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NAVAL WARFARE ANALYSIS GROUP

STUDY 64

Study of Land/Air Trade-Offs (Short Title: SLAT)

Volume IX
Statistical Analysis of Korean War Data - Spring 1951

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NAVAL WARFARE ANALYSIS GROUP

OF

THE CENTER FOR NAVAL ANALYSES

STUDY NO. 64

STUDY OF LAND/AIR TRADE-OFFS

(SHORT TITLE: SLAT)

VOLUME IX

STATISTICAL ANALYSIS OF KOREAN WAR DATA - SPRING 1951

The work reported here was conducted under the direction of the Center for Naval Analyses and represents the opinion of the Center for Naval Analyses at the time of issue. It does not necessarily represent the opinion of the Department of the Navy except to the extent indicated by the comments of the Chief of Naval Operations.

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ABSTRACT

Battle data of divisions engaged in the Korean War during the spring of 1951 is analyzed by statistical methods for relations among strengths, firepower, and casualties.

Graphs are presented covering this and other data for U. N. divisions over time. Equations are derived for friendly and enemy casualties and for ground gained, in terms of strength and firepower. The equations are not symmetrical for opposing forces because of differences in firepower, and in policy covering ground gains and reinforcements.

Major factors are the opposing strengths and artillery, which proved important during enemy attacks. Techniques were developed for compiling air sorties from all services for the divisions supported and relating them to results: the effects are usually not very evident in the numbers of enemy troops killed, except when the U. N. force was attacking.

Though the work is indicative, it is limited by the poor quality of information about enemy strength. Perhaps for this reason, little difference is evident among five forms of the Lanchester equations tested.

Empirical equations among strengths, firepower, and casualties are derived for low- and high-intensity battles in which each side was attacking. The equations based on divisional level do not appear useful for planning.

This work is related to SLAT studies of firepower potential; in those, engagements were conducted on a smaller scale.

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SUMMARY

This volume, concerned with the Korean War, describes attempts to relate the strengths of the opposing U. N. North Korean/Chinese Communist forces, including fire support, with the ground gained and casualties on both sides.

The objective is to determine, by statistical techniques, relationships among various kinds of battle data recorded each day by divisions. These were intended to provide guidance in the TWSP simulation, which rests on the assumption that the Lanchester equations are correct.

The methods include plots, a review of the data for validity, formulation of proposed relationships, and testing by multiple regression.

The relationships between casualties and the opposing strengths and firepower are derived, both empirically and by Lanchester equations. The resulting equations are then examined for plausibility of the military inferences that may be drawn.

The data consists of daily friendly and enemy strengths and casualties in the IX and X Corps in central Korea, air and artillery support of U. S. and ROK divisions, and changes in territory. The enemy had no artillery or air support in this sector during this period. The data on small arms, machine guns, mortars, tanks, etc., though not available for either side in division records, is assumed to be proportional to the strengths and to such ordnance expended in World War II. Reserves were introduced continually by U. N. forces, intermittently in the form of new divisions by the Communists.

The data for May 1951 is used because it is readily available and includes various kinds of combat in a period when sectors were relatively quiet, the thrust of the Chinese Communist Army near the Soyang River and the counterattack of U. N. forces. In this period, there were no large break-throughs or amphibious landings.

Other variables recorded and included in the analysis were the weather, the effects of break-throughs on adjacent units and the commanders' major decisions (advance, withdraw, or hold). Terrain information was inadequate, and data about air interdiction was not tested.

Three approaches have been followed in the analysis of data:

- Display by graphs, plots, and histograms so that interrelationships can be observed by the casual reader and as a preliminary step for selecting appropriate statistical treatment.

- Statistical treatment for the best empirical explanations for enemy and friendly casualties and ground gained on the basis of strength, firepower, mission, weather, events occurring in adjacent units, and delayed effects of interdiction.

- Testing of Lanchester equations of various forms and comparison of the results with the empirical relationships.

DISPLAY OF THE DATA

Strengths, casualties, ground gained, and firepower for the center of the Korean front are displayed graphically. The battle was relatively quiet for the first half of the month; then the Chinese Communists launched an attack on the Republic of Korea (ROK) divisions and drove them back before a U.N. counterattack was mounted.

A program has been devised for relating the points of impact of air-delivered ordnance to the positions of the divisions, thus permitting a compilation of air ordnance in support of each division. The air support, extensive in the western and eastern sections, where other data was missing, serves as a measure of battle intensity there.

On the western flank, armed reconnaissance air sorties were apparently flown against enemy troops who were moving to reinforce their forces near Seoul. Weather and flight records show that in May there was a 7/8 reduction in the number of sorties about one-tenth of the time, and activity was inhibited about one-third of the time. With all-weather equipment at bases and near the battle area, weather is likely to have less effect in future wars, but it is still a factor to be weighed more heavily than has been customary in past simulations.

The battle was intense for as many as 6 divisions; often more than one thrust was underway at a time, with relative quiet for the divisions between. When a battle is intense, the strengths, casualties, and firepower increase drastically. In some instances, the study group has equalized the difference in the numbers of men per division by measuring the strength per mile of front and the numbers of miles advanced under various situations and with various amounts of fire delivered.

STATISTICAL TREATMENT OF THE DATA

Algebraic equations relating casualties with the strength and firepower of opposing forces and with ground gained were fitted by multiple regression. The residuals were examined for possible causes for variation; when they were large the new variable was inserted, leading to new equations. This process increased the amount of explained variation, measured by R^2. Still more variables that may be significant were recognized, but not all could be included because the sample size decreased as the number of variables increased. The collinearity between rounds of artillery shells and numbers of light and heavy bomber sorties was eliminated by conversion to the tonnages of ordnance delivered by all methods. Still another limitation of statistical analysis is inaccuracy in reported data. Subsequent investigation (reported in appendix B of volume VIII) has shown that some of the strengths reported by corps were delayed reports from heavily engaged divisions; this data has not been examined adequately.

TESTING OF LANCHESTER EQUATIONS

The linear, squared, and logarithmic forms were examined in differential forms and in special combinations, such as:

$$\text{Enemy casualties} = K_o + K_h \text{ (fire support)} + K_i \text{ (friendly strength)} + K_j \text{ (enemy strength)}$$

where K_o, K_h, K_i, and K_j are constants derived by least squares.

In these equations, there is almost no difference from one form of Lanchester law to another (table IX).

The empirically derived equations did not fit the usual Lanchester equations. The equations for casualties always contain the strengths of the forces; the exponents, however, are not 1.0, ranging instead from -2.5 to 2.8. Light artillery terms appear consistently. Only if the negative terms are considered secondary and negligible effects do the equations take the form of Lanchester's square law type; some linear and some logarithmic laws have been found. The variations appear to reflect the unequal firepower of the opponents.

Helmbold's "bitterness ratio" is tested, but does not prove interesting.

The equations are summarized in appendixes D and E.

FINDINGS

There are relationships between casualties, on the one hand, and opposing strengths and firepower on the other. However, the relationships depend on the intensity of battle, on whether the forces were advancing or retreating, and on the mathematical model used.

Any conclusions must be drawn cautiously because changes in the mathematical model or in the data base produce different significant terms.

The following findings appear consistently throughout the analysis.

- According to this analysis, simple Lanchester-type equations – linear, square, or logarithmic – do not appear to work well for units as large as divisions.

- Different tactical situations require different equations.

- The equations for casualties show, as expected, that strengths of forces are very important where troops are advancing, but unimportant during static periods.

- Firepower, expressed in both total tonnages of ordnance delivered and the types of weapons used, is important in the fitting of equations to historical reality.

- Approximately equal tonnages of ordnance were fired per mile of front in World War II, as exemplified by the battle of Guam, and the Korean War, as represented by the Soyang River battle.

- Lanchester-type equations are not symmetrically applicable to both sides in the Soyang River battle. The differences can probably be ascribed to differences in available fire support: The enemy had no air or artillery support, relying on manpower and small arms, instead; the U. N. forces, on the other hand, had both artillery and air support.

- Fire support provided the three forces considered in this volume – United States, ROK, and NKPA/ChiCom – had differing results, as measured in casualties inflicted and ground gained. Empirically derived equations of the Lanchester type, relating ground gained and casualties (suffered or inflicted) to the strengths of the opposing sides, differ according to the amounts of fire support received.

- The equations for enemy casualties, after classification according to ground gained and firepower, explained more than two-thirds of the variation; artillery and air ordnance and friendly strength were important when the U. N. was advancing, and artillery ordnance was significant when the enemy attacked.

• Enemy strength figures alone proved inadequate as a basis for predicting friendly casualties. The reasons may be: the poor data about strength, the lack of knowledge about the fraction of friendly forces engaged, and the U. N. command's general policy of inflicting maximum casualties on the enemy at minimum cost in friendly casualties.

• Strengths aside, the casualties suffered by enemy forces increased with increases in the amount of U. N. light artillery ordnance used during heavy enemy attacks. During patrols, however, their casualties increased with heavy artillery and light bomber sorties. The increases in friendly casualties were related to a deficiency of heavy bomber sorties, but there was no evidence of shortages in other ordnance.

• Ground gained correlated most closely with casualties, and fewer friendly losses occurred during advances than during retreats. Gains by friendly forces were associated with increases in air sorties during both patrols and heavy attacks. The poorer equations (low R^2) for friendly gains may reflect differences in enemy resistance either from dug-in positions or during rapid evacuations. Enemy gains occurred during a deficiency of heavy bomber sorties. They also occurred when our forces were massed and when our forces received the support of light bomber sorties - probably measures of our response to the anticipated attack. A possible program is presented for extending this work to examination of data from smaller units than divisions and to examination of other battles.

ANALYSIS OF KOREAN WAR DATA

INTRODUCTION

This analysis - using statistical techniques - is an attempt to bridge the gaps between descriptive accounts of battles, the recorded data, and theoretical relationships between strengths and losses of opposing forces as originally proposed by Lanchester.

The analysis of Korean War data assesses the effectiveness of various mixtures of forces and fire support, both air and artillery, in different types of ground combat situations. Equations are derived to relate battle outcomes to force levels, based on war records.

There are major problems·

1) Though some items are very precise, some data is inexact. The poorest data is concerned with enemy strength, enemy casualties, and attrition inflicted by deep support.

2) It is difficult to decide which data has military significance and how to incorporate data that is related because of preplanning.

3) Some division records are useless for analysis, or even totally missing; mutual support, therefore, cannot be estimated to the extent desired.

This analysis parallels the simulations of the main text of the study; it has not been used directly.

An account of the war in winter and spring of 1951 gives command policies, character of the opposing forces, the nature of the battles, and the battlefield conditions, because all of these influence the mathematical formulation of the relationships. The command policies and political objectives merit special consideration because they establish the tempo of conflict and troop exposure to fire.

BATTLE SITUATION

The statistical analysis was confined to the spring of 1951; the month of May was subjected to detailed analysis. A major change occurred in the 8th Army command on 12 April, when General MacArthur was replaced by General Ridgway, who was, in turn, replaced by General Van Fleet. To place the data in perspective, we paraphrase the account of General Ridgway (reference (a)), who then commanded the 8th Army.

The strength of the Chinese forces at the start of their offensive in late November 1951 was estimated at 300,000. After the massive withdrawal of U.N. forces from near the Manchurian border in December 1950 to the narrow waist of Korea, the line was set about 50 miles below Seoul. The strength of the enemy in front was unknown, even after extensive reconnaissance.

On January 25, our forces launched a probing offensive, which attained positions above the 38th parallel by 22 April 1951, after a series of Chinese counterattacks.

The enemy thrust south on 22 April and again on 17 May. Usually the primary targets were ROK units that sometimes gave way under massed night attacks. Multi-pronged attacks were also made. These attacks were conducted during bad weather.

The U.N. objective during this period, according to General Ridgway, was to improve positions markedly, inflict maximum damage on the enemy with minimum loss to ourselves, maintain all major units intact, and carefully avoid being sucked into an enemy trap that could destroy our forces piecemeal.

The U.N. forces limited their pursuit to points where powerful support could be provided or a timely disengagement and local withdrawal could be effected. The maximum movement was about 75 miles on the central and eastern sectors. In the west, after the recapture of Seoul, the decision was made to hold rather than take territory because the battle line would have been extended unduly: "Acquisition of terrain in itself is of little or no value."

In the period covered in the analysis, a thrust of 6000 Chinese across the Han River to the peninsula near Seoul was launched on 29 April. Our aircraft inflicted heavy casualties, and defending ground forces prevented the survivors from crossing. This action extended into May.

The 8th Army returned to the offensive with the intention of pushing back to the Kansas Line, menacing the Iron Triangle (Pyongyang-Chorwon-Kumhwa) and harassing the retreating Chinese. By the second week in May, enemy resistance stiffened, and Chinese logistic and troop movement southward were noted despite air attacks. U.N. defenses were strengthened.

The attack started on 15 May, with 21 Chinese divisions and 9 North Korean divisions against the U.S. X Corps and ROK III Corps. The weather was bad; few sorties were flown. Even with air support on the 17th, the ROK 5th and 7th Divisions crumpled; the U.S. 2nd Division and 1st Marine Division were moved to the east, and for a time supply lines were severed. On the 18th, a planned withdrawal and redeployment of units was executed. Massive fire support was provided.

The Chinese conducted two additional attacks – one in the east, another in the west. The former gained ground and captured the equipment of several divisions. The ROK III Corps became ineffective and was deactivated, after which its units were divided between the U. S. X Corps and ROK I Corps.

The U. N. force counterattacked on 20 May against enemy resistance that was especially heavy where terrain was favorable. Bad weather slowed tanks and grounded many aircraft in the last week, enabling the enemy to retreat with much of his force and supplies intact. While on the offensive, enemy forces suffered approximately 25,000 casualties; later, in the U. N. counterattack, they lost an estimated 17,000 dead plus 17,000 prisoners, a total of 59,000. South Korean losses amounted to 11,000 killed, wounded, missing, or sick.

The data used in this study (tables III and IV, pages 20 and 22) reports Chinese offensive losses of 84,557 for 16-23 May, and an additional 70,750 to the end of the month – a total of 155,307. In the same period, the latter half of May, U. N. losses amounted to 16,470.

METHODS OF ANALYSIS

Examination of the Data

The data has been collected on infantry division-sized units on a daily basis because:

- Close air support was allocated on a division basis during most of the war.
- Available resources and time permitted the extraction of detailed data on a division level and daily basis only; several man-years would have been required for study of battalion or regimental records or consideration of shorter periods than one day. The poor quality of enemy strength data limits the conclusions, and further study is unlikely to improve it.
- Units larger than divisions cannot be regarded as tactical entities.

The continuing infantry battle is described by plots of each type of data. All types of data considered are discussed briefly. Most of the data is taken from annex A-2 of volume VII.

New air sortie data was compiled and, with other information, was plotted by computer, with the programs described in reference (c). The analytical techniques developed for this portion of the study may prove useful in future, similar studies.

Measures of Effectiveness. The outcome of a tactical situation is described in terms of the dependent variables:

- Ground gained: This is the amount of area (square kilometers) acquired by a friendly unit. Enemy gains are shown as a negative value–a loss of the friendly unit. In some cases, the division advance has been obtained by division of area by front length.
- Friendly battle casualties: Non-battle casualties are excluded.
- Enemy casualties: Some records distinguish between casualties caused by air action and those caused by ground action; regression analyses, however, have dealt with total casualty figures.

Preliminary Examination

A sample situation map of the Eighth Army (for 19 May 1951) is presented in figure 1. This period marked the peak of the Soyang River spring offensive of the Chinese Communists. The map displays division front lines, division and corps boundaries, enemy positions, and enemy unit identifications.

The changes in strengths, fire support casualties, and ground gained and lost by the U. S. Army's 2nd Division were plotted daily for February through May 1951 in figures A-1 through A-7 of appendix A. There were two periods of hard-fought X Corps battles: mid-February and the latter half of May. The buildup in strength of forces, fire support, casualties, and movement during these periods was pronounced.

Most of the analysis centered on the data for May 1951, which covers the Soyang River battle, because the data for air sorties by all services was readily obtainable for this period only. Later battles could be examined after transcription of Air Force data for computer compilation. Study of earlier battles would require transcription of original Navy and Marine Corps records, as well.

Position of Divisions and Front Length. The front length and positions shown in table I, were measured from the Army daily situation maps (see figure 1). The difference in lengths of front held varied from 3 to 17.5 miles, the smaller distance being held by smaller forces. Fronts were longer when large divisions were on the line and when the enemy broke through. The redeployment of divisions along the battle line, too, can be established from the table. The relative positions were obtained by serial numbering of the divisions according to their location on the battle line, starting from the western end.

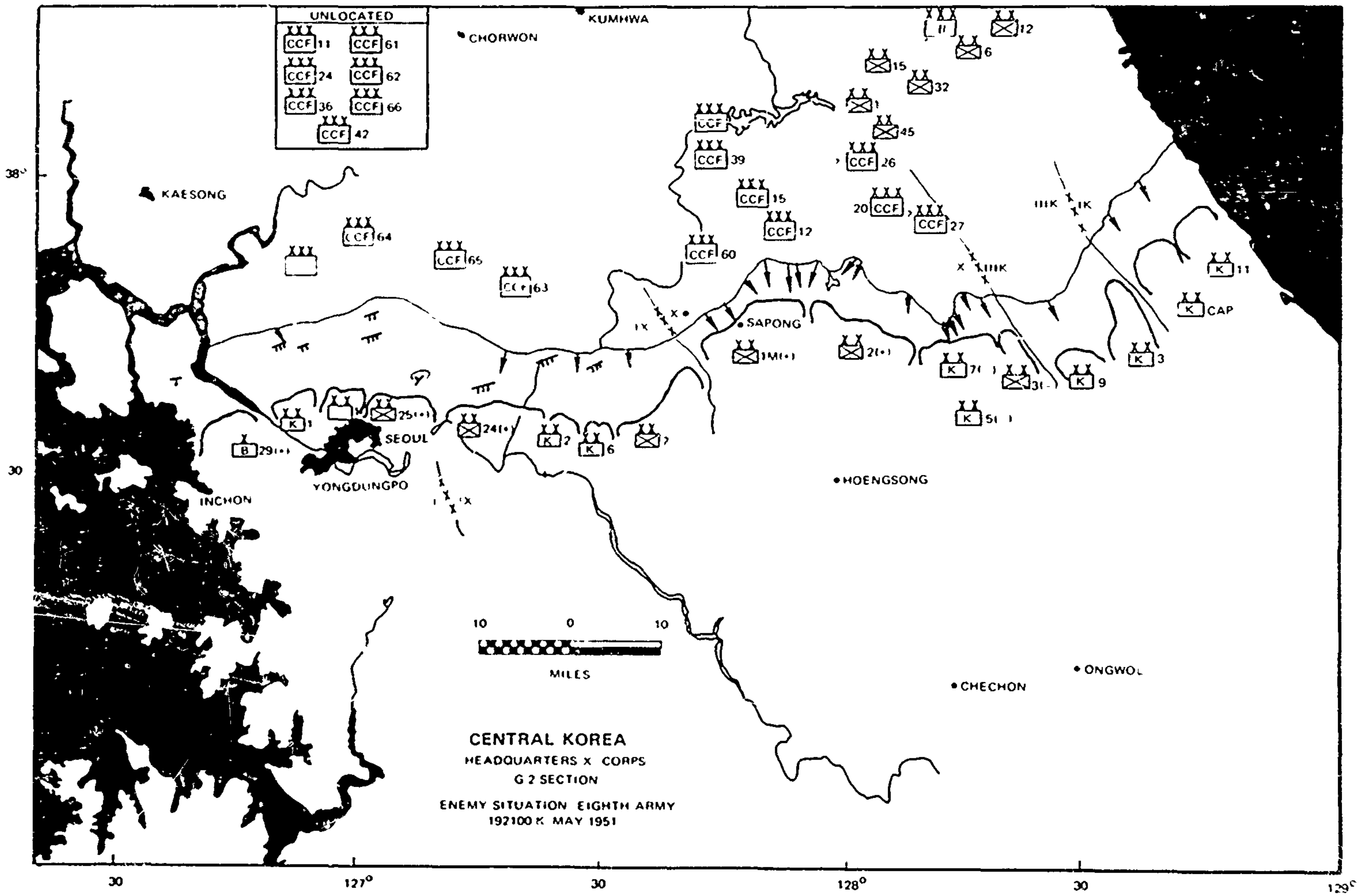


FIG. 1: SAMPLE DAILY SITUATION MAP

TABLE I

POSITIONS OF CORPS AND DIVISIONS AND LENGTHS OF FRONT HELD (STATUTE MILE) (1–16 MAY 1951)

Corps		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
I	R20	5	6	10	10	5	6 2	5	6	5	5	4 5	4 5	4 5	4 5	4 5	4 5
						B29 5	4 1	5			8 9	8	9 5	7 8	7	7	6 5
	R01	5	5	5	5	4 5			4	8 5	4 1	6	6 2	7	6	6 1	6
							U03 4 5	4	4 6	4 6						--	--
	C01	4	4	4	7	4	4	8	4 5	4 5	4 3	4 5	4 2	5 5	4 5	4 5	4 5
	U25	8	8	7 4	5	8	75		8	8	7 5	7 5	7 5	8	8	7 6	8
IX	U24	16 5	13 2	13 2	10	12 5	12	12 5	2 6	12 8	12 9	12 5	12 3	12 5	12	11 9	12 5
	R06	9 9	R02 3 6	4	5 1	3	3 6	3 3	3 5	3 8	3 9	4 8	3 5	4	3 5	3 5	4
	R02	4	R06 3 9	4	4 1	3 6	3 9	3 5	3 6	4	3 9	4 8	[illegible] 5	3 5	3	3	3 9
			U07 8	8	8 5	7	6 7	7 1	7 5	[illegible] 2	6 8	6 9	7 1	7 5	7	7	7
X	M01	9	9	9 6	8	8 5	8 2	8 5	8 5	8 5	8	7 5	8	8	7	7 4	7 5
	U02	10 5	10 9	10 9	10 1	11	10 8	11	1	8 9	8 5	8 3	8	8 9	8	9	8 6
	R05	6	6	5 8	5 2	5	5	5	4	4	3 5	3 4	3 3	3	3	3	3
	R07	4 9	4 7	5	4	4 4	4	4 5	3 9	5 2	5 5	5 2	5 3	5	5	5 5	5 5
ROK III	R09	4 5	4	4	4	4	4 6	5	4 5	5 3	5	4 6	4 5	4 3	4	4	4
	R03	5	5	6	5 5	5 5	5	5	6 5	5	5 1	5	5	4 5	4 5	7	7
ROK I	R04	7	7	6 8	6	6 3	6 1	6 9	5 5	6	4 5	4 5	5 5	5 6	5	5	6 5
	R11	5 9	5 6	4 2	4 8	4 9	4 8	4	5 8	5 2	6 5	6 5	8	7 3	7	5 8	6
		17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
I	R20	4 5	4 5	4 5	4 5	4 5	4 5	6	6 5	7	5	5	5	6	6 7	5 5	
	B29	7	7	6 6	5 6	4	5 5				--	-	--	-	--	--	
	R01	6	5	5 1	5	11	9	11	10	10	9	10	12	12	11 5	13	
	C01	6	4 5	4 5	4 8	7	6 5	6 5	5	8 5	7	7 5	9	12	11	11	
	-												-	- -	--	U03 7 5	
	U25	8 5	7 8	8	7 5	6 5	7	10	9 5	7 5	11 5	13 3	14	15	16	4	
IX					-				R02 7 5	7	7 5	6	6	6	5	4 5	
	U24	11 8	11 5	11 5	12 2	11	11	11	6	9	6 5	U07 5	U24 6 3	5	6	6 5	
	R02	4	4	4	3 8	4	5	5	-	-	-	--	U07 7	6	4	5	
	R06	3 2	3 5 •	3	3 8	4 8	5	7 5	7 2	5 5	5	5	6	7	7	6 3	
	U07	7 5	9	10	8 3	7 5	6 5	7 5	7	6	5	--	--	-	--	--	
X	M01	7 5	8	11	12	11	11 2	8	9	10 5	13 6	13 5	14	9 5	13	13 5	
	U02	7 5	10	12	14 2	5	U18 3	11 5	1 5	12 5	15	13	13 5	13 5	13	11	
	R05	10 1	2 5	8 5	R07 7 5	6	U02 12	TFA 4 5	5 5		-	-	--	--	--	--	
			U03 8	4 5	6 8	8	10 8	R05 4				--	--	--	R08 9	9	
	R07	6 8	4				--	U03 11 8	1 2	17 5	13 6	16	13	11	--	--	
ROK III	R09	6	5	5 5	5	4 8	--	--		- -	-	-	--	--	--	--	
	R03	6	5	5 5	4	2	-	- -	-		-	- -	-	--	--	--	
ROK I	--						R03 7	7	-	-			--	--	--	R03 8	
	R04	5 0	5 5	4	4	4 5	5	5	8	7	4	6	6	R11 7 5	9	4 5	
	R11	5 5	5	4	6	5 2	6	11	9	9	7 5	6 3	5	R04 4	5	4 5	

Ground Gained

The ground gained (in area) was measured by planimeter from the change that occurred from one day to the next for each division. We obtained the average gain or loss by dividing the area by the front length.

A summary of ground gained and lost by the various divisions is shown in figure 2. The major events were the Chinese attack on 15 May and the U. N. counterattack on 20 May. The U. S. 2nd Division fought beside the ROK divisions. The hard Chinese thrust against the ROK divisions forced them back; the units next to them also fell back, maintaining a continuity of line. When the U. N. forces regained the offensive on 20 May, all forces moved forward at about the same rate, except for the U. S. 7th Division, which advanced much faster.

Independent Variables

Friendly strength is the net number of troops in and attached to a U. N. division, including the changes resulting from non-combat casualties, replacements, and returned-to-duty categories. As shown in figure 3, divisions were not of the same size; there were changes in strength during battles, though the changes were not large.

During the Chinese offensive, the strength of the ROK 5th and 6th Divisions decreased and the enemy strength opposing them increased (figure 4).

Because the strengths in figures 3 and 4 are not expressed in the same units, a comparison has been made (employing table I) showing that U. S. forces had more troops deployed than the enemy per mile of front - generally by a small margin, but occasionally by a margin of 3 or 4 to one. These opportunities were not always exploited. The 1st MarDiv deployed 2000 to 4000 men per mile, the 24th U. S. Army Division — 1300 to 2800; the ROK divisions had 900 to 3200 men per mile.

Enemy Strength. Enemy troop density remained about 1000 to 2000 men per mile, except before the initial assault, when it increased to as much as 5500 (figure 3).

Enemy strength was measured from the enemy order of battle, as derived by intelligence sources. Only the elements of the Communist division opposing each friendly division are available. Unfortunately, information about the strength of the enemy unit opposing the ROK 6th Division is either fragmentary or missing entirely for the period of greatest interest. In contrast to the U. N. practice of daily replacement of casualties, the North Koreans replaced a division at a time. Thus, actual strength increased suddenly, diminished as casualties were suffered, and then suddenly jumped again.

In our calculations the elements of divisions were considered full divisions; each was multiplied by 8000 to give the number of enemy troops.

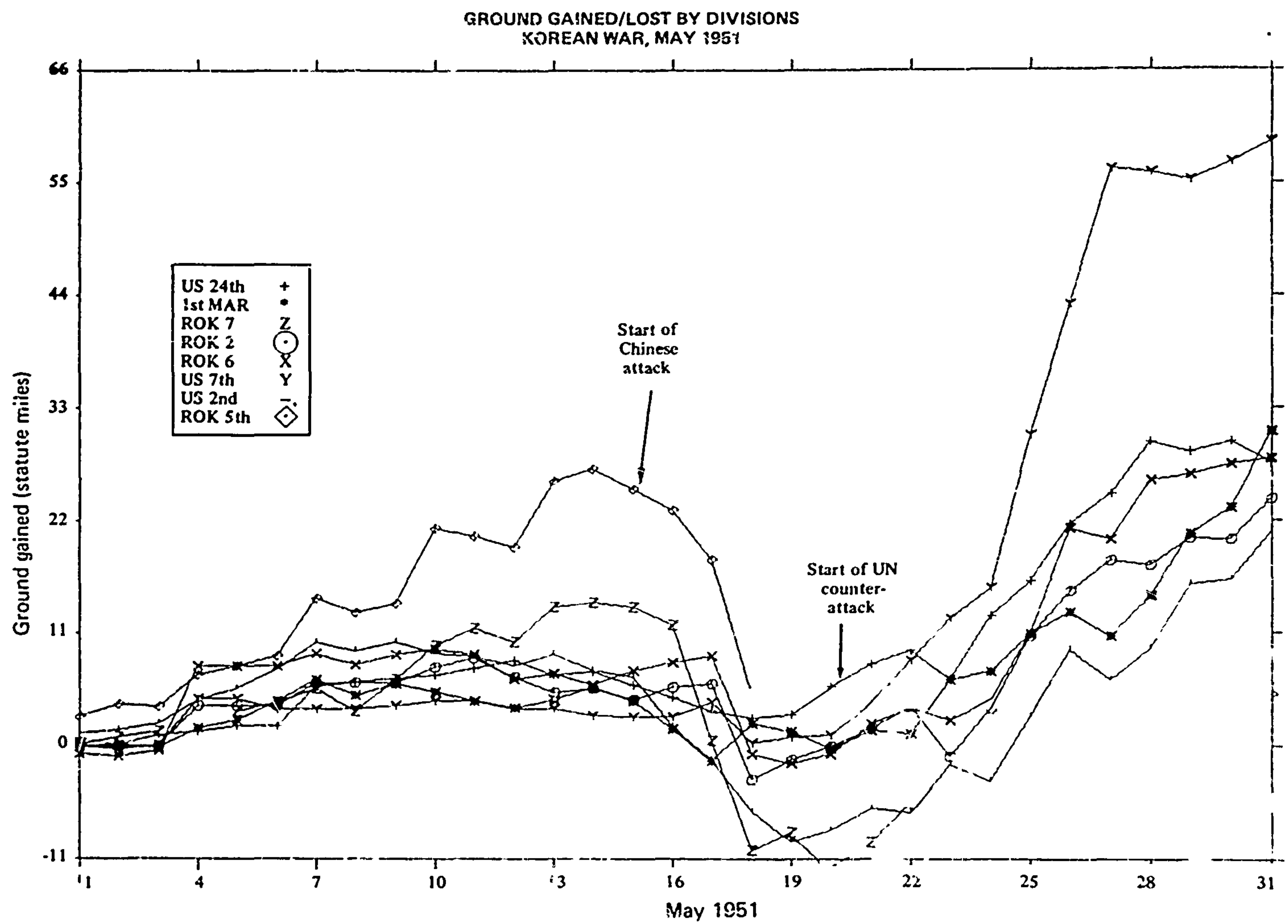


FIG. 2: GROUND GAINED/LOST BY DIVISIONS, MAY 1951

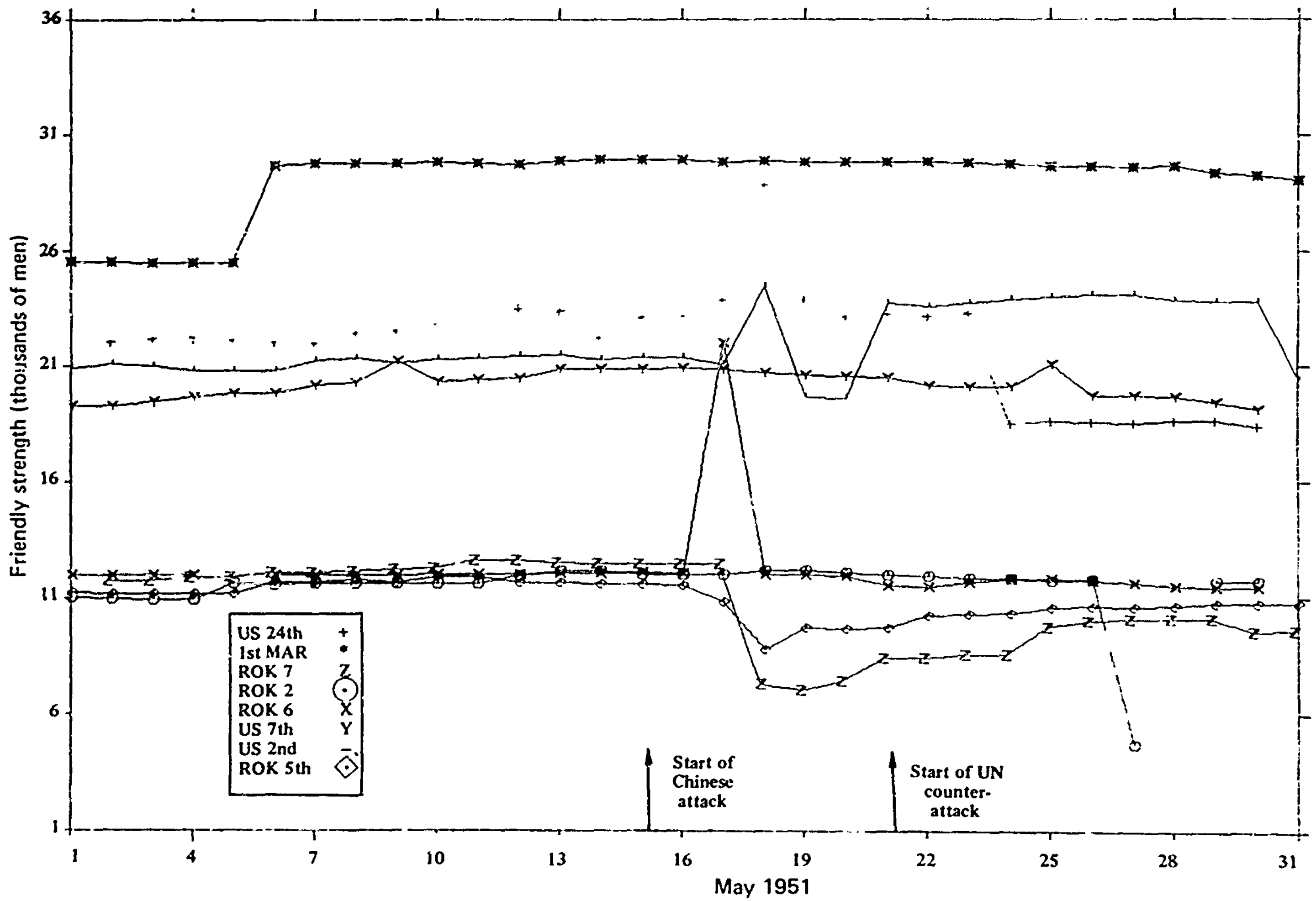


FIG. 3: U. N. DIVISION STRENGTHS, MAY 1951

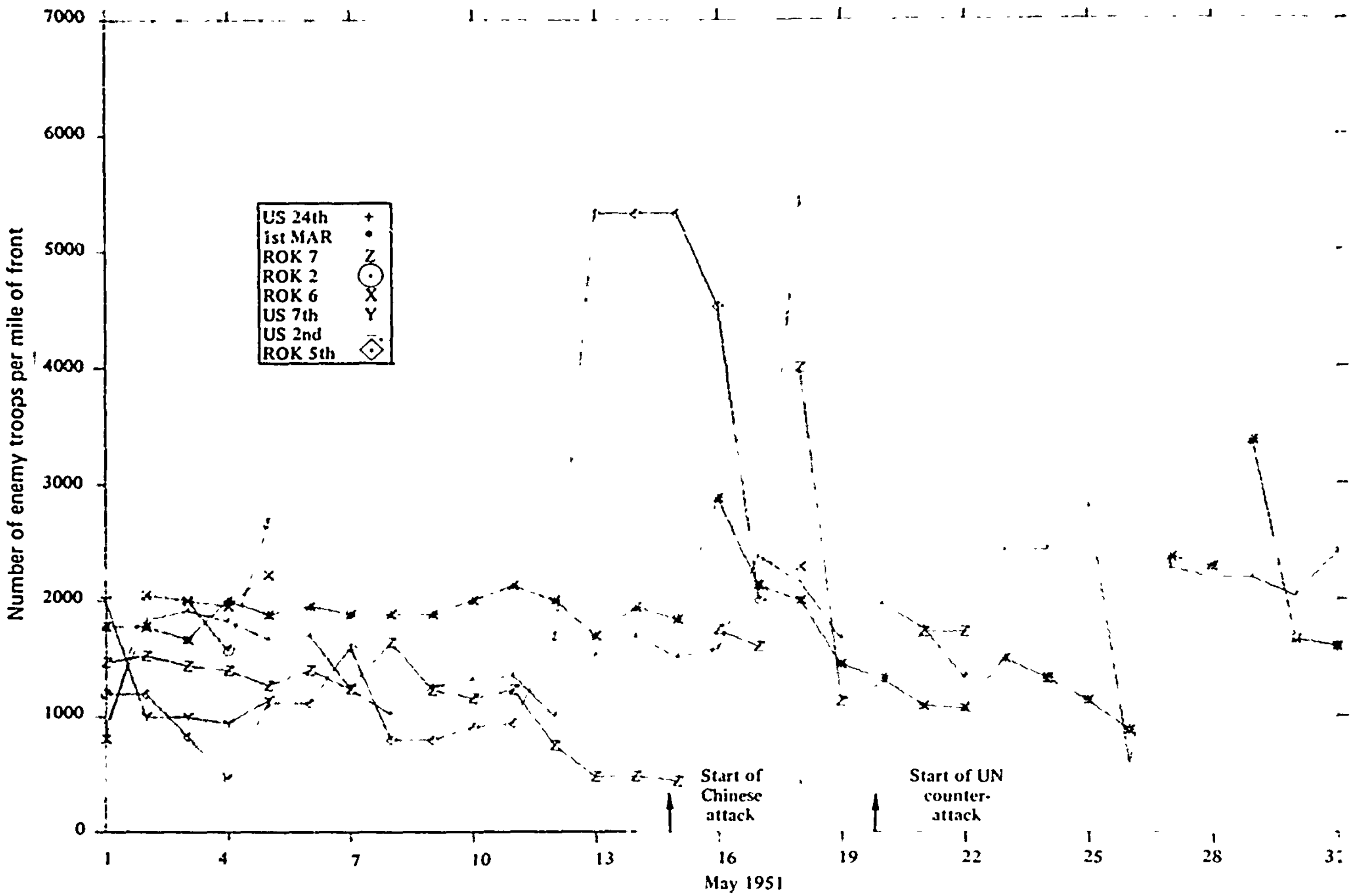


FIG. 4: ENEMY STRENGTH PER MILE OF FRONT OPPOSING U. N. DIVISIONS, MAY 1951

This enemy strength data was poor. The order of battle was derived from captured prisoners; when the enemy was attacking, little or no data was available because no prisoners were taken. Even where such intelligence was available, it was often inaccurate because the enemy troops were sometimes transferred and did not know the designations of their new units when interrogated. Analysis after the war disclosed that the number of enemy troops was frequently underestimated because the Chinese Communists were trained for long night marches, averaging 15 to 20 miles a night for two or three weeks, and were taught to stand stock-still when aircraft were heard or sighted. U. N. forces discounted sightings by South Korean civilians (reference (n)). Finally, known enemy casualties were not subtracted from the order of battle.

Three approaches were made toward improving the quality of information about enemy strength, but none was successful:

- Simple subtraction of casualties from strength figures led to inconsistencies.

- Improved enemy strength data was generated from battalion and company records in the hand-played reconstruction of the Soyang River battle.

- Multiple regression analyses were made to predict enemy strength from other battlefield data (R^2 = .38 to .45); this procedure was not used in further analyses. The equations are given in table E-IV.

Fire Support. "Fire support" here refers to the artillery and air ordnance delivered in support of each division. The positions of adjacent units are pertinent because the heavy artillery assigned to corps could sometimes assist adjacent units. Air support for corps was taken to be the ordnance dropped between 15 and 30 miles from the battle line. Interdiction/strategic bombing was measured in ordnance dropped beyond 30 miles, in support of the entire front.

Artillery Support. Light artillery is expressed in numbers of rounds of ammunition 105mm and larger. Artillery support of the divisions, in tons per mile of front, is displayed in figure 5. Support to a maximum of 50 to 60 tons per mile was received by the 1st Marine Division and 2nd U. S. Army Division; the ROK 5th and 6th Divisions received a maximum of 25 tons per mile during the intense phase of the battle. During patrol periods, the artillery ordnance amounted to less than 10 tons per mile of front. The ROK units that collapsed received less fire support and faced a far higher ratio of enemy-to-friendly forces than the units that held; their morale and training may also have contributed.

Air Support. Three types of data were collected: "close air support," "sorties by light bombers," and "sorties by heavy bombers."

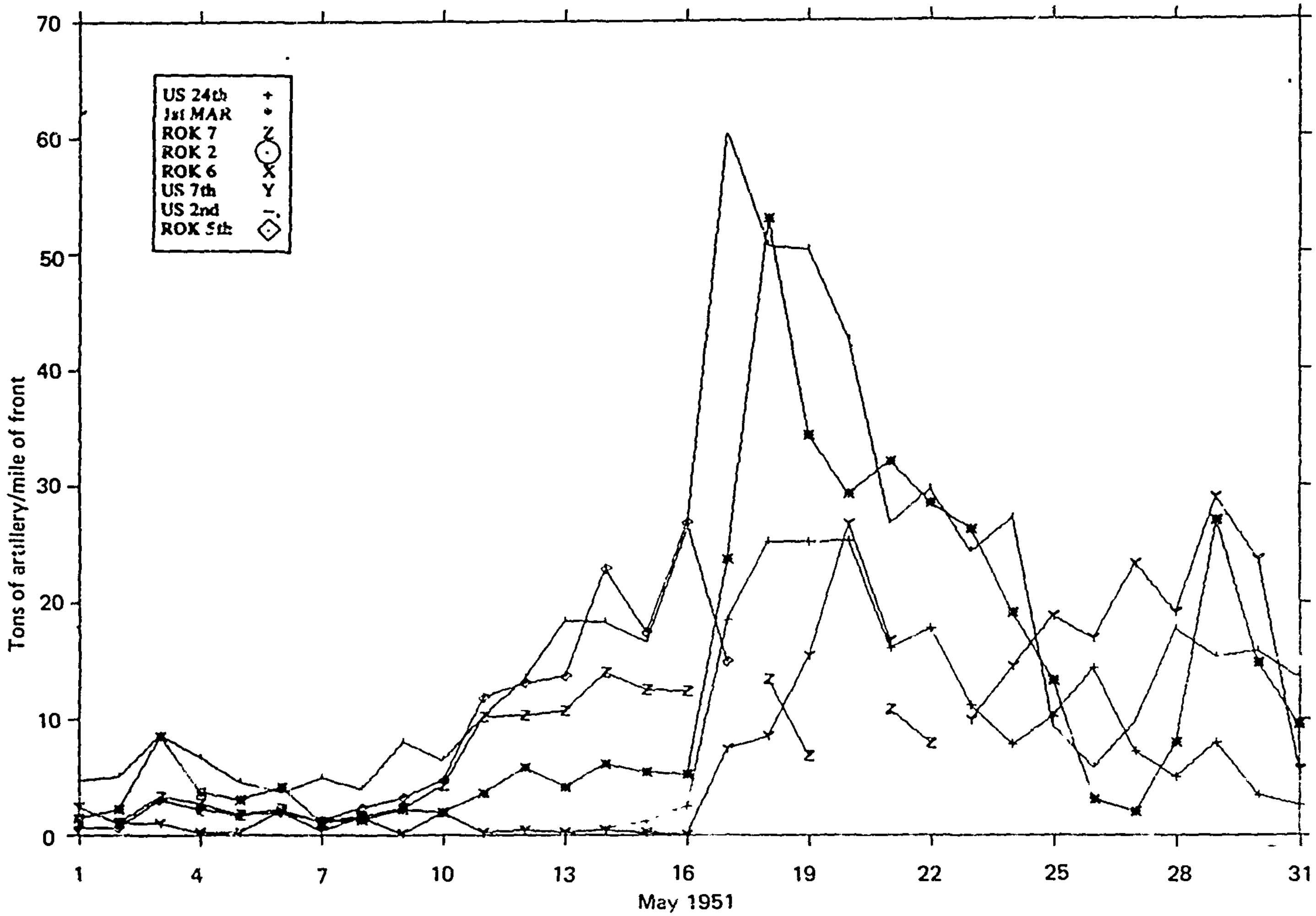


FIG. 5: ARTILLERY ORDNANCE FIRED (TONS) IN SUPPORT OF U. N. DIVISIONS PER MILE OF FRONT

"Close air support" consists of ordnance sorties delivered, under ground or air control, to targets beyond the forward bomb line. The numbers of air sorties recorded in the Army corps records were used in correlations in appendix D. However, the close air support records from various sources were not in agreement - and fragmentary, in any event.

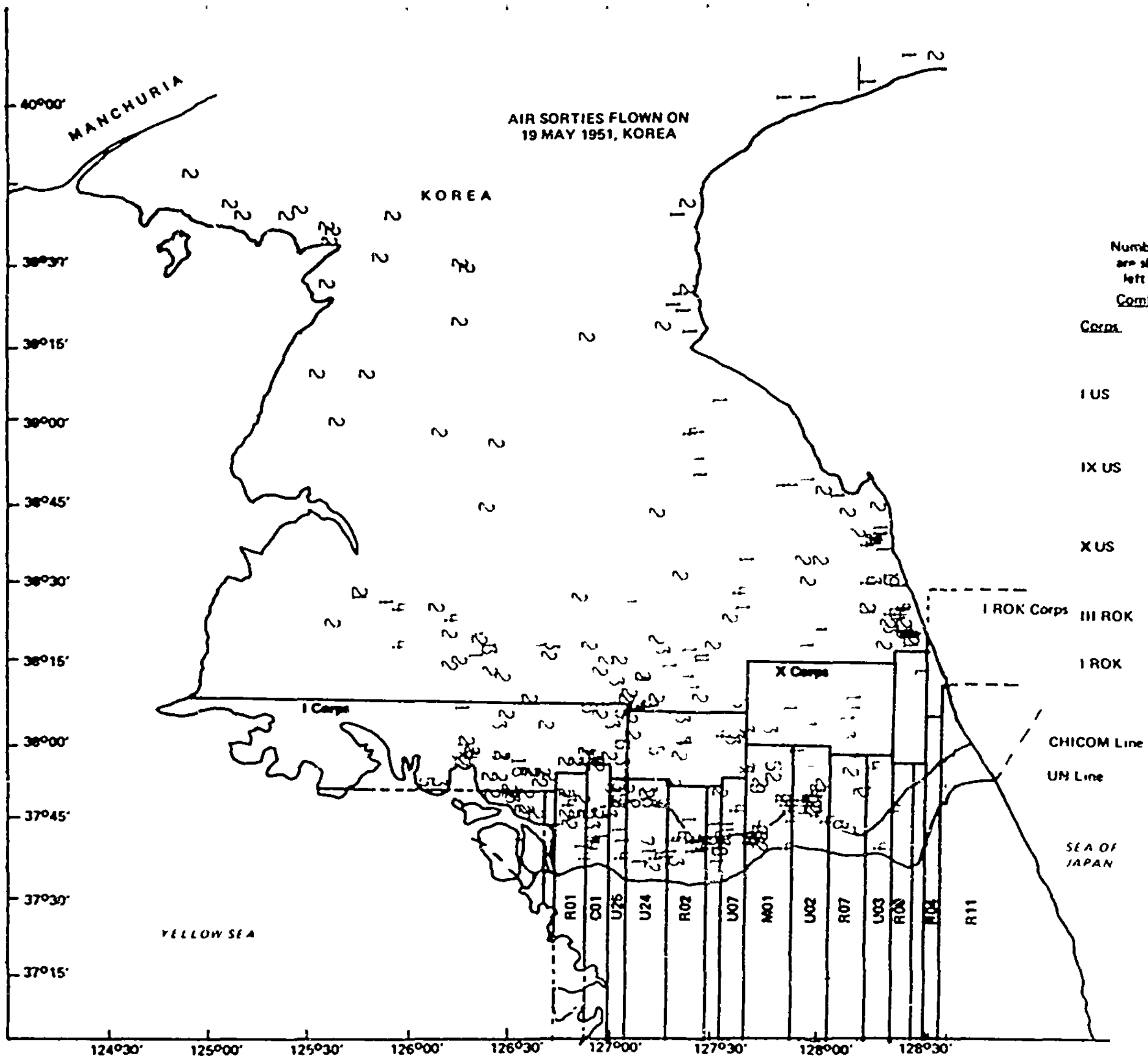
The reasons probably stem from the fact that the Army recorded the controlled sorties. Considerable battlefield support was not recorded, especially along the logistical supply routes close to the battle lines, during intense battles, when more sorties were available than could be controlled.

The summaries prepared by the Air Force, Navy, and Marine Corps do not relate the sorties to the ground forces being supported, but the impact points of the ordnance were obtained from pilot debrief forms and Air Force records (reference (w)). The computer programs for compiling this data are, as noted, described in reference (c).

Air Support by Light Bombers (AS_0). The air ordnance delivered within 15 miles of the battle line in support of each division was taken as fire in support of ground troops. Data was also obtained concerning ordnance delivered in support of corps and army. For example, air support provided to the ground troops in the May 19, 1951 battle is plotted in figure 6. The numbers of sorties are printed near the points where the ordnance was dropped. When aircraft attacked several positions, the sorties were listed as fractional and accumulated for each target. In this report the fractions are rounded to whole sorties.

This figure illustrates some of the problems in correct assessment of the air support. The ranges of 0-15 miles, 15-30 miles, and more than 30 miles north from the U. N. main lines for division, corps, and army support were chosen after examination of a preliminary plot. Fifteen miles is a greater distance than is usually considered for close air support, but the presence of large forces in outpost positions (in the western sector) required air support both in front and in back of the outpost positions.

There is no clear distinction between the effects of air support with various distances from the lines. In some cases the air attacks on enemy units across rivers were credited to support of the nearest friendly unit on the same side of the river. In other cases, as in the enemy's attempt to cross the Han, the assistance was given to the defending forces across the river. Some of the sorties in the "more than 30 miles" sector were flown along supply routes between mountains. These routes are indicated by the continuity of sorties. Drawing the boundary lines between units northward neglected such terrain effects, but the loss does not appear serious. The correlation analysis reported later in this volume used only the 0-15-mile data; no attempt was made to determine the delayed effects of deeper strikes on the battle.



Numbers of sorties on each target are shown on the point at lower left of the sortie figure (i.e. 2)

Combat units from West to East

Corps	Divisions	Abbreviations
I US	1st ROK	R01
	1st US Cavalry	C02
	25th US Infantry	U25
IX US	24th US Infantry	U24
	2nd ROK	R02
	5th ROK	R06
	7th US Infantry	U07
X US	1st US Marines	MC1
	2nd US Infantry	U02
	7th ROK	R07
	3rd US Infantry	U03
III ROK	9th ROK	R09
	3rd ROK	R03
I ROK	4th ROK	x04
	11th ROK	R11

FIG. 6: SAMPLE OF AIR SORTIE DATA EXTRACTED FROM ORDNANCE IMPACT POINTS

Heavy Bomber Support (HBS): 0 to 15, 15 to 30, and over 30 miles: B-29 sorties supporting the divisions, corps, and army were derived from Air Force records that cited ordnance delivery coordinates. Data covering sorties by heavy bombers was derived in the same manner as for light bomber sorties and is shown in parentheses in table II. Each heavy bomber carried about 10 times as much as light bombers and often attacked several targets in one mission; fractional values for heavy bomber sorties per target appear in parentheses in the table.

Air Sorties for May 1951. The May data collected on air sorties, now the most complete of any category of data, is displayed in table I. Details of the forces supported are shown in table A-I. Table I shows the extent of battles in time, geographical expanse, and intensity over the entire front. It therefore provides some insight into the intensity of battles for the eastern and western sectors, for which no other data is available. In division support, the cross-hatched data shows the U. N. divisions that received more than 10 sorties per day; the outlined figures identify those that received more than 50 sorties per day. The corps support is cross-hatched when more than 50 sorties were flown. The number of light bomber sorties per day ranges from 138 to 806, the variation being primarily the result of bad weather at the ship/base or in the target area. The weather data is discussed in appendix B.

The B-29 bomber sorties were often flown at night or in bad weather under MPQ-2 radar control, supporting the units that received the largest amounts of air support that day. The numbers of sorties are given in parentheses in table I. Support of an adjacent unit may indicate a shifting enemy objective. Five daily CAS sorties were usually scheduled, but from 19-22 May the B-29's delivered 15 to 21 sorties in close support. Since the B-29 capacity has about 10 times the ordnance load of the usual bombers, these strikes were important and some enemy attacks were broken.

When the ground action was most intense, most of the air sorties were flown in close support of troops taking the brunt of enemy attacks. These attacks apparently were on fronts from 1 to 6 divisions wide. Usually the probing attacks were multi-pronged; some units between the positions attacked were rather inactive. The western front near Seoul was under heavy attack from 1 to 9 May when the central sector was relatively quiet. Activity in the central sector was moderate starting about the 6th, and intense from the 17th to the 20th. The eastern sector was quiet until the U. N. forces counterattacked. Then air was used to attack the retreating enemy, starting on the 22nd. Interdiction sorties were flown along the east coast throughout the Soyang battle; the supply routes that were attacked can be traced by the pattern of sorties given in figure 3 for 19 May. The effectiveness and timeliness of the air sorties and artillery ordnance were not assessed. Analyses made at the time are available, covering the numbers of requested air missions that were met and time delays that were incurred (reference (f)).

TABLE II

BOMBER SORTIES FLOWN IN SUPPORT OF U. N. DIVISIONS IN MAY 1951
(Heavy bomber sorties in parentheses)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Sorties division support ordnance delivered at 0 to 15 miles from battle line																	
I	R 20	0	15	6	45 (3)	0	5	4	0	1	4	2	9	0	4	0	3
	B 29	-				17 (2)	1	21	-	--	5	8	4	0	1	0	8
	R 01	57 (3)	28	22	3 (5)	28		13	16	29 (2)	10	14	1	6	0	4q	0
	U 03					-	25	14	23 (13)	8 (4)		-	--	-	--	--	--
	C 01	32	25	4	4 (3)	1	0	19	4	0	8	1	2	0	0 (2)	2	4
	U 25	17	16 (3)	13	1	29	21		17	4 (2)	13	3	9	6	41	0	7 (6)
IX	U 24	13 (6)	30 (8)	28 (5)	22	6	28	16 (3)	20 (7)	26	18	25 (7)	37	32	29	12 (3)	19
	R 02		0	6 (3)	3	2	4	0	0	0	0	3	0	0	3	0 (8)	0
	R 06	5 (8)	0 (3)	5	0	0 (2)	2	0	0	0	0	0 (5)	3	0	0	0	0
	U 07	2	0	7	0	3	1	0	2	3	9	5 (8)	2	4	9	0	0
X	M 01	25 (11)	38 (5)	23	7	17 (14)	35	13	13	0	40 (3)	25	13	36	0 (10)	2	0
	U 02	33	8 (8)	6 (5)	6 (1)	4 (5)	10	2	7 (10)	8	10 (12)	14 (14)	14 (7)	20	0 (7)	3	2
	R 05	7 (3)	0	2	0	0	5 (6)	12	10 (5)	2	4 (12)	0	0 (7)	5	0	0	0 (4)
	R 07	7	10	8 (3)	2 (4)	2	16 (20)	40 (17)	0 (5)	10	11 (3)	7	12 (7)	4	4	1	6
ROK III	R 09	10	0	0	0	9	32 (24)	51 (17)	13	5	24 (3)	1	0	15	0	0	0
	R 03	1	7 (3)	16	0 (5)	12	20	13	27	0	3	25	30	18	0	0	0
ROK I	R 04	0	0	0	0	0	2	0 (4)	12	0	5	9	5	0	0	0	0
	R 11	3	0	2	0	0	0	1	0	0	0 (3)	2	13	0	0	0	0
Corps support number of sorties delivering ordnance 15 to 30 miles from battle line																	
I		53 (1)	116 (1)	98 (3)	28 (4)	45 (4)	50 (3)	48	23 (1)	19 (11)	80	72 (4)	42	68	58 (7)	39 (12)	66 (6)
IX		18	2 (3)	22 (1)	9	8 (2)	22	10 (9)	8 (1)	12	37 (15)	18 (20)	78 (21)	72 (47)	89 (17)	9 (17)	17 (30)
X		31	36	36	6	6	20	45	26	6 (7)	17 (11)	26 (12)	37 (7)	7	32 (7)	1	0 (4)
	ROK 3	9	0	2	2	9	7	7	0	0	2	2	0	3	4	0	0
	ROK 1	0	2	0	3	1	13	7	8	0	0	0	3	0	0	3	6
Army support number of sorties delivering ordnance more than 30 miles from battle line																	
	Over 30	448 (12)	472 (13)	473 (13)	79 (13)	325 (8)	429 (8)	355 (13)	219 (11)	327 (4)	331 (114)	426 (12)	363 (12)	327 (14)	170 (15)	61 (12)	109 (14)
Total sorties		765 (17)	806 (18)	779 (18)	221 (19)	525 (17)	747 (16)	694 (18)	447 (17)	461 (13)	633 (18)	689 (19)	679 (17)	622 (19)	446 (20)	138 (17)	241 (19)

		17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	R 20	42	20	40	33	13	9	11	2	4 (5)	9	1	21	33	0	20
	B 29	0	2 (4)	0	5	0	0						-	--	--	
	R 01	8 (5)	5	21	25	26 (5)	72	7	19	1 (3)	2	5	12	15	0	27
	C 01	5 (5)	13	34	6	10		12	7	12 (3)	5	13	22	16	0	30
	U 03									-			--	--	--	8
	U 25	17	32	17	17	1	21	12	70	28 (3)	15	13	39	43	3	3
	R 02								28	13	0	7	19	27	1 (4)	1 (7)
	U 24	35	70	36	64	24	52	30	0	4	0 (8)	12	31	6	2	19
	R 07	0	8	12	12	7	14	0	1			U 07 21	U 07 5	25	3 (9)	22 (7)
	R 06	0	0	4	4	11	0	2		21	0	30 (2)	19	(3)	1	0
	U 07	12 (5)	8 (9)	36	44	4 (9)	8	0	24	36 (13)	3	-	--	--	--	--
	M 01	24	31 (37)	75 (18)	57 (6)	23 (44)	14 (34)	27 (13)	44 (3)	63 (3)	10 (21)	66 (7)	18	32 (17)	13 (13)	44 (7)
	U 02	60 (15)	63 (52)	65 (133)	91 (173)	10 (97)	U18 5 (0)	U0220 (7)	57 (16)	69 (3)	18 (17)	35 (9)	141 (1)	98	19	34 (29)
	R 05	63 (5)	13	20	14 (24)	0 (14)	U0224 (44)	TFA 7	4 (8)	-			40	--	--	--
	U 03		25	4	39	25 (28)	R0717 (78)	R05 0	--	U03 33 (7)	0	0	--	--	R 08 0	0
	R 07	21 (5)	47	0 (14)		0	U03 12 (64)	23	38				-	--	--	--
ROK III	R 09	18	8	4	34	10	-	-	-	-			--	-	--	--
	R 03	0	0	0	0	0	4	36	-	--			-	-	--	--
	R 04	2	0	0	0	0	0		4	0	0	3	4	R 11 5	0	0
	R 11	0	1	0	0	4 (4)	0	24	2	3	0	0	0	R 04 0	0	0
Corps 15 30																
I		70	70	114	75	49	56	126	94	96 (1)	18 (1)	64 (2)	133	157	21	152 (1)
IX		24	42 (4)	42	17	16 (9)	35	51	65 (3)	62 (3)	10	28	50	70	11	10
X		55	77	13 (5)	36	29	55	71	66 (21)	47 (7)	1 (4)	15	33	9	0	9
	ROK 3	4	3	1	6	4									--	--
	ROK 1	0	0 (10)	0	2	4	0	11	15	0	0	0	2	0	0	0
	Over 30	228 (12)	178 (11)	240 (6)	219 (6)	225 (0)	245 (0)	263 (10)	185 (8)	148 (13)	9 (12)	28 (15)	148 (15)	203 (15)	93 (2)	98 (11)
	Total	668 (17)	715 (23)	778 (17)	800 (21)	493 (21)	676 (22)	725 (12)	674 (11)	642 (12)	100 (18)	282 (17)	737 (16)	763 (17)	169 (5)	478 (17)

Average ordnance per sortie for all aircraft, except B-29's, was 0.80 tons*; the B-29's carried 8.54 tons per sortie. (See appendix C for the derivation of aviation ordnance per sortie.)

Ordnance Delivered. The daily rounds of ammunition and the aviation were compared on total ordnance (figure 7) and on a per-mile-of-front-per-division basis (figure 8). There was a small increase in ordnance delivered before the Chinese attack, but a great increase as the attack started.

Total fire support per mile when the battle reached its climax amounted to 60 tons per mile, in addition to the ordnance fired by the ground forces. The latter data was available from information about the weapons fired at Guam in World War II, where maximum fire from rifles, mortars, machine guns, flamethrowers, and tanks amounted to about 12 tons per mile of front per day. The total fire support in intense land battles was equal in Korea and World War II, under the assumption that the Soyang River and Guam battles are typical. These totals, however, are only about one-tenth as much as was fired on D-day during the landing. During patrol periods, the expenditure was less than 10 tons of artillery and aviation ordnance - often 1 or 3 tons per mile. The calculations are given in appendix C.

The predominant tonnage of ordnance was delivered by artillery. The relation between casualties and tonnage was examined in the regression equations to be discussed later.

Though the tonnage of ordnance delivered is only a partial measure of fire support, it was the only measure readily available from historical records. It measures the degree of logistical support. The lethalities of various weapons against various targets could not be examined because target information and bomb damage assessments were not available. Tonnage is also a poor measure for estimating the amount of ammunition required for new weapons of improved effectiveness. Past experience should be taken into account with new weapons, however, because neutralization, harassing, and interdiction fire, for example, is directed against area targets to keep the enemy from using his weapons or his supply routes freely. These targets and targets of uncertain location have consumed large amounts of ordnance in the past; tonnage figures give some measure of the total amount likely to be fired in future wars.

Dependent Variables

Enemy Casualties. The enemy casualties inflicted by U. N. divisions on the front from west to east are shown in table III; they correspond to the data on air sorties in table II.

*In some compilations, where noted, an estimate of 0.747 tons per bomber sorties was used. The revised value, 0.80, would not alter the results significantly.

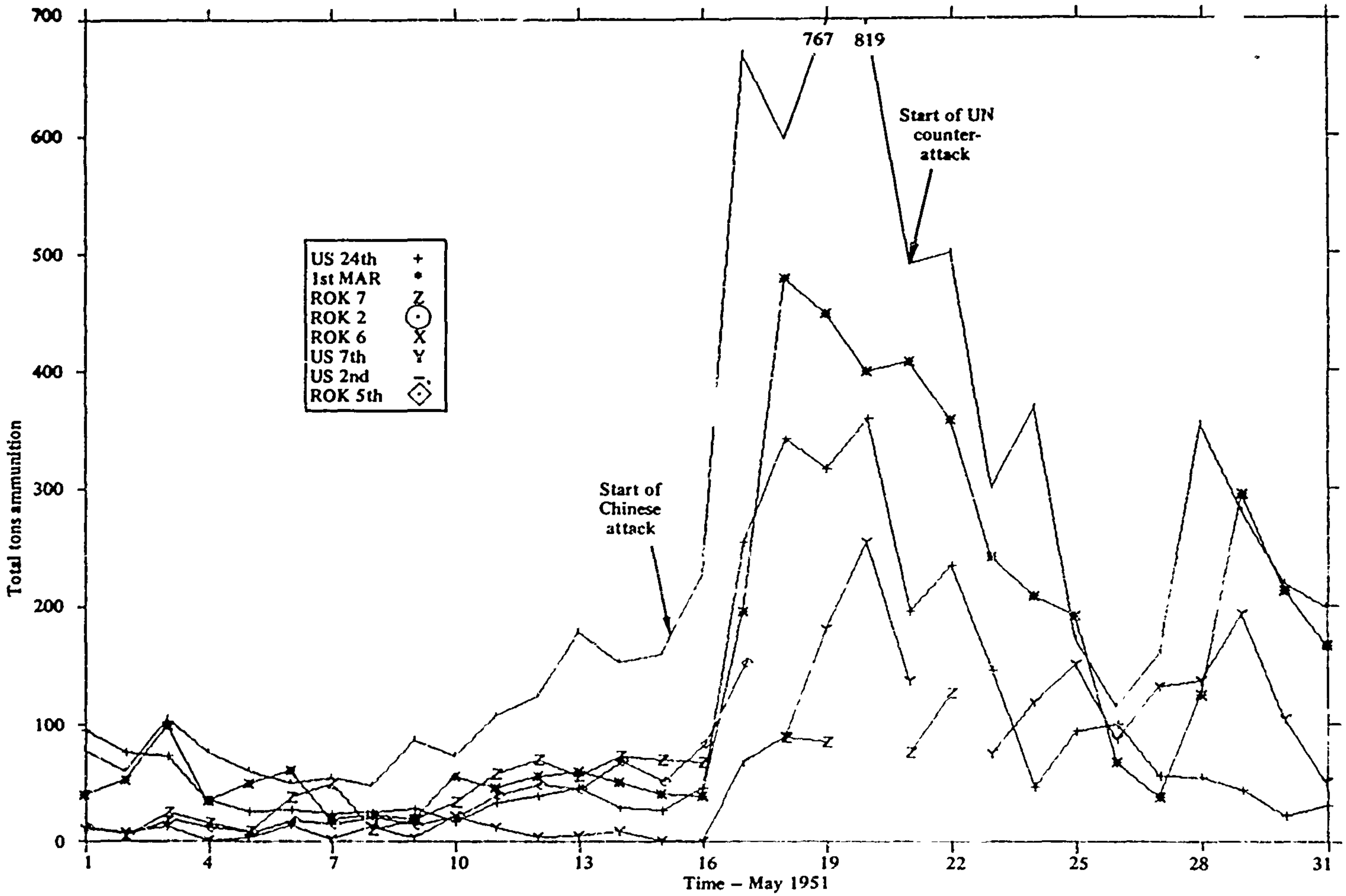


FIG. 7: TONS OF AIR AND ARTILLERY ORDNANCE FOR VARIOUS DIVISIONS
KOREAN WAR, MAY 1951

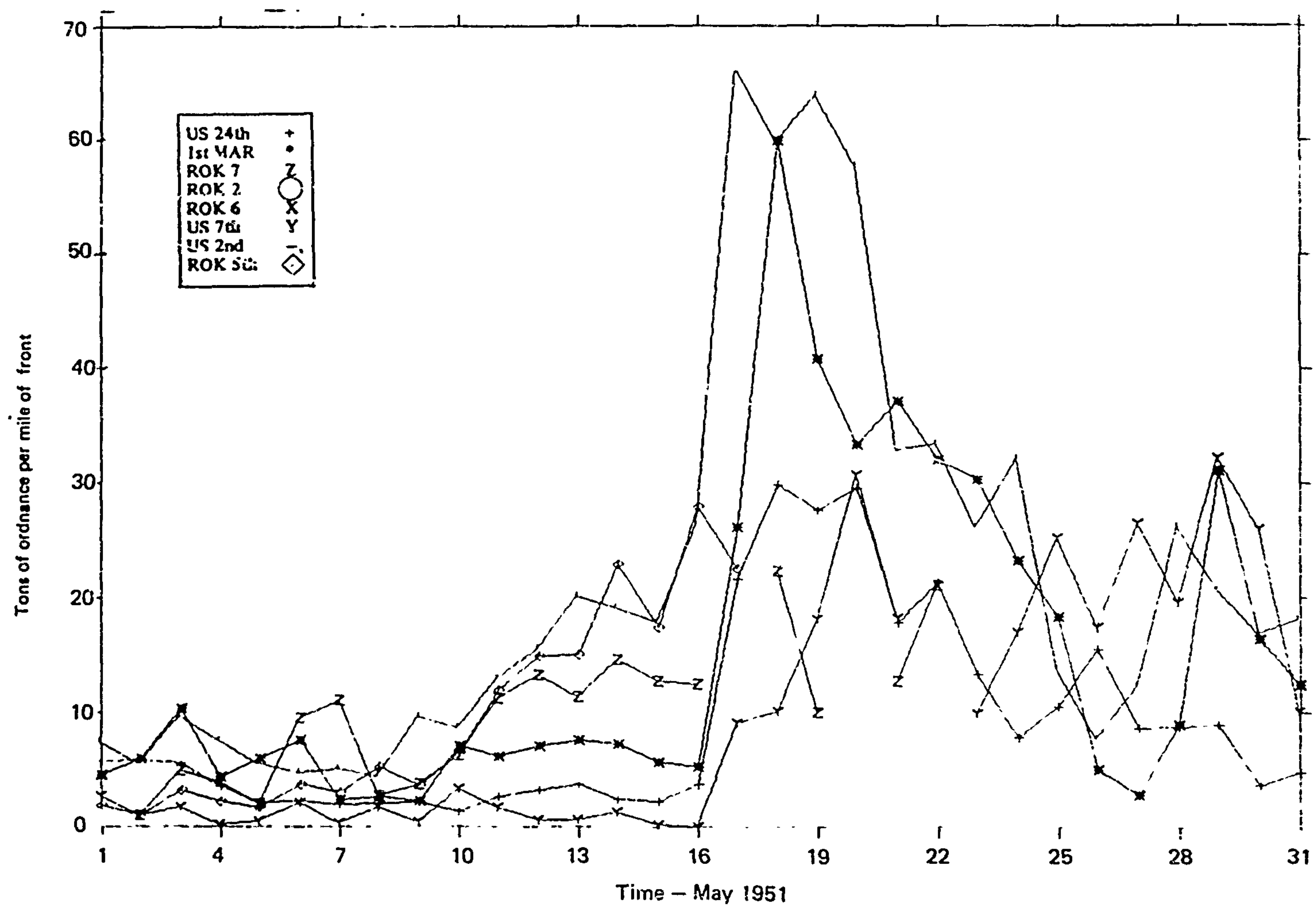


FIG. 8: TOTAL AIR AND ARTILLERY AMMUNITION TONS PER MILE FRONT
KOREAN WAR, MAY 1951

TABLE III

ENEMY CASUALTIES IN FORCES OPPOSING U. N. DIVISIONS*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
R 20	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
B 29	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
C 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 25																
U 24	0	18	57	8	4	5	1	4	16	3	0	40	25	12	55	96
R 02		0	0	0	0	0	1	1	0	0	0	0	3	9	1	1
R 06	0	1	0	0	0	0	0	24	0	18	0	0	0	5	0	0
U 07	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0
M 01	0	0	217	43	0	125	1	2	5	0	23	4	2	3	1	35
U 02	670	270	267	124	30	157	194	109	39	372	661	748	776	197	818	1984
R 05	0	2	0	0	0	0	0	1	0	0	0	0	0	16	5	0
R 07	0	2	6	0	0	0	2	4	4	13	0	0	1	0	452	2
R 09	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 04	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 11	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
R 20	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
B 29	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
C 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 25	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 02								581	353	157	140	32	270	0	469	
U 24	3746	2455	1608	1192	1527	3454	419	540	664	645	4504	2402	518	1729	462	
R 02	375	344	35	863	156	13	1999				U07 3027	2503	3263	49	202	
R 06	246	440	7420	4949	526	5	111	206	469	1120	7312	4786	1035	1411	195	
U 07	125	113	930	2371	522	501	654	529	607	2051						
M 01	948	38	236	2067	476	844	123	1109	2259	29	164	755	1224	1502	318	
U 02	8195	8422	3766	22685	7685	U 187 M	U02 1184	3487	5249	1992	3199	1642	1025	815	968	
R 05	20	1	M	R07 1	16	U02 2203	TFA M	M								
U 03		M	0	20	847	R07 248	R05 M							R08 M	17	
R 07	M	M	M	M	M	U03 536	667	19	639	972	512	285	44			
R 09	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	
R 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	
R 4	M	M	M	M	M	M	M	M	M	M	M	M	R11M	M	M	
R 11	M	M	M	M	M	M	M	M	M	M	M	M	R 4 M	M	M	

* *Casualty data is taken from volume VII; division positions are taken from table I.*

M = missing data.

Data is missing for the western and eastern fronts, and casualties were not recorded by the ROK units that were under heaviest attack on May 17 and 18. Correct assessment of enemy casualties is difficult because the battles are continuous, and it is not usually possible to count actual bodies on the battlefield. Even these figures do not take into account those who were carried from the battlefield. The exceptionally high level of casualties reported by the 2nd U.S. Army Division for 20 May (22,685) is shown (appendix B of volume VIII) to be a delayed report, covering 19 May and before. The policy for reporting casualties appears to be different between divisions. A preferred source would be the North Korean and Chinese Communist records, which are, of course, not available. Enemy casualties are expressed as casualties for regression analyses.

Friendly Casualties. The friendly casualty data is given in table IV in the same form as the enemy casualty and air sortie data. Except for the heavy casualties sustained on 17 and 18 May by the ROK divisions, the U.N. casualties were much lighter than those of the enemy that opposed them. This result should be expected because the U.N. forces fighting with ground force weapons received extensive artillery and air support, while the North Koreans and Chinese Communists fought almost entirely with rifles, mortars, rockets, and machine guns. Logarithms were also taken of friendly casualties, for reasons given in the paragraph above and explained more fully in appendix D.

The data provides some clues about the adequacy of fire support. This relationship was sought in equations relating casualties, strength, ground gained or lost, and firepower. Since all arms usage increases when battle is joined, it was appropriate to try the effect of total fire support with the effects of each type separately.

The cumulative casualties for each division are shown in figure 9. The form of plotting - namely, the logarithm of the casualties plotted against the logarithm of time - was chosen to accentuate battle periods.

During the Chinese attack the heaviest casualties were suffered by the 5th and 7th ROK Divisions and the 2nd U.S. Division; casualties inflicted on the 7th U.S. Division and the 1st Marines increased slightly. Similarly, during the U.N. counterattack, the ROK 6th, ROK 2nd, and U.S. 7th suffered the greatest increases in casualties.

Ground Gained, another dependent variable has already been discussed.

Regression Analysis of the Battle Data

The battles are described by variables already mentioned - the strengths, casualties, and firepower of the opposing sides - and such others as weather, terrain, tempo of fighting, the effects of pre-bombardment and of movement of adjacent units, and the decisions of the opposing commanders to attack, withdraw, or hold.

TABLE !V

U. N. CASUALITES

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
West coast																
R 20	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
B 29	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
C 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 25	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 24	7	1	1	0	0	1	3	0	2	4	0	0	1	1	2	1
R 02	0	0	0	0	0	0	3	1	21	0	0	0	0	1	0	0
R 06	2	0	3	11	0	0	4	2	1	0	7	0	0	3	1	0
U 07	0	0	2	0	0	8	1	0	1	0	0	0	0	0	0	2
M 01	M	4	6	0	1	5	0	0	3	26	1	4	3	2	5	5
U 02	1	3	3	1	1	0	1	5	6	20	4	25	7	2	7	9
R 05	7	0	0	0	1	1	2	3	4	24	0	5	1	19	7	12
R 07	M	0	0	3	0	0	1	0	0	1	11	4	2	2	4	3
R 09	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 04	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 11	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
East coast																

	17	18	19	20	21	22	23	24	25	26	27	28	229	30	31
West coast															
R 20	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
B 29	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
C 01	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
U 25	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 02								1	32	32	19	M	3	22	M
U 24	50	52	45	16	54	65	30	4	7	20	56	11	17	22	M
R 02	0	6	67	61	33	15	51				U 07 2	36	115	78	M
R 06	1	0	5	41	416	30	2	8	2	125	38	107	67	M	M
U 07	3	1	2	34	47	24	57	32	16	63					
M 01	42	3	9	26	12	6	1	11	42		14	26	184	61	155
U 02	430	1092	595	141	35	U 187 M	U 02 49	48	162	39	11	214	333	81	98
R 05	741	2236	326	153	262	U 02 63	*FA M	M							
U 03		M	11	3	67	R 07 1	R 05 7							R 08 10	2
R 0*	482	4945	415	19	29	U 03 55	103	73	468	165	82	16	0	M	M
R 05	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 03	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 04	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
R 11	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
East coast															

* Casualty data is taken from volume VII division pos on are taken from tabe

M missing data

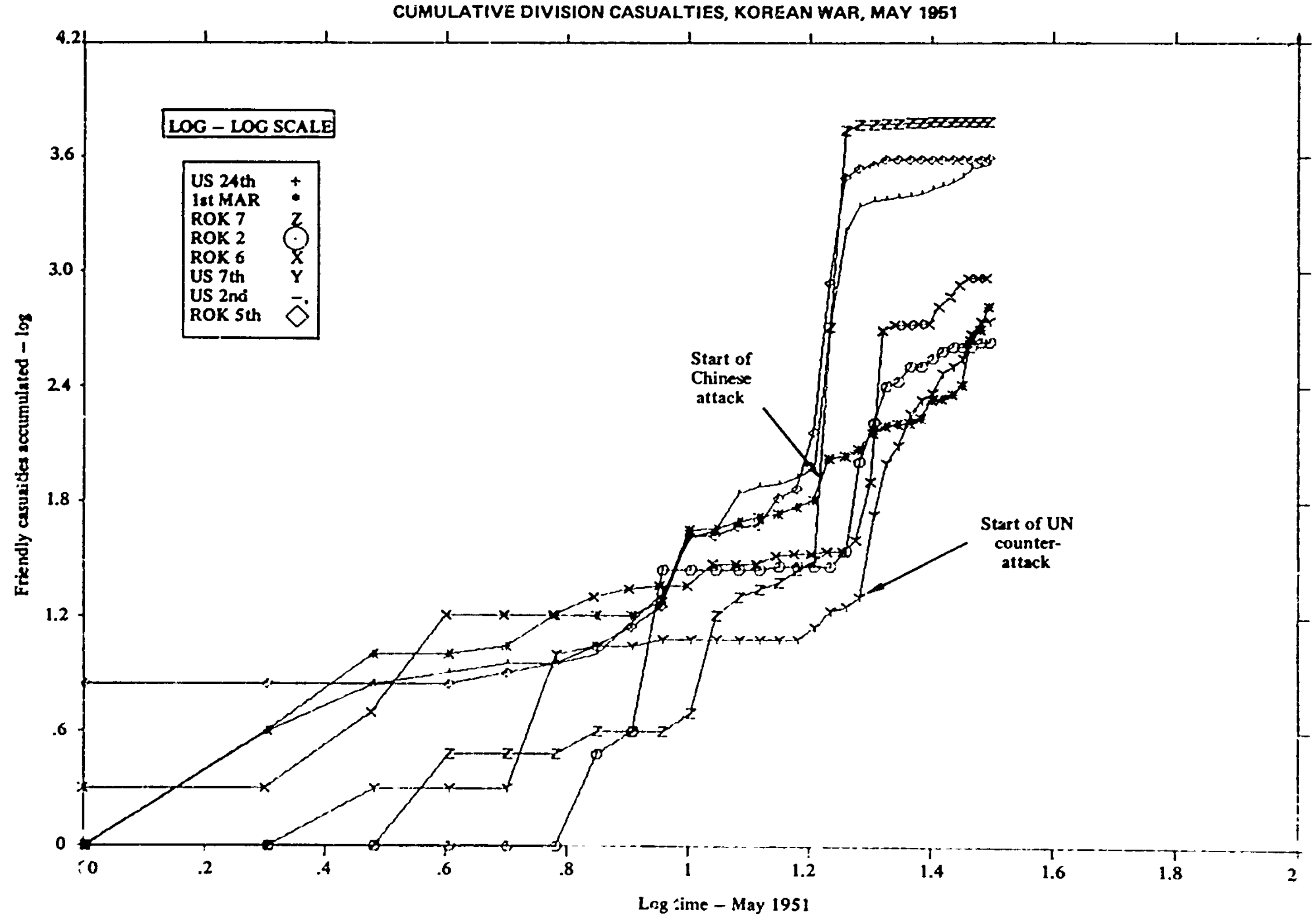


FIG. 9: DATA FOR MAY 1951 – PLOTTED AS THE LOGARITHMS OF CASUALTIES AND TIME

The techniques of regression analysis are discussed in many texts; two of these are Draper and Smith (reference (t)) and Snedecor and Cochran (reference (u)). The techniques are used to:

- Construct an equation in the independent variables, the X's, that gives the best prediction of the Y's.

- When there are many subsets of X's, find the one that gives the best linear prediction equation.

- Discover which variables are related to Y, and, if possible, rate the variables in order of importance.

These equations do not reveal cause and effect, the relationships of greatest interest. Any inferences of cause-and-effect relationship must be based on external knowledge, and are necessarily risky. In regression analysis, many relationships are found to have a common cause. For example, when preparing for attack, a commander will normally mass troops and conduct preliminary bombardment and air strikes. Eventually, both sides suffer more casualties, and ground may be gained. In reference (v), Johnson refers to this simultaneous change in independent and dependent variables that are highly correlated with each other as "multi-collinearity." Attempts have been made to eliminate some of the collinearity between the rounds of light and heavy artillery and light and heavy bomber sorties by converting them to the tonnages of ordnance delivered. The reader's attention is drawn to equations that are attributable to collinearity and where cause-and-effect relations may possibly exist.

Computer Programs

These are the programs used:

- The ERSU econometrics program has the advantage of permitting 2-way plots between residuals (the difference between individual data points and fitted curves) and other variables. The program described in reference (d) requires postulation of specified relationships.

- The BIMED - 34 stepwise multiple regression program, described in reference (e), is somewhat more flexible, especially when the analyst is selecting new combinations of variables and testing whether data is related to previous time periods.

A print-out of significant coefficients from the BIMED-34 program is illustrated in figure E-1. The example shows the significant coefficients in the multiple regression equation for enemy casualties when the enemy is attacking; ln enemy casualties = .002 ground light artillery + 1.89 LN friendly strength - 1.61 ln friendly strength of the previous day. After specification of the independent variable to be examined, the program selects the variable having the greatest effect and then computes and lists the coefficient and the standard error of the estimate. The coefficient divided by the error is the t- value, a statistical measure that serves as a guide in the decision whether to retain the coefficient or not. For normally distributed data in large amounts, a "t" value of 2.0 was selected as a criterion for inclusion of new terms.

The process is repeated to select the next best coefficient until all have been entered. At each step a new equation is computed for all the variables. The fraction of variability explained by the regression equation is the R^2 value given for each step. We have condensed the output in appendix E in this appendix by recording the coefficients for the significant terms. Others tested but not significant are listed in a separate column. The R^2 value for significant terms is followed by the R^2 value, for all terms, in parentheses.

RESULTS

Summaries of Equations From the Regression Analysis

The summaries of the multiple regression equations appear in appendix E. The data given is the significant coefficients and the values for R^2. Two approaches were used:

- Equations were fitted empirically, with no preconceived notion of the form of the equation except that R^2 should be high for the equation chosen among several alternatives for predicting dependent variables. Collinearity was reduced by conversion of numbers of ordnance rounds, aircraft sorties, and numbers of troops to the tonnages of ordnance delivered by each; a new variable is thus substituted for the group. Similarly, reduction of the values of troop strengths and ordnance expended to densities per mile of front, removed the variation in size between units.
- Lanchester equations of various types were tested.

Empirical Study

The original data collection provided information about the effects of pre-assault bombardments, interdiction efforts, and mutual support of adjacent units. All of these have some effect on battles, but the quantitative assessment has been poor because of missing data and the poor quality of information about enemy strength.

When the strength and casualties figures are plotted in the original form and as logarithms, the latter points are distributed more normally and the transformation reduces the variance of the residuals and improves the curve-fitting process. These plots appear in appendix D.

Summary of Calculation Results on the April-May 1951 Data

The initial calculations reported in appendix F were made for days in April and May 1951, when selected U. N. divisions were attacking. The air sorties were those taken from Army records, which did not reflect the entire support. The details of the significant terms are displayed in tables E-I and E-V.

Friendly casualties (equations 1, 4, 5, and 6). The equations vary according to the size of the coefficients and the statistically significant terms. The important terms are those reflecting friendly strength, which varies inversely with friendly casualties and directly with the ground gained; this is to be expected, because troops are exposed to fire while attacking. When 3-day averages were used to eliminate variations in enemy strength and reduce the effect of missing data, the enemy strength became important. In all equations, increases of friendly casualties were related to greater use of light artillery—which is clearly not a cause of the casualties but reflects greater fire support during attack. Equations 4 and 5 are the most plausible but contain the light artillery term and, at best, have low R^2 (0.36, 0.34).

Enemy Casualties (equations 2, 7, 8, and 9). The enemy casualties are related to rounds of light artillery but not to force strengths. When all variables were tested but only the significant ones included, pre-bombardment was significant too. There is little effect shown in casualties as the U. S. gained ground. The value of R^2 lies between 0.56 and 0.66 . In the latter case, the friendly casualties should have been deleted.

Ground Gained (equations 3, 10, 11, and 12). The important terms were friendly strength, ground gained by adjacent units, and friendly casualties. The last is the largest term, and where it was deleted, friendly strength appeared important. Fire support was not significant in any equation. The highest R^2 was 0.59 for the 3-day averaged data.

When the data on troop strengths was averaged over 3-day periods, the 5th ROK Division received less fire support than U. S. units and was a less effective fighting force. It did not gain as much ground or inflict as many casualties as the U. S. division, though it suffered as many casualties.

Summary of May 1951 Data

The data base for May 1951 was augmented by the compiled air sortie data, and front widths which permitted calculation of average miles advanced, as well as strengths, casualties, and firepower per mile of front. These calculations were made, and new variables were entered. As previously mentioned, this analysis was confined to a 1-month period.

We made the first run by inserting all variables and data. As expected, the results were meaningless because many different battle situations were lumped together. The values of R^2 for friendly and enemy casualties were 0.28 and 0.55. These provided a basis for judging the effect of introducing new variables.

The effects of separating the data into battle phases and into missions were tested, yielding R^2 values of 0.43 and 0.44 for friendly casualties – better than previous single-day results. Two battle phases (labelled 17 and 18 by historians) occurred in May – before and during the Soyang battle. The missions recorded are subjective judgments of the mission by CNA analysts who examined the battle records, and do not, in some instances, agree with the orders of the day. Table V gives the number of missions for May 1951. These were divided into three groups because the sample size was not large enough for finer separation.

- Offense and advance by U. N. forces (29 observations).
- Offense and advance by enemy forces (20 observations).
- Static or patrol periods, including time when either side was patrolling or in assembly areas (75 observations).

The sample sizes are small because of missing data. Data was complete, for these 124 division-days, out of the 205 division-days of table V. There were 464 division-days on line for the entire battle front. Because of the smaller data base, we could not conduct adequate tests of mutual support among adjacent units and the effects of continuity from day to day.

The regressions were run to relate the causes for friendly and enemy casualties and the ground gained. In this series the logarithms of the strengths and casualties were used as before and the stepwise regressions were made, with the constant eliminated. This forces the fitted curve to pass through the origin. Table VI contains the regression coefficients for 4 variations on 9 equations, namely, for (a) the untransformed or normal fire support data and the logarithm of the data and (b) the effect of lagging and not lagging (that is, whether the previous day's results were considered or not). R^2 values are highest when fire support is not transformed and lagged data is included. The ground-gained equations did not include terms for casualties, and the results are so poor that they are not worth further discussion, except that they tend to bear out the tentative conclusion that firepower and strengths do not in themselves gain ground. Apparently, a decision to advance and take casualties is required to take ground. In one run (#26), weather was included and found to increase R^2 somewhat, in agreement with General Ridgway's observation that the Chinese and North Koreans attacked when our aircraft could not fly very effectively.

The equations for friendly and enemy casualties corresponding to regression coefficients of table VI are given in table VII, in which the values of R^2 are also repeated. As shown above, the enemy casualties are explained better than the friendly casualties. There are 4 equations for each situation; when R^2 is high, the same types of terms tend to appear in all 4 equations, lagged and unlagged data of a given kind are regarded as the same. When R^2 is lower, the terms in the equations are less stable, and should serve as a warning not to take the equations very seriously. An example is the situation when the U. N. forces were attacking; the friendly casualty equations contain very different types of terms.

After this warning against strict interpretation, the types of equations that result when the logarithms are removed are shown in table VIII.

TABLE V

NUMBER OF DIVISION–DAY OBSERVATIONS FOR CATEGORIES OF FRIENDLY AND ENEMY MISSIONS MAY 1951

Friendly mission	1. O-Advance	2. O-Patrol	3. D-Fixed	4. D-Withdraw	5. Assembly
1. O-Advance			14	43	4
2. O-Patrol	1		55	3	56
3. D-Fixed	22		1		
4. D-Withdraw	4				
5. Assembly			2		

TABLE VI

REGRESSION COEFFICIENTS* FOR EQUATIONS RELATING CASUALTIES AND GROUND GAINED TO STRENGTHS AND FIRE SUPPORT BY MISSION FORCE MAY 1951

	Lagged data not considered		Lagged strength and fire support included		
	Fire support + In strengths**	In Fire support + In strengths **	Fire support + In strengths**	In Fire support + In strengths**	Number of observations
Friendly casualties					
U.N. attacking	.27 (.35)	.425 (.48)	.41 (.54)	.42 (.61)	29
Enemy attacking	.57 (.61)	.61(.65)	.57 (.84)	.49 (.81)	20
Patrolling/static	.56 (.57)	.25 (.33)	.70 (.73)	.37 (.43)	75
Enemy casualties					
U.N. attacking	.42 (.49)	.55 (.66)	.67 (.71)	.55 (.78)	29
Enemy attacking	.70 (.72)	.46 (.61)	.78 (.86)	.70 (.87)	20
Patrolling/static	.57 (.59)	.37 (.39)	.61 (.63)	.45 (.48)	75
Ground gained					
U.N. attacking	.05 (.12)	.06 (.11)	.05 (.30)	.06 (.28)	29
Enemy attacking	.23 (.42)	.29 (.39)	.15 (.66)	.29 (.70)	20
Patrolling/static	.14 (.23)	.02 (.13)	.41 (.43)	.28 (.32)	75

* *The constant was suppressed in these equations.*

** *First R^2 is for significant terms in the equation and R^2 in parentheses is for all terms tested, including insignificant ones.*

TABLE VII A

COMPARISON OF EQUATIONS FOR FRIENDLY CASUALTIES

Friendly casualties	Run	Lagged	Logged fire support	Equation for ln FC	R^2
U. N. attack	24-1-1	No	No	= .27 ln ES + .013ASo	.27 (.35)
	24-2-2	No	Yes	= .37 ln LA - .67 ln HBS	.425 (.48)
	25-1-1	Yes	No	= -.55 ln FS_{d-1} + 0.88 ES_{d-1} + .0063 LA	.41 (.54)
	25-1-2	Yes	Yes	= .37 ln LA - .65 ln HBS	.42 (.61)
Enemy attack	24-1	No	No	= -2.5 ln FS + 2.8 ln ES + .037 ASo - .14 HBS	.57 (.61)
	24-2	No	Yes	= -2.8 ln FS + 3.1 ln ES + .72 ln ASo - .87 ln HBS	.61 (.65)
	25-2-1	Yes	No	= -2.5 FS + 2.8 ES + 0.037 ASo -.14 HBS	.57 (.84)*
	25-2-2	Yes	Yes	= -1.9 FS + 2.0 ES + .4 ln AS + .2 ln AS_{d-1}	.49 (.81)
Patrol	24-1	No	No	= .0012 LA + 0.014 ASo	.56 (.57)
	24-2	No	Yes	= .084 ln LA + .36 ln HBS	.25 (.33)
	25-3-1	Yes	No	= .026 ln ES_{d-1} + .0009 LA + .0008 LA_{d-1} + .008 AS - .008 ASo_{d-1}	.70 (.73)*
	25-3-2	Yes	Yes	= -.077 ln HA_{d-1} + .11 ln LA + .31 ln HBS + .45 HBS_{d-1}	.37 (.43)

* *Equations are given anti-log form in table XIII.*

TABLE VII B

COMPARISON OF EQUATIONS FOR ENEMY CASUALTIES

Enemy casualties	Run	Lagged	Logged fire support	Equation for ln EC	R^2
U. N. attack	24-1	No	No	= .56 ln ES + .0016 LN	.42 (.49)
	24-2	No	Yes	= .65 ln LA	.55 (.66)
	25-1-1	Yes	No	= -1.3 ln FS_{d-1} + .73 ln ES + 1.2 ln ES_{d-1} + .0017 LA_{d-1}	.67 (.71)*
	25-2-1	Yes	Yes	= .65 ln LA	.55 (.78)
Enemy attack	24-3	No	No	= .24 ln FS + .0024 LA	.70 (.72)
	24-4	No	Yes	= .62 ln LA	.46 (.61)
	25-2-1	Yes	No	= 1.9 ln FS - 1.6 FS_{d-1} + .002 LA	.78 (.86)*
	25-2-2	Yes	Yes	= -1.4 ln ES + 1.36 ln HA + .41 ln LA + .84 ln HBS	.70 (.87)
Patrol	24-5	No	No	= .02 HA + 0.023 ASo	57 (.59)
	24-6	No	Yes	= .12 ln FS + .47 ln LA + .89 ln HBS	.37 (.39)
	25-3-1	Yes	No	= .018 HA + .0010 LA_{d-1} + .014 ASo	.61 (.65)*
	25-3-2	Yes	Yes	= -.20 ln FS + .34 ln LA	.45 (.48)

* *Equations are given anti-log form in table XIII.*

TABLE VIII

SUMMARY OF R^2 VALUES FOR VARIOUS LANCHESTER LAWS

Missions	Dependent variables	Method of including firepower	Square law		Linear with constants		Log with constants	
			No constants	With constants	I	II	I	II
U. N. attack	Enemy casualties	Separate	.20	.25	.39	.25	.39	.39
		Total	0	.02	.22	.05	.28	.28
	Friendly casualties**	Separate	.03	.07	.07	.07	.07	.07
Enemy attack	Enemy casualties	Separate	.58	.61	.61	.61	.61	.61
		Total	.31	.32	.56	.51	.69	.68
	Friendly casualties**	Separate	.01	.01	.08*	.01	.08*	.08*
Patrols/limited action	Enemy casualties	Separate	.80	.81	.81	.81	.81	.81
		Total	.57	.66	.58	.77*	.61	.61
	Friendly casualties**	Separate	.12	.18	.22	.18*	.22	.22
		Total	.29	.34	.32	.18	.14	.14
Computer run number			33	32	34	36	35	37

* *Constant is significant.*

** *Total ordnance runs are too low to be reported.*

Abbreviations

ES or ENST = Enemy strength

FS or FRST = Friendly (U. N.) strength

EC or ENCA = Enemy casualties

FC or FRCA = Friendly (U. N.) casualties

GG = Ground gained

LA = Light artillery rounds

HA = Heavy artillery rounds

AS = Close air support from army records

AS_o = Compiled air sorties delivering ordnance, 0-15 miles from the battle line

AS_{15} = Ditto, 15-30 miles from the battle line

AS_{30} = Ditto, over 30 miles from the battle line

HBS_o, HBS_{15}, HBS_{30} = Heavy bomber (B-29) sorties in the three range bands

Subscript t-1 or d-1 = data for the previous day

FL = Front length

Lanchester Laws

In 1914, Lanchester proposed a square law and a linear law (reference (g)); Morse and Kimball showed several modifications of these laws in reference (h). Weiss and Peterson have proposed logarithmic laws in references (i) and (j).

The Lanchester laws take numerous forms. We chose to use derivative forms relating casualties per day and the strengths at the beginning of the day. The time of one day is not stated in the equations to follow but was used throughout.

Contrary to practice in most discussions of Lanchester laws, the following discussion considers firepower:

Square Law. Lanchester's square law can be represented by the following two equations:

$$ENCA = \Sigma K_{iF} \cdot FRST$$

$$FRCA = \Sigma K_{iE} \cdot ENST \ .$$

Since our information about friendly strengths and firepower is far superior to our knowledge of enemy strength and firepower, it was to be expected that enemy casualties would be predicted more accurately than friendly casualties. This is true for all of the other laws as well.

Linear Law. Similarly, the linear law can be written as:

$$ENCA = \Sigma K_{iF} \cdot FRST \cdot ENST$$

and

$$FRCA = \Sigma K_{iE} \cdot ENST \cdot FRST$$

A variation on the linear law, described by these equations, was also tested:

$$ENCA = \Sigma K_{iF}$$

$$FRCA = \Sigma K_{iE} \ .$$

Logarithmic Law. Two forms of a logarithmic law were explored. One relationship is:

$$ENCA = \Sigma K_{iF} \cdot ENST$$

and

$$FRCA = \Sigma K_{iE} \cdot FRST \ .$$

The second law is represented by:

$$ENCA = \Sigma K_{iF} \cdot ENST \cdot \ln FRST$$

and

$$FRCA = \Sigma K_{iE} \cdot FRST \cdot \ln ENST$$

Firepower is treated as part of each coefficient: ΣK_{iE} and ΣK_{iF} are the series of terms for various kinds of firepower on the two sides are E enemy and F friendly and the K_i are the coefficients fitted by least squares.

The assumptions made in the square, linear, and logarithmic "laws" are these:

Square law	Linear law	Logarithmic law
1. Each force is attacking the other	1. Same as in square law	1. One opponent cannot not bring his weapons to bear and incurs losses.
2. Opposing units may have different kill rates	2. Same as in square law	2. Near limiting case for one side
3. Each side knows the location of the opposing units and changes targets as soon as the engaged target is destroyed.	3. One side knows only the general area occupied by the enemy and does not know whether targets have been destroyed.	3. The kill rate of one side increases with the size of the enemy, but its effectiveness decreases with the size of the enemy.
4. Fire is distributed uniformly over the area occupied by the enemy.	4. Fire from surviving units is distributed uniformly over the area occupied by the enemy.	

Historical. Attempts to apply the laws are as follows:

Battles	Author	Reference
Iwo Jima, WW II	Engel	(k)
Crete, WW II	Karns	(l)
Civil War battles	Weiss	(i)
92 historical ba[illegible]s (over 250 years)	Helmbold	(m)

Engel and Karns found rather good fits [illegible]er single battles for the square law. Weiss and Helmbold looked at initial an[illegible] final strengths over battles of varying duration and concluded that particular kinds of battle had to be defined if the laws were to hold. Weiss found differences between attacks on fortified positions and meeting engagements, for example. Helmbold found poor fits for linear and logarithmic laws and somewhat better fits for the square law.

Tests of the Lanchester Equations

The equations tested are listed on page 33. The five different expressions were tested under two methods of combining the ordnance:

- Casualties = (k_i · ordnance type) (strength(s)) <u>Type I</u>
- Casualties = k · (sum of ordnance tonnage) (strength(s)) <u>Type II</u>

The difference between the equations is that Type I gives coefficients for each kind of ordnance, and Type II gives only one coefficient for the weight of all ordnance, The same "causalties" and "strengths" are used to describe the various expressions in the five forms of the equations.

The usual constant, k_o, which appears in regression equations:

$$Y = K_o + K_i x_i + \text{error}$$

was both included and eliminated in separate series. The constant was significant in only 5 cases out of 45.

The ordnance weights used in these trials were as follows:

Light artillery	18 tons per 1000 rounds
Heavy artillery	48 tons per 1000 rounds
Air support	0.755 tons per sortie
Heavy bomber	8.54 tons per sortie
Ground forces	40 pounds per man (estimate includes machine guns, rifles, mortars, tanks, etc.)*

A summary of the regression coefficients for the five Lanchester Laws appears in table VIII. Several main conclusions can be drawn:

- The equations using the separate forms of ordnance gave better fits than total ordnance; these equations are marked "separate" and "total" in the table.
- There is little difference in results among the various Lanchester equations.

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* Subsequent tests were made on the amount of ground force ordnance. Extremes of none and 400 pounds per man were tested. The large amount produced poorer regression coefficients. There was some indication that 40 pounds was too much but that complete omission of the term made little difference. The derivation of the tonnage of air ordnance is given in appendix C.

- Confirmation was given to the previous conclusion that enemy casualties are predicted with much greater accuracy than friendly casualties.

- Inclusion of the constant K_0 improves R^2 only marginally.

- Prediction is best for enemy casualties during patrols (.8 R^2), next best when the enemy was attacking (.6), and poorest when the U.N. forces attacked (.39).

The significant terms in the equations in table IX appeared to be the same for all the laws though, in many cases, the coefficients had different numerical values, as might be expected from the different functions.

Subsequent Runs

The remaining runs were confined to finding the reasons for low regression coefficients and then rearranging the data for new runs. The enemy casualties incurred during enemy attacks had a regression coefficient of 0.6. In plotting the residuals from Lanchester log law I against the distance advanced, it was found that 6 of 20 observations were sorted incorrectly: An enemy mission to advance ended in a retreat, instead. The entire deck was rearranged and the cards classified according to ground gained or lost, rather than by mission. The improvement in R^2 over the earlier results of table VIII is shown in table X.

Lagged fire support was included, but no significant new terms appeared and the regression coefficients remained the same. A residual plot was also made, to show whether the area gained had any effect on casualty/strength equation; no new factors were found. The equations in table XI correspond to table X log law I.

These equations were the best derived thus far for enemy casualties. The poor equations for enemy casualties can be improved by the separation of data cards, but the poor equations for friendly casualties are thereby made still poorer. The only point of interest was that when the enemy attacked and the ROK 5th Division collapsed (on 18 May), there was a large residual for that day, but residuals for the other data points were small. This suggests that the equations apply only to fighting units that maintain their integrity.

The log law that proved best is in agreement with the actual situation, namely, that the U.N. forces were attacking and destroying enemy forces before they could bring their weapons to bear.

Friendly Casualty Equations

Since the equations investigated above showed only weak association with Lanchester relationships in which U.N. ordnance was important, we returned to the earlier "best" equation in which a linear law was used and the effect of

TABLE IX

SIGNIFICANT (AT 95% LEVEL) TERMS IN THE LANCHESTER EQUATIONS

		Type equation	Terms in equations
U. N. attack -	Enemy casualties	I	Air sorties (enemy strength)
		II	Total firepower
	Friendly casualties	I and II	None significant
Enemy attack -	Enemy casualties	I	Light artillery and heavy bombers
		II	Total firepower
	Friendly casualties	I and II	None significant
Patrols -	Enemy casualties	I	Light and heavy artillery
		II	Total firepower
	Friendly casualties	I	Enemy firepower and small negative coefficient for friendly firepower
		II	Enemy firepower

The coefficients for the equations are given in appendix E.

TABLE X

REGRESSION COEFFICIENTS FROM DATA SORTED ON MISSION AND ON GROUND GAINED OR LOST (LANCHESTER LOG LAW I)

	Firepower equation	U. N. attack: Mission	U. N. attack: Ground gained	Enemy attack: Mission	Enemy attack: Ground gained	Patrol: Mission	Patrol: Ground gained by U.N.	Patrol: Ground gained by enemy
Enemy casualties	Separate	.39	.67	.61	.67	.81	.80	.83
	Total	.28	.61	.69	.75	.61	.62	.76
Friendly casualties	Separate	.07	.06	.08	.11	.22	.01	.06
	Total	.07	.06	.01	.11	.14	.01	.06

TABLE XI

REGRESSION EQUATIONS FOR CASUALTIES FOR TWO INTENSITIES OF CONFLICT WHEN GAINING GROUND
(See Series 41 in table E-2)

Conditions	Battle level	Equation	R^2
U. N. advancing	Intense	FC 2295 + (47 AS_0 + 750 HB_0)	.67 (.69)
Enemy advancing	Intense	EC - (.27 LA) EC = (.0004 total ord)	68 (.74)
U. N. advancing	Static	EC -58 + (.12 LA + 0.10 HA)	.78 (.80)
Enemy advancing	Static	EC = + (.10 LA + .16 HA)	.83 (.85)

* *Where ord = tons of ordnance of the type expended.*

TABLE XII

EQUATIONS FOR FRIENDLY CASUALTIES FROM STRENGTHS AND GROUND GAINED
(Square kilometer – Series 48)

The equations were

Situation	R^2
U N battle attack – Friendly casualties = 1.5 ground gained by U. N.*	20 (.24)
Enemy battle attack – Friendly casualties = -15.0 ground gained by U. N.**	.17 (27)
U N patrol attack – Friendly casualties = 2 6 ground gained by U. N.*	.64 (.65)
Enemy patrol attack – Friendly casualties = .0009 enemy strength	.14 (14)

* *Enemy strength, the next term to be entered, is not statistically significant.*

** *Friendly strength the next term to be entered, is not statistically significant.*

Strengths and ground gained were forced into the equation, but firepower was not Three of the four equations have such low R^2 that they are not very credible. However, the signs of the coefficients are proper. The ground gained was obtained at a cost of 1.5-2.6 men per square kilometer, but lost at a cost of 15 men per square kilometer.

ground gained was included. (An enemy advance is recorded as a negative gain for the U.N. forces.) The equations appear in table XII.

Conversion of Empirical Equations to Lanchester-Type Equations

The independent variables tested in all equations were friendly and enemy strength, light and heavy artillery, and sorties of light and heavy bombers - and, in some equations, the same data for the preceding day.

TABLE XIII

REGRESSION EQUATIONS TRANSFORMED INTO LANCHESTER TYPE EXPRESSIONS
(From the equation denoted by an asterisk in table VII)

Friendly casualties

U.N. attack $FC = FS_{d-1}^{-.55} \cdot ES_{d-1}^{0.88} \cdot \exp(0.00063LA)$.

Enemy attack $FC = FS^{-2.5} \cdot ES^{2.8} \cdot \exp .037AS \cdot \exp(-.14HBS)$.

Patrol $FC = ES_{d-1}^{.026} \cdot \exp(.0009LA + .0008LA_{d-1} + .008AS_O - .008AS_{O,\, d-1}.)$

Enemy casualty equations

U.N. attack $EC = FS_{d-1}^{-1.3} \cdot E.S.^{.73} \cdot ES_{d-1}^{1.2} \exp(0.0017LA_{d-1})$.

Enemy attack $EC = FS^{1.9} \cdot F.S._{d-1}^{-1.6} \cdot \exp(.002LA)$.

Patrols $EC = \exp(.018HA + .014LA)$.

These equations in table XIII resemble the Lanchester equations, except that the exponents for friendly and enemy strengths are far from the 1.0 value that is generally used. The enemy strength exponents range from 0.73 to 2.8 during attacks, but are small or absent from the equations for patrols. The negative exponents for friendly strength are interpreted to mean that, as friendly strength increases, friendly losses decrease.

The R^2 values are higher for all but one of the empirical equations than for the Lanchester equations (see table XIV).

TABLE XIV

COMPARISON OF R^2 VALUES FOR LANCHESTER LINEAR LAW I AND BEST EMPIRICAL EQUATIONS

Friendly casualties	Lanchester linear I law	Empirical equations*
U. N. advance	.07	.42
Enemy advance	.08	.57
Patrol	.22	.70
Enemy casualties		
U. N. advancing	.39	.67
Enemy advancing	.61	.78
Patrol	.81	.61

*From tables VII and XIII.

An attempt was made to classify the equation into an appropriate Lanchester equation. If the negative exponents are regarded as secondary effects, the friendly casualties follow the square law; if the presence of both friendly and enemy strengths is the criterion (even though the coefficients are far from 1.0), however, the linear law applies. Similarly, the numbers of enemy casualties inflicted during U. N. attacks follow the logarithmic law (neglecting negative exponents) or a modified square law. In patrols, the enemy casualties follow one of the linear laws.

The fire support terms that appear in friendly casualty equations have small coefficients (relative to those for enemy casualties) and probably are, simply, collinear effects. One should not draw the absurd conclusion that the more U. N. fire support there is, the heavier are the casualties.

Other Comparisons

Several measures have been used to measure battle intensity. Two are illustrated:

- Constants are derived from Lanchester's square laws:

  Friendly casualties (FC) = K · Enemy strength
  Enemy casualties (EC) = K' · Friendly strength

The constant $K'' = \frac{K'}{K} = \frac{ES}{FS} \cdot \frac{EC}{FC}$ was computed for the May 1951 data in table XIV. An extreme high value of 228, representing heavy enemy attrition, was inflicted on May 20 by the 2nd U. S. Division. The situation was the reverse on 18 May: The value for the ROK 5th Division, which collapsed, was lower than the ratio for the ROK 7th Division, which survived.

Table XV also demonstrates the paucity of data that precluded meaningful analysis of the effect of adjacent units. During the Soyang battle, there were often only 2 adjacent divisions providing data.

In the search for regression expressions for conflict results, a number of plots were made of the battle results of May 1951. Helmbold (reference (m)) proposed a number of measures, such as "advantage," geometric mean of the "activity ratio," "bitterness" and "intensity."

The following derivation was used to convert the available daily strengths and casualties of opposing forces to "bitterness."

Helmbold's Bitterness:

$$\frac{X_o}{Y_o} = \text{initial force ratio of attacker to defender}$$

$$\frac{X}{X_o} = a = \text{surviving fraction of attacker}$$

$$\frac{Y}{Y_o} = d = \text{surviving fraction of defender}$$

For the Korean War data, where replacements occurred but the numbers of replacements were unknown, a method was devised for treating data on a daily basis: Friendly and enemy force strengths (FS and ES) at the beginning of each day are recorded; friendly and enemy casualties (FC and EC) are then subtracted to give the strengths at the end of the day. We follow the convention that the enemy always attacks. Then X and Y are the strengths at the end of the day and are equal to (FS-FC) and (ES-EC). Helmbold defined a quantity μ as follows:

$$\mu^2 = \frac{1 - a^2}{1 - d^2}$$

TABLE XV

VALUES OF AN EXCHANGE COEFFICIENT $K = \frac{ES \cdot EC}{FS \cdot FC}$ FROM LANCHESTER'S SQUARE LAW, MAY 1951 KOREAN WAR

Divisions	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
ROK 1																
CO 1																
U 25																
U 24	13	9 66	26 2	8 1	4 5											
ROK 2	--	--	--	--	--											
ROK 6	22	1 34	17	05	( )	( )										
U 07	41	41	13	41	40											
M 01	( )	13	13 5	27 6	31	11 3	1 1	1 6	0 8	0 02	6 45	54	34	60	15	
U 02	154 0	69 0	66 4	55 3	13 7	140 0	62 4	9 6	3 0	9 34	69 4	10 7	61 4	42 2	65 0	
ROK 5	0 08	1 9	42	21	25	24	22	14	.09	011	27	0 08	68	1 16	1 03	
ROK 7		1.84	4 29	12	47	46	70	2 6	2 6	3 64	04	04	13	064	17 4	
U 03																
ROK 4																
ROK 9																
ROK 4																
ROK 11																
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
ROK 1																
C 01																
U 25																
U 24																
ROK 2																
ROK 6			292 7													
U 07		11 0														
M 01	4 3	11 8	5 2	12 7	41 1	13 2	56 7	24 6	37 3	21 3	4 0	11 8	30 2	7 2	17 9	1.5
U 02	126 1	21 6	6 8	6 4	228	82 9	29 2	27 9	85 4	48 4	7 2	34 8	9 5	3 8	11 0	12 6
ROK 5	0 02	35	12 82													
ROK 7	58	0017	0014	( )	13	69	152 6	( )	( )							
U 03					1 69	6 0	3 79	2 34	14	66	1 7	2 0	5 6	15 0		
ROK 4																
ROK 9																
ROK 4																
ROK 11																

Substituting $\frac{ES-EC}{ES}$ for a and $\frac{FS-FC}{FS}$ for b, the equation becomes:

$$\mu^2 = \frac{1-\left[\frac{ES-EC}{ES}\right]^2}{\left[\frac{FS-FC}{FS}\right]^2}$$

$$\mu^2 = \frac{FS^2}{ES^2}\left[\frac{2FS \cdot EC - EC^2}{2FS \cdot FC - FC^2}\right] \tag{3}$$

Helmbold's equation 5 is:

$$\frac{D}{A} = \left[\frac{1-a^2}{1-d^2}\right]\left[\frac{X_o}{Y_o}\right]^2 = \mu^2 \left[\frac{ES}{FS}\right]^2 \tag{4}$$

where D and A are the coefficients for the Lanchester square law.

Substituting (3) into (4):

$$\frac{D}{A} = \left[\frac{2ES \cdot FS - EC^2}{2FS \cdot FC - FC^2}\right] \tag{5}$$

From this expression, D and A are equal to the numerator and denominator respectively.

Helmbold gives a relationship

$$\lambda = \sqrt{AD} = \epsilon \cdot t^{-1}$$

where λ is the geometric mean of the defender and attacker activity, and ϵ is the bitterness. On a daily basis, where $t = 1$, the equation

$$\frac{\lambda}{t} = \epsilon = \sqrt{\frac{AD}{t}} \quad \text{becomes} \quad \epsilon = \sqrt{AD} \tag{6}$$

or by substituting A and D from (5) into (6) ,

$$\epsilon = \sqrt{(2\,ES \cdot EC - EC^2)\ (2FS \cdot FC - FC^2)} \tag{7}$$

A plot of this function against time for various divisions is figure 10. When these results are compared with plots or other views of the battle, "bitterness" generally is higher when ground is changing hands and ordnance usage is high, but the results are not spectacular. Omission of firepower from the equation is believed to be a serious deficiency.

Comments

The equations for a given dependent variable and combat condition agree in general form when the R^2 values are high, even though the equations may have been cast in different forms, such as the Lanchester equations. When the explanation is poor, the terms for different types of equation are inconsistent and therefore worthless.

- Conclusions are drawn from various equations, but equations containing all terms could not be confirmed simultaneously.
- Divisional data is too aggregated to give very high R^2 values, probably because of differences in engaged forces. This is demonstrated by the variation in exponents on force strengths from near -3 to near +3.
- The enemy casualties can be forecast reasonably well from the intensity of battle, the exposure of troops (as indicated by advancing, holding, or retreating), force strengths, and firepower.
- Air is most important when U. N. forces advanced, and artillery was more important for defense.
- Friendly casualties cannot be forecast well, primarily because the casualties relate directly to enemy strength data, which is very poor. Another reason is the care taken by the U. N. command to minimize casualties among friendly troops. The enemy had no air or artillery in the sector during this period.
- Casualty equations are different for different intensities of battle. In patrol periods, strengths are not important.
- When ROK units collapsed, the losses were greater than predicted by the equations which applied to intact forces.
- Relationships are clearer when the data is based on a per mile of front or per mile advanced.
- The effect of prebombardment, and of supporting adjacent units, were demonstrated but not very well.

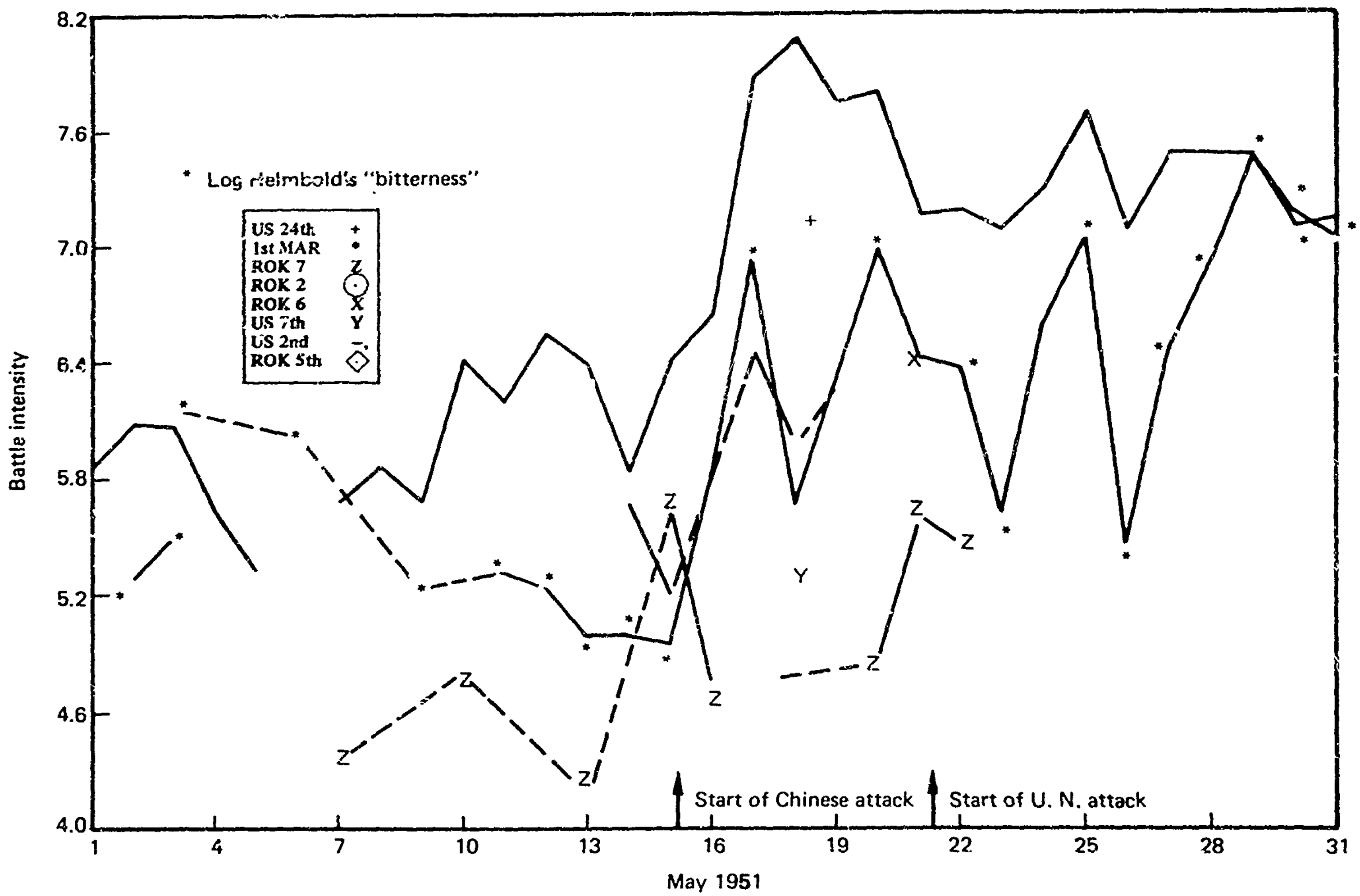

FIG. 10: BATTLE INTENSITY* FOR DIVISIONS KOREAN WAR, MAY 1951

- The effect of air interdiction of the enemy approaching the battle lines was not evaluated. The shifting emphasis in objectives from day-to-day by troop commanders and lack of knowledge of how to combine the results obtained so far for various phases of battle, were the limiting factors.

If the work on regression is continued, then several steps should be taken to improve the data and test the validity over a wider range of conditions:

- Several battles could be selected in which forces differ markedly in composition such as – Khe Sanh in Vietnam.
  - The El Alamen tank battle of World War II.
  - U.S. Army's Italian peninsula campaign
  - Inchon invasion, Korea.
- The enemy strength and casualty data should be examined more critically to see whether estimates are made in the same manner by various forces. Apparently the 2nd Army Division estimated greater casualties than the 1st Marine Division. The casualties may be estimated from the quantity of ordnance fired and may be fallacious.
- Future correlation analyses should be tried on units smaller than divisions, thus providing better estimate of truly engaged forces. The data is in appendix B of volume VIII which was not available until the regression analyses were completed.
- Improvement in the models should be made so that interdiction is accounted for.
- More data is needed about the terrain in which the battles were fought.
- Firepower scores might be introduced if the ratio of aimed fire and harassing and interdiction fire data were available.
- Data for collapsed units should be treated separately but the number of instances in May 1951 is too small to attempt characterization by regression. If a larger data base provided more cases, the characteristic leading to failure could be examined.
- When ROK units collapsed, the losses were greater than predicted by the equations applicable to intact forces.

- Relationships are clear when the data is based per mile of front or mile advanced.

- The effects of pre-bombardment and of supporting adjacent units were demonstrated, but not very well.

- The effect of air interdiction of the enemy approaching the battle lines was not evaluated. The limiting factors were the shifting emphasis in objectives from day-to-day by troop commanders, and our lack of knowledge of how to combine the results obtained thus far for various phases of the battle.

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(p) "USAF Far Eastern Air Force Bomber Command VIII," Part 4, Feb-Jun 1951, "Final Mission Summaries," (2-5593-5) (borrowed from Maxwell AFB, Ala.) Unclassified

(q) Weather records obtained from the Naval Weather Environmental Detachment, Ashville, N. C., via the Commander Naval Weather Service Command:

- Synoptic coded weather for Korea and Japan stations at 12-hour intervals, May 1951
- Weather maps derived from the synoptic codes, May 1951
- Nine reels of 35mm microfilm records for surface weather at 7 stations in Korea, Tokyo Weather Central maps for surface and the following upper air charts: 850, 700, 500, 300 and 200 mb and U. S. Weather Bureau sea level extended forecast for most of the Korean War

(r) Reference (r): Not used.

(s) USAF "Final Gun Evaluation Report," Confidential 1 Oct 1955

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APPENDIX A

DATA SOURCES

The data for U. N. and Chinese/North Korean strengths, casualties and fire support during the Korean War is taken from annex A-2 of volume VII.

The data for the 2nd U. S. Army Division for the period from February to June 1951 is plotted in figures A-1 through A-7. This information, used in early regression analyses, shows the trends over a longer time than is discussed in the body of the report.

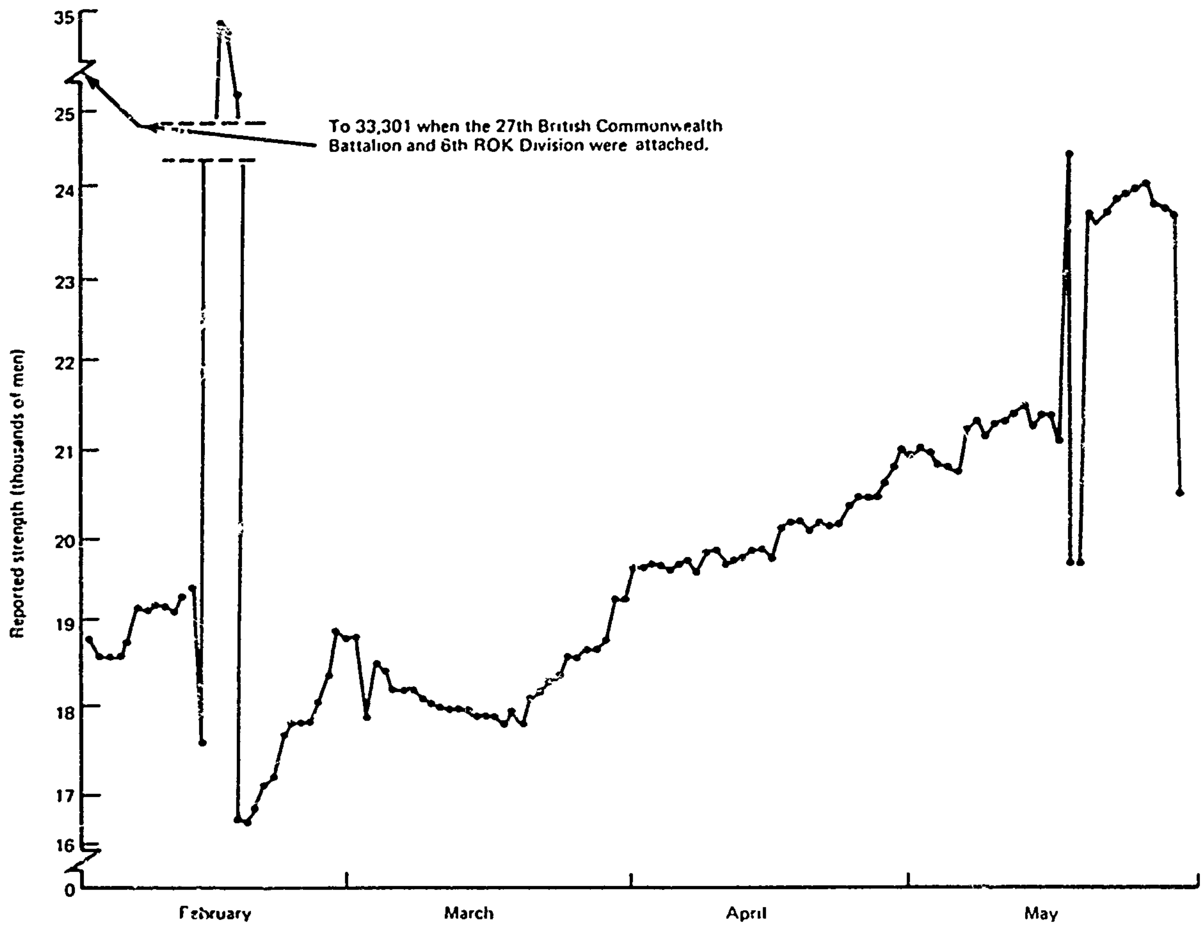


FIG. A-1: DAILY STRENGTH OF THE 2ND DIVISION ON THE KOREAN FRONT FEBRUARY THROUGH MAY, 1951

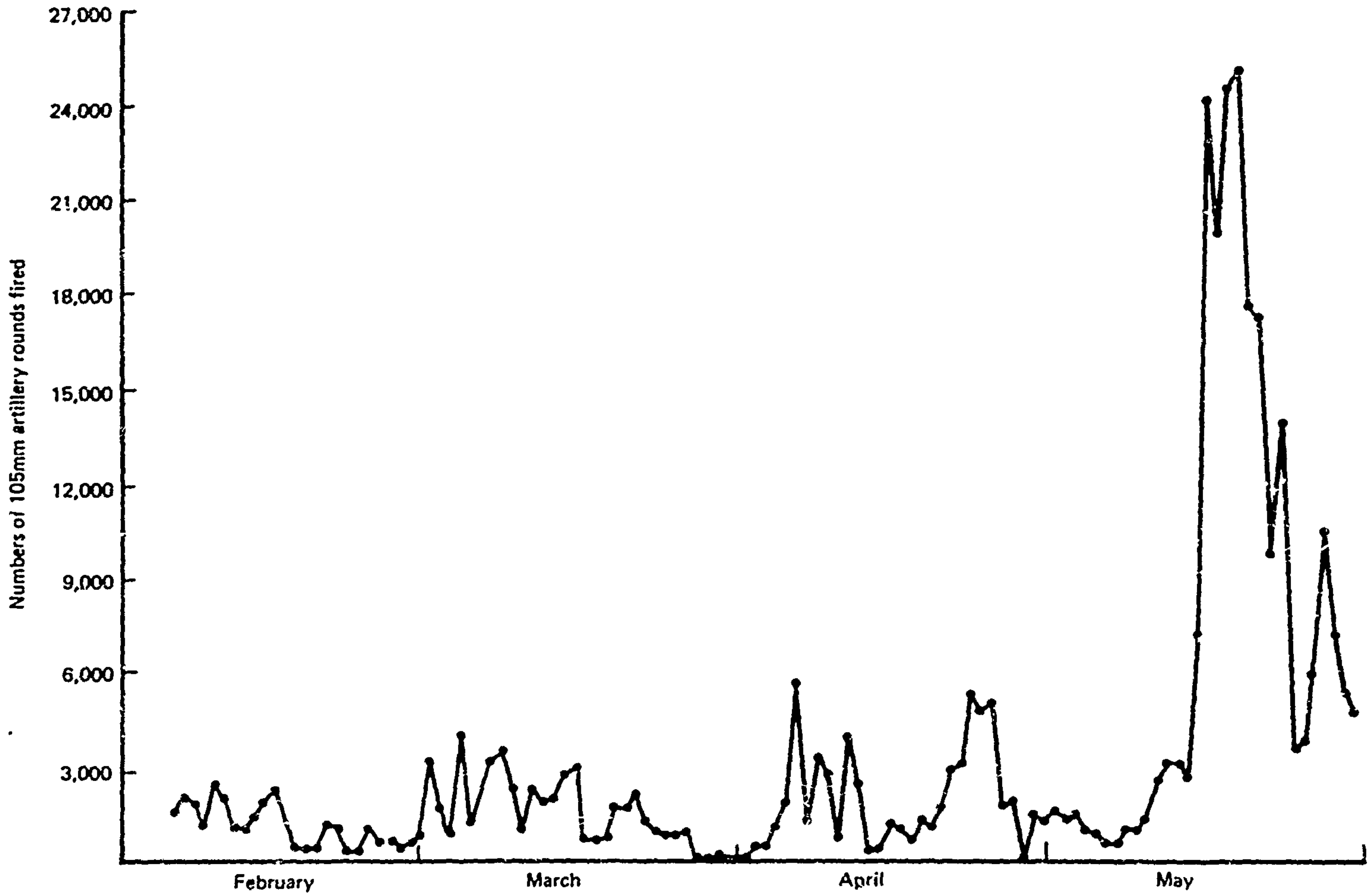


FIG. A-2: ROUNDS OF LIGHT ARTILLERY FIRED IN SUPPORT OF 2ND DIVISION FEBRUARY THROUGH MAY, 1951

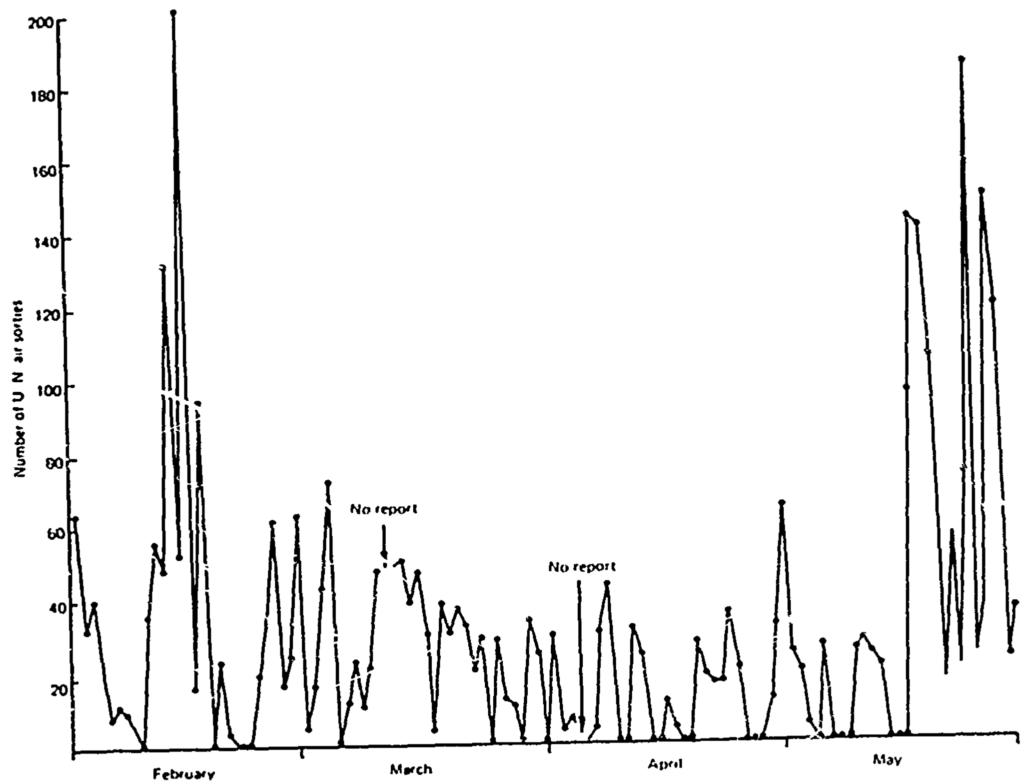


FIG. A-3: CLOSE AIR SUPPORT SORTIES REPORTED IN 2ND DIVISION'S DAILY RECORDS FEBRUARY THROUGH MAY, 1951

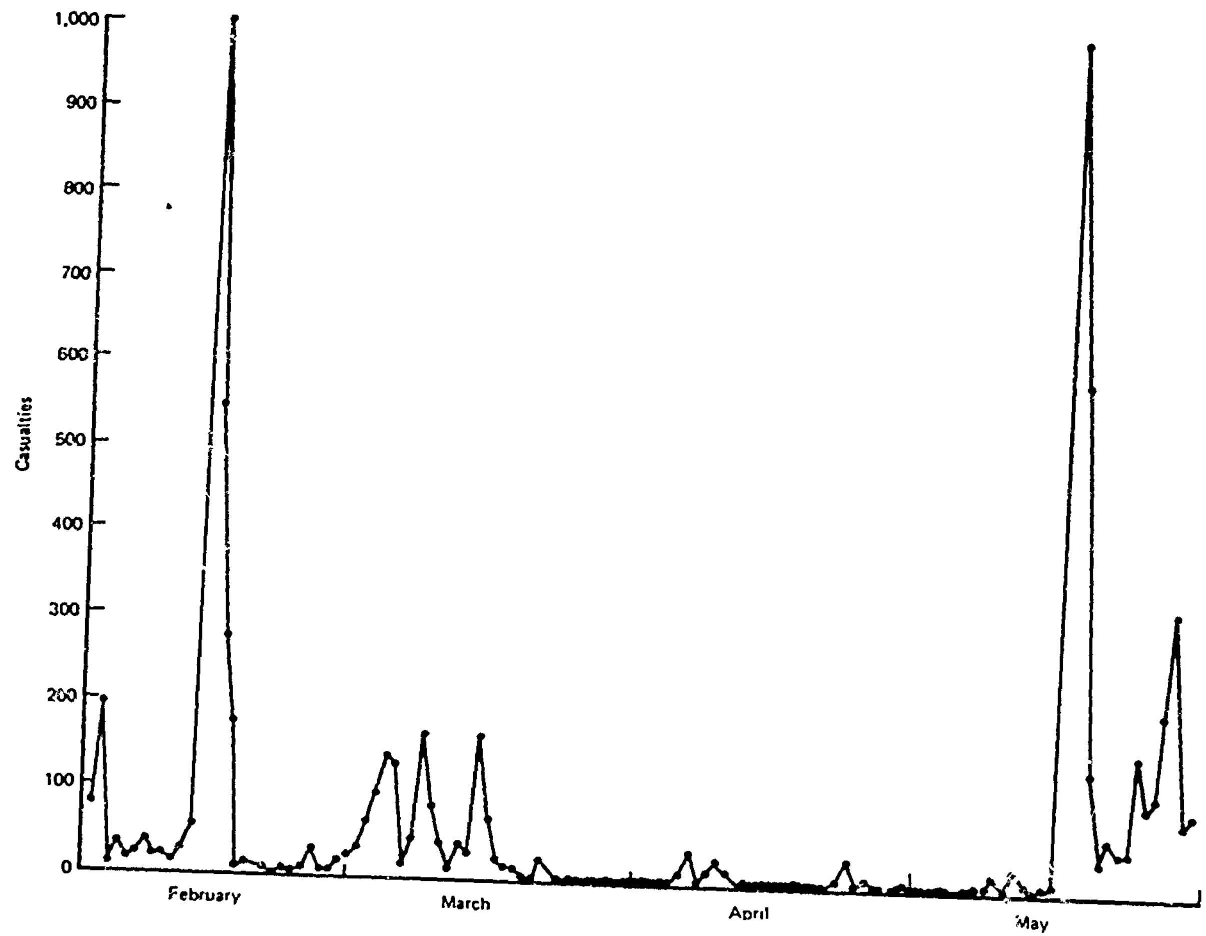


FIG. A-4: CASUALTIES SUFFERED BY THE 2ND DIVISION
FEBRUARY THROUGH MAY, 1951

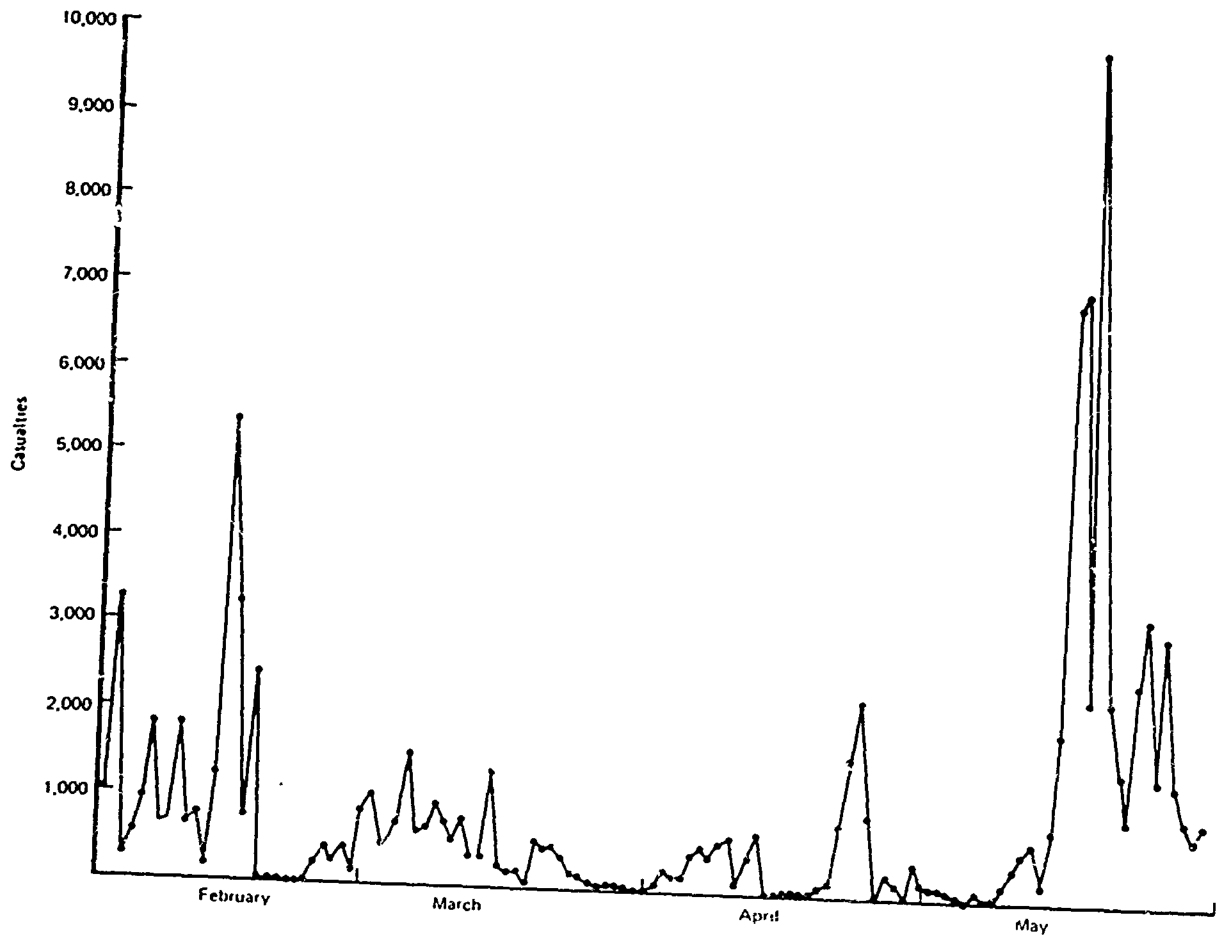


FIG. A-5: ENEMY CASUALTIES CAUSED BY GROUND ACTION ON THE 2ND DIVISION FRONT FEBRUARY THROUGH MAY, 1951

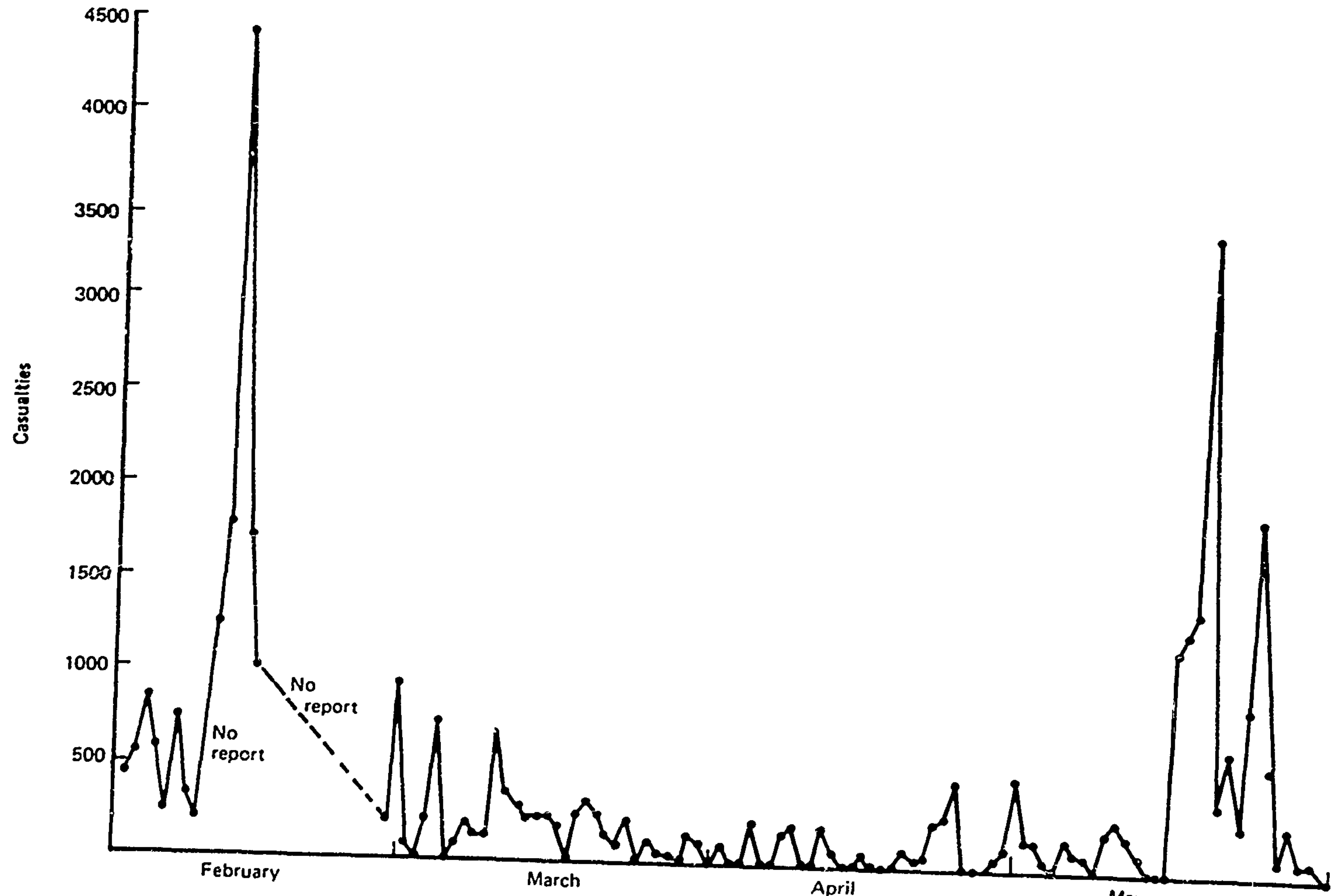


FIG. A-6: ENEMY CASULATIES INFLICTED BY AIR MISSIONS SUPPORTING THE 2ND DIVISION FEBRUARY THROUGH MAY, 1951

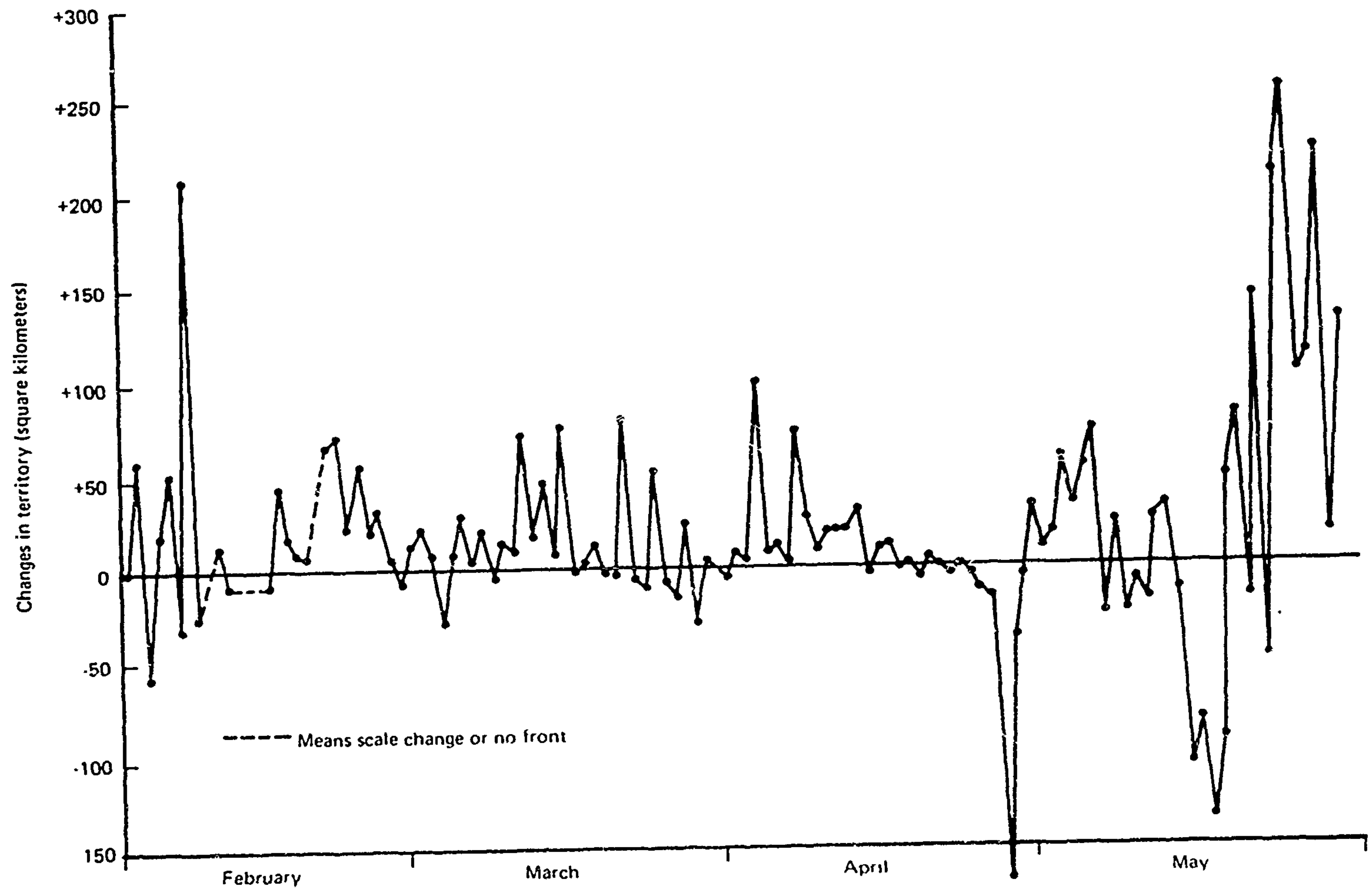


FIG. A-7: GROUND GAINED OR LOST EACH DAY BY THE 2ND DIVISION FEBRUARY THROUGH MAY, 1951

APPENDIX B

EFFECT OF WEATHER ON AIR OPERATIONS IN THE KOREAN WAR, May 1951

HISTORICAL RECORD

General Ridgway, (reference (a)) discussed the effect of weather on military operations several times. Pertinent excerpts follow:

p. 104 - "The worst of the winter would be over by March and the heavy rains and heavy cloud cover normally expected in June, July, and August would turn large areas into mudholes, make many roads impassable, wash out culverts and bridges in mountains and reduce the effectiveness of close air support."

p. 114 - (On Operation Ripper for recapture of Seoul, opening on 7 March to 15 March 1951.)
"Weather and terrain gave us more trouble than enemy action, particularly in the central zone, where mountain peaks thrust into the clouds and precipitous slopes dropped into valleys hardly wide enough for a cart road."

p. 116 -(On same operation on trying to trap forces by air drops and tank assault.)
"Heavy rains and melting snows mired our tanks so that they could do nothing but pull out and return to Seoul. And by the time the 187th RCT reached the commanding heights, the enemy had pulled back still farther north."

p. 118 -In discussing poor reports, Ridgway noted a lack of weather data.

p. 160 -On Operation Dauntless, April 1951.
"A few days of bad weather would make many of the roads useless and cut down on our air support, perhaps making it necessary to stop the attack or even, if opposition were very strong, to pull back in places."

p. 166 -Extract from the letter of instructions to the Commanding General, 8th Army.
"...the U.S.S.R. may at any time elect to exercise present capabilities by the direct military intervention of its armed forces...coordinated with the...Chinese Communist and North Korean Peoples Army military forces, all so timed as to take maximum advantage of weather and of its effect on terrain."

p. 179 -"In the last week of May (1951), the weather came to the enemy's rescue, too slowing our armor, almost wiping out many of the roads, and grounding our aircraft. As a result, the enemy was again able to trade space for time and make off with much of his force and supplies intact."

QUANTITATIVE ASSESSMENT

Thus, weather was apparently the main cause of drastic reductions in sorties during the month of May, as is shown in table B-I and in figure B-1. The fighter/light bomber sorties varied from a maximum of 806 to a minimum of 91 a day. Sometimes the aircraft could not operate from the airfields or the carriers, (references (o) and (p)). Furthermore, primary targets sometimes could not be attacked because of fog, low ceilings, mountainous terrain, or low visibility in the target area, and secondary or tertiary ("last resort") targets were struck, instead. Studies of this kind generally pay little attention to the effects of weather; this data was therefore of particular interest for the preparation of estimates of future capability.

The drastic and frequent changes in the airpower that could be brought to bear are summarized in figure B-1, which shows both total sorties and those flown in support within 30 miles of the front lines. The "rain" notations were taken from the records of only a few of the divisions. Although low flight activity was always associated with rain, many rainy days were marked by good support. Furthermore, close support missions had highest priority during bad weather, but relatively few sorties were flown.

Estimates were made of the total numbers of sorties available during the first half of the month and the latter half because the former period was relatively static and the latter was a period of intense battle on the Soyang River. The data of figure B-1 was replotted in figure B-2 with the total number of sorties plotted against the 0-30 mile support sorties. Curves were faired through the data, and the medians of table B-I were obtained.

TABLE B-I

AIR SORTIES UNDER VARIOUS CONDITIONS, MAY 1951

		Number of days	Number of daily sorties		Median number of air sorties	
			Total	0-30	Total	0-30 mi.
Static period 1-14 May	No rain	11	806 (max)	334 (max)	700	300
	Rain	3	130 (min)	78 (min)	450	150
Heavy attack 15-31 May	No rain	10	800 (max)	581 (max)	720	180
	Rain	7	91 (min)	77 (min)	450	130

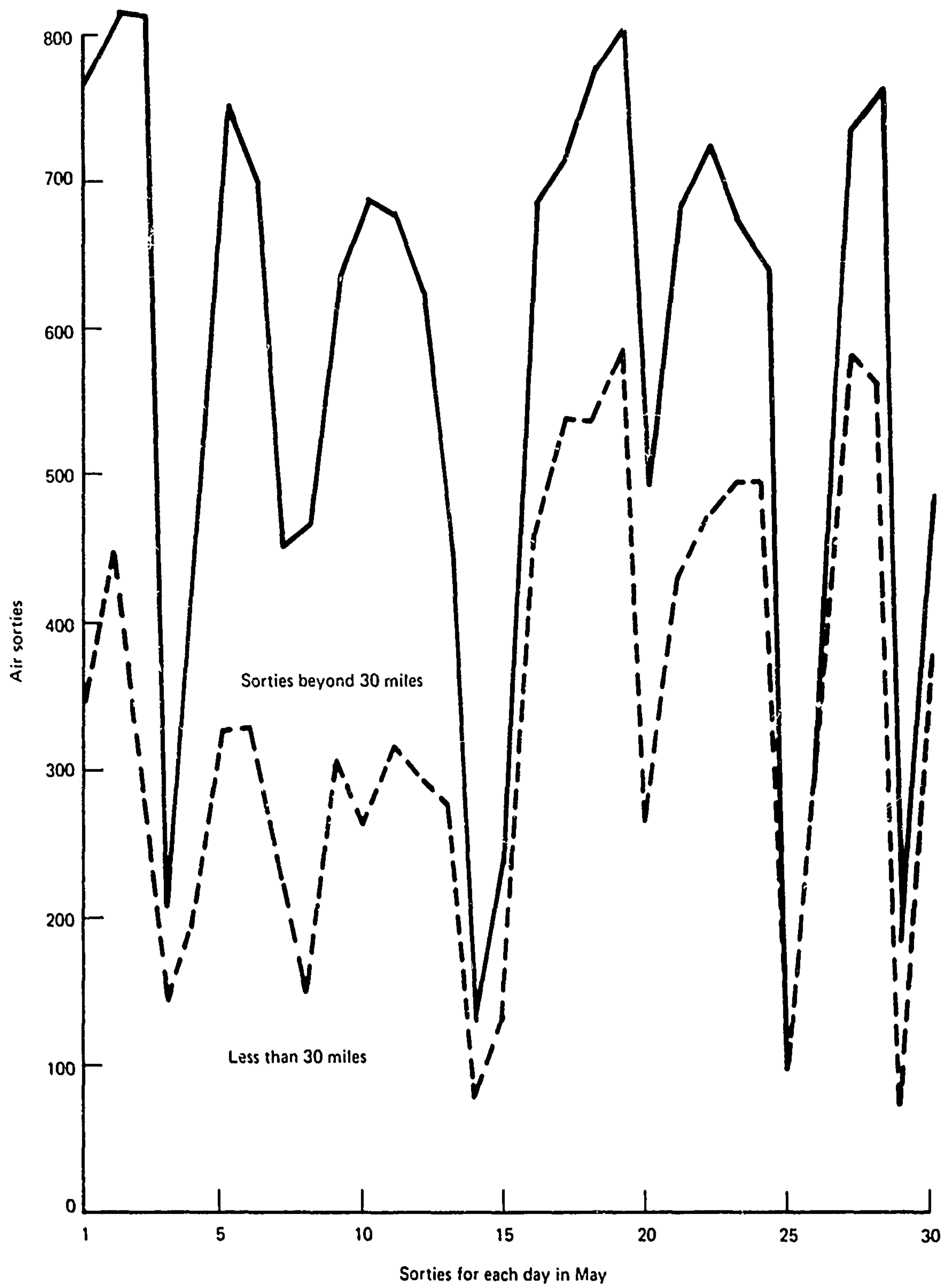


FIG. B-1: ATTACK SORTIES BY ALL SERVICES, KOREA, MAY 1951

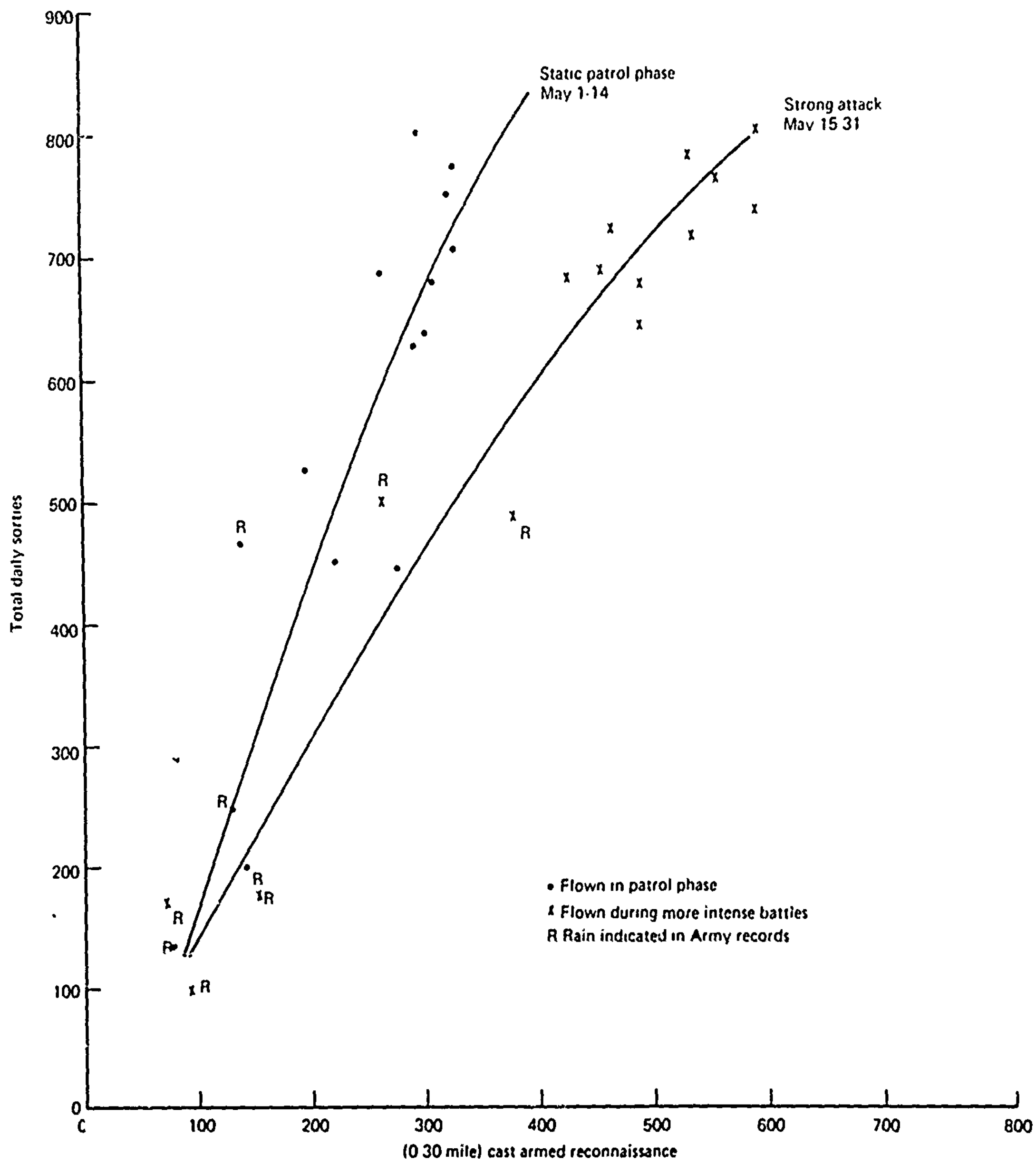


FIG. B-2: AIR SUPPORT SORTIES BY MISSION AND WEATHER

This data on frequency of rainy weather for May is in agreement with information about Korean weather and climate. The weather is worse in the summer and considerably better in December. Extensive weather records were obtained from the Naval Weather Service Environmental Detachment, Asheville, N. C., for May 1951 (reference (q)). Cursory examination suggested that weather front movement through or near the Korean theater was responsible for the reduction in the number of support missions. However, further analysis is required to determine the extent to which effects of weather at the front or at bases influence support to each ground unit.

The effect of weather on the missions also can be compiled from references (n) and (o), which show the curtailment of air activity attributable to weather near carriers and B-29 bases and at B-29 targets.

The reduction in sorties used to test the effects of weather on the sensitivity of TWSP results reported in appendix C of volume VI were derived from this source.

The great reduction in support during bad weather shows the value of all-weather capability at the bases on land and on ships and for radar-controlled close support missions. During this period the MPQ-2 radars were being introduced and were used to control all B-29 and some B-26 close support missions. An improved radar, the TPQ-10, currently in use in Vietnam, controls many Marine Corps close support missions. The all-weather capability for CVA operations has been improved drastically since the Korean War; future estimates must take these factors into account.

APPENDIX C

ESTIMATING ORDNANCE CAPABILITIES

Ordnance for Air Sorties

During battles, ordnance usage increases, and there is a high correlation between ordnance types. To estimate the consequences of eliminating this correlation, we added the ordnance tonnage delivery by aircraft and artillery. The best form of trade-off between ordnance would be based on the cost of destroying a target; with most historical data, however, the delivery accuracy and weapon lethality for each fire mission or air sortie are not available for aggregation. Nor can it be determined whether ordnance was delivered on an ill-defined target. In spite of these reservations, the tonnage delivered is a clear measure of the decision to provide fire support, and the logistic consequences of that decision.

For estimating the daily tonnage of air-delivered weapons, the ordnance weight per sortie was averaged over all aircraft in the theater. Table C-I lists deliveries of air ordnance from carrier and land-based Marine aircraft in May (see reference (o)).

TABLE C-I

NAVY-MARINE CORPS ORDNANCE, MAY 1951

Ordnance	Number of rounds	Estimated unit weight (tons)	Total weight (tons)
100 bombs	7,756	0.05	388
250 bombs	1,520	0.125	190
500 bombs	2,941	0.25	735
1000 bombs	1,129	0.50	564.5
2000 bombs	260	1.0	260
100 fragmentation bombs	246	0.05	12
220 fragmentation bombs	980	0.11	108
260 fragmentation bombs	13,768	0.13	1,790
1000 SAP	3	0.5	1.5
Napalm tanks	4,335	.375	1,620
ATAR rockets	3,639	.10 est.	364
5" and less HVAR rockets	12,030	.0575	692
Torpedoes	8	1.0	8
50 caliber	1,727,000	(.125T per 1000 rounds)	216
20 mm	1,563,000	(.03T per 1000 rounds)	470
	Total weight		7,419
	Total sorties		10,105

* *These figures for ammunition weights per sortie were taken from reference(s).*

** *Napalm was placed in Japanese external fuel tanks, each tank arrying about 100 gallons, according to pilots who flew these missions.*

Air Force ordnance expenditures for close support sorties were taken from data in volume VII. Air Force and Navy ordnance expenditures per sortie were similar--0.87 and 0.73 tons, with an average of 0.80 tons for all sorties. The B-29's carried 8.54 tons each.

TABLE C-II

AIR FORCE ORDNANCE, MAY 1951

	Close support sorties	Avg. tons per sortie*	Total Weight
F-51	3685	.98	3611.3
F-80	3898	.55	2143.9
F-84	1679	.455	763.9
B-26	1696	1.75	2968
Total Air Force	10,958		9487
Total Navy/Marine Corps	10,105		7419
Total	21,063		16,906

The ordnance expended by Navy/Marine Corps air on 19 May 1951 was close to these figures (0.73 overall and 0.74 for sorties close to the battle line), as shown in table C-III.

In May, almost all heavy artillery was 155mm with the exception of a few 8-inch rounds. The difference in weight between 155mm and 8-inch is insignificant compared with the complete omission of the expenditures of tank, mortar, machine gun, and small arms ammunition. The latter are assumed to be proportionate to troop strength.

In estimating the quantity of ordnance delivered, we multiplied these factors in table C-IV by the number of rounds fired and totaled them into a new column of data for the regression equations. Results of sample calculations that include both air-delivered and artillery ordnance are displayed in table C-V.

*Weights of bombs, 5" FFAR and 50 caliber ammunition, and NAPALM were included.

TABLE C-III

ROUNDS OF AMMUNITION EXPENDED IN NAVY SORTIES
19 MAY 1951

Dist. from battle line (statute miles)	Aircraft type	Sorties	HVAR	ATAR	100#	220#	260/265#	500	1000 #	Napalm	20mm	50 cal.	Total tons	Tons/ sortie
0-15	F4U-4/4B	106	67	4	37	–	49	4	–	111	–	115		
	AD-4/4N	11	–	–	–	–	17	11	–	11	13	–		
	F9F2/2B	6	–	6	–	–	–	–	–	–	6	–		
	F4U-5/5N	17	8	9	–	–	2	11	–	17	17	–		
	AD2/3-2	32	–	–	–	–	49	23	–	54	49	–		
	Total	172	75	19	37	0	117	49	0	193	85	115		
15-30	F4U-4/4B	8	8	–	–	–	8	–	–	8	–	12		
	F9F-2/2B	4	2	6	–	–	–	–	–	–	8	–		
	F7F-3N	3	2	–	–	–	–	2	–	1	4	–		
	F4U-5/5N	3	–	–	–	–	3	2	–	1	1	–		
	Total	18	12	9	0	0	11	4	0	10	13	12		
Over 30	F4U-4/4B	37	37	2	74	–	–	2	–	69	–	40		
	AD-4/4N	7	–	–	–	2	5	5	–	7	7	–		
	F9F-2/2B	35	20	50	–	–	–	–	–	–	60	–		
	F7F-3N	5	4	–	–	–	–	–	–	3	3	–		
	F4U-5/5N	16	4	6	–	8	6	17	1	11	21	–		
	AD-2/3-2	14	–	–	–	–	26	16	–	22	14	–		
	Total	114	65	58	74	10	37	42	1	112	105	40		
Tons of ammunition (with cases) by type														
0-15		172	4.3	1.9	1.9	0	15.2	12.3	0	72.5	5.5	13.3	126.9	.738
15-30		18	.7	0.6	0	0	1.4	1.0	0	3.8	0.8	1.0	8.4	.466
Over 30		114	3.7	5.8	3.7	1.0	4.8	10.5	0.5	42.0	6.4	4.6	83.0	.730
	Total	304								118.3			218.3	.72

TABLE C-IV

ARTILLERY ORDNANCE WEIGHTS

Type	Nominal round	Weight (tons)
Light artillery	105mm(36#)	18 t./1000 rounds
Heavy artillery	155mm(96#)	48 t./1000 rounds

Ground Force Weapons

The fire from rifles, machine guns, rockets, and mortars was estimated at 40 pounds--or 0.002 tons--per man. This was about the amount fired at Guam in WW II; no data was obtained for the Korean battles.

CALCULATION OF ORDNANCE PER MILE OF FRONT

Comparison of ordnance used in the Soyang River battle (Korea) and WWII.

The maximum total quantity of ammunition expended per mile of front in the two battles--the Soyang River battle (May 1951) and Guam (WWII)--was about 50 to 80 tons per mile, as shown in figure 8* and table C-V.

The Guam data was used to estimate that a maximum of 12 tons ordnance was fired by ground forces per mile of front. The figures are only approximate because measurements differ as a result of different interpretations of discontinuities in the battle line. Front lengths for the various divisions in the Soyang battle on 19 May 1951 were:

Statute miles	Statute miles
7.5	13
7	13
5	9
8.5	7
11.5	6
4	8
3.5	6
14	7

These measurements are consistent with Guam data (see table C-VI). In some instances, because discontinuities between units are included, these measurements are slightly larger than those reported in table I. Overall, the Guam campaign used 43 tons per mile of front, and peak requirements are as shown.

* Total air + artillery tonnage (figure 8) plus estimate of ground force weapons estimate.

TABLE C-V

SAMPLE CALCULATION OF ORDNANCE SUPPORT

	1st Marine Division				2nd U. S. Army Division				
	May 7	17	18	19	16	17	18	19	20
Rounds artillery									
Light	129	3,373	11,754	11,787	7,393	24,021	19,833	24,346	25,044
Heavy	152	2,437	4,429	3,420	1,917	3,749	3,107	3,448	3,168
Air sorties									
Bombers	13	24	31	75	2 (WX)	60	63	65	91
Heavy bombers	0	0	4	2	0	2	5	12	17
Artillery ordnance, tons									
Light	2.3	81	211	212	133	434	357	438	451
Heavy	7.3	117	212	165	92	180	149	165	153
Total	9.6	198	423	377	225	614	506	603	604
Air ordnance (0-15 miles), tons									
Bombers	10	18	23	56	2	48	50	52	73
Heavy bombers	0	0	34	17	0	17	43	103	145
Total	10	18	57	73	2	65	93	155	218
Total ordnance, tons	19.6	216	480	450	227	679	599	758	822
Percentage air ordnance	54	8	12	16	0.7	11	16	20	16
Front length, miles	8.5	8.3	8	11	8.6	10.3	10	12	14.2
Total ordnance, tons per mile front									
Artillery	1	24	53	34	26	60	51	50	43
Artillery and air	2.3	26	60	41	26	66	60	63	58

TABLE C-VI

ORDNANCE FIRED -- GUAM

Month	Date	Length of front (miles)	Total ammunition	Infantry weapon	Tons fired per mile of front: Infantry weapons	Fire support	Total	Remarks
July	21	5.1	2400	90				
	22	9.4	580	90	10	53	63	
	23	12.2	610	90	7.4	42.6	50	
	24	14.7	770	110	7.5	44.5	52	
	25	14.0	730	130	9.3	42.7	52	Attack against
	26	12.8	1000	130	10.1	68	78	dug-in Japanese
	27	14.7	990	130	8.8	56	67	
	28	13.1	660	130	9.9	40	50	
	29	12.2	480	90	7.4	32	39	
	30	12.2	210	20	1.6	15	17	Lull in fighting
	31	6.1	220	25	4.1	32	36	
						41		
August	1	4.8	220	25	5.2	41	46	
	2	4.8	250	20	4.2	48	52	
	3	6.1	370	70	11.5	49	61	Jungle
	4	5.1	640	120	23.6	101*	125	
	5	8.1	450	90	11.4	45*	56	warfare
	6	7.7	440	90	11.7	55	57	
	7	9.3	550	110	11.8	43	55	
	8	13.2	380	80	6.0	23	29	
	9	13.2	310	90	6.8	16	23	
	10	13.2	40	10	.8	2	3	

* Discontinuous lines

APPENDIX D

INTRODUCTION TO REGRESSION ANALYSIS AND A REPORT OF EARLY ANALYSIS ON MARCH, APRIL, AND MAY 1951 DATA

Regression analysis is used to fit curves between two or more variables in a collection of data. The curves may be linear or complex, and there may be families of curves if extra variables are recognized. When the fit is perfect and all of the variations are explained by the curve, the squared regression coefficient (R^2) is 1.0; if no relationship is found, R^2 is zero. In short, R^2 is a measure of the fit of the battle data.

The approach used in the regression analysis was one of trial-and-error, but following several definite lines:

- Obtain militarily useful results from division records, if possible.

- Obtain equations that explain a high proportion of variability in the data.

- Examine the effects of variables that have been believed to cause large effects such as mission, weather, terrain, adjacent units and test relationships noted from plots of original data and of residuals.

- Attempt to delete correlations that were high but of no value for planning by combining the correlated variables properly.

- Test the Lanchester equations, both linear and square and logarithmic versions, seeking to learn how best to introduce fire support.

Initial Regression Analysis

In the first battle situation, the U. N. forces were on the offensive, attempting to gain ground. A sample of X Corps data comprising 63 such division-days was taken from April and May 1951 for the following units:

2nd Division	for	April 5-15
7th Division	for	April 5-16
5th ROK Division	for	April 6-16
2nd Division	for	May 22-31
3rd Division	for	May 21-28
1st Marine Division	for	May 21-31

Plots of various kinds of data were made to determine the best statistical transformation for fitting a line through the scattered points.* The scatter plots show that the variability of the data increases significantly with the size of the effect being measured. This is seen clearly in figures D-1 and D-2, in which friendly casualties are plotted against air sorties, and enemy ground

*The three symbols in each figure represent the numbei of identical data points.

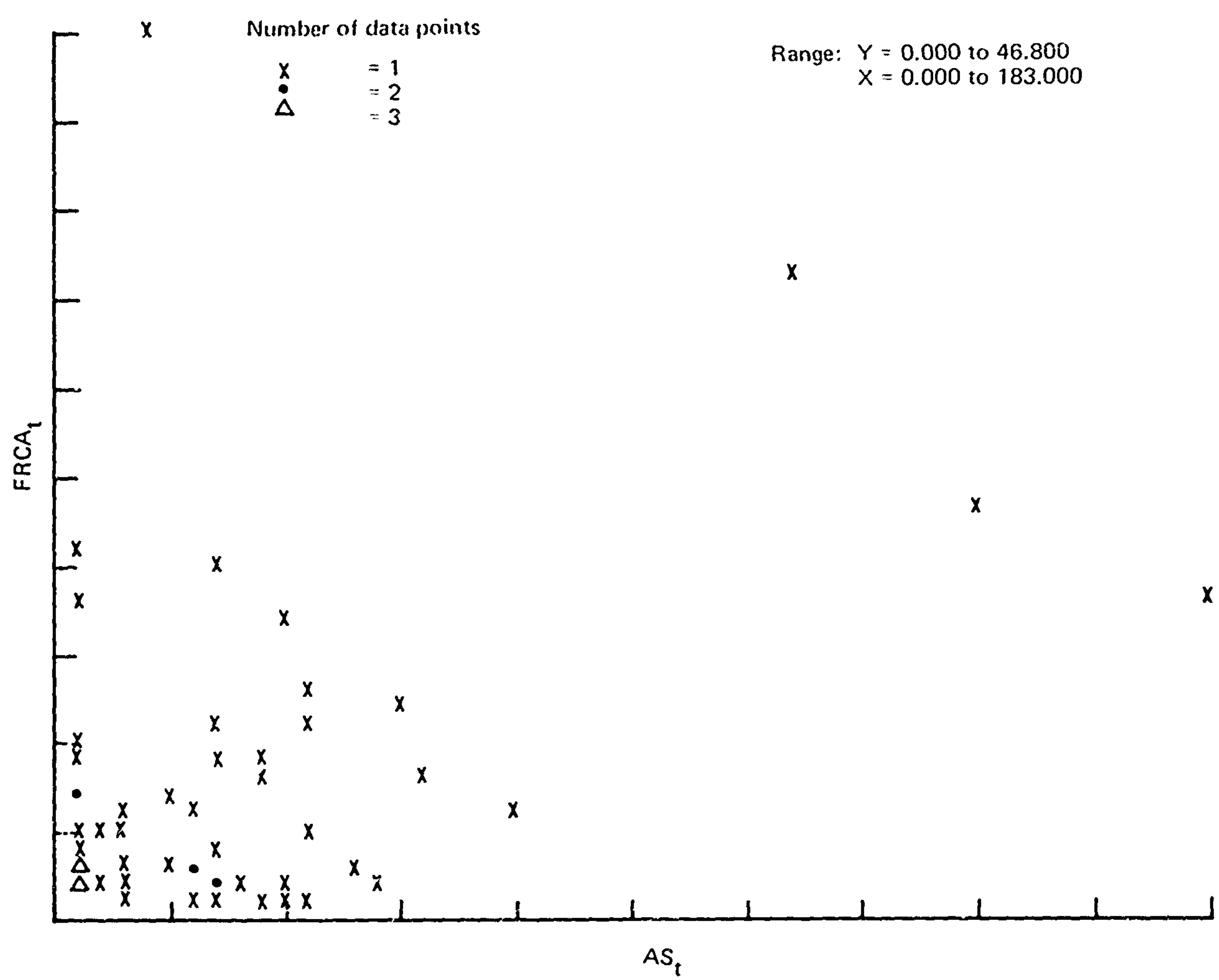


FIG. D-1: SCATTER PLOT OF FRIENDLY CASUALTIES AND AIR SORTIES
(BEFORE TRANSFORMATION)

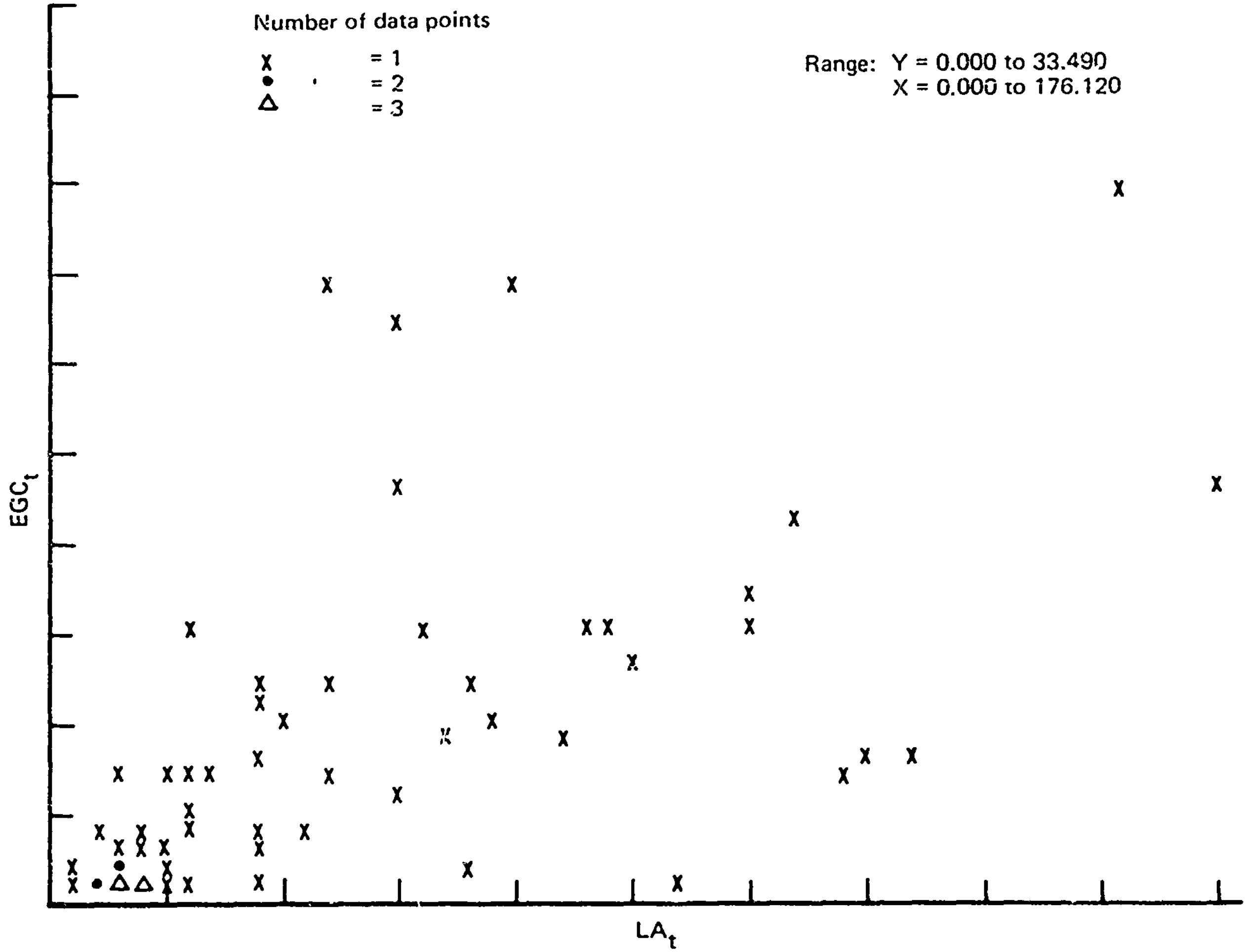


FIG. D-2: SCATTER PLOT OF ENEMY CASUALTIES AND LIGHT ARTILLERY
(BEFORE TRANSFORMATION)

casualties are plotted against artillery. The greater variability in larger numbers degrades the effectiveness of the least-squares method; hence, the transformation was made to equalize the variability over the range.

Log transformations* were performed to even out the variability in observations. This also had the effect of decreasing the contribution of the relatively few large values of variables, which were also the least accurate. In figures D-3 and D-4, log friendly casualties are plotted against log sorties, and log enemy ground casualties against log artillery. It can be seen that the variability in the data is now more constant and the patterns are clearer. In addition, the observations were scaled by quantities that render all factors about the same size. These changes increase numerical accuracy.** Inspection shows that the regression lines will not pass through the origins.

In table D-I, the unstarred variables refer to the original observations, the starred variables to the transformed. The subscript to a variable refers to the day or D-th data point in the sample and D-1, D+1 refer to the preceding and succeeding days (which may or may not be included in the sample).

A series of equations was obtained; it is summarized in table D-II. The equation derived from the first line of figures in the table is as follows:

$$FC = (19.2)\ (FS^{-1.38})\ (\exp\ (0.06GG))\ (LA^{.56})\ (\text{error})$$

where 19.2 is the antilog of the constant, 2.95, and the non-significant terms are omitted. The degrees of freedom are: 63-4 = 59.

This equation explains 46 percent of the variation in data, indicated by the R^2 of 0.46. The standard errors of the coefficient appear under each coefficient in table D-II. Unless the coefficient is about twice as large as the standard error (95 percent significant by the statistical t - test), coefficient may be error and might possibly be omitted without much loss in predictive value. In this case, therefore, the terms for enemy strength, heavy artillery and air sorties drop out, leaving the underlined terms to explain friendly casualties. In general, enemy casualties casued by aircraft are proportional to the number of the sorties flown, and enemy ground casualties are related to the amount of artillery fired. None of the combination offers good predictions of ground gained.

The appearance of the residuals offers no strong clues as to how use of the data could help improve the fits substantially. In an attempt to improve the fits, the data was aggregated into 3-day periods.

*The actual transformation used is slightly more complicated. The data was divided by constants to reduce the largest numbers to the same order of magnitude and to avoid rejection of data when there were no casualties inflicted by air sorties. The value of 1 was added to all observations; the logarithms, therefore, ranged from 0 to n (which permitted inclusion of data which were zeros). The transformations are given in annex D-1.

**By avoiding rounding errors in the computer that can occur if one set of data is much larger than the other–which is true for strengths and casualties.

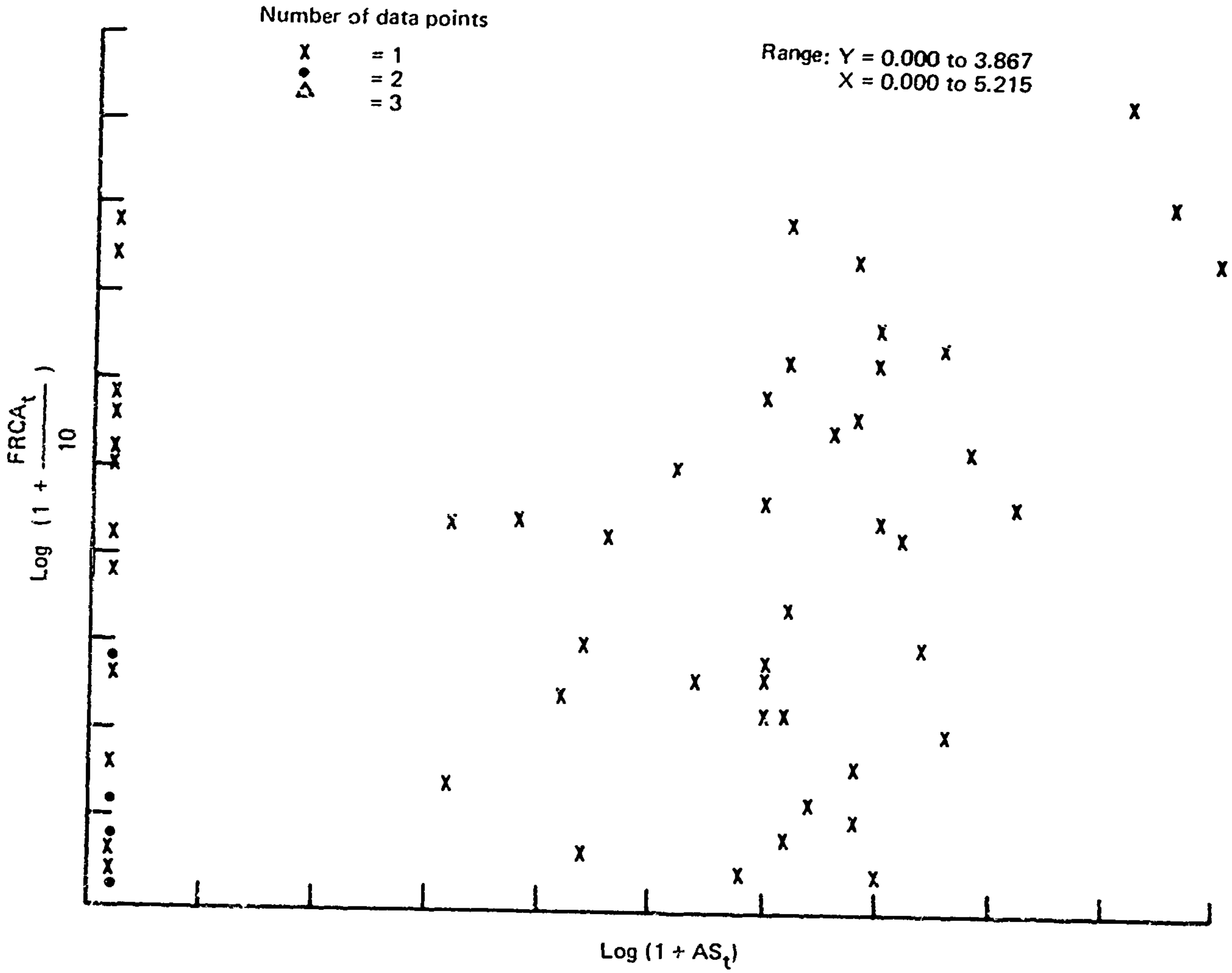


FIG. D-3: SCATTER PLOT OF FRIENDLY CASUALTIES AND AIR SORTIES (AFTER TRANSFORMATION)

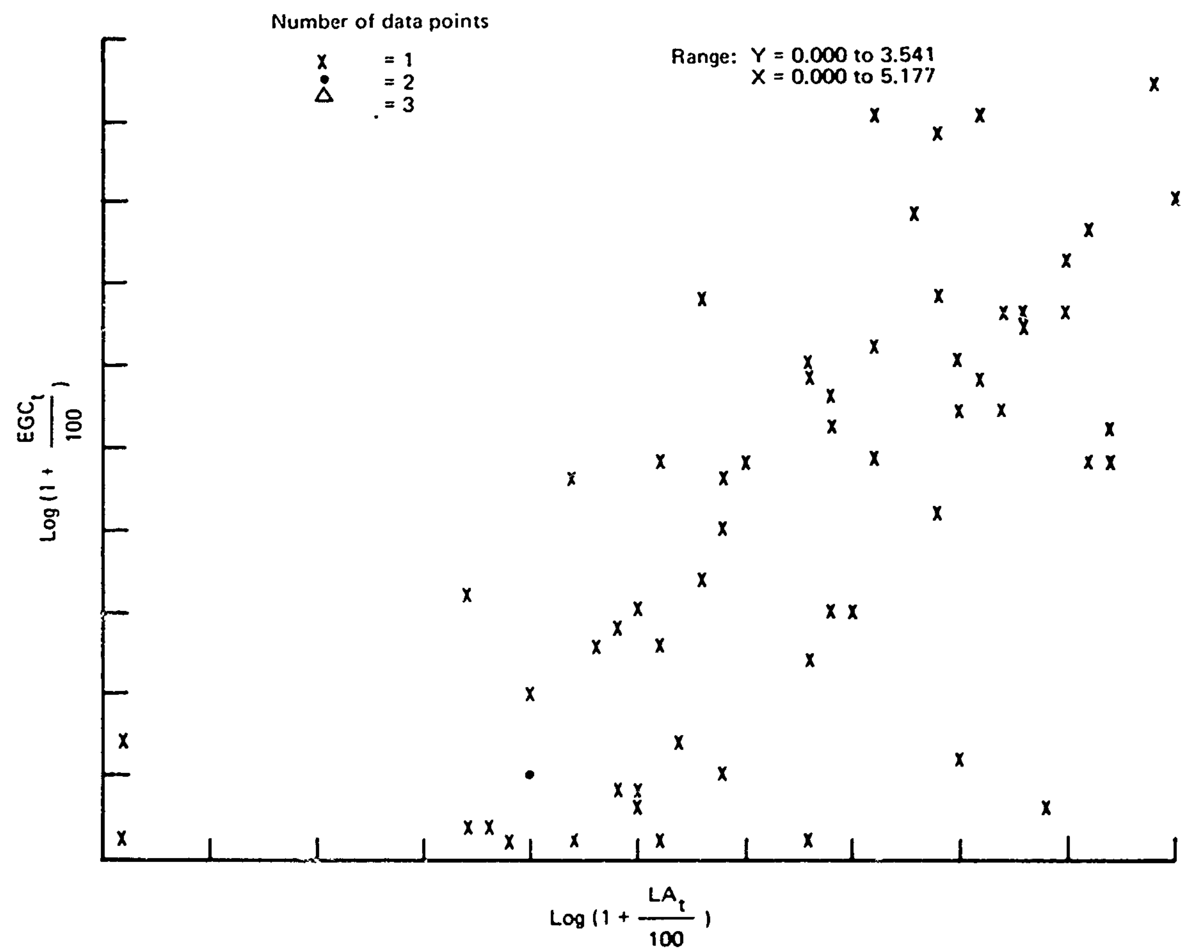


FIG. D-4: SCATTER PLOT OF ENEMY CASUALTIES AND LIGHT ARTILLERY (AFTER TRANSFORMATION)

TABLE D-I

SUMMARY OF TRIAL REGRESSION EQUATIONS

	FS_t^*	ES_t^*	HA_{t-1}^*	LA_{t-1}^*	AS_{t-1}^*	GG_{t-1}	GGA_t	GG_t	HA_t^*	LA_t^*	AS_t^*	$FRCA_t^*$	EAC_t^*	EGC_t^*
1. $FRCA_t^*$	*	*						*	*	*	*	Y		
2. EAC_t^*	*	*									*		Y	
3. EGC_t^*	*	*						*	*	*				Y
4. GG_t^*	*	*				*	*	Y		*	*	*		
5 HA_t^*	*	*	*					*	Y			*		
6. LA_t^*	*	*		*				*		Y		*		
7. AS_t^*	*	*			*			*			Y	*		

TABLE D-II

FITTED REGRESSIONS FOR X CORPS DIVISIONS – OFFENSIVE DAYS IN APRIL AND MAY 1951

Dep. variable	Independent variables[2]														Durbin Watson statistic
	C	FS_t^*	ES_t^*	GG_{t-1}	GGA_t	HA^*_{t-1}	LA^*_{t-1}	AS^*_{t-1}	GG_t	HA^*_t	LA^*_t	AS^*_t	$FECA^*_t$	R^2	
FRCA*	2.95** (1.23)[1]	-1.38** (.50)	.28 (.19)						.06** (.01)	-.01 (.18)	.56** (.17)	-.09 (.07)		.46	1.87
EAC*	-.41 (1.60)	.08 (.61)	-.05 (.26)									.68** (.10)		.50	1.66
EGC*	-1.08 (1.23)	.12 (.50)	.25 (.19)						.018 (.014)	-.039 (.186)	.45** (.16)			.46	1.75
GG*	-25.7** (10.2)	9.82** (4.14)	1.06 (1.46)	-.12 (.11)	.34** (.15)				Y_t^*		-2.73 (1.22)	.53 (.59)	3.57** (.99)	.46	2.49
HA*	-1.13 (.88)	.74** (.36)	-.09 (.13)			.48** (.10)			-.013 (.011)				.17** (.09)	.50	1.96
LA*	-2.59** (1.09)	1.22** (.45)	.25 (.15)				.39** (.10)		-.029** (.013)				.25** (.11)	.69	1.98
AS*	-2.95 (2.23)	.99 (.84)	.62 (.36)					.22 (.13)	-.005 (.031)				.07 (.24)	.25	1.91

1 Standard errors of regression coefficients are indicated in parentheses.

2 Variables are transformed in Annex D-1.

* Log transformation

** Significant at a level of approximately 95 percent.

TABLE D-III

FITTED REGRESSION EQUATIONS[3] FOR OFFENSIVE OPERATIONS OF IX AND X CORPS DIVISIONS IN MARCH, APRIL, MAY 1951 (3-DAY AVERAGES)

Y_t Definition	Run	Method	Constant	FS_t $\ln(1+\frac{FS}{100})$	ES $\ln(1+10\ ES)$	GG_{t-1} $\frac{GG_{t-1}}{10}$	$GGAJ_t$ $\frac{GGAJ_t}{10}$	HA_{t-1} $\ln(1+\frac{HA_{t-1}}{100})$	LA_{t-1} $\ln(1+\frac{LA_{t-1}}{100})$	AS_{t-1} $\ln(1+\frac{AS_{t-1}}{100})$	GG_t $\frac{GG}{10}$	HA_t $\ln(1+\frac{HA}{100})$	LA_t $\ln(1+\frac{LA}{100})$	AS_t $\ln(1+AS)$	FC_t $\ln(1+\frac{FC}{10})$	EC $\ln(1+\frac{EC}{100})$	R^2 Regression coefficient	Durbin Watson statistics
Friendly casualties	4	Structural[1] (1)	4 32 (2 02)	-1 69* (1 87)	58* (.32)						028* (010)	.19 (26)	41* (20)	04 (.16)	Y_t		36	1 5
	5	Structural[2] (2)	1 92 (1 93)	-0 88 (0.88)	78* (.34)						--- ---	-28 (29)	473* (21)	007 (17)	Y_t		34	1 55
		All factors	5 08* (2 2)	-2 54* (93)	57* (.31)	-009 (010)	017 (019)	-43* (.21)	-51* (19)	12 (.16)	016 (012)	098 (21)	31 (21)	-19 (17)	Y_t	088 (17)	65	1 69
	6	Significant[2] (2)	5 50* (1.70)	-1 82* (67)					28* (14)		028* (009)					29* (13)	50	---
Enemy casualties	7	Structural[1] (1)	1 05 (2 14)	-71 (92)	.45 (34)						01 (01)	-.32 (28)	61* (21)	25* (.17)		Y_t	.50	1 05
	8	Structural[2] (2)	104 (1 93)	39 (.86)	53 (.33)						--- ---	36* (28)	64* (.21)	23* (16)		Y_t	.56	1 15
		All factors[2] (2)	-1.24 (2 66)	-1 27 (1 14)	43 (.34)	005 (011)	-012* (020)	-.099 (27)	41* (21)	-013 (*	016 (012)	-34 (30)	.54* (21)	091 (.18)	098 (.21)	Y_t	.71	1.89
	9	Significant[2] (2)	1 62* (61)		54* (27)		-026* (012)		28* (.12)				35* (14)		29* (.14)		86	---
Ground gained	10	Structural[1] (1)	-25 6 (31 6)	7.36 (12.8)	12 (4 2)	.15 (13)	61* (21)				Y_t		-05 (2 5)	-64 (1 73)	4.22* (2.12)		.53	2 25
	11	Structural[2] (2)	-64 1* (30 1)	28.8* (13 5)	7 0 (5 1)						Y_t	-3 13 (4 34)	2 13 (5 24)	-1.66 (2 56)			.34	1 81
		All factors[2] (2)															67	2 46
	12	Significant[2] (2)	-6 1 (4 0)				77* (14)				Y_t				4 15* (1 70)		59	---
Heavy artillery		Structural[1] (1)	-3 02 (1 58)	1 59 (65)	15 (23)			51* (12)			-0077 (0085)	Y_t			049 (12)		67	1 65
		All factors[2] (2)	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.86	2.10
		Significant[2] (2)	55 (.38)		-34* (16)			33* (10)				Y_t	33* (10)	25* (09)			81	---
Light artillery		ERSU	5 54* (2 07)	2 85* (86)	071 (.30)				093 (148)		-011 (011)		Y_t		28 (18)		55	1.71
		All (Step)	---	---	---	--	---	---	---	---	---	---	---	---	---	---	80	1 85
		Significant	1 13* (0 29)				015* (008)					63* (10)				.37* (06)	.77	---
Air sorties		ERSU	3 7 (2 3)	1 52* (83)	.39 (.36)					55 (13)	-008 (013)			Y_t	03 (19)		66	1 98
		All (Step)															.83	2 22
		Significant (Step)	1 03* (59)		51* (24)					35* (10)		73 (15)		Y_t			.79	---

* Statistically significant (95% level) relationship is indicated. Student t test criteria

(1) Using Econometrics ERSU program
(2) Using BIMED 34 stepwise multiple regression
(3) Where the number in parentheses is the standard error of estimate

Three-Day Aggregates

A sample of 41 3-day aggregates of data were taken from IX and X Corps reports from March, April, and May 1951 during periods of heavy conflict when U. N. forces were advancing. As the IX Corps reports did not separate enemy casualties between air and ground categories, the ground and air casualties from the X Corps reports were added together so that the data would coincide with that of IX Corps.

The variable, enemy casualties (EC), is the sum of EGC_t and EAC. The relations for enemy air and ground casualties were combined into one relation:

$$EC_t^* = a + bFS_t^* + cES_t^* + dGG_t^* + eHA_t^* + fLA_t^* + gAS_t^* + \text{error}$$

where a, b, c, d, e, f, and g are coefficients fitted by least squares for the strengths and firepower terms defined in annex D-1.

The data now included some points from the Fifth ROK Division. Scatter plots show clearly that the Fifth ROK Division received much less fire support than the U. S. units and was a less effective fighting unit, perhaps as a consequence. Scatter plots (figures D-5 and D-6) of heavy artillery and air sorties show that the ROK unit did not receive as much heavy fire support as the U. S. units. Figures D-7 and D-8 show that the ROK's did not gain as much ground or inflict as many casualties as the U. S. units. Figure D-9, however, shows that the ROK's suffered as many casualties as the U. S. units.

The suspicion that other factors are important was borne out by subsequent runs, reported in tables D-II and D-III. The regression coefficients are given in table D-IV.

TABLE D-IV

SUMMARY OF R^2 FOR VARIOUS RUNS

			IX and X Corps – 3-day aggregates ROK unit with other forces		
	X Corps single days	ROK unit out	Not lagged	Lagged: significant	Lagged: all terms
Friendly casualties	.46	.53	.36	.50	.65
Enemy casualties	.50 Air .46 Ground	.65	.50	.66	.71
Ground gained*	.46	.55	.55	.59	.67

*Friendly casualties are included; this term has the largest coefficient.

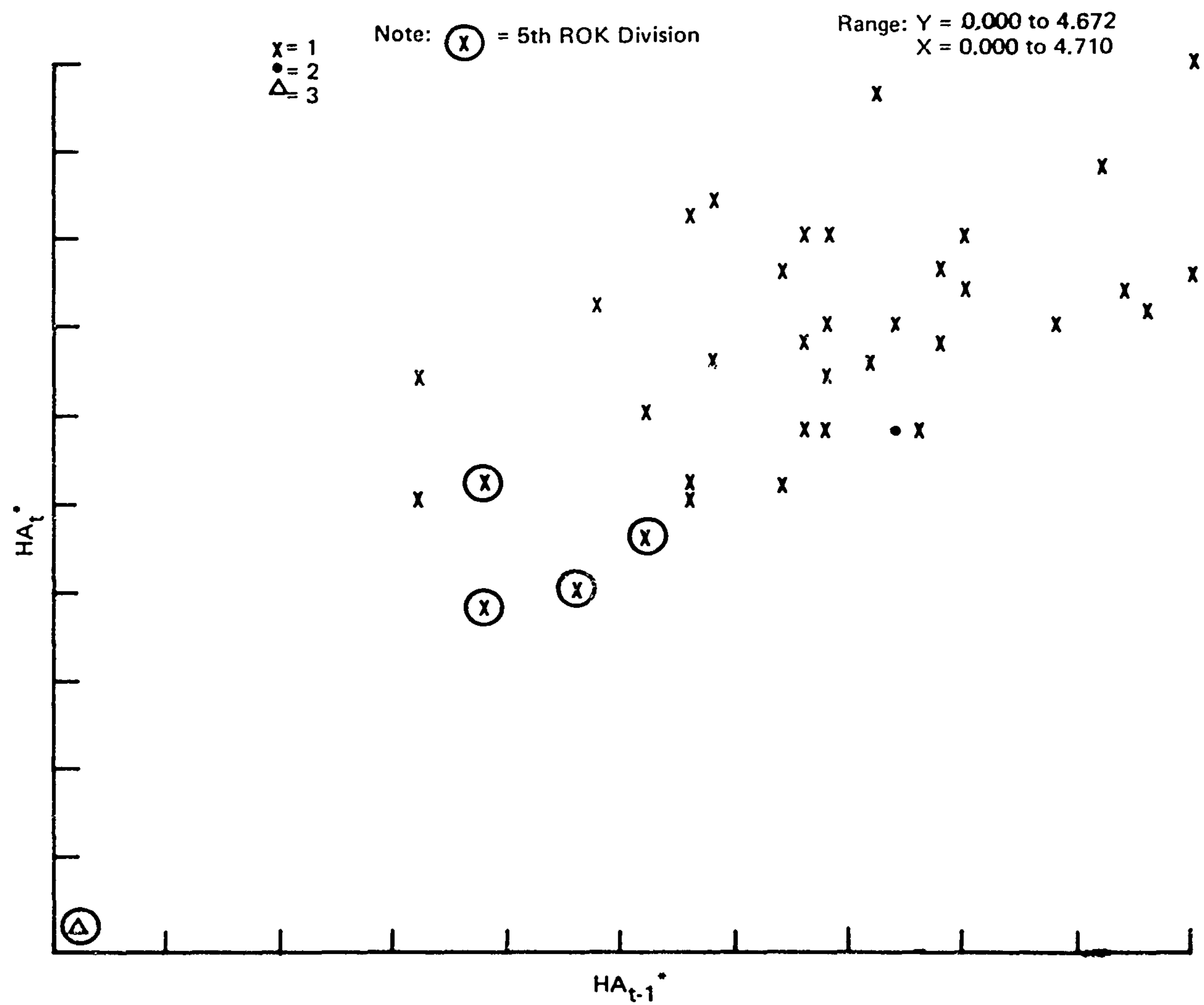


FIG. D-5: SCATTER PLOT OF HEAVY ARTILLERY AND HEAVY ARTILLERY ON PREVIOUS PERIOD (3 DAY AGGREGATES-AFTER TRANSFORMATION)

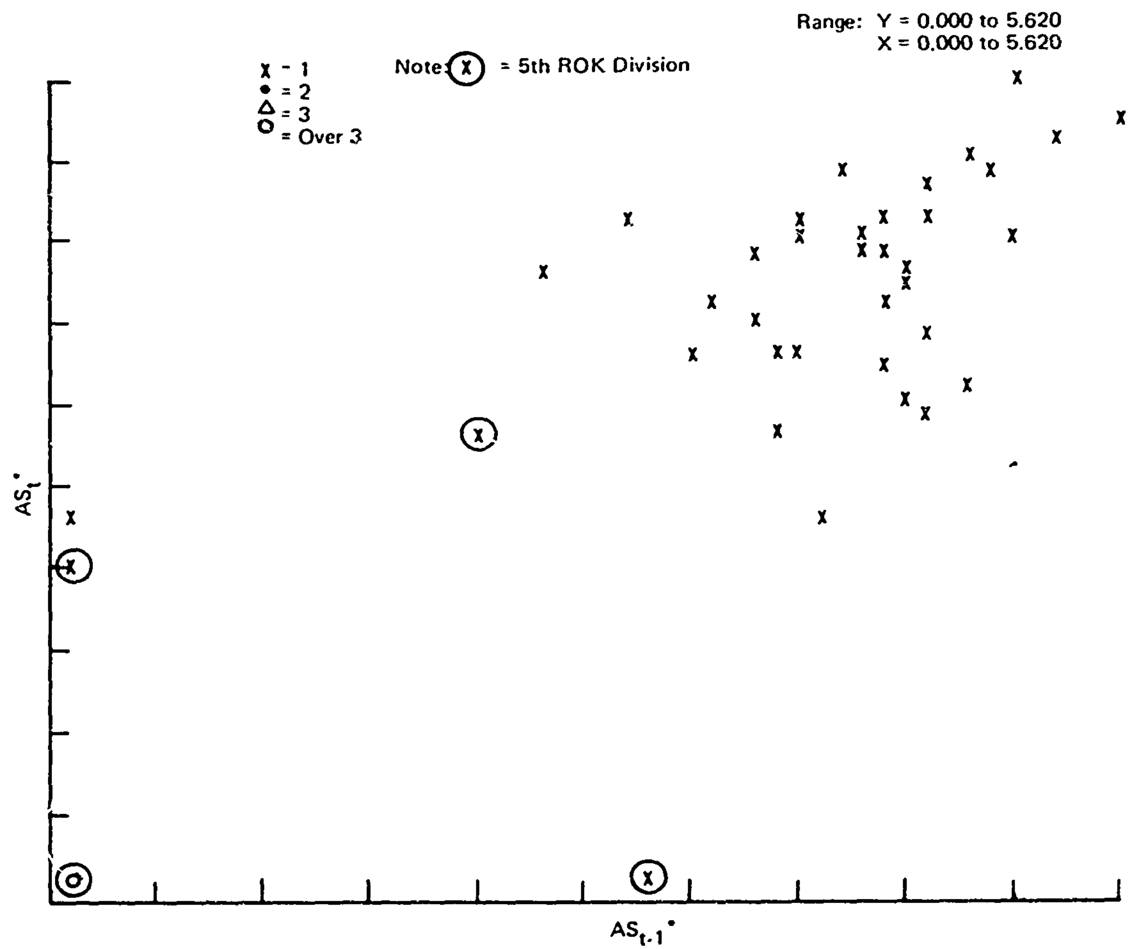


FIG. D-6: SCATTER PLOT OF AIR SORTIES AND AIR SORTIES ON PREVIOUS PERIOD (3 DAY AGGREGATES-AFTER TRANSFORMATION)

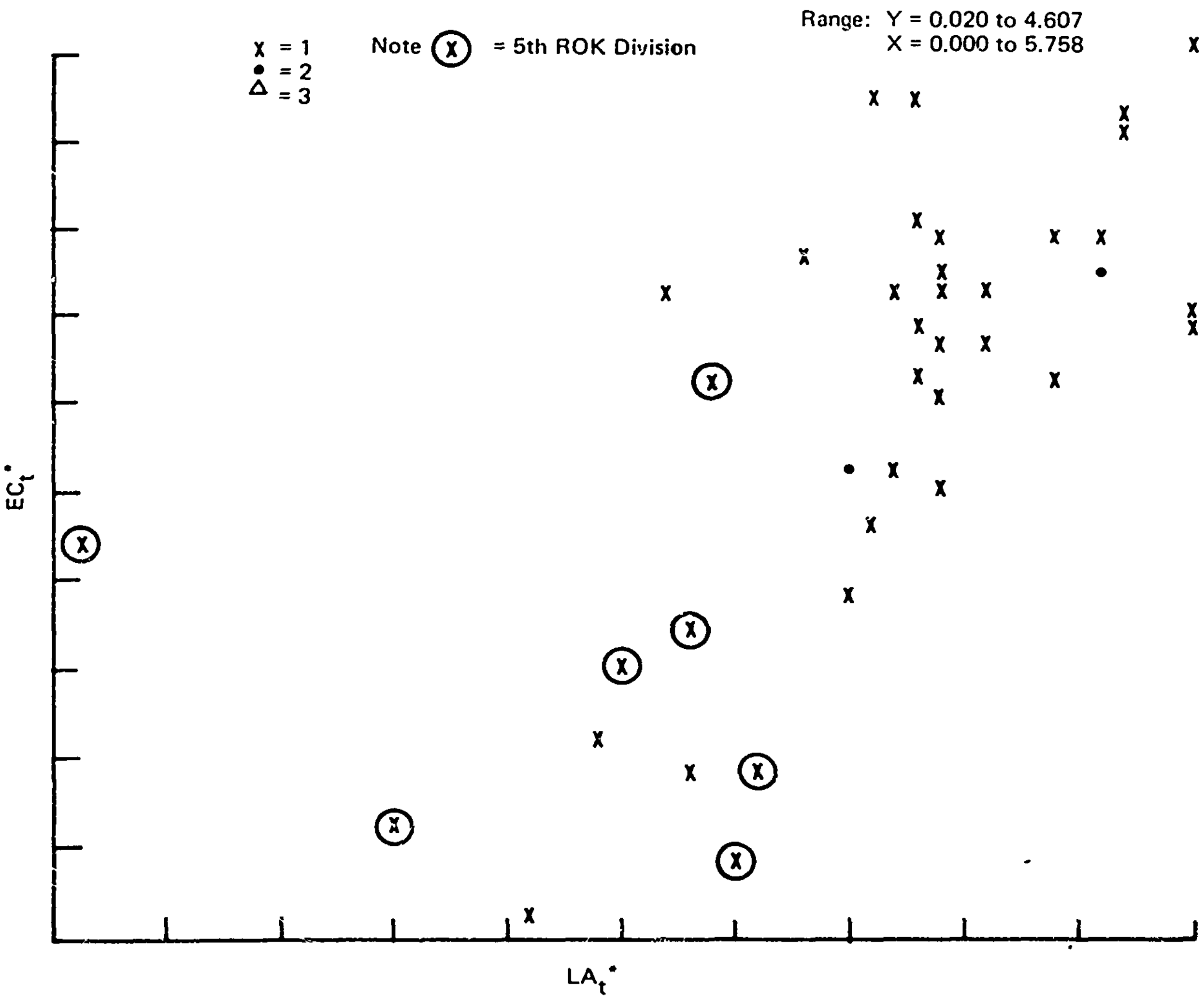


FIG. D-7: SCATTER PLOT OF ENEMY CASUALTIES AND LIGHT ARTILLERY
(3 DAY AGGREGATES-AFTER TRANSFORMATION)

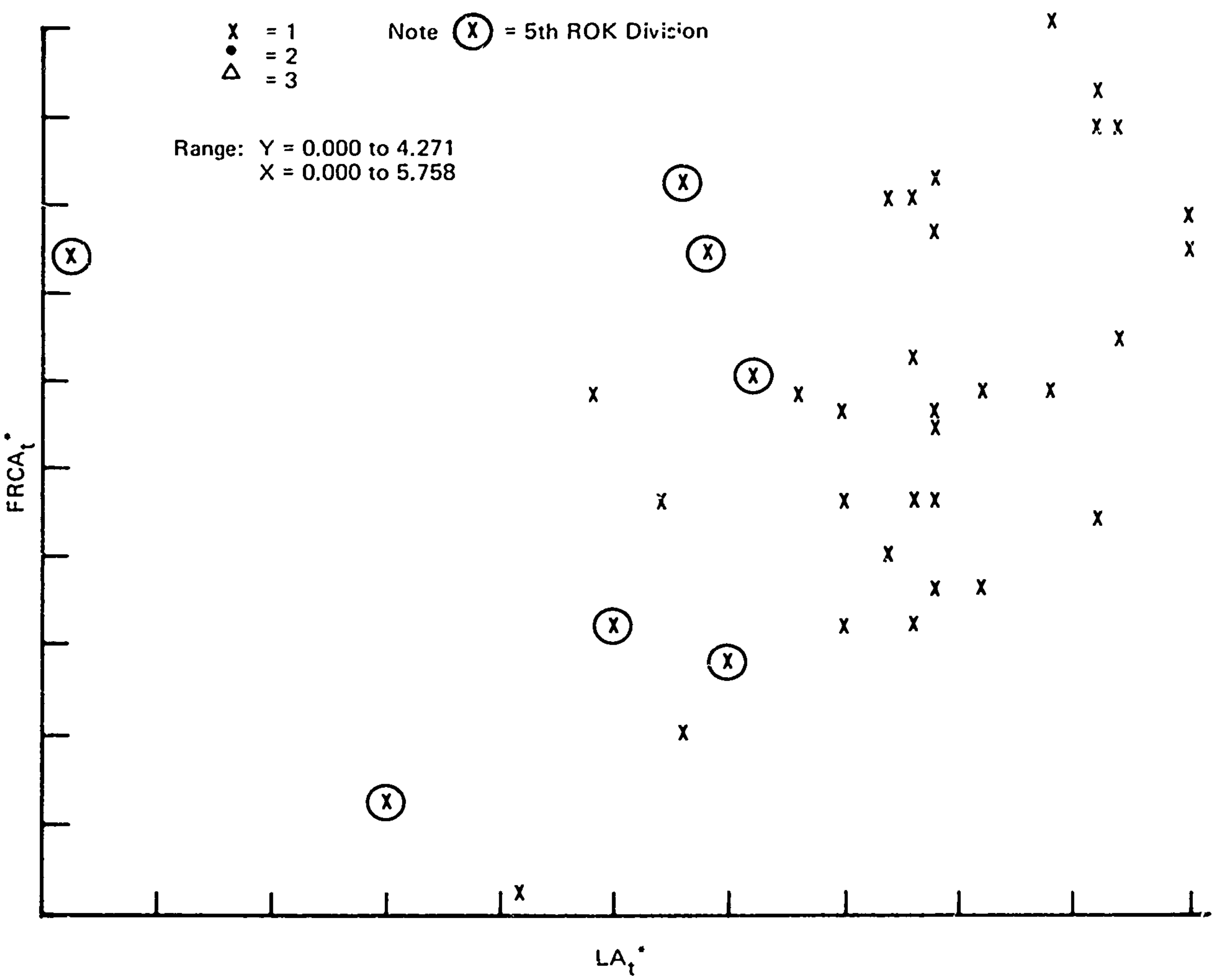


FIG. D-8: SCATTER PLOT OF FRIENDLY CASUALTIES AND LIGHT ARTILLERY (3 DAY AGGREGATES-AFTER TRANSFORMATION)

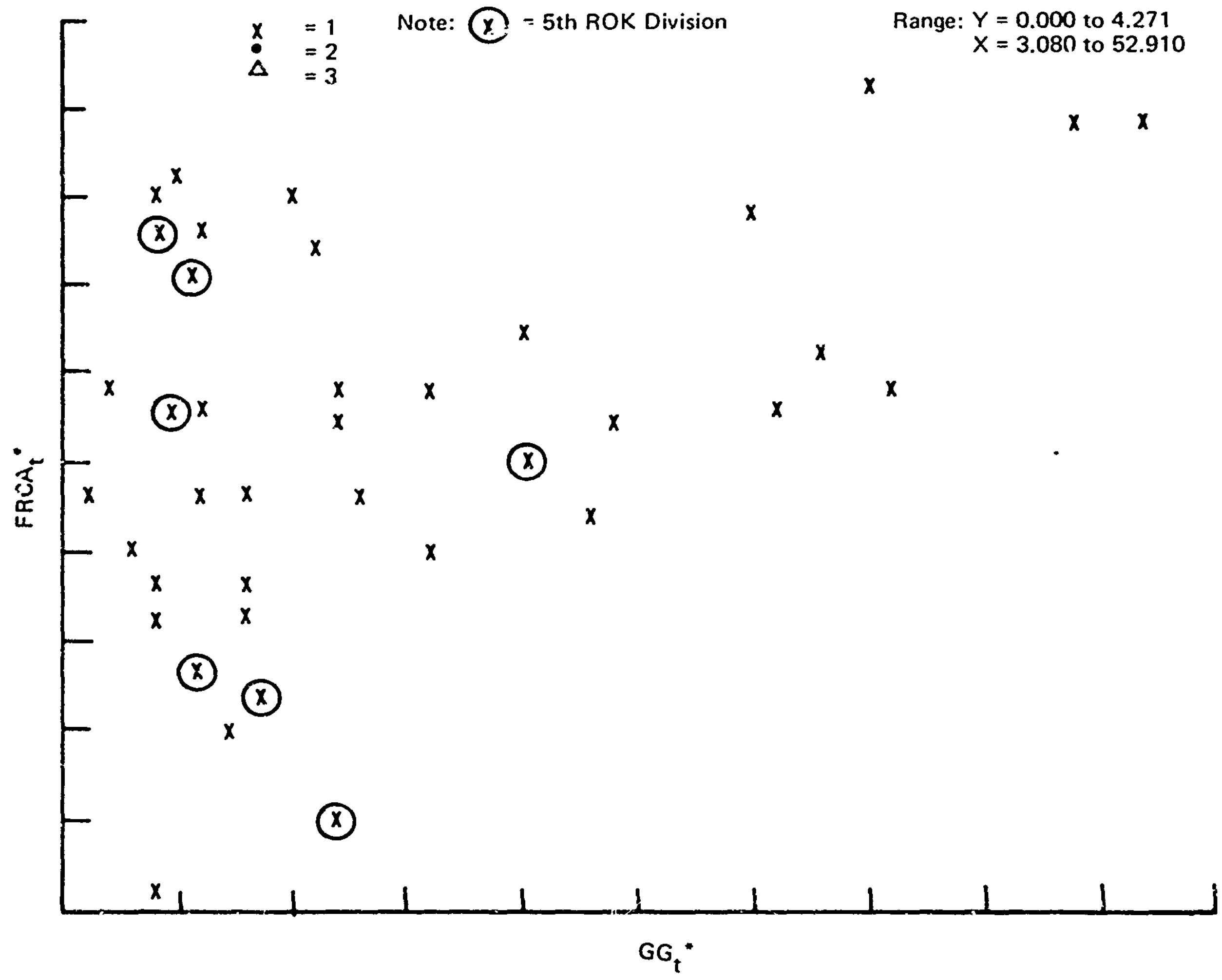


FIG. D-9: SCATTER PLOT OF FRIENDLY CASUALTIES AND GROUND GAINED (3 DAY AGGREGATES-AFTER TRANSFORMATION)

Three effects are shown:

- Removal of the data for the ROK unit increases R^2. The residuals show differences between U. S. divisions, but the differences are smaller and have not been tested for their effect on R^2.

- The artillery fired during the previous period increases R^2.

- Enemy casualties are influenced additionally by light artillery fired and by the loss of ground by adjacent units.

Low R^2 values showed that elements not in the equation – poor data, a large random error, or other factors – influenced results. These factors were sought. The plotted data and the Durbin Watson statistics* show that the preparatory artillery and air ordnance are serially related from one day to the next and that the battle extends over several divisions. When a division is overrun, the flank of the adjacent unit is exposed; it must be reinforced or withdraw. Similarly, some units may differ from others in support and effectiveness.

Ground gained is influenced by friendly casualties and by the ground gained by adjacent units.

A possible interpretation of this phenomenon is that the fighting capability of the ROK unit differed from that of the U. S. units and that the same model, or at least the same coefficients, cannot describe the behavior of both. U. S. units, however, received more heavy fire support than ROK units. This difference is one of the main sources of variation; it disappears when the ROK data points are removed. The effect exists to a much smaller extent in the allocation of light artillery.

Revised Data for May 1951

We prepared the complete regression coefficients for the 18 equations for the three categories of missions described previously – U. N. attacking, enemy attacking, and static/patrol – using only the fire support data and the logs of the force strengths. One set of 9 equations used untransformed fire support data; the other set uses the logs of artillery and air support. Values of R^2 for two cases are given: first for significant terms, then for all terms. The R^2 value is smaller for the former. Coefficients for firepower and strengths differ in the two cases.

*The Durbin Watson statistic, abbreviated DW in appendixes D and E, has a normal value of 2. When less, it indicates that the data from one day is related to the next day.

ANNEX D-1

EXACT FORMS OF VARIABLES USED IN THE REGRESSION ANALYSIS

1. Friendly strength (FS): FS_d is measured in numbers of troops.

$$FS^*_d = \log\left(1 + \frac{FS_d}{1000}\right)$$

2. Enemy strength (ES): ES_d is measured in Chinese divisional equivalents.

$$\widetilde{ES}_d = \frac{ES^{1}_{d-1} + ES_d + ES_d + 1}{3}$$

$$\widetilde{ES}^*_d = \log\left(1 + 10\ ES_d\right)$$

3. Heavy, light artillery (HA, LA): HA, LA are measured in numbers of rounds.

$$HA^*_d = \log\left(1 + \frac{HA_d}{100}\right), \quad LA^*_d = \log\left(1 + \frac{LA_d}{100}\right)$$

4. Close air support (AS): AS_d is measured in numbers of sorties.

$$AS^*_d = \log\left(1 + AS_d\right)$$

5. Friendly casualties (FRCA): FRCA are the numbers of troops killed, wounded, or missing during the period under consideration.

$$FRCA^*_d = \log\left(1 + \frac{FRCA_d}{10}\right)$$

6. Enemy air and ground casualties (EAC, EGC): EAS, EGC are the numbers of troops killed or captured during the period under consideration. These are estimates, of unknown accuracy. Sometimes air and ground casualties are not separated in the periodic intelligence reports. In that case, we use the variable enemy casualties (EC).

---

[1] Data on enemy strength was averaged over 3-day periods to smooth over uncertainties in the estimates.

$$EAC_d^* = \log\left(1 + \frac{EAC_d}{10}\right)$$

$$EGC_d^* = \log\left(1 + \frac{EGC_d}{100}\right)$$

$$EC_d^* = \log\left(1 + \frac{EC_d}{100}\right)$$

7. Ground gained, ground gained by adjacent unit (GG, GGA): GG_d, GGA_d is the territory measured in square kilometers, acquired or relinquished by a friendly unit during the time period under consideration.

$$GG_d^* = \frac{GG_d}{10}$$

$$GGA_d^* = \frac{GGA_d}{10}$$

This is the only variable that was not logged, because not logging GG produced better fits in many instances.

APPENDIX E

RESULTS FOR MAY 1951 DATA

This appendix contains a description of the runs and a summary of some regression equations and the non-significant terms. These results are tabulated as follows:

Table E-II:	Enemy casualties
Table E-III:	Friendly (U. N.) casualties
Table E-IV:	Ground gained
Table E-V:	Enemy casualties
Table E-VI:	Lanchester laws with firepower

Most of this volume is devoted to a discussion of the basic data and these regression equations.

The results presented are obtained by means of the BIMED-34 program.

The output of this stepwise multiple regression, which provides many alternatives, is usually restricted to some of the following tabulations:

- The original and transformed data
- Means, standard deviation, and variances of each variable
- Variance-co-variance matrix
- Correlation matrix
- A series of equations with 1, 2 .... n terms with the coefficients derived by ordinary least squares at each step, the standard error of the coefficients, t-tests, and analyses of variance and significance of the equation by t-tests

Plots of residuals and computations of Durbin-Watson test for serial correlation are sometimes obtained.

The program is flexible because variables can be transformed or combined into new variables, which can be used in testing. When a variable such as missions of forces affects the regression heavily, as shown by early introduction of these terms into regression equations, the data deck is sorted into the categories of the variable (such as the missions), and separate regressions are obtained for each. Obviously, if some of the equations are similar, they should be combined.

A sample print-out of one equation appears in figure E-1. Most attention is focused on the print-out for the "multiple" regression equation. The

```
STEP NUMBER  3
  VARIABLE ENTERING        37
  MULTIPLE R                0,88131
  STD ERROR OF Y,X          1,31879
  R SQUARE                  0,77671
```

MULTIPLE REGRESSION EQUATION

VARIABLE		COEFFICIENT	STD. ERROR	T VALUE	PARTIAL COR.	ELASTICITY	
LNENCA =							
LA	X-10	2,0596-003	3,7733-004	5,4847+000	7,9933-001	5,5239-001	*
LNFRST	X-23	1,6943+000	6,9168-001	2,7386+000	5,5329-001	2,9544+000	*
LLFRST	X-37	-1,6122+000	6,7072-001	-2,4036+000	-5,0364-001	-2,5030+000	*

ANALYSIS OF VARIANCE FOR REDUCTION IN SS DUE TO VARIABLE ENTERING

SOURCE	DF	SS	MS	F
DUE REGRESSION	3	3,23673+002	1.07891+002	6,20349+001
DEV. FROM REG.	17	2,95664+001	1,73920+000	
TOTAL	20	3,53239+002	1.76620+001	

PARTIAL COR. FOR VAR. NOT IN EQUATION

VARIABLE		COEFFICIENT	F FOR SELECTION
WET	X- 3	9,8837-002	1,5784-001
HA	X-11	-1,5314-001	3.8424-001
AS-0	X-15	2,4696-001	1.0392+000
HBS-L	X-20	2,8995-001	1.4686+000
LMENST	X-27	-4,0887-002	2.6792-002
LBLA	X-33	3,3954-001	2.0849+000
LBHA	X-34	1,9861-001	6.5706-001
LBAS-0	X-35	2,8431-001	1.4070+000
LBHB-0	X-36	-5,4622-002	4.7869-002
LLENST	X-38	-1,8232-001	5,5017-001

FIG. E-1: EXAMPLE OF PRINT-OUT USING THE BIMED-34 PROGRAM (MULTIPLE REGRESSION EQUATIONS FOR THE LOGARITHM OF ENEMY CASUALTIES DURING ENEMY ATTACK (RUN 26)) independent variable LNENCA (or ln enemy casualties) is equated to three independent variables in the next three lines. The rounded coefficients for each give the equation:

$$\ln \text{enemy casualties} = 0.002 \text{ light artillery} + 1.9 \ln \text{friendly strength} - 1.6 \ln \text{friendly strength on } t-1$$

The coding for the independent variables is given under the designation X-10, X-23, and X-37 at the first part of the print-out. This may lead to equations of differing appearance, as discussed in appendix D.

For purposes of this report, there are two important criteria:

- One is that the terms must be statistically significant. This is determined by the T-value, which is provided by division of the coefficient by the standard error. The coefficients are included, as long as all are above 2. 0, which is approximately the 95 percent significance level, considering sample size and multivariable choice. If the constant is not significant, it is omitted from tables E-II through E-VI.

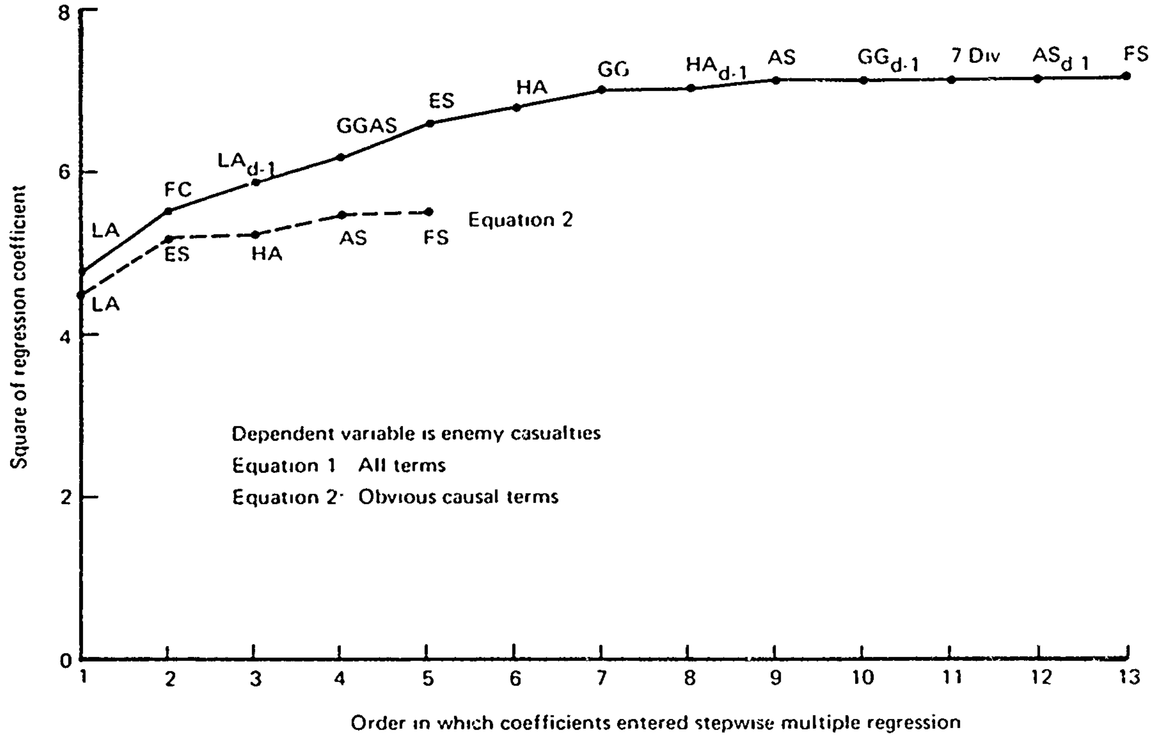
- The second criterion, the fraction of variation explained by the data, must be fairly large; it is represented by $R^2 = .77671$, which is rounded to 0.77 . The interpretation of lagged data is difficult. For instance, strength on the day of battle and strength on the preceding data are both significant, but of opposite sign. The true effect may be approximately the sum of the two coefficients: +1. 9 – 1. 6 = 0. 3. No further interpretation has been attempted.

As new terms are added in the stepwise equations, the successive R^2's are plotted (figure E-2). The last terms add little to the explanation. The complexity of relations among the variables is shown by the partial correlation coefficients in the correlation matrix.

To focus attention on the equation, the tables give two values for R^2. The first is for the equation; the R^2 value in parentheses is for the equation with all terms and should not be used. Usually, these two R^2 values are close, indicating that a few terms explain most of the variation. This compromise permits more rapid scanning of results.

Terms in the following descriptions of runs are defined in table E-I.

The equations of appendix E are not converted to the original form in all cases and are not necessary for judgments of relative worth.



EC = Enemy casualties
LA = Light artillery
AS = Air sorties
HA = Heavy artillery
ES = Enemy strength
FS = Friendly (U N.) strength
FC = Friendly casualties
GG = Ground gained
adj = Adjacent units
d-1 = Previous day

Correlation matrix

		LA	AS	HA
	EC	45	35	20
LA_{d-1}	44	44	55	45
ES	34	26	29	12
AS_{d-1}	32	38	64	48
FC	30	13	05	01
FS	25	53	48	53
HA_{d-1}	20	46	58	67
CG	17	14	07	05
GG_{adj}	07	18	10	08
GG_{d-1}	05	02	02	01

FIG. E-2: EXAMPLES OF R^2 FROM THE STEPWISE MULTIPLE REGRESSION PROGRAM

DESCRIPTION OF RUNS ON REVISED MAY 1951 DATA

Run #1 the initial run, had 206 observations. The dependent variables are the log of friendly casualties (LNFRCA), the log of enemy casualties (LNENCA), and the ground gained (GG). The variables forced into the equation are the log of the friendly strength (LNFRST), the log of the number of light artillery rounds fired (LNLA), the log of heavy artillery rounds fired (LNHA), the log of the air sorties (from Army records), the front length, and the two variables used as dependent variables in other subproblems. This run has residuals plotted after each step (values were scaled and a one added before the log is taken).

Run #2 differs from run #1 only in that 2 new variables, day and phase, are forced in.

Run #3 contains data for Phase 17 only, the static period from 1 to 16 May (112 observations). In this run, several variables are divided by the front length, and the front length is dropped from the list of independent variables.

Run #4 examines the offensive actions of 17-31 May 1951, designated Phase 18. The run is identical with run #3, except that the observations differ. There are 94 observations for Phase 18.

Run #5 is similar to runs #3 and #4 and contains cases for the data of both Phase 17 and Phase 18. Also, the GG is divided by FL, an additional variable which does not appear in runs #3 and #4.

Run #6 differs from run #5 in that it has cases for Phase 18 only and that LNAS (Army record of air sorties) is deleted from the list of independent variables, and the new data on air sorties LNAS-0 and LNAS-15 is added to the list.

Run #8 In this run, the dependent variable is the log of enemy strength (LNEST). It contains cases for Phases 17 and 18, but many observations are dropped because there is no data for LNENST. The independent variables are LNFRST, GG, LNHA, LNLA, LNAS-0, LNAS-15, LNFRCA, LNENCA, and DAY. These independent variables and the dependent variables are divided by FL.

Runs 9A, 9B and 10 In these runs for friendly and enemy casualties and ground gained, new variables – the missions of each side and heavy bomber sorties – are added. The missions are highly significant; separation of the data by missions is therefore indicated.

Runs 11-16 Separate equations are obtained for three missions – U.N. attacking, enemy attacking, and static/patrol – by separation of the cards into three decks and a rerun. The constant is included. However, the meaning of the large constants that are statistically significant is not understood. The friendly-casualty equation shows that ground gained, front length, friendly strength, and light artillery are all significant.

The interpretation of the equation, including the signs of the coefficients, is that during friendly attacks there are more friendly casualties when friendly strength is low or on an extended battle front, or if more ground is gained. The increase in friendly casualties with more light artillery in equation 11A appears to be a collinear - rather than cause-and-effect - relationship. The increase in friendly casualties during enemy attacks seems to be as expected: Increases in casualties with increases in heavy artillery are not the result of causal relationships. When ground gained is the dependent variable, all variables are divided by the front length (Equations 12, 14 and 16) to yield equations for relationships per mile of front. The values of R^2 are highest for the entire set of equations tested to date. All contain large and significant constants.

Runs 17A, B, C, to 19A, B, C These runs are similar to the last set, except that the ground gained is divided by front length in the casualty equations. The results are not as good as in the set #11 to 16.

Runs 20 to 22 One-day lagged fire-support variables are substituted for the fire-support variables. (Inclusion of both was intended.) The explained variation (R^2) is only a little lower than in the unlagged set. Proper inclusion is expected to increase the value and, perhaps, make other terms significant.

Run #23 is a first attempt to test the Lanchester square law. A dependent variable K is calculated,

$$K = \frac{ENST \quad * \quad ENCA}{FRST \quad * \quad FRCA} \quad \text{for each of 3 types of combat.}$$

For the three major problems, the data in the categories shown in table V is combined as follows: (1) FRM = 1, ALL ENM (2) ENM = 1, ALL FRM, and (3) FRM = 2-5, ENM = 3-5. Two sets of variables are forced in (1) LA, HA, AS-0, HBS - 0 and (2) LA, HA, AS-0, HBS-0, and FRST.

Run #24 For the same three major problems and for the three dependent variables, LNFRCA, LNENCA and GG, 2 sets of independent variables are forced in (1) LNENST, LNFRST, LA, HA, AS-0, HBS-0 and (2) LNENST, LNFRST, LNLA, LNHA, LNAS-0, LNHB-0.

Run #25 The main problems and dependent variables are the same as in run #24. The following 2 sets of variables are forced in (1) LNFRST, LNENST, LA, HA, AS-0, LLFRST, LGLA, LGHA, LGAS-0, LGHB-0 and (2) LNFRST, LNENST, LNLA, LNHA, LNAS-0, LNHB-0, LLFRST, LLENTST, LGLNLA, LGLNHA, LLAS-0, LLHB-0

[LN = log, LG = lag and LL=logged lag]

Run #26 differs from run #25 only in the forcing-in of the first set of variables (not logged) with the addition of WET (weather).

Run #27 is the start of work to remove collinearity by use of ordnance weights. The aim is to use a modification of the linear law, taking ordnance into account. A dependent variable K is calculated as follows:

$$K = \frac{ENCA}{FRCA} - FRST - ENST - .7548 \ AS\text{-}0$$
$$-6 \ (HBS\text{-}0) - .018 \ LA - .048 \ (HA).$$

Each term on the right of the equation, except EC/FC, is then forced into the multiple regression equation with the constant suppressed. The coefficient for HBS, 6.0, is in error and corrected in run #28 to 8.45 tons ordnance per heavy bomber sortie.

Run #28 with ENCA/FRCA as the dependent variable, K = FRST + ENST + .7548 (AS-0) + 8.45 (HBS-0) + .018 (LA) + .048 (HA) is calculated and K is forced in. The constant is not suppressed.

Run #29 is the same as run #28, except that the equation for K is changed slightly to scale the FRST and ENST by .5, i.e., K = .5 FRST + .5 (ENST) + .7458 (AS-0) + 8.54 (HB-0) +.018 (LA) + .048 (HA) to account for troop ordnance.

Run #30 attempts to test the linear law with the tonnage of ammunition fired by friendly and enemy troops (40 pounds per man = 0.002). With ENCA/FRCA as the dependent variable, .002 FRST, .002 ENST, .7458 AS-0, 8.54HB-0, .018 LA and .048HA are forced into the equation for each of the same three main problems.

Run #31 is similar to run #30, except that (ENCA · ENST) / FRCA · FRST) is the dependent variable. The square law is thus tested.

Run #32 Another attempt to verify or test the square law is made, with ENCA as the dependent variable. Two subproblems are run for each of the 3 main problems (1) forced in .002 FRST, .7458 AS-0, 8.34 HB-0. .018 LA, .048 HA and FRST, one at a time (2) calculated K = (.002 FRST + .7458 AS-0 + 8.54 HB-0 + .018 LA + .048 HA) FRST.

Run #33 is the same as run #32, except that the constant is suppressed in this run. Run #32 with the constant yields better results.

Run #34 is an attempt to test the linear law for the 2 dependent variables – (1) ENCA (2) FRCA – for each of the 3 main problems. Two sets of values are forced in for each dependent variable. For dependent variable ENCA (1) forced in .002 FRST, .7548 AS-0, 8.54 HB-0, .018 LA, .048 HA, FRST and ENST and (2) calculated K = .002 FRST + .7548 AS-0, + 8.54 HB-0 + .018 LA + .048 HA · FRST · ENST. For FRCA (1) forced in .002 ENST, FRST, ENST and (2) calculated K = .002 ENST · FRST · ENST (.002 ENST) represents the enemy firepower. The enemy has no air and almost no artillery support (100 rounds in 2 months), relying on mortars, rockets, and a few small anti-tank guns.

Run #35 is similar to runs #33 and #34, but Form I of the log laws is tested. As before, the dependent variables are ENCA and FRCA. For each of the 3 main cases, 2 sets of independent variables are forced in for each dependent variable.

For ENCA the sets are: 1) .7548 AS-0, 8.54 HB-0, .018 LA, .048 HA, .002 FRST, ENST, and 2) K = (.002 FRST + .7548 AS-0 + 8.54 HB-0 + .018 LA + .048 HA) ENST.

For FRCA the sets are: 1) .002 ENST, FRST and 2) K = .002 ENST · FRST.

Run #36 differs from run #35 in the independent variables forced in and represents a second linear law. For ENCA the independent variables are: 1) .7548AS-0, 8.54HB-0, .018LA, .048HA and .002 FRST and 2) K = .002 FRST + .7548AS-0 + 8.54HB-0 +.018LA + .048HA. For FRCA the independent variables are: 1) .002 ENST and 2) K = .002ENST. Thus, for FRCA, both subproblems are identical, since we can only estimate the enemy firepower from enemy strength.

Run #37 tests the second form of the log law. Again, it differs only in the sets of independent variables forced in. For ENCA they are: 1) .002FRST, .7548AS-0, 8.54HB-0, .018LA, .048HA.

Runs #38-40 Each of the laws is tested under an assumption of 40 pounds of ammunition per man, equivalent to 0.002. The next effort is to vary this value. This term is used with the strengths to account for the small arms fire. In these runs, only one law, the log law I, is used, duplicating the previous run (#35), except that in run #38 the factor is .02 and in run #39 the factor is .2 . In run #40, the strength factor is zero, thus taking out the firepower factor. Although the best results appear in run #35, the use of .002 is continued because, though there is little difference, it recognizes that small arms have some effect, which may show up in subsequent work.

Run #41 After further analysis of the data, it is found that in several cases, when the friendly mission is to advance, the data records the loss of friendly ground, the same being true for the enemy; therefore, the data decks are re-organized and rerun for log law I (rerun of #35 with new arrangement of data).

Now there are 4 main cases: Case I–friendly advance (27 observations), Case II–Enemy advance (22 observations), Case III–static period with friendly ground gained (49 observations), and Case IV–static period with enemy ground gained (26 observations).

Results show an improvement in R^2 for ENCA as the dependent variable, compared with the values in run #35.

Run #42 again tests the log law I (as in run #41), except that artillery factors are lagged and FRCA is dropped as dependent variable, leaving only ENCA dependent. Also, after study of the residual results of run #41, data is again shifted slightly, in an attempt to explain why several data points fit poorly and give large residuals. This changes the number of observations to I-30, II-23, III-46, IV-25. It is not clear whether lagging or new data arrangement is responsible for the poorer results.

Run #43 is a change from #42 in that the artillery is not lagged. The results are better than in run #42 (with lagged artillery). The observations or decks are in the same order as run #42.

Run #44 The data is put back in the order of run #41 and the artillery factors, both lagged and unlagged, are forced in. Since no lagged artillery factors appear significant in the regression, the results are the same as for run #41. These results are better than those for run #43; therefore, the data is left in the order established in run #41.

Regression Analyses on Friendly Casualties as the Dependent Variable

Runs #45-46 Since the ENCA has reached a fairly high correlation coefficient, an attempt is made to improve the correlation coefficient for FRCA as dependent variable. The decks are divided according to the amount of ground gained. Run #45 contains the cards with $GG > 2.0$ miles, and run #46 contains the observations with $GG \leq 2.0$. Nothing significant is learned from this effort. The linear II law is used.

Run #47 This run is similar to run #41 with the following changes: FRCA is the only dependent variable, and the linear law is used.

Run #48 is the same as run #47, but with GG added as independent variable. Results improve. The rationale for including ground gained is that troops are exposed more and are therefore more vulnerable during such attacks.

Run #49 is made to see whether the enemy strengths derived in the detailed study of the Soyang River battle improve the regression coefficient. The EC values used before are used here, even though they are slightly different from the newly derived values. The FRCA is the dependent variable tested both with the factors of the linear law and as independent variables. Results are not as good.

TABLE E-I

EQUATIONS FOR ENEMY CASUALTIES MAY 1951

Usual transformations:

$$\ln FS = \ln\left(1 + \frac{FS}{100}\right) \qquad \ln AS = \ln(1 + AS)$$

$$\ln EC = \ln\left(1 + \frac{EC}{10}\right) \qquad GG_{(\text{in sq. mi.})} = GG\ .386$$

$$\ln LA = \ln\left(1 + \frac{LA}{10}\right) \qquad \ln HBS = \ln(1 + HBS)$$

$$\ln HA = \ln\left(1 + \frac{HA}{10}\right) \qquad \ln ES = \ln(1 + 10\ ES)$$

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2(R^2 total)	N
1B	All data	ln EC = -.12 - .3 ln HA + .64 ln LA + .21 ln AS + .52 ln FC	ln FS, GG, FL	Low R^2 expected	.55(.55)	205
9	Data per mile, battle	ln EC = -1.95 + .45 ln LA + .19 FL + missions	ln FS, ln ES, GG, ln HA, ln all air sorties	Shows data needs to be separated by missions	.62(.65)	129
11, 20-1, 17	U.N. advance, GG÷FL	ln EC = -4.3 + .93 ln ES + 0.57 ln LA	ln FS, GG, ln HA, ln AS_o, ln HBS, FL	Different causes for enemy casualties depending on U.N. or enemy mission	.64(.71)	27
13,18, 20-2	Enemy advance, GG÷FL	ln EC = -7.5 + .98 ln HA + 39 FL - 0.016 GG ln EC = -9.3 + 1.6 ln HA + .44 FL	ln FS, ln ES, ln AS_o, ln HBS_o	Different causes for enemy casualties depending on U.N. or enemy mission	.79(.82) .73(.82)	22
15 & 20-3, 19	Patrol	ln EC = -1.31 + .38 ln LA + .14 FL	ln FS, ln ES, GG, ln HA, ln AS, ln HBS		.39(.42)	76
22-1	U.N. attack, lagged	ln EC = -6.20 + 1.65 ln ES + .31 ln LA_{d-1}	Lagged ln HA, ln AS, ln HBS, GG	Limited lag test	.55(.61)	27
22	Enemy attack	ln EC = -10.8 + 1.7 ln ES + .55 FL	All lagged terms, ln FS, GG	Limited lag test	.70(.79)	22
22	Patrol	ln EC = -1.55 + .26 ln LA_{d-1} + .19 FK + .89 ln HBS_{d-1}	ln HA_{d-1}, ln $AS_{o,d-1}$, ln FS, ln IS	Limited lag test	.31(.41)	76

* See pages E-6 through E-12 for further explanation.

TABLE E-I (Cont'd)

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2 (R^2 total)	N
24-1	U.N. attack, unlogged ord.	ln EC = .56 ln ES + .0016 . LA	ln FS, AS_o, HBS_o, HA	Constant suppressed GG + FL deleted unlogged & logged ordnance	.42(.49)	28
24-2	U.N. attack, logged ord.	ln EC = .65 lnLA	ln FS, ln ES, ln HA, ln AS, ln HBS		.55(.66)	
24-3	Enemy attack, unlogged ord.	ln EC = .24 ln FS + .002 LA	ln ES, HA, HS_o, HBS_o		.70(.72)	19
24-4	Enemy attack, logged ord.	ln EC = .62 ln LA	ln ES, ln FS, ln HA, ln AS, ln HBS		.46(.60)	
24-5	Patrol, unlogged ord.	ln EC = .02 HA + 0.023 AS_o	ln ES, ln FS, LA, HBS		.57(.59)	74
24-6	Patrol, logged ord.	ln EC = 31 LA	ln FS, ln ES, ln HA, ln AS_o, ln HBS		.31(.39)	
25-1-1	U.N. attack, unlogged ord.	ln EC = -1.3 ln FS_{d-1} + .73 ln ES + 1.2 ln ES_{d-1} + .0017 LA_{d-1}	HA, LA AS_o, HA_{d-1}, HBS_{d-1}	Constant suppressed lagging included	.67(.71)	28
25-1-2	U.N. attack, logged ord.	ln EC = .65 ln LA	All other ord & strengths, plus lagged terms		.55(.78)	28
25-2-1	Enemy attack, unlogged ord.	ln EC = 1.9 ln FS -1.6 ln FS_{d-1} + .002 LA	ES, HA, AS, HBS & lagged terms		.78(.86)	19
25-2-2	Enemy attack, logged ord.	ln EC = -1.4 ln ES + 1.36 ln HA + .41 ln LA + .84 ln HBS	ln FS, ln AS_o, & all lagged terms		.70(.87)	19
25-3-1	Patrol, unlogged ord.	ln EC = .018 HA + .0013 LA_{d-1} + .014 AS_o	ln FS, ln ES, LA HBS & most lagged terms		.61(.63)	74
25-3-2	Patrol, logged ord.	ln EC = -.20 ln FS + .34 ln LA	ln ES, ln HA, ln AS, ln HBS, & lagged		.45(.48)	74

TABLE E-I (Cont'd)

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2 (R^2 total)	N
26-1-1	U.N. attack, unlogged ord.	Same as 25-1-1	WX & others as above	Weather Tested, but not significant.		28
26-2-1	Enemy attack, unlogged ord.	Same as 25-1-1				19
26-3-1	Patrol, unlogged ord.	Same as 25-1-1				74

Individual ordnance (tons) = .002 FS, .755 AS, 8.54 HBS, .018 LA, .048 HA*
Total ordnance (tons) = sum of the weights of each ordnance type

Lanchester equations with ordnance weights

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2 (R^2 total)	N
33-1-2	U.N. attack, square individual ord., constant suppressed	EC = 38.1 (0.755) AS_o	All strengths and other firepower		.20(.27)	28
33-1-3	U.N. attack total ord., constant suppressed	EC = 0.00012 (total ordnance tonnage)	All strengths		.06	28
33-2-1	Enemy attack, square, Ind. ord. weights	EC = 20.6 (0.012) LA	Strengths and other ordnance		.51(.66)	19
33-2-2	Enemy attack, square, total wt. ord.	EC = .00038 (total ordnance tonnage)			.31	19
33-3-1	Patrol, square, ind. ord, wts.	EC = 6.3 (.018) LA + 1.7 (.048) (HA)			.80(.82)	74

Lanchester square law with constant

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2 (R^2 total)	N
32-1-1	U.N. attack, ind. ord. wt.	EC = -28.6 (.755) AS_o			.25(.28)2	28
32-1-2	U.N. attack, total ord. wt.	EC = non-significant	All terms		.05	28
32-2-1	Enemy attack, ind. ord. wt.	EC = 46.5 (8.54) HBS + 16.2 (.018) LA			.61(.66)	19

TABLE E-I (Cont'd)

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2 (R^2 total)	N
32-2-2	Enemy attack, total ord. wt.	EC = .00046 (total ordnance tonnage)			.32	19
32-3-1	Patrol, ind. ord. wt.	EC = 6.2 (.018) LA + 2.8 + (.048) HA			.81(.83)	74
32-3-2	Patrol, total ord. wt.	EC = -162 + .00014 (total ordnance tonnage)			.66	74
	Linear law I (+ constant)					
34-1-1	U.N. attack, ind. ord.	EC = .05 ES + 17.5 (0.755) AS_o			.39(.41)	28
34-1-2	U.N. attack, total ord.	EC = + .000000009 (total ordnance tonnage)			.22	28
34-2-1	Enemy attack, ind. ord.	EC = 46.5 (8.54) HBS + 16.2 (.018) LA			.61(.71)	19
34-2-2	Enemy attack, total ord.	EC = .000000026 (total ordnance tonnage)			.56	19
34-3-1	Patrol, ind. ord.	EC = -51 + 6.18 (.018) LA + 2.8 (.048) HA			.81(.83)	74
34-3-2	Patrol, total ord.	EC = .000000006 (total ordnance tonnage)			.58	74
	Linear law II (+ constant)					
36-1-1	U.N. attack, ind, ord.	EC = 28.6 (0.755) AS_o			.25(.28)	28
36-1-2	U.N. attack, total ord.	None	All		.05	28
36-2-1	Enemy attack, ind. ord.	EC = 46.5 (8.54) HBS + 16.2 (.018) HA			.61(.66)	19
36-2-2	Enemy attack, total ord.	EC = 14.5 (total ordnance tonnage)			.51	19
36-3-1	Patrol, ind. ord.	EC = 6.2 (.018) LA + 2.8 (.048) LA			.81(.82)	74
36.3.2	Patrol, total ord.	EC = -277 + 4.1 (total ordnance tonnage)			.77	74

TABLE E-I (Cont'd)

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2(R^2 total)	N
Lanchester log law I (+ constant)						
35-1-1	U.N. attack, ind. ord. wts.	EC = 0.05 ES + 17.6 (.755) AS_0	Friendly strength & artillery		.39(.41)	28
35-1-2	U.N. attack, total ord. wts.	EC = .00018 (total ordnance tonnage)	Strengths		.28	28
35-2-1	Enemy attack, ind. ord wts.	EC = 46.5 (8.54) HBS + 16.2 (.018) LA	Strengths, AS & HA		.61(.71)	19
35-2-2	Enemy attack, total ord.	EC = .0006 (total ordnance tonnage)			.69	19
35-3-1	Patrol, ind. ord. wts.	EC = 6.2 (.018) LA + 2.8 (.048) HA			.81(.83)	74
35-3-2	Patrol, total ord. wts.	EC = .00015 (total ordnance tonnage)			.61	74
Lanchester log law II (+ constant) - (Same results as log law I)						
37-1-1	U.N. attack, ind. ord. wt.	EC = .05 ES + 17.6 (.755) AS_0			.39(.41)	28
37-1-2	U.N. attack, total ord. wt.	EC = .000018 (total ordnance tonnage)	Strengths		.28	28
37-2-1	Enemy attack, ind. ord.	EC = 46.5 (8.54) HBS + 16.2 (.018) LA			.61(.71)	19
37-2-2	Enemy attack. total ord.	EC = .00006 (total ordnance tonnage)			.68	19
37.3-1	Patrol, ind. ord. wt.	EC = 6.2 (.018) LA + 2.8 (.048) HA			.81(.83)	74
37-3-2	Patrol, total ord. wt.	EC = .000015 (total ordnance tonnage)			.61	74

38 Log law I with .02 as multiplier for enemy strength results almost identical to those of run number 35, except that total tonnage equations are poorer in R^2 when enemy attacks (0.56) and patrol (.27).

39 Ibed with .2 as multiplier for enemy strength results are simular to those of run number 35, except that the 3 total tonnage equations have R^2 = 0.22, 0.27, and 0.11.

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2(R^2 total)	N
Log law I omitting ordnance weight multiplier for enemy strength.						
40-1-1	U.N. attack, ind. ord. wt.	EC = -234 + 0.05 ES + 17.6 (.755) AS_0			.39(.40)	28

TABLE E-I (Cont'd)

Run	Type of run*	Significant terms in equation	Non-significant terms	Comment	R^2(R^2 total)	N
40-1-2	U.N. attack, total ord. wt.	EC = 0.0002 (total ordnance tonnage)			.27	28
40-2-1	Enemy attack, ind. ord. wt.	EC = 46.5 (8.54)HBS_o + 16.2 (.018) LA			.61(.70)	19
40-2-2	Enemy attack. total ord. wt.	EC = .00067 (total ordnance tonnage)			.70	19
40-3-1	Patrol, ind. ord. wt.	EC = -51 + 6.2 (.018) LA + 2.8 (.048) HA			.81 (.82)	74
40-3-2	Patrol, total ord. wt.	EC = .00018 (total ordnance tonnage)			.68	74
	Resorted cards so that advances are consistent with mission log law I, with constant					
41-1-1	U.N. advance, ind. ord.	EC = -2295 + 62.9 (.755) AS_o + 88 (8.54) HBS_o	Strengths and artillery		.67(.69)	26
41-1-2	U.N. advance, total ord.	EC = -1537 + .00067 (total ordnance tonnage)			.61	26
41-2-1	Enemy attack, ind. ord.	EC = 14.7 (.018) LA	Strengths, HA, all air		.68(.74)	21
41-2-2	Enemy attack, total ord.	EC = .00043 (total ordnance tonnage)			.75	21
41-3-1	Patrol, U.N. gained ground ind. ord	EC = -58 + 6.9 (.018) LA + 2.0 (.048) HA	Strengths, air		.78(.80)	48
41-3-2	Patrol, U.N. gained ground total ord.	EC = .00015 (total ordnance tonnage)			.62	48
41-4-1	Patrol, enemy gained ground ind. ord	EC = 5.7 (.018) LA + 3.4 (.048) HA	Strengths, air		.83(.85)	25
41-4-2	Patrol, enemy gained ground total ord.	EC = .0002 (total ordnance tonnage)			.76	25

44 series Same as 41 except that lagged and unlagged artillery were forced in. Results were identical with above and lagging was not significant.

TABLE E-II

EQUATIONS FOR FRIENDLY CASUALTIES (FC) FOR MAY 1951

Run	Type of run*	Significant terms in equation, Y = constant $+\Sigma K_i X_i$	Tested terms that were not sig. at 95%	Comment	R^2(R^2 total)	N
For transformations see table F-I.						
1A	All data	ln FC = .0002 + .13 ln HA + .21 ln EC	ln FS, GG, ln HA, ln LA, ln AS, FL	Low R^2 expected	.28(.29)	20
2A	Separation of days and place	ln FC = -23 + .11 ln HA + .08 ln FS + 1.36 phase	GG, ln LA, ln AS, ln EC, FL, day	Data should be separated by phase	.43(.44)	20
9	Test of missions	ln FC = 2.03 - .32 FR mission - .44 EN mission + .20 HA + 0.19 ln AS_o	GG, FL, ln FS, ln LA, ln ES, in	Shows data needs to be separated by mission	.44(.48)	12
11,20-1	U.N. advance	ln FC = 12.3-2.9 ln FS + .009 GG + .45 ln HA + .002 FL	ln ES, ln LA, ln AS_o, ln HBS_o	Note high R^2	.65(.70)	2
13,20-2	Enemy advance	ln FC = -3.4 ln FS + 1.91 ln ES - .035 GG + 1.7 ln HA	ln LA, ln AS_o, FL, ln HBS_o	Note high R^2	.74 (.79)	2
15,20-3	Patrols	ln FC = .26 + .02 GG + .10 ln LA + .46 ln HBS	ln FS, ln ES, ln HA, ln AS_o, FL		.45(.47)	7
17	U.N. advance, FL÷GG	ln FC = 12-6-3.0 ln FS + .10 GG (?) + .51 ln LA + .22 FL + .41 ln HBS_o	ln ES, ln HA, ln AS_o	Terms FL & GG not wanted.	.68(.70)	27
18	Enemy advance, FL÷GG	ln FC = .7 = .38 GG + .80 ln HBS_o	ln FS, ln ES, ln HA, ln AS_o	Terms FL & GG not wanted.	.44(.82)	22
19	Patrol, FL GG	ln FC = .3 + 0.09 GG + .12 ln LA + .44 ln HBS_o	ln FS, ln ES, ln HA, ln AS_o, FL	Terms FL & GG not wanted.	.37(.40)	76
22	U.N. attack	ln FC = -.7 - .43 ln LA_{d-1} + 0.2 FL	ln FS, ln ES, ln FC_{d-1}, GG	Test at lagged variables but omitted same day (error)	.42(.62)	27
22	Enemy attack	ln FC = 1.5 - 1.8 ln FS + 2.0 ln ES - .25 GG - .41 ln HA_{d-1}	All lagged ord. terms except HA		.77(.79)	22
22	Patrol	ln FC = 0.07 GG + 0.045 FL	ln FS, ln ES, all lagged ord.		.19(.55)	77
24-1	U.N. attack	ln FC = .27 ln ES + .013 AS_o	ln FS, LA, HA, HBS_o	Constant suppressed GG & FL deleted, comparison of	.27(.35)	28

TABLE E-II (Cont'd)

Run	Type of run*	Significant terms in equation $Y = \text{constant} + \Sigma K_i X_i$	Tested terms that were not sig. at 95%	Comment	R^2(R^2 total)	N
24-2	U.N. attack	ln FC = .37 ln LA - .66 ln HBS_o	ln FS, ln ES, ln HA, ln AS_o		.43(.48)	28
24-1	Enemy attack	ln FC = -2.5 ln FS + 2.8 ln ES + 0.023 AS_o + .17 HBS_o	LA, HA		.57(.61)	19
24-2	Enemy attack	ln FC = -2.8 ln FS + 3.1 ln ES + 0.71 ln AS_o - 0.87 ln HBS_o	ln LA, ln HA		.61(.65)	19
24-1	Patrol	ln FC = .0012 LA + .014 AS_o	ln FS, ln ES		.56(.56)	74
24-2	Patrol	ln FC = .084 ln LA + .36 ln HBS_o	ln LA, ln HA		.61(.65)	19
25-1-1	U.N. attack	ln FC = -0.55 ln FS_{d-1} + .88 ln ES_{d-1} + .0006 LA	ln FS, 2n ES, HA, HA_{d-1}, AS_o, $AS_{o_{d-1}}$, HBS_o, HBS_o, HBS_{d-1}	Similar to 24 but with lagging	.41(.54)	28
25-1-2	U.N. attack	ln FC = .37 ln LA - .65 ln HBS	ln strengths, ln HA, ln HA_{d-1}		.42(.54)	28
25-2-1	Enemy attack	ln FC = -2.5 ln FS + 2.8 ln ES + 0.037 AS_o - 0.14 HBS_o	HA, LA, and all lagged terms		.57(.84)	19
25-2-2	Enemy attack	ln FC = -1.9 ln FS + 2.0 ln ES + .4 ln AS_o + .2 ln AS_{d-1}	All but one lagged terms, LA, HA, HBS_o		.70(.73)	19
25-3-1	Patrol	ln FC = 0.026 ln ES_{d-1} + 0.0009 LA + .0008 LA_{d-1} + .008 AS_o - .008 AS_{d-1}	ln FS, ln FS_{d-1} ln ES, HBS_o, HBS_{d-1}	Similar to 24 but with lagging	.70(.73)	74
25-3-2	Patrol	ln FS = -.078 ln HA_{d-1} + .11 ln LA + .31 ln HBS + 0.45 HBS_{d-1}	All strengths, ln HA, ln LA_{d-1}, both AS_o terms		.37(.43)	74
26-1-1	U.N. attack	ln FC = -.55 ln FS_{d-1} + 0.88 ln ES_{d-1} + .00063 LA	Tested ln of strengths and original LA, HA, AS_o, HBS, and all lagged terms, also WX. (13 terms); insignificant ones are obtained by difference.	Test WX and lagged strength and fire support, constant suppressed.	.41(.54)	28

TABLE E-II (Cont'd)

Run	Type of run*	Significant terms in equation, $Y = \text{constant} + \Sigma K_i X_i$	Tested terms that were not sig at 95% Comment	$R^2(R^2\text{total})$	N
26-2-1	Enemy attack	$\ln FC = -2.5 \ln FS + 2.8 \ln ES + .037 AS_o - .14 HBS$		.57(.85)	19
26-3-1	Patrol	$\ln FC = .026 \ln ES_{d-1} + .0009 LA + .0008 LA_{d-1} + .008 AS - 0.008 AD_{d-1}$		.70(.73)	74
Lanchester Laws					
Square law, constant suppressed					
33-1-3	U.N. attack	FC = .004 ES	All other terms	.07	28
33-2	Enemy attack	FC = .017 ES		.01	19
33-3	Patrol	FC = .66 (.002) ES		.12	74
Square law, with constant					
32-1-3	U.N. attack	FC = .0025 (.002) ES	All other terms	.07(.10)2	28
32-1-4	Enemy attack	FC = Not significant	All terms	.01	19
32-	Patrol	FC = -.00147 FS + 1.9 (.002) ES		.22	74
Lanchester linear law, with constant					
34-1-3	U.N. attack, ind. ord.	FC = Not significant	All	.07(.10)	28
34-2-3	Enemy attack, ind. ord.	FC = 634	All	.08(.14)	19
34-3-3	Patrol, ind. ord.	FC = -0.0015 FS + 1.86 (.002) ES	Ordnance terms	.22(.22)	74
Lanchester linear law II					
36-1-3	U.N. attack, ind. ord.	FC = Not significant	All	.07	28
36-2-3	Enemy attack, ind. ord.	FC = Not significant	All	.01	19
36-3-3	Patrol, ind. ord.	FC = -21 + 1.4 (.002) ES		.18	74

TABLE E-II (Cont'd)

Run	Type of run*	Significant terms in equation, Y = constant + $\Sigma K_i X_i$	Tested terms that were not sig. at 95% Comment	R^2(R^2 total)	N
Lanchester log law I					
35-1-3	U.N. attack, ind. ord.	FC = Not significant	All	.07	28
35-2-3	Enemy attack, ind. ord.	FC = 634	All except constant	.08	19
35-3-3	Patrol	FC = -.0015 FS + 1.86 (.002) ES		.22	74
Lanchester log law II					
37-1-3	U.N. attack	FC = Not significant	All	.07	28
37-2-3	Enemy attack	FC = 634	All except constant	.08	19
37-3-3	Patrol	FC = -0.0015 FS + 1.86 (.002) ES		.22	74
Log law I with cards resorted so advances are consistent with mission (cf#35)					
41-1-3	U.N. attack	FC = Not significant	All	.09	26
41-2-3	Enemy attack	FC = 721	All except constant	.11	21
41-3-3	Patrol-U.N. gains	FC = Not significant	All	.01	48
41-4-3	Patrol-enemy gains	FC = Not significant	All	.06	25
48 series linear law, with ground gained as variable					
48-1-1	U.N. attack	FC = 1.56 sq. km. gained	Friendly and enemy strengths	.20(.24)	26
48-2-1	Enemy attack	FC = -15.1 sq.km. lost to enemy	Friendly and enemy strengths	.17(.28)	21
48-3-1	Patrol-U. N. gains	FC = -39.1 + 2.4 km. gained + .0013 ES	Friendly strength	.62(.65)	48
48-4-1	Patrol-enemy gains	FC = .0009 ES	Friendly strength and ground gained	.14(.14)	25

TABLE E-II (Cont'd)

Run	Type of run*	Significant terms in equation, Y = constant $+\Sigma K_i X_i$	Tested terms that were not sig. at 95 %	Comment	R^2(R^2 total)	N
49 series. Replacement of enemy strength and casualty data by data derived from the Soyang battle reconstruction						
49-1-1	U.N. attack	FC = 1.57 sq. km. gained	FS and ES		.20(.23)	26
49-2-1	Enemy attack	FC = -15.0 sq. km. lost to enemy	FS and ES		.17 (.27)	21
49-3-1	Patrol-U.N. gains	FC = -26. + 2.0 sq. km. gained -0.0013 SE + .0027 ES	None		.70(.70)	48
49-4-1	Patrol-enemy gains	FC = .0009 ES	FS and GG		.13(.7)	25

TABLE E-III

GROUND GAINED

Run	Type of run*	Equation, $Y = constant + \Sigma K_i X_i$	Tested terms that were not significant at 95%	Comment	R^2(R^2 total)	N
1A	All data	GG=209.7-43.3 ln HA	ln FS, ln LA, ln AS, ln FC, ln EC, FL	All variables	.01(.02)	205
2A	Tested day 1 phase	GG = -6613-45 ln HA - 65 ln EC + 399.6 phase	ln FS, ln FC	Separation by phase is less important than for FC & EC.	.05(.06)	205
10	Test mission	GG = .003 - .75 ln HA - .62 ln LA - .70 FR mission - .54 En mission	ln FS, ln ES, ln AS_o, ln EC, ln HBS_o		.29(.30)	129
12,17	U.N. advance, GG÷FL	GG = -1.92 + .89 ln HA -1.4 ln HBS = 15.6 + 3.3 ln FS	ln FS, ln ES, ln LA, ln AS_o		.14(.15) .12 (.26)	29 29
14,18	Enemy advance, GG÷FL	GG = 6.23 - 3.59 ln FS + .94 ln LA - .88 ln AS_o	ln ES, ln HA, ln HBS		.49(.60)	22
		= -4.1 + .58 ln LA - .94 AS_o + 1.9 HBS	1, FS, 1n ES, 1n HA		.52 (.58)	22
16,19	Patrol advance GG÷FL	GG = .88	ln FS, ln ES, ln HA, ln LA, ln AS_o, ln HBS	No intent to gain ground	.01(.02)	76
20-1	U.N. attack	GG = -219 + 46 ln FS	1n FS + ln HA + ln LA + ln AS_o + ln HBS		.13(.22)	27
20-2	Enemy attack	GG = -6.7 ln AS_o + 17.1 HBS	ln FS, ln ES, ln HA, ln LA		.33(.40)	22
20-3	Patrol	GG = 46 ln FS	1n ES, 1n HA, ln LA, ln AS_o, ln HBS		.13(.22)	76
24-1-1	U.N. attack	GG = 6.0 ln FS		No constant unlogged ord.	.03(.12)	28
24-1-2	U.N. attack	GG = 9.3 ln HA + ln HBS		No constant ln ord.	.05(.11)	28
24-2	Enemy attack	GG = -2.52 ln FS		No constant, unlogged ord.	.00(.42)	19
		GG = -8.6 ln AS + 14.3 ln HBS		No constant, ln ord.	.29(.39)	19
24-3	Patrol	GG = .47 AS		No constant, unlogged ord.	.14(.23)	74

TABLE E-III (Cont'd)

Run	Type of run*	Equation, $Y = \text{constant} + \Sigma K_i X_i$	Tested terms that were not significant at 95%	Comment	R^2 (R^2 total)	N
		GG = 3.4 ln AS		No constant, ln ord.	.02(.12)	74
25-1-1	U.N. attack unlogged ord.	GG = 6.7 ln ES_{d-1}	ln ES, ln FS, HA, LA, AS_o, HBS_o, most lagged terms	No constant, test lagging	.05(.30)	28
25-1-2	U.N. attack, ln ord.	GG = 7.5 ln HA	ln ES, ln FS, ln LA, ln AS_o, ln HBS, all lagged terms	No constant	.06(.28)	28
25-2-1	Enemy attack, unlogged ord.	GG = -4.2 ln FS + 2.0 HBS	All lagged terms		.15(.66)	19
25-2-2	Enemy attack, ln ord.	GG = -8.6 ln AS_o + 14.5 HBS	All lagged terms		.29(.70)	19
25-3-1	Patrol, unlogged ord.	GG = 1.7 ln ES + .27 AS_o - .29 $AS_{o,d-1}$ - 6.5 HBS + 5.8 HBS_{d-1}			.41(.43)	74
25-3-2	Patrol, ln ord.	GG = 7.4 ln ES = 5.6 ln FS_{d-1} - 9.8 ln HBS + 15.8 ln HBS_{d-1}			.28(.32)	74
26-3-1	U.N. attack (like 25 + WX)	GG = 6.7 ln ES_{d-1}	All other terms including lagged terms. Weather not significant.		.05(.38)	28
26-3-2	Enemy attack (like 25 + WX)	GG = -15.5 ln FS + .09 HA + WX		Weather significant (42.9)	.41(.73)	19
26-3-3	Patrol (like) 25 + WX)	GG = .33 AS_o - .25 AS_{d-1} + WX + 5.5 HBS + 5.9 HBS $_{d-1}$		Weather significant (4.2)	.41(.44)	74

TABLE E-IV

PREDICTION OF ENEMY STRENGTH

Enemy strength (ES)

Run	Type of run*	Equation, Y = Co + CiXi	Non-significant terms at 95%	Comment	R^2(R^2 total)	N
21	U.N. attack	ES = 3.6 + .2 ln LA	ln FS, ln ES, ln HA, ln AS_o, FL, HBS		.38(.54)	28
21-2	U.N.attack, GG÷FL	ES = 3.6 + .23 ln LA	Same		.38(.54)	28
21	Enemy attack	ES = 3.5 + .3 ln HA + .14 ln HBS			.41(.55)	22
21-1	Enemy attack, GG÷FL	ES = 2.7 + .47 ln HA - .15 AS_o + .24 HBS_o - .04 GG/FL			.58(.59)	22
21-3	Patrol, (same for for both cases)	ES = 3.5 + .13 FL			.45(.45)	77

TABLE E-V

LANCHESTER LAWS WITH ORDNANCE WEIGHTS

Square law: $K = \frac{ES \times EC}{FS \times FC} + C_i \, (w) \, (X_i)$

Run	Type of run*	Equation, Y = Co + CiXi	Non-significant terms at 95%	R^2(R^2 total)	N
31-1	U.N. attack	K = .48 (.002) (ES)	Ordnance sources: HA, AS, HBS, FS(ord.)	.24(.32)	28
31-2	Enemy attack	K = .95 (8.54) HBS		.41(.44)	22
31-3	Patrol	K = not significant		.03(.05)	74
Linear law					
30-1	U.N. attack	EC/FC = .28(0.048) HA		.21(.23)	28
30-2	Enemy attack	EC/FC = .54(8.5) HBS		.15(.20)	19
30-3	Patrol	EC/FC = not significant		.03(.05)	74