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TECHNICAL REPORT 2

TIME CORRECTION IN PASSIVE RANGING:
BREAKTHROUGH OR BOOTSTRAP?

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TIME CORRECTION IN PASSIVE RANGING:
BREAKTHROUGH OR BOOTSTRAP?[*]

Introduction and Summary

The concept of time correction in passive ranging is described
in reference (a). It was developed by Daniel H. Wagner,
Associates, who believe it to be a breakthrough in the subject
of passive ranging. The concept has been applied to the four-
bearings problem for which it was conceived and to the Ekelund
range estimate involving two bearing rates. A time-corrected
Ekelund range estimator is being considered for part of the
target motion analysis in a new fire-control system.

Does the concept of time correction represent a significant
breakthrough, as claimed, or is the apparent gain an illusion?
We have examined this question carefully and have found no
support for the claims that have been made for the method. We
have concluded that the method has no special value in passive
ranging and should be discarded. Our arguments are presented
below.

The Time-Correction Method

Time correction, as developed in reference (a), is a method for
the computation of a range R* at a time t* from the four bearings
B_1, B_2 B_3, B_4 observed at the corresponding times t_1, t_2, t_3, t_4
which are assumed to be in the order $t_1 < t_2 < t_3 < t_4$. We
outline the derivation of the equations below.

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We use the standard naval coordinate system. For $i = 1, 2, 3, 4$, let

t_i = time at the $\underline{i^{th}}$ observation

B_i = bearing of the target from the tracking submarine

R_i = range of the target from the tracking submarine

u_{ri} = range component of target velocity measured outward along B_i

u_{bi} = normal component of target velocity measured normal to B_i in the direction of positive (clockwise) angular rotation

Also, let

μ_{ij}^{k} = distance (directed) tracking submarine moves in the direction B_k from time t_i to time t_j

ν_{ij}^{k} = distance (directed) tracking submarine moves normal to B_k from time t_i to time t_j

It will be convenient to use

$$t_{ij} = t_j - t_i, \quad B_{ij} = B_j - B_i, \quad S_{ij} = \sin B_{ij}, \quad C_{ij} = \cos B_{ij}$$

Assuming linear target motion it is easy to derive the equations

$$R_2 S_{12} = u_{b1} t_{12} - \nu_{12}^{1} \tag{1}$$

$$R_4 S_{34} = u_{b3} t_{34} - \nu_{34}^{3} \tag{2}$$

$$R_2 C_{24} = R_4 - u_{r4} t_{24} + \mu_{24}^{4} \tag{3}$$

$$u_{b1} C_{13} = u_{b3} + u_{r1} S_{13} \tag{4}$$

These equations are equivalent to the equations (2-5) (2-6) (2-7) and an unnumbered equation on page 2-18 of reference (a). Eliminating R_2, u_{b1}, and u_{b3} from the above equations and solving for R_4 we obtain

$$R_4 = \hat{R}_4 + a_1 u_{r1} + a_2 u_{r4} \tag{5}$$

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where

$$\hat{R}_4 = (v_{34}^3 / t_{34} - C_{13} v_{12}^1 / t_{12} - \mu_{24}^4 \beta_{12}) / (\beta_{12} - \beta_{34}) \quad (6)$$

$$a_1 = S_{13} / (\beta_{12} - \beta_{34}), \quad a_2 = t_{24} \beta_{12} / (\beta_{12} - \beta_{34}) \quad (7)$$

$$\beta_{12} = (S_{12}/t_{12}) (C_{13}/C_{24}), \quad \beta_{34} = S_{34} / t_{34} \quad (8)$$

Equation (5) expresses the range R_4 in terms of an estimate $\hat{R}_4$ that can be computed from observed quantities, and two range components of target velocity.

In the time-correction method the two range components, u_{r1} and u_{r4}, of target velocity are eliminated by introducing two bearings, B_5 and B_6, at times t_5 and t_6 by using two equations of type (3) above. They are

$$R_5 C_{45} = R_4 + u_{r4} t_{45} - \mu_{45}^4 \quad (9)$$

$$R_6 C_{16} = R_5 C_{15} + u_{r1} t_{56} - \mu_{56}^1 \quad (10)$$

The general equation of this type is

$$R_j C_{jk} = R_i C_{ik} + u_{rk} t_{ij} - \mu_{ij}^k \quad (11)$$

Equations (3) (9) (10) are special cases of equation (11).

Eliminating R_4 and R_5 from equations (5) (9) (10) we obtain the equation

$$R_6 C_{16} = (C_{15}/C_{45}) (\hat{R}_4 - \mu_{45}^4) - \mu_{56}^1 + a_3 u_{r1} + a_4 u_{r4} \quad (12)$$

where a_3 and a_4 depend on t_5 and t_6. Let $\bar{t}$ and t^* be the values of t_5 and t_6 respectively that make $a_3 = a_4 = 0$. Also, let $\bar{B}$ and B^* be the corresponding bearings of the target at these times, and let R^* be the corresponding value of R_6. Then

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$$\overline{t} = (\beta_{12}t_2 - \beta_{34}t_4) / (\beta_{12} - \beta_{34}) \tag{13}$$

$$t^* = \overline{t} - S_{13} \cos (\overline{B} - B_1) / (\beta_{12} - \beta_{34}) \cos (\overline{B} - B_4) \tag{14}$$

$$R^* \cos (B^* - B_1) = (\hat{R}_4 - \mu_{45}^4) \cos (\overline{B} - B_1) / \cos (\overline{B} - B_4) - \mu_{56}^1 \tag{15}$$

Equations (13) (14) (15) are the same as equations (2-14) (2-18) (2-20) respectively of reference (a) when our subscript sequence (1, 2, 3, 4) is replaced by the sequence (1', 1, 2', 2) of reference (a). (It is convenient to retain the subscripts 5 and 6 in μ_{45}^4 and μ_{56}^1 to avoid ambiguity.)

The range R^* is called "the time-corrected range" and t^* is called "the best range time." The time-correction method consists of using equation (15) to compute R^* and equation (14) to compute the corresponding time t^* at which it applies.

Discussion

Does the above method offer a significant breakthrough in passive ranging? Various claims are made for the method in reference (a). It is asserted that the basis for the method is that "every ranging maneuver has associated with it a best range time", at which time the error in the range component of velocity "will have minimum effect on the accuracy of the range solution." And when t^* is in the future, it is claimed that the four bearing observations and the three distance measurements (μ_{24}^4, μ_{45}^4, and μ_{56}^1) "provide an accurate range at a future time with no additional information needed." Are these claims justified?

The derivation is deceptively simple. No optimization is needed. Only simple algebraic manipulations of well-known equations are required to eliminate the two range components, u_{r1} and u_{r4}, of target velocity from the range equation (5). It is surprising that such a simple derivation yields a range estimate that is

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superior to other range estimates, as claimed. And it is even
more surprising that the result remained undiscovered so long,
since the derivation involves only simple mathematics.

Since t^* is not obtained by optimizing a payoff function, why
is it called "best"? Apparently, t^* is called the best range
time because the corresponding range R^* has been written in
equation (15) in a form that does not involve a range component
u_r of target velocity. Immediately following the equation, which
is numbered (2-20) in reference (a), it is stated that, "The
range R^* is the time-corrected range. Note that all of the
terms in Equation (2-20) can be measured by the SSK. The time
t^* at which this applies is called the best range time..."

Can all of the terms in equation (15) above be "measured"?
Certainly, $\bar{B}$ and B^* can't be measured in the same way that B_1,
B_2, B_3, B_4 are measured by recording the observed value at the
time of occurrence. The bearings $\bar{B}$ and B^* are the bearings at
the times $\bar{t}$ and t^*. Obviously, if $\bar{t}$ and t^* fall outside the
tracking interval (t_1, t_4), the corresponding bearings $\bar{B}$ and B^*
can't be observed. Nor can they be observed, if $\bar{t}$ and t^* fall
inside the tracking interval. The values of $\bar{t}$ and t^* in
equations (13) and (14) require the values of all four bearings.
Hence, t_4 is the earliest time at which $\bar{t}$ and t^* can be computed,
and it then is too late to observe $\bar{B}$ and B^*.

Hence, $\bar{B}$ and B^* must be "estimated" from the bearing-time plot,
by interpolation when $\bar{t}$ and t^* fall inside the tracking interval
and by extrapolation when they fall outside the tracking interval.
In either case the errors in estimating $\bar{B}$ and B^* may be large.
Here, interpolation may be as inaccurate as extrapolation, since
the SSK is maneuvering during the tracking interval and bearing
observations usually are not made during the turns. However,
the resulting errors in the cosine factors, and in R^*, in
equation (15) would be small. Hence, the fact that $\bar{B}$ and B^*

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must be estimated by interpolation or extrapolation is not a major disadvantage. The error is comparable to that made in using the small-angle approximations.

Errors also will occur in estimating the two distances μ_{45}^4 and μ_{56}^1, since they also must be obtained by interpolation or extrapolation after $\bar{t}$ ($=t_5$) and t^* ($=t_6$) have been computed. These errors may produce larger errors in R^* than those produced by the errors in estimating $\bar{B}$ and B^*.

More serious questions concerning the method can be raised as follows:

    a.   What is the motivation behind the procedure? What objective are we trying to achieve? Is it merely to find an equation for the range R^* at some time t^* in a form that involves only quantities that the SSK can measure or estimate?

    b.   Does the procedure lead to a unique solution that has some desirable property? If so, what?

    c.   After we find R^* at time t^*, how will we obtain an estimate of the range R at time t when the time t at which we need a range estimate does not coincide with t^*? Will this require that we use an equation of type (11) above that requires a range component u_r of target velocity? If so, what have we gained by first computing R^*?

If the objective of the method is to obtain an equation for range in a form that involves only quantities that the SSK can measure or estimate, why stop at one? There are many such equations. What special properties does the "solution" in equation (15) have that it should be selected?

Other "solutions" of the type displayed above can be obtained by interchanging two subscripts. This possibility seems to have been recognized in some parts of reference (a), and used in a

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special case in chapter 4. But the implications concerning the validity of the general method and the uniqueness of the solution were not explored.

Some of the "solutions" obtained in this way reduce to the same forms for t^* and R^* in the small-angle approximations, and others do not. For example, if we replace equations (9) and (10) by the equations

$$R_5 \, C_{15} = R_4 \, C_{14} + u_{r1} \, t_{45} - \mu_{45}^1$$

$$R_6 \, C_{46} = R_5 \, C_{45} + u_{r4} \, t_{56} - \mu_{56}^4$$

and repeat the steps of the solution, the equations for t^* and R^* are not identical to equations (14) and (15) but reduce to the same small-angle forms that equations (14) and (15) become. The equation for $\overline{t}$ does not do so. On the other hand, if we interchange the subscripts 2 and 3 in the original solution, we get a solution for which the small-angle approximations for t^* and R^*, as well as that for $\overline{t}$, do not reduce to the same small-angle forms obtained in the original solution. This solution is written in full below:

$$\overline{t} = (\beta_{13} \, t_3 - \beta_{24} \, t_4) \, / \, (\beta_{13} - \beta_{24}) \tag{13a}$$

$$t^* = \overline{t} - S_{12} \, \cos(\overline{B} - B_1) \, / \, (\beta_{13} - \beta_{24}) \, \cos(\overline{B} - B_4) \tag{14a}$$

$$R^* \cos(B^* - B_1) = (\hat{R}_4 - \mu_{45}^4) \, \cos(\overline{B} - B_1) \, / \, \cos(\overline{B} - B_4) - \mu_{56}^1 \tag{15a}$$

where now

$$\hat{R}_4 = (\nu_{24}^2 \, / \, t_{24} - C_{12} \, \nu_{13}^1 \, / \, t_{13} - \mu_{34}^4 \, \beta_{13}) \, / \, (\beta_{13} - \beta_{24}) \tag{6a}$$

$$\beta_{13} = (S_{13} \, / \, t_{13}) \, (C_{12} \, / \, C_{34}), \quad \beta_{24} = S_{24} \, / \, t_{24} \tag{8a}$$

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It is evident that we can't call the time t^* in equation (14) the best range time, since it is neither unique nor optimal in any demonstrated way. It may be a good range time, but the property, if any, that makes it better than an arbitrary time has not been displayed in reference (a) or elsewhere, to our knowledge. The property that led to its introduction - that it permits us to write an equation for the corresponding range R^* in a form that involves only quantities that the SSK can measure or estimate - is not a sufficient reason for its use. Any arbitrary time has this property.

Under the assumption of linear target motion, on which the derivations above and in reference (a) are based, four bearing observations (made on a ranging maneuver that doesn't consist of a single linear path) are sufficient for a complete TMA. Hence, the range R at any arbitrary time t can be written as a function of t, the four observation pairs (t_i, B_i), i = 1, 2, 3, 4, and components of own-ship motion. It is not necessary to compute two times $\bar{t}$ and t^* and then estimate the corresponding bearings $\bar{B}$ and B^* by interpolation or extrapolation from the bearing-time plot. The only bearings involved are B_1, B_2, B_3, B_4.

The TMA requires only the solution of four simultaneous linear equations in a set of target parameters, such as x_4, y_4, u_x, u_y, where u_x and u_y are the components of target velocity. The solution has been obtained many times. In fact, Spiess' form of the solution is discussed in chapter 5 of reference (a). By means of the TMA we can write an equation for the range R at an arbitrary time t, in a form that involves only quantities that the SSK can measure or estimate.

The solution for R is simple when t coincides with one of the observation times. For example, when $t = t_4$, the solution becomes

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$$R_4 = \frac{v^1_{14}\, t_{24}\, t_{34}\, S_{23} - v^2_{24}\, t_{14}\, t_{34}\, S_{13} + v^3_{34}\, t_{14}\, t_{24}\, S_{12}}{- t_{13}\, t_{24}\, S_{12}\, S_{34} + t_{12}\, t_{34}\, S_{13}\, S_{24}} \tag{16}$$

where, as before,

v^k_{ij} = distance SSK moves normal to B_k in the interval $(t_i,\ t_j)$

S_{ij} = sin $(B_j - B_i)$

t_{ij} = $t_j - t_i$

We do not recommend the use of the estimator (16) for R_4. However, equation (16) is a simple equation for the range R_4 in terms of the four $(t_i,\ B_i)$ pairs and three components of own-ship motion. Also, it is exact, that is, it will yield the exact value for the range at time t_4 in the absence of errors in the measured quantities. And, certainly, the SSK can measure all the quantities in equation (16). Do these facts make t_4 a "good" range time? If not, what property does t^* possess that makes it better? Will any gain that is obtained from using t^* and R^* be lost in computing R_4 from R^*, if t_4 is the time at which a range estimate is needed.

It is surprising that nothing is said in reference (a) about the problem of converting from the estimate R^* at time t^* to an estimate of the range at the time at which it is needed. How is this conversion to be made? By solving for the TMA? If so, won't we finish with the same deterministic relationships, but in a more circuitous form, with many more chances for errors in the measurements? Will the estimate of R_4 by such a method be more accurate than that obtained from equation (16)?

A possible way of avoiding these questions is to state that the submarine commander can control the value of t^* by his choice of ranging maneuvers and hence can make t^* equal to, or close to, the desired time, say t_4. In fact, this point is offered in

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reference (a) as a major advantage of the time-correction method. We contend that control of the value of t^* is not a practical solution to this problem. The commander of own submarine has only partial control over t^* and that control is rather tenuous. The commander of the target submarine must cooperate by maintaining constant course and speed. Even when he does so, the problem of the selection of maneuvers and times t_1, t_2, t_3, t_4 that will make t^* fall at t_4 is not an easy one to solve, since the value of t^* can't be computed until the last bearing B_4 has been recorded. It is the type of problem that can be solved after the fact but is very difficult to solve in real time and sequence. Even if it can be done, there remains the problem of how to assure linear target motion in the relevant time interval.

Criteria For Selection of a Range Estimator

The equations derived above have been based on two important assumptions, as follows:

(1) The target submarine maintains constant course and speed throughout the relevant time interval, which is the maximum interval spanned by the times involved in the computation.

(2) There are no errors in the measurements of bearings, including those obtained by interpolation or extrapolation, or in the measurement of own-submarine motion.

Both assumptions are questionable, at best, and we should consider the effects of departures from these assumptions.

The argument so far has been based solely on formula accuracy under these assumptions. The main argument for the use of R^* in equation (15) seems to be that equation (15) is exact and contains only quantities that the SSK can measure or estimate;

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whereas some other equations for a range estimator are inexact,
such as equation (6) as an estimator for R_4; or they contain
quantities that the SSK can't measure or estimate, such as
equation (4). More precisely, equation (4) contains two factors,
u_{r1} and u_{r4}, that the analyst hasn't taken the trouble to express
in terms of quantities that the SSK can measure or estimate.
If this had been done, equation (5) would have reduced to
equation (16) when simplified.

Formula accuracy is only one characteristic of a range estimator,
and not the dominant one in most applications. The main
characteristics of a range estimator are:

    (a) Formula accuracy

    (b) Ruggedness to target maneuvers

    (c) Sensitivity to error measurements

It is not likely that an estimator will dominate all other
contenders in all respects. For example, the estimator for R_4
in equation (16) is exact, and hence superior in formula
accuracy to the estimator $\hat{R}_4$ for R_4 in equation (6), and to the
estimator R_E obtained by dropping the μ_{24}^4 term in equation (6),
and to the Ekelund estimator R_{EK} (a limiting form of R_E), since
$\hat{R}_4$, R_E, and R_{EK} are biased estimators. However, R_4 in (16)
probably is not as rugged to target maneuvers as some of the
other estimators, and undoubtedly is more sensitive to random
errors than $\hat{R}_4$, R_E, or R_{EK}. It is impossible to estimate the
ruggedness and sensitivity, or even the formula accuracy, of an
estimator for R_4 that starts with R^* in equation (15), since it
depends on the procedure used to calculate R_4 from R^*.

If there is no dominant estimator, how do we select one? We
believe that this decision requires a careful examination of the
use that will be made of the range estimate and the selection of
a suitable performance measure or payoff function. We then

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compute the payoff function for possible target maneuvers and
error distributions, select the dominant estimator, if it exists,
or use a weighted mean otherwise.

In many applications formula accuracy is relatively unimportant,
particularly when likely target maneuvers or measurement errors
will produce range errors that are much larger than the inherent
bias in the formula. Under such conditions it is absurd to select
an estimator on the criterion of formula accuracy, alone or
predominantly.

Comparisons of range estimators, or complete TMA estimators,
often (usually?) have been made on a partial basis. We believe
that all characteristics of the estimator should be considered.
It should be possible to analyze and compare leading contenders
in respect to the major characteristics listed above. When
this has been done, and done impartially and completely, we
will have available for the first time the facts that are
necessary for the intelligent selection of an estimator. And
the selection should be made in consideration of the use that
will be made of the estimate and the characteristics of the
estimator that are important for that application.

In many applications of passive tracking the entire TMA is
needed. For this reason we believe that the analysis should be
extended to TMA estimators, as well as range estimators.

A complete analysis of the type described above seldom has been
attempted, and perhaps never has been done satisfactorily. An
analysis of several range estimators (the Ekelund and a new
three bearing-rate estimator that removes the bias in the Ekelund
estimator) was made in reference (b). Reference (b) also includes
analyses of several TMA estimators (CHURN and a new one developed
by the writer), using acquisition probability as the payoff
function. While these analyses cover all the major characteristics

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of the estimators, they are not extensive enough to be
considered complete.

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     Manual (U), Volume III THEORY, Dec. 1969 Confidential.

(b) ADL Report NUWRES #12, "Control Modes and Acquisition
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13. ABSTRACT

A review is made of the method of time correction in passive ranging. It is concluded that there is little, if any, support for the claims made for the method.

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