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Spectral Analysis of Signals by Using The Z-Transform Algorithm

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17 August 1970

New London Laboratory

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ABSTRACT

A Z-transform algorithm, developed for the spectral analysis of signals, allows one to get closer to the poles of a signal and effectively reduces the signal's bandwidth and sharpens its peak point. It can give a high resolution, narrow-band frequency analysis with frequency spacing $\Delta f \leq 1/T$, where T = total length of the analysis interval. This algorithm also enhances (1) the signal poles that lie on circular or spiral contours that begin at almost any point in the Z-plane and (2) the angular spacing of points in an arbitrary constant. Since this algorithm takes advantage of high-speed convolution, it is almost as fast as and more flexible than the Fast Fourier Transform (FFT).

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The algorithm described in this report is particularly useful for spectral analysis of active sonar signals where high resolution is essential. It has also the ability to evaluate high-resolution spectra over a passband that can be located anywhere within the total band of interest.

Benjamin F. Cron, Research Associate in the Acoustic Research Branch of the Ocean Sciences Division, was the Technical Reviewer for this report.

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SPECTRAL ANALYSIS OF SIGNALS BY USING THE Z-TRANSFORM ALGORITHM

INTRODUCTION

A Z-transform algorithm is applied to the spectral analysis of signals whose poles and zeros are either on or off the imaginary axis of a complex S-plane. This algorithm is flexible and can be applied either on or off the imaginary axis and can enhance the poles in the spectral analysis of signals. It is also useful in evaluating high resolution, narrow-frequency-band spectra.

The signals that are to be analyzed by using the digital technique are considered at discrete values of t , usually $n\Delta t$, where $n=0, 1, 2, \dots$, and Δt is a fixed positive number usually referred to as a sampling period. In Fig. 1, a continuous function of time, $f(t)$, is shown, and its values at $t = n\Delta t$ are indicated.

Let Δt be the fixed positive number and let $f(t)$ be defined for $t > 0$. The Z-transform of $f(t)$ is the function

$$F(z) = \sum_{n=0}^{\infty} f(n\Delta t) Z^{-n} , \quad (1)$$

where

$Z = \text{complex variable.}$

For finite sum, we can write

$$F(z) = \sum_{n=0}^{N-1} f(n\Delta t) Z^{-n} . \quad (2)$$

If $Z = e^{S\Delta t}$, where $S = \sigma + j\omega$, then Eq. (2) stands for the Laplace transform of a train of equally spaced impulses of magnitude $f(n\Delta t)$. The Laplace transform of a train of impulses repeats its value in a horizontal strip of the S-plane of width $2\pi/\Delta t$ in every strip parallel to it. The Z-transform maps each such strip into an entire Z-plane or, conversely, the entire Z-plane corresponds to any horizontal strip of the S-plane, e.g., the regions $-\infty < \sigma < \infty$ and $-\pi/\Delta t \leq \omega \leq \pi/\Delta t$,

where $S = \sigma + j\omega$. The $j\omega$ axis of the S-plane along which we generally equate the Laplace transform with the Fourier transform is the unit circle in the Z-plane, and the origin of the S-plane corresponds to $Z = 1$. The interior of the unit circle of a Z-plane corresponds to the left half of the S-plane, and the external corresponds to the right-half plane. Straight lines in the S-plane correspond to a circle or a spiral in the Z-plane, as shown in Fig. 2.

Usually the Z-transforms are computed along the path corresponding to the $j\omega$ axis; i. e., the unit circle on the Z-plane gives a discrete equivalent of the Fourier transform.

There are many applications of the Z-transform on and off the unit circle,^{1,2} e. g., spectra, filtering, interpolation, etc. This report presents the spectral estimation of signals for which one has some a priori knowledge of the locations of poles by using the Z-transform algorithm. Moreover, this report investigates the effect of additive white noise on the Z-transform algorithm when signal spectrums are evaluated.

THEORETICAL APPROACH

For convenience, the equation for the finite sum (Eq. (2)) is written again^{1,2} :

$$F(z_k) = \sum_{n=0}^{N-1} f(n\Delta t) Z_k^{-n} \quad (3)$$

Let

$$Z_k = AW^{-k}, k = 0, 1, 2, \dots, M-1 \quad (4)$$

where M is an arbitrary integer and both A and W are arbitrary complex numbers:

$$A = A_0 e^{j2\pi\theta_0} \quad (5)$$

and

$$W = W_0 e^{j2\pi\theta_0} \quad (6)$$

The starting point of the contour is determined by A, and the resolution is determined by W (Fig. 3).

If $A = 1$, $W = e^{-j2\pi/N}$, and $N = M$, then Eq. (3) corresponds to the discrete Fourier transform. If N is a power of two, the algorithm can be implemented as a Fast Fourier Transform (FFT).

The general Z-plane contour begins with point $Z = A$ and, depending on the value of W , spirals in or out with respect to its origin. If $W_0 = 1$, the contour is an arc of a circle. The angular spacing of the samples in the Z-plane is $2\pi\phi_0$.

Since A and W are arbitrary complex numbers, we see that the points

$$S_k = S_0 + k(\Delta\sigma + j\Delta\omega) = \frac{1}{\Delta t}(\ln A - k \ln W) \quad , \quad (7)$$

where

$$k = 0, 1, 2, \dots, M-1 \text{ and}$$

$$S_0 = \sigma_0 + j\omega_0 = \frac{1}{\Delta t} \ln A \quad ,$$

lie on an arbitrary straight-line segment of arbitrary length and arbitrary sampling density.

The contour $Z_k = AW^{-k}$, where $k = 0, 1, 2, \dots, M-1$, is considerably more general than that to which the FFT applies (Fig. 3). It is seen from Eq. (3) that the computation of a Z-transform along the general contour requires NM multiplications and additions, since the special symmetry of $e^{j2\pi k/N}$, which is exploited in deriving the FFT, is absent in more general cases. However, by manipulating the role of $W^{n^2/2}$ in calculation, it can be shown that the computations can be reduced.

Substituting the value of $Z_k = AW^{-k}$, we see that Eq. (3) yields

$$\begin{aligned} F(Z_k) &= \sum_{n=0}^{N-1} f(n\Delta t) (AW^{-k})^{-n} \\ &= \sum_{n=0}^{N-1} f_n A^{-n} W^{nk} \quad , \end{aligned} \quad (8)$$

where

$$f_n = f(n\Delta t) \quad ,$$

$$n = 0, 1, 2, \dots, N-1, \text{ and}$$

$$k = 0, 1, 2, \dots, M-1 \quad .$$

Substituting the value of $nk = n^2 + k^2 - (k-n)^2/2$ for the exponent of W in Eq. (8), we get

$$F(Z_k) = \sum_{n=0}^{N-1} f_n A^{-n} W^{\frac{n^2 + k^2 - (k-n)^2}{2}}$$

$$\begin{aligned}
&= \sum_{n=0}^{N-1} f_n A^{-n} W^{n^2/2} W^{k^2/2} W^{-(k-n)^2/2} \\
&= \sum_{n=0}^{N-1} f_n A^{-n} W^{k^2/2} \left(W^{-(k-n)^2/2} \right) \left(W^{k^2/2} \right) .
\end{aligned} \tag{9}$$

Let

$$Y_n = f_n A^{-n} W^{n^2/2}, n = 0, 1, 2, \dots, N-1, \tag{10}$$

and

$$V_n = W^{-n^2/2}, n = 0, 1, 2, \dots, N-1. \tag{11}$$

Now convolving Y_n with V_n , we can write

$$g_k = \sum_{n=0}^{N-1} Y_n V_{k-n}, k = 0, 1, 2, \dots, M-1. \tag{12}$$

Multiplying Eq. (12) by $W^{k^2/2}$, we can write

$$F(Z_k) = g_k W^{k^2/2}, k = 0, 1, 2, \dots, M-1, \tag{13}$$

which is evaluated as follows:

a. The input signals in sequence are multiplied by $A^{-n} W^{n^2/2}$ to get Y_n , which requires N multiplication.

b. The symbol g_k is the convolution of Y_n and V_n . This convolution is equivalent to multiplying the Fourier transform of Y_n and V_n . The g_k is evaluated by taking the inverse FFT of the product of the individual FFT of Y_n and V_n . These operations take approximately a time proportional to $(N+M) \log(N+M)$.²

c. The next step is to multiply g_k by $W^{k^2/2}$ to get $F(Z_k)$, which requires only M multiplication. (The whole operation is summarized in Fig. 4.)

SPECTRAL ANALYSIS OF SIGNALS

The advantage of the Z-transform algorithm over the FFT is the algorithm's ability to evaluate the Z-transform at points inside and outside the unit circle. By evaluating the transform that is outside the

unit circle, one can get closer to the poles and the zeros of the signal, thus effectively reducing the signal's bandwidth and sharpening its peak point. This will be demonstrated by the simulated examples 1 and 2.

Example 1:

$$f(t) = \sum_{i=1}^4 A_i e^{a_i} \cos 2\pi f_i t, \quad 0 < t < T = 78 \text{ msec},$$

$$= 0 \quad T < t < 128 \text{ msec},$$

where

$$A_i = 1, \quad i = 1, 2, 3, \text{ and } 4$$

$$a_1 = -0.0038 \quad f_1 = 50 \text{ Hz}$$

$$a_2 = -0.0164 \quad f_2 = 200 \text{ Hz}$$

$$a_3 = -0.023 \quad f_3 = 300 \text{ Hz}$$

$$a_4 = -0.029 \quad f_4 = 370 \text{ Hz}$$

and $1/\Delta t = \text{sampling frequency} = 1 \text{ kHz}$.

The signal $f(t)$ is shown in Fig. 5. For this example, only the pole positions of the upper half of the S-plane are shown in Fig. 6.

In Example 1, the Z-transform of 128 sample data points is evaluated on two spirals outside the unit circle; one spiral is on the unit circle and the other is inside the unit circle. Figure 7 shows the four contours as they would appear in the S-plane. The Z-transform is evaluated at 51 equally spaced points, from 0 through 500 Hz (frequency spacing $\Delta f = 10 \text{ Hz}$), that have a corresponding value of $\phi_0 = -1/100$. The evaluation of the magnitude response of simulated signals without noise on contours 1, 2, 3, and 4 of Fig. 7 is shown in Figs. 8A through 8D, respectively. Figures 8A and 8B show that peak values at higher dominant frequencies are smaller than the peak values at lower dominant frequencies because the signal poles are farther away at higher frequencies than at lower frequencies. Figure 8C shows that the peak values at all dominant frequencies are almost equal. This is due to the fact that the gradient of contour 4 of Fig. 7 relative to contour 3 is constant. As expected, Fig. 8D shows that the sharpening of magnitude response in the region of the poles is quite pronounced and higher compared with Figs. 8A, 8B, and 8C since the poles are very close to contour 4. For comparison, a direct FFT of simulated signals is evaluated and shown in Fig. 9. The resolution or frequency spacing is $\Delta f = 1/N\Delta t \cong 7.8 \text{ Hz}$, whereas the frequency spacing in the case of a Z-transform is 10 Hz.

To see the effect of noise on the Z-transform algorithm, white noise is added to the signal. Now, the Z-transform of the composite signal (signal plus noise) is evaluated, as mentioned above, on the same contours at signal-to-noise ratio $(S/N) = +8.87$ dB and at $S/N = -0.6$ dB. The S/N is defined as

$$\frac{S}{N} = 10 \log \frac{\text{average signal power}}{\text{average noise power}}$$

Figures 10A through 10D and 11A through 11D, which show the magnitude response of composite signals at $S/N = 8.87$ dB and -0.6 dB, respectively, indicate that the magnitude response in the pole regions is more pronounced. Since the noise is white, its effect is noted by the bias increase of the whole spectrum. For comparison, direct FFT evaluation of composite signals is shown in Figs. 12A and 12B.

Example 2: One very useful advantage of the Z-transform algorithm is that it enables evaluating high resolution, narrow frequency band spectra efficiently, whereas the frequency resolution in the standard FFT technique is limited by the total length of the signal. However, if one requires resolution, Δf , where $\Delta f \leq 1/N\Delta t$ and where $\Delta t =$ sampling period and $N =$ number of sample points, then the number of sample points, N , must be increased. In other words, the small Δf implies a large value of N .

Often, however, a high resolution for a limited range of frequencies and low resolution for the rest of the spectrum is desired, i. e., a detailed view of the spectrum in the passband and only a general view outside the passband.

The following simulated results will show that the Z-transform algorithm allows (1) independent selection of initial frequency or starting point and (2) frequency spacing that is independent of the number of time samples. (For the standard FFT, the frequency spacing is not independent of the number of time samples.) Hence, high resolution can be obtained over a narrow frequency range.

To illustrate these points, a signal in the form of

$$f(t) = \sum_{i=1}^5 \cos 2\pi f_i t, \quad 0 < t < T = 78 \text{ msec},$$

$$= 0 \quad T < t < 128 \text{ msec} ,$$

where

$$f_1 = 230 \text{ Hz}$$

$$f_2 = 240 \text{ Hz}$$

$$f_3 = 250 \text{ Hz}$$

$$f_4 = 260 \text{ Hz}$$

$$f_5 = 270 \text{ Hz} ,$$

is generated as shown in Fig. 13.

The FFT for 128 points of this signal is calculated at various S/N's, as shown in Figs. 14A, 14B, and 14C. In all cases, most of the signal energy is located between 200 through 300 Hz.

To investigate the passband and the transition regions more carefully, the Z-transform algorithm was used to give a 2-Hz resolution from 200 through 300 Hz, as shown in Figs. 15A, 15B, and 15C. The contour is the same as in the case of the FFT. To achieve this resolution, one would require 500 time-sample values instead of only the 128 points that have been used in the Z-transform algorithm. This shows that this algorithm is a powerful tool for the close examination of small frequency bands.

CONCLUSION

Signal simulation results show that the Z-transform algorithm can enhance the signal poles, which, in turn, sharpen the peak of the dominant frequency and reduce the bandwidth. The algorithm provides a number of output points that are independent of the number of input points; i. e., the output points may be larger or smaller than the input points. It also has the added advantage of enabling one to start at an arbitrary point in the Z-plane. This algorithm will be very useful for a close look at the filter passbands.

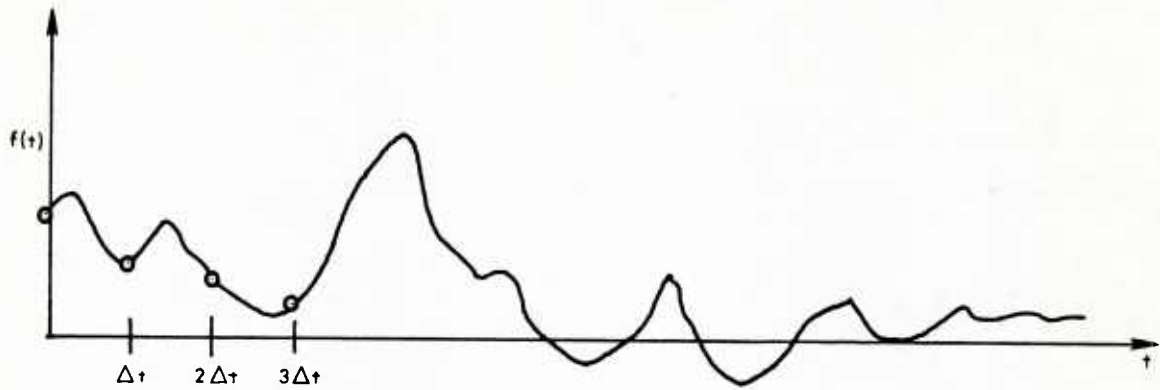


Fig. 1. A Discrete and Continuous Time Function

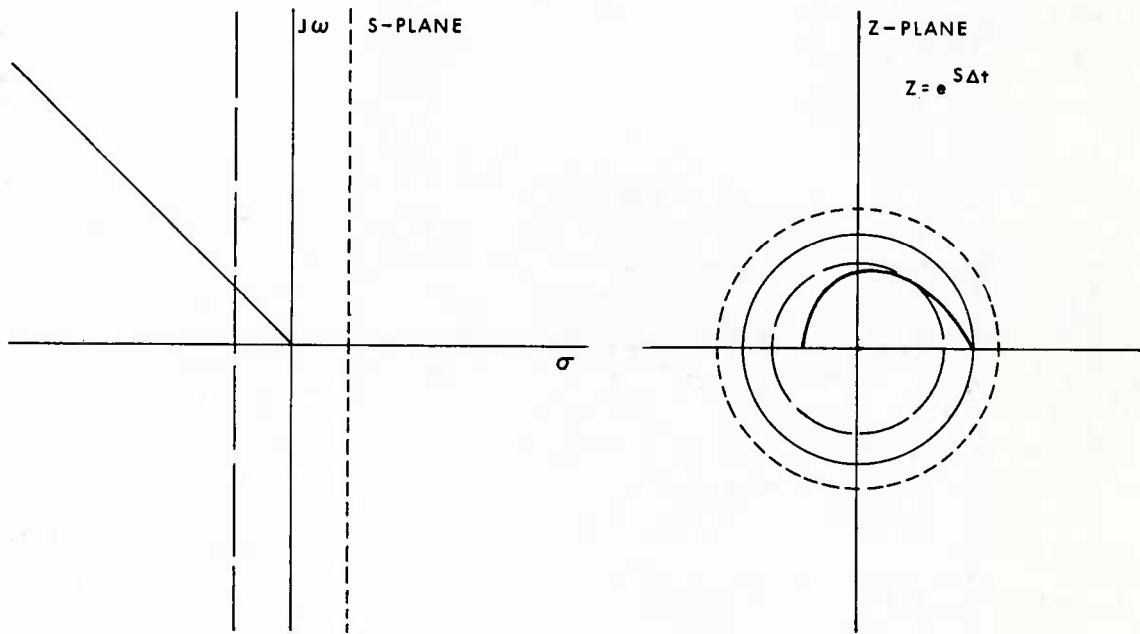


Fig. 2. Correspondence of a Z-Plane Contour with an S-Plane Contour

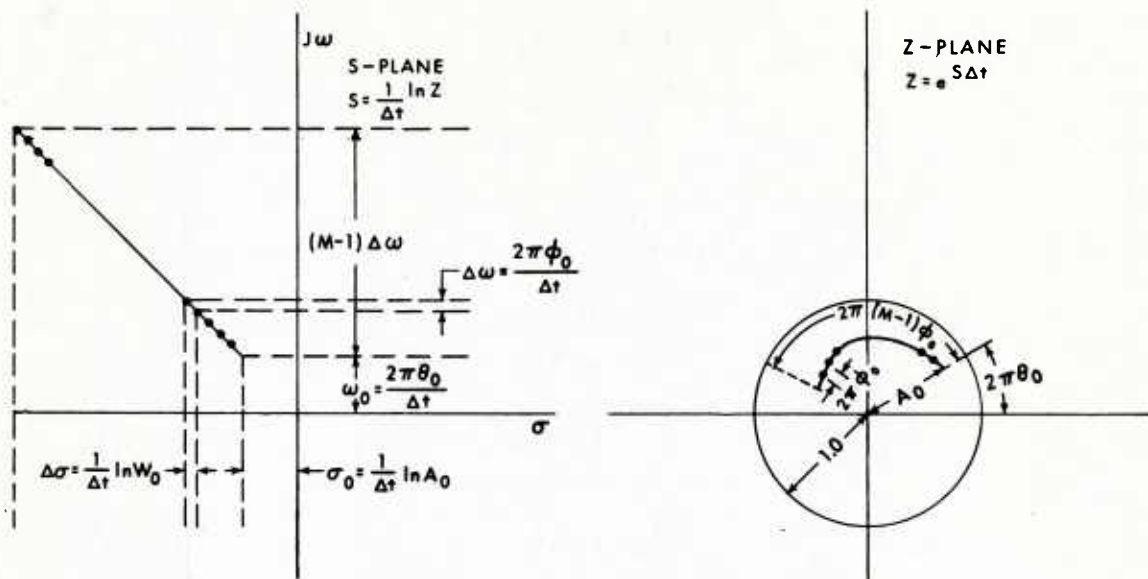


Fig. 3. Correspondence of Z- and S-Plane Contours with Various Parameters

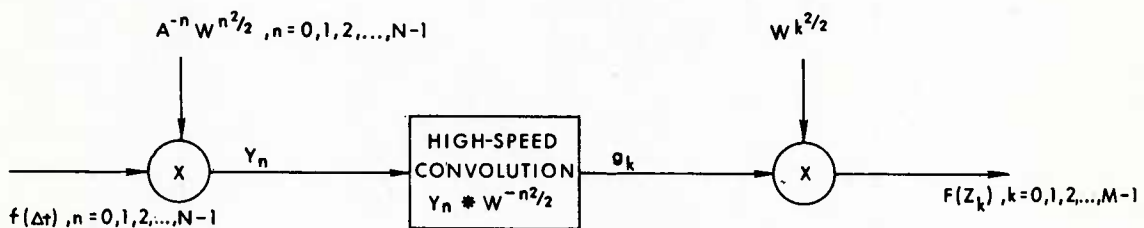


Fig. 4. Steps Involved in Evaluating a Z-Transform of Time Function

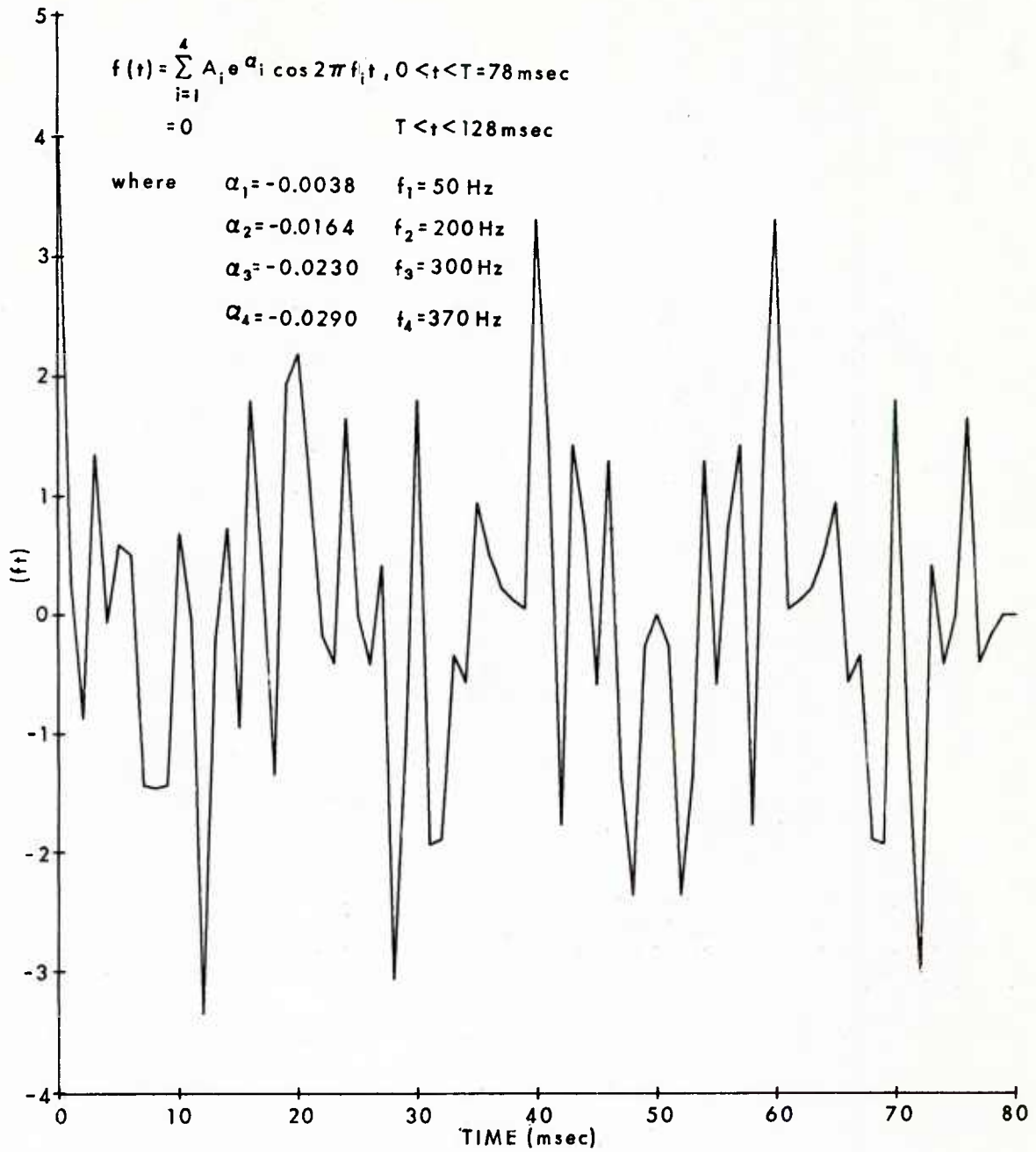


Fig. 5. Simulated Signal

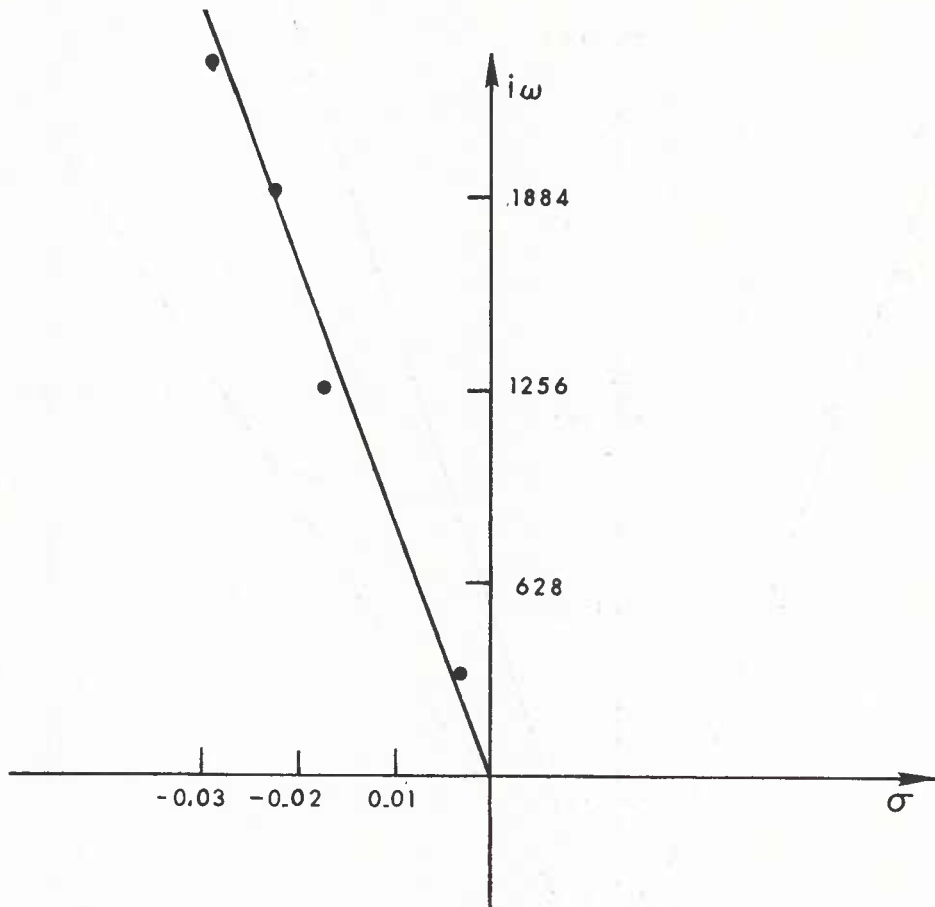


Fig. 6. Pole Positions of a Simulated Signal in the Left Upper-Half of an S-Plane

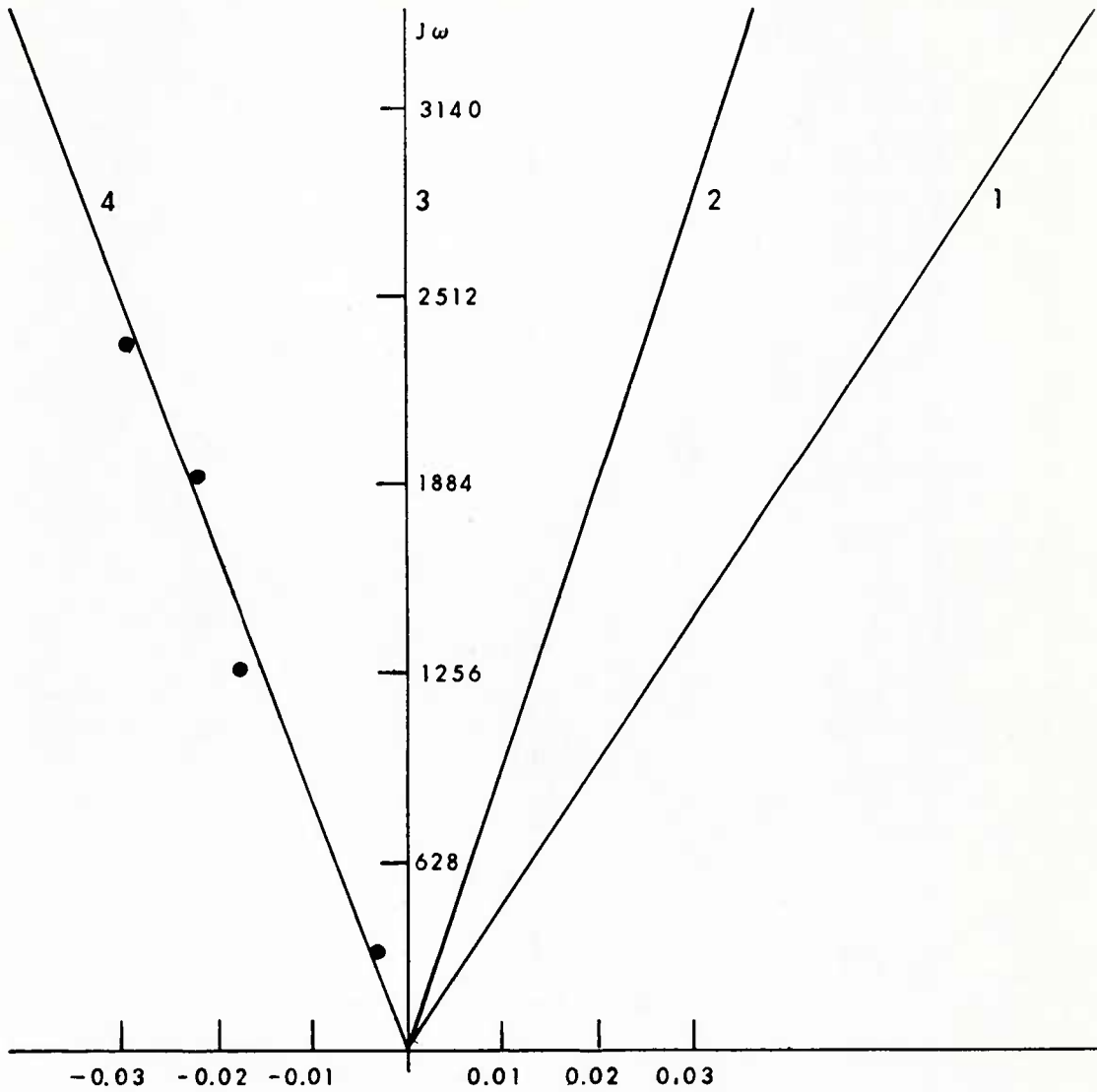


Fig. 7. Four Contours Used to Evaluate a Z-Transform

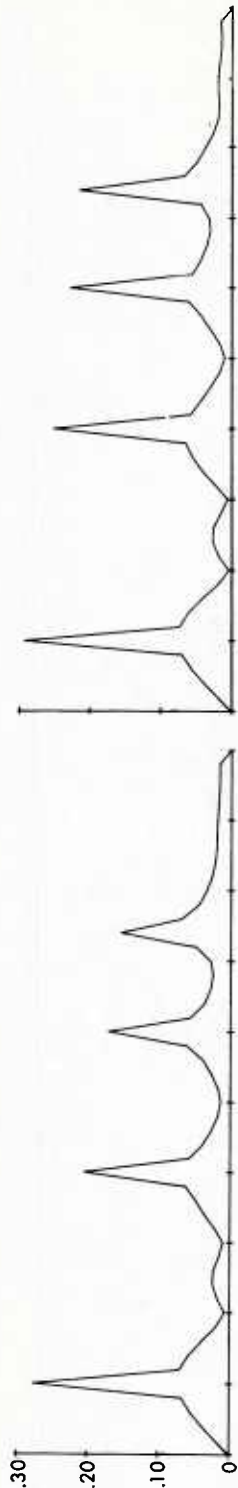


Fig. 8A. Contour 1

Fig. 8B. Contour 2

ABSOLUTE VALUE OF SPECTRUM

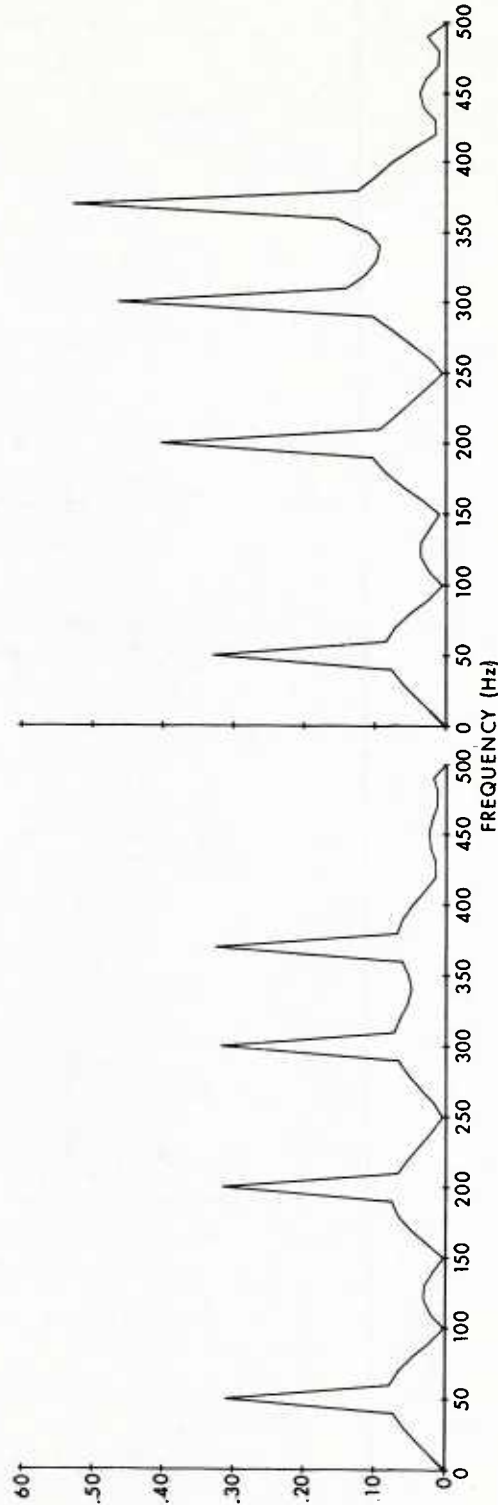


Fig. 8C. Contour 3

Fig. 8D. Contour 4

Fig. 8. Z-Transform Absolute Magnitude Response of Simulated Signals Evaluated on the Four Contours of Fig. 7 at $S/N = \infty$ dB

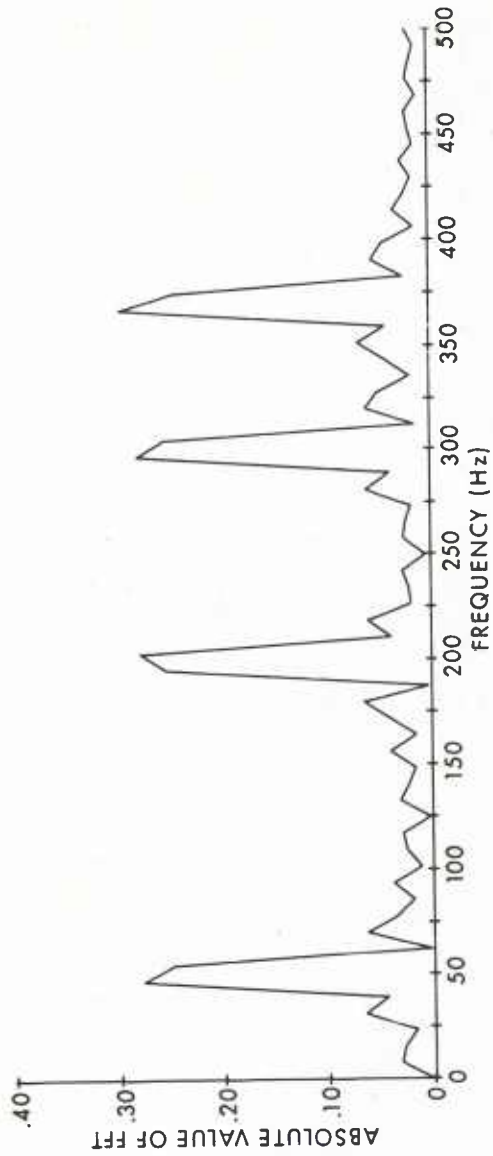


Fig. 9. FFT Absolute Magnitude of Simulated Signals at $S/N = \infty$ dB

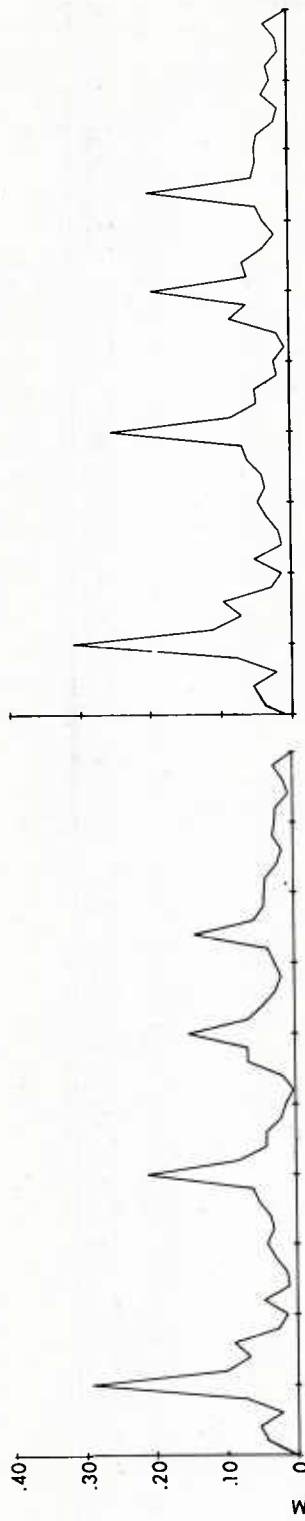


Fig. 10B. Contour 2

Fig. 10A. Contour 1

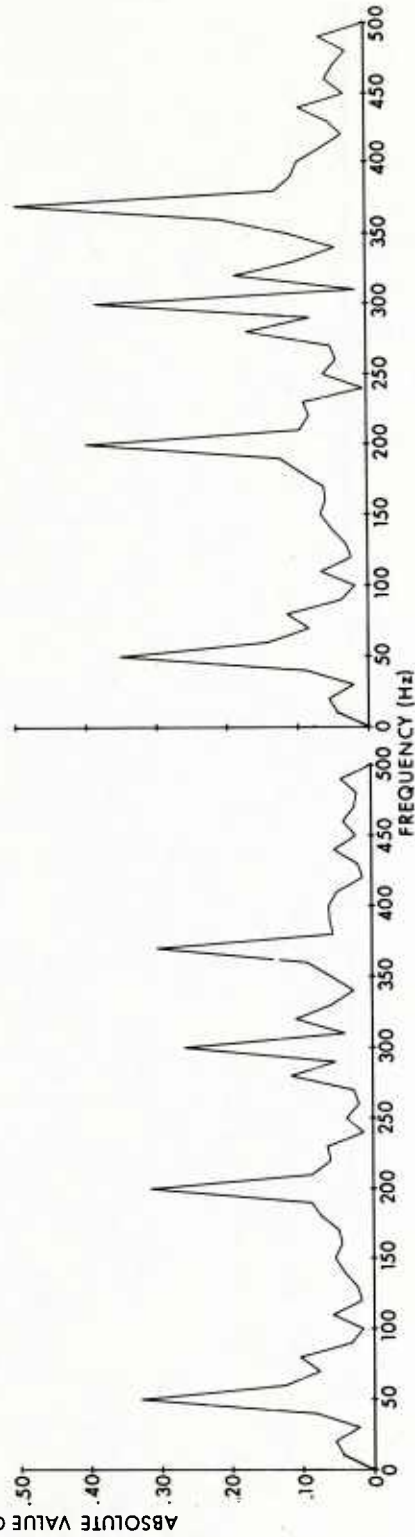


Fig. 10D. Contour 4

Fig. 10C. Contour 3

Fig. 10. Z-Transform Absolute Magnitude Response of Simulated Signals Evaluated on the Four Contours of Fig. 7 at $S/N = 8.87$ dB

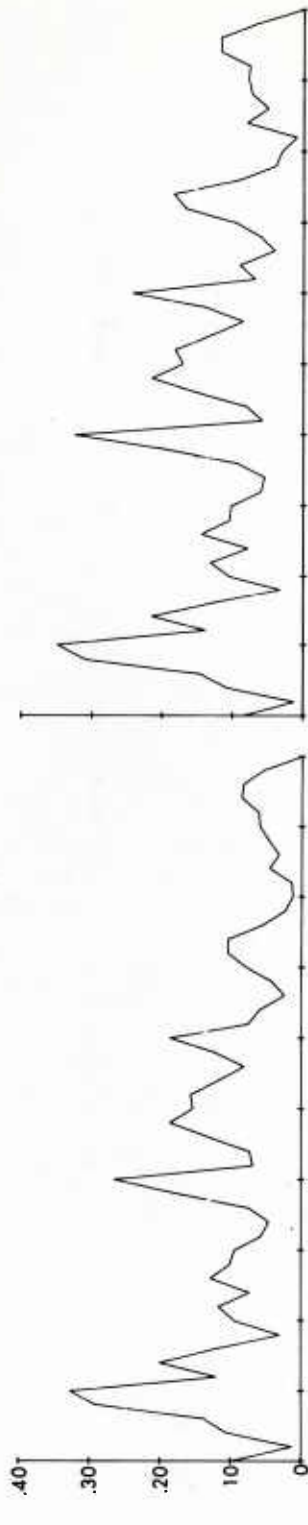


Fig. 11A. Contour 1

Fig. 11B. Contour 2

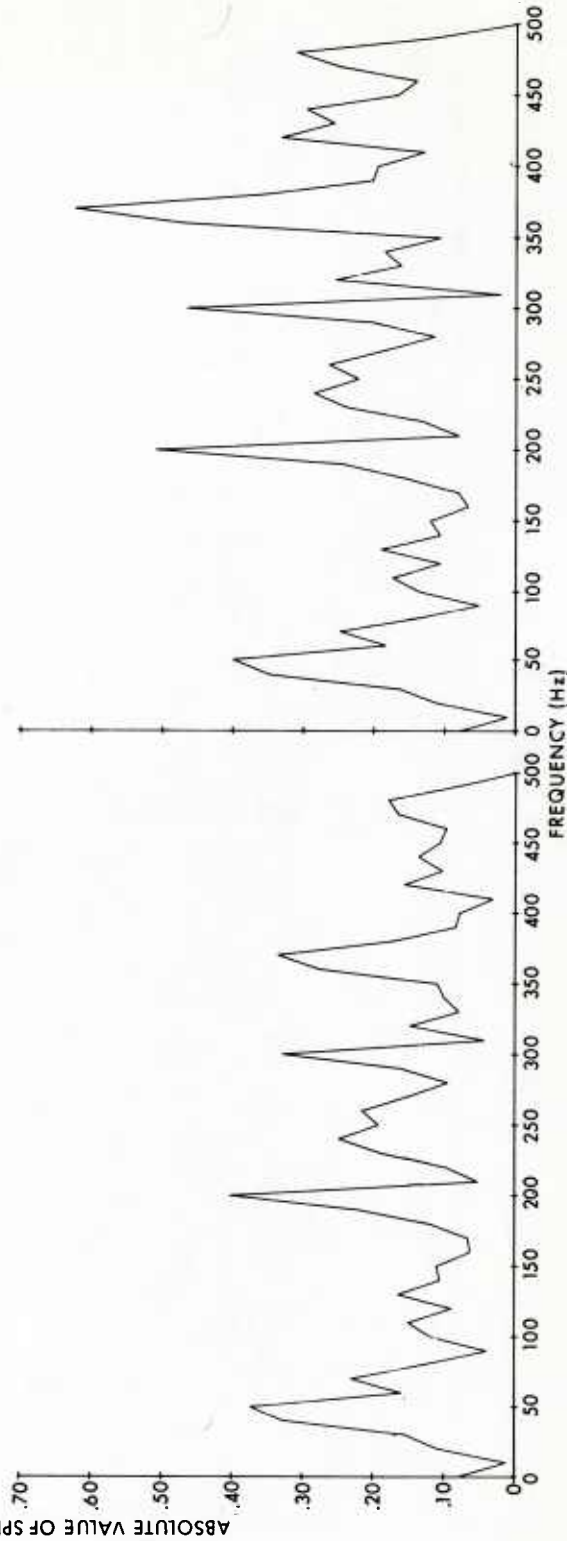


Fig. 11C. Contour 3

Fig. 11D. Contour 4

Fig. 11. Z-Transform Absolute Magnitude Response of Simulated Signals Evaluated on the Four Contours of Fig. 7 at $S/N = -0.6$ dB

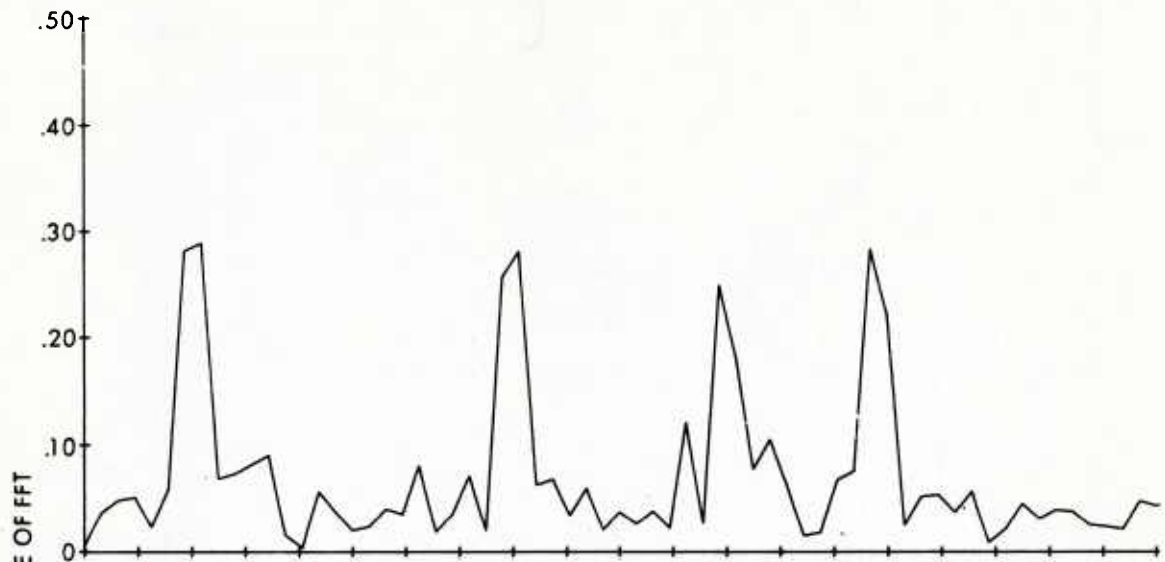


Fig. 12A. $S/N = 8.87$ dB

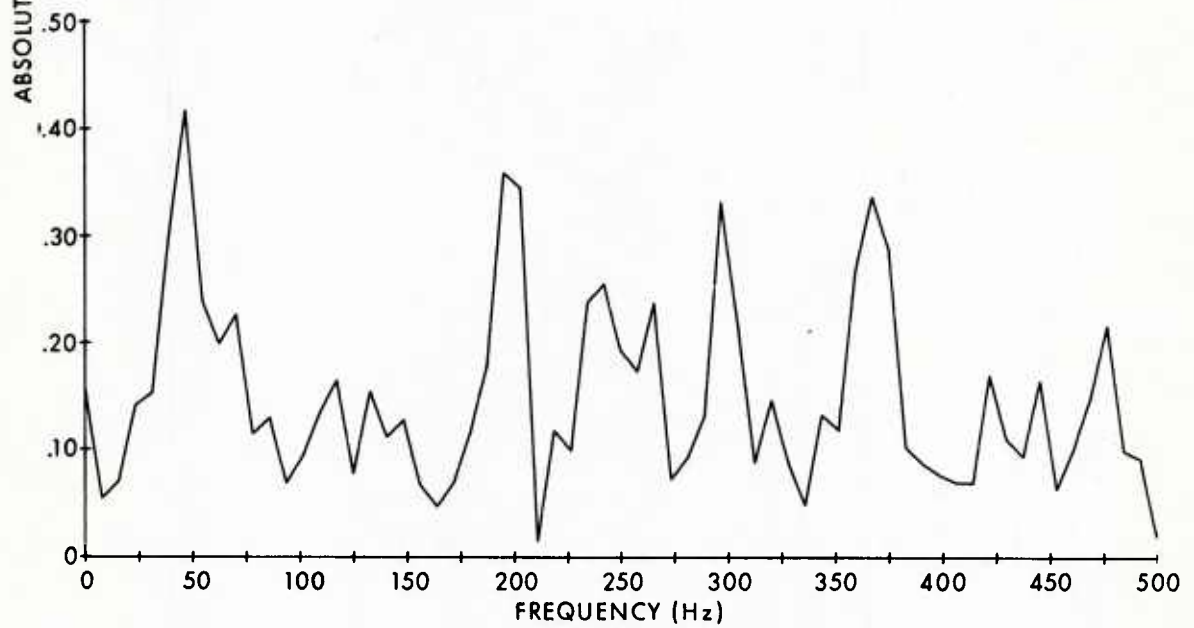


Fig. 12B. $S/N = -0.6$ dB

Fig. 12. FFT Absolute Magnitude of Simulated Signals at $S/N = 8.87$ dB and $S/N = -0.6$ dB,

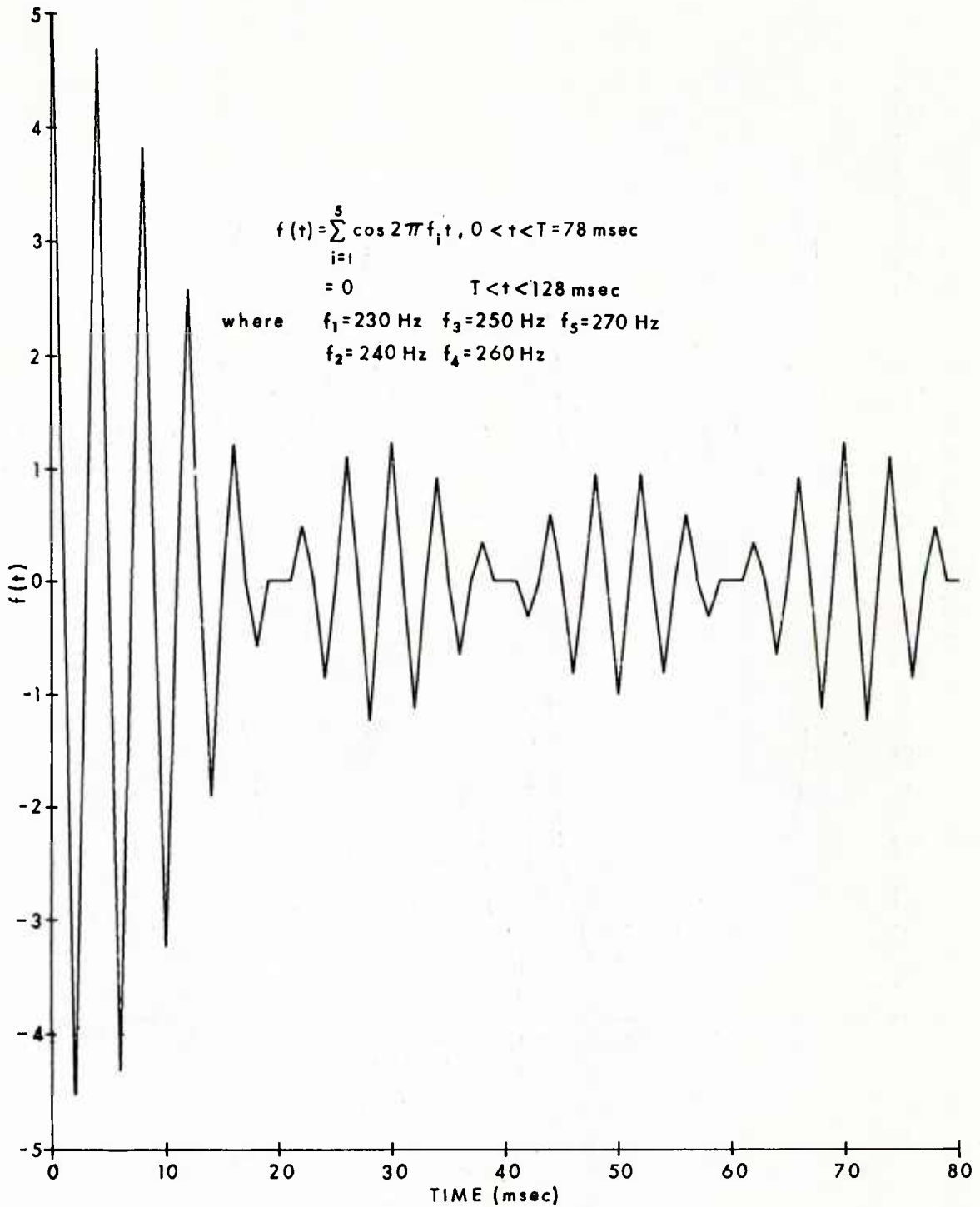


Fig. 13. Simulated Signals for Passband Analysis

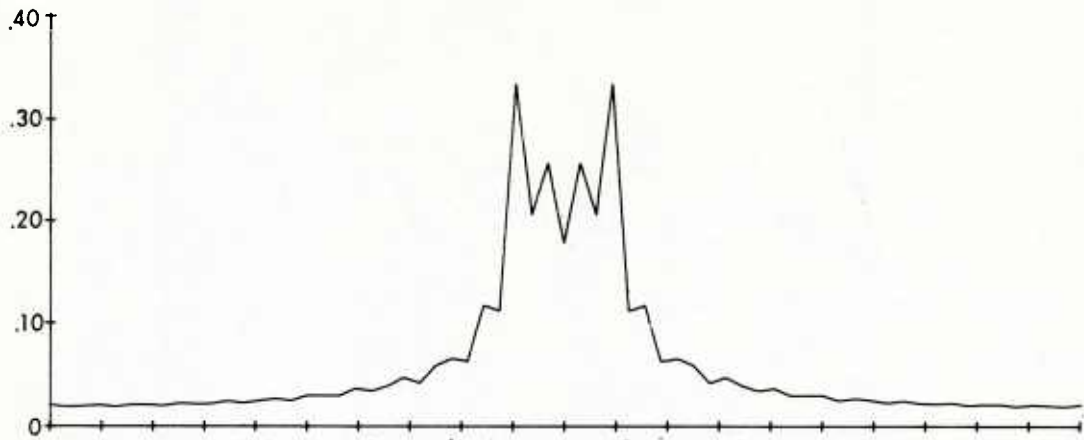


Fig. 14A. $S/N = \infty$ dB (Frequency Resolution $\Delta f = 7.8$ Hz)

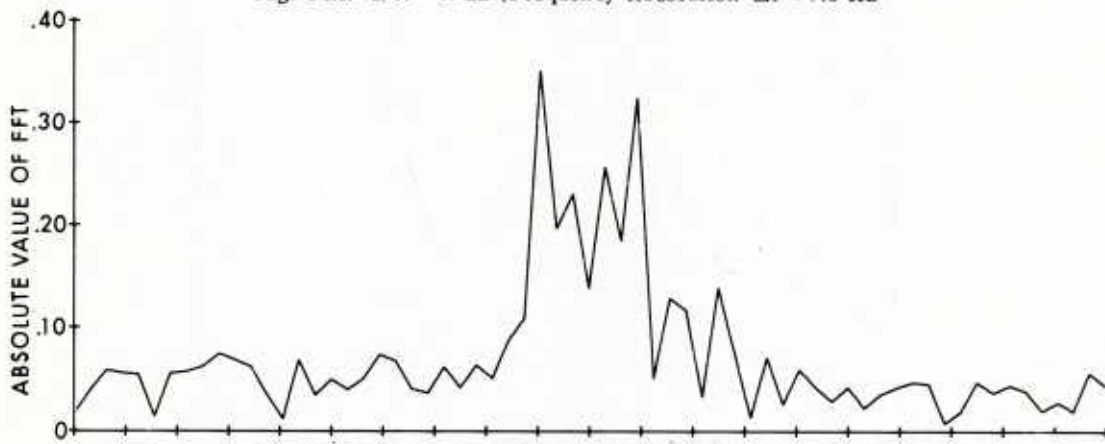


Fig. 14B. $S/N = 8.47$ dB (Frequency Resolution $\Delta f = 7.8$ Hz)

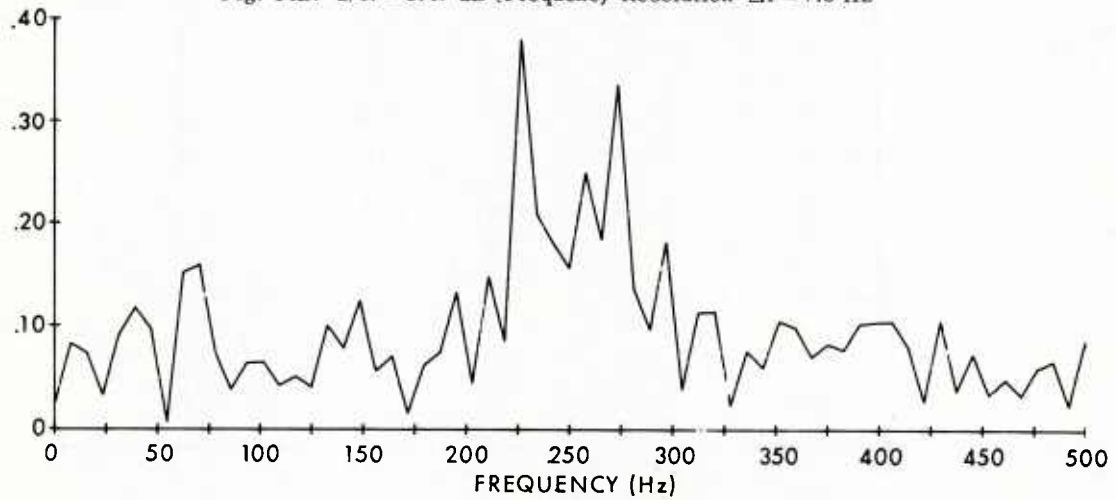


Fig. 14C. $S/N = 3.96$ dB (Frequency Resolution $\Delta f = 7.8$ Hz)

Fig. 14. FFT Absolute Magnitude of Signal (Fig. 13) of 128 Input Points at $S/N = \infty$ dB, $S/N = 8.47$ dB, and $S/N = 3.96$ dB (Frequency Resolution of $\Delta f = 7.8$ Hz)

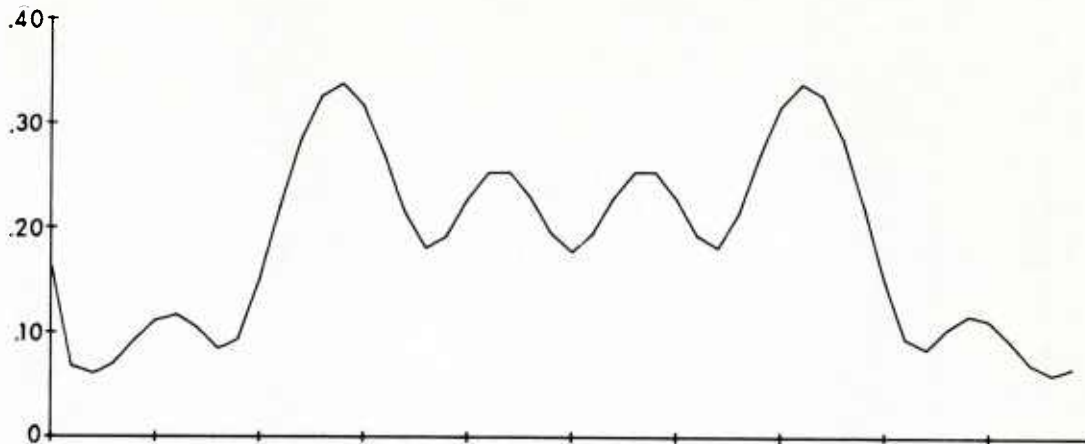


Fig. 15A. $S/N = \infty$ dB

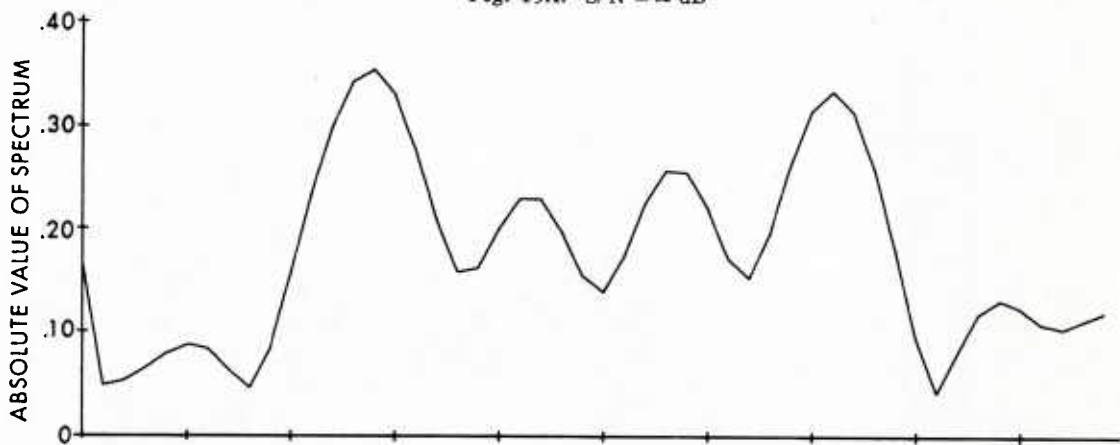


Fig. 15B. $S/N = 8.47$ dB

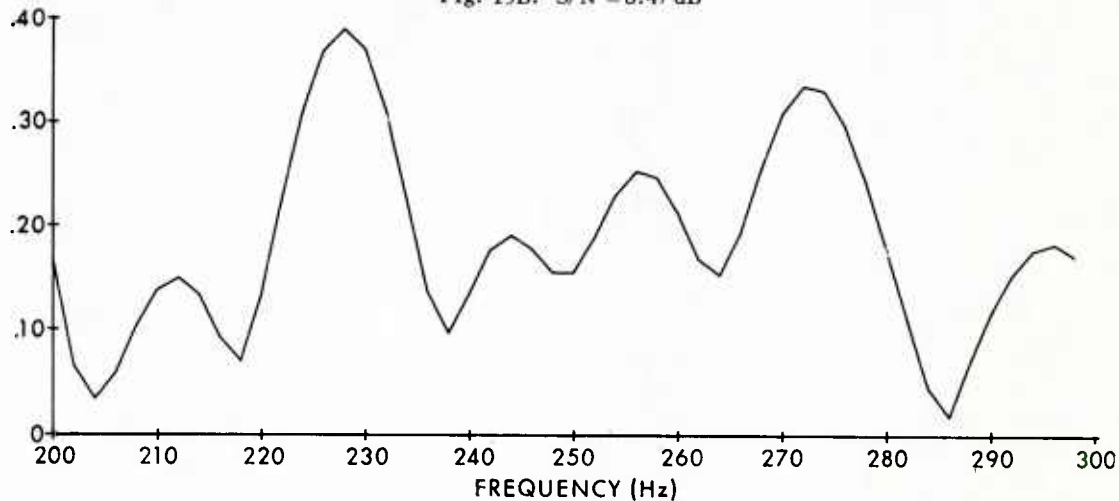


Fig. 15C. $S/N = 3.96$ dB

Fig. 15. Z-Transform Absolute Magnitude Response of 128 Input Point Simulated Signals (Fig. 13) Evaluated Along Contour 3 (Fig. 7) for 101 Output Points (Passband 200 through 300 Hz; Frequency Resolution $\Delta f = 2$ Hz; and $S/N = \infty$ dB, $S/N = 8.47$ dB, and $S/N = 3.96$ dB)

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- Analysis

- I. Quazi, A.H.
- II. Title
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