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Project Serial No. SF11-121-106
Task 8523
TRACOR Project 035 001
Document No. T73-AU-9520-U

FINAL REPORT

COMPUTER AIDED DETECTION FOR WIDEBAND PASSIVE SONAR SYSTEMS

by

H. A. Reeder

Submitted to

Commander
Naval Ship Systems Command
Department of the Navy
Washington, D. C. 20362

Attention: [redacted]
SHIPS 302-4

14 February 1973

TRACOR

TRACOR 6500 TRACOR LANE, AUSTIN, TEXAS 78721

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Attention: Mr. Steve McBurnett
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Approved:

W. C. Moyer
Division Vice President

Submitted:

H. A. Reeder
Project Director

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ABSTRACT

A computer-aided detection processor has been developed for a wideband passive sonar system. The processor may be applied to a variety of passive systems. In addition to the individual processor, a general framework for a computer system that accepts and analyzes the vast quantity of data generated a modern sonar suite has been developed. The output of this computer system is an array of alerting functions that measure the likelihood that a given coordinate vector is the location of a target. The computer-aided detection processor for wideband passive sonar systems is based on the Sequential Likelihood Ratio (SLR) hypothesis test. Methods were developed in this study to choose critical parameters of the processor and to analyze its performance. The wideband passive SLR processor outperformed a conventional processor in the sense of reducing the required input signal-to-noise ratio required for 0.5 probability of detection at a given fixed probability of false alarm by several dB. The SLR processor is ideally suited for implementation on the output of passive sonar systems.

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1.0 INTRODUCTION

Two of the major problems faced by modern sonar systems are the consolidation and presentation of a potentially large quantity of data and the ability of the sonar operator to effectively assimilate and process these data. Potentially, a modern sonar suite could be capable of delivering thousands of channels of information to the operator who in turn, even in an alerted state, could not process all of this information. Moreover, operators do not typically perform in an alert manner when required to search for extended periods of time, especially when the incidence of contacts is low. The result of this is that submarines may go undetected for longer than necessary, and when detected, the resultant time available for classification and tracking is reduced--possibly to an extent that seriously degrade the ASW system's performance. Our approach to the solution of this problem is to develop a computer processing system which can handle the vast quantity of data and in so doing operate in a near optimum manner by virtue of an automatic tracking and integration algorithm that operates from update-to-update on the outputs of passive sonar search receivers.

Work under this contract has produced a general framework for a computer system that will accept and analyze this vast quantity of data. The output of this system will be an array of alerting function values that measures the likelihood that a target occupies each of the coordinate positions within the search volume of the sonar. Information concerning the target track is stored in the computer bank. During this study a computer processor which analyzes the output of a wideband passive sonar system has been developed. Previously, computer processors were developed to analyze the outputs of two active processors, high- and low-Doppler. The outputs of these processors were then combined into a single output channel which could be used for decision purposes.

The processing is based on Sequential Likelihood Ratio (SLR) procedures that have been investigated previously. Briefly, the SLR processor combines a statistical decision test (Wald's Sequential Probability Ratio Test) and a basic tracking program. The tracking program selects target tracks that have motion consistent with that of a submarine and the SLR test is used to decide whether the track should be rejected or retained, and, if retained, possibly displayed. The testing procedure operates much like an alert operator but without the variability of an operator who is, of course, susceptible to fatigue, subjectivity, boredom, poor training, and a host of other deterrents to ideal, time invariant detection performance. More important than this perhaps is the fact that the information handling capacity of this computer process is far in excess of that of the operator and is also subject to expansion as computer technology improves, whereas the capacity of the human operator is unlikely to be expanded.

2.0 SUMMARY OF RESULTS

Several major items have been accomplished during this study. This report is concerned with two of the principal items:

1. Development of a general framework for combining multiple sonar receiver outputs to form alerting functions.

2. Development of a Sequential Likelihood Ratio (SLR) processor for the output of a wideband passive sonar system.

Other items accomplished during this study are reported elsewhere.* These include the development of high- and low-Doppler SLR processors and the combination of the two outputs and the conduction of a display observer study using active low-Doppler sea data and injected targets.

The primary steps for combining multiple sonar receiver outputs include SLR processing of each individual processing channel, weighting the results depending on expected processor performance, combining channels that have overlapping performance envelopes, e.g., wideband and narrowband passive processors, and choosing the maximum output among the individual and combined processors. Provision must be made for dimensionality mismatch and varying resolution cell sizes of the separate processing channels. The multireceiver processor is described in detail in Section 3.0.

A major task of this contract was to adapt SLR processing to wideband passive receiver outputs. In order to do this, certain receiver parameters and statistics must be assumed

*H. A. Reeder, "Simultaneous Likelihood Ratio Processing for Two Active Receivers," TRACOR Document T71-AU-9594-U, Vol. I, 25 August 1971; H. A. Reeder and D. W. Hamm, "Computer Aided Detection for Active and Passive Sonar Systems," TRACOR Document T73-AU-9519-U, 14 February 1973.

for actual implementation. For active search receivers, treated previously, the parameters and statistics of a typical modern system were assumed. In keeping with this tradition, a typical modern wideband passive system is postulated. This system consists of a linear preformed-beam beamformer, square law detectors followed by a finite time, perfect averagers. Moreover, it is assumed that the averager output waveforms are stationary processes, that is, time normalized. A block diagram of the wideband system with the associated SLR processor is shown in Fig. 2-1.

The implementation of the wideband passive SLR processor is described in detail in Section 4.0. Briefly, the input data are subjected to initial thresholding and converted to a log likelihood ratio based on an assumed signal-to-noise ratio. A simple tracking process is carried out as illustrated in Fig. 2-2. The purpose of the tracking algorithm is mainly to eliminate target tracks whose movements are inconsistent with those of an actual maneuvering submarine, rather than to achieve an accurate estimate of the target's position and motion. For simplicity the diagram shows only one event crossing the initial input threshold on update i. The shaded area shows the expected allowable target movement. On update i+1 a single update event occurs within the tracking window. The joint log likelihood ratio is formed by adding the two individual log likelihood ratios. The results are subjected to the sequential decision test which uses two thresholds based on the probabilities of incorrect decision. If the joint log likelihood ratio is below the lower decision threshold, T_L, the track is deleted from the computer; that is, the decision is made that the track is noise alone. If the joint log likelihood ratio is above the upper decision threshold, T_D, the track is displayed to the operator; that is, the decision is made that the track is signal plus noise. Also, the track is retained by the computer. If the joint log likelihood ratio is between the thresholds, the computer retains the track for further processing. This decision process is illustrated in Fig. 2-3.

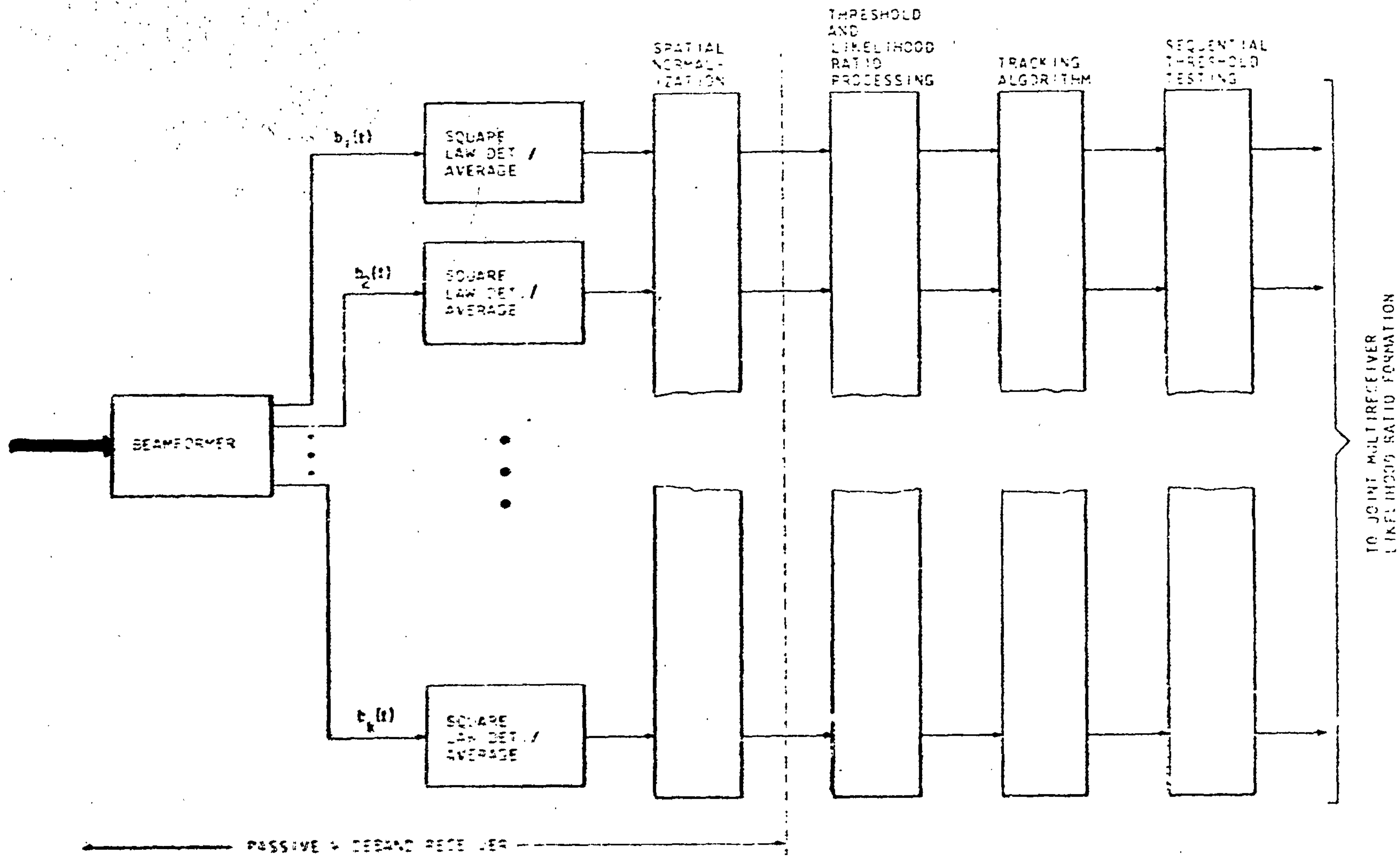


FIG. 2-1 - BLOCK DIAGRAM OF A WIDEBAND PASSIVE RECEIVER FOLLOWED BY A SEQUENTIAL LIKELIHOOD RATIO TRACKING ALGORITHM

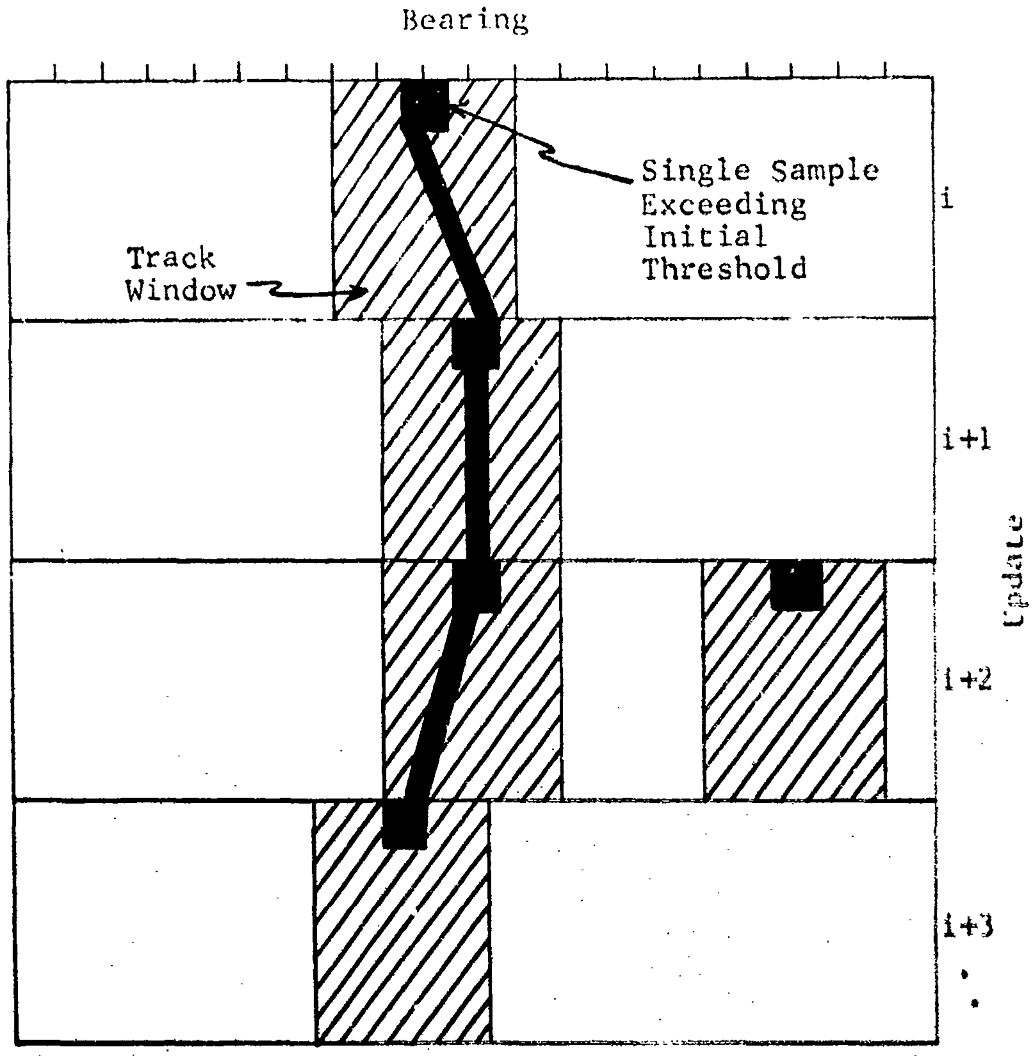


FIG. 2-2 - ILLUSTRATION OF TRACKING ALGORITHM FOR WIDEBAND PASSIVE SLR PROCESSOR

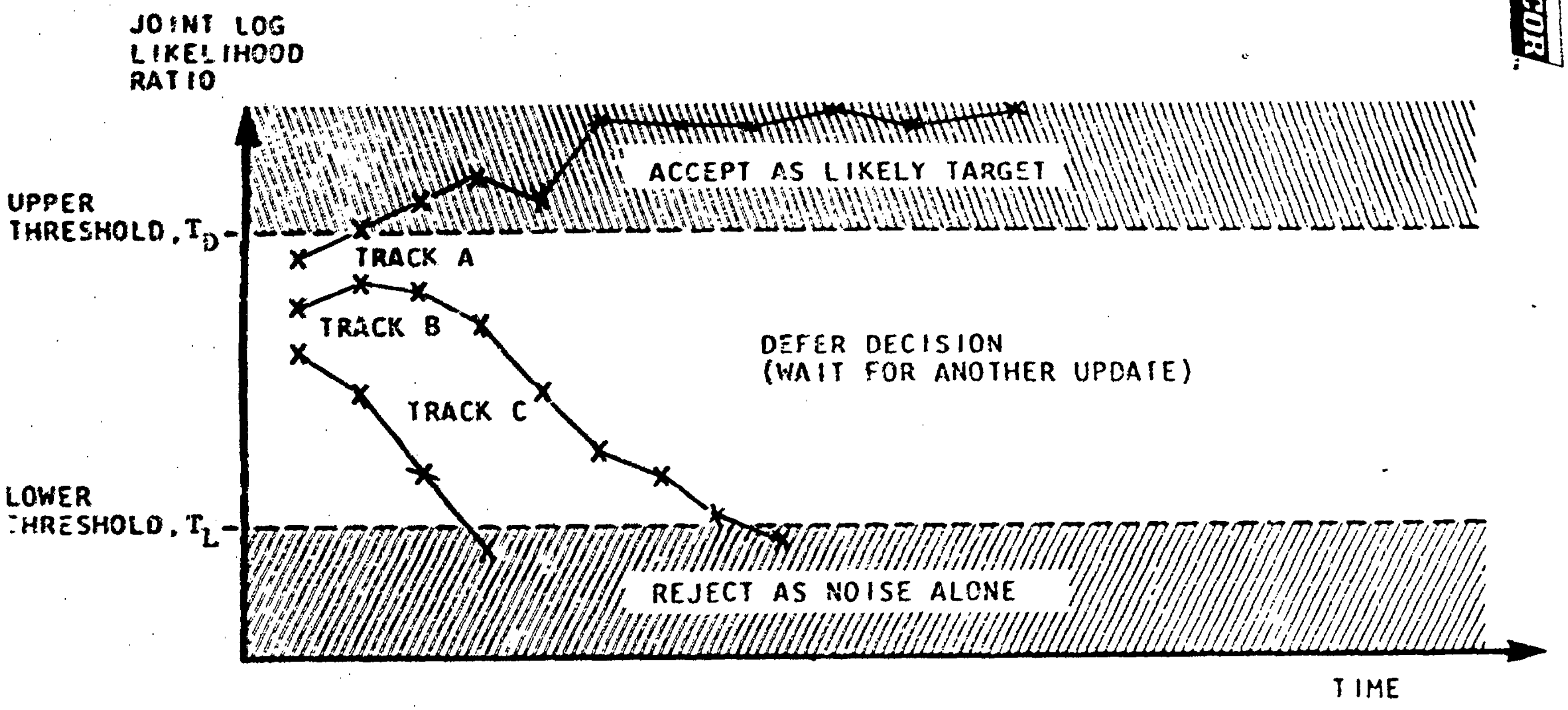


FIG. 2-3 - REPRESENTATION OF THE SEQUENTIAL THRESHOLD TESTING PROCEDURE

This tracking and decision procedure is carried out for as long as the joint log likelihood ratio of the track remains above the lower decision threshold. Typically, thousands of tracks may be processed on any update and tracks may begin on any update.

The results of implementing the SLR processor on the output of a wideband passive system is shown in Fig. 2-4. The solid curves represent the probability of exceeding threshold vs input signal-to-noise ratio for various number of updates. The threshold is chosen to yield a specified probability of false alarm, in this case 10^{-3}, under steady state noise only conditions. A similar curve for conventionally (non-SLR) processed data is shown for one update. Note, that the performance of the SLR processor on the second update is roughly equivalent to the performance of the non-SLR processed data on a single update. This is caused by the fact that the SLR is assumed to be operating continuously on a noise field (which would be the case in an operational implementation); hence, some spurious noise tracks have had the opportunity to integrate up for a few updates. This causes a slight rise in the noise background. This rise is overcome after a target track has been integrated over two updates. After integration over more updates occurs, significantly better performance is achieved. It is illustrative to calculate the required input signal-to-noise ratio for 0.5 probability of threshold crossing as a function of the number of updates integrated. This information is presented in Fig. 2-5 for both the SLR and non-SLR. Again, the probability of false alarm has been set at 10^{-3}. After several updates, the SLR can clearly enhance the detection of marginal signal-to-noise ratio tracks.

In each of the above illustrations the SLR processed data has been compared to non-SLR data, but always on a multi-update basis against a single update basis. In order to assess

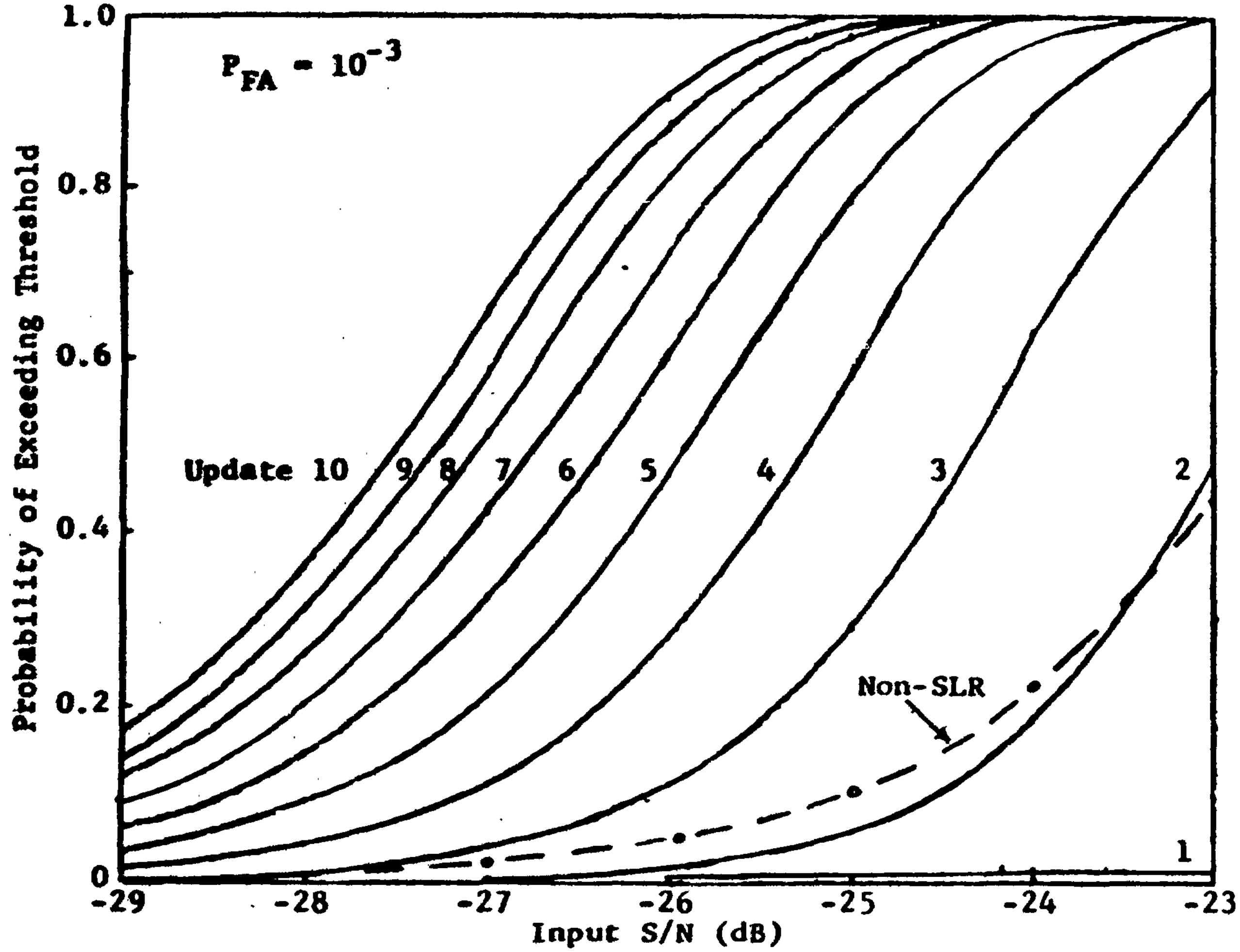


FIG. 2-4 - PROBABILITY OF EXCEEDING THRESHOLD VS INPUT S/N FOR WIDEBAND PASSIVE RECEIVER. INTEGRATION TIME 2 MINUTES PER UPDATE

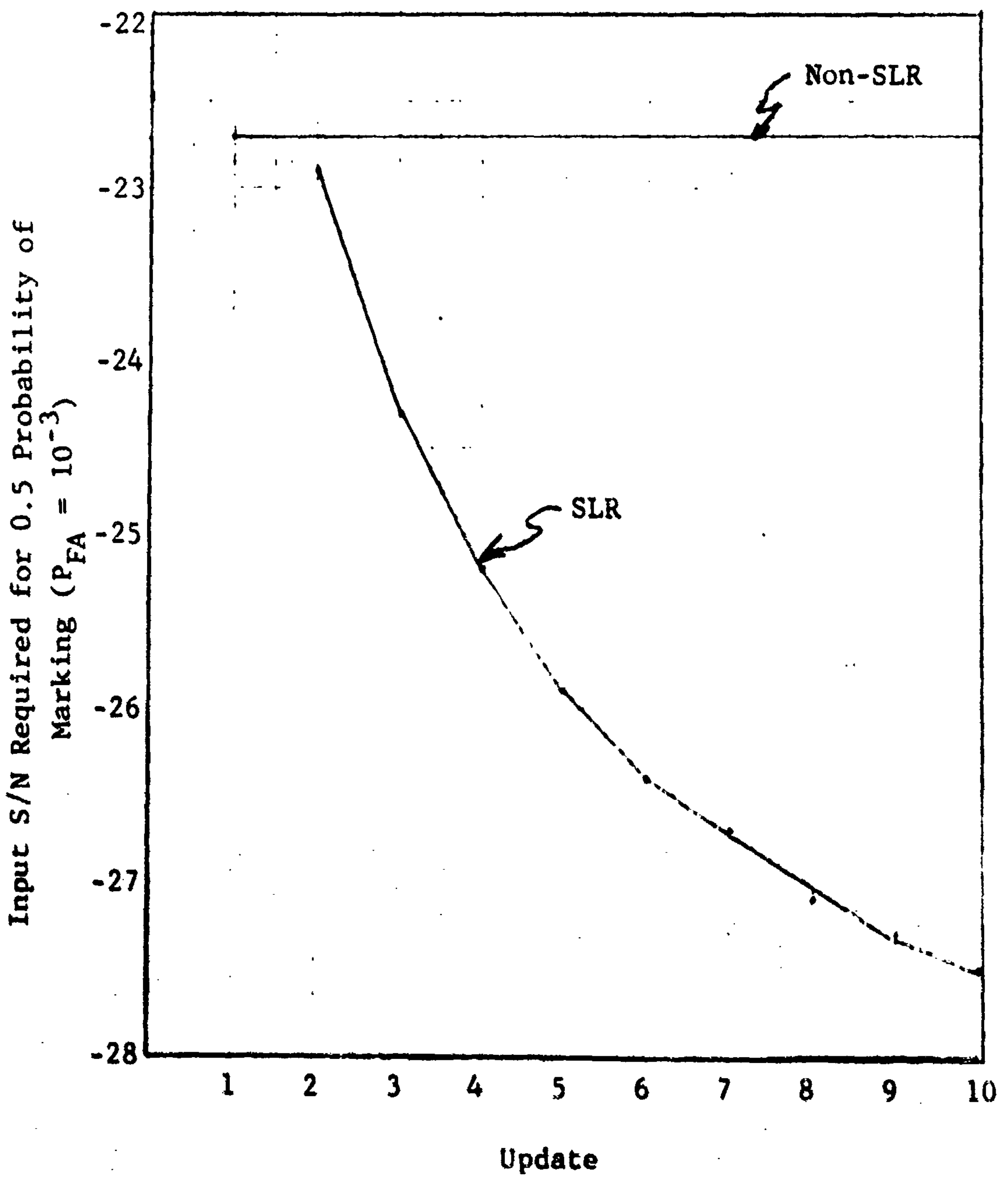


FIG. 2-5 - INPUT S/N REQUIRED FOR 0.5 PROBABILITY OF DISPLAY MARKING AFTER N UPDATES. PROBABILITY OF FALSE ALARM 10^{-3}. WIDEBAND PASSIVE SYSTEM INTEGRATION TIME IS 2 MINUTES the difference on an equal multi-update basis, the following comparison was made. Realizing that an operator would not call a target detection on a single update basis, a sonar display was hypothesized that presents to the operator the last five updates; the operator is assumed to call a detection if at least three marks appear on the display during any of the five consecutive updates. After each update, the display is pushed down one, eliminating the oldest update, and the next update is presented at the top of the display, creating a "waterfall" effect. The previous results were used to calculate the probability of satisfying this criterion of three marks out of the last five possible marks at least once during ten updates. The results for both SLR and non-SLR are presented in Fig. 2-6. The input signal-to-noise ratio required for 0.5 probability of detection is approximately 4 dB less for the SLR processor than the conventional processor after ten updates.

Another experiment was conducted where the integration time was reduced by a factor of 10 and the signal track integrated for 50 updates. The results are presented in Fig.2-7 as signal points at a -25.9 dB input signal-to-noise ratio. The point 5 is the results after 50 updates and corresponds in time to the curve marked 5 for the longer integration time. This result shows that decreased sonar processor integration time and increased SLR integration time yield better results. At time 5 the increase in probability of detection is approximately 40%. Apparently with faster updates, the SLR processor is able to reject noise tracks in a shorter average time; hence, the noise background is reduced but the integration of signal-plus-noise tracks remains approximately the same. This yields the improved performance.

In summary, the SLR processor is ideally suited for implementation on the output of passive sonar systems. Available gains due to SLR processing of passive sonar data appear to be

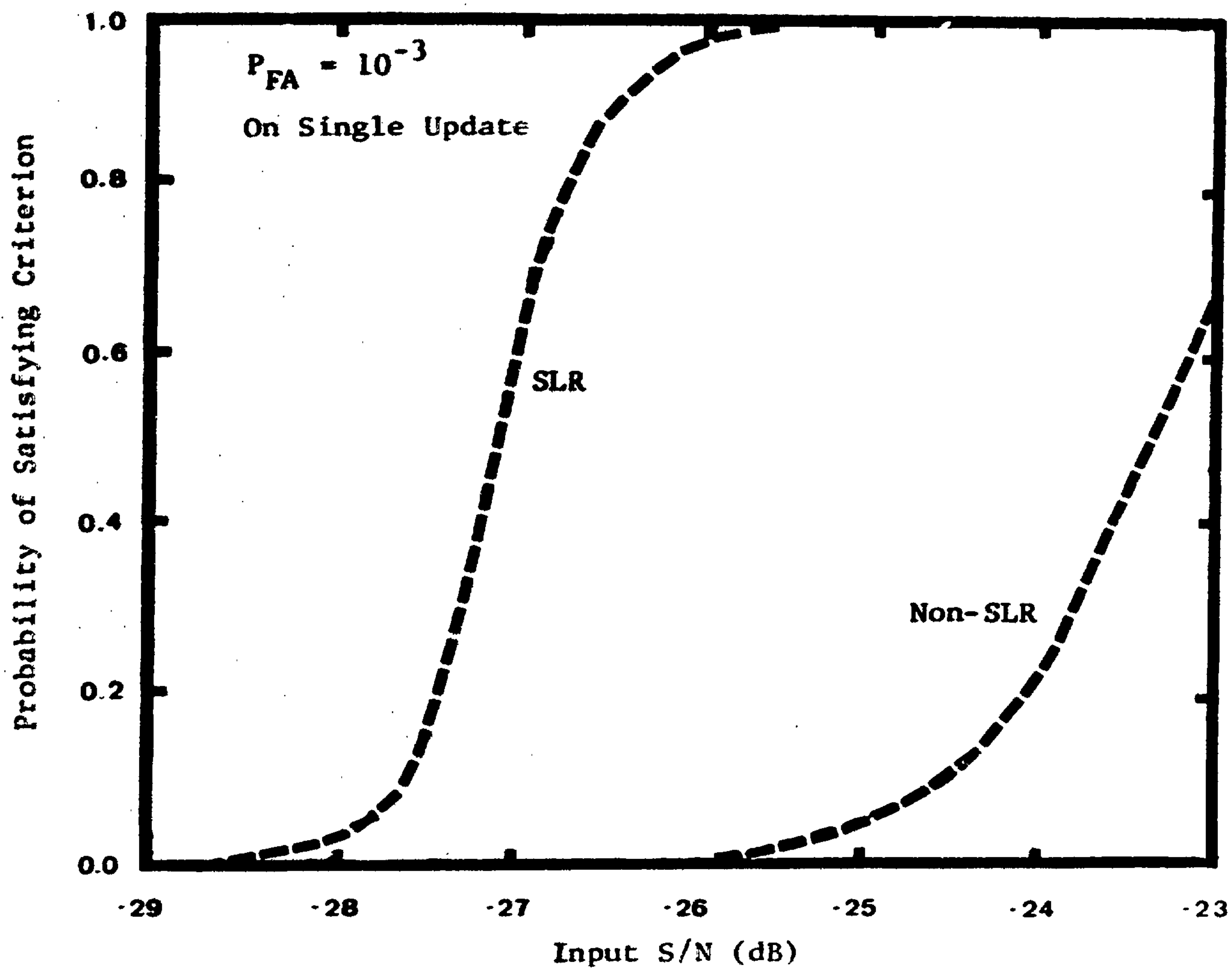


FIG. 2-6 - PROBABILITY OF SATISFYING CRITERION VS INPUT S/N. CUMULATIVE PROBABILITY OF AT LEAST 3 OUT OF 5 POSSIBLE MARKS FOR 10 UPDATES. INTEGRATION TIME 2 MINUTES PER UPDATE

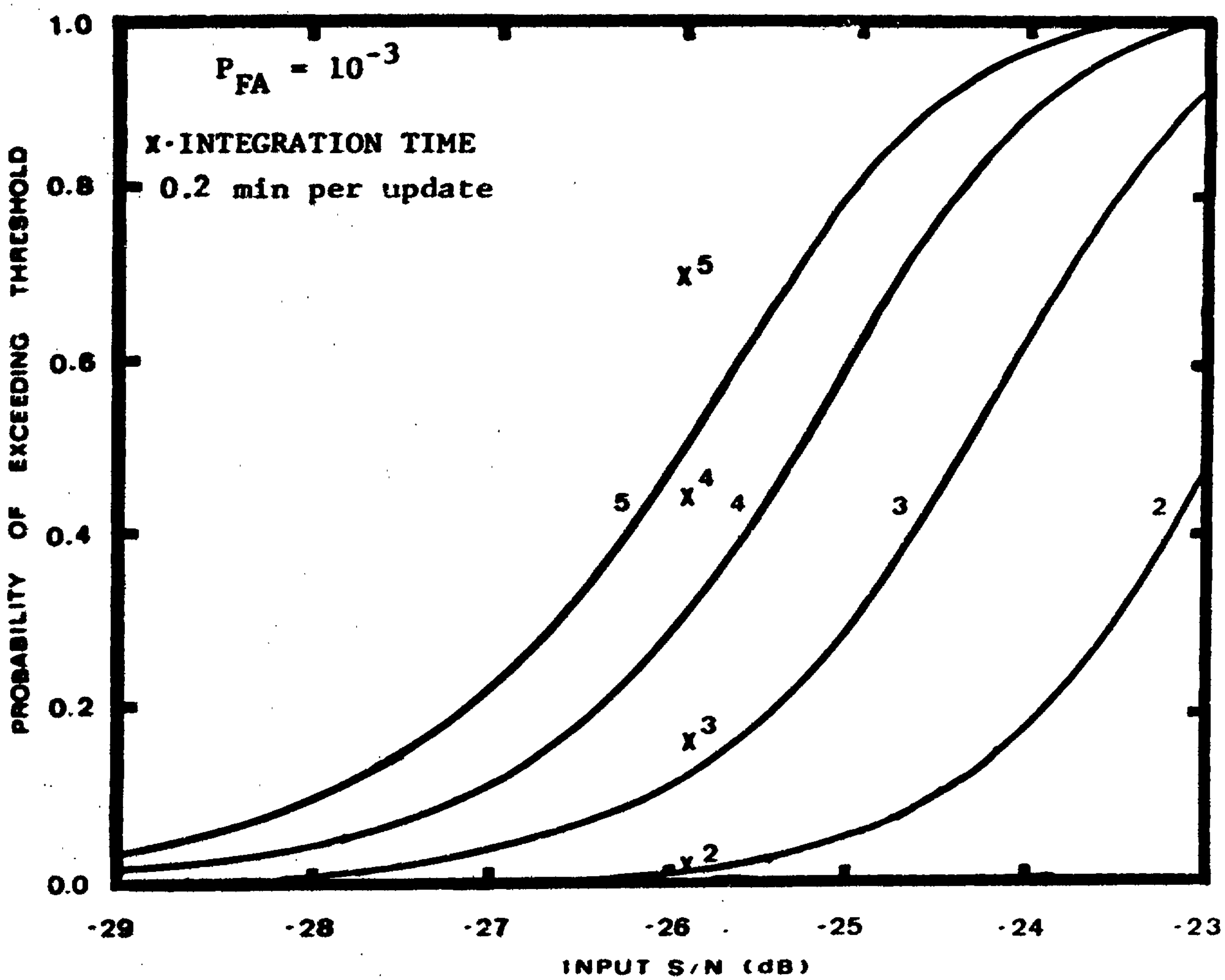


FIG. 2-7 - PROBABILITY OF EXCEEDING THRESHOLD VS INPUT S/N.
SOLID CURVES EQUAL INTEGRATION TIME 2 MINUTES PER UPDATE

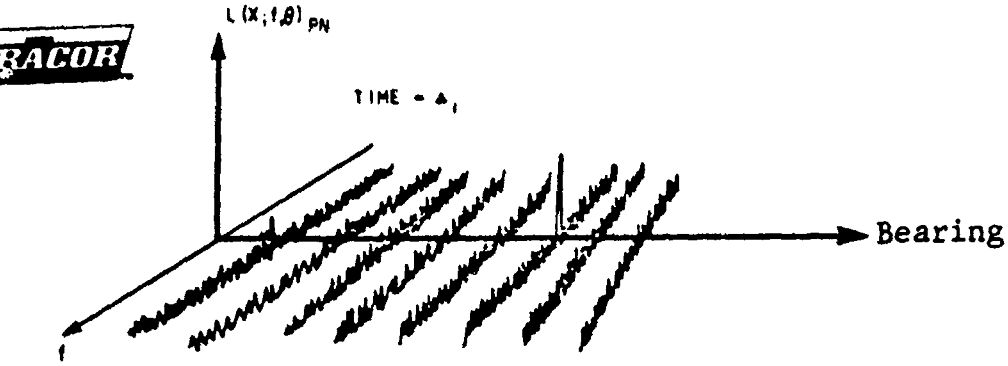
better than with active sonar systems since time between updates generally may be much less than the ping repetition rate. In addition, an important step has been taken in the development of computer based system to integrate the outputs of wideband and narrowband passive sonar systems in a logical, meaningful manner.

3.0 FRAMEWORK FOR INCLUSION OF MULTIPLE SONAR RECEIVER OUTPUTS IN SLR PROCESSING

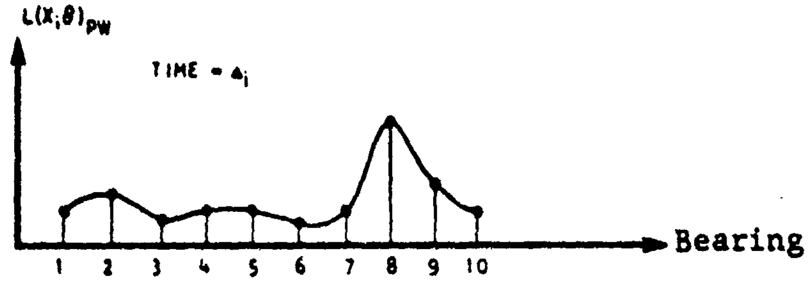
3.1 Formation of Multireceiver Joint Likelihood Ratios - Alerting Functions

The objective of this section is to develop a multichannel alerting algorithm for various input channels such as high- and low-Doppler active search receiver outputs and wideband and narrowband passive search receiver outputs. It is assumed that each separate channel has been subjected to a sequential likelihood ratio process that yields a likelihood ratio for each resolution cell. For purposes of illustration four processors will be considered: a low-Doppler processor whose output is indexed by range and bearing; a high-Doppler processor, indexed by range, bearing, and Doppler shift; a wideband passive processor, indexed by bearing; and a narrowband passive receiver, indexed by bearing and line frequency. These four processors demonstrate a variety of problems inherent in combining likelihood ratios with different dimensionality and resolution cell sizes.

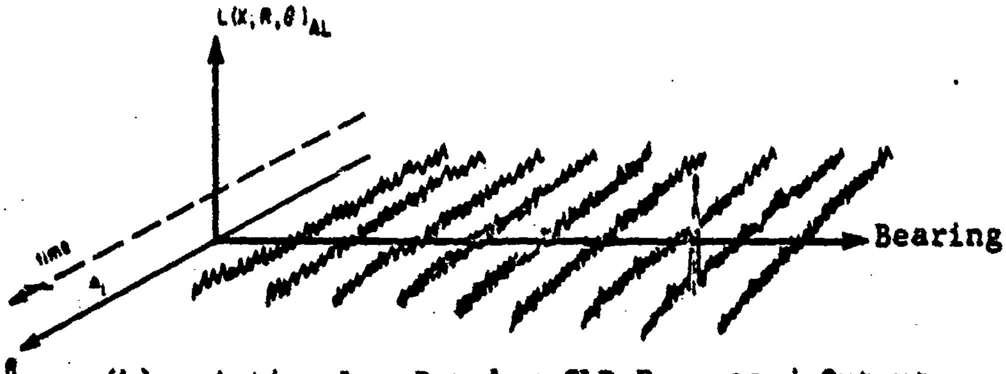
When all of the SLR ping-to-ping and update-to-date tracking is accomplished on each receiver output, there will exist the information shown in Fig. 3-1. For convenience, this figure shows the processed data in a continuous and unthresholded form, although in reality, each output is sampled and thresholded so that the actual quantity of data will be less than that shown. The task before us now is to adopt a method for combining these outputs to form a measure of the likelihood that a target occupies a given range-bearing cell. This will be approached by first combining the two processed active outputs, then (conceptually) combining the two passive system outputs and finally (again conceptually) combining the joint active and passive outputs.



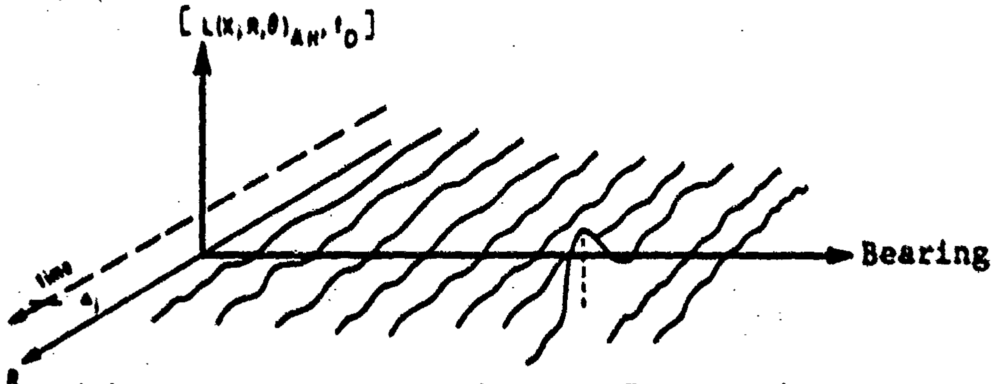
(d) - Passive Narrowband SLR Processed Output



(c) - Passive Wideband SLR Processed Output



(b) - Active Low-Doppler SLR Processed Output



(a) - Active High-Doppler SLR Processed Output

FIG. 3-1 - REPRESENTATIONS OF THE SLR PROCESSED ACTIVE AND PASSIVE RECEIVER OUTPUTS

The general idea here is to first combine those processors that are most similar in processed data form and then combine the results of these combinations.

In combining the outputs of two or more processors we face problems related to different resolution cell size and dimension mismatch. For example, in the active high- and low-Doppler receivers we have tracks developing in the range, bearing and Doppler dimensions of the former while tracks in the latter develop in only the range and bearing dimensions. In addition, it is possible to have a range cell size mismatch between these two processor outputs since resolution in the low-Doppler receiver is determined by the coded pulse bandwidth while range resolution in the high-Doppler receiver is determined by the duration of a long CW pulse. These differences are depicted in Figs. 3-1a and 3-1b where in the case of the high-Doppler output the ordinate is described by a pair of numbers, one giving the track likelihood ratio, the other giving the associated Doppler. Clearly, for any given range-bearing cell in the high-Doppler system there can be more than one (likelihood ratio, Doppler) pair since tracking is occurring in the Doppler dimension. To reduce the dimensionality to the range-bearing dimensions, the computer will retain only the maximum likelihood ratio (and its associated Doppler) in each range-bearing cell. This eliminates the problem of equalizing the dimensionality between the two active systems.

3.1.1 <u>Combination of Low- and High-Doppler Active System Outputs</u> - The next problem is one of mismatch between resolution cell size. Figure 3-2 shows a representation of the outputs of the SLR processed high- and low-Doppler active systems from preformed beams steered in the same direction. The outputs have

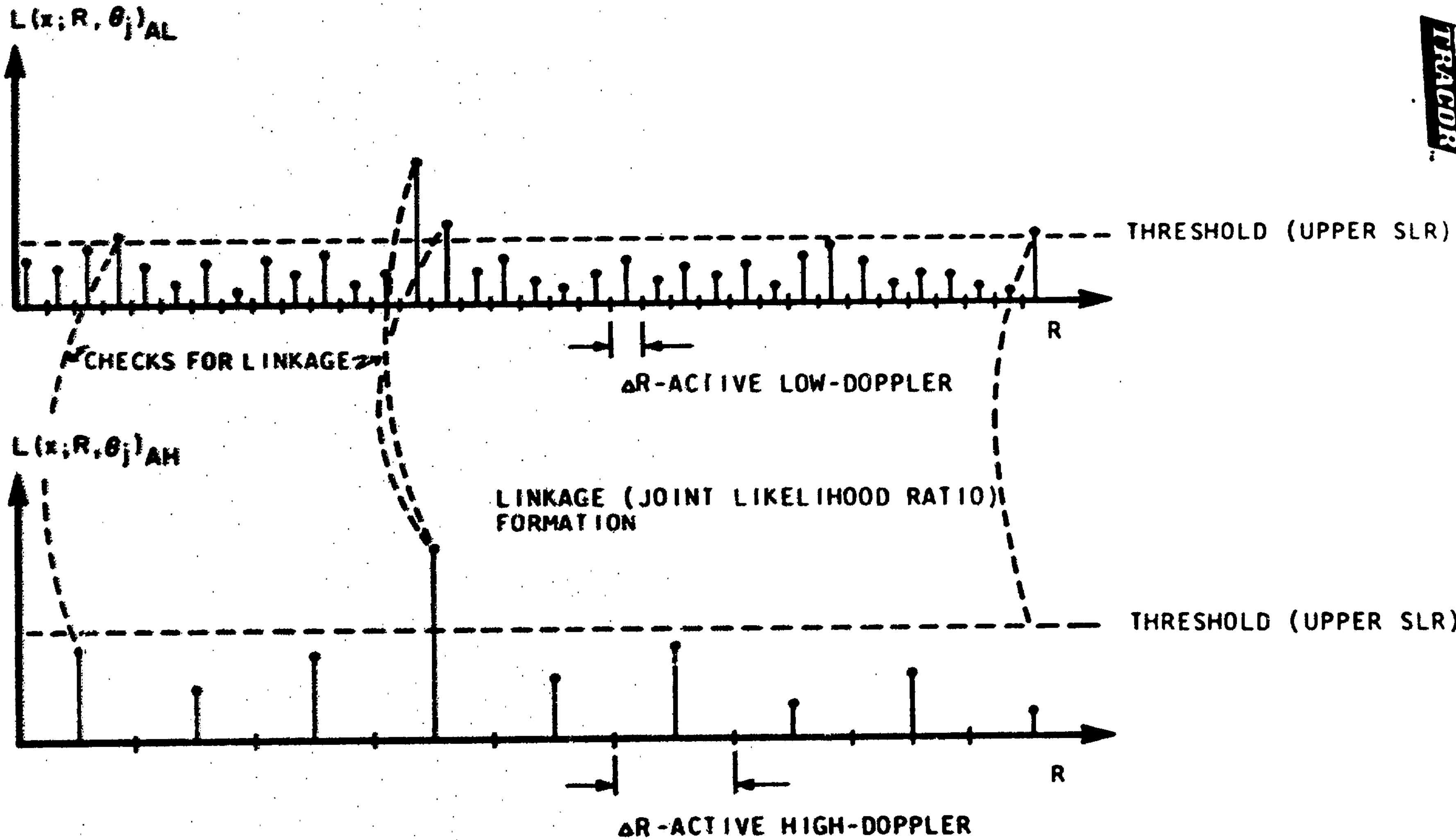


FIG. 3-2 - REPRESENTATIONS OF THE JOINT LIKELIHOOD RATIO FORMATION BETWEEN HIGH- AND LOW-DOPPLER SLR PROCESSED OUTPUT DATA been time delayed as necessary so that each time resolution cell covers the same range. The output from the low-Doppler SLR processor has been serially OR-ed* over a range gate corresponding to the approximate length of a target or a display resolution cell. This is done because eventually the output of the combined SLR must be matched to a display for presentation and the serial OR at this point reduces subsequent processing with almost no degradation in performance. In any case the low- and high-Doppler outputs are combined as shown in Fig. 3.2. This figure shows that in the low-Doppler system four threshold exceedings have occurred during the echo cycle shown. Each of these events leads to a check for a threshold-exceeding event in the high-Doppler SLR processed system output within high-Doppler range resolution cells which encompass the range cell containing the low-Doppler event. If threshold-exceedings do not occur on both system outputs within a common high-Doppler range cell, then no track linkage is made during the echo cycle under consideration. In this way the SLR processed data from both active receivers is combined into a common measure of the likelihood of target.

3.1.2 <u>Combination of Wideband and Narrowband Passive System Outputs</u> - It is desirable to find a single quantity which represents both channels of passive information, or, more generally, a quantity that represents the likelihood of a target being on a particular bearing only. The first problem faced in finding a joint measure of passive information is a dimensionality mismatch. For any given time, the narrowband information is indexed by frequency and bearing while the wideband information is indexed by bearing only. Probably, the best way to approach this problem is to adopt the same procedure as in the active case and reduce the dimensionality of the narrowband data. This may be done by a Max-OR process. A similar procedure has been extensively analyzed

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*Serially OR'ed is a phrase that describes the process of selecting ("discarding all those except") the sample whose magnitude is maximum amongst a set of serially occurring samples.

and found to be an effective method of reducing the rate of data out of a narrowband processor.*

Once the dimensionality has been matched, a combination process such as the active case may be carried out. This involves adding log likelihood ratios that correspond to the same bearing resolution cell and that exceed certain thresholds. The results are joint log likelihood ratios that form a measure of the probability there is a passively-detected target associated with that bearing resolution cell.

3.1.3 Combination of Active and Passive SLR Processed Data - It is assumed that each active and passive beam will have a joint log likelihood ratio as a function of time associated with it. The task is now to combine active and passive information. Since each information channel is obtained in a different way and in separate frequency bands, it is reasonable to assume tentatively that they are statistically independent; hence, the joint active-passive log likelihood ratio is just the sum of the two individual log likelihood ratios. The essential problem in this case is the dimensionality mismatch mentioned previously. The passive information is indexed by bearing and time only while the active is indexed by range, bearing and time.

Recall from Fig. 3-1 that for the passive systems we have log likelihood ratios indexed by bearing and time. This means that when the joint log likelihood ratios are formed, we will have for the passive receivers a single measure of the likelihood of target for each resolvable bearing and at each instant of time. Similarly, for the active systems we will have a set of joint log likelihood ratios indexed by range and bearing, and by time or ping

*J. J. Dow, B. M. Brown, and W. B. Butler, "Determination of the Detection Performance of the Several Passive Narrowband Multibeam Search Receivers (U)," TRACOR Document T72-AU-7072-S, Vols. I and II, 14 July 1972.

number. These arrays of numbers are shown diagrammatically in Fig. 3-3. This figure shows the output of the joint SLR processed passive data which exists at time Δ_i, as well as the joint SLR processed active data during a ping cycle which exists over a time period that encompasses the time Δ_i. Since no range information is currently available from the passive search system, the selected active joint log likelihood ratios will be enhanced by summing them with the last available and relevant passive log likelihood ratios that occur on the same bearing. By "selected" we mean that a threshold test would be applied to the active data before linking to it the thresholded passive data. The levels at which these thresholds will ultimately be set will depend upon the available computer capacity. That is, ideally it would be desirable to perform no thresholding until all of the joint active and passive log likelihood ratios have been formed. However, from a realistic viewpoint there must be the capability for reducing the amount of data stored in the shipboard computer.

The computer algorithm which will perform sequential likelihood ratio tracking and joint log likelihood ratio formation on the outputs of active high- and low-Doppler and passive narrowband and wideband outputs is shown in block diagram form in Fig. 3-4.

This figure shows each of the active and passive SLR processed outputs being applied to threshold circuits and then to weighting circuits. The purpose of the threshold circuits is primarily to control computer loading. This will be accomplished in the following manner. The outputs of each of the SLR processors shown in Fig. 3-4 will be unthresholded tracks and thus will contain numerous spurious noise tracks. Ideally, we would prefer to defer any decision with regard to threshold as late in the processing as possible so that low signal-to-noise ratio tracks will be enhanced as much as possible by the process of joint tracking and

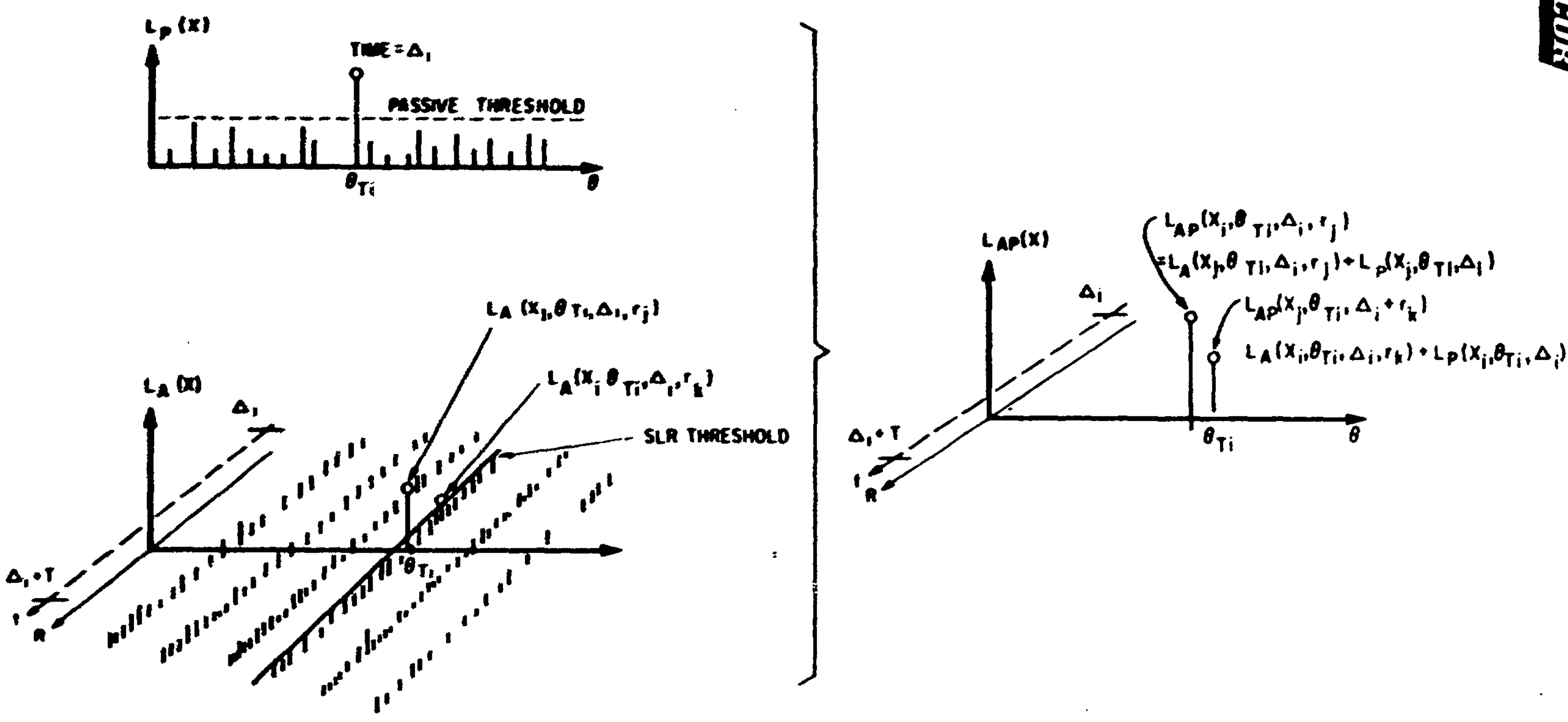

FIG. 3-3 - DIAGRAMMATIC REPRESENTATION OF THE JOINT ACTIVE-PASSIVE LOG LIKELIHOOD RATIO FORMATION PROCESS

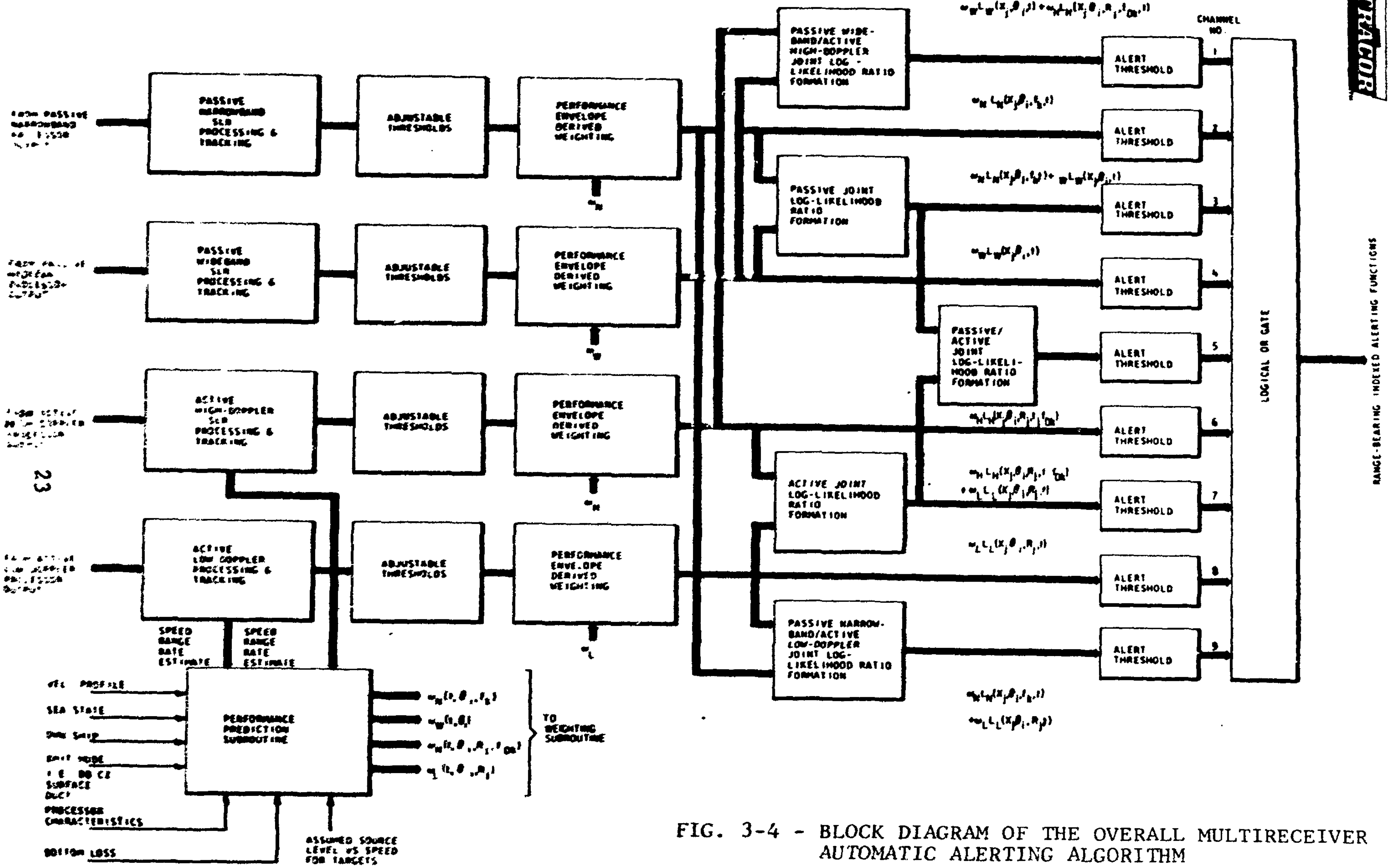

FIG. 3-4 - BLOCK DIAGRAM OF THE OVERALL MULTIRECEIVER AUTOMATIC ALERTING ALGORITHM likelihood ratio formation. However, it will be necessary to provide a means of controlling the number of tracks so that the computer storage capacity is not exceeded. This then is the function of the adjustable thresholds shown in Fig. 3-4.

The thresholded data are then passed to circuits which weight the log likelihood ratios according to a predicted performance envelope. The weighting of the SLR processed output of each sonar receiver is necessary from the standpoint of performance. When one can predict that a certain processor will not perform beyond a certain range under a specific set of conditions then it is obvious that the inclusion of that processor's output in the final joint log likelihood ratio will only serve to degrade otherwise potentially sound tracks. As an example of this use of weightings, consider the situation where the speed of a given target in the active system is estimated to be near zero. There are other conditions such as own ship speed, sea state, and propagation loss which when combined with sufficiently low target speed would result in a very low probability of threshold crossing in the passive wideband receiver at all but very small target ranges. Such a prediction will of course require that one assume some radiated spectrum level versus speed function for the potential target. This may be accomplished by considering those enemy submarines of interest which exhibit the highest radiated noise level as a function of speed and obtaining an average relationship for these targets to be used as input to the performance prediction subroutine. This approach will result in a maximum average detection envelope and thus will result in weightings for the wideband passive system output which will ensure (statistically) that all targets of interest will be processed. When in error, as for example when a quieter submarine is actually present, the overall log likelihood ratios will suffer some degradation as a result of the conservative manner in which weightings are derived. However, the individual receiver log likelihood ratios will not be degraded.

These weights will, without any other data available, be either zero or one, and will be indexed by range and bearing angle. Fig. 3-5 shows an example of the weighting function for the wideband passive system as well as the manner in which it is derived. As shown in this figure, the probability of exceeding threshold as a function of range is used to determine some performance threshold and hence a maximum performance range R_{max}. This information is used to form the weighting function which acts, in fact, as a performance envelope gate for the data emerging from each channel of the multireceiver system.

At the output of the weighting circuits the four channels of data are combined not only into active and passive joint log likelihood ratios and a joint active/passive log likelihood ratio, but also in other various ways. Namely, in addition to the joint log likelihood ratios just mentioned, the individual log likelihood ratios are preserved separately as well as combined into other joint functions. Specifically, the joint log likelihood ratios may be based on passive wideband and active high-Doppler outputs as well as passive narrowband and active low-Doppler outputs. In this way the system consists of nine different channels of output SLR data.

Consider for a moment the reasons for producing an output of this type. To begin with, if we could predict exactly for a given target and environment, the performance envelope of each of the four receivers, and thus quite accurately accept or reject those receiver outputs which should or should not contribute constructively to the overall joint log likelihood ratio, then there would be no need for a multichannel output. To some extent we can do this--in particular according to the weighting mechanism described earlier. However, it is not possible to forecast a priori the exact performance envelope for every target type and environment

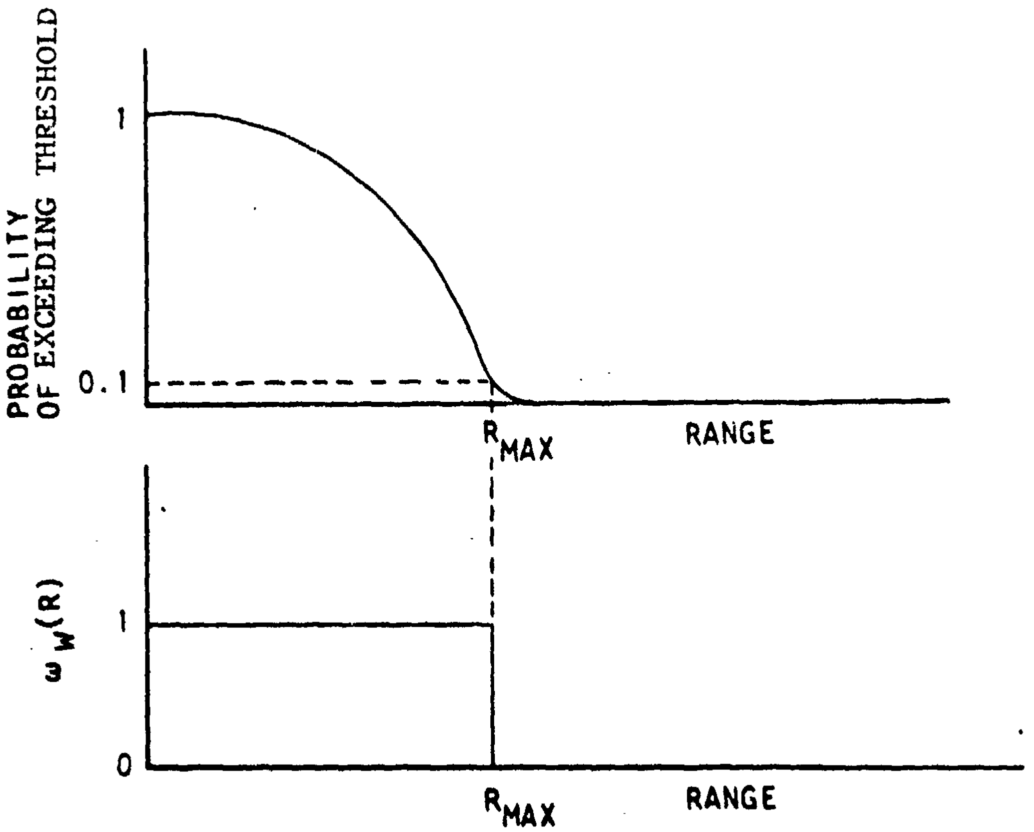

FIG. 3-5 - PERFORMANCE ENVELOPE AND RESULTANT WEIGHTING FUNCTION FOR THE WIDEBAND PASSIVE SYSTEM and then select the proper envelope and weight. Since we have chosen to use performance envelopes based on best case characteristics (e.g., a noisy target) there will arise cases where a receiver has no chance of detecting the target even though the weight is equal to unity (e.g., a very quiet target). In this instance the wideband passive receiver will contribute a noisy output to the joint likelihood ratio. In general there may very well be cases where one channel by itself could cause an alarm but when combined with other nonperforming, noisy channels of data, the chance to detect is lost. This possibility can be dealt with by allowing each channel to generate an alarm on its own.

Actually, the situation we have here is quite analogous to the problem of detecting a narrowband signal in a wideband noise background when the carrier frequency of the signal is not known a priori.

It is known that the optimum approach to detecting this signal is to design a bank of contiguous filters, each having a center frequency which the signal carrier frequency may conditionally take on.* Thus each channel of this system is optimum for detecting the signal under the condition that the carrier frequency of the signal is equal to the center frequency of the channel. The remaining part of the processing in this case is to select the filter output whose likelihood ratio is maximum. By analogy then we can view our multichannel system as consisting of several channels each of which is optimum under some condition which cannot be predetermined. That is, if the performance curves of each of the four input channels overlap for some set of

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*E. J. Kelly, I. S. Reed, and W. L. Root, "Detection of Radar Echoes in Noise, I," Journal of the Society of Industrial Applied Mathematics, pp 309-341, Vol. 8, No. 2, June 1960.

conditions, then the overall joint active/passive log likelihood ratio channel will be the optimum channel and will control the output of the OR-gate shown in Fig. 3-4. If, on the other hand, the target is a high speed, bow aspect target such that these and other conditions render the active high-Doppler and passive wideband combination optimum, then this channel will dominate the OR-gate output and lead to an alerting function.

The price that is paid by taking this approach is an increased number of opportunities to false alarm which can be compensated for by increased thresholds which in turn leads to some decrease in detection capability. However, theoretical considerations indicate that there will be a net gain by this approach.

It will be recognized in Fig. 3-4 that not all combinations of the four receiver outputs are considered. There are in fact fifteen different combinations which could be formed from the original four channels of data. We have selected certain channels which appear reasonable. That is, we have the overall joint active/passive channel which will be optimum when all performance curves overlap. There are also the individual log likelihood ratio channels, one of which can produce an alarm should the other three channels be inoperable by virtue of the constructed performance envelopes. The combination of both passive channels results in a detection channel which should be effective against torpedoes where active receiver performance is seriously degraded by small target strengths. The combined active channels are effective against a deep, quiet submarine running at a speed greater than zero knots but just below the cavitation inception speed. The combination of active high-Doppler and passive wideband is a reasonable choice for a high speed sub, while the combination of active low-Doppler and passive narrowband receiver outputs may be the optimum means of detecting a very low speed sub.

What must be established is the performance of different configurations of the automatic alerting system under a variety of input conditions which represent real world situations. This is necessary so that we can make an intelligent choice of a single configuration with respect to both detection performance and computer requirements.

4.0 WIDEBAND PASSIVE SLR PROCESSOR

4.1 Introduction

One of the major tasks of the present contract is to adapt SLR processing to wideband passive receiver outputs. In order to do this certain receiver parameters and statistics must be assumed. For active search receivers, the parameters and statistics of a specific modern sonar have been assumed. In keeping with this tradition, a typical modern wideband passive receiver system is assumed. This system consists of a preformed-beam beamformer, square law detectors and averagers followed by a normalizer subsystem which is assumed to provide a stationary noise background.

4.2 System Description

A block diagram of the wideband system with the associated SLR tracker is shown in Fig. 4-1. The first section of the system is a beamformer that takes stave outputs and forms 48 beams for 360° of azimuthal coverage. Each beam is bandlimited from 150 to 3000 Hz. The output of the beamformer is square law detected and integrated for two minutes. The integration is followed by a spatial normalizer. For the purposes of this study, it is assumed that this normalization has been carried out and the system is receiving isotropic noise that is Gaussianly distributed.

At this point there exist processed preformed beam outputs which are conventionally displayed on a bearing-time recorder. Figure 4-2 gives representations of the outputs of the various preformed beams at successive times Δ_1, Δ_2, and Δ_3. These samples are approximately Gaussianly distributed with zero mean for those beams containing no signal, i.e., the noise mean has been removed. For those beams whose outputs are influenced by

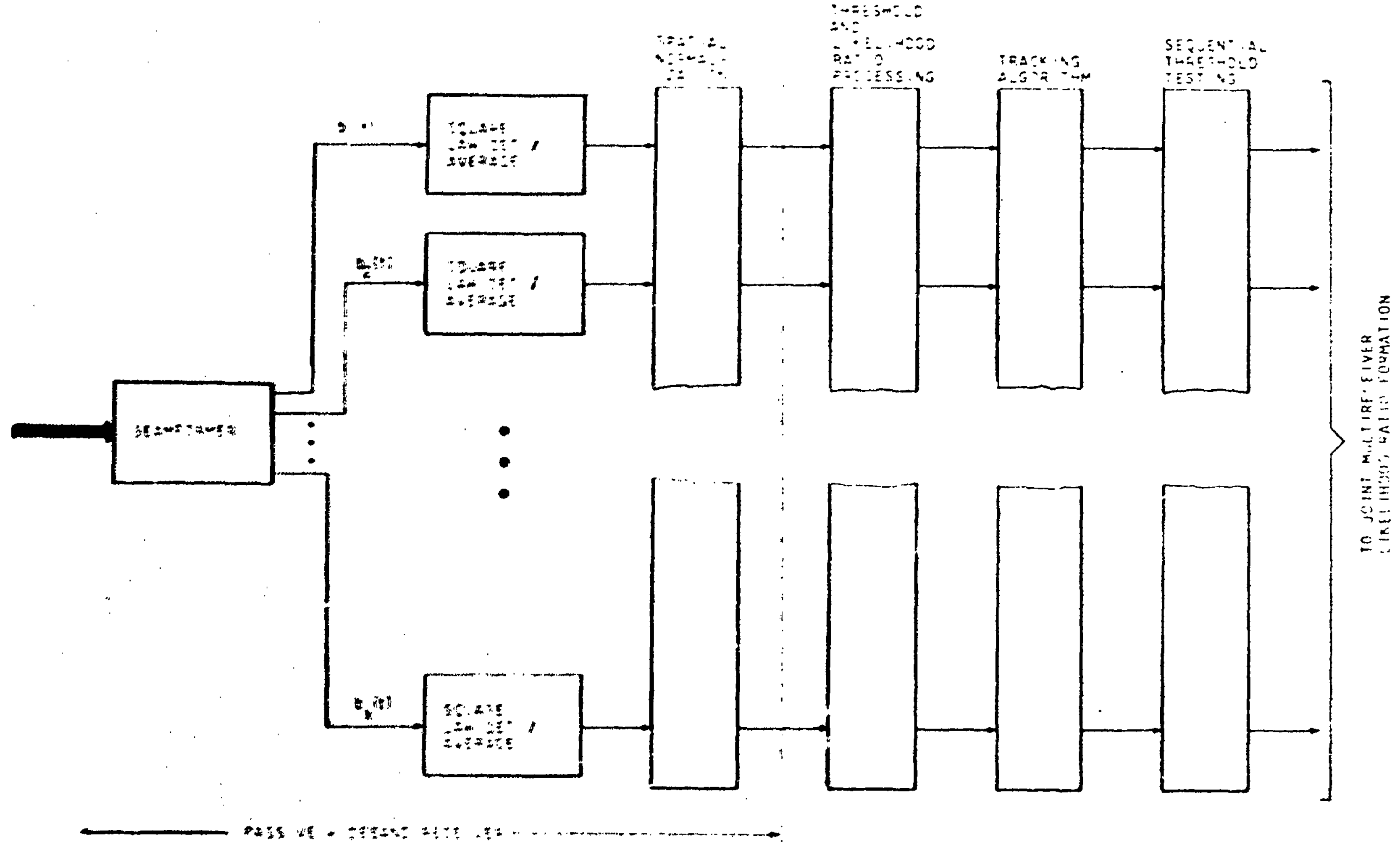

FIG. 4-1 - BLOCK DIAGRAM OF A WIDEBAND PASSIVE RECEIVER FOLLOWED BY A SEQUENTIAL LIKELIHOOD RATIO TRACKING ALGORITHM

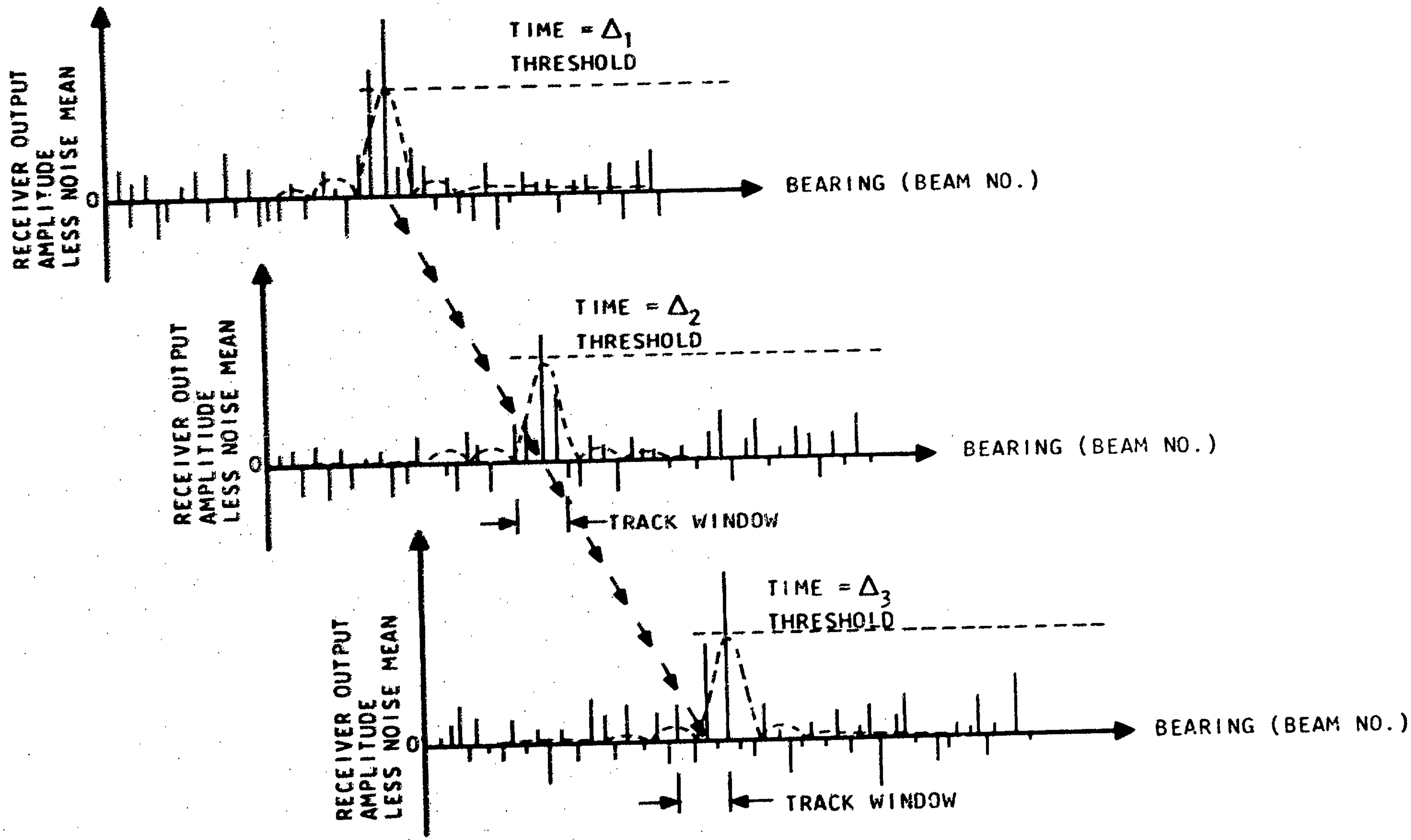

FIGURE 4-2. REPRESENTATIONS OF THE OUTPUT OF A WIDEBAND PASSIVE RECEIVER AT SEQUENTIAL TIMES

the presence of a signal the samples are also approximately Gaussianly distributed but with nonzero mean. This effect is shown by the dashed curves which represent the mean value of the receive beam outputs.* In the vicinity of a target signal the beam outputs not only exhibit a nonzero mean, but are also more correlated than those away from a target signal. However, as shown later, the input signal-to-noise ratios for marginal detection are on the order of -27 dB, and at this level the side lobes contribute very little to the detectability of the signal. For this reason the simple process of using only the main signal lobe was adopted.

Once this processing method had been selected the next step is to track the processed data. Briefly, as shown in Fig. 4-2, each sample--actually the processed sample amplitude shown by the right-hand ordinates--which exceeds a preset threshold is passed to the tracking section of the computer. Its processed amplitude and bearing are noted and a bearing for the next time, Δ_2, is projected, along with a track window whose width is set initially by the expected maximum target dynamics and the time between updates (Δ_2 - Δ_1). If a sample at time Δ_2 exceeds threshold and falls within the window, the two successive processed amplitudes are summed. The position (beam number) of the new sample is noted and an expected position is extrapolated to the next time, Δ_3, along with a new track window.

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*The particular shape of these mean value curves is determined by the signal-to-noise ratio, the spectral density function of the signal and the array beam patterns.

Tracks that are generated in this fashion are next subjected to sequential threshold testing. Those tracks which exceed the upper threshold are then passed to the joint multi-receiver likelihood ratio formation portion of the computer processor.

4.3 Likelihood Ratio for Decision Making

The Sequential Likelihood Ratio processor decision test uses the track joint log likelihood ratio to accept or reject candidate tracks as signal-plus-noise or noise alone. The likelihood ratio $\ell(x)$ is defined by

$$\ell(x) \triangleq \frac{p_1(x)}{p_o(x)} ,$$

where $p_1(x)$ and $p_o(x)$ are the probability density functions associated with the hypotheses H_1 and H_o, respectively. The likelihood ratio is shown graphically in Fig. 4-3. In the case at hand, H_1 is the hypothesis that the track is signal-plus-noise and H_o is the hypothesis that the track is noise alone. The likelihood ratio plays a fundamental role in statistical decision theory. It may be shown that when the costs of incorrect decisions* are unknown, the probability of detection may be maximized for a fixed false alarm probability by a threshold test on the likelihood ratio.**

*The possible incorrect decisions are deciding that the track is signal plus noise when the track is noise or vice versa.

**C. W. Helstrom, "Statistical Theory of Signal Detection," MacMillan, New York, 1960.

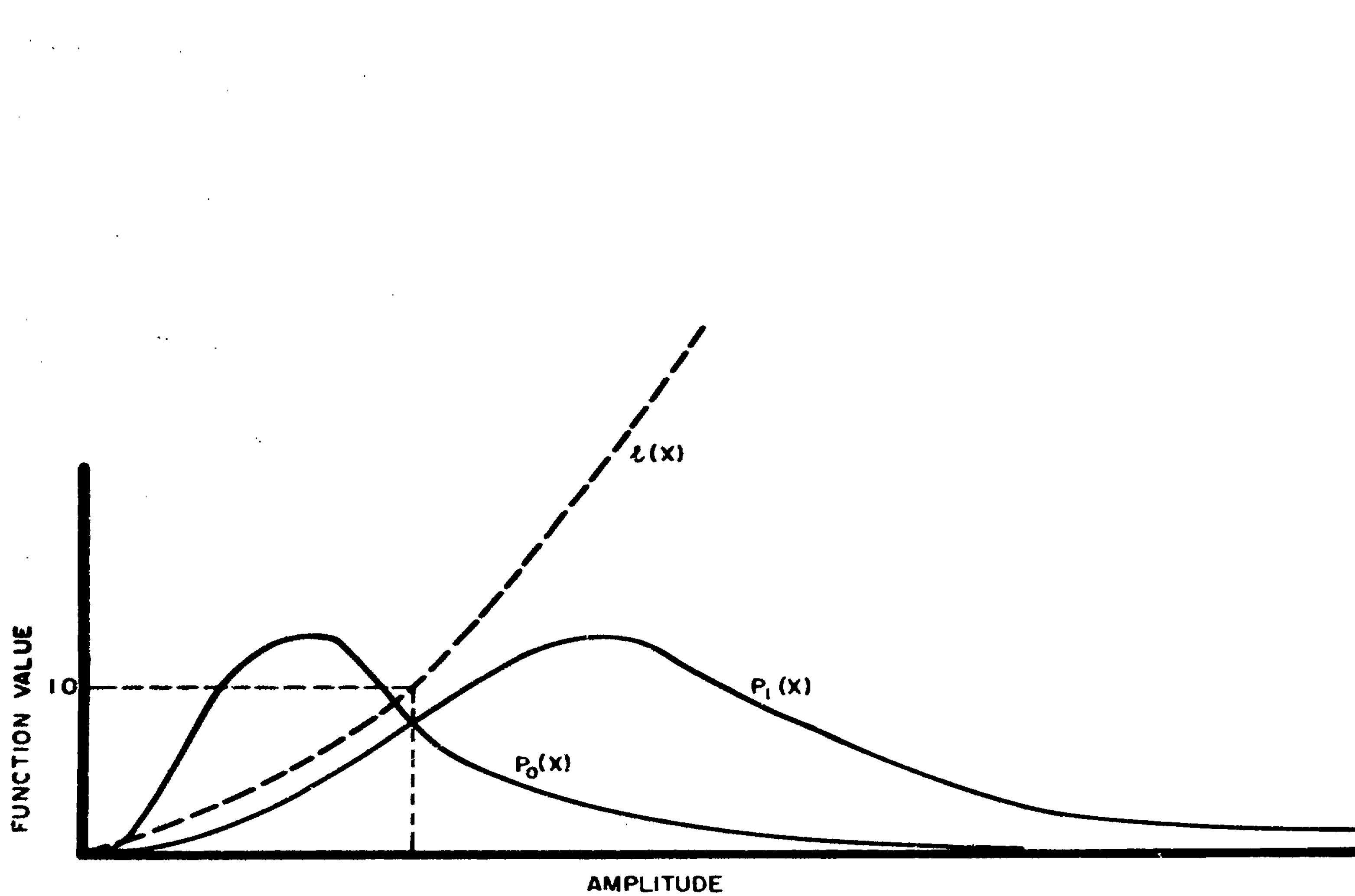

FIG. 4-3 - TYPICAL PROBABILITY DENSITY FUNCTIONS AND LIKELIHOOD RATIO

If n observations of the quantity x are to be made at separate points in time, or specifically after successive integration times, thereby resulting in the sequence $(x_1, x_2, x_3, \ldots, x_n)$, then a joint likelihood ratio, $\ell(x_1, x_2, x_3, \ldots, x_n)$, can be defined based upon the n dimensional probability density functions, $p_1(x_1, x_2, x_3, \ldots, x_n)$ and $p_o(x_1, x_2, x_3, \ldots, x_n)$, similar to $p_o(x)$ and $p_1(x)$. The joint likelihood ratio is then

$$\ell(x_1, x_2, x_3, \ldots, x_n) \triangleq \frac{p_1(x_1, x_2, x_3, \ldots, x_n)}{p_o(x_1, x_2, x_3, \ldots, x_n)} .$$

If the observations $(x_1, x_2, x_3, \ldots, x_n)$ can be considered statistically independent, then the appropriate multidimensional probability density function can be described as the product of the individual probability density functions; thus,

$$\ell(x_1, x_2, x_3, \ldots, x_n) = \frac{p_1(x_1) \cdot p_1(x_2) \cdot p_1(x_3) \cdot \ldots \cdot p_1(x_n)}{p_o(x_1) \cdot p_o(x_2) \cdot p_o(x_3) \cdot \ldots \cdot p_o(x_n)} .$$

This yields a significant simplification in the determination of processor output statistics, and leads to the suggestion of the log likelihood ratio, $L(x_i)$, which is formed by taking the logarithm of $\ell(x_i)$, thus

$$L(x_i) \triangleq \text{Log}\,[\ell(x_i)] = \text{Log}\left[\frac{p_1(x_i)}{p_o(x_i)}\right],$$

$$L(x_i) = \text{Log}\,[p_1(x_i)] - \text{Log}\,[p_o(x_i)], \text{ and}$$

$$L(x_1, x_2, x_3, \ldots) = L(x_1) + L(x_2) + L(_3) + \ldots, + L(x_n).$$

The procedure of adding rather than multiplying lends itself quite well to a digital computer; however, the process of taking logarithms can be time consuming. Fortunately, for the assumed processor the log likelihood ratio is a linear function of the input data. For more general systems where the input statistics are Gaussian the log likelihood ratio is approximately a linear function of the input data. These results are discussed in the next section.

4.4 Log Likelihood Ratio Equation

4.4.1 Derivation of the Log Likelihood Ratio for the Wideband Passive System - The statistical distribution of the output of a square law envelope detector operating on a zero-mean Gaussian signal is well known.* If the bandwidth-averaging time ($\beta\tau$) is equal to unity, and the output is distributed as an exponential density, then

$$f_{S+N}(x) = \frac{1}{\sigma_N^2(1+\rho)} e^{-x/(1+\rho)\sigma_N^2} \quad x>0 \tag{4-1}$$

where $\rho = \sigma_S^2/(\sigma_N^2)$ is the power signal-to-noise ratio. It is mathematically convenient to set $\sigma_N^2 = 1$. This simply means that the output is scaled by the reciprocal of the noise variance.

In the wideband passive receiver M independent envelope samples are integrated to generate one output sample, y. The sum of M exponential random variables is a Gamma random variable and the density function is

$$f_{S+N} = \frac{y^{M-1}}{(1+\rho)^M (M-1)!} e^{-y/(1+\rho)}, \quad y>0. \tag{4-2}$$

*Helstrom, op. cit.

In this case M is equal to the β--product of the wideband receiver, M = (3000 - 150)·120 = 342,000.

The noise density function may be obtained by setting $\rho = 0$ in Eq. (4-2). The likelihood ratio, $\ell(x)$, is easily found to be

$$\ell(y) = f_{S+N}(x)/f_N(x)$$

$$= \frac{y^{M-1}}{(1+\rho)^M (M-1)!} e^{-y/(1+\rho)} \Big/ \frac{y^{M-1}}{(M-1)!} e^{-y}$$

$$= e^{y\rho/(1+\rho)} \Big/ (1+\rho)^M \quad .$$

and the log likelihood ratio, L(y), is

$$L(y) = \log \ell(y) = y\rho/(1+\rho) - M \log (1+\rho). \qquad (4\text{-}3)$$

The log likelihood ratio equation is a simple linear equation of the output of the passive receiver. This simplifies the initial SLR processings.

4.4.2 Generality of Log Likelihood Ratio Equation - A particular wideband passive processor has been assumed and analyzed. A question arises as to the applicability of the results obtained using this processor and its associated statistics. It turns out that the results may be applied to a wide variety of processors. Since the integration time is long and the bandwidth large, the number of independent samples summed is very large. By the central limit theorem of probability theory one can assume the output will be Gaussian to a very good approximation. This will be true for almost any type of distribution of the input into the integrator.

Hence, it is seen that the output statistics will be of similar form regardless of the type of envelope detector, although means and standard deviations may vary from processor to processor. To obtain a comparison with previous results, it is necessary to make the important assumption that the standard deviation at the output of the envelope is the same for noise alone as for signal-plus-noise. For small input signal-to-noise (less than -20 dB) this has been found to be approximately true of a variety of processors.* At higher signal-to-noise ratios this approximation is bad; however, it is the small signal-to-noise ratios that the SLR will aid in detecting while the higher signal-to-noise ratios will be detected regardless. As a practical matter, this assumption does not impair the ability of the SLR processor.

For purposes of comparison with Section 4.4.1, let the noise standard deviation be equal to unity and furthermore let the output be shifted so the output noise samples have mean M. If the signal-to-noise ratio is ρ after envelope detection, that is, equal to the signal-to-noise used in Section 4.4.1, then the signal-to-noise ratio after integrating is

$$\rho_{out} = \sqrt{M}\,\rho = \frac{\mu_{S+N} - M}{\sqrt{M}} .$$

Solving for the output signal mean,

$$\mu_{S+N} = (1 + \rho)M.$$

The output signal-plus-noise density is given by

$$f_{S+N}(y) = \frac{1}{\sqrt{2\pi M}}\, e^{-\frac{(y - (1+\rho)M)^2}{2M}} \qquad (4\text{-}4)$$

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*Leon Camp, "Underwater Acoustics," Wiley-Interscience, New York, 1970.

and the output noise density by

$$f_N(y) = \frac{1}{\sqrt{2\pi M}} e^{-\frac{(y-M)^2}{2M}} \quad . \tag{4-5}$$

The likelihood ratio may be found from

$$\ell(y) = f_{S+N}(y)/f_N(y)$$

$$= e^{\rho y - M(2\rho + \rho^2)/2}$$

and the log likelihood ratio is

$$L(y) = \rho y - M(\rho + \frac{\rho^2}{2}) \, .$$

If ρ is very small, and it is for the region of interest,

$$L(y) = \rho y - M\rho \tag{4-6}$$

by neglecting the second order term in ρ. In comparison, Eq. (4-3) for the assumption of a very small ρ yields

$$L(y) = \rho y - M\rho$$

since $\log(1 + \delta) = \delta - \frac{\delta^2}{2} + \frac{\delta^3}{3} - \ldots.$

Therefore, the same approximate likelihood ratio is obtained and the statistics (after scaling and mean shift) are nearly the same. Since the above properties, output statistics and log likelihood ratios, determine the SLR performance, the use of the square-law model should not be too restrictive.

4.5 Sequential Likelihood Ratio Testing

As stated previously statistical tests based on the likelihood ratio, or equivalently the log likelihood ratio, are optimal in the sense of maximizing the probability of detection for a fixed false alarm probability. When multiple track samples are considered, there is another possible improvement. The technique is known as sequential testing, and requires that two thresholds, T_L and T_D be established. In the case at hand, the threshold T_L, with $(\underline{x}) = (x_i, x_{i+1}, \ldots, x_{j-1}, x_j)$, is chosen such that if the value of the log likelihood ratio, $L(\underline{x})$, falls below T_L, the decision is made that H_o is true, that no target is present. Thus, the track is rejected as noise, and the testing chain stops.

Similarly, T_D is chosen such that if $\ell(x)$ exceeds T_D, the decision is made that H_1 is true that a target is present. This completes the detection process in a sense, but in our application the testing procedure does not stop. Rather, the sequential testing continues and forms an automatic track. If the value of $L(x)$ lies between the thresholds, that is, if

$$T_L < L(\underline{x}) < T_D ,$$

then the decision is made to retain the track, but not display it. Following this, another sample is taken, $L(\underline{x})$ is updated, and the new $L(\underline{x})$ is compared with T_L and T_D.

This process is very similar to the random walk problem, and it can be shown that eventually, with probability 1, one of the two thresholds will be crossed and a decision will be reached. Three possible outcomes are shown in Fig. 4-4. For the sequential test described above, the average number of samples required to reach a decision is less than the number required for

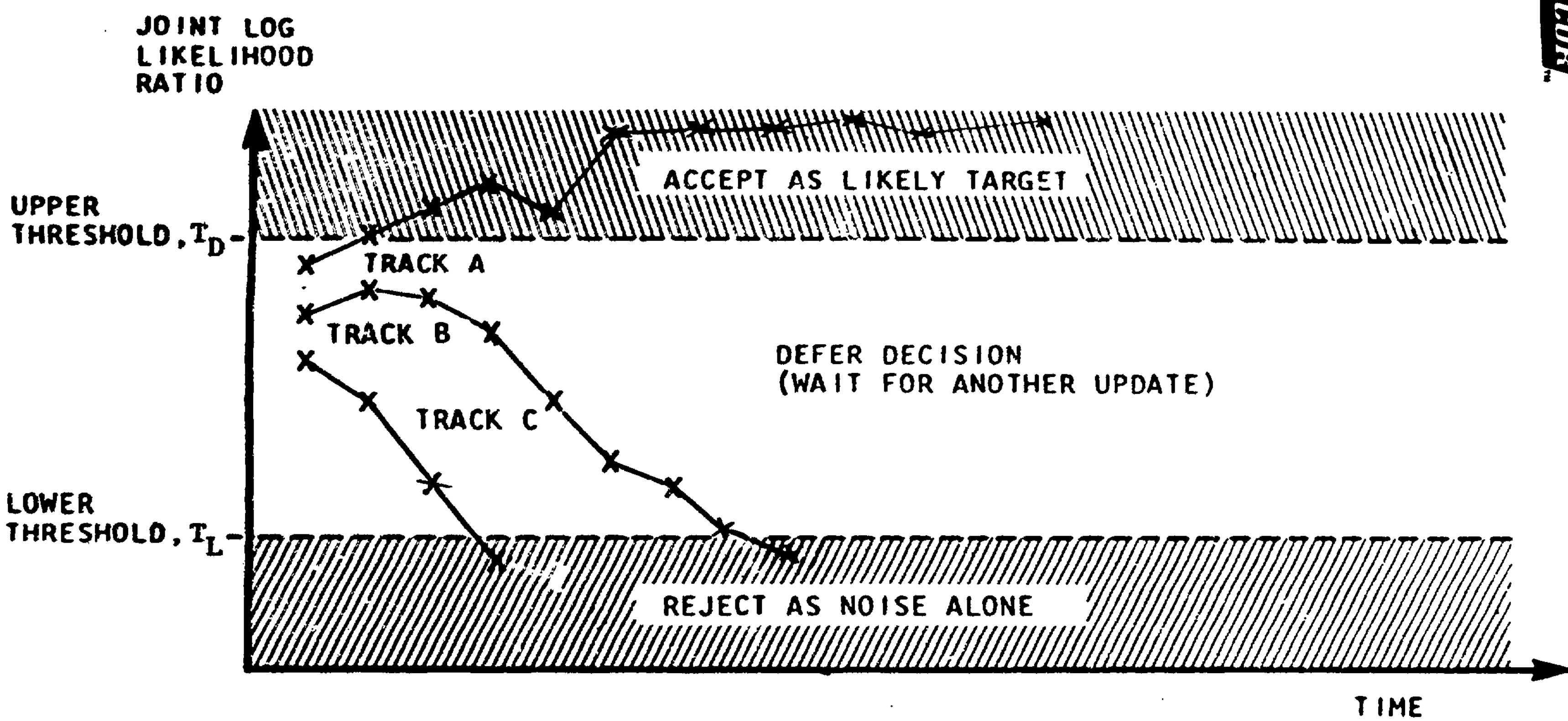

FIG. 4-4 - REPRESENTATION OF THE SEQUENTIAL THRESHOLD TESTING PROCEDURE a fixed sample-size test with the same probabilities of error, α and β. If the track is a target, a greater number of samples may be required for a decision.

It is this lower average number of samples that constitutes an improvement over a rigid, fixed sample-size test. In fact, the conventional processor which uses fixed integration time, and then displays, is equivalent to a fixed sample-size test. The SLR processor is essentially a viable time integrator that will generally integrate signal-plus-noise tracks for longer periods of time than noise tracks, yielding improved performance. That is, for any fixed amount of time (many independent samples), the conventional processor will always integrate all the independent samples, whether noise or signal-plus-noise. In the SLR processor, if several updates are considered in this same period of time, some noise tracks will be rejected, reducing the average integrated noise level. A few signal-plus-noise tracks will be rejected also (the number depends on track signal-to-noise ratio, design signal-to-noise ratio, etc.); however, much fewer signal-plus-noise tracks will be rejected than noise tracks. The integrated signal-plus-noise level will be virtually unchanged on the average for signal-to-noise ratios at or above the design signal-to-noise ratio. When this integrated signal-plus-noise level is compared with the reduced noise level, the gain in performance is realized.

4.6 Tracking Algorithm

The tracking procedure has been described briefly in Section 4.2. The primary purpose of the tracking algorithm is to eliminate target tracks whose movements are inconsistent with those of an actual maneuvering submarine, rather than to achieve an accurate estimate of the target's position and motion. The tracking process is shown diagrammatically in Fig. 4-5. For simplicity of illustration, the figure shows only one event that crosses the input

TRACOR

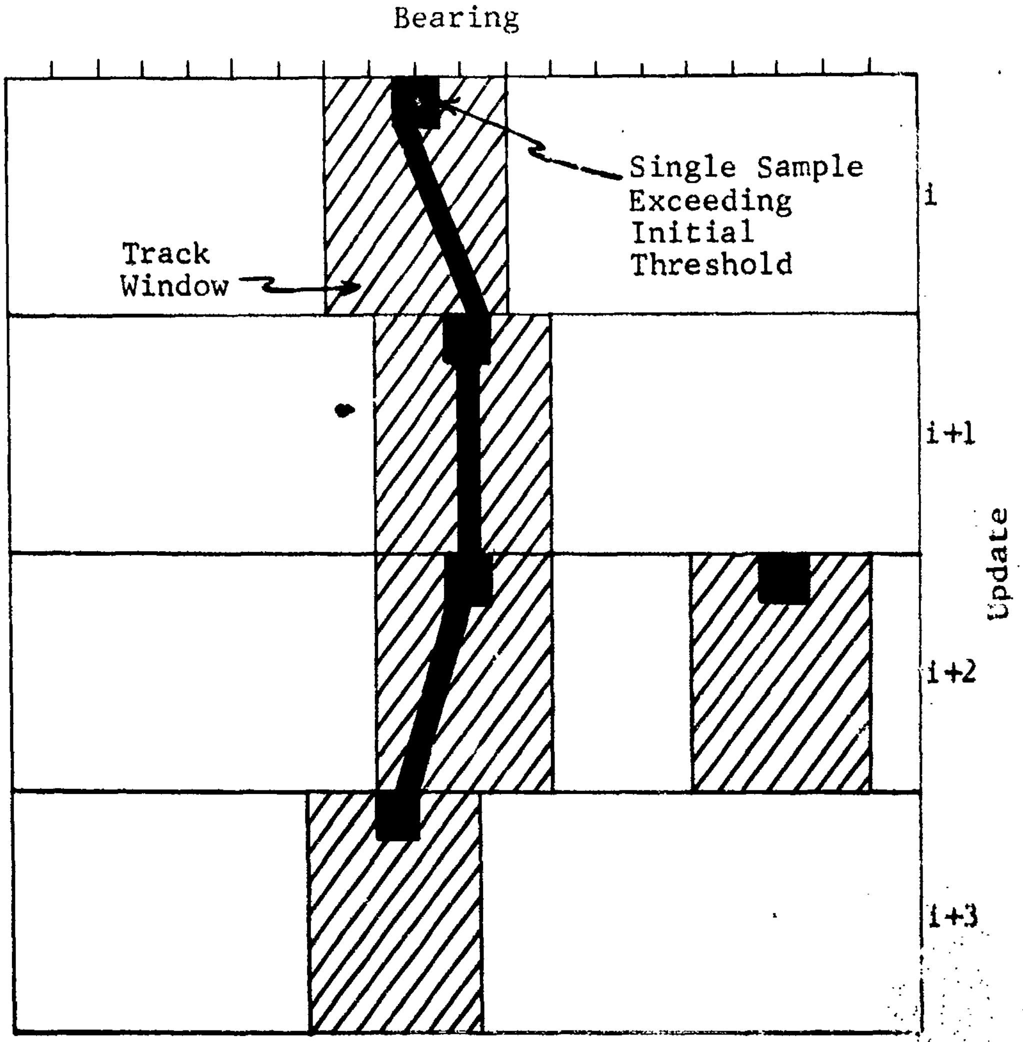

FIG. 4-5 - ILLUSTRATION OF TRACKING ALGORITHM FOR WIDEBAND PASSIVE SLR PROCESSOR

threshold, that is, its single sample log likelihood ratio crosses the lower decision threshold, T_L. The expected allowable target movement in bearing is shown by the shaded area. On update i+1 another single update event is observed within the established track window. The joint log likelihood ratio is calculated for the two track samples. If the track statistic is above the lower decision threshold, the track is continued and another tracking window established. If the statistic is below the threshold, the track is rejected and deleted from the computer. The track is shown in Fig. 4-5 as it continues through several updates. If a single update event does not occur within an established track window due to signal fluctuations, the track is not automatically deleted. Rather the track log likelihood ratio is degraded and if it is still above the lower decision threshold, the track is projected ahead. Hence, track continuity may be maintained on a fluctuating target signal.

In the actual implementation many tracks may start on any update and be tracked simultaneously. The tracking procedure is sufficiently simple that a modest* computer may be used to accomplish all necessary operations in real time.

4.7 Summary

4.7.1 Introduction - This section describes the computer process designed to accomplish SLR processing on multibeam output data from a wideband passive sonar signal processor. This computer process is implemented on a UNIVAC 1108 digital computer. The characteristics of this particular implementation are such that the process may be implemented on a reasonably modest, state-of-the-art digital computer, such as can be found on board newer ships, or alternatively, on special purpose digital hardware.

*By modest, we mean a computer similar to the AN/UYK-7 computer with approximately 20% of core and execution time dedicated to the wideband passive SLR system.

The overall purpose of the SLR process is to produce a sonar display with reduced noise background marking, wherein the digital processor can perform long term integration for any single update events which are not large enough to display initially. In this manner, the sonar operator may remain alerted for longer periods of time, as well as becoming alerted earlier than with only the conventional processor. This process has been designed as a function which can be inserted into a conventional wideband passive sonar processing system between the output of the signal processor and the cathode ray tube (CRT) display or as input to the multireceiver automatic alerting system. The primary requirement for its implementation is a digital computer with sufficient capacity, or specialized digital hardware.

The information flow in the SLR computer process is shown in Fig. 4-6. The remainder of this section is devoted to a more detailed explanation of the process.

4.7.2 Preliminary Data Reduction - For the purpose of this explanation it is assumed that the output of the sonar signal processor is time and bearing normalized.* Thus, the normalized data from the current update are processed first by the Preliminary Data Reduction section. This section has three purposes. First, the data received are grouped into single-update event packages and, if necessary, are converted from analog to digital format. These single-update event packages contain bearing and amplitude information of each data point from the processor output. In order to facilitate digital computer processing with the SLR method, the parameters which describe a single update event package are divided into bearing resolution cells, normally beam outputs.

*This is a basic assumption and is necessary in order to evaluate correctly the likelihood ratio of the output sample for each time and bearing resolution cell.

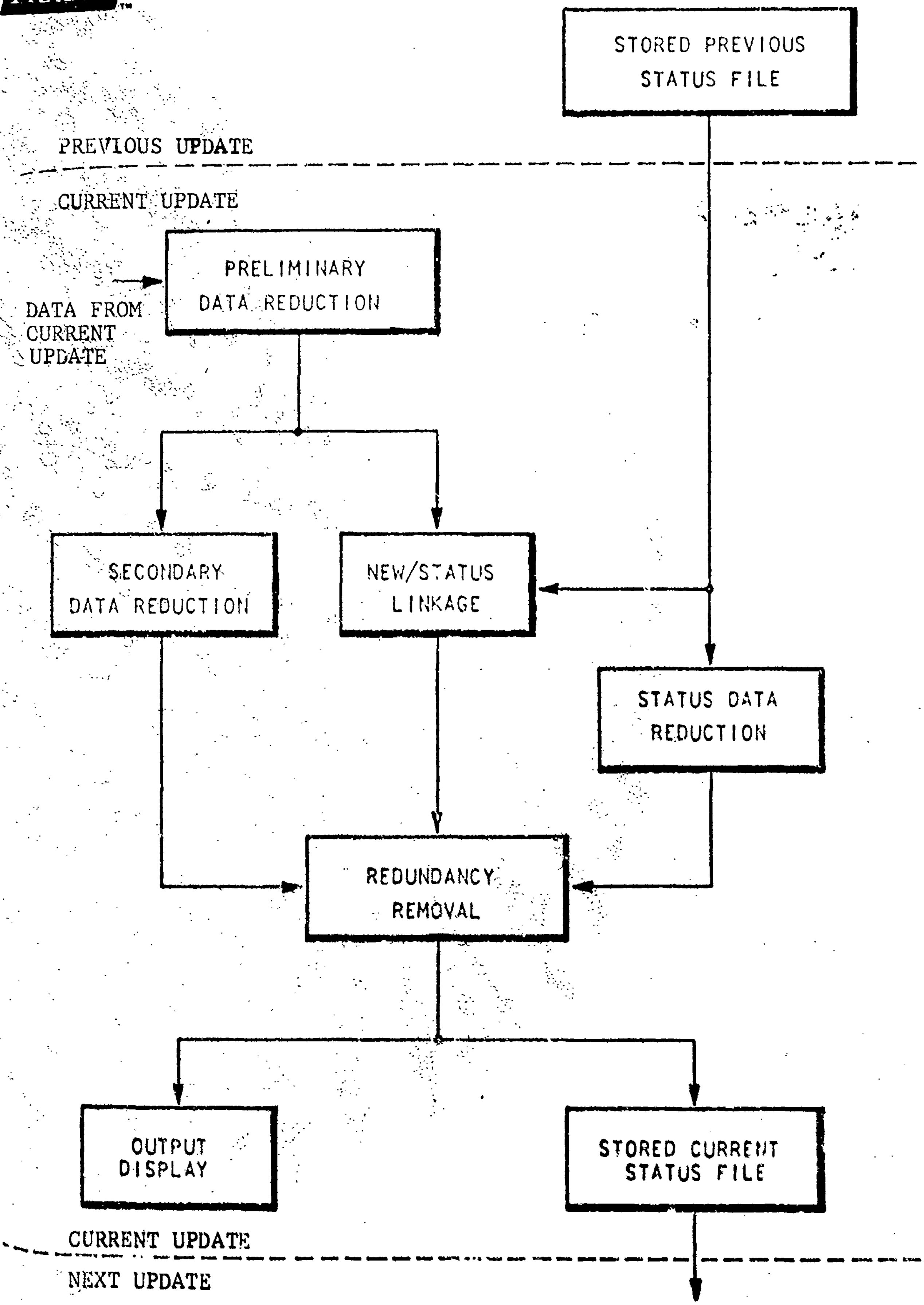

FIG. 4-6 - GENERAL DATA FLOW IN THE SLR PROCESS

Second, the section performs a preliminary or initial thresholding function mentioned earlier. Third, the amplitude of each data point which passed the preliminary threshold is mapped to the logarithm of its likelihood ratio, using the linear equation derived previously. The output of the Preliminary Data Reduction section is passed to two sections, New/Status Linkage, and Secondary Data Reduction.

4.7.3 New/Status Linkage - The New/Status Linkage section receives two inputs; one is the reduced single update sonar output from the Preliminary Data Reduction section, and the other is the series of multiupdate event packages containing the joint log likelihood ratio, beam number, and projected search window from the previous status file. Each event package is called one status unit.

4.7.3.1 Status File - Each event package, or status unit stored in the status file is represented by four functional quantities which are listed below:

1. The event position vector made up of the coordinates from the preceding update;
2. The expected event position vector made up of the predicted coordinates for the current update;
3. The search area or volume for the current update; and
4. The joint log likelihood ratio resulting from the previous updates.

The number of dimensions of position vectors and search areas depends upon the sonar system. That is, the number of dimensions depends upon the number of coordinates that can be measured for each sample by the sonar system, in this case bearing only. The search area or volume defines the region centered about the expected position vector within which legitimate linkages can occur during the current update with the event logged in the status unit.

4.7.3.2 Linkage Process - The New/Status Linkage section compares each status unit with the single update event packages from the reduced sonar output. If the single update event position vector lies within the search area of the status unit, the single update event is said to be linked with the status file entry. When this situation occurs, the joint log likelihood ratio of the new multiupdate event is formed by the process described earlier in this section and is then tested against the lower decision threshold, T_L.

If this new joint log likelihood ratio is greater than T_L, then a new status unit is formed, with information from the old status unit being processed in conjunction with the single update event package to generate a new event position vector, a new estimated position vector, and a new search area for the new status unit. If the new joint log likelihood ratio is less than T_L, then hypothesis H_0 is chosen and the track linkage is discarded, precluding the calculation of a new status unit.

A status unit is allowed to link with all events which fall within its projected search area. Similarly, a single update event can fall within the search areas of several status units and hence be linked in several ways. This procedure allows many incorrect linkages, but since all incorrect linkages will yield a noise track, the process will decrease the log likelihood ratio and the track will eventually be dropped. The process will reach a steady-state condition in which as many noise tracks are being discarded as are being added, on the average.

4.7.4 Secondary Data Reduction - The reduced sonar output from the Preliminary Data Reduction section is processed by the Secondary Data Reduction section. This section tests the log likelihood ratio of the single update event package against the lower decision threshold, T_L, and makes the appropriate decision. If the single update event exceeds the threshold, a new status unit is created on a single update basis.

Upon initialization of the SLR computer process, there are no previously acquired status units, hence the Secondary Data Reduction section is the only section capable of producing a status unit. In each update, it is here that new tracks are started. Note that the entire process does not prevent a single large update sample from being entered into the status file and being placed upon the output display immediately.

4.7.5 Status Data Reduction - The status file information is utilized in two ways in the SLR computer process. As described above, each status unit is furnished to the New/Status Linkage section to determine linkages and form target tracks. Also, the entire status file is passed through the Status Data Reduction section. The purpose of this section is to maintain a strong target track even though the current update did not produce a linkage with this track.

This function is accomplished by assuming that each status unit is linked with a small single update event whose log likelihood ratio was just below T_L, and whose position vector was the same as the expected position vector of the status unit being processed. The search area is enlarged to accommodate the increased uncertainty of target position, and a possible new status unit is formed. The log likelihood ratio of the new status unit is tested against T_L, and the appropriate decision is made.

If the new status unit exceeds the threshold, it is passed to the next processing section. This procedure helps to avoid losing a well-established track because of a single miss, yet a noise track is discarded quickly because of the degradation.

4.7.6 Redundancy Removal - From the above discussion, it can be seen that there are three sections in the SLR process capable of producing status units to be entered into the current status file. The three sections are listed below:

1. New/Status Linkage;
2. Secondary Data Reduction; and
3. Status Data Reduction.

Since these three sections operate independently in generating possible status units, there is a possibility that some of the status units will be redundant, that is, several may have the same predicted location vector and the same present location vector, in terms of resolution cells. This redundancy can be caused in a number of ways. For example, a single update entry may be formed, a linkage may be formed with the single update entry and a track propagation entry may be formed, all with the same present and expected position vectors. The redundancy removal section scans all entries to determine these redundancies and removes all except the status unit with the largest log likelihood ratio.

The output of the Redundancy Removal section is the new status file for the current update. This is placed in storage for the next update, and is made available to the Output Display.

4.7.7 Output Display - The Output Display section is assumed to be part of the original sonar system. Hence, the operator should have control of the display threshold, T_D. By increasing this threshold, the operator can reduce the noise marking to a more acceptable rate with the SLR and retain the same target information as without the SLR processor. When the operator becomes alerted, he can lower the display threshold in order to look at the status file in more detail, since a change in in the display threshold immediately changes what information is displayed. There is no need to wait for past events to accumulate on the display, since the accumulation has already occurred and is stored in the status file.

Note that the SLR processor does not include a fundamental specification of the number of updates over which integration will be carried. Rather, a single status unit could represent a track that has been carried for an indefinite number of updates. Note also that a change in the lower decision threshold does not affect the degree of clutter on the display, but only the amount of processing and storage. Hence there is significant improvement over conventional approaches which allow update-to-update integration only through the operat[illegible] looking at the display, in which it is necessary to operate with a noise marking rate sufficient to allow small signals to mark the display so that the update-to-update integration process may begin.

5.0 ANALYSIS OF THE PERFORMANCE OF THE WIDEBAND PASSIVE SLR PROCESSOR

5.1 Introduction and Summary

After implementation of the SLR processor, its output was analyzed to determine proper parameter values to maximize performance and to measure actual performance. The parameter values that were determined were the design signal-to-noise ratio and the tracking window width. A method was developed to find analytically the design signal-to-noise ratio which is consistent with computer loading and detection performance. A previous study* indicated that reduction of the track window can dramatically increase performance gains. A study was undertaken to assess the performance of the wideband passive SLR with and without bearing tracking.

It was found that for a representative passive array, beam patterns were such that, due to overlap, target track amplitudes built up quickly through the sidelobes of the beams adjacent to the beam occupied by the target. Hence, it is not necessarily advantageous to track between beams. This result does depend on the tactical situation. If the target bearing rate is fast enough, the target will remain in any one beam for only two or three updates, then this conclusion is not valid and linkage between beams is definitely desirable. However, to achieve this bearing rate, the target must be moving at a high rate of speed or be very near the sonar receiver, in either case detection on a single update basis should not be a problem. The performance of the SLR processor was compared to a non-SLR (conventional) processor on the basis of required input signal-to-noise ratio to achieve 0.5 probability of detection at a fixed false alarm rate, both on

*H. A. Reeder, "Simultaneous Likelihood Ratio Processing for Two Active Processors," T71-AU-9594-U, 25 August 1971.

a single update basis and on a "three-out-of-last-five-marks" detection criterion.

5.2 Choice of Design Signal-to-Noise Ratio

In section 4.4 the log likelihood ratio for the wideband passive sonar processor was derived. The key parameter in the equation is the assumed signal-to-noise ratio, $\rho = \sigma_s^2/\sigma_n^2$, at the input to the square law detector. The value of this parameter determines the proper choice between the two alternative hypotheses in the sequential test. That is, the test is conducted between two alternatives: H_0, that the track is noise alone, $\rho = 0$, and H_1, that the track is signal plus noise with signal-to-noise ratio ρ_0. The most desirable test would be that of test between $\rho = 0$ and $\rho > 0$; however, this test is difficult to implement. Wald* has shown that the test used in the SLR processor, that is, $\rho = 0$ versus $\rho = \rho_0$, results in an effective test of $\rho = 0$ versus $\rho > \delta$ where $0 < \delta < \rho_0$. This phenomenon has been observed previously** where it was pointed out that targets with signal-to-noise ratios 3 to 4 dB less than the design signal-to-noise ratio were detected with greater than 0.5 probability. This phenomenon was again observed in this study.

In previous studies the design signal-to-noise ratio was chosen empirically on the basis of observed computer loading and desired detection performance. However for this study a procedure for choosing the design signal-to-noise was developed that takes into account the tracking window size and the probability of detection. The basic procedure may be applied to other previously developed SLR processors. This procedure ignores computer loading

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*A. Wald, Sequential Analysis, John Wiley and Sons, Inc., Second Printing, 1948.

**H. A. Reeder, "Computer Utilization of Sequential Hypothesis Testing for Detection and Classification of Sonar Signals," TRACOR Document No. 67-717-U, 27 October 1967.

which is not a problem in the wideband passive SLR since the number of resolution cells (beams) is limited.

The procedure is essentially to adjust the design signal-to-noise ratio to minimize the signal-to-noise ratio that will give a positive log likelihood value with probability 0.5 after the logarithm of the track window size is subtracted (This is necessary to adjust for noise branching.*). The motivation of this procedure is that any track with a greater signal-to-noise ratio will give a positive log likelihood ratio (adjusted for tracking) with probability greater than 0.5 and, therefore, will tend to integrate above any preset upper threshold.** This is not the sharpest criterion one may establish. For instance a more desirable criterion would be to choose the design signal-to-noise ratio which minimizes the track signal-to-noise ratio that gives 0.5 probability of crossing for a threshold preset to yield a given probability of false alarm. However, this latter criterion is dependent on the probability of false alarm and is more difficult to analyze. The positive log likelihood ratio criterion is very easy to analyze and appears to give nearly the same answer in the wideband passive case.

The log likelihood ratio equation of the assumed wideband passive sonar systems is given by Equation (4-3),

$$L(Y) = Y\rho/(1 + \rho) - M \log(1 + \rho) .$$

Adjusting for a tracking window with N independent samples in it, yields

$$L'(Y) = Y\rho/(1 + \rho) - M \log(1 + \rho) - \log N \qquad (5\text{-}1)$$

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*H. A. Reeder, "Reduction of Computer Requirements for the Sequential Likelihood Ratio Processor," TRACOR Document T70-AU-7242-U.

**This may be related to the Gambler's Ruin problem where the gambling house has infinite resources and a greater than 0.5 probability of winning. Over an arbitrary number of trials, the house will ruin any gambler with finite resources.

Using the fact that the Y's are approximately Gaussianly distributed, a sample from a signal-plus-noise distribution with parameter ρ' will be greater than Z with probability 0.5 if Z is chosen such that

$$\frac{Z - M(1 + \rho')}{\sqrt{(1 + \rho')M}} = 0$$

or

$$Z = M(1 + \rho') \quad .$$

That is, samples from a Gaussian distribution are greater than the mean with probability 0.5. Substituting this value into equation (5-1) yields

$$L(Z) = \frac{M(1 + \rho')\rho}{1 + \rho} - M \log (1 + \rho) - \log N = 0 \ . \qquad (5\text{-}2)$$

Note that using this value we are guaranteed that the

$$\Pr(L(Y) \geq L(Z)) = 0.5$$

for samples from a distribution with signal-to-noise ratio parameter ρ'. Solving Eq. (5-2) for ρ' gives

$$\rho' = (M(1 + \rho) \log(1 + \rho) + (1 + \rho) \log N)/(M\rho) - 1 \ . \quad (5\text{-}3)$$

Expanding the log terms in a power series about $\rho = 0$ and neglecting all terms of power three or greater yields

$$\rho' \approx \frac{\rho}{2} - \frac{\rho^2}{6} + \frac{\log N}{M} (1/\rho + 1) \quad . \qquad (5\text{-}4)$$

We now minimize ρ′ by setting the first derivative of Eq. (5-4) equal to zero.

$$\frac{d\rho'}{d\rho} = \frac{1}{2} - \frac{\rho}{3} - \log N/(M\rho^2) = 0$$

or

$$\rho^2/2 - \rho^3/3 = (\log N)/M \qquad (5\text{-}5)$$

As presently configured, the wideband passive SLR processor allows linkages to the two adjacent beams in addition to the beam on which the target was last observed for a total of three possible linkages; hence, N = 3. As previously derived M = 342,000, the $\beta\tau$ product, of the wideband passive system. Using these parameter values, Eq. (5-5) was solved. The proper design signal-to-noise ratio, converted to dB is -25.9 dB. It is interesting to note that based of a quick empirical survey* the design signal-to-noise ratio had been tentatively set at -25 dB, less than 1 dB difference from the analytically derived design signal-to-noise ratio.

5.3 Performance of the Wideband Passive SLR Processor

The performance of the wideband passive SLR processor was assessed under a variety of operating conditions. Cases considered included tracking in bearing or not, variation of input thresholds, different sonar processor integration times, and variation of design signal-to-noise ratio. For low bearing rate targets (more than 3 updates occurring in each beam), it appears that improved performance may be obtained by not tracking between beams. Also, increased performance may be obtained by using shorter integration times by the sonar signal processor and allowing the SLR processor to do the long term integration.

*H. A. Reeder, "Eighth Quarterly Progress Report," TRACOR Document No. T72-AU-9569-U, 11 August 1972.

5.3.1 Beam Tracking - A study was undertaken to assess the value of tracking between beams. The study considered a marginal signal-to-noise ratio track that starts in one beam and proceeds to the next beam at a rate of 0.5 degrees per update. The signal processor considered is one with similar output statistics to the wideband passive system and is applicable to the case at hand. Figure 5-1 shows the two typical beam patterns that exhibit different overlapping coverage. The tracking experiment was conducted for these two beam pattern cases. The results are presented in Figs. 5-2 and 5-3. The curves give probability of crossing a decision threshold for the SLR operating on the left beam only, right beam only, and both beams. In addition, the probability of marking at least one beam of the single beam only tracker is shown. To calculate this latter probability, statistical independence of the beams was assumed. This is not a good assumption, and the performance of the single beam only implementations will fall between this probability and the maximum of the performance of the left and right beam cases (complete dependence). From Figs. 5-2 and 5-3 it appears that tracking across beams does not yield improved performance. As previously discussed in Section 5.1, this latter conclusion may not be valid for higher bearing rate targets where only two or three marks are observed in any one beam, but this high bearing rate would probably indicate that the target was very close or maneuvering at a high rate of speed. Under these conditions the single update probability of detection should be high.

5.3.2 Effects on Performance of Tracking Window Size and Input Threshold - The effect of tracking in bearing was studied in a slightly different manner than above, as well as the effect of changes in input threshold. The results are presented in the form of curves of probability of exceeding threshold versus input signal-to-noise ratio, where the threshold is set to achieve a specified probability of false alarm. Curves for one to ten updates are

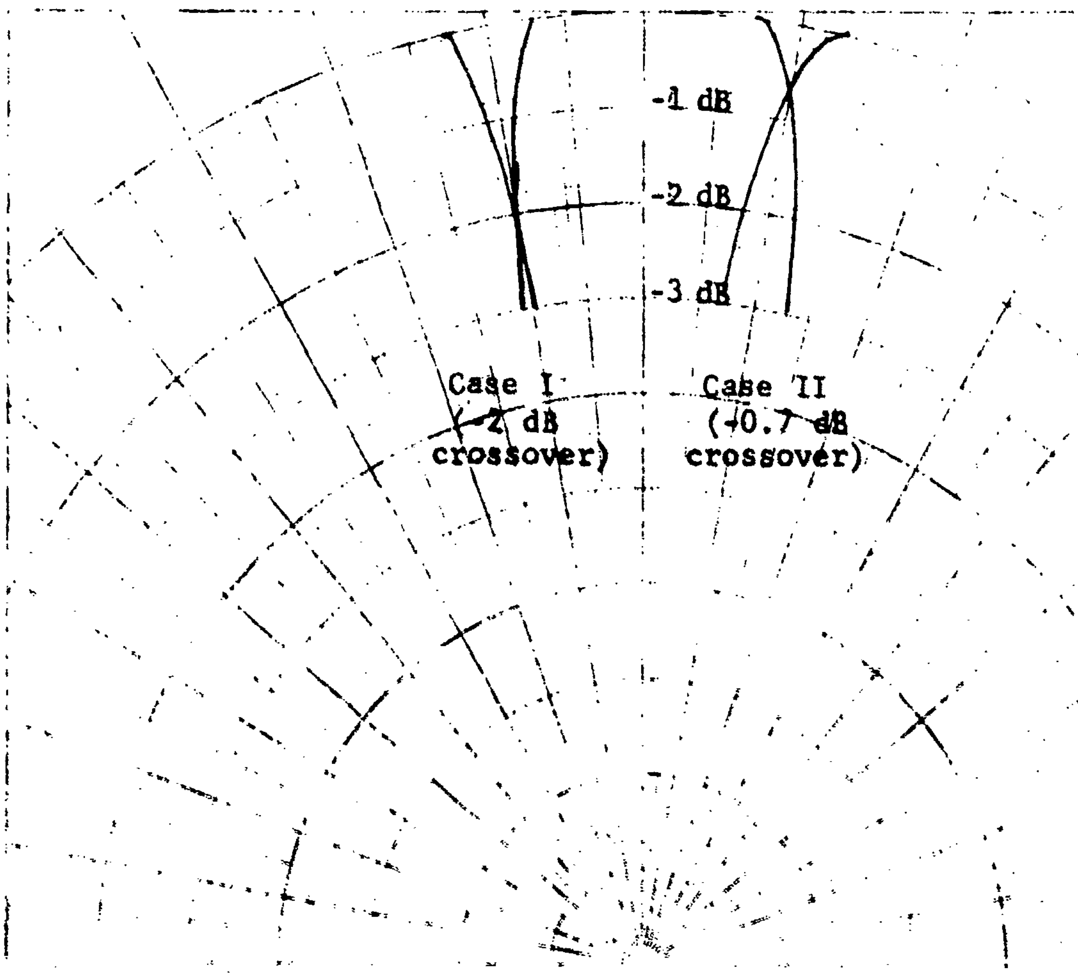

FIG. 5-1 - TYPICAL BEAM PATTERN USED IN TRACKING TEST

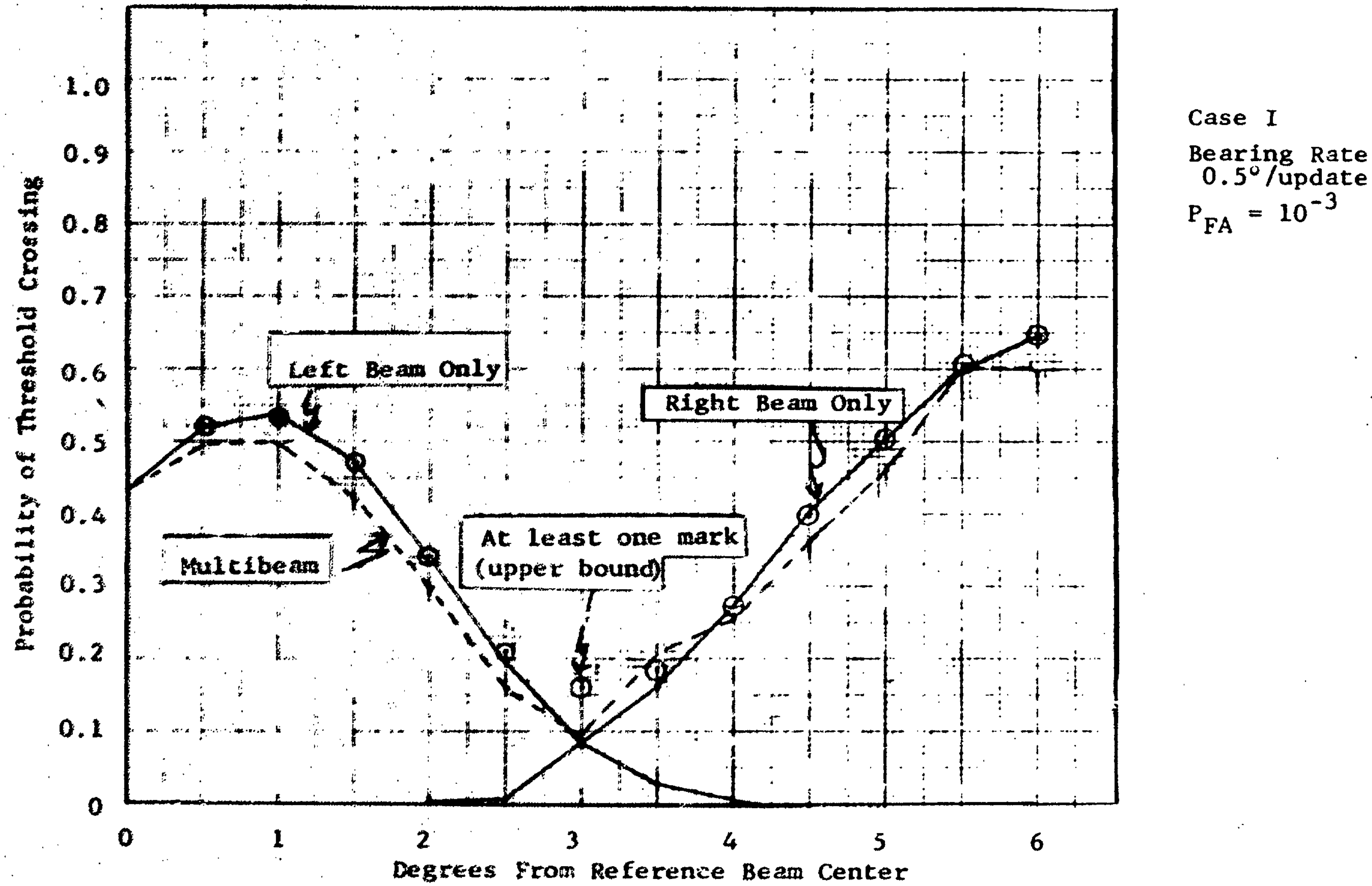

FIG. 5-2 - PROBABILITY OF MARKING FOR MOVING TARGET

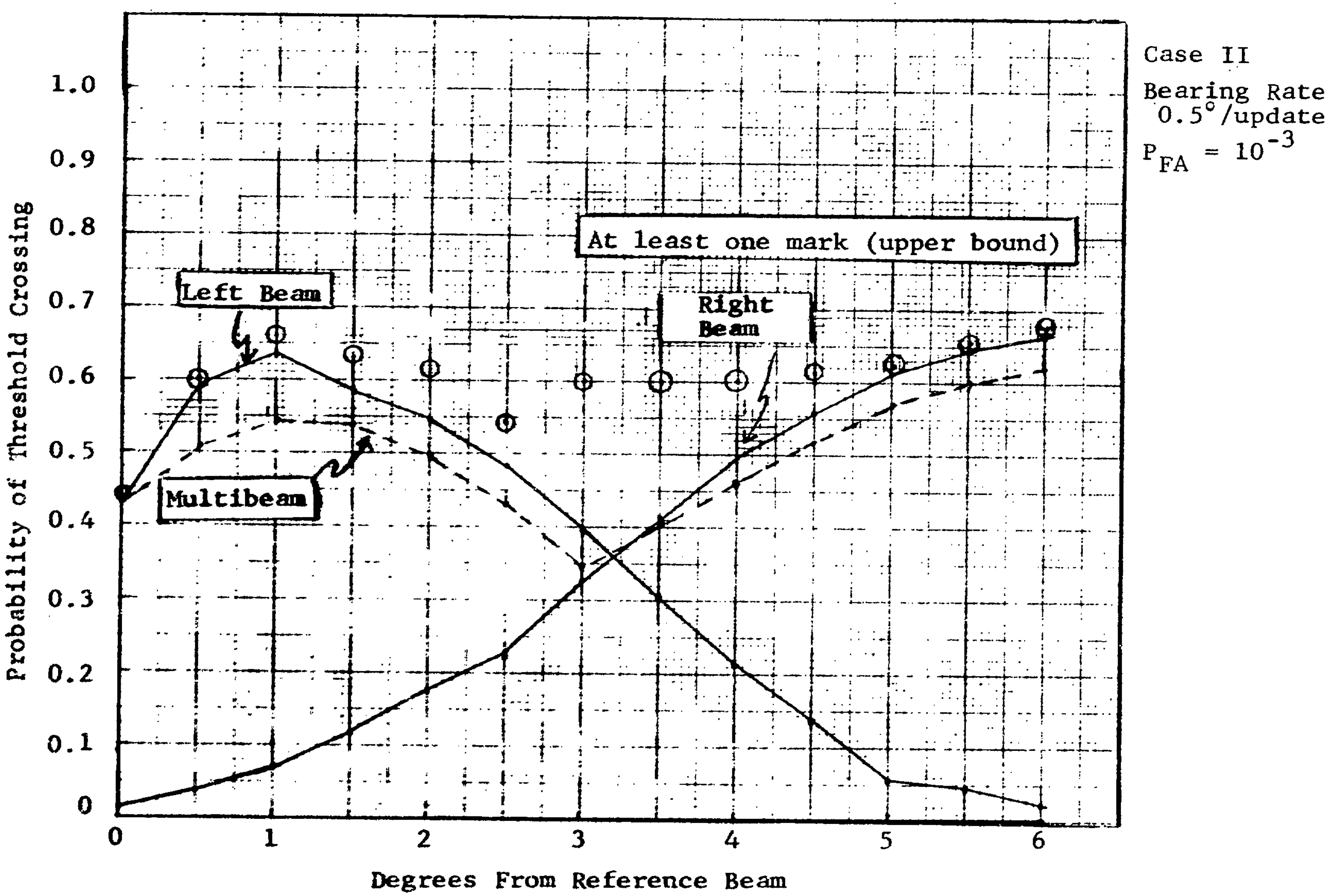

FIG. 5-3 - PROBABILITY OF MARKING FOR MOVING TARGET presented as well as a dashed curve for the probability of threshold crossing for a conventional non-SLR processor on a single update basis. Performance improvement may be assessed by the difference between SLR and non-SLR curves. Figure 5-4 shows performance when a tracking window is used and the input data are thresholded so that only 30% of the noise samples are entered into the computer. Due to the thresholding and the log likelihood ratio tracking correction, performance at lower signal-to-noise ratios is not as good as desired.

By not tracking in beam, i.e., no tracking window, performance may be improved as shown in Fig. 5-5. Strictly speaking, when the tracking window is removed, a new design signal-to-noise ratio should be derived, but for comparison purposes it was desirable to hold this parameter constant. If the analysis in Section 5.2 is carried out for no tracking windows, it indicates that an arbitrarily small design signal-to-noise ratio should be chosen. This is undesirable because the time to reject noise tracks as noise tracks becomes longer and longer. Hence, the design signal-to-noise ratio should be chosen to reject noise tracks in a reasonable length of time. A design signal-to-noise ratio of -25.9 dB accomplishes this.

The complete removal of the input threshold increases performance for the lower input signal-to-noise ratios even further as shown by Fig. 5-6. It might be well to comment here on why the non-SLR outperforms the SLR on a one update basis as shown by the difference in the curve marked (1) and the non-SLR dashed curve. The SLR integrates noise alone tracks for more than one update on the average; therefore, the noise background is integrated to a higher level, increasing the decision threshold the SLR signal track statistic must exceed. For the present configuration, two SLR updates give approximately the same performance as the non-SLR and further updates give much superior performance.

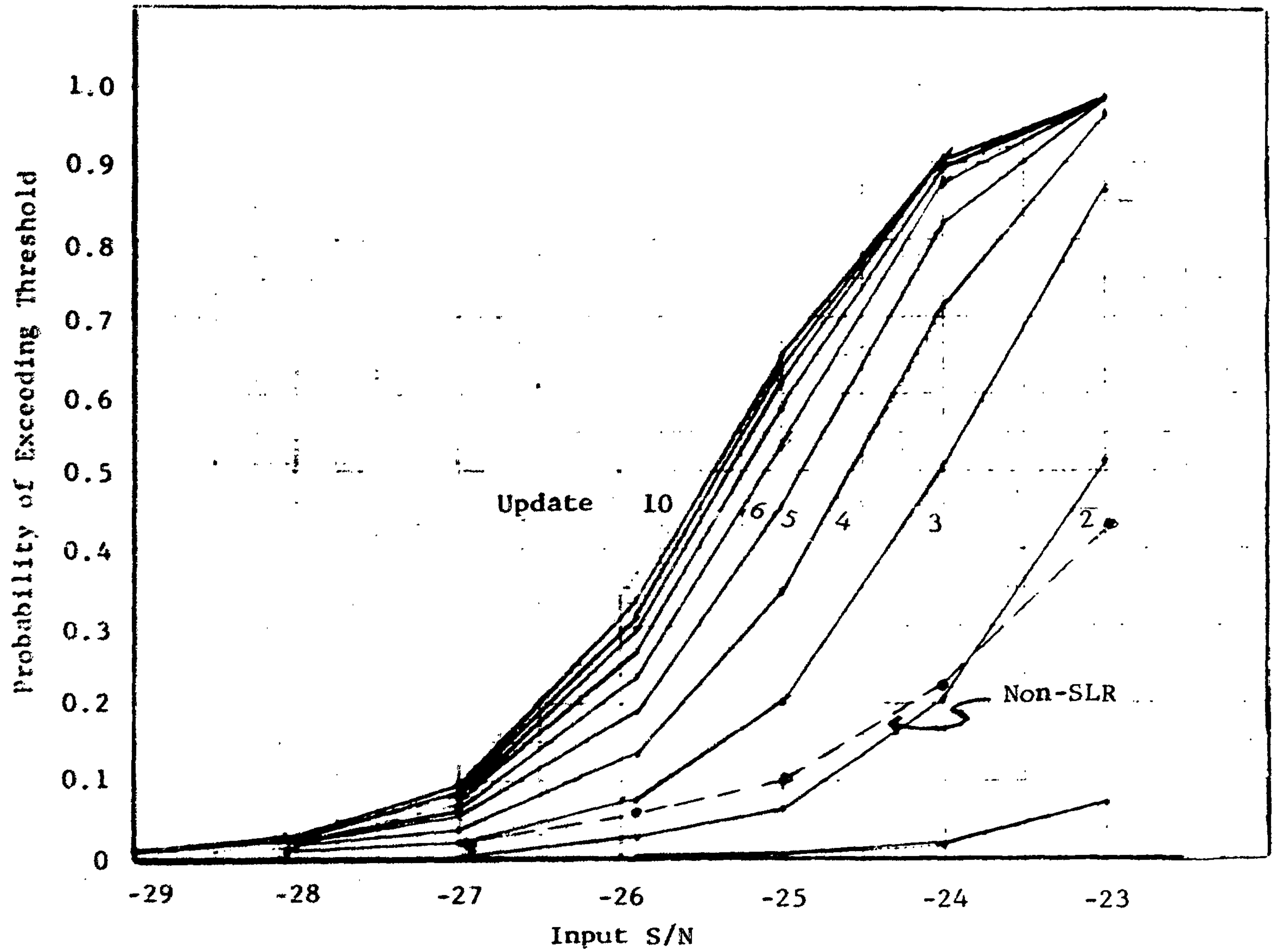

FIG. 5-4 - PROBABILITY OF EXCEEDING THRESHOLD VS INPUT S/N-INTEGRATION TIME 2 MIN/UPDATE-TRACKING WINDOWS-THRESHOLD 30%-S/N_d=-25.9

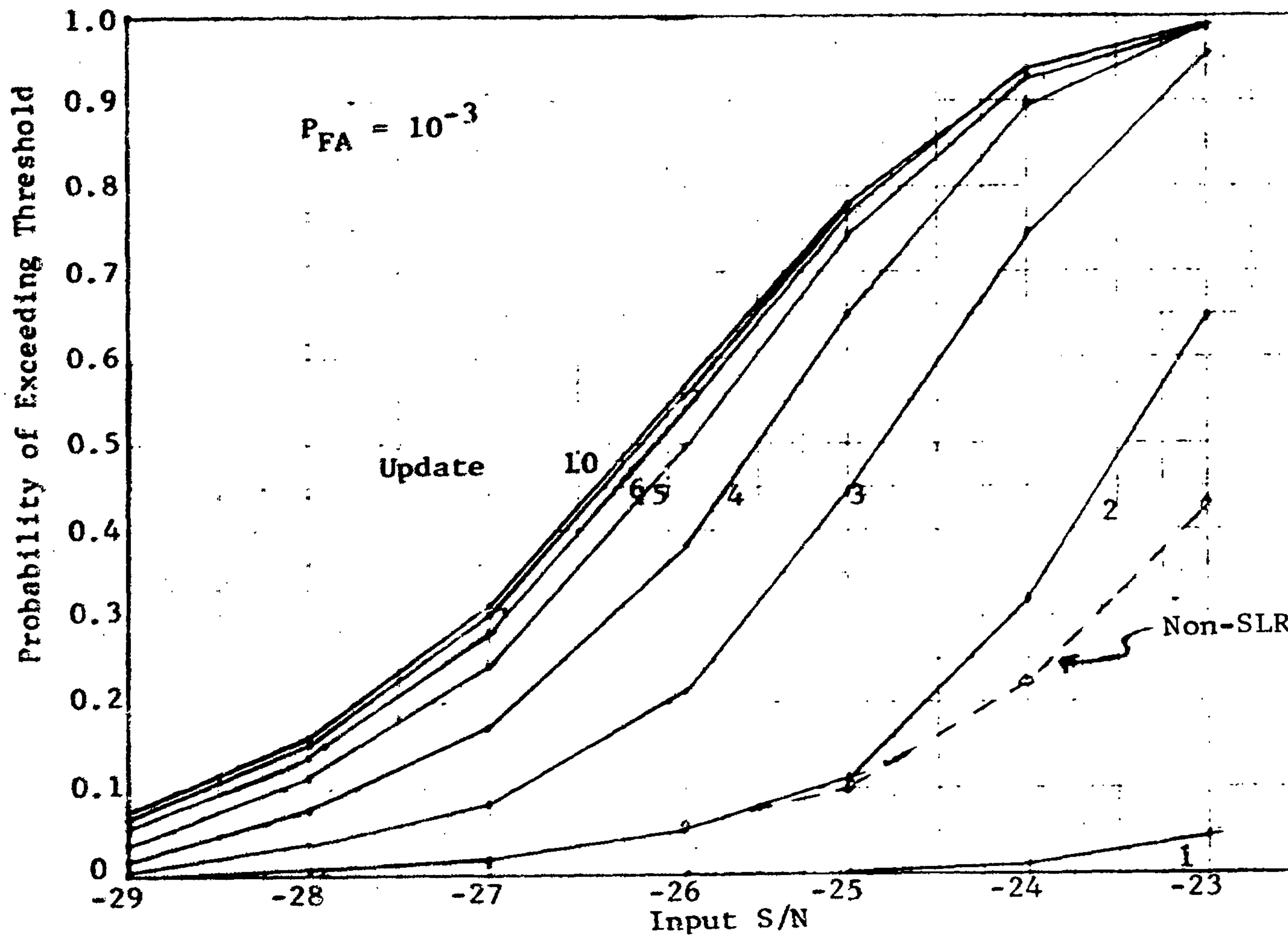

FIG. 5-5 - PROBABILITY OF EXCEEDING THRESHOLD VS. INPUT S/N

Integration time 2 minutes per update. No tracking window. Threshold 30%. $S/N_d = -25.9$

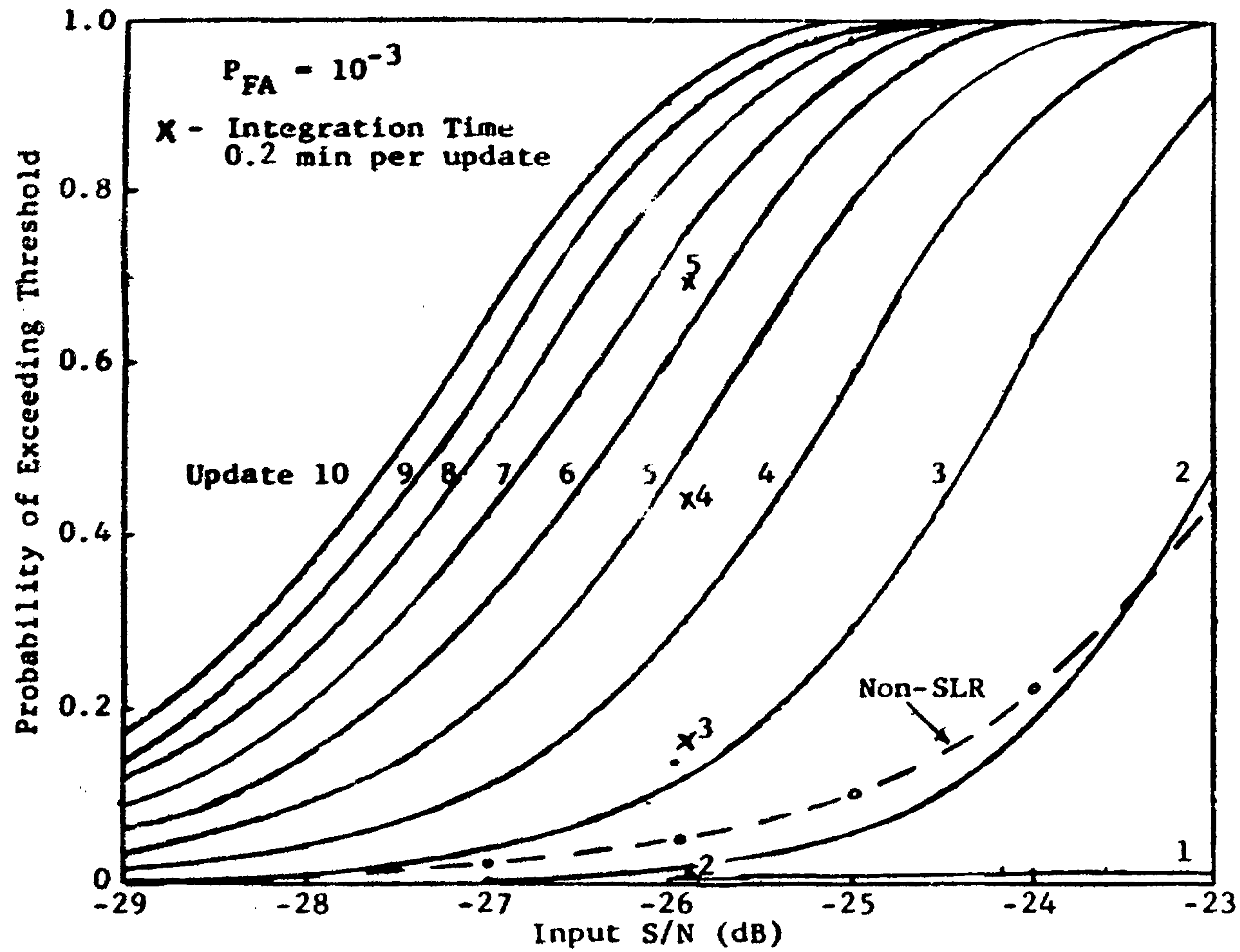

FIG. 5-6 - PROBABILITY OF EXCEEDING THRESHOLD VS. INPUT S/N

Integration time 2 minutes per update. No tracking window. No threshold. S/N_d = -25.9 dB

Another experiment was performed by reducing the integration time of the wideband passive signal processor by a factor of 10 to 0.2 min and allowing the SLR to accomplish the long term integration. Effectively, by considering more rapid updates, noise tracks degrade more quickly in time although the average number of samples to rejection may increase somewhat. The net result is a decrease in the noise integration with respect to the signal-plus-noise track integration. The results are shown for a -25.9 dB signal-to-noise ratio track on Fig. 5-6. The mark numbered (5) is after 50 updates and therefore corresponds in time with curve 5 of the 2 min integration time implementation. Based on this study, it appears that shorter integration times in the sonar signal processor, combined with the SLR processor to accomplish the long term integration, is advantageous.

Figure 5-7 is the same case as Fig. 5-6 except the probability of false alarm has been decreased to 10^{-5}. The results are much the same; however, more updates are required to cross the higher threshold with the same probability.

In Figs. 5-4 through 5-7, the results of SLR processing have been compared with non-SLR processing on a single update basis. When an operator is using a system, he is unlikely to call a detection based on a single update but will use a criterion such as "at least three marks out of the last five consecutive updates" to call a detection. This criterion while somewhat arbitrary appears to be reasonable for the probability of false alarms considered here. Using a method developed by Greenberg,* the probability of satisfying the criterion of at least three marks out of the last five updates for a total of

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*I. Greenberg, "The First Occurrence of n Successes in N Trials," <u>Technometrics</u>, August 1970, page 627.

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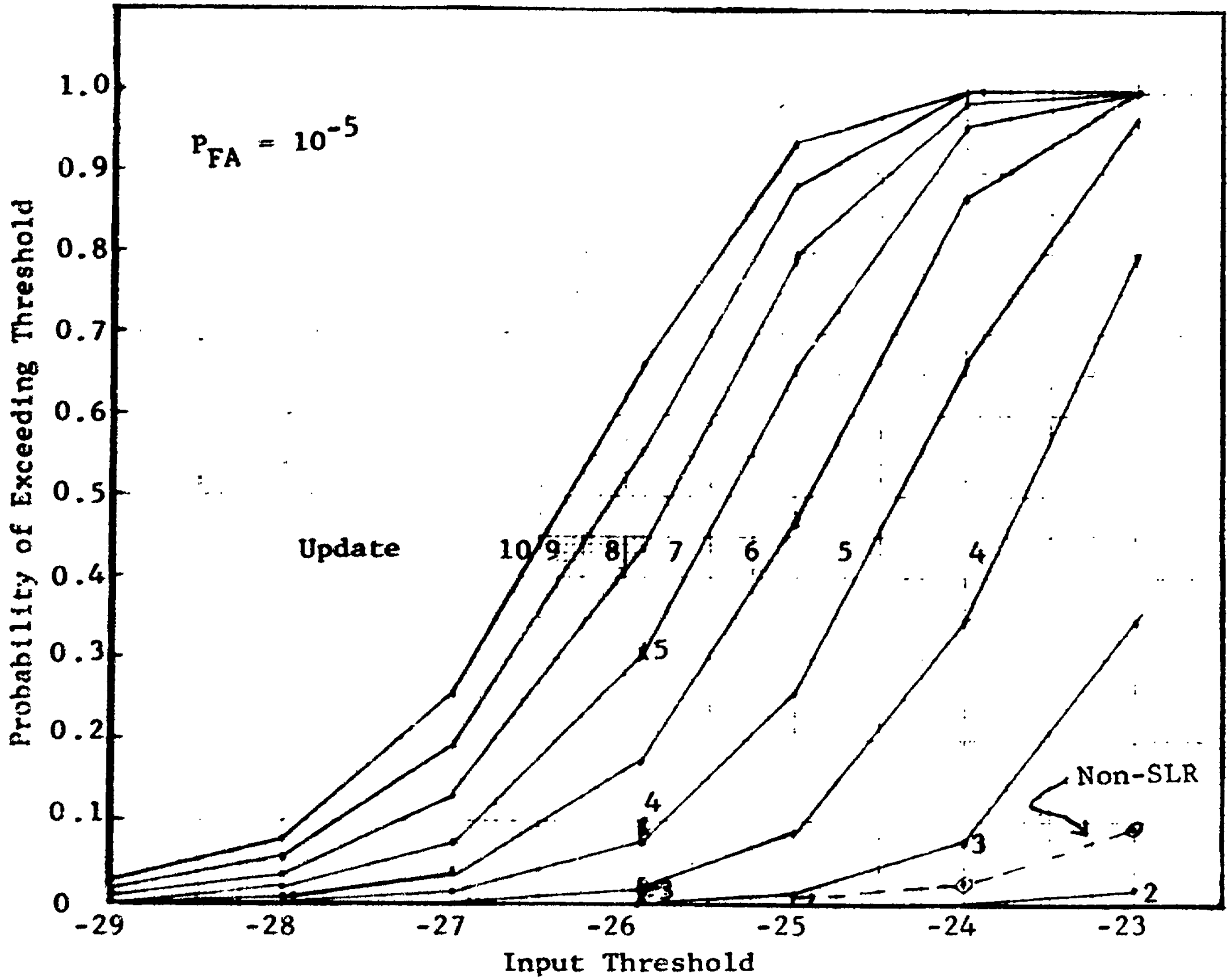

FIG. 5-7 - PROBABILITY OF EXCEEDING THRESHOLD VS. INPUT THRESHOLD.

Integration time 2 minutes per update. No tracking window. No threshold. S/N_d = -25.9 dB ten updates was calculated. The results are presented in Fig. 5-8 as the dashed curves, both for SLR and for non-SLR processed data. The solid curves are the same as Fig. 5-6. Figure 5-8 shows that the probability of an operator calling a detection using this criterion on SLR output data is considerably more on the non-SLR processed data. Another way of looking at these results is to note the required input signal-to-noise ratio for 0.5 probability of detection. For the SLR processor, this input signal-to-noise ratio is -27.1 dB and for the non-SLR processor, -23.3 dB. For this situation, SLR processing gains approximately 3.8 dB considering equivalent detection performance.

5.4 Conclusions

A method has been developed to allow a general purpose digital computer to survey the output of a wideband passive sonar system, select promising target tracks, and integrate them for a variable length of time, and achieve improved performance. This process is based on the sequential likelihood ratio hypothesis test which has many desirable statistical features.

Although the study concentrated on an SLR processor for wideband passive sonar systems, techniques applicable to other SLR implementations were developed. For instance, a method was developed to determine analytically the design signal-to-noise ratio of the statistical hypothesis test. This method is particularly applicable to the wideband passive SLR processor where computer loading is light; however, it has applications in the design of other SLR processors.

The benefits of tracking between beams was studied extensively. It was found that if a target may be expected to remain in a beam for several updates, then tracking between beams is not advantageous. This simplification of the processor reduces

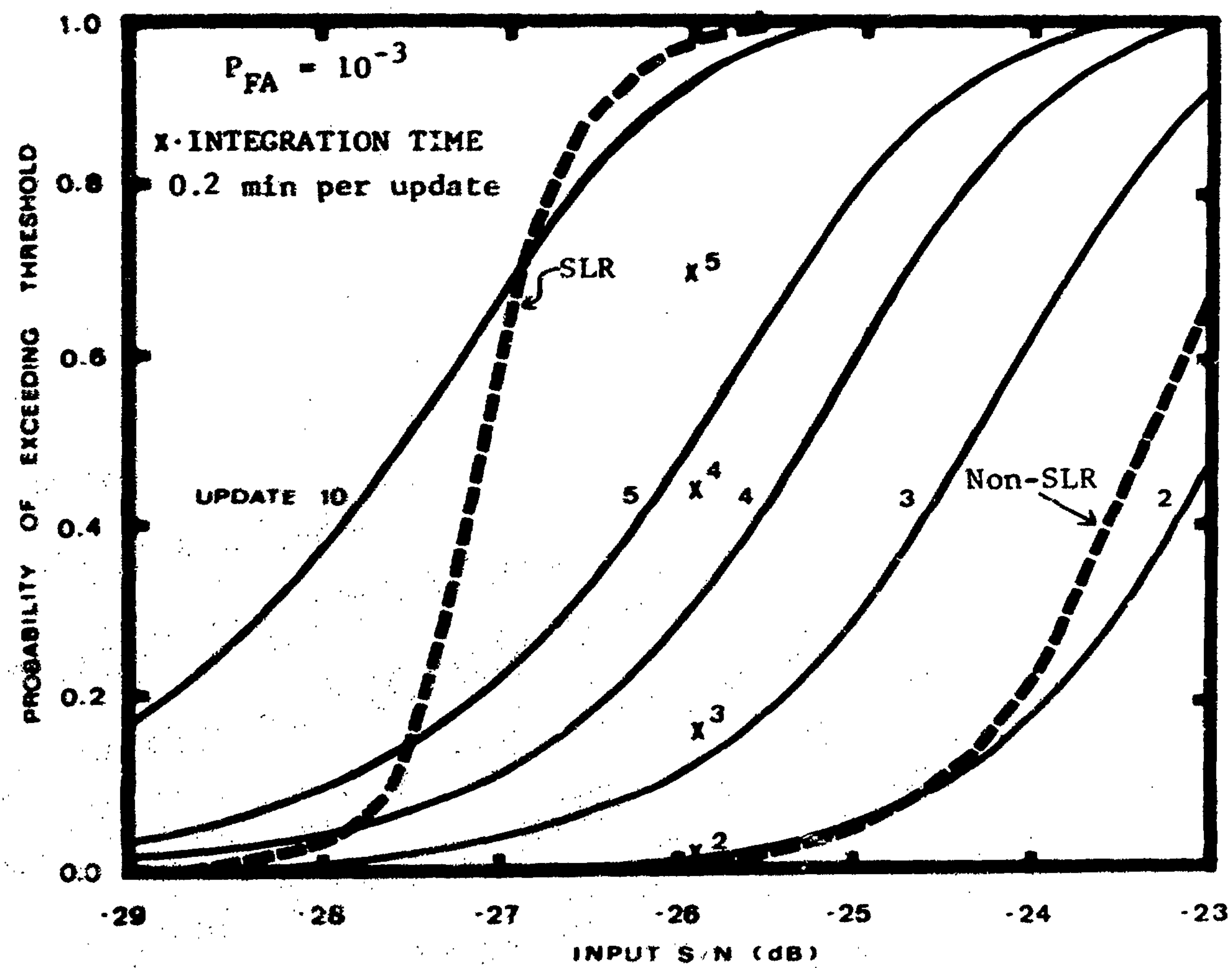

FIG. 5-8 - PROBABILITY OF EXCEEDING THRESHOLD VS. INPUT S/N

Integration time 2 minutes per update. Dashed curves cumulative probability of at least 3 out of 5 possible marks for 10 updates.

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the possibility of spurious noise tracks and yields improved performance.

The most interesting and, potentially, the most important development under this contract was the discovery of the improved performance due to decreasing the sonar processor integration time and allowing the SLR processor to do the long term integration over the increased number of updates. In previous applications it was not possible to take advantage of this fact because the data input was limited to the ping repetition rate of an active sonar system. In a passive system, the update rate is limited only to the time between independent samples. In essence, the sequential test allows the integration of signal-plus-noise tracks for the same length of time as a given fixed integration time, but the noise tracks are integrated for a shorter time on the average. This gives a greater relative difference between signal-plus-noise tracks and noise alone.

In summary, the SLR processor is ideally suited for implementation on the output of passive sonar systems. Available gains appear to be better than with active sonar systems. Also, an important first step has been taken to develop a computer based system to integrate the outputs of wideband and narrowband passive sonar systems in a logical and meaningful manner.

6.0 RECOMMENDATIONS

The SLR processor has progressed from a relatively crude one-dimensional processor into a sophisticated multidimensional, multichannel processor capable of processing large amounts of varied data types. The present emphasis is on adapting SLR processing to the output of passive systems. It is ideally suited for use in a passive system where its full capability of update-to-update integration may be utilized by varying the sonar processor's integration time. In an active implementation the update time is limited by the ping repetition rate which, in turn, is determined by the target's range. In a passive system many more track samples may be considered, exploiting the capabilities of the sequential testing. This present study developed a SLR processor for wideband passive sonar systems. The next logical extension is to apply the technique to the output of a narrowband sonar processsor and TRACOR is under contract* to do this. Also, under that same contract the outputs of the wideband and narrowband SLR processors will be combined as outlined in Section 3.0 of this report.

After successful testing of the combined processor, the next phase should be a laboratory implementation of a complete hardware system to do multichannel narrowband/wideband analysis, normalization, SLR processing, data rate reduction, and display. To accomplish this, careful analysis of system requirements must be done. Hardware may then be obtained and interfaced. A representative sea data base should then be analyzed to choose system software parameters. The ultimate goal of this project would be a real time

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*Contract N00024-73-C-1201.

demonstration of the computer aided processor. From this demonstration, the effectiveness of the computer aided processor to alert a fleet sonar operator may be assessed. This, in turn, will measure the impact of this system on the fleet.

Upon satisfactory completion of this laboratory demonstration, the total hardware system should be installed in fleet unit for an extensive at-sea test. During this operational test the computer aided alerting system should show its greatest utility. When target incidence is low, operators tend to be less alert and it is precisely this situation where the automatic passive alerting system will be most useful.

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13. ABSTRACT

A computer-aided detection processor has been developed for a wideband passive sonar system. The processor may be applied to a variety of passive systems. In addition to the individual processor, a general framework for a computer system that accepts and analyzes the vast quantity of data generated a modern sonar suite has been developed. The output of this computer system is an array of alerting functions that measure the likelihood that a given coordinate vector is the location of a target. The computer-aided detection processor for wideband passive sonar systems is based on the Sequential Likelihood Ratio (SLR) hypothesis test. Methods were developed in this study to choose critical parameters of the processor and to analyze its performance. The wideband passive SLR processor outperformed a conventional processor in the [illegible] reducing the required input signal-to-noise ratio required for 0.5 probability of detection at a given fixed probability of false alarm by several dB. The SLR processor is ideally suited for implementation on the output of passive sonar systems.

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