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TECHNICAL REPORT 1

CORRECTED-INTERCEPT CONTROL OF TORPEDO MK 48 WITH CENTROID TRACKING

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CORRECTED-INTERCEPT CONTROL OF TORPEDO MK48 WITH CENTROID TRACKING*

Introduction

The acquisition models that are described and developed in reference (a) are based on the assumption that tracking either is prevented by the torpedo noise (complete masking) or can proceed without degradation, depending on the positions of the torpedo and target submarine relative to the tracking submarine. In the present report we modify the models to include centroid tracking, in which the sonar operator tracks a composite signal at a bearing that is a weighted average of the bearings of the torpedo and the target submarine. The weight depends on the relative intensities of the two signals at the sonar receiver.

As the distance of the torpedo from the tracking submarine increases and no control is exercised, the composite bearing line shifts gradually from a position near the torpedo bearing line to a position near the target bearing line, under most tracking conditions. The motion of the composite bearing line when control is exercised depends on the control mode and the procedure for computing the changes in the torpedo course angle. In this report we examine the motion of the composite bearing line and the effects of this motion when corrected-intercept control is exercised and the corrections to the torpedo course angle are computed by our new method described in reference (a). Acquisition probabilities are computed and compared with those obtained in reference (a) with dog-leg unmasking.

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Summary and Conclusions

The main conclusion reached from the study is that our new TMA and the associated method of computing lead angles will counteract the adverse effects of torpedo radiated noise, provided that the sonar tracks the centroid of the received signals and the signals are merged sufficiently (both signals in the main tracking lobe) when post-launch tracking begins. By simply tracking the centroid bearing, accurate torpedo tracks for interception are generated in the corrected-intercept control mode, even when the target maneuvers radically and the torpedo noise intensity at the source exceeds the target sound intensity at the source. Acquisition probabilities are comparable with those obtained in reference (a) with dog-leg unmasking.

The good results that are obtained with the corrected-intercept control mode from bearing data obtained in centroid tracking will not be obtained with other known methods of target motion analysis, such as CHURN, and the associated methods of computing fire control orders. The favorable outcome stems from the motion of the centroid bearing line, the effect of this motion on the apparent bearing rate, and the strong use of current bearing rate to compute the lead angle in our solution.

Centroid Tracking

We assume, as before, that the target is completely masked and tracking is impossible, when the torpedo is very close to the tracking submarine. If desired, we can use the square spreading law to express the range at which complete masking terminates in terms of the range of the target, the difference in signal strengths at the source, the difference in sonar sensitivities in the two directions, and the difference in signal strengths at the receiver that corresponds to the end of complete masking. Thus, let

ΔS = difference in received signal strengths (target minus torpedo)

ΔH = corresponding difference in signal strengths at the source

ΔG = corresponding difference in directional sensitivities

R = range of the target from the tracking submarine

R_T = range of the torpedo from the tracking submarine

Then

$$\Delta S = \Delta G + \Delta H - 20 \log_{10} (R/R_T), \quad (1)$$

from which we obtain

$$R_T = R\, e^{0.115 (\Delta S - \Delta G - \Delta H)} \quad (2)$$

Here ΔS, ΔG, and ΔH are in decibels, and ΔG is positive when the target is in the major lobe and the torpedo is in a side lobe. In equation (1) we have omitted the attenuation term because it is small relative to other terms at the ranges of interest.

To use equation (2) to find the range at which complete masking ends we need to specify a value for ΔS that corresponds to this condition. The value, $\Delta S = 0$, usually is considered to be the value at which the target signal ceases to be merged with the torpedo signal, so that the two signals might be tracked separately. However, it is difficult to estimate the value of ΔS that represents the passage from no tracking to "some" tracking of the target.

We can avoid this difficulty by assuming that the sonar operator always tracks a composite signal at a bearing that is a weighted average of the two separate bearings. Let

B = bearing of the target

B_T = bearing of the torpedo

The apparent bearing of the composite signal is the weighted average,

$$B^* = (B + \rho B_T) / (1 + \rho), \tag{3}$$

where

$$\rho = 10^{-\Delta S/10} \tag{4}$$

Substituting equation (1) into equation (4) we obtain

$$\rho = (R/R_T)^2 \, e^{-0.23 \, (\Delta G + \Delta H)} \tag{5}$$

If the torpedo and target submarine are in the major lobe, we usually assume that $\Delta G = 0$. This assumption is acceptable when the sensitivity is nearly constant over a large fraction of the lobe. If the major lobe is sharply peaked, better estimates of ΔG and ρ can be obtained from the sensitivity function. Thus, let $G(b)$ be the sensitivity at angle b from the axis. Then

$$\Delta G = G(B - B^*) - G(B_T - B^*)$$

Using equation (3), ΔG becomes

$$\Delta G = G(\rho \Delta B/(1 + \rho)) - G(-\Delta B/(1 + \rho)), \tag{6}$$

where

$$\Delta B = B - B_T \tag{7}$$

We then find the values of ρ and ΔG from equations (5) and (6) by iteration. Start with $\Delta G = 0$, find ρ from (5) and substitute in (6) to obtain a new value of ΔG; repeat until no change is obtained.

From equations (3) and (4) it is seen that $\rho = 1$ and the bearing angle B^* is the mean of the two bearing angles when $\Delta S = 0$. Also, it is seen from (6) that $\Delta G = 0$ when $\rho = 1$, if the function $G(b)$ is symmetrical about the axis. Hence, if we put $\Delta G = 0$ when both target and torpedo are in the major lobe, the estimate of ρ will be accurate in the vicinity of the cross-over; it will be too large when $\rho < 1$ and too small when $\rho > 1$. That is, the apparent bearing line will be closer to the mean bearing line when ΔG is set equal to zero than would be the case when ΔG is estimated from the sensitivity function.

For example, consider the sensitivity function

$$G(b) = G_1 + G_o \cos(\pi b/2B'), \quad |b| < B'$$
$$= G_1 \quad , \quad |b| \geq B'$$

where B' is the half-angle of the major lobe, G_1 is the sensitivity in the minor lobes, and G_o is the gain in sensitivity on the axis of the main lobe over that in the minor lobes. If both the target and the torpedo are in the major lobe,

$$\Delta G = G_o \left[\cos(\pi\delta/2(1 + \rho^{-1})) - \cos(\pi\delta/2(1 + \rho))\right], \tag{9}$$

where

$$\delta = |\Delta B| / B' \tag{10}$$

The expression (9) for ΔG is obtained directly from equations (6) and (8). It applies when both arguments are less than or equal to $\pi/2$, which is equivalent to the condition

$$\delta \leq 1 + \min(\rho, \rho^{-1}) \tag{11}$$

It is evident that $\Delta G>0$ when $\rho<1$, and $\Delta G<0$ when $\rho>1$.

We estimate ρ by iteration from equations (5) and (9). The first approximation is

$$\rho_1 = (R/R_T)^2 \, e^{-0.23\Delta H} \tag{12}$$

with $\Delta G = 0$. The next estimate of ΔG is

$$\Delta_1 G = \Delta G \text{ in (9) with } \rho = \rho_1 \tag{13}$$

and the corresponding approximation for ρ is

$$\rho = \rho_1 \, e^{-0.23\Delta_1 G} \tag{14}$$

If condition (11) is not satisfied, the sonar either is tracking the target or the torpedo, depending on the signal strengths, the skill of the operator, and other circumstances. In general, if $\rho_1 \ll 1$, it is the target; and if $\rho_1 \gg 1$, it is the torpedo. For ρ_1 close to 1, either one could be tracked, depending on which one happened to be in the major lobe when the separation between the bearing lines became large enough that condition (11) is not satisfied.

Condition (11) certainly is satisfied, if $\beta \leq 1$; and is not satisfied, if $\beta > 2$. For $1 < \beta \leq 2$, the condition is satisfied for some values of ρ and is not satisfied for others. We must replace the cosine term for which the inequality is not satisfied by 0. Thus, if we put

$$\gamma = \min\,[1, \beta/(1+\rho)], \quad \gamma^1 = \min\,[1, \rho\beta/(1+\rho)], \tag{15}$$

the equation

$$\Delta G = G_o\,[\cos(\pi\gamma^1/2) - \cos(\pi\gamma/2)] \tag{16}$$

applies for $\beta \leq 2$. In making the approximation, we put $\rho = \rho_1$ in (15), compute $\Delta_1 G$ by putting the corresponding values of γ and γ^1 in (16), and then substitute in (14) to estimate ρ.

If $\beta>2$, it should be possible to detect the separate signals. If there is any doubt as to which one is the target, the doubt can be resolved by dead-reckoning the torpedo. Hence, we assume that the major lobe will be put on the target, and that consequently $\Delta G = G_o$ for all values of ρ when $\beta>2$. This assumption might be questionable when $\rho_1>>1$, since then the received target signal with no gain is much weaker than the corresponding received torpedo signal. A possibility exists that under these conditions - and under some other conditions, as well - the operator may track on the wrong null in the returned target signal, which would introduce a large bias. Our acquisition models do not include this possibility.

Bearing Rates

The motion of the centroid bearing line depends on the motions of the target bearing line and the torpedo bearing line, and on the rate of change of ρ. From equation (3) we obtain

$$\dot{B}^* = (\dot{B} + \rho\ \dot{B}_T) / (1 + \rho) - \Delta B\ \dot{\rho}/(1 + \rho)^2 \qquad (17)$$

The first term on the right-hand side of equation (17) is the weighted mean bearing rate, while the second term is produced by the changing weight. It was expected that the first term would be the dominant term, in general. However, in a sample computation the magnitude of the second term was larger than that of the first term at the start of the post-launch tracking, when $|\Delta B|$ was large. This term has the same sign as ΔB, since $\dot{\rho}$ is negative.

We can write ρ in the form

$$\rho = gh\ (R/R_T)^2 \qquad (18)$$

where

$$g = e^{-.23\Delta G}, \; h = e^{-.23\Delta H} \tag{19}$$

and ΔG is the difference in sensitivity gain and ΔH is the difference in signal strengths at the source. The factor h is constant and the factor g decreases as t increases, since ΔG increases. Also, R/R_T decreases as t increases. From (18) we have

$$\dot{c} = h \, \dot{g} \, (R/R_T)^2 + 2 \, gh \, (R/R_T^{\,3}) \, (R_T\dot{R} - R\dot{R}_T) \tag{20}$$

Both terms on the right-hand side are negative, since $\dot{g} < 0$, $\dot{R} < \dot{R}_T$, and $R_T < R$ for the control interval. The $\dot{g}$ term may be significant at the start of tracking, as in our example; it is difficult to express in term of the rates of change of the bearings and ranges, since g depends on c in a transcendental form. At any rate, c usually decreases fast enough to give $\dot{B}^*$ in equation (17) a large pseudo component toward the bearing B of the target.

The Role of the TMA and Correction Computations

The effect of the large component of pseudo bearing rate, in the direction from the torpedo bearing toward the target bearing, will depend very strongly on the target motion analysis. For a method, such as CHURN, in which the target parameters are obtained from a simultaneous solution based on all the observed bearings, the effect of a small number of additional bearing observations after launch will have a small effect on the solution. The effect will be large, and difficult to predict, when the number of post-launch observations is comparable to the number made before launch; it will depend on the target motion.

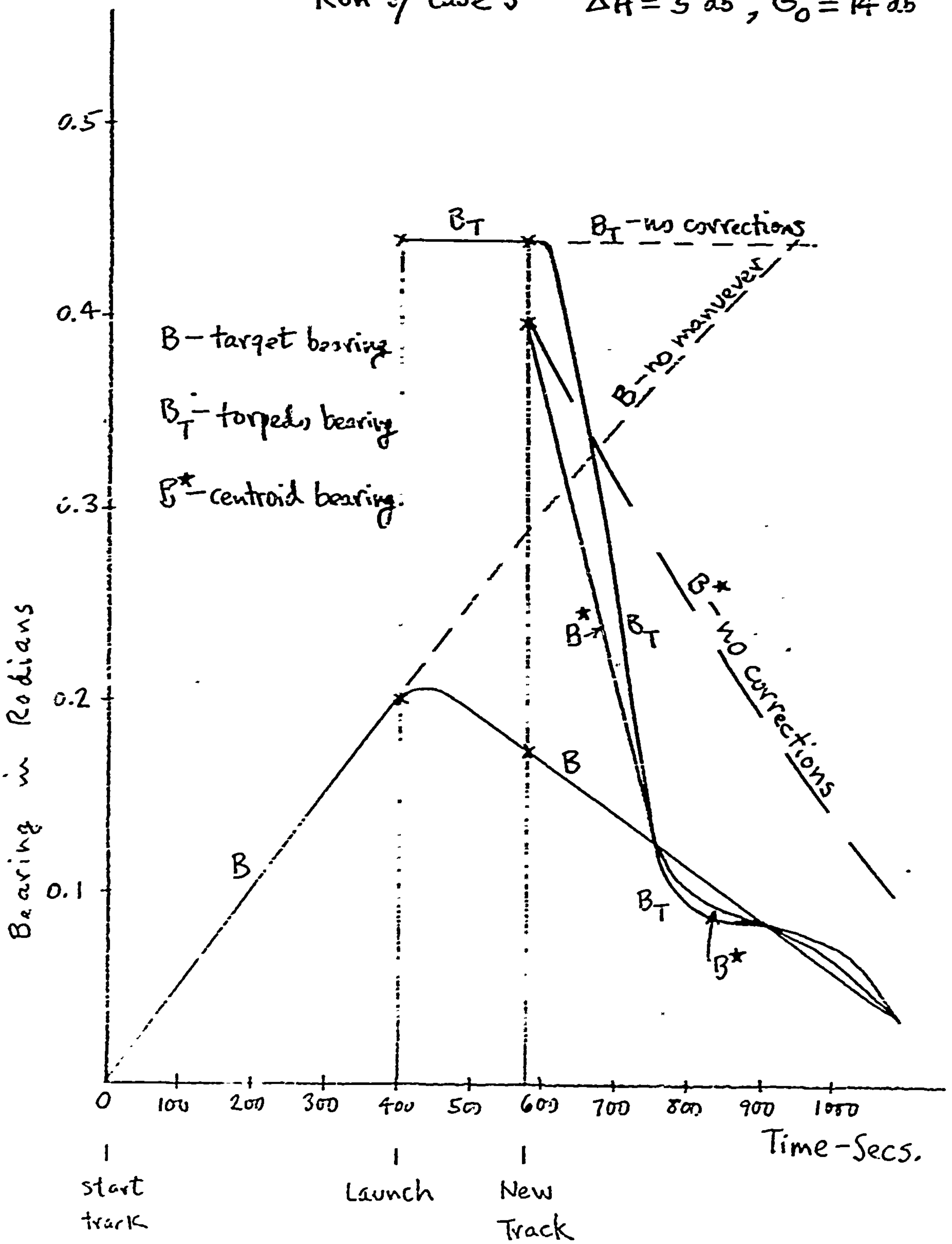
For example, consider the target motion in the check calculation, run 3/case 3 of reference (a): the target runs on course 90 degrees at 5.6 yds/sec. for 400 seconds and then turns 120 degrees to course -30 degrees. The bearings of the target and torpedo, relative to the position of own submarine at the start of tracking (that is, subtracting off the effect of own submarine motion on the bearings), are shown in Figure 1. Also shown are the centroid bearing and the torpedo bearing after post-launch tracking begins, assuming no corrections are made.

The tracks of the torpedo bearing β_T and the centroid bearing B^* will depend on the corrections that are made to the torpedo gyro course, and the corrections will depend on the TMA. If the CHURN TMA is used, there will be little change in B_T and B^* for the first few opportunities, since we have assumed that corrections will be made at intervals of 20 seconds, which is only 5 percent of the pre-launch tracking interval. When the post-launch tracking interval becomes appreciable - say, 25 percent or more - it is difficult to anticipate what the CHURN solution will do to the torpedo course, without making a detailed computation. Certainly, when the post-launch tracking interval is at least 50 percent that of the pre-launch interval, CHURN will have a difficult problem in trying to fit a single tangent curve to these bearing observations. In fact, the problem would be difficult even if the target did not maneuver.

A similar result is obtained for most methods of target motion analysis, including the equal-segment method, and the partially-mechanized version of it known as MATE. A few observations in post-launch tracking will have a small effect. The effect of a large number of observations in post-launch tracking will depend on the relative weights that are given to the post-launch data and the pre-launch data.

The motion of the torpedo when corrections are computed from our TMA is quite different. At the first opportunity a large

Fig. 1. Bearings of Target, Torpedo, and Centroid
Run 3 / Case 3 $\Delta H = 5$ db, $G_0 = 14$ db



correction is applied in the direction toward the target. Additional corrections in this direction are applied at later opportunities, until the torpedo bearing is close to the target bearing and $|\Delta b|$ is small. In the absence of damping, the torpedo usually swings to the opposite side of the target bearing line before corrections are applied to drive it back. Thus, oscillations occur, unless some form of damping is applied.

Two simple forms of damping were found to be effective. The first one often is called proportional navigation in guided-missile applications. The second one is a simple limitation on the magnitude of the change in course angle that can be made at any correction. With either form of damping the oscillations can be controlled - even eliminated entirely, if desired - to obtain accurate interception courses.

The reason for the fast reaction to the apparent target motion in the post-launch phase is the emphasis put on the current bearing rate in our TMA and corresponding solution for the lead angle. We use only the post-launch data in computing the bearing rate, and we use exponential weighting to keep it current. We then compute the component u_b of target velocity normal to the sight line, and the corresponding lead angle, using the estimate of range made at launch and assuming that the range component of velocity is zero.

The large component of $\dot{B}^*$ obtained from the $\dot{\rho}$ term in equation (17) may yield unrealistic values for u_b for a few corrections. (In our example we obtained a value of approximately - 30 yds./sec. for two corrections.) To avoid over-correcting and to keep the computer happy,* we restrict $|u_b|$ to be no more than some

*Unless $|u_b|$ is less than the torpedo speed, the formula used in computing the lead angle θ_L may produce a value of sin θ_L that exceeds 1.0 in absolute value, which causes the computer to complain and sometimes to react in a spiteful way.

reasonable limit, such as 15 yards/sec., that is less than the torpedo speed.

An attempt was made to study the motion of the torpedo by writing the differential equations of motion, under the assumption that corrections are applied continuously. The equations are difficult to solve analytically. A numerical solution could have been obtained by a number of methods, but it would have been a less accurate description of the true motion than that which we obtain from the deterministic computer simulation that is embedded in our analytic simulation model.

Centroid Capture by the Torpedo

An important effect of the $\dot{\rho}$ term in $\dot{B}^*$, when $\dot{B}^*$ is used in our TMA, is to avoid having the centroid bearing lock onto the torpedo bearing. The biggest danger occurs at the start of post-launch tracking.

If $|B^* - B|$ decreases at the start of post-launch tracking, centroid capture by the torpedo will not occur immediately and is not likely to occur later. Now, $|B^* - B|$ will decrease, if $\dot{B}^* - \dot{B}$ has the same sign as $B - B^*$, which is the same as the sign of $B - B_T$. Hence, to avoid capture we want $(\dot{B}^* - \dot{B}) / (B - B_T) > 0$. From equation (17) we obtain

$$\frac{\dot{B}^* - \dot{B}}{B - B_T} = \frac{-\rho}{1 + \rho} \left(\frac{\dot{B} - \dot{B}_T}{B - B_T} \right) - \frac{\dot{\rho}}{(1 + \rho)^2}$$

Hence, we have the condition

$$\dot{\rho} \left(\frac{1}{1 + \rho} - \frac{1}{\rho} \right) - \left(\frac{\dot{B} - \dot{B}_T}{B - B_T} \right) > 0,$$

or

$$\frac{d}{dt} \log \left[(1 + 1/\rho) / |\Delta B| \right] > 0,$$

which is satisfied if and only if

$$(1 + 1/\rho) \ / \ |\Delta B|$$

is an increasing function of t. Since the numerator is an increasing function of t, the only danger of capture is when $|\Delta B|$ is increasing, and at a fractional rate that is greater than that of $(1 + 1/\rho)$.

The Simulation Model

The simulation model for the corrected-intercept mode that is described in reference (a) was revised to remove the dog-leg unmasking and to insert the simulation of centroid tracking. The required changes in the computation of the acquisition probability also were made. The revised computational procedure is outlined in the Appendix, with only the revised sections written in detail.

The revised procedure for centroid tracking was programmed. Acquisition probabilities were computed for the same runs used in reference (a). Some of the new parameters were varied to determine their effects, but most of them were assigned one arbitrary value. The results are given below.

Comparison Conditions: Runs and Parameter Values

The five run types are maneuvers from an initial course of 90 degrees and target speed of 10 knots, starting at a range of 10,000 yards. The maneuvers are as follows:

Run	Maneuver
1	60 degrees turn away
2	60 degrees turn towards
3	120 degrees turn away
4	120 degrees turn towards
5	Decelerate from 10 knots to 4 knots and accelerate back to 10 knots

Own submarine runs at 5 knots on alternate courses of 90 degrees and -90 degrees for tracking legs of 100 seconds, with 50 seconds allowed for the turns. The maneuvers are started at the following times:

Case	Time	Position of Own Submarine
1	200 secs.	Middle of second tracking leg
2	300 secs.	Start of third tracking leg
3	400 secs.	End of third tracking leg
4	500 secs.	Middle of fourth tracking leg, if made
5	600 secs.	Start of fifth tracking leg, if made

The tracking and fire-control procedure described in Reference (a) uses tests at the end of the first three tracking legs to determine whether or not to accept the range solution that has been generated and launch the torpedo, or to track for an additional leg. The test consists essentially of a comparison of our new estimate $\hat{r}$ based on three legs with the Ekelund estimate $\tilde{r}$ based on the last two legs. It is

$$r_{min} < \tilde{r} < r_{max}, \; r_{min} < \hat{r} < r_{max}$$

$$a_r < \tilde{r}/\hat{r} < 1/a_r, \; 0 < a_r < 1$$

We test to see if both estimates are in a reasonable range interval and are close together. If all inequalities are satisfied, we accept the estimate $\hat{r}$. If any inequality is not satisfied, we track for an additional leg and try again. Simulation runs, reported in Reference (a), show that the test usually rejects a poor solution and usually accepts a good solution when $a_r = 0.7$, $r_{min} = 1000$ yards, $r_{max} = 25{,}000$ yards. The values of r_{min} and r_{max} are not critical.

If the solution at the end of three legs is accepted, as usually occurs, the maneuver is made during tracking in cases 1 and 2, at the end of tracking in case 3, and after tracking has terminated in cases 4 and 5.

The parameter values listed in Table 3.1 of reference (a) were retained, except that $r_{min} = 2\ d_g$ was used to avoid the slight possibility of accepting a range estimate less than d_g; this change had no effect on the comparisons.

Some of the parameters listed in Table 3.1 of reference (a) were eliminated when the dog leg was eliminated, and other parameters were introduced in the centroid tracking. Additional parameters, mostly new, and the values assumed for them in the initial comparison, are listed in Table 1. After making computations for all runs and cases with the parameter values listed in Table 1, additional computations were made to explore the effects of some of the parameters, such as ΔH.

TABLE 1. ADDITIONAL PARAMETER VALUES

Parameter	Units	Value	Definition
f_1	fraction	0.4	Start post-launch tracking: fraction of estimated range
f_2	fraction	0.5	Enable: fraction of estimated range (new definition)
ΔH	decibels	5.0	Target sound above torpedo noise
G_o	decibels	14.0	Gain at center of major tracking lobe over side lobes
B^1	radians	0.3	Half angle of major tracking lobe
β^1	ratio	2.0	Discrimination ratio of separation angle to the angle B^1
f_4	fraction	0.4*	Proportion of correction in Proportional Navigation
$\Delta\theta_{max}$	radians	0.2*	Limit on corrections
max $\lvert u_b \rvert$	yds/sec.	15.0	Limit on normal velocity component

*not used simultaneously; the set ($f_4 = 1.0$, $\Delta\theta_{max} = 0.2$) and the set ($f_4 = 0.4$, $\Delta\theta_{max} = 1.0$) were used.

Another fraction f_3, with a value of 0.9, had been inserted to cut off the control corrections, to avoid wild oscillations near expected passage. However, after damping (proportional navigation or limited corrections) had been applied, it was found that control cutoff was not needed and sometimes stopped corrections too soon. Hence, it was removed, in effect, by putting $f_3 = 20$.

Before choosing values for the damping parameters, f_4 and $\Delta\theta_{max}$, a partial sensitivity analysis was made for run 3/case 3. With $\Delta\theta_{max} = 1.0$, f_4 was varied from 0.2 to 0.6 inclusive; with $f_4 = 1.0$, $\Delta\theta_{max}$ was varied from 0.1 to 0.3 inclusive. The large value of the parameter that was not varied had the effect of removing the corresponding form of damping. For both forms of damping the acquisition probability remained almost constant when the damping parameter was varied, indicating that the chosen interval was on the plateau of the curve, since very low acquisition probabilities were obtained with no damping and with excessive damping. From these results the following two sets of damping parameters were chosen:

Proportional Navigation: $f_4 = 0.4$, $\Delta\theta_{max} = 1.0$

Limited Correction : $f_4 = 1.0$, $\Delta\theta_{max} = 0.2$

In each case the correction applied to the gyro course is

$$+ f_4 \min(|\Delta\theta|, \Delta\theta_{max}), \text{ if } \Delta\theta \geq 0$$

$$- f_4 \min(|\Delta\theta|, \Delta\theta_{max}), \text{ if } \Delta\theta < 0$$

where $\Delta\theta$ is the correction that is computed from the TMA. If proportional navigation alone is used, the correction is simply $f_4 \Delta\theta$.

Results

Some results are shown in Tables 2 and 3. Acquisition probabilities for all runs and cases are shown in Table 2, for the parameter values listed in Table 1, including the two forms of sampling. Also shown for comparison purposes are the acquisition probabilities from Table 3.4 of reference (a) when no control is exercised and when a dog leg is used to unmask.

Acquisition probabilities are shown in Table 3, for runs 3 and 4 only, with ΔH equal to 5, 0, and -5 decibels, to show the effect of changes in the target sound intensity and the torpedo noise. The value of ΔH is the sound intensity of the target above the torpedo noise.

TABLE 2. ACQUISITION PROBABILITIES-CORRECTED INTERCEPT MODE

Run	Case	No Control	Dog Leg	Centroid Tracking: $\Delta H = 5$ 40% Correction	Centroid Tracking: $\Delta H = 5$ 12^{o} max Correction
1	1	.34	.46	.45	.43
	2	.49	.56	.52	.50
	3	.47	.54	.57	.55
	4	.50	.57	.61	.60
	5	.54	.59	.64	.64
2	1	.77	.79	.82(a)	.81(a)
	2	.75	.79	.81	.80
	3	.69	.75	.79	.79
	4	.67	.74	.77	.77
	5	.65	.72	.75	.75
3	1	.32	.42	.41	.41
	2	.34	.46	.45	.45
	3	.33	.51	.56	.55
	4	.42	.50	.62	.60
	5	.51	.53	.66	.66
4	1	.80	.84	.86	.86
	2	.78	.84	.86	.86
	3	.58	.74	.73	.73
	4	.63	.71	.72	.75
	5	.65	.66	.73	.74
5	1	.59	.67	.67	.68
	2	.58	.61	.68	.67
	3	.59	.68	.68	.69
	4	.59	.68	.69	.69
	5	.59	.68	.69	.69

Note: (a) These acquisition probabilities are obtained when the torpedo is enabled at the time at which the laminar point has reached the 50 percent point ($f_2 = 0.5$) of the estimated range. If enable is delayed until $f_2 = 0.7$, the laminar point has almost certainly passed the target and the acquisition probabilities are very small. See the section, Discussion of Results, for an explanation.

TABLE 3. ACQUISITION PROBABILITIES: EFFECTS OF TORPEDO AND TARGET NOISE

		40% Correction			12° max. Correction		
Run	Case	$\Delta H^{(a)}=5$	0	-5	5	0	-5
3	1	.41	.50	.50	.41	.50	.49
	2	.45	.49	.48	.45	.48	.37
	3	.56	.55	.49	.55	.55	.55
	4	.62	.61	.60	.60	.60	.60
	5	.66	.64	.65	.66	.62	.63
4	1	.86	.86	.84	.86	.80	.64
	2	.86	.85	.84	.86	.83	.67
	3	.73	.80	.73	.73	.78	.79
	4	.72	.76	.79	.75	.70	.75
	5	.73	.72	.76	.74	.70	.64

Note: (a) ΔH = Target sound intensity at source minus Torpedo noise at source

Discussion of Results

From Table 2 it is seen that when the additional parameters have the values listed in Table 1 the acquisition probabilities with centroid tracking of the degraded signal are comparable with those obtained by unmasking. In some cases, such as cases 4 and 5 of run 3, they are significantly higher. These results suggest that centroid tracking probably is significantly better than dog-leg unmasking when the target turns away after being alerted by the launching noises, and may be somewhat better against most maneuvers at longer ranges. These questions can be explored by more extensive computations.

It also appears from Table 2 that there is little to choose between the two forms of damping with the values chosen for the damping parameters. Results in Table 3 suggest that proportional navigation is slightly better than limited correction. However, the values chosen for the damping parameters are not necessarily optimal for a particular run and case, or for a mixture of runs and cases. Again, more extensive computations are needed to answer these questions.

The enable fraction f_2 was first chosen to be 0.7 to delay enable until the laminar point is 70 percent of the distance to the expected interception point. For case 1 of run 2 the acquisition probability was 0.01 for both forms of damping. Examination of the details revealed that enable had been delayed too long and the laminar point had passed the target (with high probability) when enable occurred. For this type of maneuver and the assumed path of own submarine during tracking, both range estimates $\hat{r}$ and $\tilde{r}$ are too large when three tracking legs are used, but are close enough together to be accepted by our test. The 70 percent value of the accepted range estimate exceeds the true range to passage by the laminar point, particularly since the target path is such as to decrease the range. In case 2 of run 2 enable occurs barely in time.

The late enable time that occurred in case 1 of run 2 does not occur in case 1 of run 4, even though the target again turns toward own submarine at the execution time of 200 seconds. The 120 degrees change in course, rather than the 60 degrees in run 2, produces underestimates of range by both methods. The errors are large, but the two estimates again are close enough to be accepted. An understanding of this paradox can be obtained by a detailed examination of bearing rates on the tracking legs and their effects on the range estimates.

To avoid late enable, such as that which occurred in case 1 of run 2, we reduced f_2 to 0.5 for these computations. It is questionable whether the avoidance of an occasional late enable more than offsets the delay in passage that would occur when the torpedo runout speed exceeds the search speed. The question is difficult to answer, even with extensive computations, since it involves distributions of the engagement range and the target course and speed.

A natural question is the extent to which the results depend on the assumed values for the parameters that control the centroid tracking, particularly the difference ΔH in sound output from the target and torpedo. The comparison in Table 3 shows that there is little change, and not always downward, in the acquisition probability on runs 3 and 4 for either form of damping when ΔH changes from 5 db to 0 db. With proportional navigation there are no significant changes when ΔH is dropped to -5 db. With the 12 degrees limitation on the correction, decreases occur in a few cases.

In some cases, such as case 1 of run 3, the torpedo captures the tracking beam. For this case the range estimates $\tilde{r}$ and $\hat{r}$ at the end of three legs are only about 40 percent of the true range, and are close enough together to be accepted. With the resulting small range estimate post-launch tracking starts soon after launch, and the value of the weighting factor ρ is very large, even for ΔH = 5 db. Since $|\Delta B|$ is increasing and at a fractional rate that exceeds that of $(1 + 1/\rho)$, the torpedo captures the tracking beam at the start of post-launch tracking. However, the large value of $\dot{\rho}$ eventually contributes a sufficient component to the bearing rate in the direction of the target to generate corrections to the gyro course that break the capture condition.

The conditions, if any, under which capture persists, after it occurs, are difficult to determine, since the condition depends in a complex way on the motions of the target and torpedo relative to own submarine, the range estimate, the ranges to the torpedo and target, and many control parameters. The critical question is whether the early course "corrections" that are generated move the torpedo bearing closer to the target bearing or farther away. The question of the conditions under which torpedo capture of the tracking beam persists is an important question in the use of corrected-intercept control with our TMA against targets that are quieter than the torpedo. If the target is quieter than the torpedo, the torpedo usually captures the tracking beam at the start of post-launch tracking. The use of our TMA apparently permits capture to be broken before the laminar point passes the target.

When the separation angle $|\Delta B|$ becomes large enough the sonar operator can detect the separate signals and presumably shift the tracking beam to the target. An allowance is made for this shift in our model: if $\beta > \beta^1$, the gain G in sensitivity shifts to $+ G_o$ from the value near $- G_o$ it had when the torpedo was tracked. Here, $\beta = |\Delta B| / B^1$ is the half angle of the tracking lobe. We assume that the operator can discriminate between the two signals when the separation angle is β^1 times the half angle. In our computations we used B^1 = 0.3 radians and β^1 = 2.0, which is equivalent to a separation requirement of 34 degrees. A smaller separation requirement could permit a shift to be made sooner. The effect on the acquisition probability depends on many factors. A shift in the value of B^1 to 0.15 radians yielded lower acquisition probabilities for case 1 of run 3.

Reference

(a) ADL Report NUWRES #12, "Control Modes and Acquisition Probabilities for Torpedo MK 48 (U), "Contract No. N00140-68-C-0278, January 1970, Final Report Confidential, Technical Appendices (bound separately) Unclassified.

APPENDIX A

COMPUTATIONAL PROCEDURE FOR THE C-I MODE WITH CENTROID TRACKING

The computational procedure for the corrected-intercept mode when a dog leg is used to unmask is given in Appendix E of Reference (a). We list below only the changes that are needed to replace the unmasking maneuver by centroid tracking.

1. TMA for Torpedo Launch

2. Initial Course and Launch

These two sections are not affected by the masking assumptions, and are equivalent to section E.2 of Appendix E of Reference (a).

3. Post-Launch Tracking

We start with the coordinates (x_{To}, y_{To}) of the pseudo origin, the initial gyro angle Θ_o, the estimate $\hat{r}^{(o)}$ of range, and the number $N^{(o)}$ of time steps in the pre-launch tracking interval.

f_1 = factor for initial leg (input)
f_2 = factor for the enabling step (input)
s = torpedo speed during runout (input)
s' = torpedo speed made good during search (input)
j_1 = integral part of $[f_1 (\hat{r}^{(o)} - d_g) / (s\, \Delta t)]$
j_2 = integral part of $[f_2 (\hat{r}^{(o)} - d_g) / (s\, \Delta t)]$

$$s_j = \begin{cases} s & , j \leq j_2 \\ s' & , j > j_2 \end{cases}$$

For superscript j = 1, 2, 3, ...

$$N^{(j)} = N^{(o)} + j_1 + (j - 1)\, C$$

For subscript $j = 1,2,\ldots, N^{(2)} - N^{(o)}$

$$\Theta_j = \Theta_o$$

$$x_{Tj} = x_{Tj-1} - s_j\ \Delta t \sin \Theta_j,\ y_{Tj} = y_{Tj-1} + s_j\ \Delta t \cos \Theta_j$$

$$\Theta^{(o)} = \Theta^{(1)} = \Theta_o$$

Next, we compute the "corrected" gyro course angles $\Theta^{(2)}$, $\Theta^{(3)}$, ..., $\Theta^{(\max j)}$ by large loops, for which the one for $\Theta^{(m)}$ is as follows:

For subscript $j = N^{(m)} - N^{(o)} - C + 1, \ldots, N^{(m)} - N^{(o)}$

$$\Theta_j = \Theta^{(m-1)}$$

$$x_{Tj} = x_{Tj-1} + s_j\ \Delta t \sin \Theta_j$$

$$y_{Tj} = y_{Tj-1} + s_j\ \Delta t \cos \Theta_j$$

For $j = j_1, j_1 + 1, \ldots, \max j$ compute

$$x_j^{\ 1} = x_{j+N^{(o)}} - \xi_{j+N^{(o)}}\ ,\ y_j^{\ 1} = y_{j+N^{(o)}} - \eta_{j+N^{(o)}}$$

$$x_{Tj}^1 = x_{Tj} - \xi_{j+N^{(o)}}\ ,\ y_{Tj}^1 = y_{Tj} - \eta_{j+N^{(o)}}$$

$$r_j = [(x_j^1)^2 + (y_j^1)^2]^{1/2}\ ,\ r_{Tj} = [(x_{Tj}^1)^2 + (y_{Tj}^1)^2]^{1/2}$$

$$\sin B_j = x_j^1 / r_j,\ \cos B_j = y_j^1 / r_j$$

$$\sin B_{Tj} = x_{Tj}^1 / r_{Tj}\ ,\ \cos B_{Tj} = y_{Tj}^1 / r_{Tj}$$

Find B_j and B_{Tj}, $-\pi < B_j \leq \pi$, $-\pi < B_{Tj} \leq \pi$

B^1 = half-angle of major lobe (input)

G_o = gain in sensitivity in major lobe (input)

ΔH = difference in signal strengths at the source (input)

$$\beta_j = |B_j - B_{Tj}| / B^1$$

$$\rho_j = (r_j / r_{Tj})^2 \exp(-0.23\ \Delta H)$$

$$\gamma_j = \min[1, \beta_j / (1+\rho_j)],\quad \gamma_j^1 = \min[1, \rho_j \beta_j / (1+\rho_j)]$$

$$\Delta G_j = G_o [\cos(\pi\gamma_j^1/2) - \cos(\pi\gamma_j/2)],\quad \beta_j \leq \beta^1$$

$$= G_o \qquad\qquad ,\quad \beta_j > \beta^1$$

$$\rho_j^1 = \rho_j \exp(-0.23\ \Delta G_j)$$

$$B_j^* = (B_j + \rho_j^1 B_{Tj}) / (1 + \rho^1{}_j)$$

$$\dot{B}^{(m)} = \frac{\sum_{i=1}^{(m-1)C-1} \left(B^*_{N^{(m)} - N^{(o)}} - B^*_{N^{(m)} - N^{(o)} - i} \right) \exp(-i\Delta t/T)}{\Delta t \sum_{i=1}^{(m-1)C-1} i^2 \exp(-i\Delta t/T)}$$

$$\ell^{(m)} = N^{(m)} - N^{(o)}$$

$$\dot{v}^{(m)} = S_{N^{(m)}} \left(\sin\phi_{N^{(m)}} \cos B^*_{\ell^{(m)}} - \cos\phi_{N^{(m)}} \sin B^*_{\ell^{(m)}} \right)$$

$$\hat{u}_b^{(m)} = \dot{v}^{(m)} + \hat{r}^{(o)} \dot{B}^{(m)}$$

$$x_T^{(m)} = x_{T,N^{(m)} - N^{(o)}}, \quad y_T^{(m)} = y_{T,N^{(m)} - N^{(o)}}$$

$$x_G^{(m)} = x_T^{(m)} + d_g \sin \Theta^{(m-1)}$$

$$y_G^{(m)} = y_T^{(m)} + d_g \cos \Theta^{(m-1)}$$

$$x_v^{(m)} = \xi_{N^{(m)}} + \hat{r}^{(o)} \sin B^*_{N^{(m)} - N^{(o)}} - x_G^{(m)}$$

$$y_v^{(m)} = \eta_{N^{(m)}} + \hat{r}^{(o)} \cos B^*_{N^{(m)} - N^{(o)}} - y_G^{(m)}$$

$$r_v^{(m)} = [(x_v^{(m)})^2 + (y_v^{(m)})^2]^{1/2}$$

$$\sin \hat{\psi}^{(m)} = x_v^{(m)} / r_v^{(m)}, \quad \cos \hat{\psi}^{(m)} = y_v^{(m)} / r_v^{(m)}$$

$$\cos B_v^{(m)} = \cos B^*_{N^{(m)} - N^{(o)}} \cos \hat{\psi}^{(m)} + \sin B^*_{N^{(m)} - N^{(o)}} \sin \hat{\psi}^{(m)}$$

$$h = j_2, \quad \ell^{(m)} = N^{(m)} - N^{(o)}$$

$$s^{(m)} = \begin{cases} \dfrac{(s^2 (h - \ell^{(m)}) \Delta t + s^1 (\hat{r}^{(o)} - hs\Delta t))}{(\hat{r}^{(o)} - s\,\ell^{(m)} \Delta t)}, & \ell^{(m)} < h \\ s^1, & \ell^{(m)} \geq h \end{cases}$$

$$\sin \Theta_L^{(m)} = (\hat{u}_b^{(m)} \cos B_v^{(m)}) / s^{(m)}$$

$$\cos \Theta_L^{(m)} = (1 - \sin^2 \Theta_L^{(m)})^{1/2}$$

$$\sin \Theta_g^{(m)} = \sin \Theta_L^{(m)} \cos \hat{\psi}^{(m)} + \cos \Theta_L^{(m)} \sin \hat{\psi}^{(m)}$$

$$\cos \Theta_g^{(m)} = \cos \Theta_L^{(m)} \cos \hat{\psi}^{(m)} - \sin \Theta_L^{(m)} \sin \hat{\psi}^{(m)}$$

Find $\Theta_g^{(m)}$ in radians, $-\pi < \Theta_g^{(m)} \leq \pi$.

Compute

$$\Delta\Theta^{(m)} = \Theta_g^{(m)} - \Theta^{(m-1)}$$

f_4 = proportion in Proportional Navigation (input)

$\Delta\Theta_{max}$ = limitation on correction (input)

$$\Theta^{(m)} = \Theta^{(m-1)} + f_4 \min (|\Delta\Theta^{(m)}|, \Delta\Theta_{max}), \text{ if } \Delta\Theta^{(m)} \geq 0$$

$$\Theta^{(m)} = \Theta^{(m-1)} - f_4 \min (|\Delta\Theta^{(m)}|, \Delta\Theta_{max}), \text{ if } \Delta\Theta^{(m)} < 0$$

This completes the loop to find $\Theta^{(m)}$. Starting with $\Theta^{(o)} = \Theta^{(1)} = \Theta_o$, repeat the loop to find $\Theta^{(2)}, \Theta^{(3)}, \ldots, \Theta^{(max\ j)}$.

4. Remainder of Computations

The remainder of the computations are made by the procedures described in Sections E.4, E.5, and E.6 of Appendix E of Reference (a).

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13. ABSTRACT
Equations are developed to describe centroid tracking, in which the sonar operator tracks a composite signal at a bearing that is a weighted average of the bearings of the torpedo and target submarine. A study is made of the motion of the apparent bearing line and the effects of this motion when corrected-intercept control is exercised and corrections are computed by a recently developed procedure. Acquisition probabilities are computed by suitable modifications to a previously developed simulation model, and compared with the corresponding probabilities obtained when a torpedo maneuver is used to unmask the target.

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