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18 January 1951

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Subj: Operations Evaluation Group Study No. 430 (Addendum to OEG Study No. 368) - forwarding of

Ref: (a) CNO Conf ltr ser [illegible]4 of 24 Nov 1948

Encl: (1) OEG Study No. 430 (Addendum to OEG Study No. 368) "Computation of Probability of Visual Detection in Air Interception"

1. Reference (a) forwarded OEG Study No. 368, "Visual Detection in Air Interception". Enclosure (1), an addendum to that study prepared by the Operations Evaluation Group, is forwarded for your information and retention. It is an extension of the work presented in Study No. 368, and may be used in conjunction with the theory contained therein. It presents a method and associated working graphs which enable one to calculate the probability of visual detection of a target aircraft by an airborne observer, under daylight conditions of illumination, for a wide range of the parameters involved.

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OPERATIONS EVALUATION GROUP
STUDY NO. 430

ADDENDUM TO OEG STUDY NO. 368

COMPUTATION OF PROBABILITY OF VISUAL DETECTION IN AIR INTERCEPTION.

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Prepared by
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21 November 1950

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OPERATIONS EVALUATION GROUP
STUDY NO. 430

ADDENDUM TO OEG STUDY NO. 368

COMPUTATION OF PROBABILITY OF VISUAL DETECTION IN AIR INTERCEPTION

Ref: (a) OEG Study No. 368 "Visual Detection in Air Interception" Conf 24 Nov 1948

ABSTRACT

This addendum is an extension of work previously presented in reference (a), and is to be used in conjunction with the theory contained therein. It presents a method and associated working graphs which enable one to calculate the probability of visual detection of a target aircraft by an airborne observer, under daylight conditions of illumination, for a wide range of the parameters involved.

INTRODUCTION

In reference (a) a theory of visual detection of target aircraft by an airborne observer under daylight conditions of illumination was developed. In addition, a method was presented for computing the probability of visually detecting the target by the time the observer had closed on a collision course to any given range. Working graphs obtained by this method were presented and were applicable in a limited number of specific cases. Since the publication of reference (a), the computational method has been improved and graphs have been obtained which apply under a much wider variety of operational conditions. This later work is presented in this addendum; the theory and assumptions on which it is based are presented in detail in reference (a).

1

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In general, the type of problem is as follows:

An interceptor is being vectored by CIC (during daylight) to close within sighting range of an enemy bomber so that he may then position himself for a firing run. It is desired to determine the probability that he will sight the target at a range which will be great enough to allow this attack manouver. As discussed in reference (a), this is a function of the relative closing speeds, the apparent size of the target, its brightness relative to the background, the visibility at flight altitudes, and the size of the field scanned by the interceptor pilot.

TERMINOLOGY

More specifically, the desired probability is a function of the following parameters:

C_o - intrinsic contrast of the target (%)

V - meteorological visibility (mi.)

R_o - maximum range in absence of haze (mi.)

W - gross weight of target (lbs.)

α - aspect angle of the target (degrees)

ϕ - elevation scanning angle (degrees)

Θ - azimuth scanning angle (degrees)

A - presented area of the target (sq. ft.)

R_m - maximum range at which detection is possible, under given conditions (mi.)

v - relative velocity (kts.)

R_1 - range at which search is begun (mi.)

CALCULATION OF DETECTION PROBABILITIES

The method of computing these probabilities will be illustrated by working out a specific example. Suppose the enemy is assumed to be a medium bomber weighing approximately 45,000 lbs., unpainted aluminum, and is heading due north at a speed of 400 knots. The interceptor is on a collision course, 135° true heading, travelling at 500 knots, and the pilot is scanning 20° on either side of the expected position of the target and 3° above and below the expected relative altitude (i.e., $\Theta = 20^{\circ}$, $\phi = 3^{\circ}$). The meteorological visibility at flight altitude is about 20 miles. ($V = 20$ mi.). The pilot scans in a regular and methodical fashion from the time search begins until he sights the target. The computation of detection probabilities will be illustrated first for the case in which the pilot begins his search while still outside visual range of the target, (i.e., $R_1 \geq R_m$), and secondly, for the case in which search is not begun until the pilot is within visual range (i.e., $R_1 \leq R_m$).

The probability of having detected the target by range R if search was begun at range R_1 is given by the equation

$$P(R) = 1 - e^{-\frac{R_o}{V}\left[I(R) - I(R_1)\right]},$$

where $I(R)$ is a quantity obtained graphically (from Figure A2) and is a function of the parameters Θ, ϕ, C_o, and $\frac{R_o}{V}$. If search is begun at or beyond the maximum possible detection range, i.e., $R_1 \geq R_m$, then $I(R_1) = 0$ and the probability can be written as

$$P(R) = 1 - e^{-\frac{R_o}{V} I(R)}$$

In either case, the computation would proceed as follows:

(1) The target and interceptor courses are plotted on a maneuvering board, the relative motion vector drawn, and the aspect angle and relative speed measured directly (See Figure (1)). For the given conditions, the aspect angle α is found to be 82.5°, and the relative speed v to be 356 knots.

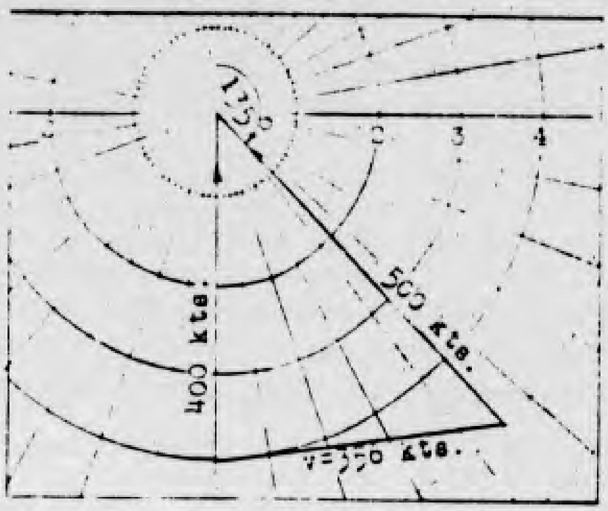

FIG. 1

(2) In reference (a), the intrinsic contrast, C_o of unpainted aluminum is given as 27%. For the accuracy desired, it will be sufficient to use C_o = 30% and avoid interpolation for C_o.

Using the nomograph, Figure A1, and entering the known values C_o = 30% and W = 45,000 lbs., one finds that the corresponding value of R_{oo} (maximum range for bow aspect) is 15.0 miles, and that for aspect angle α = 82.5°, the value of $R_{o\alpha}$ is 24.0 miles. ($R_{o\alpha}$ will now be referred to simply as R_o).

(3) Thus, $R_o/V = 24/20 = 1.2$; $R_o/v = 24/356 = .0674$.

(4) In Figure A2, choose the graph corresponding to the parameters Θ = 20°; ϕ = 3°; C_o = 30%.

(5) The minimum value of R/R_o for which the computation can be carried, is seen to be, in this case, $R/R_o = .05$.

(6) When search is begun outside visual range of the target, it is necessary to determine the value of R_m, the range at which detection first becomes possible. This can be obtained from Figure A2, since R_m/R_o is the value of R/R_o at which $I = 0$. The values of R_m/R_o for the $R_o/V = 1.0$ and $R_o/V = 1.5$ curves can be read directly from the graph, and are respectively, $R_m/R_o = .43$ and $R_m/R_o = .35$. Using linear interpolation (sufficient for the accuracy desired in this problem) the value of R_m/R_o for $R_o/V = 1.2$ is found to be .40. Hence $R_m = R_o \left(\frac{R_m}{R_o}\right) = 24\ (.40) = 9.6$ miles.

(7) It is now desired to find values of the detection probability, $P(R)$, for several values of R/R_o within the range $.05 \leq R/R_o \leq .40$ so that a graph of $P(R)$ vs. R can be drawn. For any particular value of R/R_o, say $R/R_o = .25$, the corresponding probability is found as follows:

(a) $I(R)$ is found by using linear interpolation. When $R/R_o = .25$, the values of $I(R)$ for $R_o/V = 1.0$ and $R_o/V = 1.5$ as read from the graph are respectively $I = 14.5$ and $I = 5.0$ (Note: The ordinate of Figure A2 is $I + 10$; ∴ subtract 10 from the ordinate to read I). Therefore, for $R_o/V = 1.2$, one obtains $I = 14.5 - .4(14.5-5.0) = 10.7$.

(b) $\therefore \frac{R_o}{V} I(R) = .0674\ (10.7) = .72$

(c) $\therefore P(R) = 1 - e^{-.72} = .51$

(d) The absolute range is $R_o(R/R_o) = 24 \times .25 = 6$ mi.

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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A graph of P vs. R is shown [illegible]. [illegible] at a single [illegible] visual [illegible] the interceptor will have [illegible] found the [illegible] to range R_1 [illegible] example, a [illegible] reached when the target has closed to about [illegible] mi.

If [illegible] does not begin his visual search until some time after [illegible] maximum visual range of the target, the probability of detection will be somewhat reduced. To illustrate: suppose now that the interceptor we have [illegible] does not begin his visual search until he has closed to a range of [illegible] miles, all other factors being held constant.

Steps 1-3 of the computation still apply.

The range of R/R_m is now restricted to $.05 \leq R/R_m \leq .25$ since $P = 0$ when $R/R_m \geq .25$ (i.e. $R \geq 6$ mi.) Since $R_1 < R_m$, $I(R_1)$ has to be subtracted from all the values of $I(R)$. $I(R_1)$ is the value of I for $R/R_m = .25$ and has previously been found to be 10.7.

A graph of $P(R)$ vs. R is then obtained by calculating $P(R) = 1 - e^{-\frac{F}{V}[I(R)-10.7]}$ for several values of R/R_m within the above range. Steps in the calculation are tabulated below. $R/R_m = .225$ was included to get a value of P between 0 and .5.

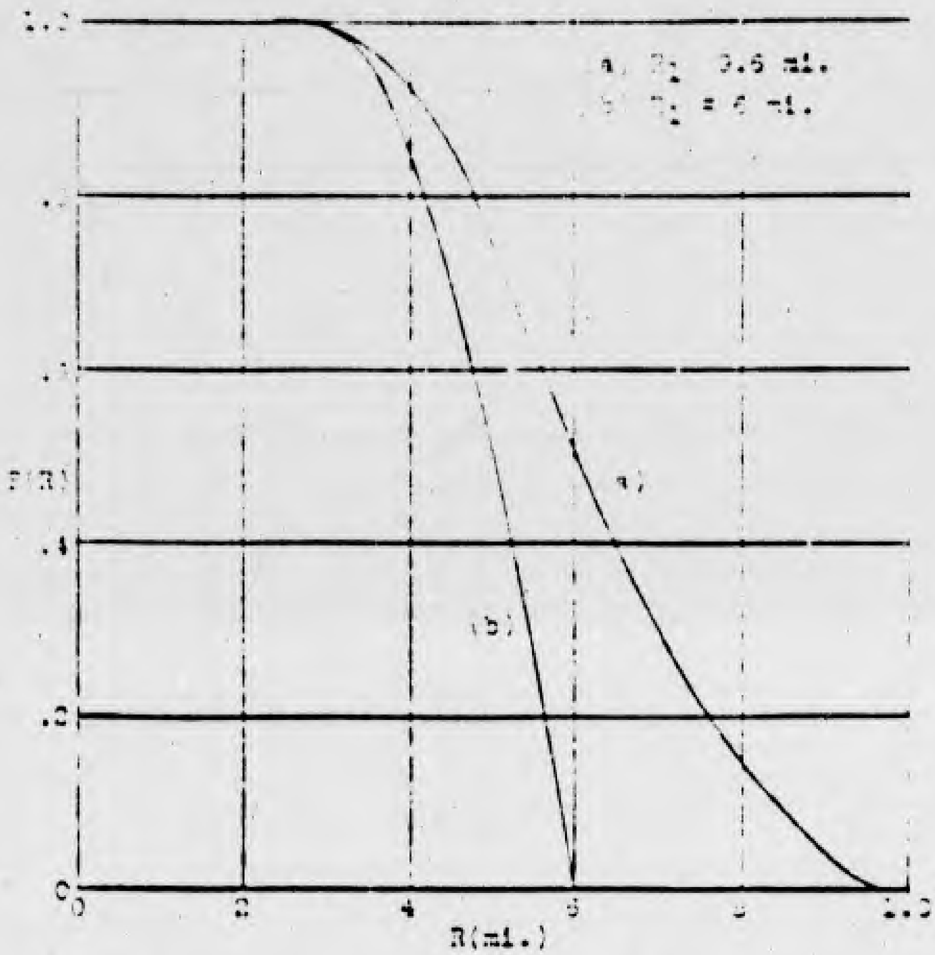


FIG. 2: PROBABILITY OF VISUAL DETECTION

R/R_o	.25	.225	.20	.15	.10	.05
$I(R)$	10.7	16.4	24	54	114	240
$I(R) - I(R_1)$	0	5.7	13	43	103	229
$\frac{R_o}{V} I(R)-I(R_1)$	0	.38	.88	2.30	6.9	15
$P(R)$	0	.32	.59	.94	1.0	1.0
R	6.0	5.4	4.8	3.6	2.4	1.2

$P(R)$ vs. R for this case is shown in (b) Figure 2. It shows, for example, that a probability of detection of .75 is reached at a range of 4.4 mi. when search is begun at 6 mi.

Submitted to the Director:

E. MARCUSE
Mathematical Services Section
Operations Evaluation Group

Approved by:

E. S. LAMAR
Deputy Director
Operations Evaluation Group

APPENDIX A

CALCULATION OF FIGURES A1 AND A2

Since the probability of detection P is dependent on the value of R_o, the maximum range in the absence of haze, our first problem is to determine R_o as a function of given parameters. If the presented area of the target, A, is known, use can be made of the relation

(1) $R_o = .1655 \sqrt{C_o - 1.565)\, A}$, which is presented in nomographic form in Figure 8 of reference (a). However, the area is seldom given in aircraft specifications, so that it is desirable to determine R_o as a function of gross weight of target. In reference (a), the assumption that for a given aspect, square root of presented area is proportional to cube root of gross weight, was tested for a variety of aircraft, and appeared to be a sufficiently accurate assumption for purposes of computation. (See Figure 7, reference (a)). For bow aspect, this approximation is given by

(2) $A^{\frac{1}{2}} = .4733\, W^{1/3}$.

Substitution of (8) in (7) will give us the maximum range at bow aspect, R_{oo}, in terms of gross weight and intrinsic contrast, namely,

(3) $R_{oo} = .0793 \sqrt{C_o - 1.565}\, W^{1/3}$

The effect of target aspect on the maximum range is defined in reference (a) by the relationship

(4) $R_{o\alpha} = R_{oo} \sqrt{\cos \alpha + 2.4 \sin \alpha}$, where $R_{o\alpha}$ is the value of R_o for aspect angle α.

The nomograph, Figure A1, combines equations (3) and (4) and thus enables us to solve for R_o ($= R_{o\alpha}$), given C_o, W, and α. This replaces the use of Figures 6 and 8 of reference (a).

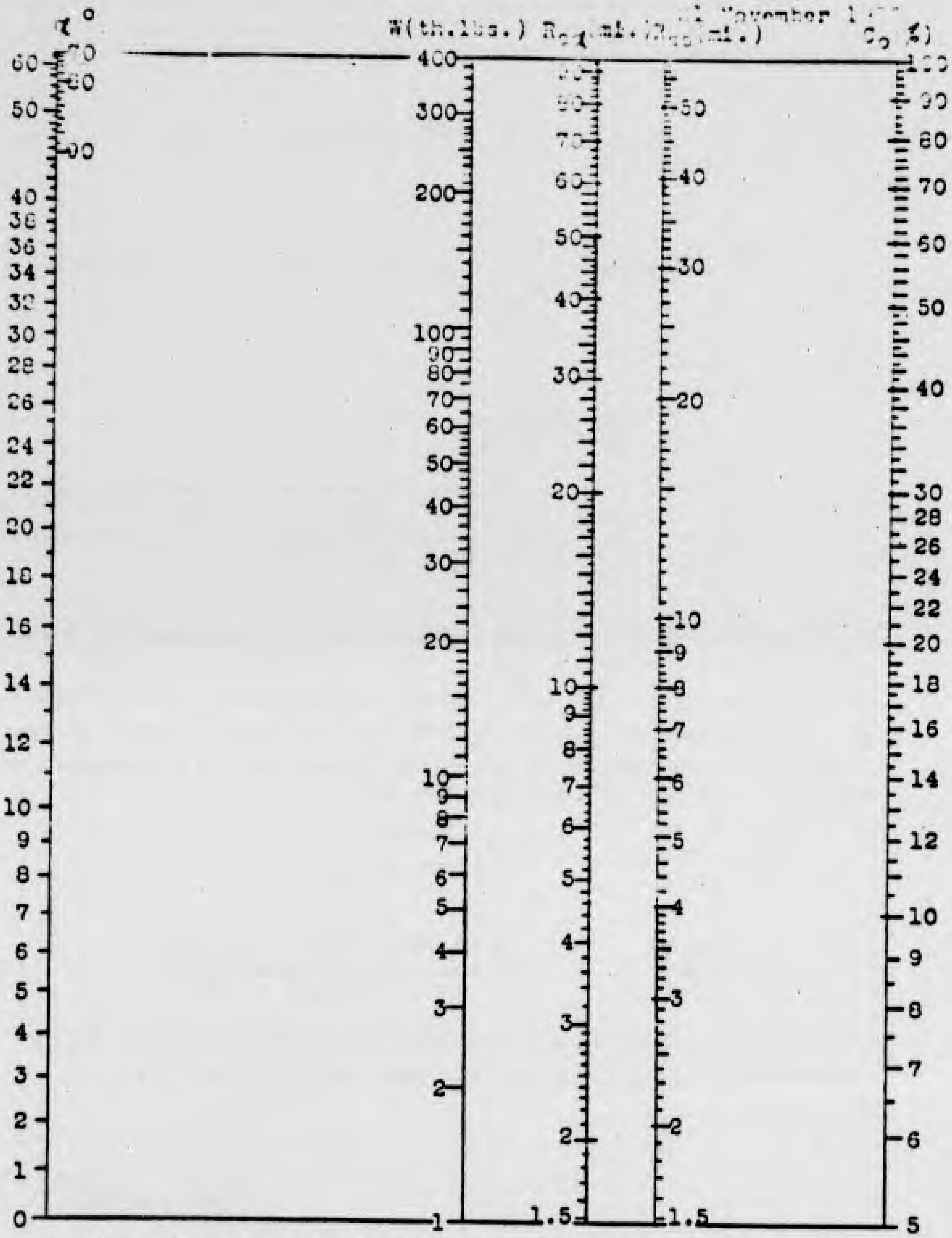


FIG. A1: NOMAGRAPH CONNECTING THE PARAMETERS C_o, W, α, R_{oo}, AND $R_{o\alpha}$

As derived in reference (a), the probability of visual detection of a target by an interceptor on a collision course with the target, which has been searching from range R_1 to R_2, is given by

$$(5) \quad P(R_1,R_2) = 1 - e^{-\frac{R_0}{V} I(R_1,R_2)}, \text{ where}$$

$$(6) \quad I(R_1,R_2) = \int_{R_2/R_0}^{R_1/R_0} -2.21(10)^3 \ln(1-g)\, d(R/R_0),$$

$$R_L \le R_2 < R_1 \le R_m$$

$$(7) \quad g = \frac{\theta^2}{[(H+\theta)][\phi+\theta]}, \text{ and}$$

$$(8) \quad \theta = \frac{-1.75 + \sqrt{3.0625 - 4.936\, C_0(C_0-1.565)\, e^{-3.44R/V}(R/R_0)^2}}{2.498\,(C_0-1.565)\,(R/R_0)^2}$$

The detection lobe in dimensionless form is described by θ vs. R/R_0, θ being the polar angle. The probability of detection in one glimpse is g. R_m and R_L are solutions of the transendental equations obtained by setting $\theta = .8^\circ$, 90° respectively in the equation of the detection lobe (which applies only between these values). These equations are respectively,

$$(9) \quad \left(\frac{R_m}{R_0}\right)^2 = \frac{C_0 e^{-(3.44R_0/V)R_m/R_0} - 1.565}{C_0 - 1.565} \quad \text{and}$$

$$(10) \quad \left(\frac{R_L}{R_0}\right)^2 = \frac{C_0 e^{-(3.44R_0/V)R_L/R_0} - 16.602}{112.4\,(C_0 - 1.565)}$$

The integral $I(R_m R)$ was computed for a set of values of the parameters $C_0, R_0/V$, H, and ϕ. For $R_1 < R_m$, it is a simple matter to obtain $I(R_1,R)$ by using the relation

(11) $I(R_1,R) = I(R_m,R) - I(R_m,R_1)$

The values of the integral I were obtained in the following way:

1) For each combination of the parameters R_o/V and C_o, the transcendental equations (9) and (10) were solved, using the Newton-Raphson method of successive approximations, to determine the values of R_m/R_o and R_L/R_o.

2) For each case, the range band $R_L/R_o \leq R/R_o < R_m/R_o$ was divided into ten equal intervals, the last three of these being subdivided in two.

3) For each set of 13 values of R/R_o obtained in this way, corresponding values of θ were computed, using equation (8).

4) For every combination of values of Θ and ϕ, using the values of θ computed above, g was computed, using equation (7).

5) For each value of g, $-\ln(1-g)$ was obtained.

6) Integration was done numerically, using Simpson's rule.

7) Multiplication by an appropriate constant was performed to obtain I.

(Steps 4,5,6, and 7 were done on IBM equipment)

8) Graphs were plotted on semi-log paper of $I + 10$ vs. R/R_o, each page containing one combination of the parameters C_o, Θ, and ϕ, and the complete range of R_o/V.

Note: $I + 10$ was chosen as the ordinate rather than I in order that the whole range of values could be included on the semi-log scale.

Contents:

(I) Figure A1, page 14: Nomograph connecting the parameters C_o, W, α, R_{oo}, and $R_{o\alpha}$. To be used in determining the value of R_o corresponding to given values of C_o, W, and α. Corresponding values of W, C_o, and R_{oo} are colinear; likewise, corresponding values of α, R_{oo}, and $R_{o\alpha}$.

(II) Figure A2, pages 15 to 89: Graphs to be used in determining the value of the integral I, used in equation (5) for computing the probability of visual detection of a single target by an interceptor on a collision course. These graphs are given for the following range of parameter values:

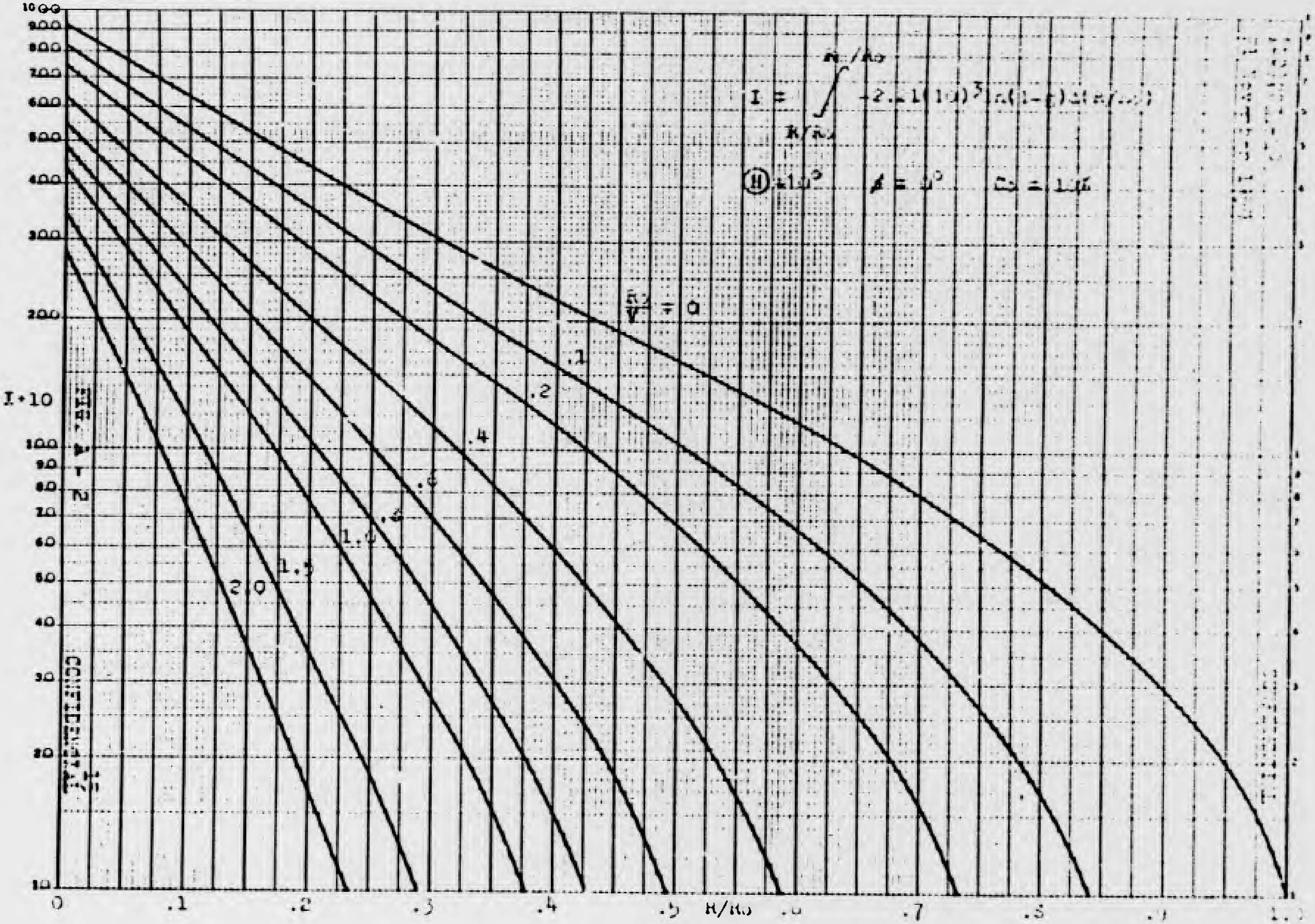
(i) Azimuth scan angles H: 10°, 20°, 30°, 45°, 60°

(ii) Elevation scan angles: 0°, 3°, 5°

(iii) Intrinsic contrast of target, C_o: 10,20,30, 50, 100%

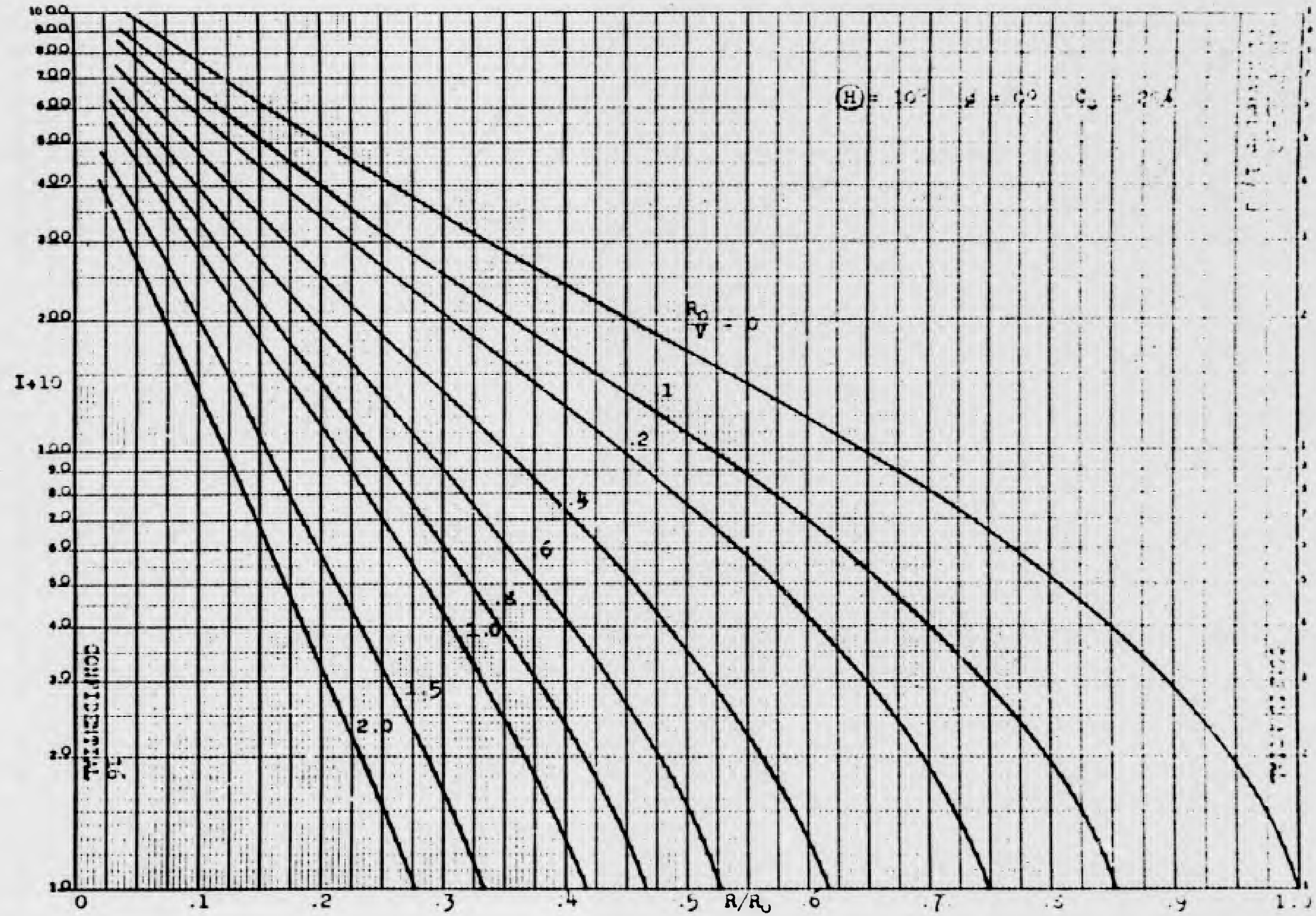
(iv) R_o/V: 0, .1,.2,.4,.6,.8,1.0,1.5,2.0

The limits of integration are from the variable lower limit R/R_o to the fixed upper limit R_m/R_o. The ordinate plotted is $I + 10$ in order to include the complete range of I on the logarithmic scale.



I*10
R/Ro
$\frac{R^2}{V} = 0$
.1
.2
.4
1.0
1.5
2.0
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.5
.7

FIG. A - 2


I+10
R/R0
Ro/V = 0
.1
.2
.5
.6
.8
1.0
1.5
2.0
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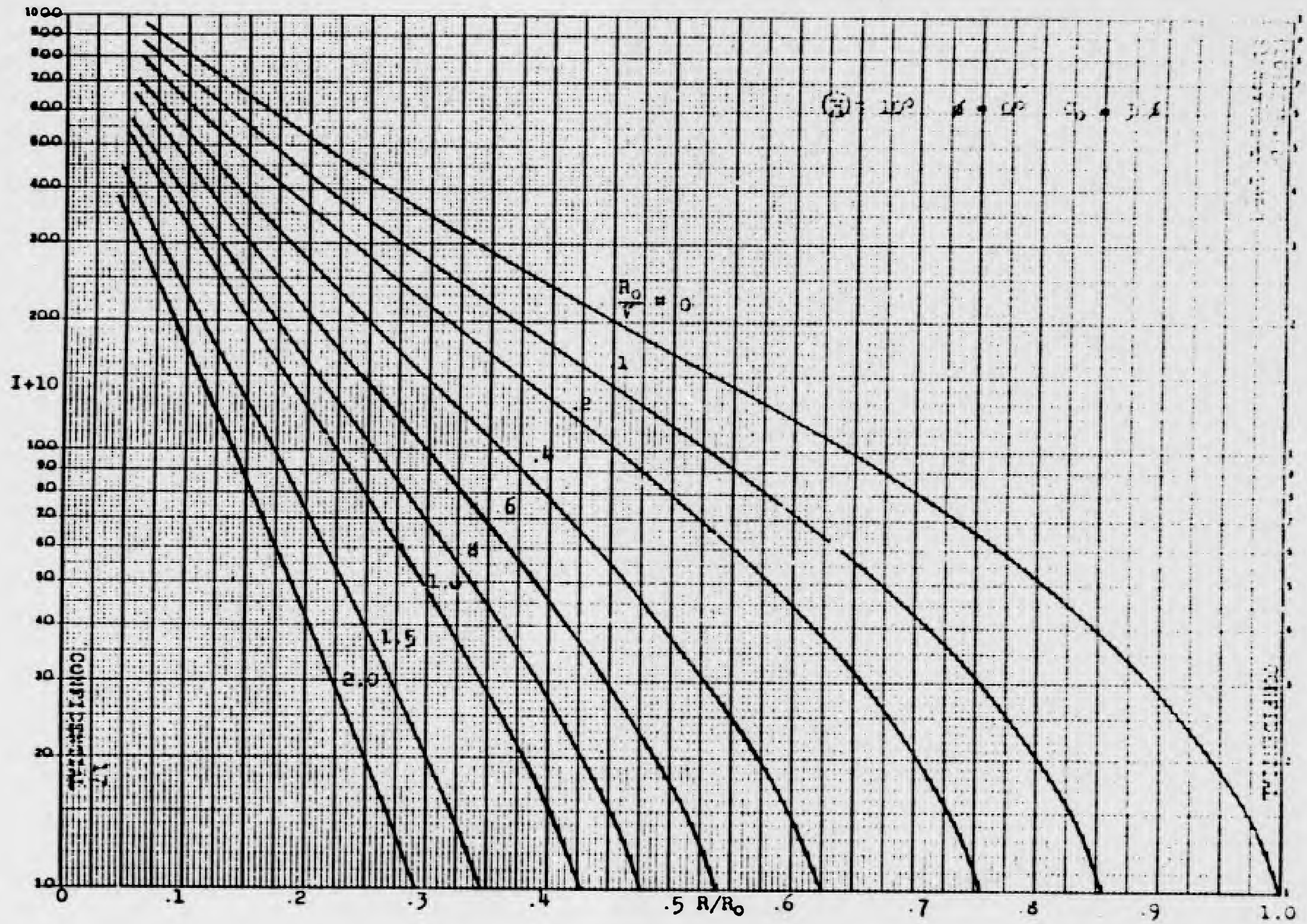
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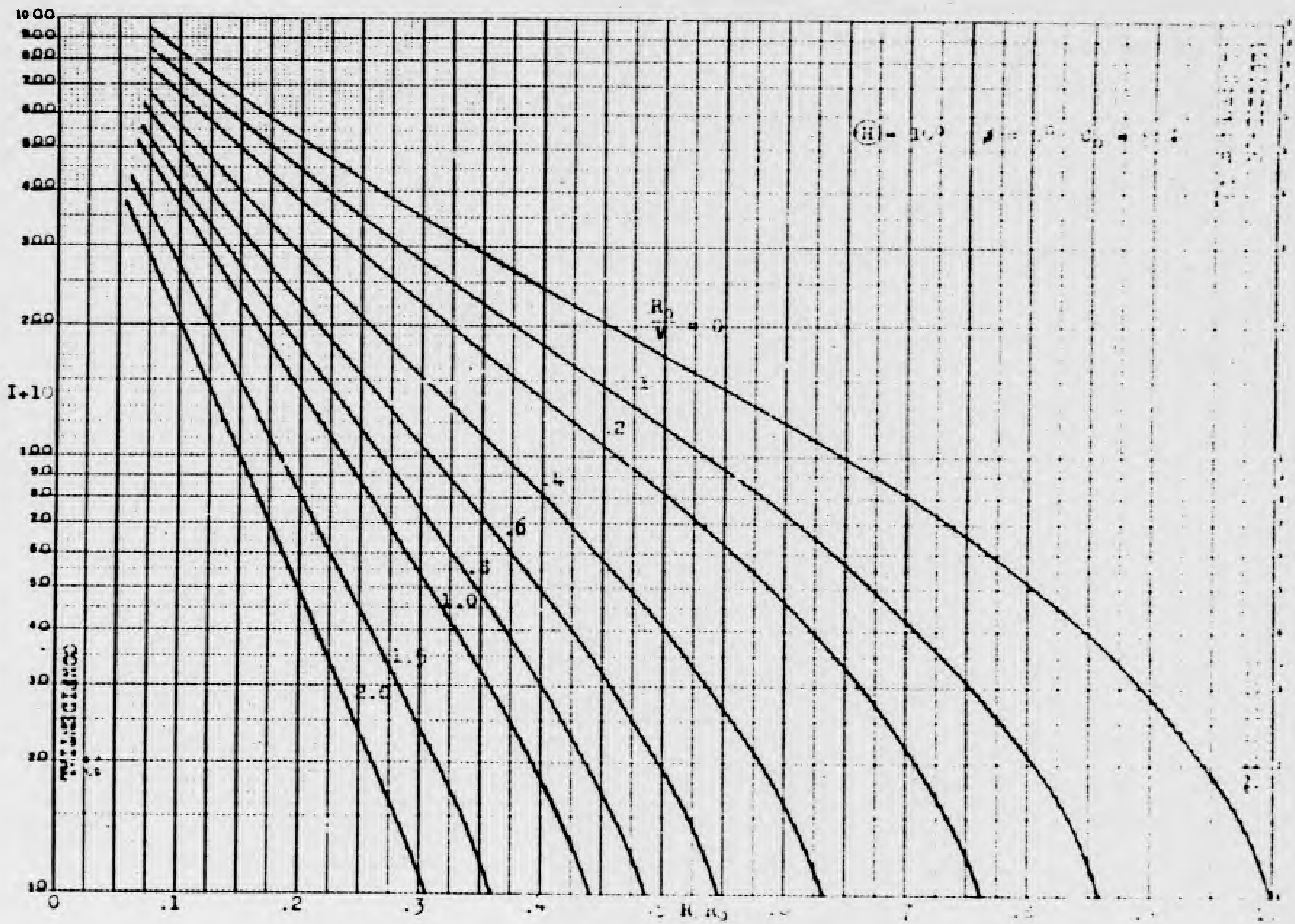
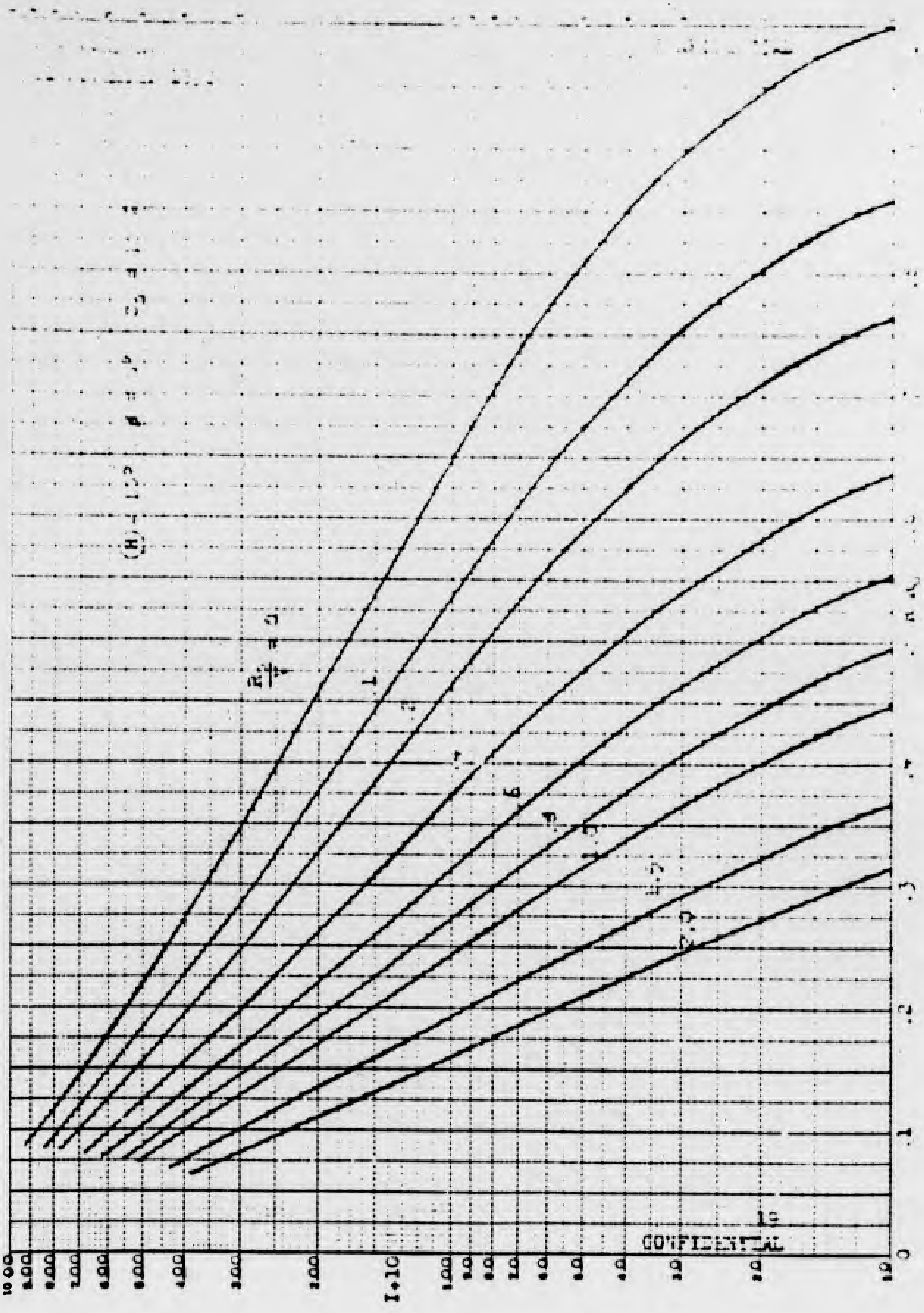
I+10
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.5 R/Ro
.6
.7
.8
.9
1.0
Ro/V = 0
1
.2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL



I+10
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.2
.4
.6
1.0
2.0
CONFIDENTIAL


I+10
0
.1
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.3
.4
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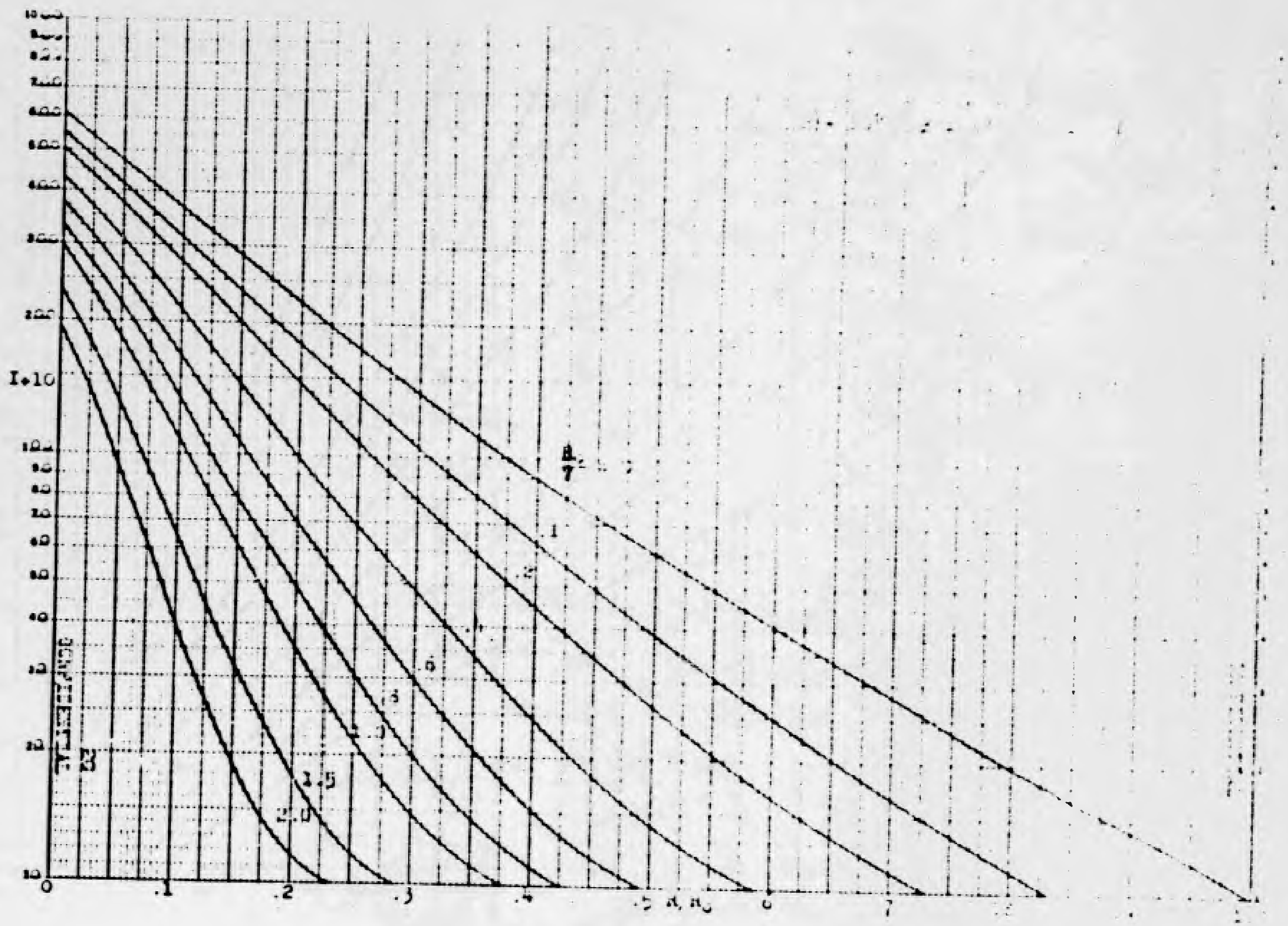
2.00
3.00
4.00
10.00
1
.2
.6
1.5
2.0



I+10
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23
1
1.5
2.0
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.1
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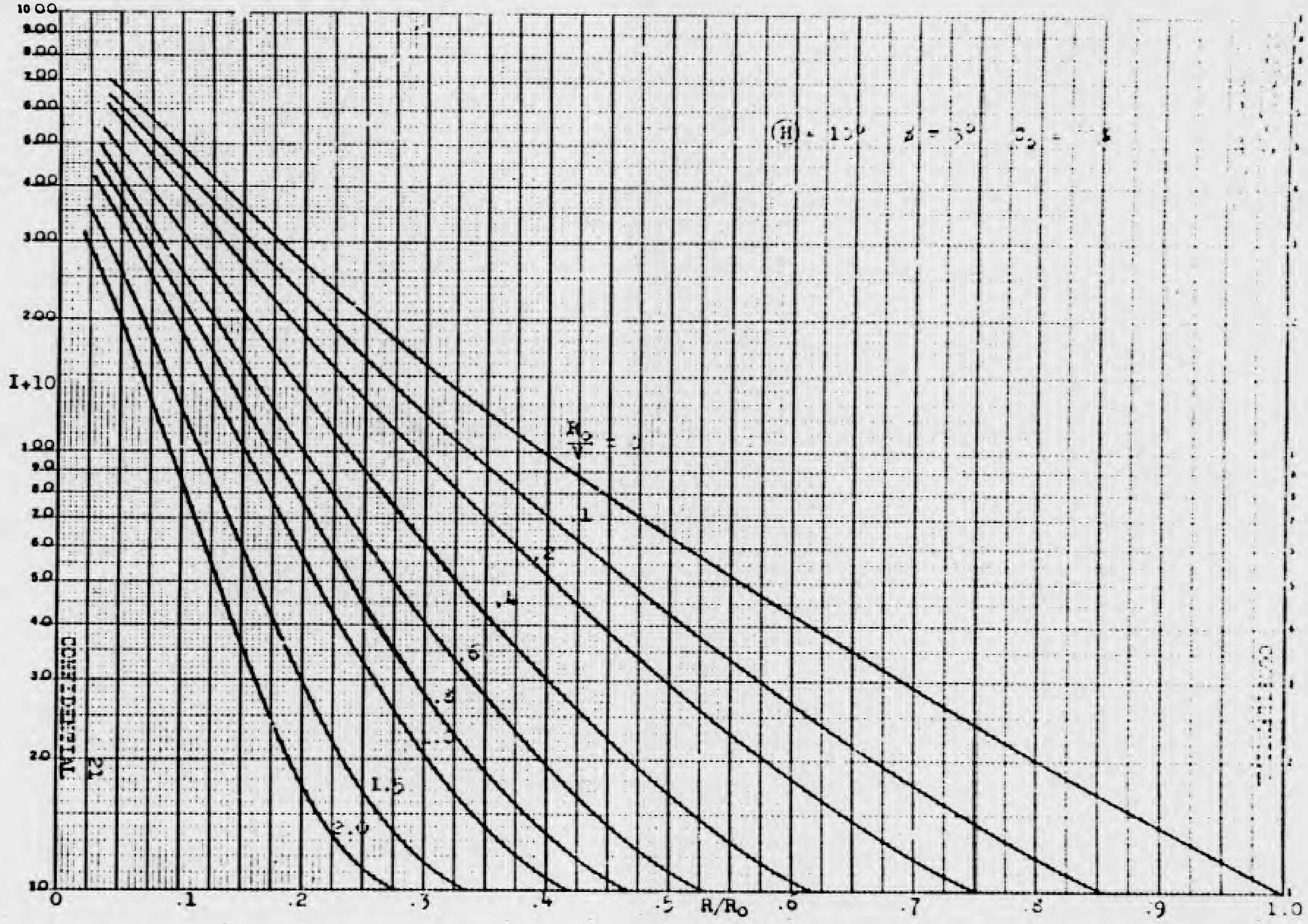

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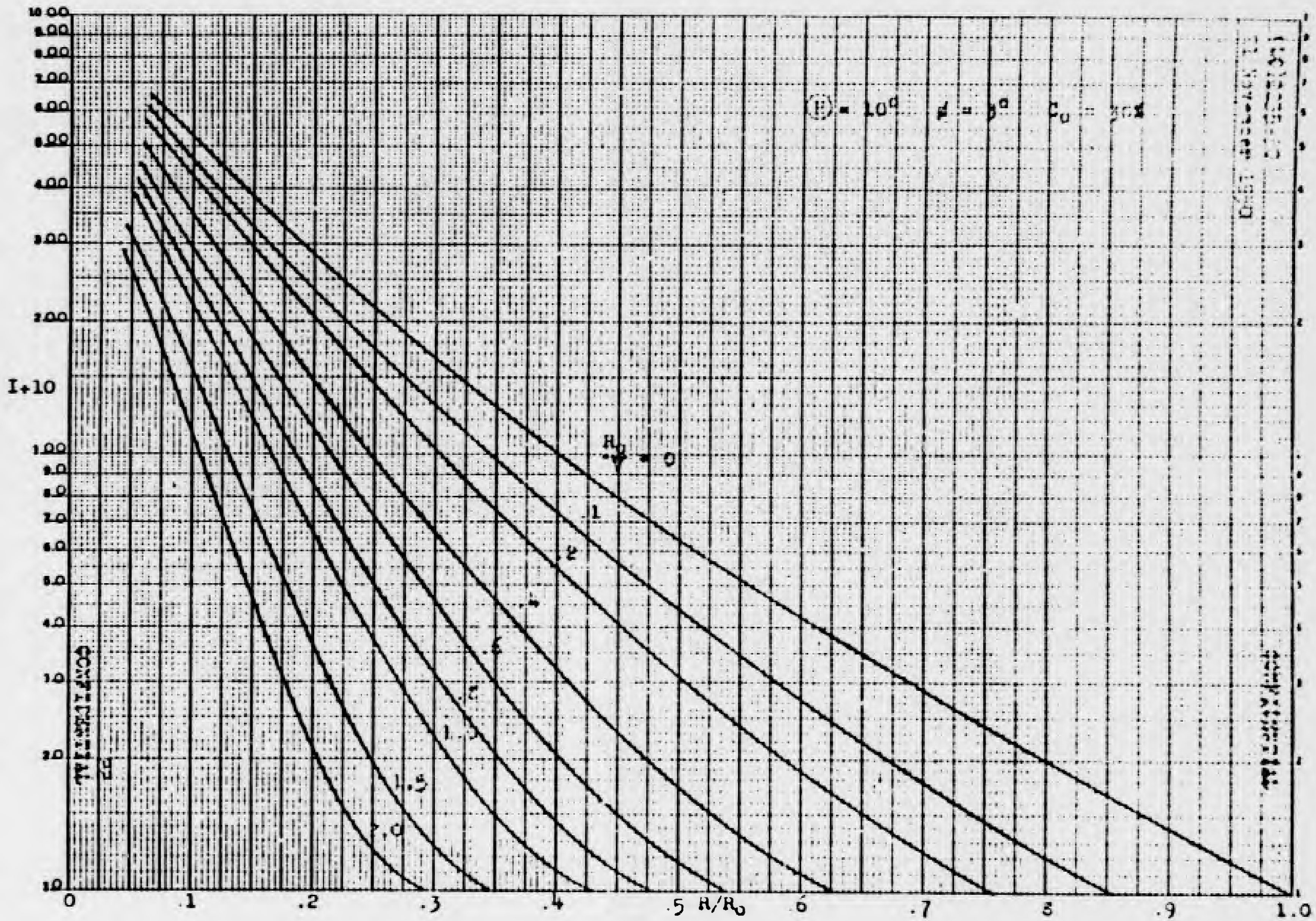
21
(H) = 15°
I+10
R/R₀
1.5
2.0


I+10
R/R0
1.0
.9
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.3
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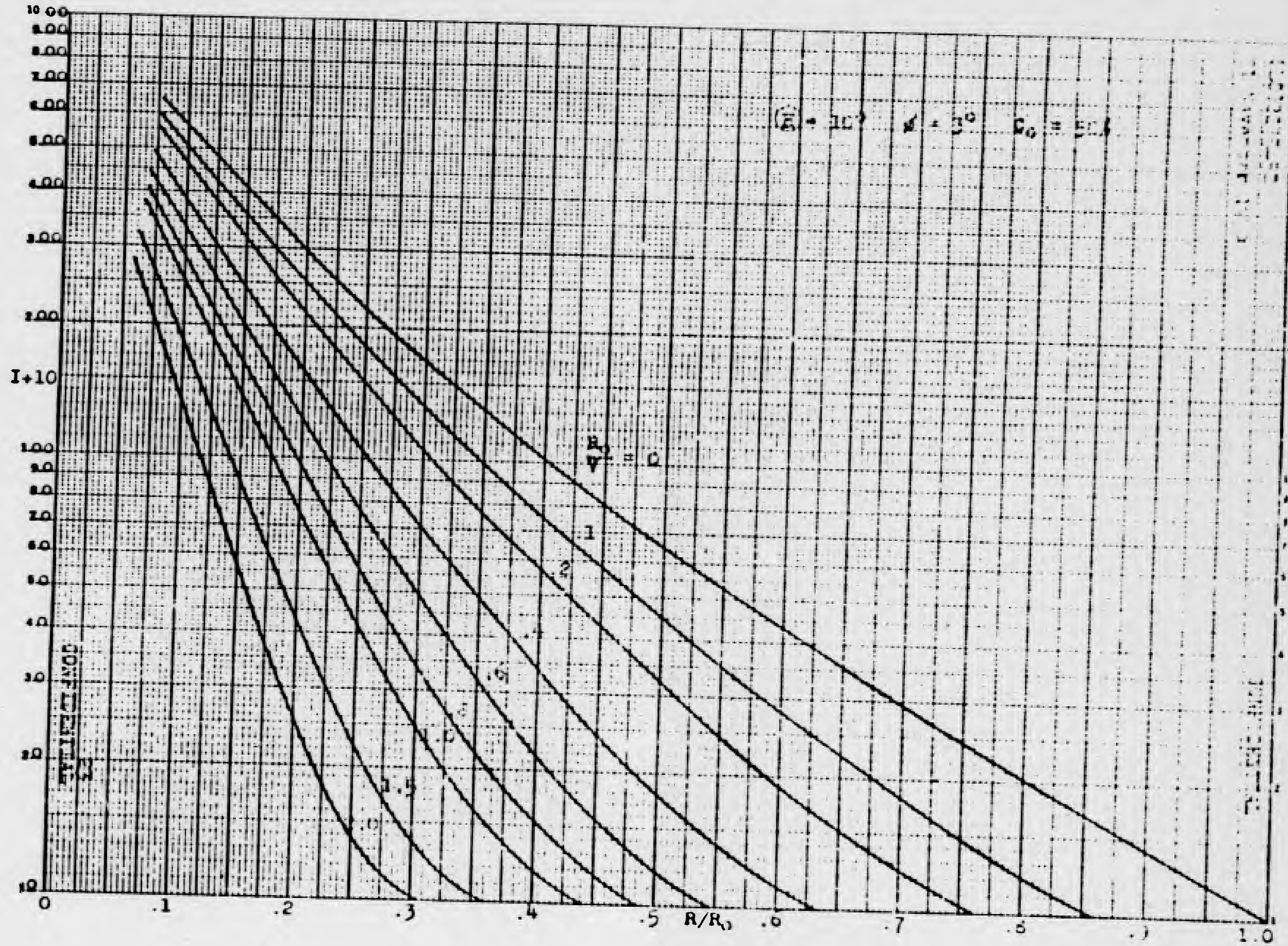
0
1.5
2.0


I+10
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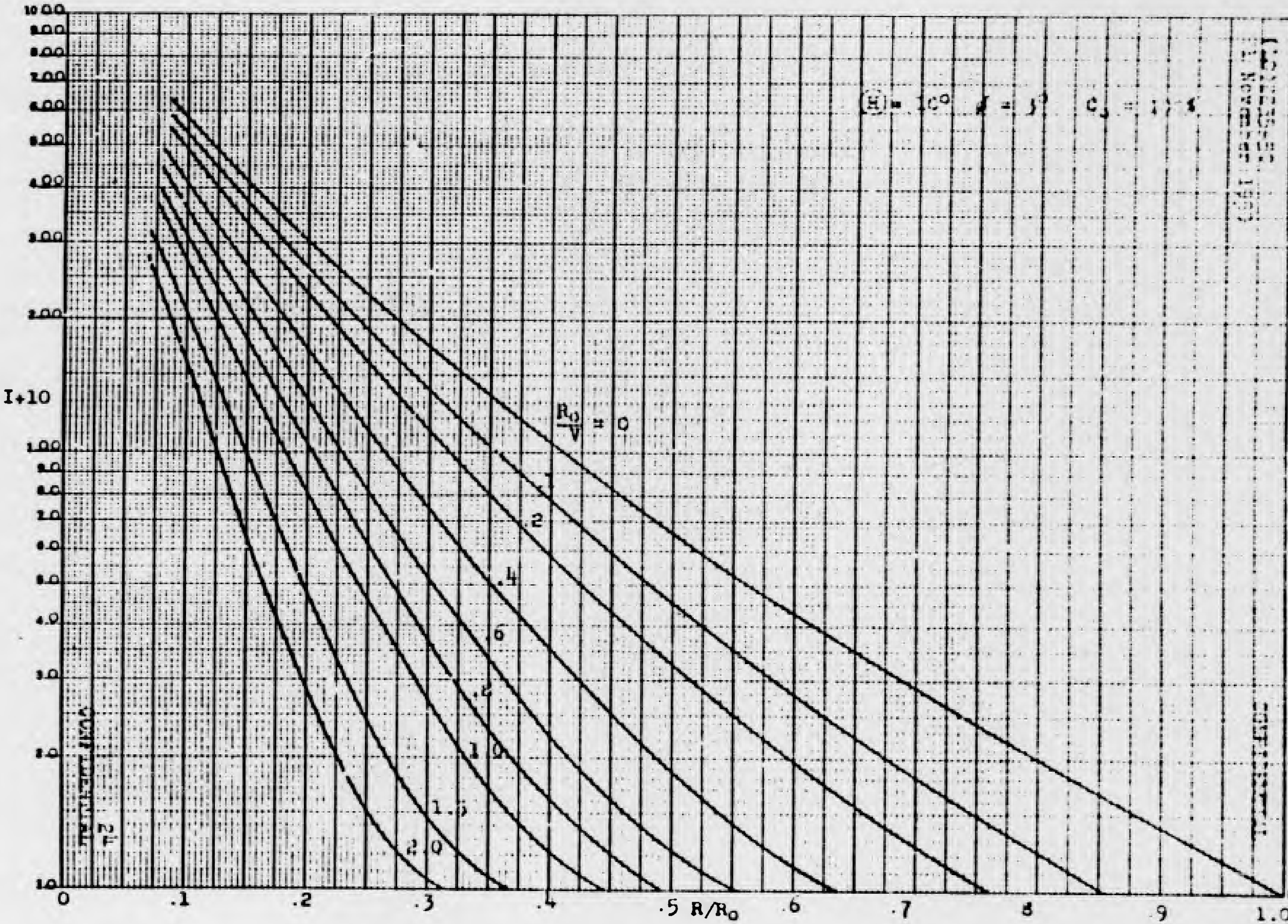
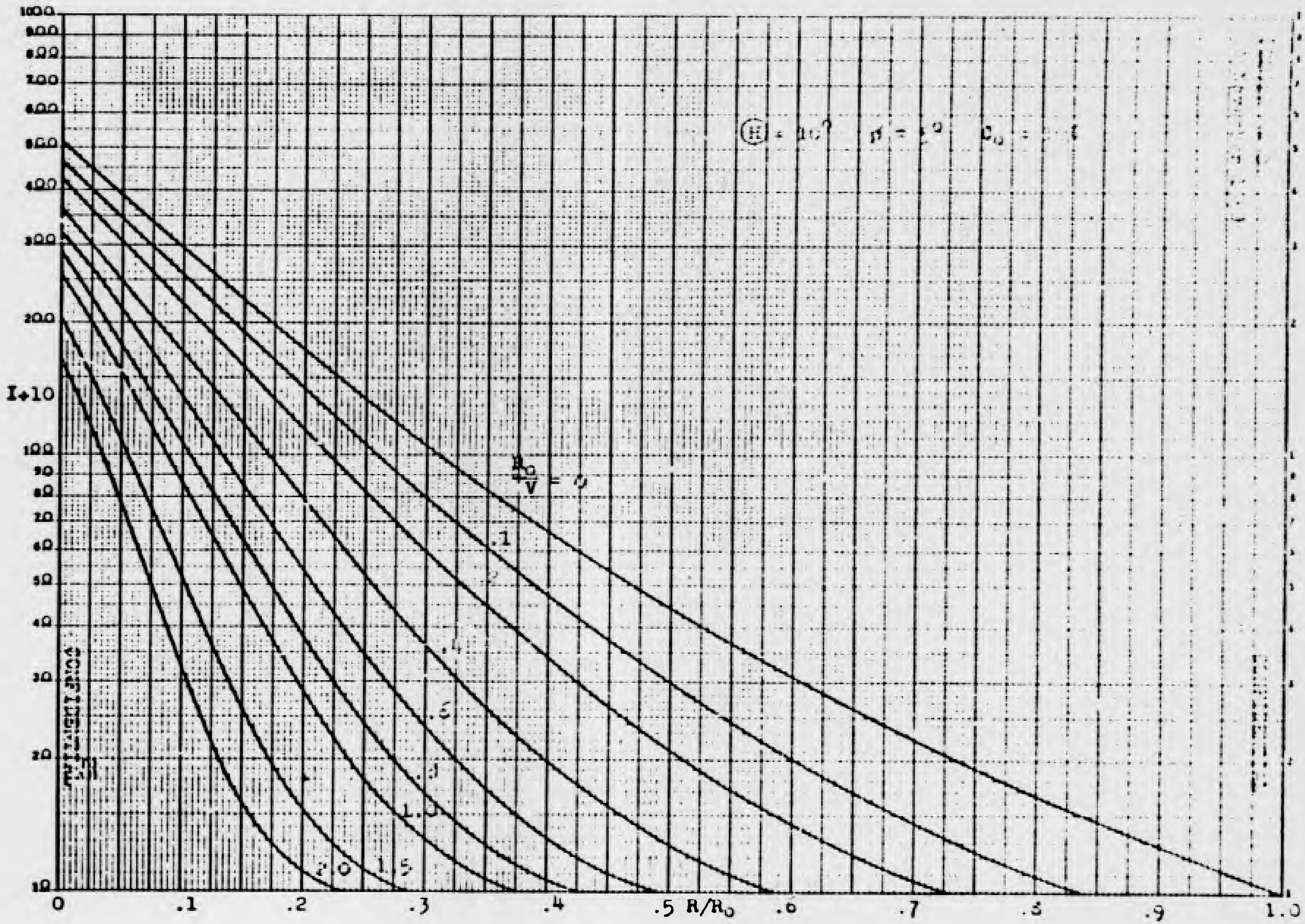
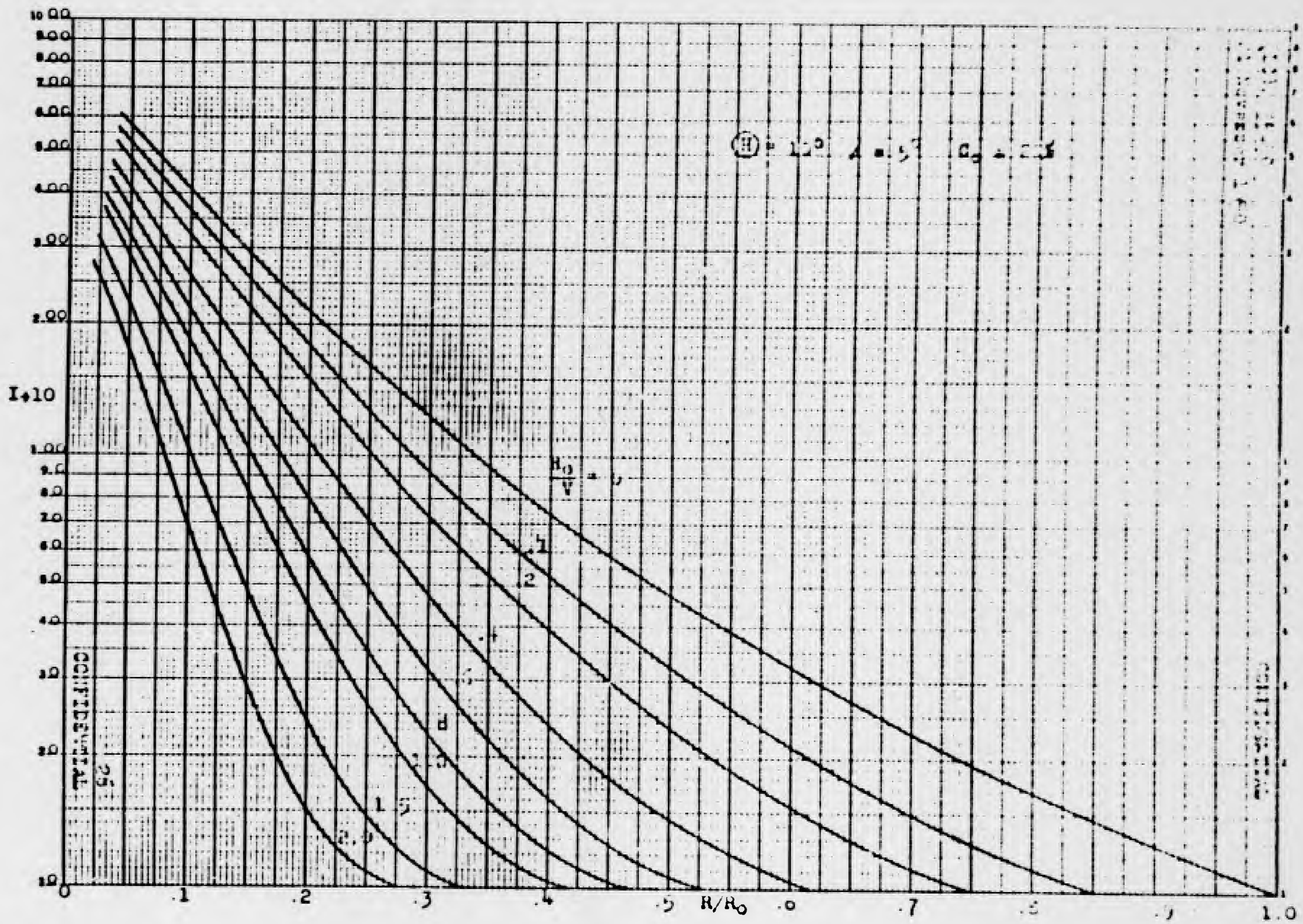
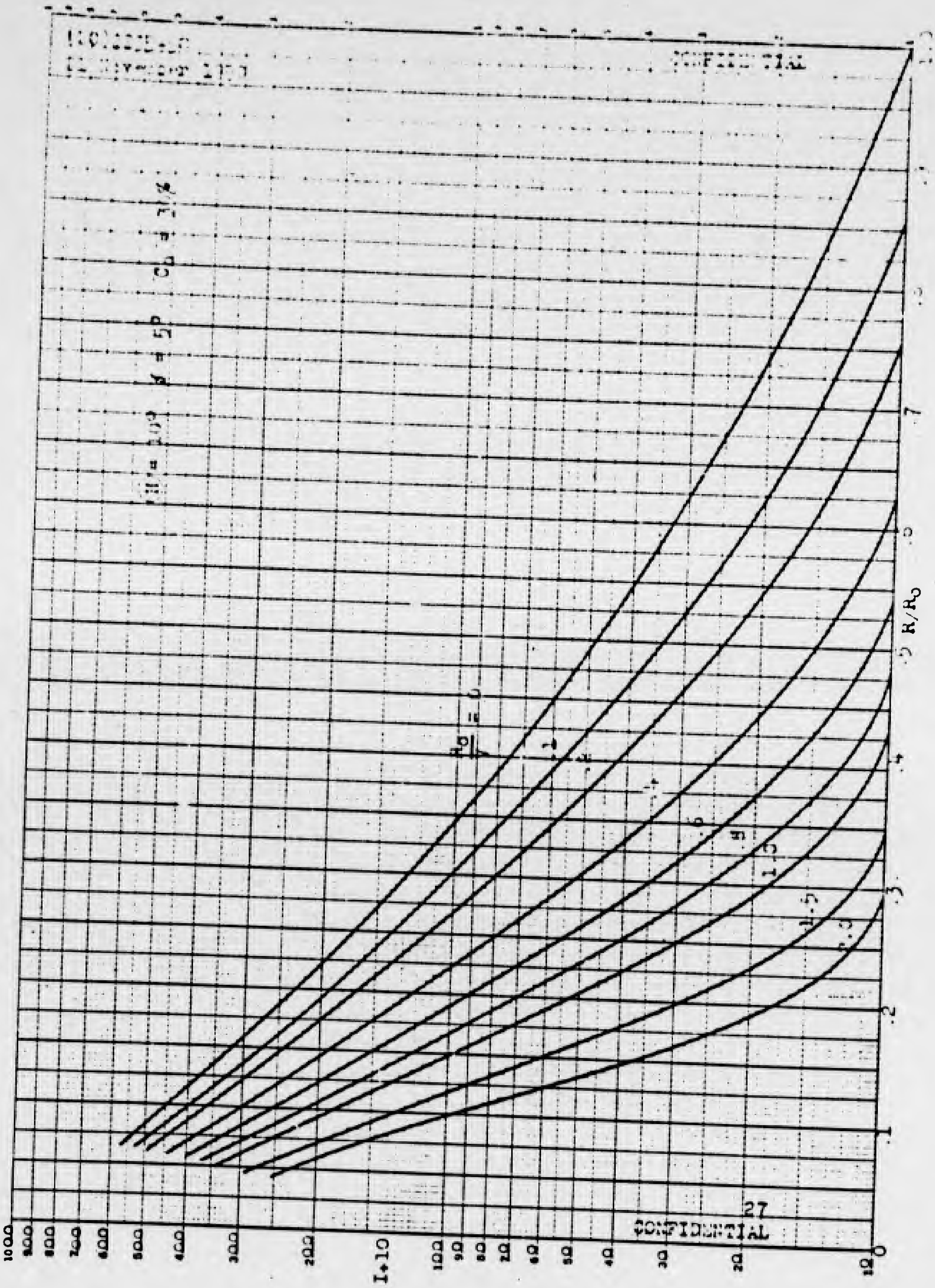


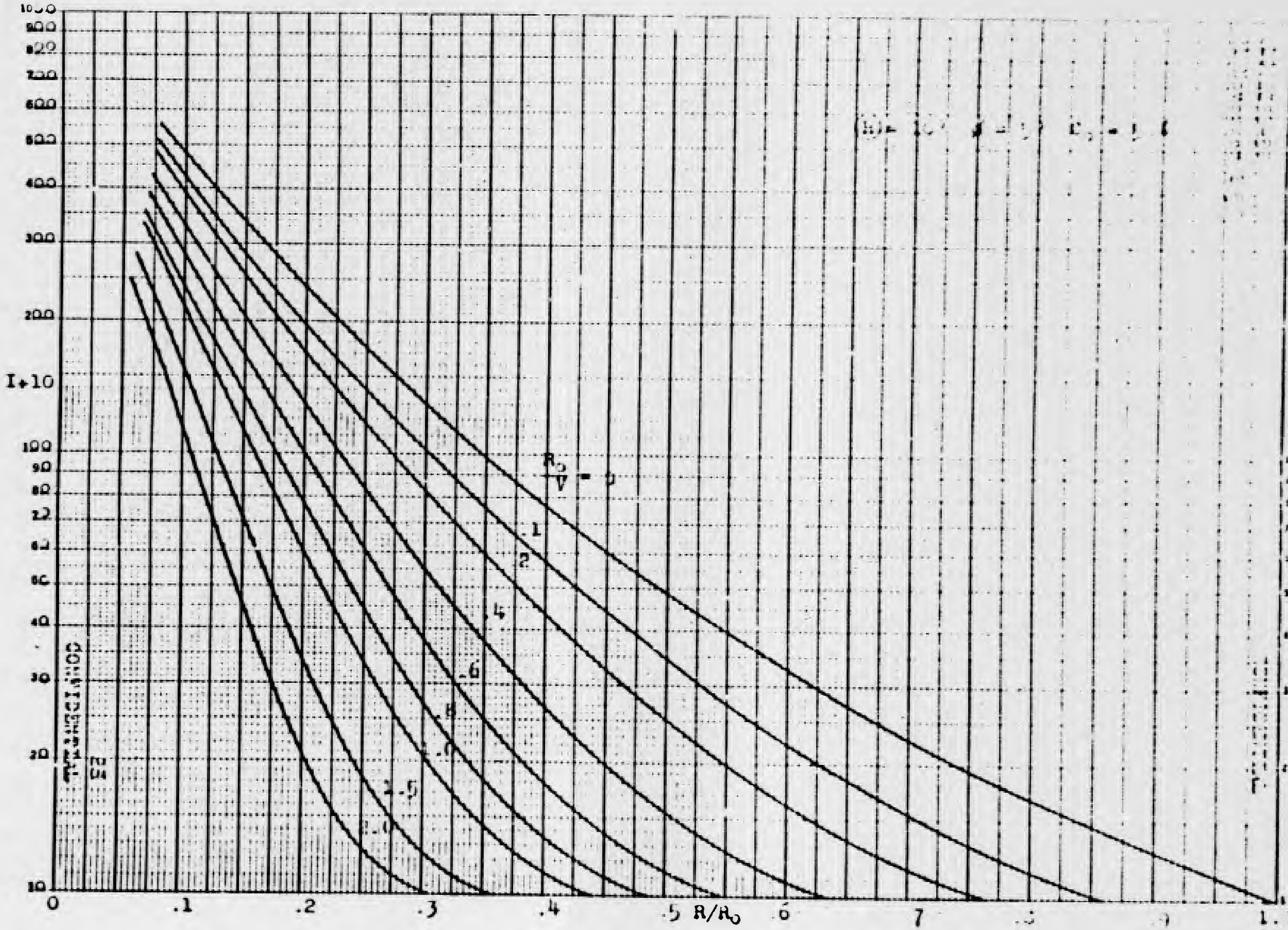
I+10
R/R₀
$\frac{R_0}{V} = 0$
.2
.4
.6
.8
1.0
1.5
2.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0


I+10
R/R0
0
.1
.2
.3
.4
.5
.6
.7
.9
1.0
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
CONFIDENTIAL

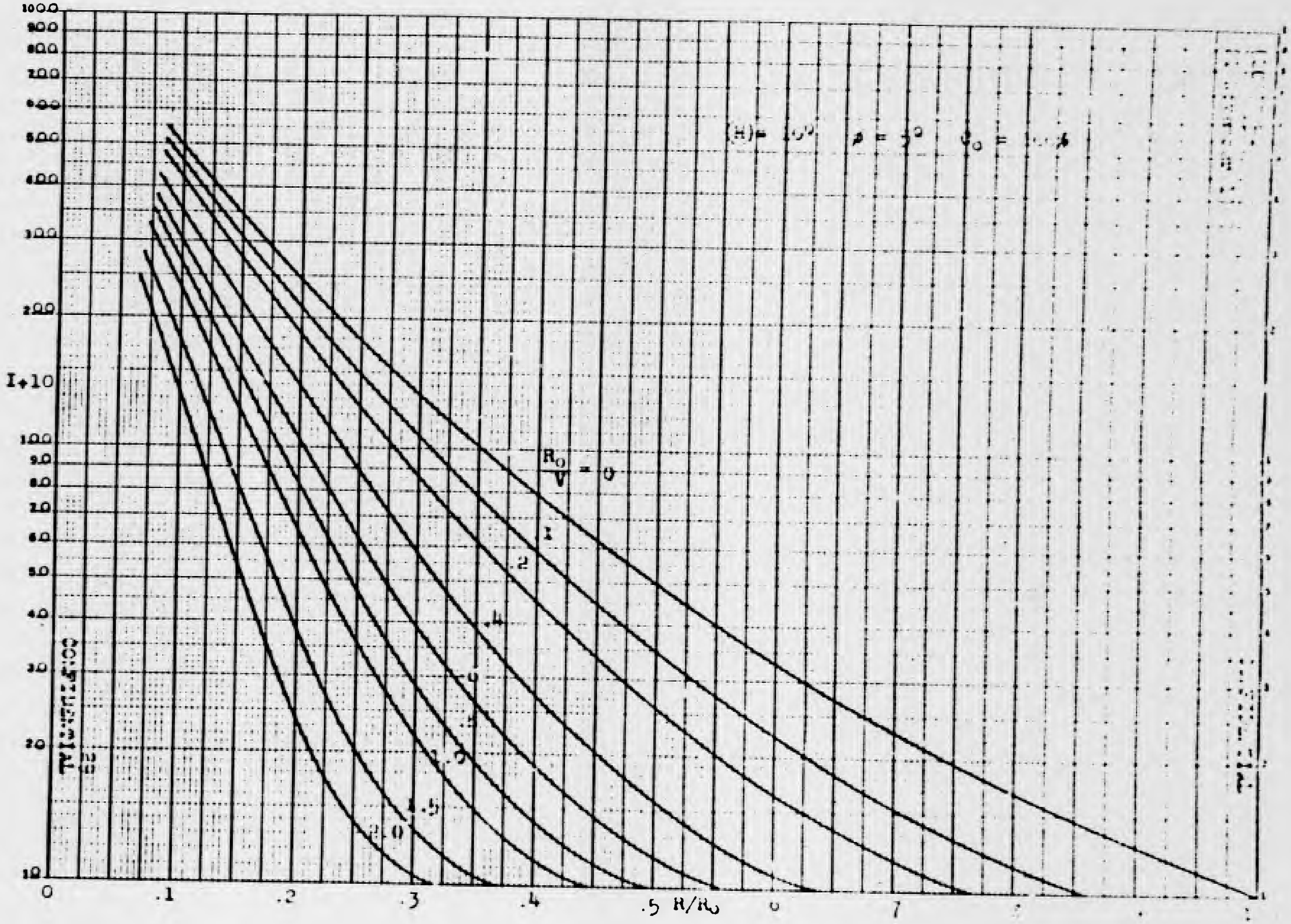

CONFIDENTIAL
25
I+10
R/Ro
0
.1
.2
.3
.4
.5
.7
1.0
.1
.2
1.5
CONFIDENTIAL


I+10
R/R₀
Hₐ/γ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
C_L = 30%

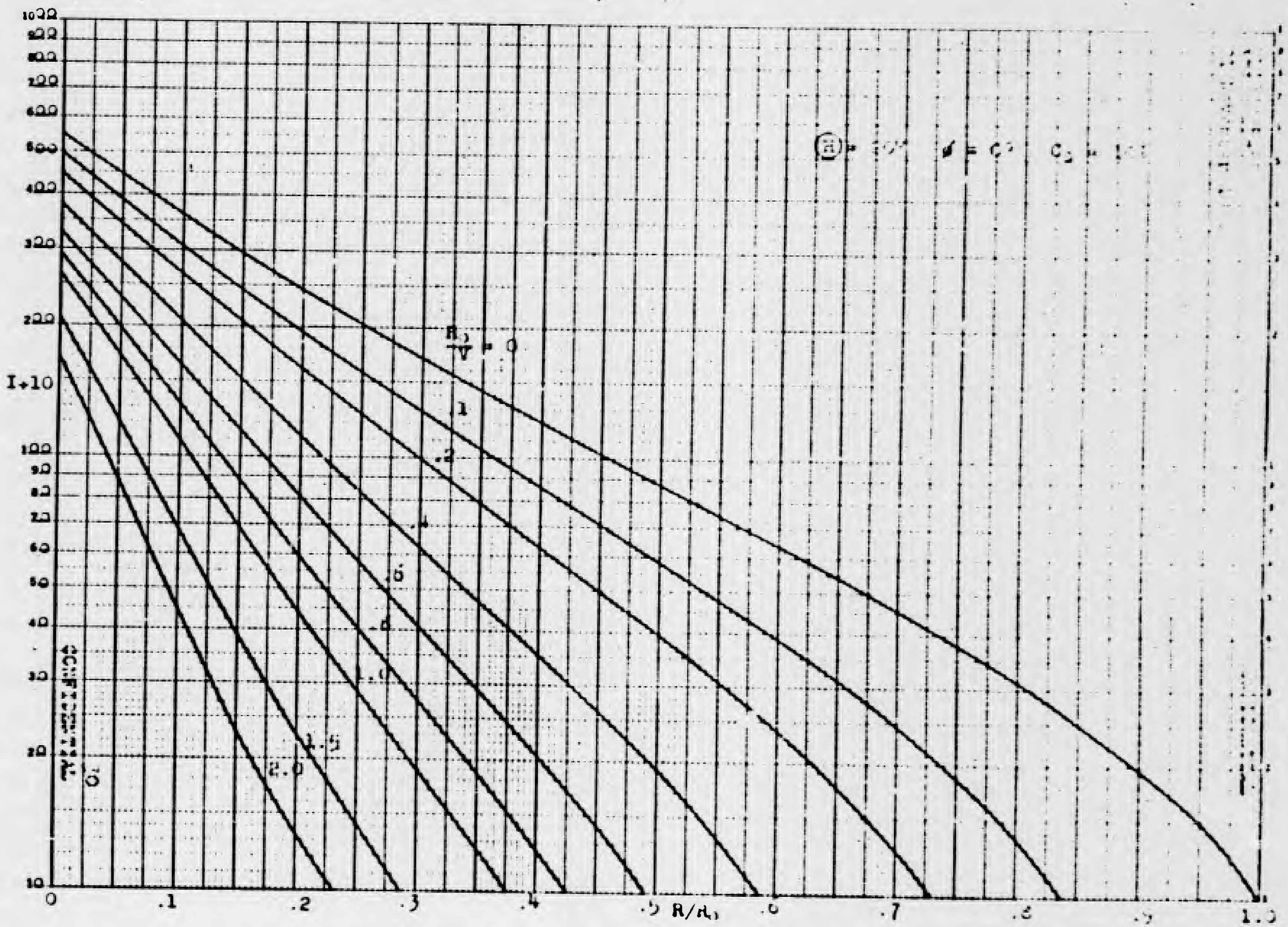
CONFIDENTIAL

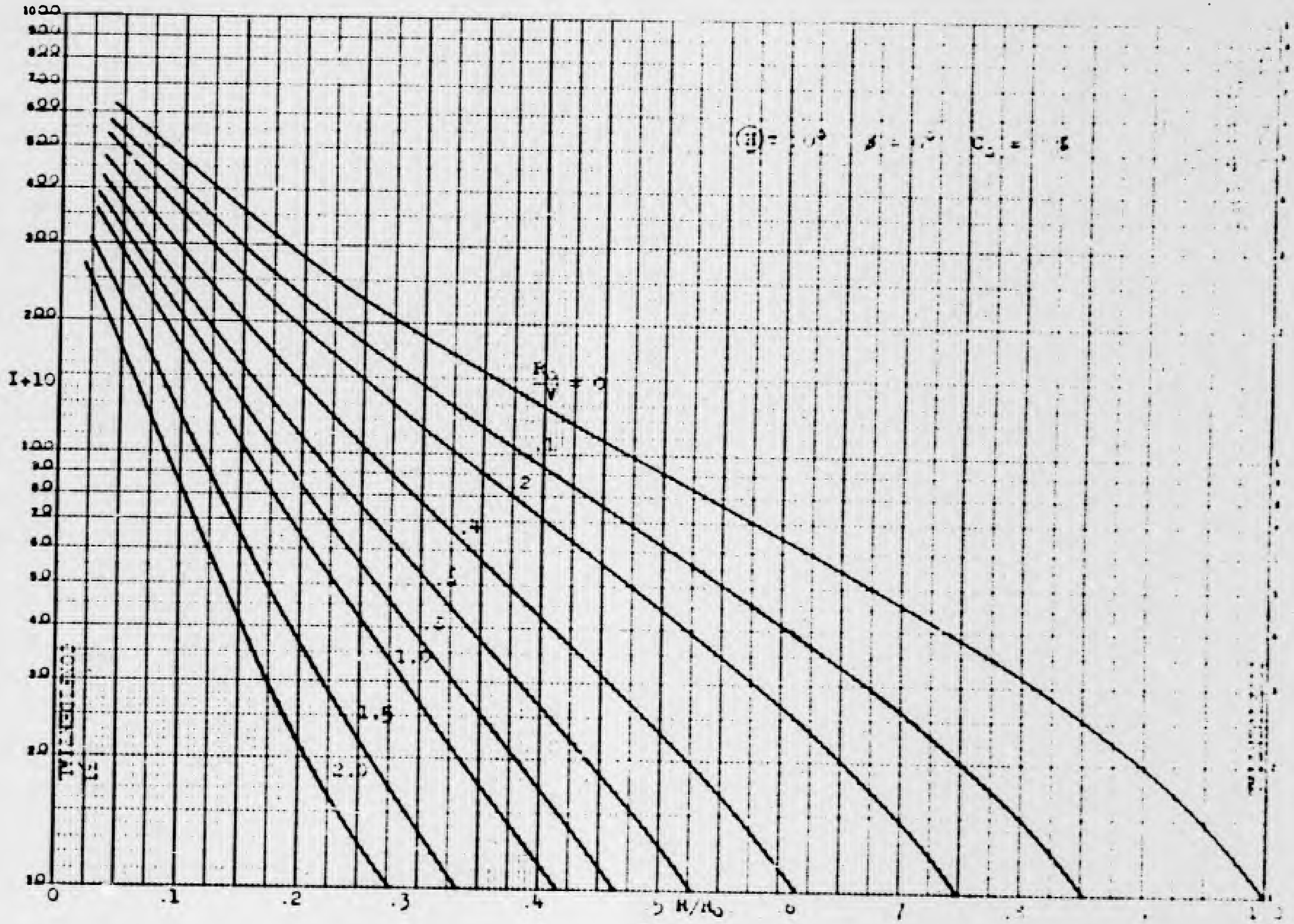


CONFIDENTIAL

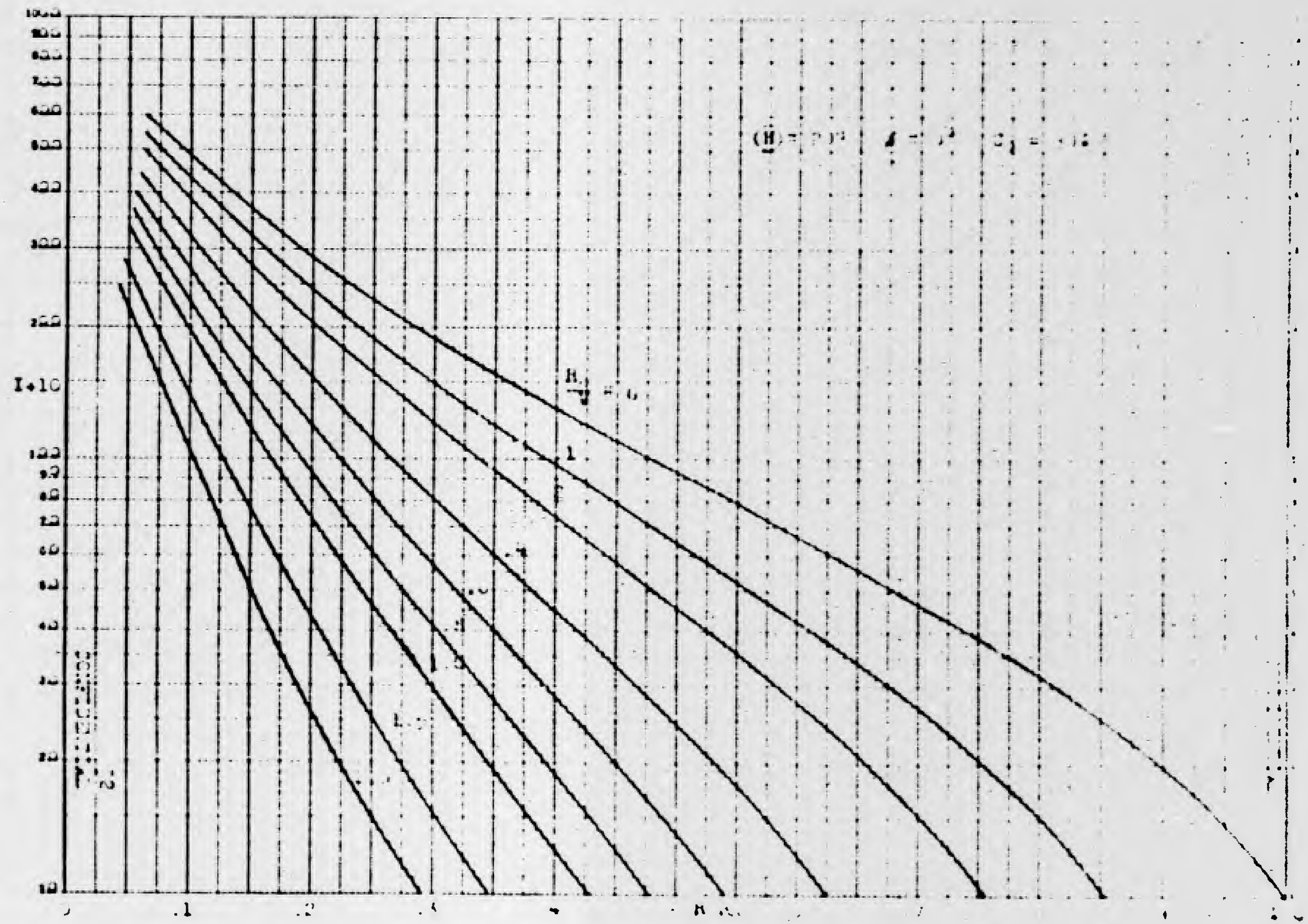


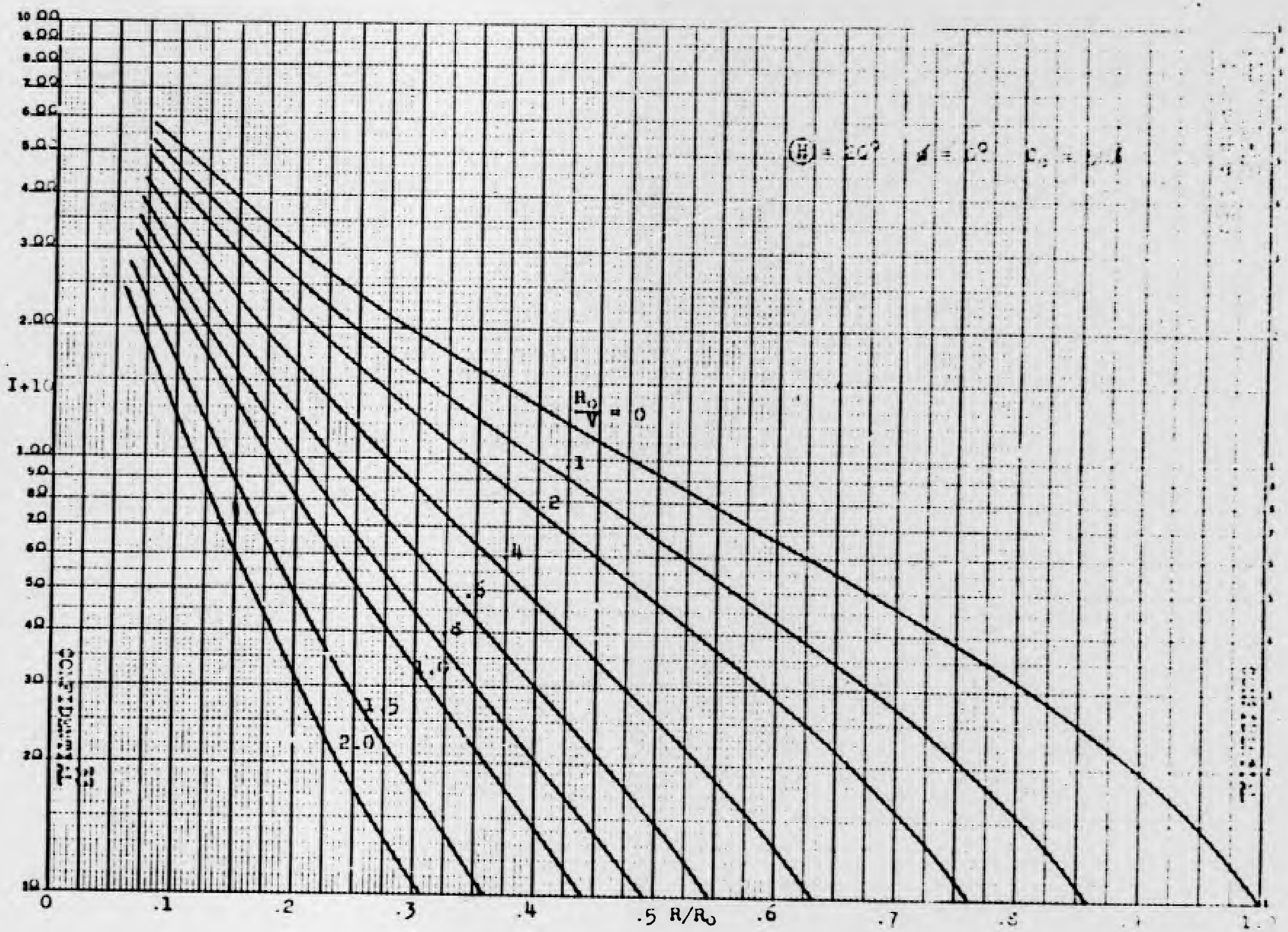
I+10
R/R₀
$\frac{R_0}{V} = 0$
.1
.2
.4
1.5
2.0
CONFIDENTIAL

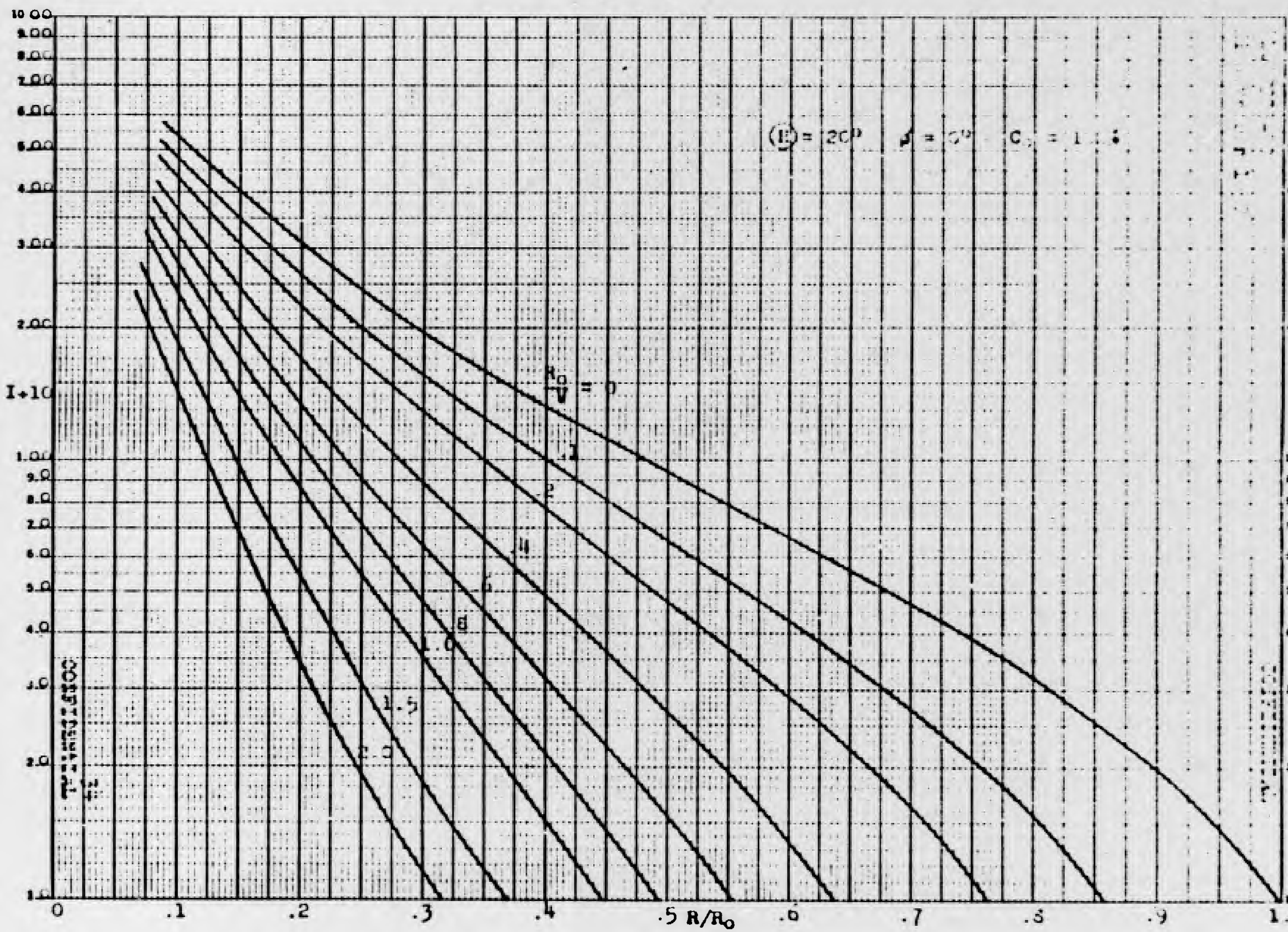

I+10
R/R₁
CONFIDENTIAL
0
.1
.2
.3
.4
.6
.7
1.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
.1
.2
.4
.6
.8
1.0
1.5
2.0

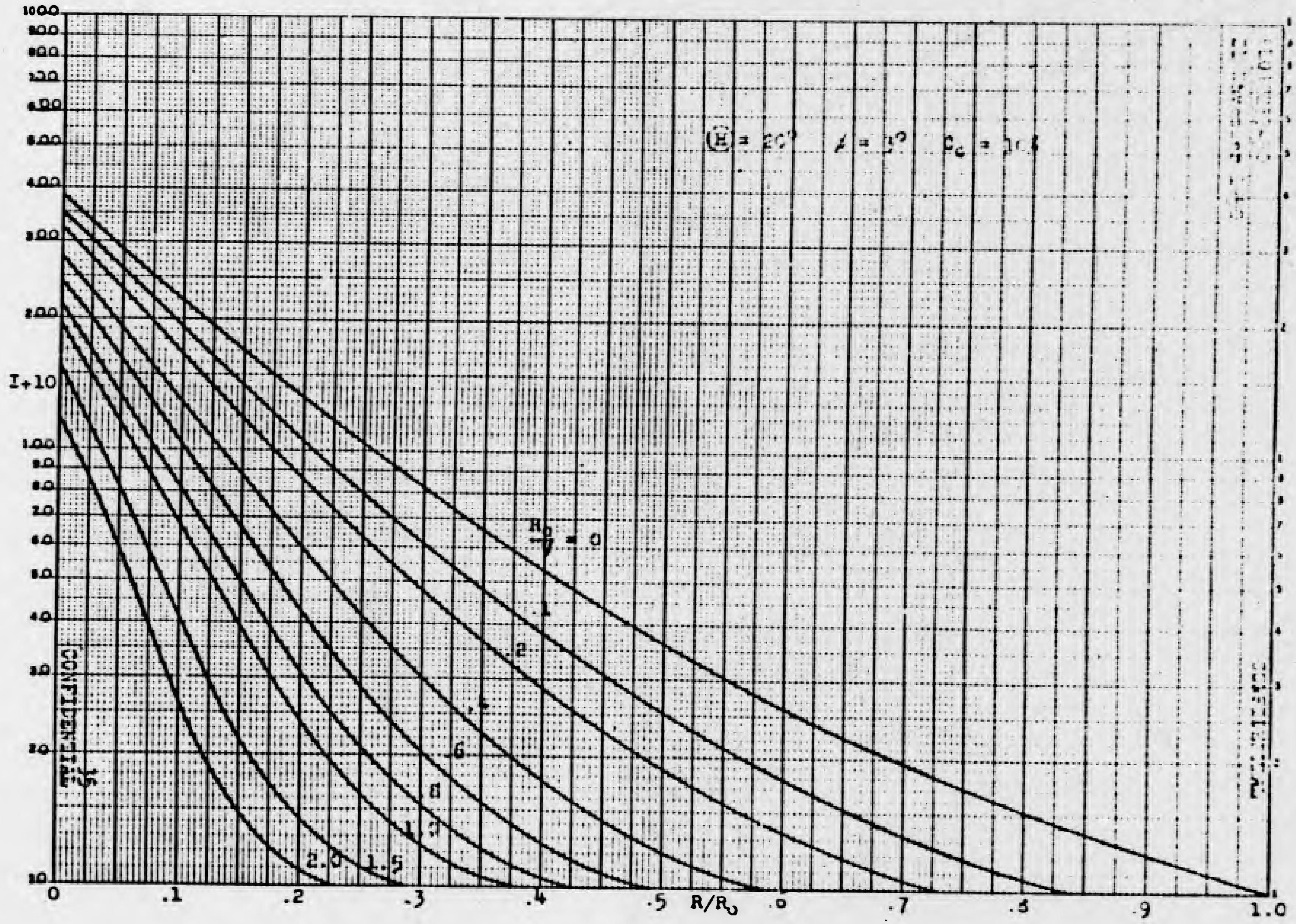


I+10
R/R₀
.1
.2
.3
.4
2
1.5
2.0
CONFIDENTIAL

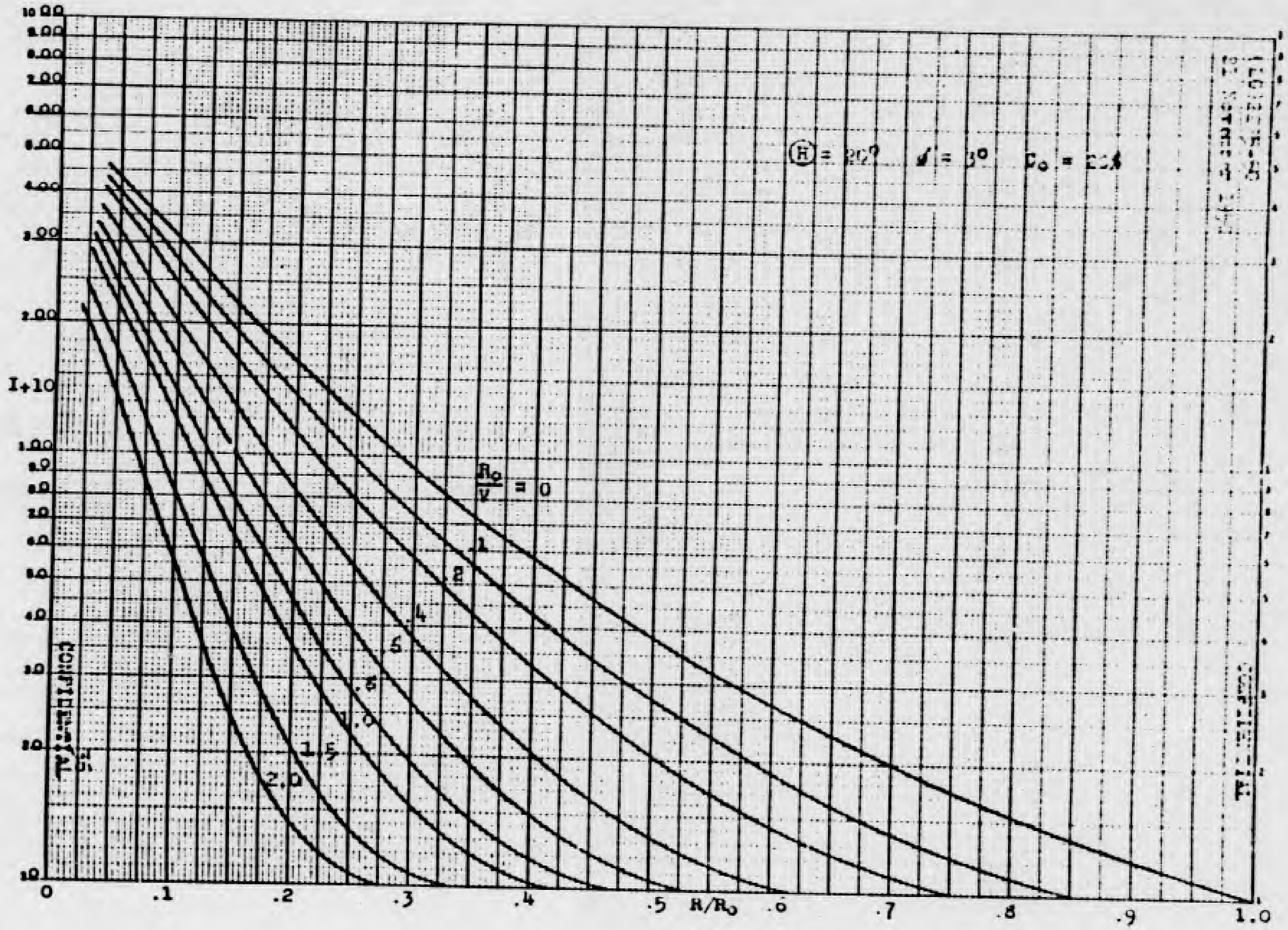

I+10
.1
.2


I+10
R/R₀
$\frac{R_0}{V}$ = 0
1
2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL

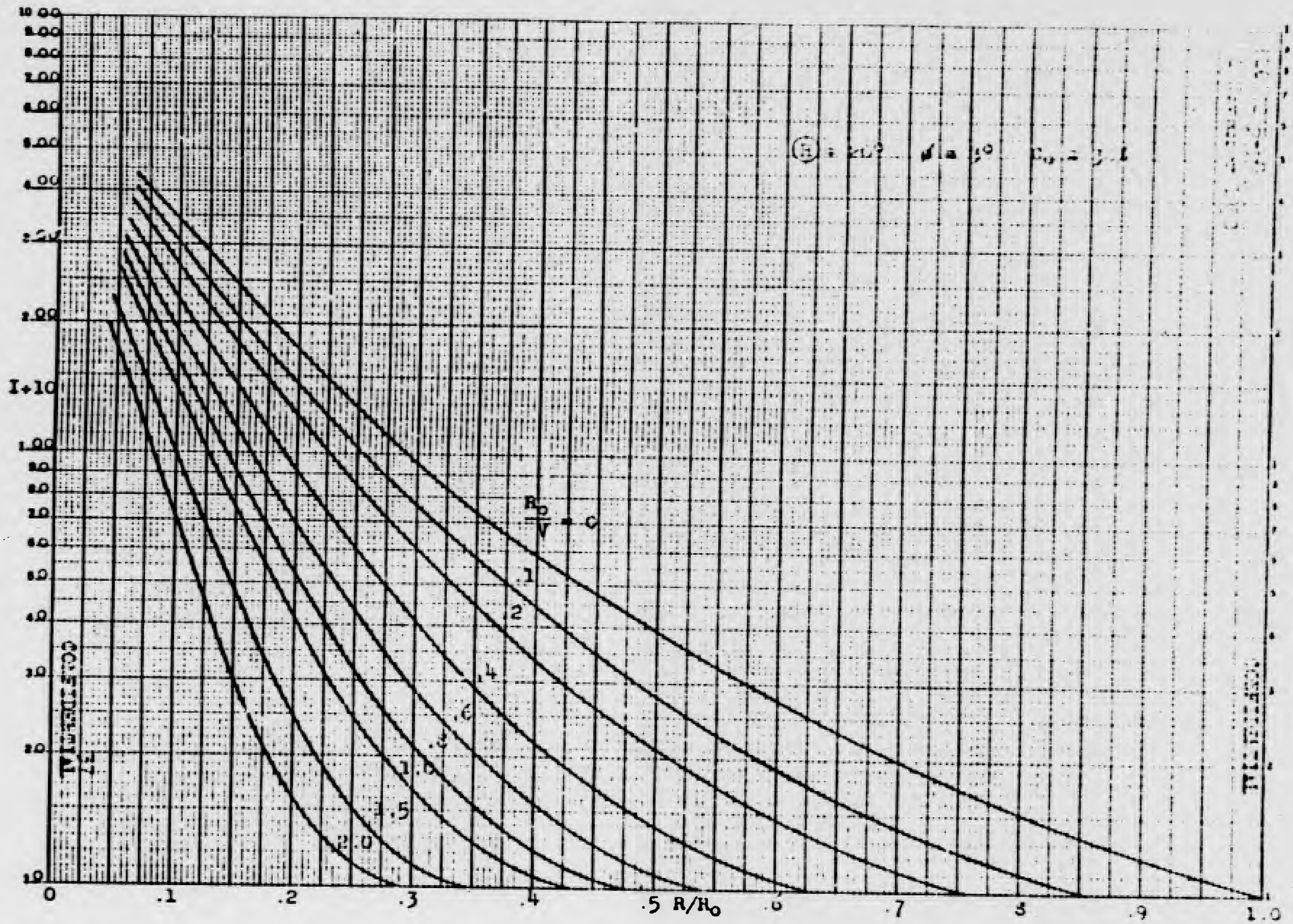

CONFIDENTIAL
I+10
R/Ro
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
2.0
1.5
1.0

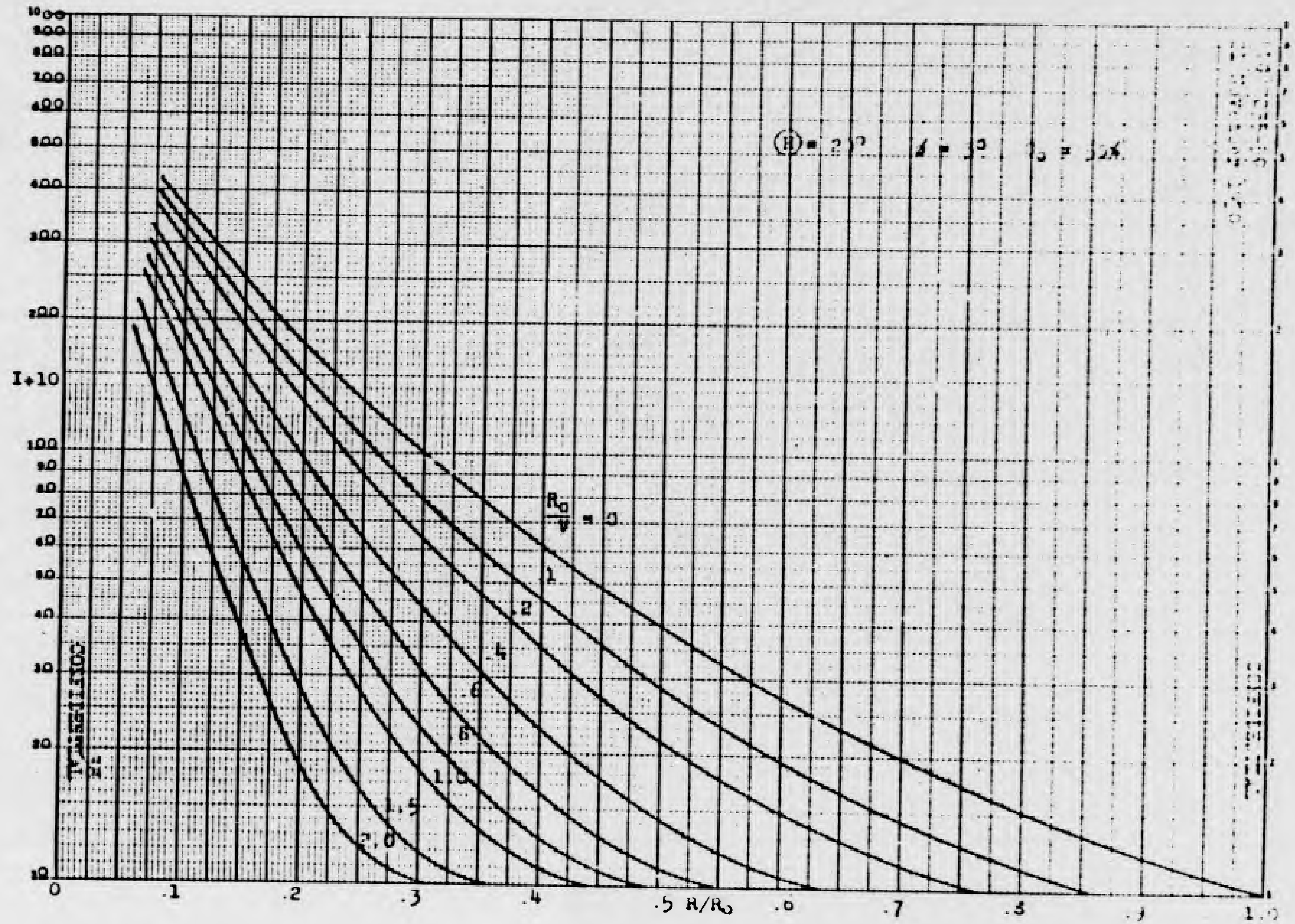


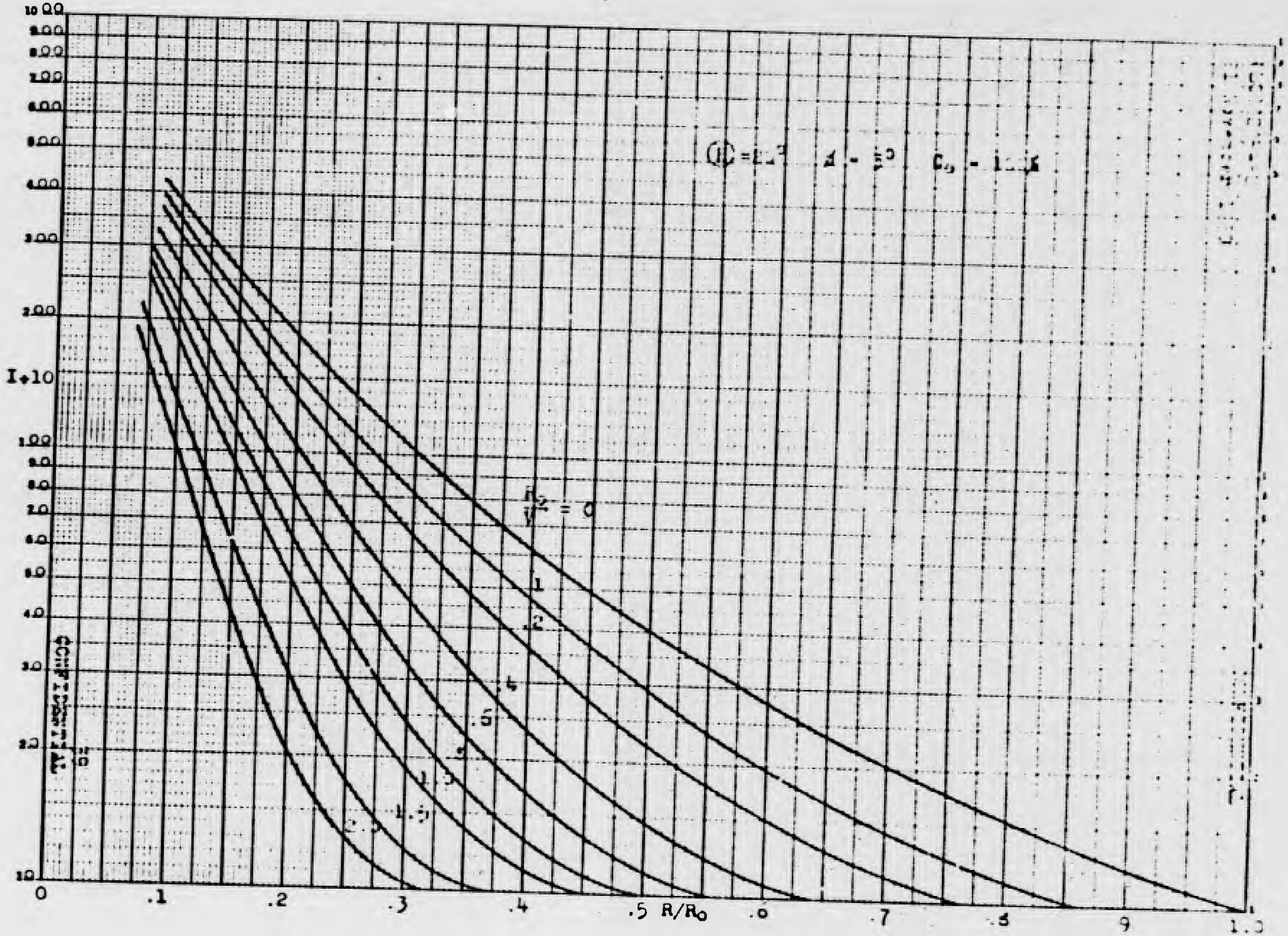
1000
900
800
700
600
500
400
300
200
I+10
100
90
80
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.5
R/R₀
.7
.9
1.0
$\frac{R_0}{T} = 0$
.1
.2
.6
.8
1.5
2.0
CONFIDENTIAL

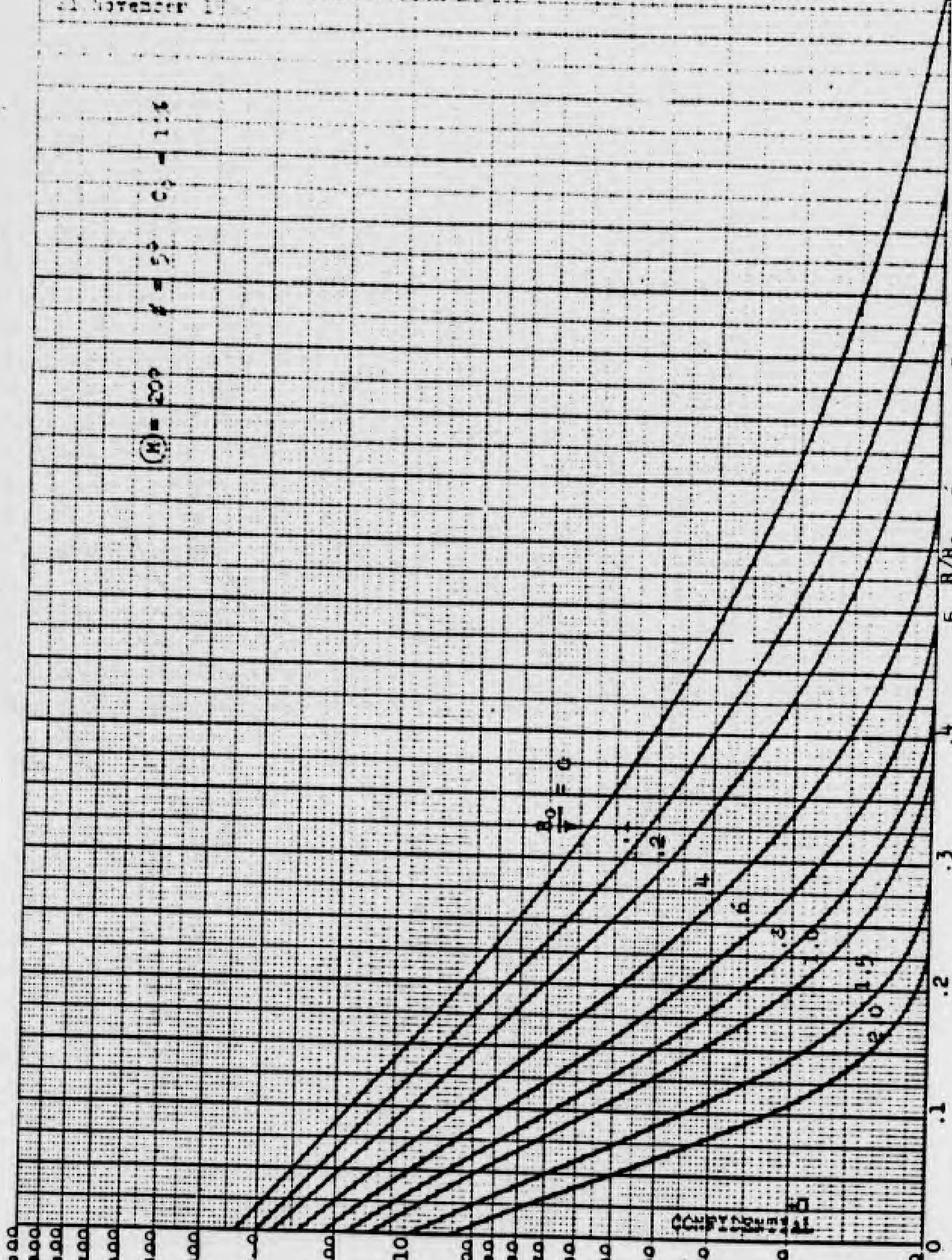


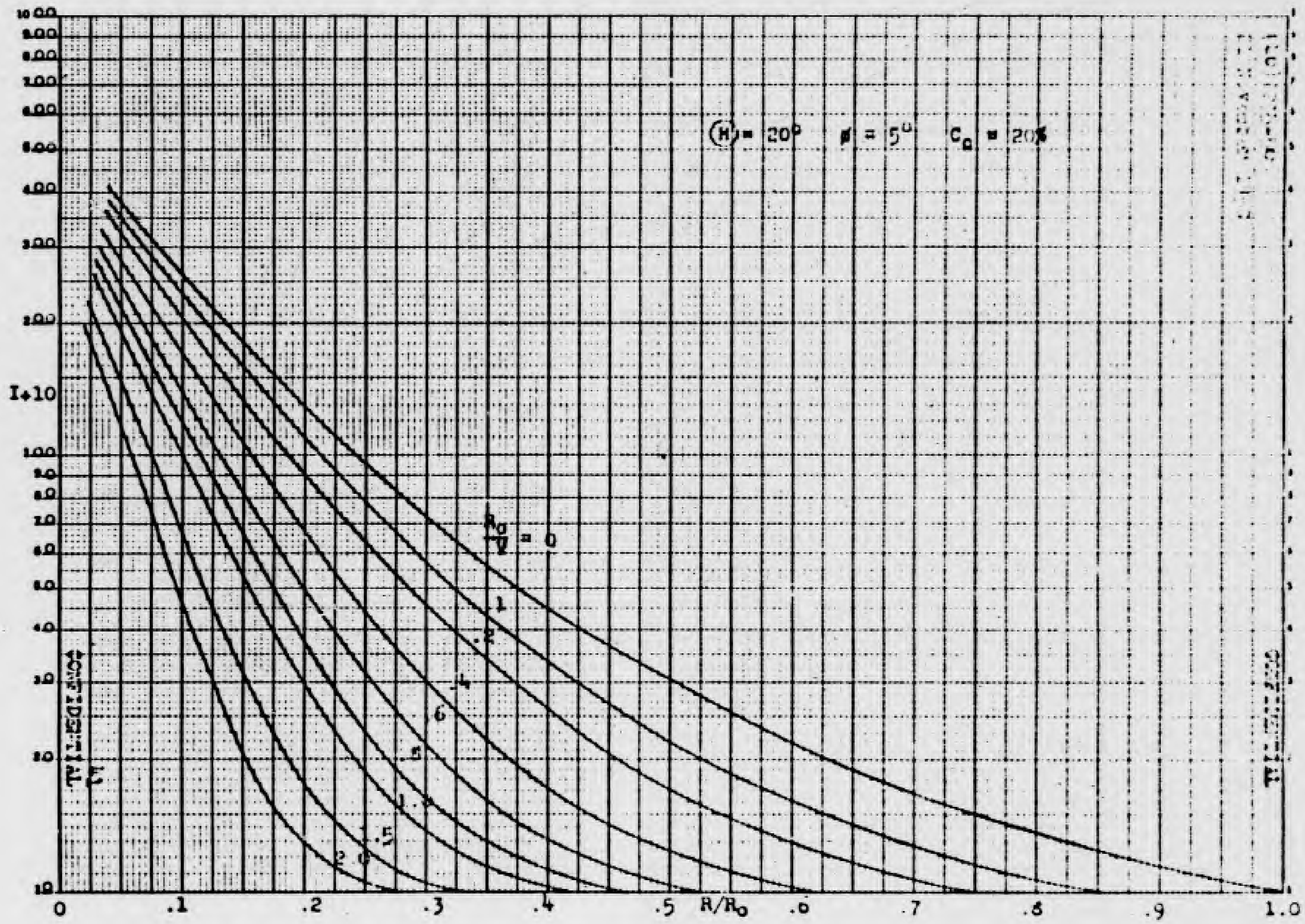
CONFIDENTIAL
$\frac{R_0}{V} = 0$
.1
.2
.4
.5
.6
1.0
1.5
2.0
I+10
R/R$_0$
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

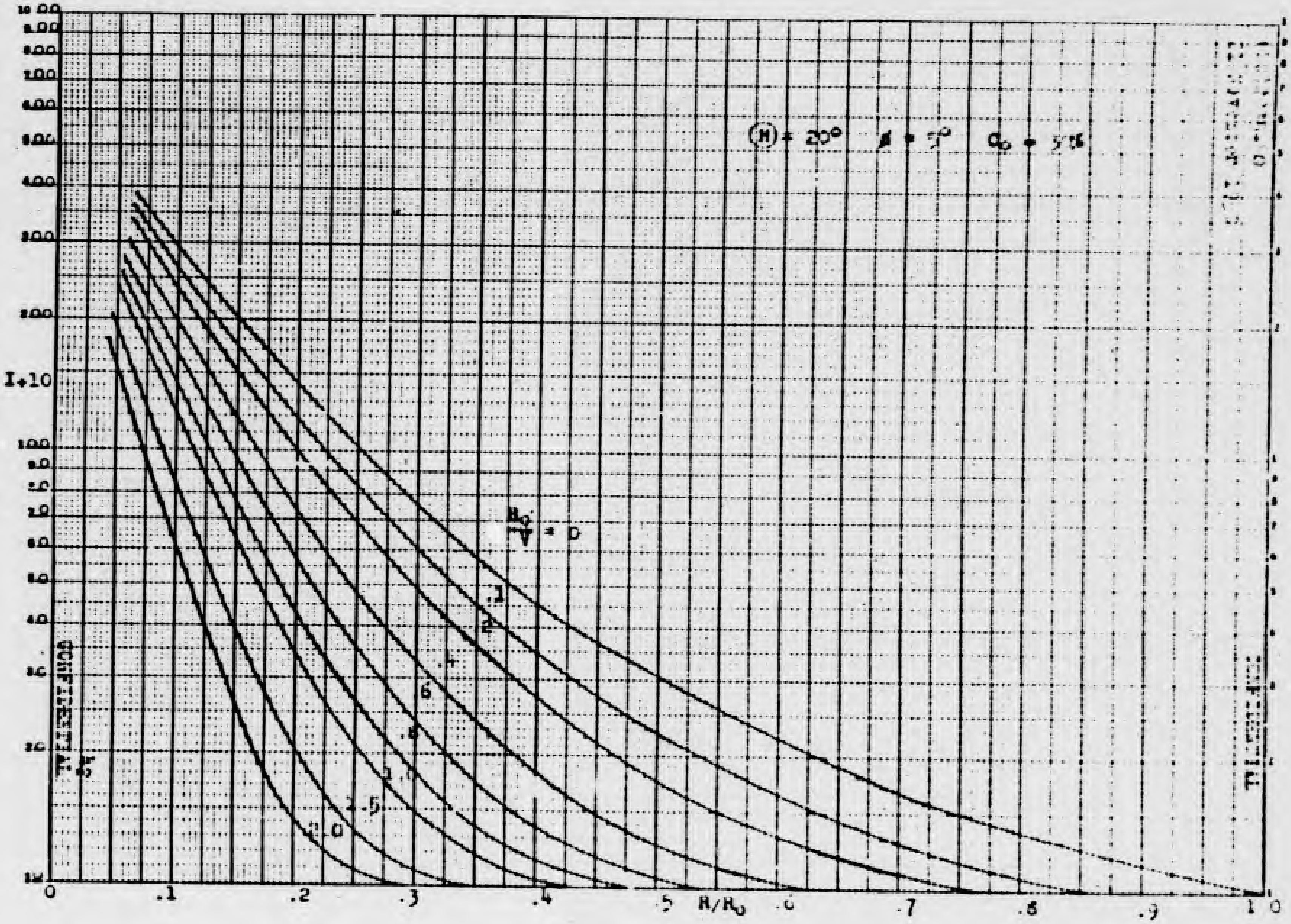

I+10
R/Ro
$\frac{R_o}{V} = C$
.1
2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL
37
CONFIDENTIAL

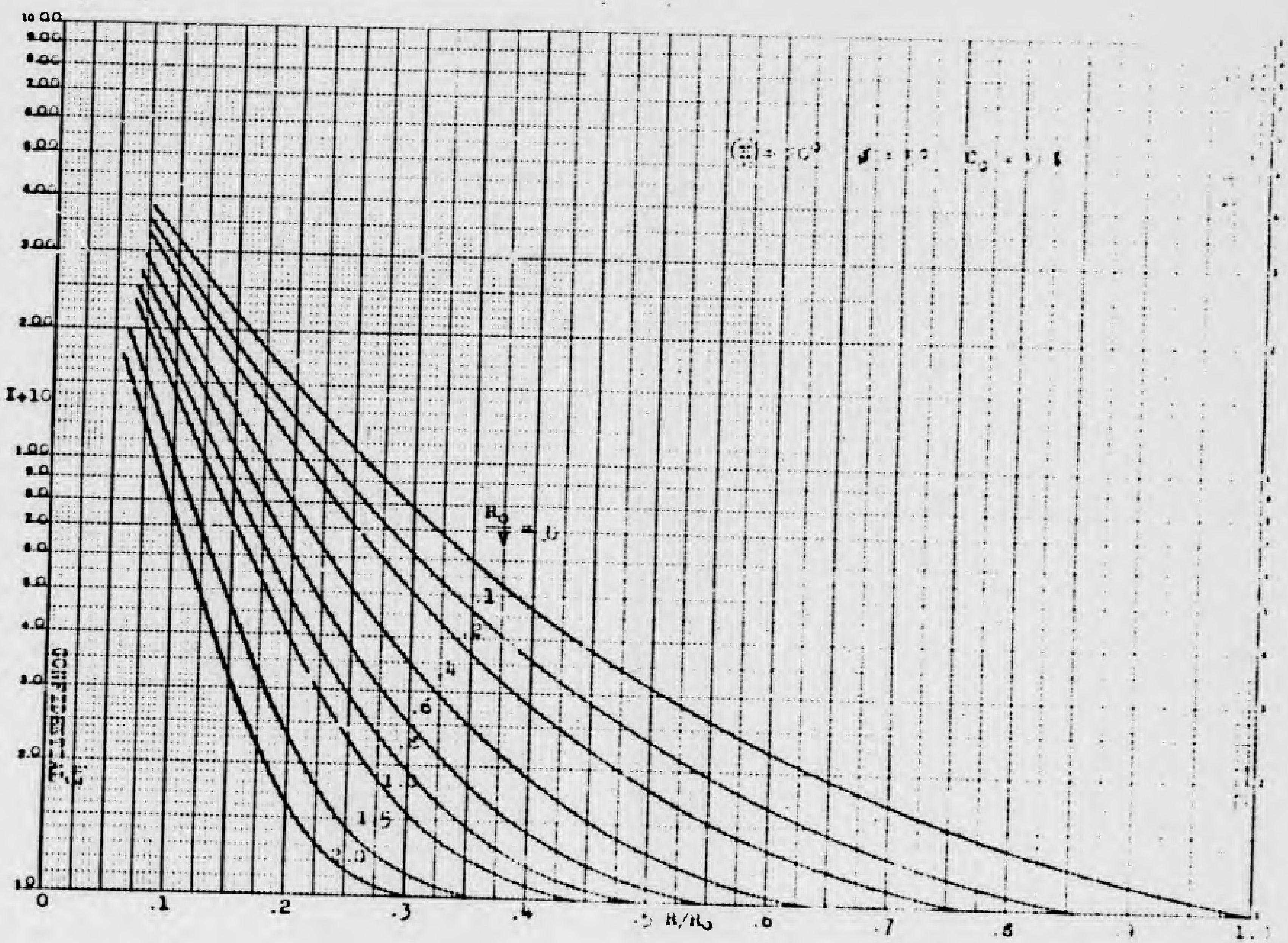

CONFIDENTIAL
I+10
R/R₀
$\frac{R_0}{V}$ = 0
1
2
.4
.6
.8
1.0
1.5
2.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
CONFIDENTIAL


I+10
R/Ro
CONFIDENTIAL

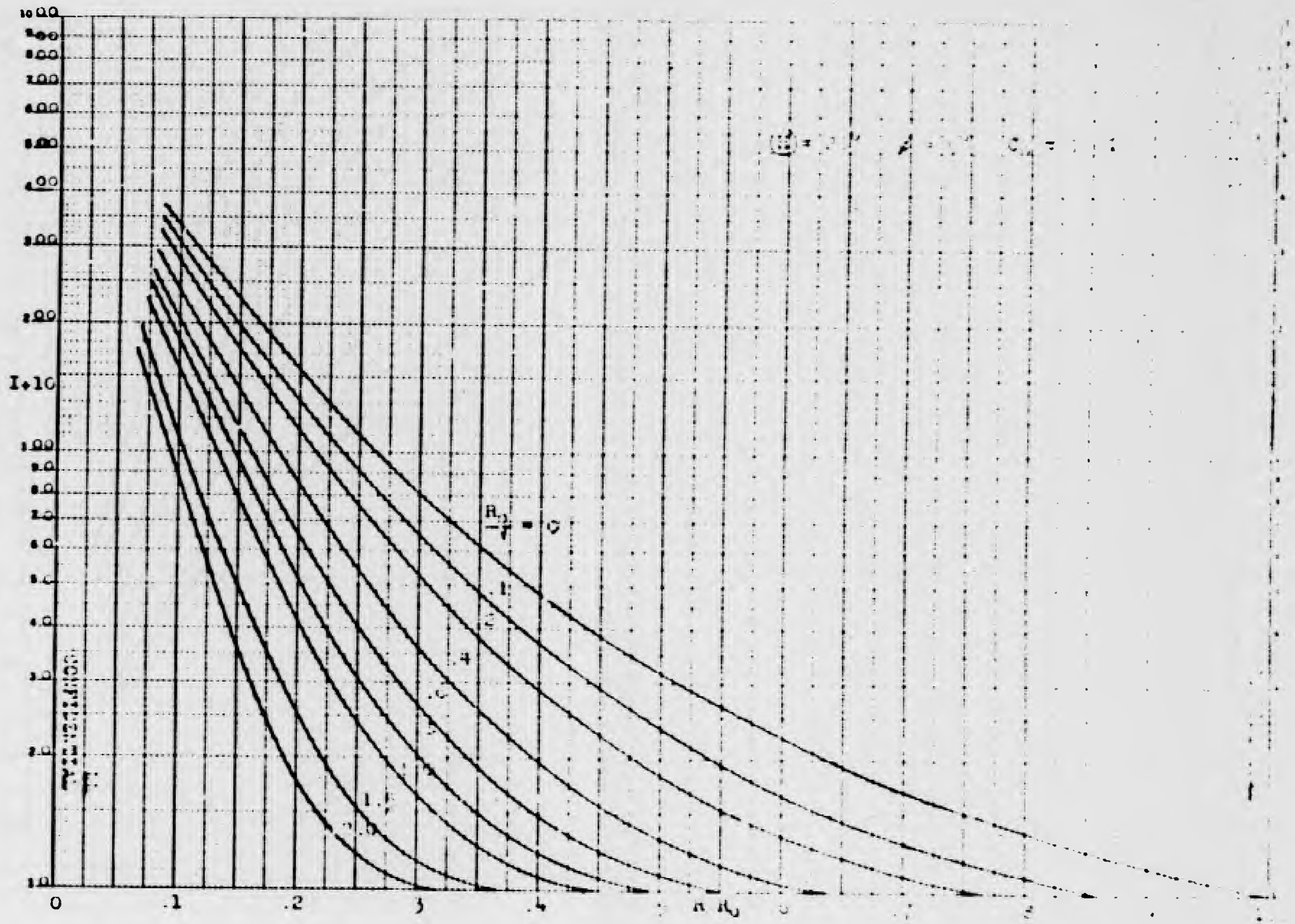

(M) = 209   ø = 5° C_5 = 1.6
I+10
R/R_o
R_o/Y = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

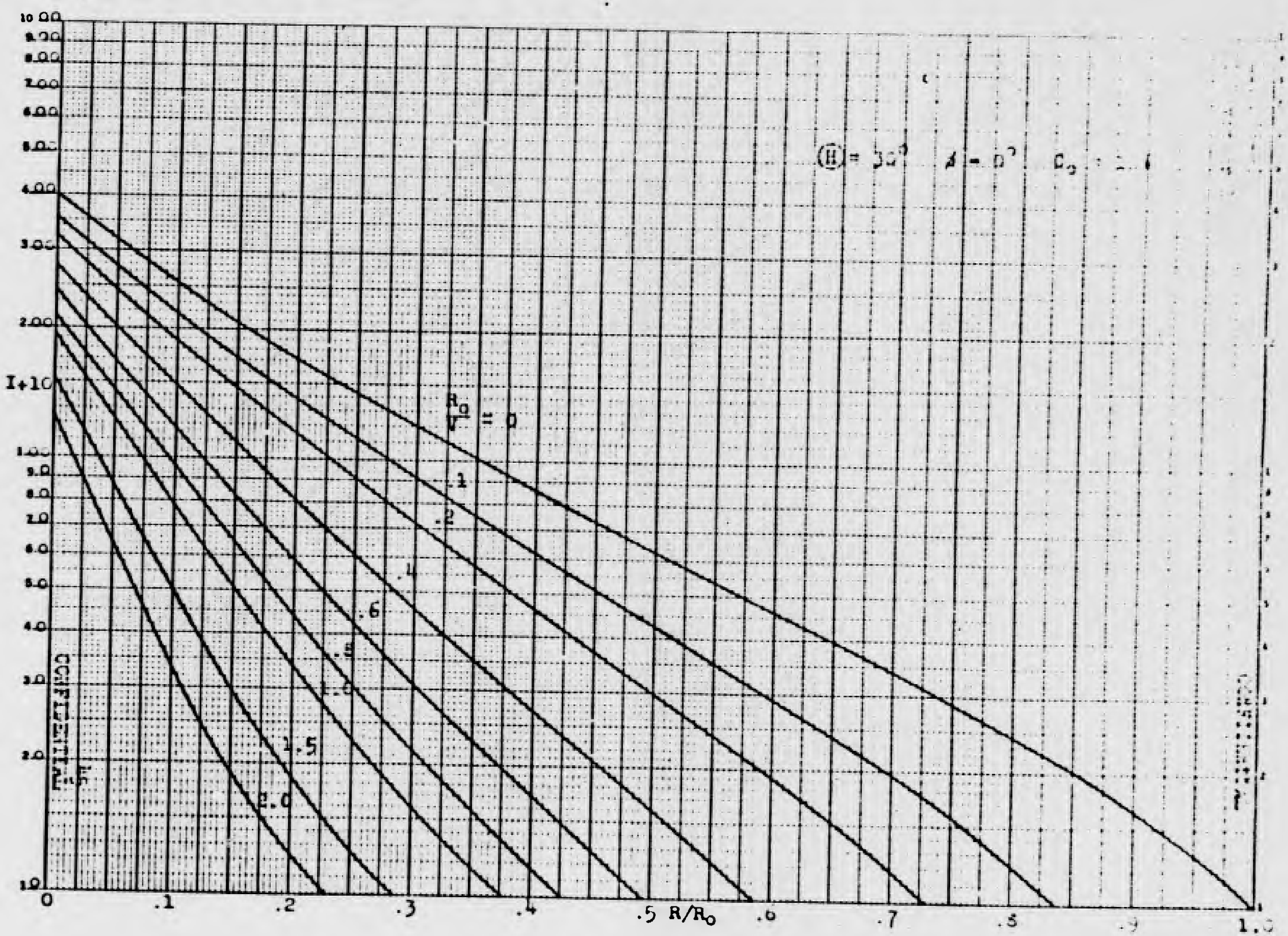

CONFIDENTIAL
(H) = 20° ø = 5° C_Q = 20%
$\frac{R_0}{V}$ = 0
1
.2
.4
.6
.8
1.0
1.5
2.0
I+10
R/Ro
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
CONFIDENTIAL
41


CONFIDENTIAL
Ⓗ = 20°   ∅ = 5° α_o = 5%
$\frac{h_o}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
I+10
R/R_o
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
CONFIDENTIAL

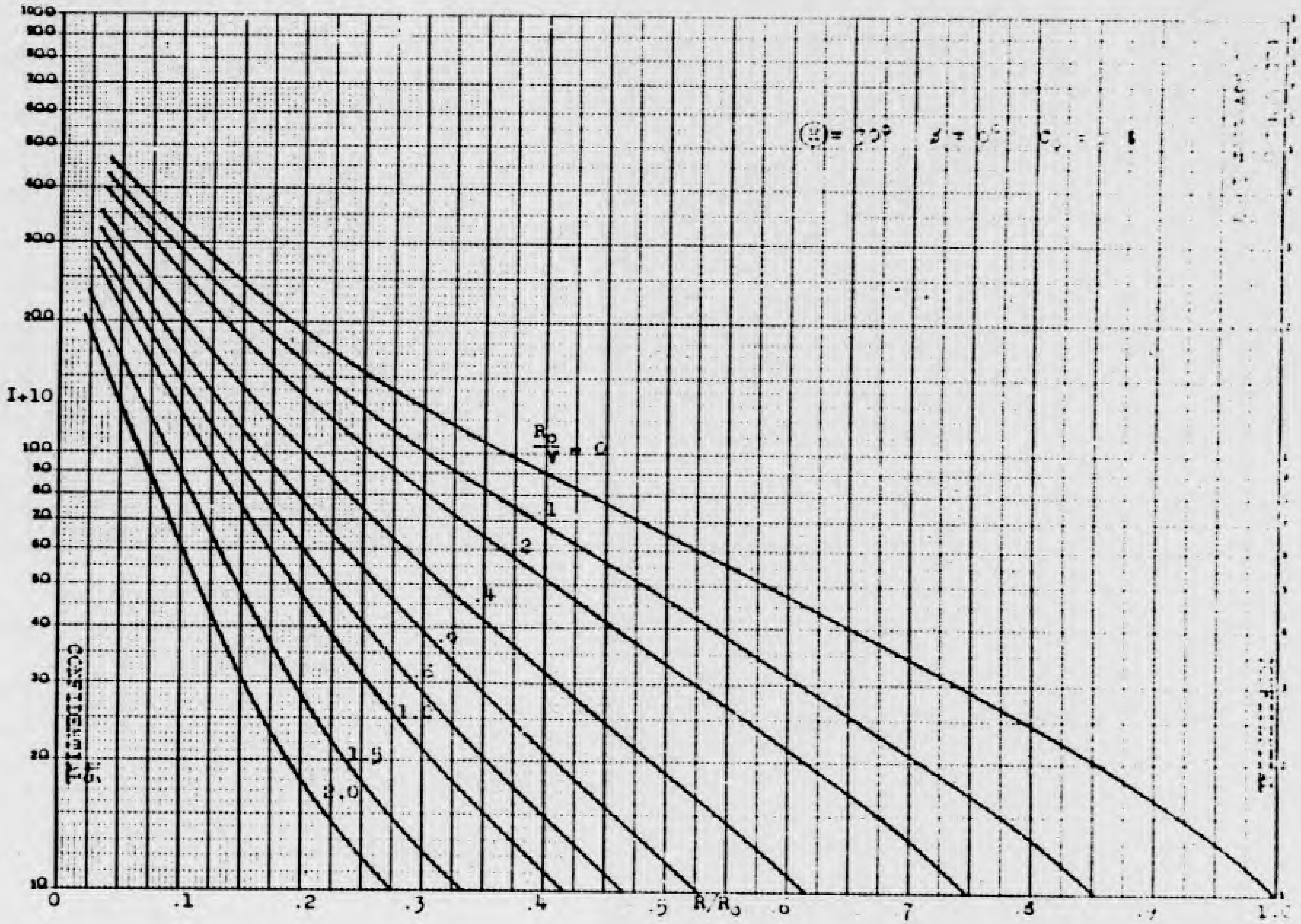


I+10
R/R0
CONFIDENTIAL
.1
.2
.4
.6
1.0
1.5
2.0

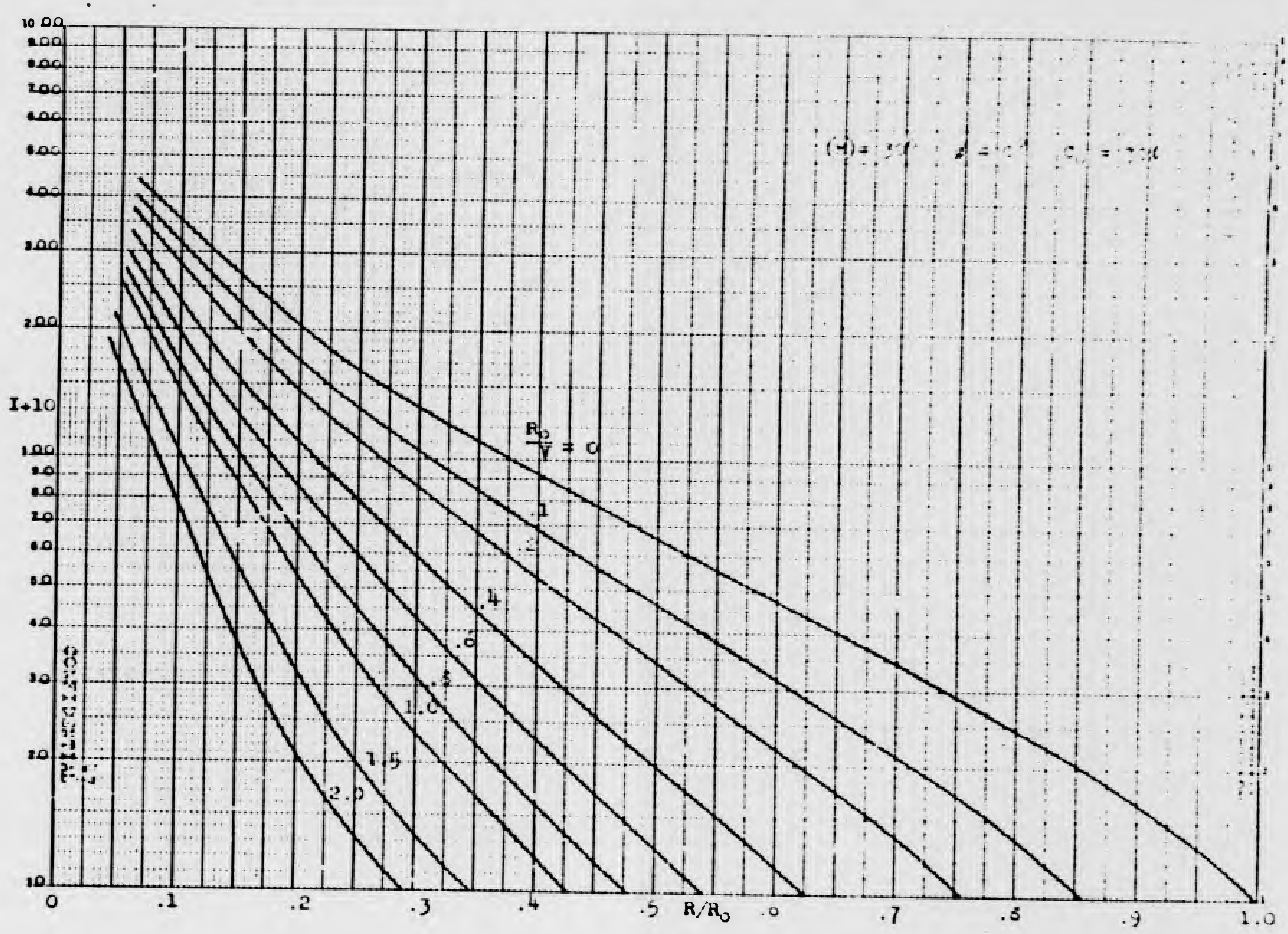

I+10
0
.1
.2
.3
.4
.1
.2
.4
.6
1.5
2.0
CONFIDENTIAL



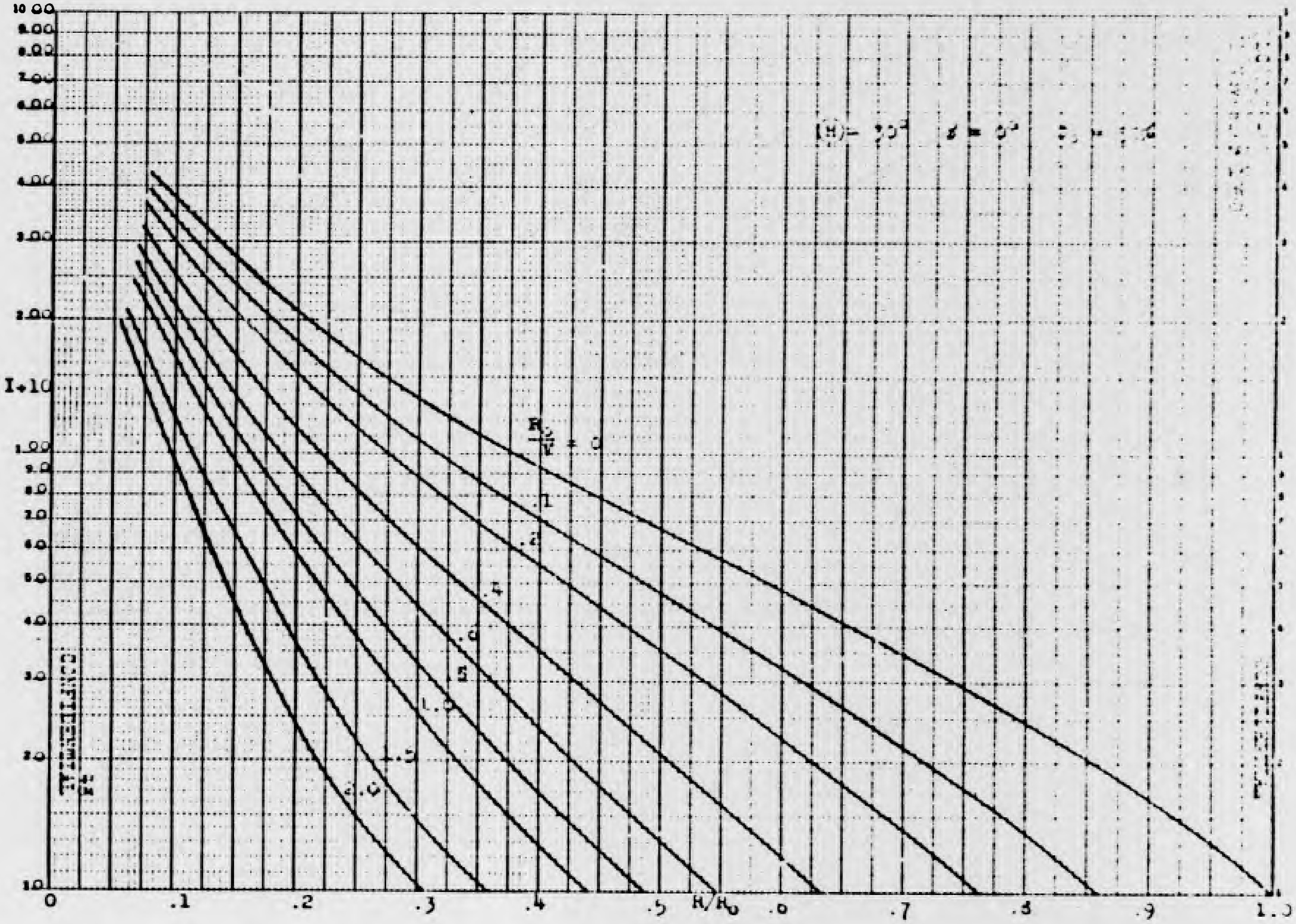
CONFIDENTIAL
I+10
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
.1
.2
.3
.4
.5 R/Ro
.6
.7
.8
.9
1.0
Ro/V = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0

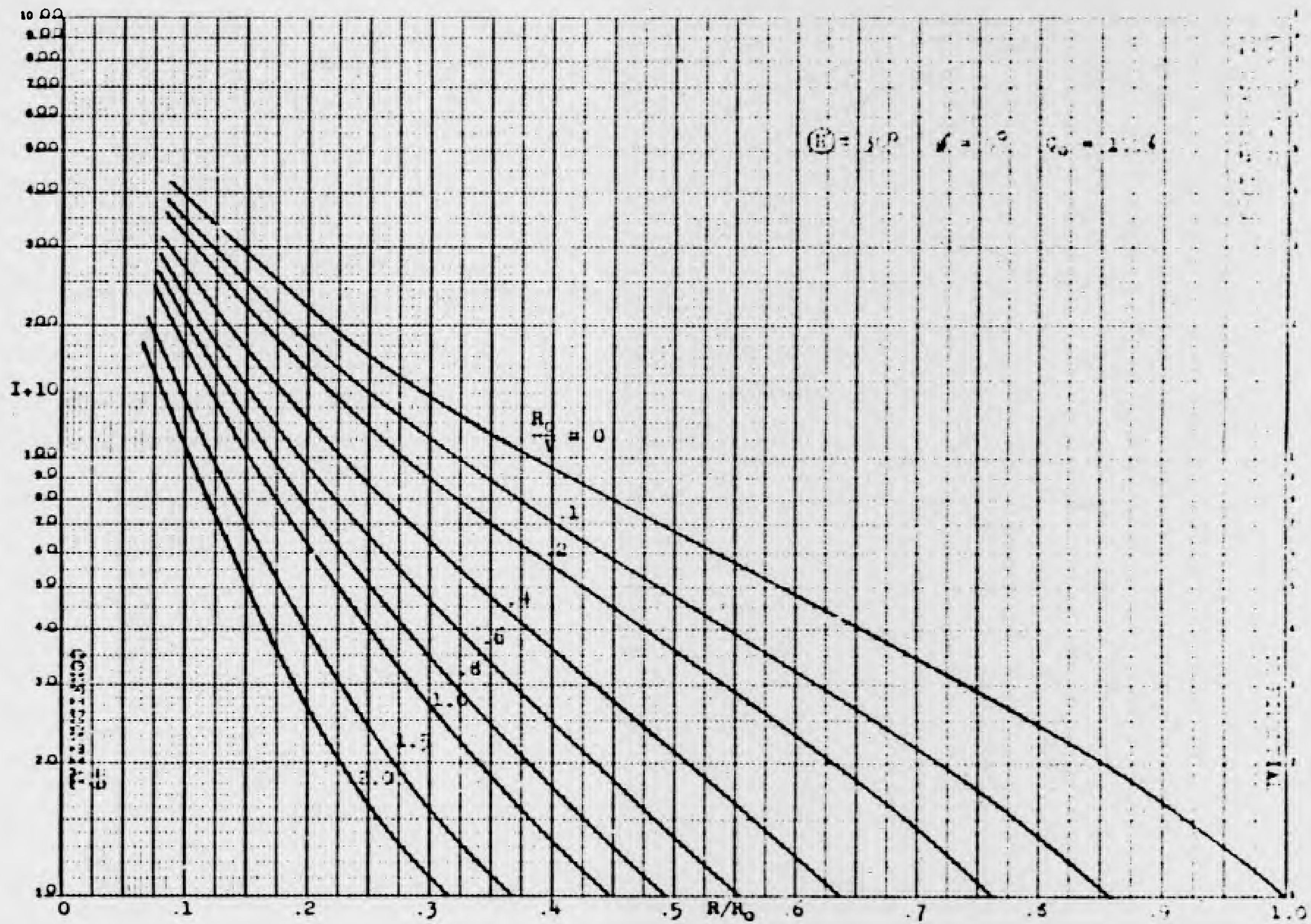


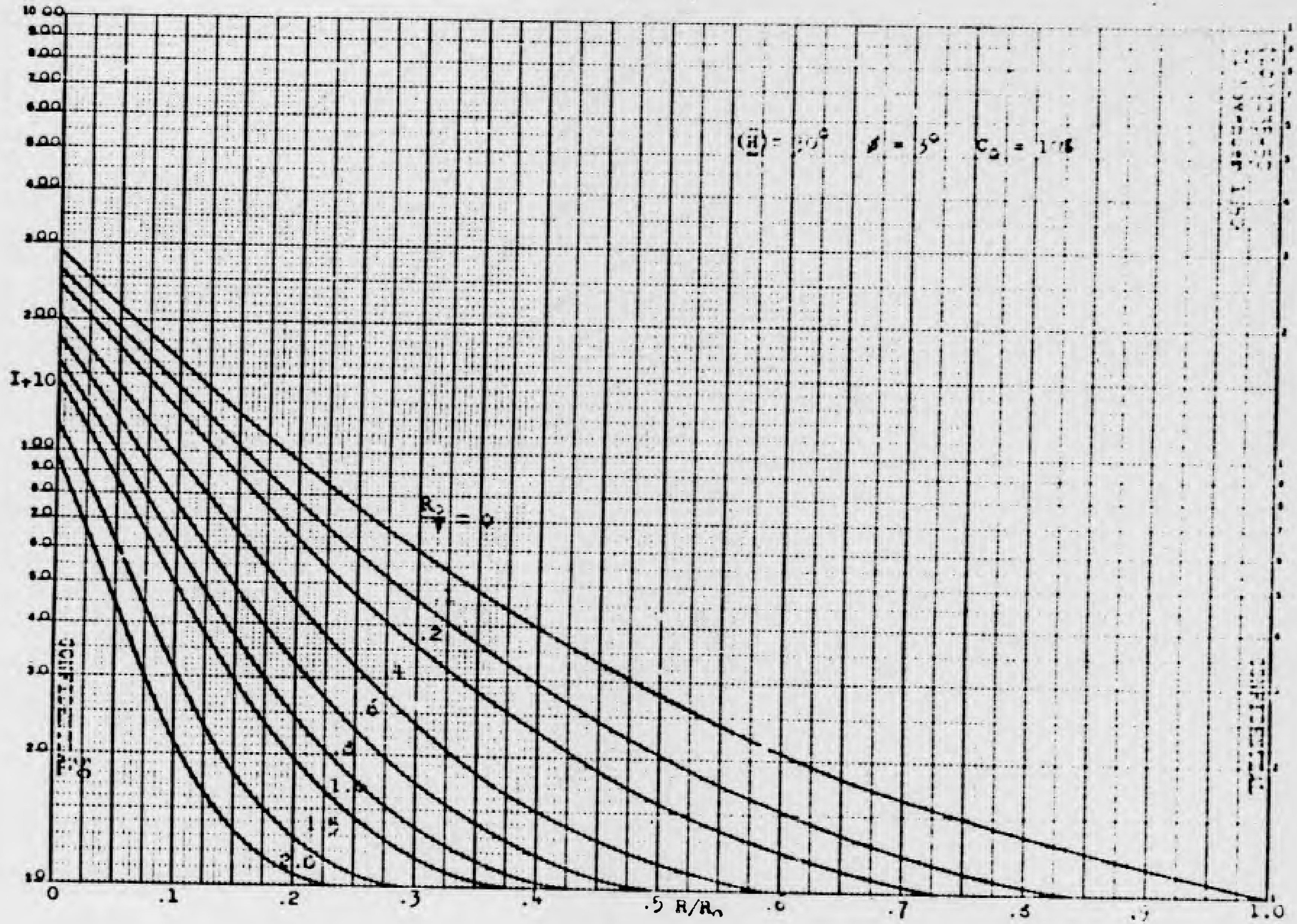
I+10
R/R0
$\frac{R_p}{V}$ = 0
.1
.2
.4
.9
.6
1.2
1.5
2.0
CONFIDENTIAL



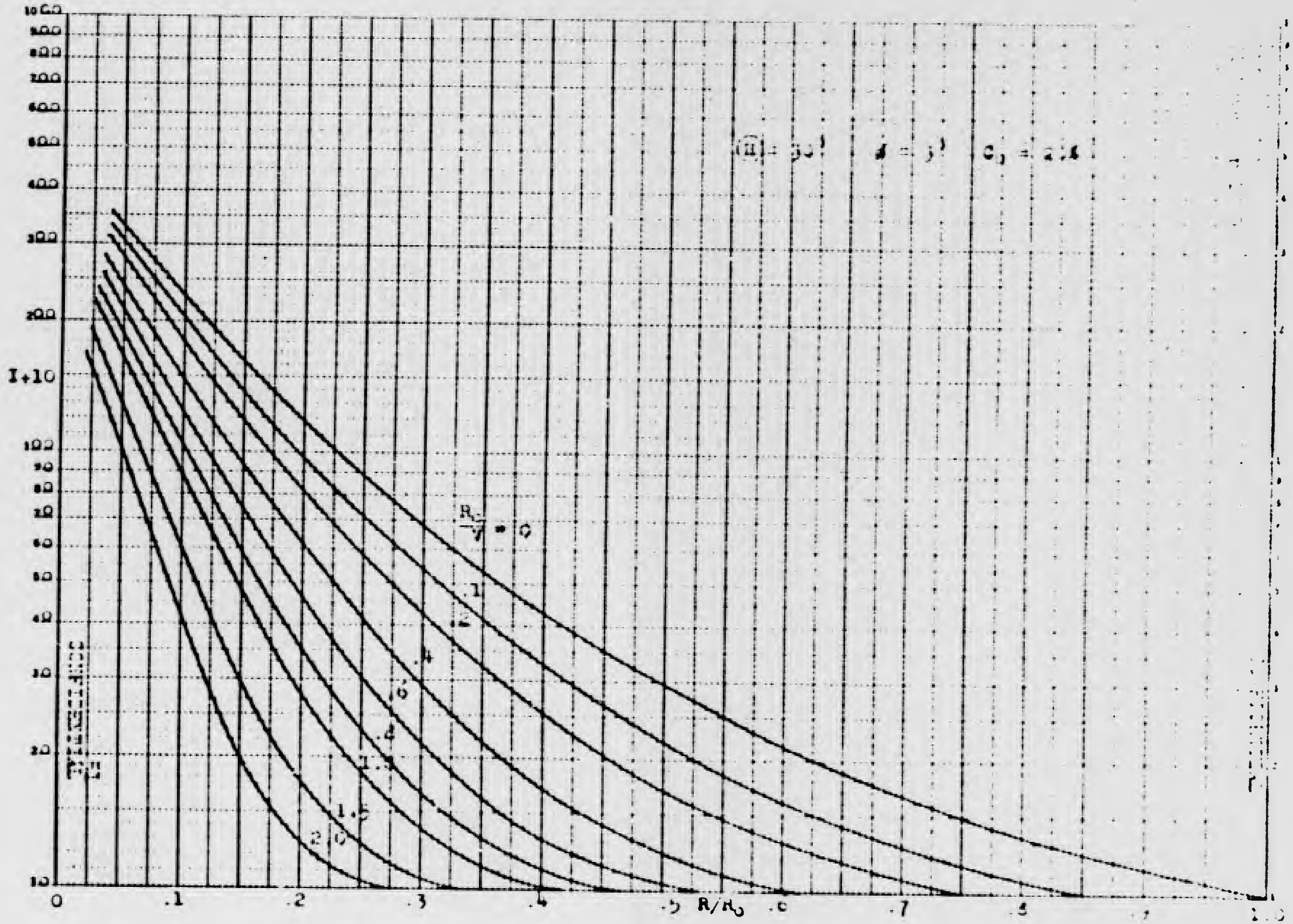
I+10
R/R₀
$\frac{R_0}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL


CONFIDENTIAL
I+10
H/H0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
.1
.2
.4
.6
.8
1.0
1.5
2.0

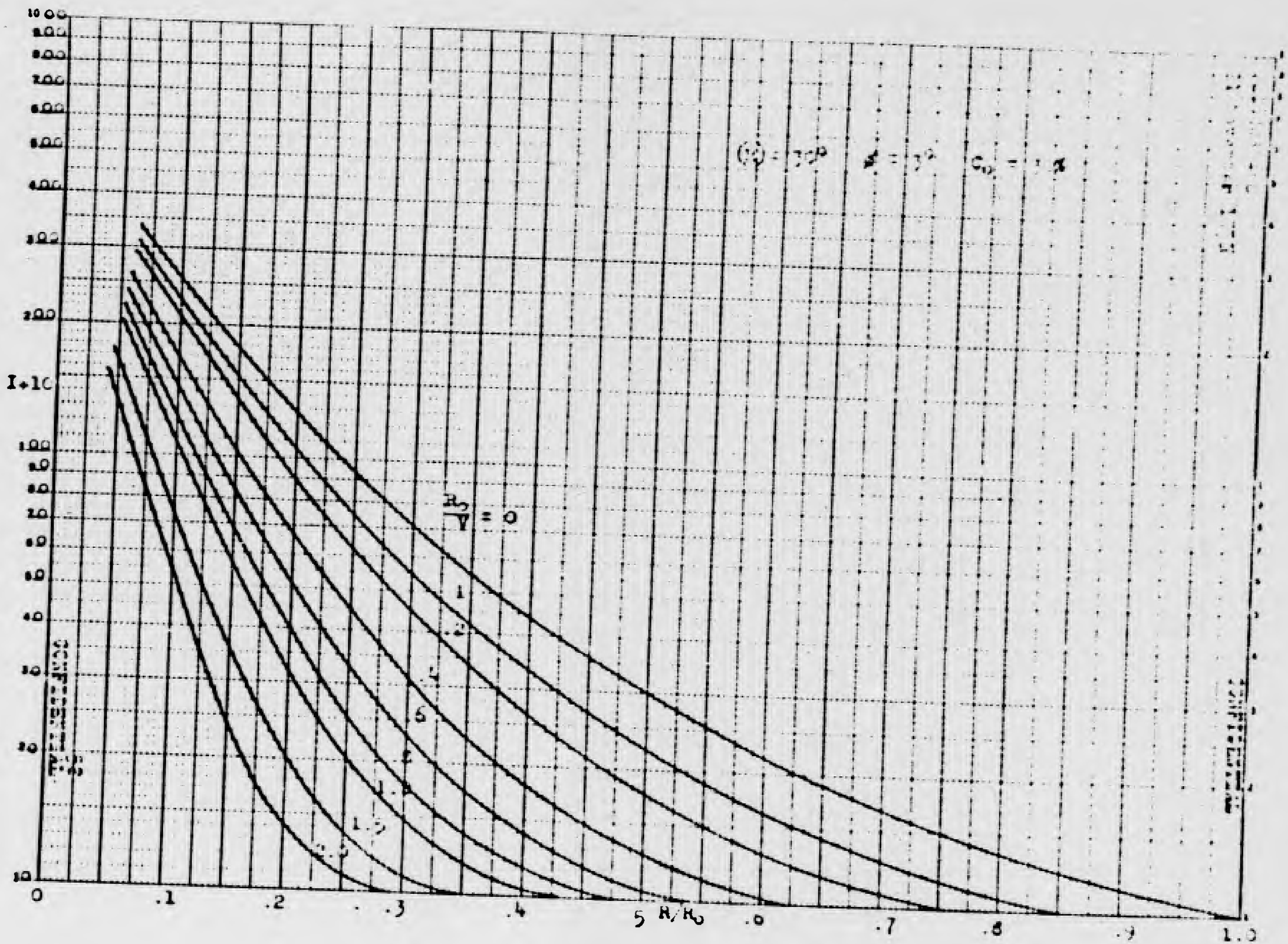

I+10
R/R0
$\frac{R_0}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0



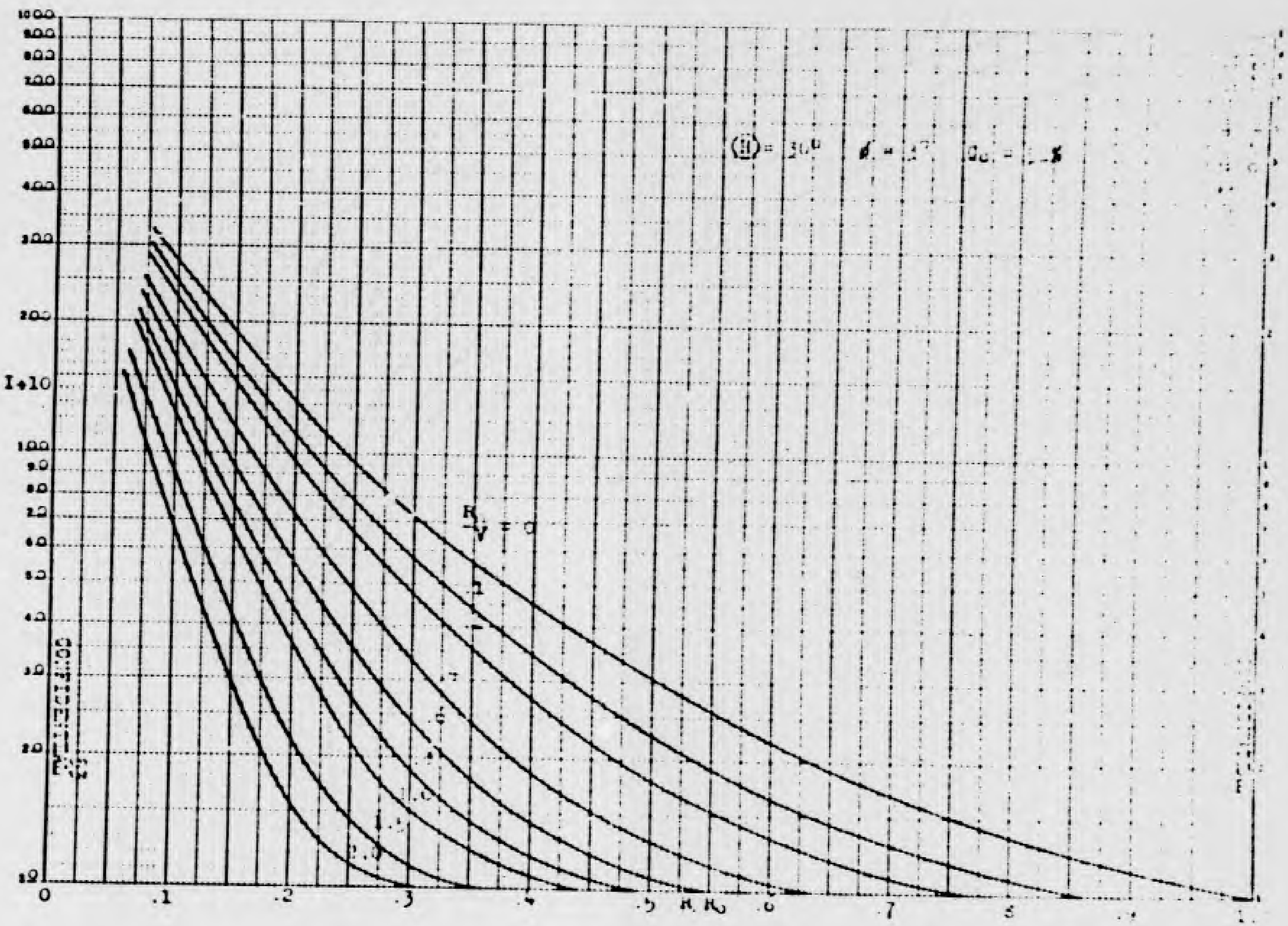
I+10
.5 R/Rn
0
.1
.2
.3
.4
.6
.7
1.0
.2
.4
.6
2.0
CONFIDENTIAL



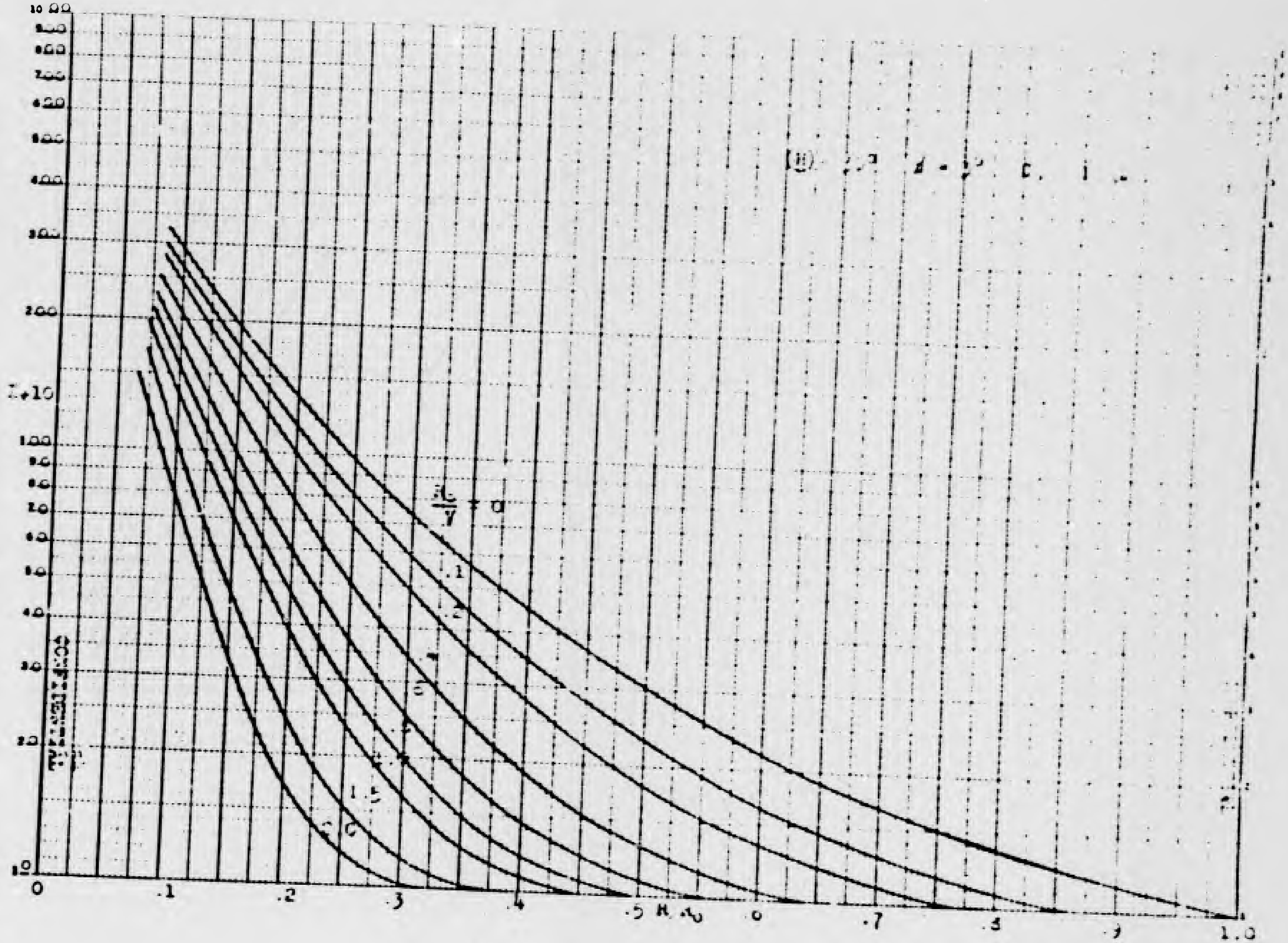
I+10
R/R₀
R₀/y = 0
.1
.2
.4
.6
.8
1.5
2.0
CONFIDENTIAL

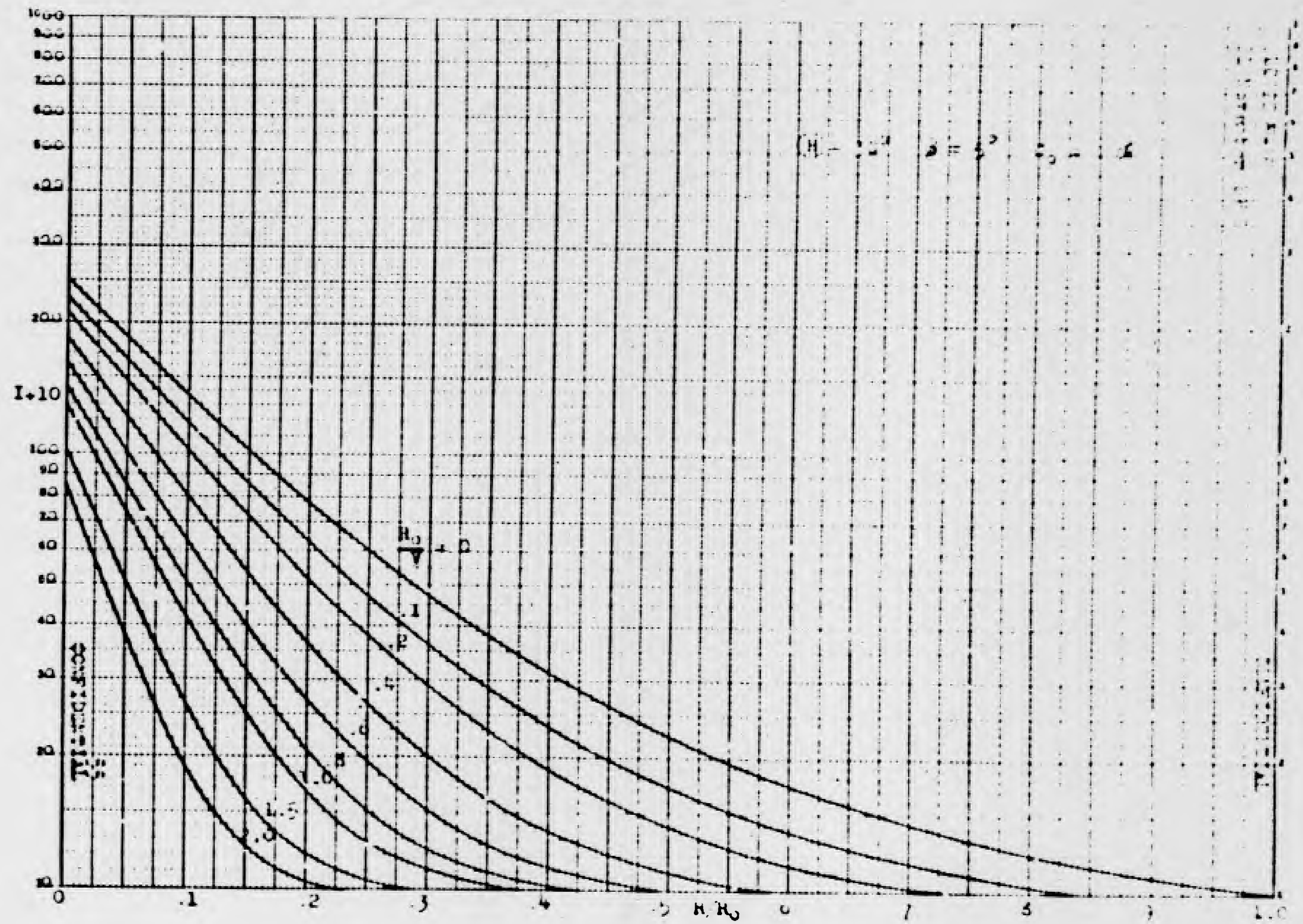


CONFIDENTIAL
I+10
R/R₀
R₀/Y = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0

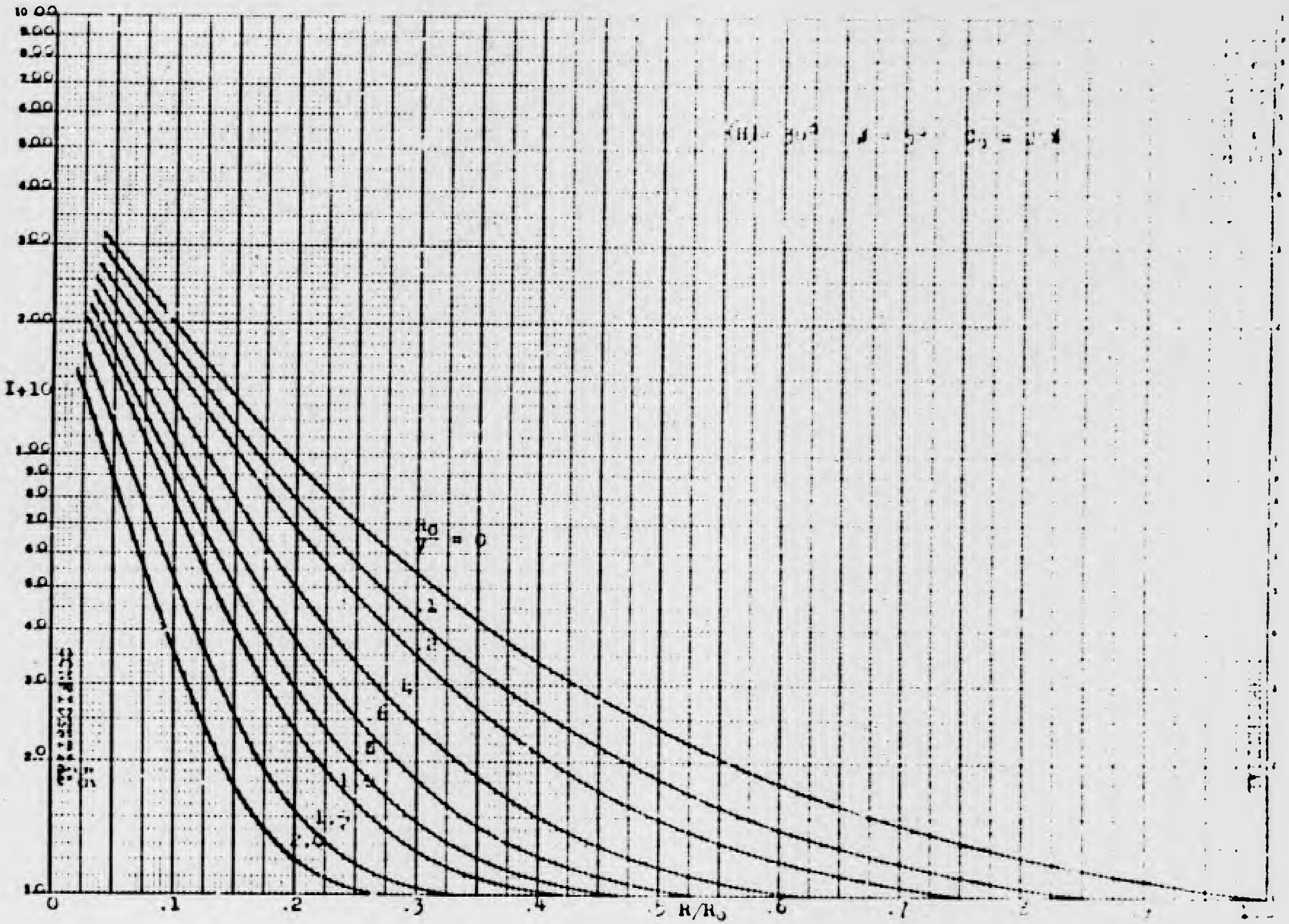


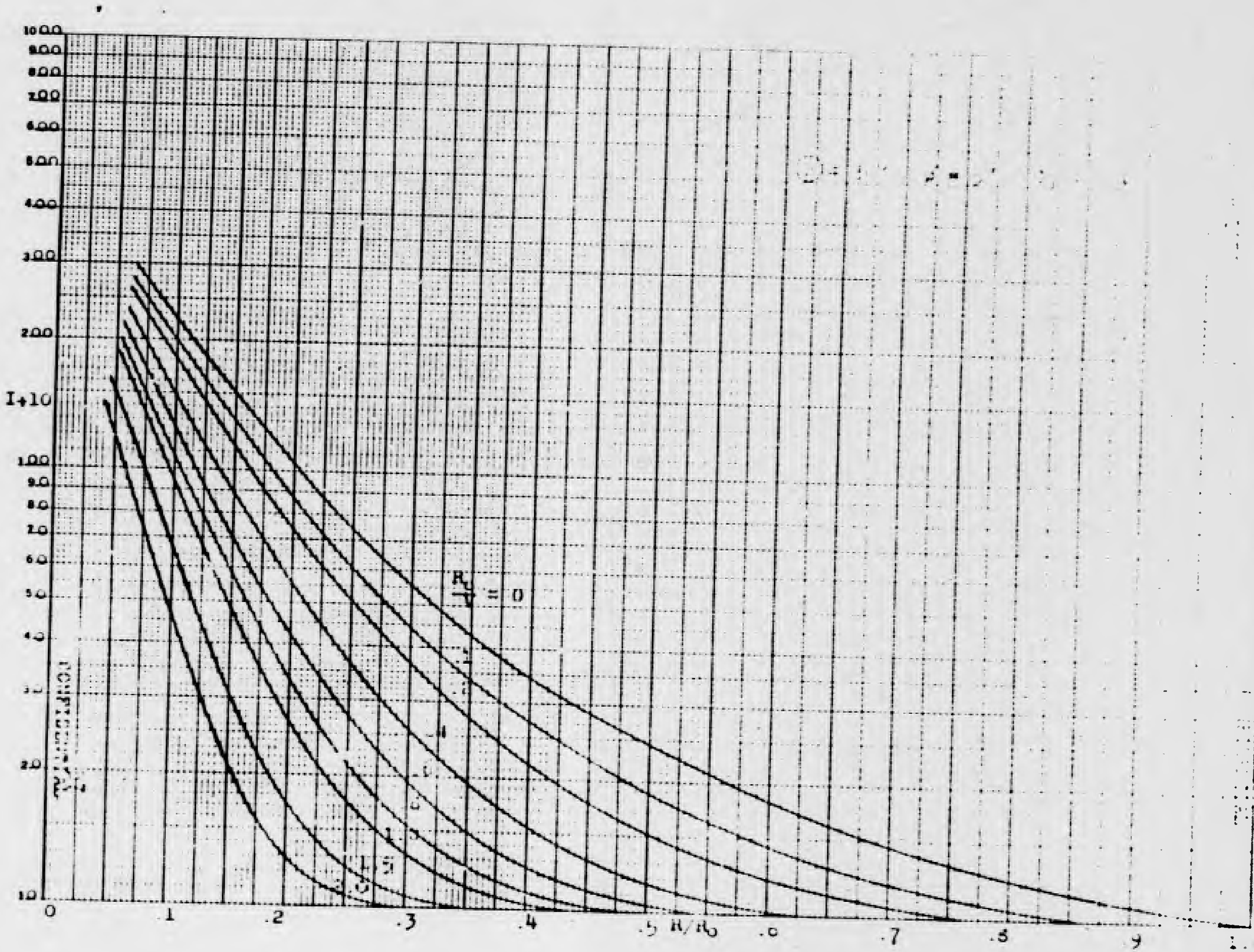
I+10
CONFIDENTIAL
$R/V = 0$
1.0
1.5
2.0
.1
.2
.3
.4
.5
.7
0


CONFIDENTIAL
.1
.2
1.5
0
.1
.2
.3
.4
.7
1.0

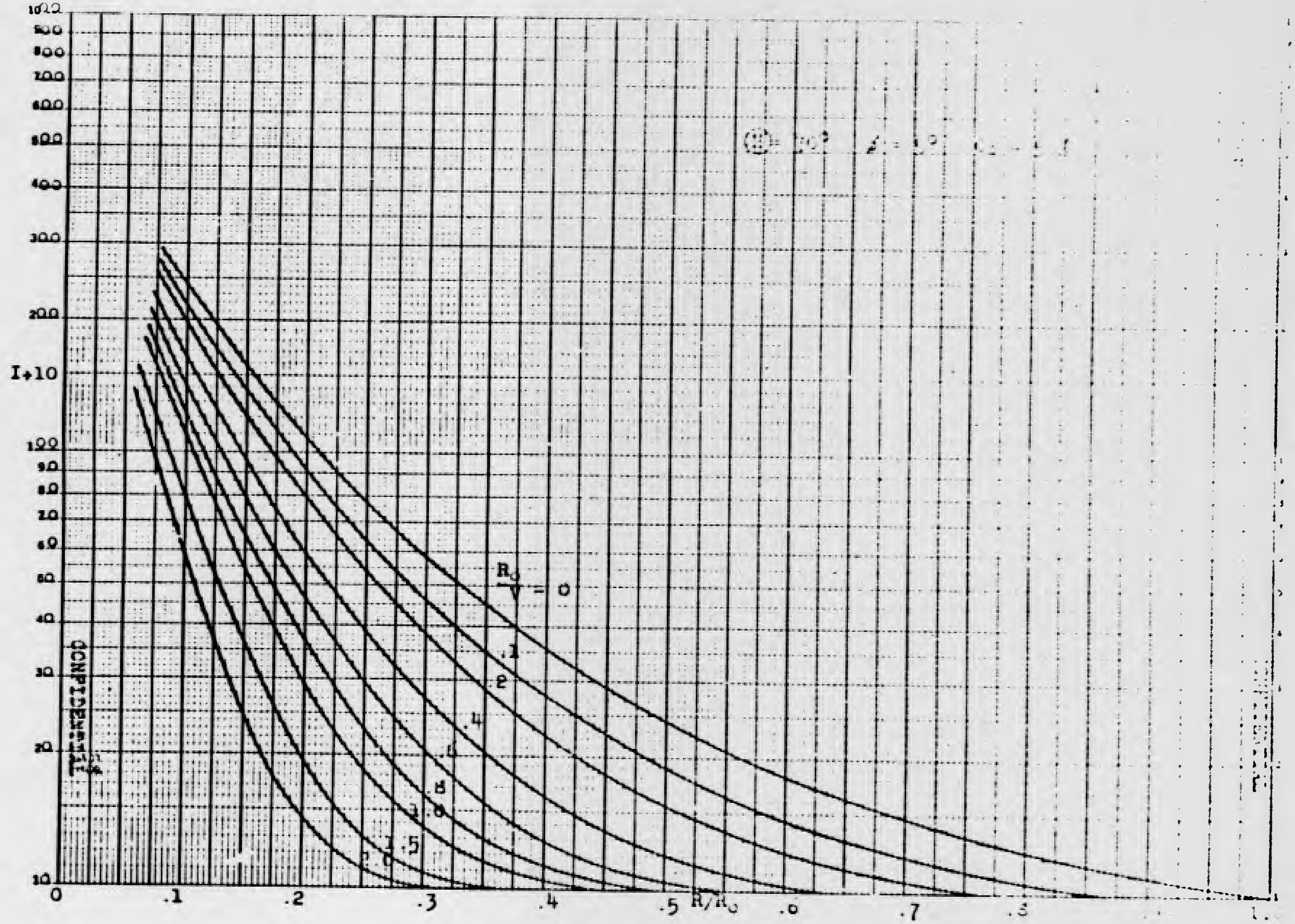


I+10
R/R₀
CONFIDENTIAL

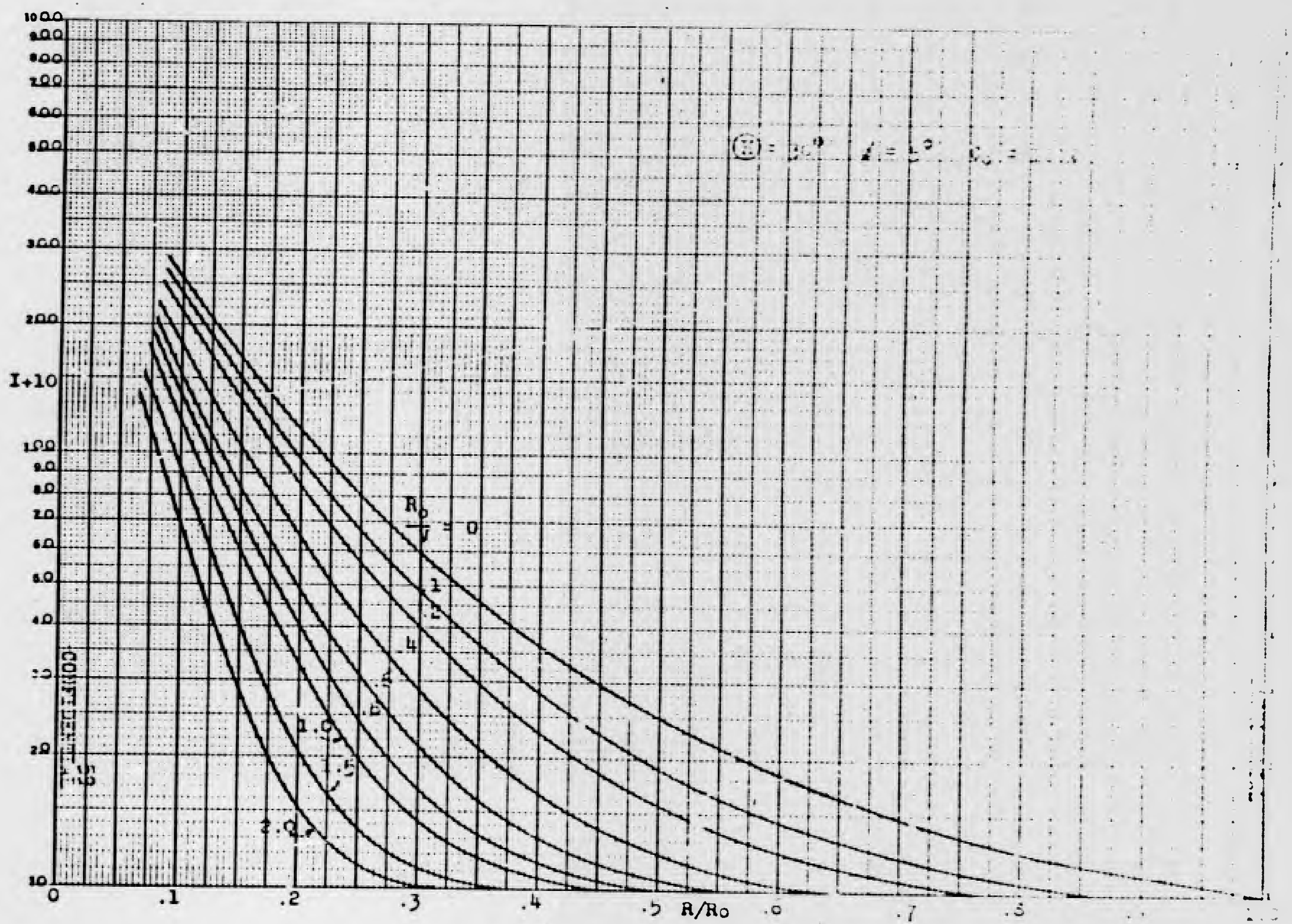

I+10
R/R₀
0
.1
.2
.3
.4
.6
.7
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
= 0
.4
.6
2.0



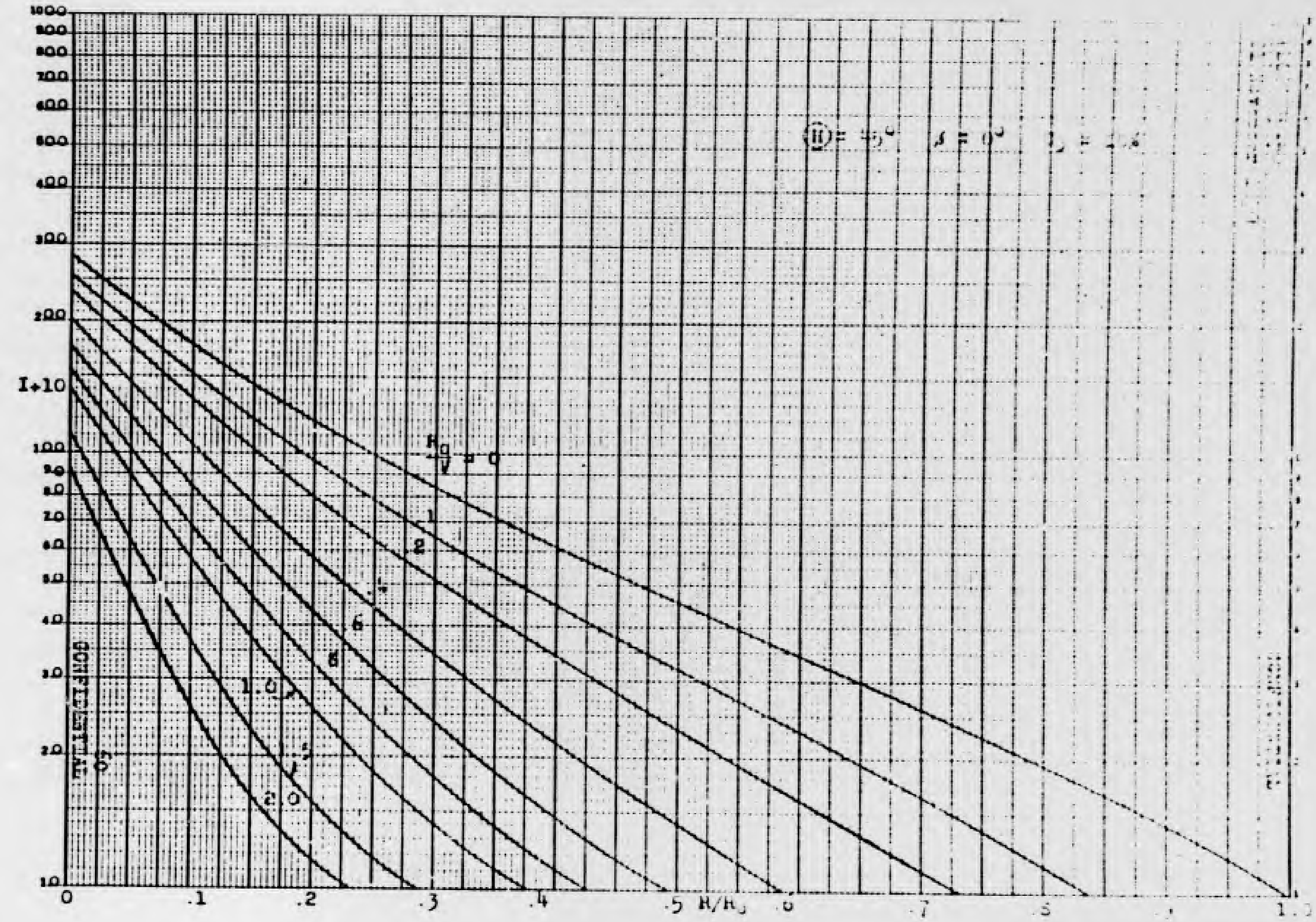
I+10
R/R₀
$\frac{R_c}{V} = 0$
.4
1.5
CONFIDENTIAL



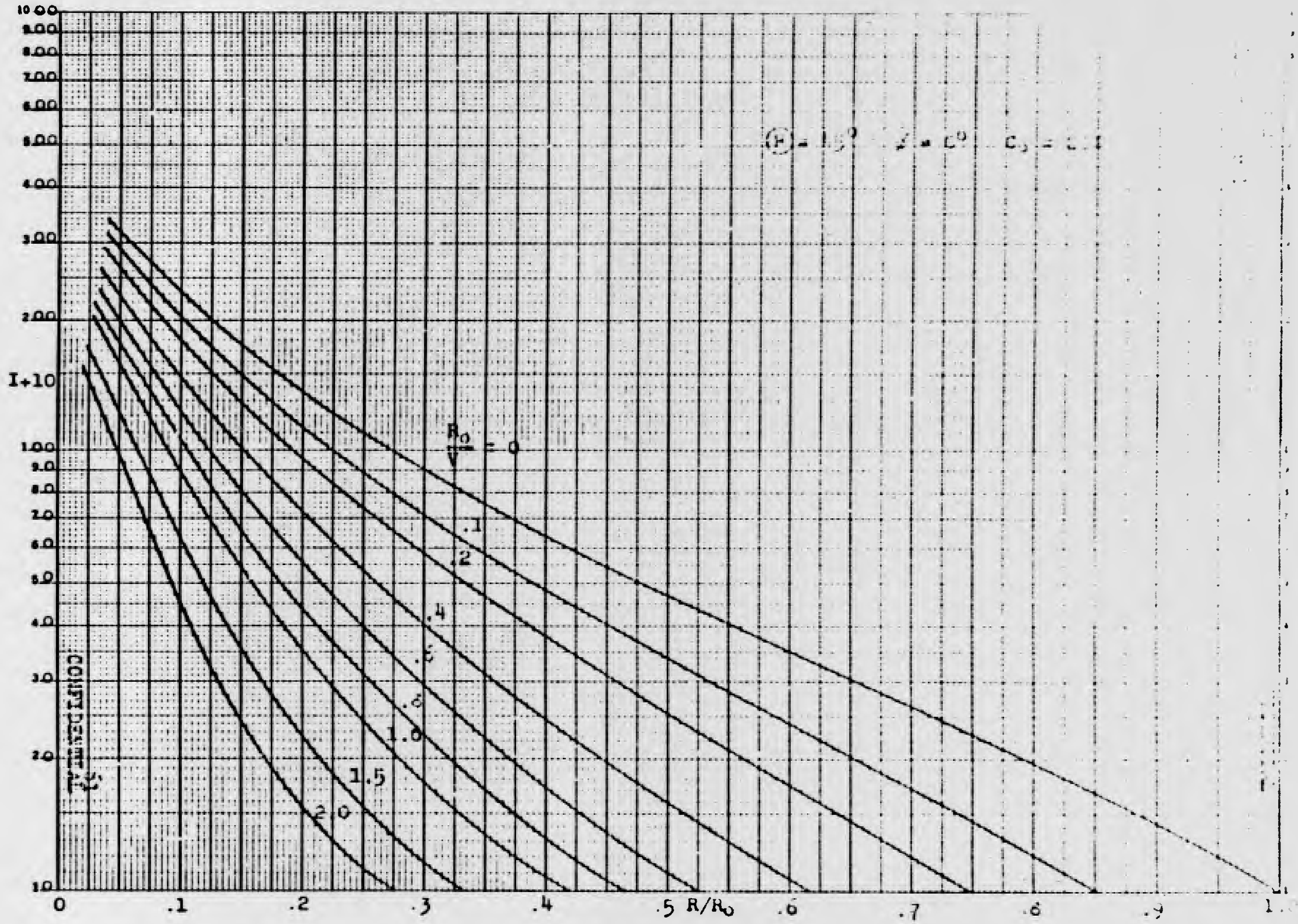
CONFIDENTIAL



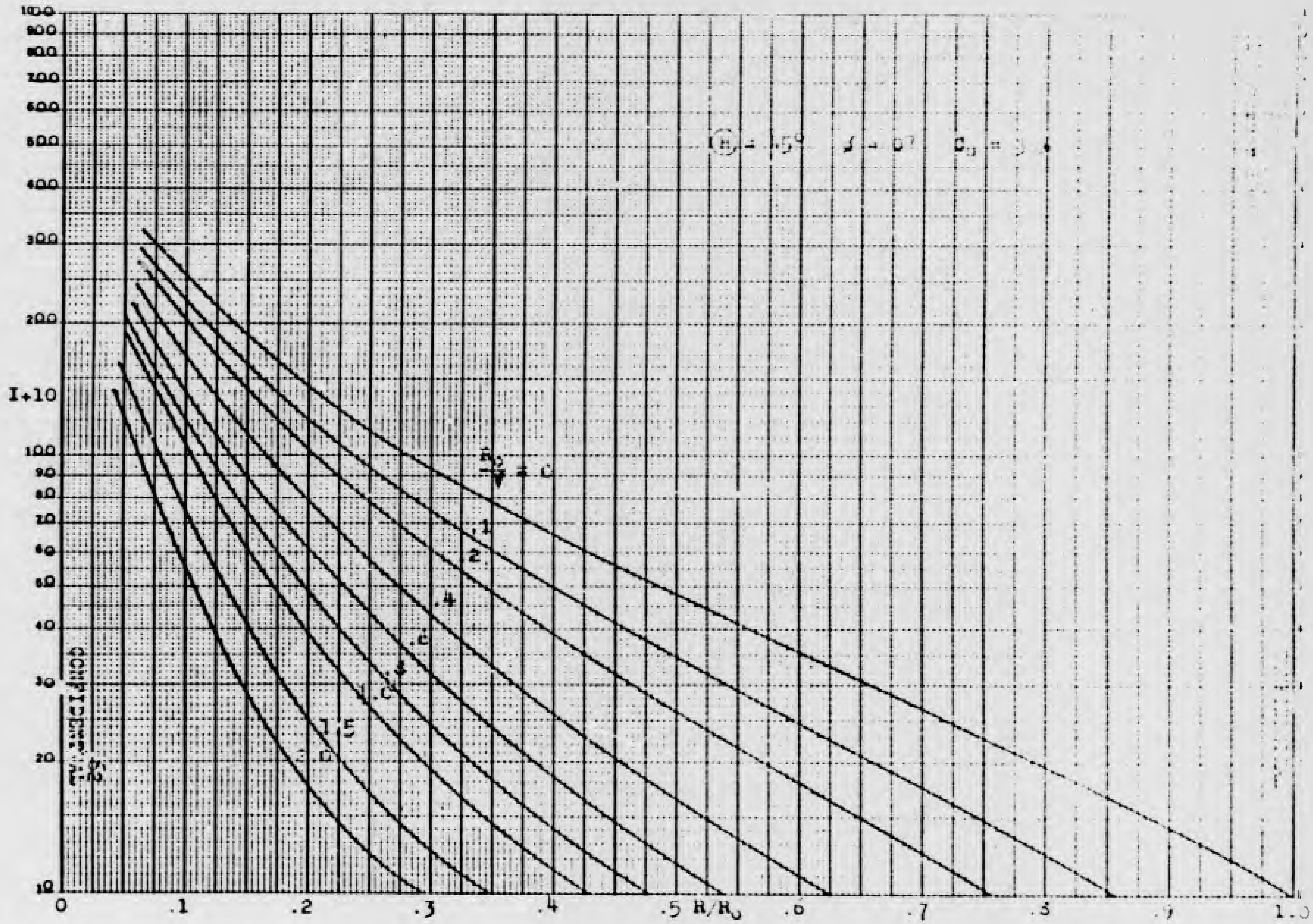
CONFIDENTIAL

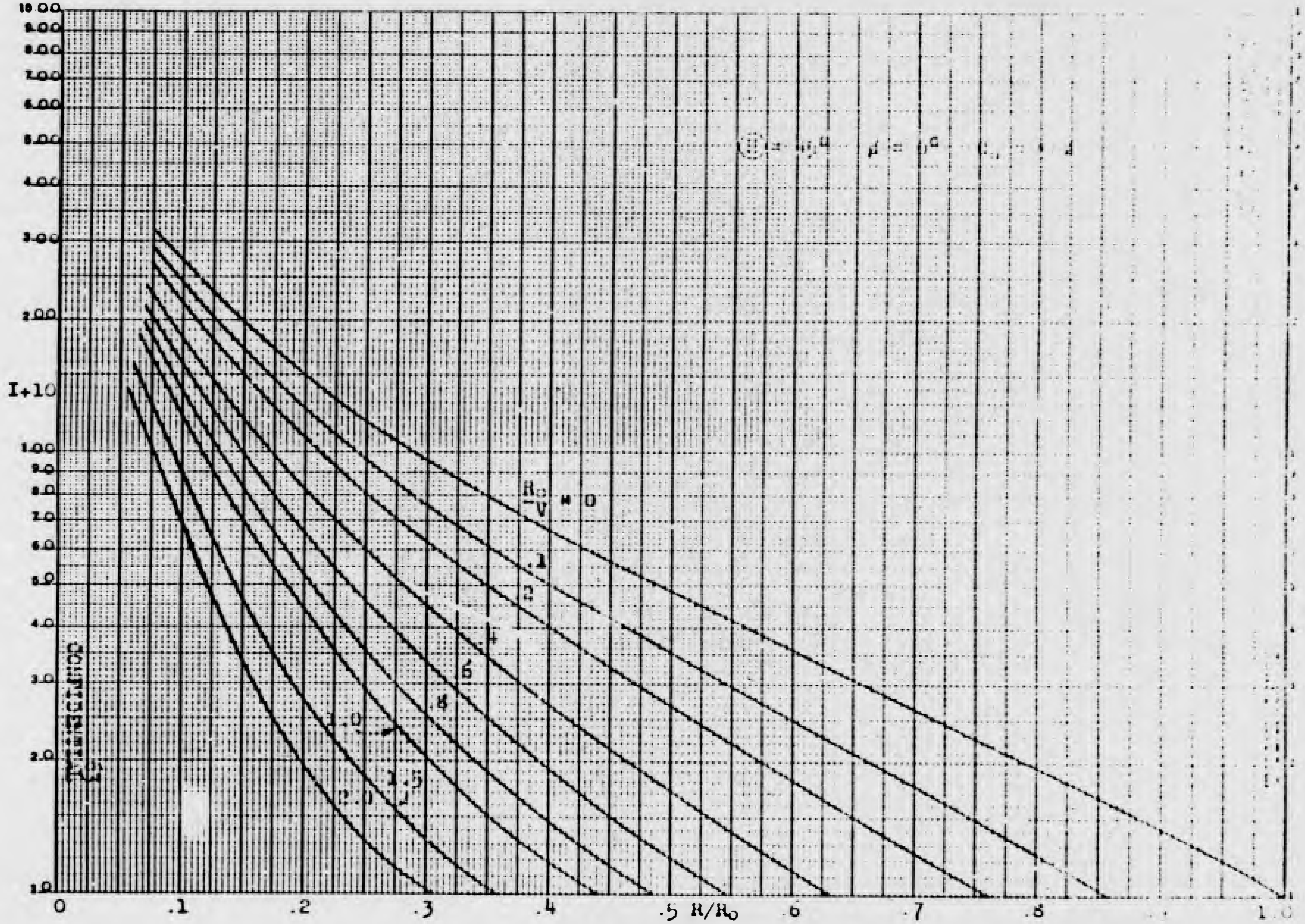


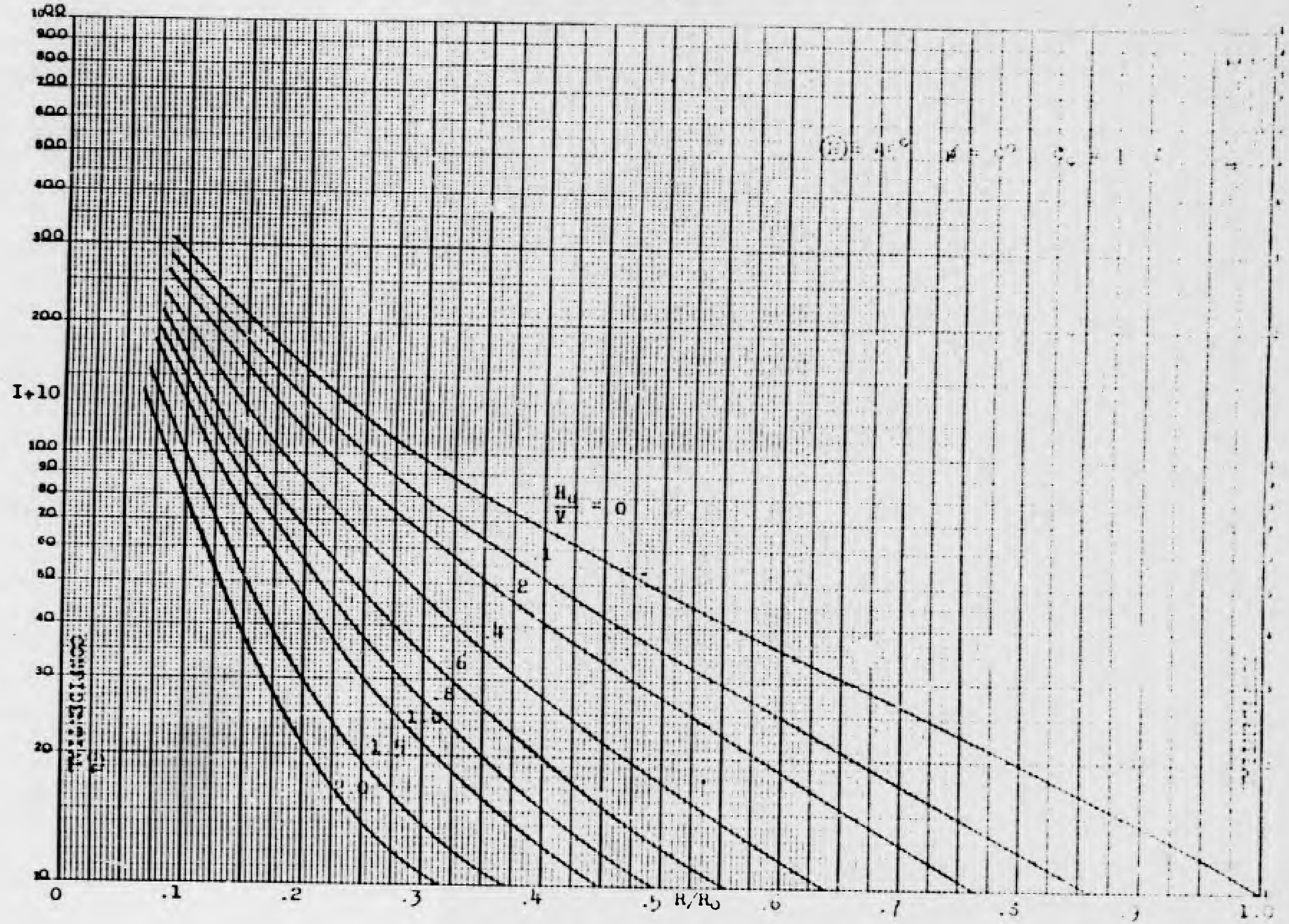
I+10
1000
300
200
100
10
0
.1
.2
.3
.4
.5 R/R₀
.6
.7
.8
1.0
1
2
6
8
1.0
2.0
CONFIDENTIAL

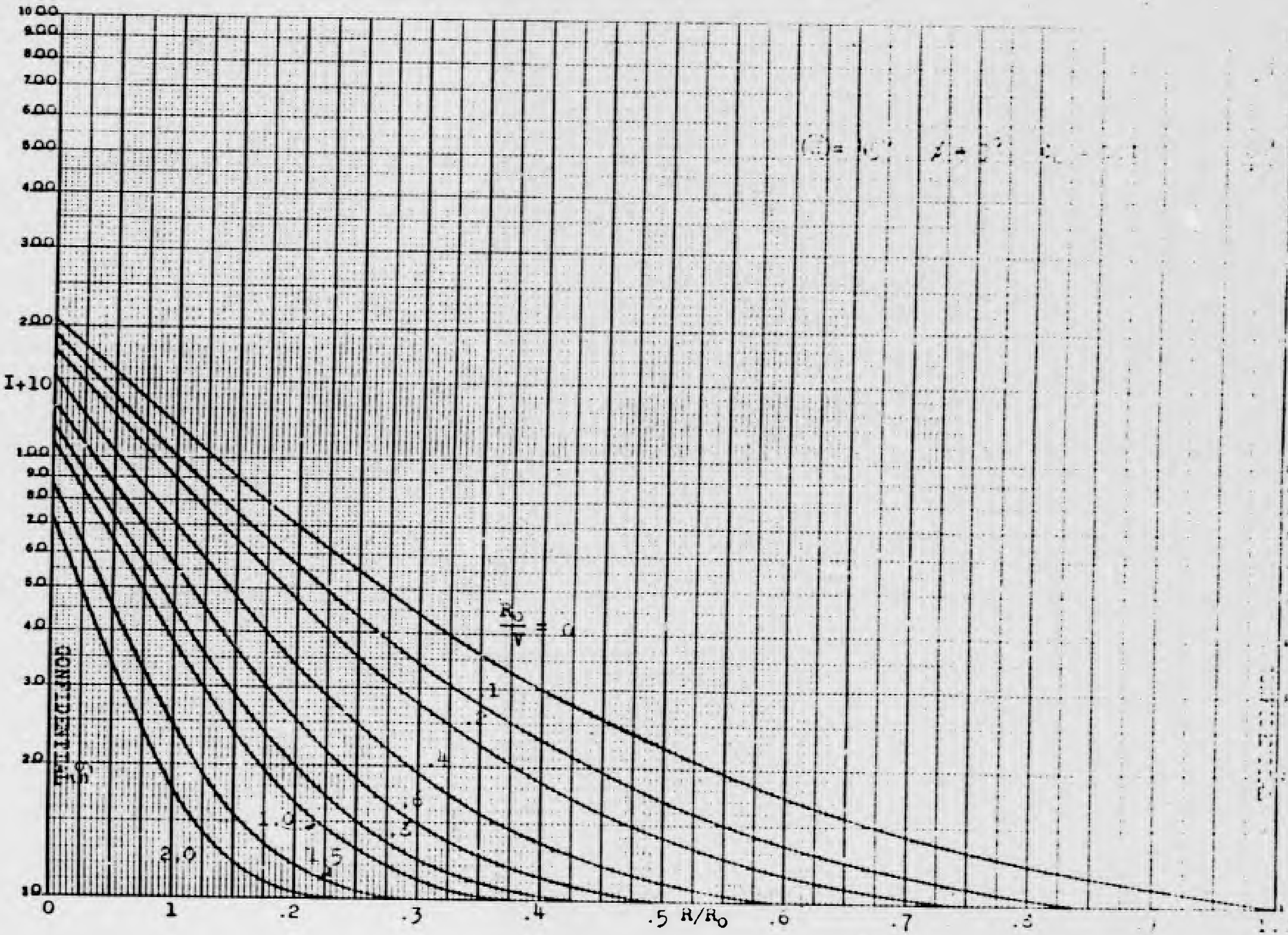


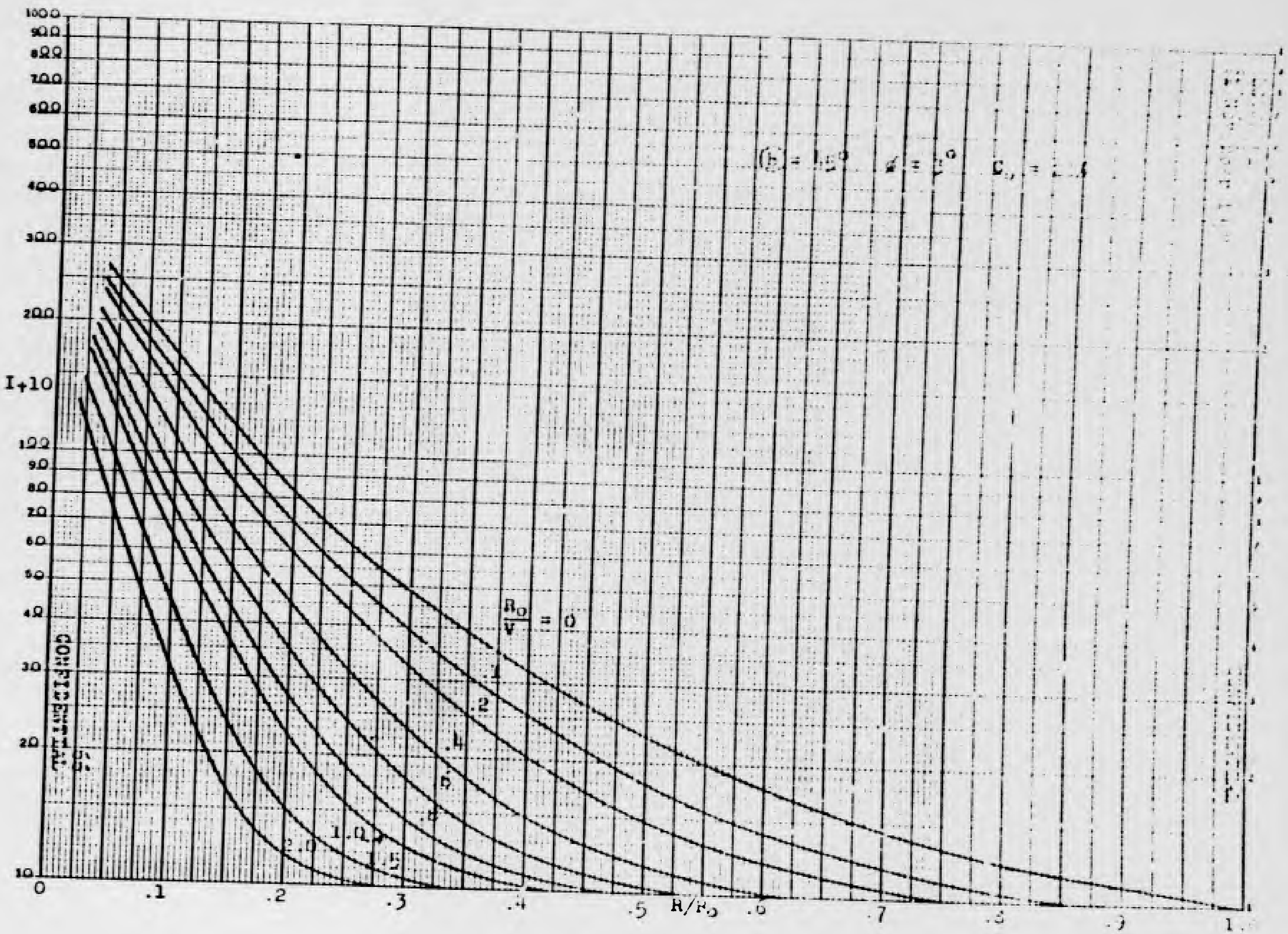
I+10
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
.1
.2
.3
.4
.5 R/R0
.6
.7
.8
.9
1.
R0/V = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL


I+10
R/R₀
0
.1
.2
.3
.4
.5
.6
.7
1.0
10
20
30
40
100
200
300
400
500
600
700
800
900
1000
.1
.2
.4
1.5

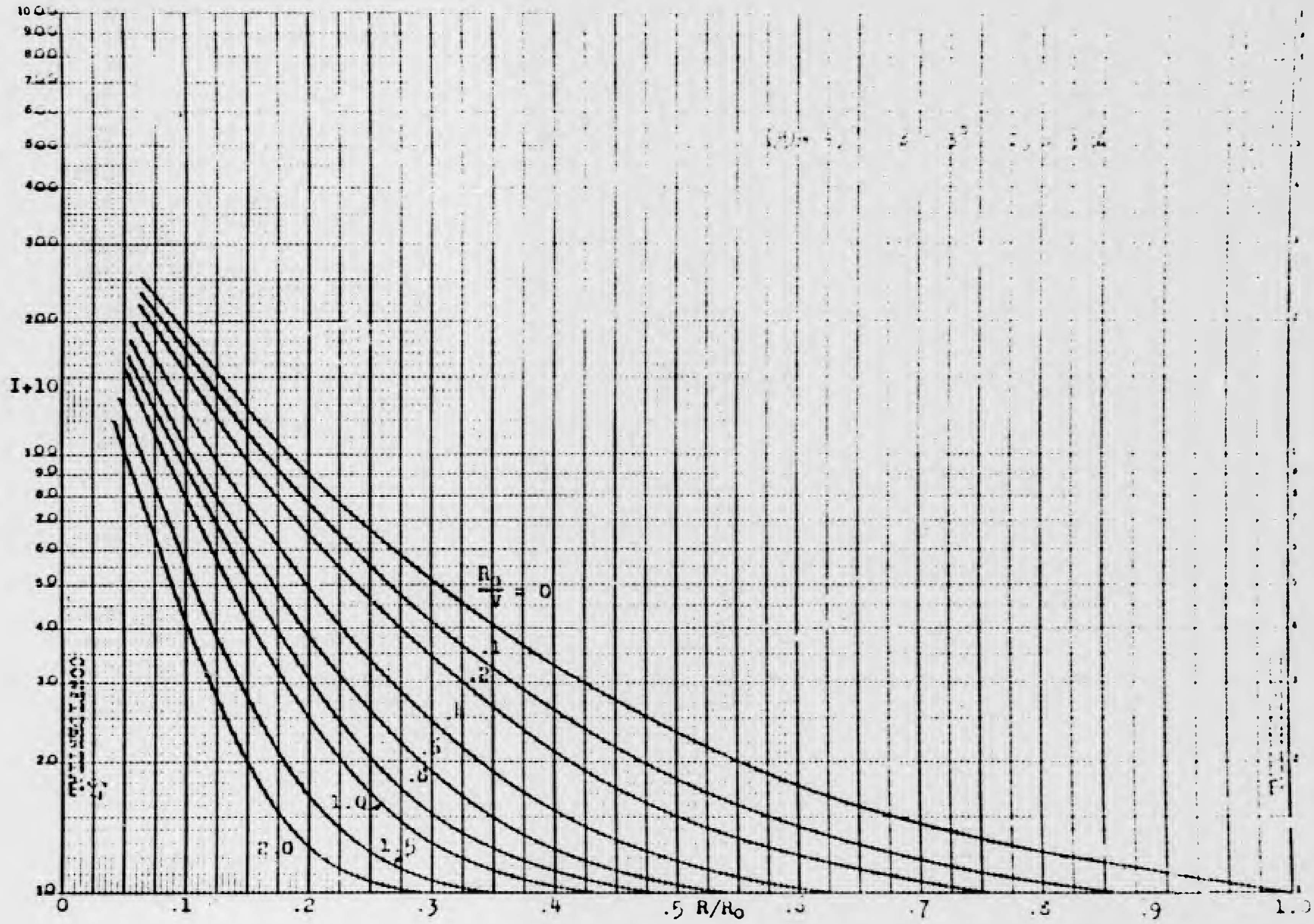

10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
I+10
1.00
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
.1
.2
.3
.4
.5 R/R₀
.6
.7
.8
1.0
CONFIDENTIAL
$\frac{R_0}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0


CONFIDENTIAL
I+10
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.5 R/R0
$\frac{H_0}{V} = 0$
.2
.4
.6
.8
1.0
1.5
2.0

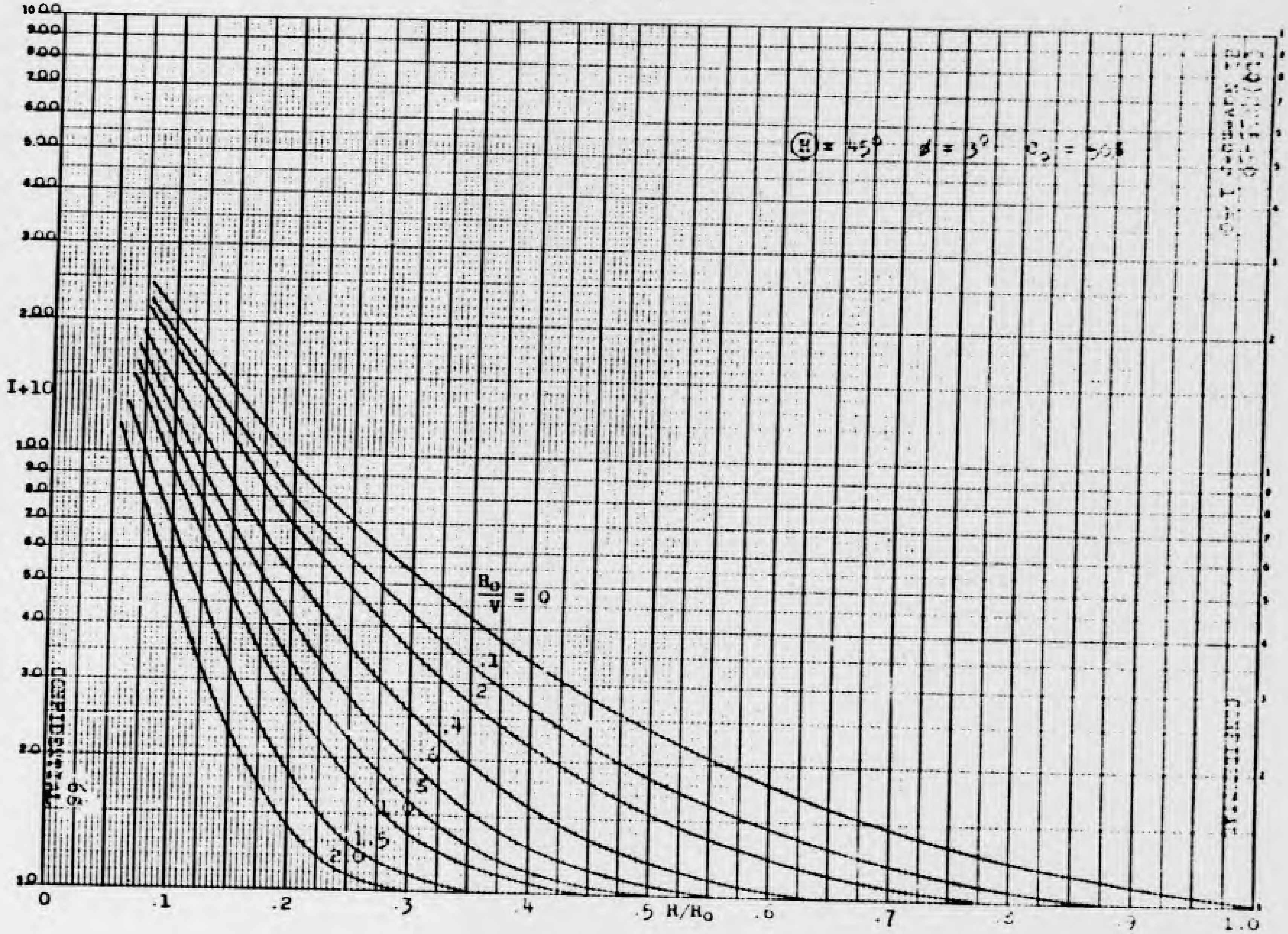

I+10
R/Ro
CONFIDENTIAL



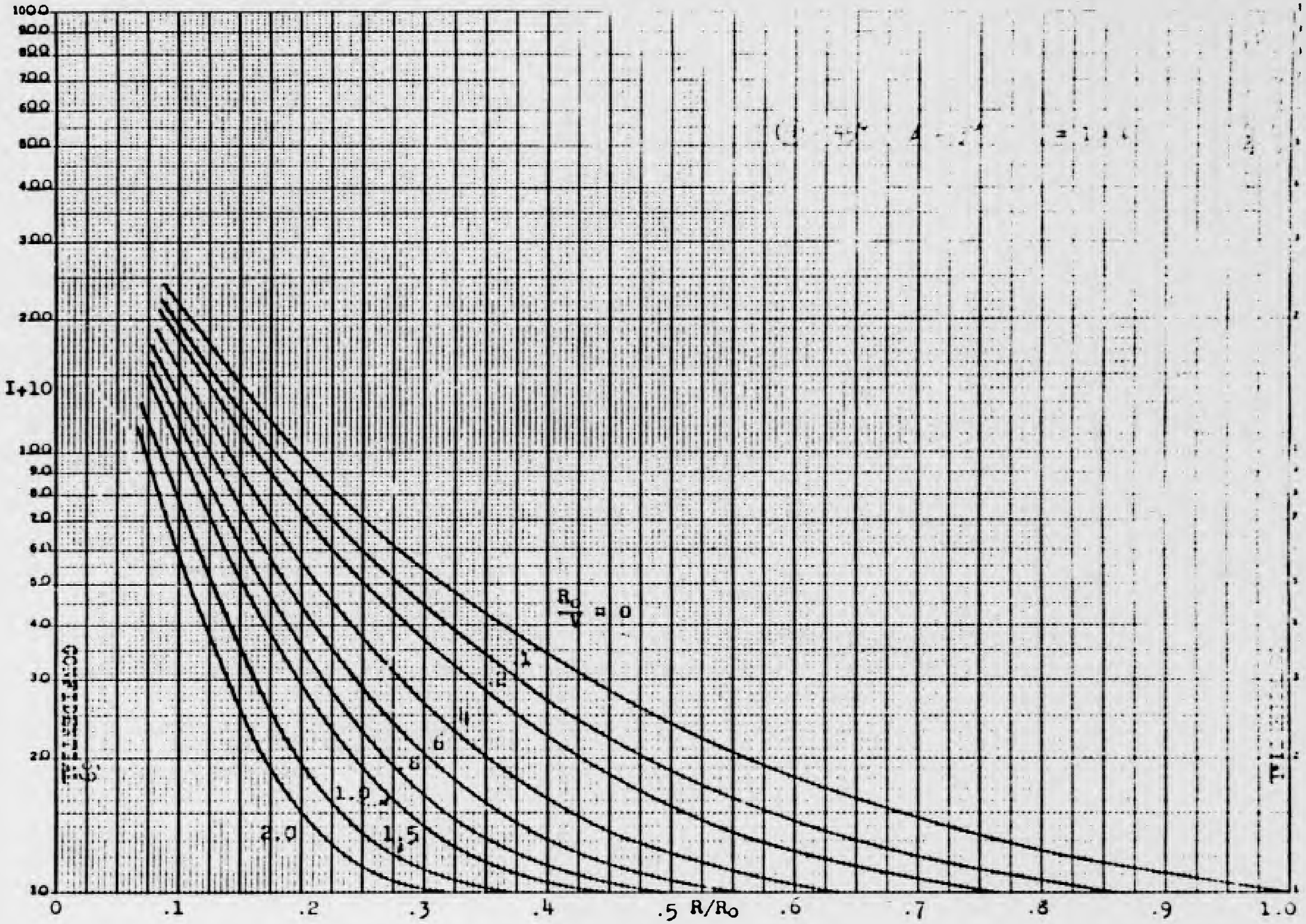
I+10
R/R₀
$\frac{R_0}{V} = 0$
.1
.2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL



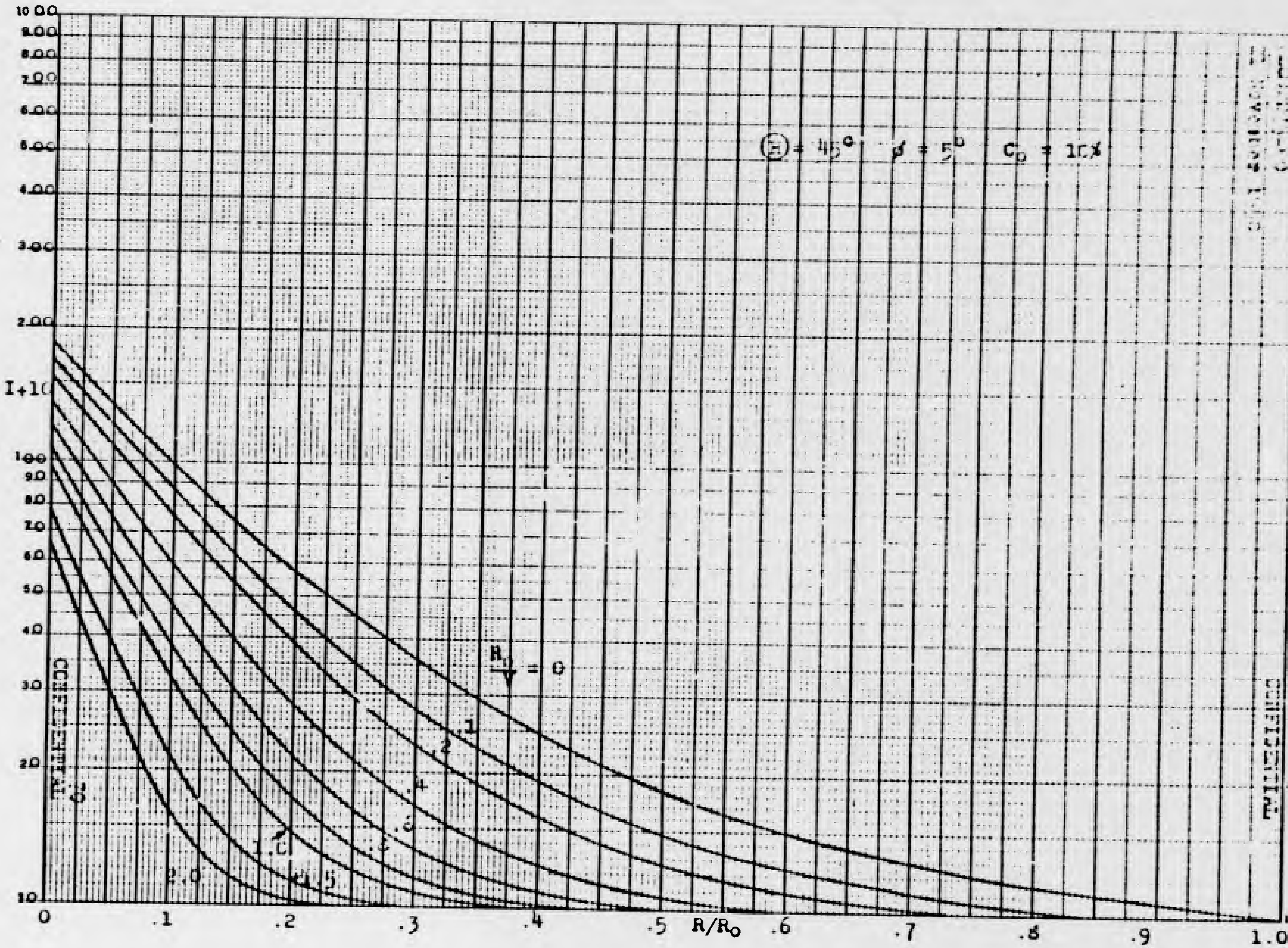
CONFIDENTIAL
I+10
1000
900
800
700
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.5 R/Ro
.7
.9
$\frac{R_0}{V}$ = 0
.1
.2
.5
1.0
1.5
2.0

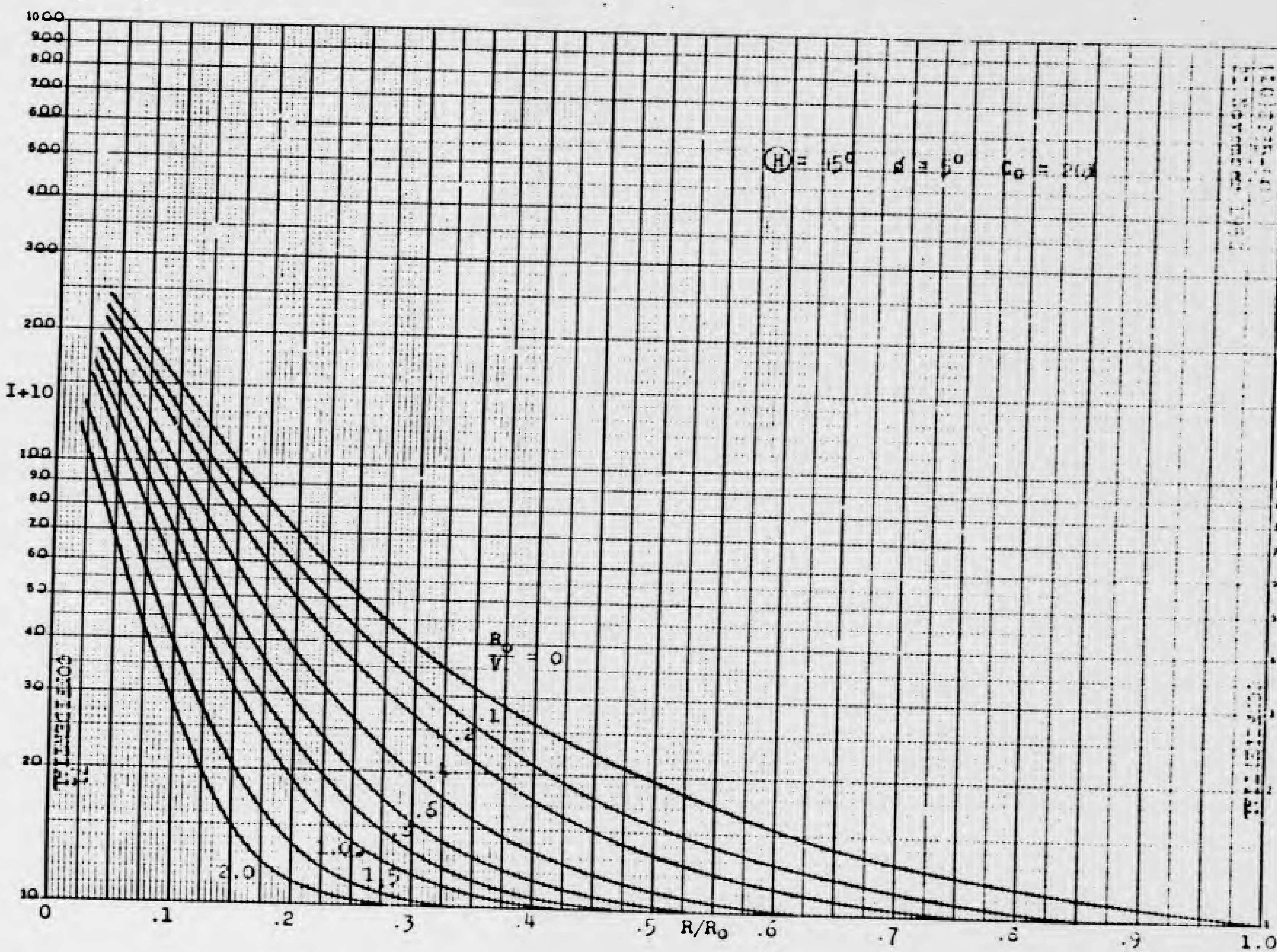


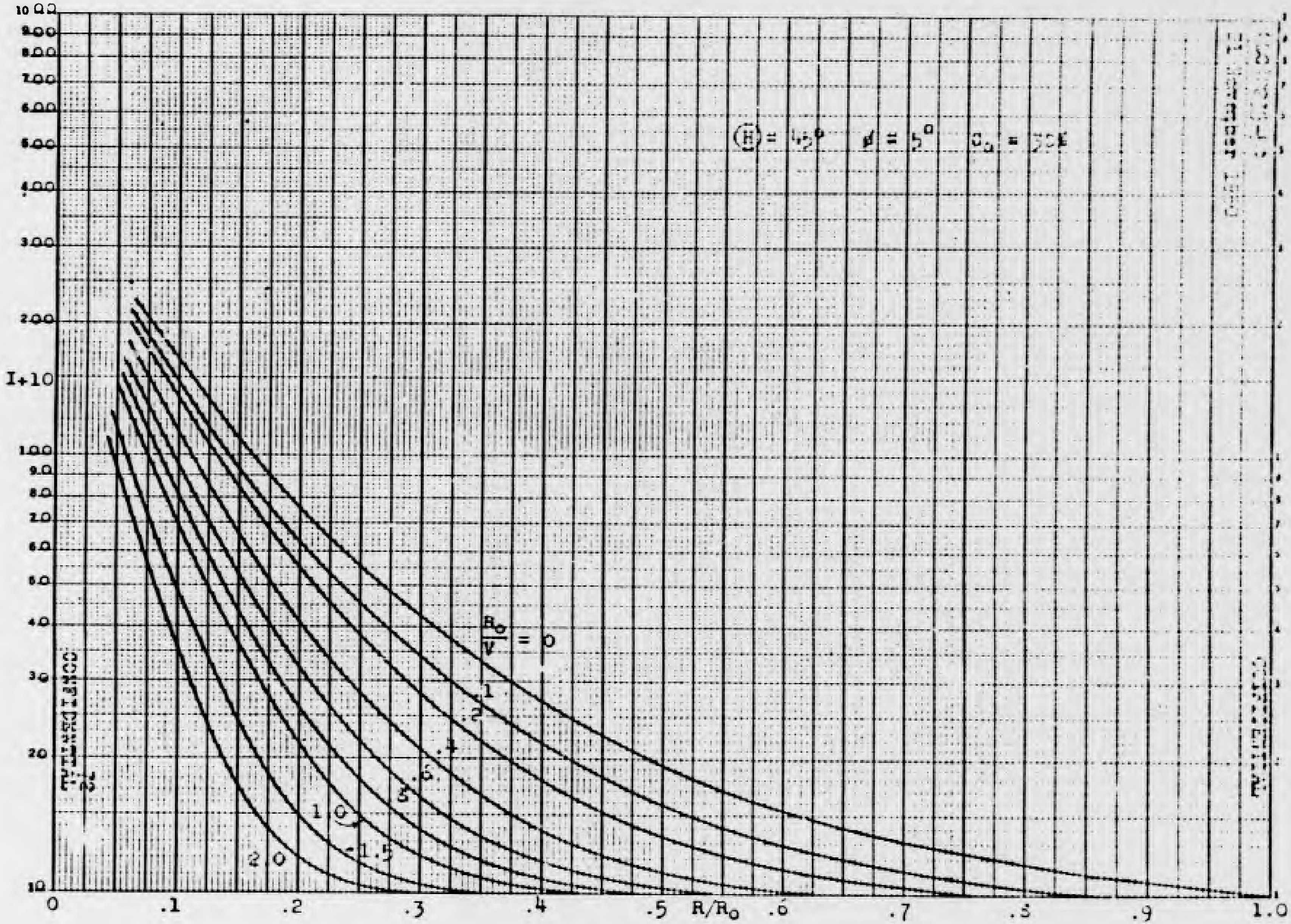
Ⓗ = 45° ∅ = 3°
$\frac{R_o}{V}$ = 0
.1
2
.4
5
1.0
1.5
2.0
I+10
R/R₀
CONFIDENTIAL

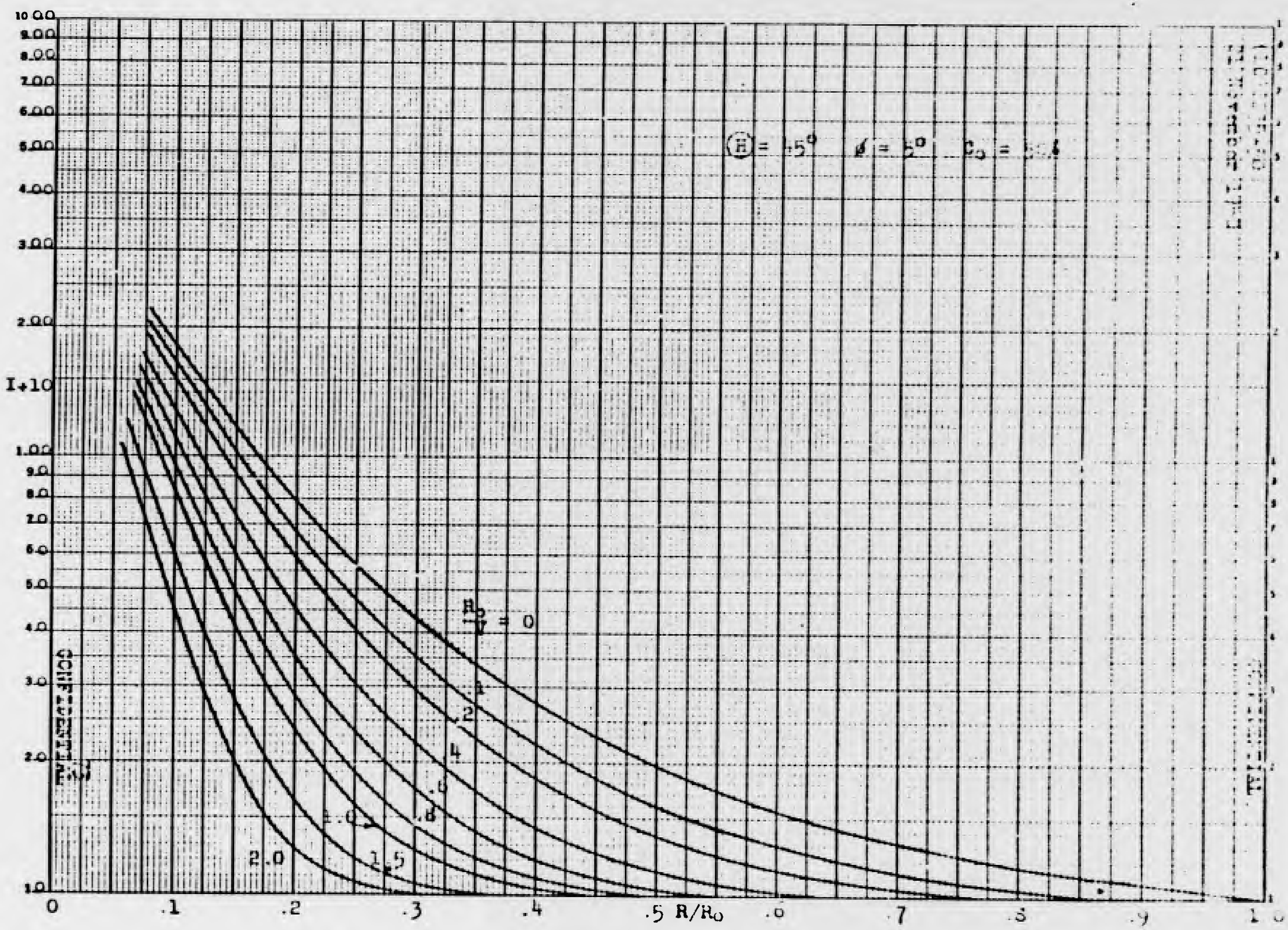


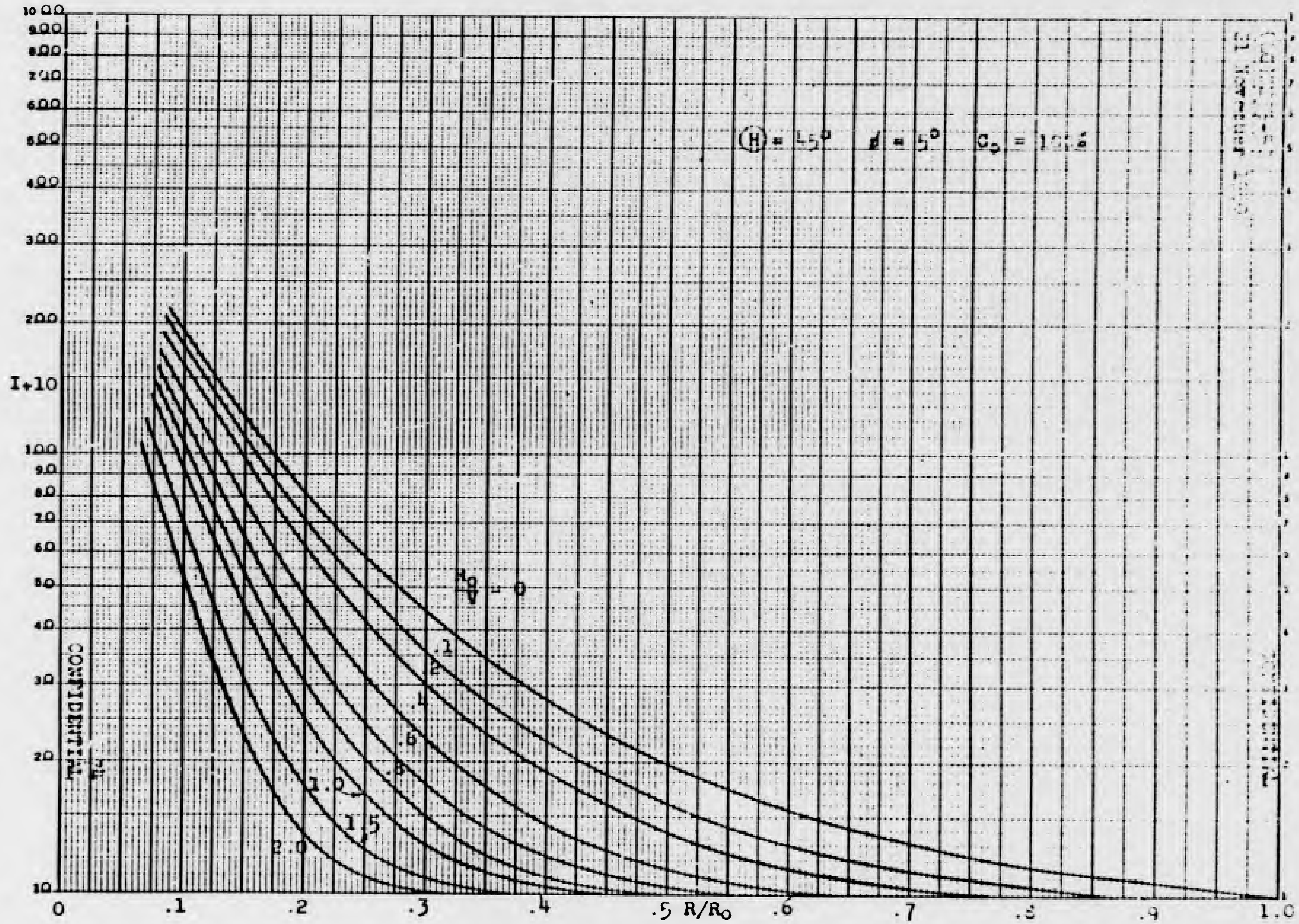
CONFIDENTIAL
I+10
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.5 R/Ro
.6
.7
.8
.9
1.0
Ro/V = 0
1.0
1.5
2.0


CONFIDENTIAL
(Θ) = 45°   ø = 5° C_D = 10%
$\frac{R_1}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
I+10
R/R_o
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
CONFIDENTIAL

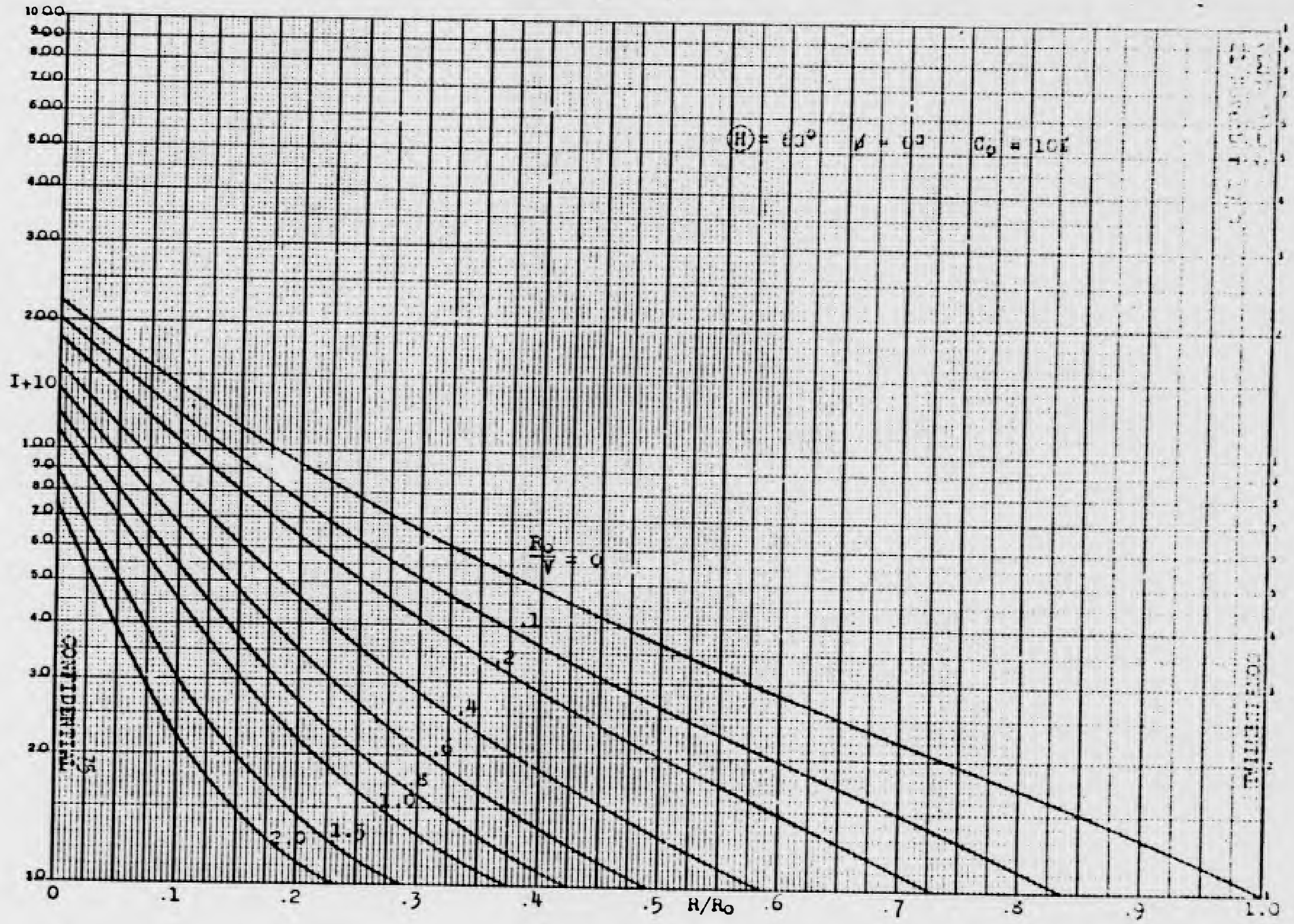

CONFIDENTIAL
CONFIDENTIAL
(H) = 15° ∅ = 5° C_o = 20
I+10
R/Ro
$\frac{R_o}{V}$ = 0
.1
.2
.4
.6
1.0
1.5
2.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000


CONFIDENTIAL
21 November 1960
Ⓗ = 45° ø = 5°
I+10
R/Ro
$\frac{R_o}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL
72

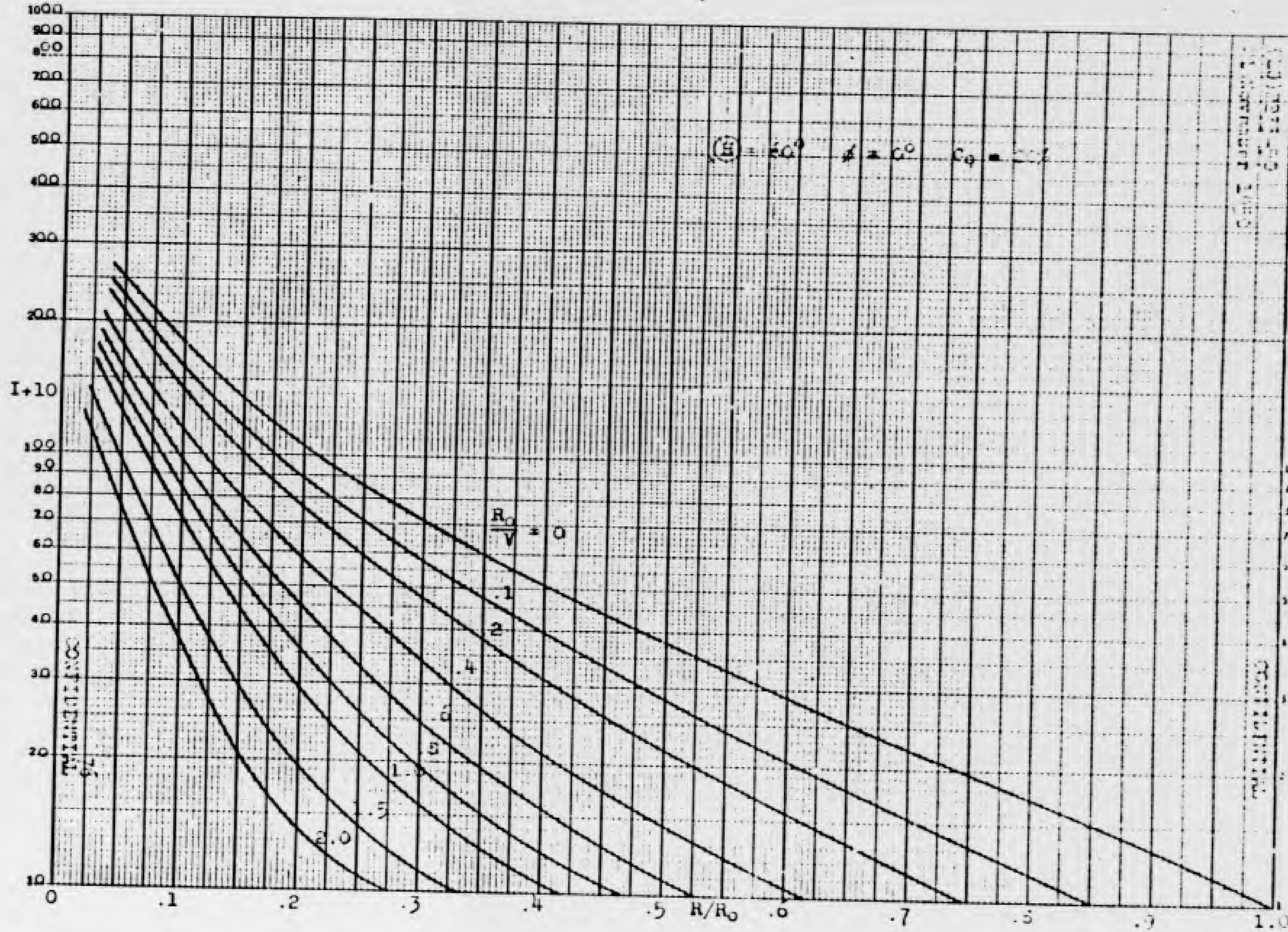

CONFIDENTIAL
I+10
R/Ro
2.0
1.5
.8
.4
.2

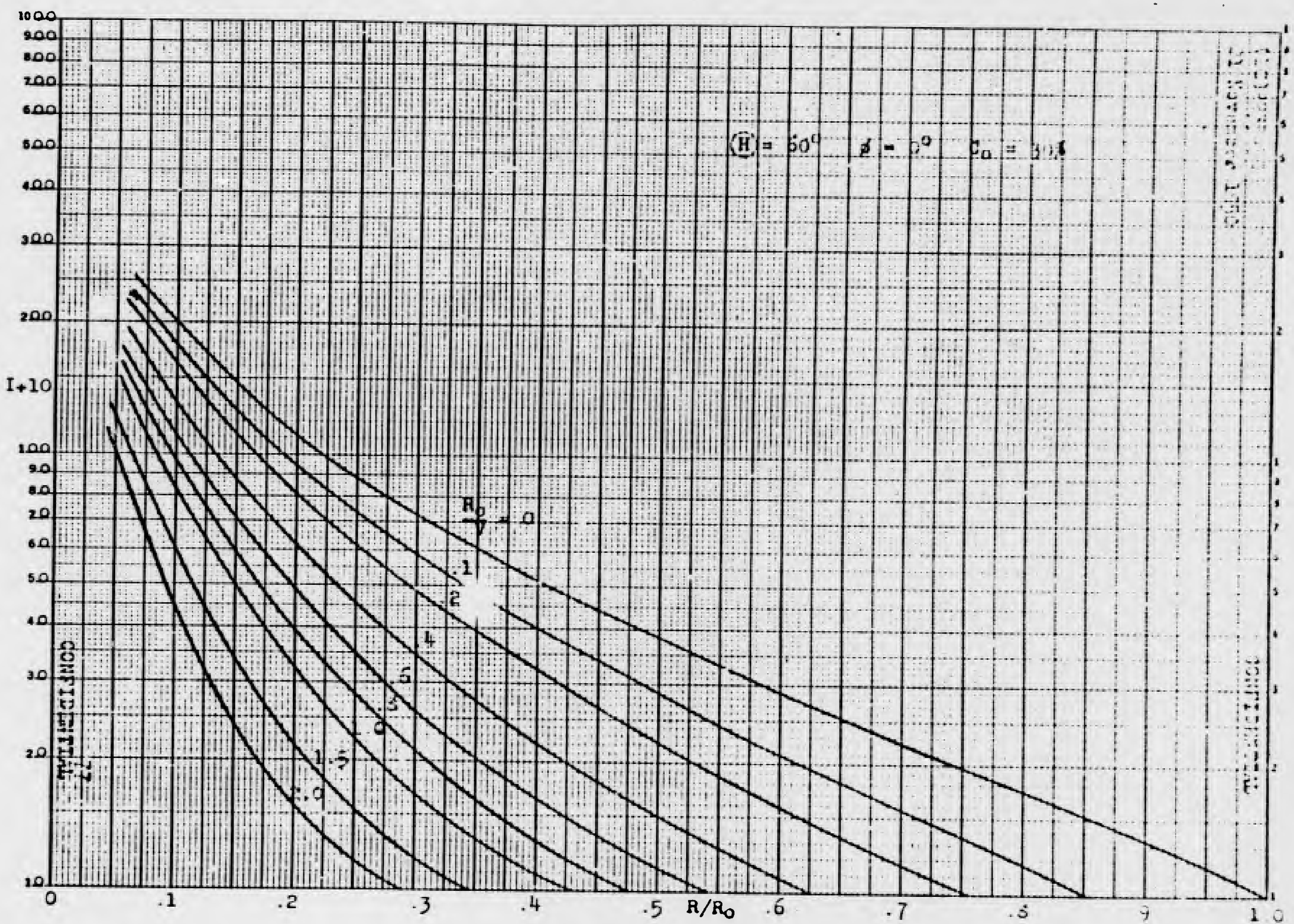


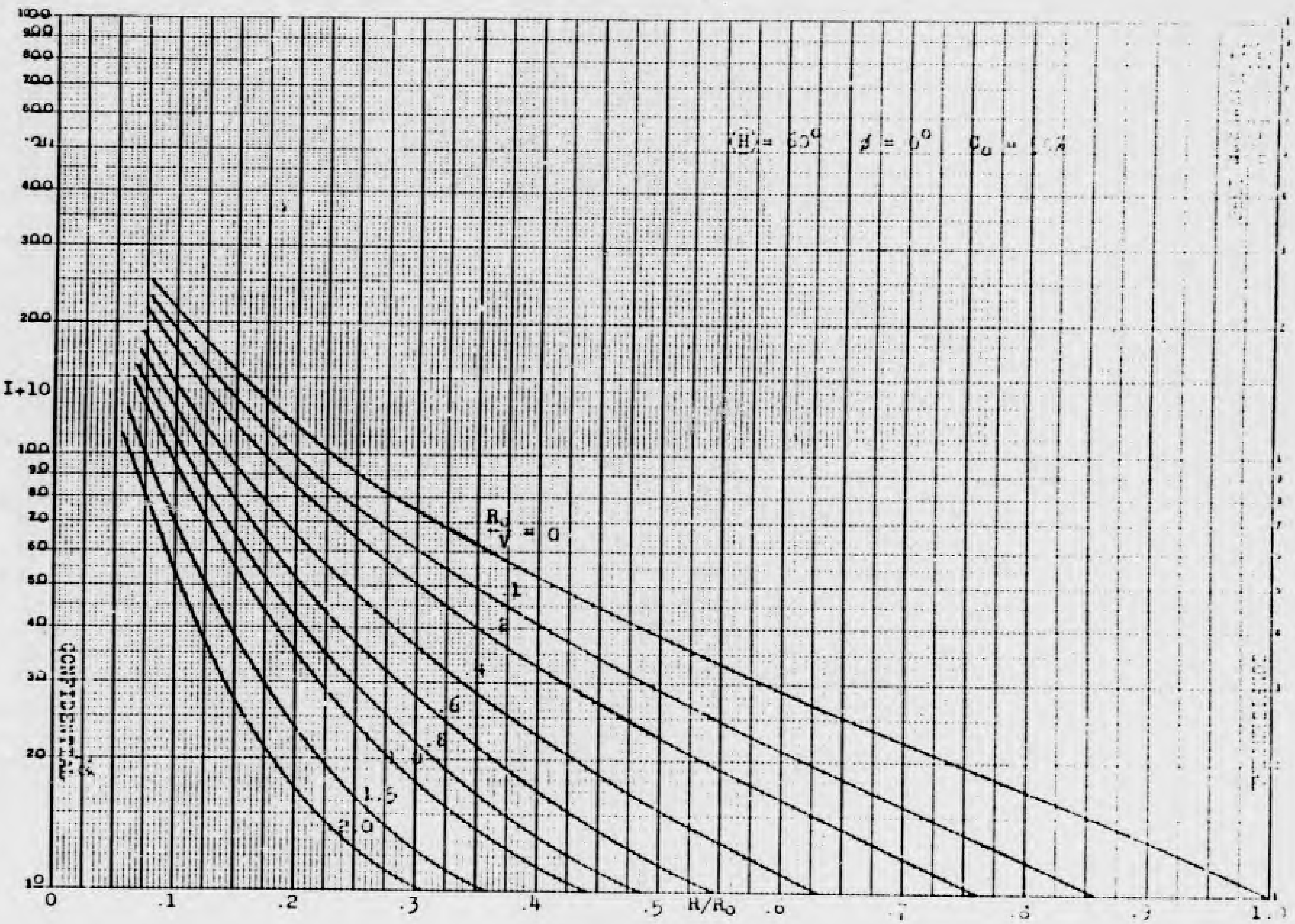
CONFIDENTIAL
Ⓗ = 45° ø = 5°
I+10
R/Ro
Ho/V = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

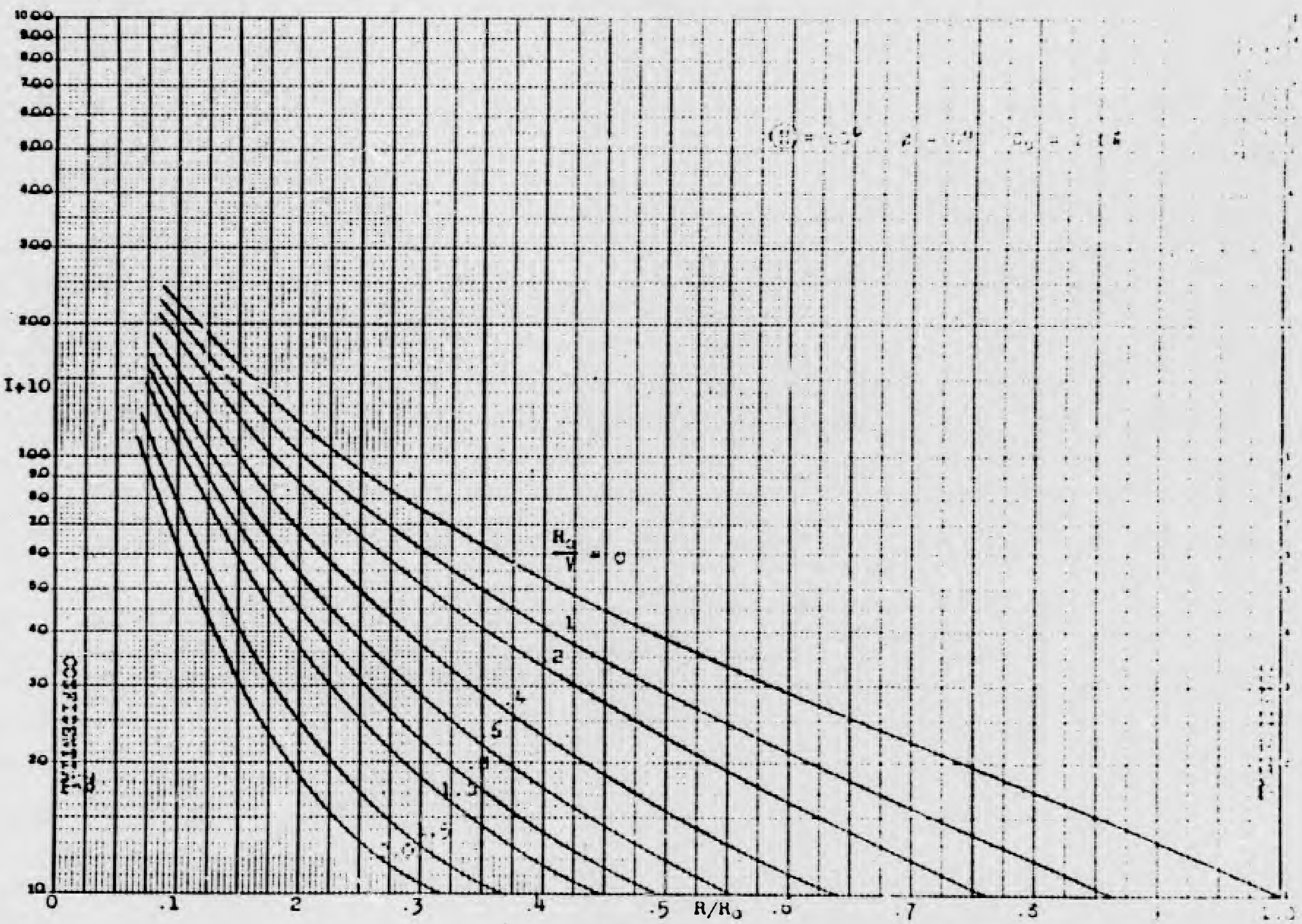


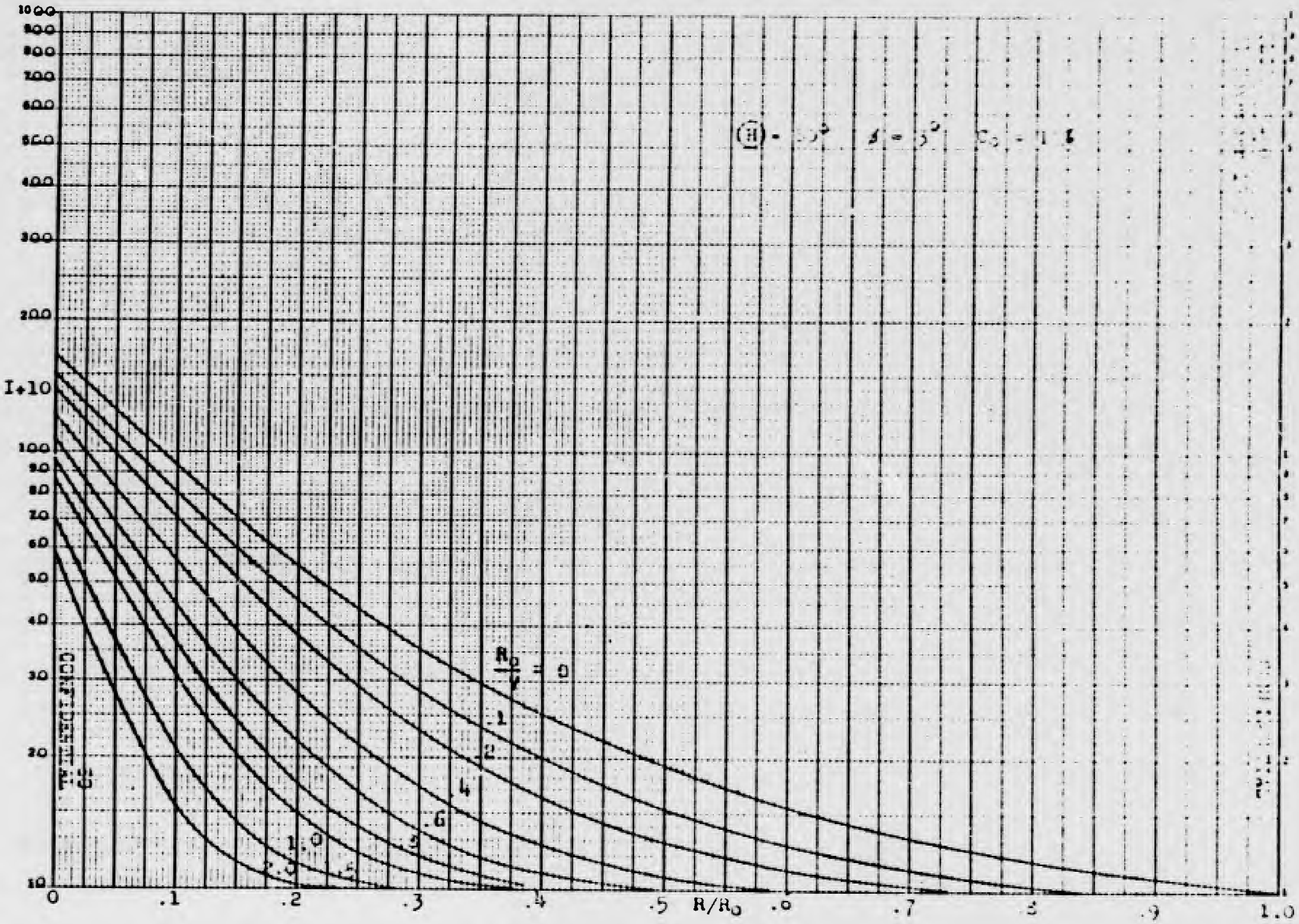
(H) = 60° ø = 0° C_D = 10%
$\frac{R_0}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
I+10
R/Ro
CONFIDENTIAL
CONFIDENTIAL
75


CONFIDENTIAL
I+10
R/Ro
$\frac{R_o}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
200.0
300.0
400.0
500.0
600.0
700.0
800.0
900.0
1000.0

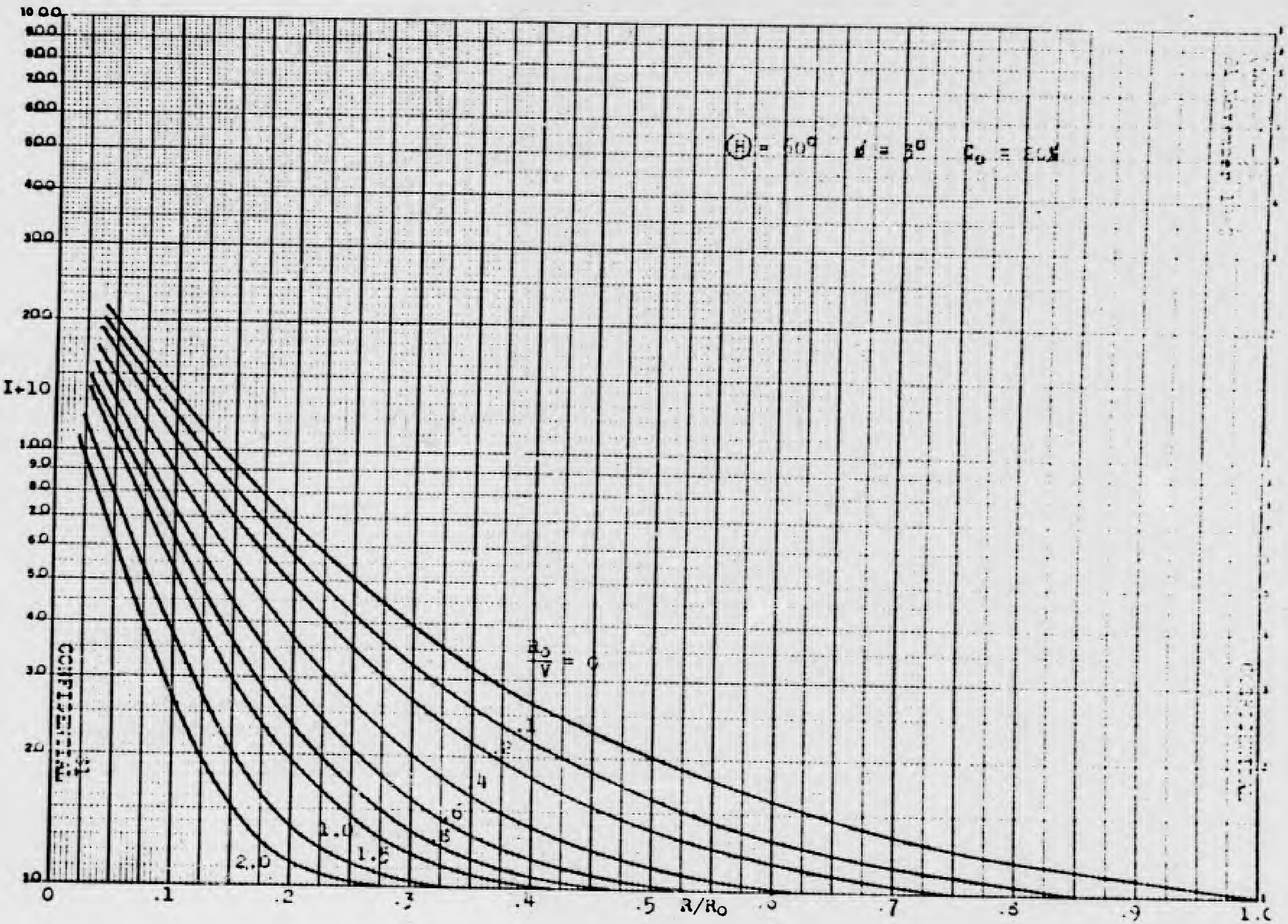

CONFIDENTIAL
(H) = 60° ø = 0° C_a = 30%
$\frac{R_o}{V}$ = 0
.1
.2
.4
.5
.8
1.0
1.5
2.0
I+10
R/Ro
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
200.0
300.0
400.0
500.0
600.0
700.0
800.0
900.0
1000.0
CONFIDENTIAL
77

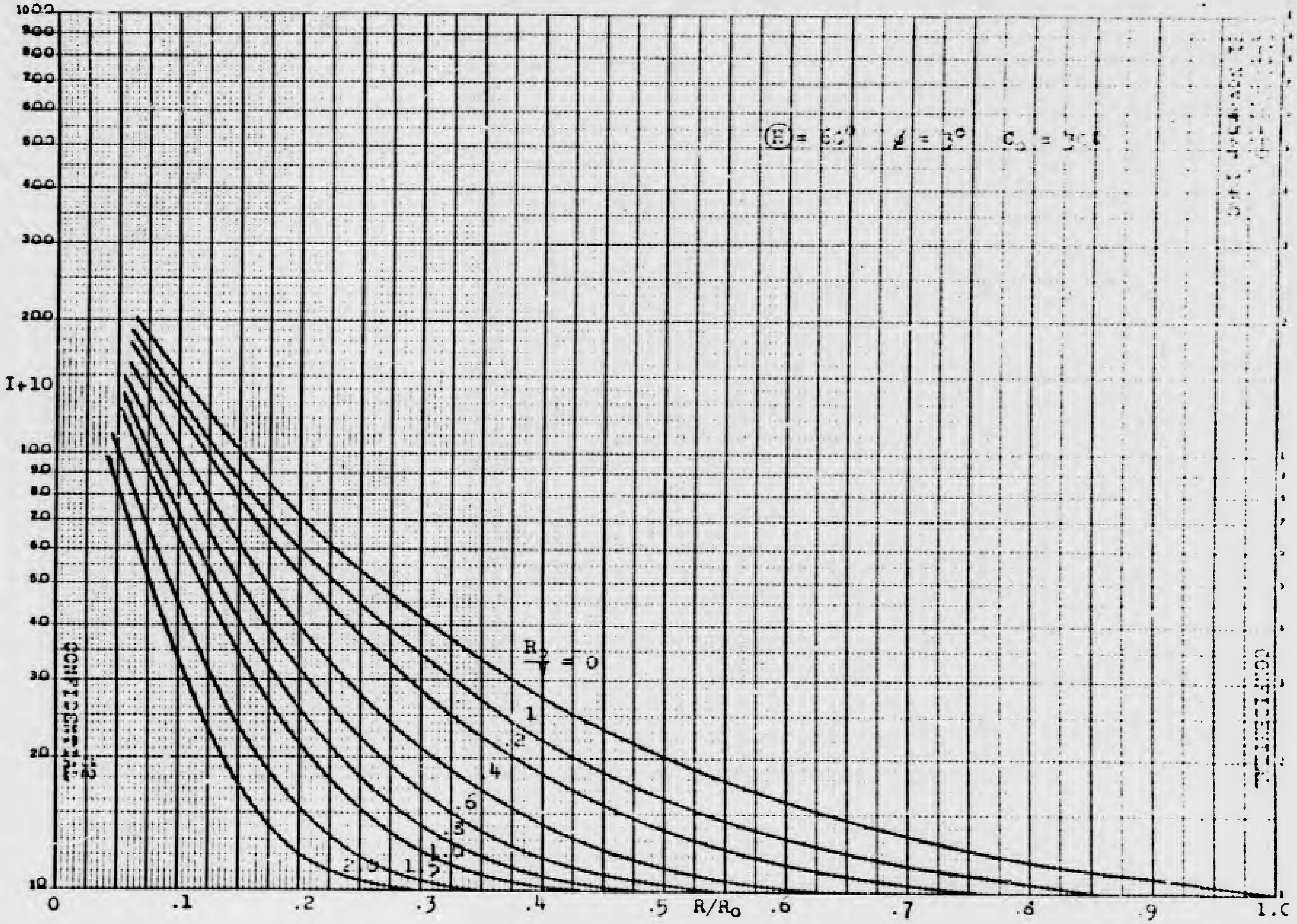

I+10
H/Ro
$\frac{R_j}{V}$ = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL

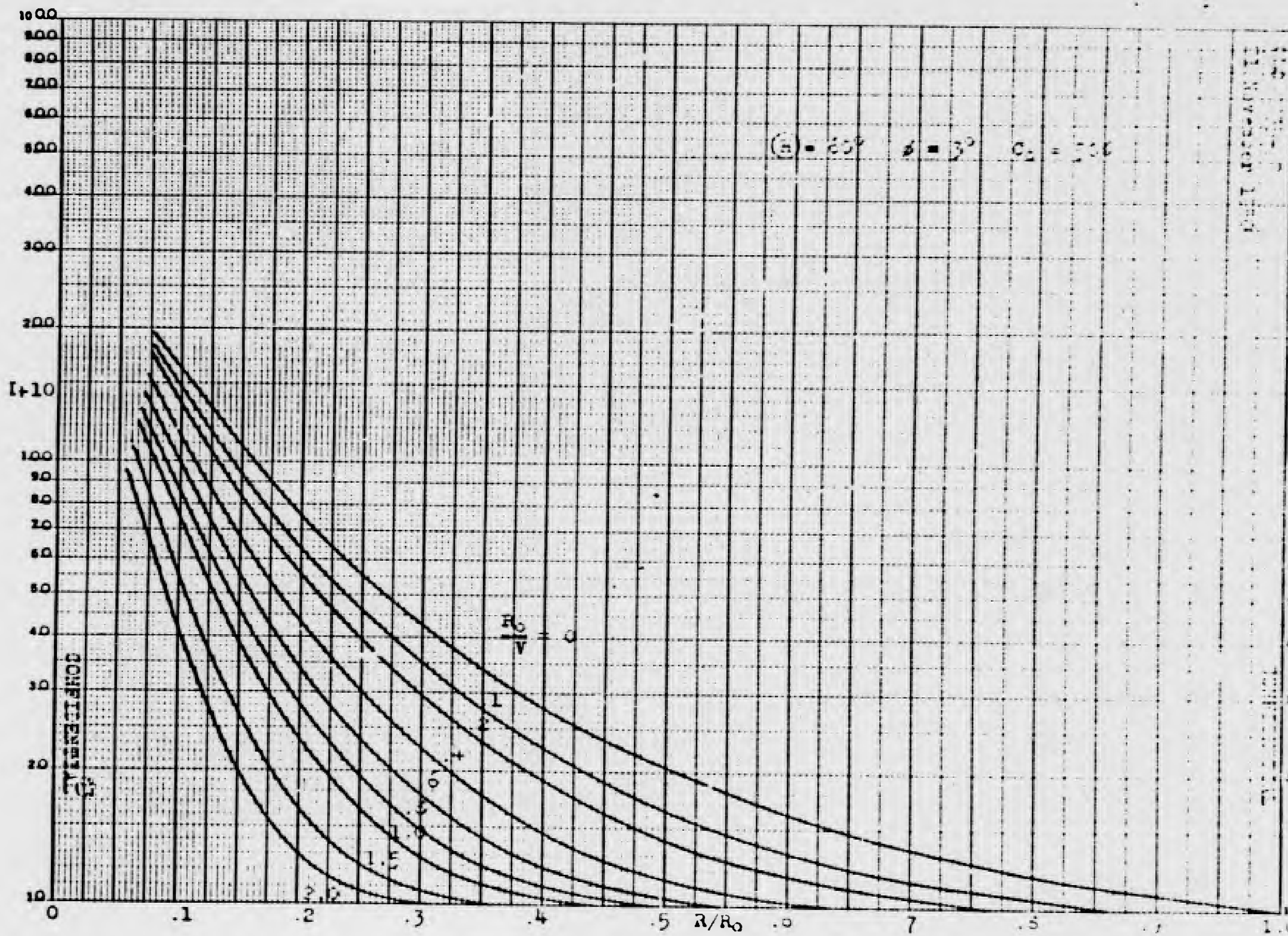

CONFIDENTIAL
I+10
R/R₀
R₁/V = 0
1
2
.4
.5
1.0
1.5
2.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0



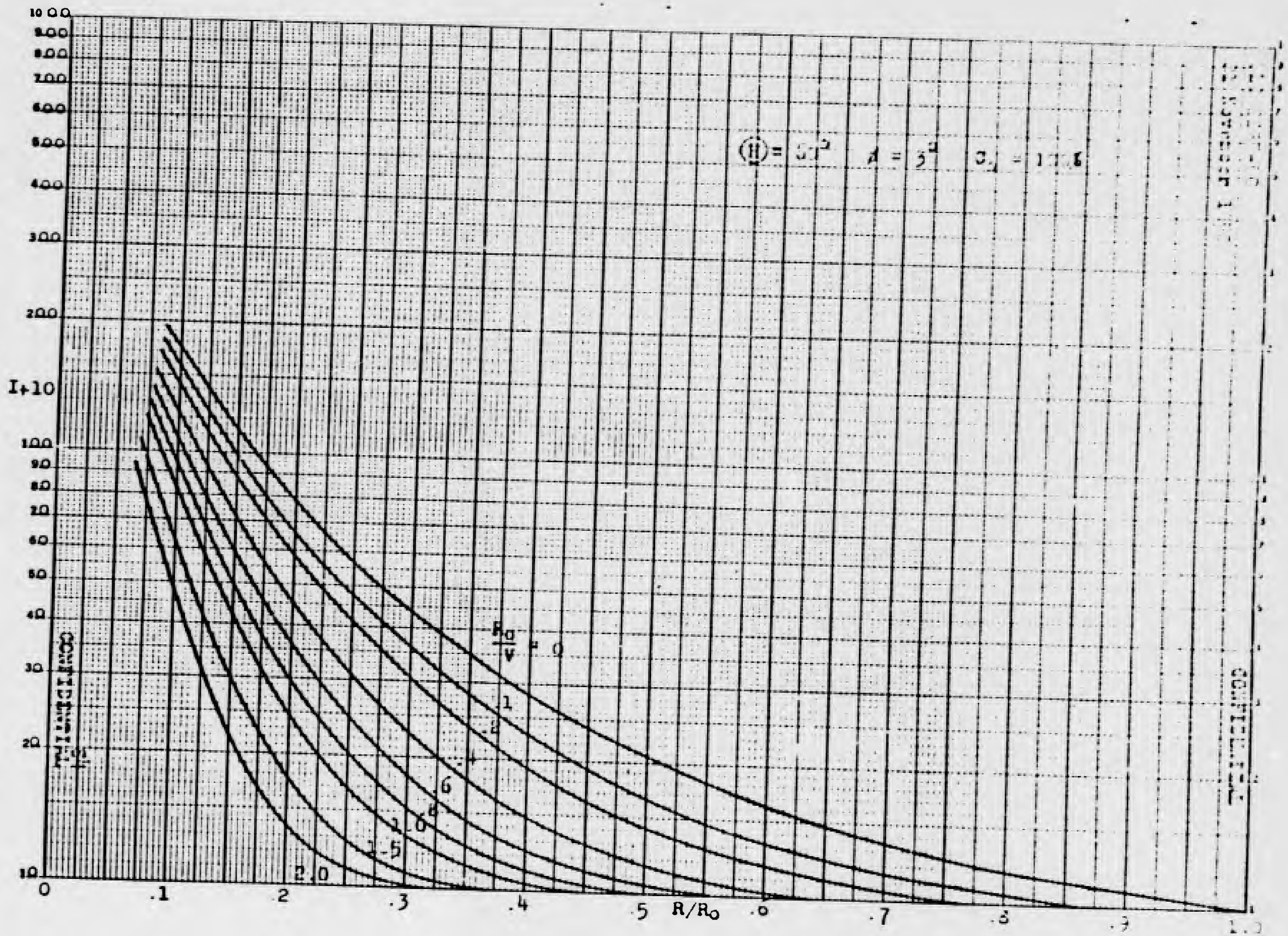
I+10
R/R0
CONFIDENTIAL
Ro/V = 0
.1
.2
.4
.6
1.0
0
.1
.2
.3
.4
.5
.7
.9
1.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000


I+10
R/Ro
CONFIDENTIAL
CONFIDENTIAL
1.0
1.5
2.0
4

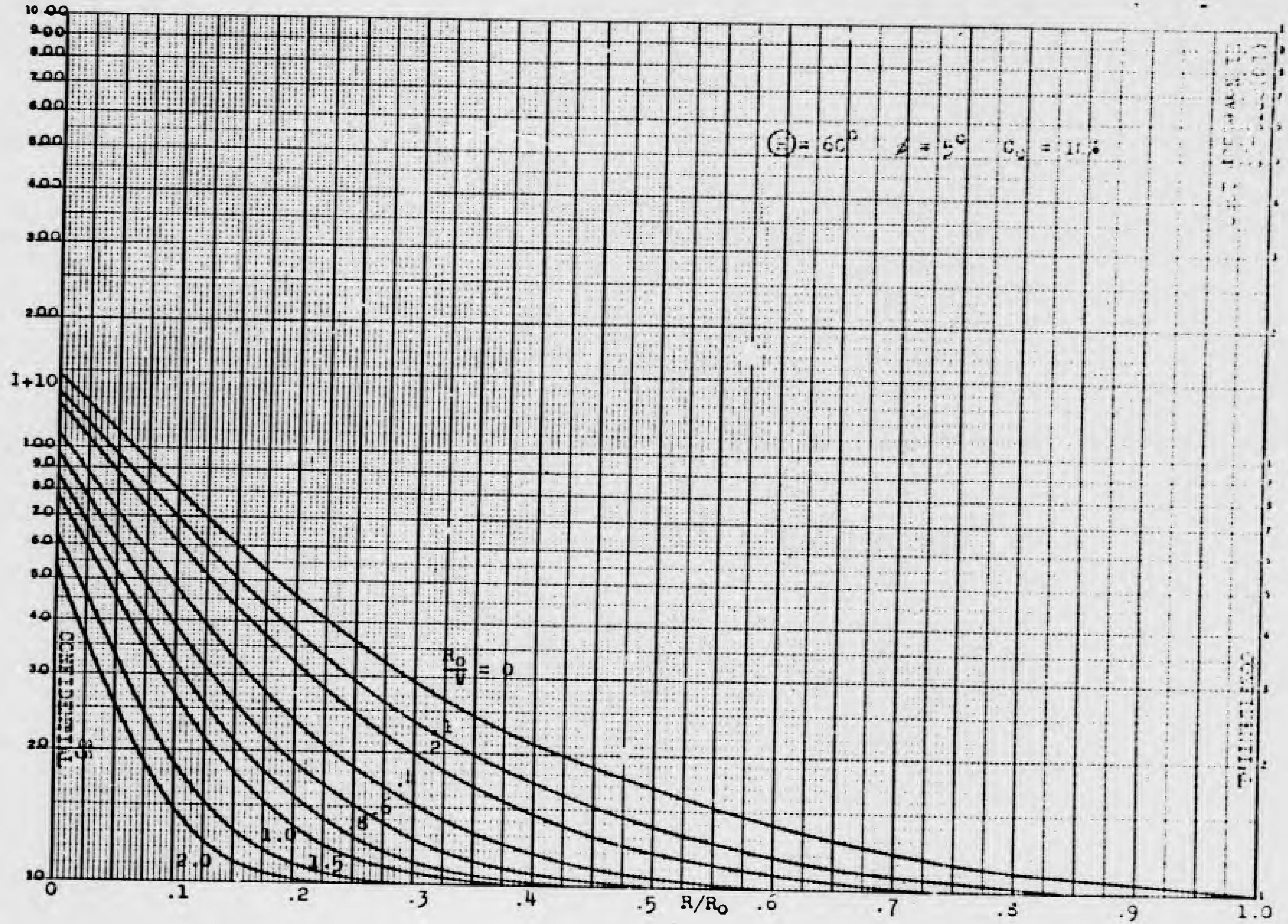

CONFIDENTIAL
I+10
R/R₀
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
1
.2
4
.6
.8
1.0
1.2
1.5
2
CONFIDENTIAL



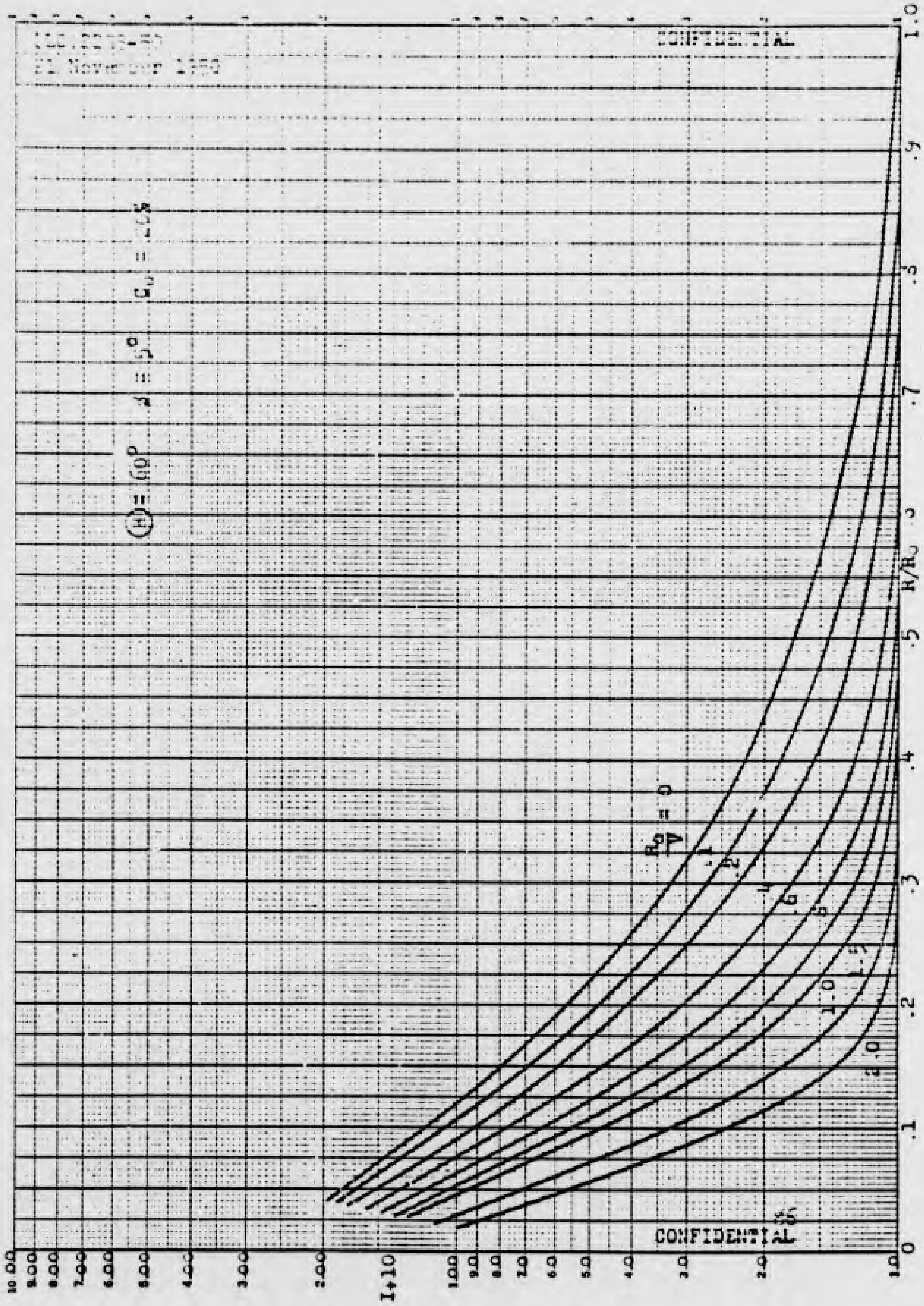
I+10
R/R_0
$\frac{R_0}{V} = 0$
1
2
1.5
2.0
CONFIDENTIAL

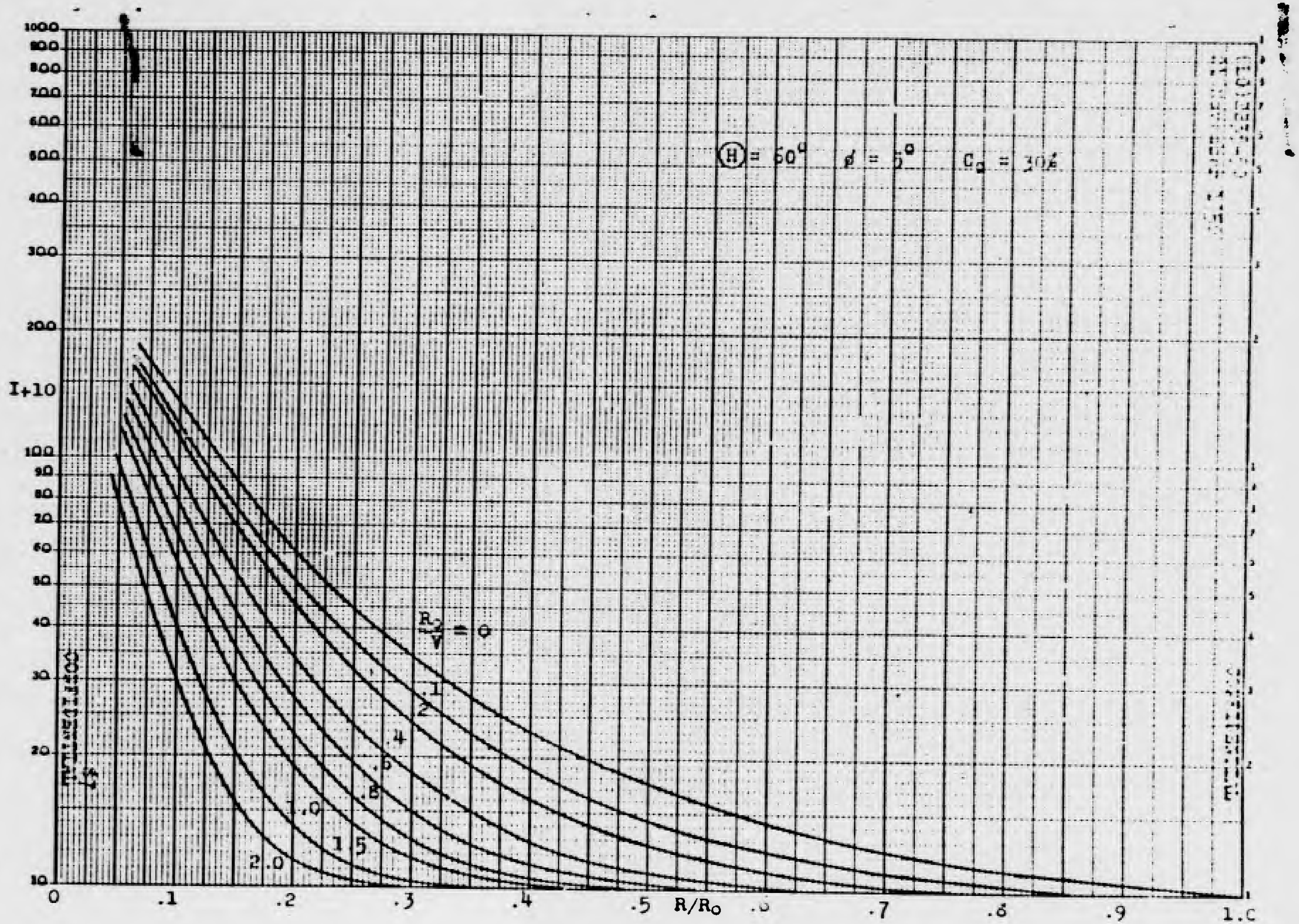


$I+10$
R/R_0
$\frac{R_0}{V} = 0$
.1
.2
.4
.6
.8
1.0
1.5
2.0
CONFIDENTIAL

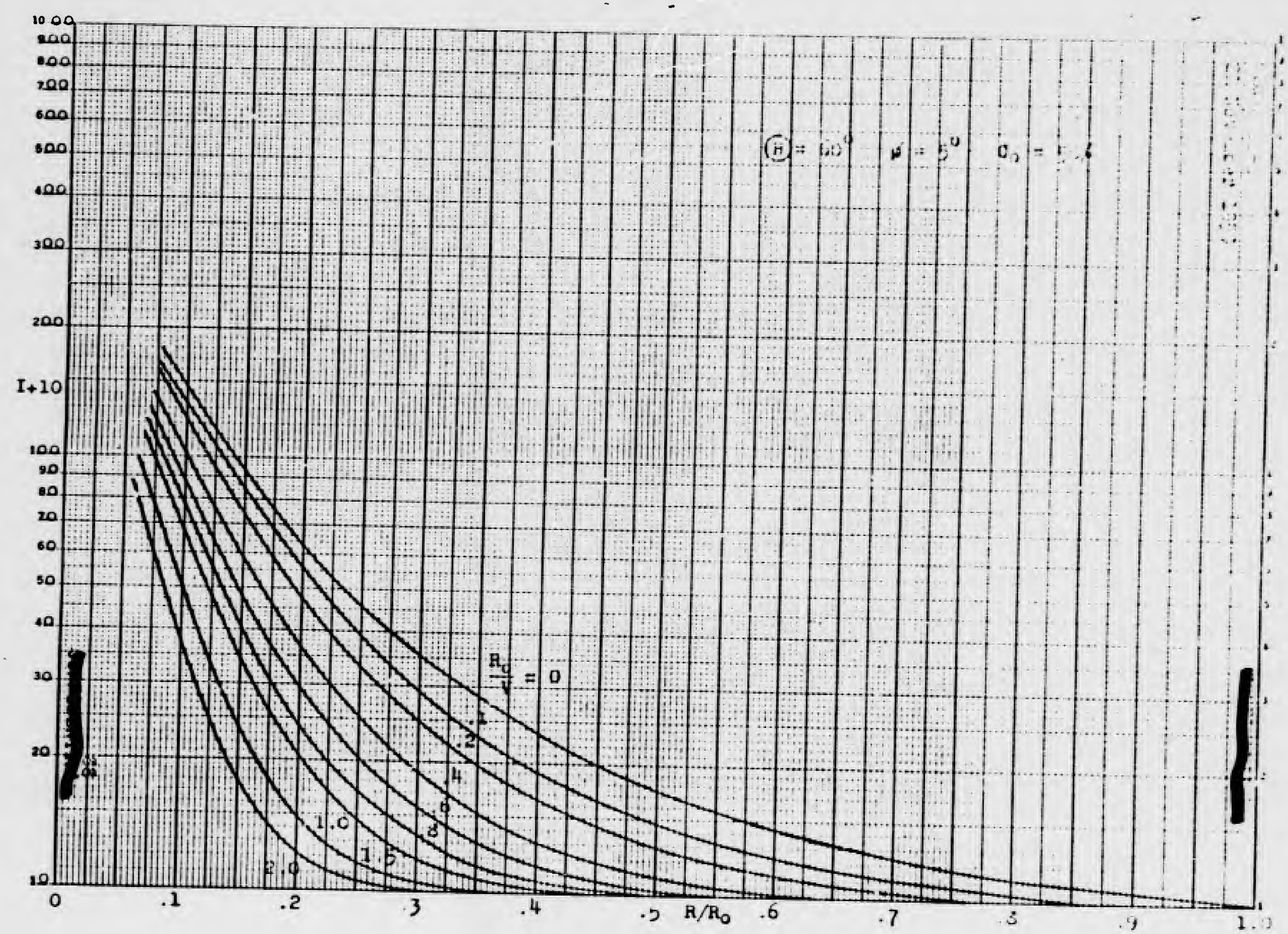


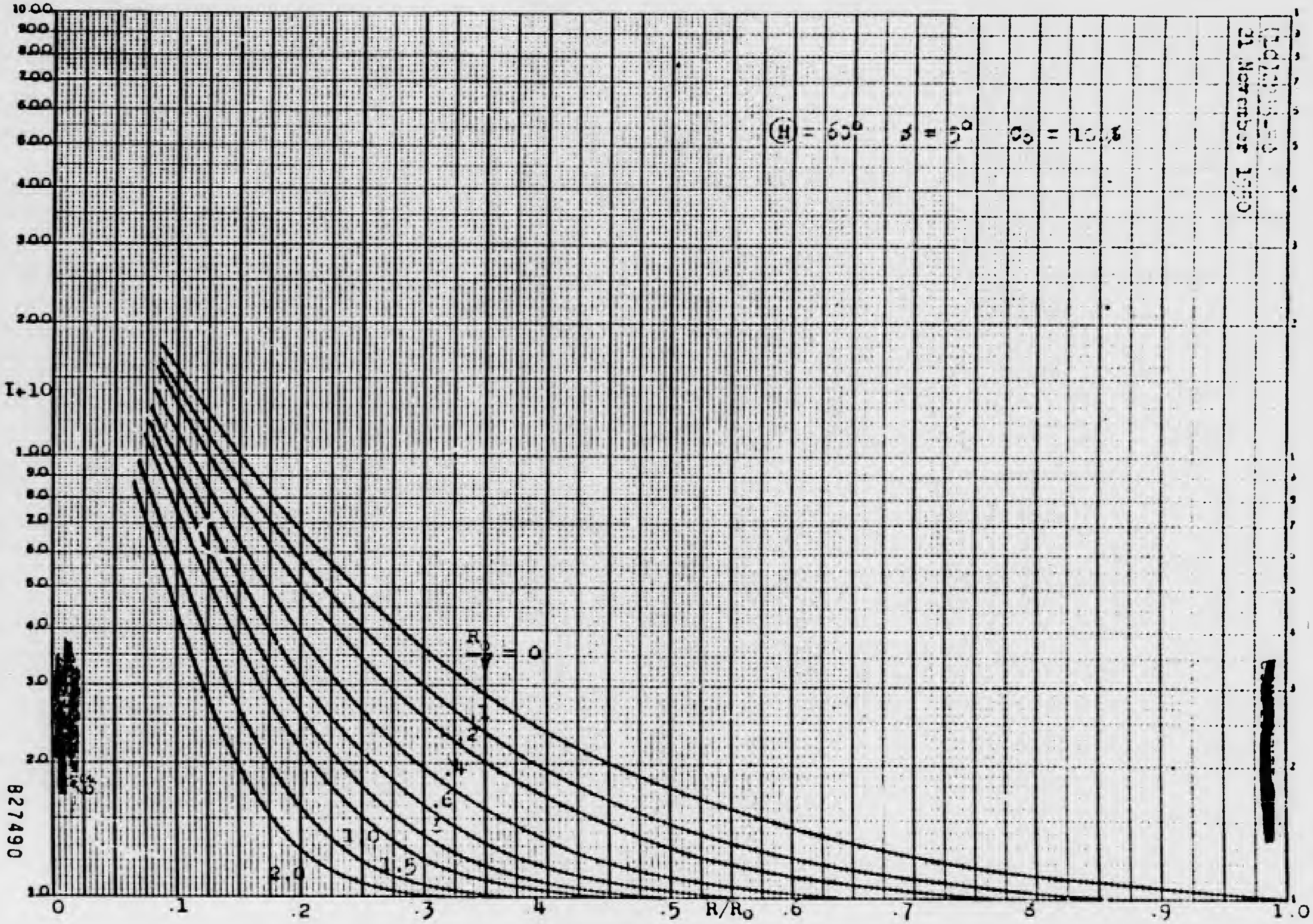
CONFIDENTIAL
CONFIDENTIAL
Θ = 60° ∅ = 5° C_v = 10%
$\frac{R_o}{V}$ = 0
.2
.1
.5
.8
1.0
1.5
2.0
1+10
R/Ro
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0


CONFIDENTIAL
(H) = 60°
$\frac{R_0}{V} = 0$
.1
.2
.4
.6
.8
1.0
1.5
2.0
I+10
R/R0
0
.1
.2
.3
.4
.5
.7
.9
1.0
10.00
5.00
4.00
3.00
2.00
1.00
CONFIDENTIAL



CONFIDENTIAL
H = 60° ø = 5° Cd = 30%
I+10
R/Ro
R/V = 0
.1
.2
.4
.6
.8
1.0
1.5
2.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
CONFIDENTIAL


I+10
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
.1
.2
.3
.4
.5
R/Ro
.6
.7
.9
1.0
$\frac{R_0}{V} = 0$
.2
.4
.6
.8
1.0
1.5
2.0


21 November 1950
[redacted]
Ⓗ = 60°   δ = 5° C_0 = 10.8
I+10
R/R₀
$\frac{R_0}{\nu}$ = 0
.1
.2
.4
.6
.7
1.0
1.5
2.0
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
[redacted]

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