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# V/STOL DYNAMICS AND AEROELASTIC ROTOR-AIRFRAME TECHNOLOGY

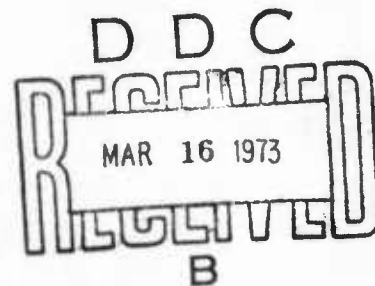
Volume I. State-of-the-Art Review  
of V/STOL Rotor Technology

H. R. ALEXANDER  
P. F. LEONE

THE BOEING COMPANY, VERTOL DIVISION

TECHNICAL REPORT AFFDL-TR-72-40,  
VOLUME I

JANUARY 1973



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# **V/STOL DYNAMICS AND AEROELASTIC ROTOR-AIRFRAME TECHNOLOGY**

**Volume I. State-of-the-Art Review  
of V/STOL Rotor Technology**

*H. R. ALEXANDER*

*P. F. LEONE*

*THE BOEING COMPANY, VERTOL DIVISION*

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## FOREWORD

This report was prepared by The Boeing Company, Vertol Division of Philadelphia, Pennsylvania, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract F33615-71-C-1310. This research is part of a continuing effort to develop new and improved techniques for defining dynamic and aeroelastic phenomena for rotor/propeller-powered V/STOL flight vehicles under the Air Force Systems Command's exploratory development program. This contract was initiated under Project 1370, "Dynamic Problems in Military Flight Vehicles," Task 137005, "Prediction and Control of Flight Vehicle Vibration." Mr. A. R. Basso of the Aerospace Dynamics Branch was the Project Engineer.

The final report is presented in three volumes. The first volume contains a state-of-the-art review of stability and blade vibratory loads in V/STOL aircraft. The second volume contains the development of the analytical methods, the correlation of analytical results with experimental data, and the results of parametric investigations. The third volume contains a user's guide to the digital computer programs including input and output formats. The third volume is not being distributed; however, it is available upon request from the Air Force Flight Dynamics Laboratory/FYS, Wright-Patterson Air Force Base, Ohio 45433.

Mr. H. R. Alexander was The Boeing Company, Vertol Division Project Engineer.

This report covers work conducted from February 1971 through February 1972. The manuscript was released by the authors in February 1972 for publication as an AFFDL Technical Report.

This Technical Report has been reviewed and is approved.

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## ABSTRACT

The aeroelastic phenomena associated with prop/rotor systems are discussed and classified. It is concluded that an acceptable technology exists in several areas, including wing/rotor divergence, whirl flutter, aeromechanical instability, and air and ground resonance.

The technology is less successful in those areas where the flow through the rotor is significantly nonaxial, e.g., tilt-rotor transition regime and high-speed helicopter flight; also when forms of intermodal blade coupling exist due to finite deflections of the blades. It is believed that, in addition to collective deflections, finite cyclical deflections of the blades produce destabilizing coupling effects in some cases. Significantly large edgewise flow in combination with nonzero blade steady-state deflections is also seen to be destabilizing.

A minimum-complexity methodology which may be expected to correlate with currently identified phenomena is defined.

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## LIST OF SYMBOLS

a	arm from nacelle gimbal pivot to rotor hub
ac	aerodynamic center
C	wing chord
$C_a$	inertial coupling coefficient between flap and pitch
$C_d$	aerodynamic stiffness coefficient coupling flap and pitch
$C_m, C_{m\alpha}$	hub pitching moment derivative
$C_N, C_{N\alpha}$	normal force derivative, $N_P/q_S$
$C_{n\alpha}$	yawing moment derivative
$C_T$	thrust coefficient
$C_V$	aerodynamic damping coefficient coupling flap and pitch
$C_{Y\alpha}$	side force derivative
c	blade chord
cg	center of gravity
$c_\theta$	nacelle damping rate
D	rotor disc diameter
$dC_L/d\alpha$	lift curve slope
$H_D$	density altitude, feet
Hz	Hertz, cycles per second
I	moment of inertia about flap axis
$I_r$	product of inertia about pitch-flap axes
$I_\theta$	moment of inertia about pitch axis
J	inflow ratio
K	reduced frequency parameter

$K_{\theta}$	torsional stiffness of wing at rotor location
L	wing lift
M	Mach number
$M_{\phi}$	blade flap frequency
$M_{\theta}$	blade pitch frequency
P	per blade revolution, i.e., 1P = one per rev
R	blade radius
r	radial distance from wing tilting line
rpm	revolutions per minute
r/R	radial station along blade length, ratio or percent
S	rotor disc area
$S_{\theta}$	pitch stiffness
$S_{\psi}$	yaw stiffness
V	velocity
$V_n$	component of circulation velocity normal to disc
$V_{\theta}$	circulation velocity
x	moment arm from rotor hub to torsional axis
$\dot{z}$	blade velocity normal to relative wind
$1/2\rho V^2$	dynamic pressure
$\alpha$	angle of attack, degrees; elastic twist of wing
$\alpha_0$	initial incidence of the wing
$\alpha_2$	rate of change of blade pitch with respect to blade lag
$\Gamma$	strength of bound vortex of wing
$\gamma$	Lock number, $2\pi\rho CR^4/I$
$\Delta\alpha$	small change in wing angle

$\delta_3$	rate of change of blade pitch with respect to blade flap
$\zeta$	lead-lag freedom
$\eta_F$	flap frequency ratio
$\theta$	nacelle pitch angle
$\theta_t$	blade twist
$\Lambda$	angle between relative wind and blade chord
$\lambda$	advance ratio, $V_{AXIAL}/V_{TIP}$
$\mu$	$V_{NORMAL TO SHAFT}/V_{TIP}$
$\rho$	density of air
$\sigma$	rotor solidity, total blade area/total disc area
$\psi$	blade azimuth position, degrees; nacelle yaw angle
$\Omega$	angular velocity of the rotor shaft; rotor speed, rpm
$\omega_b$	natural frequency of an individual blade
$\omega_F$	flutter frequency
$\omega_h$	wing-nacelle bending, torsion natural frequency
$\omega_L$	rotating natural frequency of blade lead-lag mode
$\omega_R$	rotating natural frequency of blade
$\omega_\beta$	uncoupled rotating flap natural frequency
$\omega_\theta$	uncoupled rotating pitch natural frequency
$\omega_\psi$	nacelle yaw natural frequency

# **Part I. Stability in V/STOL Aircraft**

**H. R. Alexander**



## INTRODUCTION

Until quite recently it was possible to regard the aeroelastic stability problems of helicopter rotor systems and those of fixed-wing aircraft as distinct areas of technology whose study could quite properly proceed independently. The first serious departure from this state of affairs occurred with the Electra accidents of 1959 in which the propeller dynamic effects specifically provided the unstable mechanism which led to wing failures. After certain fatigue failures in the engine mounting structure, a whirling oscillation of the propeller would occur in a mode similar to the retrograde motion of a gyroscope. In contrast to a gyroscope, which experiences only damping force, the propeller aerodynamic forces beyond a certain speed always tend to feed energy into the whirling motion, with catastrophic results in the case of the Electra.

As an outcome of these incidents, the phenomenon of propeller whirl, which had been previously recognized in the design of engine mountings, was reformulated for the total dynamic system consisting of the airframe, propeller, engine and mounting structure, including the propeller's aerodynamic forces and moments.

The helicopter phenomenon known as ground resonance was successfully formulated by Coleman (1) in terms of the interaction of the rotor system with fuselage degrees of freedom which provided significant amplitudes of vibration in the plane of rotation of the rotor. Prior to that time the aeroelastic behavior of helicopter blades had been considered independently of the rest of the system. It now became apparent that fuselage and landing gear properties could interact with the rotor to exhibit overall system instabilities. While the propeller whirl phenomenon encountered on the Electra involved propeller blades, adequate predictions could be made assuming that the blades were structurally rigid. Prediction of ground resonance behavior, however, required lead-lag blade freedoms to be accounted for while the airframe motion was generally rolling in a rigid-body mode. These phenomena then are opposite extremes in a spectrum of interaction of rotary-wing and fixed-wing dynamic behavior.

Subsequent developments in V/STOL and helicopter design have led to nearly every possible degree of interactive relationship. In tilt-wing transport aircraft detailed account must be taken of the airframe dynamic properties, but the prop/rotor blades may only need to be represented by a fundamental bending mode since they are relatively stiff, but not sufficiently so to be considered rigid. Tilt-rotor V/STOL designs generally feature large-diameter rotors and relatively low blade frequencies so that accurate representation of the rotor blade structural modes as well as the airframe is required. Hence, in tilt-rotor configurations, the analyst is concerned with preventing the occurrence of helicopter-type phenomena



such as ground and air resonance, airplane-type phenomena such as whirl flutter involving blades and wings, classical flutter of the airframe and individual blades, and a variety of instabilities caused by adverse coupling between the flap, lag, and pitch freedoms of the blades. Limit-cycle instabilities of a nonlinear nature must also be avoided.

It is evident that the conventional treatment of blade instability, wing stability, ground resonance, etc., as separate phenomena is not valid in such cases and that a comprehensive methodology is required. While most of the instability mechanisms are well understood when viewed as general physical phenomena, it is not uncommon in specific cases for an aircraft to be in serious trouble because of one or more of them. More than a general qualitative understanding is required when margins of safety are to be established. Precise quantitative methods of analysis are required.

It is the purpose of this review to assess the problems which are known to exist and the methods available for their prediction. The limitations of the methodology in relation to known behavior will be defined.

#### CLASSIFICATION OF UNDESIRABLE AEROELASTIC EFFECTS

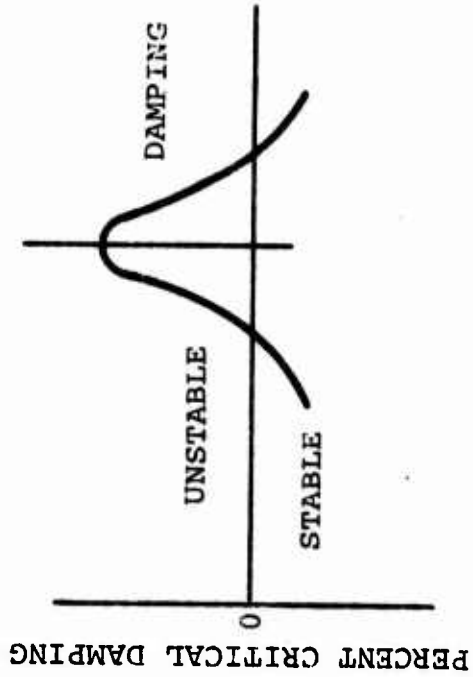
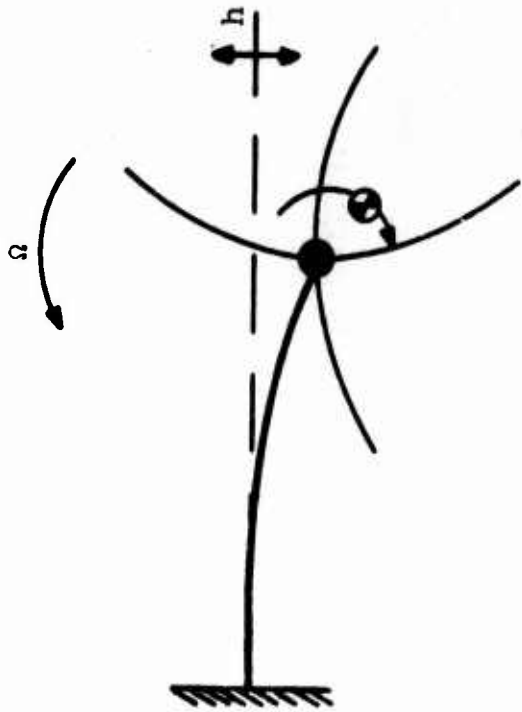
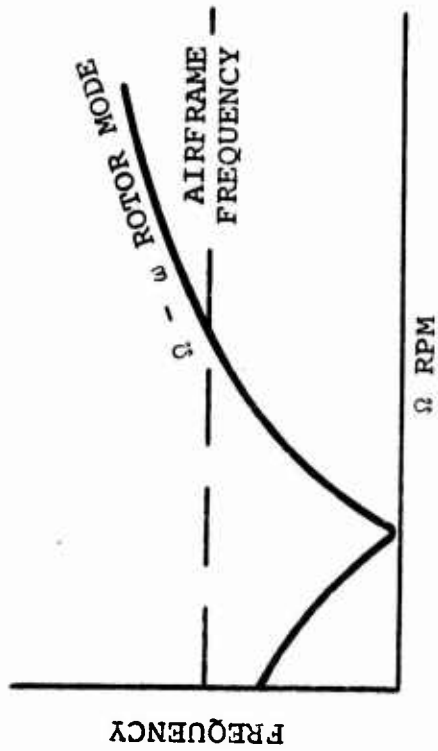
We may group the phenomena under discussion into several distinct categories.

##### Mechanical Instability (Ground and Air Resonance)

Mechanical instability is a phenomenon in which the inertial coupling between blade lagging motion and hub motion in the plane of the rotor produces a growing oscillation (see Figure 1). This may occur on the ground or in flight. The introduction of rotor aerodynamic effects may modify the severity and parametric boundaries of this type of phenomenon but the essential nature of these instabilities remains the same. Early studies (1) showed that this type of instability can occur only when the blade lead-lag frequency is below 1P and when the mounting frequency equalled  $\Omega - \omega$  blade.

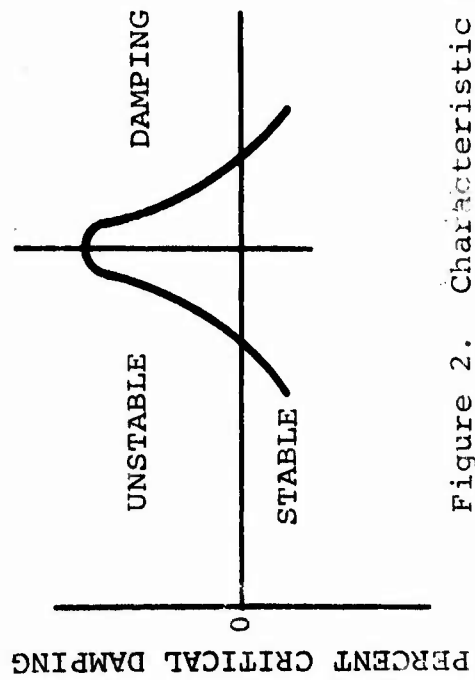
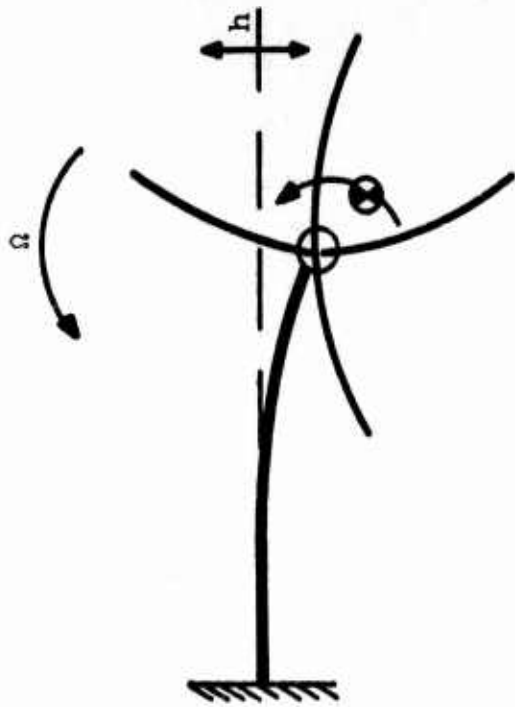
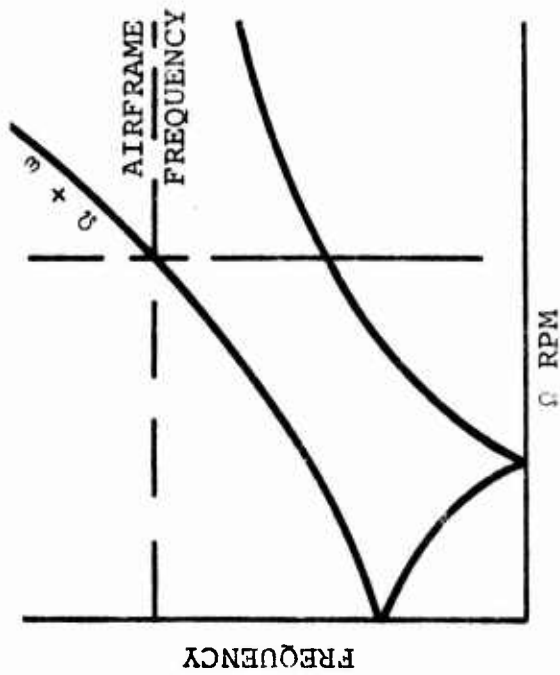
##### Aeromechanical Instability

When aerodynamic forces are present, an instability involving mechanical coupling has been experienced which involves a lead-lag mode above 1 per rev (2) (see Figure 2). This showed a similar coalescence of hub frequency as in ground resonance, but in this instance the instability occurred when the hub frequency was equal to  $\Omega + \omega$  blade.



- BLADE FREQUENCY BELOW 1P
- INSTABILITY OCCURS AROUND COALESCENCE OF LOW-FREQUENCY ROTOR MODE AND AIRFRAME FREQUENCY

Figure 1. Characteristic Features of Mechanical Instability (Air and Ground Resonance)



- BLADE FREQUENCY GREATER THAN  $\Omega$
- HIGHER ROTOR MODE INVOLVED

Figure 2. Characteristic Features of Aeromechanical Instability

## Static Divergence

At the high advance ratios associated with cruise flight in tilt-rotor V/STOL aircraft, the aerodynamic forces acting on the rotor when its axis is inclined to the freestream provide a large component of lift which is significantly destabilizing in the static aeroelastic sense. Prediction of divergence involves an analysis taking account of wing and blade structural properties and the aerodynamic effects of the net flow through the rotor. Prop/rotors generate side forces and yawing moments as well as normal forces and pitching moments when inclined to the freestream, so that yawing as well as torsional flexibility must be considered in a conservative analysis as indicated in Figure 3. Prediction capability for divergence is in a reasonably satisfactory state as indicated by Figure 3, showing predictions accurate within + 6 percent for the Boeing-Vertol Model 213 1/9-scale stowed-rotor conversion model, shown in Figure 4.

## Whirl Flutter

The term whirl flutter was originally used to describe the unstable whirling motion of propeller assemblies which occurs if inadequate stiffness and damping are present in the mounting system. The tendency toward instabilities in anti-vibration mountings of engine-propeller packages was pointed out by Taylor and Brown in 1938 (see Reference 3). The pitching and yawing motion of the engine and propeller occurred in a manner that was not significantly coupled with the rest of the airframe. While anti-vibration mountings had the advantage of reducing vibration levels, it meant that the effective inertias and stiffness of the engine-propeller were approximately the same in pitch and yaw. Since the principal coupling between pitch and yaw was gyroscopic due to propeller rotation, a disturbance in either degree of freedom would result in a whirling motion of the hub which was nearly circular. Two modes existed, one retrograde in which the direction of whirl was in the opposite direction to the shaft rotation and the other prograde with the direction of whirl in the same direction as the shaft rotation. As forward speed increases from zero, the total damping in the whirl modes changes. Initially damping in the retrograde mode increases with speed but eventually begins to reduce until finally the system becomes unstable. A definitive discussion of the classical whirl flutter phenomenon is given in Reference 3: the onset of such a flutter may be accurately predicted by analysis and has been avoided by the provision of adequate stiffness, damping, and failsafe characteristics.

In the context of large prop/rotors, the term whirl flutter has come to be habitually applied to instabilities involving blade flapping and significant amounts of airframe motion.

FOR STATIC EQUILIBRIUM  
INCREMENTAL DEFLECTIONS ARE GIVEN BY

$$\begin{bmatrix} \text{STIFFNESS} \\ \text{MATRIX} \\ \text{STRUCTURE} \\ \text{AND} \\ \text{CENTRIFUGAL} \end{bmatrix} \begin{Bmatrix} \Delta P \\ \Delta Y \\ \Delta \theta C \\ \Delta \theta S \end{Bmatrix} = q \begin{bmatrix} \text{AERODYNAMIC} \\ \text{DERIVATIVE} \\ \text{MATRIX} \end{bmatrix} \begin{Bmatrix} P_0 \\ Y_0 + \theta_{CO} \\ \theta_{SO} \end{Bmatrix} \begin{Bmatrix} \Delta P \\ \Delta Y \\ \Delta \theta C \\ \Delta \theta S \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta P \\ \Delta Y \\ \Delta \theta C \\ \Delta \theta S \end{Bmatrix} = \left[ \begin{bmatrix} \text{STIFFNESS} \\ \text{MATRIX} \end{bmatrix} - q \begin{bmatrix} \text{AERODYNAMIC} \\ \text{DERIVATIVE} \\ \text{MATRIX} \end{bmatrix} \right]^{-1} x$$

$$q \begin{bmatrix} \text{AERODYNAMIC} \\ \text{DERIVATIVE} \\ \text{MATRIX} \end{bmatrix} \begin{Bmatrix} P_0 \\ Y_0 \\ \theta_{CO} \\ \theta_{SO} \end{Bmatrix}$$

- WING AERODYNAMIC CONTRIBUTION NOT SHOWN
- P = AMOUNT OF WING AND NACELLE TWIST SENSED AT ROTOR
- Y = AMOUNT OF HORIZONTAL BENDING SLOPE SENSED AT ROTOR

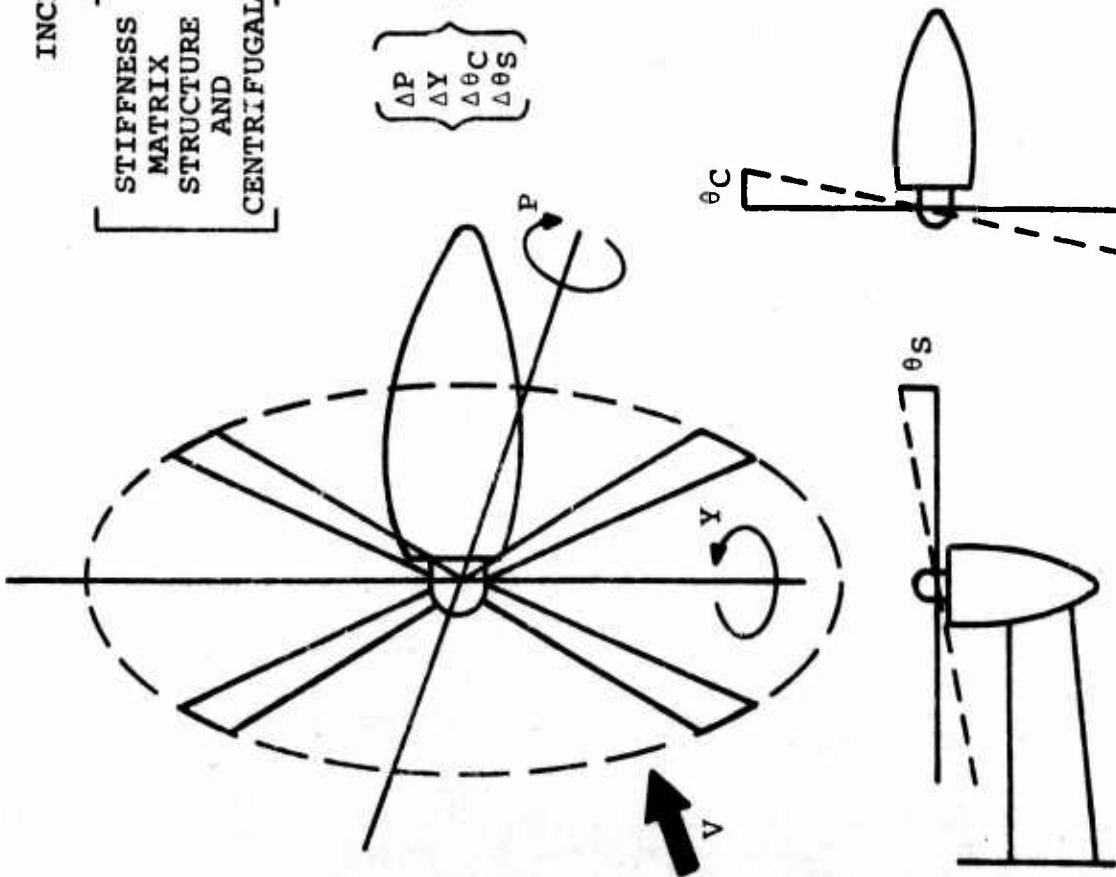
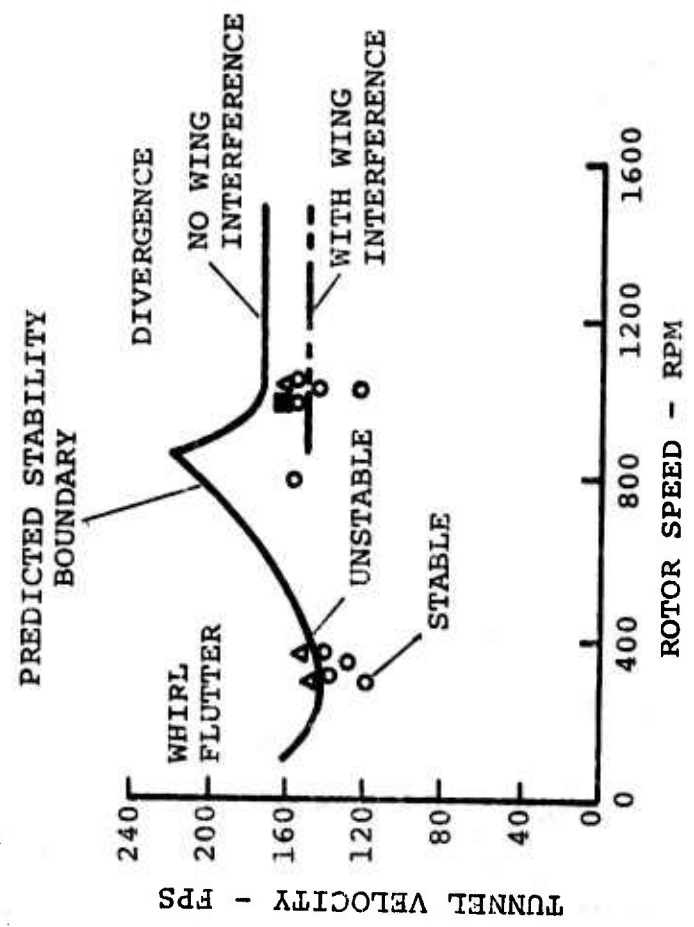


Figure 3. Mechanism of Static Divergence for Wing with Flexible Rotor

- AT 1,000 RPM DIVERGENCE PREDICTED AT 170 FT/SEC
- DIVERGENCE OCCURRED AT 160 FT/SEC
- ACCOUNTING FOR WING-INDUCED FLOW REDUCED THEORETICAL VALUE TO 150 FT/SEC
- PREDICTED VALUES CORRECT WITHIN + 6 PERCENT
- BLADE FREQUENCIES

0.76 PER REV  
1.26 PER REV



MODEL AEROELASTIC STABILITY  
VTUNNEL VS MODEL RPM

- TEST POINTS
- STABLE
  - △ FLUTTER
  - DIVERGENCE

Figure 4. Prediction of Static Divergence of M213 1/9-Scale Conversion Model with Reduced-Stiffness Wing Spar by C-40 and C-41 Computer Programs



The degrees of freedom involved may be such that no significant whirling motion of the hub is observed. Degrees of freedom participating in such a case may be wing vertical bending and torsion and blade flapping. However, the flapping behavior of the blades will be patterned such that the tip path plane is pitching and yawing and therefore, in a certain sense, whirling. Many cases correctly classified as whirl flutter may exhibit significant amounts of lead-lag blade motion; this occurs when the blades are highly twisted and the modes have strong components of both flap and lag. Lead-lag motion will also be present as a forced response when there is significant in-plane hub motion present in the airframe contribution to the whirl mode.

#### Distinguishing Features of Whirl Flutter, Mechanical Instability, and Aeromechanical Instability

It is useful to distinguish between the various instabilities on the basis of the following frequency and modal participation criteria.

Whirl flutter involves predominantly flapping blade modes: these will most often have frequencies above 1 per rev since the centrifugal stiffening will automatically ensure this unless special features such as negative  $\delta_3$  are present in the blade. Frequency coalescence of rotor natural modes with airframe modes is not a requirement since whirl flutter may occur even with rigid blades. Individual blade motion in the flutter mode will be complex but may generally be resolved into the two frequency components  $\Omega \pm \omega_F$  which are expected when a rotating system is excited at a fixed system frequency  $\omega_F$ .

In contrast, mechanical instability and aeromechanical instability will generally exhibit a clear coalescence of modal frequency in the rotor and in the fixed system. The presence of hub motion in the plane of the rotor and of lead-lag blade motion is a requirement. In twisted blades noticeable amounts of blade flapping may also be present.

Air and ground resonances involve blade lead-lag freedoms below 1 per rev; aeromechanical instability involves lead-lag freedoms above 1 per rev. In the first case an airframe frequency  $\omega$  is coalescent with the lower rotor frequency  $\Omega - \omega_D$  and in the aeromechanical case the coalescence occurs in the upper rotor frequency  $\Omega + \omega_D$ .

These frequency relationships in the unstable mode along with the predominant blade motion will give a clear indication, in almost all cases, of the mechanism of the instability. These characteristics are summarized in Table I.

TABLE I

INSTABILITIES ENCOUNTERED IN PROP/ROTOR VEHICLES

Instability	Principal Freedoms	Frequency Characteristics	Comments
Whirl Flutter	Pitch, yaw of prop, blade flapping, hub linear and angular motion	$\omega_B > \Omega$ $\omega_I = \omega_{AIRFRAME}$ $\omega_{BF} = \Omega \pm \omega_{AIRFRAME}$	$\omega_B$ = blade natural frequency $\omega_{BF}$ = frequencies of blade oscillation in the flutter mode
Ground Resonance	Blade lead-lag linear hub motion in plane of rotor	$\omega_B < \Omega$ $\omega_I = \omega_{AIRFRAME} = (\Omega \pm \omega_B)$	
Air Resonance	Blade lead-lag linear hub motion in plane of rotor	$\omega_B < \Omega$ $\omega_I = \omega_{AIRFRAME} = (\Omega - \omega_B)$	Essentially same as ground resonance. Aerodynamic damping may be stabilizing.
Aeromechanical Instability	Blade lead-lag linear hub motion in plane of rotor	$\omega_B > \Omega$ $\omega_I = \omega_{AIRFRAME} = (\Omega + \omega_B)$	Essentially same as air resonance, although blade frequency above 1/rev.
Limit-Cycle Instabilities	Blade flapping, pitching, and lead-lag plus linear and angular hub motion	$\omega_I = \omega_{AIRFRAME} = (\Omega \pm \omega_B)$	Blade deflections due to flight conditions provide flap-lag-pitch coupling.



## Blade Instabilities

Instabilities of the rotating blade are a category which involve detailed treatment of the blade internal properties such as chordwise center of gravity location relative to the elastic axis and blade deflections under load. The influence of centrifugal and coriolis forces and interactions of flap, lag, and pitch or torsion when the blade is deflected are further effects which must be considered. This means that the analysis of blade instabilities is inherently more difficult than the phenomena discussed in the preceding paragraphs.

In addition to blade classical pitch-flap flutter, pitch-lag, pitch-flap, and pitch-lag-flap mechanisms may be present due to blade deflections, actual or effective  $\delta_3$  and  $\alpha_2$ , steady thrust and torque, and coriolis coupling between flap and lag. Hub motion due to airframe vibratory freedoms will generally couple with the blades if there is an instability due to any of these mechanisms, so that the oscillatory motion of the blades will assume a pronounced overall rotor pattern with superficial similarity to ground or air resonance or whirl flutter.

The detailed discussion of blade instabilities is presented after the discussion on classical rotor airframe problems. It is concluded that a comprehensive capability requires the inclusion of blade effects normally ignored in the analysis of whirl flutter and mechanical instabilities. These effects, such as blade finite deflections, seem to provide the necessary mechanisms to explain the occurrence of limit-cycle behavior and other phenomena not falling into the traditional classifications.

### Stall Flutter

The aerodynamic nonlinearity associated with stall provides the mechanism for a single-degree-of-freedom instability in blade pitch or torsion. This may become a serious source of blade and pitch link loads if the blade is operating with a significant proportion of its span in stall. This occurs most frequently in high-speed helicopter flight where the retreating blade experiences large excursions of angle of attack. The destabilizing effect of stall and its effect on blade and pitch link loads is discussed in full detail in Part II and will not be pursued further in this section.

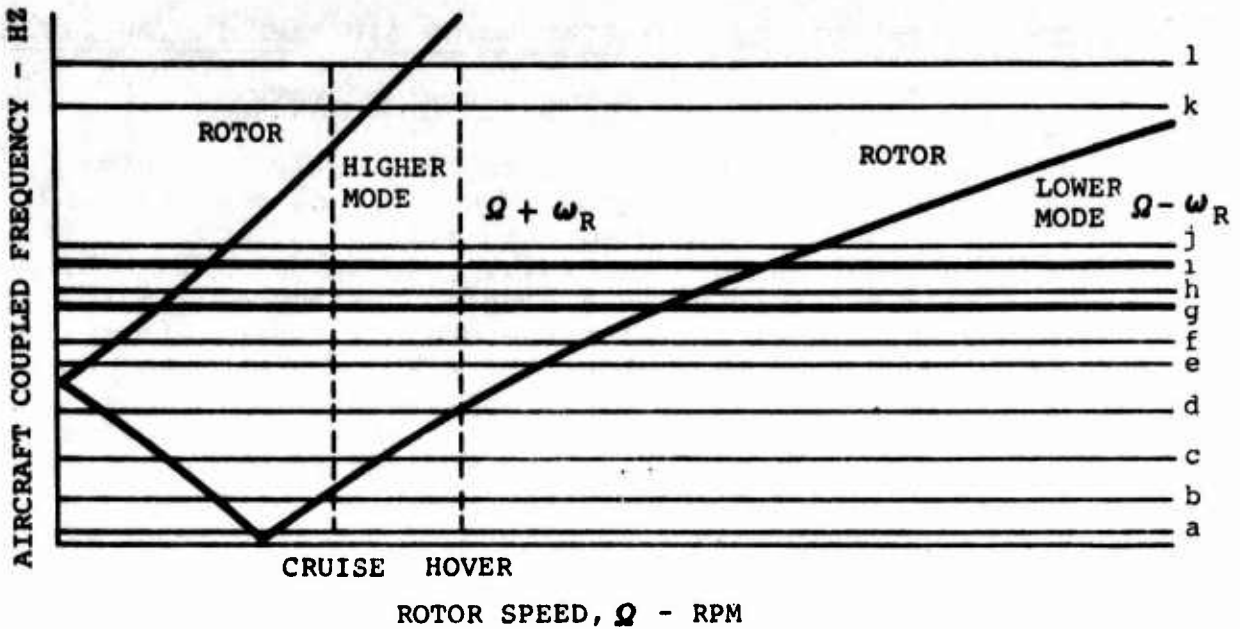
## TECHNICAL ASPECTS OF CLASSICAL ROTOR/AIRFRAME INSTABILITIES

### MECHANICAL INSTABILITY OF ROTOR/AIRFRAME SYSTEMS

Ground resonance has been the most frequently encountered form of mechanical instability experienced in helicopters, and for many years has been predicted with sufficient accuracy that preventive measures have generally been successful. The phenomenon most often involves a coupled horizontal and rolling motion of the blades in which the overall center of gravity of the rotor disc precesses about the rotor shaft in the opposite direction to shaft rotation, as indicated schematically in Figure 1. The precession of the disc center of mass creates periodic inertia forces which are reacted at the hub. This produces a forced response in the pylon and airframe. A potentially unstable situation exists if the precession frequency is near that of a normal mode of the rest of the system. However, the occurrence of an instability depends on a number of parametric conditions. These include, but are not limited to, the amount of linear motion at the rotor hub normal to the shaft, the relative magnitude of blade and airframe masses, and damping in the blade and aircraft degrees of freedom.

The frequency at which the retrograde motion of the blade center of gravity occurs is  $\Omega - \omega_b$ , where  $\Omega$  is the angular velocity of the rotor shaft and  $\omega_b$  is the natural frequency of an individual blade. Since the regression mode frequency is directly related to  $\Omega$ , a number of potentially unstable resonances may occur within the operating rpm range. In conventional helicopters the frequencies of the aircraft on the ground are those most likely to provide coalescence with the rotor frequencies. This type of instability, known as ground resonance, has been successfully prevented in conventional helicopter configurations by the use of blade lag dampers and careful choice of landing gear parameters. A definitive analytical treatment of this problem is given in Reference 1 by R. P. Coleman. This basic approach continues to be used successfully in conventional helicopter work.

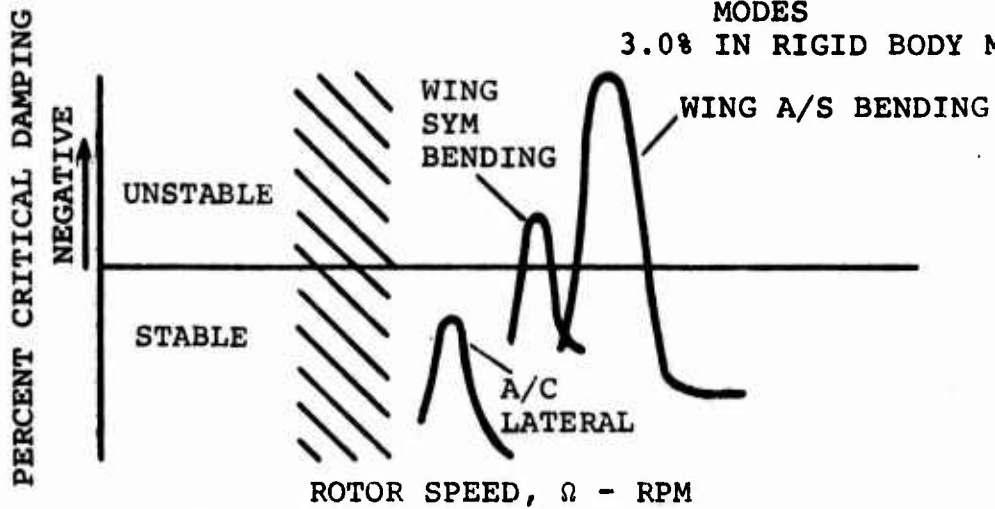
A single-rotor helicopter on its landing gear has four sources of hub displacement in the plane of the rotor; these are the two coupled lateral-roll oscillations of the fuselage and the two coupled longitudinal-pitch characteristics. In a tandem helicopter, fuselage yaw introduces an additional source of lateral hub motion which needs to be considered in ground stability analyses. The possibilities of mechanical instability might be expected to increase in tilt-rotor configurations where the rotor is mounted at the tip of the wing, since the number of potentially unstable situations expands by the number of wing modes within reasonable proximity of foreseeable rotor speeds. However, Figure 5 illustrates a typical



- LEGEND:
- |                       |                            |
|-----------------------|----------------------------|
| a. A/C ROLL           | g. WING SYM FWD BEND       |
| b. A/C YAW            | h. WING ANTI-SYM VERT BEND |
| c. A/C PITCH          | i. WING ANTI-SYM FWD BEND  |
| d. A/C VERTICAL       | j. A/C FORE AND AFT        |
| e. A/C LATERAL        | k. WING SYM TORSION        |
| f. WING SYM VERT BEND | l. WING ANTI-SYM TORSION   |

AIRCRAFT FREQUENCY SPECTRUM ON GROUND WITH QUASI-NORMAL BLADE LAG MODES

CRITICAL DAMPING ASSUMED  
 0.5% IN BLADES  
 1.0% IN FLEXIBLE AIRFRAME MODES  
 3.0% IN RIGID BODY MODES



DAMPING IN MARGINAL AND UNSTABLE MODES

Figure 5. Summary Chart of Potential and Predicted Mechanical Instabilities on a Tilt-Rotor Design

situation existing in one tilt-rotor design. The operational rpm range is stable and the aircraft lateral mode, which is troublesome in most helicopters, occurs at a frequency beyond maximum hover rpm and is stable for small nominal amounts of blade structural damping (i.e., heavy lag dampers required in most helicopter applications). The two instabilities predicted occur at frequencies well beyond the design rpm range and involve wing horizontal bending in the symmetric and anti-symmetric modes.

When the aircraft is in flight, the rigid-body degrees of freedom may usually be disregarded because their frequencies from a structural stability standpoint become approximately zero. However, the airframe frequencies persist with little change in magnitude or mode shape into the flight regime and instabilities of a mechanical nature occurring in flight are referred to as air resonance. The potential for instability due to frequency coalescence continues into the cruise regime of tilt-rotor aircraft and a changing spectrum of frequencies needs to be considered as the rotor and nacelle tilt from the vertical hover mode to the cruise mode. Blade damping increases rapidly with collective pitch and inflow velocity to the extent that conventional air resonance in the cruise mode is generally not a problem.

The predictive capability for helicopter ground resonance has been acceptable for many years. Good predictive capability has also been established in tilt-rotor work as shown by Figure 6. In this test of a 1/9-scale Boeing Folding Tilt-Rotor model (4), mechanical instability was predicted at 1,050 rpm and 1,070 rpm with wind velocities of 140 feet per second and 104 feet per second respectively. A mild air resonance condition was experienced in test at 1,050 rpm and 104 feet per second. The unstable region is associated with coalescence of the wing vertical bending frequency and the lower ( $\Omega - \omega_L$ ) rotor frequency and extends to either side of the point of intersection of the frequency curves.

#### AEROMECHANICAL INSTABILITY

The mechanical instabilities discussed in the preceding paragraph are associated with the lower ( $\Omega - \omega_L$ ) lead-lag mode of rotors having blade natural frequencies below 1 per rev. Reference 2 described an instability which involves mechanical coupling between the rotor upper lag frequency ( $\Omega + \omega_L$ ) and linear motion of the hub. The interesting feature of this case is the fact that the blade lead-lag frequency involved is significantly above 1 per rev. Predictive capability appears to be excellent as shown in Figure 7.

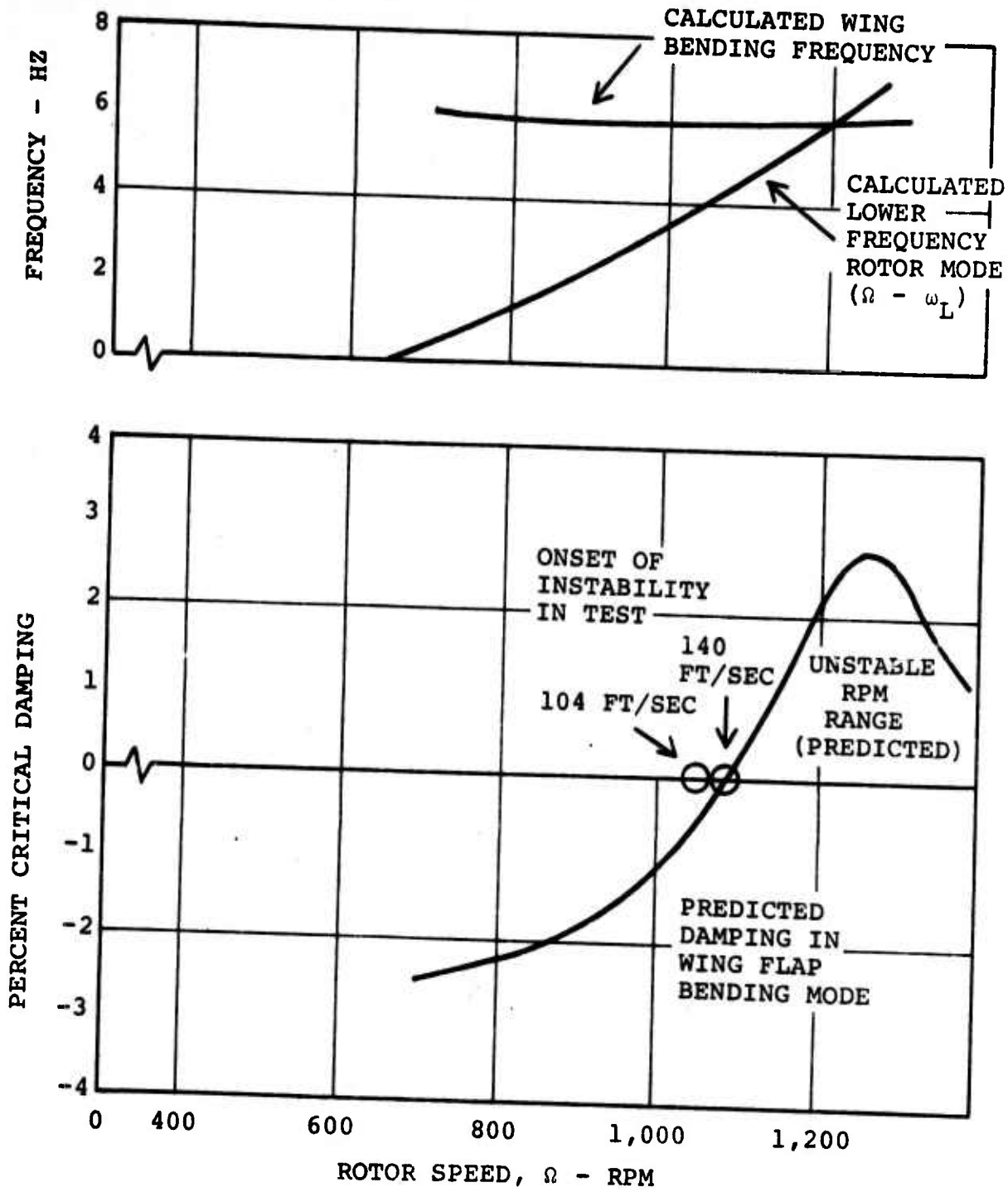
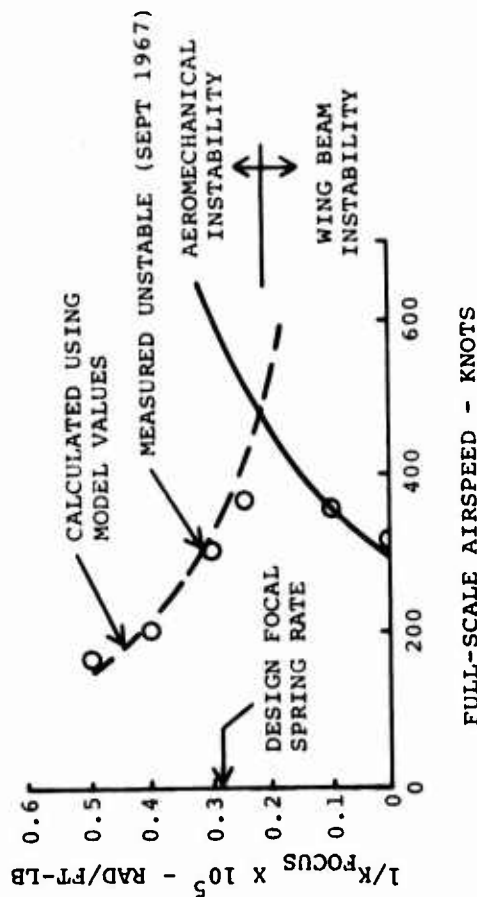


Figure 6. Correlation of Test Air Resonance Instability with Analysis





- FOCAL MAST SYSTEM PROVIDES BLADE PITCH CONTROL FEEDBACK
- MAST STIFFNESS SELECTED SO THAT MAST PITCHES IN OPPOSITE SENSE TO WING AT WING FUNDAMENTAL FREQUENCY
- AT THIS FREQUENCY SWASHPLATE AND ROTOR HAVE NO ANGULAR MOTION IN SPACE AND NO PRECESSIONAL AIRLOADS GENERATED. HOWEVER, IN-PLANE MOTION IS STILL PRESENT
- AEROMECHANICAL INSTABILITY OCCURS WHEN FOCAL MAST FREQUENCY AND BLADE LEAD-LAG FREQUENCIES COALESCE, I.E., AIR RESONANCE
- AEROMECHANICAL INSTABILITY OCCURS FOR BLADE IN-PLANE FREQUENCIES GREATER THAN 1/REV

Figure 7. Comparison of Theory and Test of Effect of Focal Mast Stiffness on Stability for Bell Model 266

## WING/ROTOR DIVERGENCE

When a wing supports large propellers or rotors, a number of factors are present which make the accurate prediction of divergence conditions more difficult than is the case with a clean wing configuration. These include the following.

### Local Flexibilities

The net compliance of the rotor in pitch is the sum of wing pitch, tilt actuator, and bearing flexibilities. This net value may be significantly less than that due to the wing alone.

### Location of Prop/Rotor

The aerodynamic forces on the prop/rotor act well forward of the wing torsional axis and, in the case of tilt-rotors, near the outer wingtip. This generally means that the static aeroelastic behavior is dominated by the rotor hub force and moment derivatives and that these must be accurately estimated if reliable divergence speeds are to be calculated.

### Technical Discussion of Factors Influencing Divergence Behavior

Elements which determine the behavior of rotor derivatives are aeroelastic as well as aerodynamic. Blade flapping natural frequency has been seen to significantly influence all hub force derivatives (see Figure 8). Lock number and advance ratio also influence the derivatives significantly.

The effect of Lock number variation is shown in Figure 9 for constant blade flap natural frequency and very high lag and torsion frequencies. These results show that the rotor derivatives are highly sensitive to blade frequencies and mass-inertia properties so that particular accuracy must be observed in these parameters in divergence prediction and correlation studies. Preliminary studies of blade lag frequency also indicate a strong influence on the rotor normal force derivative  $C_{N\alpha}$ , which is directly related to divergence properties as shown by Figure 10.

### Elementary Mathematical Formulation

The mechanism of static divergence will be illustrated by the following discussion of the behavior of a prop/rotor mounted on a wing (Figure 11). The wing and rotor are tilted to the airstream by an angle  $\alpha_0$ . The aerodynamic loading at the rotor tends to twist the wing and produce further aerodynamic force increments. These in turn produce more twist and so on. If the wing is stiff enough, the process will be a finite

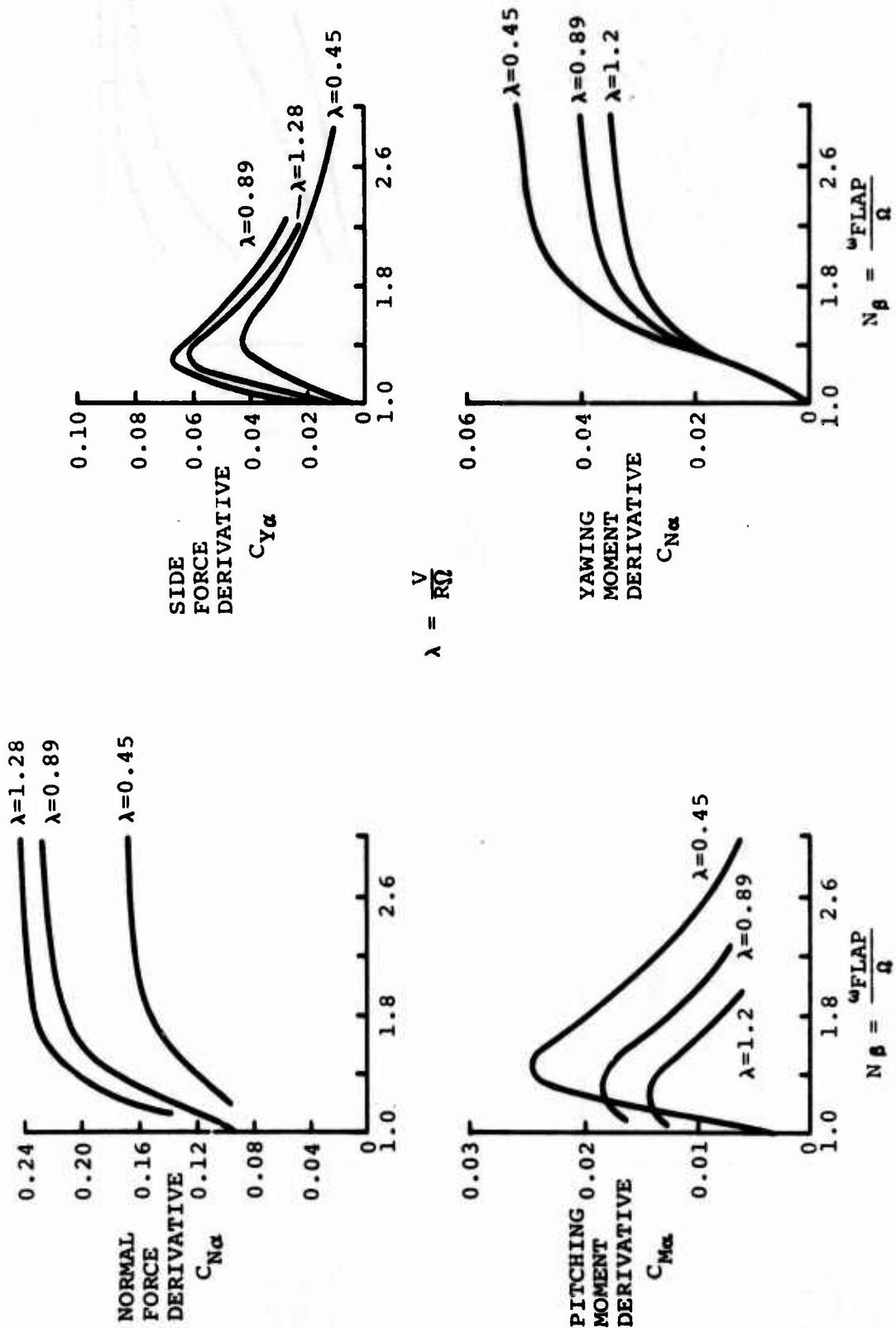


Figure 8. Variation of Rotor Derivatives With Flap Frequency and Lag Frequency High



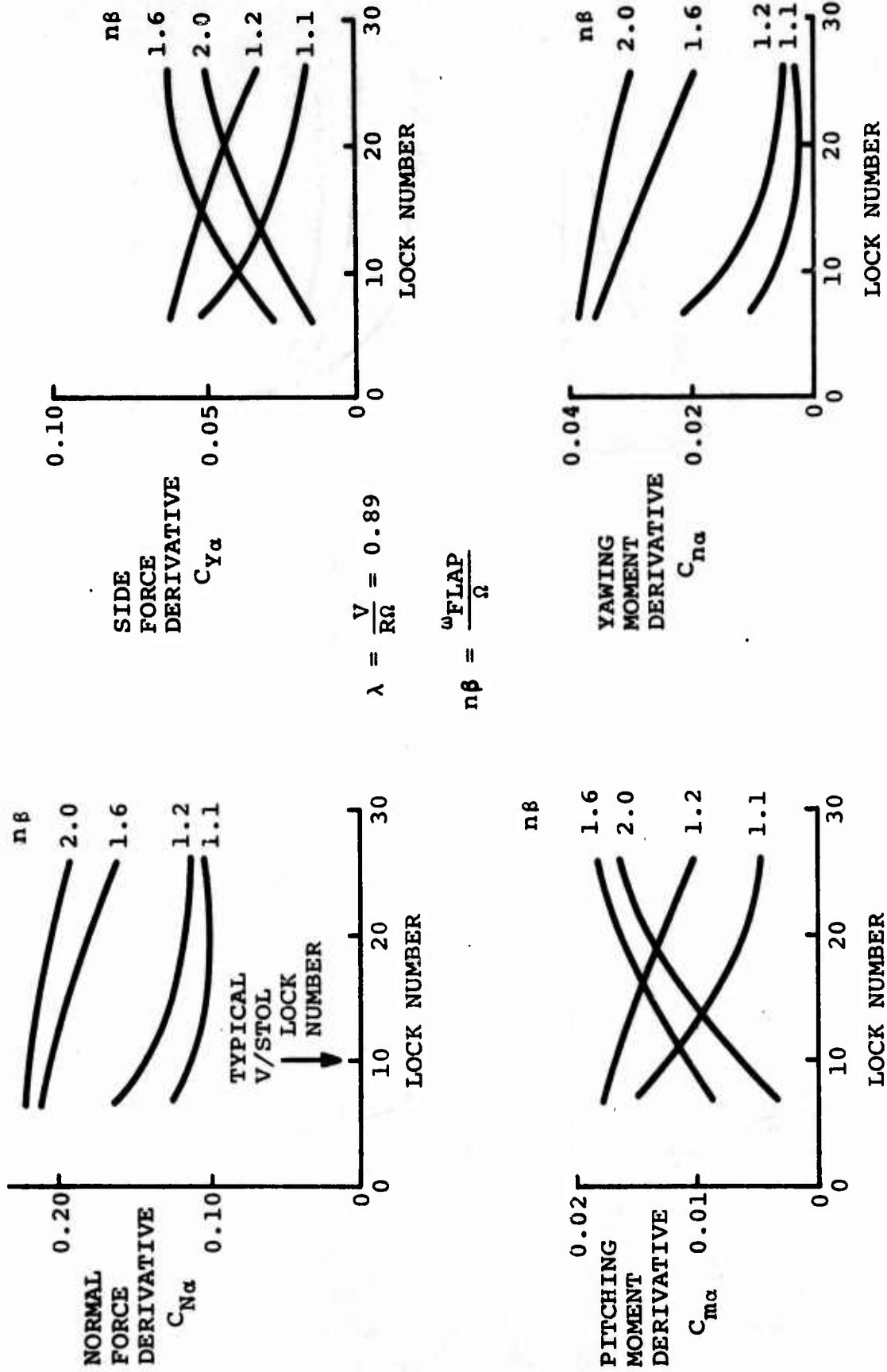


Figure 9. Effect of Lock Number on Rotor Derivatives for Flap Frequency Ratios  $n\beta = 1.1, 1.2, 1.6, \text{ and } 2.0$

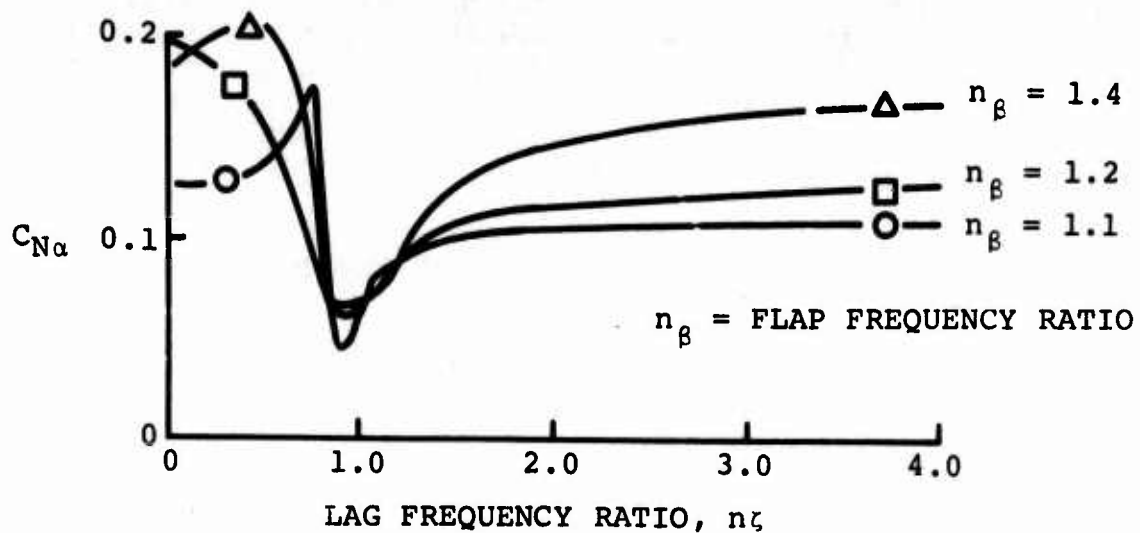


Figure 10. Variation of Normal Force With Lag Frequency Ratio Calculated With Boeing-Vertol C-41 Derivative Program

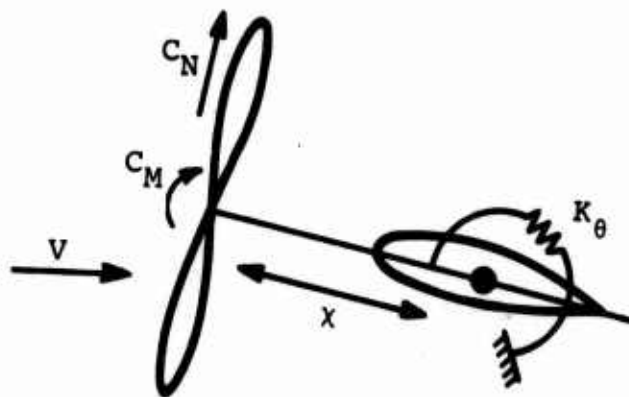


Figure 11. Simplified Mathematical Model for Prop/Rotor Divergence

amount. Ignoring for the moment the effects of wing aerodynamic force, we may express the behavior mathematically as follows: Let the initial unloaded wing and rotor setting be  $\alpha_0$ . Assume that the only aerodynamic loads present are those on the rotor and that, as a consequence of the rotor aerodynamic effects, the wing twists through an angle  $\alpha$ . Then, if the system reaches an equilibrium twist angle, we may state that

$$\begin{aligned}
 K_\theta \alpha &= \text{Aerodynamic Moment About the Torsion Axis} \\
 &= 1/2 \rho V^2 (\alpha_0 + \alpha) S \{ x C_{N_\alpha} + D C_{m_\alpha} \} \quad (1)
 \end{aligned}$$

where  $K_\theta$  = torsional stiffness of the wing at the rotor location

$1/2 \rho V^2$  = dynamic pressure

$\alpha$  = elastic twist of the wing

$\alpha_0$  = initial incidence of the wing

$C_N$  = normal force derivative

$C_m$  = hub pitching moment derivative

$S$  = rotor disc area

$D$  = rotor disc diameter

$x$  = moment arm from the rotor hub to the torsional axis

From Equation 1 we may solve for the elastic twist

$$\alpha = \{ K_\theta - 1/2 \rho V^2 S (x C_{N_\alpha} + D C_{m_\alpha}) \}^{-1} \cdot 1/2 \rho V^2 S \alpha_0 \{ x C_{N_\alpha} + D C_{m_\alpha} \}$$

So long as  $\{ K_\theta - 1/2 \rho V^2 S (x C_{N_\alpha} + D C_{m_\alpha}) \}$  remains positive, a finite solution for  $\alpha$  exists. However, when  $K_\theta \leq 1/2 \rho V^2 S (x C_{N_\alpha} + D C_{m_\alpha})$ , the net stiffness is zero or negative and the system is divergent. We define the divergence speed to be

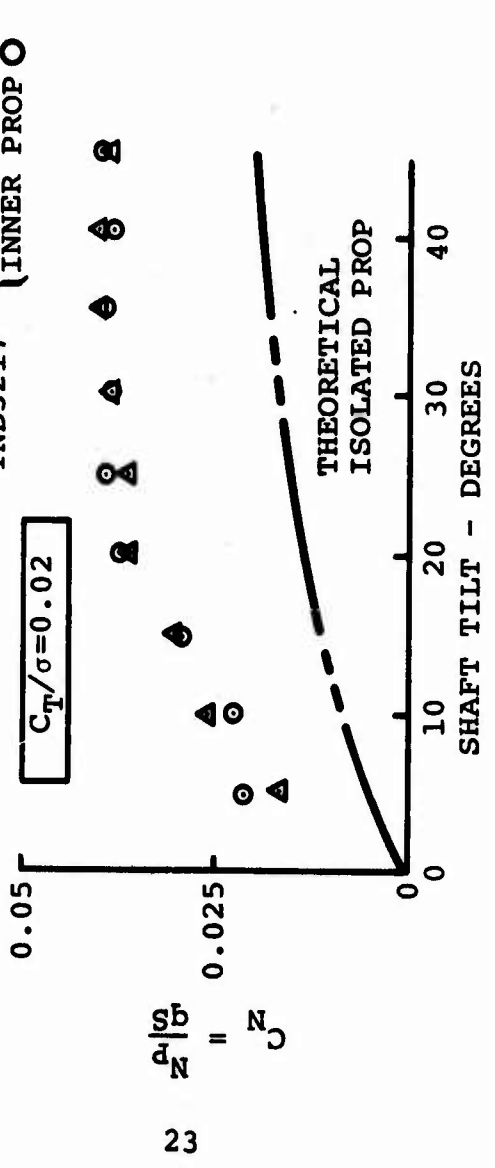
$$V_{DIV} = \left( \frac{K_\theta}{1/2 \rho S (x C_{N_\alpha} + D C_{m_\alpha})} \right)^{1/2} \quad (2)$$

#### Induced In-Flow Effects

The magnitude of rotor hub derivatives is also strongly influenced by the in-flow induced by the wing and the rotor itself. Flight experience, e.g., XC-142 experience (Figure 12) and wind tunnel test work have shown that the presence of a lifting wing behind the prop/rotor can bring about large increases in the rotor aerodynamic derivatives compared with those of an isolated rotor (Figure 13). The mechanism may be explained by reference to potential flow theory in the 2-dimensional case where the prop/rotor is ahead of a wing of high aspect ratio.

B-67 PROGRAM ————  
 TND3217 { OUTER PROP  $\Delta$   
 INNER PROP  $\circ$

- PROPELLER RADIUS SMALL RELATIVE TO WING CHORD
- MEASURED SLOPE OF  $C_N$  CURVE APPROXIMATELY TWICE THEORETICAL
- DIFFERENCE ACCOUNTED FOR BY WING CIRCULATION
- DISCREPANCY SUFFICIENTLY LARGE TO AFFECT FLYING QUALITIES



COMPARISON OF THEORETICAL ISOLATED PROPELLER NORMAL FORCE AND MEASURED VALUES

Figure 12. Effect of Wing-Induced Flow From Comparison of Theoretical and Measured Hub Normal Force on the XC-142 Aircraft

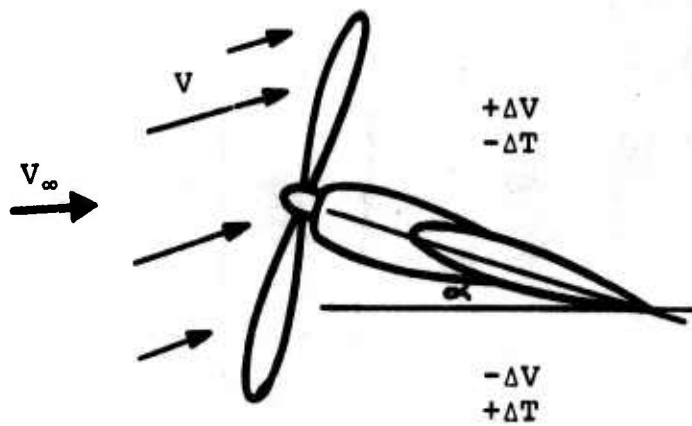


Figure 13. Incremental Velocity and Thrust Effects Due to Presence of Wing

Intuitively we recognize that the upwash produced by the wing increases the angle of attack of the rotor. This may be interpreted as an increase in the rotor derivatives, in particular that of normal force,  $C_{N\alpha}$ .

The wing lift also induces incremental velocities normal to the disc with the tendency that the flow through the upper half disc is increased and flow through the lower section is decreased. This produces an azimuthal blade angle of attack variation which is experienced as a nose-up pitching moment. Since this is proportional to the wing angle of attack, the effect may be accounted for by a change in the rotor derivative. Both of these effects,  $\Delta C_{N\alpha}$  and  $\Delta C_{m\alpha}$ , are destabilizing for normal configurations.

#### Estimates of Magnitude of Induced-Flow Velocity Component

We may usefully discuss these effects further in an elementary but quantitative manner by considering a prop/rotor in front of a wing of infinite aspect ratio.

For such a wing with uniform lift  $L$  per unit span, the circulation function is given by  $L = \rho \Gamma V_\infty$ .

$$\text{Also, } L = 1/2 \rho V_\infty^2 C \cdot C_L.$$

$$\text{Hence, } \Gamma = V_\infty C/2 \cdot C_L.$$

Hence the potential at a radial distance  $r$  from the quarter-chord point is

$$\omega = \frac{\Gamma}{2\pi} \log_e r$$

giving

$$\begin{aligned} V_\theta &= \frac{\partial \omega}{\partial r} = \frac{\Gamma}{2\pi r} \\ &= \frac{V_\infty}{2\pi} \cdot \frac{1}{\gamma} \left\{ C_{L0} + \alpha \frac{dC_L}{d\alpha} \right\} \end{aligned}$$

where  $\gamma$  is radial distance expressed in half chords and

$$\text{assuming } \frac{dC_L}{d\alpha} = 2\pi, \frac{V_\theta}{V_\infty} = \frac{1}{\gamma} \{ \alpha_0 + \alpha \}.$$

$V_\theta$  may be resolved into components parallel and normal to the rotor disc and the effect integrated over the rotor disc to produce a set of modified derivatives.

The magnitude of the effect may be estimated by considering a point directly ahead of the wing. Assuming as in Figure 14 that the rotor is located one chord length ahead of the leading edge, we have:



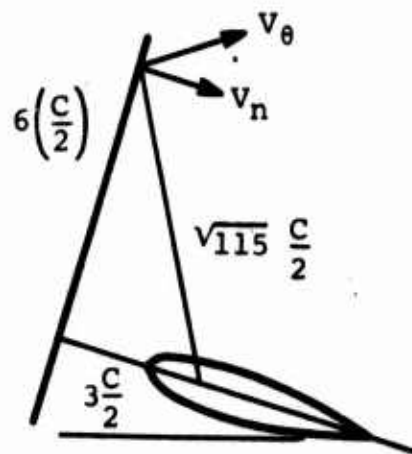
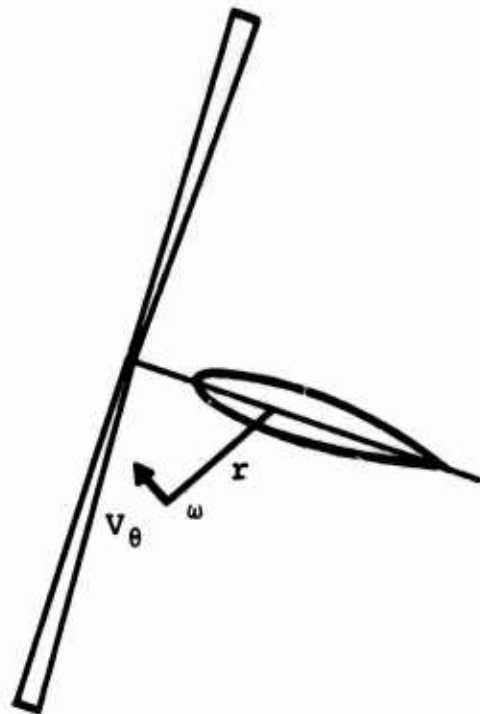


Figure 14. Induced Velocity Components  
Caused by Circulation  
(2-Dimensional Approximation)

$$\alpha_{\text{ROTOR}} = \alpha_{\text{WING}} + \frac{V_{\theta}}{V_{\infty}} = \alpha_{\omega} + 1/3 \{ \alpha_0 + \alpha_{\omega} \}$$

or the change  $\Delta\alpha_R$  in effective rotor shaft angle produced by a change  $\Delta\alpha$  in wing angle is

$$\Delta\alpha_{\text{ROTOR}} = (1 + 1/3) \Delta\alpha_{\text{WING}}$$

This is equivalent to an increase in the rotor derivatives relative to those of an isolated rotor and the effect is clearly significant.

The incremental induced flow normal to the rotor will be estimated at the 3/4R station of the blade. If the blade radius is of the order of 8 semichords and the hub is 3 semichords from the center of pressure, the radial distance is 45 semichords. The induced flow normal to the disc is

$$\begin{aligned} V_n &= V_{\theta} \cos \theta = \frac{V_{\infty}}{\gamma} \{ \alpha_0 + \alpha \} \cos \theta \\ &= \frac{V_{\infty}}{\sqrt{45}} \{ \alpha_0 + \alpha \} \frac{3}{\sqrt{45}} \end{aligned}$$

i.e., 
$$V_n = \frac{V_{\infty}}{15} \{ \alpha_0 + \alpha \}$$

and this may be expressed as an equivalent rate of pitch about the rotor hub

$$\dot{\beta} = \frac{V_n}{6 \left( \frac{C}{2} \right)} = \frac{V_{\infty}}{90 \left( \frac{C}{2} \right)} (\alpha_0 + \alpha)$$

Hence, a unit incremental change in wing angle of attack produces an induced-flow effect at the rotor of an order equivalent to pitching about the hub with angular velocity

$$\frac{V_{\infty}}{90 \left( \frac{C}{2} \right)} \text{ rad/sec.}$$

Hence, at a forward speed of 360 feet per second, we have an effect equivalent to  $\frac{8}{C}$  rad/sec pitch velocity, for each

radian of incremental wing angle of attack.

That is, we have additional induced  $C_{m\alpha}^1$  which may be estimated by

$$1/2 v^2 S DC_{m\alpha}^1 = 1/2 v^2 S DC_m \left\{ v/90 \left( \frac{C}{2} \right) \right\}$$

or 
$$C_{m\alpha}^1 = v_\infty/90 C_{mq} \times (2/C) = v/90 (C/2) \cdot C_{mq}$$

Since  $C_{mq}$  tends to vary inversely with  $J$  or  $\lambda$ , this effect will tend to remain constant over the speed range.

In actual practice the situation is more complicated since large prop/rotors are generally located at the wingtip and two-dimensional assumptions are involved. Additional effects due to the wingtip vortex, fuselage proximity, the nacelle and spinner, and thrust-induced effects are all present to a greater or lesser degree.

### Recommendations

The conclusions that may be drawn from the preceding discussion on divergence are that a satisfactory state of the art exists. Caution must be exercised, however, in the assessment of V/STOL vehicles for which the rotor hub force derivatives may dominate the behavior. These have been seen to depend critically on blade elastic and inertial properties as well as the purely aerodynamic characteristics of the rotor blade. Also, in certain cases, the effect of wing-induced inflow components may be very significant. However, if care and good judgement are used in quantifying these effects, accurate predictions of static divergence are possible.

It is also noted that divergence is a static aeroelastic phenomenon, in which the wing twists under the influence of nonvibratory loads. It is therefore important to evaluate and use the mode of twist appropriate to such loads, rather than the mode of twist associated with the fundamental vibration mode.

This implies a divergence calculation in which the basic static aeroelastic properties, rather than the vibratory modes, are used to represent the wing. Thus, if the divergence speed is sought using a flutter-oriented program, particular care must be taken to ensure that the inputs will permit the structure to adopt the nonvibratory torsional wing mode shape associated with divergence.







### WHIRL FLUTTER AND OSCILLATORY BEHAVIOR

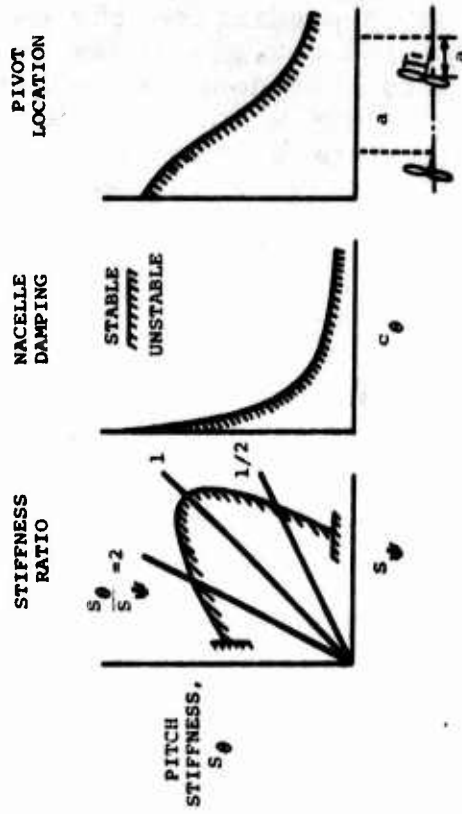
The oscillatory stability of a tilt-rotor configuration is considerably influenced by the presence of large, flexible rotors. In addition to providing gyroscopic coupling (as is the case with rigid propellers), the blade flexibility adds additional degrees of freedom whose effects must be accounted for in assessing stability. An acceptable measure of success

has been achieved in predicting the oscillatory behavior of propeller systems and certain types of flexible blade systems. Reference 3 reports that good correlation with test is achieved for propellers which are stiff enough to be considered rigid. Better correlation was reported using empirically measured propeller derivatives, but conservative results were obtained with calculated values. The principal conclusions of Reference 3 in relation to the whirl behavior of rigid propellers are presented in Figures 15 and 16.

The conclusion was that the technology for analysis of the phenomenon for rigid propellers was in a satisfactory condition. In the development of an articulated propeller system, a small model tested by Grumman exhibited instability in which blade flapping played a critical part (Reference 3). This test was of limited value because of the small scale of the model and the isotropic inertial properties of the nacelle. A carefully engineered model test conducted in 1968 (Reference 5) produced a significant amount of whirl flutter data. This model featured an articulated flapping rotor mounted at the tip of a wing spar which was scaled to be representative of the full-scale modes and frequencies of a tilt-rotor aircraft in vertical and horizontal bending and torsion (see Figure 17). The nacelle inertias and degrees of freedom were representative of full scale. Windmilling tests of this model provided many flutter data points. The testing procedure was to select a blade collective pitch setting and increase tunnel speed gradually until an unstable condition was reached or the rpm stress limits were attained. The procedure was repeated for a wide range of collective pitch settings and a flutter boundary defined. This was done for a set of parametric variations of wing spar stiffness, blade hinge offset, and blade inertia. The analytical correlation was good in that the mode of flutter was correctly predicted, although in places the prediction was overly conservative (Figure 18). Typical flutter traces from this test are shown in Figure 19. These results are in broad agreement with results reported by Bell (Reference 2) for a similar system, Figure 20. The lack of agreement noted in the flutter speeds of Figure 18 may have resulted from approximations in the mathematical model then current (C-26). This assumed a zero hinge offset with the blade frequency controlled by a restraining spring. The correlation was observed to be better with the smaller (5 percent) hinge offset than in the case of the 12-1/2 percent offset, indicating that the discrepancy in mode shape introduced by the mathematical model was important. Later programs include a general mode shape capability.

An interesting feature of the Bell results is the phenomenon described as aeromechanical instability. This is described as involving the lead-lag cyclic rotor mode and the focal mount stiffness. When the angular frequency of the rotor  $\Omega$  plus the

NATURAL VIBRATION MODES		
NONROTATING PROP	ROTATING PROP WITHOUT AIR FORCES	TRANSIENT RESPONSE WITH AIR FORCES
PITCH 	FORWARD WHIRL 	STABLE ( $V < V_{CRIT}$ ) 
YAW 	BACKWARD WHIRL 	UNSTABLE ( $V > V_{CRIT}$ ) 



NATURAL VIBRATION MODES OF SYSTEM WITH RIGID PROPELLER

RESUME OF PROPELLER WHIRL TREND STUDIES

Figure 15. Whirl Behavior of Rigid Propellers

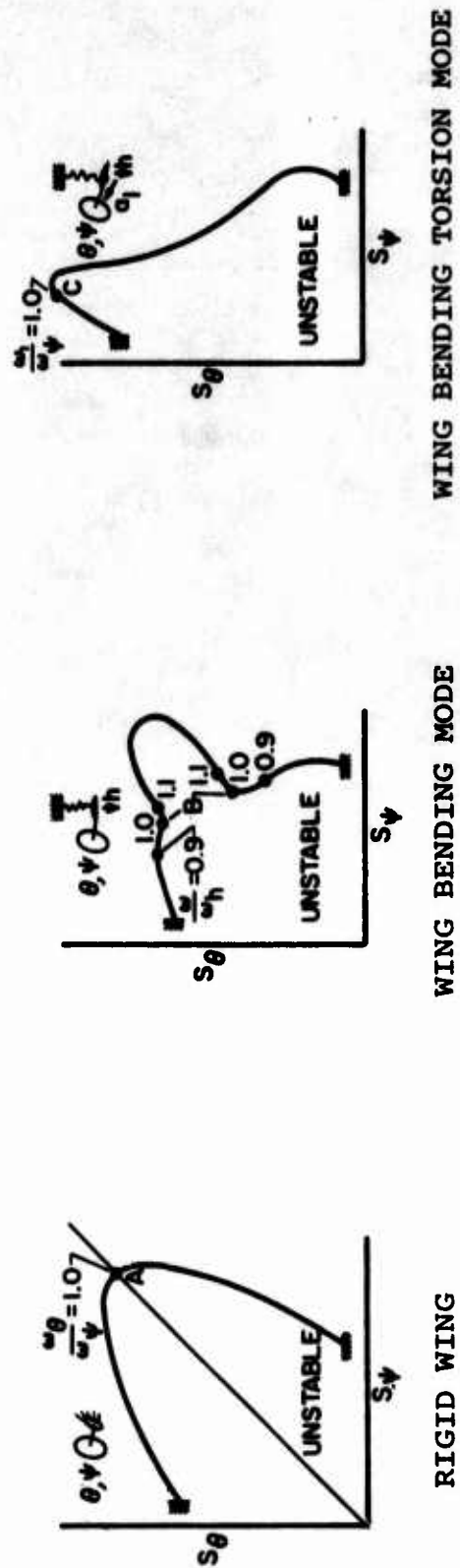
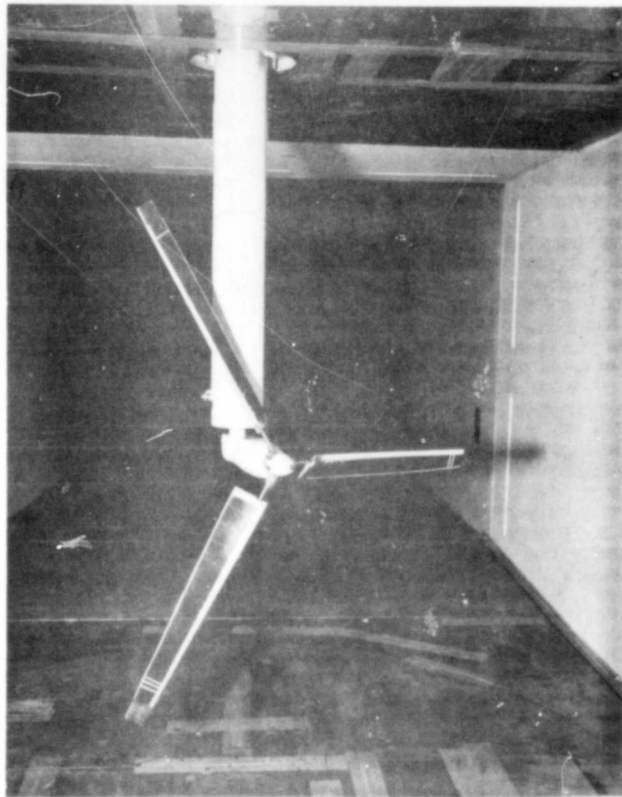


Figure 16. Influence of Wing Flexibility on Whirl Flutter

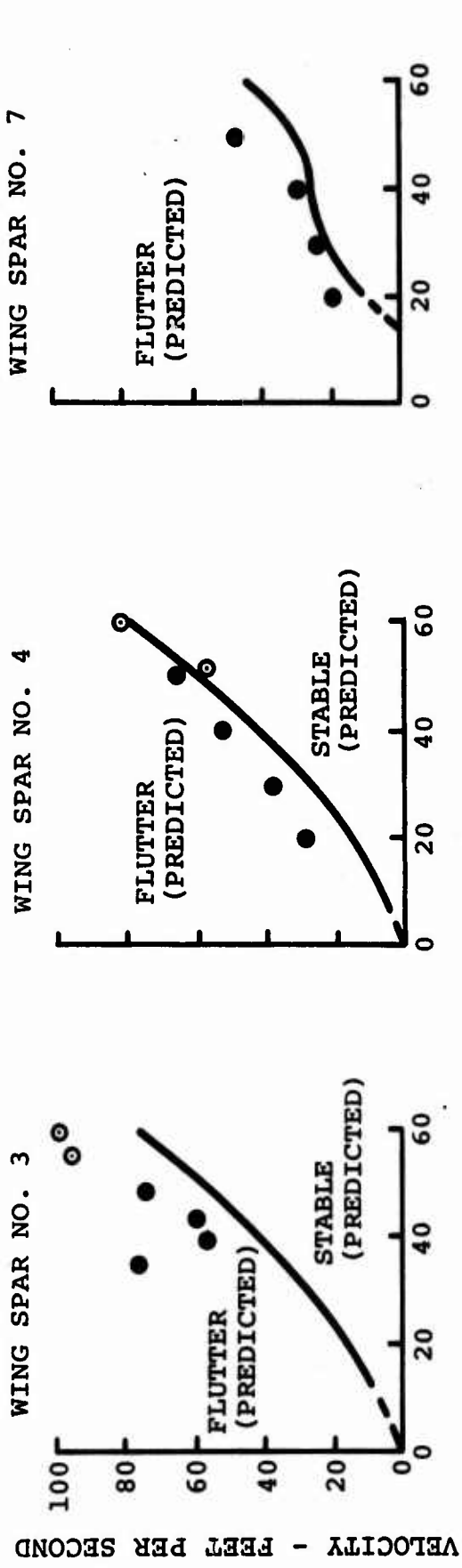




1/22-SCALE MODEL FEATURES

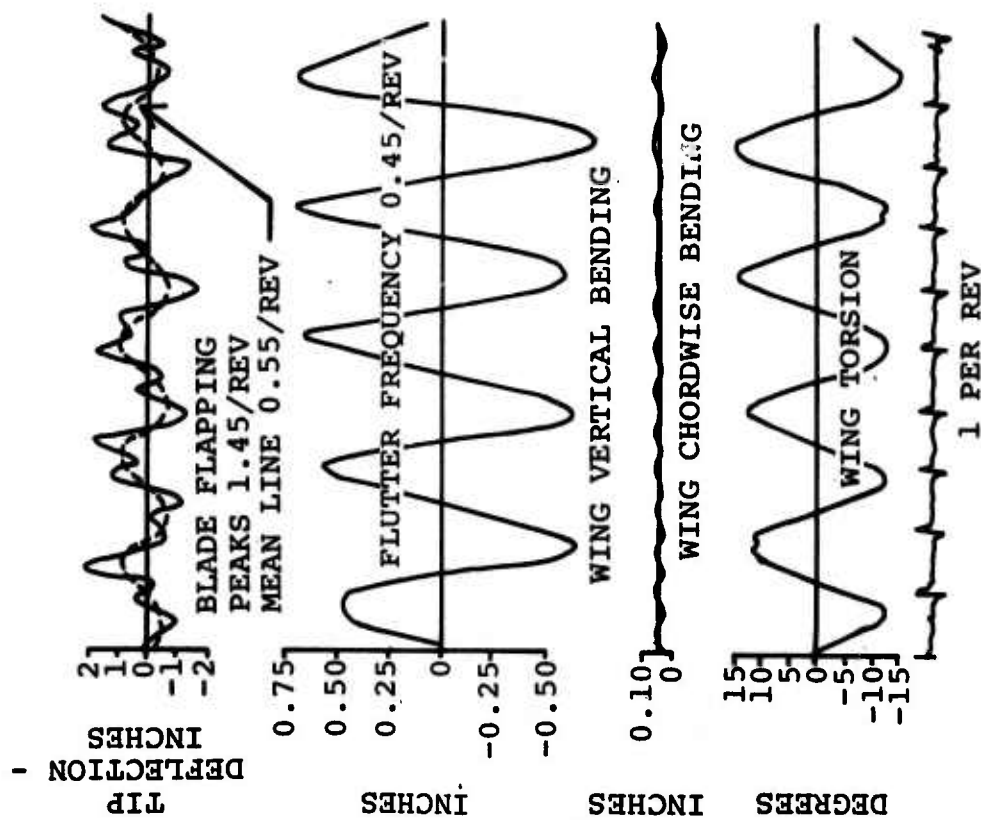
- BLADES ARTICULATED IN FLAP
- OFFSET VARIABLE
- BLADE INERTIA VARIABLE
- WING SPAR STIFFNESS VARIABLE
- WING SPAR FAIRED BY NON-STRUCTURAL AIRFOIL
- TILT RANGE 0 - 90 DEGREES

Figure 17. Vertol M160 Dynamically Similar 1/22-Scale Windmilling Model



WING SPAR NO.	3.	4.	7.
WING VERTICAL BENDING:	6.0	2.44	3.73
WING HORIZONTAL BENDING:	9.7	11.6	11.1
WING TORSION:	9.93	8.8	6.41

Figure 18. Correlation of C-26 Program Analysis With Flutter Test Points for Vertol M160 1/22-Scale Dynamically Similar Model



- FREQUENCY IN FIXED SYSTEM 0.45 PER REV
- BLADE FLAPPING FREQUENCY 0.55 AND 1.45 PER REV
- BLADE FREQUENCIES ARE  $\Omega \pm \omega$  WING

Figure 19. Typical Flutter Traces From Test of Vertol M160 1/22-Scale Dynamically Similar Model

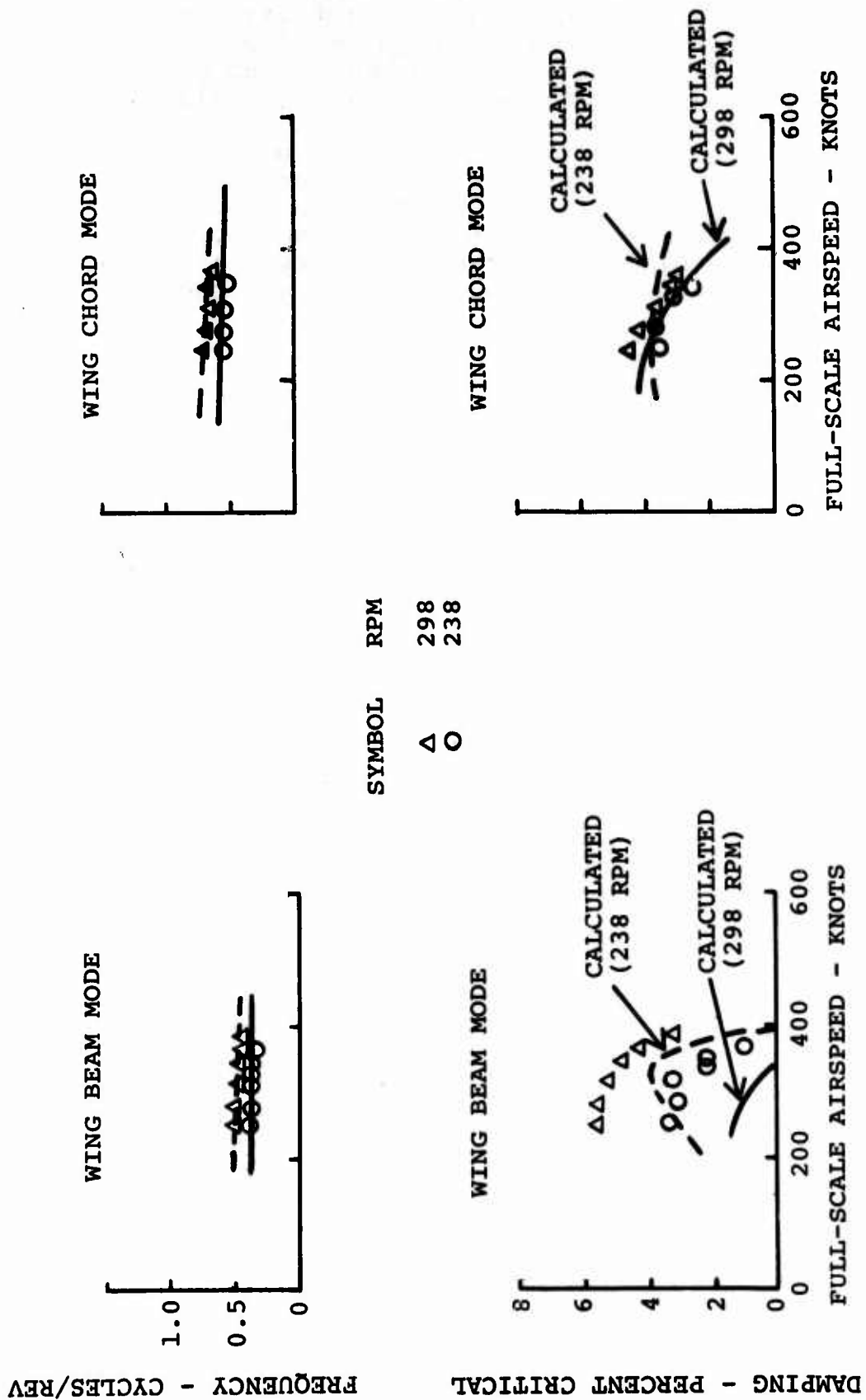


Figure 20. Whirl Flutter Stability of Bell Model 266 With Focused Rotor Locked Out

blade second mode frequency (approximately 3/rev) coalesced with the focal mount frequency, an instability occurred. The airspeed at which this occurred is proportional to the focal mast spring stiffness as shown in Figure 7 and good correlation between analysis and test is reported. This is of considerable interest because it demonstrates that mechanically coupled instabilities are not confined to blade modes with frequencies below one per rev. It also demonstrates the need for care in the application of feedback to suppress instabilities. This point is also made in Reference 6, where it is shown that rotor blade pitch feedback introduced to improve handling results in loss of damping in an air resonance mode.

#### CHARACTERISTICS OF STATE-OF-THE-ART THEORY

A number of assumptions have traditionally been made in analysis addressing propeller whirl and other airframe-rotor interactive modes of behavior. The features of a typical group of such analyses are listed in Table II. Most of these assumptions have been made around technical grounds, although simplicity and minimization of analytical derivation have also been factors. All the analyses listed are for the cruise configuration, i.e., axial flow only, on the basis that tilt and transition occur at very much lower airspeeds than maximum design dive speed and that this would therefore be the most critical case. A less plausible assumption has been that the effects of blade deflections under load could be ignored. Blade torsion has also been ignored on the assumption that blade torsional frequencies would be so high compared with the other frequencies of the system that this degree of freedom would not participate in low-frequency whirl flutter.

This assumption is frequently tenable provided the blade remains undeflected in flap and lag. However, when the blade is deflected, finite steady amounts of perturbation in either flap or lag will generate an associated twisting motion which has the same effect as  $\delta_3$  or  $\alpha_2$ . These effects may be important even if the torsional natural frequency is high relative to the flap and lag frequencies.

TABLE II

## BOEING-VERTOL STATE-OF-THE-ART V/STOL STABILITY PROGRAMS

Program Number	Degrees of Freedom	Blade Description	Aerodynamic Assumptions	Comments
C-26	6: Wing uncoupled bending, yawing, and torsion. Tip path collective pitch and yaw.	Articulated rigid blade. Out-of-plane flap only.	Axial-flow lifting line. No wing aerodynamics.	(A) Correlation good for small offset. (B) Variable gearing between blade pitch and wing torsion.
C-27	9: Wing uncoupled bending, yawing, and torsion. 2 coupled flap-lag modes per blade. Perturbations about undeflected condition.	Hingeless rotor with coupled elastic deflections in and out of rotor plane.	Axial-flow lifting line. No wing aerodynamics.	(A) Correlation good for whirl flutter and mechanical instability. (B) Correlation not good when blades are deflected in steady state.
C-40	Up to 4 rotor blades representation as in C-27. Airframe representation generalized to coupled vibration modes plus 6 rigid-body modes. Landing gear added.	As C-27	Wing and empenage aerodynamics added. Two-dimensional incompressible oscillatory aerodynamics.	(A) Correlation good for classical and whirl flutter and mechanical instability. (B) Not good for deflected blades.
C-41	Blade representation as in C-27.	As C-27	Axial-flow lifting line.	Solves for blade response and hub force and moment derivatives when rotor shaft is given small pitch rate. Correlation good.



## OSCILLATORY BEHAVIOR NOT READILY PREDICTED BY CURRENT STATE-OF-THE-ART TECHNOLOGY

It is noted from the foregoing discussion that the prediction and correlation of static and oscillatory aeroelastic behavior have been reasonably successful in a number of important applications. These applications are those where the classical assumptions of small deflections and near-axial flow were valid. See Table II for description of Boeing-Vertol state-of-the-art analyses.

A number of cases which depart from these assumptions and in which correlation is not so good are now presented.

### EFFECT OF TILT

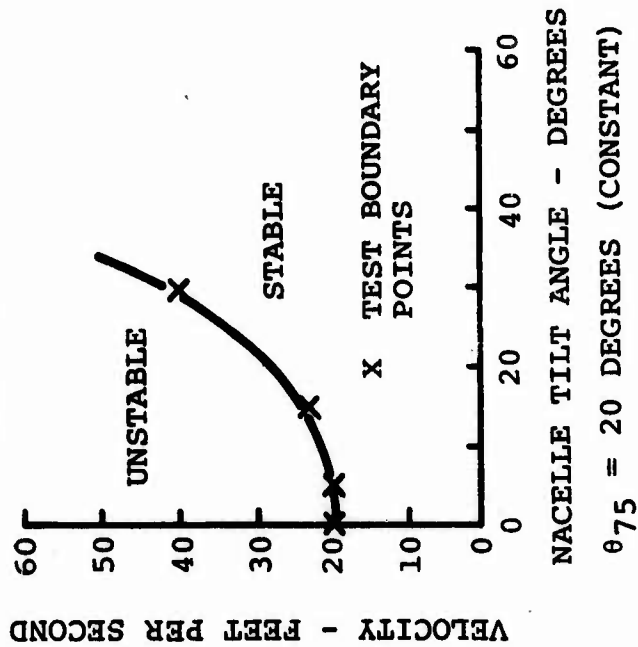
Model test results in the cruise configuration indicate a quite good predictive capability for whirl flutter in the cruise configuration. Behavior during transition does not show the same good correlation. The few published data on this topic relate to rigid-rotor behavior. Figure 21 shows data from the Boeing 1/22-scale model which indicates significant behavior variation with tilt. The blades on this model were articulated and designed to have freedom only in flap out of the plane of rotation. It is seen that, with one wing stiffness, flutter occurs with 4-1/2 degrees of tilt at much lower speeds than when the rotor shaft is untilted. With a different wing stiffness condition, the onset of flutter is increasingly delayed as the shaft is tilted.

This behavior has not been successfully correlated with existing analyses. Features present in the test which were not accounted for in the available analyses are the effects of nonaxial flow and the initial steady-state cyclic flapping of the blades.

### LIMIT-CYCLE PHENOMENA

The model test data discussed above are for a rotor whose blades have freedom only to flap or whose lead-lag frequencies are high. A later test conducted by Boeing was on a hingeless rotor model, Figure 22, whose blades had flexure mode frequencies of 0.84 and 1.2 per rev at the operating rpm and collectives. This model did not exhibit the clear-cut divergent oscillations encountered on the earlier model, but rather limit-cycle oscillations which were clearly self-sustaining. These occurred only at a combination of negative thrust and rpm conditions such that the blades were coned back significantly from the precone angle, Figure 23. The freedoms involved in these instabilities included fundamental vertical flexure torsion mode of the wing and blade flap, lag, and torsion. This type of instability appears to have something

WING SPAR NO. 7  
 5% FLAP HINGE OFFSET  
 LIGHT BLADE



WING SPAR NO. 3  
 5% FLAP HINGE OFFSET  
 LIGHT BLADE

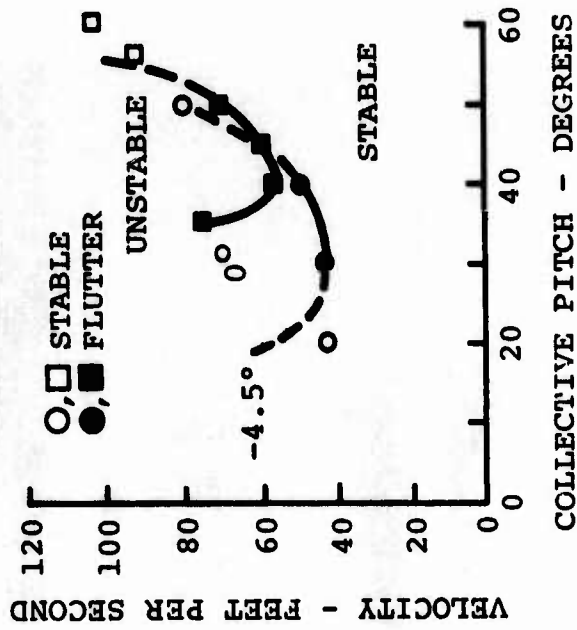


Figure 21. Effect of Nacelle Tilt on Flutter Speed

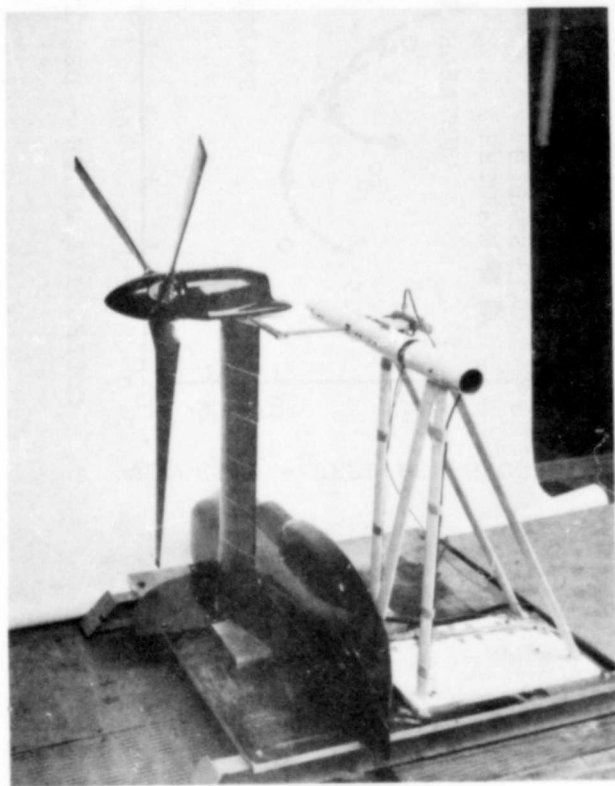
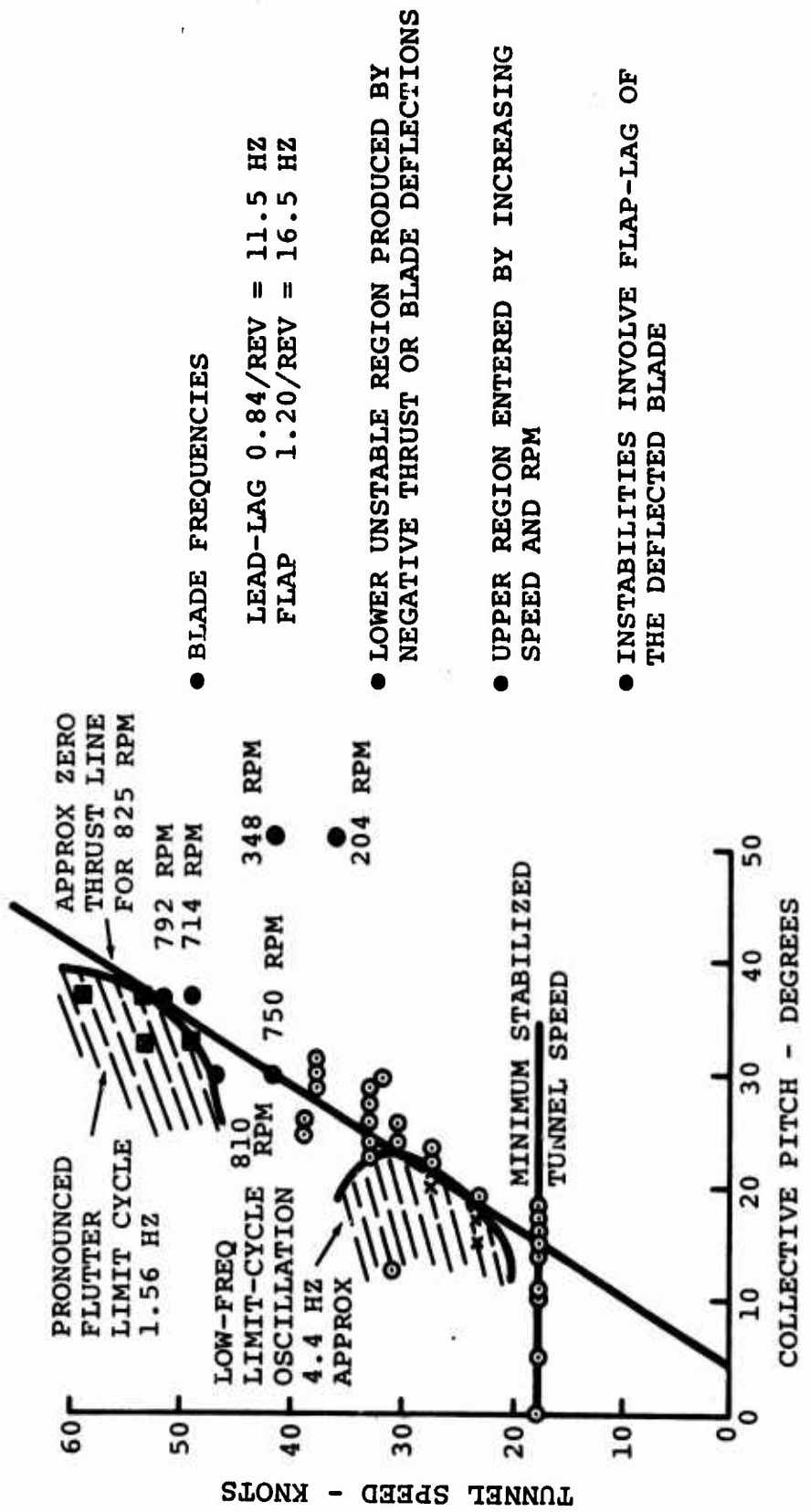


Figure 22. 1/10-Scale M160 Dynamic Model



NOTE: ALL O POINTS 825 RPM  
 X FLUTTER (825 RPM) POWERED  
 □ FLUTTER (WINDMILLING)  
 ● STABLE (WINDMILLING)

● BLADE FREQUENCIES

LEAD-LAG 0.84/REV = 11.5 HZ  
 FLAP 1.20/REV = 16.5 HZ

● LOWER UNSTABLE REGION PRODUCED BY NEGATIVE THRUST OR BLADE DEFLECTIONS

● UPPER REGION ENTERED BY INCREASING SPEED AND RPM

● INSTABILITIES INVOLVE FLAP-LAG OF THE DEFLECTED BLADE

Figure 23. Data Points and Flutter Regions for Basic Cruise Configurations of Vertol M160 1/10-Scale Dynamically Similar Model

in common with the behavior predicted in Reference 7. However, the analysis of Reference 7 is not by itself adequate to predict the behavior noted since it does not include the hub degrees of freedom, which appear to be an important component of motion in the unstable mode.

#### BLADE MOTION - AH-56 1P-2P INSTABILITY

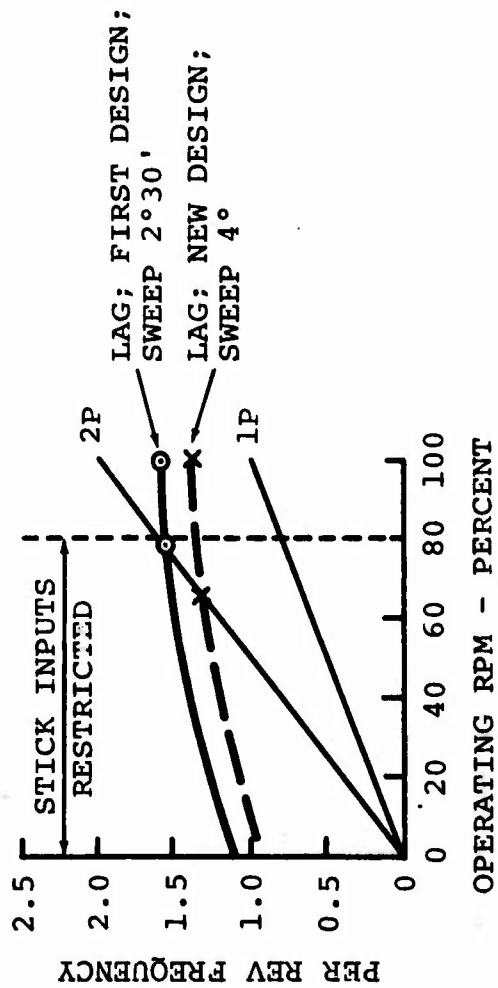
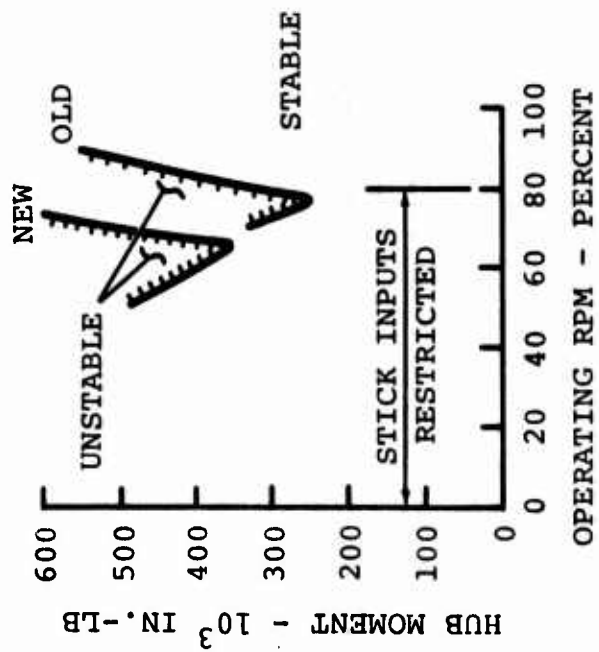
Experience with the AH-56 (8) indicates that the flap-lag coriolis-type instability is a threat in any situation where the lag impedance to in-plane forces induced by flapping is small. An unstable region was observed to exist when the blade cyclic flapping exceeded a certain limit which approached a minimum when the blade lag frequency was at 2P, Figure 24. The mechanism of this instability seems to be similar to that discussed in Reference 7 where collective flap deflection of the blade produces in-plane forces to drive the lag mode when its frequency is near 1P. In Reference 8, the mechanism involves large cyclic initial conditions of blade flapping which produce in-plane coriolis 2P loads. This creates the conditions necessary for a flap-lag instability. When the lag frequency is exactly 2P the instability becomes a slow rotor tilt divergence.

#### AIRFRAME AND ROTOR INSTABILITY IN FORWARD FLIGHT

Also reported in Reference 8 is an effect described as a 1/2-per-rev hop. This seems to be similar to effects described by Hohenemser, References 7 and 9. The phenomenon is described as occurring in high-speed forward flight and can be predicted conservatively by an analysis whose critical parameters are collective pitch stiffness, blade lag frequency, and flap-pitch coupling or  $\delta_3$ . In Figure 25, sensitivity studies conducted by Lockheed indicate that a 13-degree-of-freedom analysis predicts the behavior with a conservative margin of approximately 10 percent. An increase of 10 percent in the critical speed required a 20-percent increase in collective stiffness. The sensitivity to lag frequency depended on the rpm.

At 3 percent above operating rpm, there was no change in the critical speed for a range of in-plane frequency from 1.46 to 1.52 per rev. At 3 percent below operating rpm, the same change in lag frequency produced a shift in critical speed from 150 knots to 160 knots, i.e., to produce a 6-percent increase in speed we require a 4-percent increase in lag frequency. A 50-percent increase in lag damping was calculated to increase the critical speed by less than 2 percent.

Overall, the underestimated speed was conservative by about 10 percent.



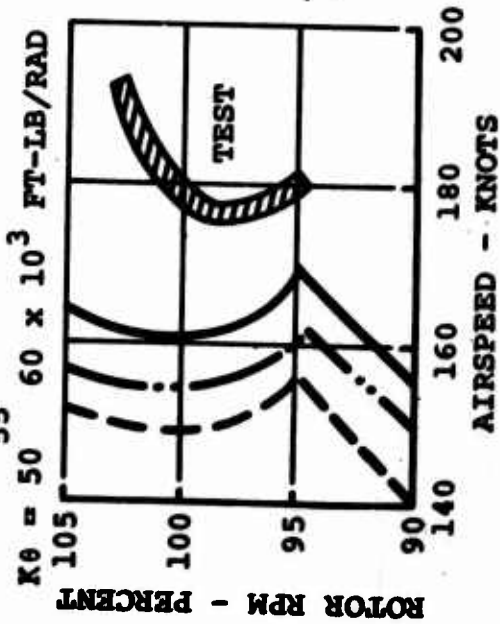
ORIGINAL DESIGN: IN-PLANE FREQUENCY = 1.6P; SWEEP 2°30'  
 0 IV DESIGN: IN-PLANE FREQUENCY = 1.4P; SWEEP 4°0'

Figure 24. Relation of Unstable Regions to Lag Frequency 2P Coalescence in AH-56A Instability

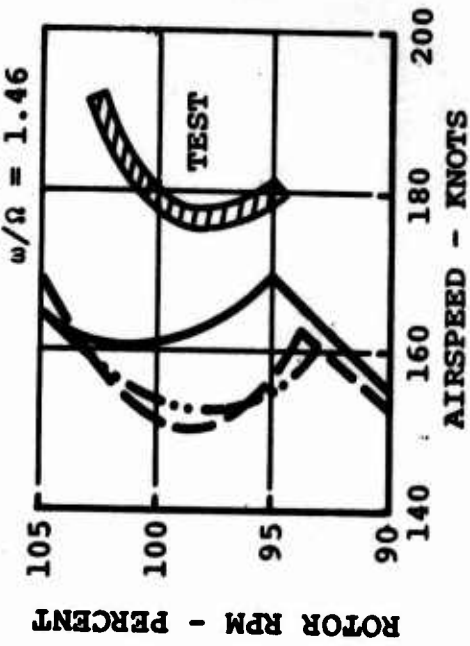


EFFECT OF COLLECTIVE STIFFNESS  
ON STABILITY BOUNDARIES

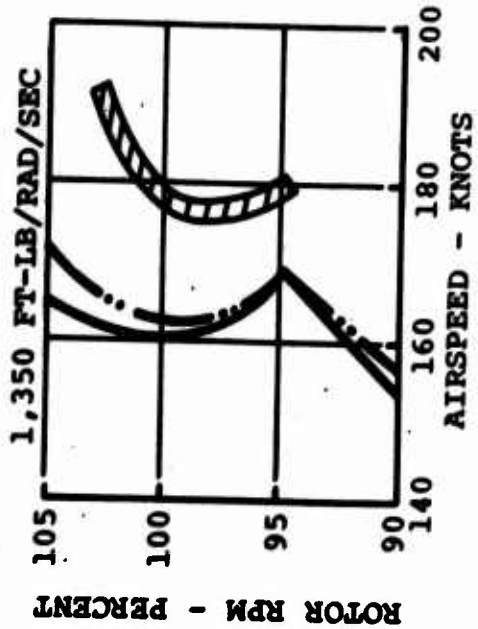
55



EFFECT OF IN-PLANE FREQUENCY  
ON STABILITY BOUNDARIES



EFFECT OF IN-PLANE DAMPING  
ON STABILITY BOUNDARIES



$\omega/\Omega = 1.46$  (dashed line)  
 $\omega/\Omega = 1.50$  (solid line)  
 $\omega/\Omega = 1.52$  (dash-dot line)

Figure 25. Correlation of Analysis and Test for AH-56A Subharmonic Hop

## SUMMARY OF ROTOR/AIRFRAME STABILITY PREDICTION CAPABILITY

At this point several things have become clear:

1. Typical analyses used to predict divergence, whirl flutter, and aeromechanical and mechanical instability are successful enough when the inherent assumptions are valid. Such assumptions have included: (a) small perturbations about zero deflection equilibrium conditions, (b) axial flow through the rotors.
2. Incidents in which the existing theory has not proved successful have involved finite blade deflections and nonaxial flow.

It is concluded that these effects must be considered if the range of correlated phenomena is to be extended. Since a considerable body of literature exists on the behavior of individual blades, this will be examined to provide guidelines as to what effects are important in the context of the overall aircraft. This study will indicate which effects must be included in an upgraded capability.

## BLADE INSTABILITY MECHANISMS

Since one of the departures of state-of-the-art analyses from reality relates to the blade idealizations used, and since a considerable body of literature exists concerning the stability of individual blades, a survey of the main mechanisms identified for individual blades will now be undertaken. This will indicate additional features to be included in an up-graded technology if correlation with a wider range of phenomena is to be attempted.

The principal mechanisms and types of instability in individual blades are listed in Table III and discussed in detail in the following paragraphs.

### BLADE CLASSICAL FLUTTER

A blade possesses all the potential for undesirable aeroelastic behavior familiar from fixed-wing experience, but even the classical problems tend to be more severe when encountered in the rotating environment. The flexure torsion flutter problem, for example, becomes more sensitive to chordwise center-of-gravity location because of the additional coupling produced by the centrifugal force field.

For hover or vertical flight, as in the propulsion mode for tilt rotors, the equations take the same form as those for classical fixed-wing flutter:

$$\begin{bmatrix} 1 & C_a \\ \overline{C_a} & 1 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\beta} \end{bmatrix} + \begin{bmatrix} M_{\dot{\theta}} & C_v \\ \overline{C_v} & M_{\dot{\beta}} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} M_{\theta} & C_d \\ \overline{C_d} & M_{\beta} \end{bmatrix} \begin{bmatrix} \theta \\ \beta \end{bmatrix} = 0$$

The equations have been divided through by the pitching and flapping mass moments of inertia  $I_{\theta}$  and  $I_{\beta}$  respectively. In the first equation, the  $\theta$  coefficient arises from aerodynamic pitch damping.

The  $\theta$  coefficient consists of three components:

1. The aerodynamic moment resulting from the center of pressure not being at the pitch axis.
2. The centrifugal restoring moment or tennis racquet effect.
3. The elastic restoring moment due to the pitch spring, i.e., control stiffness.

TABLE III  
 INSTABILITIES ENCOUNTERED IN PROP/ROTORS

Instability	Principal Freedoms	Frequency Characteristics	Comments
Classical Flutter	Flap-pitch or torsion. CG offset from torsion axis.	$\omega$ Flap $< \omega_I$ $< \omega$ torsion dynamic forces tend to produce coalescence.	Flutter mode may involve overtone bending.
Flap-Lag-Pitch $\alpha_2, \delta_3$ coupling	Two or more of flap, lag, pitch with $\alpha_2$ or $\delta_3$ present.	$\omega_I$ = frequency coalescence of participating modes due to rpm, velocity effects.	Mechanism essentially classical.
Flap-Lag-Pitch Deflected Blades	As above. Effective $\delta_3$ results from flap oscillation when blade has steady lag. $\alpha_2$ results from lead-lag oscillation when blade has steady flap deflection.	$\omega_I$ = frequency coalescence of participating mode.	Mechanism essentially same as above. Limit cycle.
Flap-Lag Coriolis Hover	Flap-lag perturbations about deflected equilibrium position.	$\omega_I = \omega_{LAG}$	Flap perturbation produces lead-lag coriolis force. Lead-lag response produces additional lift and flap response. Limit cycle.
Flap- Coriolis $\mu \neq 0$	Flap-lag about deflected equilibrium positions.	$\omega_I = \omega_{LAG}$	Azimuthal variation of lift on blade gives coupling additional to coriolis forces.

The pitching moment arising from flapping (i.e., the coefficient of  $\beta$  in the first equation) may be similarly identified. The  $\ddot{\beta}$  term arises from the product of inertia about the pitch-flap axes, the  $\dot{\beta}$  term because the lift does not in general act at the pitch axis, and the  $\beta$  term due to centrifugal forces acting on the masses which may be off both flapping and pitching axes. In the second equation, the  $\ddot{\beta}$  term is strictly aerodynamic flap-damping, and if there is no flap-spring, the  $\dot{\beta}$  term is just due to centrifugal restoring moments. The  $\dot{\theta}$  term is due to product of inertia, and the  $\ddot{\theta}$  term is the aerodynamic flapping moment arising from blade pitch. Finally, the  $\theta$  coefficient  $C_d$  has two parts: (a) a centrifugal flapping moment arising from product of inertia and (b) an aerodynamic part which exists because lift causes a flapping moment. For a rigid uniform blade, with zero  $\delta_3$ , and neglecting all wake aerodynamic effects, aerodynamic contributions to effective mass, and structural damping, the coefficients are:

$$\begin{aligned} M_{\dot{\theta}} &= \frac{\gamma}{8} \frac{I}{I_{\theta}} \frac{\Omega}{R^2} \left[ \frac{c}{2} - a \right] \left[ \frac{c}{2} - 2a \right] ; & M_{\dot{\beta}} &= \frac{\gamma}{8} \Omega \\ M_{\theta} &= \omega_{\theta}^2 - \frac{\gamma}{8} \frac{I}{I_{\theta}} \frac{\Omega^2}{R} \cdot a & ; & M_{\beta} = \omega_{\beta}^2 \\ C_a &= \frac{I R}{I_{\theta}} & ; & \bar{C}_a = \frac{I R}{I} \\ C_v &= \frac{\gamma}{8} \frac{I}{I_{\theta}} \frac{\Omega}{R} \cdot a & ; & \bar{C}_v = -\frac{\gamma}{6} \frac{\Omega}{R} \left( \frac{3}{4}c - a \right) \\ C_d &= \Omega^2 \frac{I_r}{I_{\theta}} & ; & \bar{C}_d = \Omega^2 \left( \frac{I_r}{I} - \frac{\gamma}{8} \right) \end{aligned}$$

where  $\gamma = \text{Lock Number} = 2 \pi \rho C R^4 / I$

$\omega_{\beta} =$  uncoupled, rotating flap natural frequency

$\omega_{\theta} =$  uncoupled, rotating pitch natural frequency

$I_r =$  product of inertia about pitch-flap axes

$I_{\theta} =$  moment of inertia about pitch axis

$I =$  moment of inertia about flap axis

The literature dealing with this problem for cases which involve the rigid-body flapping and pitching motions of hinged blades, with and without various amounts of elastic bending and twisting motion, is reasonably complete (see References 10, 11, 12, and 13). The principal parameter of influence in these cases is, as it is for fixed wings, the distance between

the section center of gravity and the section aerodynamic center\*, although this is not obvious in the coefficients listed above since the chordwise position of the center of gravity is integrated into the product of inertia,  $I_r$ .

Further, the critical flutter speed will tend to be a minimum where a flap-bending degree of freedom and a pitch-torsional degree of freedom are close together, for any given value of pitch frequency. When these effects are considered, the critical flutter speed will tend to be directly proportional to the pitch-torsional frequency. When flexible modes and nonuniform blade properties with span are considered, the coefficients involve integrals over the span which must be evaluated numerically. The coefficients listed above are the values of such integrals for the special case of a linear flap mode and a twist mode in which all the rotation occurs at the root (i.e., pitch).

Quasi-static aerodynamics usually are satisfactory for predicting the critical speeds, although fixed-wing type unsteady, potential flow results are often used (Reference 13, for example). The relationship is such that quasi-static aerodynamics remain if  $F$  and  $G$  are assumed to be 1 and 0, respectively, and aerodynamic effective mass terms (i.e., those which are coefficients of accelerations) are dropped. An exception to this rule is that some analysts drop the  $\dot{\alpha}$ , or effective camber terms when making the quasi-static assumption. Where the total downwash velocities through the rotor are low enough that the wake shed and trailed from oscillating blades remains close to the rotor disk, then neither fixed-wing unsteady nor quasi-static aerodynamics are adequate to the task of predicting rotor flutter. Such so-called wake flutter cases are examined, for example, in Reference 12, using the results of Reference 14. Figure 26 taken from the former paper shows that several additional branches of what appears to be flutter of a classical type can exist when a rotor blade operates close to its own wake or that of a preceding blade.

Reference 14 also shows that, at low values of inflow, single-degree-of-freedom pitch flutter can occur if the pitch axis is forward of the quarter chord. These effects apparently become negligible when the axial distance between shed vorticity in chord lengths,  $\frac{2\pi u}{n\Omega c}$ , is greater than about 3. Under these circumstances, the major differences between these rotor

---

\*This term really implies the center of pressure for changes in lift, as distinct from center of pressure including steady-moment terms which are of no consequence to classical flutter.



PITCH-FLAP RATIO = 2, VIRTUAL  $\delta_3 = 63.4^\circ$

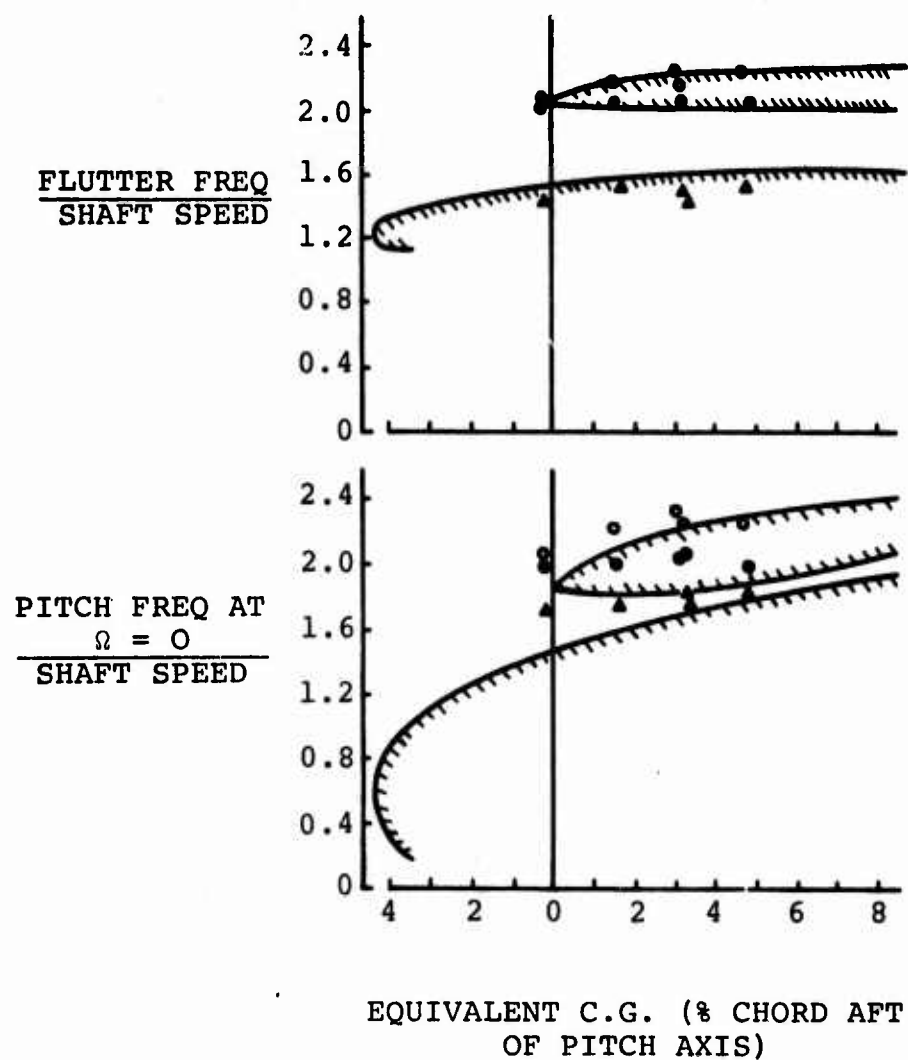


Figure 26. Pitch-Flap Flutter of a Blade Operating in Wake Effects

flutter cases and wing flutter are in the dynamic effects associated with rotation.

### FLAP-PITCH FLUTTER ( $\delta_3$ )

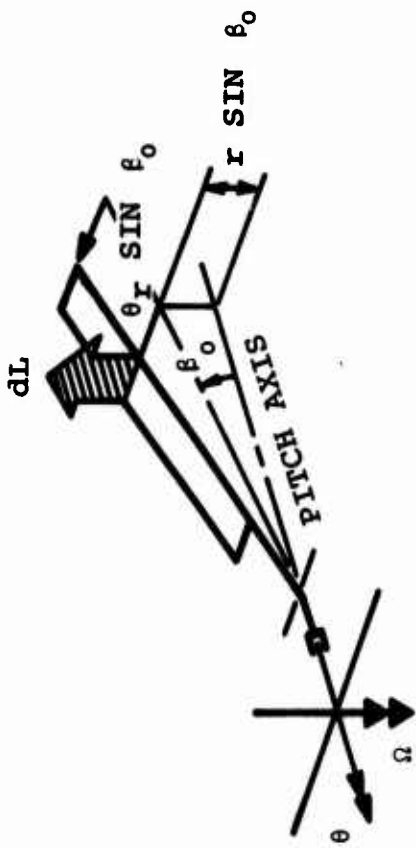
The kinematic coupling between flapping and pitching,  $\tan \delta_3$ , is shown in Reference 11 to be destabilizing for bending modes where the root bending slope is in the same direction as the tip deflection. This, of course, is an effect with no direct counterpart in the case of fixed-wing aircraft; furthermore, it is not likely to have been predicted intuitively since positive  $\delta_3$  is stabilizing from many considerations. When the root slope direction is opposite from the tip bending deflection,  $\tan \delta_3$  tends to be stabilizing.

### FINITE DEFLECTIONS, PITCH-LAG COUPLING

Large, steady bending deflections, mentioned earlier in connection with drag forces and static divergence, have potential importance for helicopter rotor blade flutter as well, as was pointed out in Reference 10. In cases where a blade section is above the extension of the pitch axis at the same radial station (as in Figure 27), then there are important additional coriolis moments introduced about the inboard pitch axis. To illustrate this effect, consider a blade with a pitch axis and flapping axis in the plane of rotation and at the centerline of the rotor. Suppose this blade was precone to a value  $\beta_0$ . In this case not only would the feathering moments of inertia about the pitch axis change as compared to a blade where  $\beta_0 = 0$ , Figure 28, but the coupling term  $C_V$  would change by the amount  $-2 \frac{I}{I_\theta} \Omega \sin^2 \beta_0$ . Further, Reference 10 points out that, in such cases, the steady lift at an outboard section vertically displaced will be laterally displaced an amount  $\theta \chi \sin \beta_0$  by rotation,  $\theta$ , about the inboard pitch axis. This component of pitching moment plus that caused by variations in in-plane aerodynamic force acting through the vertical offset moment arm about the pitch axis will change the terms  $C_V$  and  $M_\theta$  by the amounts

$$\Omega \frac{\gamma}{2} \frac{I}{I_\theta} \beta_0 \left( \frac{\theta_0}{4} - \frac{2}{3} \lambda \right) \text{ and } \Omega^2 \frac{\gamma}{2} \frac{I}{I_\theta} \beta_0 \left( \frac{\theta_0}{4} - \frac{2}{3} \lambda \right)$$

respectively. Similar steady bending effects in the plane of rotation can also introduce additional pitching moments as a result of lift variations. Since the center of gravity also moves aft, however, and it is center of gravity-aerodynamic center relationship that matters, rather than reference to the pitch axis, these steady in-plane bending effects are less likely to be important. Where concentrated masses are added, however, the effective center of gravity can move more rapidly



- BLADE SECTION DISPLACED AMOUNT  $r \sin \beta_0$  BY CONING
- LATERALLY DISPLACED BY AMOUNT  $\theta_r \sin \beta_0$  BY ROTATION
- CORIOLIS MOMENT PRODUCED ABOUT PITCH AXIS

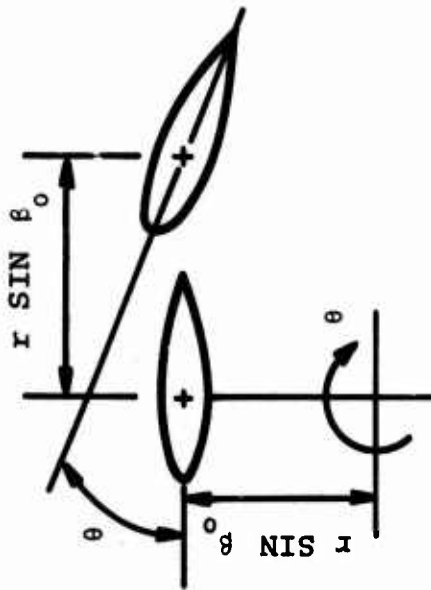
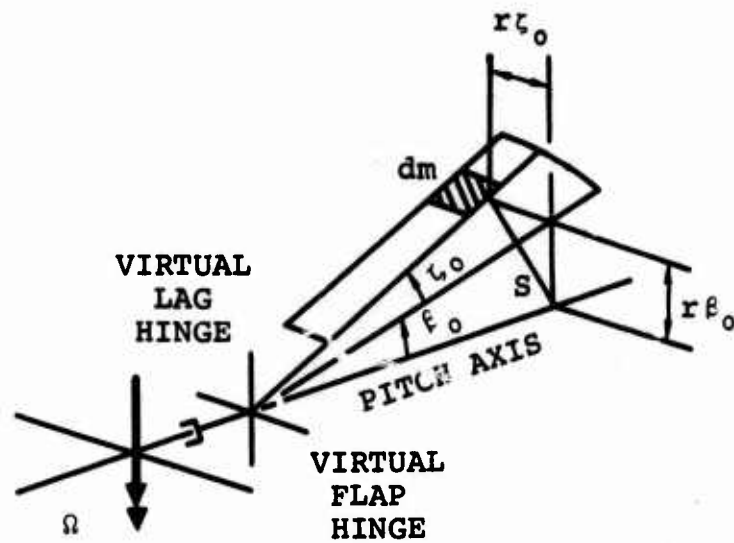


Figure 27. Mechanism of Pitch-Lag Coupling



$$\begin{aligned}
 I_{\theta} \text{ (PITCH INERTIA)} &= I_{\theta}^1 + \int S^2 dm \\
 &= I_{\theta}^1 + \int [(r\beta_0)^2 + (r\zeta_0)^2] dm \\
 &= I_{\theta}^1 + \epsilon_0^2 I_{\epsilon} + \zeta_0^2 I_{\zeta}
 \end{aligned}$$

Figure 28. Finite Deflection Effect on Blade Pitch Inertia

than the aerodynamic center, so that in such cases the effect of steady lag-bending deflections should not be neglected in stability analyses.

### PITCH-LAG AND FLAP-LAG INSTABILITIES

In the preceding section, steady in-plane deflections were mentioned as a potentially important part of the configuration when flap-pitch instabilities are considered. When lag hinges and/or substantial flexibility in chordwise bending are incorporated, such deflections must often be considered as additional degrees of freedom (Figure 29). There is under these circumstances an increase in the number of dynamic instabilities possible. An early theoretical analysis showing the possibility of a classical type instability involving lag motion appeared in Reference 15, and the first analysis of a case where such instabilities were actually encountered was given in Reference 16. The degrees of freedom involved are lagging and flapping. The most critical parameter, however, is a kinematic coupling,  $\tan \alpha_2$ , causing a pitch angle change to occur as a result of blade lag deflections (Figure 30). For this reason, the phenomenon has been called a pitch-lag instability. The equations of motion for a rotor with  $\delta_3$  but no flap hinge offset and zero twist are:

$$\begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \ddot{\begin{matrix} \xi \\ \xi \end{matrix}} + \begin{bmatrix} M_{\xi}^{\cdot} & C_V \\ C_V & M_{\xi}^{\cdot} \end{bmatrix} \dot{\begin{matrix} \xi \\ \xi \end{matrix}} + \begin{bmatrix} M_{\xi} & C_a \\ C_a & M_{\xi} \end{bmatrix} \begin{matrix} \xi \\ \xi \end{matrix} = 0$$

$$\text{where } C_V = \Omega \gamma \left[ \frac{\theta_0}{4} - \frac{\theta_0}{3} e_{\xi} \right]$$

$$C_a = -\frac{\gamma}{8} \Omega^2 \tan \alpha_2$$

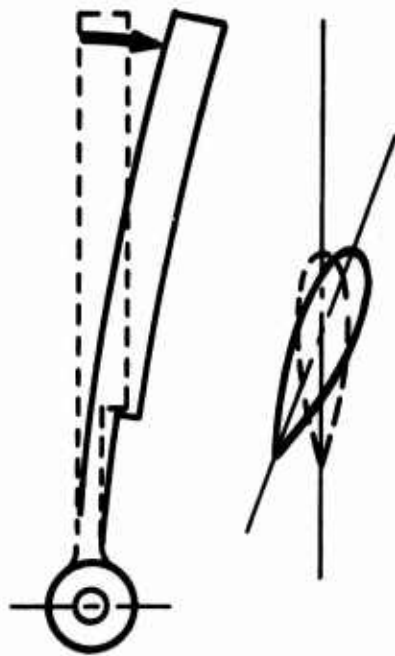
$$\bar{C}_V = \frac{2F_0}{\Omega} (\omega_{\xi}^2 + \Omega^2) - \frac{\gamma I}{I_{\xi}} \frac{\Omega}{2} \left[ \theta_0 \left( \frac{1}{4} - \frac{2}{3} e_{\xi} + \frac{e_{\xi}^2}{2} \right) + \lambda \left( \frac{1}{3} - \frac{e_{\xi}}{2} \right) \right]$$

$$\bar{C}_a = \frac{\gamma I}{I_{\xi}} \frac{\Omega^2}{2} \tan \delta_3 \left( \frac{1}{3} - \frac{e_{\xi}}{2} \right) \lambda$$

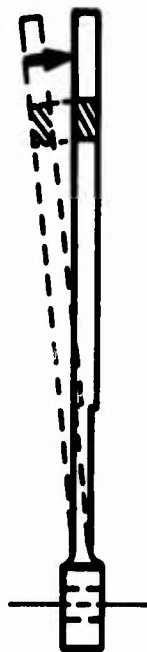
$$M_{\xi}^{\cdot} = \frac{C_{\xi}}{I_{\xi}} + \frac{\gamma I}{I_{\xi}} \Omega^2 \left\{ \tan \delta_3 \left( \frac{1}{4} - \frac{2}{3} e_{\xi} + \frac{e_{\xi}^2}{2} \right) - \frac{\theta_0 \lambda}{2} \left( \frac{1}{3} - e_{\xi} + e_{\xi}^2 \right) \right\}$$

$$M_{\xi} = \omega_{\xi}^2 + \frac{\gamma I}{I_{\xi}} \frac{\Omega^2}{2} \tan \alpha_2 \left( \frac{1}{3} - \frac{e_{\xi}}{2} \right) \lambda$$

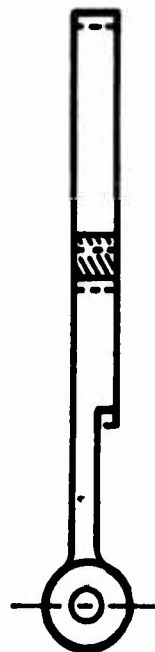
Here  $M_{\xi}^{\cdot}$  and  $M_{\xi}$  are as defined earlier but the latter has the quantity  $\frac{\gamma \Omega^2}{8} \tan \delta_3$  subtracted from it to account for the effect of  $\delta_3$ .



- BLADE LAGS
- PITCH ANGLE DECREASES DUE TO  $\alpha_2$
- NEGATIVE INCREMENT OF LIFT PRODUCED



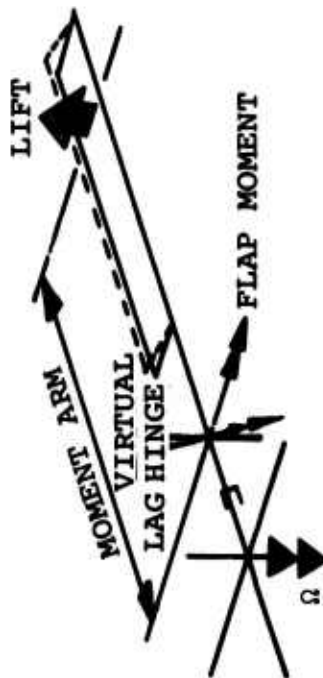
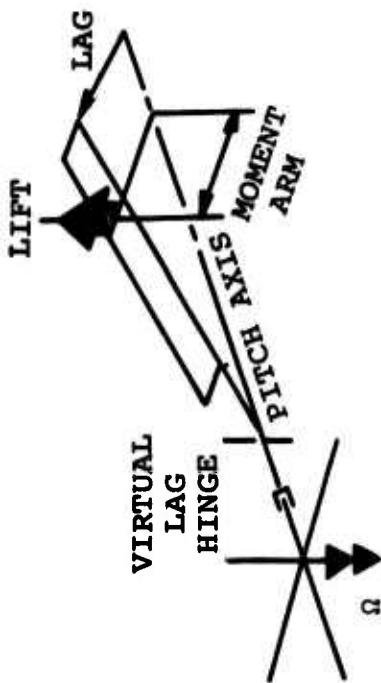
- BLADE DOWNWARD FLAPPING OCCURS DUE TO LIFT LOSS
- CORIOLIS EFFECTS PRODUCE FORCES IN LAG DIRECTION



- PHASING EXISTS BETWEEN LIFT LOSS AND FLAP RESPONSE AND BETWEEN CORIOLIS FORCES AND LEAD-LAG RESPONSES
- WITH ADVERSE SET OF FREQUENCY RELATIONS SYSTEM MAY BE UNSTABLE

Figure 29. A Mechanism of Pitch-Lag Flap Instability with Coriolis and  $\alpha_2$  Coupling





- BLADE LIFT MOVES WITH BLADE LAG DEFLECTION
- A NOSE-DOWN MOMENT ABOUT PITCH AXIS IS PRODUCED
- PITCH MOMENT PRODUCES ROTATION OF VIRTUAL LAG HINGE
- FLAP MOMENT THEN HAS COMPONENT IN LEAD DIRECTION
- PITCH-LAG INSTABILITY MAY OCCUR
- COUPLINGS DO NOT EXIST IF PITCH AXIS IS OUTBOARD OF LAG HINGE

Figure 30. Pitch-Lag Coupling in Rotor Blade

Pitch-lag kinematic coupling can arise from inclination of the lag hinge with respect to the blade axis and/or the adverse positioning of control links with respect to such axes when all the other blade angles such as lag, flap, and pitch are accounted for as initial deflections (Figure 31). If this kinematic coupling is such that the blade pitch angle increases as the blade lags forward (negative, in the convention of the above equations) and the lagging frequency,  $\omega_\xi$ , is below the flap frequency,  $\omega_\beta$ , (which also implies  $\omega_\xi < \Omega$ ), Reference 16 shows clearly that an instability can result and presents the following simple stability criterion,

$$C_\xi + \frac{\tan \alpha_2}{1 - \frac{\rho_0}{\theta_0} \tan \delta_3} - 2 \Omega I_\xi \frac{\rho_0^2}{\theta_0} > 0$$

The value of lag damping is particularly critical where the principal motion is in lagging or chordwise bending and the frequency of the unstable motion is close to the uncoupled lagging frequency. Reference 17, in further studies of this phenomenon, shows that the aerodynamic damping due to lag motion and proper accounting for blade hinge offsets must be included to be assured of a conservative analysis.

#### SPECIAL ASPECTS OF PROP/ROTORS

The coupled flap-lag analyses of References 15 through 17 are all intended for the helicopter state, in which the total inflow is relatively small compared to the rotor tip speed. When a V/STOL prop-rotor operates in high-speed airplane flight regimes, the inflow ratio,  $J$ , is no longer negligible compared to unity, and this gives rise to additional aerodynamic couplings between motions in the plane of rotation and out of the plane of rotation. In fact, flap-lag instabilities can occur, as discussed in Reference 18, for rotors with natural rotating lag-bending and flap-bending frequencies close to and higher than rotor speed. As pointed out in this reference, the need for high geometric pitch angles at high values of  $J$  rotates the low beamwise bending stiffness of the prop-rotor blades so that substantial beamwise motion (i.e., perpendicular to the local chord plane) contributes to in-plane motion, thus lowering the frequency of the first in-plane mode. Similarly, this rotation orients the chord-bending stiffness so as to have a larger component out of plane, and consequently, the first flap-bending mode tends to be raised. As expected,  $\delta_3$  has a substantial influence on this kind of instability. Reference 17 calls attention to the fact that the modes of highly twisted prop/rotor blades at high advance ratios are coupled and that a mode identified as being predominantly in-plane will have substantial out-of-plane flap bending slopes. Thus

- BLADE LEAD VELOCITY INCREASES LOCAL WIND VELOCITY AND ADDITIONAL LIFT AND FLAP MOMENT
- LEAD VELOCITY ALSO PRODUCES ADDITIONAL CENTRIFUGAL FORCE STIFFENING IN FLAP MODE
- WHEN THESE INCREMENTS ARE NOT EQUAL FLAPPING OCCURS
- FLAPPING VELOCITY PRODUCES ADDITIONAL CORIOLIS LEAD-LAG FORCES
- THE MECHANISM MAY BE UNSTABLE FOR SUFFICIENTLY LARGE VALUES OF INITIAL FLAP DEFLECTION AND APPROPRIATE FREQUENCY RATIOS OF FLAP AND LAG MODES

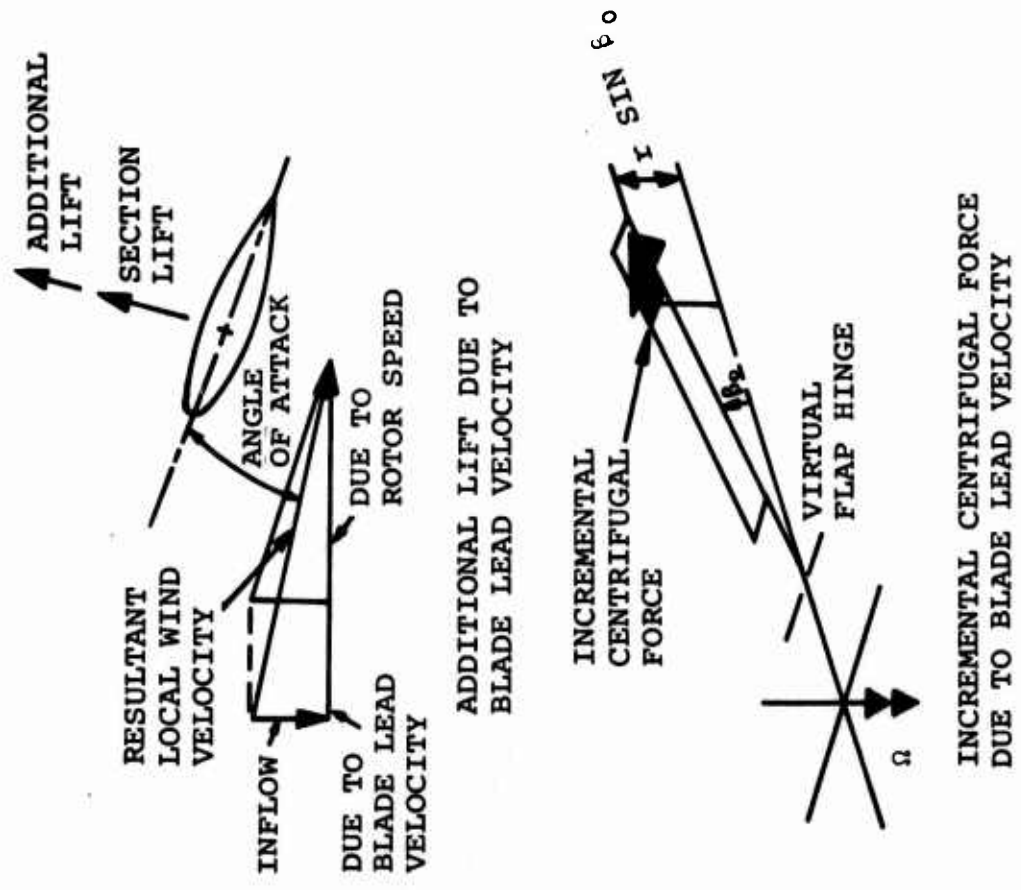


Figure 31. Mechanism of Flap-Lag Instability (Hover Case)

the presence of  $\delta_3$  provides kinematic pitch coupling for a predominantly in-plane mode, i.e., in such a situation  $\delta_3$  can provide effective pitch-lag coupling  $\alpha_2$ .

#### COUPLED FLAP-PITCH-LAG FLUTTER

It is not unusual to have natural frequencies in flap, pitch, and lag reasonably close to one another and rotor blades may be shown to be susceptible to flutter in situations where out-of-plane bending or flapping motions and in-plane bending are coupled, both to each other and to torsional or pitching motions.

An example of such an analysis is found, for example, in Reference 9. Although the hub configuration considered is unusual, many important characteristics of rotor flutter are discussed in this paper. The steady inflow, for example, was found to be important, even though the rotor is in the helicopter state; further, these studies showed the coupling between lag-bending and pitch to be one of the most important parameters. The importance of inflow in flap-lag instabilities was also noted in Reference 18 for the prop/rotor case.

The lag bending pitch coupling is associated with the effect shown in References 15 through 17 to be critical for pitch-lag instabilities. However, these references demonstrate a destabilizing effect produced by increasing angle of attack as the blade leads forward while Reference 9 indicates it to be stabilizing. The difference appears to be due to the fact that, in References 16 and 17, the lag frequency is considerably lower than rotational speed, while in Reference 9 it is not. The simplified criterion given in Reference 16 is obtained by neglecting terms associated with the lag frequency ratio on the assumption of their being small compared with other terms in the equation, particularly some multiplied by the lag damping constant. In Reference 17, this approximation was not made but the frequency ratios were of the same size so that the results of Reference 17 essentially confirmed Chou's approximation.

For blades without lag hinges, the lag frequency is higher and the damping is much lower.

Equation 21 of Reference 16 shows that, if  $\tan \alpha_2$  is positive and  $\frac{\omega_\xi}{\Omega}$  is not small compared to 1.0, then a second amplified pitch-lag stability criterion emerges; namely for stability

$$\tan \alpha_2 < \frac{\theta_0}{2f_0^2} \left[ 1 - \left( 1 - \frac{C_\xi}{I_\xi \Omega} \right) \left( \frac{\omega_\xi}{\Omega} \right)^2 + \frac{C_\xi}{I_\xi \Omega} \left( 1 + \frac{C_\xi}{I_\xi \Omega} \right) - \frac{f_0}{\theta_0} \tan \delta_3 \right]$$

Note that, here if  $C_{\xi} \rightarrow 0$  and  $\omega_{\xi} \rightarrow \Omega$ , virtually any amount of positive  $\tan \alpha_2$  will be unstable.

It is important to note that the results of Reference 9 emphasize the importance of lag-bending frequency ratio in arriving at stability criteria. The effects of steady bending deflections discussed in Reference 10 are also borne out by the results of Reference 9 where increases in blade tip weight increased the flutter stability, principally by reducing the amount of steady flap-bending and hence, the contribution to lag-pitch coupling.

#### FLAP-LAG CORIOLIS INSTABILITY

All of the instabilities discussed so far are variations on the classical flutter theme in which blade twist or pitch is replaced as an independent degree of freedom by flap or lag, with pitch appearing as a geared effect due to the presence of  $\delta_3$  or  $\alpha_2$ . Thus, while the phenomena are of great practical importance, they do not present anything unexpected in terms of classical theory.

However, References 7 and 19 identify a quite different type of instability. This is limit cycle in nature and requires only the presence of flexural degrees of freedom in flap and lead-lag, and can occur with blade pitching or twisting totally absent. The essential coupling consists of coriolis forces between flap and lead-lag perturbations which are present when the blade is deflected due to thrust. The strength of the coupling depends on the amplitude of the initial deflection and the frequency of the flapping mode. The mechanism is self-limiting since the strength of the coupling is reduced as the amplitude of oscillation grows. Reference 7 also shows that the effect of forward flight ( $u$ ) is destabilizing in this mode of behavior.

The phenomenon is of less significance for articulated rotors since the balance of aerodynamic load and blade steady flap which exists tends to eliminate the coupling mechanism. In hingeless rotors, however, this type of instability may be quite troublesome, although not catastrophic. Reference 7 shows that blade equations in flap and lag in the hover case may be written in the form:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\beta} \\ \ddot{\xi} \end{bmatrix} + \begin{bmatrix} \gamma/8 & -2\beta_0 n_{\beta}^2 \\ -2\beta_0 & 2n_{\xi} \eta \end{bmatrix} \begin{bmatrix} \dot{\beta} \\ \dot{\xi} \end{bmatrix} + \begin{bmatrix} 1 + n_{\beta}^2 & 0 \\ 0 & n_{\xi}^2 \end{bmatrix} \begin{bmatrix} \beta \\ \xi \end{bmatrix} = 0$$

where  $n_{\beta}$  is the flap nonrotating natural frequency  
 $n_{\xi}$  is the lead-lag natural frequency



It is observed that, if there is no elastic restraint in flap, i.e., if  $n_\beta = 0$ , the two equations are effectively decoupled. The physical explanation of this is that when the blade has an effective mean positive angle of attack, a rate of lead motion produces an incremental upward aerodynamic moment and an incremental downward centrifugal flapping moment. If  $\beta_0$  is obtained by the balance of thrust moment and centrifugal moment, these two opposing incremental effects are equal and no flap-lag coupling exists. If  $\beta_0$  is reduced because of elastic flapping restraint, the incremental aerodynamic flapping moment dominates and a rate of lead motion produces an up-flapping moment which can cause unstable blade behavior.

When a balance of centrifugal and thrust moments is accomplished by the use of precone, the incremental centrifugal moment is restored to the same magnitude as the incremental aerodynamic moment and the motions are decoupled. Thus small perturbations in flap and lag about the preconed position are always stable.

In forward flight, however, additional coupling terms are present and preconing will not guarantee stability. In forward flight the following additional contributions to the coefficient of the damping matrix are present:

$$\begin{bmatrix} \frac{1}{6} \gamma \mu \sin \psi & , & \bar{\alpha} \gamma \mu \sin \psi \\ 0 & , & \gamma/3 \bar{\alpha}^2 \mu \sin \psi \end{bmatrix}$$

It is seen that even if  $n_\beta = 0$ , the term  $\bar{\alpha} \gamma \sin \psi$  provides coupling which may destabilize, so that as long as the coning deflection  $\beta_0$  is non-zero the strength of the coupling between the modes will increase with  $\mu$ ; so we might expect the onset of flap-lag instability as a function of forward speed.



## REQUIRED FEATURES OF AN ENGINEERING ANALYTICAL CAPABILITY

### THE NEED FOR COMPREHENSIVE ANALYSES

When we define the requirements of an engineering capability for stability analysis, the question may pose itself as to why such an analysis or capability is required. Most of the phenomena discussed are described adequately in the literature. However, it will frequently be found that the mathematical models in published papers are selected to demonstrate the phenomenon rather than to represent a hardware system. The influence of additional members of the system is often ignored, although these may have a cumulative effect which, from a practical point of view, may be just as important as the critical parametric variations on which published material tends to concentrate. Thus, unstable behavior indicated by examination of one system element (say, the rotor blade) in isolation may be significantly improved or impaired when the influence of the rest of the system is taken into account. As an example of potential interaction of this sort, we will discuss flap-lag coriolis instability analyzed in Reference 7. In this type of instability the coriolis forces couple the rotor flapping freedom and the lead-lag freedoms and, under certain conditions of blade natural frequency and steady deflection, an instability occurs at the lead-lag natural frequency. However, we know that blade lead-lag motion will couple with airframe motion in the plane of the rotor (ground resonance or mechanical instability) so that, when the three degrees of freedom are examined together, we should not be surprised if the total system were less stable than either of its component subsets. The opposite might hold true depending on the circumstances of the particular configuration.

It is difficult to envision a heavily damped airframe mode coupling with the blade lead-lag and absorbing sufficient energy to stabilize the flap-lag systems. Speculation of this kind can only be ended by a comprehensive analysis which takes into account all the potentially significant factors. (The same objective can be achieved using a modeling technique but a prior analytical approach is justified.)

Nevertheless, valuable guidance is provided by the academic studies investigating 1- and 2-degree-of-freedom systems so far as they define critical parameters which must be considered in formulating more general analyses and provide rules which may be used effectively in preliminary design.

The other major source of information on parameters which influence stability is experience with dynamically similar models and full-scale hardware.

## ANALYTICAL REQUIREMENTS FOR PROP/ROTOR CONFIGURATIONS

An engineering analytical capability must address the problems outlined in the previous sections in a quantitative way. A modal description of blade freedoms is a primary requirement. The use of equivalent hinge offset systems is a generally understood way of demonstrating the existence of the various phenomena and their parametric sensitivity; however, for accuracy in the prediction of point design behavior, and particularly in the case of highly twisted blades, a modal approach is required, since the relationships between flap, lag, and torsion vary along the span and cannot be easily accounted for by constants such as  $\alpha_2$  and  $\delta_3$ .

We may then outline the principal features necessary for the upgraded technology designed to address the problem areas not covered adequately by current state-of-the-art capabilities (see Table IV).

- Blade Degrees of Freedom

A minimum blade representation is required which will include analytically the mode which is predominantly:

- (a) Fundamental out-of-plane flap
- (b) Fundamental lead-lag
- (c) Fundamental torsion
- (d) Pitch of the blade about the pitch axis

- The requirement is satisfied by the provision for four general mode shapes with each allowed components in flap-lag-torsion. This then permits representation of (a) through (d) in as general a form as may be required. It also permits consideration of higher modes than the fundamental if this is desired. The analysis must address the stability of perturbations about initially deflected conditions of the blades caused by aerodynamic and inertial loadings.

- Airframe Degrees of Freedom

The hub to which the blades attach must have degrees of freedom representing the rigid-body motion of the aircraft, the vibratory modes of the airframe, and local freedoms introduced by soft mounting or by gimbaling the rotor.

The number of such airframe and rigid-body modes required depends on the special circumstance pertaining in any given case.

- A minimum capability will include:
  - (a) Six rigid-body degrees of freedom
  - (b) As many general airframe modes as are required to represent:
    - (1) Wing fundamental vertical bending and torsion
    - (2) Wing fundamental horizontal bending
    - (3) Fuselage bending and torsion
    - (4) Local modes of the nacelle
  - (c) Adequate representation of landing gear dynamics to permit prediction of ground resonance effects.

- Blade and Hub Geometry

Control system representation must take account of the fact that the pitch axis may occupy a variable position with respect to the blade deflected shapes; the amount of pitch, lag-flap coupling is directly influenced by the spanwise location of the pitch axis. Precone, prelead or lag, offset, and sweep are important in various contexts and will be included.

- Aerodynamic Representations

Aerodynamic sophistication in the blade representation is not required for prediction and correlation of the phenomena discussed: typical blade chords, frequencies, and velocities are such that the blade frequency parameters,  $K = \frac{\omega b}{V}$ , are small and quasi-

static assumptions appear to be adequate. Wing-induced effects need to be included since they are seen to have a potentially powerful effect on the rotor derivatives and thus on divergence speeds. Wing and empennage properties, however, require that account be taken of frequency dependency. The range of airframe frequencies present in the airframe might lead to errors in stability boundaries if this representation were restricted to be quasi-static.

- Direction and Type of Flow Through Rotor

Small angle assumptions regarding flow are not acceptable, since the analysis will be used to investigate flight conditions ranging from the propeller cruise mode (axial flow) through transition (nonaxial high angularity) to the helicopter regime (edgewise flow). In addition, nonuniform effects caused by the wing-bound circulation must be considered.

- Blade Deflections

One of the more troublesome phenomena in recent years has been the presence of flap-lag coupling instabilities associated with deflected blades. These have been limit cycle in nature but would nevertheless be an unacceptable feature in operation. Prediction capabilities must therefore include the behavior of blades deflected under loading arising from steady centrifugal and aerodynamic forces.

TABLE IV

MINIMUM REQUIREMENTS FOR AN UPGRADED TECHNOLOGY

Airframe Degrees of Freedom	Blade Degrees of Freedom
<ul style="list-style-type: none"> <li>● Rigid-body modes</li> <li>● All vibration modes which are of same order of frequency as <math>\Omega \pm \omega_B</math></li> <li>● These may extend well beyond fundamentals</li> <li>● Local nacelle modes</li> <li>● Above objectives may be achieved by provision of sufficiently large number of general airframe modes</li> </ul>	<ul style="list-style-type: none"> <li>● Sufficient to represent fundamental pitch, flap, lag, and torsion</li> <li>● Model representation must include coupled effects so that effective <math>\delta_3, \alpha_2,</math> etc., are included</li> <li>● Dynamic and aerodynamic effects of deflections must be included</li> <li>● Aerodynamics must include effect of <math>\mu</math> as well as <math>\lambda</math></li> </ul>

## OUTLINE DESCRIPTION OF THE STABILITY ANALYSIS DEVELOPED UNDER CONTRACT AND DISCUSSED IN VOLUME II

The analytical requirements defined in the preceding paragraphs have substantially been incorporated in the aeroelastic stability analysis computer program described in Volumes II and III.

### Airframe Representation

The program permits representation of the six rigid-body degrees of freedom and an additional six elastic modes of the airframe. This is adequate to accommodate the fundamental structural modes and additional selected modes to represent local effects, such as flexibility between the engine-nacelle package and the wing.

### Rotor Representation

Two rotors are included in the analysis, each rotor having 3 or more blades. Rotor gimbaling freedoms in pitch and yaw are available, and also a shaft rotation mode. Up to four blade modal degrees of freedom may be used, each mode having flap, lag, and torsional components if required. The aeroelastic coupling effects of blade deflections are taken into account in evaluating the coefficients of the differential equation of the system.

### Aerodynamic Representation

A two-dimensional strip theory representation has been used for the wings and empennage. The rotor blade aerodynamics account for  $\alpha$  effects (optional) and for large angles of inflow such as occur in the tilt-rotor transition regime. The program will also account for nonuniform inflow such as induced by the wing, provided that the flow field is specified.

### Landing Gear Representation

Although most configurations feature two main gears and a nose gear, the program was written for systems with up to four gears, since a number of helicopters have a four-poster arrangement. An idealized linear spring and damper arrangement is used to represent each set of struts, oleos, and tires.

### Application to Articulated Rotors

The program can be used to evaluate articulated rotor systems. Blade flap and lag about a hinge and kinematic effects such as  $\delta_3$  or  $\alpha_2$  may be represented by linear mode shapes with the appropriate torsional components.

## Part II. Blade Vibratory Loads

P. F. Leone



## INTRODUCTION

### SOURCE OF AIRLOADS

#### Velocity Relative to the Airfoil

A prop/rotor blade operating in an environment in which the forward-flight velocity vector has a component in the plane normal to the rotor shaft will experience a 1/rev fluctuation in the total relative wind velocity acting at a blade section. This is the result of the fact that, in the advancing-rotor half cycle, the forward velocity vector adds to the rotational speed vector; whereas, in the retreating-rotor half cycle, the reverse is true (see Figure 32). Since the dynamic pressure acting at an airfoil section is proportional to the square of the total relative wind velocity, it results as a periodic quantity over one rotor cycle containing steady, 1/rev, and 2/rev harmonic components. The introduction of a rotor collective pitch angle is felt as a steady aerodynamic angle of attack by the section which, when multiplied by the dynamic pressure, will give rise to aerodynamic forces which are also periodic in one rotor cycle and contain steady, 1/rev, and 2/rev components. The introduction of a 1/rev cyclic pitch angle to the rotor when multiplied by the dynamic pressure will now introduce an additional vibratory term, being a 3/rev component, as well as its own steady, 1/rev, and 2/rev components (see Figure 33).

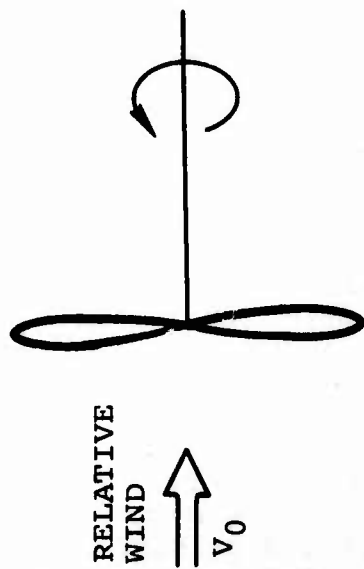
#### Blade Motion and Downwash

Blade flexibility in the flapwise, chordwise, and pitch directions, along with vorticity due to lift variations with time and blade span, generates airloads of all frequencies (see Figure 34). Pitch deflections change the mechanical angle of attack of the airfoil while flapwise and chordwise deflections induce a change in the angle of attack by altering the direction of the wind relative to the blade. Discontinuities in spanwise blade lift generate trailed vortices, with an especially strong vortex trailed at the blade tip. Changes in blade lift with time generate shed vortices from the blade. The distribution of shed and trailed vortices created by the rotor blade itself or by other rotor blades of the same rotor or other rotors operating in the vicinity of the rotor blade in question generates an induced velocity on the blade that further changes the angle of attack and blade lift.

#### Other Sources of Vibratory Airloads

These periodic aerodynamic forces are those derived from the fundamental expression for a lift force in which the section lift coefficient is a linear function of the aerodynamic angle of attack and the discussion in the preceding paragraphs is

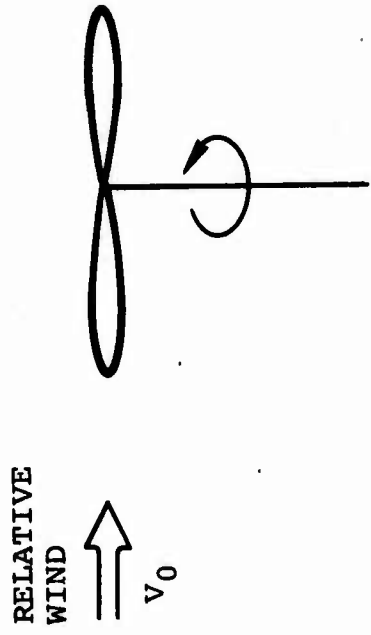
• PURE AXIAL FLOW



TANGENTIAL AND NORMAL VELOCITIES ARE CONSTANT



• PURE EDGEWISE FLOW



TANGENTIAL VELOCITY HAS A ONE-PER-REV VARIATION

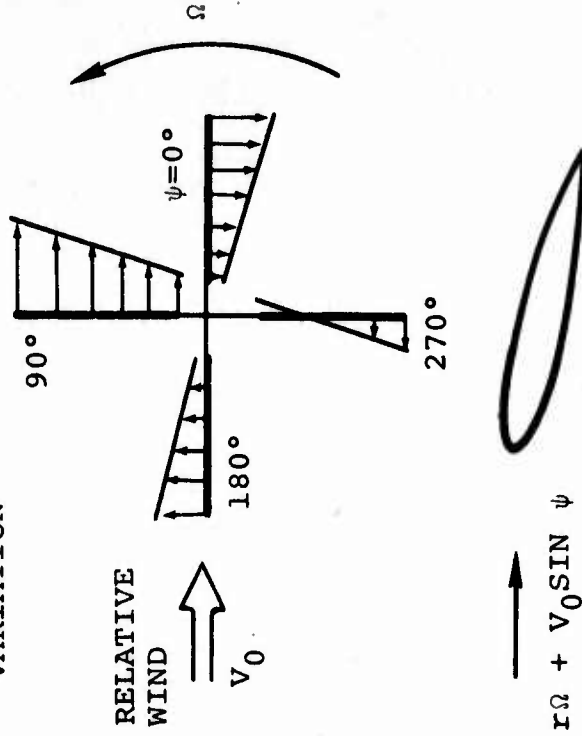
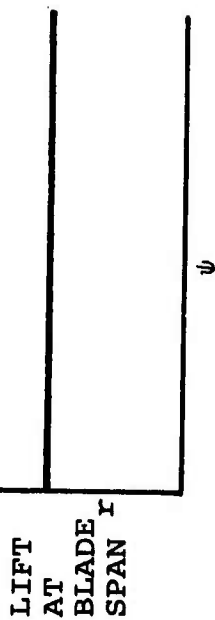


Figure 32. Sources of Prop/Rotor Airloads

• PURE AXIAL FLOW

LIFT IS PROPORTIONAL TO THE VELOCITY SQUARED - GIVES STEADY, 1/REV, and 2/REV AIRLOADS

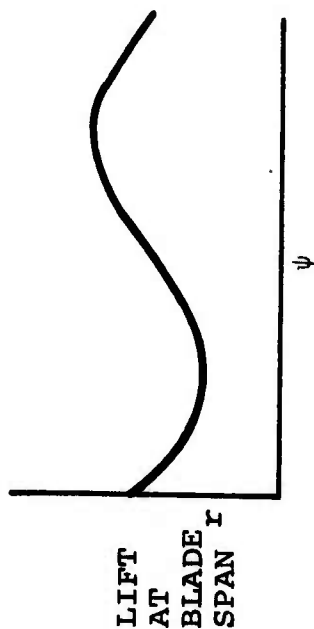
- GIVES STEADY AIRLOAD ONLY



$$L = K [r^2 \Omega^2 + V_0^2]^{1/2}$$

ADDING CYCLIC PITCH CONTROL AT ONE PER REV (I.E.,  $\theta_{CYC} = -6^\circ \sin \psi$ )

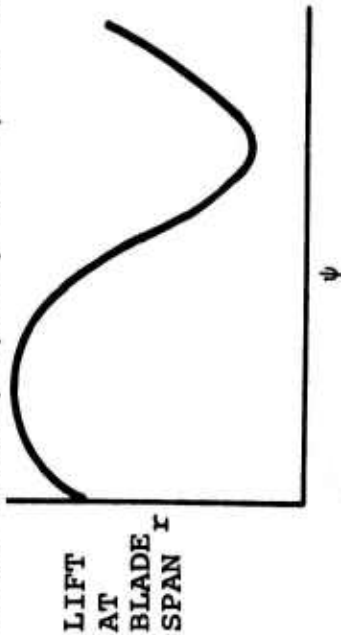
- GIVES STEADY, ONE-PER-REV, TWO-PER-REV, AND THREE-PER-REV AIRLOADS



$$L = K [r^2 \Omega^2 + V_0^2]^{1/2} + \frac{\theta_{CYC}}{\theta} K [r^2 \Omega^2 + V_0^2]^{1/2} \sin \psi - \frac{1}{4V_0^2} \sin 3\psi$$

• PURE EDGEWISE FLOW

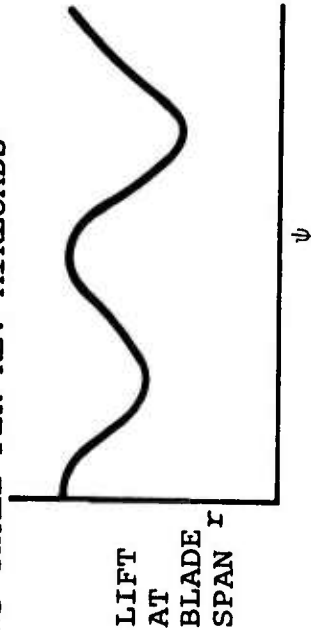
LIFT IS PROPORTIONAL TO THE VELOCITY SQUARED - GIVES STEADY, 1/REV, and 2/REV AIRLOADS



$$L = K (r^2 \Omega^2 + 1/2V_0^2 + 2r\Omega V_0 \sin \psi - 1/2V_0^2 \cos 2\psi)$$

(I.E.,  $\theta_{CYC} = -6^\circ \sin \psi$ )

- GIVES STEADY, ONE-PER-REV, TWO-PER-REV, AND THREE-PER-REV AIRLOADS

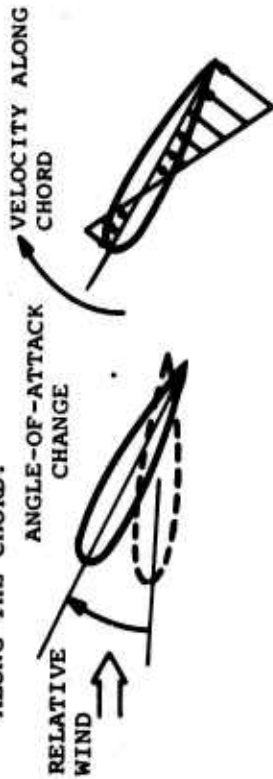


$$L = K (r^2 \Omega^2 + 1/2V_0^2 + 2r\Omega V_0 \sin \psi - 1/2V_0^2 \cos 2\psi) + \frac{\theta_{CYC}}{\theta} K [r\Omega V_0 + (r^2 \Omega^2 + 3/4V_0^2) \sin \psi - r\Omega V_0 \cos 2\psi] - 1/4V_0^2 \sin 3\psi$$

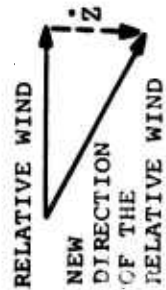
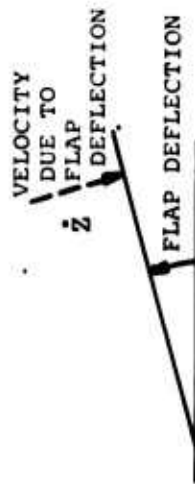
Figure 33. Generation of Periodic Airloads

- AIRLOADS OF ALL FREQUENCIES ARE GENERATED BY:  
FLAPWISE-CHORDWISE AND PITCH BLADE DEFLECTIONS

PITCH DEFLECTIONS CHANGE THE MECHANICAL ANGLE OF ATTACK AND INDUCE A VELOCITY ALONG THE CHORD.



FLAP DEFLECTIONS CHANGE THE DIRECTION OF THE RELATIVE WIND



VORTEX-INDUCED NONUNIFORM DOWNWASH, WHICH INDUCES CHANGES IN DIRECTION OF THE RELATIVE WIND IN THE SAME MANNER AS  $\dot{z}$

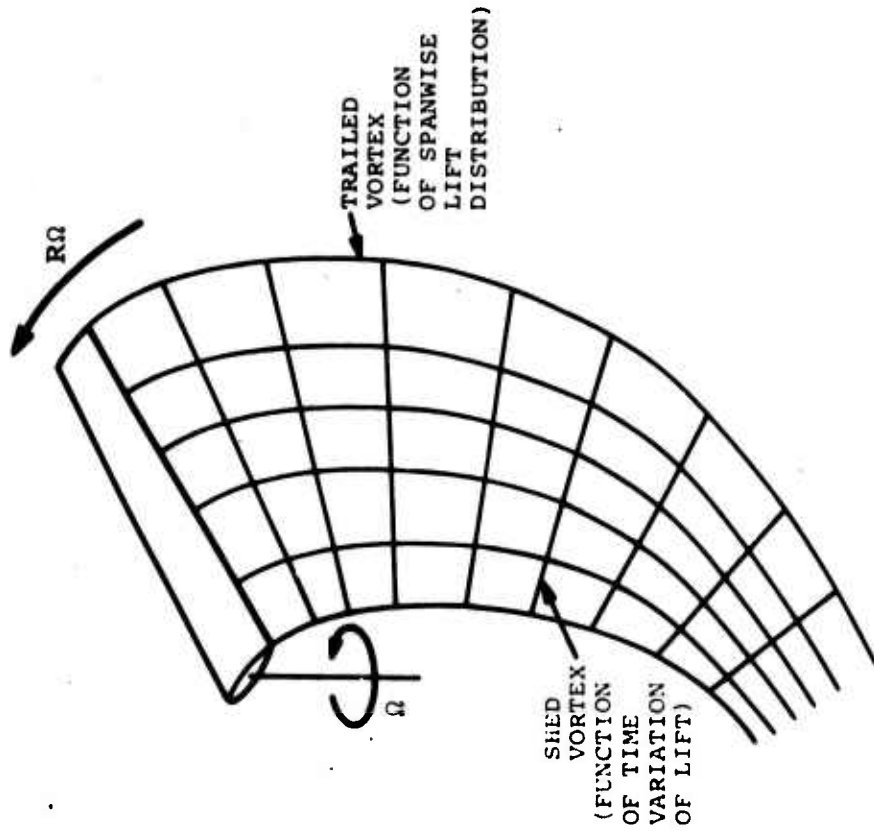


Figure 34. Airloads Due to Blade Motions and Trailed and Shed Vortices

valid for aerodynamic angles of attack up to stall. Associated with the lift there are periodic aerodynamic profile and induced-drag forces and quarter-chord moments which are generated by the periodic dynamic pressure acting at an airfoil section and derived from the section drag moment coefficients. Over and above the fundamental periodic aerodynamic section lift, drag, and moment derived from linear incompressible theory, there are additional nonlinear compressible counterparts to these section excitations which contribute to the aeroelastic response of the rotor blade as the local angles of attack vary rapidly. These nonlinear compressible components are of particular importance for those blade sections operating in a stalled environment (see Figure 35).

### The Axial-Flow Flight Condition

For a prop/rotor operating in a pure axial-flow environment, the total relative wind velocity acting at a blade section will be a constant (see Figure 32). The introduction of 1/rev cyclic pitch angle to the rotor will now yield a steady and 1/rev lift (see Figure 33). A small component of velocity in the plane normal to the rotor shaft due to the angle of attack of the rotor shaft or cyclic induced lead-lag deflections (see Figure 36) will introduce the multifrequency airloads discussed above.

### BLADE LOADS

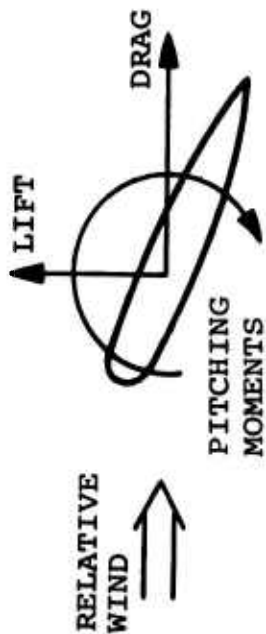
#### Components of the Load

The consequence of these periodic aerodynamic forces acting upon the prop/rotor blade is the generation of periodic bending moments which are partially relieved by centrifugal and inertial forces (see Figure 37), the remainder being expended in deforming the blade, thus producing bending strains and their associated bending stresses. These resulting periodic bending stresses are generated throughout the duration of the periodic dynamic pressure in the rotor shaft normal plane; which, as in the case of the helicopter, is the full duration of its forward-flight condition. Consequently, the structural design problem of a prop/rotor blade centers largely upon its ability to withstand these cyclic fatigue stresses for a prescribed period of time.

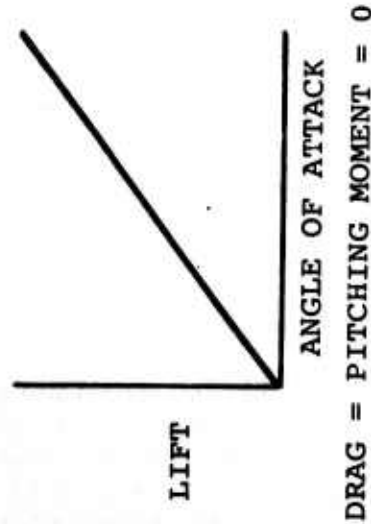
#### Effect of Blade Root Constraint

One method for minimizing this basic problem is to employ mechanical hinges at the blade root; this compels the blade bending moments to approach a zero value at the hinges. When mechanical springs and dampers are considered about the hinge axes, nonzero bending moments result that are proportional to the spring and damper rates. A hingeless rotor blade, on the

- AIRLOADS GENERATE LIFT, DRAG, AND PITCHING MOMENTS ON AN AIRFOIL

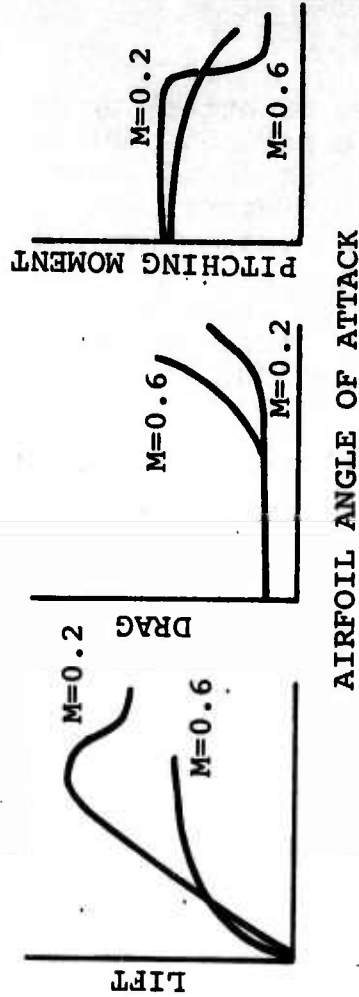


- SIMPLE THEORY ASSUMES THAT LIFT IS A LINEAR FUNCTION OF BLADE ANGLE OF ATTACK, AND THAT DRAG AND PITCHING MOMENT ARE ZERO



- THE CONSIDERATION OF DRAG AND PITCHING MOMENTS, AERODYNAMIC NONLINEARITIES (STALL), COMPRESSIBILITY, AND UNSTEADY EFFECTS FURTHER ADDS TO ALL FREQUENCIES OF AIRLOADS

STEADY-STATE, NONLINEAR, COMPRESSIBLE THEORY:



UNSTEADY EFFECTS:



Figure 35. Linear, Unsteady, and Compressibility Effects on Airloads

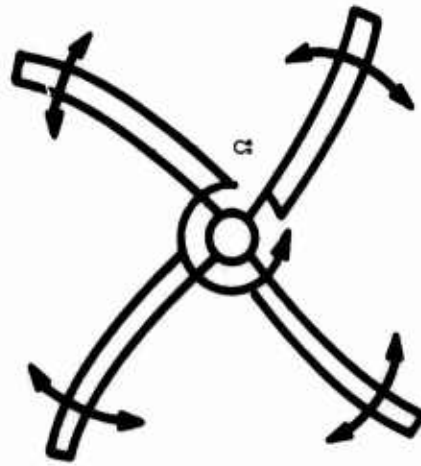


- FOR A PROP/ROTOR IN AXIAL FLOW A VARYING TANGENTIAL COMPONENT OF VELOCITY CAN BE OBTAINED BY:

AN ANGLE OF ATTACK OF THE ROTOR SHAFT WITH RESPECT TO THE WIND



LEAD-LAG BLADE MOTION



THEREFORE, AIRLOADS OF ALL FREQUENCIES CAN BE ACTING IN BOTH HELICOPTER AND PROP/ROTOR AIRCRAFT

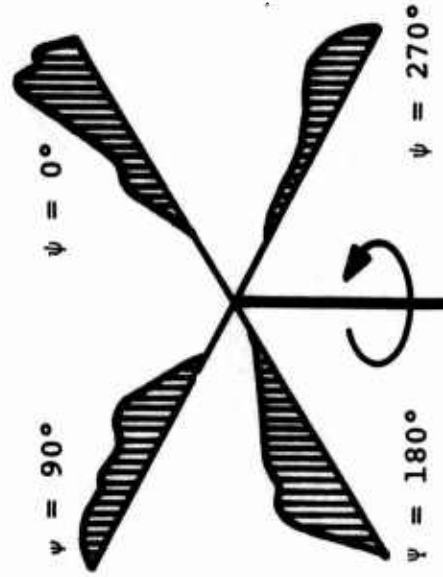
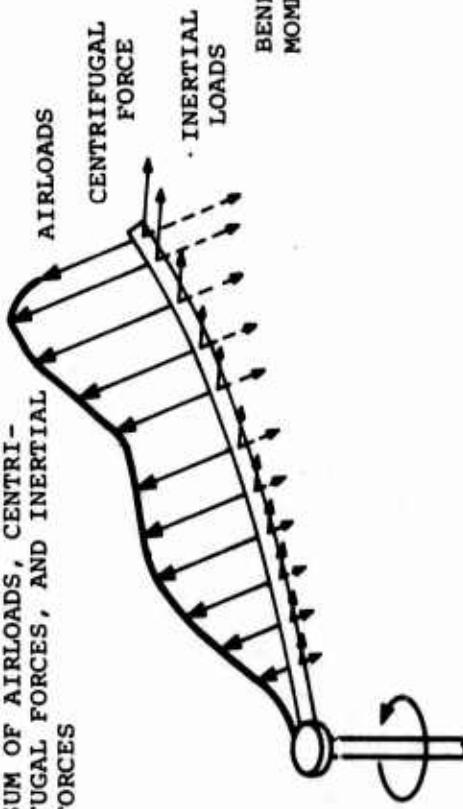
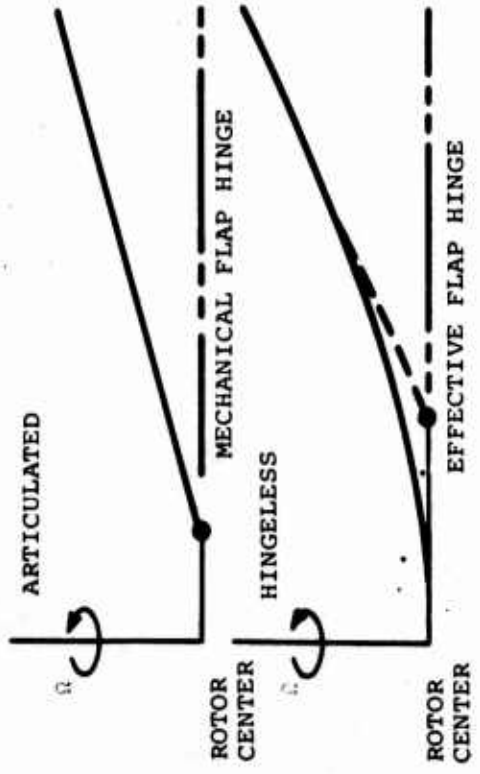


Figure 36. Sources of Alternating Wind Components in Rotor Disc

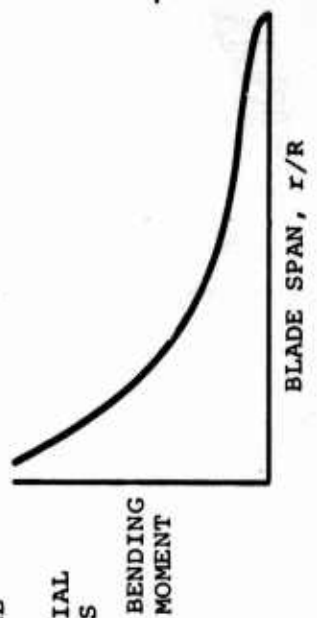
• BLADE LOADS RESULT FROM THE SUM OF AIRLOADS, CENTRIFUGAL FORCES, AND INERTIAL FORCES



• TO REDUCE BENDING MOMENTS A HINGELESS ROTOR HAS A RELATIVELY SOFT ROOT END TO SIMULATE AN ARTICULATED ROTOR



• RIGID BLADE



• ARTICULATED BLADE

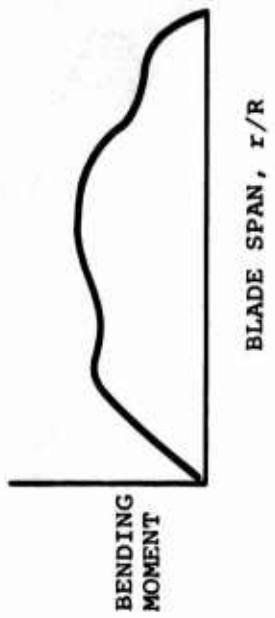


Figure 37. Blade Loads and Bending Moments

other hand, must absorb these root bending moments which rise sharply in this region, the rate being proportional to the bending stiffness at the root (see Figure 37). Consequently, the typical hingeless rotor blade employs a low bending stiffness in the inboard root area to minimize these rapidly rising bending moments. As a result, the typical hingeless rotor blade deforms in its fundamental mode in a manner similar to an articulated rotor blade having a large effective hinge offset from the rotor center in the region of minimum bending stiffness.

### Coriolis Loads

Both the articulated rotor blade with mechanical hinges and the hingeless rotor blade featuring effective hinges execute angular motions about the hinge axes which give rise to coriolis inertia forces which must be absorbed by the rotor blade (see Figure 38). These coriolis inertia forces are proportional to twice the product of the rotor angular velocity with a linear velocity of a mass point relative to the rotating frame of reference. Like the aerodynamic forces, these coriolis inertia forces are periodic in nature and contain 1/rev, 2/rev, etc., harmonic components but no steady components.

### Blade Dynamics

Because of its elasticity, the prop/rotor blade has natural modes of vibration whose individual natural frequencies vary with the rotor angular velocity (see Figure 39). Since the periodic aeromechanical exciting forces are describable in terms of harmonic components whose frequencies are integral multiples of rotor speed, it is obvious that a resonance can occur in any one natural mode for more than one rotor speed. Fortunately, aerodynamic damping forces are generated by the blade elastic motions which reduce these resonant responses (see Figure 40). However, care must be taken in the design of a prop/rotor blade to avoid such resonances in the normal operating rotor speed band; a determination of the blade natural modes is of primary importance during initial design of the rotor blade. In addition to determining the undamped natural frequencies for each natural mode, it is desirable to include the determination of the associated damped amplification factors for rotor harmonic exciting frequency. These factors are important in assessing the effects of elasticity and mode shape on the aerodynamic damping process generated by the rotor blade in its natural modes of vibration.

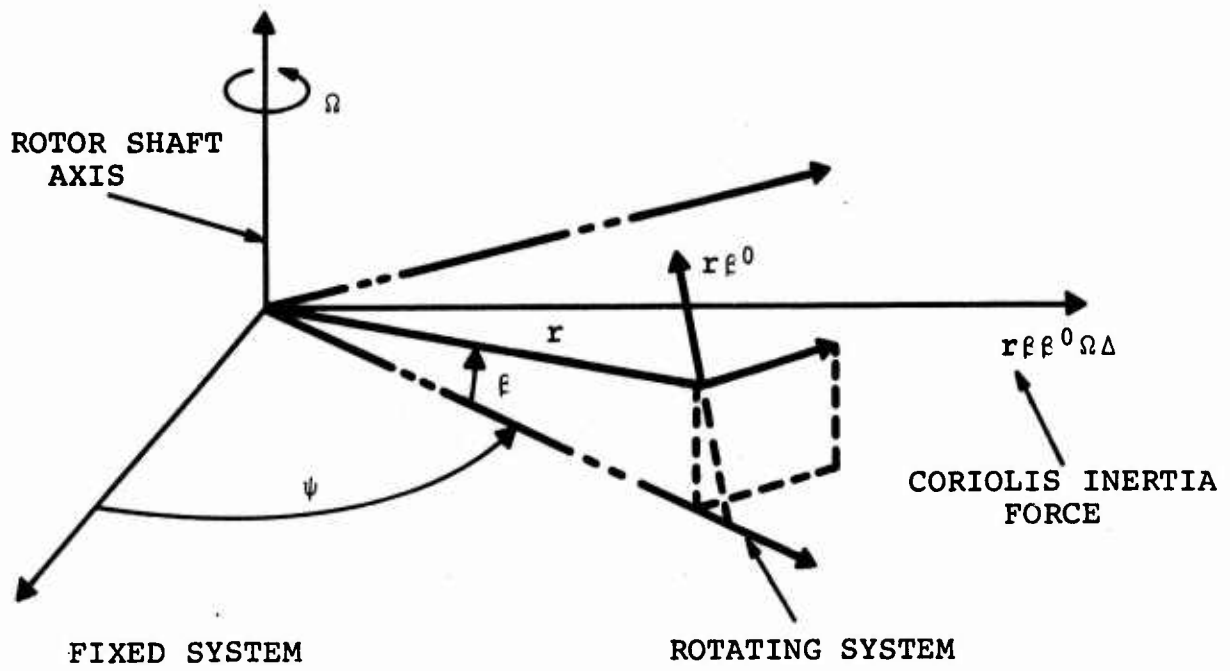


Figure 38. Coriolis Inertia Force Due to Flapwise Motion

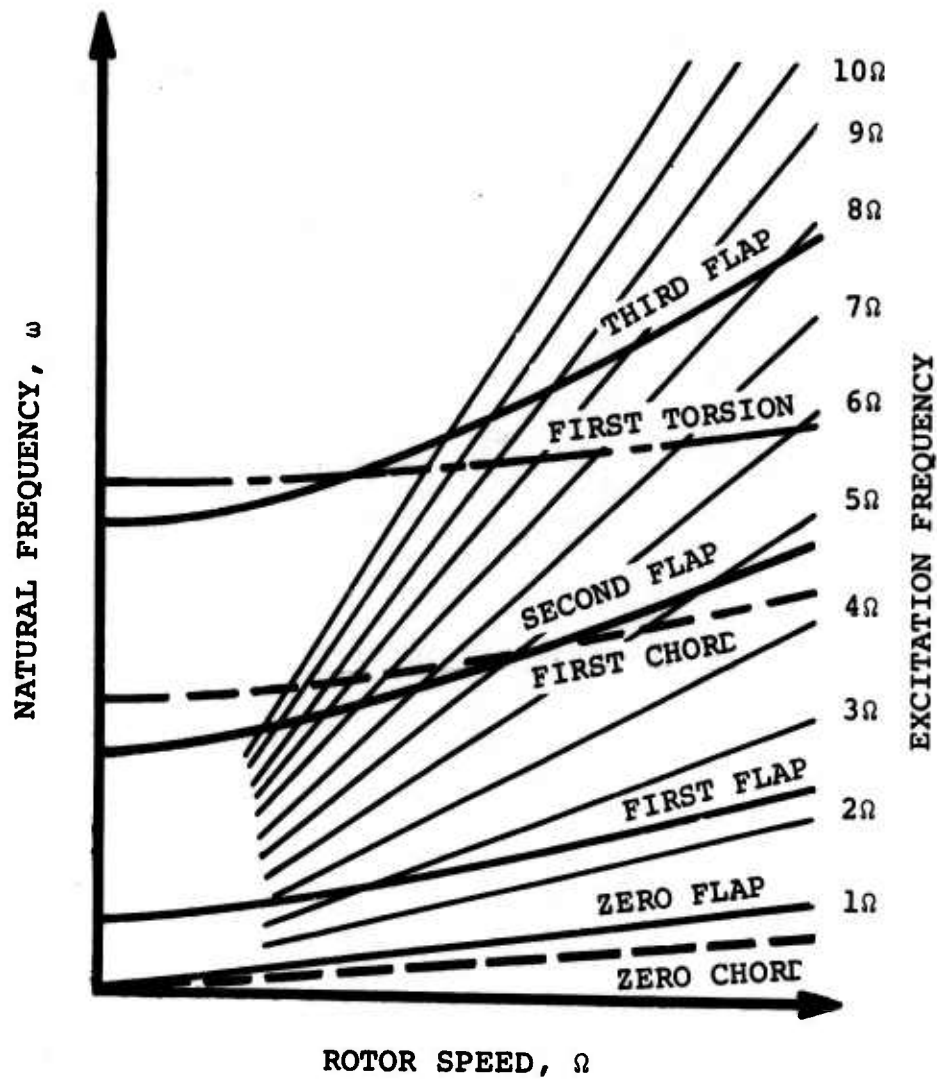


Figure 39. Natural Frequency Spectra for a Prop/Rotor Blade

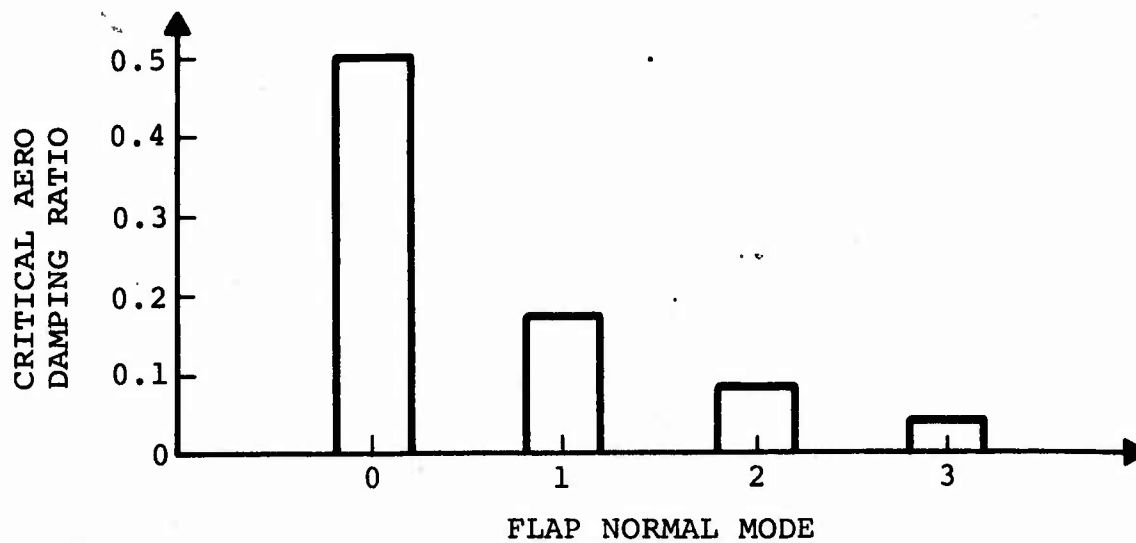
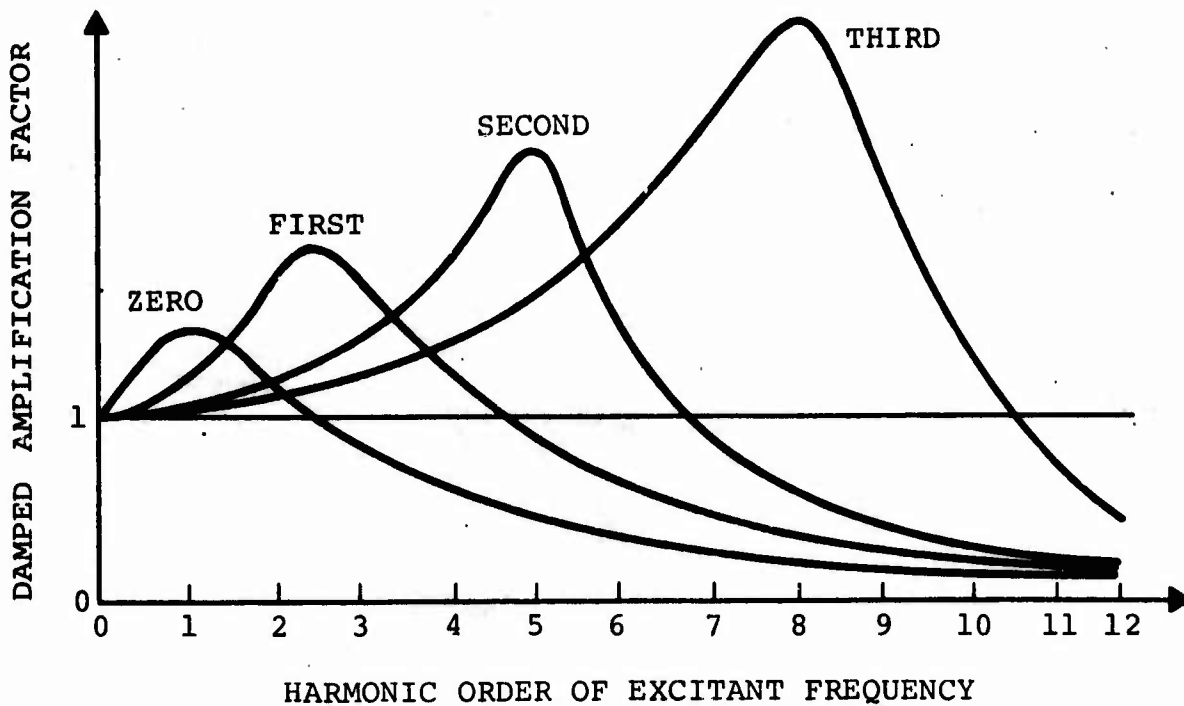


Figure 40. Flap Modes Damped Amplification Factors



## CURRENT MATHEMATICAL MODEL

### ROTOR BLADE REPRESENTATION

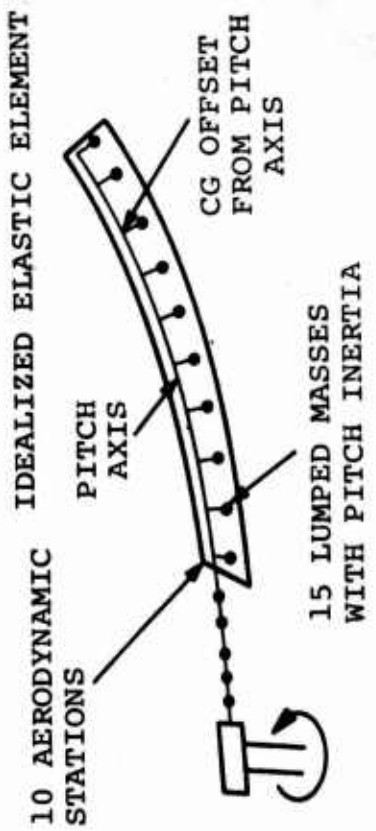
The current mathematical model in use by Boeing-Vertol to predict the aeroelastic response of a prop/rotor blade to aeromechanical excitations has for its basis the decomposition of the blade into a finite number of lumped masses interconnected by massless elastic beams (see Figure 41). This basic technique permits accounting for the nonuniform spanwise distributions of the blade aerodynamic and structural properties. Each lumped mass is subjected to its individual aeromechanical excitation, its resulting response being coupled to its adjacent lumped masses through linear difference equations which relate the local changes in deflection, slope, moment, shear, and torque. At the blade root, boundary conditions are satisfied which permit the solution for the blade deformation response at each lumped mass station. The current model accounts for dynamic and aerodynamic coupling between flap bending and torsion and uncoupled chord bending and is applicable to a low-twist prop/rotor blade, such as a helicopter rotor blade. The net result of the analysis is the solution for the flap bending, chord bending, and torsional aeroelastic response, including the total solutions for the blade deflection, slope, moment, shear, and torque, the associated control system forces, the associated rotor hub loads, and the rotor performance.

### AERODYNAMIC REPRESENTATION

Because of its use of unsteady aerodynamics, nonuniform downwash, airfoil section data, shed wake effects, and compressibility effects, the current model has been very successful in predicting pitch link load waveforms throughout the level-flight speed range, including, of particular importance, the stall regime for full-scale helicopter rotors (see Figures 42, 43, and 44). The model has had good success in predicting flap and chord bending moments and rotor lift distributions (see Figures 45, 46, and 47) from 110 to 125 knots for the full-scale H-34 helicopter rotor. In the area of highly twisted V/STOL wind tunnel model rotors, the mathematical model has been adequate for predicting flap bending moments but less successful in predicting the chord bending moments (see Figures 48 and 49) and pitch link loads. The current aerodynamic capability is reviewed in Figures 50 and 51.

### DYNAMIC REPRESENTATION

The poor prediction for the highly twisted model rotor is believed due largely to the neglect of the large twist angle of the model and its effects on the resultant elastic and aerodynamic coupling. Although the current mathematical model



- BLADE IDEALIZATION
- COUPLED FLAP-PITCH AND UNCOUPLED LAG
- SMALL BLADE PRETWIST
- VARIABLE CENTER OF GRAVITY
- COINCIDENT SHEAR CENTER, PITCH AXIS, AND VERTICAL NEUTRAL AXIS
- FLAP-PITCH DEFLECTIONS ITERATED WITH AIRLOADS

- HINGELESS, ARTICULATED, AND TEETERING ROTORS

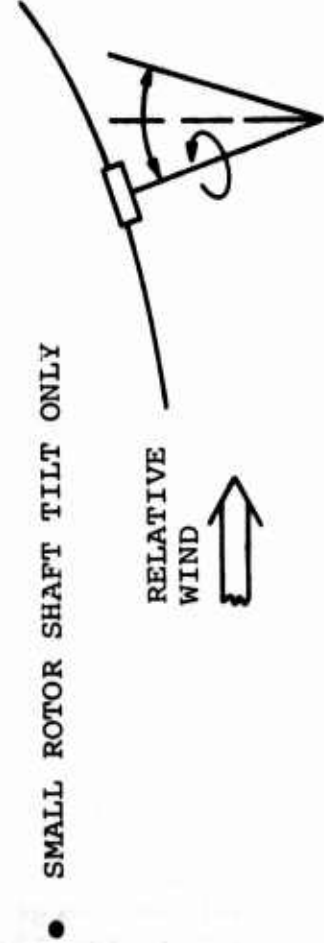
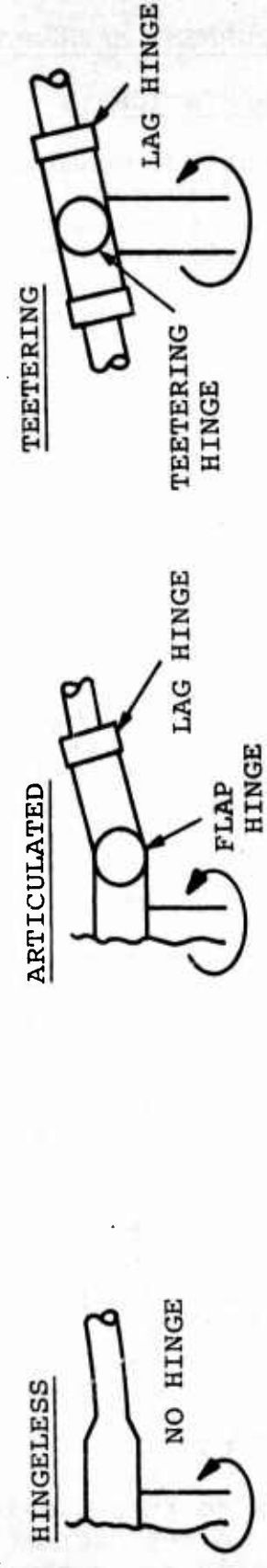


Figure 41. Current Capability in Dynamics

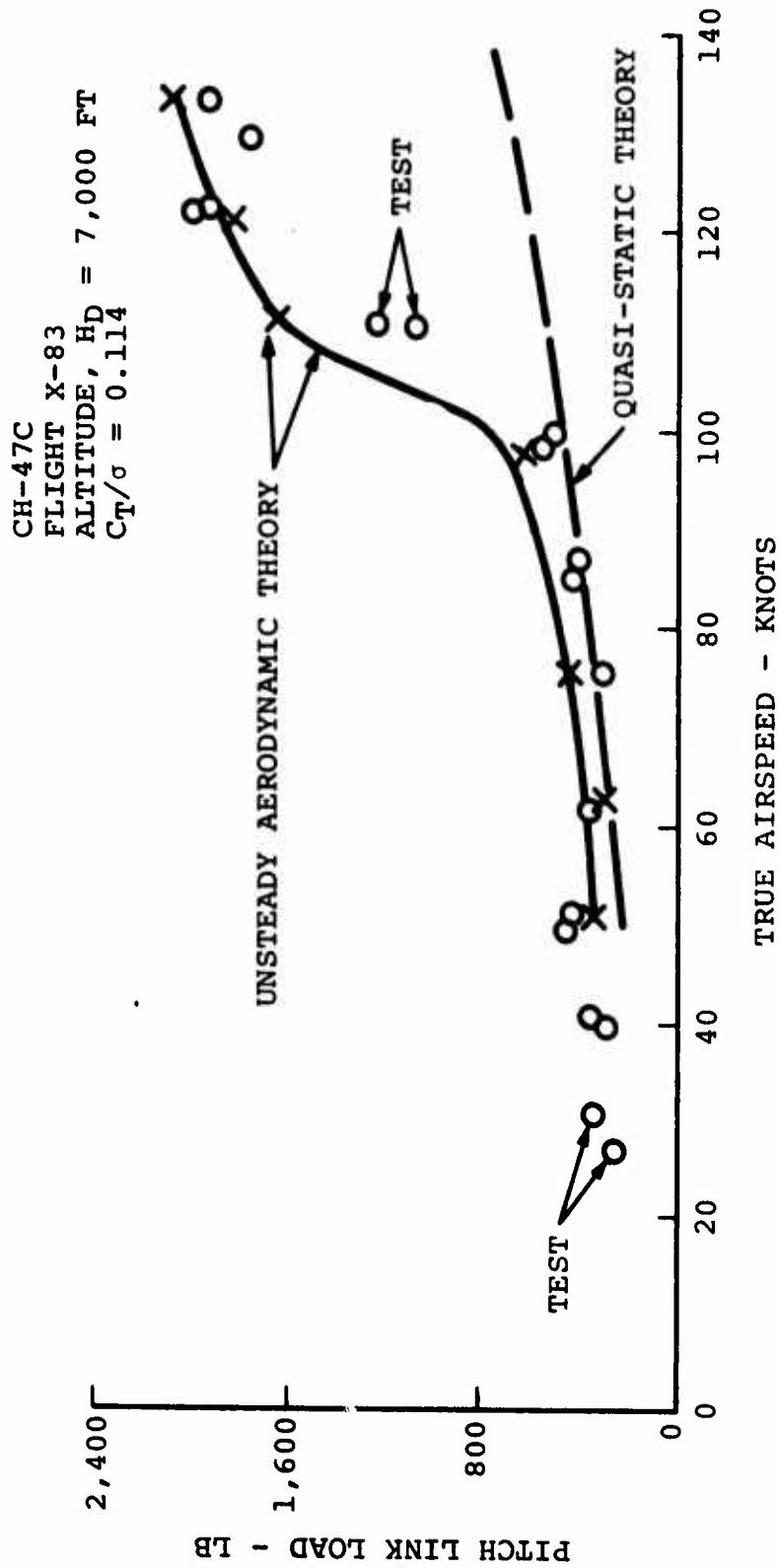


Figure 42. Comparison of Test and Analytical Pitch Link Loads for an Airspeed Sweep

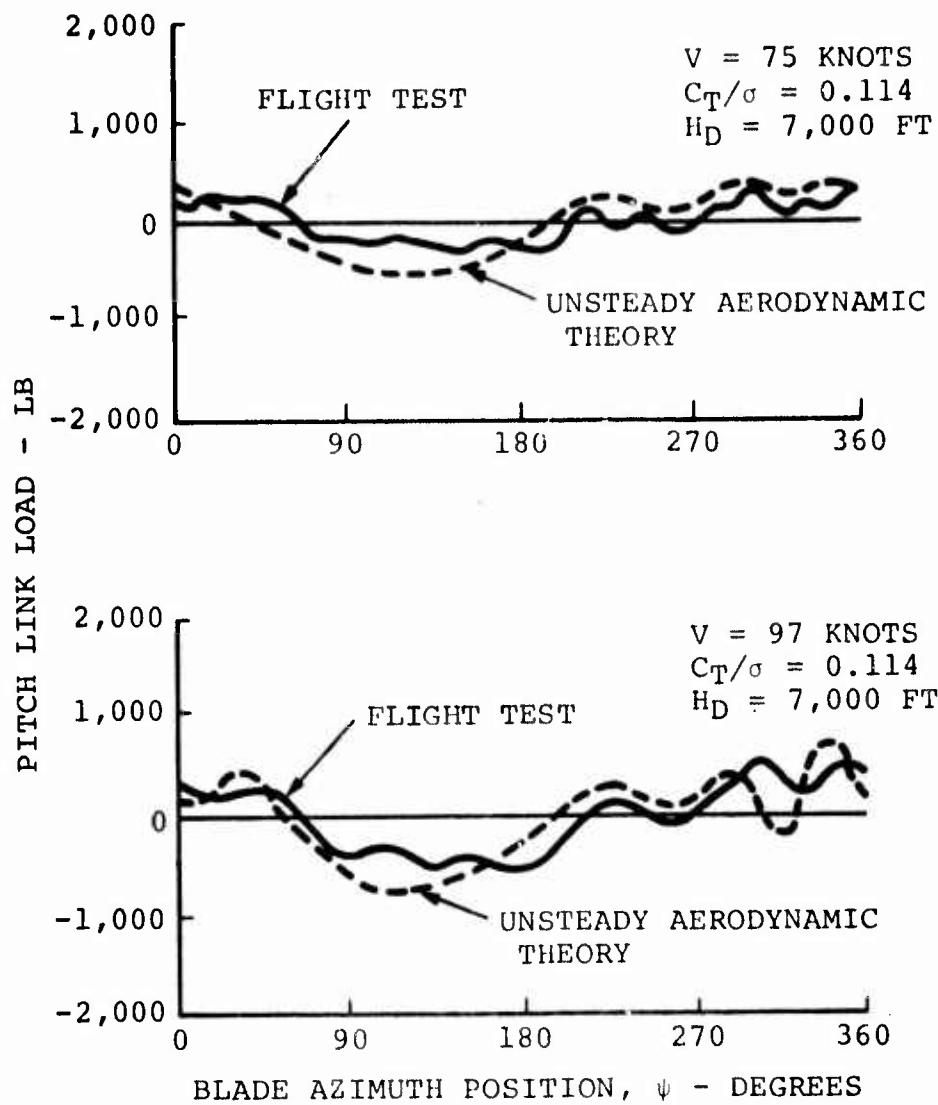


Figure 43. Low-Speed Pitch Link Load Waveforms

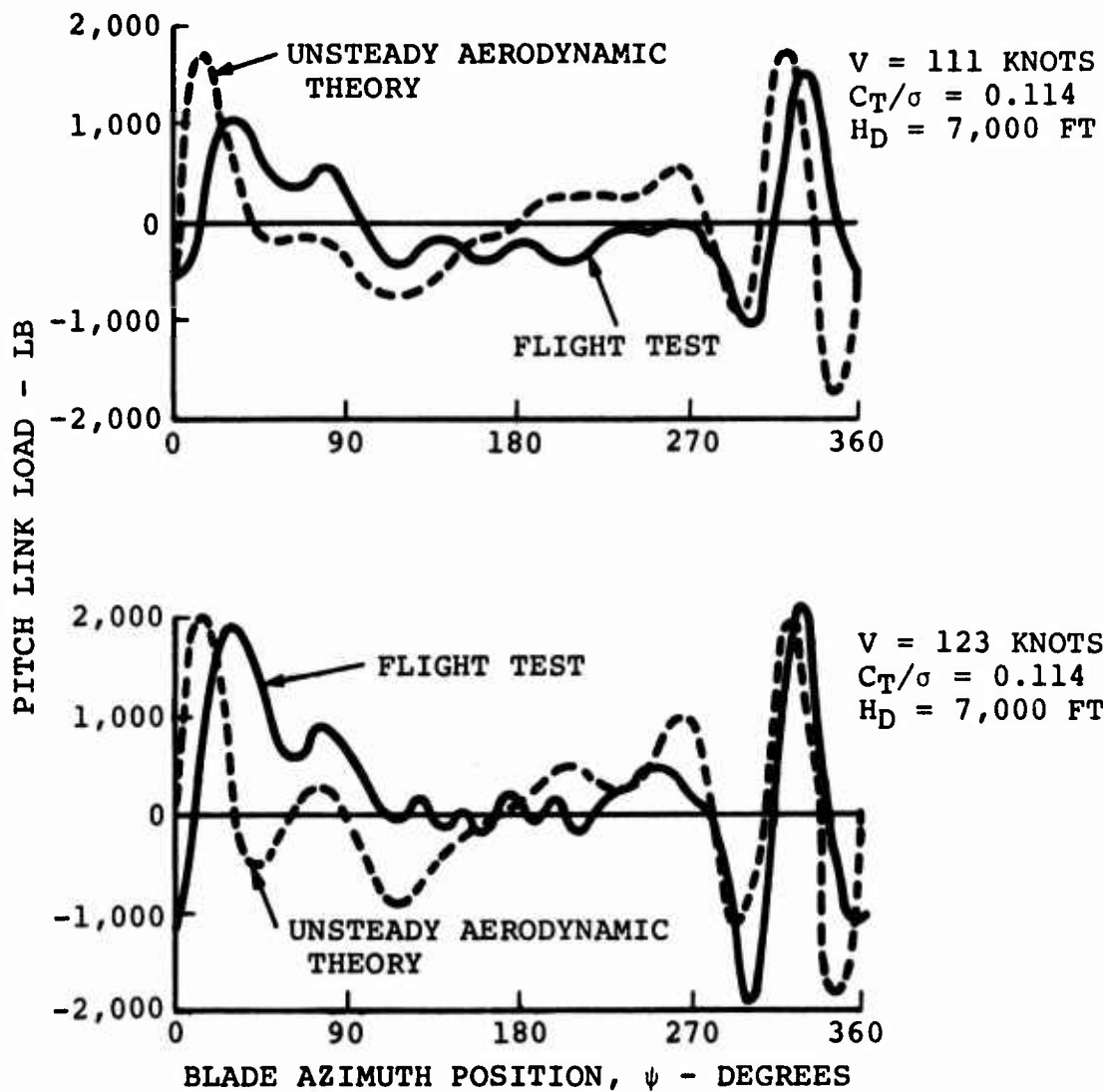


Figure 44. High-Speed Pitch Link Load Waveforms

V = 175 KNOTS  
5° FWD SHAFT TILT

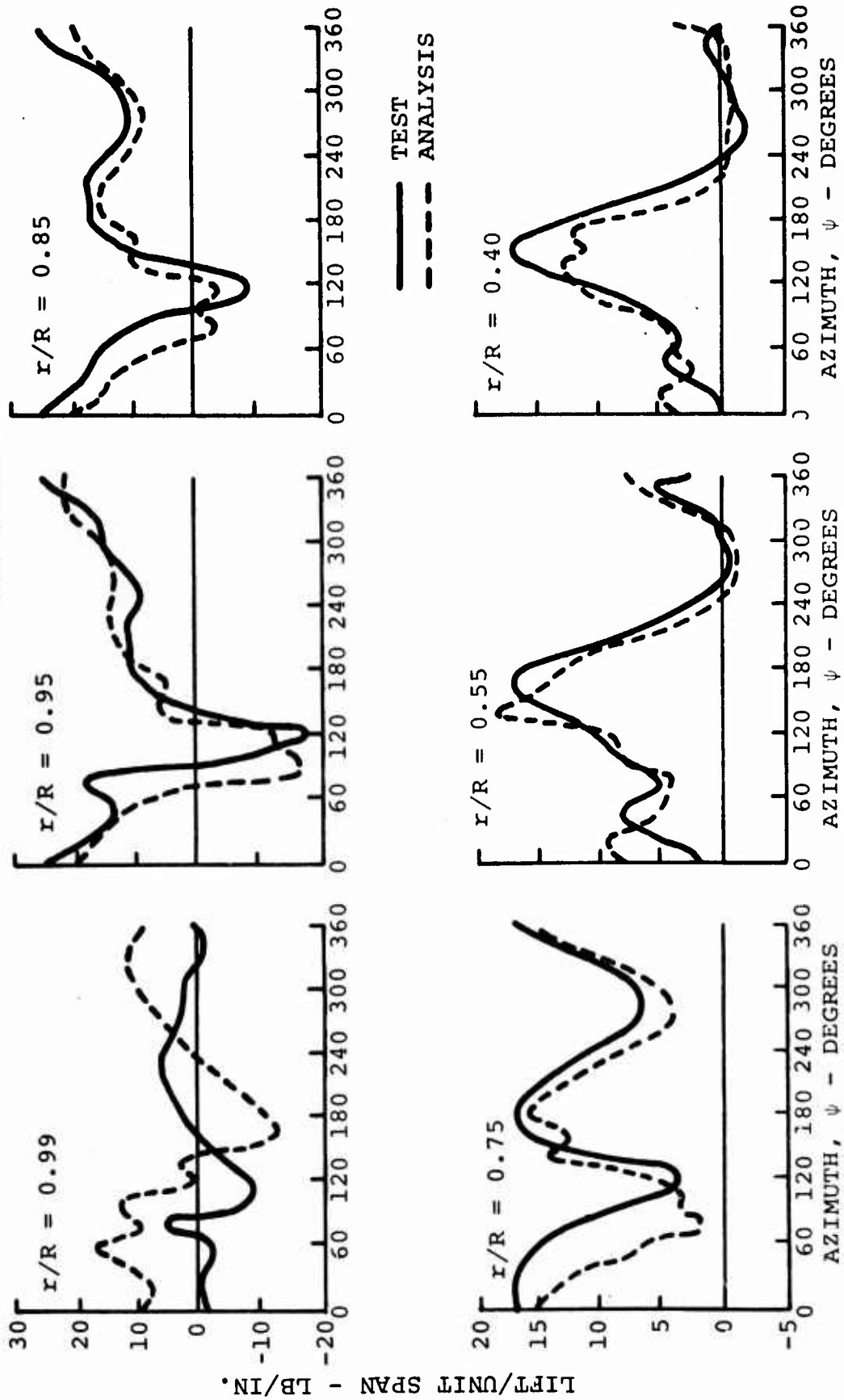


Figure 45. Typical Lift Waveform Correlation at All Radial Positions for H-34 Rotor in Ames 40- by 60-Foot Tunnel



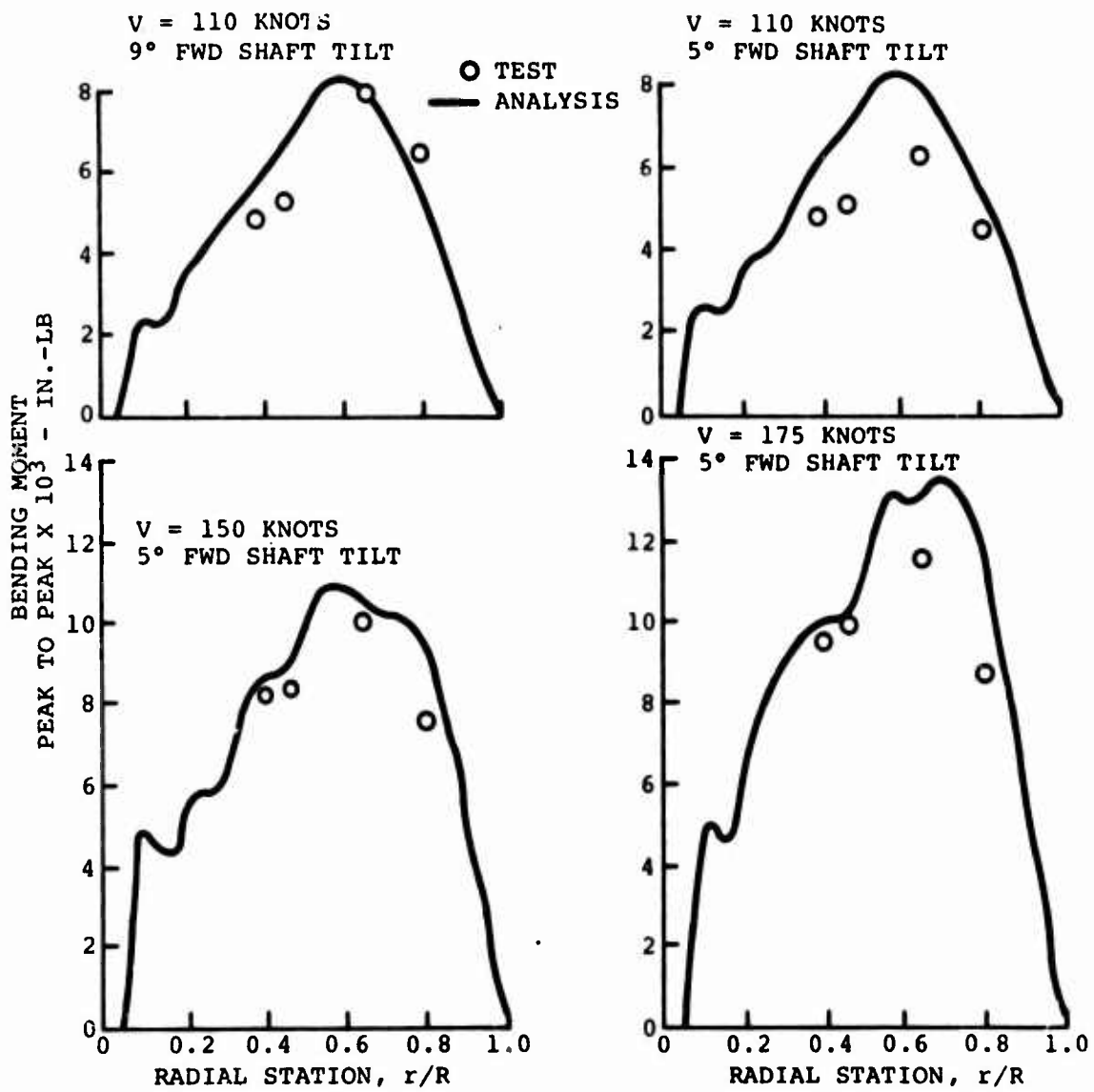
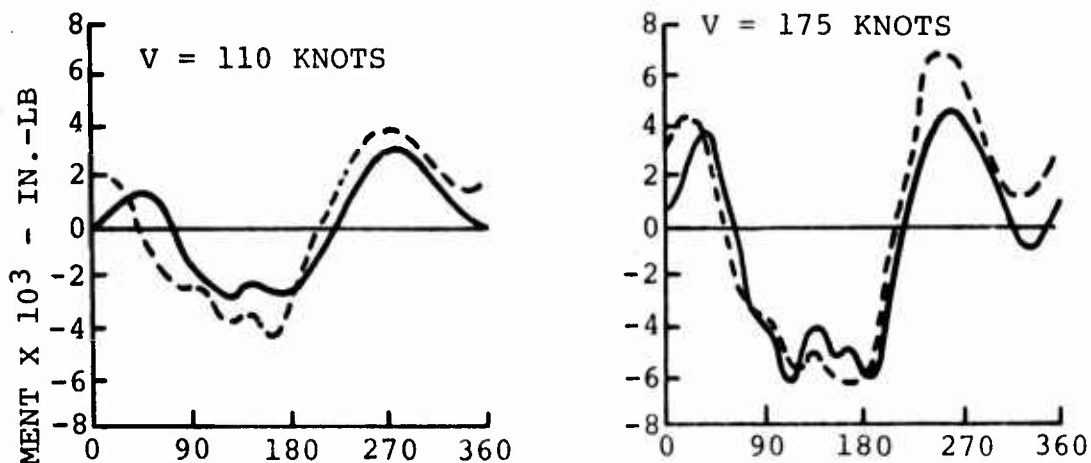


Figure 46. Analytical Flap Bending Moment Peak-to-Peak Spanwise Envelope Compared With Test for Forward Shaft Tilt on H-34 Rotor in Ames 40- by 60-Foot Tunnel

FOR  $r/R = 0.065$



FOR  $r/R = 0.375$

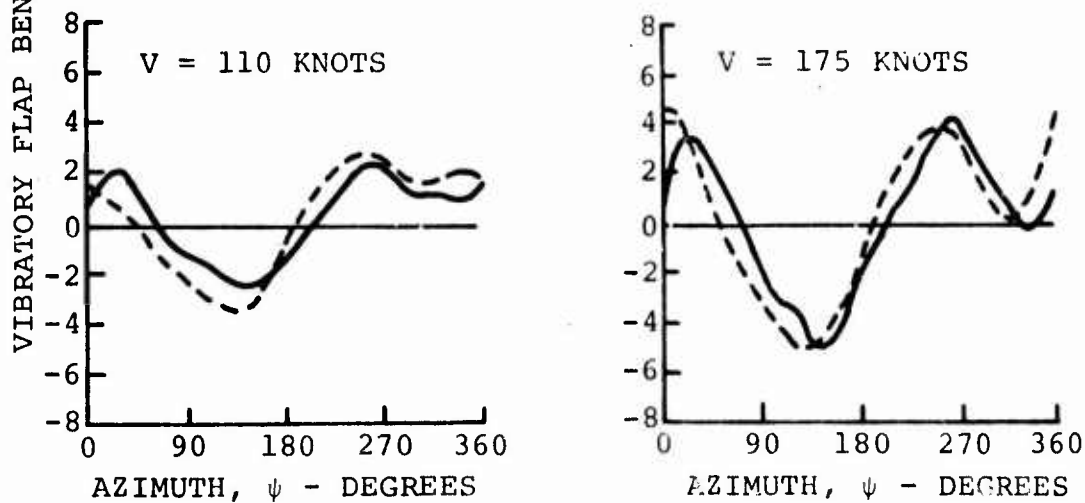
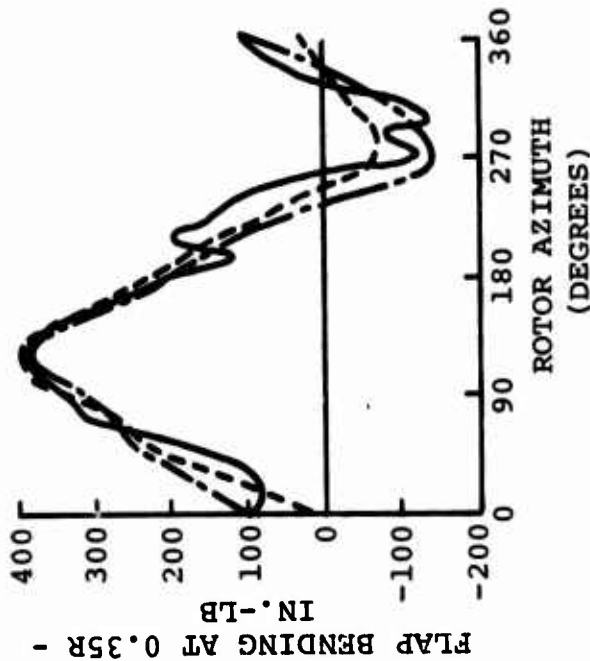


Figure 47. Typical Test and Analytical Flap Bending Moment Waveforms For 5 Degree Forward Shaft Tilt on H-34 Rotor in Ames 40- by 60-Foot Tunnel

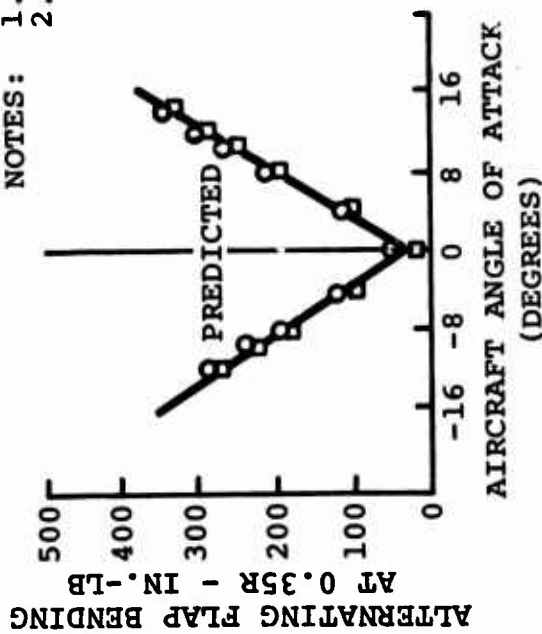
$$\theta_t = 36^\circ$$

PREDICTION ———  
 RIGHT ROTOR - - -  
 LEFT ROTOR - - -



MEASURED AND PREDICTED BLADE FLAP BENDING IN CRUISE FOR  $\alpha = 10^\circ$ ,  $\theta_{75} = 30^\circ$ ,  $V_{TIP} = 570$  FPS, AND  $V_{TUNNEL} = 220$  FPS

NOTES: 1.  $\delta$  FLAP = 0  
 2.  $\circ$  LEFT ROTOR  
 $\square$  RIGHT ROTOR



MEASURED AND PREDICTED ALTERNATING FLAP BENDING PRODUCED BY AIRCRAFT ANGLE OF ATTACK IN CRUISE AT  $\theta_{75} = 30^\circ$ ,  $V_{TIP} = 570$  FPS, AND  $V_{TUNNEL} = 220$  FPS

Figure 48. Theory and Test Flap Bending Moments for Tilt-Rotor Model 160 Performance Model

$$\theta_t = 36^\circ$$

MEASURED AND PREDICTED ALTERNATING CHORD BENDING PRODUCED BY AIRCRAFT ANGLE OF ATTACK IN CRUISE AT  $\theta_75 = 30^\circ$ ,  $V_{TIP} = 570$  FPS, AND  $V_{TUNNEL} = 220$  FPS

ALTERNATING BLADE TORSION VS RPM FOR  $i_n = 90^\circ$ ,  $\theta_2 = 3^\circ$ , AND  $h/D = 0.39$ , 1.8

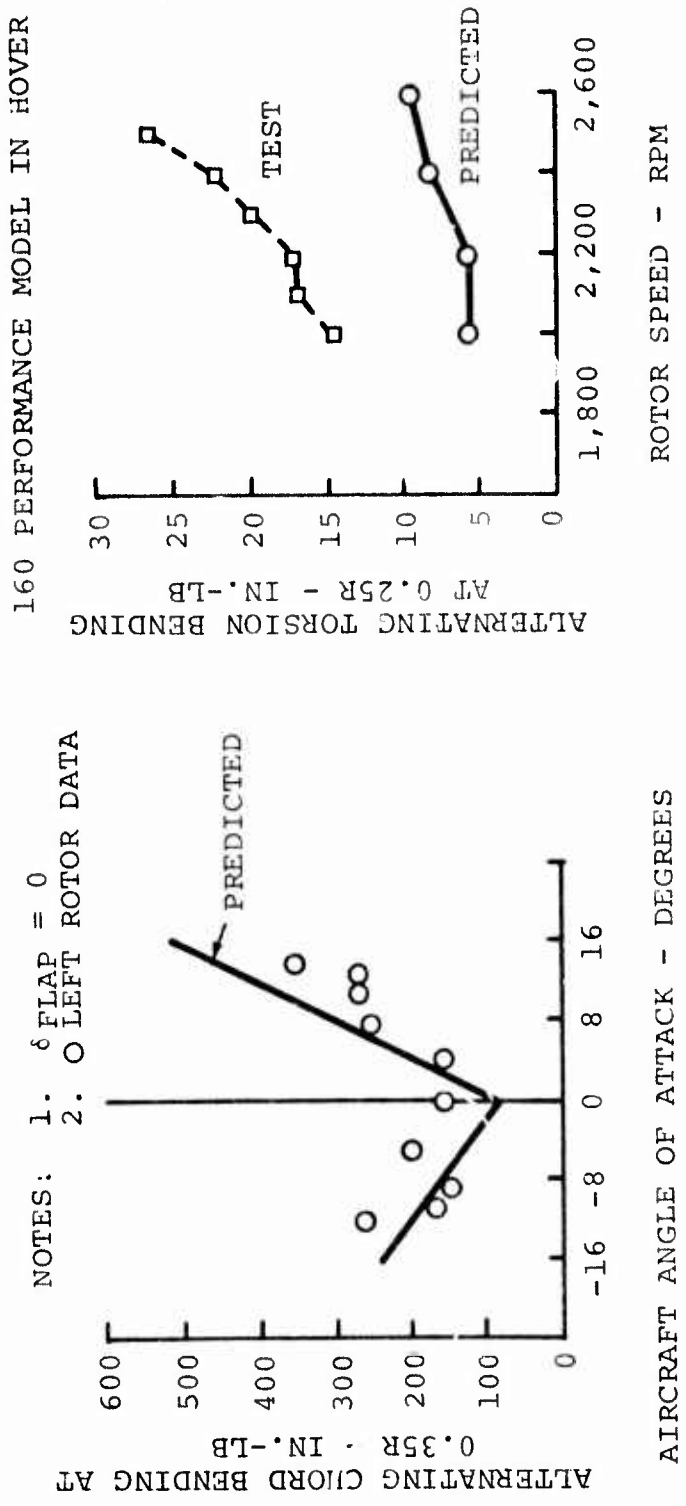


Figure 49. Theory and Test Chord Bending Moments and Torsion for Tilt-Rotor Model 160 Performance Model

● LIFTING LINE THEORY INCLUDING  
COMPRESSIBILITY AND STALL



UNSTEADY EFFECTS FOR BOTH STALL AND  
UNSTALLLED CONDITIONS

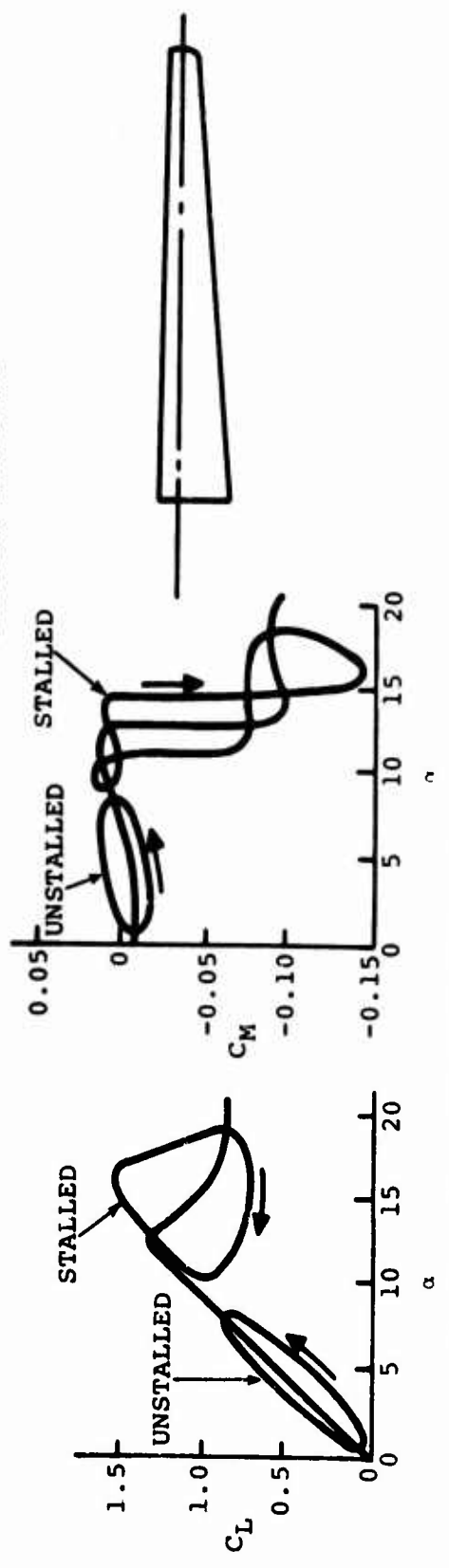


Figure 50. Current Capability in Aerodynamic Properties  
of Airfoils

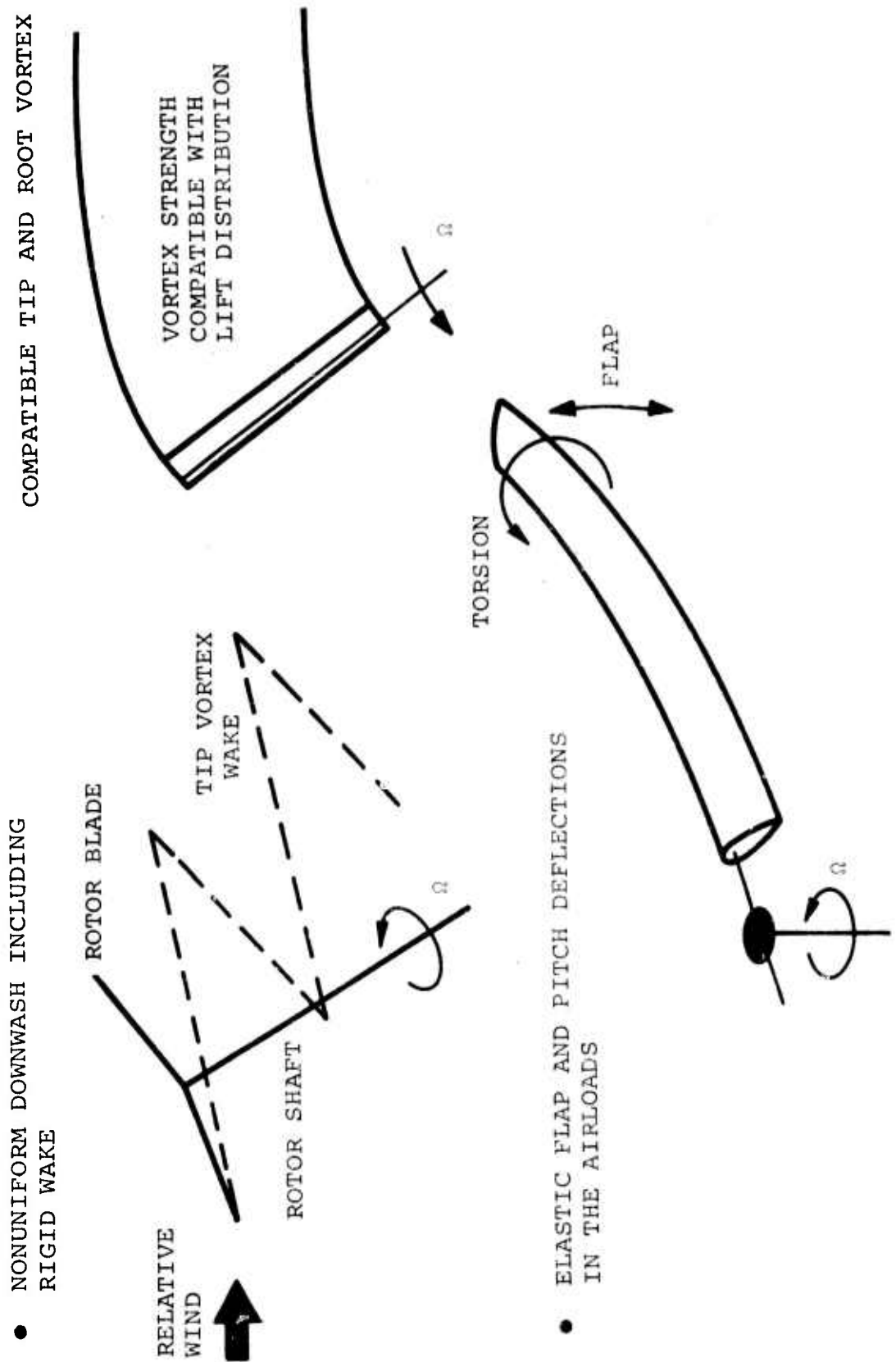


Figure 51. Current Capability in Aerodynamic Effects of Downwash and Elasticity



accounts for both the articulated and hingeless rotor configurations, it does not account for those V/STOL rotor designs whose root-end attachments, although hingeless in appearance, are actually pinned with an effective angular spring about the pin along with an effective linear spring acting outboard of the pin axis. This is a potential reason for poor bending moments for highly twisted rotors of this type. Still another reason is the aerodynamic coupling between the flap and chord bending motions which is not accounted for in the present mathematical model and which is believed to be of particular importance for soft-in-plane hingeless rotors. These major deficiencies, along with many others of less importance, will be removed in the mathematical model being developed under this contract. This new model will employ the most advanced concepts in dynamics, aerodynamics, and elasticity known at this time for a prop/rotor blade.

#### PROGRAM USAGE

Utilization of this program requires the definition of a flight condition, detailed rotor blade physical properties, and gross aircraft properties. A trim analysis must be run to define the control input, thrust, aircraft attitude, and blade initial deflections. A blade idealization program is required to lump the detailed blade properties into discrete elements. For hover conditions an additional calculation must be performed to correct the cyclic to account for the downwash due to lift dissymmetry. For other conditions where wing circulation passes through the rotor, another program must be run to define the velocity distribution due to this blade circulation. A review of the program usage is given in Figure 52.

#### PROGRAM FLOW DIAGRAM

Figure 53 shows a simple block diagram of the current loads program. The nonuniform downwash calculation is based upon the initial deflections only. The airload calculations can provide required thrust by altering the collective angle until the required thrust is obtained. The program can provide up to 10 iterations between the airloads and the coupled flap-pitch response. The uncoupled lag response is determined by the airload and coriolis calculations; the lag response is not used to alter the airloads or flap-pitch response.

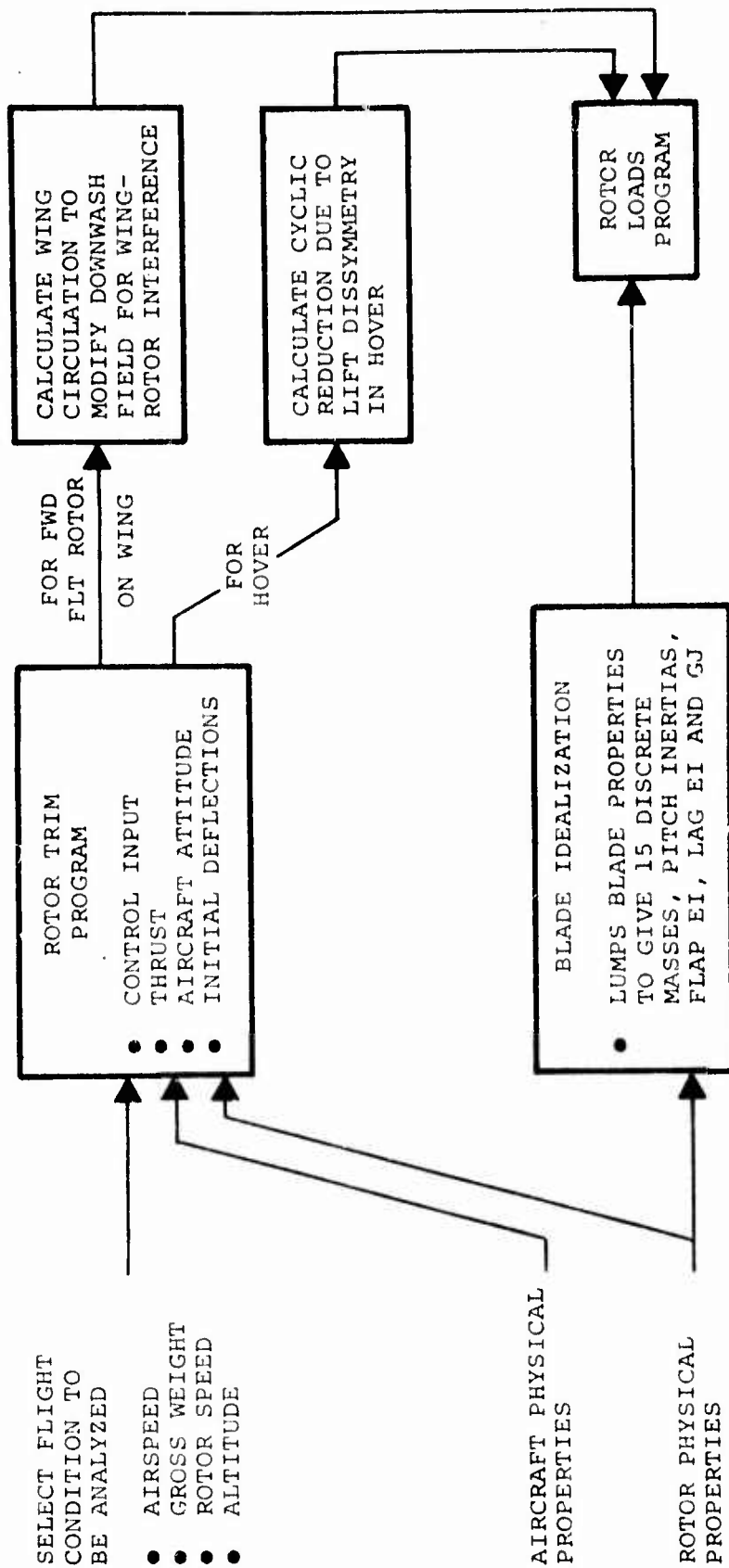


Figure 52. Program Usage

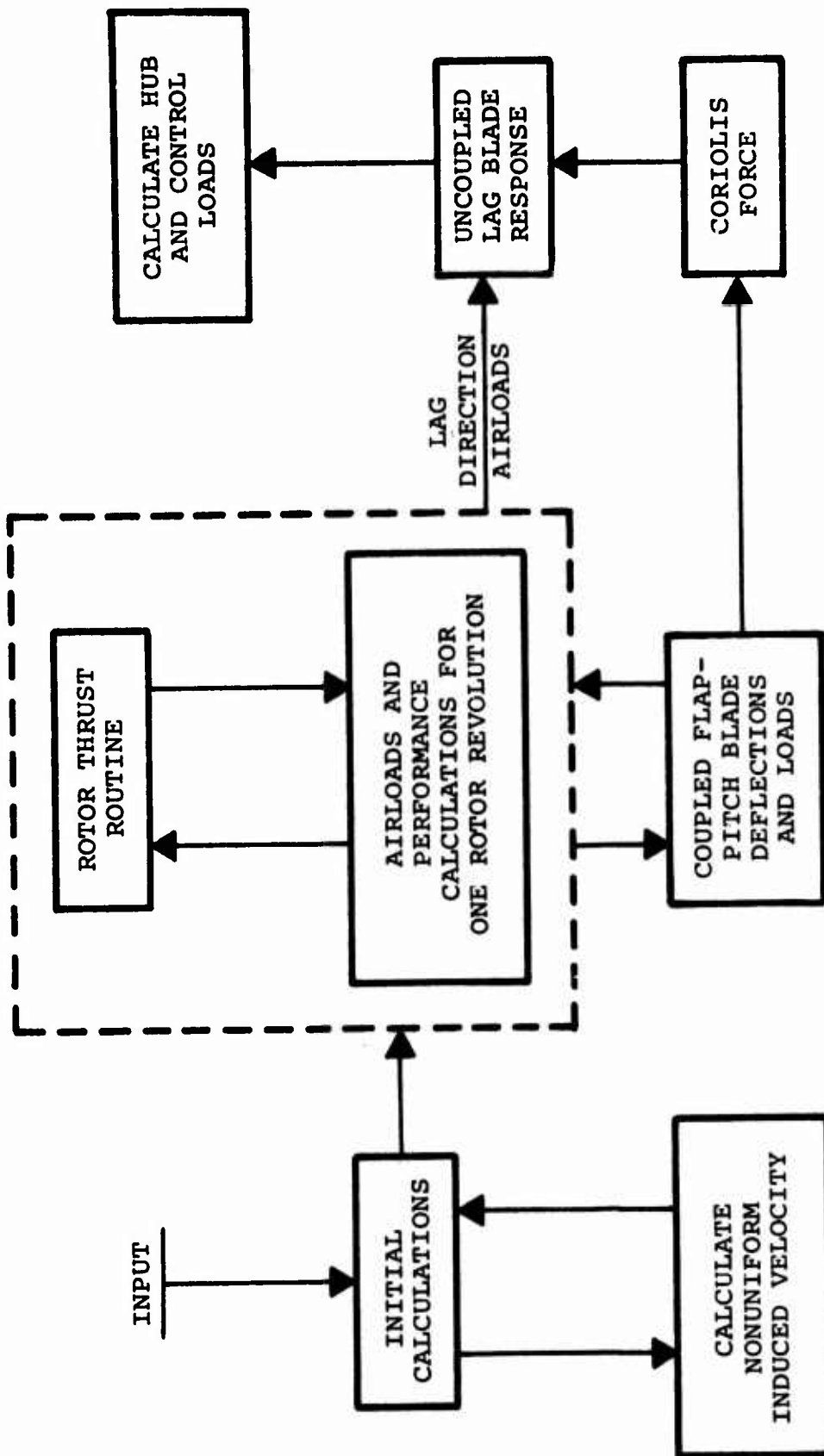


Figure 53. Flow Chart of Current Loads Program

## MATHEMATICAL MODEL UNDER DEVELOPMENT

The mathematical model being developed under this contract will include modifications to the current model which will improve its prediction capabilities and enlarge its applicability to include V/STOL-type prop/rotor blades. These major modifications to the current mode include the following items.

### BLADE IDEALIZATION

#### Number of Mass Stations

The model now has a prop/rotor blade idealization that includes 20 mass stations and 15 aerodynamic stations, thereby increasing the accuracy of the dynamic and aerodynamic force representations.

#### Coupled Flap-Chord-Pitch

A new solution technique considers a fully coupled analysis for flap bending, chord bending, and torsion. Specifically, the coupling includes dynamic, aerodynamic, and elastic forces involving the bending deflections normal and parallel to a section chord line with the section torsional deflections.

#### Other Improvements

The prop/rotor blade idealization now permits the inclusion of a large built-in twist angle, blade precone, blade prelag, variable vertical and horizontal neutral axes, variable shear center, and section inertias.

### AERODYNAMICS

An advanced unsteady aerodynamic theory which has improved the prediction of pitch link loads in the stall regime of a helicopter rotor was reported in the May 1971 Annual Forum Meeting of the American Helicopter Society.

### COMPUTER TECHNIQUES

There is a new digital computer program of the model that reflects the improved efficiency in coding techniques and data processing gained from experience with the current model.

### APPROACH

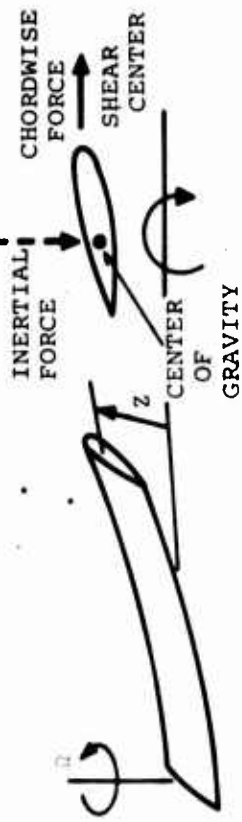
Although the mathematical model under development contains the most recent advances in the field, only those items of importance are being selected in order to avoid generating an impractical computer program because of the inability to predict input data, long running time, or numerical convergence

problems. This limiting compromise follows from the fact that the state of the art for a rotor aeroelastic analysis is determined not by advanced theories in dynamics, aerodynamics, and elasticity, but by the practical limits of computer time and numerical tractability. A review of the additional capability under development is given in Figures 54 and 55.

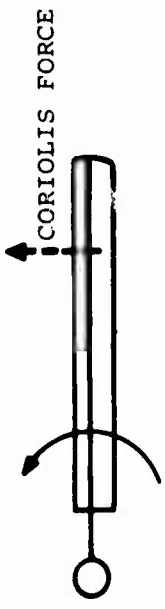
#### PROGRAM FLOW DIAGRAM

Figure 56 shows the changes in the simple block diagram of Figure 53. The program under development shows the coupled flap-lag-pitch deflection calculations iterated with the airloads. This allows the flap, lag, and pitch degrees of freedom to be included in the airloads and includes all significant couplings between flap, lag, and pitch.

- FLAP, CHORD, AND PITCH FREEDOMS WILL BE COUPLED DYNAMICALLY AND AERODYNAMICALLY  
FLAP MOTION CAUSES PITCH MOTION:



- FLAP MOTION CAUSES CHORDWISE MOTION:



- CHORDWISE MOTION CAUSES PITCH MOTION:



- PITCH MOTION CAUSES FLAP MOTION:



- LARGE TWIST



NOTE: THE FLAP AND CHORDWISE DIRECTIONS ROTATE RELATIVE TO THE DISC PLANE

- FLAP, CHORD, AND PITCH DEFLECTIONS ARE INCLUDED IN THE AIRLOADS

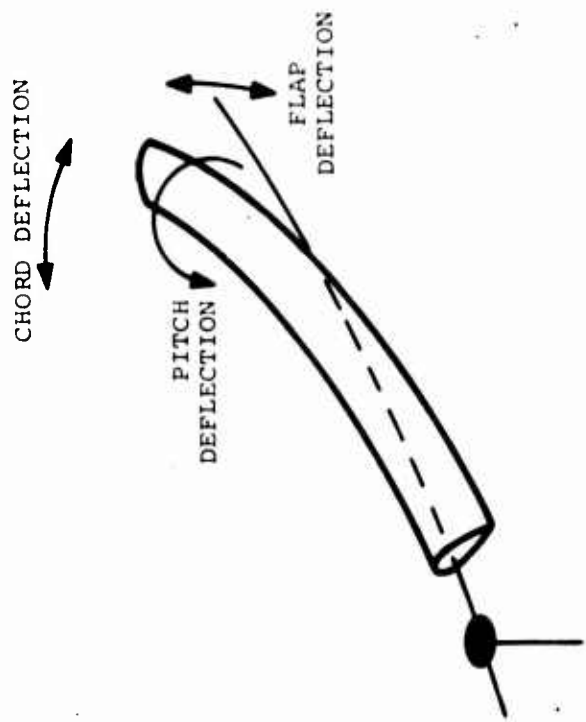
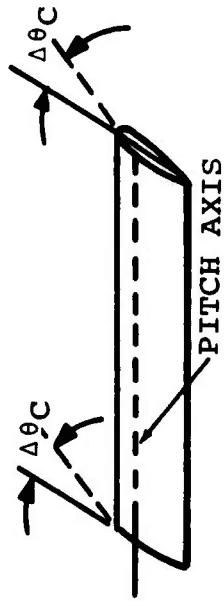


Figure 54. Coupling of Blade Motions in Improved Program

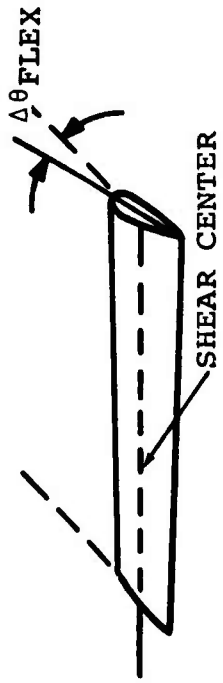


- PITCH AXIS, SHEAR CENTER, AND VERTICAL NEUTRAL AXIS ARE INDIVIDUALLY VARIABLE
- ALL SHAFT TILTS ARE CONSIDERED

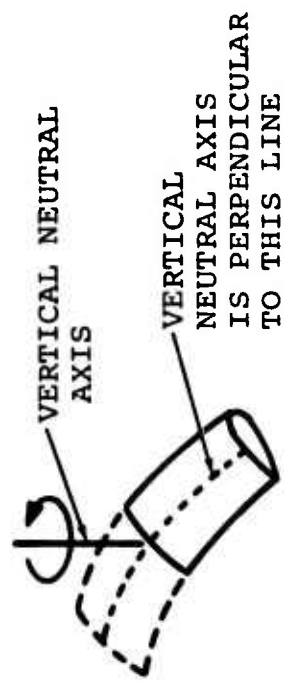
THE RIGID-BODY CONTROL INPUT IS ABOUT THE PITCH AXIS



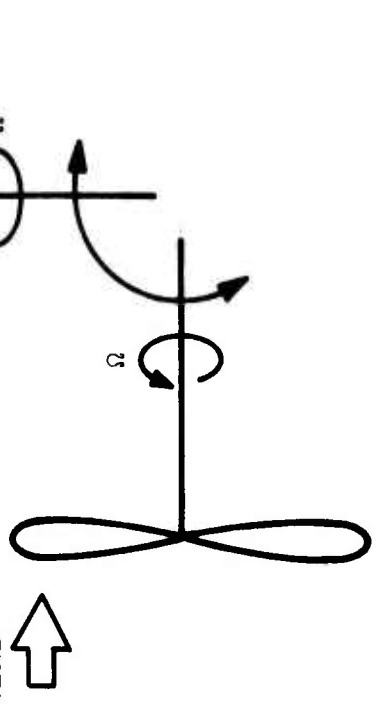
THE FLEXIBLE BLADE TWIST IS ABOUT THE SHEAR CENTER



THE CHORDWISE BENDING IS ABOUT THE VERTICAL NEUTRAL AXIS



RELATIVE WIND



- THE AERODYNAMIC STATIONS INCREASED BY 50 PERCENT

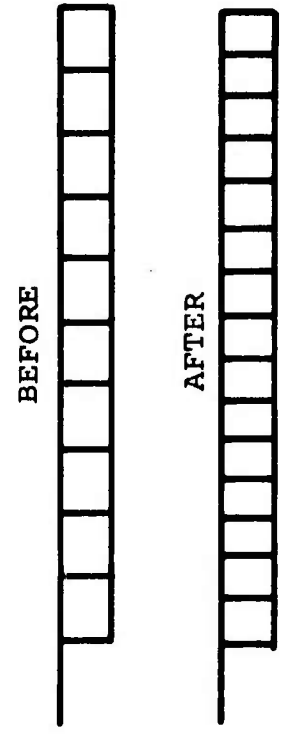


Figure 55. High Twist and Shaft Tilts in Improved Program

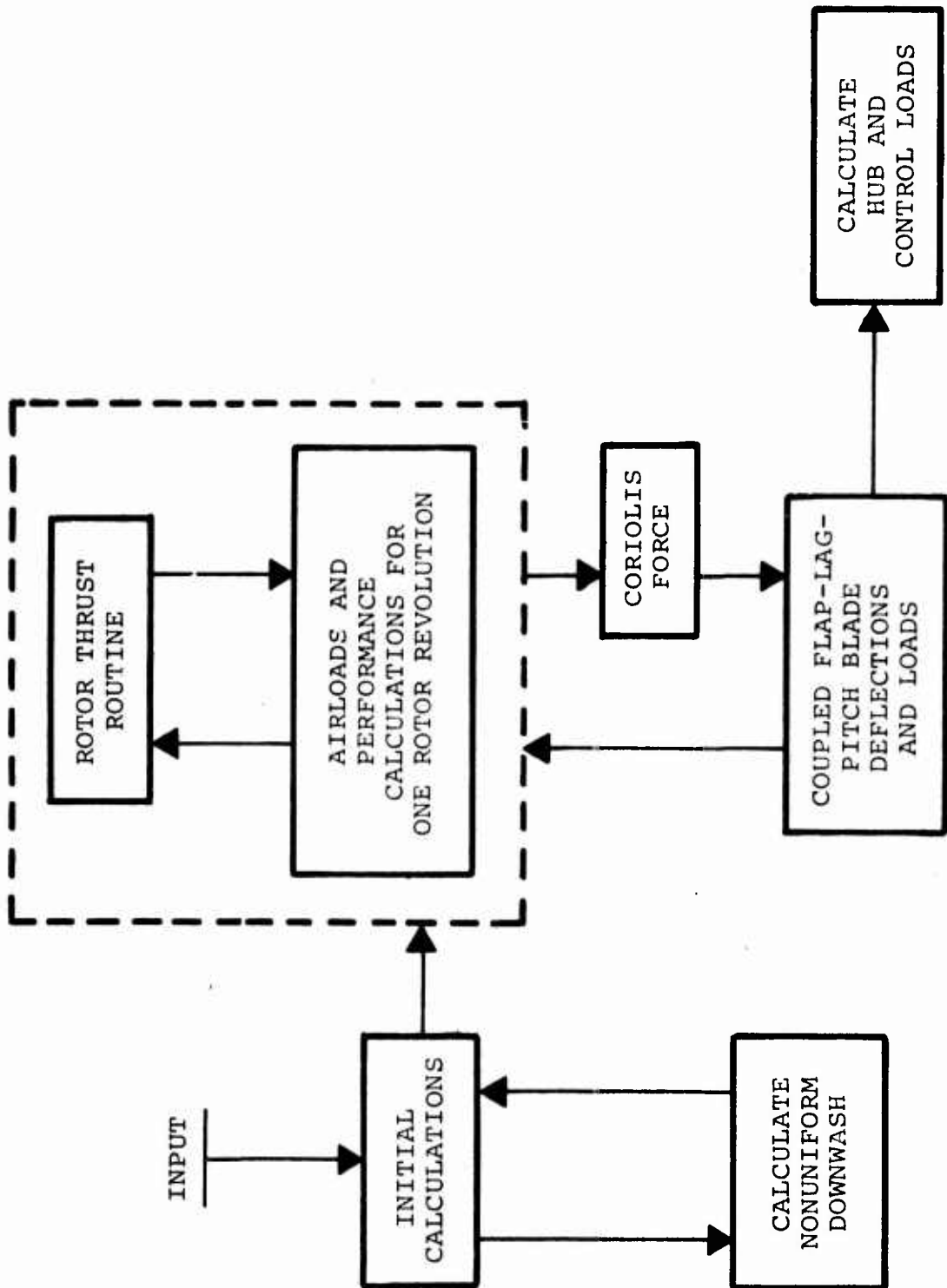


Figure 56. Flow Chart of Improved Loads Program

## INDUSTRY REVIEW

### METHOD OF SOLUTION

All of the available published data indicates that the majority of the members of the rotary-wing industry employ the lumped-mass approach in solving the rotor loads problem for a V/STOL prop/rotor blade. An alternative approach to the solution of this problem is the modal analytical method which employs the prop/rotor blade normal modes derived from a free vibration analysis. The lumped-mass approach appears to have an advantage over the modal analytical method in that it does not require preknowledge of the blade normal modes, nor does it encounter numerical convergence problems which are inherent in the modal analysis. References 20 through 38 list the details of these analytical methods and their predictive capabilities which vary in technique due to consideration of a particular rotor system where experience has shown that a particular mode of description or a particular parameter in the dynamics, aerodynamics, or elasticity provides the best correlation between theory and flight test data.

The typical rotor aeroelastic analysis first determines the actual or equivalent flapping motion of the prop/rotor blade by a step-by-step timewise integration of the inelastic flapping equation of motion; the solution will converge to a cyclic pattern when the steady-state flight condition is being analyzed. This analysis can be refined further by introducing the first flap bending normal mode and its associated equation of motion, where now the two equations are integrated on the basis of a set of starting boundary values determined from the inelastic blade solution. When a steady-state condition is being analyzed, the integration proceeds in small but finite timewise steps; after a number of rotor revolutions, the predicted motions will become cyclic within a desired tolerance. This is the usual solution sought, and the rotor performance, loads, stress, and dynamic calculations are based on these accepted cyclic motions. The airload calculations include airfoil section geometry, compressibility, stall, 3-dimensional flow, unsteady aerodynamics, and nonuniform downwash. The unsteady aerodynamic loads are calculated by various modifications of the stall loads resulting from the airfoil tables. The modifications typically include Theodorsen's shed wake function or a derivative thereof, dynamic stall effects based upon oscillating airfoil data, and yawed flow across the blade. The nonuniform downwash calculations are based on shed vortex data that ranges from tip vortex, tip and root vortex, to multiple vortices immediately behind the blade. Several different iterative schemes are used to establish vortex strength as a function of calculated blade lift. Dissipation of the vortex downstream is somewhat arbitrary in present analytical techniques and the wake in most cases is assumed

rigid with drift relative to the hub and a resultant velocity due to uniform inflow and aircraft speed. The solution for the nonlinear aerodynamic loads and the coupled blade response is performed in a series and the iteration between the airloads and blade response is used to obtain the final steady-state solution.

#### PUBLISHED CORRELATION

Figures 57 through 65 summarize the available published data on the loads-prediction capability typical of the industry. This data includes the Kaman theory correlation with the CH-34 full-scale flight test conducted by NASA-Langley as reported in Reference 35; the Sikorsky theory correlation with the CH-34 full-scale tunnel test conducted by NASA-Ames as reported in Reference 37; and the Sikorsky theory correlation with the S-61F full-scale flight test conducted by Sikorsky as reported in Reference 38. Generally speaking, the correlation of theory with test data is fair to good, indicating the obvious necessity for continued refinement of all theories presented herein.

REFERENCE 35

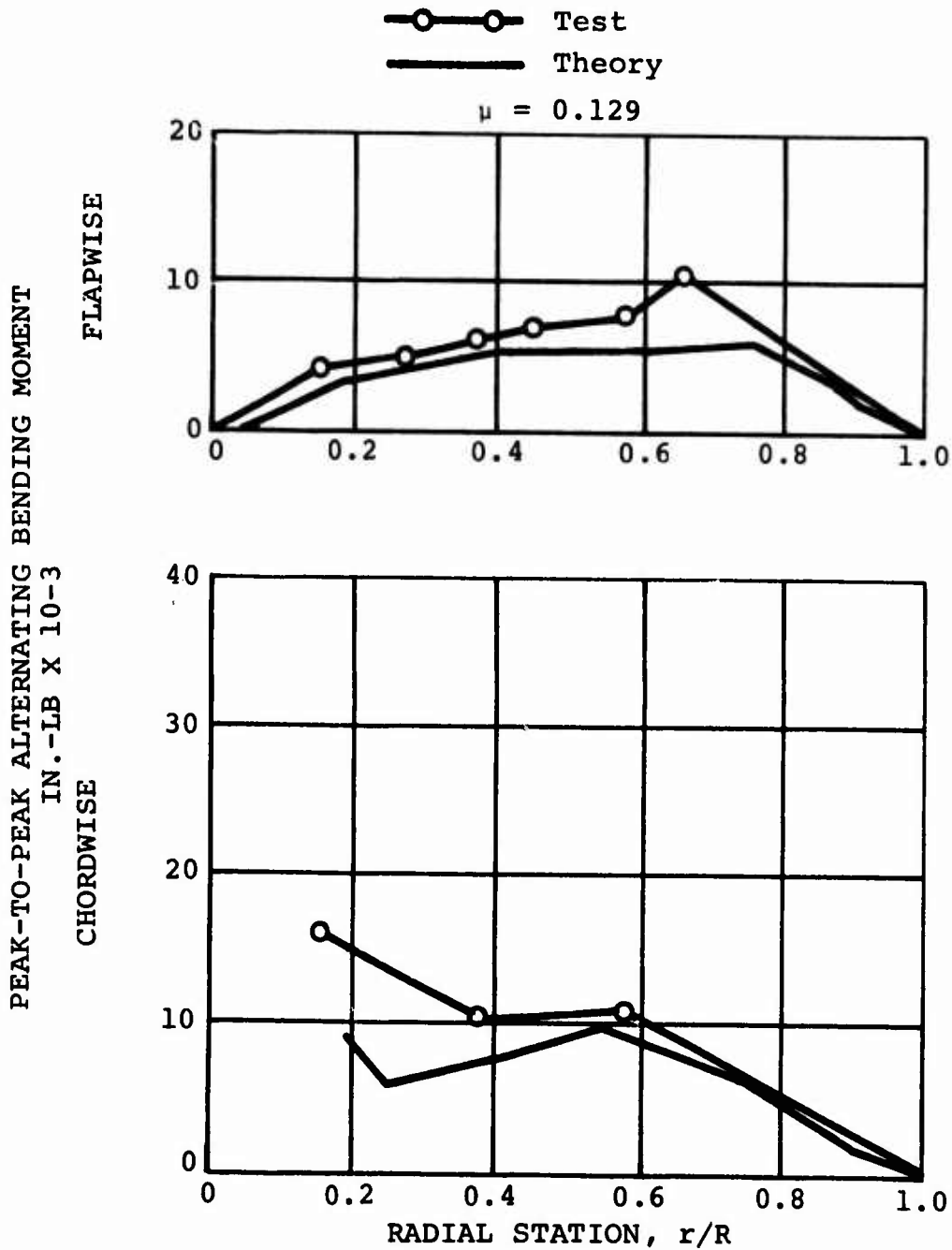


Figure 57. Transition Flight Moments from Sikorsky CH-34 Flight Data, NASA - Langley Test Data, and Kaman Theory

REFERENCE 35

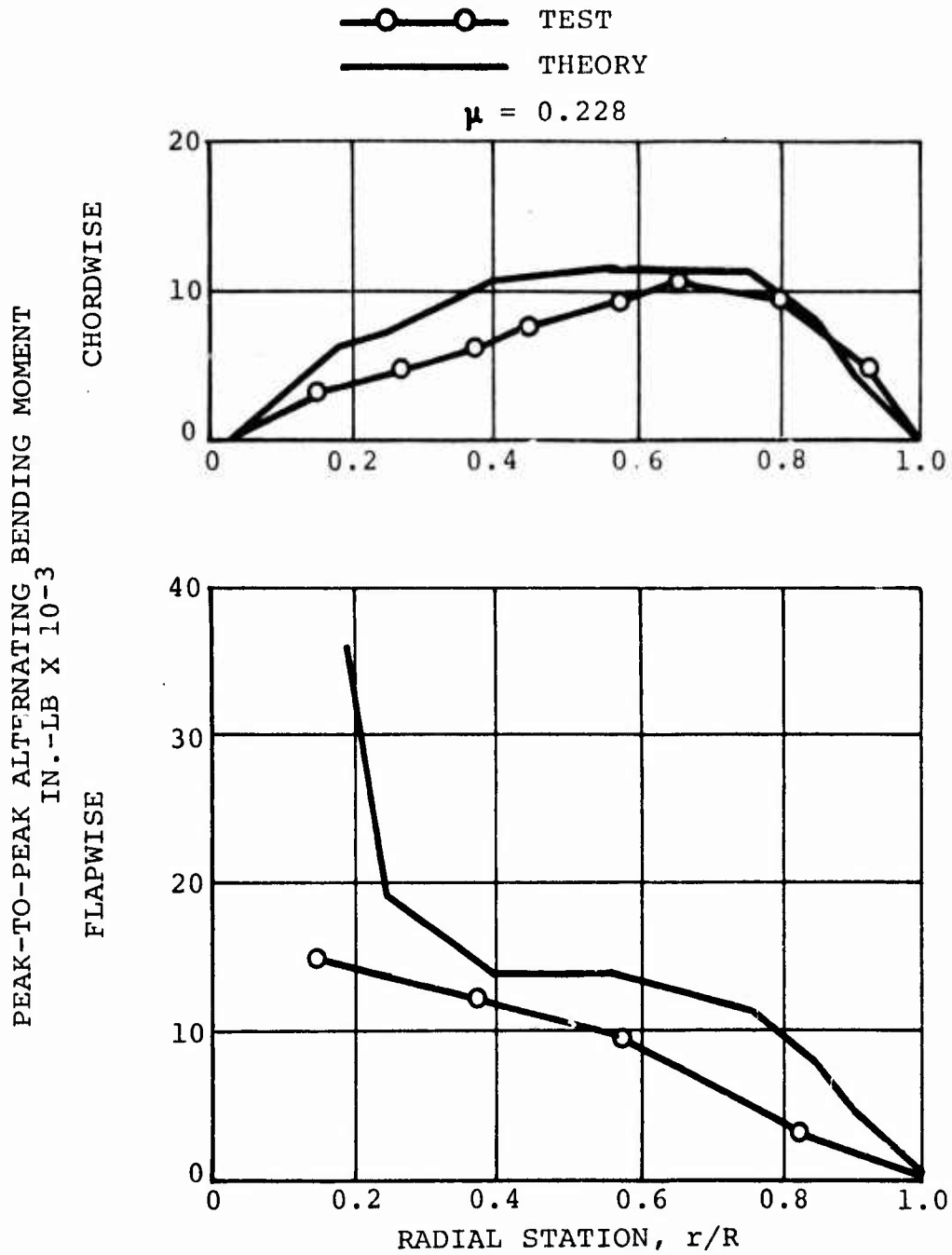


Figure 58. Cruise Flight Moments From Sikorsky CH-34 Flight Data, NASA - Langley Test Data, and Kaman Theory



REFERENCE 35

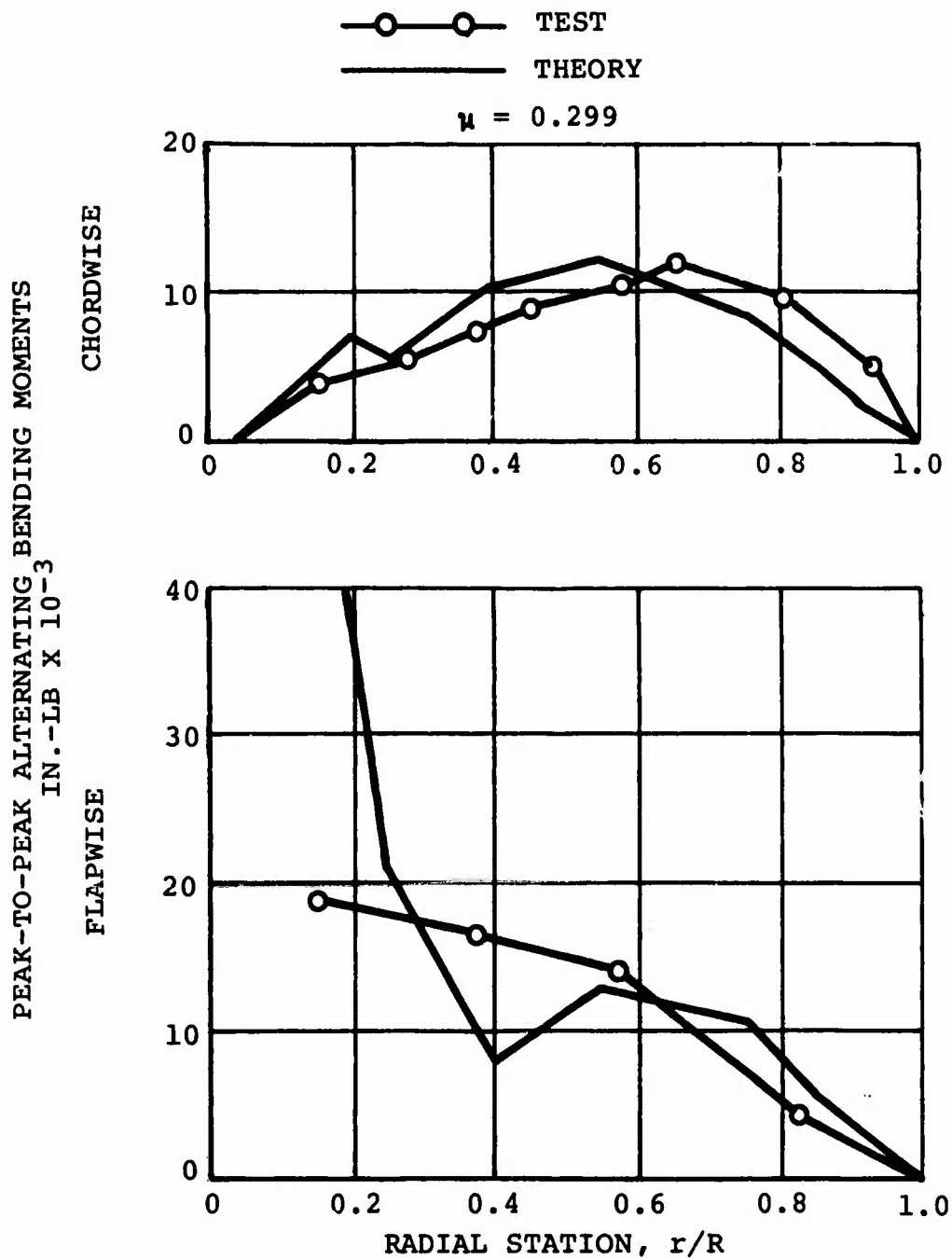


Figure 59. High-Speed Flight Moments From Sikorsky CH-34 Flight Data, NASA - Langley Test Data, and Kaman Theory

REFERENCE 35

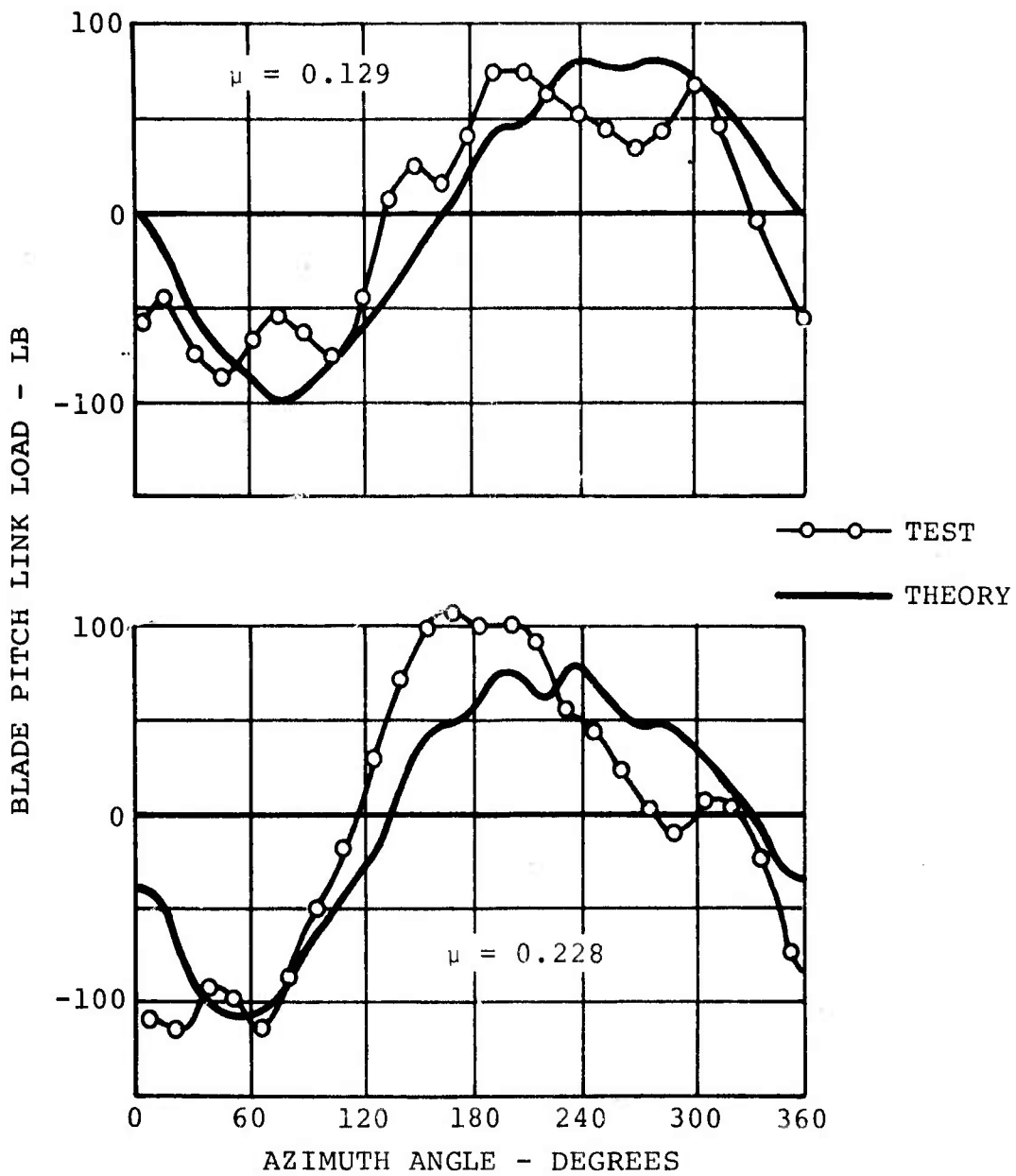


Figure 60. Cruise Pitch Link Loads From Sikorsky CH-34 Flight Data, NASA - Langley Test Data, and Kaman Theory

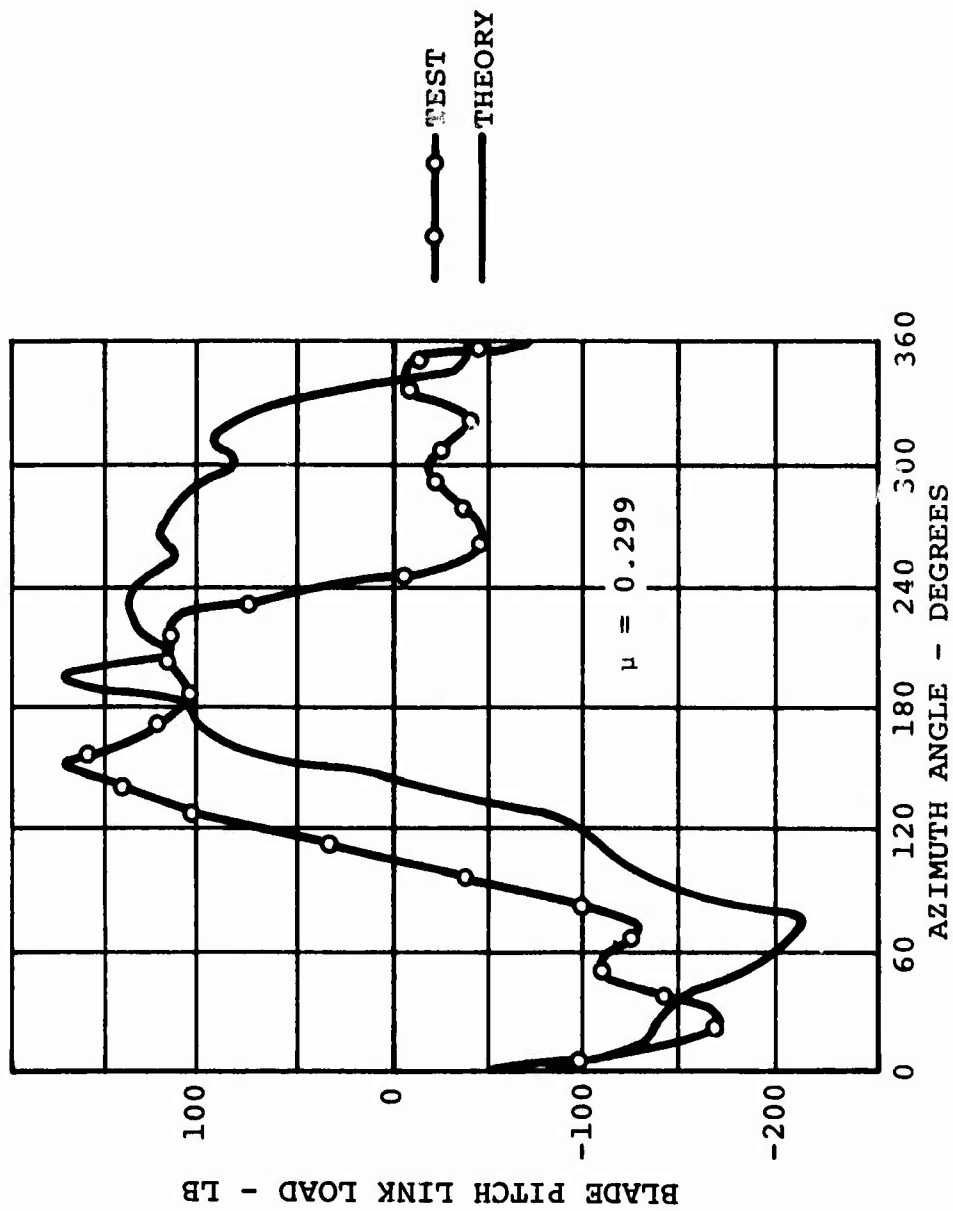


Figure 61. High-Speed Pitch Link Loads From Sikorsky CH-34 Flight Data, NASA - Langley Test Data, and Kaman Theory

—○— TEST DATA  
 - - - UNIFORM INFLOW THEORY  
 - · - VARIABLE INFLOW THEORY

$V = 110$  KNOTS  
 $\alpha_S = -5^\circ$   
 $L = 8,300$  LB

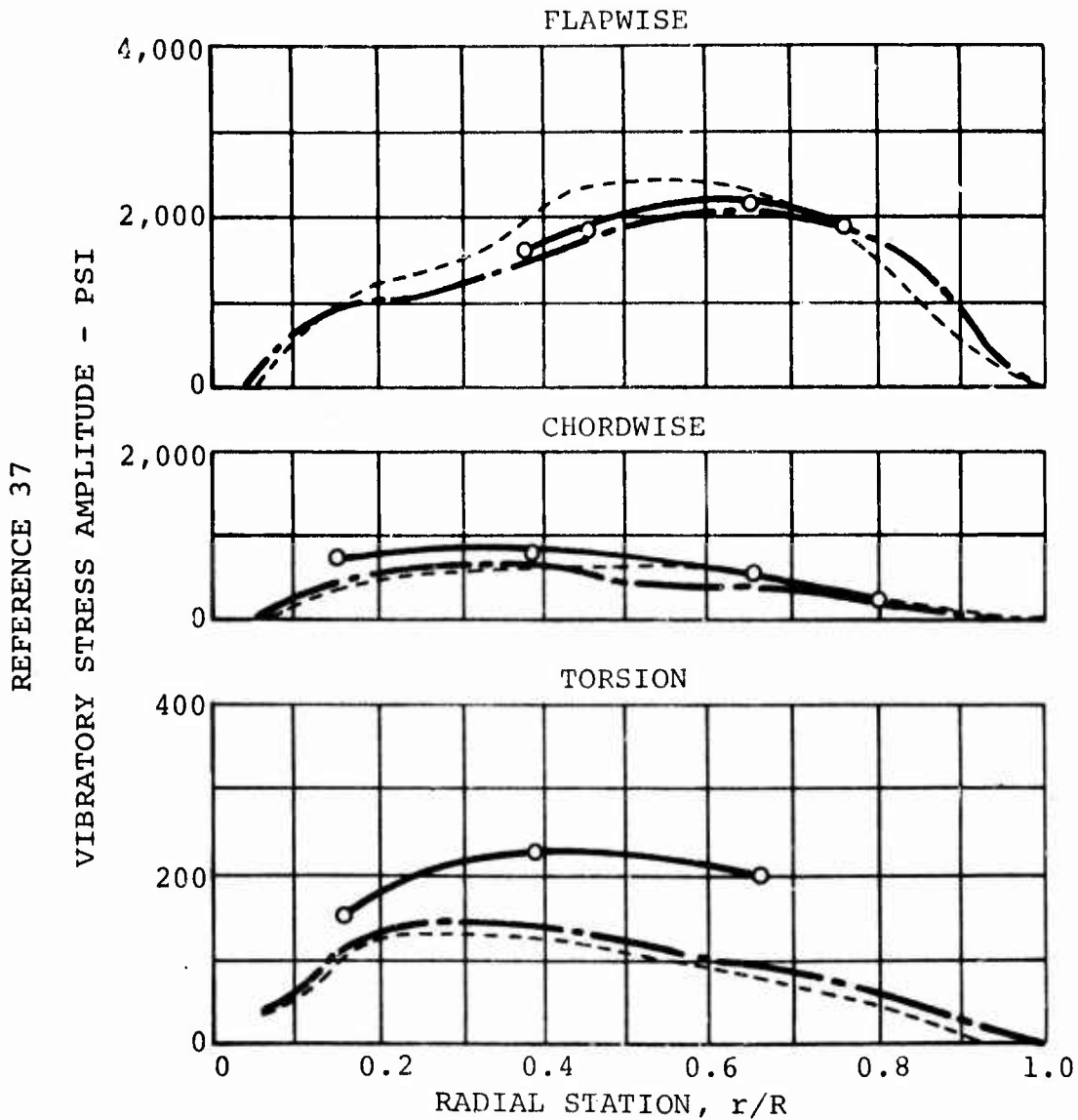


Figure 62. Transition Flight Loads From Sikorsky CH-34  
 Tunnel Data, NASA-Ames Test Data, and  
 Sikorsky Theory

○—○ TEST DATA

----- UNIFORM INFLOW THEORY

V = 150 KNOTS

- - - - VARIABLE INFLOW THEORY

$\alpha_S = -5^\circ$

REFERENCE 37

L = 8,500 LB

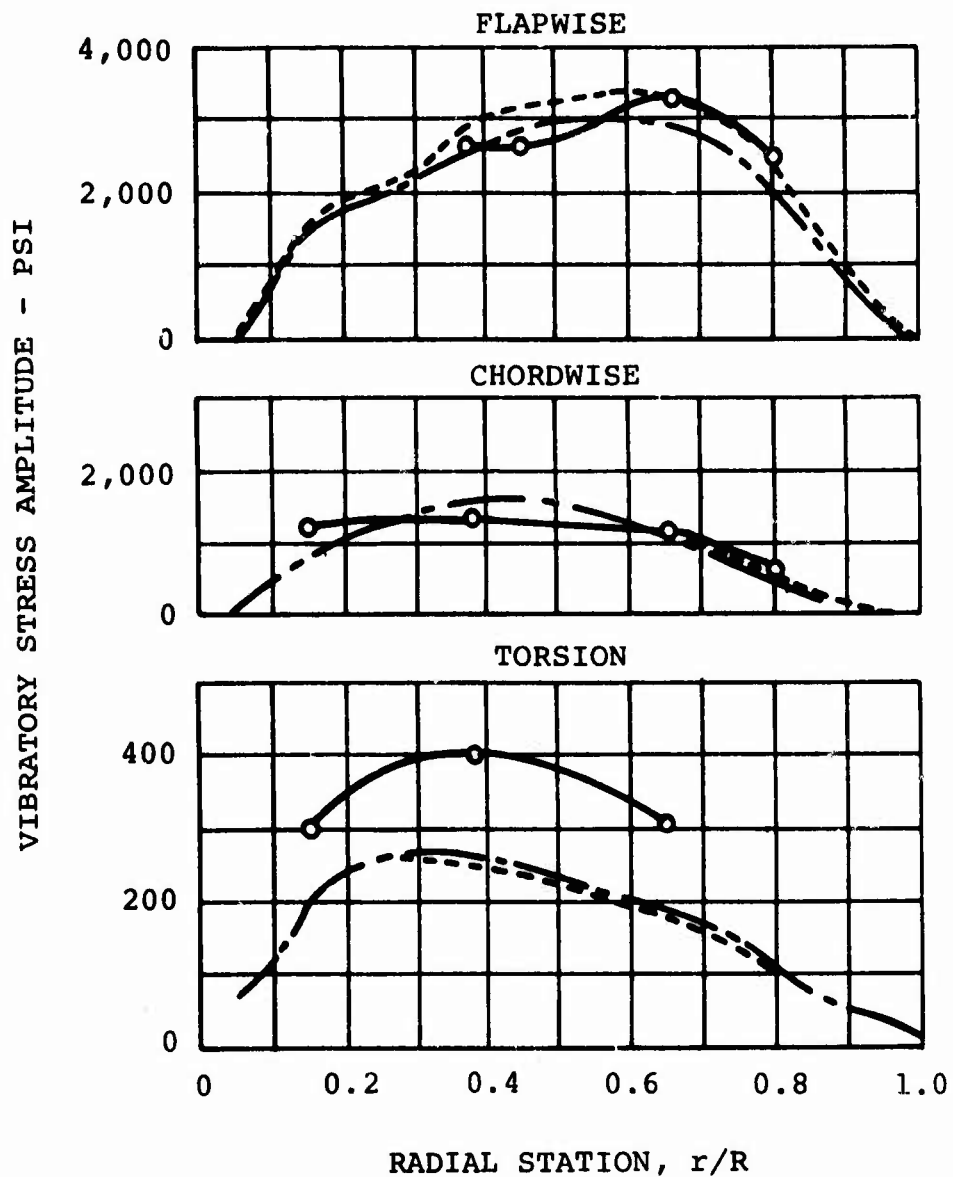


Figure 63. Cruise Flight Loads From Sikorsky CH-34 Tunnel Data, NASA-Ames Test Data, and Sikorsky Theory

—○— TEST DATA

V = 175 KNOTS

----- UNIFORM INFLOW THEORY

$\alpha_s = -5^\circ$

- - - - VARIABLE INFLOW THEORY

L = 7,100 LB

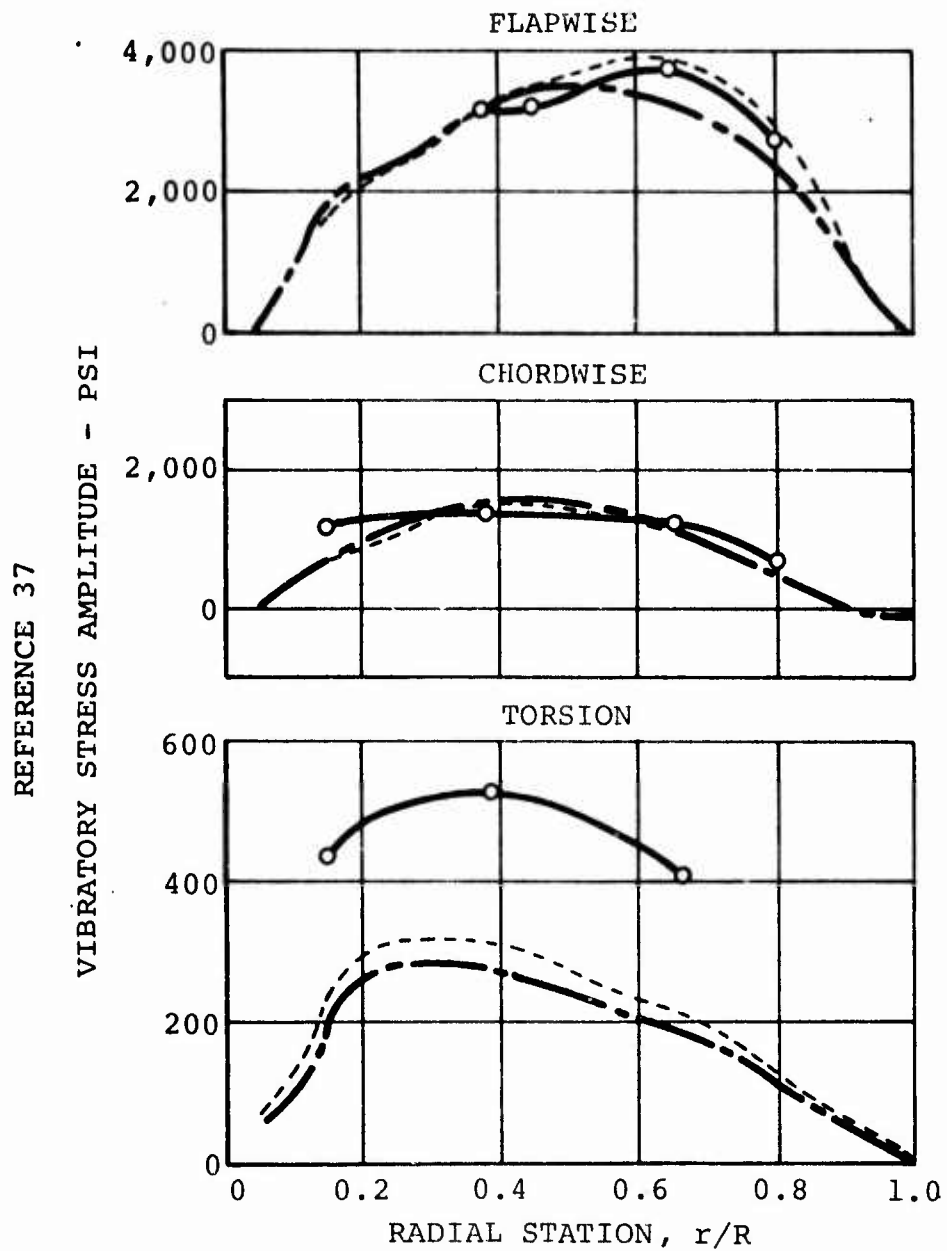
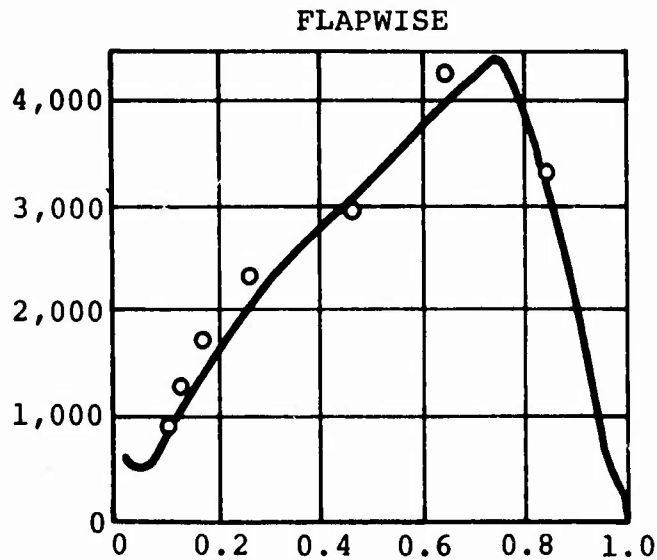


Figure 64. High-Speed Flight Loads From Sikorsky CH-34 Tunnel Data, NASA-Ames Test Data, and Sikorsky Theory

REFERENCE 38

VIBRATORY STRESS AMPLITUDE - PSI



CHORDWISE

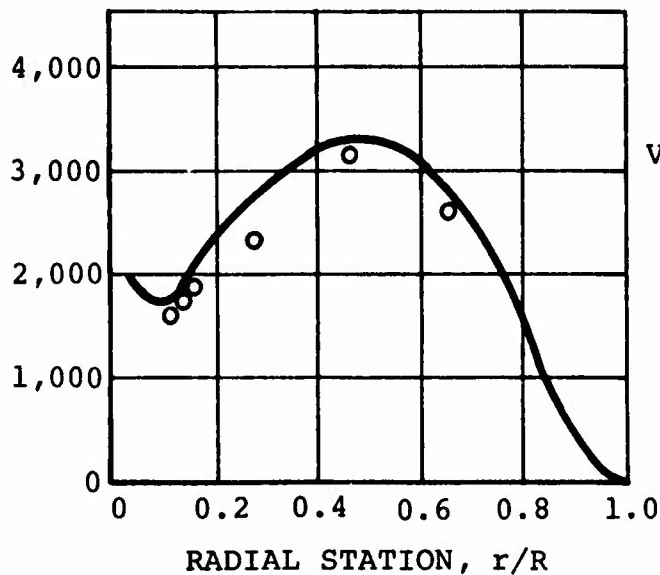


Figure 65. High-Speed Flight Moments From Sikorsky S-61F Compound Full-Scale Flight Data



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13. ABSTRACT The aeroelastic phenomena associated with prop/rotor systems are discussed and classified. It is concluded that an acceptable technology exists in several areas, including wing/rotor divergence, whirl flutter, aeromechanical instability, and air and ground resonance. The technology is less successful in those areas where the flow through the rotor is significantly nonaxial, e.g., tilt-rotor transition regime and high-speed helicopter flight; also when forms of intermodal blade coupling exist due to finite deflections of the blades. It is believed that, in addition to collective deflections, finite cyclical deflections of the blades produce destabilizing coupling effects in some cases. Significantly large edgewise flow in combination with nonzero blade steady- state deflections is also seen to be destabilizing. A minimum-complexity methodology which may be expected to correlate with currently identified phenomena is defined. The methods contained in this report are intended to be used by designers to calculate with improved accuracy, the dynamic and aeroelastic response characteristics of rotor powered V/STOL aircraft. The essential new feature of these methods is that the coupled flap-pitch-lag blade deflections are taken into account. These calculations are essential if a high level of confidence is to be had in the results.		

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	ROLE	WT	ROLE	WT	ROLE	WT
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Rotor/airframe instabilities						
Wing/rotor divergence						
Whirl flutter						
Oscillatory behavior						
Blade instability mechanisms						
Blade vibratory loads						
Source of airloads						
Blade loads						
Mathematical model						
Industry review						