimpoint offset angle

(d) weapon aimpoint offset distance

(e) weapon placement bias range error

(f) standard deviation of weapon placement random range error

(g) weapon placement bias lateral displacement error

(h) standard deviation of weapon placement random lateral displacement error

(i) weapon placement bias bearing error

(j) standard deviation of weapon placement random bearing error

(k) velocity of the weapon

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform: Attack unit - Submarine

4.1.1) Quantitative Factors:

(a) radial distance from attack unit to submarine's true position at the time attack is initiated

(b) bearing from attack unit to submarine's true position at the time attack is initiated

4.2) Sensor - Platform: Sonar - Submarine

4.2.1) Quantitative Factors:

(a) bias error of radial distance from attack unit to submarine's true position at the time attack is initiated

(b) standard deviation of random error of radial distance from attack unit to submarine's true position at the time attack is initiated

(c) bias error of bearing from attack unit to submarine's true position at the time attack is initiated

(d) standard deviation of random error of bearing from attack unit to submarine's true position at the time attack is initiated

4.3) Armament - Platform: Antisubmarine weapon - Submarine

4.3.1) Quantitative Factors:

(a) conditional probability density function of submarine damage, given weapon impact point and true target position

(b) distribution of depth of submarine when weapon is activated

STUDY REVIEW SUMMARY NO. (10)-12

A. <u>STUDY DESCRIPTION</u>

1) Originating Activity: General Precision, Inc., Librascope Group, Glendale, California
2) Report Title: <u>Target Motion Analysis and System Effectiveness</u>
3) Report Number: (AD-518 652)
4) Date: 16 December 1964
5) Classification: Confidential
6) Contract: N140(122)767053 (U.S. Naval Underwater Ordnance Station)
7) Abstract: This report describes a study of a sonar-fire control-weapon complex. The primary objective is to demonstrate the effectiveness of certain existing ASW systems and to evaluate improvements in future systems. An area of emphasis is fire control and target motion analysis. Studies are conducted to evaluate existing target motion analysis methods used in Fire Control Systems Mk 101 and Mk 112. Evaluations are made in terms of target motion analysis requirements and corresponding weapon effectiveness. The system effectiveness is determined using kill or acquisition probability as a measure. All important known sources of system degradation and error have been included in the math model whether originating in the weapon, the sonar or the fire control system. The math model for representing acquisition and kill probability is based on a first order analysis of all system errors. This model has been programmed for digital computer simulation.
8) Descriptors: Antisubmarine warfare, computer, contact prosecution, fire control, kill probability, navigation, normal density function, sonar, submarine, surface ship, target acquisition, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Mission: Contact prosecution
   3.1) Definition: An ASW fire control computer receives target information from a sonar and then transmits aiming orders to a weapon. The weapon is then fired at a submerged target.
   3.2) Criterion For Success: Destruction of submarine
   3.3) MOE's Selected:
   $(MOE)_1$ = Probability of target kill
   $(MOE)_2$ = Probability of target acquisition
   3.4) Functional Form Of MOE's:
   $(MOE)_1 = f_1(x_1, x_2)$
   $(MOE)_2 = f_2(x_1, x_8, \ldots, x_{11})$

where

x_1 = joint probability density function of the horizontal and vertical displacement of weapon relative to target at weapon impact
$= g_1(x_3, x_4, x_5)$

x_2 = weapon radius of destruction

x_3 = variance of the horizontal displacement of weapon relative to target at weapon impact
$= h_1(x_6)$

x_4 = variance of the vertical displacement of weapon relative to target at weapon impact
$= h_2(x_7)$

x_5 = covariance of the horizontal and vertical displacement of weapon relative to target at weapon impact
$= h_3(x_3, x_4, x_6; x_7)$

x_6 = mean value of the horizontal displacement of weapon relative to target at weapon impact

$= i_1(x_{12}, x_{13})$

x_7 = mean value of the vertical displacement of weapon relative to target at weapon impact

$= i_2(x_{14}, x_{15})$

x_8 = positive horizontal displacement of detection swath about the expected weapon intercept point

$= g_2(x_9, x_{17}, x_{18}, x_{26}, x_{39}, x_{40})$

x_9 = negative horizontal displacement of detection swath about the expected weapon intercept point

$= g_3(x_{17}, x_{18}, x_{26}, x_{39}, x_{40})$

x_{10} = positive vertical displacement of detection swath about the expected weapon intercept point

$= g_4(x_9, x_{17}, x_{22}, x_{34}, x_{38}, x_{41}, x_{42})$

x_{11} = negative vertical displacement of detection swath about the expected weapon intercept point

$= g_5(x_9, x_{17}, x_{22}, x_{34}, x_{38}, x_{41}, x_{43}, x_{44})$

x_{12} = horizontal weapon error

$$
= \begin{cases}
j_1(x_{16}, \ldots, x_{21}) & \text{for preset weapon guidance and burst detonation} \\
j_2(x_{16}, \ldots, x_{27}) & \text{for preset weapon guidance and acoustic search} \\
j_3(x_{16}, \ldots, x_{21}, x_{29}, \ldots, x_{33}, x_{37}) & \text{for wire guided weapon and burst detonation} \\
j_4(x_{16}, \ldots, x_{27}, x_{29}, \ldots, x_{33}, x_{37}) & \text{for wire guided weapon and acoustic search}
\end{cases}
$$

$$x_{13} = \text{horizontal target error}$$

$$= \begin{cases} j_5(x_{17}, x_{27}, x_{34}, x_{37}, x_{38}, x_{47}, x_{52}, x_{63}) \\ \quad \text{for preset weapon guidance and burst detonation} \\ j_6(x_{17}, x_{27}, x_{34}, x_{37}, x_{38}, x_{47}, x_{52}, x_{54}, x_{68}) \\ \quad \text{for preset weapon guidance and acoustic search} \\ j_7(x_{17}, x_{50}, x_{53}, x_{54}, x_{57}, x_{68}, x_{69}) \\ \quad \text{for wire guided weapon and burst detonation} \\ j_8(x_{17}, x_{50}, x_{53}, x_{54}, x_{57}, x_{68}, x_{69}) \\ \quad \text{for wire guided weapon and acoustic search} \end{cases}$$

$$x_{14} = \text{vertical weapon error}$$

$$= \begin{cases} j_9(x_{16}, \ldots, x_{21}) \\ \quad \text{for preset weapon guidance and burst detonation} \\ j_{10}(x_{16}, \ldots, x_{27}) \\ \quad \text{for preset weapon guidance and acoustic search} \\ j_{11}(x_{16}, \ldots, x_{21}, x_{29}, \ldots, x_{33}, x_{57}) \\ \quad \text{for wire guided weapon and burst detonation} \\ j_{12}(x_{16}, \ldots, x_{27}, x_{29}, \ldots, x_{33}, x_{57}) \\ \quad \text{for wire guided weapon and acoustic search} \end{cases}$$

$$x_{15} = \text{vertical target error}$$

$$= \begin{cases} j_{13}(x_{17}, x_{27}, x_{34}, x_{37}, x_{38}, x_{47}, x_{52}, x_{63}) \\ \quad \text{for preset weapon guidance and burst detection} \\ j_{14}(x_{17}, x_{27}, x_{34}, x_{37}, x_{38}, x_{47}, x_{52}, x_{54}, x_{68}) \\ \quad \text{for preset weapon guidance and acoustic search} \\ j_{15}(x_{17}, x_{50}, x_{53}, x_{54}, x_{57}, x_{68}, x_{69}) \\ \quad \text{for wire guided weapon and burst detonation} \\ j_{16}(x_{17}, x_{50}, x_{53}, x_{54}, x_{57}, x_{68}, x_{69}) \\ \quad \text{for wire guided weapon and acoustic search} \end{cases}$$

x_{16} = weapon run time (time interval from firing to enable or detonation)

$= k_1(x_{18}, x_{21}, x_{24}, x_{26}, x_{34}, x_{38}, x_{44}, x_{47})$

x_{17} = angle of relative motion of weapon and target

$= k_2(x_{18}, x_{22}, x_{34}, x_{38})$

x_{18} = deflection angle (angle which the weapon path makes with the range line)

$= k_3(x_{28}, x_{30}, x_{35}, x_{36}, x_{47}, x_{50}, x_{57}, x_{60})$

x_{19} = sum of errors in gyro bias, angle solver and transmission of gyro angle

x_{20} = weapon run speed

x_{21} = weapon dispersion due to error in run speed from the nominal

x_{22} = weapon search speed

x_{23} = weapon dispersion due to error in search speed from the nominal

x_{24} = nominal search time (time interval from enable to expected intercept)

$= k_4(x_{22}, x_{44})$

x_{25} = weapon gyro drift error

x_{26} = laminar distance for weapon with acoustic search capability

x_{27} = nominal time of weapon flight from firing to expected intercept

$= k_5(x_{16}, x_{24})$

x_{28} = own ship speed

x_{29} = error in own ship speed

x_{30} = own ship course

x_{31} = weapon displacement along the track

x_{32} = weapon horizontal velocity error

x_{33} = weapon vertical velocity error

x_{34} = estimated target speed

x_{35} = error in estimating target speed

$$= \begin{cases} k_6(x_{29}, x_{30}, x_{34}, x_{36}, x_{45}, x_{47}, x_{51}, x_{52}, x_{57}, x_{61}, x_{63}, x_{82}, x_{84}) & \text{for preset weapon using endpoint target motion analysis} \\ k_7(x_{29}, x_{30}, x_{34}, x_{36}, x_{45}, x_{47}, x_{51}, x_{52}, x_{57}, x_{61}, x_{63}, x_{82}, x_{84}) & \text{for preset weapon using range rate target motion analysis} \end{cases}$$

x_{36} = estimated target course

x_{37} = error in estimating target course

$$= \begin{cases} k_8(x_{29}, x_{30}, x_{34}, x_{36}, x_{45}, x_{46}, x_{47}, x_{52}, x_{55}, x_{56}, x_{57}, x_{61}, x_{62}, x_{63}, x_{82}, x_{83}, x_{84}) & \text{for preset weapon using three-bearing-with-speed-constraint target motion analysis} \\ k_9(x_{29}, x_{30}, x_{34}, x_{36}, x_{45}, x_{47}, x_{52}, x_{55}, x_{57}, x_{61}, x_{63}, x_{82}, x_{84}) & \text{for preset weapon using endpoint target motion analysis} \\ k_{10}(x_{29}, x_{30}, x_{34}, x_{36}, x_{45}, x_{47}, x_{52}, x_{55}, x_{57}, x_{61}, x_{63}, x_{82}, x_{84}) & \text{for preset weapon using range rate target motion analysis} \\ k_{11}(x_{29}, x_{30}, x_{34}, x_{36}, x_{45}, x_{46}, x_{47}, x_{52}, x_{55}, x_{56}, x_{61}, x_{62}, x_{82}, x_{83}, x_{84}) & \text{for wire guided weapon using three-bearing-with-speed-constraint target motion analysis} \end{cases}$$

x_{38} = target angle or angle on the bow at time weapon is fired
= $k_{12}(x_{28}, x_{30}, x_{35}, x_{36}, x_{47}, x_{50}, x_{57}, x_{60})$

x_{39} = radius of laminar region

x_{40} = enclosed angle of laminar region

x_{41} = length of travel of laminar point along weapon path after nominal intercept

x_{42} = positive "squaring off" length

x_{43} = negative "squaring off" length

x_{44} = enabling run offset

x_{45} = range from own ship to target at reference time 1
= $l_1(x_{28}, x_{30}, x_{34}, x_{36}, x_{82})$

x_{46} = range from own ship to target at reference time 2
= $l_2(x_{28}, x_{30}, x_{34}, x_{36}, x_{83})$

x_{47} = range from own ship to target at reference time 3
= $l_3(x_{28}, x_{30}, x_{34}, x_{36}, x_{84})$

x_{48} = range from own ship to target at reference time 4
= $l_4(x_{28}, x_{30}, x_{34}, x_{36}, x_{85})$

x_{49} = range from own ship to target at reference time 5
= $l_5(x_{28}, x_{30}, x_{34}, x_{36}, x_{86})$

x_{50} = range from own ship to target at reference time 6
= $l_6(x_{28}, x_{30}, x_{34}, x_{36}, x_{87})$

x_{51} = random error in range at reference time 1

x_{52} = random error in range at time weapon is fired

$$= \begin{cases} l_7(x_{29}, x_{30}, x_{34}, x_{36}, x_{45}, x_{46}, x_{47}, x_{55}, x_{56}, \\ \quad x_{57}, x_{61}, x_{62}, x_{63}, x_{82}, x_{83}, x_{84}) \\ \quad \text{for preset weapon using three-bearing-with-speed-} \\ \quad \text{constraint target motion analysis} \\ x_{88} \quad \text{otherwise} \end{cases}$$

x_{53} = random error in range at nominal intercept

= $l_8(x_{30}, x_{36}, x_{48}, x_{49}, x_{50}, x_{58}, x_{59}, x_{60}, x_{64}, x_{65}, x_{66}, x_{85}, x_{86}, x_{87})$

x_{54} = bias error in estimating range

x_{55} = bearing from own ship to target at reference time 1

= $l_9(x_{70}, x_{76})$

x_{56} = bearing from own ship to target at reference time 2

= $l_{10}(x_{71}, x_{77})$

x_{57} = bearing from own ship to target at reference time 3

= $l_{11}(x_{72}, x_{78})$

x_{58} = bearing from own ship to target at reference time 4

= $l_{12}(x_{73}, x_{79})$

x_{59} = bearing from own ship to target at reference time 5

= $l_{13}(x_{74}, x_{80})$

x_{60} = bearing from own ship to target at reference time 6

= $l_{14}(x_{75}, x_{81})$

x_{61} = own ship to target bearing error at reference time 1

= $l_{15}(x_{67}, x_{68}, x_{69})$

x_{62} = own ship to target bearing error at reference time 2

= $l_{16}(x_{67}, x_{68}, x_{69})$

x_{63} = own ship to target bearing error at reference time 3

= $l_{17}(x_{67}, x_{68}, x_{69})$

x_{64} = own ship to target bearing error at reference time 4

= $l_{18}(x_{67}, x_{68}, x_{69})$

x_{65} = own ship to target bearing error at reference time 5

= $l_{19}(x_{67}, x_{68}, x_{69})$

x_{66} = own ship to target bearing error at reference time 6

= $l_{20}(x_{67}, x_{68}, x_{69})$

x_{67} = bias error in relative bearing
x_{68} = random error in relative bearing
x_{69} = random error in gyrocompass
x_{70} = own ship course as measured by gyrocompass at reference time 1
x_{71} = own ship course as measured by gyrocompass at reference time 2
x_{72} = own ship course as measured by gyrocompass at reference time 3
x_{73} = own ship course as measured by gyrocompass at reference time 4
x_{74} = own ship course as measured by gyrocompass at reference time 5
x_{75} = own ship course as measured by gyrocompass at reference time 6
x_{76} = input from own ship's dead reckoner at reference time 1
x_{77} = input from own ship's dead reckoner at reference time 2
x_{78} = input from own ship's dead reckoner at reference time 3
x_{79} = input from own ship's dead reckoner at reference time 4
x_{80} = input from own ship's dead reckoner at reference time 5
x_{81} = input from own ship's dead reckoner at reference time 6
x_{82} = time first bearing data obtained (reference time one)
x_{83} = time second bearing data obtained (reference time two)
x_{84} = time third bearing data obtained (reference time three and weapon firing time)
x_{85} = time fourth bearing data obtained (reference time four)
x_{86} = time fifth bearing data obtained (reference time five)
x_{87} = time sixth bearing data obtained (reference time six and nominal intercept time)
x_{88} = random error in range at time weapon is fired

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) The math model for representing acquisition and kill probability is based on a first order analysis of all system errors.

(b) Own ship and target are assumed to travel straight line, constant speed paths.

(c) The weapon is fired on an intercept path.

(d) Depth effects are not represented directly.

(e) The relative displacement of weapon and target at weapon enable or detonation is bivariate normally distributed.

(f) Errors which are inputs to target motion analysis from own ship dead reckoning are expected to be small. The only error included is that which arises from a bias error in measuring own ship speed.

(g) For preset weapons,

(g.1) If the target falls within radius of destruction, the kill probability is unity. If the target falls outside the radius of destruction, the probability of kill is zero.

(g.2) Errors in target course and speed are not considered in the calculation of kill probability.

(g.3) Own ship dead reckoner and target position keeper errors are not treated, since errors originating within own ship dead reckoner are small and target position keeper errors are periodically corrected by sonar data inputs and will not degrade kill probability significantly.

(h) For wire guided weapons,

(h.1) Weapon speed change at enable is provided for the aiming equation.

(h.2) Laminar distance is provided for in the aiming equation.

(h.3) The turn radius of the weapon is assumed small compared to the target range.

(h.4) The weapon is initially fired on an intercept course and is steered such that it will always be on an intercept course. Since there is no target maneuver, the weapon traverses a nominal preset path to intercept.

(h.5) Bias error in own ship course has no over-all effect on system accuracy since it is the same as a shift in the vertical reference direction. Thus, only the random part of this error is included.

(h.6) The detection swath is approximated by a rectangular region.

C. EFFECTIVENESS MEASUREMENT

1) Physical Environment: Not stated
2) Threat Composition: Submarine
3) Friendly Force Composition: Surface ship
   - 3.1) Quantitative Factors:
     - (a) own ship speed
     - (b) error in own ship speed
   - 3.2) Sensor: Sonar
     - 3.2.1) Quantitative Factors:
       - (a) random error in range at reference time 1
       - (b) bias error in estimating range
       - (c) bias error in relative bearing
       - (d) random error in relative bearing
       - (e) random error in range at time weapon is fired
   - 3.3) Fire control computer: Target analyzer, angle solver and torpedo dead reckoner
     - 3.3.1) Quantitative Factors:
       - (a) sum of errors in gyro bias, angle solver and transmission of gyro angle
       - (b) weapon displacement along the track
       - (c) weapon horizontal velocity error
       - (d) weapon vertical velocity error
   - 3.4) Navigation: Gyrocompass, EM-Log and ship dead reckoner
     - 3.4.1) Quantitative Factors:
       - (a) random error in gyrocompass

(b) own ship course as measured by gyrocompass at reference time 1

(c) own ship course as measured by gyrocompass at reference time 2

(d) own ship course as measured by gyrocompass at reference time 3

(e) own ship course as measured by gyrocompass at reference time 4

(f) own ship course as measured by gyrocompass at reference time 5

(g) own ship course as measured by gyrocompass at reference time 6

(h) input from own ship's dead reckoner at reference time 1

(i) input from own ship's dead reckoner at reference time 2

(j) input from own ship's dead reckoner at reference time 3

(k) input from own ship's dead reckoner at reference time 4

(l) input from own ship's dead reckoner at reference time 5

(m) input from own ship's dead reckoner at reference time 6

3.5) Armament: Torpedoes

3.5.1) Quantitative Factors:

(a) weapon radius of destruction

(b) weapon run speed

(c) weapon dispersion due to error in run speed from the nominal

(d) weapon gyro drift error

(e) laminar distance for weapon with acoustic search capability

(f) radius of laminar region
(g) enclosed angle of laminar region
(h) length of travel of laminar point along weapon path after nominal intercept
(i) positive "squaring off" length
(j) negative "squaring off" length
(k) enabling run offset

3.5.2) Tactics:

3.5.2.1) Quantitative Factors:

(a) weapon search speed
(b) weapon dispersion due to error in search speed from the nominal

3.6) Deployment:

3.6.1) Quantitative Factor:

(a) own ship course

4) Friendly Force - Threat Interaction:

4.1) Sensor - Platform: Sonar - Submarine

4.1.1) Quantitative Factors:

(a) estimated target speed
(b) estimated target course
(c) time first bearing data obtained (reference time one)
(d) time second bearing data obtained (reference time two)
(e) time third bearing data obtained (reference time three and weapon firing time)
(f) time fourth bearing data obtained (reference time four)
(g) time fifth bearing data obtained (reference time five)
(h) time sixth bearing data obtained (reference time six and nominal intercept time)

STUDY REVIEW SUMMARY NO. (10)-13

A. STUDY DESCRIPTION

1) Originating Activity: Presearch Incorporated, Silver Spring, Maryland
2) Report Title: Value of Acoustic Countermeasures Employed by ASW Escorts Against Submarine Sonars
3) Authors: J.R. Penny and R.F. Waddey
4) Report Number: Technical Report No. 174 (AD-507 753)
5) Date: 15 February 1970
6) Classification: Secret (NOFORN)
7) Contract: N00024-69-C-1299 (Naval Ship Systems Command)
8) Abstract: This report summarizes the results of an analytical study directed toward determining the military value of equipping escort ships with rocket launched NAE beacons for sonar countermeasure purposes. Estimates are made of the performance requirements for such beacons to be useful in various tactical siutations.
9) Descriptors: Acoustic decoy, antisubmarine warfare, carrier, convoy defense, convoy escort, countermeasure, detection, escort ship, Monte Carlo method, screen, sonar, submarine, submarine attack, surface ship, tactics, torpedo

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Tactical Situation: Escort versus Submarine
   3.1) Definition: An escort ship in a carrier screen gains contact with a submarine and then launches one or more sonar countermeasures beacons.
   3.2) Criterion For Success: Denial of tracking information

3.3) MOE Selected: Time from countermeasures activation until tracking information is regained

3.3.1) Rationale For Selection. This provides a measure of the time to be allowed for evasion and maneuvering in order to avoid being attacked.

3.4) Functional Form Of MOE:

$$MOE = \begin{cases} f_1(x_1, x_2) & \text{if submarine attacks the escort} \\ f_2(x_1, x_{34}) & \text{if submarine attacks the carrier} \end{cases}$$

where

x_1 = a vector whose $i^{\underline{th}}$ component represents the $i^{\underline{th}}$ time increment

x_2 = a vector whose $i^{\underline{th}}$ component represents the submarine sonar signal excess at the $i^{\underline{th}}$ time increment
= $g_1(x_3, \ldots, x_{16})$

x_3 = source level of escort
= $h_1(x_{17}, x_{18})$

x_4 = source level of left countermeasure beacon

x_5 = source level of right countermeasure beacon

x_6 = source level of center countermeasure beacon

x_7 = submarine self noise

x_8 = submarine sonar receiving directivity index

x_9 = submarine sonar 50 percent probability of detection range

x_{10} = a vector whose $i^{\underline{th}}$ component represents the transmission loss to the escort at the $i^{\underline{th}}$ time increment
= $h_2(x_{19}, x_{20})$

x_{11} = a vector whose $i^{\underline{th}}$ component represents the transmission loss to the left countermeasure beacon at the $i^{\underline{th}}$ time increment
= $h_3(x_{19}, x_{21}, x_{33})$

x_{12} = a vector whose i^{th} component represents the transmission loss to the right countermeasure beacon at the i^{th} time increment

= $h_4(x_{19}, x_{22}, x_{33})$

x_{13} = a vector whose i^{th} component represents the transmission. loss to the center countermeasure beacon at the i^{th} time increment

= $h_5(x_{19}, x_{23}, x_{33})$

x_{14} = a vector whose i^{th} component represents the beam pattern correction to the left countermeasure beacon at the i^{th} time increment

= $h_6(x_{24}, x_{33})$

x_{15} = a vector whose i^{th} component represents the beam pattern correction to the right countermeasure beacon at the i^{th} time increment

= $h_7(x_{25}, x_{33})$

x_{16} = a vector whose i^{th} component represents the beam pattern correction to the center countermeasure beacon at the i^{th} time increment

= $h_8(x_{26}, x_{33})$

x_{17} = escort length

x_{18} = escort speed

x_{19} = transmission loss factor

x_{20} = a vector whose i^{th} component represents the range from submarine to escort at the i^{th} time increment

= $i_1(x_1, x_{18}, x_{28}, \ldots, x_{32})$.

x_{21} = a vector whose i^{th} component represents the range from submarine to left countermeasure beacon at the i^{th} time increment

= $i_2(x_1, x_{27}, x_{28}, x_{30}, x_{31}, x_{32})$

x_{22} = a vector whose i^{th} component represents the range from submarine to right countermeasure beacon at the i^{th} time increment

$= i_3(x_1, x_{27}, x_{28}, x_{30}, x_{31}, x_{32})$

x_{23} = a vector whose i^{th} component represents the range from submarine to center countermeasure beacon at the i^{th} time increment

$= i_4(x_1, x_{28}, x_{30}, x_{31}, x_{32})$

x_{24} = a vector whose i^{th} component represents the bearing from the submarine track to the left countermeasure beacon at the i^{th} time increment

$= i_5(x_1, x_{27}, x_{28}, x_{30}, x_{31}, x_{32})$

x_{25} = a vector whose i^{th} component represents the bearing from the submarine track to the right countermeasure beacon at the i^{th} time increment

$= i_6(x_1, x_{27}, x_{28}, x_{30}, x_{31}, x_{32})$

x_{26} = a vector whose i^{th} component represents the bearing from the submarine track to the right countermeasure beacon at the i^{th} time increment

$= i_7(x_1, x_{28}, x_{30}, x_{31}, x_{32})$

x_{27} = countermeasure beacon spacing

x_{28} = submarine speed

x_{29} = escort initial turn angle

x_{30} = submarine lead angle

x_{31} = initial range from escort to submarine

x_{32} = initial submarine bearing with respect to escort heading

x_{33} = countermeasure beacon endurance

x_{34} = a vector whose i^{th} component represents the submarine sonar signal excess at the i^{th} time increment

$= g_2(x_4, \ldots, x_9, x_{11}, \ldots, x_{16}, x_{35}, x_{36})$

x_{35} = source level of CVA

= $h_9(x_{37}, x_{38})$

x_{36} = a vector whose $i^{\underline{th}}$ component represents the transmission loss to the CVA at the $i^{\underline{th}}$ time increment

= $h_{10}(x_{19}, x_{39})$

x_{37} = CVA length

x_{38} = CVA speed

x_{39} = a vector whose $i^{\underline{th}}$ component represents the range from submarine to CVA at the $i^{\underline{th}}$ time increment

= $i_8(x_1, x_{28}, x_{30}, x_{31}, x_{32}, x_{40}, \ldots, x_{43})$

x_{40} = initial range from CVA to submarine

x_{41} = CVA initial turn angle

x_{42} = CVA second turn angle

x_{43} = time CVA executes second turn

4) MOE Usage In Study: The MOE was formulated and used as an input to an Escort/Submarine Duel Model which utilizes a Monte-Carlo technique to calculate the ratio of submarines hit to escorts hit for various tactical and equipment situations. This ratio is then used to estimate the value of sonar countermeasures.

5) Special Study Assumptions:

(a) For the case of submarine attack on the escort,

(a.1) The submarine is assumed to have detected the escort ship and has begun closing at relatively narrow bow angles. At some range the escort gains contact with the submarine, launches one or more sonar countermeasures and begins a maneuver.

(a.2) The escort drops either a single countermeasure astern or a countermeasure astern plus two additional countermeasures launched perpendicular to the ship's track at a specified distance.

(a.3) After countermeasure launch the escort either continues on his base course; turns 45 degrees to the right or left from his base course.

(b) For the case of submarine attack on the carrier,

(b.1) The maintain course tactic is not considered, and an additional 90 degree turn is added to the 45 degree evasion tactic to roughly duplicate the advance and transfer of a 135 degree turn to totally avoid the contact area.

(b.2) It is assumed that a screen ship has detected the submarine and deployed countermeasures (one or three beacons); however, the escort itself is not considered in this case.

(c) After the submarine detects the countermeasure deployment, he is assumed to continue on his original course.

(d) The 50 percent detection range is used as a "cookie cutter" range and thus a positive signal excess is taken as detection.

(e) The model implements only very simple straight line motions of the submarine and target. Thus, turns are taken to be instantaneous, and, where this assumption could introduce significant error, the motion must be broken into several straight line segments.

C. EFFECTIVENESS FACTORS

1) Physical Environment:

1.1) Quantitative Factor:

(a) transmission loss factor

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) submarine self noise

(b) submarine speed

2.2) Sensor: Sonar

2.2.1) Quantitative Factors:

(a) submarine sonar receiving directivity index

(b) submarine sonar 50 percent probability of detection range

2.3) Tactics:

2.3.1) Qualitative Factors:

(a) constant speed

(b) fixed course

2.3.2) Quantitative Factor:

(a) submarine lead angle

3) Friendly Force Composition: CVA, escort and sonar countermeasure beacons

3.1) Platform Type: CVA

3.1.1) Quantitative Factors:

(a) CVA length

(b) CVA speed

3.1.2) Tactics:

3.1.2.1) Qualitative Factors:

(a) constant speed

(b) executes two evasion turns

3.1.2.2) Quantitative Factors:

(a) CVA initial turn angle

(b) CVA second turn angle

(c) time CVA executes second turn

3.2) Platform Type: Escort

3.2.1) Quantitative Factors:

(a) escort length

(b) escort speed

3.2.2) Tactics:

3.2.2.1) Qualitative Factors:

(a) constant speed

(b) continues on course or executes 45 degree right or left turn

3.2.2.2) Quantitative Factor:

(a) escort initial turn angle

3.3) Platform Type: Sonar countermeasure beacon

3.3.1) Quantitative Factors:

(a) source level of left countermeasure beacon

(b) source level of right countermeasure beacon

(c) source level of center countermeasure beacon

(d) countermeasure beacon endurance

3.3.2) Deployment:

3.3.2.1) Quantitative Factor:

(a) countermeasure beacon spacing

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform:

4.1.1) Type: CVA - Submarine

4.1.1.1) Quantitative Factor:

(a) initial range from CVA to submarine

4.1.2) Type: Escort - Submarine

4.1.2.1) Quantitative Factors:

(a) initial range from escort to submarine

(b) initial submarine bearing with respect to escort heading

4.1.3) Type: CVA and Escort - Submarine

4.1.3.1) Quantitative Factor:

(a) a vector whose $i^{\underline{th}}$ component represents the $i^{\underline{th}}$ time increment

STUDY REVIEW SUMMARY NO. (10)-14

A. STUDY DESCRIPTION

1) Originating Activity: Commander, Destroyer Development Group, U.S. Atlantic Fleet
2) Report Title: Single Ship Search Tactic
3) Authors: F.M. Bomse and W.L. May
4) Report Number: Technical Report No. 5-72 (AD-519 666)
5) Date: 14 March 1972
6) Classification: Confidential
7) Abstract: The tactic discussed in this report is concerned with an optimum search speed to be utilized by a single destroyer conducting a short time late search against a high speed submarine. The basic tenet underlying this tactic is the optimum compromise between the range rate which exists between the destroyer and the evading submarine and the effect of destroyer speed on sonar performance.
8) Descriptors: Aircraft, antisubmarine warfare, contact investigation, destroyer, detection, Lofar, sonar, submarine

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: System
2) Function: Surface ASW
3) Mission: Contact investigation
    3.1) Definition: A single destroyer searches in the vicinity of the point of last contact for a submarine contact which has been momentarily lost.
    3.2) Criterion For Success: Detection of submarine
    3.3) MOE Selected: Maximum exposure time of the submarine
        3.3.1) Rationale For Selection: The longer the exposure time, the greater is the number of pings available for

recognition of the target; thus, the greater the chance is for detection.

3.4) Functional Form Of MOE:

$$MOE = \max_{x_1} f(x_1, \ldots, x_5)$$

where

$f(x_1, \ldots, x_4)$ = exposure time of the submarine for destroyer search speed x_1

x_1 = destroyer search speed

x_2 = submarine speed

x_3 = submarine bearing from search axis during evasion

x_4 = submarine's initial position relative to destroyer

x_5 = maximum detection range of the destroyer sonar
= $g(x_6)$

x_6 = sonar figure of merit
= $h(x_1)$

4) MOE Usage In Study: Formulation and numerical examples

5) Special Study Assumptions:

(a) The destroyer's course during the search is directed along the line connecting the destroyer's position at the initiation of the tactic and the datum point.

(b) The search plan is predicated on the availability of a single sonar equipped ship which is part of a larger ASW force. At the time of initiation of the search tactic, no other surface unit is available for aiding in the search, although an aircraft may be available.

(c) At the time the tactic is initiated, the target may be located off the search axis.

(d) The target is aware of the presence of the surface ship and evades at constant speed and direction in such a manner that it always opens the ship.

(e) The evasion speed of the target exceeds the maximum possible surface ship search speed.

(f) A logarithmic transmission loss law is assumed for destroyer sonar.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

   2.1) Quantitative Factor:

      (a) submarine speed

3) Friendly Force Composition: Destroyer

4) Friendly Force - Threat Interaction:

   4.1) Platform - Platform: Destroyer - Submarine

      4.1.1) Quantitative Factors:

         (a) submarine bearing from search axis during evasion

         (b) submarine's initial position relative to destroyer

STUDY REVIEW SUMMARY NO. (10)-15

A. STUDY DESCRIPTION

1) Originating Activity: Bell Aerosystems Company, Buffalo, New York
2) Report Title: Applications of the Surface Effect Vehicle to Anti-Submarine Warfare Missions, Volume II Mission Analysis, Final Report
3) Report Number: 7353-95002 (AD-505 481)
4) Date: July 1969
5) Classification: Secret
6) Contract: DAHC15-69-C-0231 (Advanced Research Projects Agency)
7) Abstract: The purpose of the study was to determine the possible applications of Surface Effect Vehicles (SEV) in antisubmarine warfare (ASW). This volume contains the scenario development and mission analysis of five basic ASW missions; naval task force screening, submarine trailing, open area search, contact prosecution and barriers. The basis for requirements of a family of vehicles is developed.
8) Descriptors: Antisubmarine warfare, convoy defense, dipping sonar, screen, search, sonobuoy, SOSUS, surface effect vehicle, trailing, undersea surveillance

B. EFFECTIVENESS MEASUREMENT

1) Evaluation Level: Force and System
2) Function: Surface ASW
3) Missions:
   3.1) Mission Type: Escort/screen
      3.1.1) Definition: SEV's are to protect screened units (carriers, convoy or amphibious forces ships) from attack by enemy submarines.

3.1.2) Criterion For Success: Prevention of submarine interception of screened units

3.1.3) MOE's Selected:

Case 1 - SEV screening using retrievable sonar buoys in fore and aft shuttle screening mode

$(MOE)_1$ = Minimum effective SEV speed in retrieving sonar buoys

$(MOE)_2$ = Minimum effective SEV speed in laying sonar buoys

Case 2 - SEV screening using hull-attached deep-dipped sonar systems

$(MOE)_3$ = Minimum effective SEV speed

3.1.4) Functional Form Of MOE's:

Case 1

$$(MOE)_1 = f_1(x_1, x_2)$$

$$(MOE)_2 = f_2(x_3, x_4)$$

where

x_1 = retrieval range of SEV per cycle
$= g_1(x_5, x_6, x_7)$

x_2 = retrieval time per cycle in which SEV is cruising
$= g_2(x_5, x_8, x_9, x_{10})$

x_3 = laying range of SEV per cycle
$= g_3(x_5, x_6, x_7)$

x_4 = laying time per cycle in which SEV is cruising
$= g_4(x_8, x_9)$

x_5 = number of buoys laid (retrieved) by each SEV during one complete cycle

x_6 = buoy spacing

x_7 = distance traveled by task force during one complete cycle of buoy-laying and retrieval

x_8 = period of cycle
$= h_1(x_7, x_{11})$

x_9 = time on or at mother SEV or ship for refueling

x_{10} = time on-station to retrieve one buoy

x_{11} = speed of advance of task force

Case 2

$$(MOE)_3 = f_3(x_{12}, x_{13})$$

where

x_{12} = transit time between dip stations
$= g_5(x_{14}, \ldots, x_{17})$

x_{13} = distance between dip stations

x_{14} = total cycle time

x_{15} = ping time at each dip station

x_{16} = time to reel up sonar

x_{17} = time to unreel sonar

3.2) Mission Type: Submarine trailing

3.2.1) Definition: An SEV is to maintain constant close contact with an enemy submarine and immediately attack the submarine should the enemy initiate such action.

3.2.2) Criterion For Success: Constant close contact of submarine while it is in the trailing area

3.2.3) MOE Selected: Minimum effective SEV speed

3.2.4) Functional Form Of MOE:

$$(MOE)_4 = f_4(x_{18}, x_{19})$$

where

x_{18} = SEV cruise time per cycle
$= g_6(x_{20}, x_{21})$

x_{19} = active sonar detection range

x_{20} = total time per cycle
$= h_2(x_{19}, x_{22})$

x_{21} = SEV total operational delay time at each dip point
$= h_3(x_{23}, x_{24}, x_{25})$

x_{22} = submarine maximum speed

x_{23} = time for active detection by dipping sonar

x_{24} = time to raise sonar to RAP (reliable acoustic path) depth

x_{25} = time to lower sonar to RAP depth

3.3) Mission Type: Submarine search

3.3.1) Definition: The SEV's are to search for, detect, classify and localize enemy submarines in various ocean areas where no prior knowledge exists that any submarines are operating in the particular area undergoing search.

3.3.2) Criterion For Success: Detection, classification and localization of submarine

3.3.3) MOE Selected: Minimum effective SEV speed

3.3.4) Functional Form Of MOE:

$$(MOE)_5 = f_5(x_{26}, x_{27})$$

where

x_{26} = total distance traveled by SEV while searching

$= g_7(x_{19}, x_{28})$

x_{27} = total time on-station for search

$= g_8(x_{19}, x_{21}, x_{29})$

x_{28} = total number of dips

$= h_4(x_{19}, x_{30})$

x_{29} = SEV cruise time between dips

x_{30} = total area searched

3.4) Mission Type: Contact investigation

3.4.1) Definition: The SEV's are to develop any submarine contact initially made by SOSUS to the point where a kill can be made.

3.4.2) Criterion For Success: Localization of submarine

3.4.3) MOE Selected: Target uncertainty area

3.4.4) Functional Form Of MOE:

$$(MOE)_6 = f_6(x_{31}, x_{32}, x_{33})$$

where

x_{31} = SOSUS probability area (SPA)

x_{32} = SEV time late to SPA center

$= g_9(x_{34}, x_{35})$

x_{33} = submarine cruise speed

x_{34} = range to SPA center

x_{35} = SEV speed

4) MOE Usage In Study: Formulation and evaluation of the effectiveness of SEV's to perform various missions.

5) Special Study Assumptions:

(a) In the ASW screening of task forces using fore and aft screening mode, (a.1) The fore and aft shuttle operational concept mode utilizes 2N SEV's and a mother ship which travels with the screening unit (SU). N-SEV's operate forward of the SU, each in a line parallel to the SU direction of advance. Each SEV lays a string of N retrievable sonar buoys during a cycle of operation. Simultaneously, the other N SEV's operate in the same lanes aft of the SU, each retrieving N buoys per cycle. At the end of each cycle, all 2N SEV's return to the mother SEV or ship for refueling, maintenance, etc., then the two groups of N SEV's interchange duties and start the next cycle.

(a.2) In order that steady-state operations be maintained, a constraint has been imposed that the situation at the end of a cycle is the same as at the beginning.

(a.3) There is a time delay in retrieving each buoy but there is no appreciable time delay in laying a buoy.

(a.4) The total distance traveled in the laying (or retrieving) cycle is the same for each SEV.

(b) In the ASW screen of task forces using dipping sonar,

(b.1) In the dipped sonar mode the SEV lowers its transducer by cable to RAP depth and pings for a minimum specified time. If no contacts are obtained the SEV proceeds to the next listening station to repeat the cycle.

(b.2) Equal periods of time are spent at each dip station.

(c) In the submarine trailing,

(c.1) The submarine is alerted by the SEV sonar pings and after the last ping the submarine travels at maximum speed directly away from SEV.

(c.2) The submarine is at maximum detection range of the dipped sonar when alerted.

(d) In the open ocean search,

(d.1) The search area is rectangular.

(e) In the ASW contact prosecution,

(e.1) The SOSUS probability area is elliptical.

(e.2) The SOSUS probability area is independent of submarine course and speed.

C. EFFECTIVENESS FACTORS

1) Physical Environment: Not stated

2) Threat Composition: Submarine

2.1) Quantitative Factors:

(a) submarine maximum speed

(b) submarine cruise speed

2.2) Tactics:

2.2.1) Qualitative Factor:

(a) see assumption (c.1)

3) Friendly Force Composition: Surface Effect Vehicles, task force, and SOSUS

3.1) Platform Type: Surface Effect Vehicles

3.1.1) Quantitative Factors:

(a) time on or at mother SEV or ship for refueling

(b) SEV speed

3.1.2) Sensors: Sonobuoys, dipping sonar

3.1.2.1) Type: Sonobuoys

3.1.2.1.1) Quantitative Factors:

(a) number of buoys laid (retrieved) by each SEV during one complete cycle

(b) buoy spacing

3.1.2.2) Type: Dipping sonar

3.1.2.2.1) Quantitative Factors:

(a) distance between dip stations

(b) ping time at each dip station

(c) time to reel up sonar

(d) time to unreel sonar

(e) active sonar detection range

(f) time for active detection by dipping sonar

(g) time to raise sonar to RAP (reliable acoustic path) depth

(h) time to lower sonar to RAP depth

3.1.3) Deployment;

3.1.3.1) Qualitative Factors:

(a) see assumption (a.1)

(b) see assumption (b.1)

3.1.3.2) Quantitative Factors:

(a) time on-station to retrieve one buoy

(b) total cycle time

(c) SEV cruise time between dips

3.2) Platform Type: Task force.

3.2.1) Quantitative Factors:

(a) distance traveled by task force during one complete cycle of buoy-laying and retrieval

(b) speed of advance of task force

4) Friendly Force - Threat Interaction:

4.1) Platform - Platform:

4.1.1) Type: Surface Effect Vehicles - Submarine

4.1.1.1) Quantitative Factor:

(a) total area searched

4.1.2) Type: SOSUS - Submarine:

4.1.2.1) Quantitative Factors:

(a) SOSUS probability area (SPA)

(b) range to SPA center