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END FIXITY OF COLUMNS

By

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August 1970

U. S. ARMY AVIATION MATERIEL LABORATORIES
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U S ARMY MATERIEL COMMAND

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This program was carried out under Contract DAAJ02-68-C-0035 with Stanford University as prime contractor and the Georgia Institute of Technology as subcontractor.

This research was directed toward the development of a better understanding of the fundamental processes in the buckling of shells and columns. The report reviews experimental techniques and equipment used in studying stability problems and indicates the need for new techniques in the study of realistic structures.

The report has been reviewed by the U.S. Army Aviation Materiel Laboratories. It is published for the exchange of information and the stimulation of future research.

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by

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SUMMARY

This report presents a review of the important contributions made to knowledge on the stability of elastic columns by experimentalists during the last 125 years. It demonstrates that while methods of dealing with the influence of imperfections on stability have been developed and great ingenuity has been expended in devising "ideal" end restraints to compare with those theoretically described, end fixity is still a major practical difficulty. There is not a single paper, even an isolated suggestion, on how the end fixity for a realistic column might be determined other than by a test to destruction. The report concludes that this topic should receive immediate attention.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Figure		Page

LIST OF SYMBOLS

a	$\Delta L/2$ effective increase in strut length due to end fixture, in.
a_1	initial imperfection parameter, in.
E	modulus of elasticity, lb/in.^{2}
F	lateral force at column ends, lb
I	cross-section moment of inertia, in.^{4}
L	nominal beam length, in.
L_0	distance between inflection points, in.
M	bending moment, in.-lb
M_0	moment at top end ($x = 0$) of column, in.-lb
m_0	elastic rotational restraint at the top end ($x = 0$) of the column, in.-lb
m_1	elastic rotational restraint at the bottom end ($x = L$) of the column, in.-lb
n	integer
P	axial load in the column, lb
P_1	buckling load associated with the first mode, lb
P_2	buckling load associated with the second mode, lb
P_3	buckling load associated with the third mode, lb
P_n	buckling load associated with the n^{th} mode, lb
r_1	knife edge carrier radius, in.
r_2	knife edge radius, in.
S_0	rigid carrier length at the top end ($x = 0$) of the column, in.
S_1	rigid carrier length at the bottom end ($x = L$) of the column, in.
s	dimensionless parameter defined in equation (12)
t	dimensionless parameter defined in equation (12)

W_n nth coefficient in total deflection expansion, in.

W_n' n^{th} coefficient in initial deflection expension, in.

x axial column coordinate, in.

X horizontal offset of load from initial line, in.

y lateral deflection coordinate, in.

β $\pi/2$ - angle between vertical and tangent to bearing surface at point of contact

δ elastic deflection amplitude, in.

ΔL increase in effective column length, in.

μ_0 stiffness parameter defined in equation (5)

μ_1 stiffness parameter defined in equation (5)

ν Poisson's ratio

ν $= \delta/P$, in./lb

ν dimensionless parameter defined in equation (8)

ν' dimensionless parameter defined in equation (9)

ν'' dimensionless parameter defined in equation (13)

σ stress, $lb/in.^2$

σ_{cmax} maximum compressive stress, $lb/in.^2$

ϕ $= L \frac{P}{EI}$ = length parameter

ψ_0 angle at the top end ($x = 0$) of the column

ψ_1 angle at the bottom end ($x = L$) of the column

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INTRODUCTION

Interest in the ability of a column to withstand axial compression must be almost as old as man's skill in construction. The older civilizations developed forms which were suited to the needs of their times, and there are few among us who have not admired the graceful pillars of the ancient buildings. These early structures were the results of much trial and error rather than any systematic study. In more recent times, however, there has been a great deal of systematic study, both theoretical and experimental, on the subject. It is inevitable, when a topic has the importance of the chosen one and has received the attention which has been accorded to it, that much which is of value is often lost - later to be rederived and employed - but much experience also tends to become permanently overlooked. Approaches to the subject tend to become stereotyped.

Past or present, there are, to the experimentalist, five significant factors to be accounted for in column testing:

1. The end or boundary conditions must be known.

2. The test specimen must be well prepared.

3. The actual loading action used must be accurately defined.

4. The test machine must follow the desired program with accuracy.

5. Accurate and reliable methods of determination of appropriate parameters - strain, displacement, load, etc. - must be available.

In each of these areas, the state of the art has developed to a different degree, but in each there is progress. Yet, there is no system that can ensure that all conditions are met to the fullest in every test; there is no principle which guarantees that an optimum combination will be achieved in a particular circumstance. It is unlikely that either state will ever be established. One thing, however, is certain, and that is that if we are to continue to achieve progress, we must thoroughly appreciate the successes and failures of the past and fully understand and appreciate the capabilities of the present with regard to analysis and experimentation. This report aims to satisfy this requirement, in part, by presenting a review of the important experimental studies made on columns, directing particular attention to the work on end conditions.

DEVICES FOR COLUMN END FIXITY (PAST AND PRESENT)

It is interesting to note that, as early as 1729, Musschenbroek[1] built a testing machine for the determination of the strength of columns. As a result he was able to propose an empirical column formula, a formula in which the strength was proportional to the square of the cross-section dimension and inversely proportional to the square of the length. This result was qualitatively correct, and little more is found in the literature until Euler's[2] classic analytical development of 1744. The next significant development in relation to the column was made by Thomas Young,[3] who, in 1807, derived a very simple but powerful formula which associated the applied load, the Euler critical load, the initial bow, and the elastic deformation under load. This was, in essence, the same hyperbolic law subsequently derived by Ayrton and Perry[4] and, later, by Southwell[5].

In 1840, the first intensive experimental study of column behavior was made by Hodgkinson[6]. The purpose of the study was, in Hodgkinson's words, "to supply the deficiencies of Euler's theory of the strength of pillars, if it should appear capable of being rendered practically useful; and if not, to endeavour to adapt the experiments so as to lead to useful results."

In many respects, Hodgkinson's work set the pattern for much that has followed. It is interesting to record that it was the first contract research, and it is pertinent to observe that it was not cost nor time constrained, a point on which Hodgkinson comments with some force. It is clear from Hodgkinson's paper that he appreciated the influence of end fixity. In fact, his first series of experiments compared the strengths of pillars with rounded and with flat ends. A point mentioned by Hodgkinson in regard to the rounded ends should be noted. He wrote in his paper: "It became necessary to render those which were rounded at the ends more flat there than if the ends had been hemispheres." This comment was made in relation to shorter pillars and was due to the fact that their ends split. Local action at the base of pinned columns has continued to bring difficulties, as will become apparent as this review progresses. The test machines and the end shapes of the columns developed for this study are illustrated in Figures 1 and 2.

There can be little doubt that, although Hodgkinson used a wide range of materials, the prime motivation lay in the fact that iron, wrought and cast, was beginning at that time to be an attractive material to civil engineers. Similarly, the increasing use of metal in bridges and allied structures was a motivation for much that followed.

In 1879, the Watertown Arsenal[7] installed a test machine that was notable not only for its large capacity, 1,000,000 pounds compression, but also for the fact that it was the forerunner of the modern hydraulic machine. This machine was used for a very wide range of studies reported from 1881 onward. It was in connection with studies made in this machine that the

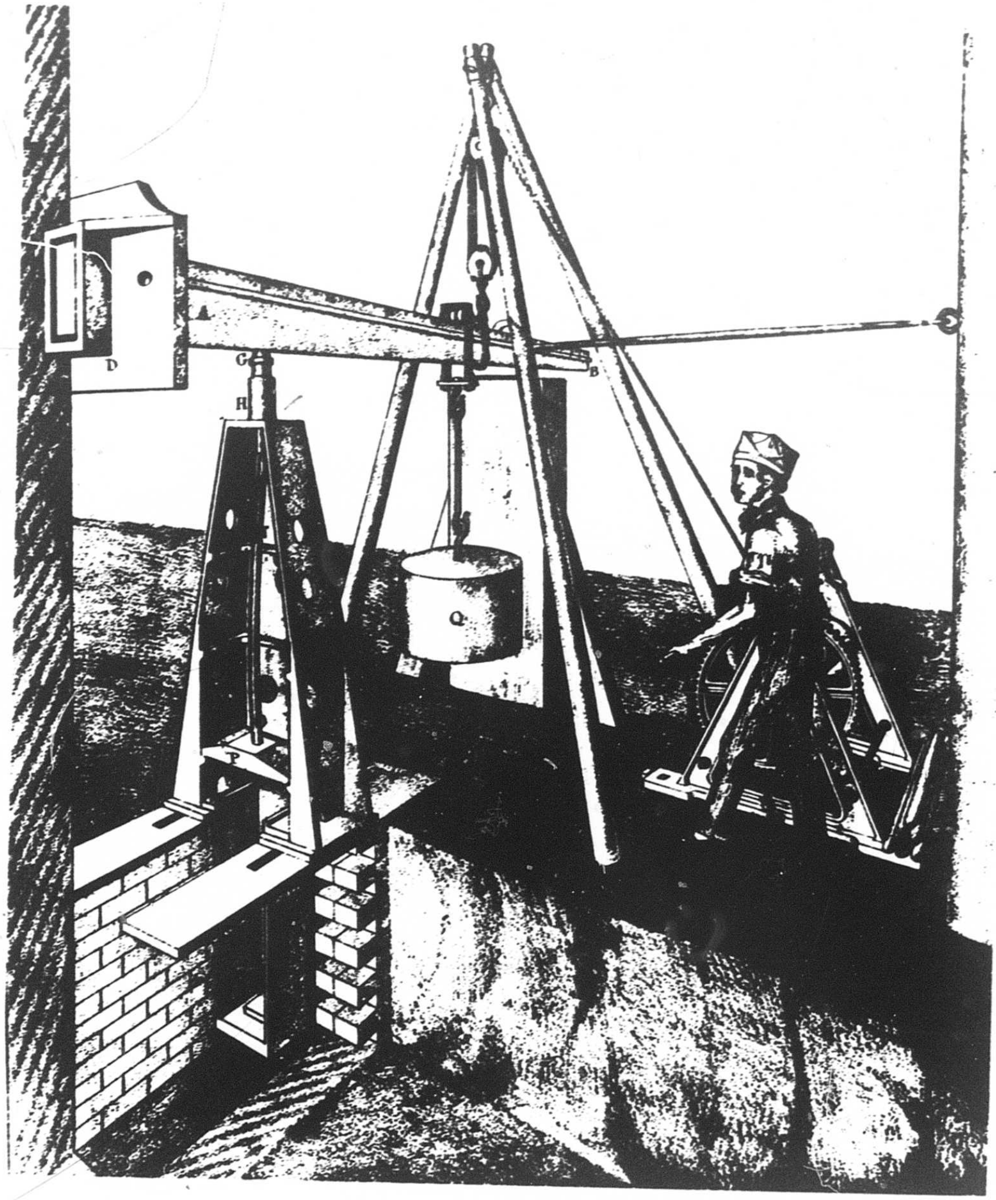

Figure 1. Hodgkinson's Test Machine.

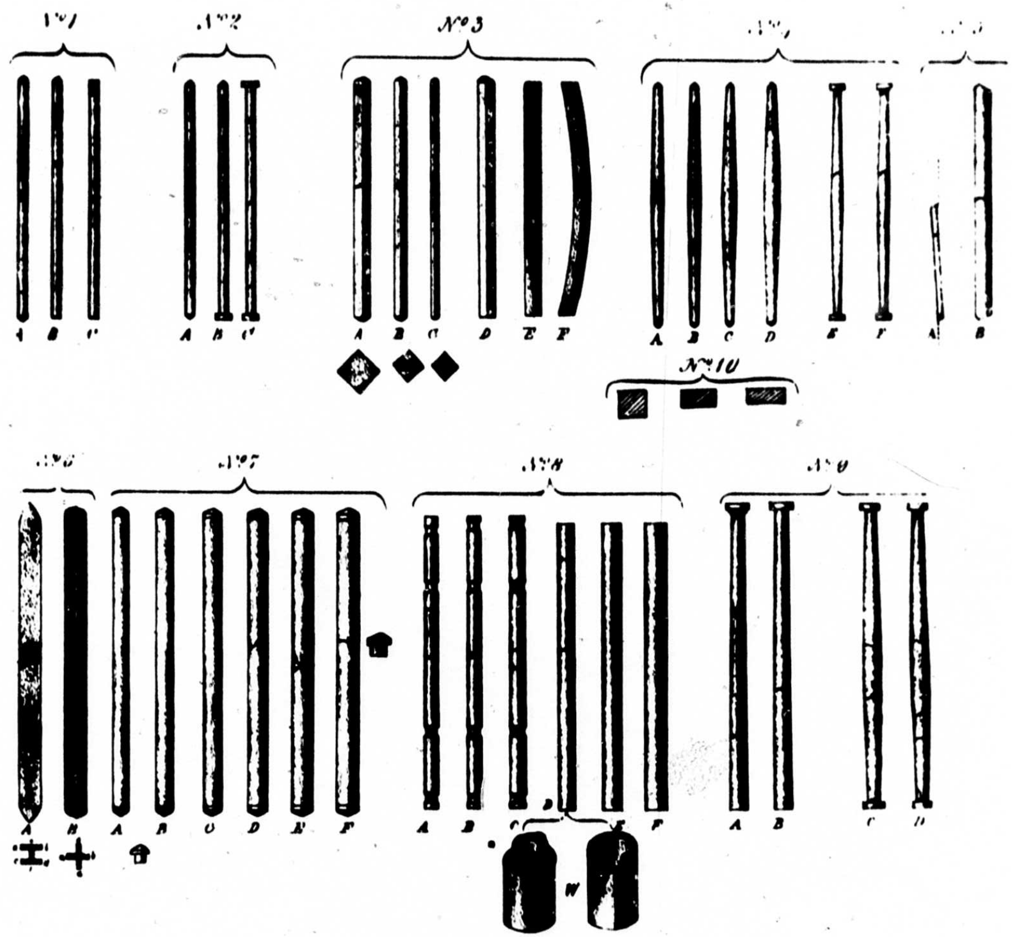

Figure 2. Hodgkinson's Specimens, Illustrating End Shapes.

end fixity arrangement depicted in Figure 3 was developed. This is a so-called pin-ended fixture, and the intent is that it should provide position but not direction fixity. Often in systems of this kind, the ideal situation is not achieved. When the diameter of the pin is relatively large (to ensure no bearing failure), and when the lubrication or surface finish is not extremely good, the fixity condition approaches that of direction fixity. Generally speaking, specimens tested with this kind of end give very varied results, unless the fixture is designed with great care.

Around 1884, Christie[8] developed a spherical seat (Figure 4) for the same purpose and conducted many tests using the device. As with the pin, so with the spherical seat: unless extreme care is exercised in fit, finish, and/or lubrication, the desired fixity condition is not met.

In 1887, Bauschinger[9] conducted a very reliable series of tests on columns, and it is in connection with this study that reference is made to the use of conical end fixtures for the first time. A similar method, Figure 5, was adopted by Tetmajer,[10] whose work was reported in 1890. Ideally, conical end fixtures allow free rotation of the column ends and ensure central application of the load, but in reality, the very high pressure inevitably developed at the point of contact results in the cone's boring into the seating.

At the turn of the century, Lilly[11,12] conducted tests on both round-ended and eccentrically-loaded columns, and it was in connection with this latter study that he used the knife edge system depicted in Figure 6.

A similar procedure was followed by von Kármán[13] for the tests that he made in 1910. It was from this study that he developed the effective EI concept and determined the reduced modulus load for the rectangular cross-section.

Von Kármán's method is, perhaps, the best of those referenced to date. Admittedly, it does not permit the column to rotate freely in arbitrary directions, but, provided the column is correctly located, the hinge operates in the plane in which buckling is desired or anticipated. In the perpendicular direction, it provides almost a fixed-ended condition.

Before proceeding to the more recent work in relation to position-fixed end restraint, it is well to note that from the very earliest investigations, there has been difficulty, also, with the direction and position fixation case. Since the work of Hodgkinson, this case has generally been tackled by making the ends flat or flanged (Figures 7 and 8). Neither of these arrangements is satisfactory, owing to the difficulty of straightening the specimens exactly and of making the end surfaces perfectly flat and parallel. In consequence, the majority of flat-ended specimens start their experimental life in the condition shown in Figure 9, with the result that their behavior pattern is frequently far from ideal. This unsatisfactory situation is, in a number of cases, made worse by motion of the test machine head or platen; i.e., tilting which can effectively create the situation of Figure 9, even for a well-prepared specimen.

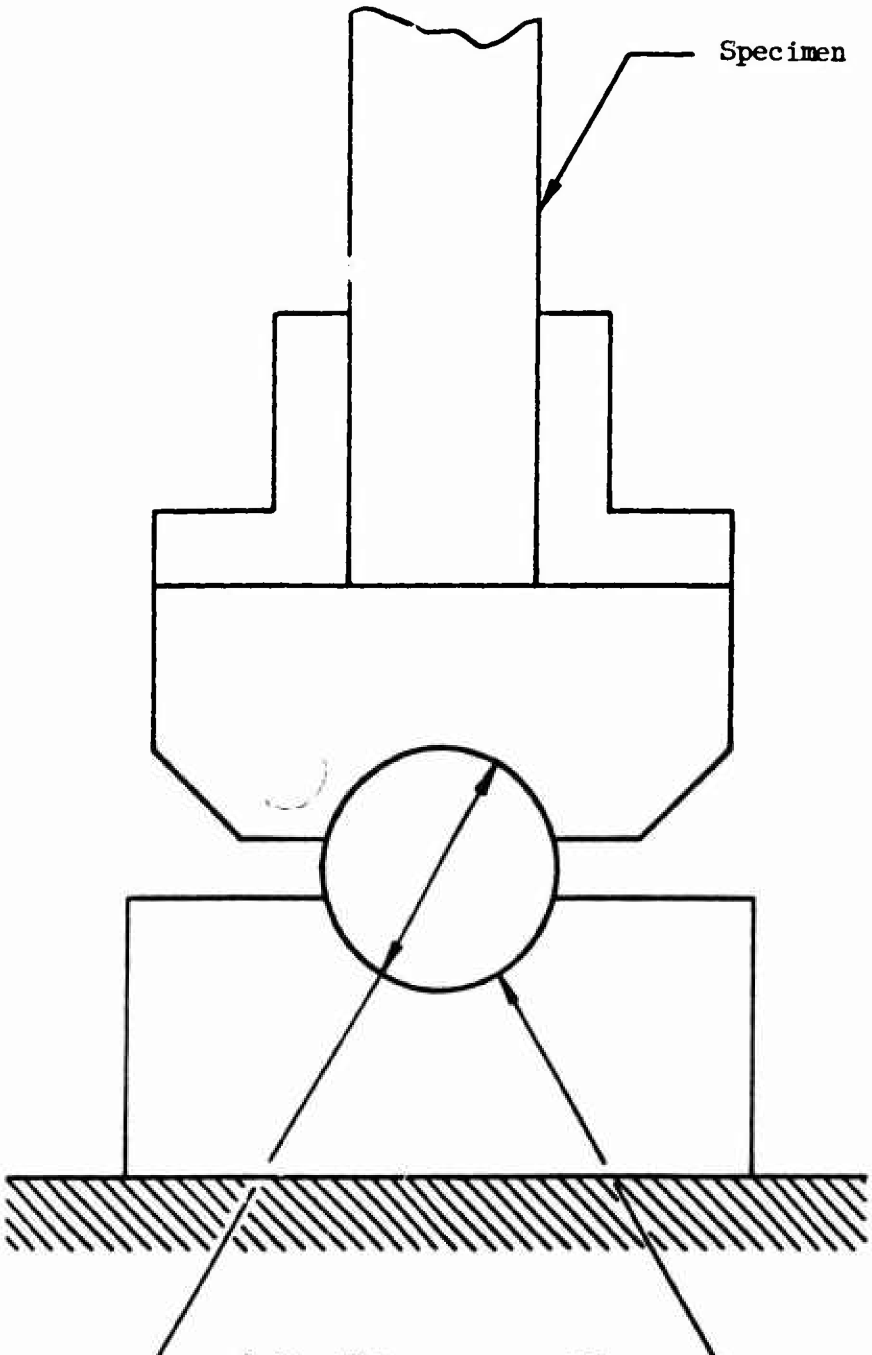

Figure 3. Watertown Arsenal Device.

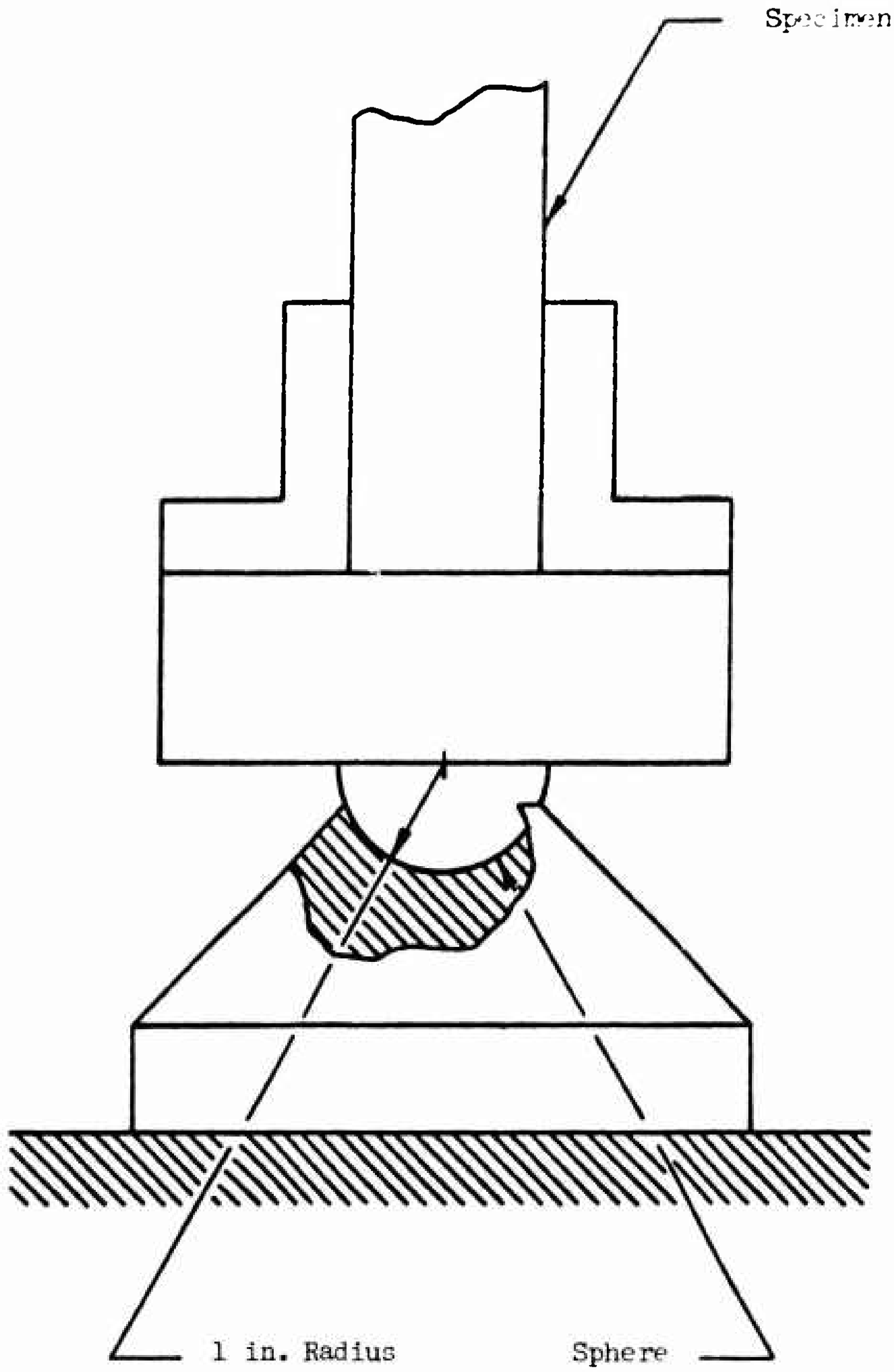

Figure 4. Christie's Spherical Seat Device.

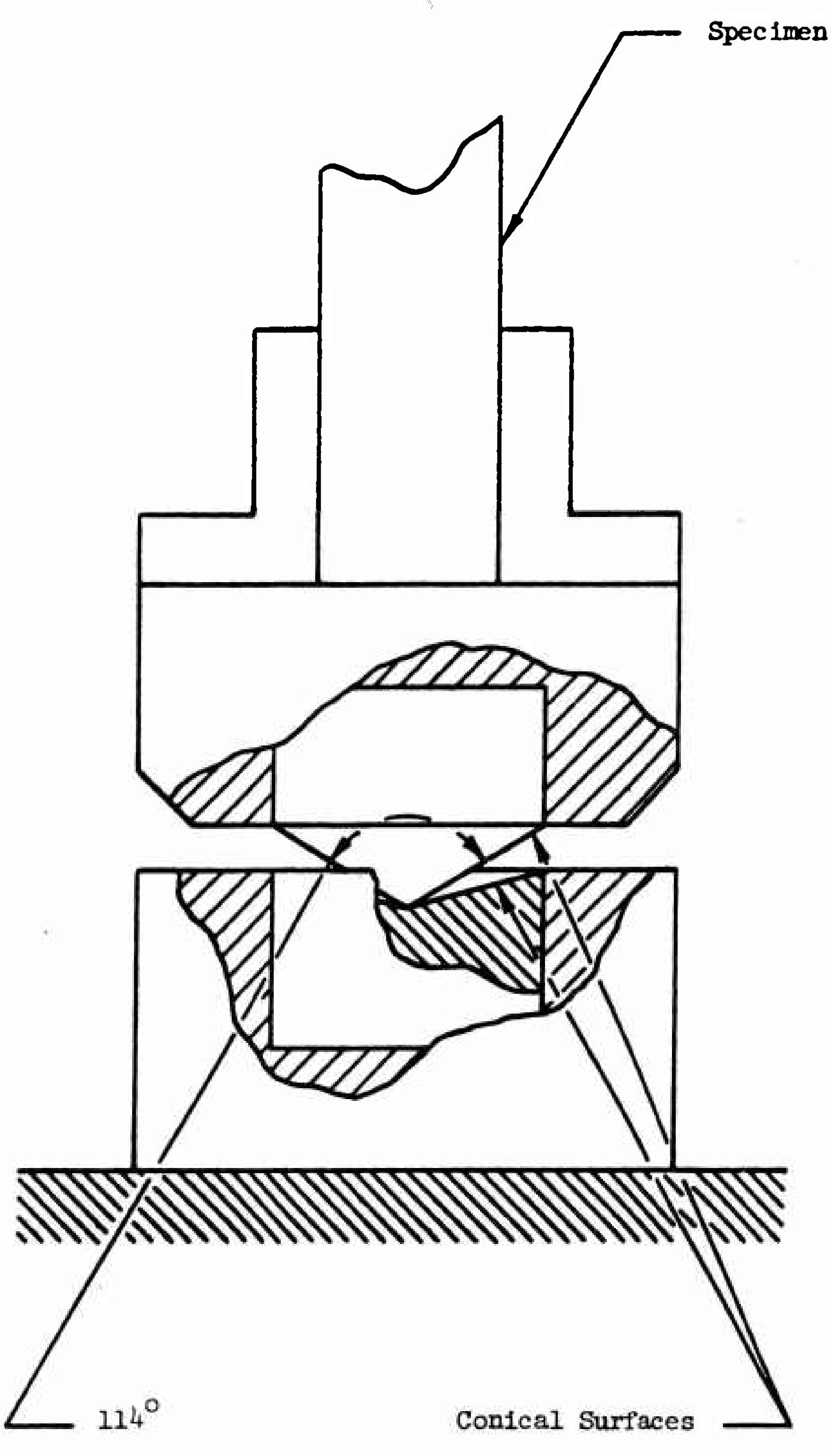

Figure 5. Tetmajer's Conical Device.

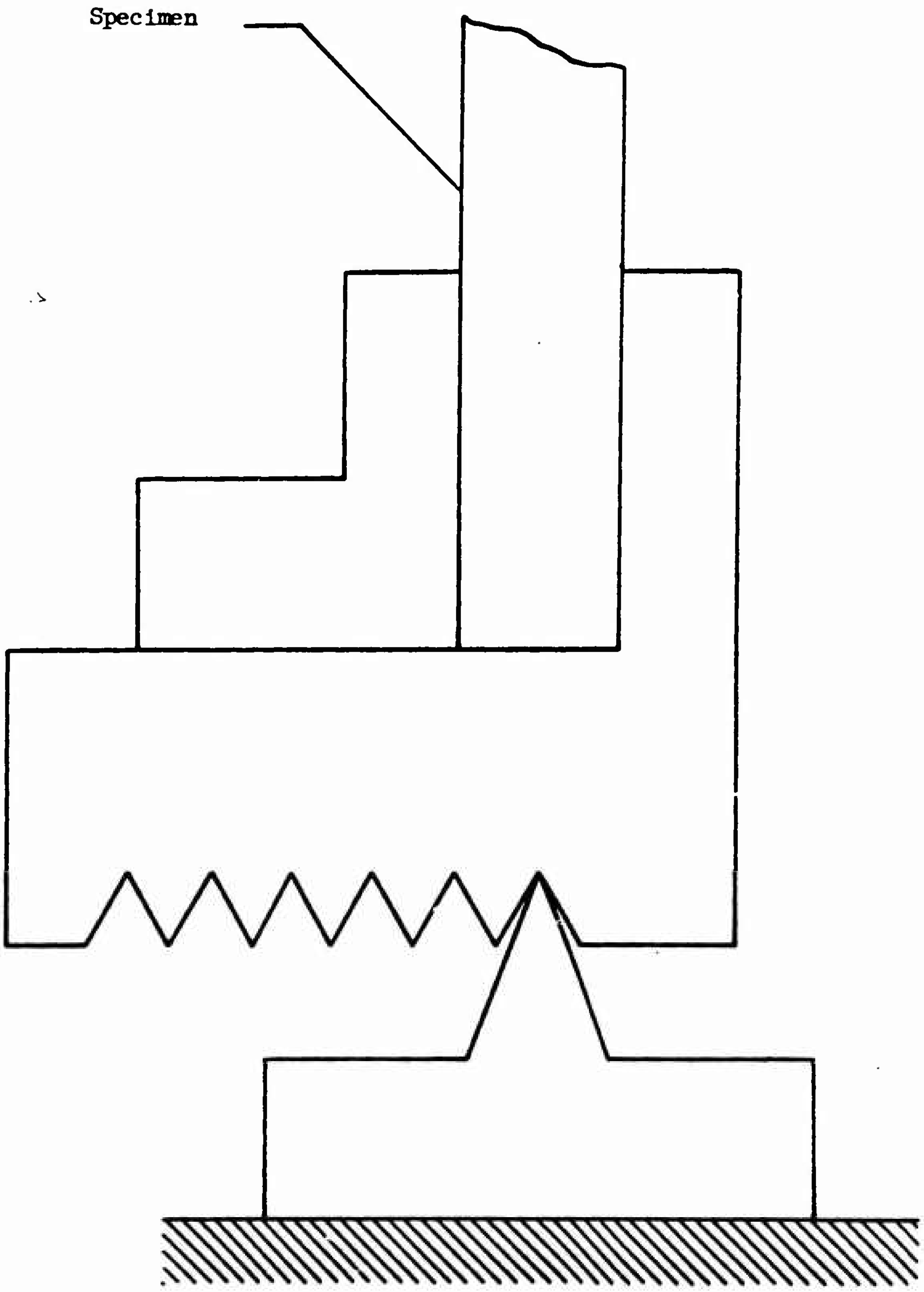

Figure 6. Lilly's Knife Edge Fixture.

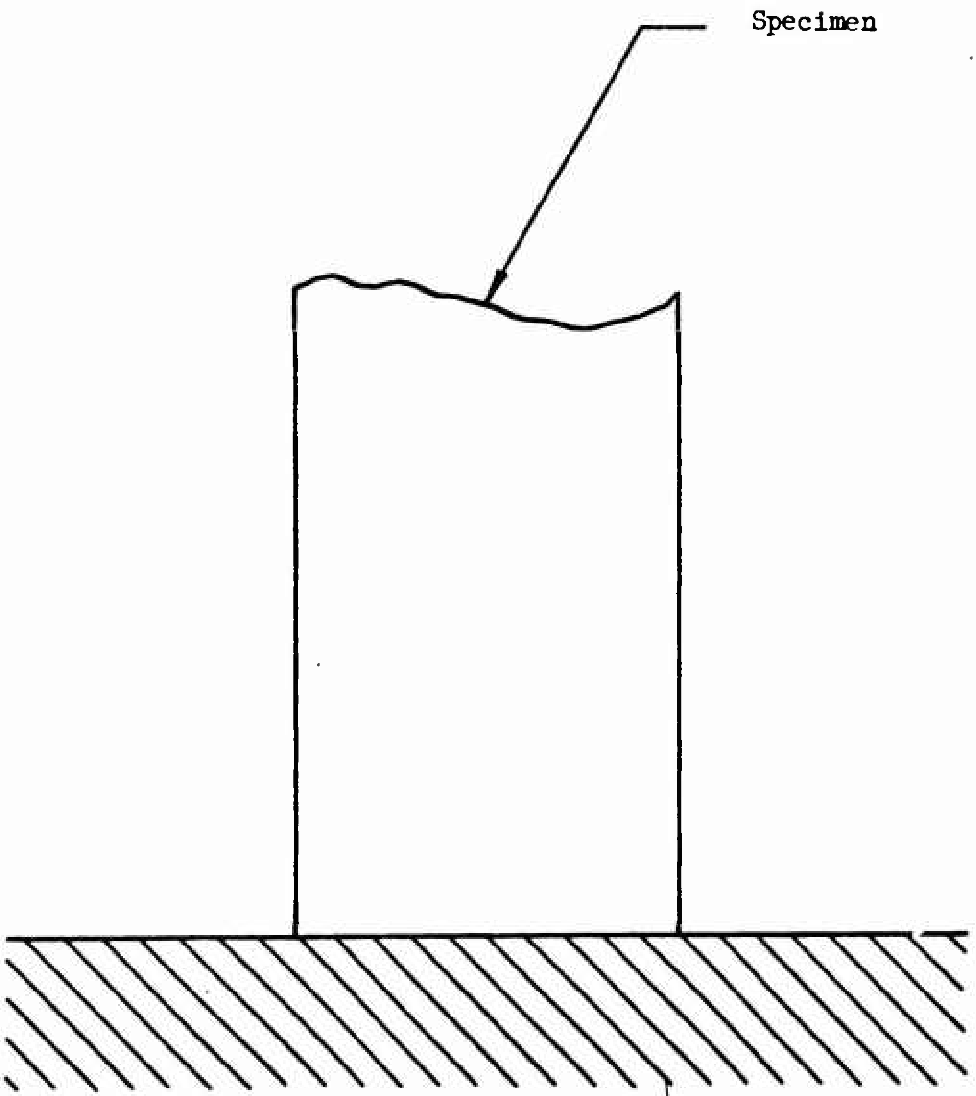

Figure 7. Flat-End Specimen Preparation.

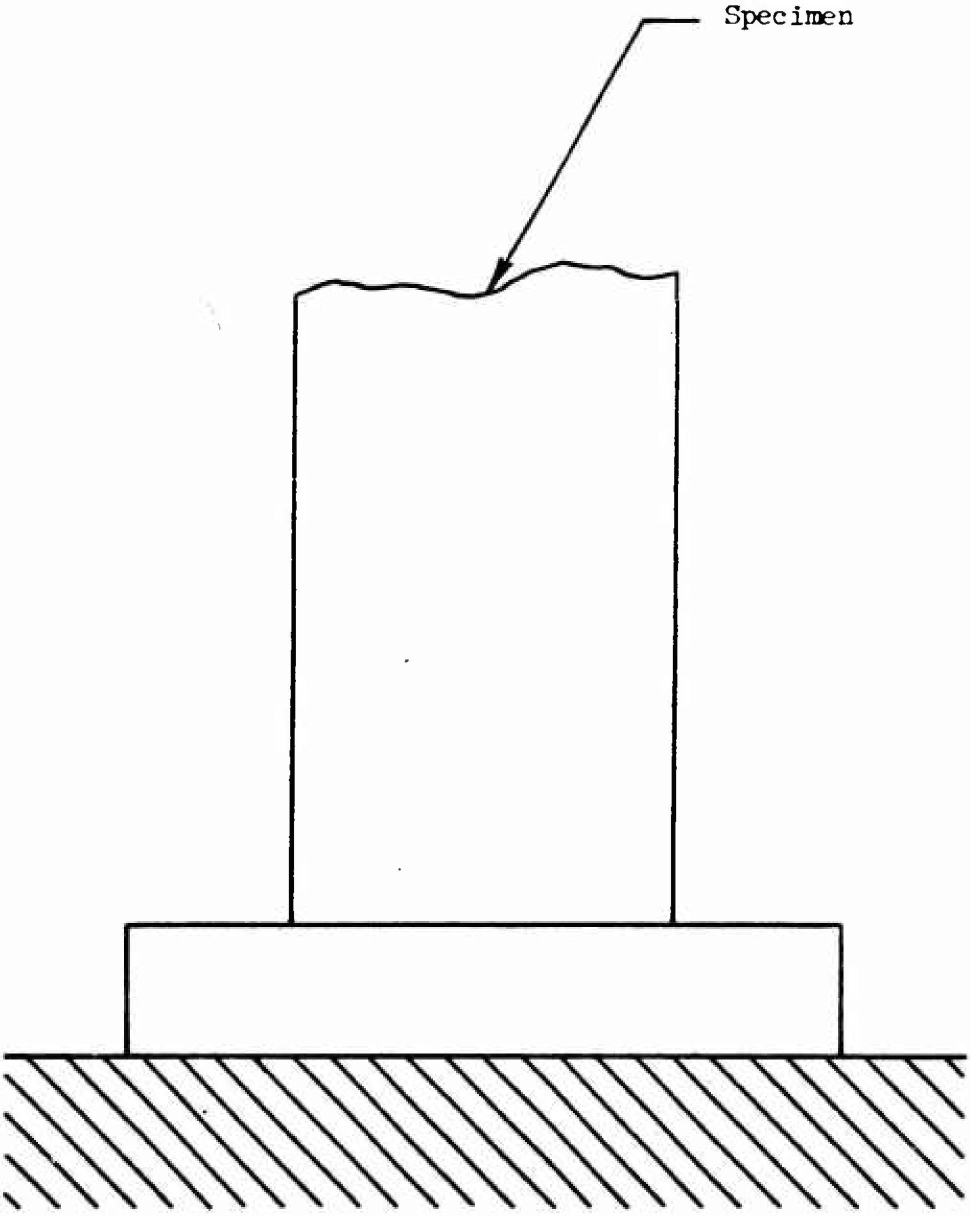

Figure 8. Flanged-End Specimen Preparation.

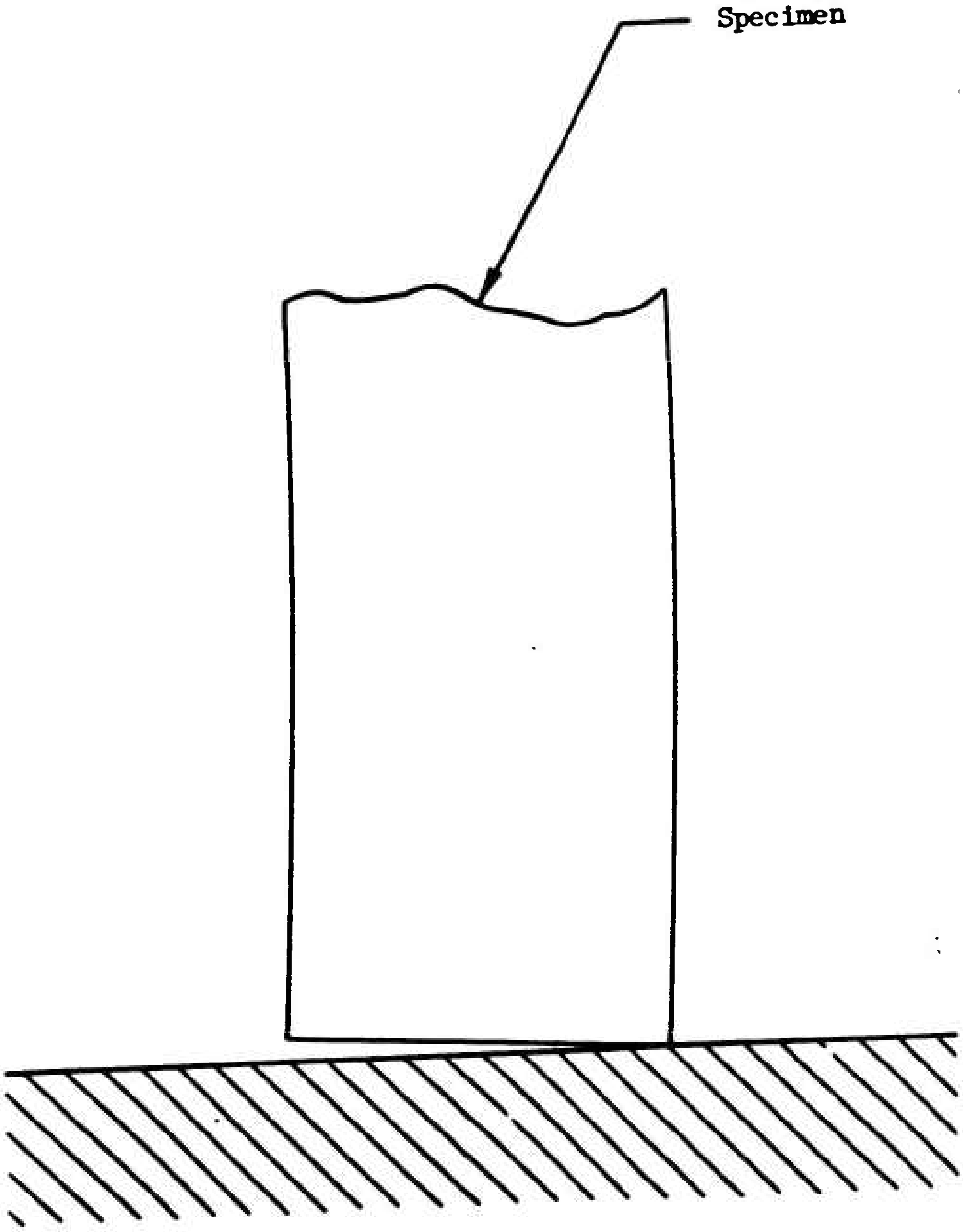

Figure 9. Actual Behavior of Specimen in Test Machine.

We have emphasized, in relation to the pin or spherical seat end-fixity device, the problems which result from friction. This is, of course, well recognized by the more proficient test engineers, and serious attempts have been made to improve this state of affairs.

A good example of the improved cylindrical-type fixture is that developed at the University of Washington in 1926.[14] This particular device, depicted in Figure 10, has a capacity of half a million pounds. As can be seen from the diagram, it consisted essentially of a semicylindrical loading bar and a concave semicylindrical base. The annular space between the two was filled with a single layer of steel rollers. The rollers were 1.5 inches in diameter and had their centers located on a circle of 6 inches radius, whose center was located at the midpoint of the face of the loading block. A friction test indicated a coefficient of moving rolling friction of 0.002. The device performed very satisfactorily. Tests show that if extreme care is exercised, it is even possible to improve on this. Tests of configuration of ground steel rollers between flat plates and of other arrangements of steel ball bearings between semicircular grooved races indicated coefficients of starting rolling friction in the neighborhood of 0.0006.

It is clear that the modern fluid bearing could be used in place of the roller system to achieve a low-friction, smooth, high-load-carrying device. A loading fixture based on this principle is shown in Figure 11.

In 1938, Osgood[15] carried out an investigation of the column strength of tubes elastically restrained against rotation at their ends. A diagrammatic sketch of the apparatus that he used for procuring this elastic restraint is shown in Figure 12. This fixture consisted essentially of a carrier with a knife edge that bore on a seat on a stationary support clamped to the head or table of the test machine. It was provided with means for holding the end of the test specimen in position in the carrier and moving it horizontally under low loads in a direction perpendicular to the knife edge. Rotation about the knife edge was restrained by the helical springs shown. The degree of restraint was varied by changing the active lengths of the springs. Wing nuts on the ends of the rods through the springs made it possible to compress the latter so that rotation of the carrier would not cause one spring to go out of action.

In 1939, Barlow[16] made a logical but simple device for testing light columns. Its essential features are depicted in Figure 13. It is seen that each unit consists of a round, hardened steel bar supported with the axis horizontal, between two ball bearing assemblies. The middle portion of the bar has a wide slot milled at its center. The surface of this slot is ground down to a plane that lies as exactly as possible in the diameters of the ball bearing assemblies. The flexural rigidity of the loading bar was such that an angular deflection of 1.2° could occur under a load of 24,000 pounds, while the bearing would not bind until this angle reached 1.5°. Since the maximum test load was 8000 pounds, the device operated well. Like the majority of knife edge or roller types, this end fixity device was restricted to a single degree of freedom. However, in addition

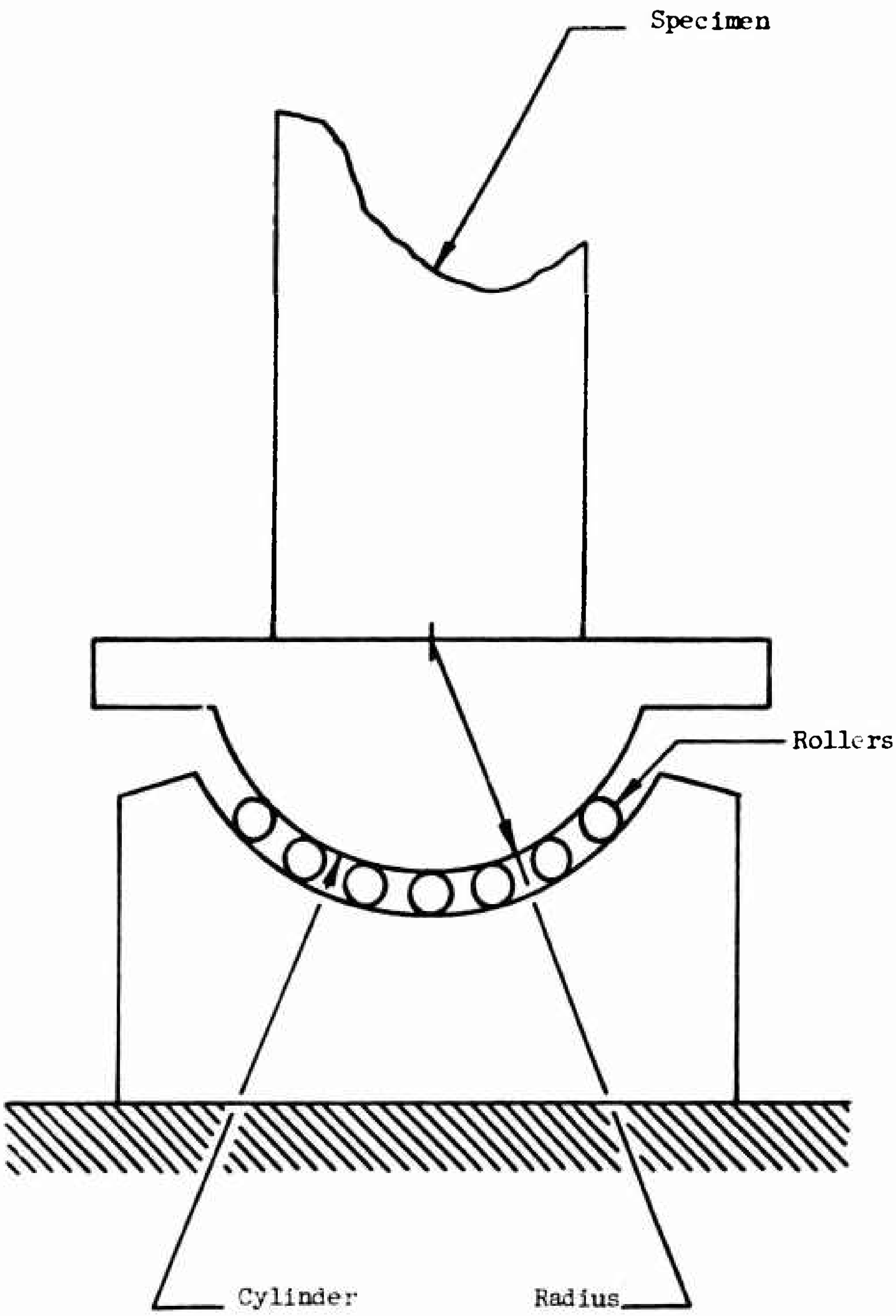

Figure 10. University of Washington Roller System.

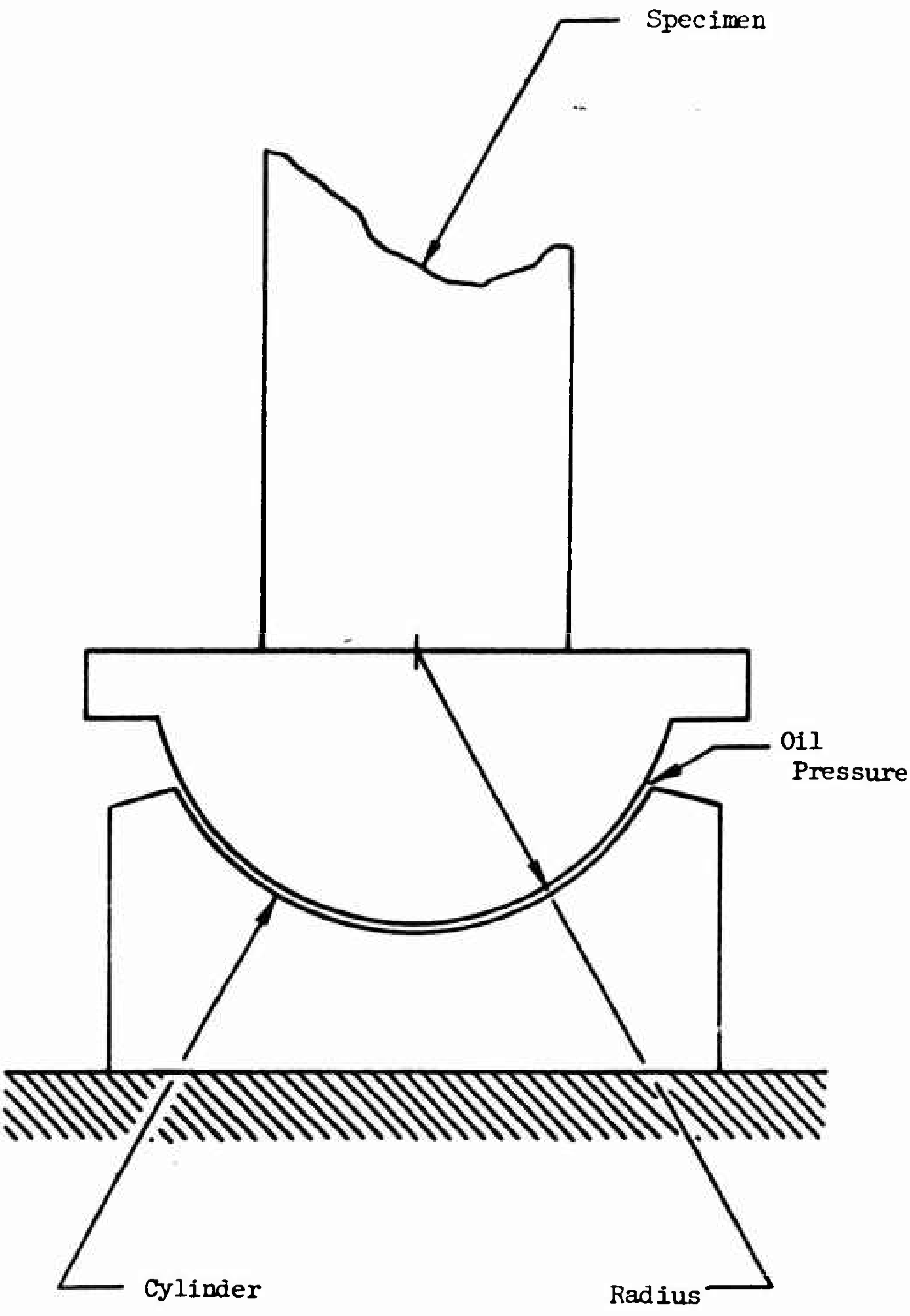

Figure 11. Hydraulic End Fixture.

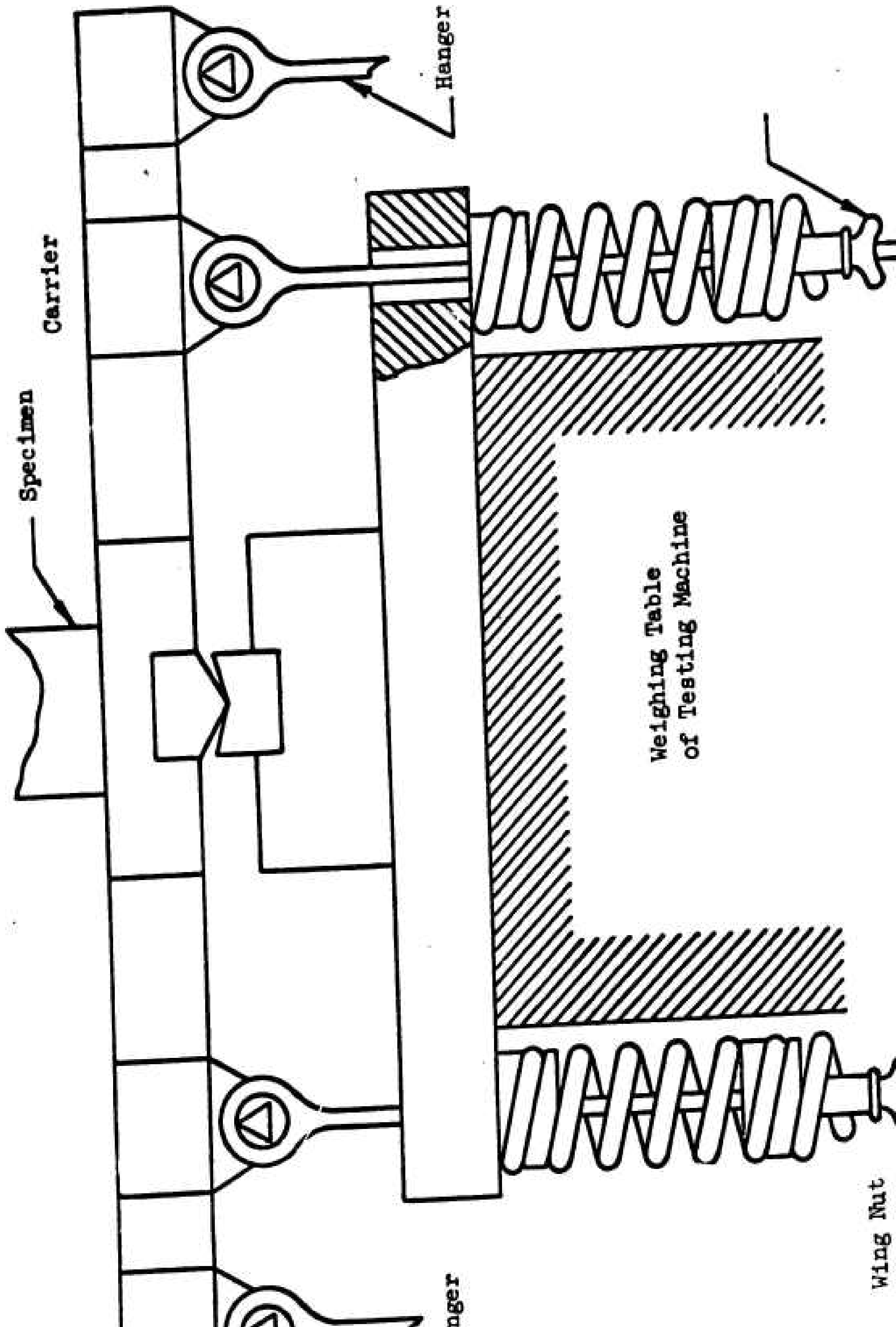

Figure 12. Diagram of Device for Procuring Elastic Restraint at Column End.

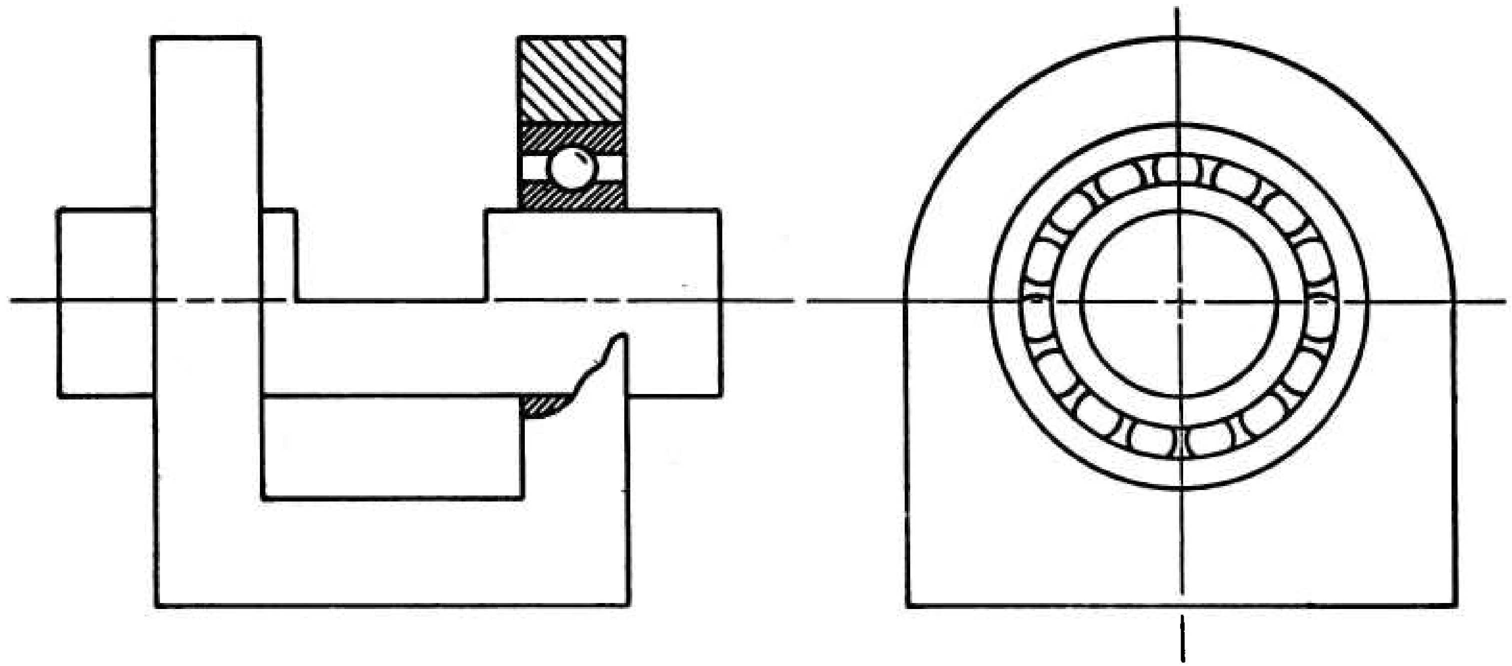

Figure 13. Barlow's Single-Degree-of-Freedom Device for Light Loads.

to the lower friction level achieved, there are several other distinct advantages obtained. One very important asset is that no corrections for effective free length are necessary; another is that the pivot axis can be counterbalanced for precision tests.

In subsequent research,[17] the concept was further developed. In the second system, two degrees of freedom were achieved and half bearings were used instead of full bearings. The device was counterweighted in order to be statically balanced about both ends of rotation. The two-degree pin-ended column test fixture developed is shown in Figure 14.

The next major development appears to have been due to Templin,[18] who devised a system of hydraulically supported, spherically seated compression testing machine platens. A general view of the testing apparatus is shown in Figure 15, and an assembly drawing for it is shown in Figure 16. In the paper that describes these developments, Templin succinctly specifies the basic requirements for suitable fixtures for making round-end tests of column specimens:

1. There should be three degrees of freedom.

2. The device should possess as low resistance to rotation as possible.

3. The method should be applicable to large total load systems.

4. Distortion should be minimal during use.

5. The device should apply either uniform axial loads or loads with prescribed eccentricities.

6. The device should be reasonable in cost, simple to install, and easy to maintain.

To a large degree, Templin's device met these aims.

The resistance to tipping under load of the device described above is illustrated in Figure 17, and there can be little doubt that the device was of good quality compared with most others which preceded it.

In 1959, Goldberg and Lenzen[19] published the description of a roller fixture for pin-ended column tests. This is an interesting development, particularly when we compare the resistance to rotation; the resistance achieved in this investigation is shown in Figure 18. A comparison with the previous figure shows that at low load levels the hydraulic bearing outperformed the roller device but that at high load values it did not. The apparatus is shown in Figure 19.

The geometric details of the fixture are clear from the drawing. Other details are as follows: the material used was Ryalloy tool steel, heated and quenched in oil to obtain a hardness of about 64 on the Rockwell "C"

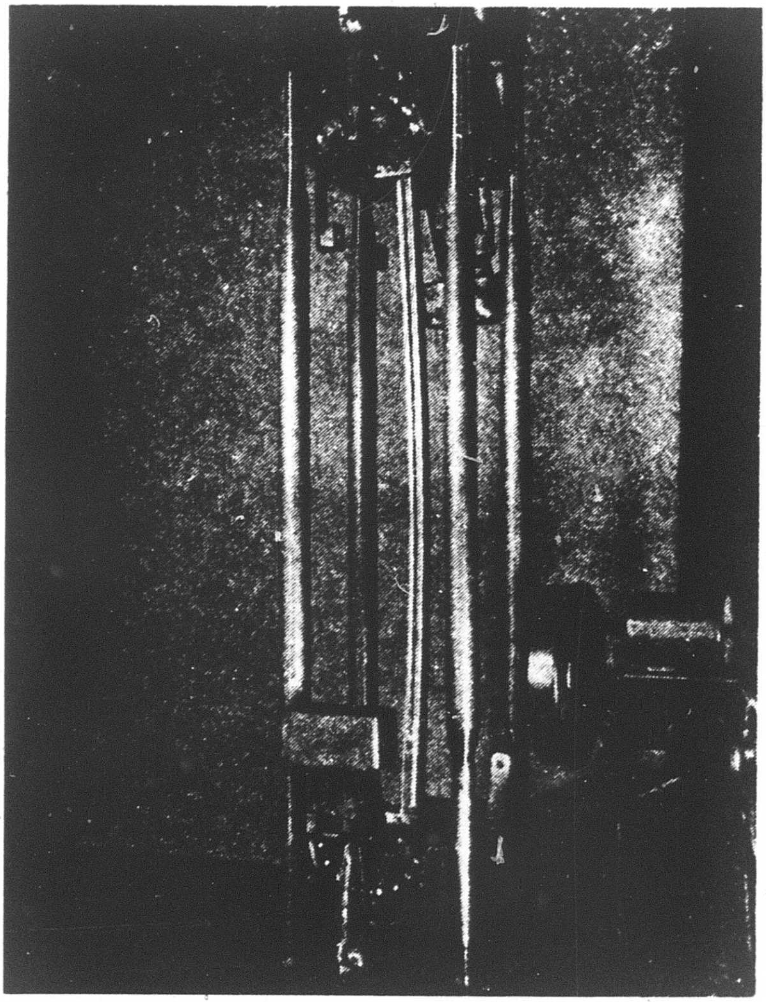

Figure 14. Barlow's Two-Degree-of-Freedom Pin-Ended Fixture.

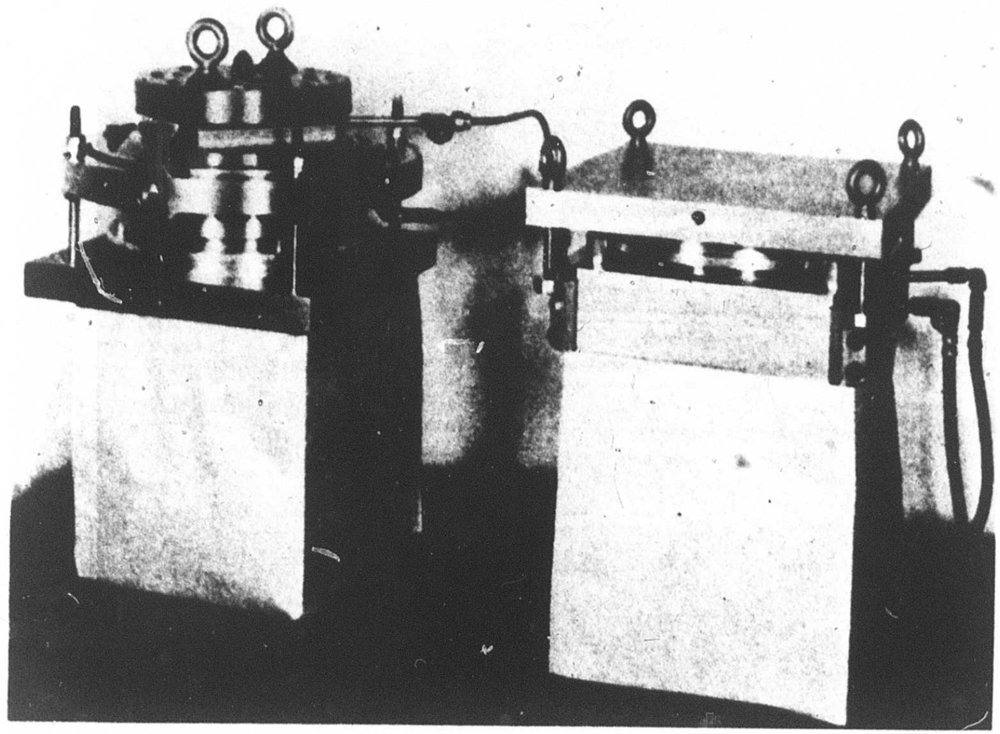

Figure 15. General View of Compression Testing Apparatus Devised by Templin.

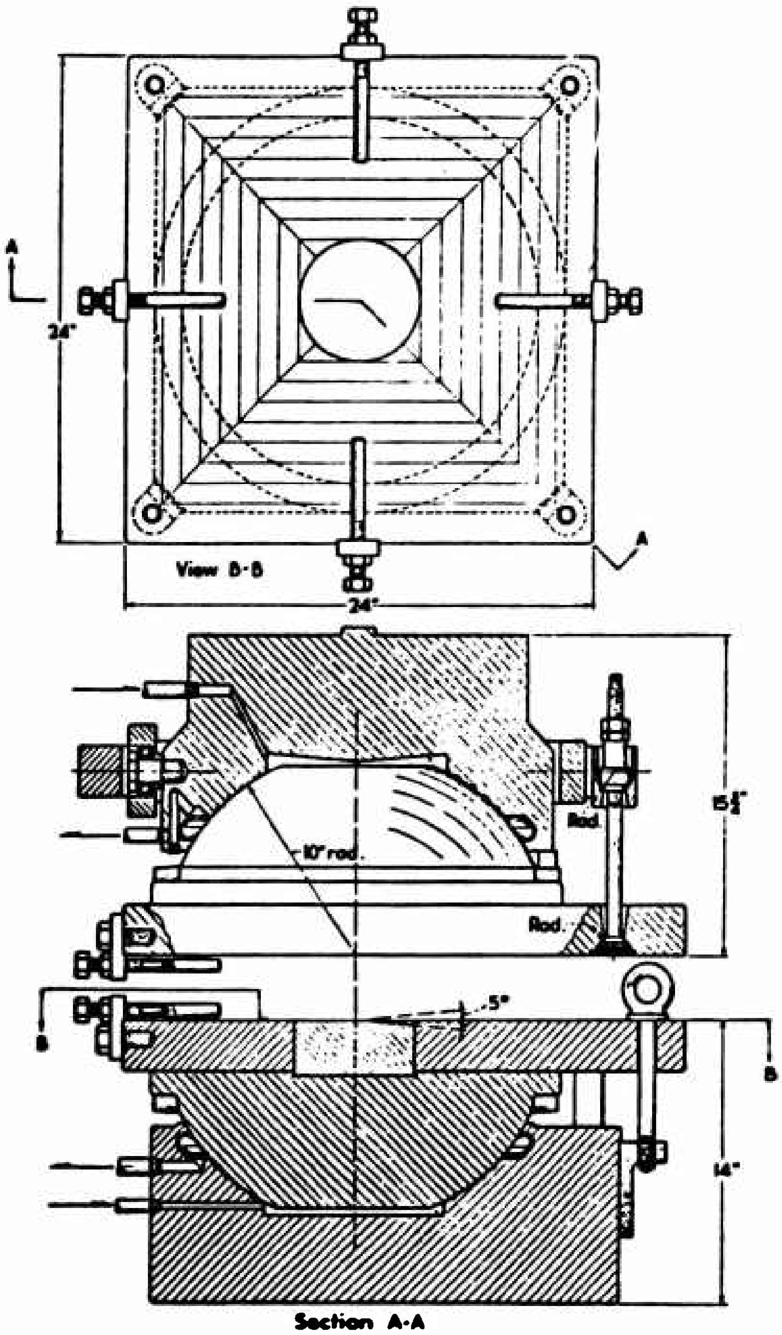

Figure 16. Assembly Drawing of Templin's Device.

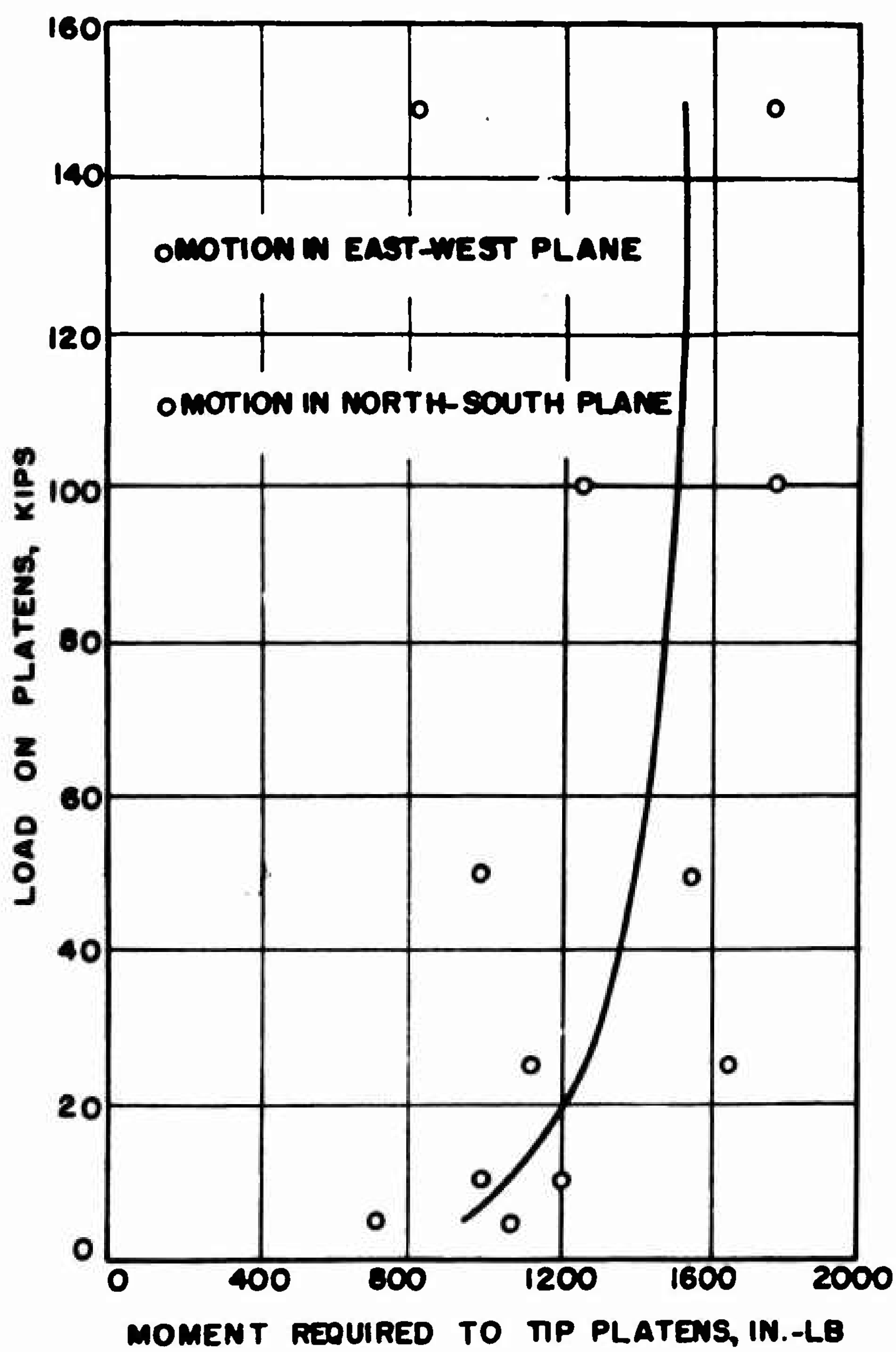

Figure 17. Resistance to Tipping of Templin Fixture When Functioning as a Complete Sphere Under Load(Templin).

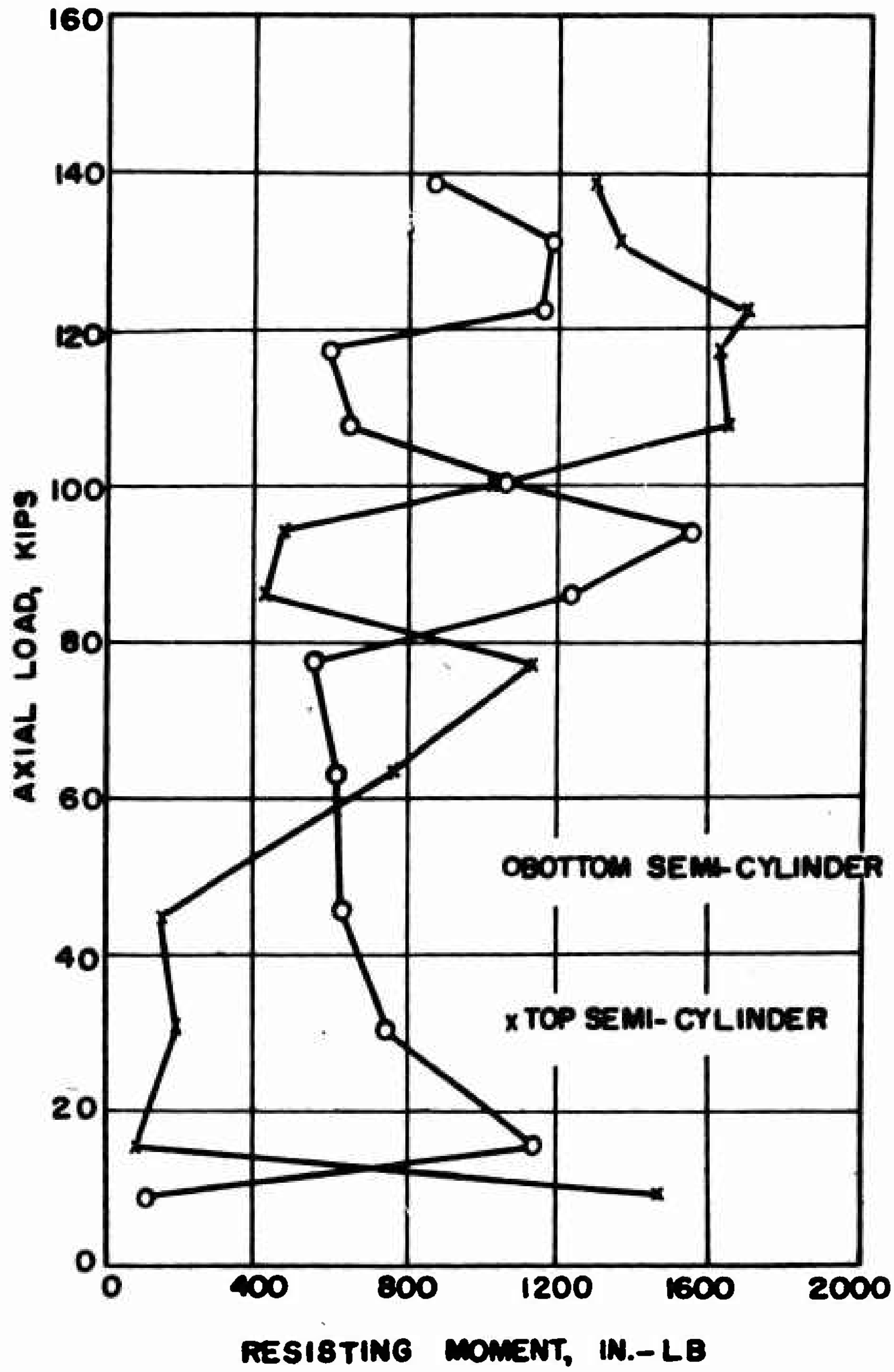

Figure 18. Resistance of Semicircular Ends to Rotation When Subjected to Axial Load (Goldberg and Lenzen).

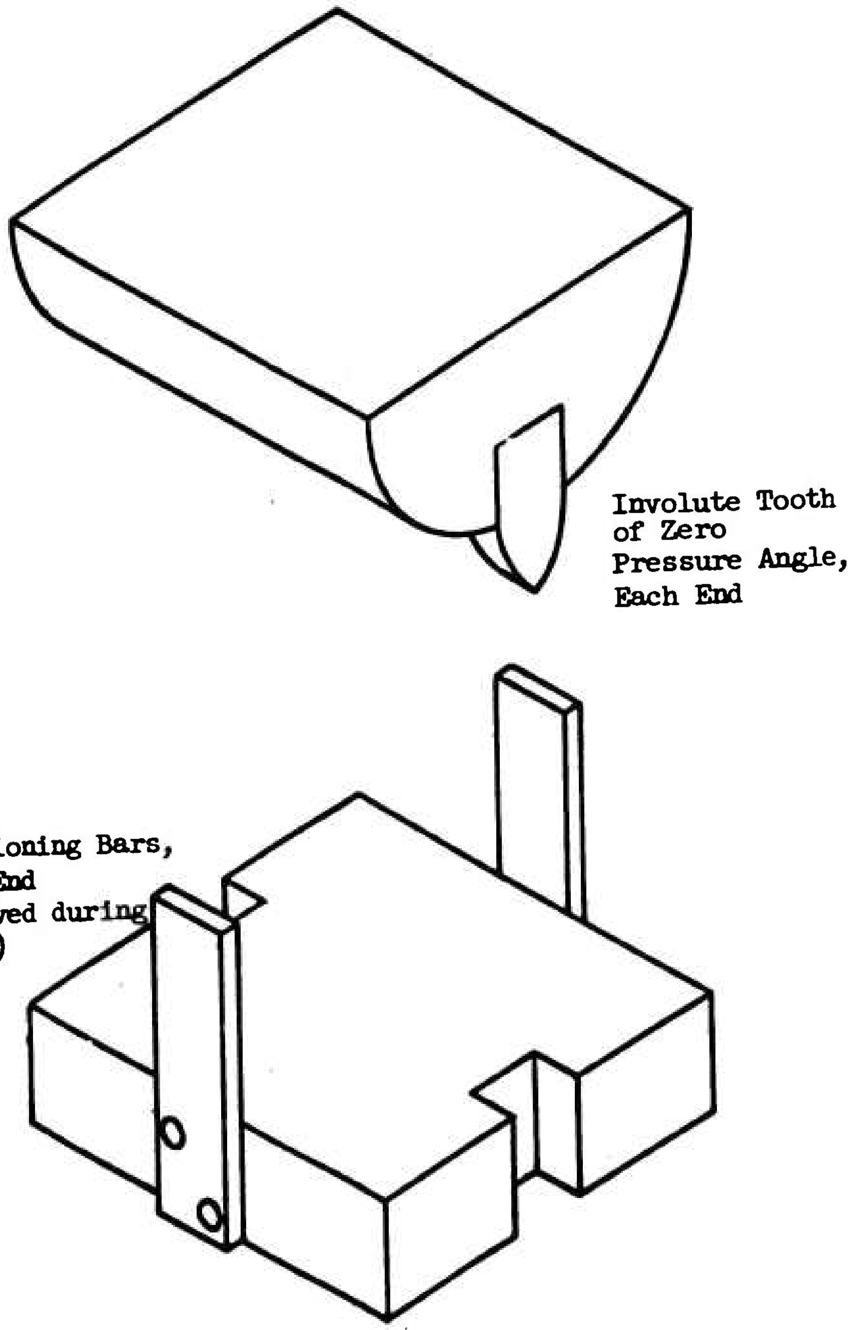

Figure 19. Goldberg and Lenzen's Semi-Cylindrical Device.

scale. Subsequent tempering at 500°F reduced the hardness to 57 on the Rockwell "C" scale. Conversion tables indicate that this hardness corresponds to an ultimate stress of about 300,000 psi. The maximum stress under a load of 300,000 pounds and with a contact length of 8 inches is determined from Föppl's formula[20] to be 215,000 psi; as a result, there was no material distress at the higher load levels. The semicylinders bore on 3-inch-thick blocks of Ryalloy steel having the same hardness as the rollers and were keyed to the blocks by a tooth at each end, these teeth being involute curves of zero pressure angle. They resist relative translation as long as the angular displacement of the roller is less than 22.5 degrees.

In more recent years, Lehigh University has carried out considerable research on steel structures for large civil engineering applications. The latest standard column end fixture used at their Fritz Engineering Laboratory[21,22] is shown schematically in Figure 20. It is interesting to note that, in essence, this device functions exactly as that of Hodgkinson, who did the first extensive testing of columns. The primary difference in the two is that the Lehigh fixture employs a cylindrical rolling surface, while Hodgkinson had spherical ends on his columns. It was pointed out that one advantage of the added constraint was that the column could be forced to buckle about an arbitrary axis; generally, this axis was that corresponding to the highest buckling load. The main reason for choosing this particular configuration was the desirability of carrying very high loads--up to 2,000,000 pounds. Thus, the sizing of the end fixture was determined primarily by the requirement of keeping the strains in the cylindrical bearing block and the bearing plate elastic. With a cylindrical radius of 10 inches and a modulus of elasticity of 30,000,000 psi, the contact stress was computed to be 209,000 psi, and the width of the bearing area was thus .506 inch under a bearing pressure of 2,000,000/24 psi per linear inch. Hence, the required length of cylinder came out to be 24 inches. The material used for the cylinder and bearing blocks was special tool steel heat-treated to 70-80 Scleroscope surface hardness.

It is evident from Figure 20 that the apparatus consists essentially of a column base plate (to which the column was welded), a fixture platen that connects the column base plate to the cylindrical bearing block, the flat bearing block, an adjusting assembly composed of sliding height-adjustment wedges and a small cylindrical bearing, and a base. Top and bottom fixtures were identical. The need for obtaining uniform contact pressure along the cylinder-bearing block interface and for neutralizing lack of normality between the column axis and the testing machine table was satisfied by the adjusting assembly. As the height of the wedge adjustment changed, the end plane of the column rotated around the cylindrical bearing. Eccentricity was minimized by sliding the column base plate relative to the fixture platen.

A series of tests was carried out to determine the rotational restraint actually present under axial load. For low load ranges, the hysteresis check was made in a load-unload cycle. No hysteresis was observed. For high load ranges, a hydraulic jack was mounted parallel to the column between the base plates, such that additional bending and axial stresses could be superposed on the existing load states. The results are shown in Figure 21.

In methods of adjustment and alignment, the above fixture is quite similar to one used earlier at Lehigh University and described in detail by Adams and Galambos[23] and by Beedle, Ready, and Johnston[24]. The main load-carrying parts of the end fixtures are dissimilar; the Beedle, Ready, and Johnston device utilized a double knife edge to carry the axial load. The test assembly is shown schematically in Figure 22 and pictorially in Figure 23. The point of departure from the past, in these tests, was that end moments were to be applied rather than minimized, and that steel structural members of full-scale building size were to be tested. The chief criterion regulating the geometry of the end fixture was that the point of end rotation and application of axial load, the center of moment, the point of lateral support (the support being necessary to equilibrate the applied end moments) and the end of the actual test specimen lie as nearly as possible in the same plane. This condition was reasonably satisfied in all but the last requirement, the end of the column being almost six inches above the knife edge. It was thus necessary to use an adjusted column length in the data analysis.

Application of moments to the end of the column was accomplished via an arm mounted at each end of the specimen perpendicular to its axis. Each arm was driven at the free end by a tension-compression hydraulic jack that was mounted on the test frame. Alignment was accomplished by centering the column carefully on the end fixtures and then positioning the fixtures equidistant from the vertical screws of the testing machine. The knife edge seats tended to automatically position the knife edge blocks, which could be moved with respect to the adjusting wedges. Alignment was checked with strain gages and levels; levelling under load was used sparingly. It was deemed that alignment was not as critical as in other tests because of the presence of the applied end moments.

Displacements were initially measured by observing the relative motion of a graduated scale fixed to the column and a taut wire hanging from a fixed point. Subsequently, dial gages were used for both lateral deflection and, in conjunction with a micrometer lead screw and a levelling bar, end rotation measurement.

Three different tests were described: axial load alone, with both ends free to rotate; axial load in combination with a moment applied at one end, with the opposite end pinned; and axial load with one applied end moment, with the opposite end clamped. In the last case, clamping was obtained by having the hydraulic jack supply enough force to the end of the moment arm to drive the measured end rotation to zero. The general testing technique was to apply a given axial load and to hold it constant while varying the applied end moment. Obviously, a moment applied at the upper end will result in a change in axial load, and in testing machine indication, equal to the shear force at the end of the moment arm; hence, an open-loop system is inadequate to hold a given axial load. To circumvent this difficulty, axial load and applied moment were alternately adjusted

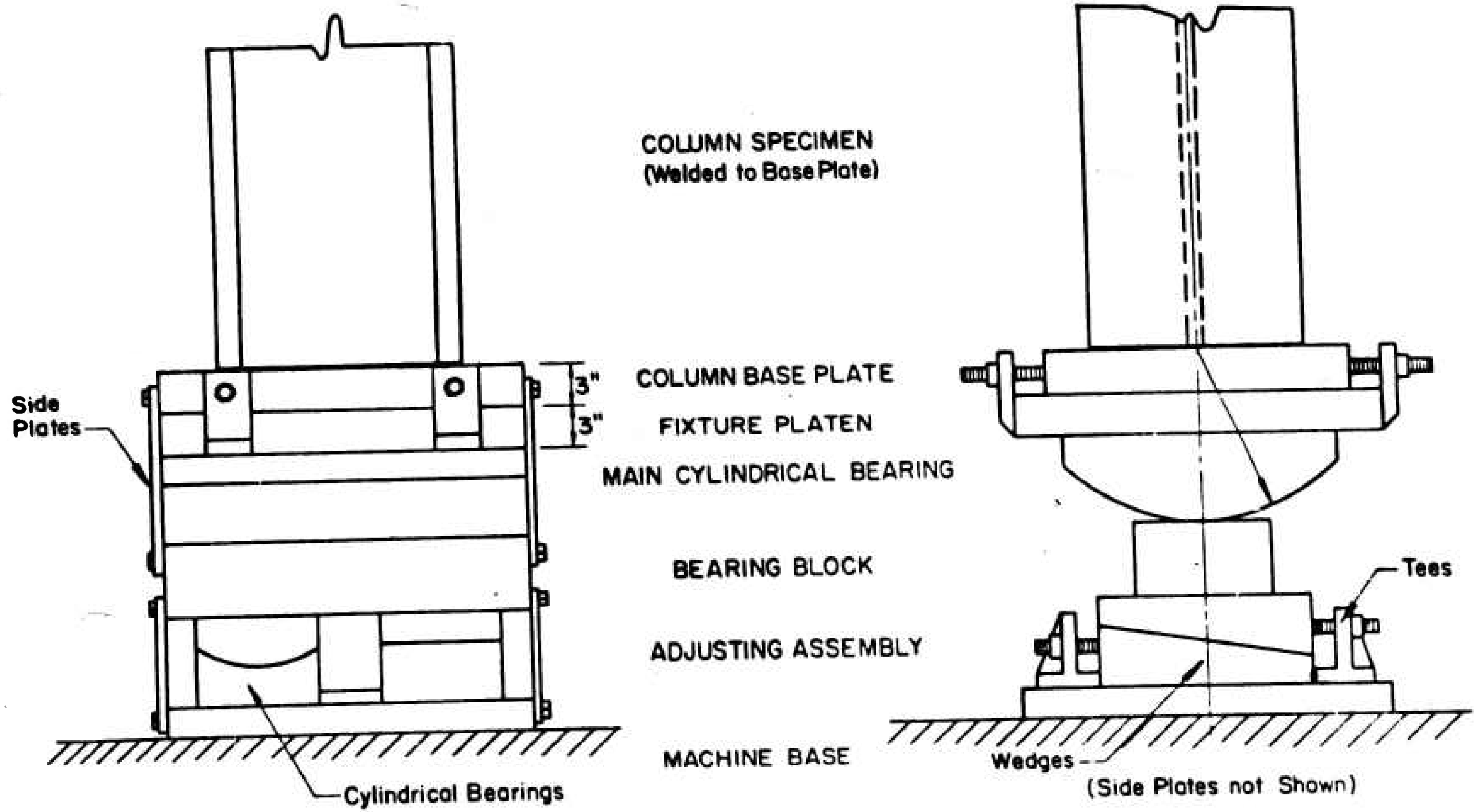

Figure 20. Standard Column End Fixture at Fritz Engineering Laboratory.

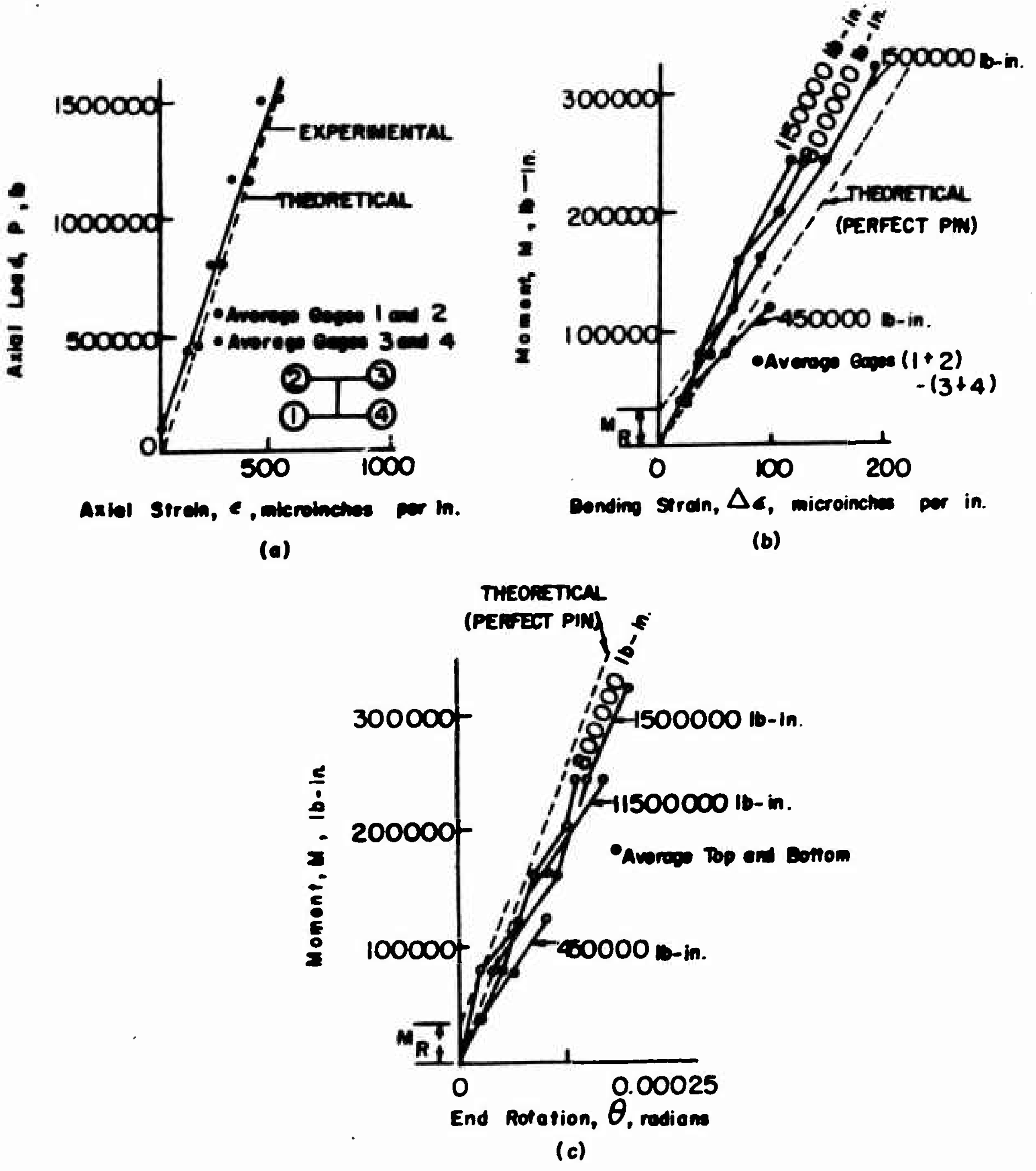

Figure 21. Performance of the Apparatus Shown in Figure 20.

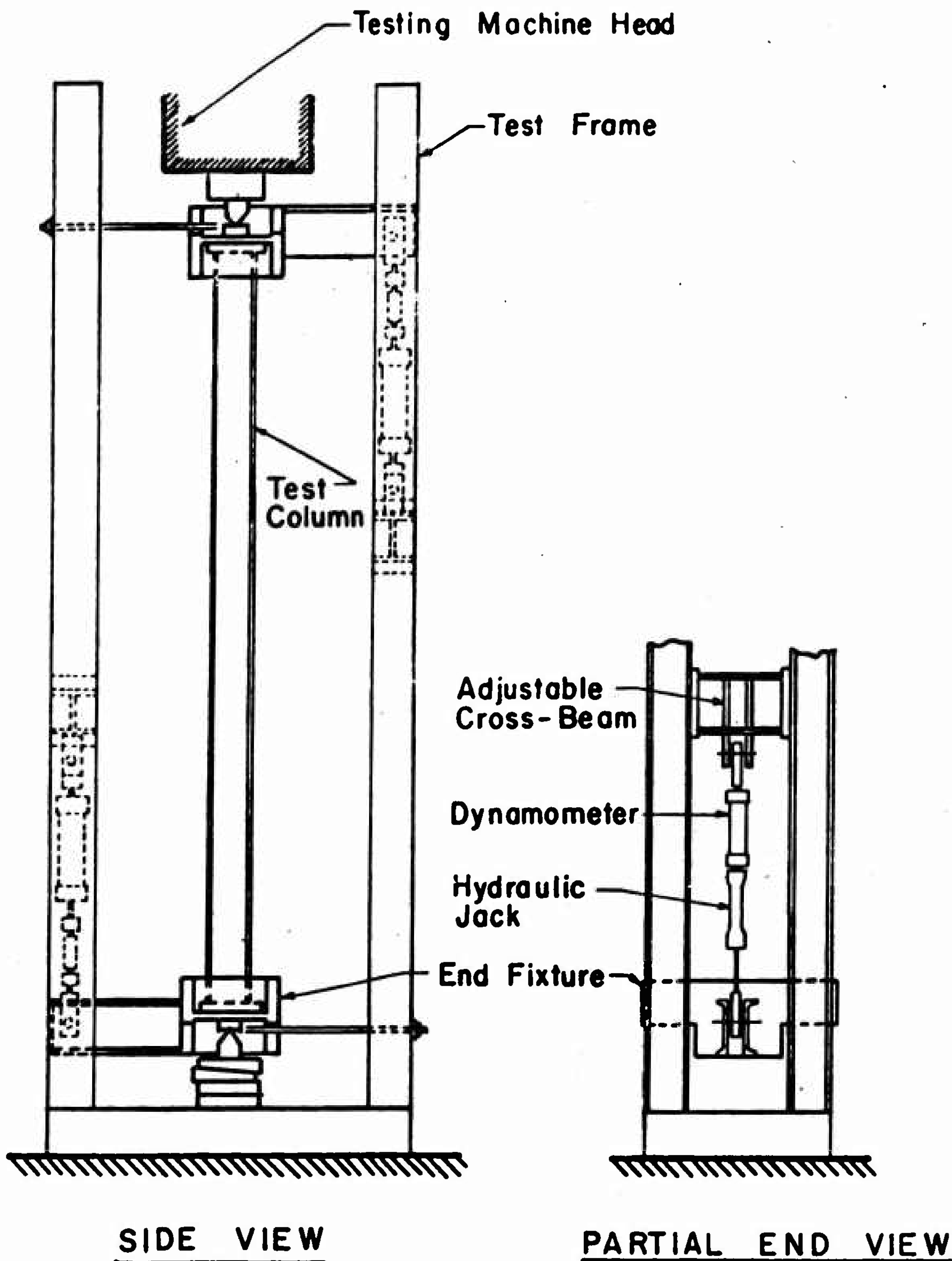

Figure 22. Test Assembly for Unbraced Columns (Adams and Galambos).

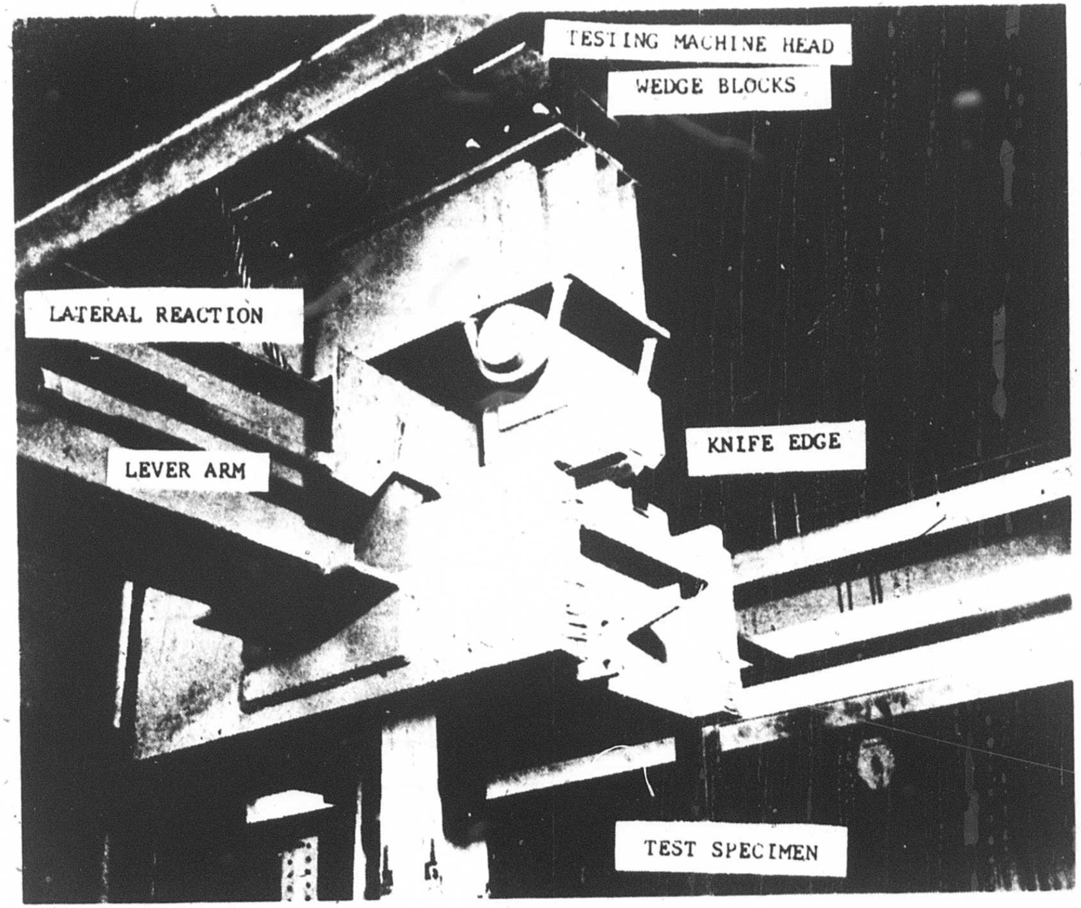

Figure 23. Picture of Upper End Fixture Shown in Figure 22. (Adams and Galambos).

until each attained the desired values. Then, the moment was incremented and the process was repeated. Lateral displacements at the end of the beam were neutralized by applying appropriate forces to the lateral supports.

One of the main objectives of the test series was to compare measured carry-over moments with those predicted by analysis. Inasmuch as the agreement was good, the authors concluded that friction in the knife edges was minimal. To verify this conclusion, they conducted moment-reversal tests at the upper knife edge and observed no detectable hysteresis, as shown in Figure 24.

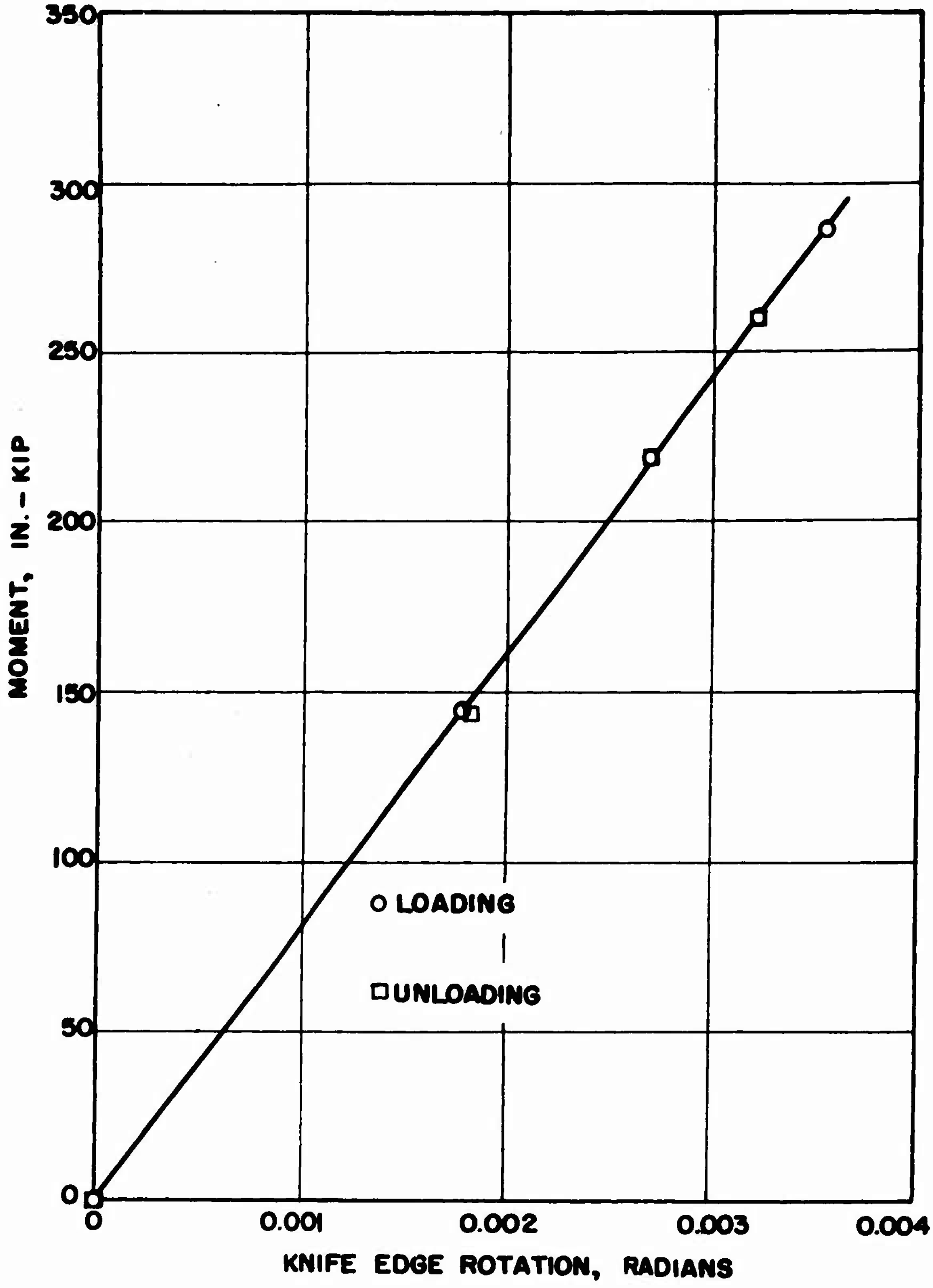

Figure 24. Performance of the Device Shown in Figures 22 and 23.

DISCUSSION OF THE PROBLEMS WHICH ARISE IN THE DEVICES FOR COLUMN END FIXITY

We begin this discussion with the round-ended column and consider the case when the round end is hemispherical and the contact surface is plane. The first issue of importance here arises from the contact stresses which can cause rupture of the ball or bearing failure of the surface or both. It will be recalled that Hodgkinson experienced difficulty with this problem. Some information with regard to actual values of contact stresses can be obtained from the work of Hertz.[20] It is instructive to choose, as an example, the case considered by Goldberg and Lenzen. They compute from the Hertz formula that if the material of the ball and plane has the characteristics hereafter defined, namely,

Modulus of Elasticity	$= E = 30 \times 10^6$ psi
Poisson's Ratio	$= \nu = 0.3$
Max. Compressive Stress	$= \sigma_{cmax} = 210{,}000$ psi

then, for an applied load of 300,000 pounds, the spherical diameter must be 82.5 inches. Such an unusal, large diameter presents difficulties, difficulties both of manufacture and of operation. There are problems with regard to the precise definition of effective length, and there are problems due to sideslip as the end rotation increases. Moreover, frictional forces, particularly at higher load levels, are of such magnitude that the rotational conditions aimed at with this device are not, by any means, met. In fact, this type of end restraint more nearly approaches direction fixed than direction free at high load levels. According to Wagner,[25] agreement with the Euler curves cannot be obtained. With long struts, the experimental points may lie 100% above the Euler curve, if the latter is based upon the length of the strut between the center of the balls.

It is apparent from the Hertz equation that one method of reducing the high contact stresses is to make the bearing block a hemispherical surface rather than a plane. In this case, if the diameters of the ball and socket are equal, the contact stresses are reduced for a given diameter so the ball size can become reasonable. However, the frictional forces become very large. It is for just this reason that the hydraulic fluid bearing becomes so attractive when a hemispherical seating is used. In this case, the hemisphere size can be substantially reduced and error in determination of the actual column length minimized.

In many respects, the semicylindrical roller on the plane surface possesses less problems. In the first case, the contact stress problem is not so difficult. To illustrate this point, we determine from Föppl's equation the diameter of the cylinder appropriate to the loading and material conditions which were used in the calculations of the hemisphere size. We find here that a diameter of 8.88 inches suffices. From a manufacturing point of view, this is much more reasonable. There are, of course, some difficulties of sideslip and friction, but these can be overcome as was clearly evidenced in the description of Goldberg's and Lenzen's roller bearing. It is consideration of this kind which gave rise to the combination semi-cylinder shell bearing developed by the University of Washington. Both this system and the Goldberg and Lenzen roller bearing have excellent overall characteristics; both appear preferable to the Lehigh device, in which sideslip is permitted. In this latter case, there is some problem with reference to the precise definition of strut length.

In tests using knife edges, clear and mathematically applicable relationships can be derived.[25] The two radii of the knife edge bearing must, of course, be chosen, as before, in correspondence with the Hertzian equations. It is desirable that the length of the knife edges be made so large that the two radii can differ substantially. The reason for this second condition can be made clear by reference to Figure 25. If the strut yields laterally, by the angle φ, as a result of the loading, the line of direction of the force ρ intersects the axis of the strut at the point A which is at a distance of

$$\frac{\Delta L}{2} = 1 / \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (1)$$

from the contact point of the knife edges. The length $(L - \Delta L)$ is, therefore, to be regarded as the buckling length. It is to be noted that if the difference between the radii r_1 and r_2 is too small, the angle β may be greater than the angle of friction. If this becomes the case, the knife edges will slip on each other and the otherwise simple relation will be obscured.

When there are rigid elements or carriers at the ends of the specimen, the situation becomes somewhat more complex. A rational method for dealing with rigid portions of equal length at freely supported ends has been presented by Engesser.[26] Nater,[27] Usinger,[28] and Leduc[29] have treated the problem of the straight elastic column elastically restrained against rotation at its ends, while Bleich[30] has made similar studies for the inelastic case. The treatment given here is due to Osgood.[15]

With the coordinate axes taken as in Figure 26, the differential equation of the deflected center line of the column is for small deflections,

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad (2)$$

where M is the bending moment at any section

$$M = M_o + F(S_o \quad x) - Py \qquad (3)$$

Integration of equation (2) and substitution of the boundary conditions,

$$x = 0: \frac{dy}{dx} = \psi_o,\ y = S_o\psi_o;\ x = 1: \frac{dy}{dx} = \psi_1, y = S_1\psi_1 \qquad (4)$$

yields four homogeneous linear equations in ψ_o and ψ_1 and two constants of

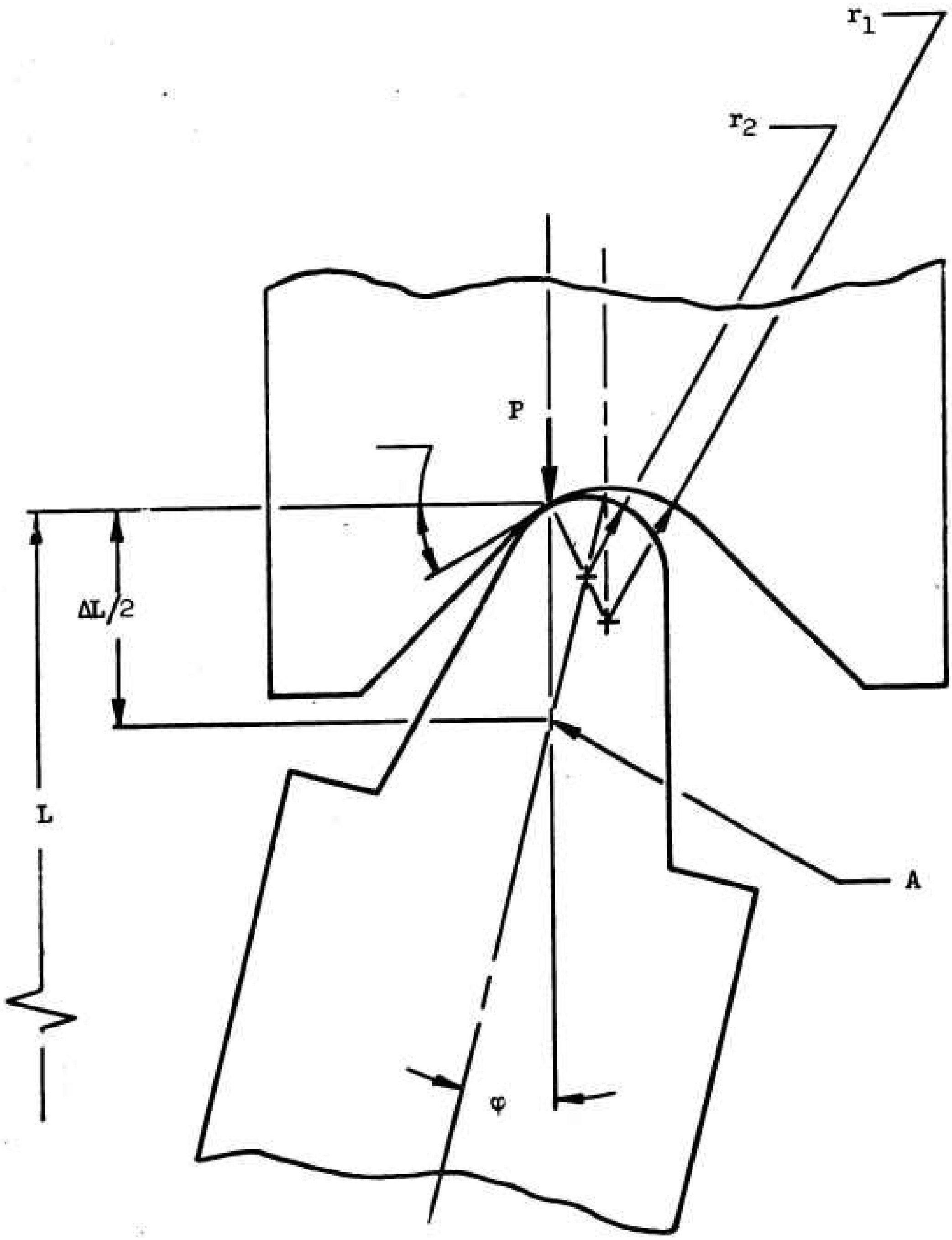

Figure 25. Geometry of Knife Edge System.

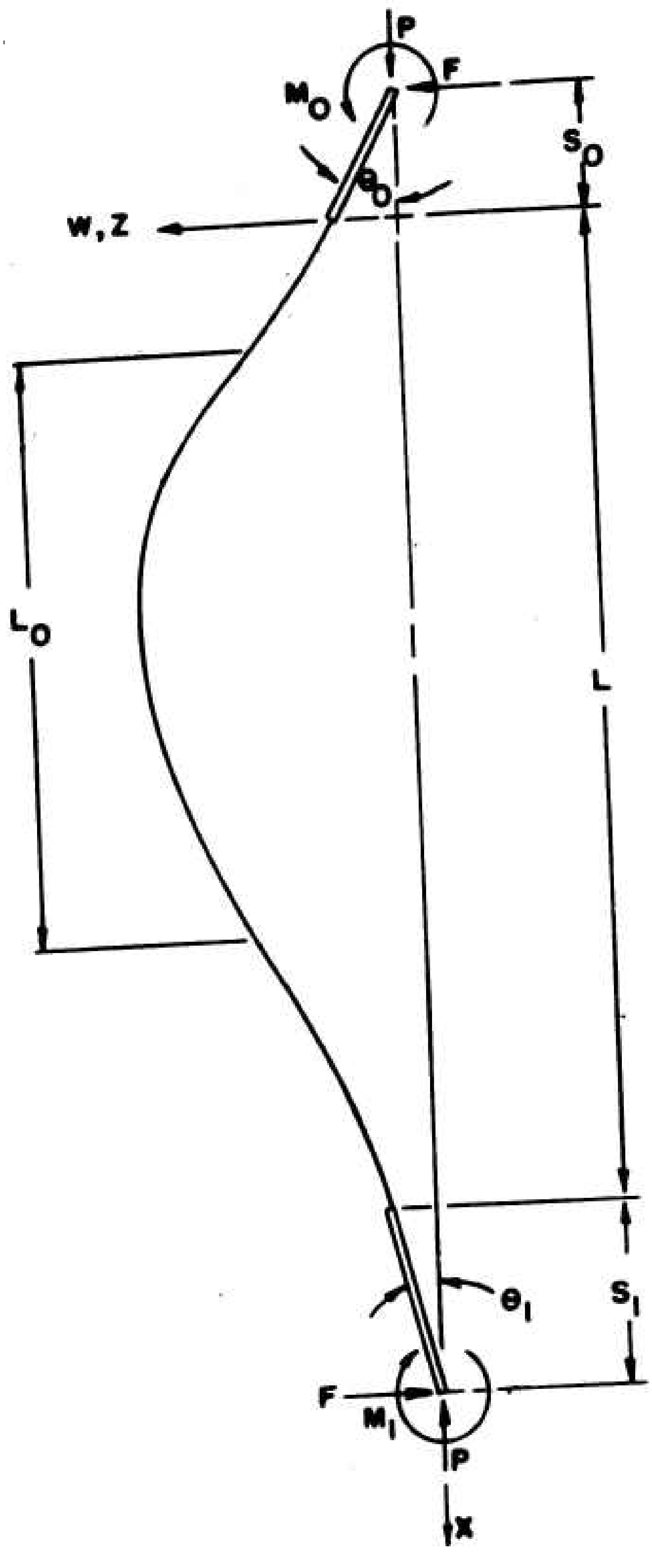

Figure 26. Deflected Center Line of Column.

integration. The deflection y becomes indeterminate when the determinant of the coefficients in these equations is equal to zero; hence, the buckling load is defined by the equation

$$\text{Det} = 0$$

Upon expansion of the above equation there is found in terms of the non-dimensional variables

$$\phi = L\sqrt{\frac{P}{EI}}$$

$$\mu_o = \frac{m_o}{PL\left(1 + \frac{S_o}{L} + \frac{S_1}{L}\right)} \quad \text{and } \mu_1 = \frac{m_1}{PL\left(1 + \frac{S_o}{L} + \frac{S_1}{L}\right)} :$$

$$\left\{ 1 + \mu_o + \mu_1 - \phi^2 \left[\frac{S_o S_1}{L^2} - \frac{S_1}{L}\left(1 + \frac{S_1}{L}\right)\mu_o - \frac{S_o}{L}\left(1 + \frac{S_o}{L}\right)\mu_1 \right.\right.$$

$$\left.\left. + \left(1 + \frac{S_o}{L} + \frac{S_1}{L}\right)\mu_o\mu_1 \right]\right\} \sin\phi - \phi \left[\mu_o - \frac{S_o}{L} + \mu_1 - \frac{S_1}{L} \right.$$

$$\left. + 2\left(1 + \frac{S_o}{L} + \frac{S_1}{L}\right)\mu_o\mu_1 \right] \cos\phi + 2\left(1 + \frac{S_o}{L} + \frac{S_1}{L}\right)\mu_o\mu_1\phi = 0 \qquad (5)$$

If the length of a freely supported column having the same strength as the given column is denoted by L_o, then

$$P = \frac{\pi^2 EI}{L_o^2} \qquad (6)$$

(the original double-modulus equation), and this equation makes it possible to write

$$\phi = \frac{\pi L}{L_o} \qquad (7)$$

Equations (5) and (7) determine ϕ and L_o when the other quantities are known.

It should be noted that the determination of the free length L_o does not require a knowledge of the value of E, the free length being determined solely by the lengths L, $S_o = S_1 = S$ and $\mu_o = \mu_1 = \mu$. Introduce the non-dimensional variables

$$\nu = 1 + \frac{2S}{l}\mu - \frac{S}{L} = \frac{m}{PL} - \frac{S}{L} \qquad (8)$$

and

$$\nu' = \frac{\mu - \frac{S}{L}}{1 + \frac{2S}{L}} = \frac{\frac{m}{PL} - \frac{S}{L}\left(1 + \frac{2S}{L}\right)}{\left(1 + \frac{2S}{L}\right)^2} \qquad (9)$$

and by factoring the left-hand side obtain -

$$\left[\frac{1 + \cos\phi}{\sin\phi} + \nu\phi\right]\left[(1 + 2\nu')(1 - \cos\phi) \quad {}' \phi \sin\phi\right] = 0 \qquad (10)$$

It may be noted that if $\phi = 2n\pi$, where n is an integer, the left hand side of this equation becomes indeterminate. Substitution of this value of ϕ into the original equation, (5), shows that $\phi = 2n\pi$ is not a solution. If the first of the factors on the left-hand side of equation (10) is equated to zero, the solution obtained corresponds to the case $\Psi_o = \Psi_1$. The first solution obtained yields the smallest value of ϕ and is the only one of practical interest. There is obtained, then, in a convenient form

$$\cot\frac{\phi}{2} + \nu\phi = 0 \qquad (11)$$

Equation (11) may also be written in terms of the trigonometric functions

$$s = \frac{\phi}{\sin\phi} - 1 \text{ and } t = 1 - \frac{\phi}{\tan\phi} : t + s = -\frac{1}{\nu} \qquad (12)$$

Where tables of t + S, Reference 31, are available, equation (11) will be the most convenient form for use.

If, when $S_o = S_1 = S$ and $\mu_o = 0$, there is introduced the nondimensional variable

$$\nu'' = \frac{\mu_1 - \frac{2S}{L}}{1 + \frac{2S}{L}} = \frac{\frac{m_1}{PL} - \frac{2S}{L}\left(1 + \frac{2S}{L}\right)}{\left(1 + \frac{2S}{L}\right)^2} \tag{13}$$

equation (5) for the determination of ϕ reduces to

$$t = - \frac{\phi^2 S}{L}\left(1 + \frac{S}{L} + \frac{1}{\nu''}\frac{S}{L}\right) - \frac{1}{\nu''} \tag{14}$$

which may be solved by trial with the aid of Table VII, Reference 15.

Finally, if $S_o = S_1 = 0$, equation (5) may be written in the form

$$\mu_o\mu_1 (t^2 - s^2) + (\mu_o + \mu_1)\, t + 1 = 0 \tag{15}$$

Zimmermann[31] gives this equation and Prager[32] gives it in a modified form, but they assume it to apply for elastic buckling only. It has also been presented in a paper by Osgood.[33] The equation may be solved directly by means of the nomogram, Figure 27, the idea for which is attributed to L. B. Tuckerman by Osgood.

In order to use the nomogram, a straight line is run through the points of the circle determined by the values of μ_o and μ_1 read on the circular scale. This line will intersect the spiral curve in at least one point. The value of ϕ/π corresponding to this point, or the lower value if there are two intersections, read on the scale of the spiral curve will be the lowest value for which buckling can occur.

In the case of two points of intersection, the higher value of ϕ/π corresponds to an unstable condition of equilibrium.

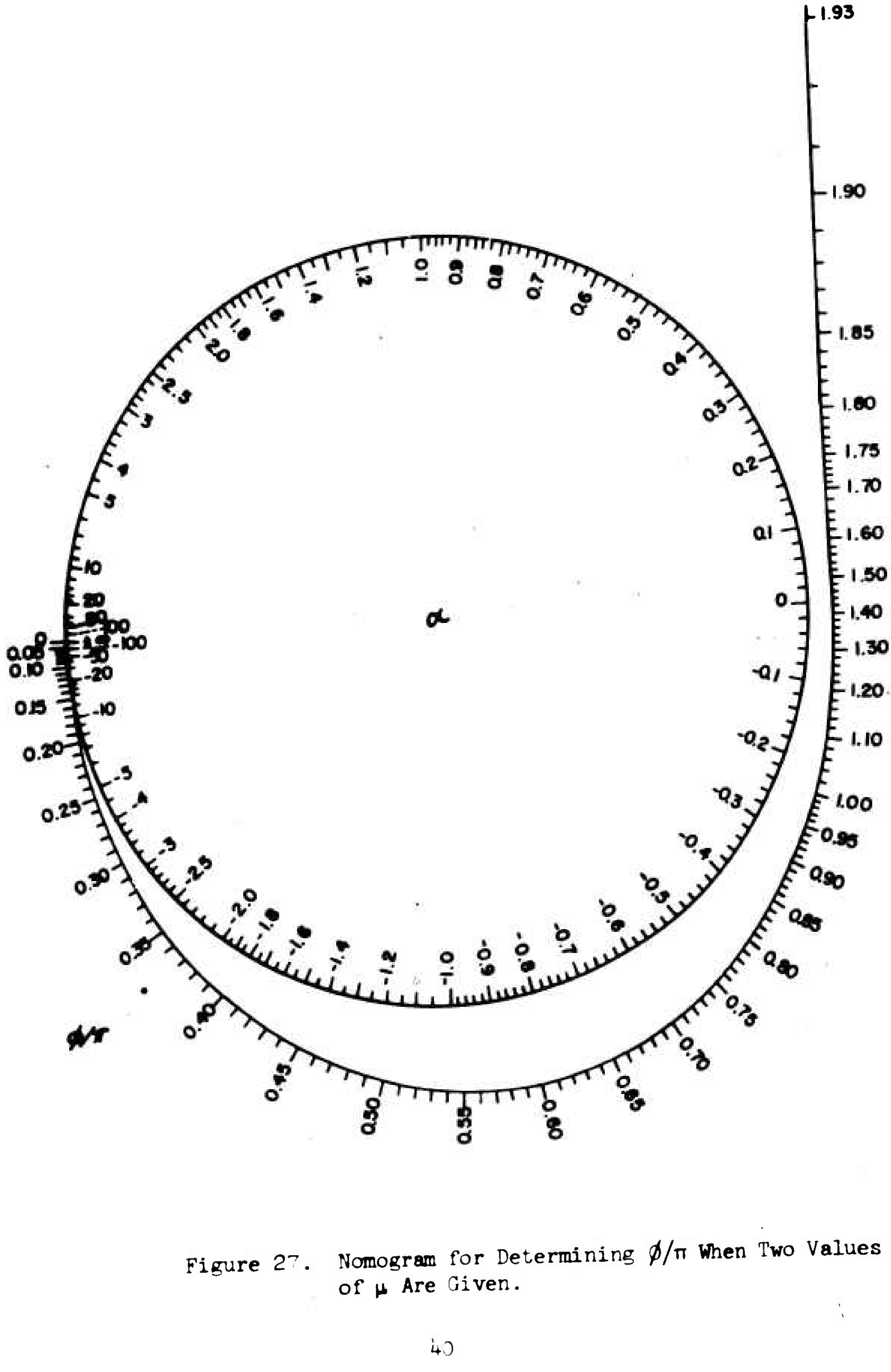

Figure 27. Nomogram for Determining ϕ/π When Two Values of μ Are Given.

IMPERFECTIONS OF FORM AND LOADING ACTION

Except in the case of very small test specimens which can be prepared with ultra-care, there are always difficulties of specimen straightness, slight imperfections of geometric form, minor irregularities in material, and slight eccentricities of loading action. This lack of ideality has been generally recognized by analysis since Young and, of course, by all experimentalists. Christie appears to have been the first to compensate for imperfections in a test column by shifting the ends of the column relative to the supports, while Considère appears to have been the first to employ centering under load systematically in a series of column tests. The well-known and often-referenced tests by von Kármán[13] show that slight imperfections such as small initial curvatures do not effect the strength of freely supported columns when centered under load. Zimmermann[31] has also shown this result theoretically, and Rein[34] in a careful series of tests has confirmed his work.

However, in this particular aspect of the problem, honors must go to the use of the relationship between the elastic lateral deflection at the mid-point of an imperfect column under axial compression, the compressive force, and the classic instability load for the perfect strut. This relationship was first derived in 1807 by Young. It had become common engineering knowledge by the end of that century. In 1886, Ayrton and Perry[4] used the formula to interpret test data on columns. In so doing, they provided the first verification of Euler's[2] work. Their paper was largely ignored, and it was not until R. V. Southwell[5] published his work that the methodology became routine for the strut problem.

The analytical derivation of the displacement formula is most straightforward for this case. It consists of equating the coefficients for various harmonics when similar Fourier series for initial and total deflection are inserted in the appropriate differential equation of equilibrium. Thus, it can be shown that

$$W_n = \frac{W'_n}{1 - P/P_n} \qquad (16)$$

and, consequently, if P_1, P_2, P_3 are widely separated and P is a substantial portion of P_1 ,

$$\delta\left(\frac{P_1}{P} - 1\right) = \text{Constant} \qquad (17)$$
$$= \text{Initial eccentricity}$$

Now, equation (17) is that of a rectangular hyperbola and can be written in a linear form, thus:

$$P_1 \nu - \delta = a_1 \qquad (18)$$

where

$$\nu = \delta / P \qquad (19)$$

and when the data are plotted in this form, the slope of the line gives the critical load and the intercept with the δ axis gives the initial imperfection. Other linear representations are possible, but the given one is the most common.

To confirm his formula and its use, Southwell analyzed van Kármán's data on centrally compressed columns. The results, Figure 28, were excellent: in no case did the critical load derived by his technique differ by more than 2.5% from the classic load. Similar agreement was reached when deliberate eccentricities of loading were introduced. The experiments were made by Robertson.[5]

Although this approach was specifically generated for the strut, it is much broader. A general review of its applicability to columns and plates was given by Horton, Cundari and Johnson[35], while an outline of its potential applicability to shell bodies has been presented by Horton and Cundari.[36]

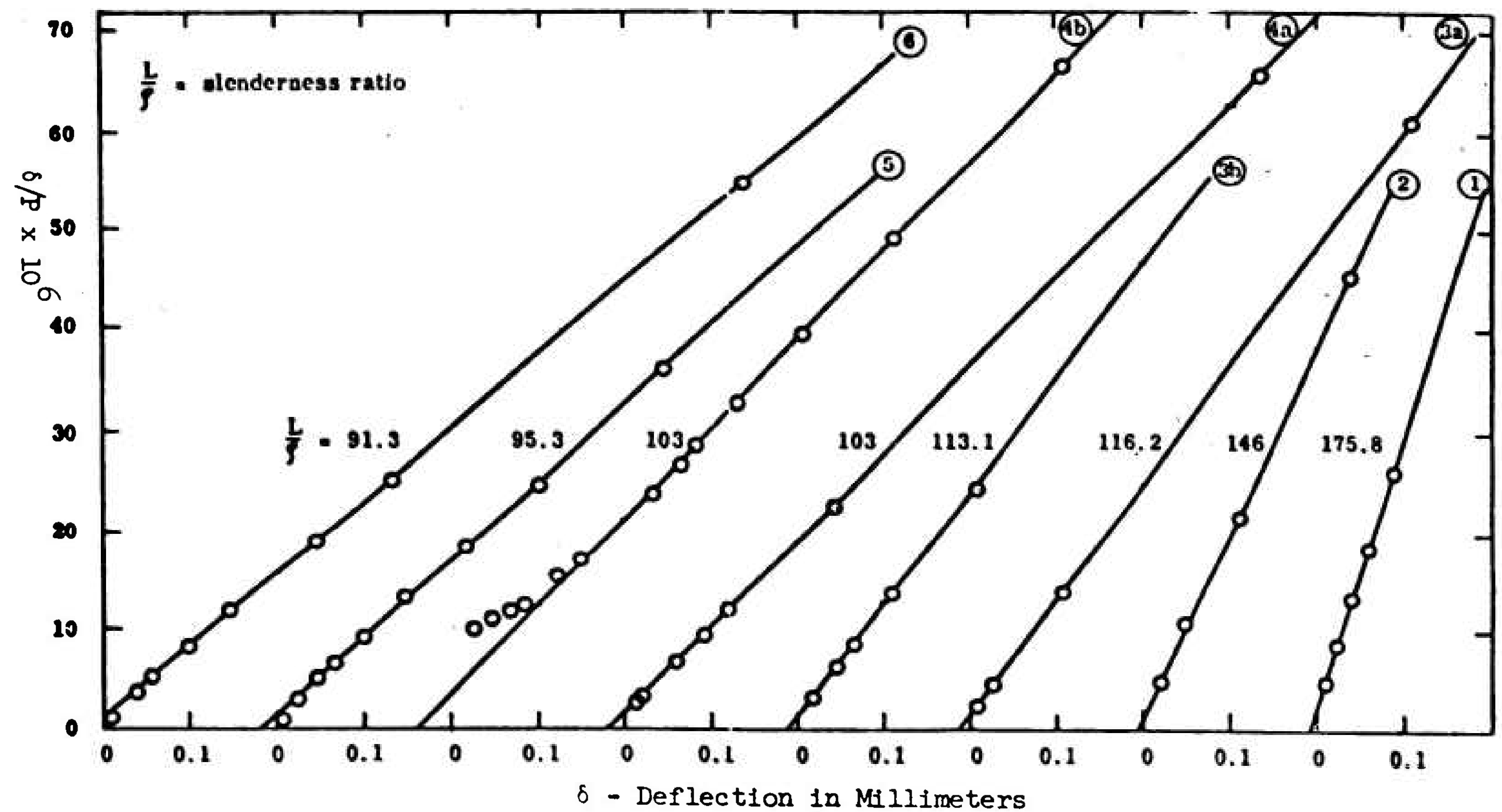

Figure 28. von Kármán data on Compressed Columns Plotted in the Linear Form by Southwell.

TEST EQUIPMENT

Generally speaking, today, there are test machines, transducers, and allied instrumentations capable of handling the most exacting demands. The best test machines have excellent weighing devices, good control, and very little rock of head or platen. For very large structures where modular loading systems rather than standard units are frequently used, there is no shortage of excellent systems and components. For the determination of strain or displacement, there are a wealth of sensitive, accurate, reliable transducers--both of the contacting and noncontacting types-- and an equally impressive range of matching recorders.

There are two minor points in regard to this aspect of the technology which are worth recording. The first is that in model studies, particularly very small scale, great care must be exercised in the choice of transducer. It is all too easy to get interference between the measured quantity and the transducer. The second point is that, while the test machine type does not influence the stability load level, it certainly influences the post-buckling behavior. Screw-jack machines, for example, are strain devices; hydraulic or pneumatic machines are load devices.

CONCLUSIONS

This review has highlighted the difficulties which the experimentalist encounters in reproducing in the laboratory the conditions specified by the theoretician in his analysis. It has indicated that the state of the art of testing columns is well developed, but, at the same time, it has clearly shown that the laboratory conditions must be quite remote from those experienced in reality. Thus, the question must be raised as to whether or not a more practical approach to column investigations would be, at this time, to seek a simple nondestructive test process which would yield data pertinent to the instability of columns with realistic end restraint. General success along this line would certainly be of immediate and immense value to practicing engineers.

We cannot too strongly emphasize our concurrence with the comment made by Salmon in the conclusion of his work on columns:

> "The most pressing point for future research on the subject of columns is undoubtedly the degree of imperfection common in practical fixed ends; in short, what value of K should be assumed for such ends? A complete answer to this question is difficult, but at present the designer has no real data whatsoever regarding practical end conditions."

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13. ABSTRACT

This report presents a review of the important contributions made to knowledge on the stability of elastic columns by experimentalists during the last 125 years. It demonstrates that while methods of dealing with the influence of imperfections on stability have been developed and great ingenuity has been expended in devising "ideal" end restraints to compare with those theoretically described, end fixity is still a major practical difficulty. There is not a single paper, even an isolated suggestion, of how the end fixity for a realistic column might be determined other than by a test to destruction. The report concludes that this topic should receive immediate attention.

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