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REPORT NO. RT-TR-70-I

TEST EVALUATION REPORT

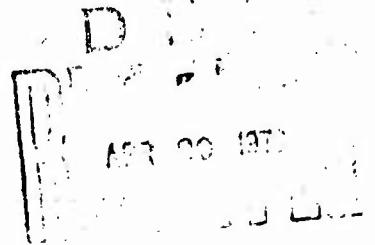# SENSITIVITY TESTING OF ONE-SHOT ITEMS

by

Carl Edward Roberts

February 1970

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama

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5 February 1970

TEST EVALUATION REPORT

SENSITIVITY TESTING
OF ONE-SHOT ITEMS

by

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Test and Reliability Evaluation Laboratory
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Redstone Arsenal, Alabama 35809

ABSTRACT

Sensitivity testing as applied to one-shot items consists of subjecting individual one-shot test items to discrete levels of a test environment, operating the test item and noting whether or not the item functions properly. From this type of test a direct measurement of the strength of the test item is not obtained, only the information that the strength is either greater than or less than the test level used. In order to obtain more meaningful information from these "quantal response" data it is necessary to use special techniques or procedures both to conduct the tests and to analyze the data.

This investigation was directed toward testing one-shot items associated with Army missiles, e. g. , thermal batteries, squibs, explosive, etc. The specific objectives of the investigations were (1) to evaluate available techniques for conducting sensitivity tests of one-shot items, (2) to improve existing techniques or develop an improved one, (3) to identify the best technique, and (4) to identify and evaluate any limitations of the best technique as identified from (3) above.

The approach taken was to simulate sensitivity testing with the different techniques on a digital computer. A simulation program was developed and the different techniques programmed for testing under closeby controlled conditions. In this way it was possible to control all inputs, providing accurate data for comparison and evaluation of the testing techniques.

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The Bruceton or Up and Down technique, and a technique developed by an Army missile contractor referred to as the missile technique, were the two techniques given primary consideration. The investigations revealed that the missile technique was superior to the standard Bruceton in providing estimates of the mean failure strength of test items. The missile technique estimates were from 1 to 32 percent more efficient than the Bruceton. Other characteristics of both techniques were evaluated and specific limitations of each technique identified.

ACKNOWLEDGMENTS

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CHAPTER I

INTRODUCTION AND BACKGROUND

1. The Requirement for Sensitivity Testing

Sensitivity testing consists of testing items at increasingly severe test levels and noting the effects on the operation or functioning of the item. A variation of this technique is the test-to-failure (TTF) approach wherein the test levels are deliberately increased to the point at which failure occurs, thus determining the inherent test item strength. Usually in this situation it is possible to operate the test item long enough to hunt for the exact value of the test environment which causes failure. But for a certain class of hardware, known as "one-shot" items, this is not possible due to their extremely short operating lives. To conduct sensitivity tests of these items it is necessary to select a single, discrete value or level of the test environment, subject the item to this test level, then operate the item and note whether or not it functions properly. A direct measurement of the strength of the test item is not obtained, only the information that the strength is either greater than or less than the test level used. This type of data is known in various forms as "all or none" data, "sensitivity" data, and "quantal responses." In order to obtain more meaningful information from quantal response data it is necessary

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to use special techniques or procedures to both conduct the tests and analyze the data.

The primary objective of testing as described above is to obtain a measure of the strength of the test item and the variation of the strength within a group or population of test items. Testing must therefore produce data from which statistical estimates of the mean strength, $\bar{x}$, and standard deviation, s, can be calculated. These estimates may then be used for various evaluations of the test items as required. One of the earlier requirements for sensitivity testing of one-shot items and analysis of the quantal responses appeared in the field of biological assay.[1] The requirement here was to estimate the mean response of a subject (some living matter such as an animal, a piece of animal tissue, a plant, or a bacterial culture) to a stimulus (a vitamin, a drug, or a fungicide). In this situation the intensity of the stimulus is usually varied by varying the measured dosage of the stimulus. The mean response is expressed as the dosage which produces the specified response. The specified response could be weight gain, an analytical value such as sugar count or pulse rate, death, or some other measurable bodily characteristic.

In the general area of explosives research, sensitivity testing with quantal responses assumes a major role. Interest in the sensitivity of explosives has been heightened during recent years by the increased use of solid propellant rockets in military applications and liquid propellant rockets in both military and space exploration applications, and the associated safety aspects. It should also be noted that the safety problems associated with tactical military rockets are greatly complicated by the inherent field handling environment to

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which such rockets are exposed.

This greatly increased concern with the safety aspect of explosives has resulted in the development and use of many techniques for the testing and measuring of explosive sensitivity.[2] The great majority of these sensitivity testing techniques result in quantal response data.

Usually these data are analyzed to provide a statistical estimate of the mean value of some physical stimulus which produces explosions in the test specimen. This information is then used to establish a qualitative evaluation of the particular explosive being tested. The evaluation usually consists of comparing the mean value of the stimulus with similar results for other explosives, establishing a relative sensitivity ranking. The stimuli used in these sensitivity tests have included impact of a weight falling from a measured height,[3] sympathetic detonation by a donor charge of a specified size and type operating through a controlled medium,[4,5] and container diameter for detonation propagation.[6]

Another technological area which has generated requirements for the statistical analysis of quantal data is the development of electro-explosive devices (EED). This area has also been stimulated in recent years by the increased emphasis on military and space rocket programs.

Electro-explosive devices are devices used on rockets to perform functions such as initiation of the main propulsion system and warheads, separation of stages, initiation of power supplies, and operating valves. It is evident that if an EED which initiates the propulsion system or warhead of a rocket is inadvertently initiated at any time after manufacture, a disastrous accident

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could result. It is known that EED's can be initiated by ambient electromagnetic energy from miscellaneous sources in the vicinity if these energy levels are high enough.[7] It is therefore necessary that the sensitivity characteristics of EED's be well defined and only those which are compatible with the electromagnetic environment be used.

The analysis of quantal response data also plays an important role in reliability evaluation. There are many items, both commercial and military, which exhibit one-shot performance and produce quantal response data when tested. Here, as in the associated area of quality assurance testing, the primary emphasis is on calculating the mean critical response value above which the item fails and below which it operates properly. It is also necessary to analyze the data to obtain an estimate of the standard deviation which is needed to establish confidence levels for reliability estimates and for hypothesis testing as used in quality assurance activities.

2. Sensitivity Testing of Missile and Rocket Hardware

The technological areas described to this point generated the original requirements for analysis of quantal response data. It is pertinent to note that all these technologies, with the exception of bio-assay work, are included in rocket and missile development. As a result, the analysis of quantal response data has received a tremendous amount of attention by the industries working in this area as well as by the responsible government military and space agencies. The technical complexity of the hardware being developed and the

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corresponding high cost of this hardware have placed emphasis on being able to completely understand its operation and reduce the possibility of performance failures.

A further complicating factor for military hardware is the use environment to which the hardware is exposed. This environment includes all the conditions present from the time the hardware leaves the production line until it is expended in the field. The use environment of army equipment is thought to be the most severe, presenting the greatest challenge to the developer of such hardware. Thus the testing problem presented by one-shot items used in army missiles is especially significant.

Generally, development testing of missile hardware has the following broad objectives: (1) to determine/verify design characteristics, (2) to assess reliability, and (3) to determine safety characteristics. For one-shot items, test-to-failure data may be used to satisfy all three of these objectives. To do this the concept of "safety margins" is used.

The use of safety margins was first advocated by Robert Lusser.[8] The advantage of this philosophy as opposed to qualification testing, for example, is that statements regarding very high reliability can be made with reasonable confidence on the basis of relatively small samples.

Design evaluation by safety margin is accomplished by testing the critical components of the missile in their most critical operating environment. The test-to-failure establishes the mean level of the environment at which the component will fail. This value is then compared with the environmental level at which the component is required to operate and the safety

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margin established. Since the standard deviation can also be calculated from the test-to-failure data, the safety margin can be expressed as so many standard deviations, or sigmas. The greater the number of sigmas, the better the design. The components which have safety margins of only 2 or 3 sigmas would be considered marginal and would have to be redesigned. A component with too large a safety margin could be considered over-designed, and redesigned if desired.

The same information (i.e., safety margins expressed as a number of sigmas above a required level) could be evaluated for reliability by referring to tables for the standardized Normal deviate. By doing this it is possible to express the reliability of the component for operation at the required environment level. Safety assessment may be accomplished by using either of the two procedures.[9]

3. Program Objectives

The particular aspect of testing one-shot items of army missile hardware and evaluating the results as described above is the general subject of this paper. More specifically, investigations of the statistical analysis of the quantal response data within the constraints imposed by army missile hardware are described. The specific objectives of these investigations were (1) to evaluate available techniques for conducting sensitivity tests, (2) to improve existing techniques or develop an improved one, (3) to identify the best technique, and (4) to identify and evaluate any limitations of the best technique as

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identified from (3) above.

The approach taken was to simulate sensitivity testing with the different techniques on a digital computer. A simulation program was developed and the different techniques programmed for testing under closely controlled conditions. In this way it was possible to control all inputs, providing accurate data for comparison and evaluation of the testing techniques.

4. Selection of Techniques for Evaluation

If quantal data analysis techniques which are nonparametric, such as the Spearman-Karber[10] and Robbins-Monro[11] methods, are ruled out because of the relatively large sample sizes required, the choice of which technique to use is one between the Probit[12,13] and Bruceton.[14,15] Investigations by Gayle[16] in which the Probit and Bruceton techniques were used to analyze identical sets of data under closely controlled conditions reveal that the Bruceton estimates are, in general, superior to the Probit estimates for equal sample sizes. Since test costs (and hence sample sizes) are the predominate factors in any sensitivity test program (except perhaps in the field of medicine), the Probit technique was eliminated from further consideration.

In rejecting the nonparametric techniques and the Probit technique solely on the fact that more samples of test hardware are required, more elaboration on this point is indicated. The primary field of sensitivity testing being considered is the aerospace field, more specifically military missile and rocket components. Included are such items as thermal batteries, explosive

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switches and relays, special infrared and visible light sources, gyroscopes, and solid propellant rocket motors. These items, especially when produced in small, development-sized quantities, can be two or three orders of magnitude more expensive than laboratory specimen or test animals used in dosage mortality and biological assay work. Further, the actual testing of the afore-mentioned hardware (e.g., vibration testing of a gyro) can cost orders of magnitude more. Thus any reduction in sample size at all can result in a very substantial savings of test program costs. One further point is that laboratory tests in which the Probit technique is used are usually more conducive to concurrent testing of individual test specimens; that is, the testing of twenty mice by injecting them with a serum is more practical than testing them one at a time. The opposite is generally true with aerospace hardware. There usually are not enough facilities or the tests are too hazardous to permit testing more than a few items at a time.

Thus, a technique for analyzing quantal data is desired which will provide reasonably accurate statistical estimates from relatively small samples of test hardware, and which will allow tests to be conducted essentially one at a time. Based on the author's personal experience it is felt that "relatively small samples" should be interpreted as meaning sample sizes of up to 40 to 50 specimens maximum allowed, and samples of 15 to 25 specimens desired. Based on the information discussed to this point, the Bruceton technique more nearly meets these restrictions.

There is another technique for sensitivity testing which was designed for and used during development and production testing of a U. S. Army

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missile system. [17] This technique, which will be called the Missile technique, permits one-at-a-time testing and can be used with sample sizes as small as 15 specimens. It appeared to be superior to the Bruceton in sample size requirements and thus more suitable for aerospace sensitivity testing. Thus, the choice of which technique to use was between the Bruceton and Missile techniques, with the ultimate answer being based primarily on sample size required and accuracy.

There are a number of similarities between the Bruceton and Missile techniques. Both require the assumption that the critical responses be distributed Normally. Both also provide maximum likelihood estimates of the statistical parameters. The test levels required by both techniques are generated sequentially, the levels being determined partially from earlier results as testing progresses through the sample.

The most apparent difference between the Bruceton and Missile methods is the manner in which the individual test levels are generated as the test program progresses. The test level for a particular Bruceton test item is set at a constant increment above or below the previous test level, depending upon whether or not the previous test was a success or a failure. A test level for the Missile method is also dependent upon the results of the previous item tested. In this method, however, the increment by which the previous test level is changed is not a constant, but is varied according to a simple algorithm.

A further basic difference between the two methods is that the Missile method requires no a priori assumptions regarding the mean or standard

deviation of critical responses. The Bruceton method requires estimates of both before the tests can be planned. The accuracy of these estimates has a direct influence on the efficiency of the test program.

5. The Assumption of Normality

Since these were basically the only techniques considered for further investigation, the Normality assumption common to both should be discussed in more detail. The requirement of this assumption is that the critical response or some transformation of it be distributed Normally. This requirement is not considered to be particularly restrictive since there are many functions which might provide the required transformation. Also the Normal distribution is the most common one found in nature and could reasonably be expected to occur frequently with aerospace hardware. That this is true has been well documented by test results from the biological assay field, [1] many tests of EED's, [9] explosive shock sensitivity test data, [3,4,5,18] and metal fatigue testing. [19]

The alternatives to an assumption of Normality (or any other form of distribution) are either (1) the use of nonparametric (distribution free) techniques or (2) conducting enough tests to determine the form of the distribution. Neither of these is considered practical because of the large sample sizes required. [20] Hence, the situation is that an assumption be accepted about the form of the distribution, or no practical technique is available at all.

The effects of using the Normal assumption with non-Normal data have

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been investigated by Gayle[16] and Hampton.[21] The general conclusion from these investigations is that the Bruceton tests provided results which were "optimistic." That is, the standard deviation estimates were smaller than the actual population standard deviation. By using the concept of "safety margins" as described by Lusser,[8] this problem is minimized.

6. Other Characteristics of the Bruceton and Missile Techniques

In considering the choice between the Bruceton and the Missile techniques, it is pertinent to realize that the Bruceton has been in use much longer than the Missile technique. The Bruceton was first described in open literature in 1944 and since that time has been used many times by investigators in various experimental situations. It would thus appear that the Bruceton technique should be better defined and its capabilities well documented. This does indeed appear to be true, but the basic technique has not changed appreciably from that originally described in 1944. This is particularly significant when the availability of high-speed computing equipment is considered. No such capabilities were available in 1944, when the Bruceton technique was developed, but the general field of quantal data analysis has since been the subject of many computer-aided investigations.[22,23] Similar investigations of the Bruceton technique have been somewhat limited, and that is one of the reasons for this investigation.

Probably the most significant problem or weak point of the Bruceton technique is the requirement to have an estimate of the mean and standard

deviation prior to starting the tests. The mean is used to establish the initial

test level, and the standard deviation is used to establish the constant incre-

ment by which test levels are changed to obtain subsequent test levels. The

accuracy of these estimates influences the results depending on (1) the posi-

tion of the population mean relative to the testing levels and (2) the ratio of the

test increment, d, (i.e., the estimated standard deviation) to the population

standard deviation, σ.

From his investigations of the Bruceton technique, Hampton[21] concluded

that the correlation between the estimated standard deviation, s, and the

population standard deviation, σ, was poor for sample sizes of 25. This fact

has been substantiated by others and must be considered in any evaluations

of Bruceton data.

There are other more subtle points made by Hampton from his investi-

gations, but these are considered insignificant so far as this particular paper

is concerned. The other conclusions discussed above are considered more

pertinent and were used during this investigation.

CHAPTER II

DESIGN AND DEVELOPMENT
OF A SENSITIVITY TEST SIMULATION PROGRAM

1. General Philosophy

The use of a digital computer simulation program with a simple Monte Carlo technique to generate artificial test results was selected as the only practical means of obtaining the desired information. The ideal way would be to test actual hardware of the type being considered and use these results in conjunction with further analytical work. Since this approach is impractical from the standpoint of time and cost, the simulation approach was selected as the most practical alternate. It was possible, however, to make use of results available from limited sensitivity tests of appropriate test hardware in establishing certain conditions required for the simulation program.

In considering the total simulation effort three phases were planned. The first was a comparison/evaluation phase in which the two techniques were used to analyze the same data under closely controlled conditions. This permitted a direct comparison of the two techniques and provided results for a detailed evaluation of each. The results from this phase were also used to identify any weaknesses in technique capability and to guide subsequent attempts at improvement. The next phase of simulation consisted of attempts

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to improve the two techniques by modification of the earlier programs. These modifications were based on the demonstrated weaknesses and were guided by statistical approaches revealed by literature research in the field of sensitivity testing and quantal data analysis. The last phase consisted of evaluating the final version of the best technique. The input data for this phase were the same as the initial simulation, permitting a direct comparison and demonstration of any improvement.

In designing the simulation program it was decided that all conditions and inputs would be held as closely as possible to practical, real-world conditions. This was done to prevent getting results which might be good in an ideal sense but would be impractical for implementation in the real-world environment.

For assistance in following the development of the simulation program, a glossary is presented on page 124 and brief descriptions of the Bruceton and Missile techniques are presented in Appendixes I and II, respectively. For more detailed information reference should be made to Princeton Statistical Research Group[14] and Dixon and Mood[15] for the Bruceton technique and Langlie[17] for the Missile technique. The simulation was programmed on an IBM 7094 computer in FORTRAN IV computer language. The computer program listing is presented in Appendix III.

Basically, the simulation program was designed to generate a sample of "test hardware," as represented by a set or sample of appropriate random numbers. The "test item" was then "tested" according to the Bruceton and then the Missile technique. This procedure was continued until the complete

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sample of "hardware" was tested. The mean and standard deviation was then computed according to the respective techniques and recorded. The sample mean and standard deviation were also computed in the usual manner for Normally distributed data and recorded for comparison. The required number of samples was generated, tested, and analyzed in this manner, and then the standard error of all the sample estimates was calculated. These standard errors were used for final evaluation of the techniques. Additionally, correlation coefficients between the Bruceton and Normal and between the Missile and Normal standard deviation estimates were calculated and recorded. This procedure was followed, in general, for all simulation work.

The test conditions for the simulation work were determined based on the range of sample sizes to be investigated and on actual results of sensitivity tests of missile component hardware. The test conditions selected are shown in Table 1, and the test data used in determining the conditions are in Table 2.

Table 1. Simulated Test Conditions

		Sample Sizes		
		15	35	55
Sample Populations	Population No. 1 $\mu = 100$ $\sigma = 0.02$	T.C.* No. 1 100 Samples	T.C. No. 4 100 Samples	T.C. No. 5 100 Samples
	Population No. 2 $\mu = 1.00$ $\sigma = 0.09$	T.C. No. 2 100 Samples	T.C. No. 6 100 Samples	T.C. No. 8 100 Samples
	Population No. 3 $\mu = 1.00$ $\sigma = 0.26$	T.C. No. 3 100 Samples	T.C. No. 7 100 Samples	T.C. No. 9 100 Samples

* T.C. ≡ Test condition

Table 2. Missile Component Test Data

Missile Component	High Temperature (°F)		Low Temperature (°F)		Vibration (g²/Hz)		Mechanical Shock (g)	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
Thermal battery No. 1	208.6	15.14	-104.9	7.82			742	0.171*
							551	0.112*
Thermal battery No. 2	199.5	20.4	-133.9	7.49				
	345.9	6.01						
Infrared source No. 1	271.7	10.33	-125.3	4.5	0.228	0.1095*		
Infrared source No. 2					0.300	0.9747*		
					0.310	0.7063*		
					0.160	0.7193*		

2. Establishing Starting Conditions

After the simulation test conditions were established, the next step was to program the two techniques for the computer. For the Bruceton technique it is necessary to have preliminary estimates of the mean and the standard deviation in order to determine the initial test level and the constant increment by which subsequent test levels should be changed. The decision to be made is to what accuracy could the mean and standard deviation be estimated. Since

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*In terms of natural logarithms.

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these estimates would ordinarily be based on similar data from other test programs, it was decided to use the data in Table 2 for this purpose. A simple statistical analysis of these data was performed which was considered to represent that which would be done under "real-world" conditions. The "estimates" thus derived as inputs are shown below.

Population No.	Estimated μ	Estimated σ
1 (1, 0.02)	1.07	0.035
2 (1, 0.09)	1.07	0.135
3 (1, 0.26)	1.07	0.390

The Bruceton technique estimate of the standard deviation, s, is calculated by first computing a value of the expression (Appendix I)

$$M = \left(\Sigma\, i^2\, n_i\right)/N - \left(\Sigma\, i\, n_i/N\right)^2 .$$

The value of M is then used to enter Table 3 or Figures 1 and 2 for a value of S. The value of s is now determined from

$$s = D\,S,$$

where D is the test level increment.

For values of M greater than approximately 0.325 the graph used is linear (Figure 1). For values of M less than 0.325, however, a family of four curves must be considered (Figure 2). The curve used is selected based on the relative location of the estimated mean to one of the test levels; i.e., if the mean were located half-way between two test levels, one specific curve would be used. Since only four curves are presented, interpolation between them is required.

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Table 3. Table of s for Obtaining the Sample Standard Deviation

M	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.30	0.5102 + 239	0.5316 + 168	0.5518 + 113	0.5711 + 70	0.5897 + 37	0.6077 + 13	0.6253 − 5	0.6426 − 18	0.6597 − 28	0.6765 − 34
0.40	0.6932 − 37	0.7098 − 39	0.7263 − 39	0.7427 − 37	0.7591 − 36	0.7754 − 33	0.7917 − 31	0.8079 − 27	0.8242 − 24	0.8405 − 22
0.50	0.8567 − 19	0.8729 − 16	0.8892 − 14	0.9054 − 12	0.9216 − 10	0.9379 − 9	0.9541 − 7	0.9703 − 6	0.9866 − 5	1.0028 − 4
0.60	1.0190 − 3	1.0353 − 3	1.0515 − 2	1.0677 − 2	1.0839 − 1	1.1001 − 0	1.1164 − 1	1.1326 − 1	1.1488 − 0	1.1650 − 0
0.70	1.1812 − 0	1.1974 − 1	1.2135 − 0	1.2297	1.2459	1.2621	1.2783	1.2944	1.3106	1.3267
0.80	1.3429	1.3590	1.3752	1.3913	1.4075	1.4236	1.4397	1.4559	1.4720	1.4881
0.90	1.5043	1.5204	1.5365	1.5526	1.5687	1.5848	1.6009	1.6170	1.6331	1.6492
1.00	1.6653	1.6814	1.6975	1.7135	1.7296	1.7457	1.7618	1.7779	1.7939	1.8100
1.10	1.8261	1.8422	1.8582	1.8743	1.8904	1.9064	1.9225	1.9386	1.9546	1.9707
1.20	1.9867	2.0028	2.0188	2.0349	2.0509	2.0670	2.0830	2.0991	2.1151	2.1312
1.30	2.1472	2.1633	2.1793	2.1953	2.2114	2.2274	2.2435	2.2595	2.2755	2.2916
1.40	2.3076	2.3236	2.3397	2.3557	2.3717	2.3878	2.4038	2.4198	2.4358	2.4519
1.50	2.4679	2.4839	2.4999	2.5159	2.5320	2.5480	2.5640	2.5800	2.5960	2.6121
1.60	2.6281	2.6441	2.6601	2.6761	2.6921	2.7082	2.7242	2.7402	2.7562	2.7722
1.70	2.7882	2.8042	2.8202	2.8362	2.8523	2.8683	2.8843	2.9003	2.9163	2.9323
1.80	2.9483	2.9643	2.9803	2.9963	3.0123	3.0283	3.0443	3.0603	3.0763	3.0923
1.90	3.1083	3.1243	3.1403	3.1563	3.1723	3.1883	3.2043	3.2203	3.2363	3.2523
2.00	3.2683	3.2843	3.3003	3.3163	3.3323	3.3483	3.3643	3.3803	3.3963	3.4123
2.10	3.4283	3.4443	3.4603	3.4763	3.4923	3.5083	3.5243	3.5402	3.5562	3.5722
2.20	3.5882	3.6042	3.6202	3.6362	3.6522	3.6682	3.6841	3.7001	3.7161	3.7321
2.30	3.7481	3.7641	3.7801	3.7961	3.8121	3.8280	3.8440	3.8600	3.8760	3.8920
2.40	3.9080	3.9240	3.9400	3.9560	3.9720	3.9879	4.0039	4.0199	4.0359	4.0519
2.50	4.0678	4.0838	4.0998	4.1158	4.1318	4.1477	4.1637	4.1797	4.1957	4.2117
2.60	4.2277	4.2436	4.2596	4.2756	4.2916	4.3075	4.3235	4.3395	4.3555	4.3715
2.70	4.3874	4.4034	4.4194	4.4354	4.4514	4.4673	4.4833	4.4993	4.5153	4.5313
2.80	4.5472	4.5632	4.5792	4.5952	4.6111	4.6271	4.6431	4.6591	4.6750	4.6910
2.90	4.7070	4.7230	4.7390	4.7549	4.7709	4.7869	4.8029	4.8188	4.8348	4.8508
3.00	4.8668									

THIS CURVE GIVES THE VALUE OF s CORRESPONDING TO THE
NUMBER M (PAGE 71). TO BE USED IN DETERMING THE ESTIMATE
OF THE STANDARD DEVIATION, σ. MORE ACCURATE VALUES
CORRESPONDING TO THIS CURVE MAY BE OBTAINED FROM TABLE 3.
AN ENLARGEMENT OF THE PORTION IN THE DOTTED RECTANGLE
IS GIVEN IN FIGURE 2 ON A LARGER SCALE, WHICH SHOULD BE
USED IF $M < 0.40$.
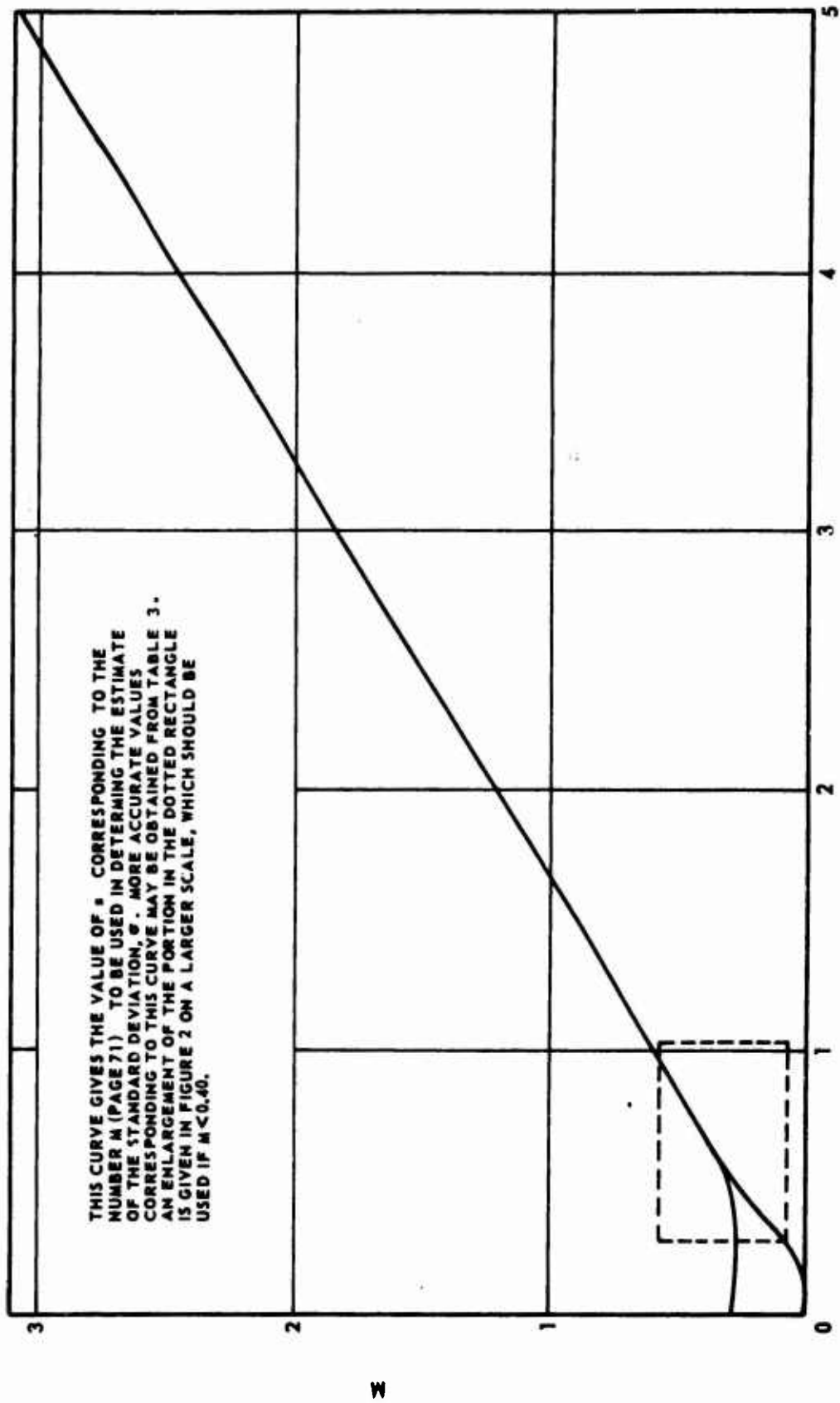
Figure 1. Curves for Estimating the Standard Deviation

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THESE CURVES GIVE THE VALUE OF s CORRESPONDING TO THE NUMBER M (PAGE 71). THE ESTIMATE OF THE STANDARD DEVIATION IS THEN σ = ds WHERE d IS THE DISTANCE BETWEEN TWO TESTING HEIGHTS. THE CURVE MARKED m - h = 0 IS FOR THE CASE WHERE THE MEAN IS AT ONE OF THE TESTING HEIGHTS, THE CURVE MARKED m - h = 0.5d IS FOR THE CASE WHERE THE MEAN IS ONE HALF OF d AWAY FROM A TESTING HEIGHT, ETC. THIS GRAPH MAY BE USED FOR SMALL VALUES OF M (0.05 TO 0.40).

VALUES OF s FOR LARGER M MAY BE OBTAINED FROM FIGURE 1 OR TABLE 3.

Figure 2.   Curves for Estimating the Standard Deviation (M <0. 40)

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To find exact mathematical functions for these four graphs and a technique for interpolating between them, all suitable for the computer, presented a formidable, if not impossible, task, and an alternate procedure was necessary. For values of M greater than 0.325 an expression for the linear portion of the graph is used as follows[15]:

$$s = D S,$$

where $S = 1.620 (M + 0.029)$. Again, this solution is only good when M is greater than 0.325.

For values of M less than 0.325 a different method was used to compute both the estimated mean, $\overline{x}$, and the estimated standard deviation, s. This method is described in Appendix IV by Dixon and Mood, [15] and, while not specifically intended for use for the reasons given here, was evaluated and found to be acceptable. The method provides simultaneous solutions of the two maximum likelihood equations by iteration. The iteration procedure was set up and checked out to obtain solutions to an accuracy of ±0.0001.

There is only one initial condition to be set for the Missile technique, the test interval. The test interval is described as an interval of the test environment or stimulus large enough to contain all values of the critical responses of the test items. To obtain realistic values for this interval the test data of Table 2 were again analyzed, and the results were:

Population No.	Test Interval
1 (1.0, 0.02)	0.364 to 2.20
2 (1.0, 0.09)	0.364 to 2.20
3 (1.0, 0.26)	0.000 to 2.63

The input conditions for the simulation program are summarized in Table 4 for the initial nine test conditions shown in Table 1. The last three items listed under "Program Input" are concerned with the Bruceton iterative solution and random number generation. These inputs will be discussed subsequently in another section of this document.

Up to this point in the simulation, efforts have been described for generating the artificial test items and testing these items using the Bruceton and Missile techniques. At this point the simulation program was satisfactorily designed to accomplish this, and subsequent programming for computations necessary in the evaluation of the results were formulated. Since the basic objective was to compare and evaluate the two techniques, a basis for comparison was necessary.

Each technique estimates the mean and standard deviation using the quantal responses. Since the actual values of the critical responses (i.e., the RNN) are available, it is possible to estimate the mean and standard deviation of the samples using this information in the standard manner (i.e., using the equations for continuous, Normally distributed data). It can be shown that the sample estimates using the Normal equations are the best obtainable, and hence could be used as a basis for comparing the other estimates. Thus the Bruceton and Missile estimates can only approach the Normal estimates in accuracy and repeatability. Any comparison of estimates must consider the magnitude by which the estimate differs from the true value as well as the repeatability of the estimate. The means of the estimated means, $\overline{X}_{\overline{x}}$, and the estimated standard deviation, $\overline{X}_s$, should thus be used. Also, the

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Table 4. Simulation Program Computer Inputs

Program	Test Conditions								
Input	1	2	3	4	5	6	7	8	9
PXB (population mean)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PS (population standard deviation)	0.02	0.09	0.26	0.02	0.02	0.09	0.26	0.09	0.26
H (initial test level — Bruceton)	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07
DELTA (test level increment — Bruceton)	0.030	0.135	0.390	0.030	0.030	0.135	0.390	0.135	0.390
NS (sample size)	15	15	15	35	55	35	35	55	55
NO (number of samples)	100	100	100	100	100	100	100	100	100
BETA (standard deviation bias correction — Missile)	0.762	0.762	0.762	0.865	0.875	0.865	0.865	0.875	0.875
ULIM (test interval upper limit — Missile)	2.20	2.20	2.63	2.20	2.20	2.20	2.63	2.20	2.63
LLIM (test interval lower limit — Missile)	0.364	0.364	0.0	0.364	0.364	0.364	0.0	0.364	0.0
XMULT ($\bar{x}$ increment for Bruceton iterative process)	0.05	0.20	0.40	0.05	0.05	0.10	0.20	0.05	0.10
SMULT (s increment for Bruceton iterative process)	0.25	0.33	0.50	0.20	0.15	0.25	0.35	0.20	0.25
MRT (random number generator)	18974 35261	19847 53021	19847 53623	18974 35263	18974 35265	19847 53625	19847 53627	29874 53621	29874 53625

standard deviations of the estimated means, $S_{\bar{X}}$, and the estimated standard

deviations, S_s, should be used to evaluate repeatability.

Therefore, in addition to the sample means and standard deviations

estimated by the Normal, Bruceton, and Missile techniques for each sample

of test items, the means of the estimated means and standard deviations from

the 100 different samples as well as the standard deviations of the data were

computed for each analysis technique.

It was also an objective of this simulation program to investigate the

degree of correlation between the individual sample Bruceton and Missile

estimates of the standard deviation with the corresponding Normal estimates.

The correlation between the sample Bruceton mean estimates was reported by

Hampton[21] to be very good, i.e., closely and positively correlated with the

calculated coefficient of correlation approaching a value of +1.0. Therefore,

no further investigation of mean correlation was necessary. The coefficient of

correlation of the standard deviations for the 100 samples from each test

condition was calculated by the formula given by Hoel. [24]

Checkout of the computer simulation program was accomplished with

two objectives in mind. First of all, it was necessary to see that the computer

had been programmed properly to accomplish the desired calculations in

correct sequence and to see that the program logic was operating as intended.

Second, if the calculations and program logic were correct, it was necessary

to see that all operations on the computer were being accomplished as effi-

ciently as reasonably possible. This was done to reduce the cost of conducting

the program and was aimed prim   reducing the computer running time.

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The checkout was accomplished by running the simulation through a small number of small sample size tests. The computer was programmed to print out the results of significant intermediate steps which would not ordinarily be required. With this information it was possible to follow the simulation step by step and check the results by manual computations on a desk calculator.

These initial efforts did indicate excessive computer run time, and certain changes were necessary. The most significant of these changes was concerned with the Bruceton iteration technique described earlier (Appendix IV). Analysis of the test results was requiring too many iterations for convergence of the solutions. The iterative procedure used by the Bruceton technique requires trial solutions on both sides (positive and negative) of zero in order to interpolate for the solution at zero (the maximum likelihood value). The computer logic required is to first determine whether the initial trial solution is positive (or negative) and then to increment this trial value by such an amount to get a solution with the opposite sign. This process is repeated until the absolute magnitude is sufficiently small to permit accurate solutions at zero from the linear interpolation.

The logic originally designed into the computer program required that the initial values of $\overline{x}$ and s (obtained by the standard Bruceton calculations) be incremented by an amount equal to the a priori estimates of $\overline{x}$ and s, i.e., 1.07 $\overline{x}$ and 1.5 s. (Initial conditions have been discussed earlier.) After evaluating the initial runs with this logic it was concluded that these increments were too large.

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In determining the size of the required increment two conflicting considerations are present. A large increment is desired in order to get a sign change, but at the same time a smaller increment is desired to get more accurate results. As a compromise the following reasoning was used. It is known that the standard error of estimating σ is a direct function of $\sqrt{n}$ (where n is the sample size). By using s at n = 15 as the base, the ratio of $\sqrt{n}$ at the different sample sizes was obtained. This, when multiplied by the increment at n = 15, should give similar increments for the larger sample sizes. The initial increments of s for the three values of σ were determined based on analysis of the preliminary runs as 25 percent of s for σ = 0.02, 33 percent for σ = 0.09, and 50 percent for σ = 0.26. These percentages are essentially estimates of the maximum error anticipated in the Bruceton estimates of the population standard deviation. Increments for $\bar{x}$ were calculated in a similar manner. The above calculations are summarized in Table 5.

Table 5. Δs and $\Delta\bar{x}$ Calculations

Δs Calculations				
n	Standard Error Ratio	Increment (Δs) @σ = 0.02	@σ = 0.09	@σ = 0.26
15	1 (base)	0.25	0.33	0.50
35	0.67	0.20 (0.67) 0.25	0.25	0.35
55	0.53	0.15 (0.53) 0.25	0.20	0.25
$\Delta\bar{x}$ Calculations				
n	Standard Error Ratio	Increment ($\Delta\bar{x}$) @σ = 0.02	@σ = 0.09	@σ = 0.26
15	1	0.05	0.20	0.40

Table 5. Δs and $\Delta \bar{x}$ Calculations (Concluded)

		$\Delta \bar{x}$ Calculations		
n	Standard Error Ratio	Increment $(\Delta \bar{x})$ @σ = 0.02	@σ = 0.09	@σ = 0.26
35	0.428	0.05	0.10	0.20
55	0.275	0.05	0.05	0.10

The increments in Table 5 were programmed into the computer as initial inputs, SMULT (s multiplier) and XMULT ($\bar{x}$ multiplier). Checkout runs with these inputs provided satisfactory results and this approach was made a permanent part of the simulation program.

3. Summary of Computer Simulation Program

With the required changes finalized and checked out the computer simulation program design was complete. It consisted of a main computer routine and the subroutines summarized below. The complete program is presented in Appendix III, and is structured as shown in the diagram below.

Main Program Routine

Bruceton Technique Subroutine (BRUCMT)

└Failure Level Analysis Subroutine (CHEK)

└Bruceton Iteration Subroutine (INTERP)

└Interpolation Subroutine (ITR)

Missile Technique Subroutine (MISSMT)

└Integration Subroutine (IGRAT)

└Solution of Simultaneous Equation Subroutine (SESOMI)
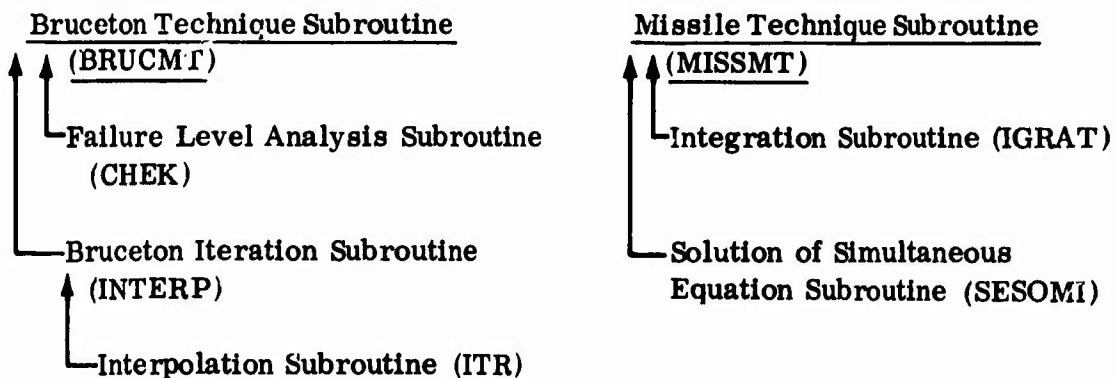
The main program routine accepts the test condition inputs, generates

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the random Normal numbers, and calculates the overall statistical param-
'eters using the Normal, Bruceton, and Missile estimates. The Bruceton
technique subroutine (BRUCMT) accepts the random Normal numbers from
the main routine, generates test levels, accomplishes the tests, tabulates
results, and computes estimates of μ and σ which are returned to the main pro-
gram routine. The Missile technique subroutine (MISSMT) operates in a
similar manner.

The Failure Level Analysis subroutine (CHEK) was used to analyze the
failure/success/test-level data and compute certain parameters from these
data required for the Bruceton estimates. The Bruceton Iteration subroutine
(INTERP) was used where the values of M were less than 0.325 and required
the iteration procedure for estimating. The Interpolation subroutine (ITR)
was used for interpolating between values in two tables used in the Bruceton
iteration procedure.

The Integration subroutine (IGRAT) was used in the Missile technique
to integrate the expression

$$G = \int_{-\infty}^{t} \frac{1}{2\pi} e^{\left(\frac{-t^2}{2}\right)} dt.$$

SESOMI is a subroutine used in the Missile technique to solve systems of
simultaneous equations. [a]

CHAPTER III

COMPARISON OF THE STANDARD BRUCETON
AND MISSILE TECHNIQUES

The first use of the simulation program was to compare the performance of the two basic techniques as they are presently used and described in the literature. [14,15,17] This was accomplished by running the test conditions shown in Table 1. The results of this particular simulation effort were used initially to determine the relative performance of the two techniques and to identify more detailed characteristics of their performance. The results also provided the primary guidance for subsequent efforts described in Chapter IV to improve the two techniques, and ultimately to define restrictions of the techniques (Chapter V).

Evaluation of the simulation results was accomplished in two phases. The first, preliminary, evaluation was accomplished using a relatively simple estimating efficiency approach designed to provide quick, quantitative answers to guide subsequent activity. The second phase, or final analysis, was based on standard statistical analysis techniques, and only those conclusions which result from the statistical analysis are considered in the final evaluation of the project. The detailed statistical analysis is presented in Appendix V.

In developing the estimating efficiency approach for the preliminary

evaluation it was recognized that the Normal estimates of μ and σ are the best available and could be used as a basis for comparison of the other two techniques. Thus, if the efficiencies of the three different estimators were calculated on the same basis, the Bruceton and Missile efficiencies could be compared directly to see which was better and the Normal efficiencies used as a measure of the degree of potential improvement possible in the other two.

The following two expressions were chosen as expressing the desired efficiency characteristic:

$$E_{\overline{X}} = \left(\mu - |\mu - \overline{X}_{\overline{X}}| - S_{\overline{X}} \right) \Big/ \mu$$

$$E_{s} = \left(\sigma - |\sigma - \overline{X}_{s}| - S_{s} \right) \Big/ \sigma \; ,$$

where

μ and σ = the true population parameters as programmed into the computer

$\overline{X}_{\overline{X}}$ = the mean of the means from the 100 samples of test results

$\overline{X}_{s}$ = the mean of the standard deviations from the 100 samples of test results

$S_{\overline{X}}$ = the standard deviation of the means estimated from the 100 samples

S_{s} = the standard deviation of the standard deviations estimated from the 100 samples.

In deriving the above expressions it was reasoned that the accuracy of any estimator is measured by two factors, (1) the amount by which the estimate differs from the true value of the parameter being estimated and (2) the repeatability of the estimator.

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These two factors were defined as the displacement and deviation, respectively, and are expressed as

$$\text{Displacement} = |\mu - \overline{X}_{\overline{X}}|, \text{ and } |\sigma - \overline{X}_{s}|$$

$$\text{Deviation} = S_{\overline{X}} \text{ and } S_{s}.$$

Since the "perfect" estimator would have values of zero for both of these factors, the efficiency expressions given earlier would have an efficiency of 1.0 for the perfect estimator, and anything less than perfect would have a value less than 1.0. It is recognized that the expressions are not efficiencies in the true sense of the word since, under certain conditions, it is possible to calculate negative values. It is not felt that this possibility significantly detracts from its use.

Data from the comparison simulation are presented in Table 6. These data were calculated from the 100 individual sample estimates of μ and σ by each of the three techniques. From the data in Table 6, efficiencies were computed (Table 7) and graphs plotted. The graphs are shown as **Figure** 3 for $\overline{E}_{\overline{X}}$ and Figure 4 for $\overline{E}_{s}$. The efficiencies $\overline{E}_{\overline{X}}$ and $\overline{E}_{s}$ are the means of the efficiencies obtained for the individual test conditions of the same sample size, or population.

A review of Figures 3 and 4 result in the following observations:

1. All the Normal estimates are better than the corresponding Bruceton or Missile estimates. This verifies the statement made earlier that the Normal estimates are the best possible.

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Table 6. Summary Analysis of 100 Sample Estimates

Test Conditions No. (μ, σ, N)	Technique	$\overline{X}_{\overline{x}}$	$S_{\overline{x}}$	$\overline{X}_s$	S_s
1 (1.0, 0.02, 15)	Normal	1.00044	0.00496	0.01965	0.00384
	Bruceton	0.99124	0.01951	0.01854	0.00728
	Missile	1.00406	0.01774	0.00445	0.01429
2 (1.0, 0.09, 15)	Normal	0.99935	0.02086	0.08744	0.01567
	Bruceton	0.96131	0.08037	0.08332	0.03003
	Missile	1.00155	0.04309	0.07653	0.06176
3 (1.0, 0.26, 15)	Normal	0.99410	0.06653	0.25533	0.04683
	Bruceton	0.95598	0.26766	0.28946	0.07630
	Missile	0.98708	0.10306	0.22876	0.16474
4 (1.0, 0.02, 35)	Normal	1.00034	0.00361	0.01961	0.00237
	Bruceton	0.98146	0.01483	0.02093	0.00532
	Missile	1.00005	0.00796	0.01559	0.01015
5 (1.0, 0.02, 55)	Normal	1.00024	0.00294	0.01984	0.00191
	Bruceton	0.98240	0.01590	0.02015	0.00366
	Missile	1.00021	0.00522	0.02023	0.00677
6 (1.0, 0.09, 35)	Normal	1.00009	0.01544	0.08920	0.01020
	Bruceton	0.93108	0.08667	0.09032	0.02076
	Missile	0.99814	0.02341	0.08379	0.03209
7 (1.0, 0.26, 35)	Normal	0.99889	0.04277	0.25828	0.03111
	Bruceton	0.92697	0.31330	0.30910	0.16804
	Missile	0.99664	0.06430	0.24926	0.09518
8 (1.0, 0.09, 55)	Normal	0.99947	0.12460	0.09037	0.00974
	Bruceton	0.94012	0.09557	0.09241	0.01531
	Missile	0.99815	0.02075	0.09151	0.03210
9 (1.0, 0.26, 55)	Normal	1.00591	0.03656	0.25757	0.02252
	Bruceton	1.00920	0.28431	0.29013	0.04480
	Missile	1.00437	0.05123	0.26647	0.07416

Table 7. Calculated Efficiencies of Techniques

Test Conditions No. (μ, σ, N)	$E_{\bar{x}}$			E_s			E		
	N	B	M	N	B	M	N	B	M
1 (1.0, 0.02, 15)	0.99460	0.97173	0.97820	0.79050	0.56300	0	0.84152	0.66516	0.24455
2 (1.9, 0.09, 15)	0.97849	0.38044	0.95536	0.79744	0.59211	0.16411	0.84270	0.66432	0.36192
3 (1.0, 0.26, 15)	0.92757	0.68832	0.88402	0.80192	0.59323	0.24623	0.83333	0.61700	0.40568
4 (1.0, 0.02, 35)	0.99605	0.96663	0.99199	0.86200	0.68750	0.27200	0.89551	0.75728	0.45200
5 (1.0, 0.02, 55)	0.99682	0.96650	0.99457	0.89650	0.80950	0.65000	0.92158	0.84875	0.73614
6 (1.0, 0.09, 35)	0.98447	0.84441	0.97473	0.87778	0.76578	0.57444	0.90445	0.78544	0.67451
7 (1.0, 0.26, 35)	0.95612	0.61367	0.93239	0.87373	0.16485	0.59261	0.89433	0.27705	0.67756
8 (1.0, 0.09, 55)	0.98701	0.84455	0.97740	0.88744	0.80311	0.62655	0.91233	0.81347	0.71426
9 (1.0, 0.26, 55)	0.95753	0.62396	0.94440	0.80403	0.71180	0.68988	0.91740	0.68984	0.75351

N	$\bar{E}_{\bar{x}}$			$\bar{E}_s$			$\bar{E}$		
	N	B	M	N	B	M	N	B	M
15	0.96689	0.84700	0.93919	0.79662	0.58278	0.13678	0.83918	0.64883	0.33738
35	0.97888	0.80824	0.96637	0.87118	0.53938	0.47968	0.89810	0.63738	0.60136

Table 7. Calculated Efficiencies of Techniques (Concluded)

N	$\overline{E}_{\overline{x}}$			$\overline{E}_s$			$\overline{E}$		
	N	B	M	N	B	M	N	B	M
55	0.98379	0.81167	0.97212	0.89599	0.77480	0.65548	0.91710	0.78402	0.73464
All	0.97652	0.82230	0.95923	0.85460	0.63232	0.42348	0.88479	0.69008	0.55779

σ	$\overline{E}_{\overline{x}}$			$\overline{E}_s$			$\overline{E}$		
	N	B	M	N	B	M	N	B	M
0.02	0.99582	0.96829	0.98835	0.84967	0.68667	0.30733	0.88620	0.75707	0.47756
0.09	0.98332	0.85663	0.96916	0.85422	0.72033	0.45503	0.88649	0.75441	0.58356
0.26	0.94707	0.64198	0.92027	0.85989	0.48996	0.50957	0.88169	0.55875	0.61225
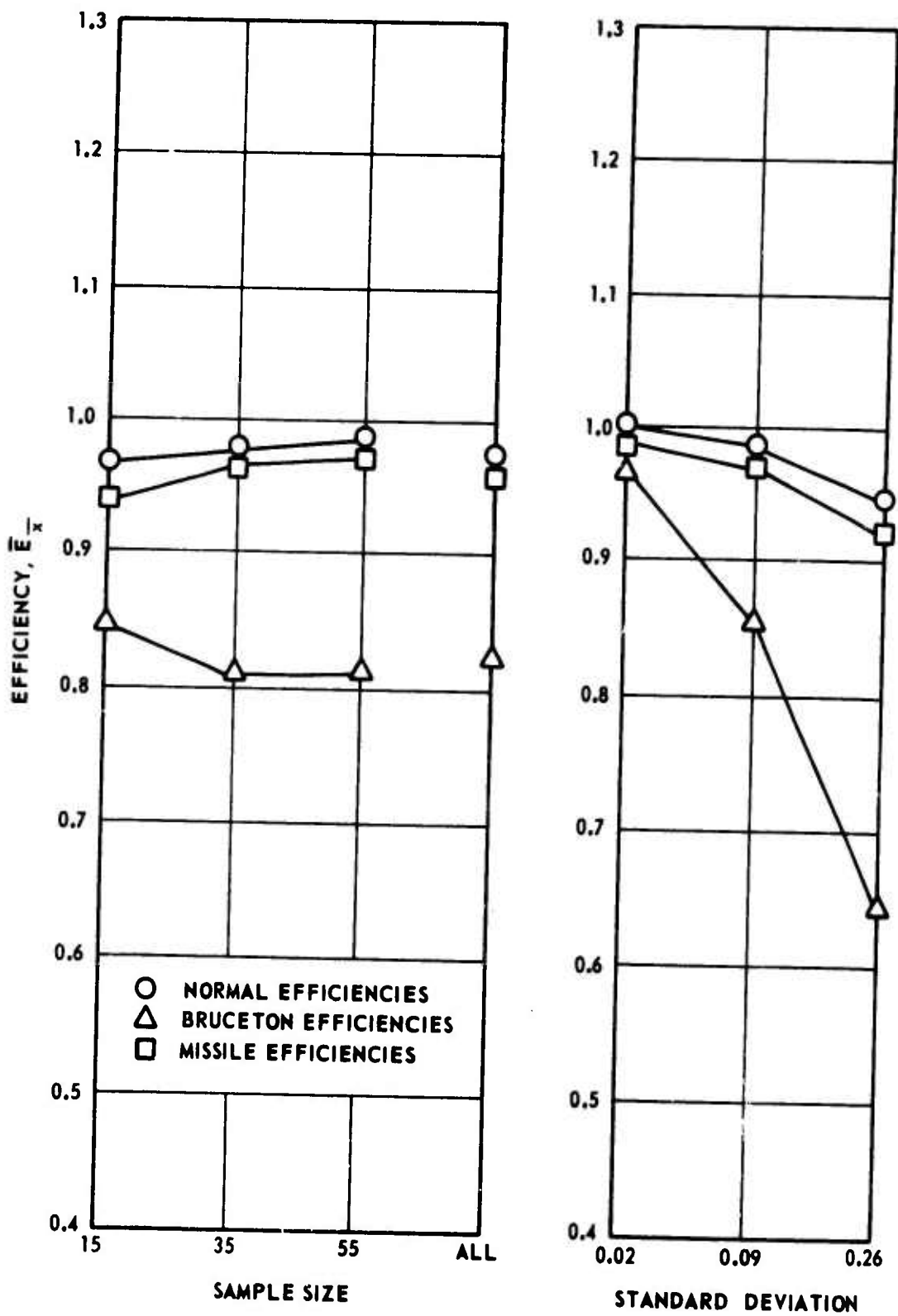
Figure 3. Mean Estimating Efficiency Versus Sample Size
and Standard Deviation
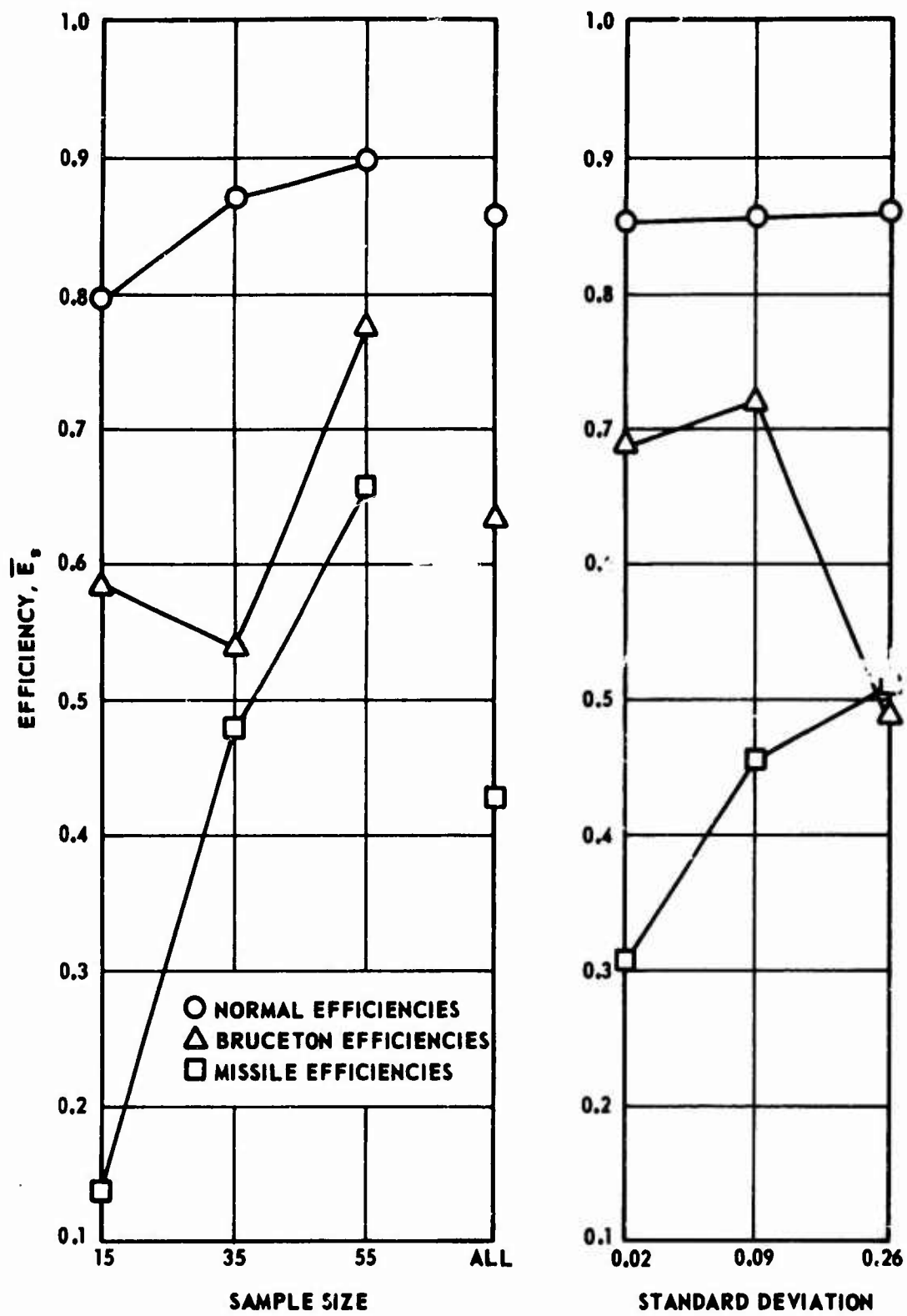
Figure 4. Standard Deviation Estimating Efficiency Versus
Sample Size and Standard Deviation

2. The Missile estimates of μ are better in all instances than the Bruceton estimates. Further, there is very little difference between the Missile and Normal estimates, indicating very little potential for improving the Missile estimate of the mean.

3. The Bruceton estimates of σ are better than the Missile estimates except for the population of $\mu = 1.0$ and $\sigma = 0.26$, specifically for a sample size of 35 items (Table 7). Estimates of σ from the $\mu = 1.0$ and $\sigma = 0.26$ population with sample sizes of 15 and 55 are both more efficient than for a sample of 35.

4. Estimates of μ are fairly insensitive to different sample sizes when samples from the three different populations are considered. This can probably be explained by noting the decrease in efficiency with increasing σ, and reasoning that this decrease offsets the increased efficiency expected with larger sample sizes.

5. The Bruceton estimates, of both μ and σ, are not statistically consistent, whereas the others, Missile and Normal, are.

Based only on the above observations, it is not possible to conclude which of the two techniques is best for estimating μ and σ, the basic objective. To do this requires consideration of the relative "weights" of μ and σ estimating efficiencies which must be given in combining them into an overall technique efficiency, E. To establish a basis for combining the statistical estimates of μ and σ it was reasoned that the estimates are usually used to define an interval in which the true population parameter will lie. This interval may be of varying widths ($\overline{X} \pm 1S$, $\overline{X} \pm 2S$, etc.), as desired, with a "3-sigma"

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interval (i.e., $\bar{X}$ ±3S) typical for aerospace testing. By substituting the asso-ciated efficiency in place of the respective parameter in the interval expression (since the interval is the usual ultimate objective), an expression for overall technique efficiency is possible; i.e., $F = \left(E_{\bar{X}} + 3\,E_s\right)/4$. This is a weighted average in which E_s is given three times the weight of $E_{\bar{X}}$. Obviously an infinite number of weight combinations are possible, although the one shown above was used predominately throughout this project.

By using the above expression, the overall efficiency, F, for each individual test condition was calculated, and from these values mean overall efficiencies, $\bar{F}$, for each sample size and population were calculated. The individual and mean overall efficiencies are presented in Table 7 and the mean overall efficiencies are shown plotted versus sample size and population in Figure 5. Finally, all the overall efficiencies (i.e., for all sample sizes and populations) were averaged for each technique. The overall mean efficiency thus calculated for the Missile technique was 0.55779 and for the Bruceton technique was 0.69009.

Data on the coefficient of correlation are presented in Table 8 and shown plotted in Figure 6. These results show that the Bruceton estimates of σ, when all the data are considered, are only slightly better correlated with the Normal estimates than are the Missile estimates ($\overline{\text{TAUB}}$ = 0.27064 versus $\overline{\text{TAUM}}$ = 0.26100, Table 8). Further review of the data indicates that the Bruceton is superior to the Missile at fairly small values of σ (i.e., σ = 0.02 and 0.09); otherwise, the Missile estimates have stronger correlation. Thus, the correlation data indicate that the Bruceton technique is probably equal

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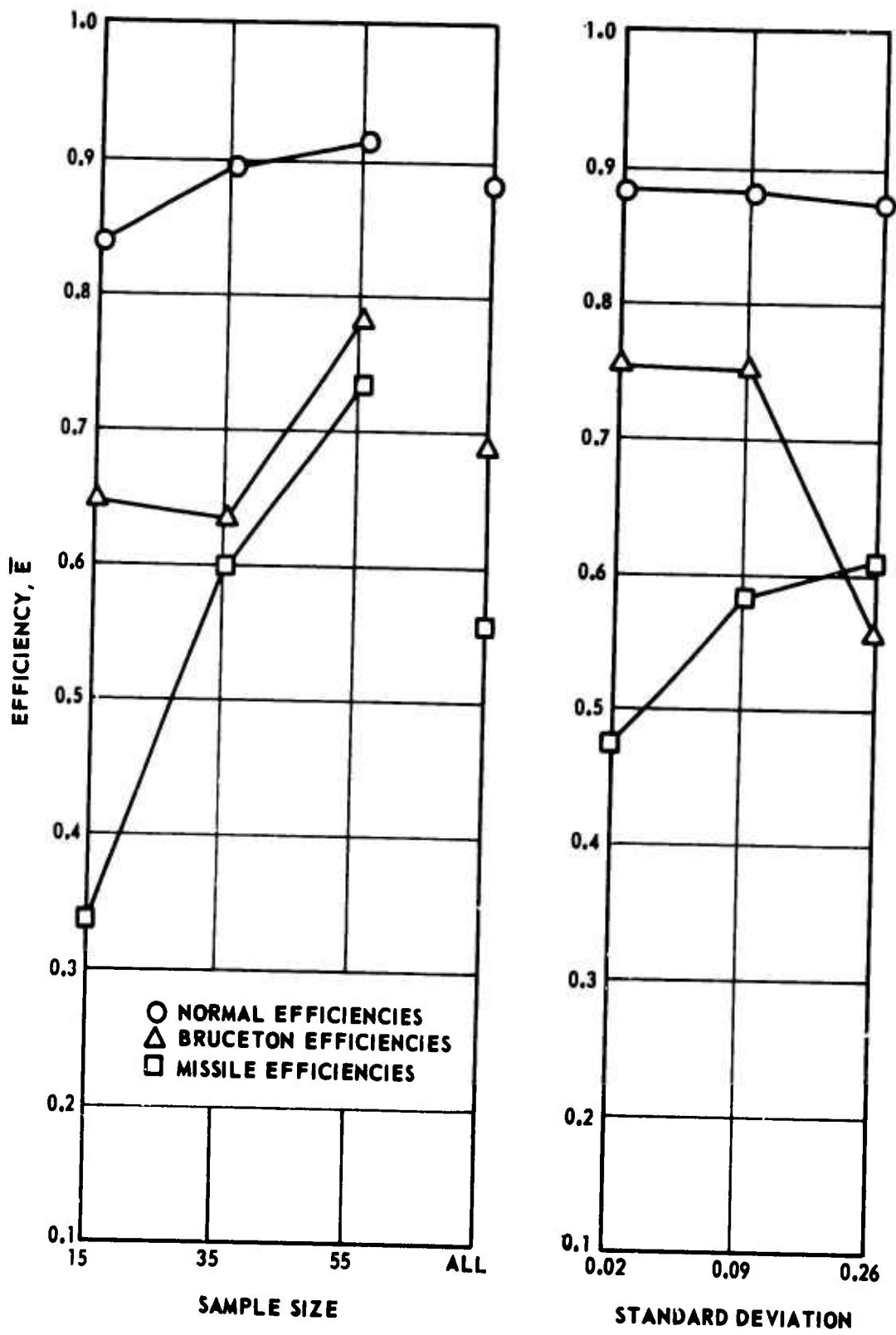
Figure 5. Overall Estimating Efficiency Versus
Sample Size and Standard Deviation

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Table 8. Correlation Coefficients

Test Conditions No. (μ, σ, N)	TAUB*	Significant at 90-Percent Confidence	TAUM**	Significant at 90-Percent Confidence
1 (1.0, 0.02, 15)	0.27107	No	0.22563	No
2 (1.0, 0.09, 15)	0.38936	Yes	0.19008	No
3 (1.0, 0.26, 15)	0.05514	No	0.36246	Yes
4 (1.0, 0.02, 35)	0.27362	Yes	0.28438	Yes
5 (1.0, 0.02, 55)	0.43154	Yes	0.26276	Yes
6 (1.0, 0.09, 35)	0.29722	Yes	0.15984	No
7 (1.0, 0.26, 35)	0.04650	No	0.34815	Yes
8 (1.0, 0.09, 55)	0.44835	Yes	0.30630	Yes
9 (1.0, 0.26, 55)	0.36763	Yes	0.25950	Yes
N	$\overline{\text{TAUB}}$	Significant at 90-Percent Confidence	$\overline{\text{TAUM}}$	Significant at 90-Percent Confidence
15	0.23852	No	0.25939	No
35	0.20578	No	0.26412	Yes
55	0.36763	Yes	0.25950	Yes
All	0.27064	Yes	0.26100	Yes
σ	$\overline{\text{TAUB}}$	Significant at 90-Percent Confidence	$\overline{\text{TAUM}}$	Significant at 90-Percent Confidence
0.02	0.32541	Yes	0.25759	Yes
0.09	0.37831	Yes	0.21874	No
0.26	0.10821	No	0.30668	Yes

* TAUB $\equiv$ correlation coefficient of Bruceton estimates

** TAUM $\equiv$ correlation coefficient of Missile estimates.
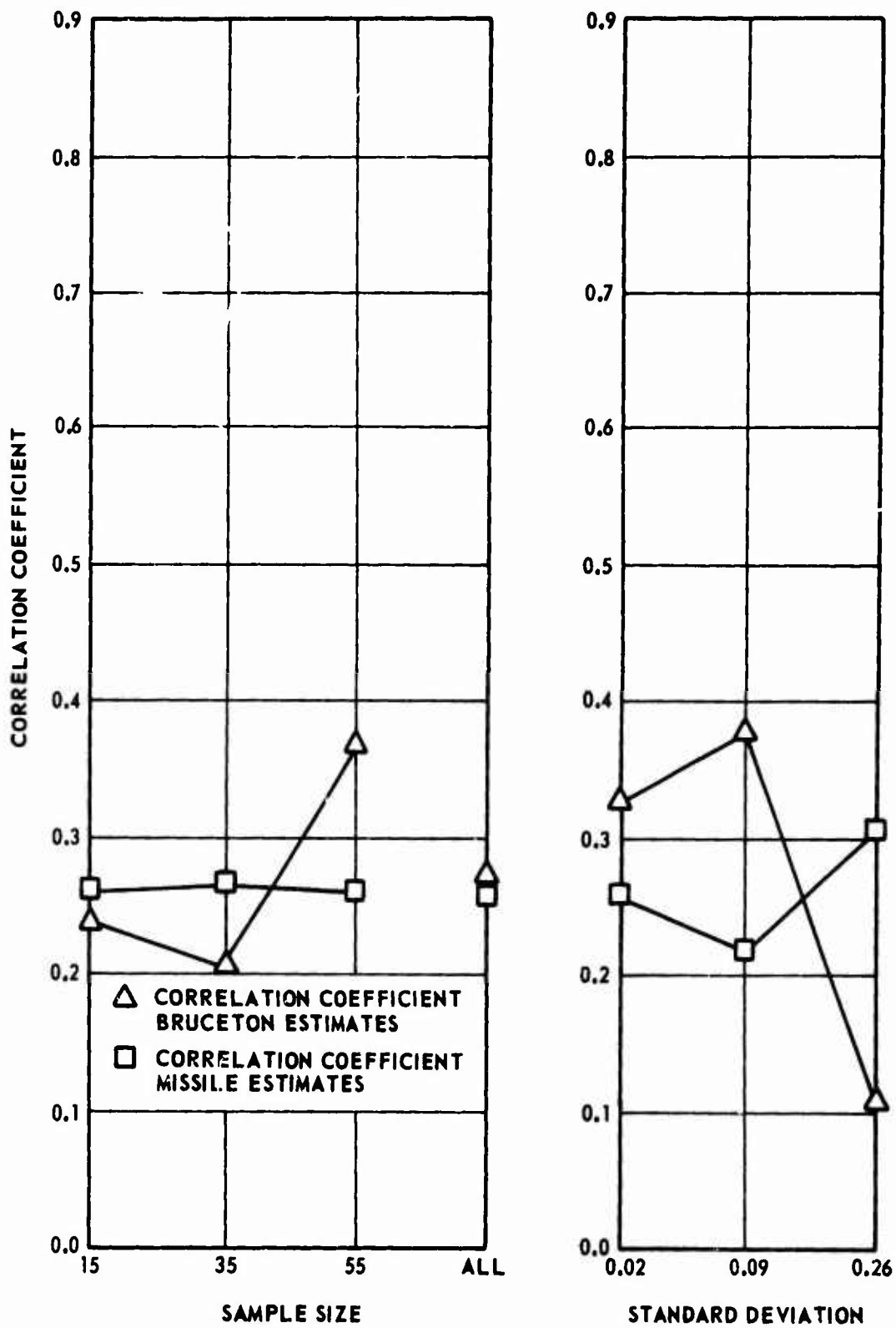
Figure 6. Correlation Coefficients of Standard Deviation
Estimates Versus Sample Size and Standard Deviation

to the Missile in the degree of correlation between estimates of σ by the Normal technique. The column in Table 8 headed "Significant at 90-Percent Confidence" indicates those correlation coefficients which were determined to be statistically significant as described in Appendix V on page 120. Further evaluation of these coefficients is presented in Chapter V.

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CHAPTER IV

TECHNIQUE IMPROVEMENT

The primary objective of the comparison simulations described in
Chapter III was to determine the better of the two techniques. Additionally,
these results were used to guide subsequent effort toward technique improve-
ment. The approach taken for technique improvement was to concentrate first
on improving the standard deviation estimates, and then on improving the
mean estimates. Technique improvement of both the mean and standard
deviation estimates simultaneously was not attempted, per se. In addition to
the many attempts made at improving the Bruceton and Missile estimates of
μ and σ, several totally new techniques were developed and evaluated. Only
the more pertinent portion of the improvement effort will be described here.

In evaluating the different modified techniques the efficiency approach
as described earlier was used. More specifically, the modifications were
checked out by changing the simulation program as required and running a
small number (5 to 15) of samples at certain of the test conditions specified in
Table 1. The test conditions were selected to permit the greatest amount of
improvement. For instance, if an improvement in Bruceton standard devia-
tion estimating was being evaluated, test condition No. 7 would be run since

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(Table 7) for this particular test condition the Bruceton standard deviation

estimating efficiency was minimum. If the modification being evaluated

produced improvement, it was further evaluated at other of the test conditions

by running more samples (usually 100).

The expression "test condition" in Chapter III had two implied mean-

ings. First it specified the values of μ, σ, and N (sample size) used in the

simulation program (i. e. , test condition No. 1 implied values of $\mu = 1.0$,

$\sigma = 0.02$, and N = 15) (Table 1). Second, "test condition" was used to refer to

a certain specific portion of the simulation effort; i. e. , test condition No. 1

referred to the 100 tests simulated at the specified conditions. As used in this

Chapter the expression test condition will refer only to the specific portions

of the simulation effort described i Chapter III. Similarly the expression

"re-test" will refer to simulation effort accomplished subsequent to that

described in Chapter III. This distinction is desired for convenience in

distinguishing between the results of simulation work described in this chapter

when compared with the results of work described in the preceding chapter.

The first attempt at technique improvement resulted from the realiza-

tion that the Bruceton technique required more a priori information than did

the Missile, and it was decided to take this information and use it to establish

a "tighter" test interval and different "degenerate solution" logic for the

Missile technique.

Degenerate solutions for the estimated standard deviation (i. e. , $\hat{\sigma}$

<0. 0) sometimes result from the Missile technique when the true value of σ

is small. Langlie[17] states that when degenerate solutions occur the actual value

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of σ is small and thus the estimate should be set equal to zero. This logic was followed in the simulation. It was reasoned, however, that perhaps the rough estimate of σ used for the Bruceton might be a better estimate for the degenerate cases. Therefore, this was programmed into the computer along with logic to set the test interval limits at $\overline{X} \pm 4S$, where $\overline{X}$ and S are the a priori Bruceton estimates. These changes result in the Missile technique using the same a priori information as the Bruceton.

Fifteen samples at $\mu = 1.00$, $\sigma = 0.02$, sample size 15 were run with these program changes as re-test No. 1 (RT No. 1). The Missile estimates from this run are presented below together with Missile technique estimates from test condition No. 1 (TC No. 1) for comparison.

	Re-Test No. 1	Test Condition No. 1
E	0.44806	0.24455
$E_{\overline{X}}$	0.99276	0.97820
E_{s}	0.28650	0

All criteria increased in magnitude, indicating improvement in all estimates. It was noted that RT No. 1 had only 47 percent degenerate solutions as compared with 90 percent for the Chapter III results. This reduction is apparently the result of the smaller test interval used, and is the reason for the improvement in estimates of μ and σ. The improvement in E_{s} is particularly significant, and it is concluded that these two changes should be adopted for the Missile technique whenever possible. These two changes were not, however, adopted for further simulation work described in this report. This decision was made to permit a more valid determination of improvement of subsequent

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changes in the basic technique.

1. Improvement in Estimating the Standard Deviation

Efforts to improve the estimates of σ were based primarily on principles developed by Friedman.[25] Friedman proved that minimum variance estimates of σ result when sensitivity tests are conducted at two test levels of $\mu \pm 1.57\sigma$. This principle was investigated thoroughly by several different approaches, with no practical, favorable results. A theoretical improvement in estimating σ was demonstrated using this principle, however, and will be described.

This approach used the Friedman principle in conjunction with the Missile technique. Logic was developed to cluster the Missile test levels around the values of $\overline{X} \pm KS$ as follows:

1. Calculate the standard Missile test level (MTL).

2. If $MTL \geq \overline{X}$, calculate $TL = \dfrac{MTL + \overline{X} + KS}{2}$.

   If $MTL < \overline{X}$, calculate $TL = \dfrac{MTL + \overline{X} - KS}{2}$.

3. Use the calculated value of TL as the test level.

This logic clustered all the test levels within ± 15 percent $\overline{X}$ of the desired values $(\overline{X} \pm 1.57\sigma)$.

This modified Missile technique was investigated with the simulation program at various values of $\overline{X}$, K, and S. Only when $\overline{X}$ was set at 1.07μ, and KS at 1.6σ was improvement in E_s demonstrated. These values were run as RT No. 8 for the $\mu = 1.0$, $\sigma = 0.26$, N = 35 condition, and the results are shown

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below together with the results of TC No. 7 for comparison.

$$\frac{\text{RT No. 8} \quad \text{TC No. 7}}{E_s \quad 0.80596 \quad 0.59376}$$

The results show a rather significant improvement in E_s. It is not only better than the standard Missile estimate of 0.59376, but also approaches the Normal estimate of 0.88346. There was also a slight improvement in $E_{\bar{x}}$ (0.90200 versus 0.89433).

Although these results indicate significant improvement in E_s, it must be pointed out that actual population values of μ and σ were used in establishing the test levels. These values would not be known in an actual testing situation, only rough estimates. For this reason the demonstrated improvement in E_s must be described as "theoretical."

Other approaches at improving estimates of σ were also unsuccessful, and it was concluded that the Bruceton estimate of σ is the best available.

2. Improvement in Estimating the Mean

During his discussion of the Missile technique, Langlie[17] stated that the accuracy of the estimates of the standard deviation should be improved with more precise estimates of σ for use as the initial value, σ_0, in the iterative solution of the Missile technique. It was reasoned that Bruceton estimates of σ would be very good as initial values for this purpose. To accomplish this, the Bruceton technique must be used to conduct the test and obtain estimates of σ, which would then be used by the Missile technique as values of σ_0 in conducting the final analysis.

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The necessary changes were made in the simulation program and four simulation runs conducted as RT No. 22-25 at values of μ, σ, and N corresponding to TC No. 1, 3, 2, and 6, respectively. The results of these re-tests were very similar and only the results of RT No. 22 (a nine-sample run) will be presented. These results are shown in Table 9.

The results show that for four of the nine tests, the Missile iteration procedure could not be successfully applied. This was because the first value of μ calculated by the procedure was negative, and the computer logic was set up to return the last, non-negative value of μ. The values of the estimated mean and standard deviation shown in Table 9 are those calculated by the Normal, Bruceton, and Missile techniques for each sample tested. The Bruceton estimates were used as initial values in the Missile iterative solution procedure, and for those samples where iteration was not accomplished (i.e., I = 0), the Bruceton and Missile estimates are identical. Of the five samples where iteration occurred, only one (No. 5) resulted in an improved estimate of σ over that started with initially.

As stated before, RT No. 25 was run at $\mu = 1.00$, $\sigma = 0.09$, and N = 35, one of the poorer conditions for making Bruceton estimates. However, during review of the RT No. 25 data it was noted that Bruceton estimates of μ were much closer to the Normal estimates whenever $M \le 0.325$. It was further recognized that the Bruceton simulation contained logic which resulted in an iterative solution whenever $M \le 0.325$. The reasons for this logic were given in Chapter II. Since both iterative solutions (iterative Bruceton and Missile) solve the maximum likelihood equations, it was at first

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Table 9. Results of Re-Test No. 22

Sample No.	Technique	Estimated Mean	Estimated Standard Deviation	I*	M
1	Normal	1.00010	0.02109		
	Bruceton	0.96500	0.03381	0	0.66667
	Missile	0.96500	0.03381		
2	Normal	0.99231	0.01546		
	Bruceton	0.98002	0.01721		0.25000
	Missile	0.98000	0.02701	13	
3	Normal	0.99535	0.01450		
	Bruceton	0.98953	0.01083		0.13889
	Missile	0.98449	0.00609	13	
4	Normal	1.00261	0.01912		
	Bruceton	0.97000	0.02436		0.47222
	Missile	0.97000	0.02436	0	
5	Normal	0.99356	0.02593		
	Bruceton	0.99501	0.01944		0.28571
	Missile	0.99500	0.02032	4	
6	Normal	1.00896	0.01810		
	Bruceton	1.01507	0.01568		0.22222
	Missile	1.01101	0.00735	11	
7	Normal	0.99829	0.01362		
	Bruceton	0.96500	0.01761		0.33333
	Missile	0.96500	0.01761	0	
8	Normal	0.99233	0.02285		
	Bruceton	0.96929	0.02125		0.40816
	Missile	0.96929	0.02125	0	
9	Normal	0.99625	0.02019		
	Bruceton	1.00497	0.01563		0.22222
	Missile	1.00404	0.00909	1	

*I ≡ number of iterations accomplished in the modified Missile technique.

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thought that the two procedures were the same with only some variations between them. Further consideration ruled this out since iteration is used with the Bruceton technique only for about 50 percent of the time (i.e., only when $M \leq 0.325$), but it is used with the Missile technique all the time. Also, the Bruceton iterative procedure is being used only because of the difficulty of programming the standard technique for the $M \leq 0.325$ situation.

To obtain further evaluation of iterative Bruceton solutions, the original nine 100-sample runs (Chapter III) were reviewed. Although no particularly significant information was obtained from this review, it was noted, at least for the TC No. 7 data, that for those samples where two iterative solutions were obtained the answers were very similar. For the samples where only one iterative solution was obtained (i.e., $M > 0.325$ and thus only the Missile estimate was an iterative solution), the iterative solution was the most accurate. Therefore, it appeared that the iterative Bruceton estimates of μ were as good as the Missile estimates and that both approached the Normal estimates in accuracy. Obviously, data were needed which allowed evaluation of the iterative Bruceton solutions for the situation where $M > 0.325$.

The additional simulation was required at the larger values of M, since, evidently, the noniterative or standard Bruceton estimates produced greater inaccuracies at these values. Further review of the nine 100-sample runs indicated that TC No. 7 would be satisfactory. Thus RT No. 26 was run at $\mu = 1.0$, $\sigma = 0.26$, and $N = 35$, providing Normal, standard Bruceton, and Missile estimates of μ and σ, as well as estimates by the iterative Bruceton technique of all samples regardless of the value of M. The results of RT No.

26 are presented in Table 10.

Table 10. Results of Re-Test No. 26

	Bruceton Technique	Iterative Bruceton	Missile Technique	Normal Technique
$E_{\overline{x}}$	0.62461	0.92651	0.93360	0.95241
E_s	0	0	0.60000	0.84630

It is seen that, as before, the Missile estimates of μ are better than the Bruceton. There is also a significant improvement of the iterative Bruce-ton over the standard Bruceton. No conclusion can be made concerning the estimates of σ other than to say that the Missile estimates are better than the others (excluding the Normal) for this particular test condition. This was also true in the earlier simulations.

In summary, it appeared that the iterative Bruceton estimates of μ were about as accurate as the Missile estimates. If this could be verified over all test conditions, then an improvement in overall technique could be obtained by combining the iterative Bruceton estimates of μ with standard Bruceton estimates of σ.

The technique combining iterative Bruceton estimates of μ and standard Bruceton estimates of σ was designated the Improved Bruceton technique. To obtain data for a complete evaluation of this technique it was necessary to simulate the nine test conditions as before (Table 1) and obtain iterative Bruceton estimates of μ. The standard Bruceton estimates of σ would be the same as those obtained earlier during the comparison simulations of Chapter III, and those data (Tables 6 and 7) were used to complete the evaluation of the

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Improved Bruceton technique. Thus, eight additional 100-sample runs were required (the ninth having already been run as RT No. 26). These were run as RT No. 27 and 28 and RT No. 31-36 at values of μ, σ, and N corresponding to TC No. 9, 3, 1, 2, 4, 5, 6, and 8, respectively.

The nine re-tests were analyzed as described previously and the results are presented in Table 11 and plotted in Figure 7. It is seen that the iterative Bruceton estimates of the mean are very good over all sample sizes and standard deviations, varying from a value of 0.91486 for the 0.26 population up to 0.99259 for the 0.02 population. The standard deviation estimates are not nearly so efficient, varying from a low value of 0.10374 for the 0.26 population up to a value of 0.55270 for the 0.09 population. Thus the iterative Bruceton mean estimates are much better than the standard Bruceton mean estimates while the standard deviation estimates are not as good. It is also noted that the iterative Bruceton estimates of the mean are more consistent than the standard Bruceton, and are fairly constant over the conditions simulated.

Based on these results the final, best technique should be as described before, the standard Bruceton test procedure with iterated Bruceton estimates of the mean, and standard Bruceton estimates of the standard deviation. This will be referred to as the Improved Bruceton technique.

Thus, while the attempts at technique improvement described in this chapter were not totally successful in that no improvement in standard deviation estimates was developed, the improvement demonstrated for estimating the mean was significant and represents further accomplishment towards the

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Table 11. Summary Analysis and Calculated Efficiencies of the Iterative Bruceton Estimates

RT No.	(TC No.)	$\bar{X}_{\bar{X}}$	$S_{\bar{X}}$	$\bar{X}_s$	S_s	$E_{\bar{X}}$	E_s	E	σ; N
31	(1)	1.00235	0.00851	0.02089	0.01021	0.98914	0.44500	0.58104	0.02; 15
32	(2)	0.99950	0.03514	0.08866	0.03963	0.96436	0.54478	0.64468	0.09; 15
28	(3)	0.99502	0.10715	0.40124	0.22774	0.88787	0.0	0.24728	0.26; 15
33	(4)	1.00127	0.00522	0.02415	0.00738	0.99351	0.42350	0.56600	0.02; 35
34	(5)	1.00070	0.00418	0.02240	0.00517	0.99512	0.62150	0.71341	0.02; 55
35	(6)	0.99811	0.02441	0.10360	0.02747	0.97370	0.54369	0.65116	0.09; 35
26	(7)	1.00885	0.06464	0.44984	0.26659	0.92651	0.0	0.23163	0.26; 35
36	(8)	1.00067	0.01861	0.10573	0.02300	0.98072	0.56967	0.67243	0.04; 55
27	(9)	1.01312	0.08667	0.34890	0.08946	0.93021	0.31423	0.46822	0.26; 55

N	$\bar{E}_{\bar{X}}$	$\bar{E}_s$	$\bar{E}$
15	0.93712	0.32993	0.49267
35	0.96457	0.32238	0.48293
55	0.96868	0.51180	0.61802
All	0.95679	0.38804	0.53121

σ	$\bar{E}_{\bar{X}}$	$\bar{E}_s$	$\bar{E}$
0.02	0.99259	0.49667	0.62015
0.09	0.97293	0.55270	0.65776
0.26	0.91486	0.10374	0.21571
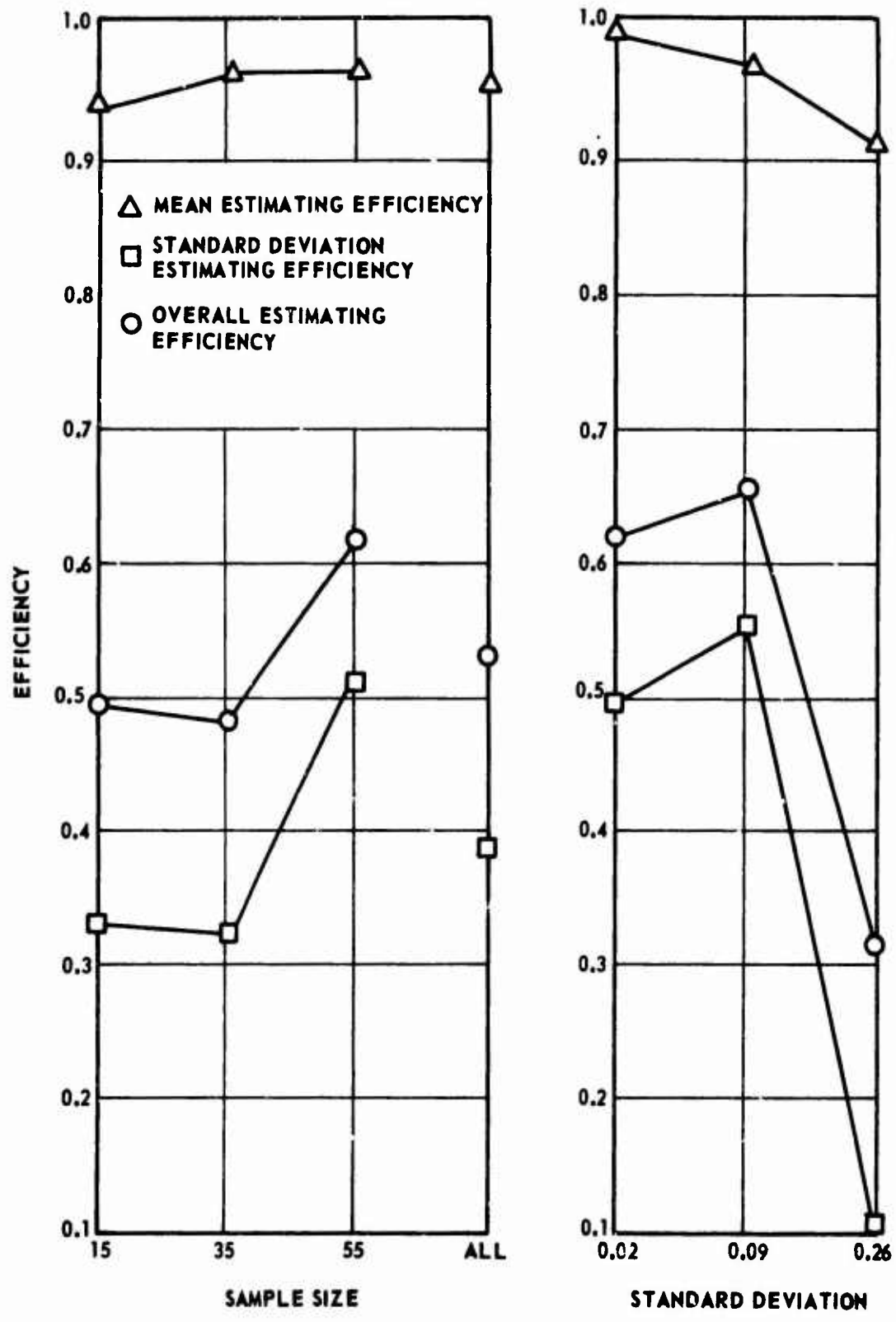
Figure 7. Iterative Bruceton Technique Estimating Efficiencies
Versus Sample Size and Standard Deviation

overall project objectives. The Improved Bruceton technique is further analyzed and evaluated in the next chapter.

CHAPTER V

FINAL EVALUATION AND CONCLUSIONS

Evaluation of the Improved Bruceton technique to this point has consisted only of analysis of the portion concerned with estimating the mean. It remains to be determined how well the overall technique survives further evaluation. In this chapter simulation data from Chapters III and IV will be combined so that the evaluation can be completed.

The data presented in Table 11 and Figure 7 relate only to the mean estimating capability of the Improved Bruceton technique. To complete the overall evaluation, data are required on the standard deviation estimating capability also. Since the standard deviation estimating procedure is the same as that of the standard Bruceton technique, the Bruceton data obtained during the comparison simulations described in Chapter III can be used. These data (Tables 6 and 7) have been appropriately combined with those obtained from the iterative Bruceton simulations of Chapter IV (Table 11) and are presented in Table 12 and Figures 8 through 10. These data are thus the same as those which would have resulted had the complete Improved Bruceton technique been simulated and the original nine test conditions run. The final evaluations of the technique will be based on these data.

Table 12. Summary Analysis and Calculated Efficiencies of the Improved Bruceton Estimates

TC No.	$\bar{X}_{\bar{X}}$	$S_{\bar{X}}$	$\bar{X}_s$	S_s	$E_{\bar{X}}$	E_s	E	σ; N
1	1.00235	0.00851	0.01854	0.00728	0.98914	0.56300	0.66954	0.02; 15
2	0.99950	0.03514	0.08332	0.03003	0.96436	0.59211	0.68517	0.09; 15
3	0.99502	0.10715	0.28946	0.07670	0.88787	0.59323	0.66689	0.26; 15
4	1.00127	0.00522	0.02093	0.00532	0.99351	0.68750	0.76400	0.02; 35
5	1.00070	0.00418	0.02015	0.00366	0.99512	0.80950	0.85590	0.02; 55
6	0.99811	0.02441	0.09032	0.02076	0.97370	0.76578	0.81776	0.09; 35
7	1.00885	0.06464	0.30910	0.16804	0.92651	0.16485	0.35526	0.09; 35
8	1.00067	0.01861	0.09241	0.01531	0.58072	0.80311	0.84751	0.09; 55
9	1.01312	0.05667	0.29013	0.04480	0.93021	0.71180	0.76640	0.26; 55

N	$\bar{E}_{\bar{X}}$	$\bar{E}_s$	$\bar{E}$
15	0.93712	0.58278	0.67387
35	0.96457	0.53938	0.64567
55	0.96868	0.77480	0.82327
All	0.95679	0.63232	0.71427

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Table 12. Summary Analysis and Calculated Efficiencies of the
Improved Bruceton Estimates (Concluded)

σ	$\overline{E}_X$	$\overline{E}_s$	$\overline{E}$
0.02	0.99259	0.68667	0.76315
0.09	0.97293	0.73033	0.78348
0.26	0.91486	0.48996	0.59618

Review of Figure 10 indicates that, based on efficiency only, the Improved
Bruceton estimates are the best of the one-shot techniques. Statistical analysis
(Appendix V) of these results does not completely substantiate this conclusion.
This analysis shows that both the Missile and Improved Bruceton techniques are
superior to the Bruceton, and no statistically significant difference between the
Missile and Improved Bruceton techniques was shown for estimating the mean.
The improvement in overall performance by the Improved Bruceton technique
is indicated as only approximately 2.5 percent, but it should be remembered
that in calculating the overall efficiencies the standard deviation efficiency is
given three times more weight than the mean efficiencies. All of the technique
improvement is the result of improvement in estimating the mean, which was
actually 13 percent. This improvement becomes more significant for testing
items which have only a small amount of variation in strength (i.e., small σ
as compared with μ). In this situation, errors in estimating the standard
deviation are not nearly as significant.

Correlation of the Improved Bruceton standard deviation estimates
remains the same as for the Bruceton estimates discussed before, since the
estimates are the same. The values for TC No. 1, 3, and 7 (Table 8) indicate

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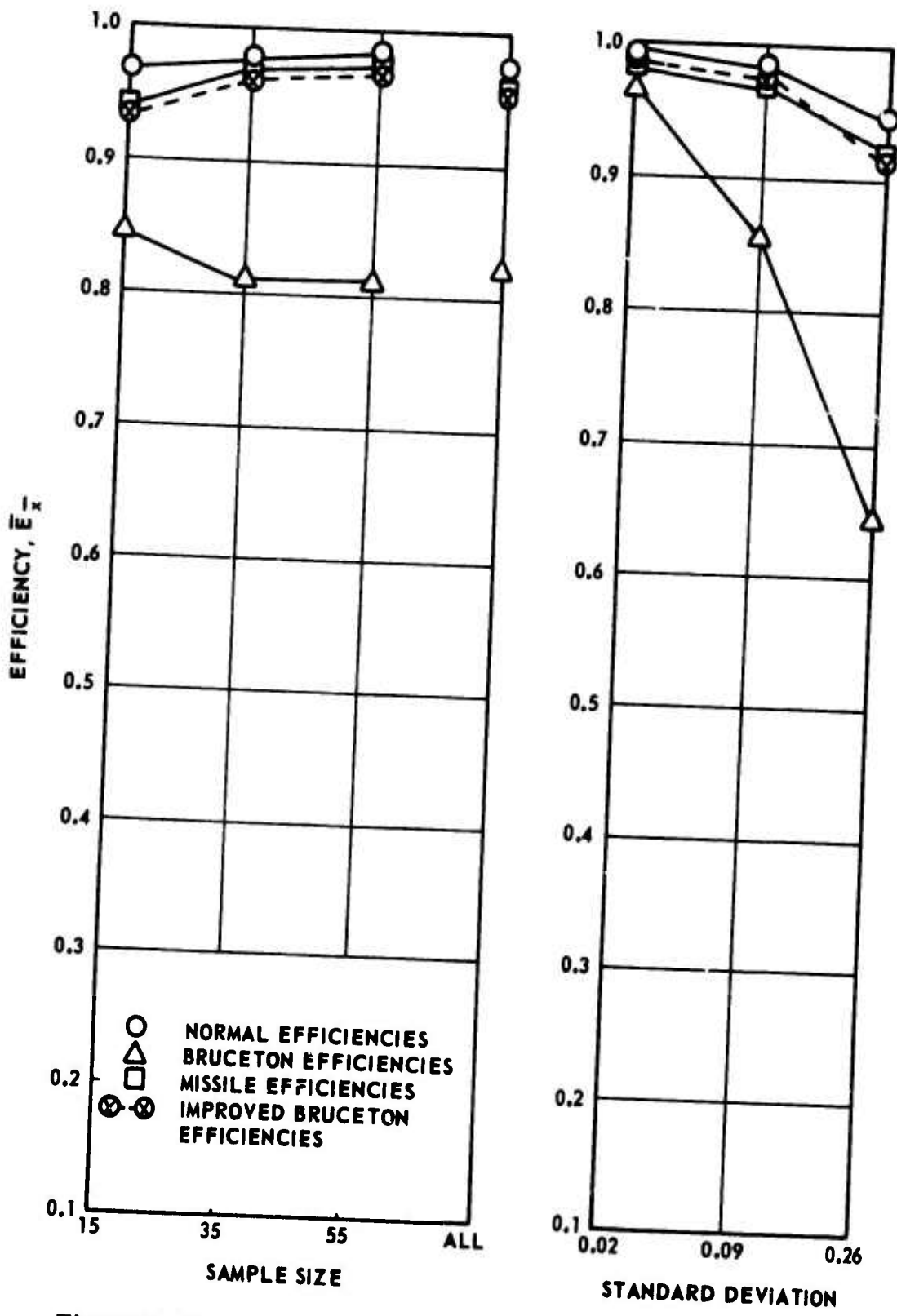
Figure 8. Comparison of the Mean Estimating Efficiency of the
Four Techniques
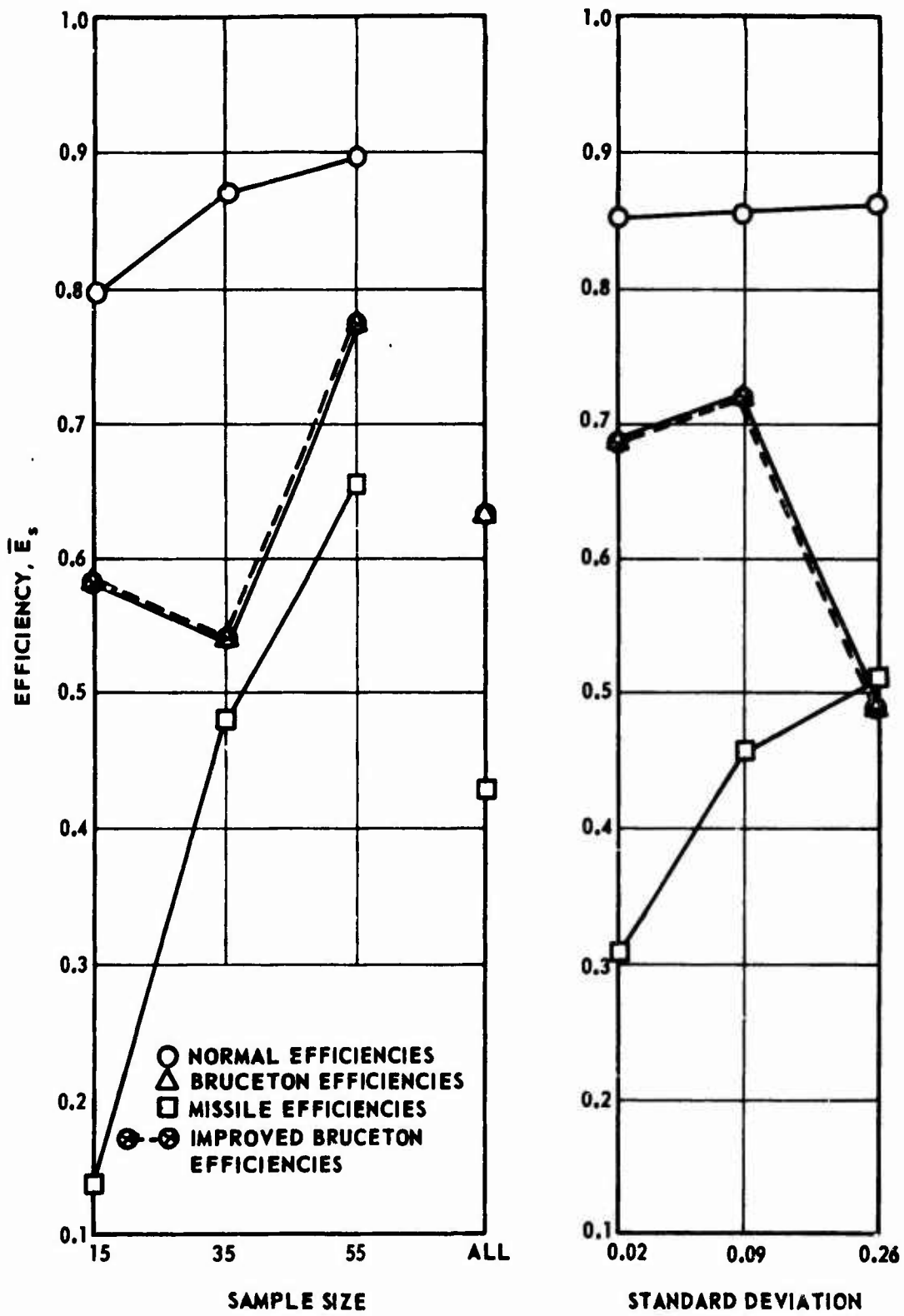
Figure 9. Comparison of the Standard Deviation Estimating
Efficiencies of the Four Techniques

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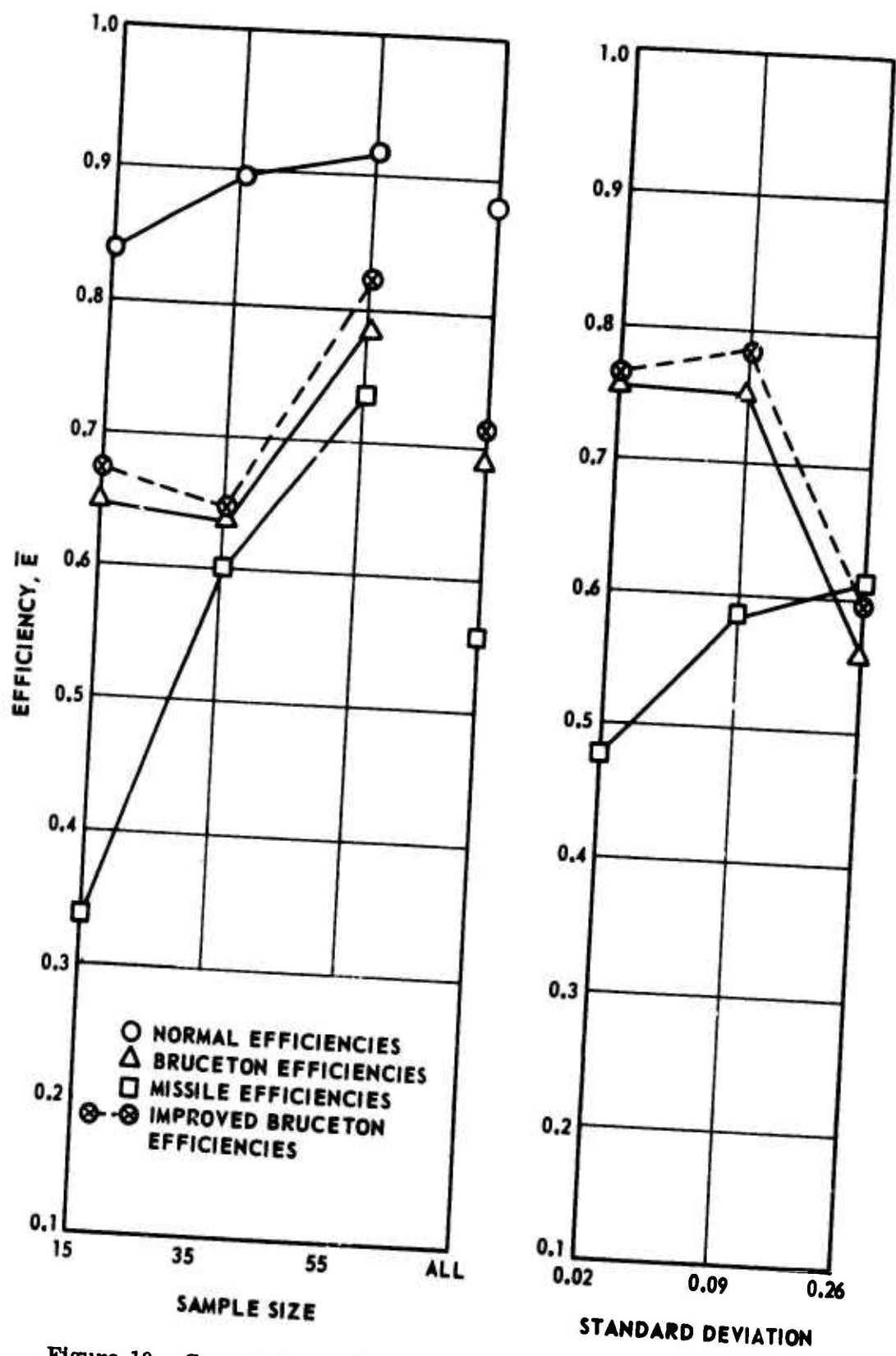
Figure 10. Comparison of the Overall Estimating Efficiencies
of the Four Techniques

zero correlation for estimates of σ. This means that estimates of the standard deviation at these conditions should be disregarded. Test condition No. 3 and 7 are both conditions with tests from the 0.26 test population with samples of the two smaller sizes. Since TC No. 9 is also from the $\sigma = 0.26$ population but at the largest sample size, it is concluded that smaller size is causing the lower degree of correlation. Thus, in order for the σ estimates to have a reasonable degree of correlation, the minimum sample size for testing hardware with relatively large variation in strength should be 55 items.

The above discussion of TC No. 1, 3, and 7 is also substantiated from the standpoint of estimating efficiency. The values of E_s for TC No. 1, 3, and 7 are generally lower than the others (Table 12). Thus, of the nine conditions investigated, restrictions on sample sizes should be placed on four. The restrictions, in effect, are that sample sizes smaller than 35 should not be used for estimating σ of any population, and for populations with relatively large σ the sample size should be increased to a minimum of 55. These same restrictions apply to the Improved Bruceton technique.

In continuing the individual test condition evaluation further, the Table 7 data were compared with the Table 12 data. The Improved Bruceton technique efficiency for estimating the mean is greater for all nine conditions, ranging from a minimum improvement of 1.7 percent at TC No. 1 to a maximum of 31.3 percent at TC No. 7. Based only on the Table 7 data, the standard Bruceton estimates of the mean of TC No. 3, 7, and 9 would not have been acceptable, and the estimates of TC No. 2, 6, and 8 would be marginal. This indicates that the standard Bruceton technique probably has a weakness for estimating the

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mean of any hardware which has relatively large variation in strength (i.e.,
$\sigma/\mu \geq 0.09$). The Improved Bruceton technique completely eliminates this
particular weakness, a very significant improvement.

So far as the Missile technique is concerned the data in Table 7 reveal
that the Missile mean estimator is more efficient than the standard Bruceton,
with the opposite generally true for the standard deviation. This is substanti-
ated by the statistical analysis presented in Appendix V. Review of the Missile
correlation coefficient data (Table 8) shows that standard deviation estimates
at TC No. 1, 2, and 6 are not correlated and thus should be disregarded. These
are essentially small sample/small σ conditions and thus a minimum sample
size of 35 should probably be specified and increased to 55 for TC No. 6.

Thus, of the nine test conditions investigated, four (TC No. 1-3, and
6) have been restricted for use with the Missile technique, and also four (TC
No. 1-3, and 7) for the standard Bruceton and Improved Bruceton techniques.

The basic objective of this investigation was originally stated as that of
providing an answer to the question, "Which technique, Bruceton or Missile, is
the better for sensitivity testing of one-shot items with sample sizes ranging
from 15 to 50 items?" It was also planned to determine whether one technique
should be preferred over another under different situations. Finally, efforts
were planned for improving either one of the two techniques or developing a
new, improved technique. Investigations have been completed as described
previously towards all these objectives with varying degrees of success.

It is felt that the question as to which of the two techniques, Bruceton or
Missile, is the better for conducting sensitivity tests was satisfactorily

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answered. Based on the analysis of variance results and sign tests described in Appendix V, it must be concluded that between the standard Missile and Bruceton test techniques, the Missile technique is the better. It was determined that limitations should be placed on the use of both techniques as described earlier.

Improvement was demonstrated for the Bruceton technique. An iterative solution is proposed for calculating the estimates of the mean. These solutions were shown to be significantly better than those provided by the standard Bruceton technique and equal to those of the Missile technique. However, since no improvement over the standard Bruceton estimates for σ was developed, the standard Bruceton estimate must be used with the limitation that for estimating σ, minimum sample sizes of 35 items should be used under any conditions, and increased to 55 for situations similar to TC No. 6. It was not possible to determine any statistically significant differences between the Improved Bruceton and Missile techniques. Mean estimates of both techniques were shown to be significantly better than the standard Bruceton estimates (both the estimates, $\overline{X}_{\overline{x}}$, and their standard deviations, $S_{\overline{x}}$, were significantly better). Analysis of the standard deviation estimates of the three techniques was inconclusive. The means of the Missile standard deviation $\left(\overline{X}_s \right)$ were shown to be better, but the standard deviation of the standard deviation estimates $\left(S_s \right)$ were shown to be larger than the standard Bruceton (or Improved Bruceton) estimates.

Thus, two test techniques of apparently equal capability are available for small-sample sensitivity testing of one-shot items. Both, however, offer certain advantages under different situations. The Improved Bruceton technique

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is amenable to manual computations, while the Missile technique requires a computer for analysis of the test data. On the other hand, the Missile technique requires no prior estimates of μ and σ, whereas the Bruceton technique does. Further, if such prior information is available and used with the Missile technique, some improvement in the estimates can be expected.

Finally, some comments are in order regarding approaches for further investigation. It is felt that the mean estimates, since they now approach the Normal in efficiency, are probably as good as they could ever be. It is therefore concluded that any future investigations should be directed toward improving the efficiency and correlation of the standard deviation estimates, especially for those conditions described as limitations. More specifically, it is thought that further effort toward utilizing the Friedman theory[25] in conjunction with some test level clustering method has good potential.

APPENDIX I

THE BRUCETON TECHNIQUE FOR SENSITIVITY TESTS

1. ## The Experimental Procedure

    The technique consists of the following steps:

    1. Choose a test level, h, at which the first specimen will be tested, and an interval, d, which will be the distance between testing levels.

    2. If the first specimen fails when tested at test level h, the second specimen will be tested at h-d. If the first specimen does not fail, the second specimen will be tested at h+d.

    3. In general a specimen will be tested at a test level d below the level at which the previous specimen was tested if that specimen failed and d above the level at which the previous one was tested if it did not fail.

    In this manner one will obtain a sequence of failures and successes which may be recorded as in Figure 11. The x's denote failures and the 0's denote successes.

    Here the first specimen did not fail, so the second one was tested at h+d; the second did fail, so the third was tested at h, the level just below h+d;

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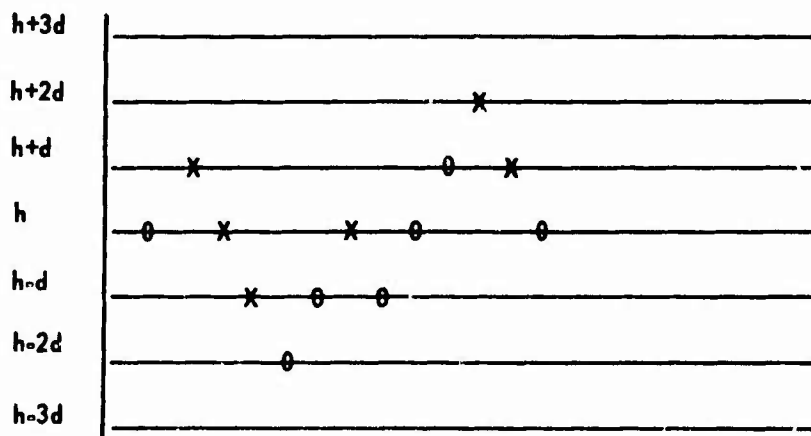
TEST LEVELS

Figure 11. Example of Bruceton Test Levels and Results

the third failed, so the fourth was tested at h-d; the fourth failed, so the fifth

was tested at h-2d; the fifth did not fail, so the sixth was tested at h-d, the

level just above h-2d.

On the basis of past experience it is usually possible to make rough

estimates of the mean and standard deviation for the item to be tested. Such

estimates are used in choosing h and d for the experiment to be performed.

If a and σ are the estimated mean and standard deviation respectively, then

choices for h and d are simply h = a and d = σ.

2. Statistical Analysis

The methods provided for estimating the mean and standard deviation

are based on the assumption that the critical responses (or some known func-

tion of them) have a Normal probability distribution. It is important that

this assumption be reasonably well satisfied. If the critical responses are

not Normally distributed, it is desirable to transform to a new variable which

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will have a Normal distribution. In biological research it is usually found that
the logarithms of the critical concentrations rather than the concentrations
themselves are Normally distributed. There is some evidence in explosives
research that logarithms of testing heights are also more nearly Normally
distributed than the heights themselves, but the evidence is not so conclusive
or so generally accepted in this field. This is a question, therefore, which
will usually require some preliminary investigation. Any effort devoted to
finding if the responses are Normally distributed, and if not, what function
of the responses is Normally distributed, will be well worthwhile. It is
assumed that there is known or has been found by preliminary investigation a
variable, x, which is Normally distributed. Any reference to test levels in
the following discussion will be to these "Normalized" levels. The original
testing level, h, and the interval, d, will be measured in terms of the Normal-
ized level. If, for example, the Normalized level is the logarithm of the
actual level and the Normalized interval is one, then corresponding to normal-
ized level -1, 0, 1, 2, 3, would be actual testing levels of $\frac{1}{10}$, 1, 10, 100, and
1000.

The statistical methods will be described by illustrating their use for
an actual drop test of an explosive. In this situation the drop height is the
critical response variable. Figure 12 is a chart showing the results of the
test of 100 specimens of the explosive. On the left the lines on which there
were tests are numbered from 0 to 4. In the table below the chart are given
the line numbers, the actual testing heights (in centimeters), the logarithms
of the testing heights, the number of explosions (failures), and the number of
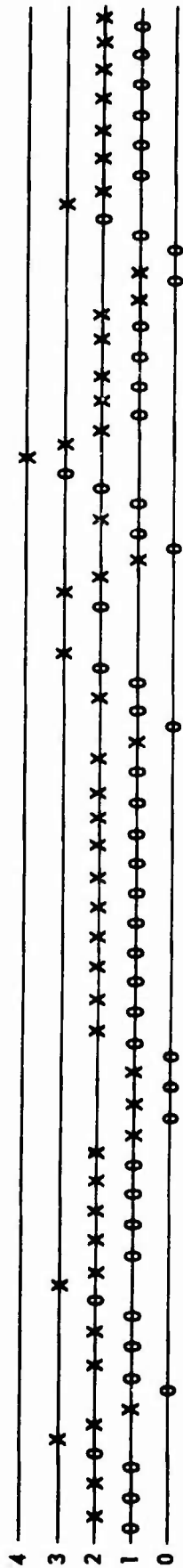
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Figure 12. Results of a 100-Sample Bruceton Test of an Explosive

nonexplosions (successes) at each level. The set of numbers is the same in both columns, as will always be the case when there are equal numbers of 0's and x's. When the total number of 0's is unequal to the total number of x's, the number of 0's at a given height will not differ by more than one from the number of x's at the next greater height.

The actual heights were chosen so that their logarithms were equally spaced, since previous investigation of similar explosives revealed that the logarithms of the critical heights could be assumed to be Normally distributed. The interval $h = 0.093$ in log units was used because another experiment with a similar explosive gave this value for the standard deviation.

The mean and standard deviation will be estimated from either the 0's or the x's depending on which occur in fewer numbers. In the particular example given in Figure 12, either the 0's or the x's mav be used, but had there been 49 0's and 51 x's, the 0's would have been used.

Let n_0, n_1, n_2, ... be the number of 0's (or x's as the case may be) on the 0th, 1st, 2nd ... lines, respectively, and let N be the total number of 0's (or x's). Let c be the normalized height of the lowest line on which there was a test recorded. In the example, the x's will be used and the 0's disregarded; the numbers just defined will then be

$$N = 50, \quad c = 0.928, \quad n_0 = 0, \quad n_1 = 8, \quad n_2 = 35, \quad n_3 = 6, \quad n_4 = 1.$$

The formula for the estimate of the mean is

$$m = c + d \left(\frac{1}{N} \Sigma \, i n_i + \frac{1}{2} \right) \tag{1}$$

if the 0's are used, or

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$$m = c + d \left(\frac{1}{N} \Sigma \ in_i - \frac{1}{2} \right) \tag{2}$$

if the x's are used.

The standard deviation will be determined by computing

$$M = \frac{\Sigma \ i^2 n_i}{N} - \left(\frac{\Sigma \ in_i}{N} \right)^2 \tag{3}$$

then using this number to find s from Table 3 or from Figure 1. After s is found, the estimated standard deviation is

$$\sigma = ds. \tag{4}$$

The sums appearing in Equations (1), (2), and (3) can easily be computed in a tabular form,

i	n_i	in_i	$i^2 n_i$
0	0	0	0
1	8	8	8
2	35	70	140
3	6	18	54
4	1	4	16

$$N = 50 \quad A = 100 \quad B = 218 \qquad ,$$

where the data of the example in Figure 12 have been used and the two sums $\Sigma \ in_i$ and $\Sigma \ i^2 n_i$ have been represented by A and B. In terms of A and B, the formulas (1), (2), and (3) may be written

$$m = c + d \left(\frac{A}{N} \pm \frac{1}{2} \right) \tag{5}$$

$$M = \frac{NB - A^2}{N^2} , \tag{6}$$

where in Equation (5) the plus sign is to be taken if the 0's are used and the minus sign taken if the x's are used.

All computations involving the heights are to be done in terms of the Normalized heights and only final results transformed to actual heights. Thus, for the particular example, the mean is

$$m = 0.928 + 0.093 \left(\frac{100}{50} - \frac{1}{2} \right)$$

$$= 1.068, \tag{7}$$

and the number M is

$$M = \frac{50(218) - (100)^2}{50^2}$$

$$= 0.360. \tag{8}$$

From Table 3 or Figure 1 it is found that the value of s corresponding to M = 0.360 is s = 0.625; the standard deviation is therefore estimated as

$$\sigma = 0.093 \ (0.625) = 0.058, \tag{9}$$

which is considerably smaller than the value (0.093) anticipated and used for the interval between heights.

In terms of actual heights, the mean m = 1.068 corresponds to 11.69 centimeters, and this is not, strictly speaking, the mean height but the median height; that is, the height at which there is an even chance that a particular specimen will or will not explode. The term σ must always be used in Normal units (log units in this case), so there is no point in transforming it to centimeters.

The value of s depends on the position of the mean relative to the testing height, but this dependence is not important unless M is less than 0.35. Figure 2 provides curves giving the value of s as a function of M for various

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positions of the mean when M is small. The curve marked m–h = 0 is for the case where the mean is at one of the testing heights; the curve marked m–h = 0.5d is for the case where the mean is midway between two testing heights; the curve marked m–h = 0.25d is for the case where the mean is one quarter of d away from the nearest testing height.

If, for example, M in Figure 2 had been 0.20 instead of 0.36, then s would have been determined as follows: The mean m is 1.068 and is 0.046 units away from the nearest testing height, 1.114, and since d = 0.093, the mean is $\frac{0.046}{0.093}$ = 0.495d units from the nearest testing height. This happens to be so near 0.5d that the interpolation is hardly worthwhile, but it will be carried through as an illustration. The interpolation is between the 0.25d curve and the 0.5d curve in Figure 2. From the 0.25d curve, s' = 0.382, and from the 0.5d curve, s'' = 0.409. By using linear interpolation,

$$s = s' + \frac{0.495 - 0.25}{0.5 - 0.25} \ (s'' - s')$$

$$= 0.382 + 0.98 \ (0.027)$$

$$= 0.408. \tag{10}$$

Table 3 may be employed for values of M greater than 0.30. The values in the table are for the case where the mean falls on one of the testing heights. The differences inserted for M between 0.30 and 0.72 give the amounts by which the tabulated values are changed when the mean falls midway between two testing heights. The differences are 0 (to four decimal places) beyond M = 0.72. When the mean does not fall on a testing height, or falls midway between two of them, interpolation may be employed as above. Thus,

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if M were 0.33 and if the mean were 0.2d from the nearest testing height, s would be

$$s = 0.5711 + \frac{0.2}{0.5} (0.0070)$$

$$= 0.5739. \tag{11}$$

APPENDIX II

THE MISSILE TECHNIQUE FOR SENSITIVITY TESTS

1. Determining the Test Interval

The first step to be performed in this technique is the determination
of the test interval which is used as the basis for generating the stress or test
levels. This interval should be selected large enough to include all possible
ranges of strengths (test environment) of the parts to be tested. This interval
can be made conservatively large, since the Missile method has been designed
to cause the stress levels to be generated in the vicinity of interest (i.e., in
the vicinity of the distribution of strengths) as the test proceeds. As a sample
illustration, the range for a drop height test for glass containers designed to
withstand say, a 6-inch drop, could be chosen to have a lower limit of 0 and an
upper limit of 3 feet. The method of analysis of the data is such that the
particular choice of the endpoints of the test interval does not have an appreci-
able effect on the results for sample sizes of 15 or more. In the event that
the test interval turns out, as the test proceeds, to be inappropriately chosen,
then the stress levels will tend to converge towards one limit or the other. In
Figure 13 are represented the results of an actual one-shot test on thermal
batteries to determine their reliability with regard to high temperature. In
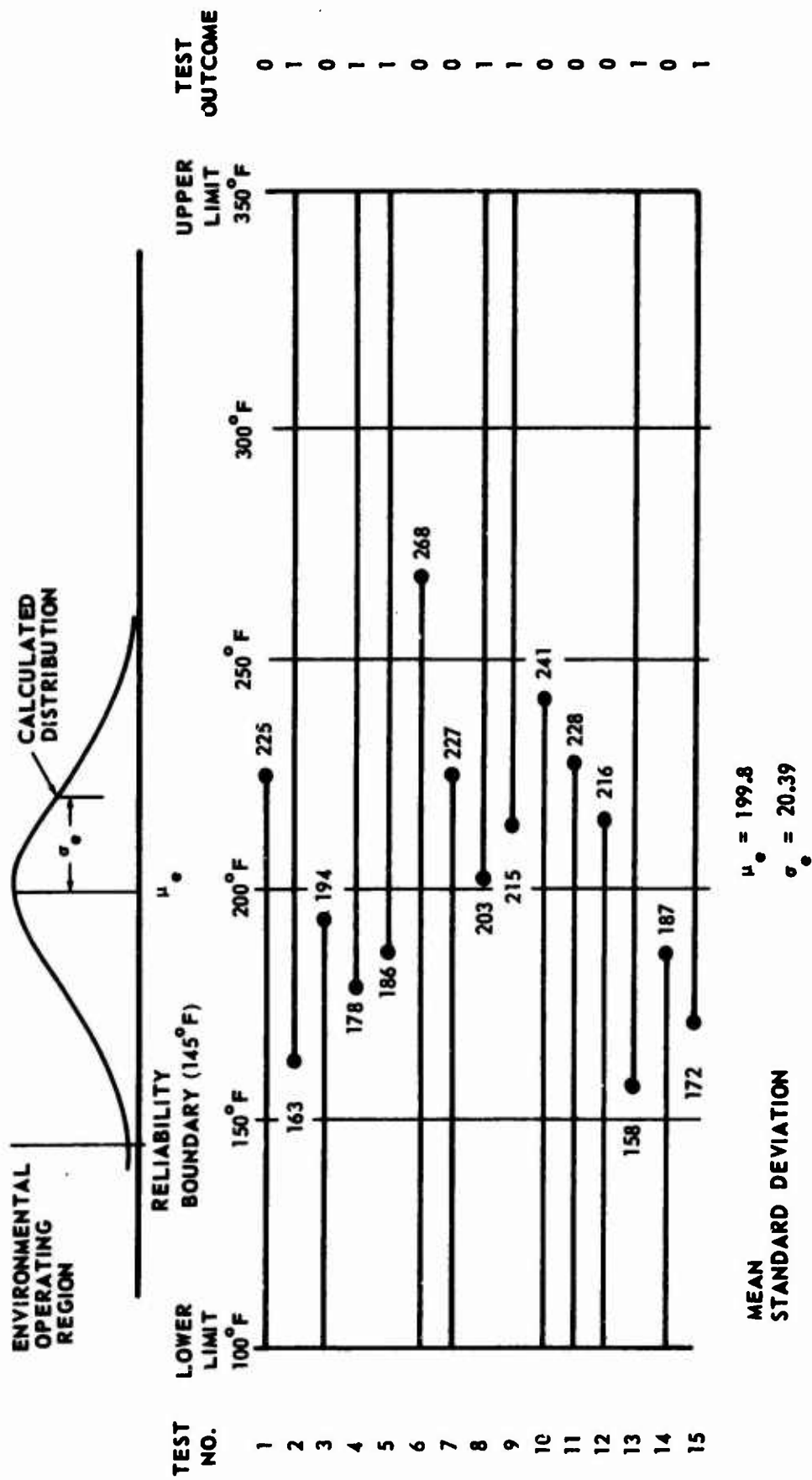
Figure 13. Sensitivity Test of Thermal Batteries in High Temperature

this instance, the batteries were designed to perform reliably at 145`F. On the basis of conservative engineering judgement and some limited development test data, the lower limit was selected to be 100°F (the level at which all thermal batteries would be expected to perform satisfactorily) and the higher limit was selected to be 350°F (the level at which all thermal batteries would be expected to fail).

2. Selecting the Test Levels

Once the test interval has been established, the test is begun by selecting the first test level at the midpoint of the interval. After the first specimen is exposed to this environmental level and activated, a 1 or 0 is recorded to indicate the outcome as a success or failure, respectively (Figure 13).

The general rule for obtaining the $(n + 1)^{st}$ test level, after completing n trials, is to work backward in the test sequence, starting at the nth trial, until a previous trial (call it the pth trial) is found such that there are as many successes as failures in the pth through the nth trials. The $(n + 1)^{st}$ test level is then obtained by averaging the nth test level with the pth test level. If there exists no previous test level satisfying the requirement stated above, then the $(n + 1)^{st}$ test level is obtained by averaging the nth test level with the lower or upper limits of the test interval according to whether the nth result was a failure or a success. To illustrate, suppose it is desired to find the second test level in Figure 13. Since there was only one previous observation (i.e., first unit failed), it is not possible to find a level where all

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intervening results even out. That is, the second test level is obtained by averaging the first with the lower limit. To find the eighth test level, it is observed that results from tests 4 through 7 (i.e., the last four results) cancel each other out. Thus, the eighth test level is obtained by averaging the fourth level with the seventh.

The test level is designated by the letter s and the outcome is designated by the letter u. The lower limit of the test interval is designated A and the upper limit is designated B. Upon the conclusion of the test, the test levels $\left(s_1, s_2, \ldots, s_N\right)$ and the corresponding outcome $\left(u_1, u_2, \ldots, u_N\right)$, where N equals the test sample size, are used to perform the analysis.

3. Performing the Analysis

The stress levels $\left(s_1, s_2, \ldots, s_N\right)$ and the outcomes $\left(u_1, u_2, \ldots, u_N\right)$ are analyzed to calculate the <u>sample mean</u> and <u>standard deviation</u> $\left(\mu_e \text{ and } \sigma_e\right)$. This appendix describes the procedure for performing these calculations. Included also is an analysis, by way of illustration, of the thermal battery data presented in Figure 13.

In summary, the steps to be followed in performing the analysis are (1) calculate the mean and standard deviation of the sample and (2) correct the calculated standard deviation for bias.

4. Calculating the Sample Mean and Standard Deviation

The maximum likelihood equations for μ_e and σ_e are as follows:

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$$p\left(\mu_e, \sigma_e\right) = \Sigma\ gh = 0$$

$$q\left(\mu_e, \sigma_e\right) = \Sigma\ tgh = 0, \tag{1}$$

where the sum is taken over N samples and where t, g, and h are given for each sample by

$$t = \frac{s_i - \mu_e}{\sigma_e} = \text{normalized stress deviate}$$

$$g = (2\pi)^{-\frac{1}{2}} \exp\left(\frac{-t^2}{2}\right) = \text{Gaussian ordinate for t}$$

$$h = \frac{u}{1 - G} - \frac{1 - u}{G} = \text{"outcome" weighting parameter}$$

and

$$G = \int_{-\infty}^{t} g\,dt = \text{Gaussian area from } -\infty \text{ to t.}$$

Tables for G and g are available in almost any engineering or mathematical handbook (for example, Handbook of Mathematical Tables and Formulas by R. S. Burington, Handbook Publishers, Inc., Sandusky, Ohio). The problem is to find values of μ_e and σ_e which, when used with the stresses and outcomes obtained in the laboratory, cause the summations in Equation (1) to be 0. The procedure used is to obtain a Taylor's expansion about the solution point $\left(\mu_e, \sigma_e\right)$ for p and q and iterate until the sums in Equation (1) become sufficiently close to 0. The formulas for this analysis are given subsequently. This method requires that an initial approximation (μ_0, σ_0), be made of the solution point so that t, g, and h can be calculated in the iterative process. The initial approximation is made as follows:

1. Denote the highest stress for which a success occurred as ψ_1.

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2. Denote the lowest stress for which a failure occurred as ψ_0.

3. Count the number of remaining stress levels which fall in between ψ_0 and ψ_1. Denote this as n_ψ.

4. Calculate

$$\mu_0 = \tfrac{1}{2}\,(\psi_1 + \psi_0)$$

$$\sigma_0 = \frac{N\,(\psi_1 - \psi_0)}{8\left(n_\psi + 2\right)} \; .$$

5. If σ_0 is negative, the calculation terminates. The outcome is said to be degenerate and the conclusion should be as follows:

μ_e lies between (ψ_1) and (ψ_0)

$\sigma_e = 0$ (i.e., too small to be measured).

A degenerate outcome does not preclude the possibility that the population mean might be outside the interval ψ_1 to ψ_0. However, if a degenerate outcome occurs for samples of $N = 15$ or more, it is indicative of very small variation in strength (failure point) from sample to sample.

Example

From Figure 13,

$$\psi_1 = 215 \quad \psi_0 = 187 \quad n_\psi = 2,$$

therefore

$$\mu_0 = \tfrac{1}{2}\,(215 + 187) = 201$$

$$\sigma_0 = \frac{15\,(215 - 187)}{8\,(2 + 2)} = 13.0 \; .$$

Given the values μ_0 and σ_0, t, g, and h can be calculated for each stress

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level-outcome pair. After the corrections $\Delta\mu$ and $\Delta\sigma$ are obtained, new values of t, g, and h, and sums (1) are calculated to obtain still closer approximations to μ_e and σ_e until convergence is reached.

Once the maximum likelihood estimates of μ and σ are calculated, it is necessary to correct the estimate, σ_e, for bias. The unbiased standard deviation, denoted $\hat{\sigma}$, is given by

$$\hat{\sigma} = \sigma_e \big/ \beta \ ,$$

where β = bias, and is the ordinate of Figure 14.

5. Calculation Methods

In order to calculate values of μ and σ satisfying Equations (1), a Taylor's expansion about the solution point $\left(\mu_e, \sigma_e\right)$ was used. For points (μ, σ) in the neighborhood of $\left(\mu_e, \sigma_e\right)$, the following formulas hold approximately:

$$0 = p\left(\mu_e, \sigma_e\right) = p\left(\mu, \sigma\right) + \Delta\mu \ p_\mu + \Delta\sigma \ p_\sigma$$
$$0 = q\left(\mu_e, \sigma_e\right) = q\left(\mu, \sigma\right) + \Delta\mu \ q_\mu + \Delta\sigma \ q_\sigma \ , \tag{2}$$

where p_σ, etc., indicate partial derivatives. These partial derivatives are given as

$$p_\mu = \sigma^{-1} \ [\Sigma \ t \ g \ h - \Sigma \ (g \ h)^2] = \sigma^{-1} \ [p - \Sigma \ (g \ h)^2]$$

$$p_\sigma = \sigma^{-1} \Sigma \ t \ g \ h \ (t - g \ h)$$

$$q_\mu = \sigma^{-1} \ [\Sigma \ t \ g \ h \ (t - g \ h) - \Sigma \ g \ h] = p_\sigma - p\sigma^{-1}$$

$$q_\sigma = \sigma^{-1} \ [\Sigma \ t^2 \ g \ h \ (t - g \ h) - q].$$

By using Equations (1) and the above expressions for the partial derivatives,
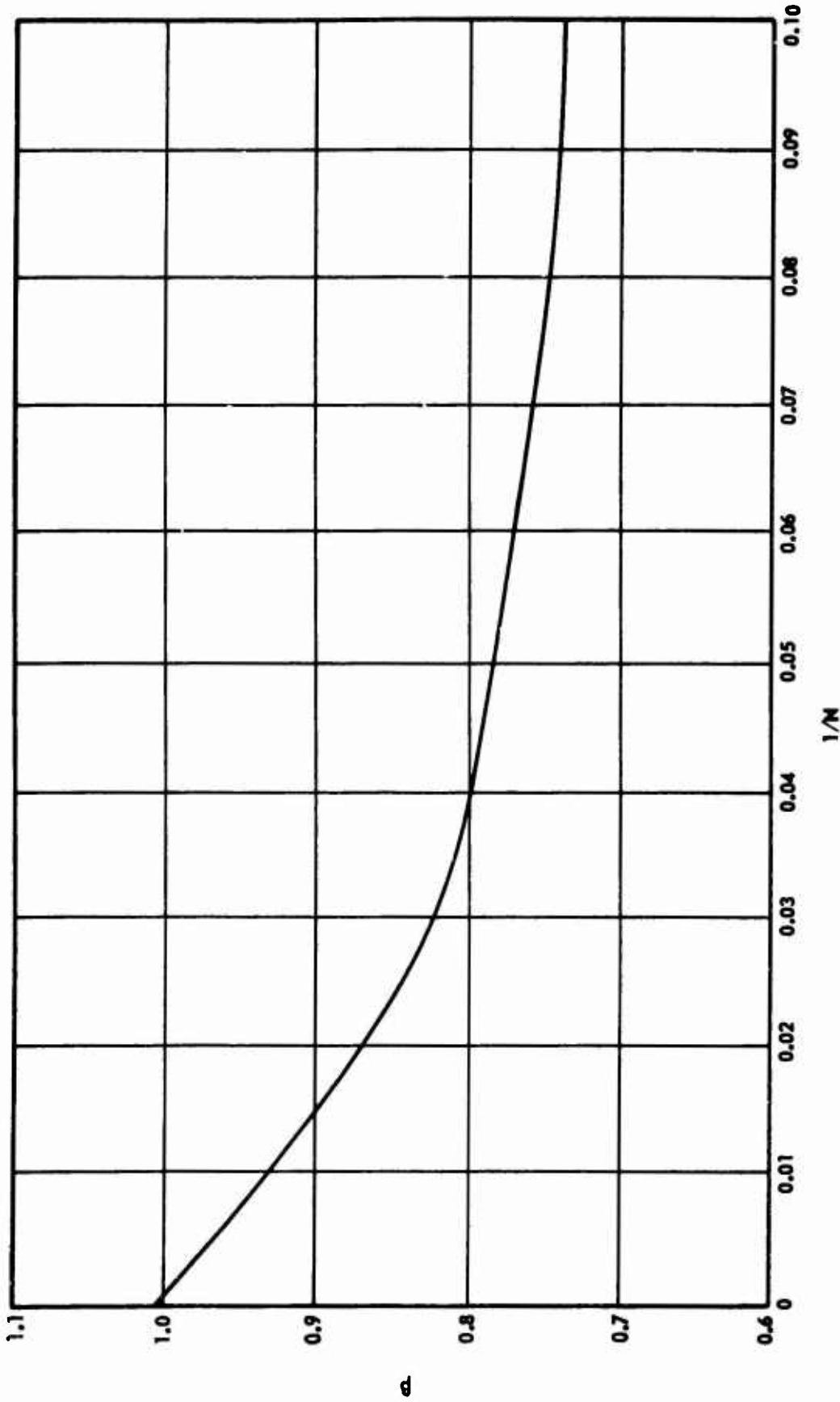
Figure 14. Bias Correction Coefficient for the Missile Standard Deviation Estimate

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Equations (2) can be solved for $\Delta\mu$ and $\Delta\sigma$, for each given outcome, u.
Then, since

$$\mu_{adj} = \mu + \Delta\mu$$

$$\sigma_{adj} = \sigma + \Delta\sigma, \qquad (3)$$

the adjusted values can be resubstituted into (2) to obtain still closer approximations. Any suitable criterion of comparison can be used to terminate the calculation, such as:

$$|\Delta\sigma| + |\Delta\mu| < 5.10^{-5}.$$

6. Sample Analysis

Figure 13 presents the results of a high temperature sensitivity test for fifteen thermal batteries. The three columns to the left in Table 13 contain the results in tabulated form suitable for calculation purposes. By using the initial estimates for the mean and standard deviation calculated earlier in the example, a high speed digital computer can be used to iterate by using the equations given in the preceding section until the sums of gh and tgh are 0 to four decimal places. To illustrate the calculation of these quantities, columns 1 through 5 are tabulated for $\mu = \mu_e$ and $\sigma = \sigma_e$. The items appearing in the first row are discussed separately below: (see formulas for reference).

Column 1: $t = (225 - 199.8)/20.39 = 1.234$

Column 2: $g = (2\pi)^{-1/2} \exp - (1.234^2/2) = 0.1863$

Table 13. Data Summary for Figure 13

Trial	s	u	Column 1 t	Column 2 g	Column 3 1/h	Column 4 gh	Column 5 tgh
1	225	0	1.234	0.1863	-0.8914	-0.2090	-0.2579
2	163	1	-1.807	0.0780	0.9646	0.0809	-0.1461
3	194	0	-0.286	0.3829	-0.3873	-0.9887	0.2832
4	178	1	-1.071	0.2248	0.8579	0.2620	-0.2806
5	186	1	-0.679	0.3169	0.7514	0.4217	-0.2852
6	268	0	3.343	0.0015	-0.9996	-0.0015	-0.0050
7	227	0	1.332	0.1643	-0.9086	-0.1808	-0.2409
8	203	1	0.155	0.3942	0.4384	0.8991	0.1394
9	215	1	0.744	0.3026	0.2286	1.3240	0.9843
10	241	0	2.019	0.0520	-0.9782	-0.0532	-0.1073
11	228	0	1.381	0.1537	-0.9164	-0.1577	-0.2317
12	216	0	0.793	0.2914	-0.7860	-0.3707	-0.2938
13	158	1	-2.052	0.0486	0.9799	0.0496	-0.1018

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Table 13. Data Summary for Figure 13 (Concluded)

Trial	s	u	Column 1 t	Column 2 g	Column 3 1/h	Column 4 gh	Column 5 tgh
14	187	0	-0.630	0.3272	-0.2644	-1.2370	0.7791
15	172	1	-1.365	0.1571	0.9139	0.1719	-0.2347
						$\Sigma = 0.0000$	$\Sigma = 0.0000$

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Column 3: $1/h = -$area $(-\infty$ to $t)* = -0.8914$

Column 4: $gh = ($column 2$)/($column 3$) = -0.2090$

Column 5: $tgh = ($column 1$) \times ($column 4$) = -0.2579$.

From Table 13, it can be observed that columns 4 and 5 run to 0, thereby indicating that $\mu_e = 199.8$ and $\sigma_e = 20.39$ are truly maximum likelihood estimates.

The final step in the analysis is to eliminate the bias in σ_e. From Figure 14, $\beta = 0.76$ for $N = 15$. Hence,

$$\hat{\sigma} = 20.39/0.76 = 26.83.$$

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*Area $(-\infty$ to $t)$ means area under the Normalized curve from $-\infty$ to t calculated for $\mu = \mu_e$ and $\sigma = \sigma_e$.

APPENDIX III

BASIC COMPUTER SIMULATION PROGRAM

```
C       SIMULATION OF SENSITIVITY  TESTS
        REAL   M,MME,MDE,LLIM
        DIMENSION  RNN(60),A(3),B(3),SIGN(60)
        COMMON RNO(60),H,DELTA,VS,BETA,U(60),ULIM,LLIM,PS,TAB(410) ,SMULT,
       1    XMULT
        DATA  A,B/2.515517,.802853,.010328,1.432788,.189269,.001308/
        NAMELIST /IN/ PXB,PS,H,DELTA,VS,RO,BETA,MMT,JLIM,LLIM,SMULT,XMULT
        NAMELIST /OUT/RNO,RNN,SIGN
      1 READ(5,IN)
        WRITE(5,IN)
        TAB = 0.
        DO 5 I=2,410
        TAB(I) = TAB(I-1) + .01
      5 CONTINUE
        SXBRN = 0.
        SXBRN2 = 0.
        SSRN = 0.
        SSRN2 = 0.
        SXBM = 0.
        SXBM2 = 0.
        SXB3 = 0.
        SXB32 = 0.
        SSB = 0.
        SSB2 = 0.
        SSM = 0.
        SSM2 = 0.
        SSBSRN = 0.
        SSMSRN = 0.
        WRITE(5,2005)
        WRITE(5,2000)PXB,PS,VS
        DO 40 I=1,VO
      9 SUM1=0.
        SUM2 = 0.
        DO 20  J=1,VS
        SIGN(J) = 1.
        RAND = ABS(RN(MMT))
        IF(.5 .GT. RAND)GO TO 10
        RAND = 1. - RAND
        SIGN(J) = -1.
     10 TM = SQRT(ALOG(1./RAND**2))
        RNN(J) = (TM - (A(1)+A(2)*TM+A(3)*TM**2)/(1.+B(1)*TM+B(2)*TM**2+
       1       B(3)*TM**3)) * SIGN(J)
        RNO(J) = PXB + RNN(J) * PS
        SUM1 = SUM1 + RNO(J)
        SUM2 = SUM2 + RNO(J)**2
     20 CONTINUE
        CALL BRJCMT(XB3,SB,      M)
        CALL MISSMT(XBM,SM, M9)
        XBRN = SUM1/FLOAT(VS)
        SRN = SQRT(ABS((SUM2 - SUM1**2/FLOAT(VS))/FLOAT(VS-1) ) )
        SXBRN = SXBRN + XBRN
        SXBRN2 = SXBRN2 + XBRN**2
        SSRN = SSRN + SRN
        SSRN2 = SSRN2 + SRN**2
        SXBM = SXBM + XBM
```

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```
      SXBM2 = SXBM2 + XBM**2
      SXBB = SXBB + XBB
      SXBB2 = SXBB2 + XBB**2
      SSB = SSB + SB
      SSB2 = SSB2 + SB**2
      SSM = SSM + SM
      SSM2 = SSM2 + SM**2
      SSBSRN = SSBSRN + SB*SRN
      SSMSRN = SSMSRN + SM*SRN
      WRITE(5,2001)I,XBRN,XBB,XBM,SRN,SB,SM,M
 2001 FORMAT(11X,I5,7(3X,F11.5))
   40 CONTINUE
      FNM1 = NO - 1
      FNO = NO
      XBXBM = SXBM/FNO
      XBXBB = SXBB/FNO
      XBXBRN = SXBRN/FNO
      STXBRN = SQRT(ABS((SXBRN2 - SXBRN**2/FNO)/FNM1) )
      STXBB = SQRT(ABS((SXBB2 - SXBB**2/FNO)/FNM1))
      STXBM = SQRT(ABS((SXBM2- SXBM**2/FNO)/FNM1) )
      XBSRN = SSRN/FNO
      XBSB = SSB/FNO
      XBSM = SSM/FNO
      STSRN = SQRT(ABS((SSRN2 - SSRN**2/FNO)/FNM1) )
      STSB = SQRT(ABS((SSB2 - SSB**2/FNO)/FNM1 ))
      STSM = SQRT(ABS((SSM2 - SSM**2/FNO)/FNM1))
      TAUB = (FNO*SSBSRN - SSB*SSRN)/SQRT( (FNO*SSB2-SSB**2)* (FNO*
     1      SSRN2 - SSRN**2) )
      TAUM = (FNO*SSMSRN - SSM*SSRN)/SQRT((FNO*SSM2 - SSM**2)* (FNO *
     1      SSRN2 - SSRN**2) )
      PXBB = ABS(PXB-XBXBB)
      PXBM = ABS(PXB-XBXBM)
      PXBRN = ABS(PXB- XBXBRN)
      PSXSB = ABS(PS - XBSB)
      PSXSM = ABS(PS - XBSM)
      PSXSRN = ABS(PS - XBSRN)
      BME = PXBRN/PXBB
      BDE = PSXSRN/ PSXSB
      MME = PXBRN/PXBM
      MDE = PSXSRN/PSXSM
      WRITE(5,2002)XBXBRN,STXBRN,XBSRN,STSRN,PXBRN,PSXSRN,XBXBB,STXBB,
     1      XBSB,STSB,PXBB,PSXSB,XBXBM,STXBM,XBSM,STSM,PXBM,PSXSM
      WRITE(5,2003)BME,MME,BDE,MDE,TAUB,TAUM
      GO TO 1
 2000 FORMAT(15X,10HPOPULATION,10X,6H MU  =F8.5,3X,7HSIGMA =F8.5/11X,
     1      18HTEST  SAMPLE  SIZE,6X,I5////11X,5H TEST,6X,6HXB(RN), 9X,
     2      5HXB(B), 9X,5HXB(M), 9X,5HS(RN), 9X,4HS(B),10X,4HS(M),12X,
     3      1HM //)
 2002 FORMAT(/////11X,9HPROCEDURE,7X,8HXB(MEAN), 8X, 7HS(MEAN), 9X,
     1      8HXB(S.D.),8X,7HS(S.D.),10X,5HMU-XX,11X,6HSIG-XS///11X,
     2      6HNORMAL,3X,6(5X,F11.5)/11X,8HBRJCETON,1X,6(5X,F11.5)/11X,
     3      7HMISSILE,2X,6(5X,F11.5) )
 2003 FORMAT(/////11X,5HBME =F11.5,10X,5HMME =F11.5//11X,5HBDE =F11.5,
     1      10X,5HMDE =F11.5//11X,5HTAUB=F11.5,10X,5HTAUM=F11.5)
 2005 FORMAT(1H1)
      END
```

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```
      SUBROUTINE BRUCMT(XB,S,N )
C     ROUTINE  FOR  BRUCETON  METHOD
      REAL N
      COMMON RNO(60),H,DELTA,NS,BETA,U(60),ULIM,LLIM,PS,TAB(410) ,SMULT,
     1      KNJLT
      DIMENSION STSL(60),FOS(60),SL(60),TXB(75),TS(75),EQ1(75),EQ2(75),
     1      XI(60),TABZQ(410),TABZP(367)
      NAMELIST/OT/SLM,SLMX,FN,J,STSL,S
      NAMELIST/OT1/N,SL,FOS,TX3,TS,EQ1,EQ2,XI,AN1,AN2
      DATA (TABZP(I),I=1,256)/.798,.792,.785,.779,.773,.766,.760,.754,
     1 .748,.741,.735,.729,.723,.717,.711,.705,.699,.693,.687,.681,.675,
     2 .669,.663,.657, .652,.645,.640,.634,.629,.623,.617,.612,.606,.6,
     3 .595,.589,.584,.578,.573,.567,.562,.556,.551,.546,.540,.535,.530,
     4 .525,.519,.514,.509,.504,.499,.494,.489,.484,.479,.474,.469,.464,
     5 .459,.454,.449,.445,.44,.435,.43,.426,.421,.417,.412,.407,.403,
     6 .399,.394,.389,.385,.381,.376,.372,.368,.363,.359,.355,.351,.346,
     7 .342,.338,.334,.33,.326,.322,.318,.314,.31,.306,.303,.299,.295,
     8 .291,.288,.284,.28,.277,.273,.269,.266,.262,.259,.255,.252,.249,
     9 .245,.242,.239,.235,.232,.229,.226,.223,.219,.216,.213,.21,.207,
     A .204,.201,.198,.195,.193,.19,.187,.184,.181,.179,.176,.173,.171,
     B .168,.165,.163,.16,.158,.155,.153,.15,.148,.146,.143,.141,.139,
     C .137,.134,.132,.13,.128,.126,.124,.121,.119,.117,.115,.113,.111,
     D .11,.108,.106,.104,.102,.1,.098,.097,.095,.093,.092,.09,.088,.087,
     E .085,.083,.082,.08,.079,.077,.076,.074,.073,.072,.07,.069,.068,
     F .066,.065,.064,.062,.061,.06,.059,.058,.055,.055,.054,.053,.052,
     G .051,.05,.049,.048,.047,.046,.045,.044,.043,.042,.041,.04,.039,
     H .039,.038,.037,.036,.035,.034,.034,.033,.032,.031,.031,.03,.029,
     I .029,.028,.027,.027,.025,.025,.025,.024,.024,.023,.023,.022,.022,
     J .021,.02,.02,.019,.019,.019,.018,.018,.017,.017,.016,.016,.016/
      DATA (TABZP(I),I=257,367)/.015,.015,.014,.014,.014,3*.013,3*.012,
     1 3*.011,4*.01,4*.009,5*.008,5*.007,5*.006,7*.005,8*.004,11*.003,
     2 16*.002,31*.001,0./,(TABZQ(I),I=1,191)/.798,.804,.811,.817,.824,
     3 .830,.836,.843,.849,.855,.863,.869,.876,.882,.889,.896,.902,.909,
     4 .915,.923,.929,.936,.943,.95,.957,.964,.97,.977,.984,.991,.998,
     5 1.005,1.012,1.019,1.025,1.033,1.040,1.047,1.054,1.062,1.069,
     6 1.075,1.083,1.090,1.097,1.105,1.112,1.119,1.126,1.134,1.141,
     7 1.149,1.156,1.163,1.17,1.178,1.185,1.193,1.2,1.207,1.215,1.222,
     8 1.23,1.237,1.245,1.253,1.26,1.268,1.275,1.283,1.29,1.298,1.306,
     9 1.313,1.321,1.329,1.336,1.344,1.352,1.36,1.367,1.375,1.383,1.391,
     A 1.399,1.406,1.414,1.422,1.43,1.438,1.446,1.454,1.461,1.469,1.477,
     B 1.485,1.493,1.501,1.509,1.517,1.525,1.533,1.541,1.549,1.557,
     C 1.565,1.573,1.581,1.59,1.598,1.606,1.614,1.622,1.63,1.638,1.646,
     D 1.655,1.663,1.671,1.679,1.687,1.696,1.704,1.712,1.72,1.729,1.737,
     E 1.745,1.754,1.762,1.77,1.779,1.787,1.795,1.804,1.812,1.82,1.829,
     F 1.838,1.846,1.854,1.862,1.871,1.879,1.888,1.896,1.905,1.913,
     G 1.922,1.93,1.938,1.947,1.955,1.964,1.972,1.981,1.99,1.998,2.007,
     4 2.015,2.024,2.033,2.041,2.05,2.058,2.067,2.076,2.084,2.093,
     I 2.102,2.11,2.119,2.128,2.136,2.145,2.154,2.162,2.171,2.18,2.188,
     J 2.197,2.206,2.215,2.223,2.232,2.241,2.25,2.258,2.267,2.276,2.285/
      DATA(TABZQ(I),I=192,403)/2.294,2.303,2.311,2.32,2.329,2.338,2.346,
     1 2.355,2.364,2.373,2.381,2.39,2.399,2.408,2.417,2.426,2.435,2.444,
     2 2.453,2.462,2.47,2.479,2.488,2.497,2.506,2.515,2.524,2.533,
     3 2.542,2.551,2.56,2.569,2.578,2.587,2.596,2.605,2.614,2.623,2.632,
     4 2.641,2.65,2.659,2.668,2.677,2.687,2.696,2.705,2.714,2.723,2.732,
     5 2.741,2.75,2.759,2.768,2.777,2.786,2.795,2.805,2.814,2.823,2.832,
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6 2.841,2.85,2.859,2.868,2.878,2.887,2.896,2.905,2.914,2.923,2.932,
7 2.942,2.951,2.96,2.969,2.978,2.987,2.997,3.006,3.015,3.024,3.033,
8 3.043,3.052,3.061,3.07,3.079,3.089,3.098,3.107,3.116,3.126,3.135,
9 3.144,3.153,3.163,3.172,3.181,3.19,3.2,3.209,3.218,3.227,3.23
A 3.245,3.255,3.265,3.274,3.283,3.292,3.302,3.311,3.32,3.33, 3.3
B 3.348,3.358,3.367,3.376,3.386,3.395,3.404,3.413,3.423,3.432,3.4
C 3.451,3.46,3.47,3.479,3.488,3.498,3.507,3.516,3.526,3.535,3.54
D 3.554,3.563,3.573,3.582,3.591,3.601,3.61,3.62,3.629,3.638,3.648,
E 3.657,3.667,3.676,3.685,3.695,3.704,3.714,3.723,3.732,3.742,
F 3.751,3.761,3.77,3.78,3.789,3.799,3.808,3.817,3.827,3.836,3.846,
G 3.855,3.865,3.874,3.884,3.893,3.902,3.912,3.992,3.931,3.94,3.95,
H 3.959,3.969,3.978,3.988,3.997,4.007,4.016,4.026,4.035,4.045,
I 4.054,4.064,4.073,4.083,4.092,4.102,4.111,4.121,4.13,4.14,4.149,
J 4.159,4.169,4.178,4.188,4.197,4.206,4.216,4.226,4.235,4.2437
  DATA (TABZQ(I),I=404,410)/4.254,4.264,4.273,4.283,4.292,4.302,
1 4.312/
  IKBCON = 0
  ISCON = 0
  HH = 4
  NF = 0
  NSS = 0
  SLM = HH
  SLMX = HH
  DO 15 I=1,NS
  IF(RND(I) .GE. HH)GO TO 5
  STSL(I) = HH
  J(I) = 0.
  NF = NF + 1
  HH = HH - DELTA
  GO TO 10
5 J(I) = 1.
  STSL(I) = HH
  HH = HH + DELTA
  NSS = NSS + 1
10 SLM = AMIN1(SLM,STSL(I))
  SLMX = AMAX1(SLMX,STSL(I))
15 CONTINUE
  FN = AMIN0(NF,NSS)
  DZ = 1.
  IF(NF .LE. NSS)DZ = 0.
  CALL CHEK(SLM,SLMX,DZ,STSL,FOS,K,SL)
  NDZ = DZ + 1.
  SUM1 = 0.
  SUM = 0.
  DO 25 I=1,K
  FIM = I - 1
  SUM = SUM + FIM*FOS(I)
  SUM1 = SUM1 + FIM**2*FOS(I)
25 CONTINUE
  GO TO (30,35),NDZ
30 X3 = SLM + DELTA *(SUM/FN + .5)
  GO TO 40
35 X3 = SLM + DELTA *(SUM/FN - .5)
40 S = 1.520 * DELTA *((FN*SUM1 - SUM**2)/FN**2 + 0.029)
  M = SUM1/FN -(SUM/FN)**2
  IF(M .GE. .325)RETURN
```

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```
      M = 1
      IF(NJZ .EQ. 1)GO TO 50
      JJ = K
      DO 45 JN=1,K
      FOS(JJ+1) = FOS(JJ)
      JJ = JJ - 1
   45 CONTINUE
   50 TXB(N) = XB
      TS(N) = S
      EQ1(N) = 0.
      EQ2(N) = 0.
      DO 85 I=1,K
      XI(I) = (SL(I)-XB)/S
      IF(I .EQ. 1)GO TO 85
      NX = 1
      NXM1 = 1
      IF(XI(I-1) .LT. 0.)NXM1 = 2
      IF(XI(I) .LT. 0.)NX = 2
      GO TO(55,60),NXM1
   55 AN1 = 4.312
      IF(XI(I-1) .GT. 4.09)GO TO 65
      CALL INTERP(XI(I-1),TAB,TABZQ,410,4,AN1,NERR)
      GO TO 65
   60 AN1 = 0.
      IF(ABS(XI(I-1)) .GT. 3.66)GO TO 55
      AAA = ABS(XI(I-1))
      CALL INTERP(AAA      ,TAB,TABZP,367,4,AN1,NERR)
   65 GO TO(70,75),NX
   70 AN2 = 0.
      IF(XI(I) .GT. 3.66)GO TO 80
      CALL INTERP(XI(I),TAB,TABZP,367,4,AN2,NERR)
      GO TO 90
   75 AN2 = 4.312
      IF(ABS(XI(I)) .GT. 4.09)GO TO 80
      AAA = ABS(XI(I))
      CALL INTERP(AAA      ,TAB,TABZQ,410,4,AN2,NERR)
   80 EQ1(N) = EQ1(N) + FOS(I)*(AN1 - AN2)
      EQ2(N) = EQ2(N) + FOS(I) *(XI(I-1)*AN1 - XI(I)*AN2)
   85 CONTINUE
      IF(N .NE. 1)GO TO 90
      IF(X3 .GT. 1.)X3=XB - (XMULT*XB)
      IF(X3 .LE. 1.)X3=XB + (XMULT*XB)
      S = S + (SMULT * S)
      N = N + 1
      GO TO 50
   90 IF(IX3CON .EQ. 1)GO TO 105
      CALL ITR(N,EQ1,TXB,XB,$100)
      IF(TX3(1) .GT. 1.)XB=XB - (XMULT*XB)
      IF(TX3(1) .LE. 1.)XB=XB + (XMULT*XB)
  100 IF(ABS(XB-TXB(N)).GT. .0001)GO TO 105
      IXBCON = 1
  105 IF(ISCON .EQ. 1)GO TO 110
      CALL ITR(N,EQ2,TS,S,$106)
      S = S + (SMULT*S)
  106 IF(ABS(S-TS(N)) .GT. .0001)GO TO 115
      ISCON = 1
```

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110 IF(ISCON .EQ. 1 .AND. IXECON .EQ. 1)RETURN
115 N = N + 1
    GO TO 50
    END
```

```
      SUBROUTINE CHEK(SMIN,SMAX,J2,STSL,FOS,K,SL1)
C
      DIMENSION FOS(60),STSL(60),SL1(60)
      COMMON RNO(60),H,DELTA,NS,BETA,U(60),ULIM,LLIM,PS,TAB(410) ,SMULT,
     1    KMULT
      K = 1
      SL = SMIN
    1 FOS(K) = 0.
      DO 5 I=1,NS
      IF(ABS(SL -STSL(I))/ABS(SL) .GT. .00001)GO TO 5
      IF(U(I) .NE. OZ)GO TO 5
      FOS(K) = FOS(K) + 1.
    5 CONTINUE
      SL1(K) = SL
      K = K + 1
      SL = SL + DELTA
      IF(SMAX .GE. SL)GO TO 1
      K = K - 1
      RETURN
      END
```

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      SUBROUTINE INTERP(ARG,XTAB,YTAB,NX,NPTS,ANS,NERR)              TRP10010
C                                                                    TRP10020
C                        1. ARG  THE INDEPENDENT VARIABLE FOR THE DESIRED  TRP10030
C                                UNKNOWN.                            TRP10040
C                        2. XTAB TABLE OF INDEPENDENT VALUES. MUST BE IN   TRP10050
C                                INCREASING ORDER.                   TRP10060
C                        3. YTAB TABLE OF DEPENDENT VALUES.          TRP10070
C                        4. NX   NUMBER OF POINTS IN XTAB.           TRP10080
C                        5. NPTS NUMBER OF POINTS USED IN THE INTERPOLATINGTRP10090
C                                FORMULA. (NPTS-1) IS THE DEGREE OF THE    TRP10100
C                                INTERPOLATION FORMULA USED.         TRP10110
C                        6. ANS  THE DEPENDENT VALUE CORRESPONDING TO THE  TRP10120
C                                VALUE OF ARG.                       TRP10130
C                        7. NERR WILL BE SET UNEQUAL ZERO IF ARG IS NOT ON TRP10140
C                                XTAB. ROUTINE WILL EXTRAPOLATE.     TRP10150
C                                NERR = 1, ARG .LT. XTAB(1)          TRP10160
C                                NERR = 2, ARG .GT. XTAB(NX)         TRP10170
C                                                 10 SEPT 64         TRP10180
C                                                 RALPH SELLERS      TRP10190
      DIMENSION XTAB(NX),YTAB(NX)                                    TRP10200
      NERR = 0                                                       TRP10210
      NP = NPTS                                                      TRP10220
      IF(NX .LT. NP) NP = NX                                         TRP10230
      NS = (NX+169)/26                                               TRP10240
      IH = NP/2                                                      TRP10250
      I = 1                                                          TRP10260
      IF (XTAB(I) - ARG)30,20,10                                     TRP10270
   10 IH = 0                                                         TRP10280
   12 NERR = 01                                                      TRP10290
      GO TO 70                                                       TRP10300
   13 NERR = 2                                                       TRP10310
      GO TO 70                                                       TRP10320
   20 ANS = YTAB(I)                                                  TRP10330
      GO TO 999                                                      TRP10340
   30 I = NX                                                         TRP10350
      IF (XTAB(I) - ARG)13,20,50                                     TRP10360
   50 L = IH + 1                                                     TRP10370
      IS = NS+L                                                      TRP10380
      IF(NX-IS)58,58,52                                              TRP10390
   52 DO 54   I=IS,NX,NS                                             TRP10400
      IF(XTAB(I)-ARG)54,20,56                                        TRP10410
   54 L = I                                                          TRP10420
      GO TO 58                                                       TRP10430
   56 L = I - NS                                                     TRP10440
   58 DO 60   I=L,NX                                                 TRP10450
      IF (XTAB(I) - ARG)60,20,70                                     TRP10460
   60 CONTINUE                                                       TRP10470
   70 K = I - IH                                                     TRP10480
      N = K + NP - 1                                                 TRP10490
      ANS = 0.0                                                      TRP10500
      IF (N - NX)90,90,80                                            TRP10510
   80 N = NX                                                         TRP10520
      K = NK-NP+1                                                    TRP10530
   90 DO 120   J=K,N                                                 TRP10540
      P = 1.0                                                        TRP10550
```

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```
      DO 110  I=K,N                                          TRP10560
      IF (I-J)100,110,100                                    TRP10570
  100 P = P * (ARG - XTAB(I)) / (XTAB(J) - XTAB(I))          TRP10580
  110 CONTINUE                                               TRP10590
  120 ANS = ANS + YTAB(J) * P                                TRP10600
  999 RETURN                                                 TRP10610
      END                                                    TRP10620
```

```
      SUBROUTINE  ITR(N,X,Y,ANS, * )
      DIMENSION X(30),Y(30)
      NM1 = N - 1
      DO 10 J=1,NM1
      JK = N - J
      IF(X(N) * X(JK) .LT. 0.)GO TO 15
10 CONTINUE
   RETURN
15 ANS = Y(JK) + ((0.-X(JK))/(X(N)-X(JK)) * (Y(N)-Y(JK))
   RETURN  1
   END
```

```
      SUBROUTINE MISSMT(XB,SMC, * )
C     SUBROUTINE FOR THE MISSILE METHOD
      REAL   LLIM,MISL,MXSL,NT,NJ,NCO
      DIMENSION  PQ(2,3),SL(60)
      COMMON RNO(60),H,DELTA,NS,BETA,U(60),ULIM,LLIM,PS,TAB(410) ,SMULT,
     1    XMULT
      NAMELIST /OT/ MISL,MXSL,NJ,SM,SL,U,NT,J
      SIGN = -1.
      SL = (ULIM + LLIM)/2.
      MISL = 1.E+10
      MXSL = 0.
      DO 50 I=1,NS
      U(I) = 0.
      IF(RNO(I) .GE. SL(I))U(I)=1.
      MJ = J(I) + 1.
      IF(I .EQ. 1)GO TO 20
      NCO = 1.
      JJ = 2
   10 IF(JJ .GT. I)GO TO 20
      J = I
      NJ = 0
      DO 15 K=1,JJ
      NJ = NJ + U(J)
      J = J - 1
   15 CONTINUE
      IF(NJ .EQ. NCO)GO TO 18
      JJ = JJ + 2
      NCO = NCO + 1.
      GO TO 10
   18 SL(I+1) = (SL(I) + SL(J+1))/2.
      GO TO (30,40),MU
   20 GO TO (25,35),NJ
   25 SL(I+1) = (SL(I) + LLIM)/2.
   30 MISL = AMIN1(MISL,SL(I))
      GO TO 50
   35 SL(I+1) = (SL(I) + ULIM)/2.
   40 MXSL = AMAX1(MXSL,SL(I))
   50 CONTINUE
      NT = 0.
      DO 55 I=1,NS
      IF(SL(I) .GT. MISL .AND. MXSL .GT. SL(I))NT = NT + 1.
   55 CONTINUE
      XB  = .5*(MXSL + MISL)
      SM = (FLOAT(NS) * (MXSL - MISL) )/(8.*(NT + 2.))
      J = 0
      IF(SM .GT. 0.)GO TO 60
      SMC= 0.
      RETURN
   60 SJM = 0.
      SJM1 = 0.
      SJM2 = 0.
      PQ(1,3) = 0.
      PQ(2,3) = 0.
      DO 65 I=1,NS
      T = (SL(I) - XB )/SM
```

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      GS = 1./SQRT(6.2831853) * EXP(-T **2/2.)
      T1T = 0.
      T2T = T
      IF(T .GT. 0.)GO TO 61
      T1T = T
      T2T = 0.
   61 CALL IGRAT(T1T,T2T,.1, BG,1)
      IF(T .GT. 0.)BG=BG+.5
      IF(T .LE. 0.)BG=.5-BG
      HM = J(I)/(1.-BG) - (1.-J(I))/BG
      PQ(1,3) = PQ(1,3) + GS * HM
      T1 = T * GS * HM
      T2 = T - GS*HM
      PQ(2,3) = PQ(2,3) + T1
      SUM = SUM + (GS * HM)**2
      SUM1 = SUM1 + T1*T2
      SUM2 = SUM2 + T**2 * GS * HM * T2
   65 CONTINUE
      PQ(1,1) = 1./SM * (PQ(2,3) - SUM)
      PQ(1,2) = 1./SM * SUM1
      PQ(2,1) = PQ(1,2) - PQ(1,3) * 1./SM
      PQ(2,2) = 1./SM *(SUM2 - PQ(2,3))
      PQ(1,3) = PQ(1,3) * SIGN
      PQ(2,3) = PQ(2,3) * SIGN
      CALL SESOMI(PQ,2,1,0,2,3,D1,R1,E)
      IF(E .EQ. 0.)GO TO 70
      WRITE(5,2000)R1
 2000 FORMAT(19H  ERROR  IN  SESOMI, E15.8)
      RETURN 1
   70 X3 = X3 + PQ(1,1)
      SM = SM + PQ(2,1)
      IF(ABS(PQ(1,1)/XB) .LT. .0001   .AND. ABS(PQ(2,1)/SM) .LT.
     1        .0001  )GO TO 75
      J = J + 1
      IF(J .LE. 100)GO TO 60
      WRITE(5,2001)
 2001 FORMAT(27H  MAX.  NO.  OF  ITERATIONS )
   75 SMC = SM/BETA
      RETURN
      END
```

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```
      SUBROUTINE IGRAT(LL,UL,DELTA,ANS,NOEQ)              IGRT0010
C                      1 LL    LOWER LIMIT OF INTEGRATION(FLOATING PT.) IGRT0020
C                      2 UL    UPPER LIMIT OF INTEGRATION              IGRT0030
C                      3 DELTA STEP SIZE FOR SOLUTION                  IGRT0040
C                      4 ANS   VALUE OF INTEGRAL                       IGRT0050
C                      5 NOEQ  ALL EQUATIONS TO BE INTEGRATED MUST BE  IGRT0060
C                            COMPILED IN A SUBROUTINE CALLED INTEQS.   IGRT0070
C                            NOEQ IS TO BE USED BY THIS SUBROUTINE     IGRT0080
C                            TO SELECT THE PROPER EQUATION FOR         IGRT0090
C                            INTEGRATION IE. NOEQ=1 WILL BE TO INTEGRAIGRT0100
C                            EQUATION 1, NOEQ=2 TO INTEGRAT EQUATION   IGRT0110
C                            2, ETC. THE FORM OF INTEQS MUST BE...     IGRT0120
C                                   INTEQS(X,Y,NOEQ)  WHERE            IGRT0130
C                                  1 X     INDEPENDENT VARIABLE        IGRT0140
C                                  2 Y     DEPENDENT VAR. IE. Y=F(X)   IGRT0150
C                                  3 NOEQ SAME AS ABOVE. PROBABLY      IGRT0160
C                                        USED IN A COMPUTED GO TO.     IGRT0170
C                                               11 SEPT 64             IGRT0180
C                                               RALPH SELLERS          IGRT0190
      DIMENSION F(6),U(6),R(3)                            IGRT0200
      DATA (U(I), I=1,6) / .11930959,-.11930959,.33060469,-.33060469,  IGRT0210
     A .46623476,-.46623476 /                             IGRT0220
      DATA (R(I), I=1,3) / .23395697,.18038079,.85662246E-1 /   IGRT0230
      REAL LLIM,MULT,LL                                   IGRT0240
      ULIM = UL                                           IGRT0250
      LLIM = LL                                           IGRT0260
      MULT = 1.0                                          IGRT0270
      IF(ULIM .GE. LLIM)   GO TO 5                        IGRT0280
      TMP1 = LLIM                                         IGRT0290
      LLIM = ULIM                                         IGRT0300
      ULIM = TMP1                                         IGRT0310
      MULT =-1.0                                          IGRT0320
    5 A = LLIM                                            IGRT0330
      DEL = ABS(DELTA)                                    IGRT0340
      LAST = 1                                            IGRT0350
      ANS = 0.0                                           IGRT0360
      IF(ABS (ULIM-LLIM) - .00001)80,80,10                IGRT0370
   10 B = A + 4.0*DEL                                     IGRT0380
      IF(B-ULIM)40,30,20                                  IGRT0390
   20 B = ULIM                                            IGRT0400
   30 LAST = 2                                            IGRT0410
   40 DO 50   I=1,6                                       IGRT0420
      X = (B-A)*U(I) + .5*(A+B)                           IGRT0430
      CALL INTEQS(X,F(I),NOEQ)                            IGRT0440
   50 CONTINUE                                            IGRT0450
   60 ANS = ANS + (B-A)*(R(1)*(F(1)+F(2)) + R(2)*(F(3)+F(4)) + R(3)*  IGRT0460
     A (F(5)+F(6)))                                       IGRT0470
      GO TO (70,80),LAST                                  IGRT0480
   70 A = B                                               IGRT0490
      GO TO 10                                            IGRT0500
   80 ANS = ANS * MULT                                    IGRT0510
      RETURN                                              IGRT0520
      END                                                 IGRT0530
```

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```
C                                                                    SES00010
      SUBROUTINE SESOMI(X,N,NB,MS,MN1,MN2,D,R,E)                      SES00020
C                                                                    SES00030
C                       1 X     FIRST LOCATION OF INPUT ARRAY,DIM AS X(MN1,MN2) SES00040
C                       2 N     NUMBER OF EQUATIONS                   SES00050
C                       3 NB    NUMBER OF RIGHT HAND COLUMN VECTORS   SES00060
C                               NB=N FOR INVERSE                      SES00070
C                       4 MS    CHECK FOR SIM. EQS OR INVERSE         SES00080
C                               MS=0 FOR SIM. EQ.                     SES00090
C                               MS=1 FOR INVERSE                      SES00100
C                       5 MN1   ROW DIMENSION OF X IE.,X(MN1,MN2)     SES00110
C                       6 MN2   MN2 = MN1+NB                          SES00120
C                       7 D     VALUE OF THE DETERMINANT              SES00130
C                       8 R     RANK OF THE MATRIX                    SES00140
C                       9 E     ERROR RETURN                         SES00150
C                               E=0 FOR NO ERROR                     SES00160
C                               E=1 FOR ERROR                        SES00170
C                                                                    SES00180
      DIMENSION X(MN1,MN2),WORK(99),SAVR(50)                         SES00190
      DIMENSION IHLD(50)                                             SES00200
C     DOUBLE PRECISION  X,WORK,SAVK,SAVEB,Y,D,SUM,TEST               SES00210
      E=0.                                                           SES00220
      R=0.                                                           SES00230
      DO 27 I=1,N                                                    SES00240
   27 SAVR(I)=X(1,I)                                                 SES00250
      DO 21 I=1,N                                                    SES00260
   21 IHLD(I)=I                                                      SES00270
      IF(MS)5,4,6                                                    SES00280
    6 NV=N+N                                                         SES00290
      NB=N                                                           SES00300
      MN=N+1                                                         SES00310
      DO 14 I=1,N                                                    SES00320
      DO 14 J=MN,NV                                                  SES00330
   14 X(I,J)=0.00                                                    SES00340
      DO 15 I=1,N                                                    SES00350
      J=I+N                                                          SES00360
   15 X(I,J)=1.D0                                                    SES00370
      GO TO 16                                                       SES00380
    4 NV=N+NB                                                        SES00390
   16 JJ=NV                                                          SES00400
      SAVEB=X(1,N+1)                                                 SES00410
      NNV=N-1                                                        SES00420
      D=1.D0                                                         SES00430
      DO 5 I=1,N                                                     SES00440
      KK=N-I                                                         SES00450
      IF(KK)10,10,26                                                 SES00460
   26 LL=KK+1                                                        SES00470
      IJJ=1                                                          SES00480
      L=I                                                            SES00490
      WORK=K                                                         SES00500
      DO 17 II=1,LL                                                  SES00510
      DO 17 J=1,LL                                                   SES00520
      IF(ABS(WORK)-ABS(X(II,J)))18,17,17                            SES00530
   18 WORK=X(II,J)                                                   SES00540
      L=J+I-1                                                        SES00550
```

```
       IJJ=J                                          SES00560
17 CONTINUE                                           SES00570
   IF(IJJ-1)2,2,19                                    SES00580
19 DO 20 II=1,N                                       SES00590
   Y=X(II,1)                                          SES00600
   X(II,1)=X(II,IJJ)                                  SES00610
20 X(II,IJJ)=Y                                        SES00620
   IY=IHLD(I)                                         SES00630
   IHLD(I)=IHLD(L)                                    SES00640
   IHLD(L)=IY                                         SES00650
   D=-D                                               SES00660
 2 DO 1 L=1,KK                                        SES00670
   IF(ABS(X)-ABS(X(L+1,1)))7,1,1                      SES00680
 7 D=-D                                               SES00690
   DO 9 J=1,JJ                                        SES00700
   Y=X(1,J)                                           SES00710
   X(1,J)=X(L+1,J)                                    SES00720
 9 X(L+1,J)=Y                                         SES00730
 1 CONTINUE                                           SES00740
10 JJ=JJ-1                                            SES00750
   IF(X)11,8,11                                       SES00760
11 D=D*X                                              SES00770
   R=R+1.                                             SES00780
   DO 12 J=1,JJ                                       SES00790
12 WORK(J)=X(1,J+1)/X                                 SES00800
   KK=JJ+1                                            SES00810
   IF(NNV.EQ.0)GO TO 33                               A7  0815
   DO 3 K=1,NNV                                       SES00820
   DO 3 J=2,KK                                        SES00830
 3 X(K,J-1)=X(K+1,J)-X(K+1,1)*WORK(J-1)               SES00840
33 DO 5 J=1,JJ                                        A7  0845
 5 X(N,J)=WORK(J)                                     SES00860
   NNV=N-1                                            SES00870
   IF(NNV.EQ.0)GO TO 34                               A7  0875
   DO 22 I=1,NNV                                      SES00880
   L=I+1                                              SES00890
   DO 22 J=L,N                                        SES00900
   IF(IHLD(I)-IHLD(J))22,22,23                        SES00910
23 IY=IHLD(I)                                         SES00920
   IHLD(I)=IHLD(J)                                    SES00930
   IHLD(J)=IY                                         SES00940
   DO 25 K=1,NB                                       SES00950
   Y=X(I,K)                                           SES00960
   X(I,K)=X(J,K)                                      SES00970
25 X(J,K)=Y                                           SES00980
22 CONTINUE                                           SES00990
34 SUM=0.                                             A7  0995
   DO 28 I=1,N                                        SES01010
28 SUM=SUM+X(I,1)*SAVR(I)                             SES01020
   TEST=ABS((SAVEB-SUM)/SAVEB)                        SES01030
   IF(TEST-.00001)13,13,8                             SES01040
13 RETURN                                             SES01050
 8 E=1.                                               SES01060
   GO TO 13                                           SES01070
   END                                                SES01080
```

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APPENDIX IV

BRUCETON ITERATION PROCEDURE

When the chosen testing interval is larger than 2σ, or when the intervals are of unequal size, it is necessary to solve Equations (1) and (2) for μ and σ.

$$\Sigma \, n_i \left(\frac{z_{i-1}}{q_{i-1}} - \frac{z_i}{p_i} \right) = 0 \tag{1}$$

$$\Sigma \, n_i \left(\frac{x_{i-1} \, z_{i-1}}{q_{i-1}} - \frac{x_i \, z_i}{p_i} \right) = 0 \, . \tag{2}$$

The intervals will be of unequal size, for example, when the normalizing transformation is unknown in advance of the experiment and must be deduced from the results of the experiment itself. A method of trial and error is probably as good as any other for solving the equations. One would first choose preliminary estimates, say m and s, of the roots. These preliminary estimates would be adjusted until the equations were satisfied to the desired degree of approximation. The left side of Equation (1) will be positive when the trial value of μ is too small, and negative when it is too large. The left side of Equation (2) will be positive when $s < \hat{\sigma}$, and negative when $s > \hat{\sigma}$. Equation (1) is relatively insensitive to changes in s, while the same is true of Equation

(2) for changes in m.

In order to facilitate the computations, the accompanying tables of z/p (Table 14) and z/q (Table 15) are provided. For negative values of x, p, and q are interchanged, that is

$$\frac{z(x)}{p(x)} = \frac{z(-x)}{q(-x)} \; .$$

The computation can be illustrated by using the data of Figure 15. The normalized heights are 0.1, 0.9, 1.5, and 1.9, as indicated in the figure. The levels are numbered 0, 1, 2, and 3 beginning with the lowest level. Since there are more successes than failures, the latter are used to determine the estimates. A preliminary estimate of μ may be obtained by using the average of the midpoints of the intervals weighted by the numbers n_i; thus,

$$m_1 = 1/29 \, [2 \, (1.7) + 26 \, (1.2) + (0.5)]$$

$$\cong 1.2.$$

A rough estimate of σ may be determined by observing that the interval 0.9

NORMALIZED HEIGHT		NUMBER OF x's	0's
1.9	x x x	3	
1.5	x x x x x x x x x x 0 x x x x x x x x x x x x x x 0 x x x	27	2
0.9	0 0 0 0 0 0 0 0 0 0 0 0 0 x 0 0 0 0 0 0 0 0 0 0	1	26
0.1	0		1

Figure 15. Record of a Sample of Sixty Tests

Table 14. ... lues of z/p

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.798	0.792	0.785	0.779	0.773	0.766	0.760	0.754	0.748	0.741
0.1	0.735	0.729	0.723	0.717	0.711	0.705	0.699	0.693	0.687	0.681
0.2	0.675	0.669	0.663	0.657	0.652	0.646	0.640	0.634	0.629	0.623
0.3	0.617	0.612	0.606	0.600	0.595	0.589	0.584	0.578	0.573	0.567
0.4	0.562	0.556	0.551	0.546	0.540	0.535	0.530	0.525	0.519	0.514
0.5	0.509	0.504	0.499	0.494	0.489	0.484	0.479	0.474	0.469	0.464
0.6	0.459	0.454	0.449	0.445	0.440	0.435	0.430	0.426	0.421	0.417
0.7	0.412	0.407	0.403	0.398	0.394	0.389	0.385	0.381	0.376	0.372
0.8	0.368	0.363	0.359	0.355	0.351	0.346	0.342	0.338	0.334	0.330
0.9	0.326	0.322	0.318	0.314	0.310	0.306	0.303	0.299	0.295	0.291
1.0	0.288	0.284	0.280	0.277	0.273	0.269	0.266	0.262	0.259	0.255
1.1	0.252	0.249	0.245	0.242	0.239	0.235	0.232	0.229	0.226	0.223
1.2	0.219	0.216	0.213	0.210	0.207	0.204	0.201	0.198	0.195	0.193
1.3	0.190	0.187	0.184	0.181	0.179	0.176	0.173	0.171	0.168	0.165

Table 14. Values of z/p (Continued)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.4	0.163	0.160	0.158	0.155	0.153	0.150	0.148	0.146	0.143	0.141
1.5	0.139	0.137	0.134	0.132	0.130	0.128	0.126	0.124	0.121	0.119
1.6	0.117	0.115	0.113	0.111	0.110	0.108	0.106	0.104	0.102	0.100
1.7	0.098	0.097	0.095	0.093	0.092	0.090	0.088	0.087	0.085	0.083
1.8	0.082	0.080	0.079	0.077	0.076	0.074	0.073	0.072	0.070	0.069
1.9	0.068	0.066	0.065	0.064	0.062	0.061	0.060	0.059	0.058	0.056
2.0	0.055	0.054	0.053	0.052	0.051	0.050	0.049	0.048	0.047	0.046
2.1	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
2.2	0.036	0.035	0.034	0.034	0.033	0.	0.031	0.031	0.030	0.029
2.3	0.029	0.028	0.027	0.027	0.026	0.025	0.025	0.024	0.024	0.023
2.4	0.023	0.022	0.022	0.021	0.020	0.320	0.019	0.019	0.019	0.018
2.5	0.018	0.017	0.017	0.016	0.016	0.016	0.015	0.015	0.014	0.014
2.6	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011	0.011
2.7	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008	0.008

Table 14. Values of z/p (Concluded)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.8	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006	0.006
2.9	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005	0.005	0.005
3.0	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.003
3.1	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002
3.2	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.3	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
3.4	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.5	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.6	0.001	0.001	0.001	0.001	0.001	0.001	0.000			

Table 15. Values of z/q

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.798	0.804	0.811	0.817	0.824	0.830	0.836	0.843	0.849	0.856
0.1	0.863	0.869	0.876	0.882	0.889	0.896	0.902	0.909	0.916	0.923
0.2	0.929	0.936	0.943	0.950	0.957	0.964	0.970	0.977	0.984	0.991
0.3	0.998	1.005	1.012	1.019	1.026	1.033	1.040	1.047	1.054	1.062
0.4	1.069	1.076	1.083	1.090	1.097	1.105	1.112	1.119	1.126	1.134
0.5	1.141	1.148	1.156	1.163	1.171	1.178	1.185	1.193	1.200	1.207
0.6	1.215	1.222	1.230	1.237	1.245	1.253	1.260	1.268	1.275	1.283
0.7	1.290	1.298	1.306	1.313	1.321	1.329	1.336	1.344	1.352	1.360
0.8	1.367	1.375	1.383	1.391	1.399	1.406	1.414	1.422	1.430	1.438
0.9	1.446	1.454	1.461	1.469	1.477	1.485	1.493	1.501	1.509	1.517
1.0	1.525	1.533	1.541	1.549	1.557	1.565	1.573	1.581	1.590	1.598
1.1	1.606	1.614	1.622	1.630	1.638	1.646	1.655	1.663	1.671	1.679
1.2	1.687	1.696	1.704	1.712	1.720	1.729	1.737	1.745	1.754	1.762
1.3	1.770	1.779	1.787	1.795	1.804	1.812	1.820	1.829	1.838	1.846

Table 15. Values of z/q (Continued)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.4	1.854	1.862	1.871	1.879	1.888	1.896	1.905	1.913	1.922	1.930
1.5	1.938	1.947	1.955	1.964	1.972	1.981	1.990	1.998	2.007	2.015
1.6	2.024	2.033	2.041	2.050	2.058	2.067	2.076	2.084	2.093	2.102
1.7	2.110	2.119	2.128	2.136	2.145	2.154	2.162	2.171	2.180	2.188
1.8	2.197	2.206	2.215	2.223	2.232	2.241	2.250	2.258	2.267	2.276
1.9	2.285	2.294	2.303	2.311	2.320	2.329	2.338	2.346	2.355	2.364
2.0	2.373	2.381	2.390	2.399	2.408	2.417	2.426	2.435	2.444	2.453
2.1	2.462	2.470	2.479	2.488	2.497	2.506	2.515	2.524	2.533	2.542
2.2	2.551	2.560	2.569	2.578	2.587	2.596	2.605	2.614	2.623	2.632
2.3	2.641	2.650	2.659	2.668	2.677	2.687	2.696	2.705	2.714	2.723
2.4	2.732	2.741	2.750	2.759	2.768	2.777	2.786	2.795	2.805	2.814
2.5	2.823	2.832	2.841	2.850	2.859	2.868	2.878	2.887	2.896	2.905
2.6	2.914	2.923	2.932	2.942	2.951	2.960	2.969	2.978	2.987	2.997
2.7	3.006	3.015	3.024	3.033	3.043	3.052	3.061	3.070	3.079	3.089

Table 15. Values of z/q (Concluded)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.8	3.098	3.107	3.116	3.126	3.135	3.144	3.15?	3.163	3.172	3.181
2.9	3.190	3.200	3.209	3.218	3.227	3.237	3.246	3.255	3.265	3.274
3.0	3.283	3.292	3.302	3.311	3.320	3.330	3.339	3.348	3.358	3.367
3.1	3.376	3.386	3.395	3.404	3.413	3.423	3.432	3.441	3.451	3.460
3.2	3.470	3.479	3.488	3.498	3.507	3.516	3.526	3.535	3.544	3.554
3.3	3.563	3.573	3.582	3.591	3.601	3.610	3.620	3.629	3.638	3.648
3.4	3.657	3.667	3.676	3.685	3.695	3.704	3.714	3.723	3.732	3.742
3.5	3.751	3.761	3.770	3.780	3.789	3.799	3.808	3.817	3.827	3.836
3.6	3.846	3.855	3.865	3.874	3.884	3.893	3.902	3.912	3.992	3.931
3.7	3.940	3.950	3.959	3.969	3.978	3.988	3.997	4.007	4.016	4.026
3.8	4.035	4.045	4.054	4.064	4.073	4.083	4.092	4.102	4.111	4.121
3.9	4.130	4.140	4.149	4.159	4.169	4.178	4.188	4.197	4.206	4.216
4.0	4.226	4.235	4.245	4.254	4.264	4.273	4.283	4.292	4.302	4.312

to 1.5 appears to contain 26/29 or about 90 percent of the distribution, hence

$$1.645 \ s_1 \cong \tfrac{1}{2} \ (1.5 - 0.9) = 0.3$$

$$s_1 = 0.18$$

may be used.

In adjusting these estimates one might be tempted to adjust m_1 first by Equation (1) and then go to Equation (2) and adjust s_1 by using a good estimate of μ. It turns out, however, that the job can be done much more rapidly by considering both equations together. The following computational form may be used:

i	n_i	h_i	x_i	$n_i\left(\dfrac{z_{i-1}}{q_{i-1}} - \dfrac{z_i}{p_i}\right)$	$\dfrac{x_i z_i}{q_i}$	$\dfrac{x_i z_i}{p_i}$	$n_i\left(\dfrac{x_{i-1} z_{i-1}}{q_{i-1}} - \dfrac{x_i z_i}{p_i}\right)$
2	2	1.9	3.89	4.17		0	6.96
2	26	1.5	1.67	0.00	3.48	0.174	-9.05
1	1	0.9	-1.67	-2.08	-0.174	-3.48	3.48
0		0.1	-6.11		0		
				2.09			1.39

Note that this table is arranged so that the frequencies of either the 0's or x's will be entered in the table as though they were x's. The symbol x_i represents $\left(h_i - m_1\right)\big/s_1$ where h_i is the height and m_1 and s_1 are the first approximations to μ and σ. The other computations are defined by the column headings. Thus the figure 4.17 at the top of the fifth column is obtained as 2 (2.084 - 0.000); 2.084 being read from Table 15 at $x = 1.67$, and 0.000 being the value of z/p at $x = 3.89$, as shown by Table 14. The sums, 2.09 and 1.39, of the fifth and

eighth columns give the values of the left-hand sides of Equations (1) and (2), respectively, since both sums are positive. It may be concluded that both m_1 and s_1 are too small. By using $m_2 = 1.3$ and $s_2 = 0.19$, the above calculation is repeated.

i	n_i	h_i	x_i	$n_i\left(\dfrac{z_{i-1}}{q_{i-1}} - \dfrac{z_i}{p_i}\right)$	$-\dfrac{x_i z_i}{q_i}$	$\dfrac{x_i z_i}{p_i}$	$n_i\left(\dfrac{x_{i-1} z_{i-1}}{q_{i-1}} - \dfrac{x_i z_i}{p_i}\right)$
3	2	1.9	3.16	3.12		0.01	3.28
3	26	1.5	1.05	-5.85	1.64	0.282	-9.75
1	1	0.9	-2.11	-2.47	-0.093	-5.21	5.21
0		0.1	-6.32		0		
				5.20			-1.26

These results show that the roots are bracketed, and good estimates of μ and σ may be obtained by interpolation between the sums. Interpolating between 1.2 and 1.3 using 2.09, 0, and -5.20, one finds that $m_3 = 1.23$, similarly $s_3 = 0.185$. By doing two more calculations similar to the two illustrated above, one would verify the third figures in m and s and obtain good estimates for the fourth figures. Here, the results to three figures are m = 1.21 and s = 0.187. However, the data do not warrant any more accuracy in the roots than is given by m_2 and s_2, and one would not do the two extra computations.

APPENDIX V

STATISTICAL ANALYSIS

The simulation work described previously in this document was planned and executed under very closely controlled conditions. The specific random normal numbers selected for the 100 samples for each of the nine basic test conditions were generated such that it was possible to repeat them exactly for any subsequent retesting. Not only did this approach eliminate any simulation effects which vary with time, but it also simplified the analysis of subsequent test techniques. Thus, the initial simulations were run with essentially three test techniques: Normal, Bruceton, and Missile. Subsequently the Improved Bruceton technique was simulated over the same nine test conditions, and was combined with the earlier simulation work for analysis as a factorial experiment.

1. Analysis of the Estimated Means

The estimated means provided by the different techniques were analyzed by analysis of variance. The experimental design was a 3 × 3 × 3 factorial with test technique (T), population (P), and sample size (S) as the three factors. The levels of all factors were fixed; two factors (population and sample size) were quantitative and the technique factor was qualitative. The three techniques

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analyzed were the Bruceton, Missile and Improved Bruceton. The so-called
Normal technique was not included since it is not a sensitivity testing technique,
but only a data analysis procedure.

The experimental design arrangement and data for analysis are shown
in Table 16. The response variable selected and tabulated in the table is $\overline{X}_{\overline{X}}$
from Tables 6 (Bruceton and Missile) and 12 (Improved Bruceton). This
variable is the mean of the 100 sample means from each of the nine test condi-
tions. To simplify analysis, the data from Tables 6 and 12 were coded by first
subtracting _.00000 from e?.ch $\overline{X}_{\overline{X}}$, and then multiplying the result by 100,000.
The coded data are shown in Table 16.

Table 16.   Coded Simulation Values of $\overline{X}_{\overline{X}}$

| | Sample | Technique | | |
	Size	Bruceton	Missile	Improved Bruceton
No. 1 $\mu = 1.0,$ $\sigma = 0.02$	15	-876	+406	+239
	35	-1854	+5	+127
	55	-1760	+21	+70
No. 2 $\mu = 1.0,$ $\sigma = 0.09$	15	-3869	+155	-50
	35	-6892	-186	-189
	55	-5988	-185	+67
No. 3 $\mu = 1.0,$ $\sigma = 0.26$	15	-4402	-1292	-498
	35	-7303	-331	+885
	55	+920	+437	+1312
	$T_{.j}$	-32,024	-970	-1963
	$\overline{X}_{.j}$	-3558	-108	+218

(Population is labeled along the left side spanning all rows)

Coded $\overline{X}_{\overline{X}} = \left(\overline{X}_{\overline{X}} - 1.00000\right) 10^5$

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The mathematical model for the experimental design and analysis is given as

$$X_{ijk} = \mu + T_i + S_j + TS_{ij} + P_k + PT_{ki} + PS_{kj} + e_{ijk} \;.$$

In this model

X_{ijk} = observations on the response variable $\overline{X}_{\overline{x}}$

μ = a common eff ct in all observations

T_i = the testiing technique effect where i = 1, 2, 3

S_j = sample size effect where j = 1, 2, 3

P_k = population effect where k = 1, 2, 3

e_{ijk} = the random experimental error.

The other terms represent the interactions between the main factors T, S, and P.

The results of the analysis of variance calculations are shown in Table 17. Two of the three factors were found to be significant. Technique was found to be highly significant, the F test resulting in significance at a confidence level exceeding 99 percent. A similar test of the population effect revealed significance at the 90 percent confidence level.

Multiple-range tests of both factors were conducted according to Duncan's procedure.[26] It was found that the means of the Improved Bruceton and Missile techniques were both significantly different from (at the 99-percent confidence level) and better than the Bruceton technique. Also, no significant difference was detected between the Improved Bruceton and Missile techniques. Thus, the Improved Bruceton and Missile techniques give essentially the same

Table 17. Analysis of Variance of Simulation $\overline{X}_{\overline{X}}$ Data

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	$\hat{F}$
Technique (T)	2	78, 817, 492	39, 408, 746	24.69*
Population (P)	2	10, 148, 468	5, 074, 234	3.18**
Sample size (S)	2	6, 286, 428	3, 143, 214	1.97
T × P interaction	4	15, 954, 136	3, 988, 534	2.50
T × S interaction	4	9, 888, 806	2, 472, 201	1.55
P × S interaction	4	14, 927, 324	3, 731, 831	2.34
e_{ijk} - error	8	12, 770, 064	1, 596, 258	--
	26	148, 792, 718		

*Significant at 99-percent confidence level.

**Significant at 90-percent confidence level.

results for estimating the means, and both techniques are significantly better than the Bruceton technique.

Multiple-range tests of the population effect revealed that there was no significant difference (at the 95-percent confidence level) between the estimates of the means from populations No. 1 and 3, and populations No. 2 and 3. There was a significant difference detected between estimates from populations No. 1 and 2. These results indicate that the population effect is nonlinear since the estimates from populations No. 1 and 3 ($\sigma = 0.02$ and $\sigma = 0.26$, respectively) are more accurate than estimates from population No. 2 ($\sigma = 0.09$). This effect is probably due to not controlling the mean of the samples from the

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$\sigma = 0.09$ population closely enough to the desired value of $\mu = 1.00$.

These results indicate that the most accurate mean estimates will result when either the Improved Bruceton or Missile technique is used, and further, that accuracy will be greatest from populations of $\mu = 1.00$, $\sigma = 0.02$; $\mu = 1.00$, $\sigma = 0.26$; and $\mu = 1.00$, $\sigma = 0.09$, accuracy decreasing in that order.

To further evaluate the mean estimates, the standard deviations of these estimates $\left(S_{\bar{x}}\right)$ were analyzed by the sign test as described by Dixon and Massey.[27] The hypotheses listed below were tested:

$$H_0: \ \mu_{S(M)} = \mu_{S(N)} \qquad H_1: \ \mu_{S(M)} \neq \mu_{S(N)}$$

$$H_0: \ \mu_{S(I)} = \mu_{S(N)} \qquad H_1: \ \mu_{S(I)} \neq \mu_{S(N)}$$

$$H_0: \ \mu_{S(B)} = \mu_{S(M)} \qquad H_1: \ \mu_{S(B)} \neq \mu_{S(M)}$$

$$H_0: \ \mu_{S(B)} = \mu_{S(I)} \qquad H_1: \ \mu_{S(B)} \neq \mu_{S(I)}$$

$$H_0: \ \mu_{S(I)} = \mu_{S(M)} \qquad H_1: \ \mu_{S(I)} \neq \mu_{S(M)} \ ,$$

where the expression, $\mu_{S(M)}$, refers to the mean of the standard deviations of the Missile technique. The subscripts I, B, and N refer to the Improved Bruceton, Bruceton, and Normal estimates, respectively. Only the last null hypothesis listed above was accepted; the other four were rejected at the 99-percent confidence level. Inspection of the values of $\mu_{S(B)}$, $\mu_{S(M)}$, and $\mu_{S(I)}$ showed that $\mu_{S(B)}$ was the largest, indicating that the Improved Bruceton and Missile techniques are both better than the Bruceton.

It is concluded that the standard deviations of the mean estimates of the Missile and Improved Bruceton techniques are equivalent, and both are better (i.e., smaller) than the Bruceton mean estimate standard deviations.

2. Analysis of the Estimated Standard Deviations

The distributions of the estimated standard deviations of the three techniques are not known, and therefore only distribution-free tests were applied in the analysis of these estimates. Thus, sign tests of the values of $\overline{X}_s$ from Table 6 were conducted. Since the standard deviation estimating procedures of the Improved Bruceton and Bruceton techniques are identical, only a comparison between the Bruceton and Missile techniques was conducted. The hypothesis tested was

$$H_0: \ \mu_{S(B)} = \mu_{S(M)} \qquad H_1: \ \mu_{S(B)} \neq \mu_{S(M)} \ .$$

The null hypothesis was rejected at the 95-percent confidence level, and it was concluded that the mean of the Bruceton standard deviation estimates is significantly different from the mean of the Missile estimates. This hypothesis test did not give any information as to which of the two techniques is the most accurate, and further testing was necessary to complete the evaluation. The following additional hypothesis test on the means was conducted:

$$H_0: \ \mu_{S(M)} = \mu_{S(N)} \qquad H_1: \ \mu_{S(M)} \neq \mu_{S(N)} \ .$$

The null hypothesis was accepted at the 95-percent confidence level. Hypothesis tests of the mean standard deviation of the standard deviation estimates $\left(S_s \right)$ were also conducted. These were as follows:

$$H_0: \ \mu_{SS(M)} = \mu_{SS(N)} \qquad H_1: \ \mu_{SS(M)} \neq \mu_{SS(N)}$$

$$H_0: \ \mu_{SS(B)} = \mu_{SS(N)} \qquad H_1: \ \mu_{SS(B)} \neq \mu_{SS(N)}$$

$$H_0: \ \mu_{SS(M)} = \mu_{SS(B)} \qquad H_1: \ \mu_{SS(M)} \neq \mu_{SS(B)} \ .$$

All three null hypotheses were rejected at the 95-percent confidence level.

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Inspection of the values of $\mu_{SS(B)}$ and $\mu_{SS(M)}$ showed that $\mu_{SS(B)}$ is the smallest, and it is concluded that the Bruceton technique provides standard deviation estimates with the best repeatability (i.e., smallest variation).

In summary, the sign tests of the standard deviation estimates of the three techniques were inconclusive. The tests indicated that the means of the Missile estimates are better but that the standard deviations of the Missile estimates are not as good as the Bruceton (and Improved Bruceton).

3. Analysis of the Correlation Coefficients

The values of TAUB and TAUM tabulated in Table 8 are estimates of the true correlation coefficients ρ_B and ρ_M, respectively. Analysis of these data consisted of making the following hypothesis tests at each of the nine test conditions:

$$H_0: \rho_B = 0 \qquad H_1: \rho_B > 0$$

$$H_0: \rho_M = 0 \qquad H_1: \rho_M > 0 .$$

Similar tests were made with combined or mean estimates at the different values of sample size and for the different populations. All tests were made at a confidence level of 90 percent according to the procedures described by Dixon and Massey.[27] These procedures make use of the fact that the variable

$$Z = (0.5) \ln (1 + \gamma/1 - \gamma)$$

is normally distributed with mean and standard deviation

$$\mu_Z = (0.5) \ln (1 + \rho/1 - \rho)$$

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$$\sigma_Z = 1/\sqrt{N - 3} \ ,$$

where γ is the estimate of ρ.

The results of this analysis are indicated in Table 8. A "yes" in the column headed ' Significant at 90-Percent Confidence" indicates that the null hypothesis is rejected and the conclusion is made that there is positive correlation. A "no" indicates no correlation between the Bruceton (or Missile) estimates of standard deviation and the corresponding Normal estimate, i.e., $\rho = 0$. Standard deviation estimates at these test conditions (where $\rho = 0$) are unacceptable and these particular test conditions should be avoided.

LIST OF REFERENCES

1. D. Finney, Statistical Methods in Biological Assay, 2d ed., New York, Hafner Publishing Co., 1964.

2. F. Fedowitz, Jr., Sensitivity of Explosives and Propellants to Impact and Friction, Explosive Safety Executive Lecture Series, Cocoa Beach, Florida, TRW Systems, June 1965, pp. 4-1, 4-21.

3. B. Baber et al., Lubrication Research and Test Method Development, A Report to the U. S. Dept. of the Air Force, Air Force Systems Command, Prepared by the Southwest Research Institute under U. S. Air Force Contract AF 33 (657)-11088, San Antonio, Texas, November 1963.

4. U. S. Dept. of the Navy, Card Gap Test for Shock Sensitivity of Liquid Propellants, Prepared by the Liquid Propellant Research Agency, Johns Hopkins University under U. S. Navy Contract N Ord 7386, Silver Spring, Md., March 1960.

5. J. Savitt, A Sensitivity Test for Castable Liquid Explosives Including Results for Some New Materials, Report NAVORD 2997, U. S. Department of the Navy, U. S. Naval Ordnance Laboratory, White Oak, Md., October 1953.

6. U. S. Dept. of the Navy, Critical Diameter and Detonation Velocity Test, Prepared by the Liquid Propellant Research Agency, Johns Hopkins University under U. S. Navy Contract N Ord 7386, Silver Spring, Md., May 1964.

7. J. Ayres, L. Hampton, and I. Kabik, The Prediction of Very-Low EED Firing Probabilities, Report NOLTR 63-133, U. S. Dept. of the Navy, U. S. Naval Ordnance Laboratory, White Oak, Maryland, September 1963.

8. R. Lusser, Reliability Through Safety Margins, U. S. Dept. of the Army, Army Rocket and Guided Missile Agency, Redstone Arsenal, Alabama, October 1958.

9. H. Culling, Statistical Methods Appropriate for Evaluation of Fuze Explosive-Train Safety and Reliability, Report NAVORD 2101, U. S. Department of the Navy, U. S. Naval Ordnance Laboratory, White Oak, Md., October 1953.

10. M. Natrella, Experimental Statistics, U. S. Dept. of Commerce, National Bureau of Standards Handbook 91, Washington, U. S. Government Printing Office, August 1963.

11. H. Robbins and S. Munro, A Stochastic Approximation Method, Annals of Mathematical Statistics, Vol. 22 (1951), pp. 400-407.

12. C. Bliss, The Method of Probits, Science, Vol. 79 (1934), pp. 38-39.

13. D. Finney, Probit Analysis: A Statistical Treatment of the Sigmoid Response Curve, 2d ed., London, Cambridge University Press, 1952.

14. Statistical Research Group, Princeton University, Statistical Analysis for a New Procedure in Sensitivity Experiments, A Report to the Applied Mathematics Panel, National Defense Research Committee, U. S. War Department, AMP Report 101.1R, July 1944.

15. W. Dixon and A. Mood, A Method for Obtaining and Analyzing Sensitivity Data, Amer. Stat. Assoc. J., Vol. 43 (1948), pp. 109-126.

16. J. Gayle, Computer Simulation Study of Bruceton and Probit Methods of Sensitivity Testing, Proceedings of the Ninth Conference on the Design of Experiments in Army Research Development and Testing. ARO-D Report 64-2, U. S. Dept. of the Army, U. S. Army Research Office, Durham, North Carolina, December 1964, pp. 205-227.

17. H. Langlie, A Reliability Test Method for "One-Shot" Items, A Report to the U. S. Dept. of the Army, U. S. Army Ordnance Missile Command, Prepared by the Aeronutronic Division of Ford Motor Company under U. S. Army Contract DA-04-495-ORD-1835, Newport Beach, California, August 1962.

18. G. Svadeba, Impact Sensitivity of Primary Explosives, Report NAVORD 2647, U. S. Dept. of the Navy, U. S. Naval Ordnance Laboratory, White Oak, Md., November 1952.

19. H. Grover, S. Gordon, and L. Jackson, Fatigue of Metals and Structures, Prepared by the Battelle Memorial Institute for the U. S. Dept. of the Navy, Washington, U. S. Government Printing Office, 1954.

20. J. Gayle and C. Hopkins, Simulation Study of the Amount of Sensitivity Test Data Required to Reject the Hypothesis of Normality when the Sample Population is Nonnormal, NASA TM X-53357, George C. Marshall Space Flight Center, Huntsville, Ala., November 1965.

21. L. Hampton, Monte Carlo Investigations of Small Sample Bruceton Tests, Report NOLTR 66-117, U. S. Dept. of the Navy, U. S. Naval Ordnance Laboratory, White Oak, Maryland, February 1967.

22. G. Wetherill, Sequential Estimation of Quantal Response Curves, Journal Royal Statistical Society, Vol. B25 (1963), pp. 1-48.

23. G. Wetherill, H. Chen, and R. Vasudeva, Sequential Estimation of Quantal Response Curve; A New Method of Estimation, Biometrika, Vol. 53 (1966), pp. 439-454.

24. P. Hoel, Introduction to Mathematical Statistics, 3rd ed., John Wiley and Sons, Inc., New York, 1965.

25. M. Friedman, Chapter 11, Planning an Experiment for Estimating the Mean and Standard Deviation of a Normal Distribution from Observations on the Cumulative Distribution, ed. C. Eisenhart, M. Hastay, and W. Wallis, McGraw-Hill Book Company, Inc., New York, 1947, pp. 329-352.

26. D. Duncan, Multiple Range and Multiple F Tests, Biometrics, Vol. 11 (1956) pp. 1-42.

27. W. Dixon and F. Massey, Introduction to Statistical Analysis, 2nd ed., McGraw-Hill Book Company, Inc., New York, 1957, pp. 123-124, 200-201.

GLOSSARY

Bruceton iterative procedure — one of three procedures available for calculat-
ing estimates of μ and σ from Bruceton test data. It is not the procedure
described for the standard Bruceton technique. The procedure is of an
iterative type in that initial values are chosen and manipulated through
the mathematical procedures and more accurate values obtained. This
may be continued until the change between successive iterations is as
small as desired. This procedure is described in detail in Appendix IV.

Comparison simulations — the simulation effort described in Chapter III in
which tests at the nine basic test conditions shown in Table 1 were sim-
ulated with the standard Bruceton and Missile techniques.

Degenerate solution — under certain conditions estimates of the standard devia-
tion from test data generated by the Missile technique assume negative
values. These negative solutions for σ are called "degenerate solutions"
by Langlie, [17] and he states that when such solutions occur, the value of
$\sigma = 0$ should be used.

Improved Bruceton technique — a sensitivity test technique which combines the
standard Bruceton test procedure, standard Bruceton estimates of σ,
and iterative (as opposed to the standard) Bruceton mean estimates into
a single overall technique. This technique is the same as the standard

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Bruceton except that an iterative procedure is substituted for estimating the mean.

Missile iterative procedure — a procedure by which the test data generated by the Missile technique are analyzed and estimates of μ and σ are calculated. The procedure is of the iterative type and is the only procedure used with the Missile technique.

Re-Test — Simulation effort at one of the nine specific test conditions other than the comparison simulations. Re-tests are done primarily to evaluate changes of the standard sensitivity test techniques.

Sample — a group of random Normal numbers generated by the simulation program to represent one-shot hardware undergoing sensitivity test. The number of random Normal numbers in the sample (i.e., the sample size) is specified by the particular test condition. The "items" in a sample are also referred to as "test items."

Simulation run size — the number of samples of random Normal numbers "tested" during a particular simulation effort.

Test or simulated test — the activity of comparing an individual random Normal number of a sample to a test level generated by one of the sensitivity test techniques, and noting whether the number is greater than, less than, or equal to the test level. If the number is greater than or equal to the test level, the test is scored a success, otherwise the test is scored a failure.

Test level — a specific value of the test stimulus or environment, as determined by the sensitivity test technique, at which a particular test item is

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tested.   Test levels are represented in the simulations by appropriately

derived numbers.

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13. ABSTRACT

 Sensitivity testing as applied to one-shot items consists of subjecting individual one-shot test items to discrete levels of a test environment, operating the test item and noting whether or not the item functions properly. From this type of test a direct measurement of the strength of the test item is not obtained, only the information that the strength is either greater than or less than the test level used. In order to obtain more meaningful information from these "quantal response" data it is necessary to use special techniques or procedures both to conduct the tests and to analyze the data.

 This investigation was directed toward testing one-shot items associated with Army missiles, e.g., thermal batteries, squibs, explosive, etc. The specific objectives of the investigations were (1) to evaluate available techniques for conducting sensitivity tests of one-shot items, (2) to improve existing techniques or develop an improved one, (3) to identify the best technique, and (4) to identify and evaluate any limitations of the best technique as identified from (3) above.

 The approach taken was to simulate sensitivity testing with the different techniques on a digital computer. A simulation program was developed and the different techniques programmed for testing under closeby controlled conditions. In this way it was possible to control all inputs, providing accurate data for comparison and evaluation of the testing techniques.

 The Bruceton or Up and Down technique, and a technique developed by an Army missile contractor referred to as the missile technique, were the two techniques given primary

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ABSTRACT (Concluded)

consideration. The investigations revealed that the missile was superior to the standard Bruceton in providing estimates of the mean failure strength of test items. The missile technique estimates were from 1 to 32 percent more efficient than the Bruceton. Other characteristics of both techniques were evaluated and specific limitations of each technique identified.