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BINARY DETECTION OF RANDOMLY OCCURRING SIGNALS

Defines a statistical procedure for detecting signals occurring at unknown times; describes implementation and procedures for calculating the probability of detection.

George M. Dillard   Research and Development Report   30 November 1970

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PROBLEM

Obtain design procedures and criteria for reliable Navy communication in a hostile, an adverse, or an unstable environment, through use of statistical decision methods and other signal processing techniques. The specific problem with which this report is concerned is the detection of signals which occur at random (unknown) times.

RESULTS

1. The binary moving-window detector is a feasible method of detecting signals which occur at random times.

2. The statistics of the moving-window detector can be obtained by an exact computational method without resorting to Monte Carlo simulation.

3. The binary moving-window detector can be easily implemented for use in existing and future communications systems.

RECOMMENDATIONS

1. Investigate existing communications systems and plans for future systems for possible application of the binary moving-window detector as an inexpensive and effective means of detecting communications signals.

2. Extend the analysis to cover multilevel quantization of data.

3. Investigate methods of obtaining analog moving-window detectors (to avoid the loss due to quantization) which can be simply implemented.

4. Analyze the use of distribution-free statistical procedures in conjunction with moving-window detectors.

ADMINISTRATIVE INFORMATION

Work was performed under ZFXX-212-001 (NELC Z222) in the Decision and Control Technology Division and under J0-0768413 (NELC fellowship in the Department of Applied Physics and Information Science, University of California at San Diego). The report covers work from January to August 1970 and was approved for publication 30 November 1970. The detection device described in this report is covered by U. S. Patent No. 3,517,172, "Moving-Window Detector for Binary Integration," Navy Case No. 42,878, issued on 23 June 1970.

CONTENTS

ILLUSTRATIONS

INTRODUCTION

Much of the literature on statistical signal detection is concerned with the problem of choosing between two hypotheses about a random variable X under the assumption that, for all of the n observations $x_1, \ldots, x_n$, on X, exactly one of the hypotheses is true. In signal detection terminology, this means that either all n observations are of signal-plus-noise or none is. This situation presupposes that knowledge is available as to when a signal, if present, will occur; for otherwise it would not be possible to ensure that the above assumption is satisfied. For example, consider a communications system where knowledge is available at the receiver as to when a message, if transmitted, will be received. Then, during the time when a signal can occur, the received data will result from either signal-plus-noise or noise only (but not both). However, if no knowledge is available as to when a message will be received, the data will first result from noise only and will change to signal-plus-noise at some unknown time. (This would be the case, for example, in signal-intercept applications.) It is this latter situation which will be discussed here; that of detecting signals which occur at some unknown times. Emphasis is placed exclusively on the detection aspect; that is, the procedure is useful only for detecting if a signal is present or not. Some other data processing is necessary to determine information such as message content.

Discussion will be confined to detection methods involving "binary integration," a term used to describe detection based on data which are quantized into two levels, one and zero (ref. 1-5). In particular, a so-called "moving-window detector" (ref. 1 and 2) will be discussed, and methods for calculating the detection statistics (false-alarm probability, etc.) related to this detector will be described.

Results will be given which will show how the detection probability varies with SNR, signal duration, and sample size (the window "width"), and a brief discussion of the equipment required for implementation will be given.

THE DETECTION PROBLEM

The detection problem considered here is that of detecting signals which appear at some unknown times and remain for a fixed time. The problem can be described as follows. Let x_n, n = 1, 2, . . . , be a sequence of independent observations on the output X from a receiver, considered as a random variable with density function $f(x, \theta)$, θ an unknown parameter. Suppose that $\theta = \theta_0$ corresponds to the case where the output results from noise alone and that $\theta = \theta_1$ corresponds to the case where the output results from signal plus noise $\left(\text{of some specified SNR, } S(\theta_1)\right)$. Suppose also that $\theta = \theta_0$ for n = 1, 2, . . . , m_1, and $\theta = \theta_1$ for n = $m_1 + 1, \ldots, m_1 + L$, where m_1 is a random variable denoting the observation at which the first signal appears. For $n > m_1 + L$, $\theta = \theta_0$ until the next appearance of a signal at the random time m_2, etc. It will be assumed that two signals will be separated by at least N observations, where N is the sample size used in the test procedure (discussed later). It will also be assumed that the duration of any signal will be a known, fixed number L of observations.

As mentioned in the introduction, only detection will be discussed here. However, a detector output indicating that a signal is present can be considered by the receiver as an "alarm" which activates recognition, location (in time or frequency), or data processing stages. Figure 1 is a simplified block diagram of a complete system utilizing a moving-window detector and a time-lag recorder. The receiver output is fed simultaneously to both the moving-window detector and the time-lag recorder. If the moving-window detector decision is that a signal is present, an alarm causes the time-lag recorder contents to be read out and processed. The time-lag recorder could be a tape loop, magnetic drum, core storage (for digital data), or any convenient means of storing data collected during a short time interval. The actual length of time for which the time-lag recorder must store data is determined by such factors as the expected duration of signals to be processed and the expected time required for the moving-window detector to produce an alarm after the signal begins.
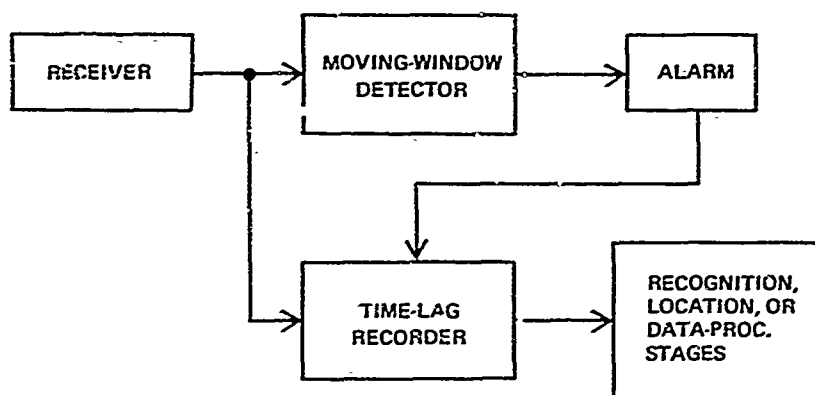

Figure 1. Simplified Block Diagram of a System Utilizing the Moving-Window Detector As An Alarm.

THE DETECTION PROCEDURE

The method of detecting signals (occurring at random times as indicated above) which will be discussed here is the so-called "moving-window detector" (also called a moving average or a sliding window) (ref. 1 and 2). The moving window of width N operates as follows. For each of the subsequences of observations $(x_1, \ldots, x_N)$, $(x_2, \ldots, x_{N+1})$, $\ldots$, $(x_k, \ldots, x_{k+N-1})$, $\ldots$, of length N, a test statistic $S_{k+N-1}(x_k, \ldots, x_{k+N-1})$ is computed and, on the basis of this statistic, a decision is made that either $\theta = \theta_0$ or $\theta = \theta_1$. The test performed on each subsequence will be called a subtest, and a specific test statistic will be discussed later.

A representative sequence of observations when a signal is present is

$$\ldots, x, x, x, x, x, \overbrace{x^*, x^*, \ldots, x^*}^{L}, x, x, x, x, x, x, \ldots$$

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where the asterisk indicates observations from the signal-plus-noise population. Note that L+N-1 consecutive subsequences (as described in the previous paragraph) will include at least one signal-plus-noise observation. This prompts the following definition of a false alarm and of a detection: If no observation in a particular subsequence is from the signal-plus-noise population and the subtest decision is that $\theta = \theta_1$ (a signal is present), then a false alarm occurs. Likewise, if for any of the L+N-1 consecutive subsequences for which at least one observation is from the signal-plus-noise population, the decision is that $\theta = \theta_1$, then a detection occurs. Optimization will be defined as the maximization of the probability of detection, with the false-alarm probability held fixed.

The false-alarm probability will be denoted by α, and the probability of detection will be denoted by $D(\cdot)$, where the parenthetical entry will determine the pertinent variables of which the detection probability is a function. For example, in the context above, $D(L,N)$ denotes the probability of detection as a function of the signal duration L and the test size N. Formulas for (or methods for calculating) α and $D(\cdot)$ will be given later. Note that since a decision occurs for each observation, the average false-alarm rate (in false alarms per observation) is equal to α.

BINARY INTEGRATION

A test which is of practical interest, due to its simplicity and ease of implementation, is the binomial test. When applied to signal detection, this test is called "binary integration," "k out of n detection," "double-threshold detection," and "coincidence detection" (ref. 1-6). For binary integration, the statistics of the moving-window detector can be obtained analytically; i.e., without resorting to Monte Carlo simulation. The calculation of the statistics of the binary moving-window detector will be discussed in the next section.

The binary moving-window detector operates as follows. The data x_n, $n = 1, 2, \ldots$, are compared with a threshold q, and the random variable y_i is given the value 1 if $x_i \geq q$, and is given the value 0 otherwise. Thus the sequence x_n, $n = 1, 2, \ldots$, is converted into the binary sequence y_n, $n = 1, 2, \ldots$. At the kth observation the statistic

$$S_k = \sum_{i=k-N+1}^{k} y_i \tag{1}$$

is calculated and compared with a threshold V and, if $S_k \geq V$, the decision is made that $\theta = \theta_1$ (a signal is present); otherwise, a decision is made that $\theta = \theta_0$.*

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*Some authors define a detection as the occurrence of a threshold crossing; that is, a signal decision occurs at the kth observation only if $S_k = V$ and $S_{k-1} < V$. (Ref. 7, page 767.)

Under the previous assumptions about the density of X, it follows that

$$P\left(y_n = 1 | \theta = \theta_0\right) = \int_q^\infty f\left(x, \theta_0\right) dx \qquad (2)$$

$$P\left(y_n = 1 | \theta = \theta_1\right) = \int_q^\infty f\left(x, \theta_1\right) dx \qquad (3)$$

Denoting these probabilities by p_0 and p_1, respectively, it follows that[*]

$$\alpha = \sum_{i=V}^{N} \binom{N}{i} p_0^i \left(1 - p_0\right)^{N-i} \qquad (4)$$

The following procedure is used to optimize the test, in the sense of maximizing $D(V,N)$ for given L, α, and SNR (ref. 6). (Here, $D(V,N)$ denotes the probability of detection as a function of the threshold V and the test size N.) Let N be fixed. For each V, $V = 1, 2, \ldots, N$, equation 4 is solved for the value $p_0(V)$ for which the given false-alarm probability α is obtained. For each $p_0(V)$, equation 2 is solved for the threshold $q(V)$, and for each $q(V)$, equation 3 is solved for $p_1(V)$. The pairs $\left(p_0(V), p_1(V)\right)$, $V = 1, 2, \ldots, N$, so obtained are used (in a method to be described in the next section) to calculate $D(V,N)$, and the value of V for which $D(V,N)$ is maximum is selected. Now N is allowed to vary and the above procedure is followed for each N.[**] The test size N is then chosen as the value of N for which $D(V,N)$ is maximized, having chosen the optimum value of V for each N. Results will be given later to show how the probability of detection varies with N, for fixed L and α.

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[*]Equation 4 is valid for any particular subtest, and the average false-alarm rate is α false alarms per observation even though the subtests are not independent.

[**]It will be seen from results presented later that usually only values of N in the neighborhood of L need be tried in the optimization procedure; that is, $N_{opt} \approx L$. Also, for many of the probability densities of interest, the optimum threshold V (for fixed N) can be determined without need of considering all possible values for V. This is the case, for example, when the detection probability $D(V)$ (for fixed L, N, and α) is unimodal. It is then only necessary to determine the first value of V for which $D(V + 1) < D(V)$. Finally, it is sometimes possible to solve equations 2 and 3 for p_1 as a function of p_0, and thus avoid the intermediate step of determining q (ref. 6, page 349).

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CALCULATING THE PROBABILITY OF DETECTION

The method used for calculating the probability of detection for the binary moving-window detector is an application of the theory of finite Markov processes[*] (ref. 8-10). This theory cannot be applied directly to S_k since $\{S_k\}_{k=1}^{\infty}$ is not a Markov process (ref. 8). However, if all possible configurations of ones and zeros in the kth subsequence are considered, then finite Markov process theory can be applied; as discussed below.

The binary moving-window can be viewed as follows. Suppose that after each observation, the contents of an N-bit shift register are shifted left by one bit and the binary datum for that observation is inserted in the right-most bit. Then, after the kth observation, the content of the shift register is the kth binary subsequence, and S_k is the sum of the 1's in the shift register. If we let R_k be the number obtained by regarding the shift register content as a binary number, then $\{R_k\}_{k=1}^{\infty}$ is a finite Markov process with possible states $0, 1, \ldots, 2^N - 1$. That $\{R_k\}$ is a Markov process follows from the fact that the possible values of R_k are determined completely by the value of R_{k-1} and y_k. Note that a left shift of an N-bit shift register is equivalent to multiplication by two, modulo 2^N. Hence, if $R_{k-1} = i$, then $R_k = (2i) \bmod 2^N$ if $y_k = 0$ and $R_k = 1 + (2i) \bmod 2^N$, if $y_k = 1$. If $p(n)$ is the probability of a 1 at the nth observation, then the transition probabilities are

$$p_{ij}(n) = P\left(R_n = j | R_{n-1} = i\right) = \begin{cases} 1 - p(n) & \text{if } j = (2i) \bmod 2^N \\ p(n) & \text{if } j = 1 + (2i) \bmod 2^N \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

The transition matrix, for the states ordered as the numbers $0, 1, \ldots, 2^N -1$, has the form

$$P(n) = \begin{pmatrix} q(n) & p(n) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & q(n) & p(n) & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & q(n) & p(n) \\ q(n) & p(n) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & q(n) & p(n) & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & q(n) & p(n) \end{pmatrix} \quad (6)$$

where $q(n) = 1 - p(n)$. Notice that the lower half of the matrix is identical with the upper half since R_n is independent of the high-order bit of R_{n-1}.

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[*]A finite Markov process is a stochastic process, which can take on only a finite number of values, such that the outcome at the kth stage of the process depends on the outcome at the $(k-1)^{st}$ stage and on no previous outcome. If, additionally, the outcome at the kth stage is independent of k, the process is called a Markov chain.

To avoid cumbersome subscripts, in the following we will let $W = 2^N$. If the components of the probability vector $\Pi_n = \left(\pi_0^{(n)}, \ldots, \pi_{W-1}^{(n)} \right)$ denote the probability that R_n has values $0, 1, \ldots, W-1$, then the corresponding probability vector Π_{n+1} can be found from

$$\Pi_{n+1} = \Pi_n P(n+1) \tag{7}$$

From the form of $P(n)$ in equation 6, note that the indicated matrix multiplication in equation 7 reduces to

$$\pi_{2i}^{(n+1)} = \left[\pi_i^{(n)} + \pi_{i+W/2}^{(n)} \right] q(n+1)$$

$$\pi_{2i+1}^{(n+1)} = \left[\pi_i^{(n)} + \pi_{i+W/2}^{(n)} \right] p(n+1) \tag{8}$$

$$i = 0, 1, \ldots, \frac{W}{2} - 1$$

For the problem treated here, it is assumed that the probability of entering a 1 into the window in the no-signal case has a constant value p_0, independent of n. It will be shown that after N observations the process reaches equilibrium in the sense that $\Pi_{N+n} = \Pi_N$, $n = 1, 2, \ldots$, until the process is disturbed by the occurrence of a signal. Specifically, if K_j is defined as the number of 1's in the binary representation of the number j, then

$$\Pi_{N+n} = \left(\pi_0, \ldots, \pi_{W-1} \right); n = 0, 1, \ldots \tag{9a}$$

$$\pi_j = p_0^{K_j} \left(1 - p_0 \right)^{N-K_j}; j = 0, 1, \ldots, W-1 \tag{9b}$$

To see that equation 9 is true for $n = 0$, note that after N observations the content of the shift register represents the results of N independent Bernoulli trials the <u>ordered</u> results of which have probabilities given by equation 9b. Next, note that $K_{i+W/2} = K_i + 1 = K_{2i+1}$ for $i = 0, 1, \ldots, \frac{W}{2} - 1$. Using this and equation 9b in equation 8, we have

$$\pi_{2i}^{(N+1)} = \left[p_0^{K_i} \left(1 - p_0 \right)^{N - \left(K_i + 1 \right)} \right] \left(1 - p_0 \right)$$

$$= p_0^{K_i} \left(1 - p_0 \right)^{N - K_i} = \pi_{2i}^{(N)}$$

$$\pi_{2i+1}^{(N+1)} = \left[p_0^{K_i} \left(1 - p_0 \right)^{N - \left(K_i + 1 \right)} \right] p_0$$

$$= p_0^{K_i + 1} \left(1 - p_0 \right)^{N - \left(K_i + 1 \right)} = \pi_{2i+1}^{(N)}$$

which implies that equation 9 is true for $n = 1, 2, \ldots$.

As stated previously, a signal-present decision occurs whenever the number S_k of ones in the window is greater than or equal to the threshold V.

Equivalently, a signal-present decision occurs whenever the outcome R_k is a member of the subset D of states such that $j \epsilon D$ if $K_j \geq V$.* (K_j is the number of 1's in the binary representation of the number j.) A state in D will be called a decision state.

From the definition of a detection, when a signal is present there are exactly N+L-1 consecutive observations for which a correct signal-present decision (not a false alarm) can occur. The probability of detection defined previously can be restated as

$$D(L,N) = 1 - P(R_k \notin D \text{ for } k = m+1, \ldots, m+L+N-1) \qquad (10)$$

where it is assumed that the first signal observation is x_{m+1}. In words, equation 10 states that $1-D(L,N)$ is the probability that the process fails to enter a decision state as the signal data "passes through" the window. Calculating the probability of detection is accomplished by the following recursive procedure.

Assume that there have been at least N observations of noise only, so that the probability vector Π_0 has the form given by equation 9. (For convenience of notation, the observations have been reindexed so that the first signal observation is x_1.) Let $\Lambda_0 = \left(\lambda_0^{(0)}, \ldots, \lambda_{W-1}^{(0)} \right)$, where $\lambda_k^{(0)} = \pi_k^{(0)}$ if $k \notin D$, and $\lambda_k^{(0)} = 0$ otherwise. Compute $\Lambda_1 = \left(\lambda_0^{(1)}, \ldots, \lambda_{W-1}^{(1)} \right)$ from Λ_0 by first using equation 8 and then setting $\lambda_k^{(0)} = 0$ if $k \epsilon D$. (That is, $\Lambda_1 = \Lambda_0 P(1)$ with all components of Λ_1 corresponding to decision states set equal to zero.) Continue computing Λ_{n+1} from Λ_n in this fashion until Λ_{N+L-1} is obtained. In these computations, p(n) (the probability of entering a 1 into the window) is given by $p(n) = p_1$ for n = 1, 2, ..., L, and by $p(n) = p_0$ for n = L+1, ..., N+L-1; corresponding to the situation where the signal data is "entering" the window and "leaving" the window, respectively. The probability of detection is given by

$$D(L,N) = 1 - \sum_{j \notin D} \lambda_j^{(N+L-1)} \qquad (11)$$

That is, $1-D(L,N)$ is the sum of the components of Λ_{N+L-1} corresponding to nondecision states

The validity of equation 11 will be proved by showing that, for n = 1, 2, ..., N+L-1, $P\left[R_n \notin D, R_{n-1} \notin D, \ldots, R_0 \notin D \right] = \sum_{j \notin D} \lambda_j^{(n)}$, with $\lambda_j^{(n)}$ obtained as indicated above. As before, $W = 2^N$. Note that

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*The notation $j \epsilon D$ means "j is a member of the set D." Similarly, $j \notin D$ means "j is not a member of the set D."

$$P\left[R_1 = j, R_0 \notin D\right] = \sum_{i \notin D} P\left[R_1 = j | R_0 = i\right] P\left[R_0 = i\right] \qquad (12)$$

$$= \sum_{i=0}^{W-1} P\left[R_1 = j | R_0 = i\right] \lambda_i(0) \qquad (13)$$

$$= \sum_{i=0}^{W-1} p_{ij}(1) \lambda_i(0) \qquad (14)$$

and (by definition) $\lambda_j(1) = P\left[R_1 = j, R_0 \notin D\right]$ if $j \notin D$ and $\lambda_j(i) = 0$ if $j \in D$. Equation 13 follows from equation 12 since $\lambda_i(0) = P\left[R_0 = i\right]$ if $i \notin D$ and $\lambda_i(0) = 0$ if $i \in D$, and equation 14 follows from equation 13 by the definition of $p_{ij}(1)$ in (5). Hence, $P\left[R_1 \notin D, R_0 \notin D\right] = \sum_{i \notin D} \lambda_i(1)$. Now, suppose $\lambda_i(m) = P\left[R_m = i, R_{m-1} \notin D, \ldots, R_0 \notin D\right]$ if $i \notin D$ and $\lambda_i(m) = 0$ if $i \in D$, $i = 0, \ldots, W-1$. Then

$$P\left[R_{m+1} = j, R_m \notin D, \ldots, R_0 \notin D\right]$$

$$= \sum_{i \notin D} P\left[R_{m+1} = j | R_m = i, R_{m-1} \notin D, \ldots, R_0 \notin D\right]$$
$$\cdot P\left[R_m = i, R_{m-1} \notin D, \ldots, R_0 \notin D\right] \qquad (15)$$

Applying the Markov property (see the footnote on page 7) to the conditional probability in equation 15 and using the definitions of $\lambda_i(m)$ and $p_{ij}(m+1)$, we have

$$P\left[R_{m+1} = j, R_m \notin D, \ldots, R_0 \notin D\right] = \sum_{i=0}^{W-1} P\left[R_{m+1} = j | R_m = i\right] \lambda_i(m) \qquad (16)$$
$$= \sum_{i=0}^{W-1} p_{ij}(m+1) \lambda_i(m) \qquad (17)$$

the right-hand side of which is an application of equation 8 to Λ_m. Setting $\lambda_j(m+1) = P\left[R_{m+1} = j, R_m \notin D, \ldots, R_0 \notin D\right]$ if $j \notin D$ and $\lambda_j(m+1) = 0$ if $j \in D$, it follows that $P\left[R_{m+1} \notin D, \ldots, R_0 \notin D\right] = \sum_{i \notin D} \lambda_i(m+1)$ and equation 11 follows by induction.

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RESULTS

Results are given for three probability densities, the Rice, Rayleigh, and Gaussian (normal) densities. The Rice density is used to describe the output X from a linear envelope detector when the SNR is constant (non-fluctuating). This density is given by

$$f\left(x, \theta_1\right) = x \, \exp\left[-\left(\frac{x^2}{2} + \theta_1\right)\right] I_0\left(x\sqrt{2\theta_1}\right) dx, \quad x \geqslant 0 \qquad (18)$$

where θ_1 is the SNR, and $I_0(\cdot)$ is the modified Bessel function of the first kind, order zero (ref. 11). The corresponding no-signal density is given by

$$f(x, 0) = x \, \exp\left[-x^2/2\right], \quad x \geqslant 0 \qquad (19)$$

which is obtained from equation 18 by setting $\theta_1 = 0 \equiv \theta_0$. Solving (2) for q, given p_0, results in

$$q = \left(-2 \ln p_0\right)^{\frac{1}{2}} \qquad (20)$$

The solution to equation 3 for p_1, given p_0, is obtained from the so-called Q-function*

$$p_1 = Q\left(\sqrt{2\theta_1}, q\right) = \int_q^\infty x \, \exp\left[-\left(\frac{x^2}{2} + \theta_1\right)\right] I_0\left(x\sqrt{2\theta_1}\right) dx \qquad (21)$$

which is tabulated (ref. 12), or can be calculated recursively (ref. 13).

The Rayleigh probability density can be used to describe the output Y from a linear envelope detector when the SNR fluctuates according to the Swerling case 2 (ref. 14). This density is given by

$$f\left(y, \theta_1\right) = \frac{y}{1+\theta_1} \exp\left[-\frac{y^2}{2\left(1 + \theta_1\right)}\right], \quad y \geqslant 0 \qquad (22)$$

where, in this case, θ_1 is the average SNR, averaged over all possible fluctuations. The corresponding no-signal distribution is again given by equation 19. The probability p_1 can be obtained in terms of p_0 directly, without the intermediate step of calculating q. Since the solution to equation 2, using equation 19, is $p_0 = e^{-q^2/2}$ and the solution to equation 3, using equation 22, is $p_1 = e^{-q^2/2\left(1+\theta_1\right)}$, it follows that

$$p_1 = p_0^{1/\left(1+\theta_1\right)} \qquad (23)$$

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*The Q-function is defined by $Q(a, t) = \int_t^\infty x \, \exp\left[-\left(x^2 + a^2\right)/2\right] I_0(ax) \, dx$.

It is noted that the results to be given for the random variable X (and Y) described above are valid also for any random variable $W = T(X)$ ($Z = T(Y)$), when T is a one-to-one monotone increasing transformation. Hence, the results obtained utilizing densities associated with a linear detector are also valid for a square-law detector, with the transformation T defined by $T(u) = u^2/2$.

The Gaussian density can describe the receiver output under a variety of situations. When a signal is present, it is assumed that the associated probability density is

$$f\left(x, \theta_1\right) = (2\pi)^{-\frac{1}{2}} \exp\left[-\left(x-\theta_1\right)^2/2\right] \tag{24}$$

where the SNR is $\theta_1^2/2$. When no signal is present, the density is

$$f(x, 0) = (2\pi)^{-\frac{1}{2}} \exp\left[-x^2/2\right] \tag{25}$$

The equation relating p_1 and p_0 (used in lieu of equations 2 and 3) is

$$p_1 = \text{erfc}\left[\text{erfc}^{-1}\left(p_0\right) - \theta_1\right] \tag{26}$$

where

$$\text{erfc}(q) = (2\pi)^{-\frac{1}{2}} \int_q^\infty e^{-t^2/2} dt \tag{27}$$

Figure 2 shows the probability of detection D(V) as a function of the threshold V for several values of the SNR, with α, N, and L fixed. For each curve, the circled point indicates the maximum value of D(V), and the corresponding optimum value of V can be read from the abscissa. These results indicate that the optimum value for V is relatively insensitive to SNR. A similar result was presented in Worley (ref. 6). Although insensitive to SNR (for each probability density), the results of figure 2 indicate that knowledge of the probability density is necessary. For example, using V = 4, which is optimum for the Rician model, significantly degrades D(V) if used for the Rayleigh model. It is expected that for large values of N (and L), the difference in the optimum values of V for the three models will become more pronounced. That is, as N increases it becomes increasingly more important that the probability density describing the receiver output be known, so that the optimum value of V can be obtained. No results are available at present to support this claim; however, related results are presented in Worley (ref. 6).
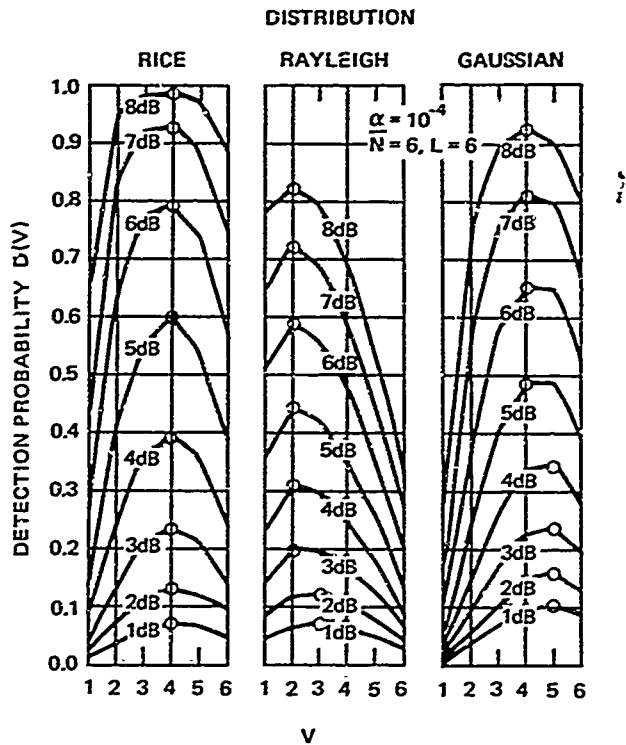
Figure 2. Probability of detection D(V) as a function of the threshold V, for several values of the SNR.

Figure 3 shows the probability of detection D(L,N) as a function of the test size N, for several values of the signal duration L where, for each N and L, the optimum V is chosen. Notice that for both the Gaussian and Rician models, D(L,N) has a distinct peak which occurs for L=N. However, for the Rayleigh model, D(L,N) is relatively flat (as a function of N for each L), with a maximum at N=1. These results again indicate the importance of knowing the probability density associated with the receiver output if optimization is to be achieved.
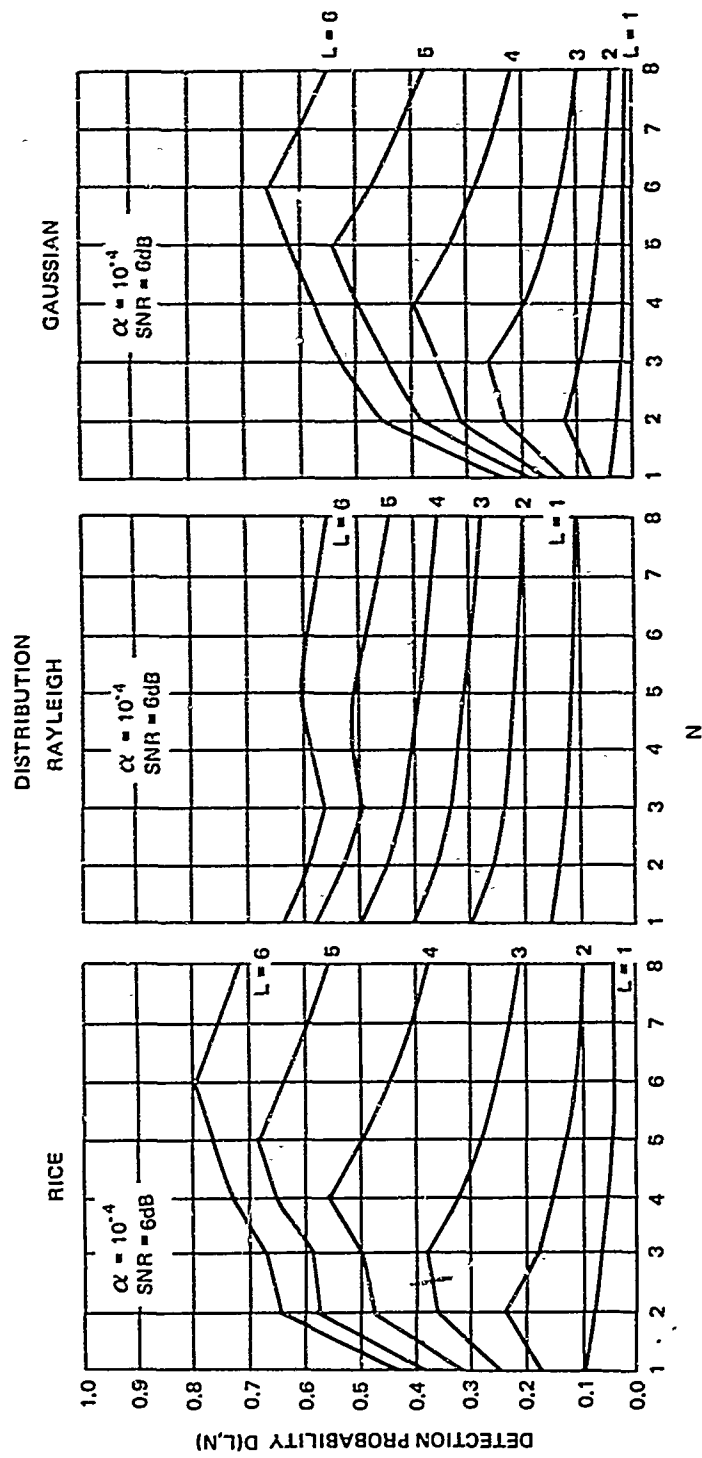
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Figure 3. Probability of Detection D(L,N) As a Function of the Test Size N.

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IMPLEMENTATION OF THE BINARY MOVING-WINDOW DETECTOR

As mentioned earlier, binary integration is of interest due to the ease in which a system for binary integration can be implemented. One very important application of the binary moving-window detector is the detection of communication signals (from both friendly and unfriendly sources) which occur at random times. If the number of data samples which must be collected and stored is large, then binary integration may well be the only feasible method of detection due to the speed at which computations must be made. A brief description of a particular method of implementing the binary moving-window detector for detecting communication signals will be given. A similar and more complete description (in terms of a multiple-range-bin radar system) is found in Dillard (ref. 2).

A block diagram of the binary moving-window detector of width L, as applied to the detection of randomly occurring communication signals, is shown in figure 4. The inputs labelled S, $\bar{S}$, and C are illustrated in figure 5. The input S (SAMPLE PULSES) is a sequence of pulses occurring at equally spaced intervals corresponding to the rate at which the receiver output is sampled and quantized. The input $\bar{S}$ is the logical complement of S when considered as a logical input to and-gate 2. (In figure 5, relatively negative voltages are considered logical "ones" and relatively positive voltages are considered logical "zeros.") The input C (CLOCK PULSES) is a sequence of pulses occurring at L times the rate of the SAMPLE PULSES, where L is the window width.

The heart of the detector is the delay line, which is a clocked (digital) recirculating memory. The total delay is adjusted to exactly L+1 CLOCK PULSE periods, where L is the window width. Logically, the delay line can be considered as an (L+1)-stage shift register which shifts at the clock rate.[*]

At each SAMPLE PULSE the receiver output X is compared with the reference q and if $X \geqslant q$ the quantizer output Q = 1; otherwise Q = 0. Also, at the occurrence of each SAMPLE PULSE, and-gate 2 is disabled by $\bar{S}$, and thus inhibits recirculation of the corresponding data bit. And-gate 1, being enabled by the SAMPLE PULSE, causes a 1 or 0 to be "written" into the delay line accordingly as Q = 1 or Q = 0. Thus, the old data which were prevented from recirculating are replaced with new data (from Q, via and-gate 1). Between occurrences of SAMPLE PULSES, and-gate 1 is disabled (S is at logical zero) and and-gate 2 is enabled, allowing the data to recirculate. Since the delay line has total delay of exactly L+1 CLOCK PULSE periods, a data bit which is inserted after the jth SAMPLE PULSE will precess by one CLOCK PULSE period for each trip through the delay line. After a total of L trips, the data bit will have precessed to the point where it appears at and-gate 2 when $\bar{S} = 0$ and is thus replaced by new data. Therefore, the input to the delay line between the occurrence of SAMPLE PULSES is the binary data from each of the previous L receiver quantizations.

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[*]In many applications it is more convenient and economical to actually use a shift register rather than a (digital) delay line. However, when L is large, the use of a shift register may be much more costly than, for example, the use of a magnetostrictive delay line.
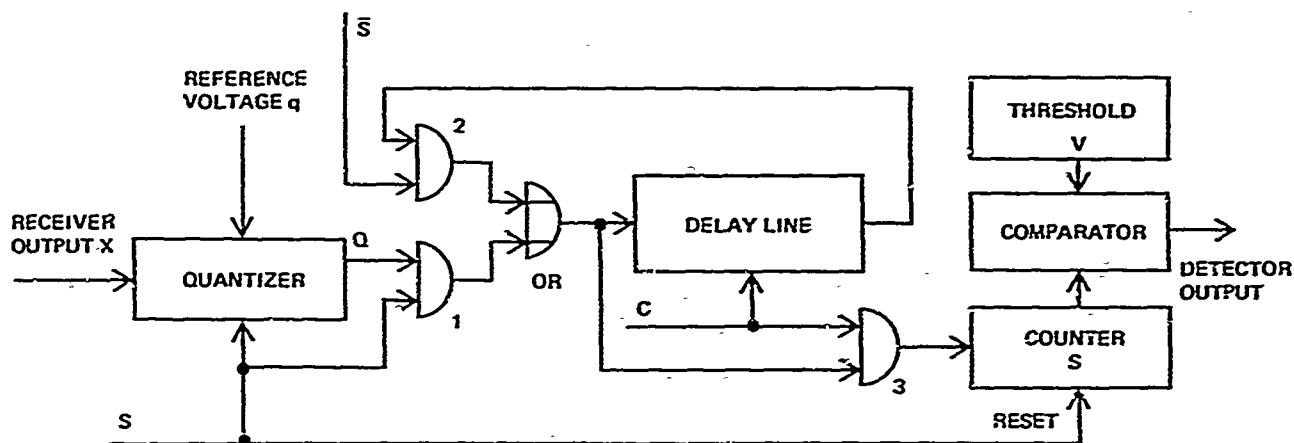
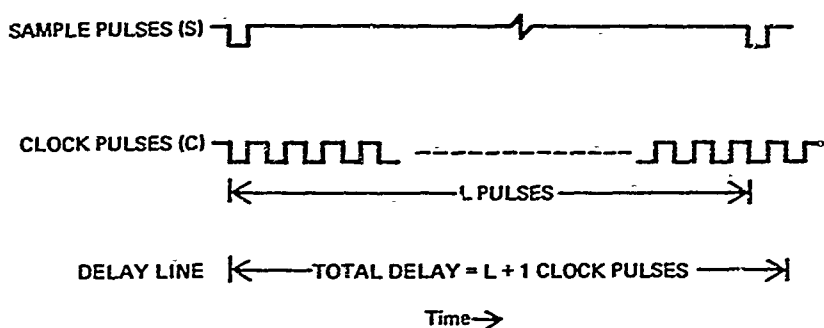Figure 4. Block Diagram of the Moving-Window Detector.

Figure 5. Timing Relationships Among the Sample Pulses,
Clock Pulses, and the Total Delay of the Delay Line.

Counter S is reset by each SAMPLE PULSE. The input to the delay
line together with the CLOCK PULSES are applied to and-gate 3. As a
result, counter S counts the total number of "ones" obtained from the L
previous quantizations of the receiver output. That is, counter S computes

$$S_k = \sum_{i=k-L+1}^{k} y_i$$

where y_i is the binary datum from the ith quantization of the receiver output.
The contents of counter S are compared with the threshold V and, if $S_k \geqslant V$,
a signal-present decision occurs. Thus, after each receiver-output quantiza-
tion, the comparator indicates if the threshold V is exceeded.

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CONCLUSIONS

The detection of randomly occurring signals can be accomplished by a binary moving-window detector. The detector is easily implemented and can be applied to existing and future communications systems, especially for use as an "alarm" to initiate location, recognition, or data processing stages.

The statistics of the binary moving-window detector can be obtained by a direct calculation, without resorting to Monte Carlo simulation. Methods for obtaining these statistics are easily programmed for computer calculation.

RECOMMENDATIONS

1. Investigate existing communications systems and plans for future systems for possible application of the binary moving-window detector as an inexpensive and effective means of detecting communications signals.

2. Extend the analysis to cover multilevel quantization of data.

3. Investigate methods of obtaining analog moving-window detectors (to avoid the loss due to quantization) which can be simply implemented.

4. Analyze the use of distribution-free statistical procedures in conjunction with moving-window detectors.

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13. ABSTRACT

Use of binary moving-window techniques for detection of randomly occurring signals was investigated. Statistical decision methods appear to offer a means for implementing a reliable signal detection and processing system in a hostile, adverse, or unstable environment. The statistics of the moving-window detector can be obtained by direct computation, without resorting to 'Monte Carlo' simulation; and are easily programmed for computer calculation.

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