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DEPARTMENT OF THE ARMY
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2531

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Author: Paul Lorenz, Berlin

Title: Variance analysis and range estimation. Two methods for the estimation of the differences between the mean values of several random samples (Varianzanalyse und Spielraumschatzung. Zwei Methoden zur Beurteilung der Unterschiede zwischen den Mittelwerten mehrerer Stichproben).

Journal: Allgemeines Statistisches Archiv 52: 245-258 (1968).

August 1969

The original form of the analysis of variance has been altered extensively in the course of time by numerous authors. However, not all of the transformations and deriviations have been acknowledged by the other theorists. Simultaneously, the analysis of variance has been burdened by a series of new ideas. Fisher's discovery concerning the range estimation was originally nothing more than a rapid method for obtaining a summary answer to the question of whether or not two "significant differences" existed between several sries of measurements.

The "range estimation" described in the following is not a derivative of the variance analysis but rather arises from another basic source.

A. THE PROBLEM

A known problem, to whose solution one is accoustomed to applying the aid of mathematical statistics is as follows: Concerning any state, several series of measurements (random measurements), which vary from each other, are displayed. These each occur under somewhat different conditions; in this connection one assumes that a biologist carries out a given number of experiments several times successively and obtains results which are different by a factor that is considered as random. The question now reads: are the differences between the series to be compared actually significant differences or can they be explained as the effect of chance? Usually one is satisfied with answering the question: if the differences between the mean values of the random samples are considered, do at least two of the averages, or perhaps more or even all, differ significantly from each other? The answer cannot be given with certainity. It should possess, however, at least a high degree of probability.

My idea is bent closely to a numerical example with r= 5 series of measurements each with n= 10 measurements, which one can represent as random samples from five basic generalities.,

m_{ij}	j-th measurement of the i-th sample
$M_i = \frac{1}{n}\sum_{j=1}^{n} m_{ij}$	mean value of the i-th sample
$\sigma_i^2 = \frac{1}{n}\sum_{j=1}^{n} (m_{ij} - M_i)^2$	variance of the i-th sample
$M = \frac{1}{r}\sum_{i=1}^{r} M_i$	mean of the means of the r-series of measurements and with that also of the whole "field".
$\sigma^2(M_i) = \frac{1}{r}\sum_{i=1}^{r} (M_i - M)^2$	Variance of the ranges of the samples-means

The corresponding parameters of the basic generalities are characterized by an additional, lowered U.

<u>Numerical Example r= 5 Samples</u>

	m_{1j}	m_{2j}	m_{3j}	m_{4j}	m_{5j}
	3	3	4	6	2
	5	4	6	5	3
	4	5	5	7	5
	5	6	5	5	4
	2	4	6	4	4
	2	3	4	5	4
	3	6	4	7	5
		4	4	8	6
	2	4	4	7	3
	3	4	5	5	4
n	10	10	10	10	10
M_i	3.3	4.3	4.7	5.9	4.0
σ_i^2	1.21	1.01	0.61	1.49	1.20

$M = 4.44 \qquad \sigma(M_i) = 0.7424$

For the solution of the problem which has been supplied, one can construct various algorithms. Here, first of all, the algorithm of the analysis of variance should be discussed briefly and then using its nucleus, variance analysis in its simplest form should be investigated. After that, a procedure for range estimation (the method of scopes) will be presented.

B. VARIANCE ANALYSIS

1. The Practical Procedure

Originally, variance analysis was a kind of rapid procedure for obtaining a summary answer to the question as to whether or not there was at least two "significant differences" between several series of measurements. In this case, it is a question of distinquishing two series of measurements as being significant if the calculations showed that they did not originate from a basic generality with a given degree of probability. One does not require higher mathematics for this and the use of the procedures employed in this connection can be easily learned. To be sure, initially there may be some difficulty in learning the techniques. However, after one has calculated a couple of examples, there is usually little difficulty and one sees how every thing fits together -"between the groups", "within the group", and "complete". Also, one begins to understand how the somewhat mysterious "degrees of freedom" fit into the calculations. As a result, most of the difficulties disappear and the table of F-distributions become as easy to employ as the the logarithm tables.

The procedure has been outlined with the aid of the example given earlier. One first of all calculates from the $r \cdot n = 50$ random samples the following four magnitudes:

(a) The n-fold summation of the squares of the differences between the group mean values and the general mean[1].

$$n \sum_{i=1}^{r} (M_i - M)^2 = 37.12$$

The designation usually used here is the "sum of the squares of the differences between groups". This is certainly impressive but not correct.

(b) The sum of the squares of the differences of measurements from their group averages, in short: "the sum of the squares of the differences within groups"

$$\sum_{i=1}^{r} \sum_{j=1}^{n} (m_{ij} - M_i)^2 = 55.20$$

(c) The number of "degrees of freedom between the groups":

$$r - 1 = 4$$

(d) The number of "degrees of freedom within the groups":

$$r(n-1) = 45$$

From these magnitidues, one can calculate two additional ones:

(e) The "average square of the differences between the groups"

$$\frac{n \sum_{i=1}^{r} (M_i - M)^2}{r-1} = \frac{37.12}{4} = 9.28$$

and

(f) The "average square of the differences within the groups"

$$\frac{\sum_{i=1}^{r} \sum_{j=1}^{n} (m_{ij} - M_i)^2}{r(n-1)} = \frac{55.20}{45} = 1.227$$

1. SEE NEXT PAGE (bottom)

Algorithms of Variance Analysis

	Sums of the squares ofthe differences	degrees of freedom	Average square of the differences
between the groups	$a) = n\sum_{i=1}^{r}(M_i - M)^2 = 37.12$	$c) = r - 1 = 4$	$\frac{a)}{c)} = \frac{37.12}{4} = 9.28$
within the groups	$b) = \sum_{i=1}^{r}\sum_{j=1}^{n}(m_{ij} - M_i)^2 = 55.20$	$d) = r(n-1) = 45$	$\frac{b)}{d)} = \frac{55.20}{45} = 1.227$

As a rule, these algorithms occur with one more row. This row is referred to by the name "complete"; "incomplete", or "total". It does not have any significance in the case of analysis of variance and serves only for control of calculations or for simplification of calculations.

In conclusion, one calculates the quotient between both of "the average squares of the differences"

$$F = \frac{9.28}{1.227} = 7.56$$

and compares it with the number of numbers on an F-table at the crossing of the "degrees of freedom". One finds for example at the level of 1 % certainity 3.78 and concludes that at least two of the groups are "very significantly different". Concerning which of the groups are these, the variance analysis says nothing. Also, it does not tell whether all or only a few of the groups differ significantly.

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1. In the case of groups of varying sizes, it must be reported: the sum obtained for the squares of the differences of the group average and the overall average.

It would naturally not fail to appear that mathematics would assume the problematics of variance analysis. The first test no doubt was that of M.S. Bartlett. He was concerned with the hypothesis that the variance of basic generalities should be the same. Later a whole series followed and an entire science around variance analysis resulted. The theorist, however, cannot be persuaded that the last tests of knowledge are the end.

Where is the difficulty then? To me the cause appears to be found in the following: If one wishes to investigate whether statistical progressions are uniform in the sense that they can arise from one and the same basic generality, then it is first necessary that no average in the series can vary widely from the common average and next one begins with observations of the mean value instead of with observations of the variances of the series and the variance of the mean values.

2. The Intellectual Nucleus of Variance Analysis

It is necessary to begin the explanation with a warning, and of a misinterpretation of the following theorems.

The variance of the fields is equal to the sum of the average variance of the series and the variance of the average values of the series

$$\frac{\sum_{i=1}^{r}\sum_{j=1}^{n}(m_{ij}-M)^2}{r\cdot n} = \frac{\sum_{i=1}^{r}\frac{1}{n}\sum_{j=1}^{n}(m_{ij}-M_i)^2}{r} + \frac{\sum_{i=1}^{r}(M_i-M)^2}{r} \tag{1}$$

or

$$\sigma^2(m_{ij}) = \frac{1}{r}\sum\sigma_i^2 + \sigma^2(M_i)$$

and specifically in our example:

$$\frac{92.320}{5\times 10} = \frac{5.520}{5} + \frac{3.712}{5}$$

or

$$1.8464 = 1.1040 + 0.7424$$

This equation, which can be generalized further[2], and can be used in suitable form for the basic generalities, is an algebraic identity; it stands always. It was used by W. Lexis in the formation of his "theory on the essential and nonessential variation components" and perhaps gave rise to the construction of variance analysis. However, it has nothing to do with the formation of basic generalities and random samples and particularly the relationship between them. Therefore, it can also not be the fundamental of variance analysis, although this utilizes the last and the next to the last part of equation (1); it utilizes these parts in a manner which is foreign to the Lexisian equation. The variance analysis interprets both parts as random sample numbers and sees in the coorresponding basic generality, into which the random samples are introduced, the real research objective. Since these, however, are only products of fantasy, they must be somehow "materialized".

This occurs fundamentally in a very primative manner, namely, by introduction and abundament use of the idea of "expected values" which are already somewhat misleading by their name. On the basis of this, one calculates whether the mean value of the basic generality is identical to the mean value of the samples and secondly whether one obtains the variance of the basic generality. Then one divides the variance of the samples by the so-called "degrees of freedom."

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[2] P. Lorenz: Anschauungsunterricht in Mathematischer Statisk, Vol. II/I Der Schluss vom Teil aufs Ganze, Leipzig, 1959, page 134.

The worst part of the introduction of the "expectation value" (symbolized by E) is that one cannot get rid of it again. Actually, the quotient, which leads to the quantity F, is as follows:

$$n E\left[\frac{\sum_{i=1}^{r}(M_i - M)^2}{r-1}\right] \Big/ E\left[\frac{\sum_{i=1}^{r}\sum_{j=1}^{n}(m_{ij} - M_i)^2}{r(n-1)}\right]$$

However, the numerators and denominators of this quotient are and remain unknown quantities, that is, one is not able to calculate them. For that reason, a power process[3] is performed and the operator E is taken out. As a result, one arrives at the following quantity:

$$F = \frac{n\sum_{i=1}^{r}(M_i - M)^2}{r-1} \Big/ \frac{\sum_{i=1}^{r}\sum_{j=1}^{n}(m_{ij} - M_i)^2}{r(n-1)}$$

In the example $9.28/1.227 = 7.56$

As a result it is left over from the apparently clever contrivance as a decision on the basis of two easily motivated "point estimations".

As a consequence of the above form of the variance analysis, it is not necessary to employ F as an estimated value. Instead of this, one returns to the question and is content with F as a test quantity. That is, an hypothesis is tested which reads as follows: it exists r series of measurements and it is assumed that they can arise from one and the same basic generality.

In order to be able to affirm or deny this hypothesis, the F value is calculated according th the analysis of variance and is compared with the value in the F-table at the corresponding number of degrees of freedom.

3. See P. Lorenz, already cited, p. 205.

1st case: F is greater than the table value. The hypothesis is not valid. At least two of the series of measurements appear to be significantly different.

2nd case: F is less than the value in the table. The hypothesis is valid. It is possible that all of the series of measurements were derived from the same basic generality.

In this form, the results are no longer objectionable but are still very informative. At best, one can lear whether at least two basic generalities are significantly (or very significantly) different from each other. One cannot tell, however, which one these are and how great the degree of difference is. How slightly effective the test quantity F is often is shown up to a certain degree in the values of the table of the F-distributions. In the case of a small number of random samples compared to each other, about r samples between 3 and 6, the value for F can be very great. (In the case of a very largen, it is greater than 3 if the degree of certainty is at least 99 %.) One will consequently frequently arrive only then at a rejection of the homogeneity hypothesis if these clearly becomme evident. If r on the other hand is greater than 6, then it signifies nothing to know only that at least two of the mean values are significantly different from each other.

A possible expedient is presented by the return from a test estimation to an estimation procedure, however, not in the form of a point estimation but rather an interval estimation. This procedure was described by Erna Weber in the fifth edition of his "Outlines"[4].

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[4] E. Weber: Outlines of Biological Statistics (Grundriss der biologischen Statisk) 5th edition, Jena, 1964, page 170.

Although the results obtained from this procedure can be openly predicted, the question still clearly remains as to which basic generalities are different from each other and to what extent when a series of measurements have been demonstrated as not being homogeneous. In these ways, one can study with the help of the t-test the collective differences between two random sample averages in order to verify the significance. That requires $\binom{r}{2}$ of such tests. Additional sufficient information, however, will allow the estimation of the ranging which is fundamentally more precise for the interesting mean values themselves. This can also be found on the basis of the t-distribution but in another way which does not take into consideration the degrees of freedom making the determination much clear and easier. This method is taken up during the second part of this contribution.

C. A PROCEDURE FOR RANGE ESTIMATION

This is a question of the direct estimation of the ranges for the mean values M_{Ui} of the basic generalities. I have published the procedure several times, most recently in a brochure entitled: "Significantly different? decision by means of estimated ranging".[5] The brochure contains the bases and explanations of methods for the following situations:

(a) Simply organized statistics

(b) Multiple organized statistics

(c) Multiple organized statistics with importance

(d) Co-variance analysis

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[5] Prof. Paul Lorenz, 1 Berlin 38, Kaiserstuhlstrasse 21.

For reasons of space, it is not possible to repeat all the details here in the bases of these procedures. Likewise, the explanations for the procedures used in situations (b), (c), and (d) must be left out. The procedure shown here for situation (a) is valid for series of observations of equal as well as unequal length. It will first of all be employed in the example given at the beginning of this paper. The practical procedure can be sketched as follows: From the summary on page 246, we assume the the group averages M_i and the group variances σ_i^2 and calculate the σ_i . Then we must determine the "projected probability" $\ddot{u}$. What this stands for will be clarified by the calculations. For example, we select for both limits of the estimated range of M_{Ui} , the lower and the upper - $\ddot{U}$, and to be sure, $\ddot{U} = 0.01$. For this $\ddot{u}$, we find in the table of (see brochure) the appropriate τ when $n = 10$. In this case, τ is 0.941. The estimated limits for the $r = 5$ basic generality mean value M_{Ui} always extends from $M_i - \tau\sigma_i$ to $M_i + \tau\sigma_i$.

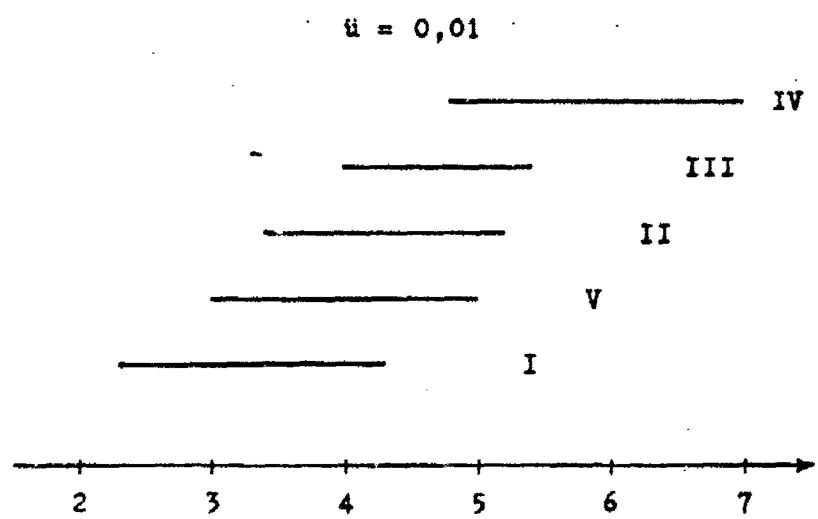
<u>NUMERICAL EXAMPLE FROM PAGE 246</u>

SAMPLE	I	II	III	IV	V
M_i	3.3	4.3	4.7	5.9	4.0
σ_i	1.100	1.005	0.781	1.221	1.095
$\tau\sigma_i$	1.0	0.9	0.7	1.1	1.0
$M_i + \tau\sigma_i$	4.3	5.2	5.4	7.0	5.0
$M_i - \tau\sigma_i$	2.3	3.4	4.0	4.8	3.0

The estimated ranges found are shown in Figure 1. Let us consider for example the estimated limits for M_{U1}. It extends from

from 2.3 to 4.3. In this range, we may presume the mean value of the first basic generality with a degree of certainity of 98 %. The probability is 0.01 that M_{U1} lies within the interval calculated for the random samples. It is to be noted that one can determine different expected probabilities u for both of the estimated limits[6]. That is expedient in many cases.

FIGURE 1
ESTIMATED LIMITS FOR THE MEAN VALUES OF BASIC GENERALITIES ON THE BASIS OF SAMPLES



Then we examine the estimated range for the M_{Ui} of the basic generalities I and V simultaneously. There are partially overlapped. That signifies there is a range in which according to the requirements made on the degree of probability M_{UI} and M_{UV} are suspected. (In the case of the choice ü = 0.01, as here, the expression "very significant" should be used; in the case of the choice ü = 0.05, the expression "significant" should be employed. Of the 5 estimated ranges for the M_{Ui}, only those for the basic generalities I and IV

[6] The so-called error probability can remain undebated in this paper.

are quite separated from each other. Only these two basic generalities are "very significantly" different. This statement is in agreement with that arrived at by variance analysis. It extends, however, much further as one can see.

For the illustration of these procedures, an additional example can be employed. The data are from the textbood of Arthur Linder who was the first to demonstrate variance analysis[7]. We wish to show these results here first. These concern the results of feeding experiments with rats with seven types of feed and a period of observation of 60 days.

FINAL WEIGHT OF 41 RATS

(in grams)

	I	II	III	IV	V	VI	VII
	m_{1j}	m_{2j}	m_{3j}	m_{4j}	m_{5j}	m_{6j}	m_{7j}
	119	123	130	144	159	139	156
	90	121	163	172	172	146	183
	102	159	159	165	210	161	146
	85	138	140	143	171	149	169
	113	178	121	179	232	124	147
	136	138	142	146	190	137	.
n_i	6	6	6	6	6	6	5
M_i	107.5	142.8	142.5	158.2	189.0	142.7	160.2
σ^2_i	302.92	402.47	219.58	208.47	634.00	130.22	198.16
Σm	645	857	855	949	1134	856	801
Σm^2	71155	124823	123155	151351	213130	122904	129311

$M = 148.7$ $\sigma^2(M_i) = 3112$

7 A. Linder: "Statistical Methods" (Statistische Methoden) 4th edition, page 110.

(1) TREATMENT BY MEANS OF VARIANCE ANALYSIS

	Sum of the squares of the deviations	degrees of freedom	mean square of the deviation
between groups	21,784	6	3,631
within groups	12,377	34	364

Thus, one obtains $F = \frac{3{,}631}{364} = 9.97$

The standard limiting value of the F-distribution in the case of 6 and 34 degrees of freedom and a degree of certainity of 99 % is very much smaller with 3.40. It indicates thus a basic difference between at least two of the seven mean averages. In order to determine which series are signfficantly different from the others, one can study them using the t-test[8]. In order to achieve completeness, one must carry out 20 paired comparisons.

(2) TREATMENT BY MEANS OF RANGE ESTIMATION

First, the "expected probabilities" $\ddot{u}$ must be selected. One can systematically select different values for $\underline{\ddot{u}}$ and for $\overline{\ddot{u}}$, that is, for the lower and upper limits of the mean values of the basic generalities. This represents an advantage of the method. In the case of the example under consideration, however, in my opinion there is no reason to assume different values for $\underline{\ddot{u}}$ and $\overline{\ddot{u}}$. We selected this time alternative $\ddot{u} = 0.01$ and 0.05 and searched for the corresponding Γ- value for both $n = 6$ and $n = 5$. The additional procedures gave the following summary.

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[8] See also A. Linder, already cited, page 111.

DETERMINATION OF ESTIMATION LIMITS FOR
THE M_{Ui}

	I	II	III	IV	V	VI	VII
σ_i	17.40	20.06	14.82	14.44	25.18	11.41	14.08
	u = 0.01	τ (n = 6) = 1.505				τ (n = 5) = 1.874	
$\tau\sigma_i$	26.2	30.2	22.3	21.7	37.9	17.2	26.4
$M_i + \tau\sigma_i$	134	173	165	180	227	160	187
$M_i - \tau\sigma_i$	81	113	120	136	151	126	134
	u = 0.05			(n = 6) = 0.901		(n = 5) = 1.066	
$\tau\sigma_i$	15.7	18.1	13.4	13.0	22.7	10.3	15.0
$M_i + \tau\sigma_i$	123	161	156	171	212	153	175
$M_i - \tau\sigma_i$	92	125	129	145	166	132	145

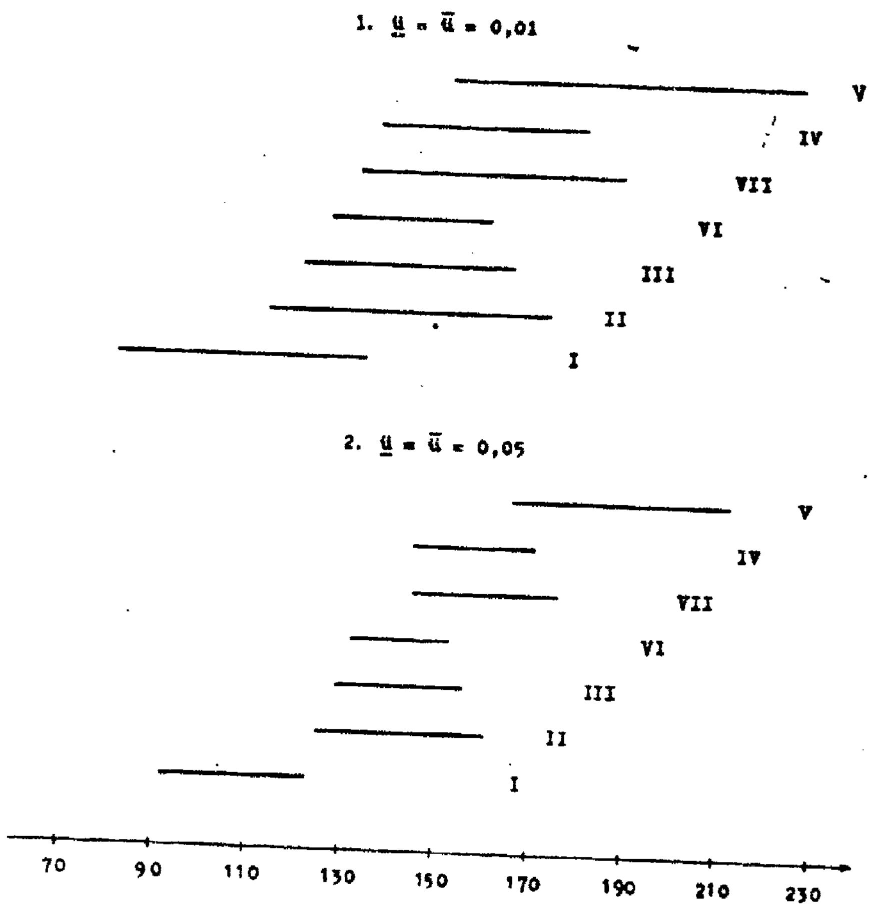
The results are presented in Figure 2. The upper part shows that a "very significant difference" exists between groups I and V. The lower part shows a variation of the differences which one can find from the diagram without resorting to calculations. Moreover, one cleary discerns a group formation (of the groups) which was already hintedat in the upper part. As a result, this figure gives a great deal more information than does variance analysis. One perceives not only which mean values are significantly different from each other but one also obtains an impression as to the probable position of the mean values in relation to each other and the probable extent of their differences.

Now it happens that one will be able to obtain the same results when the intervals are determined from the foundation of the t-distribution. However, then one has to be concerned with calculating the degrees of freedom. The explanation and calculations presented here are, on the other hand, more clearly and better explained and, if one had a table of τ values available, are quicker and easier to use.

Figure 2

FINAL WEIGHT OF 41 RATS
(in grams)

Estimated ranges for the mean values of the basic generalities



(3) SHORT MATHEMATICAL BASIS FOR THE PROCEDURES OF RANGE ESTIMATION

The starting point is the formula for the simultaneous distribution of the mean value M and the variance in sample to the extent n from a normal distribution:

$$(2)\quad \frac{2\sigma^{n-2}}{\sqrt{\pi}\,\frac{n-3}{2}!\,\left(\sigma_U/\sqrt{\frac{n}{2}}\right)^n} \cdot e^{-\left[\sigma^2+(M-M_U)^2\right]:\left[\sigma_U^2/\frac{n}{2}\right]}$$

and in polar coordinates:

$$(3)\quad \frac{\cos^{n-2}\alpha}{\sqrt{\pi}\,\frac{n-3}{2}!}\left(\frac{r}{\sigma_U/\sqrt{\frac{n}{2}}}\right)^n e^{-\left(\frac{r}{\sigma_U\sqrt{\frac{n}{2}}}\right)^2}\cdot\frac{2}{r^2} \qquad \text{ref 9.}$$

where:

$$\sigma = r\cdot\cos\alpha \quad \text{and} \quad M - M_U = r\cdot\sin\alpha$$

If one multiplies the last expression by $d\alpha\, dr$, then one obtains a probability for the number of samples which have similar α and similar r and consequently similar M and similar σ. If one integrates the expression with respect to r where $r = 0$ to $r = \infty$, then one finds the probability for the number of samples which have similar α but any r. These may be called W_α. In order to simplify the style, we substitute:

$$(4)\quad \frac{r}{\sigma_U\sqrt{\frac{n}{2}}} = \sqrt{t}$$

and obtain

$$(5)\quad W_\alpha = \frac{\cos^{n-2}\alpha}{\sqrt{\pi}\,\frac{n-3}{2}!}\, d\alpha \int_0^\infty e^{-t}\, t^{\frac{n}{2}-1}\, dt$$

9 See P. Lorenz, already cited, page 104.

The integral is the "Euler integral of the second type". It has the value $\Gamma\left(\frac{n}{2}\right)$ or $\frac{n-2}{2}!$. As a result, expression (5) is transformed into:

$$(6) \quad W_\alpha = \frac{\cos^{n-2}\alpha}{\sqrt{\pi}\ \frac{n-3}{2}!} \cdot \frac{n-2}{2}!\ d\alpha$$

The result is remarkably worthy of note. It no longer contains σ_μ either explicitly or implicitly. This condition is the fundamental basis for the procedures which have been developed. It is, therefore, unncessary to make assumptions over the variances of the basic generality and the concept of degrees of freedom is eliminated.

If one integrates expression (6) with reference to α from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, then one obtains as the total probability of all samples generally 1; as it must be.

On the basis of technical writing, let us introduce, however, an even symbol, namely, $(M - M_U)/\sigma = \tan\alpha$. M, σ , and α are considered to be variable quantities while $\ddot{u}$ is the abbreviation for the "projected probability". Then

$$(7) \quad \ddot{u} = \frac{\frac{n-2}{2}!}{\sqrt{\pi}\ \frac{N-3}{2}!} \int_{\beta}^{\frac{\pi}{2}} \cos^{n-2}\alpha\, d\alpha$$

is the probability of the number of samples for which the integration variable α is greater than the lower limit β .

If one does not consider the right side of the equation (7) as the result, but rather selected a given value for $\ddot{u}$, such as 0.01, the the equation will be the one for the determination of the lower limit of the integral and one finds for example for $\ddot{u} = 0.01$, and $n = 10, \beta = 43^{\circ}16'$ and $\tan \beta = \tau = 0.941$.

If one designates all sample for which α is greater than β as "bad" and the situation $\alpha = \beta$ as the limiting case, then $(M-M_U)/\sigma = \tan\beta = \tau$. It follows that $\underline{M_U} = M - \sigma\tau$ is the value of the lower limit of the mean average of the generality. Corresponding consideration will lead to the upper estimated limit for the mean average of the basic generality, symbolized by $\overline{M_U}$.

Result: $$\underline{M_U} = M - \sigma\tau$$

$$\overline{M_U} = M + \sigma\tau$$

SUMMARY

Two circumstances were the motive for the preceding comparision, namely:

(1) It is nearly impossible to provide an exact picture of the great number of tests dealing critically with the analysis of variance.

(2) Too little attention seems to have been paid to the method of measuring scopes for the mean values of samples (to be short: method of scopes).

Both methods are compared by means of numerical examples for simple cases, the results of the method of scopes being additionally illustrated by diagrams. To understand the computations, the usual mathematical knowledge is sufficient. To perform the calculations by means of the analysis of variance, a table of the F-distribution

is needed which can be found in any book on the analysis of variance. The calculations according to the method of scopes require a table of the function τ which is contained in the essay mentioned in the text and in the author's book "Anschauungsunterricht in mathematischer Statistik" (Perceptive Instruction in Mathematical Statistics), volumes II and III.

The mathematical essence of the method of scopes is outlined in an appendix.