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FREQUENCY-DOMAIN MAXIMUM-LIKELIHOOD
ADAPTIVE FILTERING

Technical Report No. 9

SEISMIC ARRAY PROCESSING TECHNIQUES

Prepared by

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Area Code 214, 238-6521

TEXAS INSTRUMENTS INCORPORATED

Services Group

P. O. Box 5621

Dallas, Texas 75222

Contract No. F33657-70-C-0100

Amount of Contract: \$339,052

Beginning 15 July 1969

Ending 14 July 1970

Prepared for

AIR FORCE TECHNICAL APPLICATIONS CENTER (VSC)

~~Washington, D. C. 20333~~

ALEXANDRIA, VA. 22313

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Nuclear Monitoring Research Office

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21 August 1970

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ABSTRACT

Recent intensive study of adaptive (gradient-search) filtering in the time domain has not solved the problems with rate-of-convergence problem, which is a difficulty with this technique. A recent study¹ based on a set of time-stationary synthetic data shows that the time-domain maximum-likelihood adaptive filter converges very slowly to the optimum filter. After 3300 iterations of adaption with an adaptive rate of 10 percent of maximum value, the adaptive filter is still about 4 db away from the optimum filter in the sense of mean-square outputs.

Time-domain adaptive filtering necessitates using only one convergence parameter for all filter coefficients, which may cause slow convergence for some data. Frequency-domain adaptive filtering may solve this problem, since different convergence parameters can be used for different frequency components. This report describes a frequency-domain maximum-likelihood adaptive-filtering algorithm analogous to the time-domain adaptive algorithm². This algorithm was used with a set of synthetic stationary data previously used for a time-domain adaptive-filtering study. Different filter lengths and convergence parameters were used. Results are compared with beamsteer and time-domain adaptive filters.



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SECTION I

INTRODUCTION

This report describes a maximum-likelihood filter design technique using an adaptive algorithm operating on data in the frequency domain. For a maximum-likelihood filter containing a number of points LF (length of filter) and number of channels NC , the adaptive algorithm in the time domain tries to find the minimum point on an $LF \times NC$ dimensional quadratic performance surface under a system of constraints permitting the use of only one convergence parameter for all dimensions. This may lead to slow convergence of the algorithm. It was believed that convergence might possibly be accelerated by adapting the filter in the frequency domain, where different convergence parameters can be used for different frequency components. Each frequency component, the minimum point on an NC dimensional quadratic performance surface is sought instead of the minimum on an $NC \times LF$ dimensional surface.

It is hoped that decoupling the $LF \times NC$ problem into $LF/2$ problems of NC dimension, each with its separate constraint and convergence rate, will lead to a better filter than could be obtained by applying the same amount of work to the $LF \times NC$ problem.

A frequency-domain adaption requires transforming the initial time-domain filter coefficients and time traces to the frequency domain by direct Fourier transformations. The output of the filter can then be written as the summation of individual frequency component filtering outputs. For each frequency component, an adaptive algorithm can be obtained by minimizing the square of the absolute value of the output at that frequency component, subject to a system of constraints in the frequency domain. The constraints in the frequency domain are obtained by Fourier transforming the time-domain constraints. The output of the filter can be minimized by reducing each frequency component output independently, assuming independence of the filter outputs of individual frequency components.



The frequency-domain maximum-likelihood adaptive-filtering algorithms developed in Section V, is as follows.

$$F_{i,k}^{t+1} = F_{i,k}^t + C_k y_k^t [\bar{X}_k^t - X_{i,k}^t] \quad (1-1)$$

and the output of the filter at time t is

$$y^t = y_o^t + 2 \operatorname{Re} \sum_{k=1}^{(LF-1)1/2} y_k^t \quad (1-2)$$

where

$$\bar{X}_k^t = \frac{1}{NC} \sum_{i=1}^{NC} \bar{X}_{i,k}^t$$

$$y_k^t = \frac{1}{LF} \sum_{i=1}^{NC} F_{i,k}^t X_{i,k}^{t*}$$

the output of k^{th} frequency component at time t

$X_{i,k}^t$ is the k^{th} frequency component of the i^{th} channel frequency-domain data at time t

C_k is a real or complex constant convergence parameter for the k^{th} frequency component

$F_{i,k}^t$ is the k^{th} frequency component of the i^{th} channel frequency-domain weight at time t

* means complex conjugate

Re means "the real part of"

Since the preceding algorithm is performed in the frequency domain, $F_{i,k}^t$, $X_{i,k}^t$, \bar{X}_k^t , and y_k^t numbers are all complex. The formula in Equation 1-1 is similar to the time-domain maximum-likelihood adaptive algorithm, where all variables are real numbers. Note that in frequency-domain



filtering only $(LF+1)/2$ independent frequency components correspond to an LF -point time-domain filter. This effectively makes the dimensions of a frequency-domain filter only about half those of a time-domain filter and components for the complex operations in the frequency domain. Thus if transformations of input data to the frequency domain are excluded, computations of time and frequency-domain algorithms are approximately the same.

Equation 1-1 shows that C_k should be selected to be inversely proportional to the power of input data at frequency component k in the filter. It is very difficult to choose a proper C_k if nothing is known about the power spectrum of the input data. To avoid this difficulty, Equation 1-1 can be modified to

$$F_{j,k}^{t+1} = F_{i,k}^t + \frac{C_k}{NC^2 (\bar{X}_k^t \bar{X}_k^{t*})} y_k^t [\bar{X}_k^t - X_{i,k}^t] \quad (1-3)$$

To keep the algorithm stable, the maximum value of C_k is approximately 1.

In this report, Equation 1-3 is used to adapt filters in the frequency domain with a dimension of 13 channels, 29 points, 45 points, and 59 points. After a certain number of adaptive iterations, the last filter is transformed back to the time domain and fixedly applied to the time-domain data. The mean-square output and output spectrum of frequency-domain filters are compared with those of the time-domain adaptive filters and the beamsteer processor.

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SECTION II

EXPERIMENTAL RESULTS

This experiment used a set of synthetic stationary data having the same covariance matrix as the ensemble of selected noise samples from a short-period seismic array.² The data contain 13 channels, with 3300 sample points per channel and a 72-msec sample period.

Time traces were transformed into the frequency domain according to Equation 1-3, and the initial filter was obtained by transforming a time-domain beamsteer processor to the frequency domain using Equation 6. The resulting frequency-domain data was written on magnetic tape so that the Fourier transform need not be repeated.

Three filters with lengths of 29 points, 45 points, and 59 points were tried. For each filter different sets of convergent parameters were applied and the data was passed several times. After one or more passes, the last filter was transformed back to the time domain and fixedly applied to the time traces. The output spectra were compared with those of beamsteer and time domain adaptive filtering. The results of the time-domain adaptive filtering used here as a reference were obtained by adapting 3300 points with a convergent rate of approximately 10 percent of its estimated maximum rate¹.

A. 29-POINT FILTER

A 29-point filter has 15 independent frequency component ($k=0-14$) each representing a band of about .47 Hz. The sequential transforms were taken with a 15 point overlap, thus 3300 data points provided 220 updates at each frequency. After making five passes thru the data with various converged rates the measured mean square output is 1.381×10^8 compared with 2.185×10^8 for the summation and 8.54×10^7 for the 29-point time domain adaptive filter.

These emperical results show that the frequency-domain maximum-likelihood adaptive filter (FDMLAF) does not perform better than



the time-domain maximum-likelihood adaptive filter, but the question of how well it converges to the frequency-domain maximum-likelihood optimum filter (FDMLOF) has not been answered. A comparison of the mean square output (MSO) of FDMLAF and FDMLOF at each of the sampled frequencies was made. Based on the covariance matrix of the data used for the experiment, 15 FDMLOF's can be designed, each with the frequency corresponding to one of the frequencies sampled in the 29-point FDMLAF.

The MSO's of the FDMLAF and FDMLOF at each sampled frequency are shown in Figure II-1.

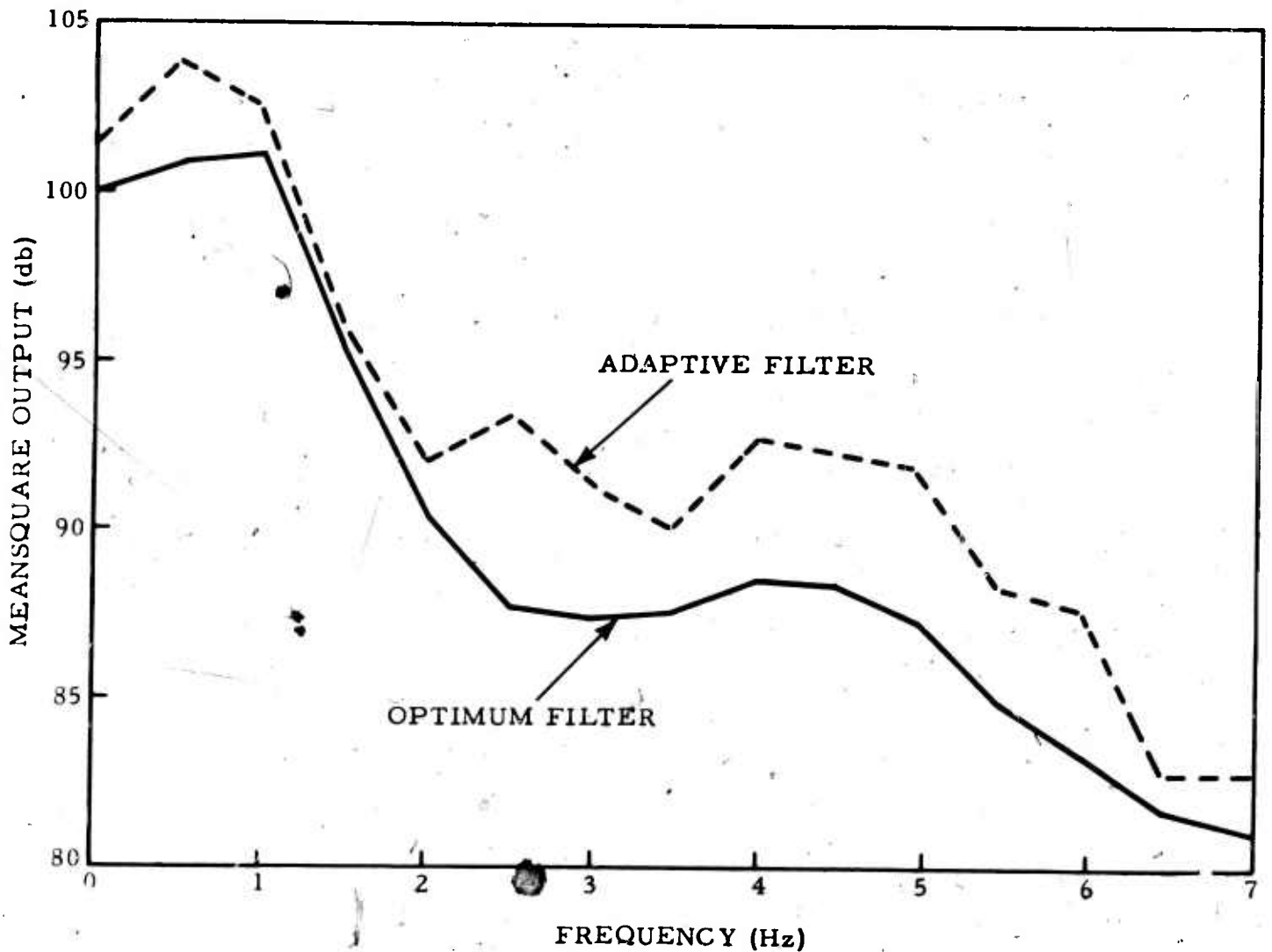


Figure II-1. MSO of FDMLOF and FDMLAF

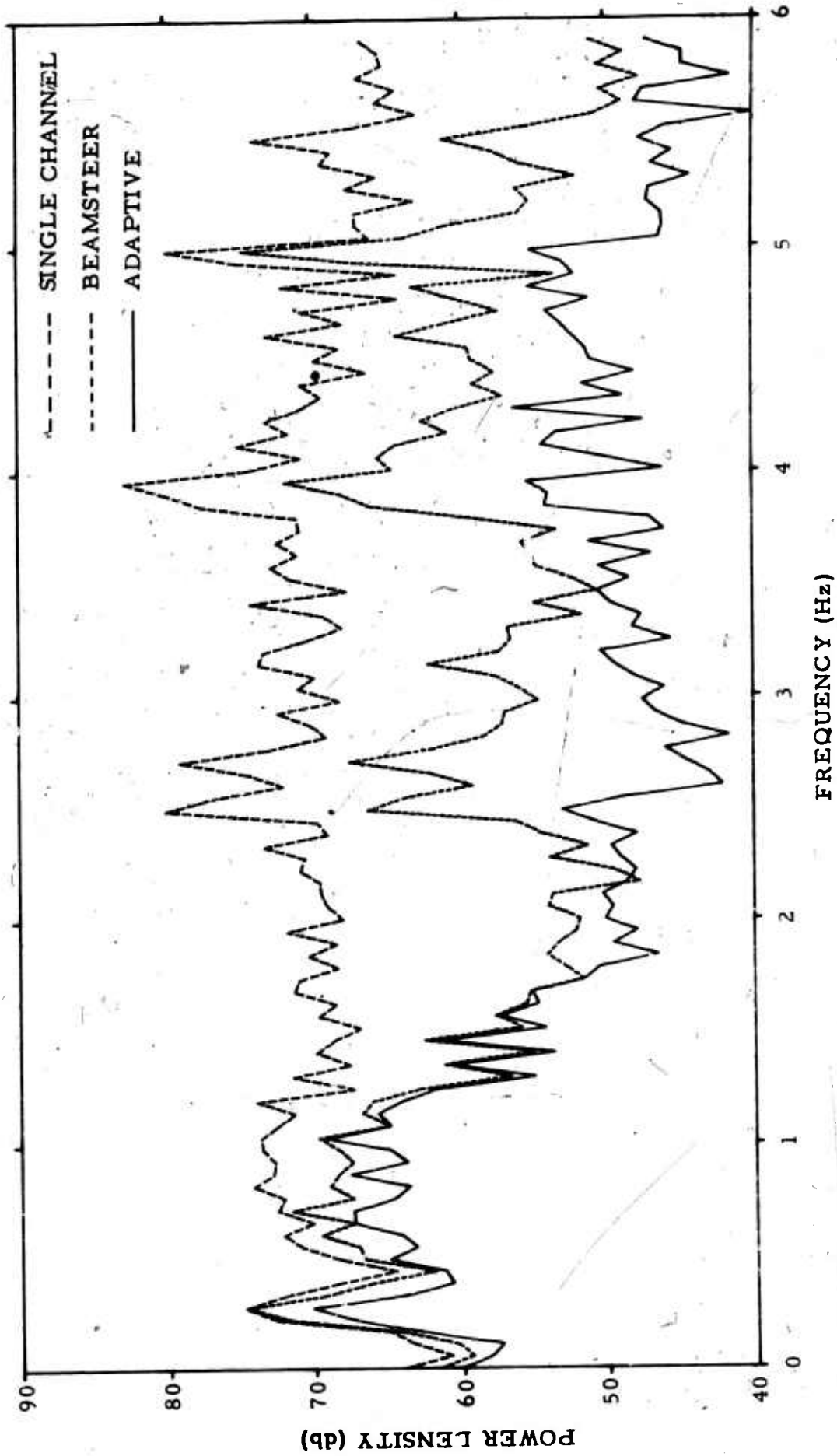


Figure II-2. Output Spectrum of Time Domain Adaptive Designed Filter

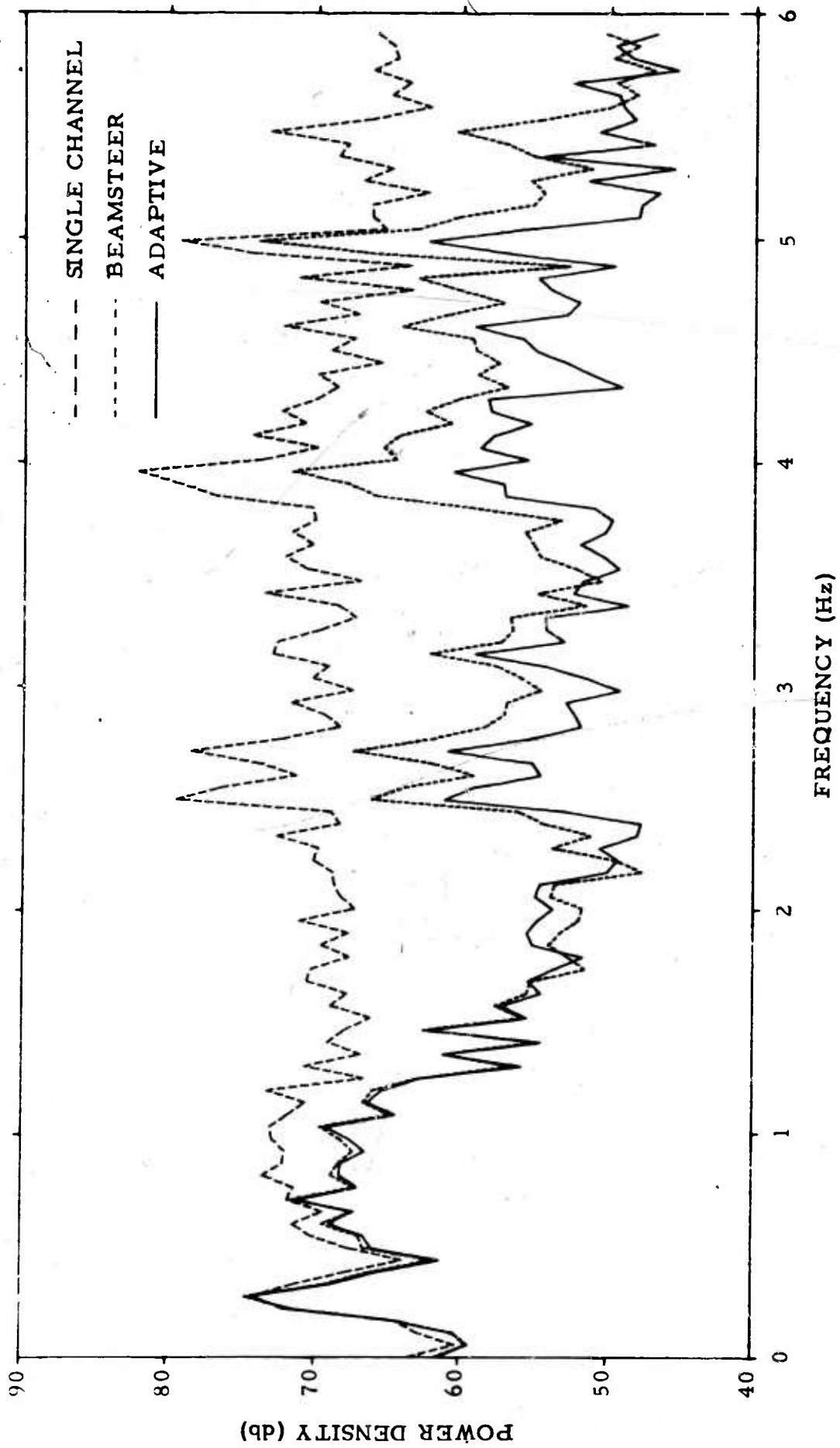
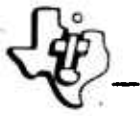


Figure II-3. Output Spectrum of F₂₉

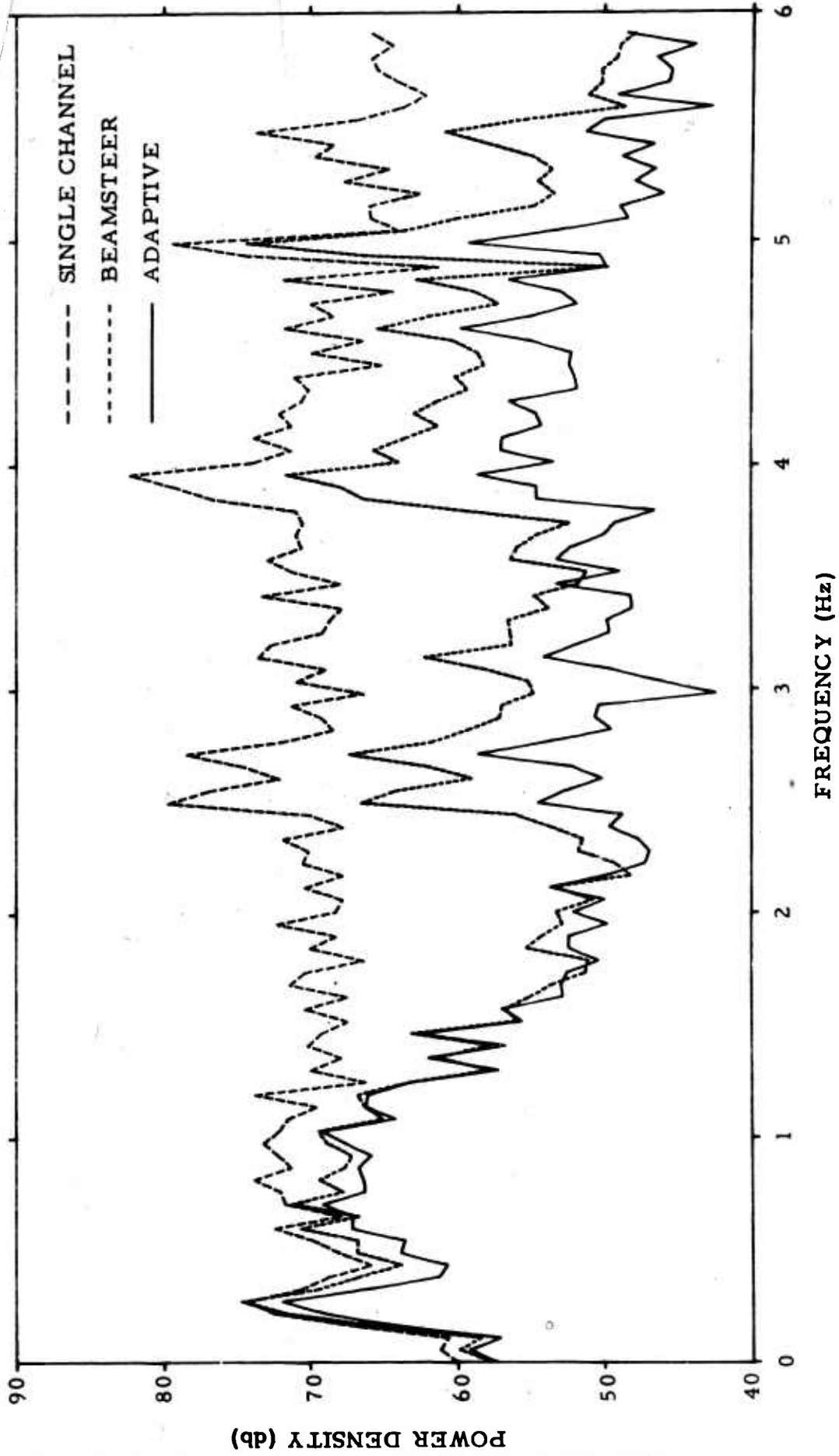
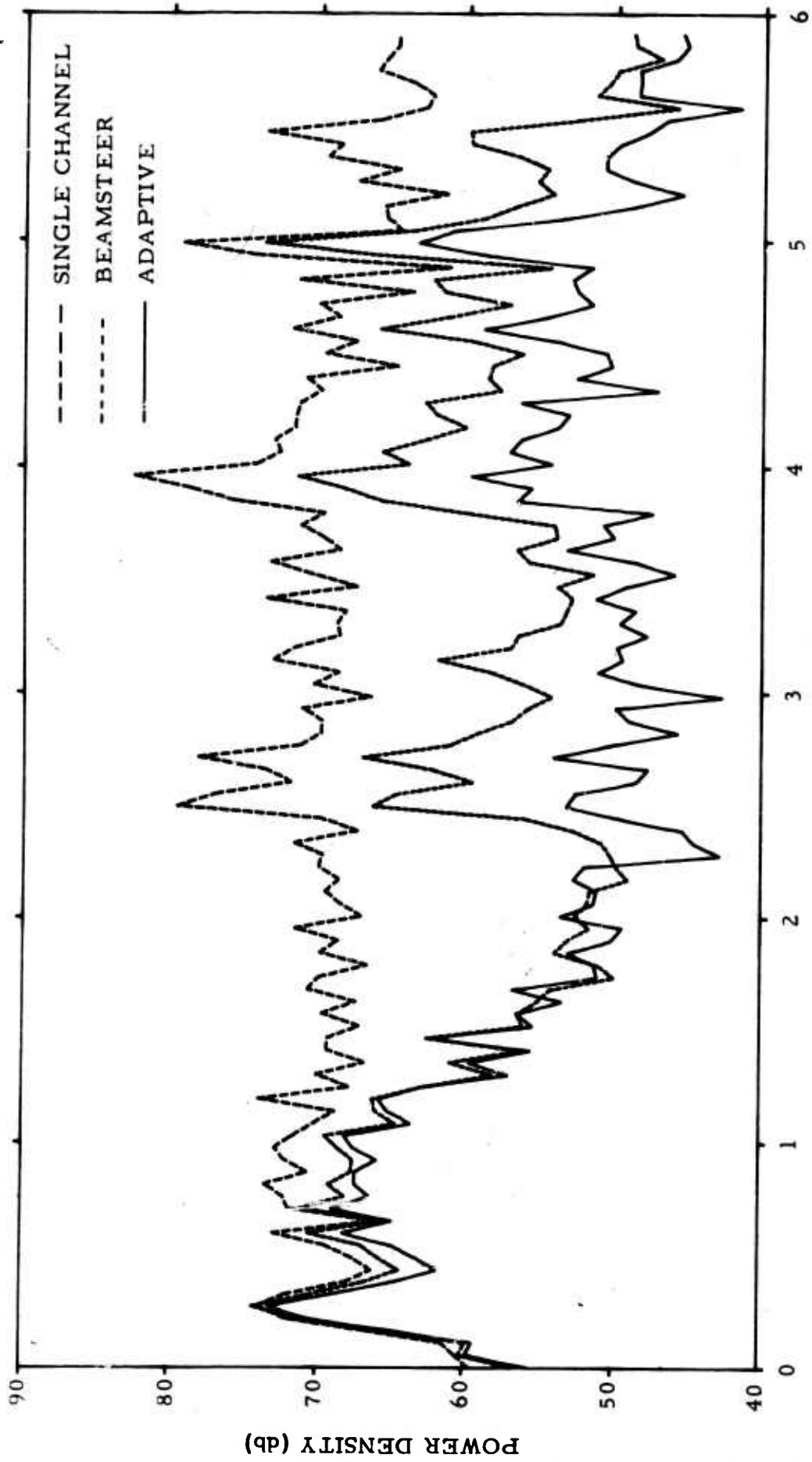


Figure II-4. Output Spectrum of F₄₅



FREQUENCY (Hz)

Figure II-5. Output Spectrum of F₅₉

POWER DENSITY (db)



Compared with FDMLOF, the MSO of the FDMLAF is 2 to 3 db worse in the 0~1-Hz band, 0.5 to 1.5 db worse in 1~2 Hz band, and 2 to 5 db worse for frequency above 2 Hz. These two curves, indicate that FDMLAF does not converge to the FDMLOF very well.

B. 45-POINT FILTER

The 45-point filter has 23 independent frequency components in the frequency domain. Fifteen data points were overlapped for two consecutive Fourier transforms of the time traces, and 140 updates were available at each frequency in one pass thru the data. Ten passes were made for this filter. Relatively larger convergent parameters are used in the 0.3- to 1.5-Hz frequency components, where beamsteer and time domain adaptive filters have very poor performance.

The best filter obtained after ten passes thru the data gave a mean square output of 1.097×10^8 as compared with 2.185×10^8 for the sum and 8.45×10^7 for the time domain adaptive filter.

C. 59-POINT FILTER

Overlapping 30 data points for two consecutive Fourier transformations of time series 100 provided update points in the frequency domain per pass thru the data. Six passes were made. At the end of the 6th pass the mean square output was 1.054×10^8 .

Figure II-2 shows the power spectra of a single channel, the summation, and the time domain adaptive filter output. Figures II-3, II-4, and II-5 show the spectra for the single channel, summation and 29 pt, 45 pt and 59 pt adaptive filters respectively. These figures indicate that the 29 point time domain adaptive filter is superior to the frequency domain filter at essentially all frequencies. This is true in spite of the fact that the time domain filter was achieved in one pass thru the data as opposed to several passes for the frequency domain filters.

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SECTION III CONCLUSIONS

The experiment using the synthetic stationary data and the selected convergent parameters on the derived frequency domain maximum-likelihood adaptive algorithm leads to the following conclusions:

- The frequency-domain adaptive filter converges slower than the time-domain adaptive filter for the convergent parameters used in this experiment. It does not converge to the frequency-domain optimum filter very well. This may be due to bias or to slow convergence. A detailed study of this "convergence" problem for both frequency and time domain algorithms would be valuable.
- For frequency-domain adaptive filtering the filter must be long enough to represent the characteristics of the spectrum of the data. The data used in this experiment requires a filter at least 45 points long to produce a reasonably good result
- The time-domain adaptive-designed filter rejects noise better on all frequency components than the frequency-domain adaption designed filter, especially for the lines
- The frequency-domain adaptive-designed filter can preserve the signal very well, since it can hold the constraints perfectly

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SECTION IV

MATHEMATICAL DEVELOPMENT OF FREQUENCY-DOMAIN
MAXIMUM-LIKELIHOOD ADAPTIVE FILTERING ALGORITHM

The time domain maximum-likelihood adaptive filter is derived by minimizing the output y^t of a linear multichannel filter \underline{f}

$$y^t = \sum_{i=1}^{NC} \sum_{j=1}^{LF} f_{i,j}^t x_{i,t-j} = \underline{f}^T \underline{x}_t \quad (4-1)$$

subject to a system of constraints

where $\underline{M} \underline{f} = \underline{\alpha} \quad (4-2)$

\underline{x}_t is the input data vector

NC is the number of channels

LF is the length of filter

$$\underline{M} = \begin{bmatrix} \overbrace{\underline{I} \quad \underline{I} \quad \dots \quad \underline{I}}^{NC} \\ \underline{I} \quad \underline{I} \quad \dots \quad \underline{I} \\ \underline{I} \quad \underline{I} \quad \dots \quad \underline{I} \\ \underline{I} \quad \underline{I} \quad \dots \quad \underline{I} \end{bmatrix} \quad (4-3)$$

$$\underline{\alpha} = (\alpha_1 \alpha_2 \dots \alpha_{LF})^T, \quad \alpha_i = \delta_{i-s}, \quad s = LF/2$$

\underline{I} is a LF by LF identity matrix

The resulting adaptive algorithm is

$$\underline{f}^{t+1} = \underline{f}^t - B y^t \left[\underline{I} - \frac{1}{NC} \underline{U} \right] \underline{x}_t \quad (4-4)$$

where \underline{I} is a LF x NC by LF x NC identity matrix, \underline{U} is a square matrix with all elements equal to 1 and dimension LF x NC, and B is a convergent parameter.

Here we derive a similar adaptive algorithm in the frequency domain.



If we take the Fourier transformation of each channel of the input data and time domain filter, and let

$$X_{i,k}^t = \sum_{r=0}^{LF-1} x_{i,t-r-1} e^{-j2\pi kr/LF} \quad (4-5)$$

$$F_{i,k}^t = \sum_{r=0}^{LF-1} f_{i,r+1}^t e^{-j\pi kr/LF} \quad (4-6)$$

where

$$k = 0, 1, 2, \dots, LF-1$$

$$i = 1, 2, 3, \dots, NC$$

then, according to Parseval's theorem, we have

$$\sum_{j=1}^{LF} f_{i,j}^t x_{i,t-j} = \frac{1}{LF} \sum_{k=0}^{LF-1} F_{i,k}^t X_{i,k}^{t*} \quad (4-7)$$

where * means complex conjugate.

Thus, Equation 4-1 can be written as

$$y^t = \frac{1}{LF} \sum_{i=1}^{NC} \sum_{k=0}^{LF-1} F_{i,k}^t X_{i,k}^{t*} = \sum_{k=0}^{LF-1} y_k^t \quad (4-8)$$

where

$$y_k^t = \frac{1}{LF} \sum_{i=1}^{NC} F_{i,k}^t X_{i,k}^{t*} \quad (4-9)$$

Let the Fourier transform (transform each row separately) of I in Equation 4-3 be N.



where

$$\underline{N} = \begin{bmatrix} R_{1,0} & R_{1,1} & \dots & R_{1,LF-1} \\ R_{2,0} & R_{2,1} & \dots & R_{2,LF-1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{LF,0} & R_{LF,1} & \dots & R_{LF,LF-1} \end{bmatrix}$$

and

$$\underline{N} = \overbrace{\begin{bmatrix} \underline{N}' & \underline{N}' & \dots & \underline{N}' \end{bmatrix}}^{NC}$$

Then, Equation 4-2 can be written as

$$\frac{1}{LF} (\underline{N}^* \underline{F}) = \underline{\alpha} \text{ or } \underline{N}^* \underline{F} = \underline{\beta}$$

where

$$\underline{\beta} = LF \underline{\alpha}$$

We assume that all frequency components of y^t are independent, i.e., y_k^t 's are independent. Then $(y^t)^2$ may be minimized by minimizing each $|y_k^t| z^k$ independently; i.e., we may minimize $y_k^t y_k^{t*}$ subject to the constraint

$$\underline{N}_k^* \underline{F}_k = \underline{\beta}_k \quad (4-10)$$

where

$$\underline{\beta}_k = (\beta_{1,k}^R \quad \beta_{1,k}^I \quad \dots \quad \beta_{LF,k}^R \quad \beta_{LF,k}^I)^T$$

$$\sum_{k=0}^{LF-1} \beta_{i,k}^R = LF \alpha_i$$

$$\sum_{k=0}^{LF-1} \beta_{i,k}^I = 0$$

$$\underline{F}_k = (F_{1,k}^R \quad iF_{1,k}^R \quad \dots \quad F_{NC,k}^R \quad iF_{NC,k}^I)^T$$



$$\tilde{N}_k = \underbrace{\begin{bmatrix} R_{1,k} & 0 & R_{1,k} & 0 & \dots & R_{1,k} & 0 \\ 0 & R_{1,k} & 0 & R_{1,k} & \dots & 0 & R_{1,k} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ R_{LF,k} & 0 & R_{LF,k} & 0 & \dots & R_{LF,k} & 0 \\ 0 & R_{LF,k} & 0 & R_{LF,k} & \dots & 0 & R_{LF,k} \end{bmatrix}}_{2 NC}$$

$F_{i,j}^R$ and $F_{i,j}^I$ are the real and imaginary part of $F_{i,j}$, respectively. Let

$$\phi_k = y_k^t y_k^{t*} + 2 (\tilde{N}_k^* \tilde{F}_k - \beta_k)^T \lambda_k$$

and

$$\nabla_k = \left[\frac{\partial}{\partial F_{1,k}^R}, i \frac{\partial}{\partial F_{1,k}^I}, \dots, \frac{\partial}{\partial F_{NC,k}^R}, i \frac{\partial}{\partial F_{NC,k}^I} \right]^T$$

Then

$$\nabla_k \phi_k = 2(\psi_k + \tilde{N}_k^* \lambda_k) \quad (4-11)$$

where

$$\psi_k = \frac{1}{LF} \begin{bmatrix} \text{Re} \left[y_k^t X_{1,k}^t \right] \\ i \text{Im} \left[y_k^t X_{1,k}^t \right] \\ \vdots \\ \text{Re} \left[y_k^t X_{NC,k}^t \right] \\ i \text{Im} \left[y_k^t X_{NC,k}^t \right] \end{bmatrix}$$

Re and Im represents "the real part of" and "the imaginary part of".



In order to find the Lagrangian multiplier λ_k , let $\nabla_k \phi_k = 0$. Then we have

$$\lambda_k = - \left[\begin{array}{cc} \tilde{N}_k^* & \tilde{N}_k^{*T} \\ \underline{N}_k & \underline{N}_k \end{array} \right]^{-1} \underline{N}_k^* \psi_k \quad (4-12)$$

Substituting Equation 4-12 into Equation 4-11, we have

$$\nabla_k \phi_k = 2 \left\{ \underline{I} - \underline{N}_k^{*T} \left[\begin{array}{cc} \tilde{N}_k^* & \tilde{N}_k^{*T} \\ \underline{N}_k & \underline{N}_k \end{array} \right]^{-1} \underline{N}_k^* \right\} \psi_k \quad (4-13)$$

After removing the redundant rows of \underline{N}_k^* , we have

$$\underline{N}_k^* = R_{1,k}^* \overbrace{\left[\begin{array}{cccccccc} 1 & 0 & 1 & 0 & - & - & - & 1 & 0 \\ 0 & 1 & 0 & 1 & - & - & - & 0 & 1 \end{array} \right]}^{2 \text{ NC}}$$

Thus,

$$\underline{N}_k^{*T} \left[\begin{array}{cc} \underline{N}_k^* & \underline{N}_k^{*T} \\ \underline{N}_k & \underline{N}_k \end{array} \right]^{-1} \underline{N}_k^* = \frac{1}{NC} \left[\begin{array}{cccccccc} 1 & 0 & 1 & 0 & - & - & - & 1 & 0 \\ 0 & 1 & 0 & 1 & - & - & - & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ 1 & 0 & 1 & 0 & - & - & - & 1 & 0 \\ 0 & 1 & 0 & 1 & - & - & - & 0 & 1 \end{array} \right] \quad \left. \vphantom{\begin{array}{c} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{array}} \right\} 2 \text{ NC}$$



and Equation 4-13 becomes

$$\nabla_k \hat{f}_k = 2 \left\{ \mathbf{I} - \frac{1}{NC} \begin{matrix} \overbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & & 1 & 0 \\ 0 & 1 & 0 & 1 & & 0 & 1 \\ \vdots & & & & & & \\ 1 & 0 & 1 & 0 & & 1 & 0 \\ 0 & 1 & 0 & 1 & & 0 & 1 \end{bmatrix}}^{2 NC} \right\} \underline{\psi}_k$$

$$= - \frac{2}{NC} \mathbf{P} \underline{\psi}_k$$

where

$$\mathbf{P} = \begin{matrix} \overbrace{\begin{bmatrix} 1-NC & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1-NC & 0 & 1 & & 0 & 1 \\ 1 & 0 & 1-NC & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & 0 & 1 & \dots & 0 & 1-NC \end{bmatrix}}^{2 NC} \end{matrix}$$

The adaptive algorithm can be written as

$$\tilde{\mathbf{F}}_k^{t+1} = \tilde{\mathbf{F}}_k^t + \frac{2 B_k}{NC} \mathbf{P} \underline{\psi}_k \quad (4-14)$$



or

$$F_{i,k}^{t+1} = F_{i,k}^t + C_k y_k^t \left[\bar{X}_k^t - X_{i,k}^t \right] \quad (4-15)$$

where

$$\bar{X}_k^t = \frac{1}{NC} \sum_{i=1}^{NC} X_{i,k}^t$$

$$C_k = \frac{2B_k}{LF}$$

Since $y_k^t = y_{LF-k}^{t*}$ and y_0^t is real, we can write Equation 5-8 as

$$y^t = y_0^t + 2 \operatorname{Re} \left[\sum_{k=1}^{(LF-1)/2} y_k^t \right] \quad (4-16)$$

Since $F_{i,k}^t = F_{i,LF-k}^{t*}$ and $F_{i,0}^t$ is real, we only need to calculate Equation 4-15 for $k=0, 1, \dots, (LF-1)/2$.



SECTION V

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13. ABSTRACT <p>Recent intensive study of adaptive (gradient-search) filtering in the time domain has not solved the problems with rate-of-convergence problem, which is a major difficulty with this technique. A recent study¹ based on a set of time-stationary synthetic data shows that the time-domain maximum-likelihood adaptive filter converges very slowly to the optimum filter. After 3300 iterations of adaption with an adaptive rate of 10 percent of maximum value, the adaptive filter is still about 4 db away from the optimum filter in the sense of mean-square outputs.</p> <p>Time-domain adaptive filtering necessitates using only one convergence parameter for all filter coefficients, which may cause slow convergence for some data. Frequency-domain adaptive filtering may solve this problem, since different convergence parameters can be used for different frequency components. This report describes a frequency-domain maximum-likelihood adaptive-filtering algorithm analogous to the time-domain adaptive algorithm². This algorithm was used with a set of synthetic stationary data previously used for a time-domain adaptive-filtering study. Different filter lengths and convergence parameters were used. Results are compared with beamsteer and time-domain adaptive filter.</p>			

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Adaptive Filtering Frequency Domain <i>R</i>						