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USAAVLABS TECHNICAL REPORT 68-73

EFFECT OF TEST MACHINE EXTENSIONAL RIGIDITY ON THE INITIAL BUCKLING LOAD FOR UNREINFORCED CIRCULAR CYLINDRICAL SHELLS IN AXIAL COMPRESSION

By

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March 1969

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

CONTRACT DA 44-177-AMC-258(T)

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This program was carried out under Contract DA 44-177-AMC-258(T) with Stanford University.

The research was directed toward the development of a better understanding of the fundamental processes in the buckling of shell bodies. The report discusses research on the influence of test machine extensional rigidity on the initial buckling load for shells.

The report has been reviewed by the U.S. Army Aviation Materiel Laboratories and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

Task 1F162204A17001
Contract DA 44-177-AMC-258(T)
USAAVLABS Technical Report 68-73

March 1969

EFFECT OF TEST MACHINE EXTENSIONAL RIGIDITY ON THE INITIAL BUCKLING LOAD FOR UNREINFORCED CIRCULAR CYLINDRICAL SHELLS IN AXIAL COMPRESSION

Final Report

By

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[redacted]

SUMMARY

This report presents the results of two series of tests which were made to investigate the commonly accepted criterion of instability for shell bodies. This criterion, as generally postulated, states that the total potential energy must be the same before and after buckling. It is an analytical consequence of this contention that the critical load at constant end shortening must be greater than at constant stress. Thus, there should be a difference between tests made in rigid and flexible machines. The experiments recorded provide very strong evidence that this is not so. In the research reported, no evidence of machine influence on initial buckling conditions could be found.

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INTRODUCTION

The influence of the test machine on initial buckling load level has received attention since the 1900's. von Kármán[1] seems to have been the first to draw attention to it. He suggested that test machine rigidity seriously influences the critical load for a column member. The influence of test machine rigidity on the critical compressive load for a circular cylindrical shell under uniform compression was first considered by Flügge.[2] He did this in an attempt to explain the difference between test values and predicted values. von Kármán and Tsien,[3] in 1941, extended the work of Donnell[4] and examined, for the first time, the postbuckling behavior of an axially compressed cylindrical shell. Their paper was, in essence, amplified by two subsequent papers published by Tsien[5,6] in 1942. Hoff[7] again examined the issue in his paper on the "Buckling of Thin Shells" presented at the 80th Anniversary Symposium for von Kármán. In a recent publication, Sobey[8] presented a very detailed theoretical study. The ideas formulated by these various authors constitute the classic opinion with regard to the subject.

According to this view, there is a certain level of load and displacement above which it is possible to snap from one equilibrium state into another. It is contended that test machine rigidity is influential in this process. The magnitude of the stress in the body which corresponds to this quasi-stable situation depends upon many parameters, including perturbations from external sources, shell irregularities, and loading inaccuracies. Two limiting cases of overloading are normally defined: overloading by a dead weight, in which case the applied load is the controlling factor, and overloading by means of a rigid test machine, in which case displacement is the deciding parameter. These two cases are illustrated in Figure 1. Normal test machines, of course, lie between these extremes.

There has been little experimental evidence to support the theoretical conjectures. Horton, Johnson, and Hoff[7] reported some experimental results which were obtained in an attempt to clarify the questions; however, because of the small number of tests and the scatter in the load values obtained, the work was most inconclusive. More recently, in 1963, Mossakovskii and Smelyi[9] carried out a more detailed experimental study of this problem. As a result of their work, they concluded that there was a noticeable difference between the behavior in rigid and elastic machines. Subsequently, Almroth, Holmes, and Brush[10] remarked, as a result of a limited program, that the evidence appeared to favor the viewpoint that the buckling of cylindrical shells was influenced more by the nature of the test specimen than by the nature of the machine. Thus, the situation is unsatisfactory, since the work appears to be more or less equally divided in the conclusions reached. Furthermore, such information as has been published is, in essence, not entirely quantitative. It does not show, for example, what the interdependence between system stiffness and structural behavior is. In view of this situation, a new program to study this phenomenon was planned. The results of this study are presented in this report.

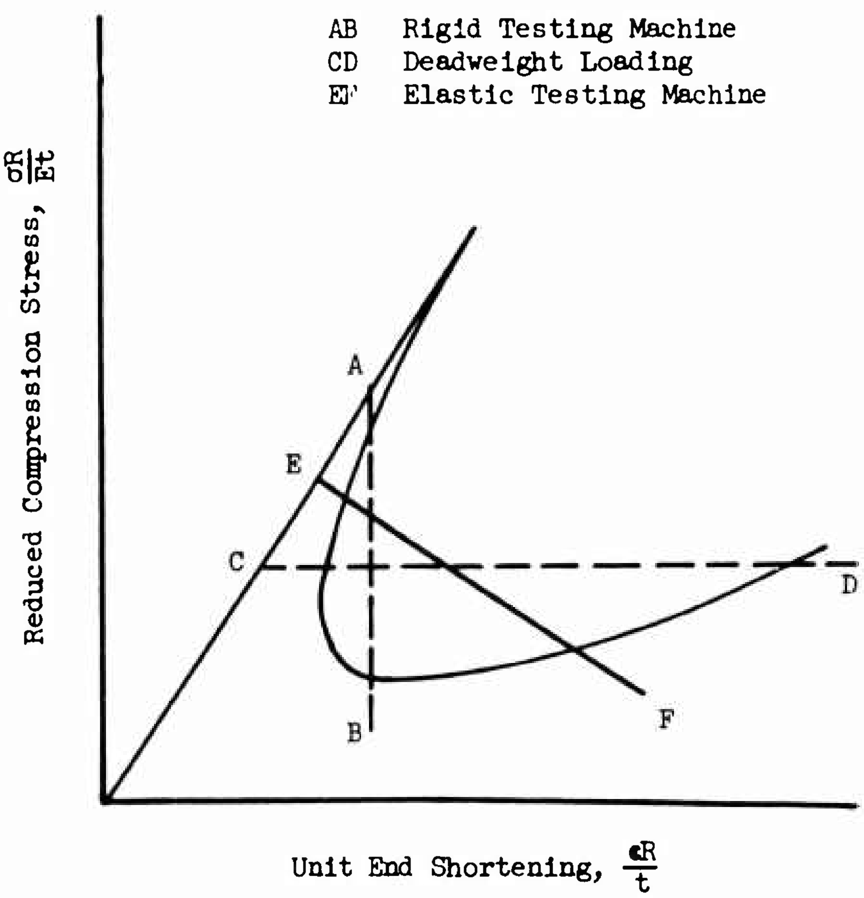


Figure 1. Equilibrium Curve for Buckling of a Thin Circular Cylindrical Shell Under Uniform Axial Compression.

EXPERIMENTAL APPROACHES

GENERAL

Because of the significance of the problem, it was decided that a conclusion should not be based upon a single experiment or series of experiments. After careful thought, two approaches to the problem were made.

A. Individual Tests on Many Specimens

B. Many Tests on an Individual Specimen

DISCUSSION OF SPECIMEN REQUIREMENTS

Approach A

It is an established principle of the science of statistics that deductions of value are more easily drawn from a large number of events than from a small number. However, the "large number of specimens" approach to a problem of experimental structural analysis is frequently unattainable because of the high cost involved. It is fortunate that an industry exists which specializes in the manufacture of shell bodies on a mass production basis and achieves a low cost with a relatively high consistency of product. The vehicles chosen for this research were beverage cans manufactured by the American Can Company. They were made in precision machinery in a fully automated process. Since they were carefully selected from the same batch of input material, and were processed in the same machine, they were consistent both in geometric form and material property. They were provided with end caps which had been made in a like manner and which were attached to the shell bodies by an automatic capping machine. In this way, the necessary large number of specimens of uniform quality and low price were available.

Approach B

The prime requirement for many tests on an individual specimen is that the specimen used must in no way deteriorate under the various loading conditions to which it is subjected during the test sequence. Until recently, an approach of this kind was totally unrealistic. Tests carried out in the normal fashion on cylindrical shells in compression inevitably result in considerable damage to the test vehicle. Even if extreme care is taken and the load is removed at the very first sign of buckling, there is still a marked drop in load-carrying capability when the process is repeated. Figure 2 shows the type of behavior generally experienced.

It has been demonstrated[11] that if a thin-walled cylindrical shell is tested in axial compression, and if the depth of the inward buckle motion is restricted by a closely fitting mandrel, the buckling process can be repeated many times in the same machine and with the same setup without any degradation of the shell Figure 3. Generally speaking, the maximum buckle depth

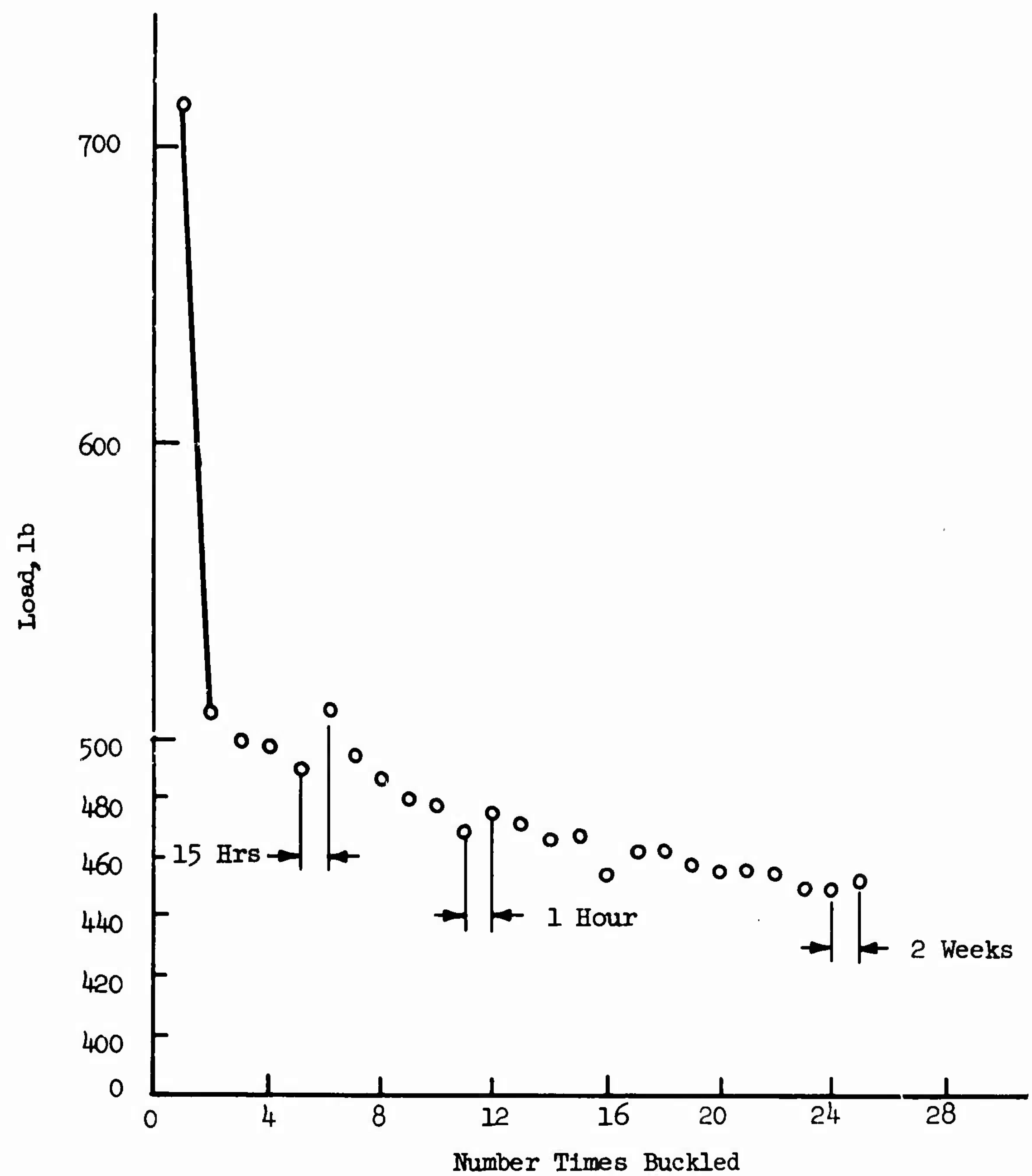


Figure 2. Repeated Buckling of a Nickel Circular Cylindrical Shell in Uniform Axial Compression.

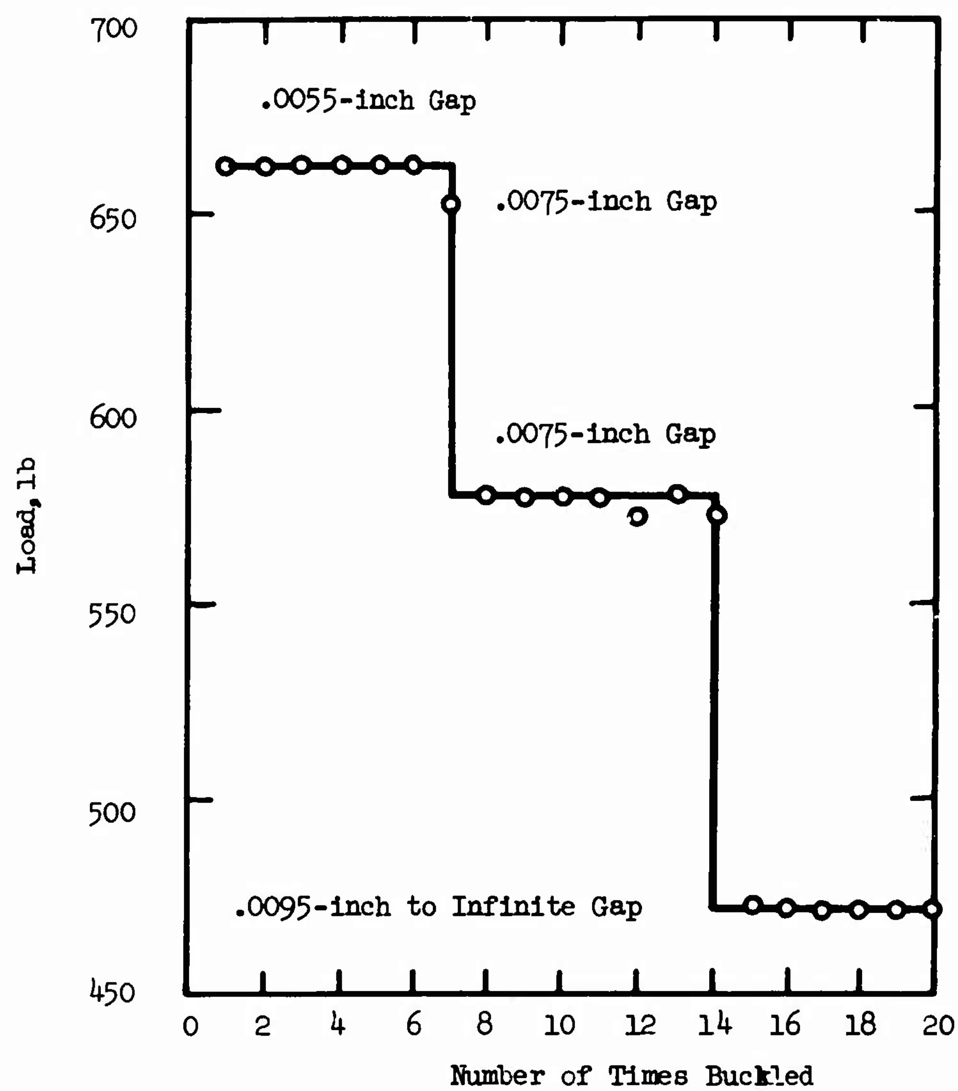


Figure 3. Plot of Buckling Load for Various Gap Settings.

must be restricted to less than the thickness of the shell, and the magnitude of the classic buckling stress should be less than 50 percent of the yield stress for the material. Thus, Approach B is based upon the use of a single-mandrel restricted specimen. The invariability of buckle load for this specimen was established before, during, and after the sequence of tests by a periodic determination of the buckling load in the base machine.

The test specimen was a thin-walled circular cylindrical shell manufactured by machining from an aluminum tube. The shell was manufactured from thick-walled tubing in the following manner: the tube was carefully bored and honed and turned until the wall thickness was 0.03125 inch. After this operation was completed, the tube was shrunk onto a ground mandrel, ready for exterior machining. The final turning operation was carried out between centers by using a carbide tip. After machining, the shell was lapped and polished to its final dimension. When the mandrel-specimen arrangement was heated slightly, the specimen was readily removed.

Upon inspection, the specimen was determined to be circular to within ± 0.001 inch and straight-sided to within the same tolerance. The wall thickness varied from nominal to no more than ± 0.0001 inch. The material was 7075-T6: the shell was 3.127 inches in diameter by 5.5 inches in length, with a nominal wall thickness of 0.005 inch. It was "locked" into stiff end plates by means of a low-temperature alloy, and the extent of inward buckle motion was limited by means of an interior mandrel which was concentric with the shell (Figure 4). The gap between the inner surface of the shell and the wall of the mandrel was arranged to be uniform at 0.005 inch. Adequate clearance between the top of the mandrel and the bottom of the upper test plate ensured that buckling could take place without the mandrel's coming into contact with the head plate.

REQUIREMENTS ON TEST MACHINE

Irrespective of whether Approach A or B is used, it is essential to pay particular attention to the test machine. Clearly, it is easy to use a variety of test machines to obtain a variation in machine stiffness; however, concurrent with the change in stiffness, there will be other nonconformities of character which may well be detrimental to our purpose. Slight discrepancies in the overall system behavior can, and frequently do, materially influence the behavior of the test specimen. To avoid such influences, all tests of a particular series were conducted in the same basic machine, and variation in stiffness was obtained by a change to this machine. This change was so planned that it did not influence any other characteristic of the test machine. As an added precaution, the test specimen was always located at the same vertical position relative to this machine.

The method used to modify the machine stiffness was simple. It relied upon the fact that if the overall test machine stiffness is S_1, then, if we insert a spring of stiffness S_2 between the specimen and one platen, the new machine defined by the combination has a stiffness whose value is $S_1S_2/(S_1 + S_2)$. The modification which accomplished this change is shown in

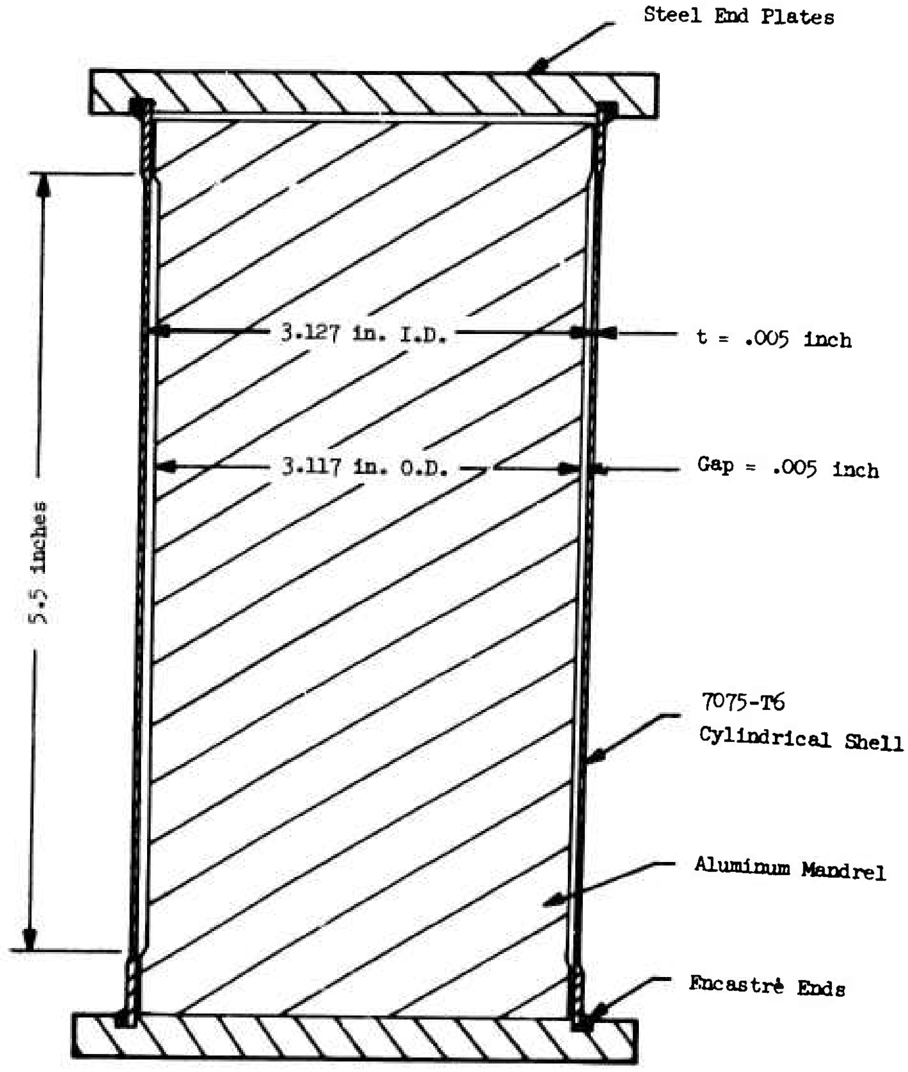


Figure 4. Cross Section of Test Vehicle Showing Restraining Mandrel.

Figure 5. It is a leaf-spring system which has 6 possible leaves and 3 possible support configurations. Variations in machine stiffness were therefore readily obtained by the alteration in the number of leaves and location of the basic support points.

REQUIREMENT TO ELIMINATE BIAS DUE TO A PARTICULAR TESTING MACHINE

To avoid any possibility that the results obtained might be influenced by the basic test machine or by the method of operation, different machines and operators were used in the two series of tests. For the first family, a standard 60,000-pound capacity, hydraulically operated Baldwin Universal Test Machine was used. It was modified by the addition of a leaf-spring system, as previously discussed. For the second group of tests, a 60,000-pound capacity Tinius-Olsen Universal Test Machine was employed. This machine was also hydraulically operated and was modified by the addition of a leaf-spring system.

RATE OF LOADING

In both series of tests, constant rate of loading was employed. However, the rate of loading in the Baldwin machine tests differed from that used in the Tinius-Olsen Machine.

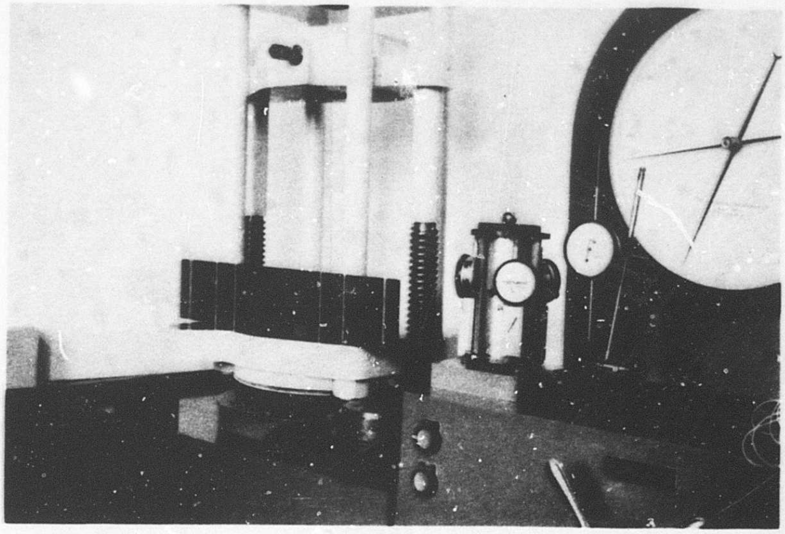
REQUIREMENT ON LOAD DISTRIBUTION

It is well recognized that the distribution of load seriously influences the crippling load for a cylindrical shell. Thus, in both test sequences, great care was exercised to ensure that the distribution of load was consistent among the various tests. For the Series A tests, a special end plate was made which tightly fitted the ends of the cylinders. Also, a loading ball was positioned between the lower end plate and the lower platen of the test machine, to avoid, as far as possible, the problems which would result from variations in parallelness of the platens which could occur with increasing load. In this way, the load distribution was made independent of magnitude. The setup is shown in Figure 6.

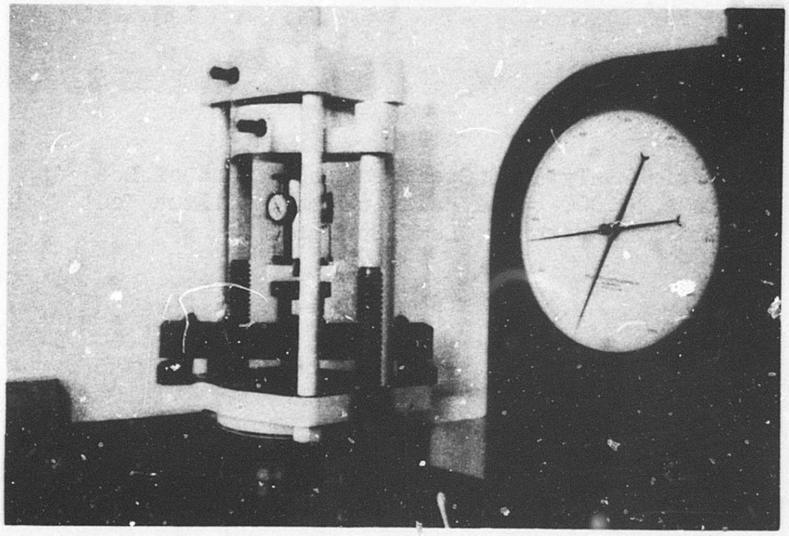
For the second series of tests, the test specimen was set in the test machine as shown in Figure 7. A loading ball system was arranged between the head of the machine and the upper end plate of the test specimen. Deflections were measured by 0.0001 inch dial gages and three stations equally spaced around the periphery of the shell. The ball system was centralized and the loading arrangements were accepted as satisfactory when the displacements recorded by these gages were equal with increasing load.

DETERMINATION OF TEST MACHINE RIGIDITY

In each case, the stiffness of the test machine and its variants was measured by pushing the loading head and platform apart by means of a 60-ton-capacity hydraulic jack. The motion which resulted and the load which was induced were measured by dial gages and the machine loading scale,



a. Detail of V-Groove Support Positions.



b. 6 Leaf Springs Supported at Outer V Groove.

Figure 5. Leaf-Spring Modification to Standard Test Machine.

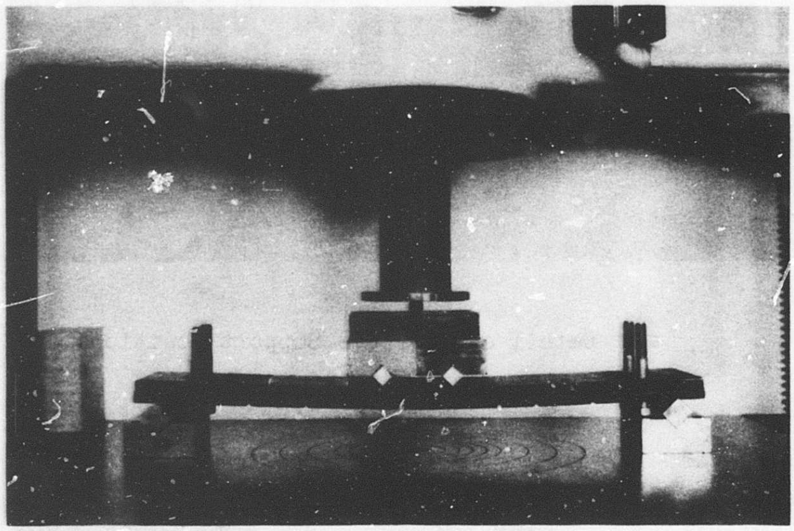
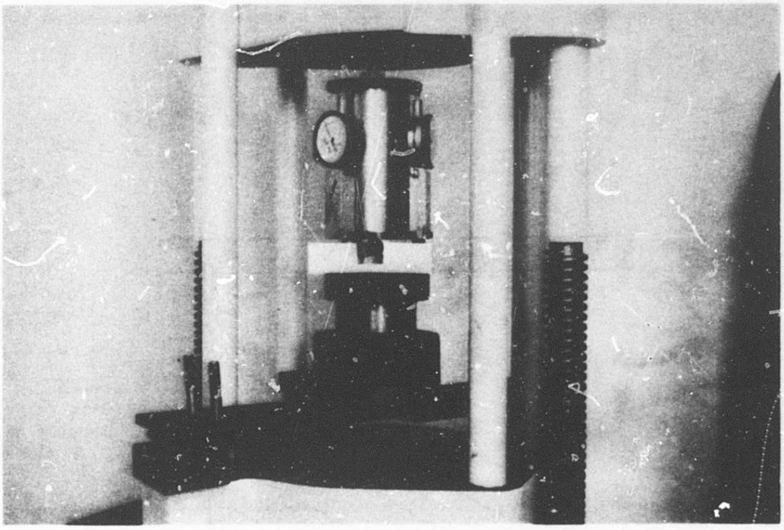
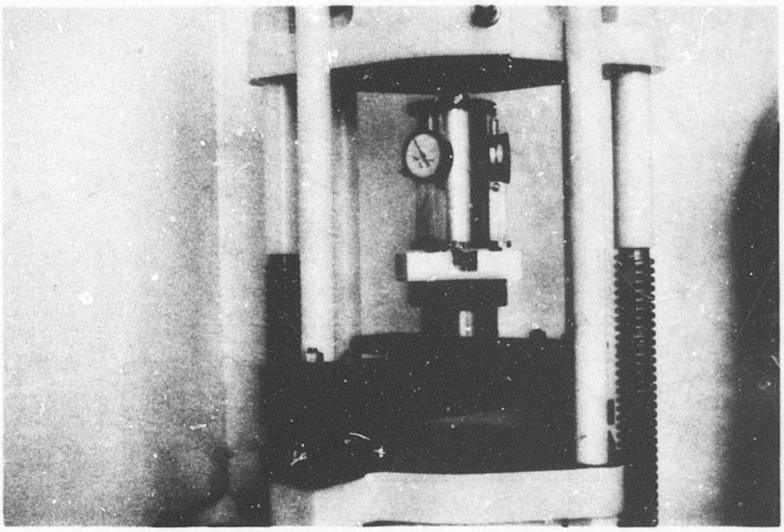


Figure 6. Series A-Typical Specimen and Method of Load Application in a 60,000-pound Baldwin-Lima-Hamilton Test Machine Modified with Two Leaf Springs.

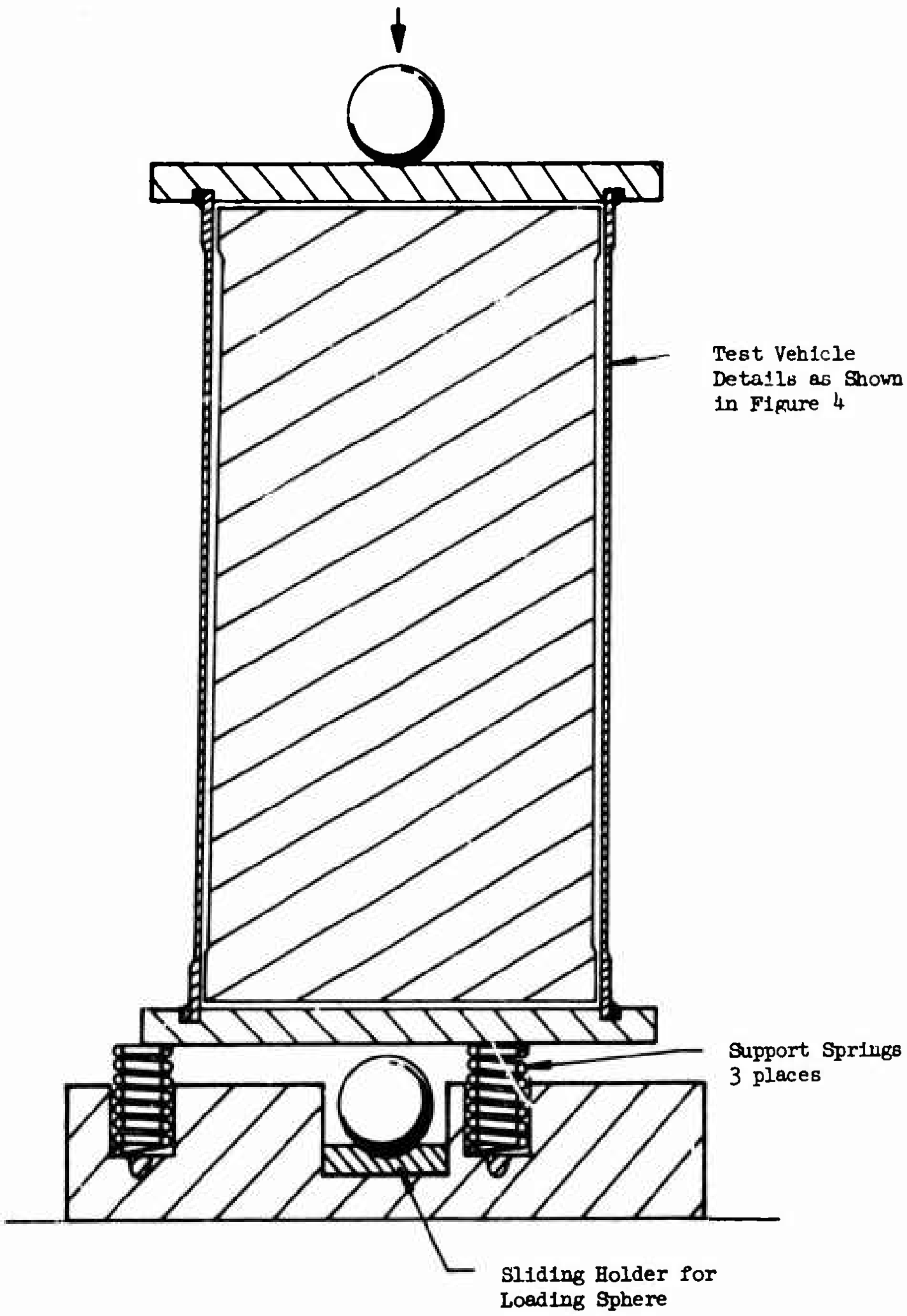


a. Detail of the 3-Load Positioning Dials.



b. Arrangement During Test.

Figure 7. Series B-Test Specimens and Loading Jig in a 60,000-pound Tinius-Olsen Test Machine Modified with Leaf Springs.



c. Cross Section Through Loading Jig for Series B Tests.

Figure 7. Continued.

respectively. Details of the test setup are clear from Figure 8. All data are given in Appendix 1.

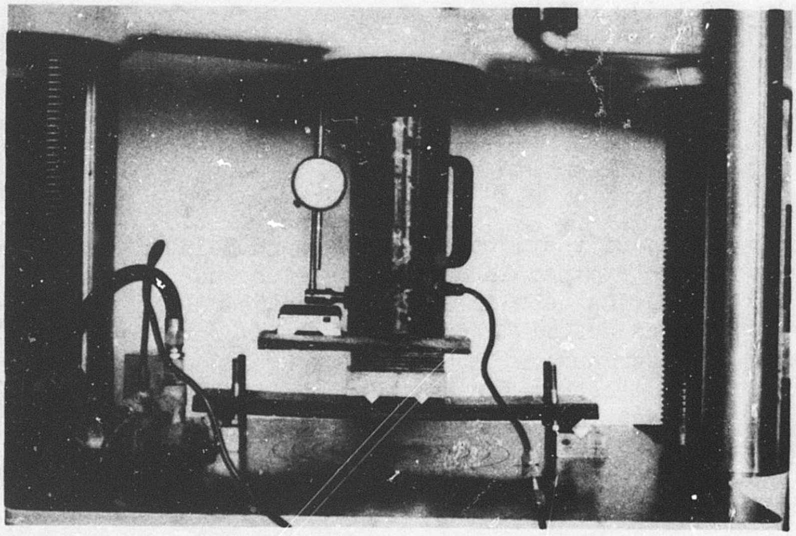
TEST PROCEDURES

SERIES A

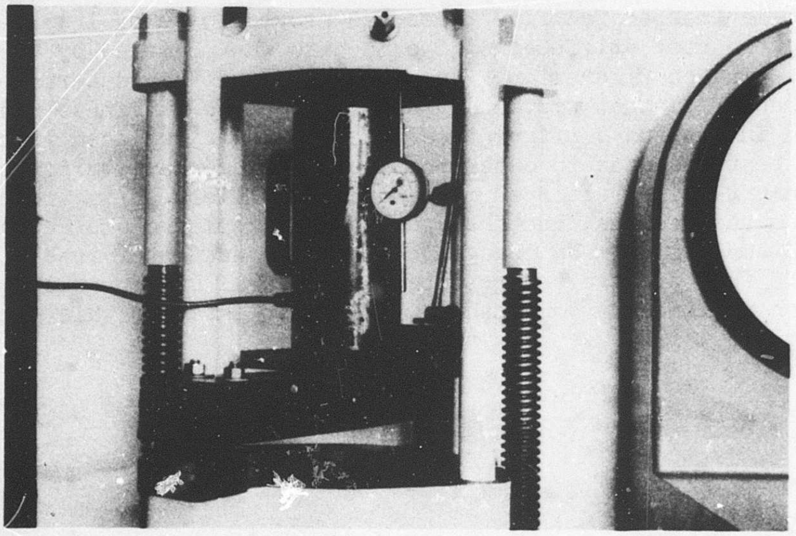
For the first family of tests, the 60,000-pound Baldwin machine was used. Its stiffness was determined in accordance with the procedure described above. Fifty cylindrical shells were then tested in this base machine. The specimens were as described previously, and provision for uniformity of load was made in accordance with the scheme outlined in "Requirement on Load Distribution". The machine was then modified by the addition of the leaf-spring arrangement and the stiffness was again determined. Fifty shells were now tested in the modified machine. In all cases, the load at the initiation of buckling was recorded. All test data obtained in this series are given in Appendix II.

SERIES B

For this family of tests, the Tinius-Olsen machine was used, as previously explained. The first step in the sequence was the determination of the stiffness of the base machine. When this had been established, the single specimen was tested five times and proved to be of invariant character. Next, the base stiffness of the machine was modified by the addition of a leaf-spring configuration. The stiffness was then determined in an analogous manner to that previously used. When the stiffness had been ascertained in the new configuration, the shell was replaced in test position and a test was made to determine its buckling characteristics in the new environment. The test specimen and the auxiliary spring were then removed from the machine, and the specimen was replaced and retested under the initial conditions. No change in character was determinable from the first test. This process was repeated for several stiffnesses. These variants were chosen in a random fashion. All test data for this series are given in Appendix III.



a. Stiffness Determination in a 60,000-pound Baldwin-Lima-Hamilton.



b. Stiffness Determination in a 60,000-pound Tinius-Olsen.

Figure 8. Determination of Test Machine Extensional Stiffness Using a Dial Gage, a Machine Load Cell, and a 60-ton Hydraulic Jack.

DISCUSSION OF TEST RESULTS

As explained in "Test Procedures", 50 nominally identical shells were tested in a soft machine (stiffness, 7,800 lb/in.) and 50 of the same type of shells were tested in a hard machine (stiffness, 589,000 lb/in.). In the Series A tests, the average crippling load for the specimens tested in the soft machine was 1,697.2 pounds, while the average load obtained in the series conducted in the hard machine was 1,700.4 pounds. The sample standard deviations were 100.8 pounds and 98.8 pounds respectively. (See Appendix II.)

The variation in load-carrying capability of the various specimens of the first family is portrayed in the probability plot of Figure 9, while that of the second family is similarly depicted in Figure 10. It is readily seen from these figures that the distributions are essentially normal. Hence, it is reasonable to compare the means by using the student's "t" test. This is done in Appendix II. The conclusion is that the hypothesis that there is no difference in the mean buckling load obtained from the hard and soft machines is acceptable at the 5-percent level of significance.

For the Series B tests, only one specimen was used. This specimen was maintained in a fully elastic state by use of the restraining mandrel technique developed by Horton and Durham[11]. Eight different levels of machine stiffness were used. These varied between 96,000 and 2,400 lb/in. (a 40:1 variation). The variation in critical load, as a function of the machine stiffness, is given in Table I. The table shows that the maximum load registered in this family of tests was 775 pounds and that the minimum was 755 pounds. The mean value was 763.75 pounds.

A linear regression analysis of the data made in Appendix III demonstrates that, to a high degree of probability, the variations from test to test may be considered accidental. The test results are shown graphically in Figure 11.

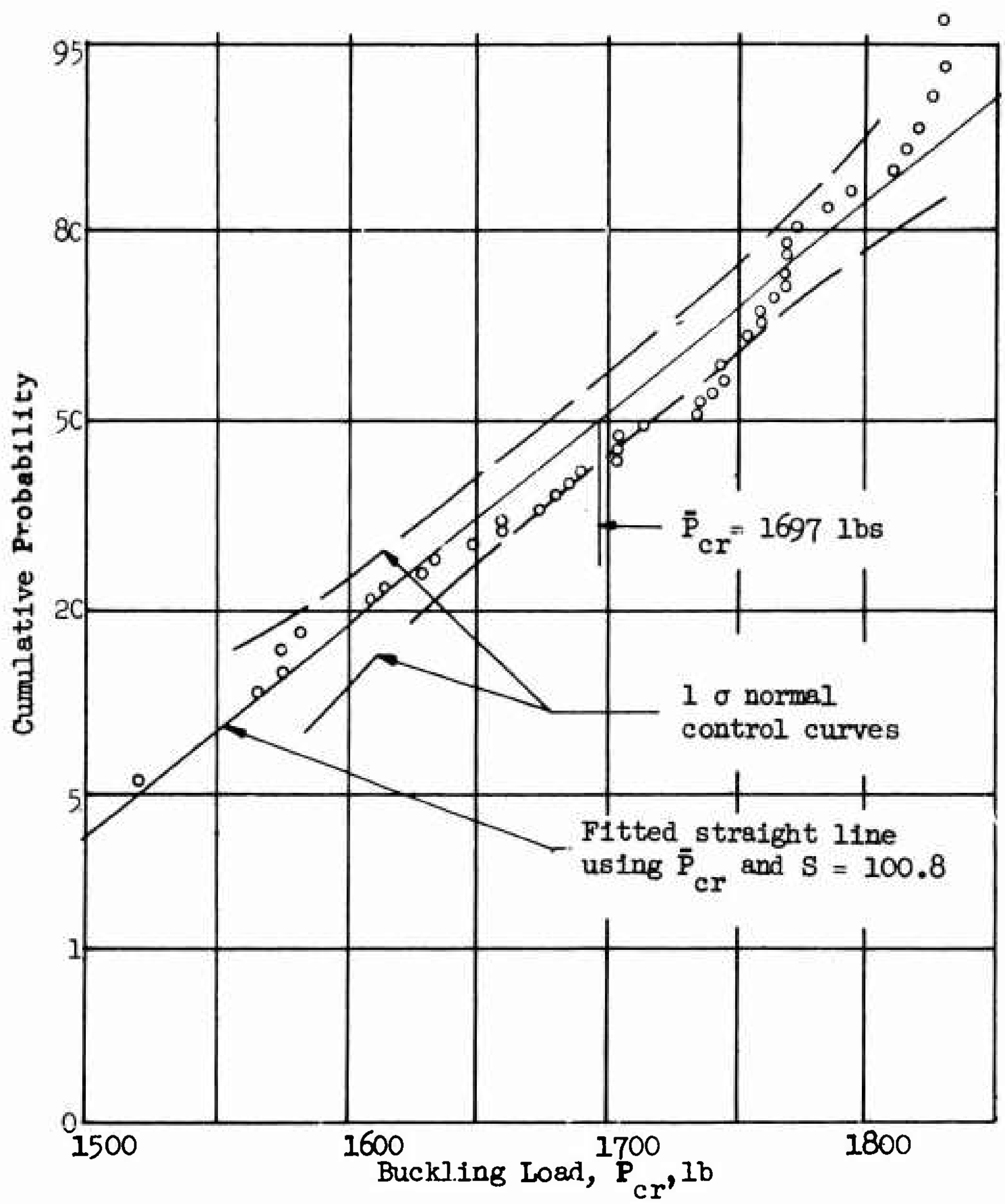


Figure 9. Distribution of Buckling Loads on Cylinders When Tested in the Modified Baldwin Test Machine (Stiffness = 7,800 lb/in.).

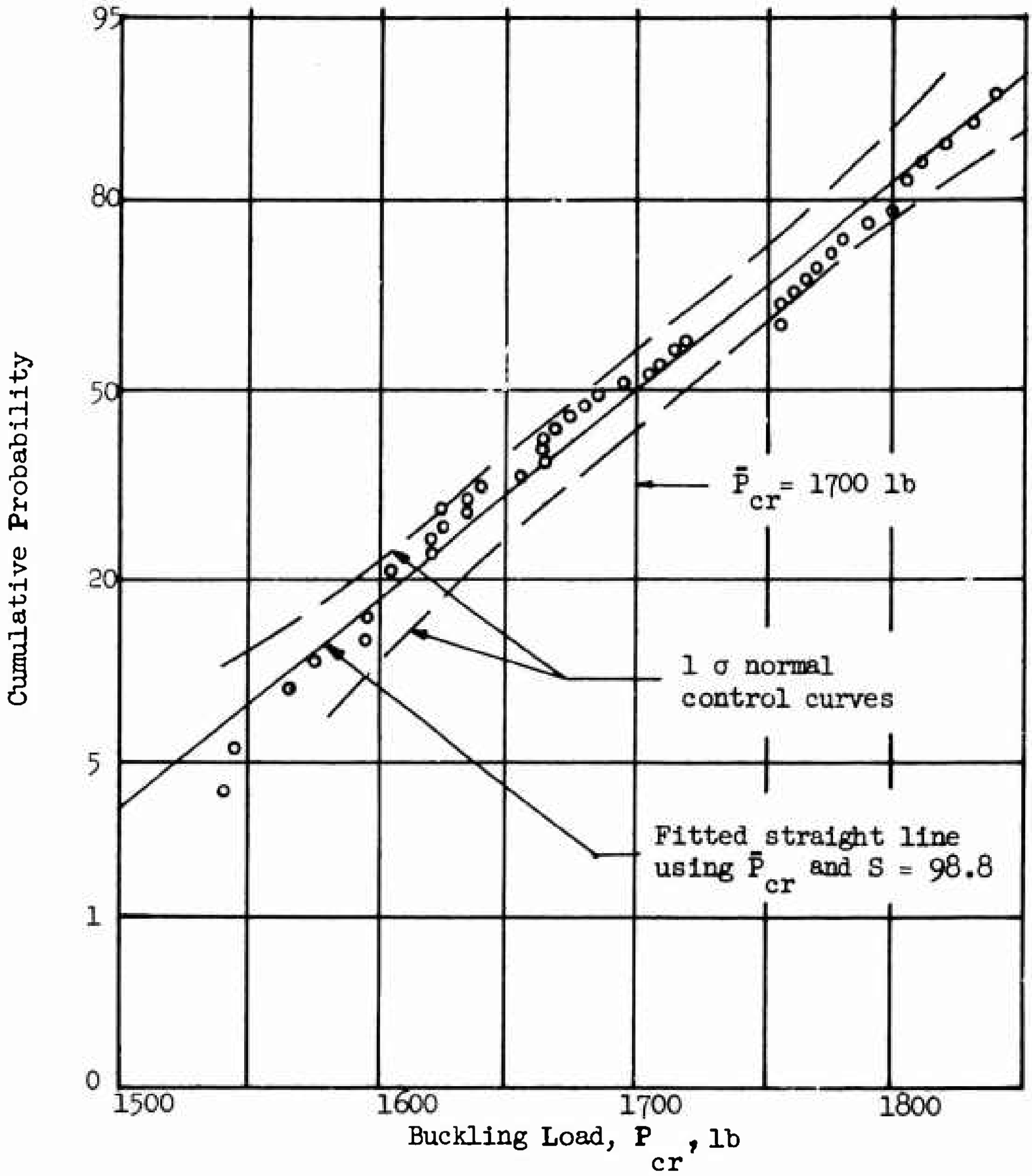


Figure 10. Distribution of Buckling Loads on Cylinders When Tested in the Standard Baldwin Test Machine (Stiffness= 589,000 lb/in.).

TABLE I. BUCKLING LOADS FROM SERIES B TESTS	
Composite Stiffness K (lb/in.)	Initial Buckling Load P_{cr} (lb)
96,000	760
53,500	755
35,600	775
17,400	755
13,500	775
8,800	760
4,300	770
2,400	760
	6,110
$\bar{P}_{cr}$ = 763.75 lb	

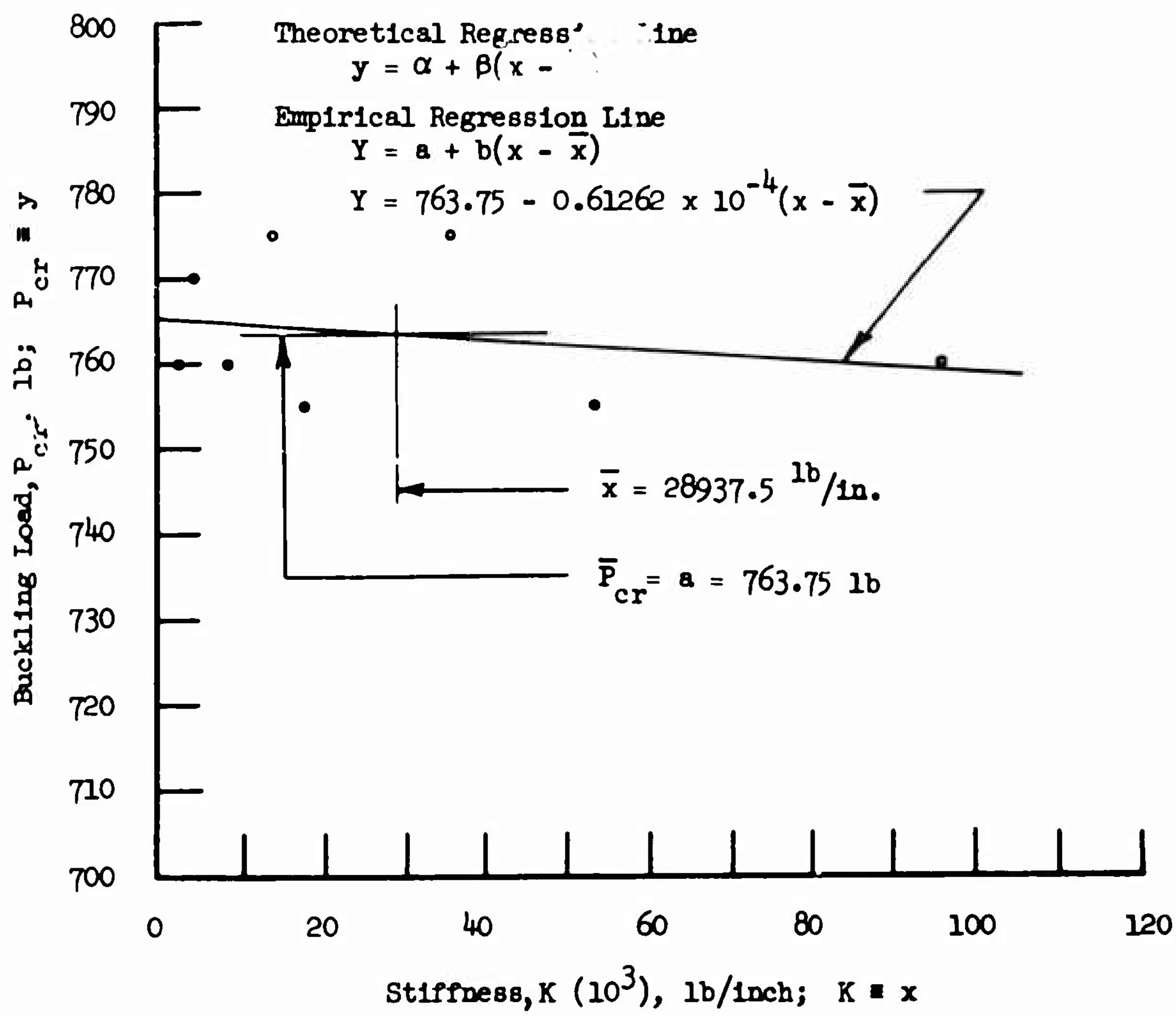


Figure 11. Regression Analysis for Second Series of Tests (Buckling Load Considered as a Linear Function of Machine Stiffness).

CONCLUSIONS

The experiments which are reported here show beyond reasonable doubt that test machine extensional stiffness does not influence the initial buckling load for a circular cylindrical shell in uniform axial compression. Thus, the criterion frequently adopted that the total potential energy (i.e., the sum of the strain energy and the potential energy of external forces) must be the same before and after buckling is invalid.

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APPENDIX I
DETERMINATION OF TEST MACHINE RIGIDITY

The rigidity of the test machines used in this investigation was obtained in accordance with the procedure outlined in "Determination of Test Machine Rigidity". The actual test data obtained are presented and analyzed in this appendix. The load deflection histories are presented in Tables II through XI, and they are displayed graphically in Figures 12 through 21. The actual machine stiffnesses are given on the appropriate figures.

The various modifications to the basic machine are described as configurations. Configuration N1 implies N springs with the support at the outermost position. Configuration N2, N springs with the support at the innermost position. Thus, in Configuration 21, there are two springs, while in 31 there are three, etc.

TABLE II. FORCE-HEAD SEPARATION DATA FOR 60,000-POUND BALDWIN-LIMA-HAMILTON TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 21	
Load x 10^2 (lb)	Deflection x 10^{-1} (in.)
1.00	0.14
2.50	0.35
4.10	0.59
6.10	0.87
8.05	1.13
9.50	1.31
11.1	1.52
12.5	1.70
14.1	1.91
15.5	2.08
17.0	2.28
18.5	2.46
20.0	2.64
Plot of data is shown in Figure 12.	

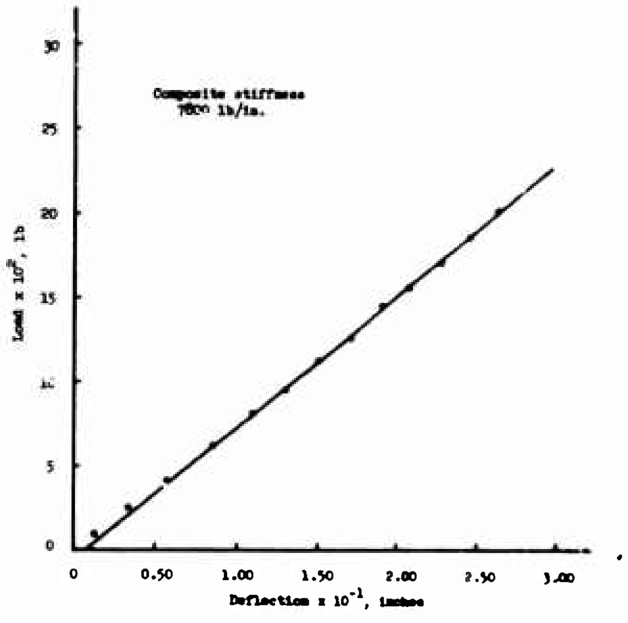

Figure 12. Stiffness Plot of 60,000-pound Baldwin-Lima-Hamilton Test Machine Modified by Leaf-Spring Configuration 21.

TABLE III. FORCE-HEAD SEPARATION DATA FOR THE BASIC 60,000-POUND BALDWIN-LIMA-HAMILTON TEST MACHINE	
Load x 10^2 (lb)	Deflection x 10^{-2} (in.)
3.25	0.88
6.15	0.91
8.60	0.97
11.60	1.03
14.10	1.07
16.5	1.13
19.5	1.17
22.5	1.21
25.8	1.27
29.0	1.33

Plot of data is shown in Figure 13.

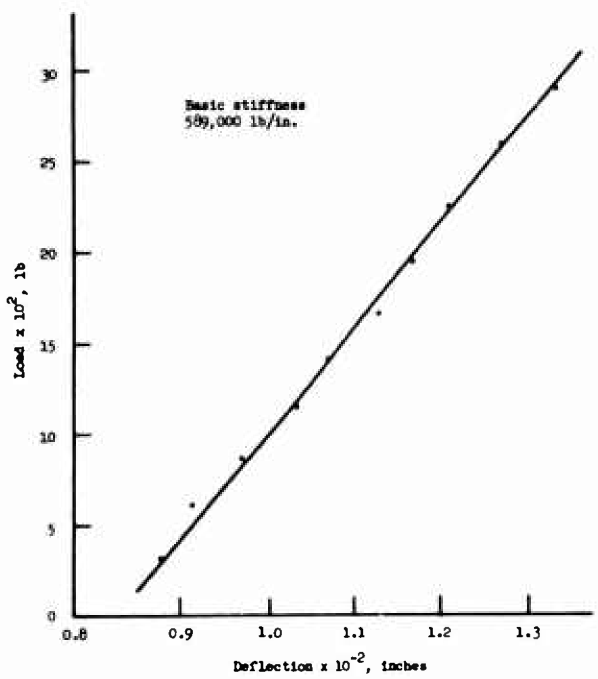

Figure 13. Stiffness Plot of the Basic 60,000-Pound Baldwin-Lima-Hamilton Test Machine.

TABLE IV. FORCE-HEAD SEPARATION DATA FOR THE BASIC 60,000 POUND TINIUS-OLSEN TEST MACHINE

Load x 10^2 (lb)	Deflection x 10^{-1} (in.)
2.00	1.73
4.00	1.75
6.00	1.77
8.20	1.80
10.10	1.82
12.00	1.84
14.00	1.86
16.00	1.88
18.20	1.90
20.00	1.92

Plot of data is shown in Figure 14.

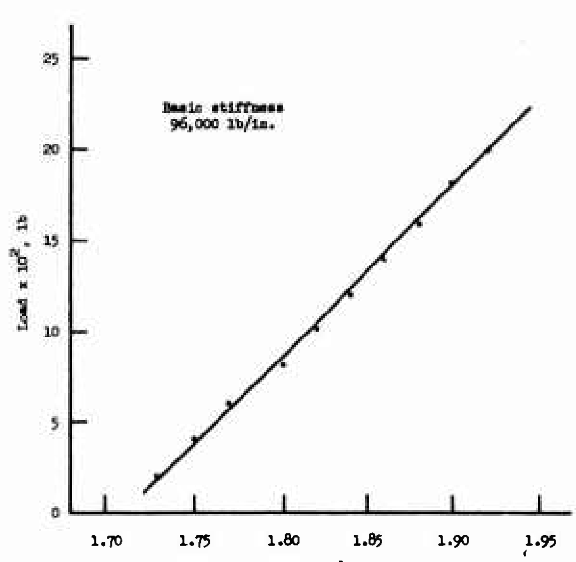

Figure 14. Stiffness Plot of the Basic 60,000-Pound Tinius-Olsen Test Machine.

TABLE V. FORCE-HEAD SEPARATION DATA FOR THE 60,000-POUND TINIUS-OLSEN TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 52

Load x 10^2 (lb)	Deflection x 10^{-1} (in.)
1.10	1.90
6.40	2.00
10.00	2.08
13.80	2.15
19.80	2.24
24.40	2.34
29.00	2.43
33.80	2.53

Plot of data is shown in Figure 15.

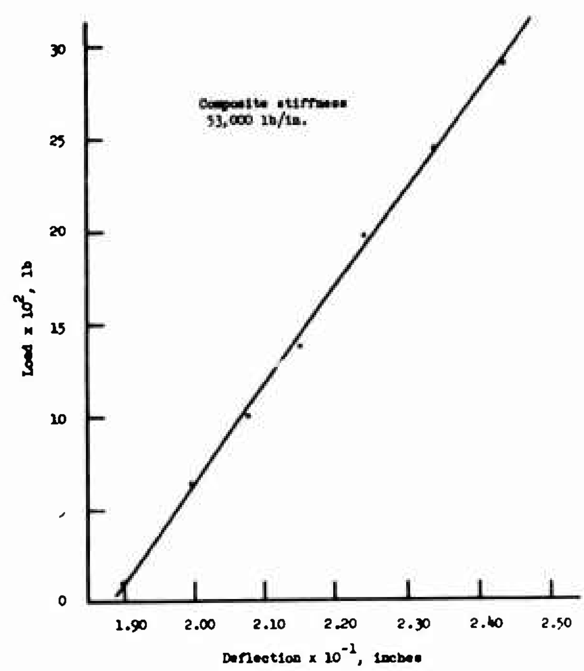

Figure 15. Stiffness Plot of 60,000-Pound Tinius-Olsen Test Machine Modified by Leaf-Spring Configuration 52.

TABLE VI. FORCE-HEAD SEPARATION DATA FOR THE 60,000-POUND TINIUS-OLSEN TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 42

Load x 10^2 (lb)	Deflection (in.)
2.20	1.39
5.00	1.40
7.30	1.41
11.60	1.42
15.70	1.43
19.60	1.44
23.20	1.45
27.00	1.46
30.00	1.47

Plot of data is shown in Figure 16.

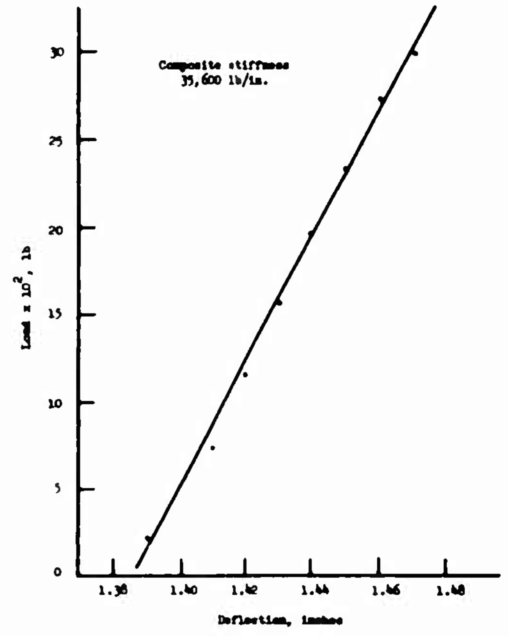

Figure 16. Stiffness Plot of 60,000-Pound Tinius-Olsen Test Machine Modified by Leaf-Spring Configuration 42.

TABLE VII. FORCE-HEAD SEPARATION DATA FOR THE 60,000-POUND TINIUS-OLSEN TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 51

Load x 10^2 (lb)	Deflection (in.)
0.80	1.42
3.00	1.44
5.20	1.45
7.60	1.47
10.60	1.48
12.60	1.49
16.40	1.52
20.20	1.54
26.20	1.57
28.60	1.58
35.40	1.62

Plot of data is shown in Figure 17.

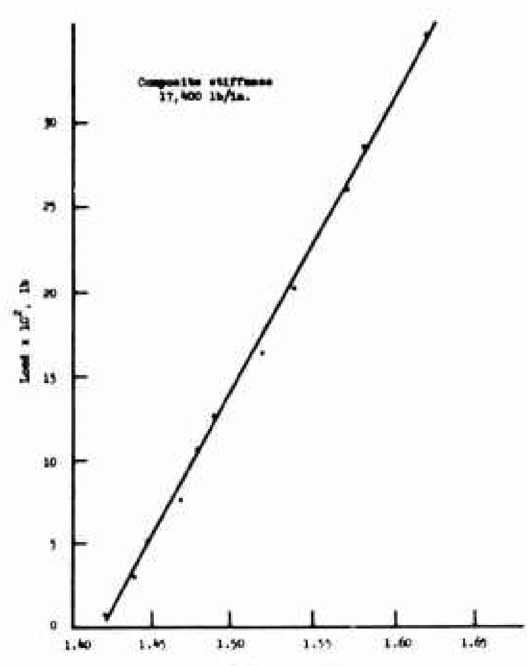

Figure 17. Stiffness Plot of 60,000-Pound Tinius-Olsen Test Machine Modified by Leaf-Spring Configuration 51.

TABLE VIII. FORCE-HEAD SEPARATION DATA FOR THE 60,000 POUND TINIUS-OLSEN TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 41

Load x 10^2 (lb)	Deflection (in.)
1.00	1.19
4.00	1.21
6.20	1.23
8.20	1.25
10.30	1.26
12.80	1.29
15.00	1.30
18.70	1.33
21.20	1.35
24.80	1.37
27.00	1.39

Plot of data is shown in Figure 18.

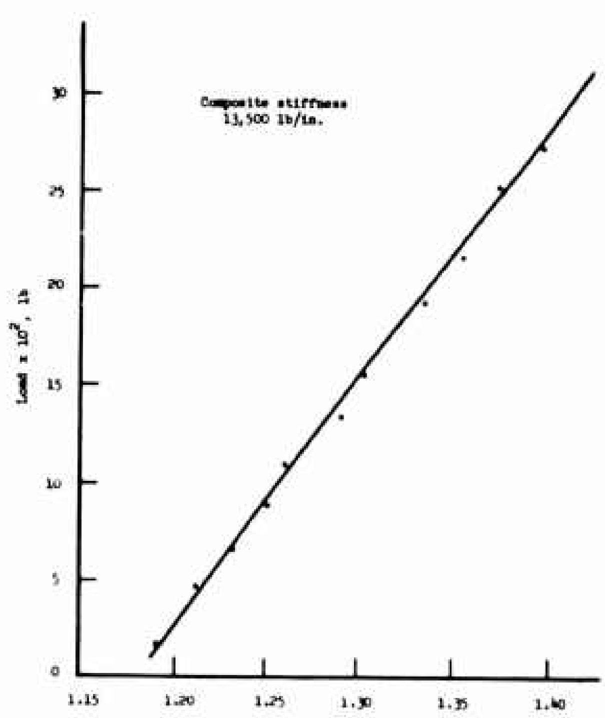

Figure 18. Stiffness Plot of 60,000-Pound Tinius-Olsen Test Machine Modified by Leaf-Spring Configuration 41.

TABLE IX. FORCE-HEAD SEPARATION DATA FOR THE 60,000-POUND TINIUS-OLSEN TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 21

Load x 10^2 (lb)	Deflection (in.)
1.00	1.32
3.33	1.34
4.80	1.36
6.10	1.38
7.50	1.40
8.90	1.42
10.60	1.44
12.60	1.46
15.00	1.48
17.10	1.50
19.10	1.52
21.00	1.54

Plot of data is shown in Figure 19.

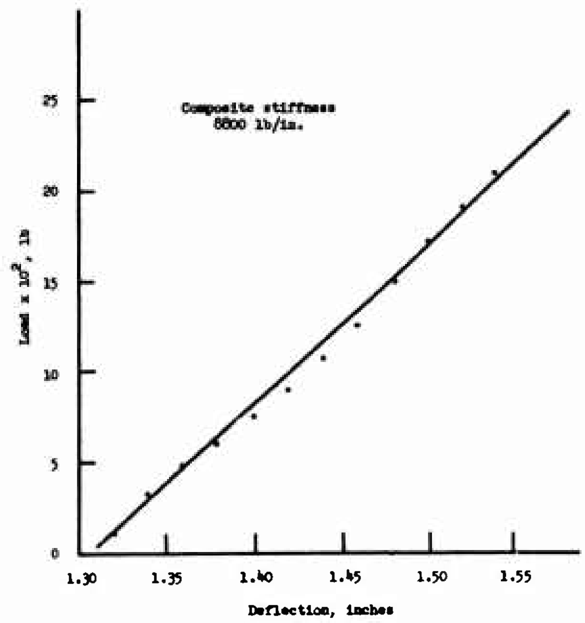

Figure 19. Stiffness Plot of 60,000-Pound Tinius-Olsen Test Machine Modified by Leaf-Spring Configuration 21.

TABLE X. FORCE-HEAD SEPARATION DATA FOR THE 60,000-POUND TINIUS-OLSEN TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 11	
Load x 10^2 (lb)	Deflection (in.)
1.00	1.10
1.90	1.12
2.50	1.14
3.20	1.16
4.00	1.18
4.80	1.20
5.70	1.22
6.80	1.24
7.80	1.26
8.70	1.28
9.60	1.30
10.40	1.32

Plot of data is shown in Figure 20.

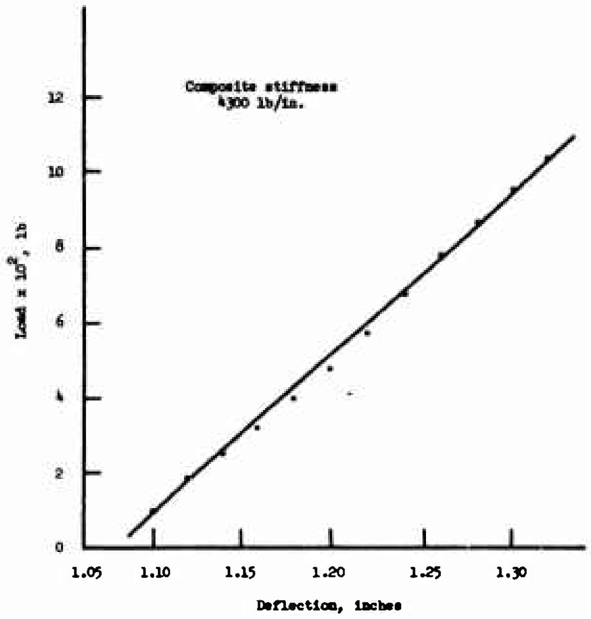

Figure 20. Stiffness Plot of 60,000-Pound Tinius-Olsen Test Machine Modified by Leaf-Spring Configuration 11.

TABLE XI. FORCE-HEAD SEPARATION DATA FOR THE 60,000 POUND TINIUS-OLSEN TEST MACHINE MODIFIED BY LEAF-SPRING CONFIGURATION 2-1

Load x 10^2 (lb)	Deflection (in.)
1.00	1.84
1.40	1.86
1.90	1.88
2.25	1.90
2.70	1.92
3.15	1.94
3.60	1.96
4.70	2.00
5.80	2.04
6.75	2.08
7.60	2.12
8.55	2.16
9.45	2.20
10.40	2.24

Plot of data is shown in Figure 21.

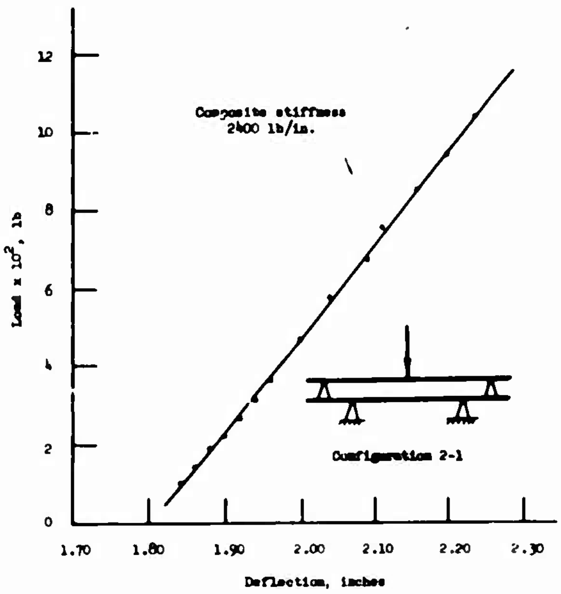

Figure 21. Stiffness Plot of 60,000-Pound Tinius-Olsen Test Machine Modified by Leaf-Spring Configuration 2-1.

APPENDIX II
STATISTICAL ANALYSIS OF THE EXPERIMENTAL DATA OBTAINED IN THE SERIES A TESTS

All test data obtained in the series A tests are presented and analyzed in this appendix. The individual critical loads for the tests performed in the soft machine (K = 7,800 lb/in.) in accordance with the procedures described in the main text are listed, in order of magnitude, in Table XII.

The buckling loads are arranged in increasing numerical sequence. In order to plot all data values, the cumulative probability of the mth observation is determined from m/n + 1.

The data appertaining to the tests made in the hard machine (K = 589,000 lb/in.) are likewise given in Table XIII. This information is graphically portrayed in the probability plots of Figures 9 and 10. It is immediately apparent from these plots that the distributions are essentially normal, and therefore the data can be analyzed by standard statistical methods.

Following these procedures with the data relevant to the soft system, we derive the mean buckling load $\bar{P}_{cr}$ as follows:

$$\bar{x} = \bar{P}_{cr} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{84860}{50} = 1697.2 \text{ lb}$$

The corresponding sample variance (s^2) and the appropriate standard deviation (s) are derived from the equation

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2$$

as follows: $s^2 = 1445330 - 2880488 = 10172$

and $s = 100.8$ lb

while the coefficient of variation ν, being the ratio of the standard deviation to the mean, is given by

$$\nu\% = \frac{s}{\bar{x}} \times 100\% = \frac{100.8}{1697.2} \times 100\% = 5.94\%$$

TABLE XII. CRITICAL LOADS FOR 50 NOMINALLY IDENTICAL CYLINDERS TESTED UNDER AXIAL COMPRESSION IN A MACHINE WITH STIFFNESS OF 7,800 LB/IN

(Values arranged in order of magnitude)

Load No. (m_i)	Critical Load P_{cr}, lb (x_i)	Cumulative Probability % $m/(n+1)$	Load No. (m_i)	Critical Load P_{cr}, lb (x_i)	Cumulative Probability % $m/(n+1)$
1	1400	1.96	26	1735	51.0
2	1485	3.42	27	1735	52.9
3	1520	5.88	28	1740	54.9
4	1545	7.85	29	1745	56.9
5	1550	9.80	30	1745	58.9
6	1565	11.8	31	1750	60.8
7	1575	13.7	32	1750	61.6
8	1575	15.7	33	1755	64.6
9	1580	17.6	34	1760	66.6
10	1580	19.6	35	1760	68.5
11	1610	21.6	36	1765	70.5
12	1615	23.5	37	1770	72.5
13	1630	25.5	38	1770	74.5
14	1635	27.4	39	1770	76.5
15	1650	29.4	40	1770	78.5
16	1660	31.4	41	1775	80.5
17	1660	33.3	42	1785	82.4
18	1675	35.1	43	1795	84.4
19	1680	37.2	44	1810	86.2
20	1685	39.2	45	1815	88.2
21	1690	41.1	46	1820	90.1
22	1705	43.1	47	1825	92.1
23	1705	45.1	48	1830	94.0
24	1705	47.1	49	1830	96.0
25	1715	49.0	50	1860	98.0

$\sum P_{cr} = 84,860$

TABLE XIII. CRITICAL LOADS FOR 50 NOMINALLY IDENTICAL CYLINDERS TESTED UNDER AXIAL COMPRESSION IN A MACHINE WITH STIFFNESS OF 589,000 POUNDS/INCHES
(Values arranged in order of magnitude)

Load No. (m_1)	Critical Load P_{cr}, lb (x_1)	Cumulative Probability % m/(n + 1)	Load No. (m_1)	Critical Load P_{cr}, lb (x_1)	Cumulative Probability % m/(n + 1))
1	1500	1.96	26	1695	51.10
2	1540	3.92	27	1705	52.9
3	1545	5.88	28	1710	54.9
4	1550	7.85	29	1715	56.9
5	1565	9.80	30	1720	58.9
6	1575	11.8	31	1730	60.8
7	1595	13.7	32	1755	61.6
8	1595	15.7	33	1755	64.6
9	1600	17.6	34	1760	66.6
10	1600	19.6	35	1765	68.5
11	1605	21.6	36	1770	70.5
12	1620	23.5	37	1775	72.5
13	1620	25.5	38	1780	74.5
14	1625	27.4	39	1790	76.5
15	1635	29.4	40	1800	78.5
16	1635	31.4	41	1800	80.5
17	1640	33.3	42	1805	82.4
18	1655	35.1	43	1810	84.4
19	1665	37.2	44	1820	86.2
20	1665	39.2	45	1830	88.2
21	1665	41.1	46	1840	90.1
22	1670	43.1	47	1860	92.1
23	1675	45.1	48	1865	94.0
24	1680	47.1	49	1870	96.0
25	1685	49.0	50	1890	98.0

$\sum P_{cr} = 85,020$

The theoretical curve for the data displayed in the probability plot of Figure 9 is

$$x = \beta + (1/\alpha)y$$

where x is the variable buckling load and y is the deviation in multiples of standard deviation about the mean cumulative probability point. These values correspond to cumulative probabilities as follows:

Deviation in σ y	Cumulative Probability % $F(x)$
+ 1	84.13
0	50.00
- 1	15.87

The classical method of least squares is applied to estimate the parameters β and $1/\alpha$. This leads to the following values:

$$\beta \approx \bar{x} = 1697.2 \text{ lb}$$

$$1/\alpha \approx \frac{s}{\sigma_n} = \frac{100.8}{0.932} = 108 \text{ lb}$$

where σ_n is the normal standard deviation as a function of sample size. With $n = 50$, $\sigma_n = 0.932$ (Reference 13, Table 1.2.9, p. 39). Thus, the empirical line is

$$x = 1697 + 108\ y$$

This is plotted in Figure 9.

The degree of fit of the straight line and the data is determined graphically by using control curves in the following manner. The standard errors $\sigma(x_m)$ of the mth observations are added to and subtracted from the mth values x_m, as determined from the fitted straight line. The points $x_m \pm \sigma(x_m)$ are joined to form these curves. The standard errors $\sigma(x_m)$ are derived from the usual statistical formula

$$\sigma(x_m) = \frac{\sigma(y_m)\sqrt{n}}{\alpha\sqrt{n}} = 15.25\ \sigma(y_m)\sqrt{n}$$

where the values $\sigma(y_m)\sqrt{n}$ are pure numbers independent of the parameters (Reference 13, Table 2.16, p. 52).

Probability	$\sigma(y_m)\sqrt{n}$	$\frac{1}{\alpha\sqrt{n}}$	$\sigma(x_m)$
0.5	1.253	15.25	19.1
0.6	1.268	15.25	19.3
0.7	1.318	15.25	20.1
0.8	1.429	15.25	21.7
0.85	1.532	15.25	23.4

Probability values ranging from 0.15 to 0.50 are obtained from symmetry.

The control curves are plotted in Figure 9.

The data for the hard system (K = 589,000 lb/in.) are treated in an identical manner to that of the soft system, and the various parameters of importance are derived as follows:

$$\text{Mean Buckling Load} = \bar{P}_{cr} = \frac{85020}{50} = 1700.4 \text{ lb}$$

$$\text{Variance} = s^2 = 2901110 - 2891360 = 9750$$

$$\text{Standard deviation} = s = 98.8 \text{ lb}$$

$$\text{Coefficient of variation } \nu\,\% = \frac{98.8}{1700.4} \times 100\,\% = 5.81\,\%$$

The coefficients β x $1/\alpha$ are given by

$$\beta \approx \bar{x} = 1700.4 \text{ lb}$$

$$1/\alpha \approx s/\sigma_n = \frac{98.8}{0.932} = 106 \text{ lb}$$

The line defined by these coefficients is plotted in Figure 10. The appropriate control curves are derived as before. The necessary values of $\sigma(x_m)$ are listed below.

$$\sigma(x_m) = \frac{\sigma(y_m)\sqrt{n}}{\alpha\sqrt{n}} = 15\,\sigma(y_m)\sqrt{n}$$

Probability	$\sigma(y_m)\sqrt{n}$	$\dfrac{1}{\sigma\sqrt{n}}$	$\sigma(x_m)$
0.5	1.253	15	18.8
0.6	1.268	15	19.0
0.7	1.318	15	19.7
0.8	1.429	15	21.4
0.85	1.532	15	23.0

These control curves are shown in Figure 10.

It is readily apparent from the probability plots of Figures 9 and 10, and their appropriate control curves, that both sets of observations can be assumed to be samples from normal distributions. The standard deviations of these distributions are extremely close and, therefore, the means are compared by using the student "t" test. The hypothesis of equality is examined.

The criterion for acceptance of this premise is

$$|t| \le t_{\alpha/2;\, n_x + n_y - 2}$$

where α is the level of significance

and $n_x + n_y$ are the sample sizes.

Hence $n_x = n_y = 50$

$$n_x + n_y - 2 = 98$$

The "t" test statistic is calculated from (Reference 12, Table 7.2, p.171)

$$t = (\bar{x} - \bar{y})\sqrt{\frac{n_x n_y \left(n_x + n_y - 2\right)}{(n_x + n_y)\left(n_x s_x^2 + n_y s_y^2\right)}}$$

where x = sample data from hard system
y = sample data from soft system

$\bar{x} = 1700.4$ $\qquad$ $\bar{y} = 1697.2$

$s_x^2 = 9750$ $\qquad$ $s_y^2 = 10172$

$n_x = 50;\ n_y = 50$ $\qquad$ $t = 3.2\sqrt{\dfrac{48}{19922}} = 0.158$

Choosing a 5-percent level of significance, a table of percentage points of the t distribution for 98 degrees of freedom gives

$$t_{0.025;\ 98} = 1.984$$

Since $|0.158| < 1.984$ the hypothesis of equality of the means is accepted at the 5-percent level.

An indication of the sensitivity of the analysis is obtained from an examination of the operating characteristic curve at this level. The curve corresponding to a sample size of 50 indicates a 95 percent probability of detecting a difference d = 0.35 (Reference 12, Figure 6.10, p. 129). For this test,

$$d = \frac{|\mu_x - \mu_y|}{2\sigma}$$

Using the average sample standard deviation from the two tests as an estimate for σ,

$$|\mu_x - \mu_y| = 0.70\ (99.8) \approx 70\ \text{lb}$$

Thus, a difference between the means as small as 70 pounds or approximately 4-percent of the average buckling load could be detected at the 95-percent level.

APPENDIX III
STATISTICAL ANALYSIS OF THE EXPERIMENTAL DATA OBTAINED IN THE SERIES B TESTS

All the test data obtained in the series B tests are presented and analyzed in this appendix. Eight levels of machine stiffness were used in this series. These stiffnesses were obtained as described in "Determination of Test Machine Rigidity", and their values are computed in Appendix I. Composite stiffnesses, together with the corresponding buckling loads, are listed in Table I.

A linear regression analysis is made on these data.[14] For this purpose, a regression line of the form

$$y = \alpha + \beta(x - \bar{x}) = A' + \beta(x)$$

is chosen where y = buckling load lb

x = machine stiffness lb/in.

The parameters α + β are estimated by the method of least squares to obtain the empirical regression line

$$y = a + b(x - \bar{x}) = A + bx$$

From the data presented in Table XIV, the mean buckling load may be computed as

$$a = \bar{v} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{6110}{8} = 763.75 \text{ lb}$$

while the mean value of machine stiffness used is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} y_i}{8} = \frac{23150}{8} = 28{,}937.50 \text{ lb/in.}$$

The slope b is defined by

$$b = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

TABLE XIV

LINEAR REGRESSION ANALYSIS-APPROACH B RESULTS

Test No.	Leaf-Spring Configuration: No. Springs	Leaf-Spring Configuration: Support Position	Composite Stiffness K lb/in. (x_i)	Initial Buckling Load P_{cr}, lb (y_i)	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$ x10^3	$(y_i-\bar{y})$	$(x_i-\bar{x})$x$(y_i-\bar{y})$	$(y_i-\bar{y})^2$
1	(Basic Machine)		96,000	760	67,063	4,497,446	-3.75	-251,486	14.063
2	5	2	53,500	755	24,563	603,341	-8.75	-214,926	76.562
3	4	2	35,600	775	6,663	44,396	11.25	74,959	126.563
4	5	1	17,400	755	-11,537	133,102	-8.75	100,948	76.562
5	4	1	13,500	775	-15,437	238,301	11.25	-173,666	126.563
6	2	1	8,800	760	-20,137	405,499	-3.75	75,514	14.063
7	1	1	4,300	770	-24,637	606,982	6.25	-153,981	39.063
8	2	(2-1)	2,400	760	-26,537	704,212	-3.75	99,514	14.063
$\sum$			231,500	6,110		7,233,279		-443,124	487.502

Ref. Figure 5
Ref. Figure 21

and for this case is given as

$$-\frac{443124}{7{,}233{,}279{,}000} = -\ 0.61262 \times 10^{-4} \text{ in.}$$

Thus, the empirical regression line becomes

$$Y = 763.75 - 0.61262 \times 10^{-4}(x - \bar{x})$$

This line is drawn through the discrete data points in Figure 11.

Now, the variability of y about the mean $\alpha + \beta(x - \bar{x})$ is described by σ^2. However, since σ^2 is unknown, it is estimated from the data by

$$s^2 = \frac{1}{n-2}\left\{\sum_{i=1}^{n}(y_1 - \bar{y})^2 - \frac{\left[\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right]^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right\}$$

Using data from Table XIV, we may write

$$s^2 = \frac{1}{6}\left[487.502 - \frac{(443{,}124)^2}{7{,}233{,}279{,}000}\right] = 76.726$$

Thus, the standard deviation is

$$s = 8.759 \text{ lb}$$

The variances of the estimate b of the slope and the estimate A of the intercept are normally distributed with means β and A', respectively. The variance of b is given by

$$\sigma_b^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

From Table XIV and the estimate for σ^2, we may write

$$\sigma_b^2 = \frac{76.726}{7{,}233{,}279{,}000} = 106.073 \times 10^{-10}$$

Thus the standard deviation is given as

$$\sigma_b = 10.299 \times 10^{-5}$$

The variance of A is given by

$$\sigma_A^2 = \sigma^2 \left[\frac{1}{n} + \frac{x^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$$

and by substituting appropriate values from Table XIV, we arrive at

$$\sigma_A^2 = 76.726 \left[\frac{1}{8} + \frac{83\text{[illegible]},378,900}{7,233,279,000} \right] = 4.298$$

In addition to the point estimates of slope and intercept, confidence intervals for β and A' may be established with confidence coefficient $1-\alpha$. They are given for β by

$$b \pm t_{\alpha/2;\ n-2}\ \sigma_b$$

and for A' by

$$A \pm t_{\alpha/2;\ n-2}\ \sigma_A$$

where $t_{\alpha/2;\ n-2}$ is the 100 $\alpha/2$ percentage point of student's "t" distribution. Choosing α = 5-percent, the 95-percent confidence interval estimate for β is

$$b \pm t_{.025;\ 6}\ \sigma_b = \left[- 0.6126 \times 10^{-4} \pm 2.447\ (1.0299) \times 10^{-4} \right]$$

$$= \left[- 3.133;\ 1.907 \right] \times 10^{-4} \text{ in.}$$

The confidence intervals for A' is given by

$$A \pm t_{\alpha/2;\ n-2}\ \sigma_A = \left[765.523 \pm 2.447\ (4.2980) \right] = \left[755.006;\ 776.040 \right] \text{ lb}$$

The theoretical slope β and the intercept A' are examined satistically to check the initial assumption that buckling load is a linear function of machine rigidity. The significance tests for these coefficients are given in Table XV.

TABLE XV. SIGNIFICANCE TESTS FOR SLOPE AND INTERCEPT OF STRAIGHT LINE (Ref. Bowker and Liberman)

$$y = \alpha + \beta(x-\bar{x}) = A' + \beta x$$

Hypothesis	Test Statistic t	Criteria For Rejection $\lvert t \rvert \geq$	Operating Characteristic Abscissa Value d
$\beta = \beta_o$	$\dfrac{b-\beta_o}{\sigma_b}$	$t_{\alpha/2;\, n-2}$	$\dfrac{\lvert \beta_o - \beta_1 \rvert}{\sigma_b \sqrt{n-1}}$
$A'=A'_o$	$\dfrac{A-A'_o}{\sigma_A}$	$t_{\alpha/2;\, n-2}$	$\dfrac{\lvert A'_o - A'_1 \rvert}{\sigma_A \sqrt{n-1}}$

Notations: β_1 + A'_1 are the variations of slope and intercept from the hypothetical values which can be detected for the given sample size and chosen probability.

In finding the value of d, the n-1 curve is used.

The empirical regression line shown in Figure 11 is almost horizontal. In addition, the 95-percent confidence interval for β includes the possibility of a zero slope. This suggests that there is no relationship between x (machine stiffness) and the mean value of y (mean buckling load) and that the small empirical slope b is due to accidental variation of the data.

This hypothesis is tested by putting $\beta_0 = 0$ in Table XV. The test statistic is

$$t = \frac{b}{\sigma_b} = \frac{0.61262}{1.0299} = -\ 0.594823$$

At the 5-percent level of significance, the criterion for rejection becomes

$$|t| \geq t_{\alpha/2;\ n-2} = t_{0.05;\ 6} = 2.447$$

Since $0.5948 < 2.447$ the hypothesis that the theoretical slope is zero is accepted at the 95-percent level.

A measure of the sensitivity of the analysis is obtained from an examination of the OC curve at this level. For a sample size of 8, there is a 95-percent probability of detecting a value of d = 1.7.

For this test,

$$d = \frac{|\beta_o - \beta_1|}{\sigma_b \sqrt{n-1}} = 1.7$$

Hence, $|\beta_1| = 1.7(10.29919 \times 10^{-5})\sqrt{7}$

$$|\beta_1| = 0.46325 \times 10^{-3}$$

Thus, the hypothesis $\beta = 0$ would be rejected at the 95-percent level if it differs from zero by as little as 0.46325×10^{-3} inch. A slope of this magnitude may be seen in better perspective if we note that it implies a change at zero stiffness given by

$$\Delta_y = \beta_1 \bar{x} = (0.46325 \times 10^{-3})(28{,}937.50) = 13.4 \text{ lb}$$

which is only $(100)\dfrac{13.4}{763.75} = 1.8\ \%$ of the average critical load.

The assumption that no relationship exists between x and the mean value of y may be tested further by assuming that the theoretical intercept A' is equal to the mean value of the buckling load. The 95-percent confidence interval for A' includes the possibility of a value of $\bar{y}$. To test this hypothesis, A'_o is set equal to $\bar{y}$ = 763.75 in Table XV. The test statistic is

$$t = \frac{A - A'_o}{\sigma_A}$$

$$t = \frac{[763.75 + (0.61262 \times 10^{-4})(28,937.5) - 763.75]}{4.298027}$$

$$t = \frac{1.7727}{4.298027} = 0.412517$$

since $0.4125 < t_{.025;6} = 2.447$ the hypothesis $A' = \bar{y}$ is accepted at the 95-percent level.

Then, from the OC curve

$$d = \frac{|A'_o - A'_1|}{\sigma_A \sqrt{n - 1}} = 1.7$$

Hence, $|A'_o - A'_1| = 1.7(4.298027)\sqrt{7}$

$$|A'_o - A'_1| = 19.33 \text{ lb}$$

Thus, the hypothesis $A' = \bar{y}$ would be rejected at the 95-percent level if the theoretical intercept differed from the mean value by as little as 19.33 pounds or only

$$100 \frac{(19.33)}{763.75} = 2.53\%$$

of the average critical load.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Stanford University, Stanford, California	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

3. REPORT TITLE

EFFECT OF TEST MACHINE EXTENSIONAL RIGIDITY ON THE INITIAL BUCKLING LOAD FOR UNREINFORCED CIRCULAR CYLINDRICAL SHELLS IN AXIAL COMPRESSION

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report

5. AUTHOR(S) (First name, middle initial, last name)

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6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1969	56	14

8a. CONTRACT OR GRANT NO. Contract DA 44-177-AMC-258(T)	9a. ORIGINATOR'S REPORT NUMBER(S) USAAVLABS Technical Report 68-73
b. PROJECT NO. Task 1F162204A17001	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

[redacted]

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY US Army Aviation Materiel Laboratories Fort Eustis, Virginia

13. ABSTRACT

This report presents the results of two series of tests which were made to investigate the commonly accepted criterion of instability for shell bodies. This criterion, as generally postulated, states that the total potential energy must be the same before and after buckling. It is an analytical consequence of this contention that the critical load at constant end shortening must be greater than at constant stress. Thus, there should be a difference between tests made in rigid and flexible machines. The experiments recorded provide very strong evidence that this is not so. In the research reported, no evidence of machine influence on initial buckling conditions could be found.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

Unclassified
Security Classification

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Buckling in Cylindrical Shells Test Techniques for Shell Buckling Effect of Test Machine on Buckling						

Unclassified

Security Classification