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THE USE OF SMALL AND LARGE DISPLACEMENT DATA FROM ESSENTIALLY ELASTIC BUCKLING TESTS ON COLUMNS AND PLATES AS A MEANS OF CORRELATING THEORY AND EXPERIMENT

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March 1969

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

CONTRACT DA 44-177-AMC-258(T)
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

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This program was carried out under Contract DA 44-177-AMC-258(T) with Stanford University.

The research was directed toward the development of a better understanding of the fundamental processes in the buckling of shell bodies. The report discusses the use of the small and large displacement theories, where a wide range of stability problems are studied.

The report has been reviewed by the U.S. Army Aviation Materiel Laboratories and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

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THE USE OF SMALL AND LARGE DISPLACEMENT DATA FROM ESSENTIALLY ELASTIC BUCKLING TESTS ON COLUMNS AND PLATES AS A MEANS OF CORRELATING THEORY AND EXPERIMENT

Final Report

By

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SUMMARY

For many years, it has been recognized that the amplitude of the normal deformation at an appropriate point on a column or on a plate could be connected with the initial deformation at that point by a simple relationship between actual load and classic load for the structure. The appropriate expression is

$$\delta\left(\frac{P_{cr}}{P} - 1\right) = \delta_o$$

This formula is of considerable importance in the interpretation of test data. However, it applies only for small displacements and requires that, in many cases, the initial imperfection be small in relation to the relevant structure parameter. This is a restriction, since it implies the need for high-quality test vehicles. A large displacement formula which does not require this constraint can be developed as follows:

$$P = P_{cr}\,(1 + \gamma\delta^2)$$

This applies, of course, only when the deflections are large and when the load to produce them is in excess of the classic critical load. A wide range of stability problems is dealt with in this report, and critical load values are determined from the appropriate test result by both formulas. The results are in excellent agreement.

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LIST OF SYMBOLS

A = deflection amplitude

b = side dimension of plate

B = flexural rigidity of column, EI

E = modulus of elasticity

f_o = Dunn's displacement parameter

I = moment of inertia, minimum

$K = \sin \frac{1}{2} \beta$

L = length of column, plate

q = number; subscripts

p = " "

n = " "

m = " "

N_x = resultant forces in middle plane of a plate

N_y = " " " "

N_{xy} = " " " "

n = number; subscript

o = denotes initial imperfection

P = actual load

P_{cr} = critical load of perfect structure

t = plate thickness

T = temperature

T_o = temperature at center of circular plate

U = internal strain energy of plate

$u = f_o/\lambda$

V = potential energy of applied load

w = displacement in z direction

x,y,z = Cartesian coordinates (z out of middle plane plate)

W_o = displacement at center of circular plate

W_{io} = initial displacement at center of plate

' = also denotes initial imperfection

GREEK LETTERS

β = angle

γ = constant dependent upon geometry and load process

δ = deflection at an appropriate point

δ_o = initial deviation

κ_1 = constant dependent on initial imperfection shape

κ_2 = constant dependent on deformation mode

λ = wavelength in Dunn displacement function

μ = Poisson's ratio

Φ = airy stress function

$$\Delta^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} = \frac{1}{r}\frac{d}{dr} \quad r\,\frac{d}{dr}$$

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INTRODUCTION

The interpretation of test data obtained from instability studies is frequently regarded as a difficult problem. Actually, a powerful method exists for its solution. The method was first given for the strut,[1,2] but subsequent research has indicated its generality.[3,4] According to this method, the deflection at an appropriate point can be related to the classic and actual loads by the equation

$$\delta\left(\frac{P_{cr}}{P} - 1\right) = \delta_o \qquad (1)$$

This is the equation of a rectangular hyperbola; when plotted with δ/P and δ as variables, it gives a straight line (a Southwell Plot)[2] whose slope is the classic load. The technique is restricted to the interpretation of observations made over a limited range of the load-displacement curve; in particular, it is restricted to displacement corresponding to loads below the critical value. Thus, it is confined to behavior within the limitations of small displacement theories.

In an attempt to describe instability behaviour more adequately, analysts have developed large deflection theories. This report is concerned with interpretations made on the basis of such treatments. It is found, as will be demonstrated herein, that there is a general relationship, applicable in the postbuckling range, associating deflection with critical and actual load parameters. Because of the well-founded analysis of the buckling and postbuckling of a column, the demonstration starts from this point.

COLUMN STRUCTURES

The load-displacement relationship for a centrally compressed column is shown in Figure 1. In this diagram, the actual and ideal motions are portrayed. The theoretical large displacement curve is approximated by the actual load-displacement relationship when the deflections become large. Now, analytically, the relation between the load and the normal displacement of the midpoint of the columns is expressed by the following equation:[5]

$$P = \frac{4[\,F(K)\,]^2\,B}{L^2} \tag{2}$$

$$\frac{\delta}{L} = \frac{K}{F(K)} \tag{3}$$

where $K = \sin \frac{1}{2}\beta$ and $F(K)$ is the appropriate elliptic integral of the first type - β being the end slope. However, provided β is less than 25°, the complex relationship given by equations (2) and (3) can be well approximated by the following parabola:[5]

$$P = P_{cr}\,(1 + \gamma\delta^2) \tag{4}$$

where

$$\gamma = \frac{\pi^2}{8L^2} \tag{5}$$

This parabola can be plotted as a straight line if the variables are taken to be P and δ^2. This line then cuts the load axis at a point which corresponds to the critical value.

Mr. E. Way carried out a test on a slender column to verify this result[20]. His data are given in Figures 2 through 4. The critical load determined from the slope of the δ/P - δ line of Figure 3 is in excellent agreement with the critical value determined by the intercept of the P - δ^2 line with the load axis seen in Figure 4. Both values are in perfect accord with the classic value computed from the formula

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{6}$$

A similar analysis made upon a more robust column of T cross section* gives an identical result. This is shown in Figure 5. It is interesting to note that this column experienced some local yielding, which is clearly evidenced in the Southwell Plot,[7] Figure 6. Nevertheless, the agreement is still excellent - 7900 pounds from the Southwell Plot as opposed to 8050 pounds from the δ^2 plot, Figure 7.

The case of a single member in axial compression can readily be extended

*Test data from Hill, Reference 6.

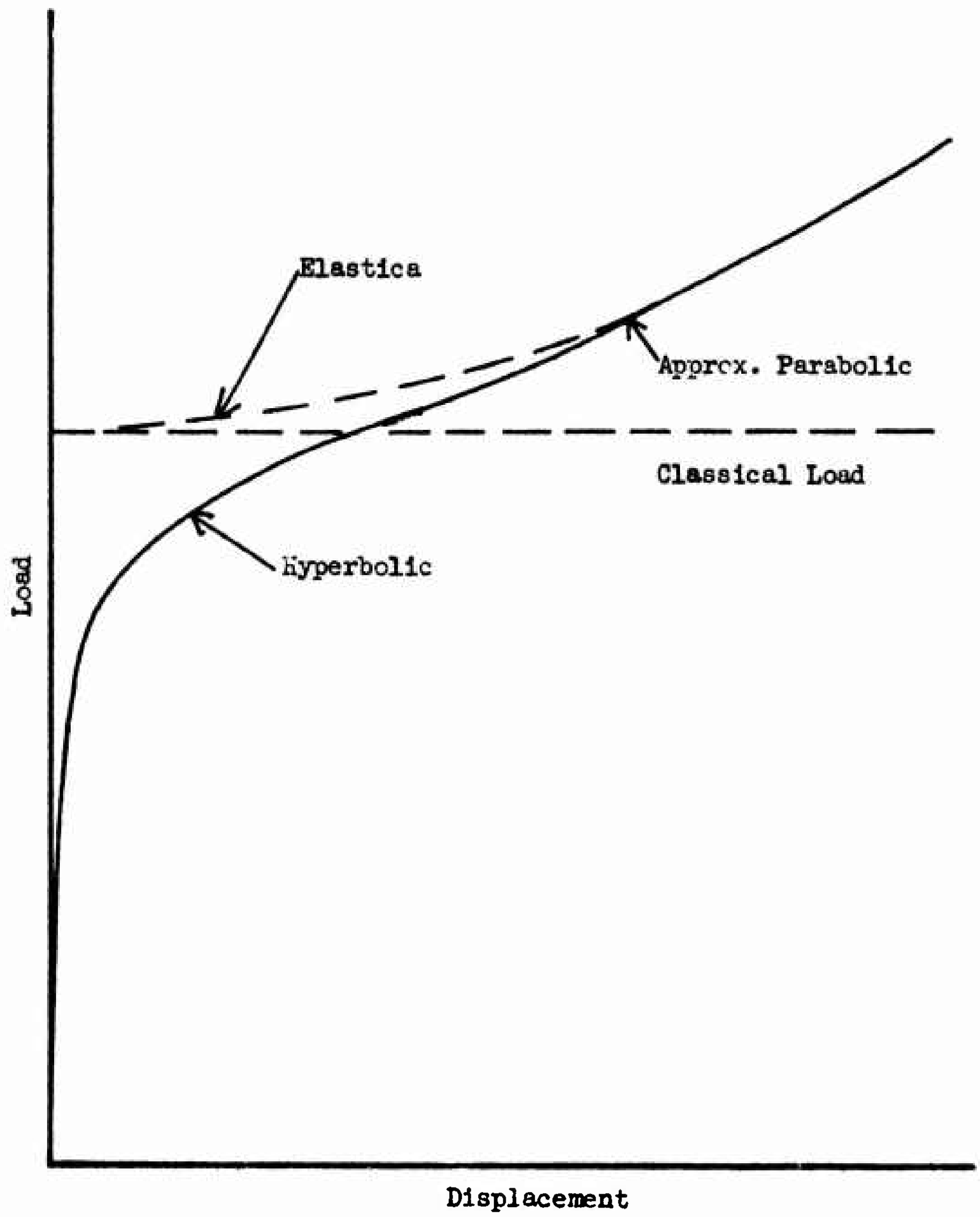

Figure 1. Column Data with Elastica Superposed.

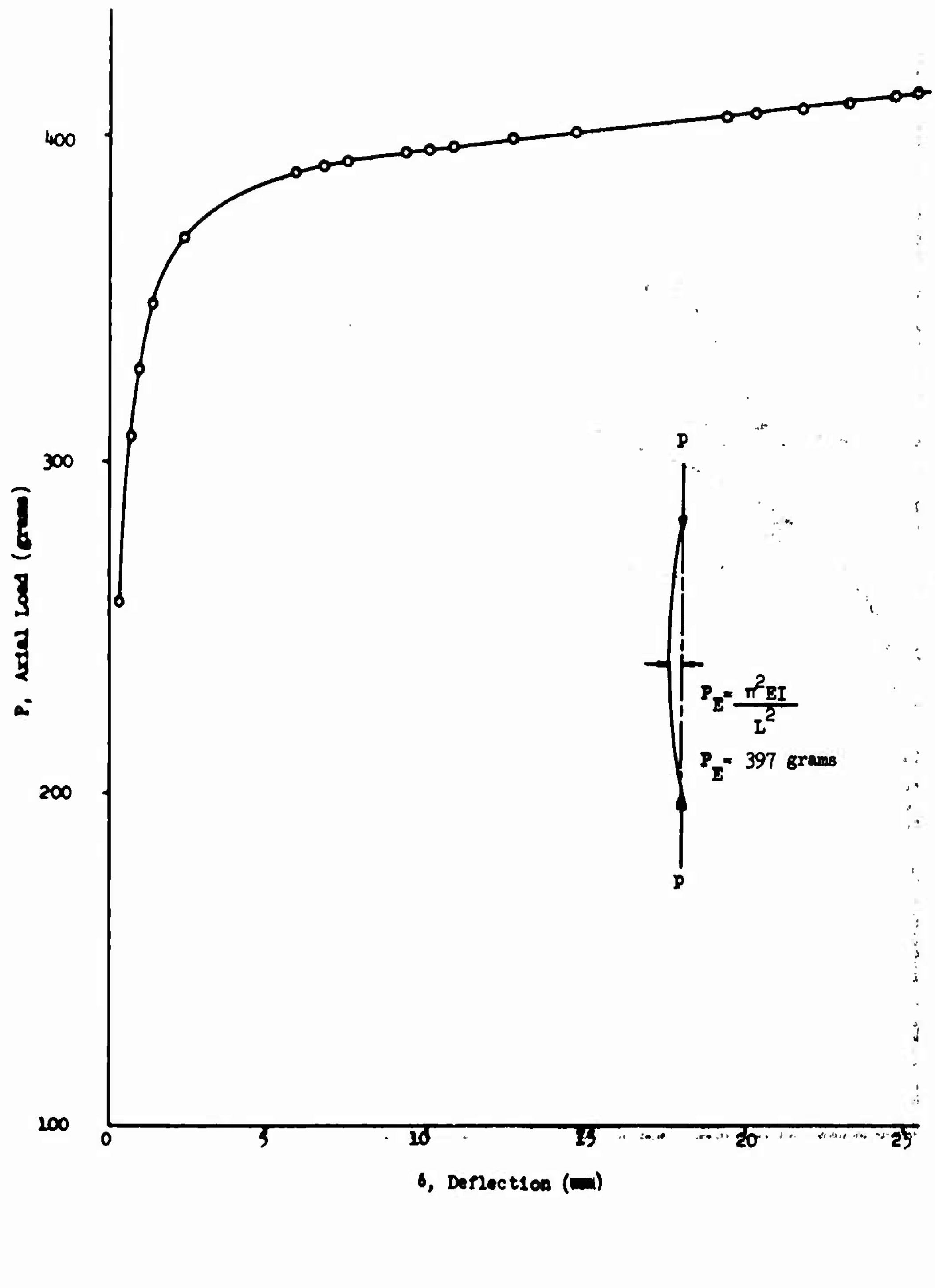

Figure 2. Load Deflection Plot of a Slender Column.

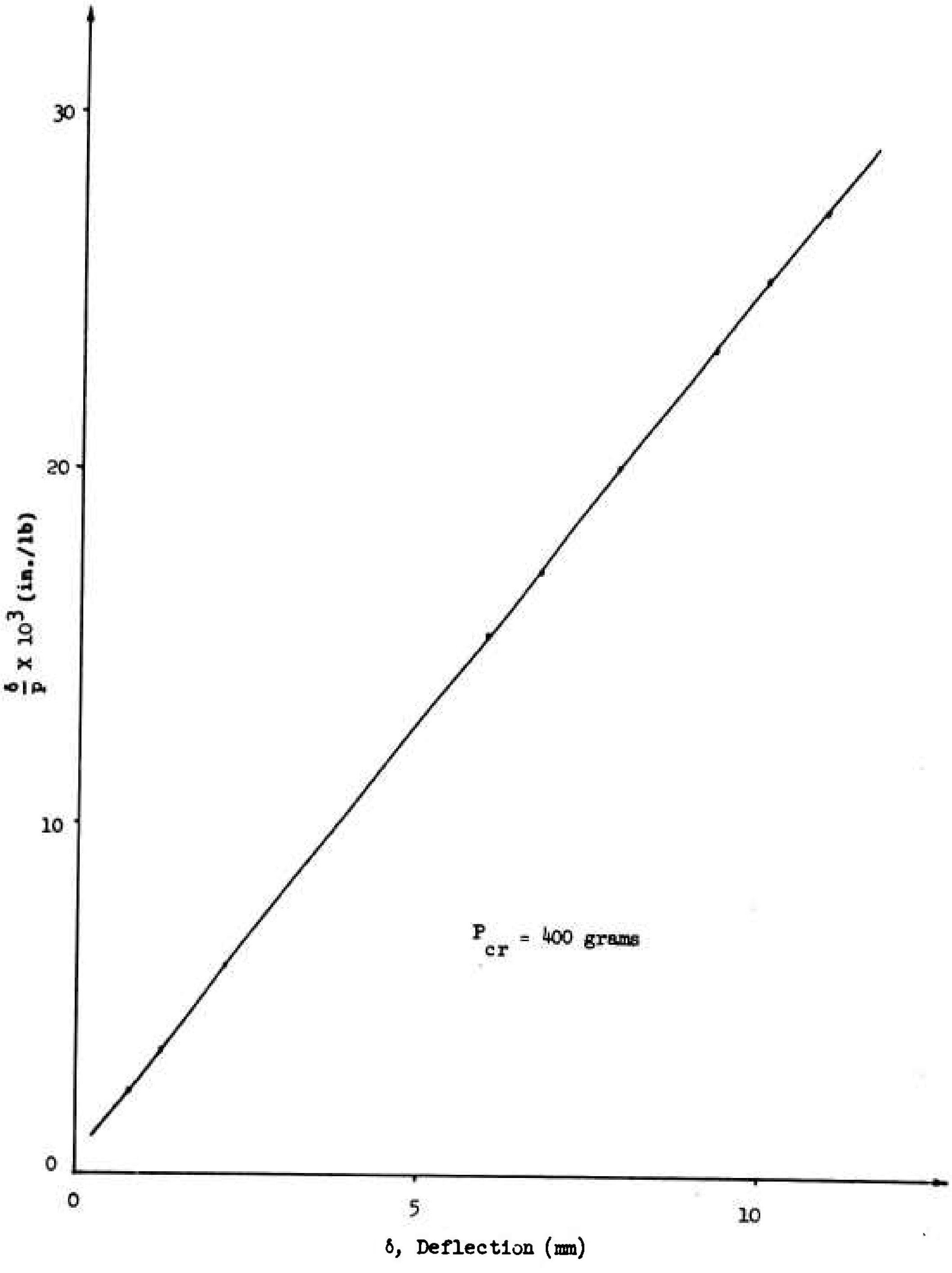

Figure 3. Southwell Plot for Column Data of Figure 2.

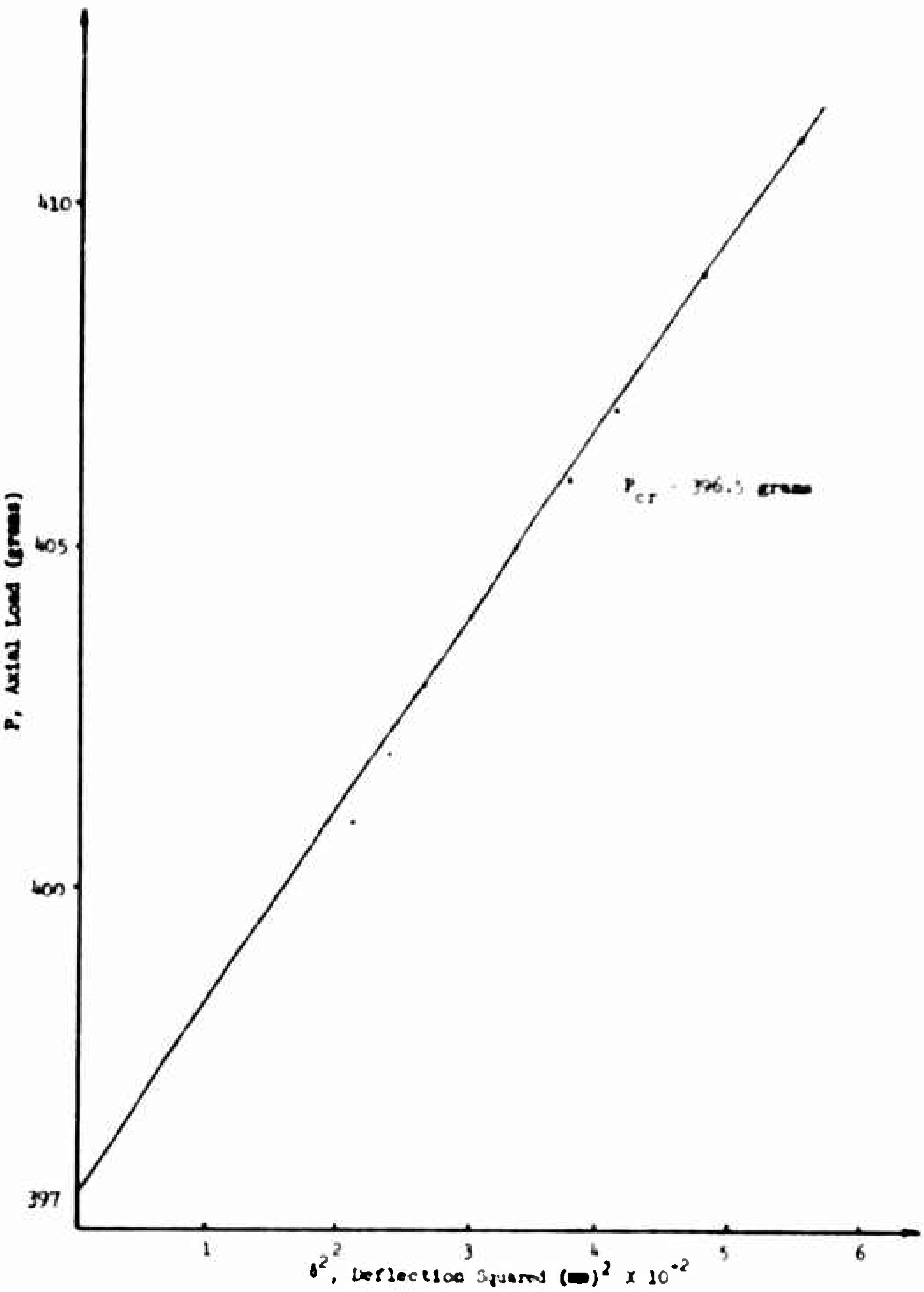

Figure 4. Plot of P versus δ^2 for Column Data of Figure 2.

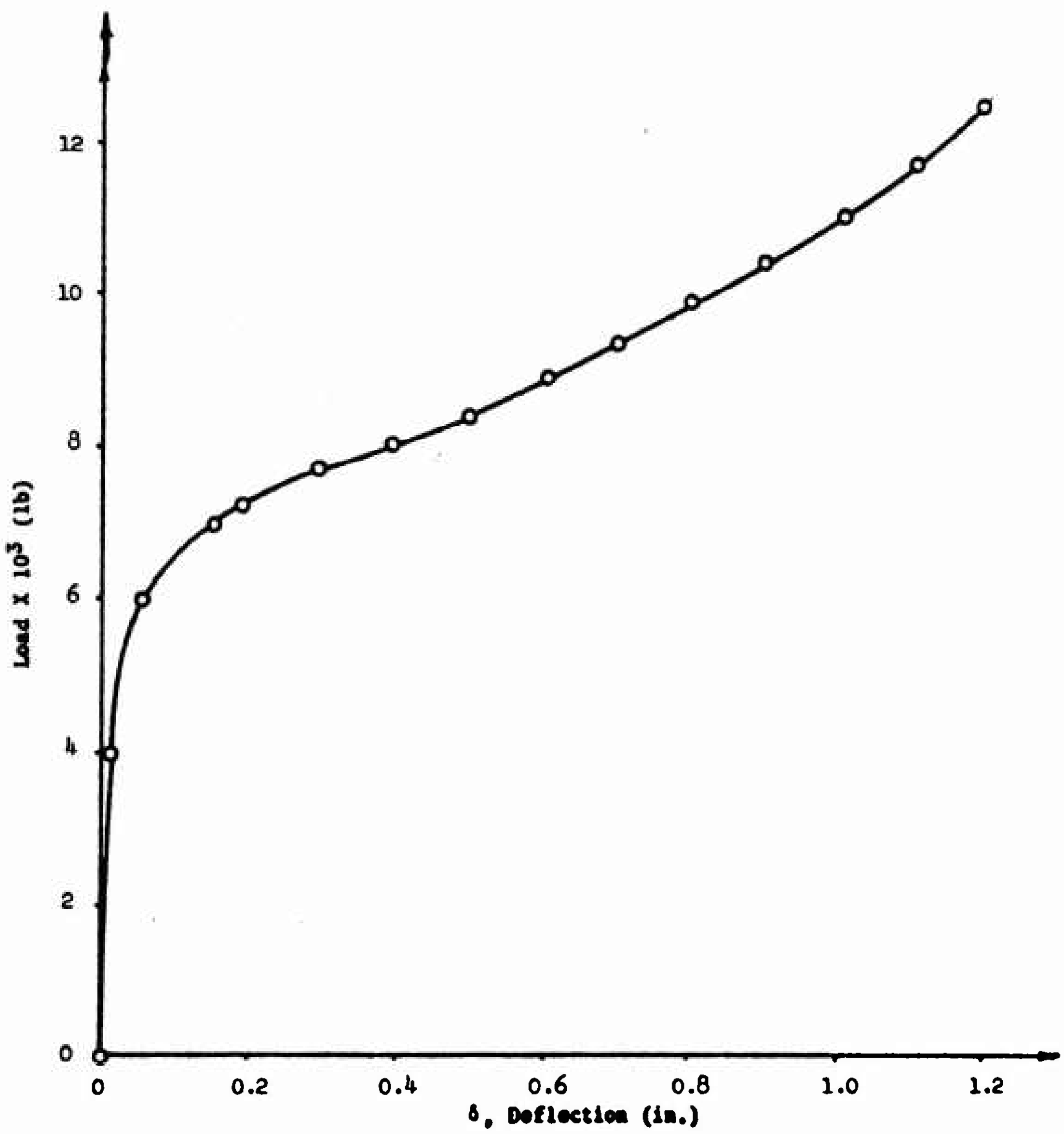

Figure 5. Test Data from Hill, an Aluminum Column Buckled Well Into Postbuckling Range, Reference 6.

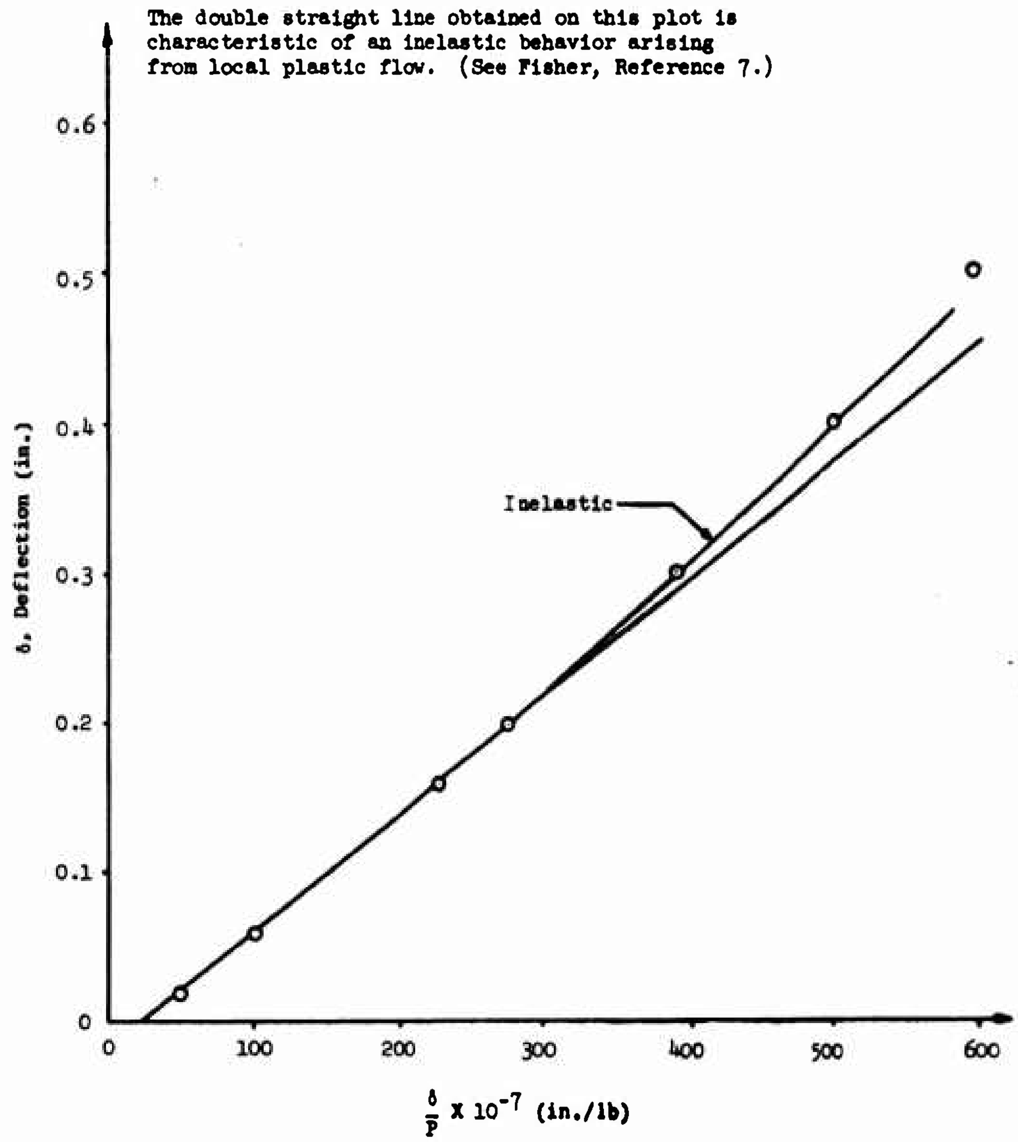

Figure 6. Southwell Plot for Column Data of Figure 5.

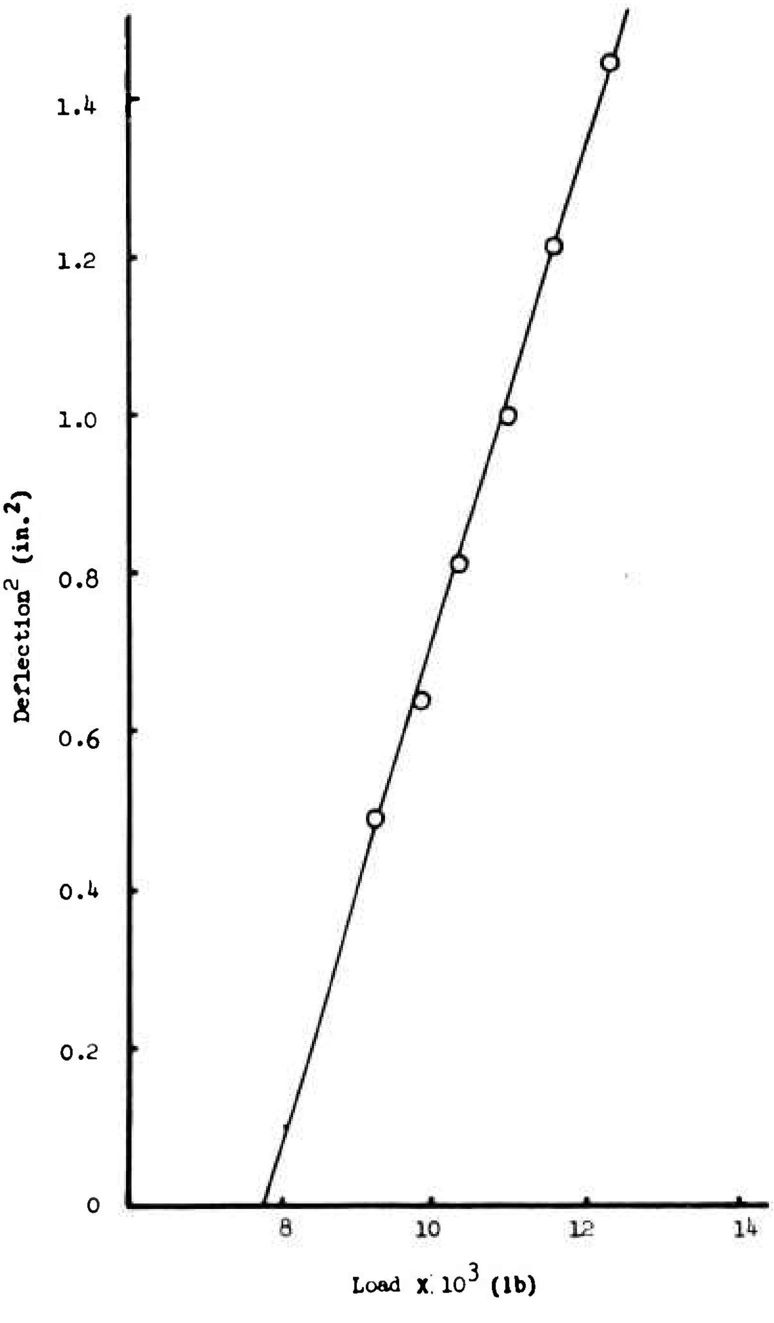

Figure 7. δ^2 Plot for the Column Data of Figure 5.

to that of the structure composed of such members. For the Southwell-type Plots, this was established by the general analysis of Westergaard[8] and by the more specific analysis and tests of Gregory.[9,10] Recently, Britvec and Chilver[11] and Roorda[12] have published papers which indicate the full applicability of the $P - \delta^2$ process for this class of structure. Their demonstrations are based upon generalized analysis and experiment. They give many examples and deal with a wide range of structural configurations. The example we have chosen to illustrate this point is taken from the work of Roorda[12] and is for the case of a two-bar frame. The details of the loading and the load-displacement relationships for clockwise and counterclockwise instabilities are given in Figure 8. When the appropriate δ/P versus δ and $P - \delta^2$ curves, Figures 9 and 10, are constructed, it is seen that excellent agreement is reached.

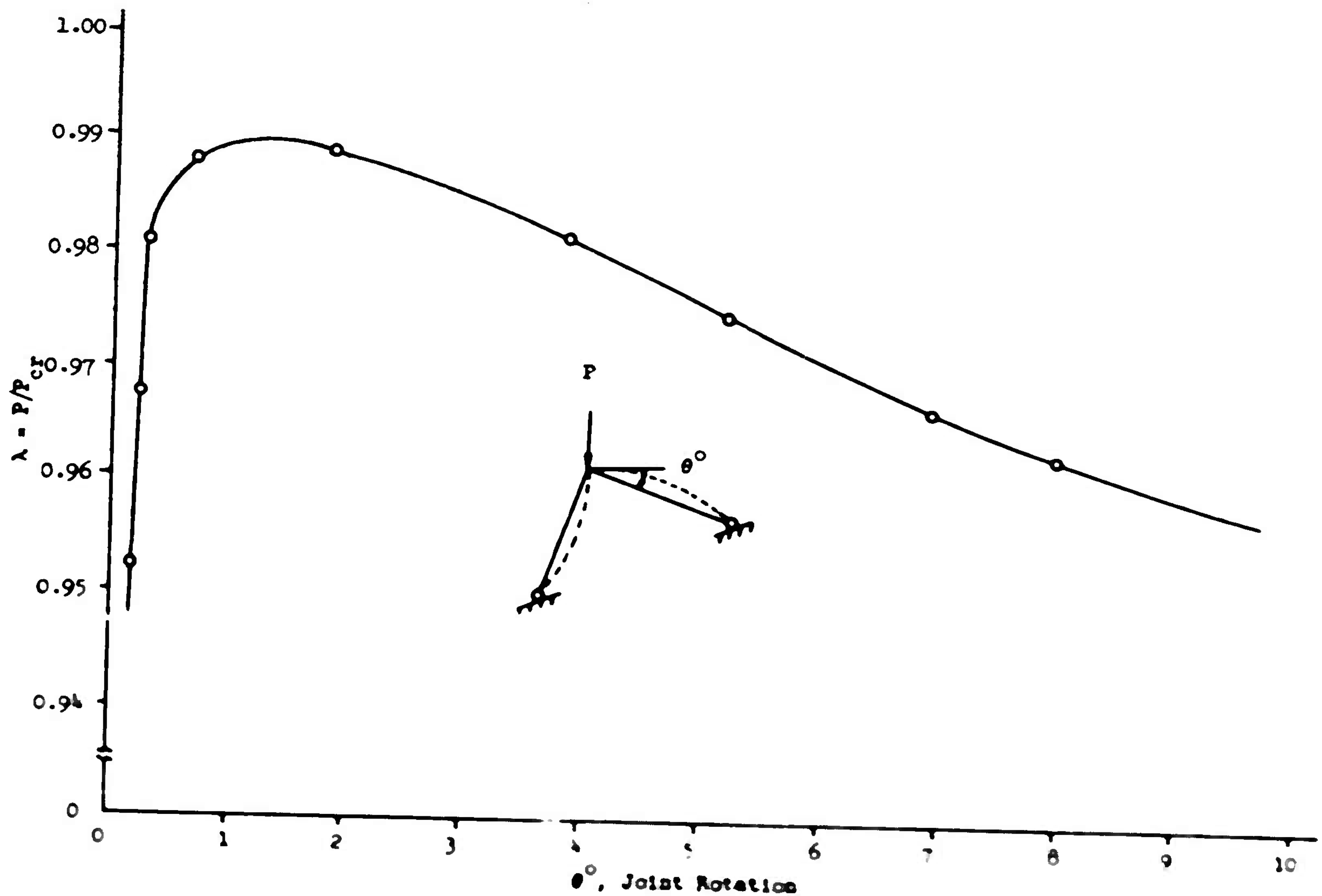

Figure 8. Load Versus Joint Rotation for Two-Bar Frame, Figure 6b of Reference 12.

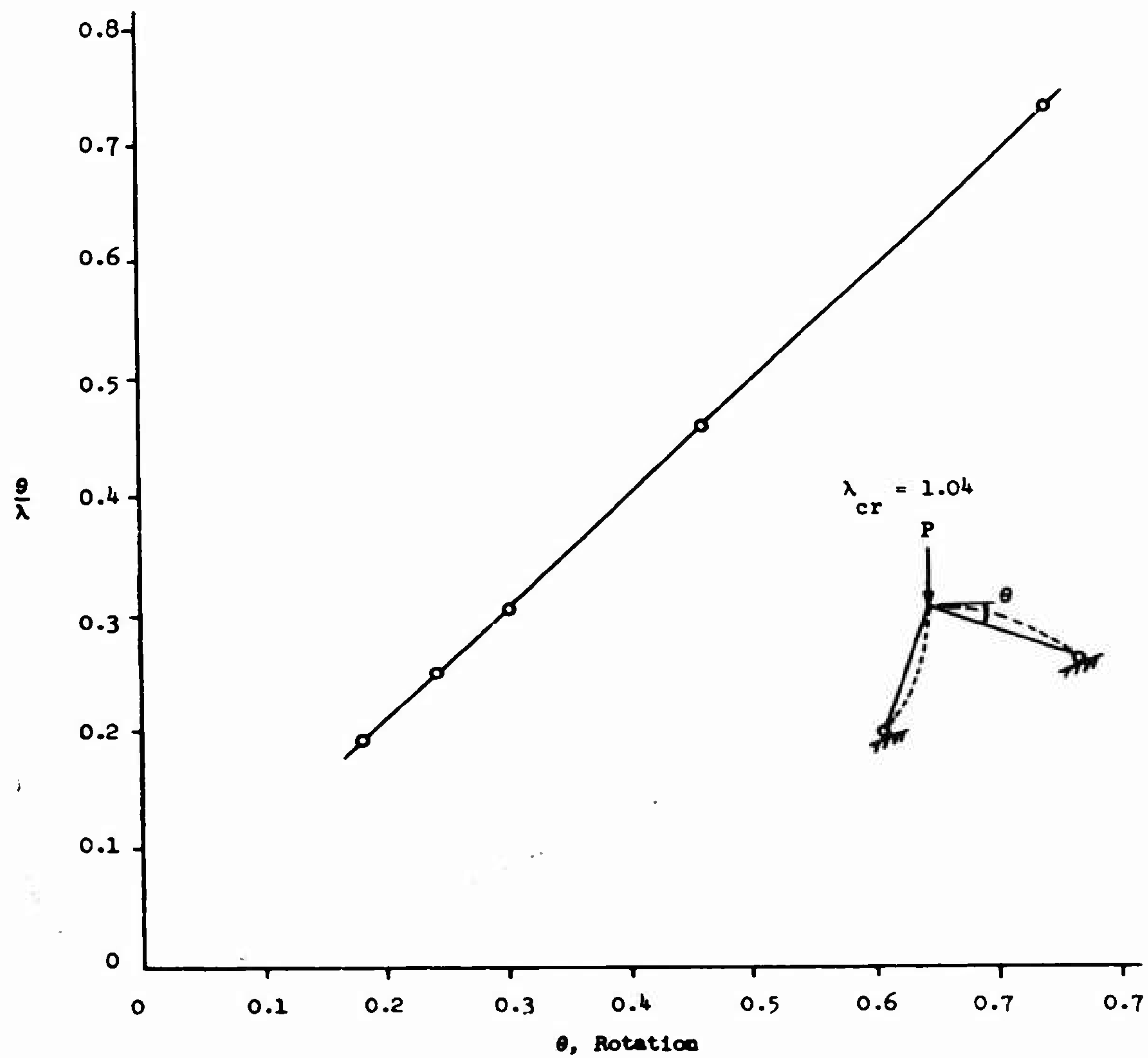

Figure 9. Southwell Plot for Data of Figure 8.

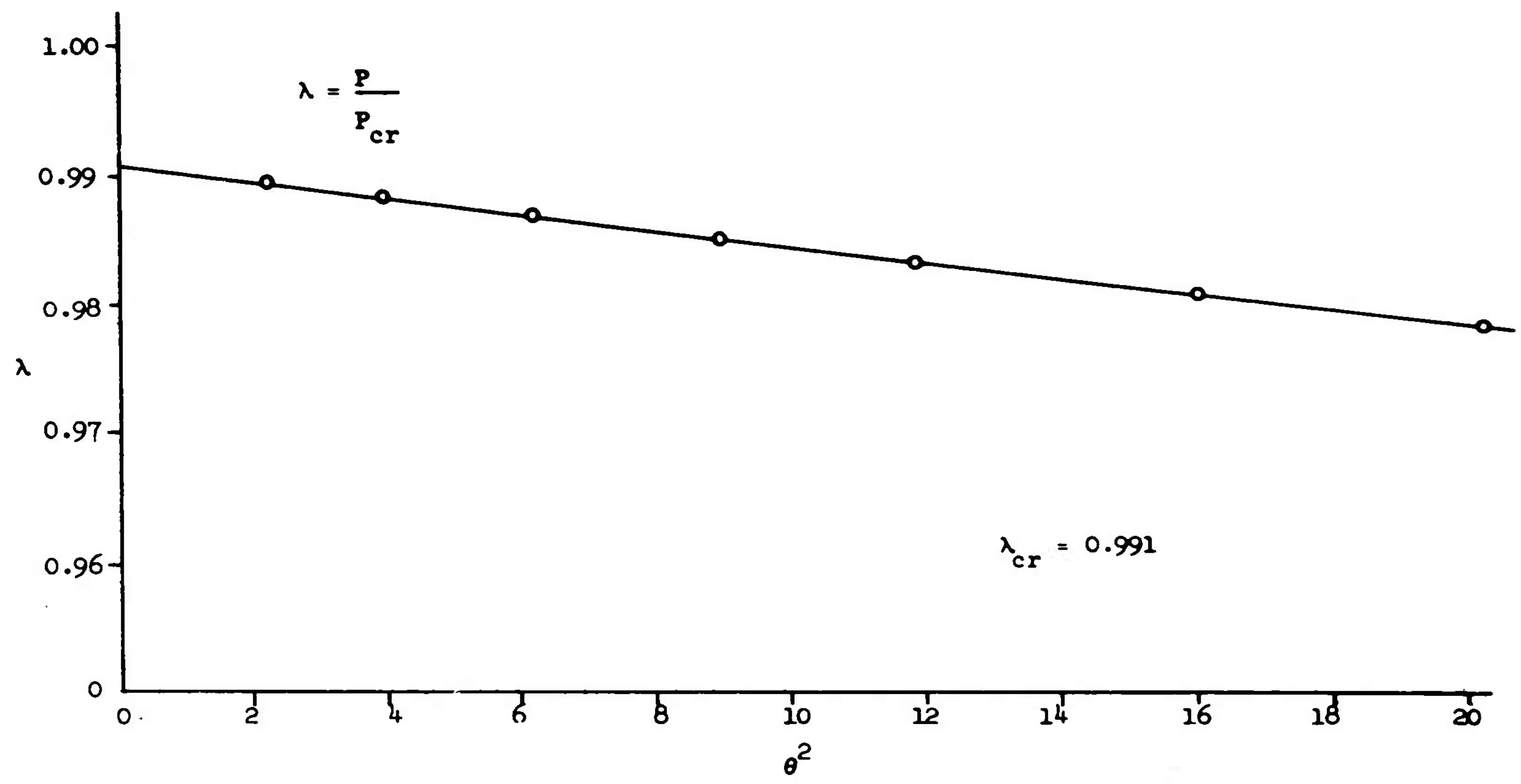

Figure 10. λ Versus θ^2 for Data of Figure 8.

PLATE STRUCTURES

Although the elastica curve has been well-known for most of this century, it must be recorded that the first use of the P - δ^2 method was made by Dunn[13] for plate studies. His analytical derivation was simplicity itself.

He wrote a functional relationship for P, the applied load, and for the displacement parameter f_o/λ. He argued that since P is independent of the direction in which the sheet buckles, P must be an even function of f_o/λ. Thus, we may write a Taylor series:

$$P = P_o + \frac{P''}{2!}(f_o/\lambda)^2 + \frac{P''''}{4!}(f_o/\lambda)^4 \tag{7}$$

Hence, if f_o/λ is defined as u and the coefficients $\frac{P''}{2!}$, $\frac{P''''}{4!}$, etc., are defined as A, B, C, etc.,

$$P = P_o + Au + Bu^2 + \tag{8}$$

Thus, for small values of u, the resulting curve is very nearly a straight line and the intercept with the load axis should be the critical load.[13]

Dunn applied his method to the analysis of data which he had obtained from compression tests on flat panels restrained by torsionally weak stiffeners. An example of his results is shown in Figure 11. No correlation was made with the Southwell process. It was, in fact, common contention at that time that the latter method is inapplicable.

Following the research of Dunn,[13] Farrar[14] made use of the method for interpreting test data on plates under axial compression. He was concerned with plates both simply supported and restrained. The next application appears to have been that of Munk, as referenced by Hemp and Griffin.[15] So far as we are aware, no other application of the method was made until recently.

It is interesting to note, however, that Donnell,[16] in his classic paper on the application of the Southwell method performed an analysis from which both the δ^2 and the δ/P versus δ methods can be obtained. He showed by the application of large displacement analysis to the square plate in compression that the relationship between the displacement functions is

$$P = P_{cr}\frac{\delta}{\delta + \delta_o}\left[1 + \frac{3(1-\mu^2)}{8t^2}(\delta + 2\delta_o)(\delta + \delta_o)\right] \tag{9}$$

If the stipulation is made that $\frac{\delta}{t}$ and $\frac{\delta_o}{t}$ should be small, then this equation reduces to

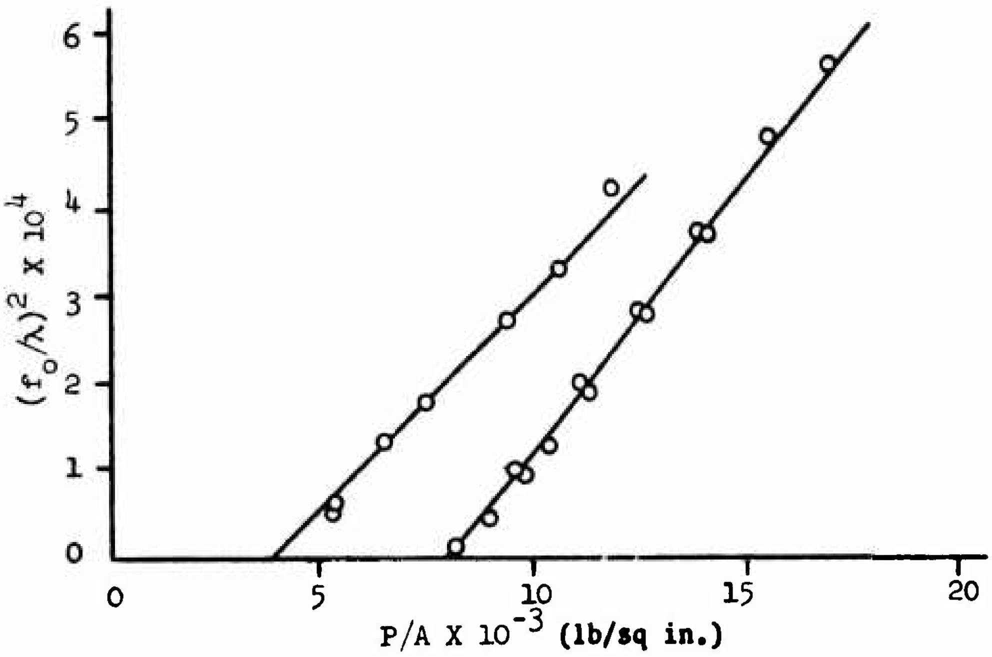

Figure 11. Dunn's Experimental Curves for Determining the Buckling Stress of the Sheet.

$$P = P_{cr} \frac{\delta}{\delta + \delta_o} \tag{10}$$

Now equation (10) is clearly identical to equation (1), the small displacement equation. When the condition that δ_o/δ be small is imposed, equation (9) simplifies in a different fashion to become

$$P = P_{cr} (1 + \gamma\delta^2) \tag{11}$$

where

$$\gamma = \frac{3(1-\mu^2)}{8t^2} \tag{12}$$

This is the large displacement formulation. It is identical in character to the parabolic approximation to the elastica.

The validity of the two relationships for the panel in axial compression was demonstrated in a series of tests made on square fiber glass resin panels. A typical load-displacement curve is given in Figure 12. When the appropriate $\delta/_P$ - δ and P - δ^2 curves are constructed, Figures 13 and 14, the linear relationship is clearly seen for both cases. Moreover, the values of P_{cr} determined from the slope and intercept, respectively, are in good agreement.

The influence of initial imperfection is readily demonstrable for this case. A panel similar to that previously described was tested with a normal force applied at its center. The several load-displacement curves are portrayed in Figure 15. The δ/P - δ and P - δ^2 curves which correspond to these load displacements are given in Figures 16 and 17. We note that the Southwell lines are parallel, indicating that the critical load is not influenced by the imperfection. The lines on the P - δ^2 plot are also seen to intersect on the P axis, indicating that the critical loads determined by the large displacement method are the same. A comparison of the values determined by the two methods indicates that they are in excellent agreement.*

It has already been shown experimentally that the Southwell Plot applies to plates in shear.[17] The applicability of the large displacement process will now be demonstrated analytically and experimentally.

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*In the experiments, bending strains were used instead of normal displacements. In this class of problem, since the units of the displacement measurement are irrelevant, the two parameters are in 1:1 correspondence.

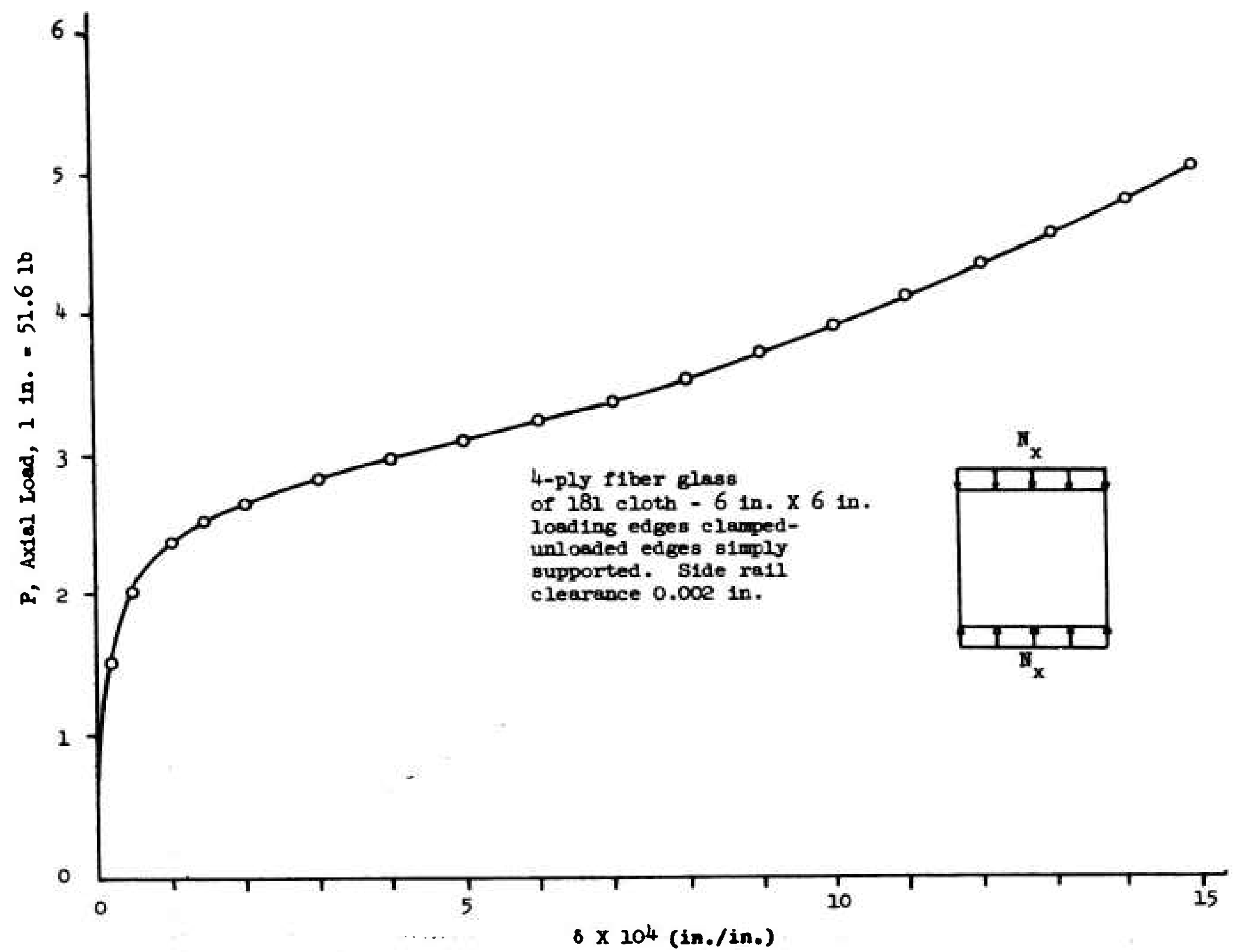

Figure 12. Axial Load Versus Bending Strain.

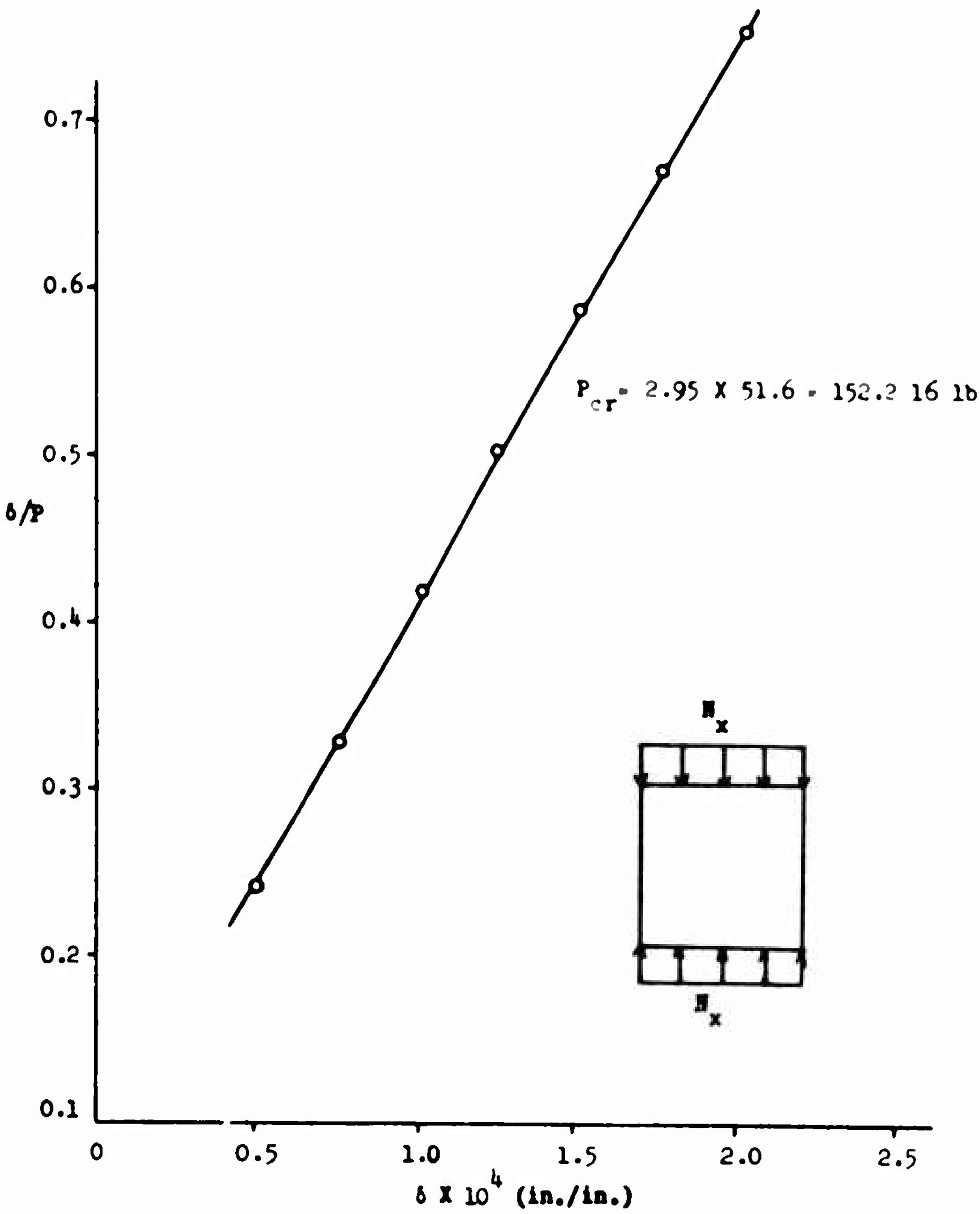

Figure 13. Southwell Plot for Data of Figure 12.

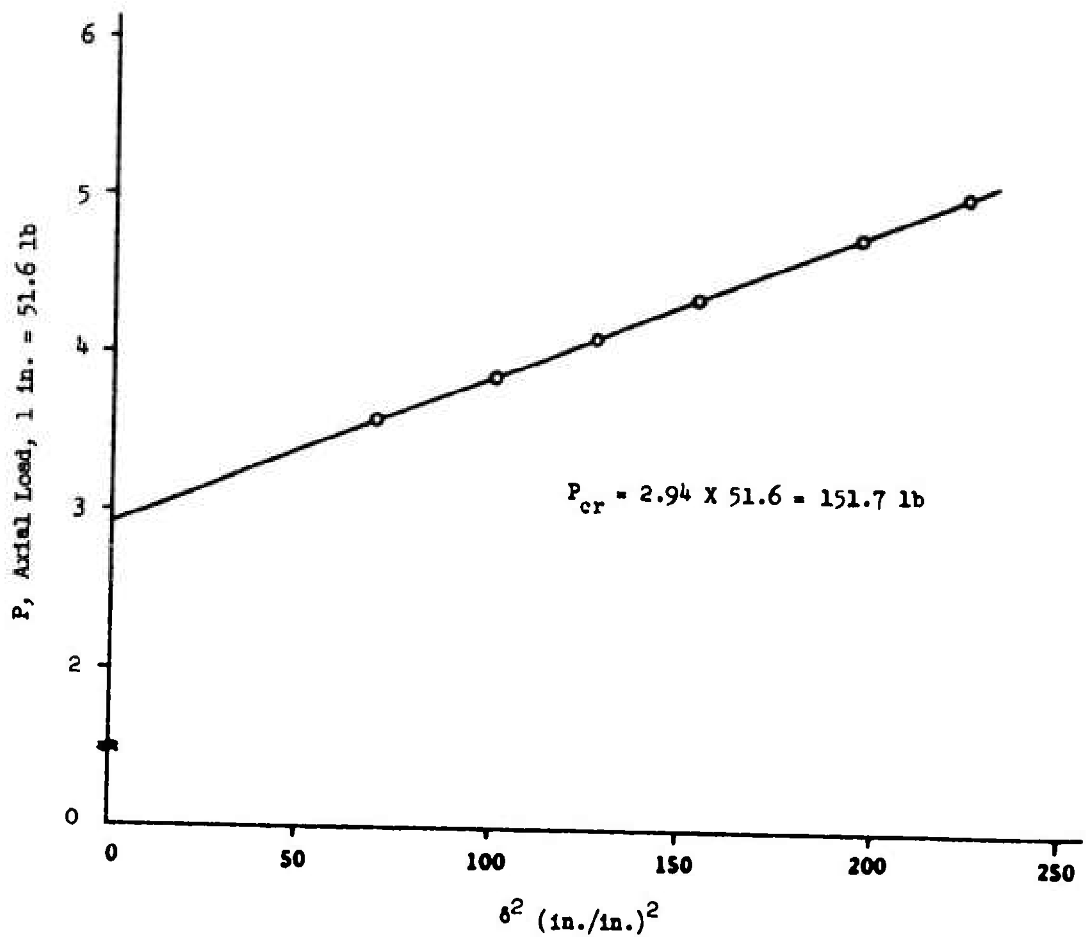

Figure 14. P Versus δ^2 for Data of Figure 12.

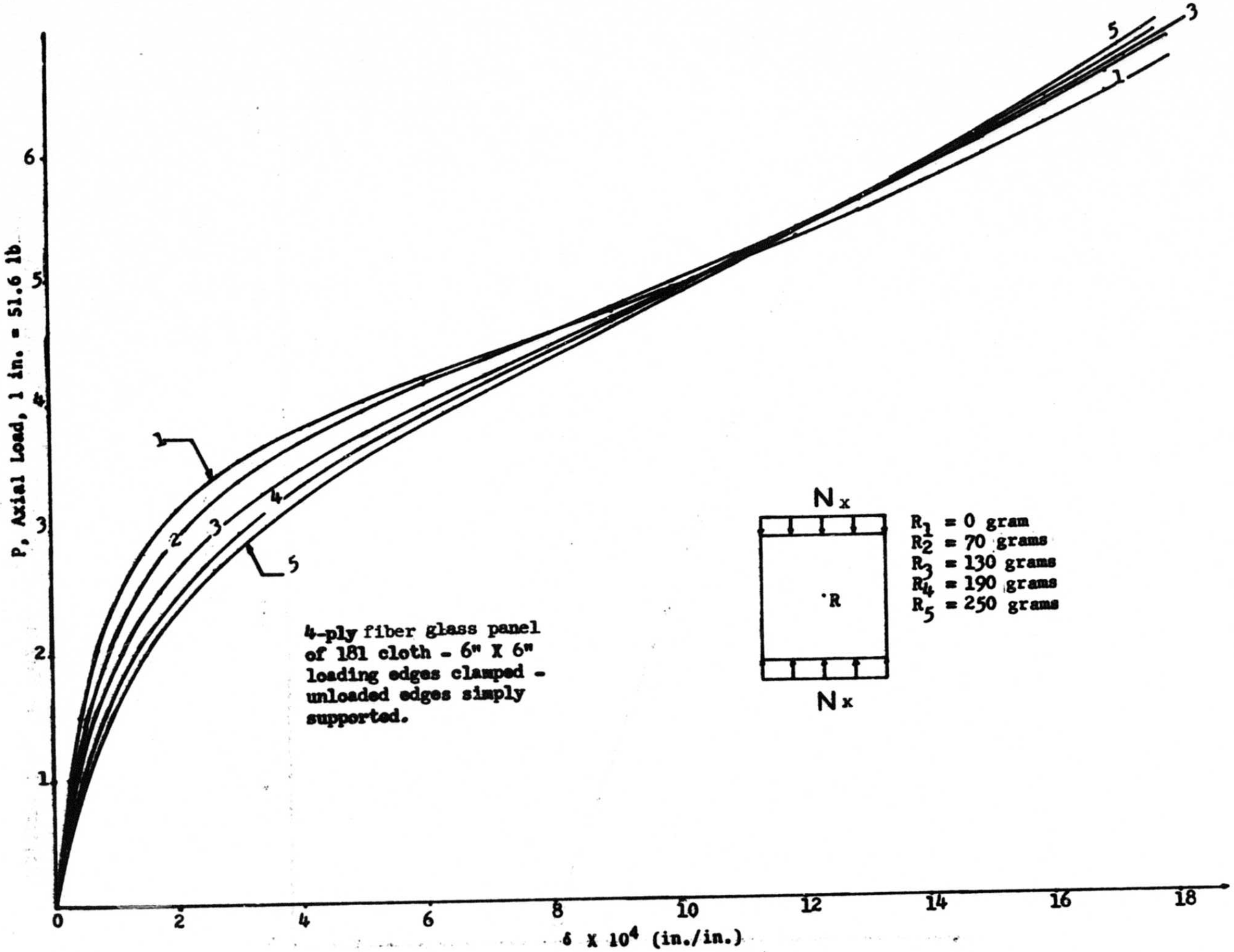

Figure 15. Load Versus Bending Strain for Tests on a Fiber Glass Panel Under Uniaxial Compression and Lateral Load Applied at Center.

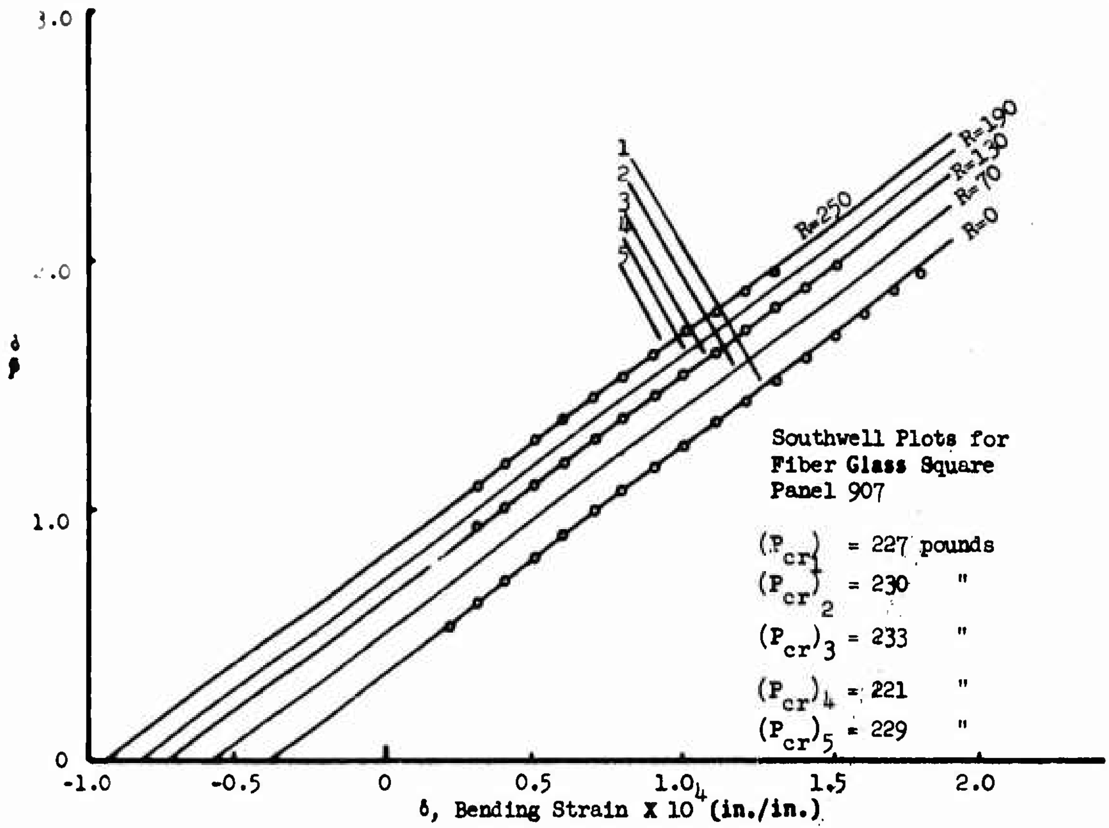

Figure 16. Southwell Plot for Data of Figure 15.

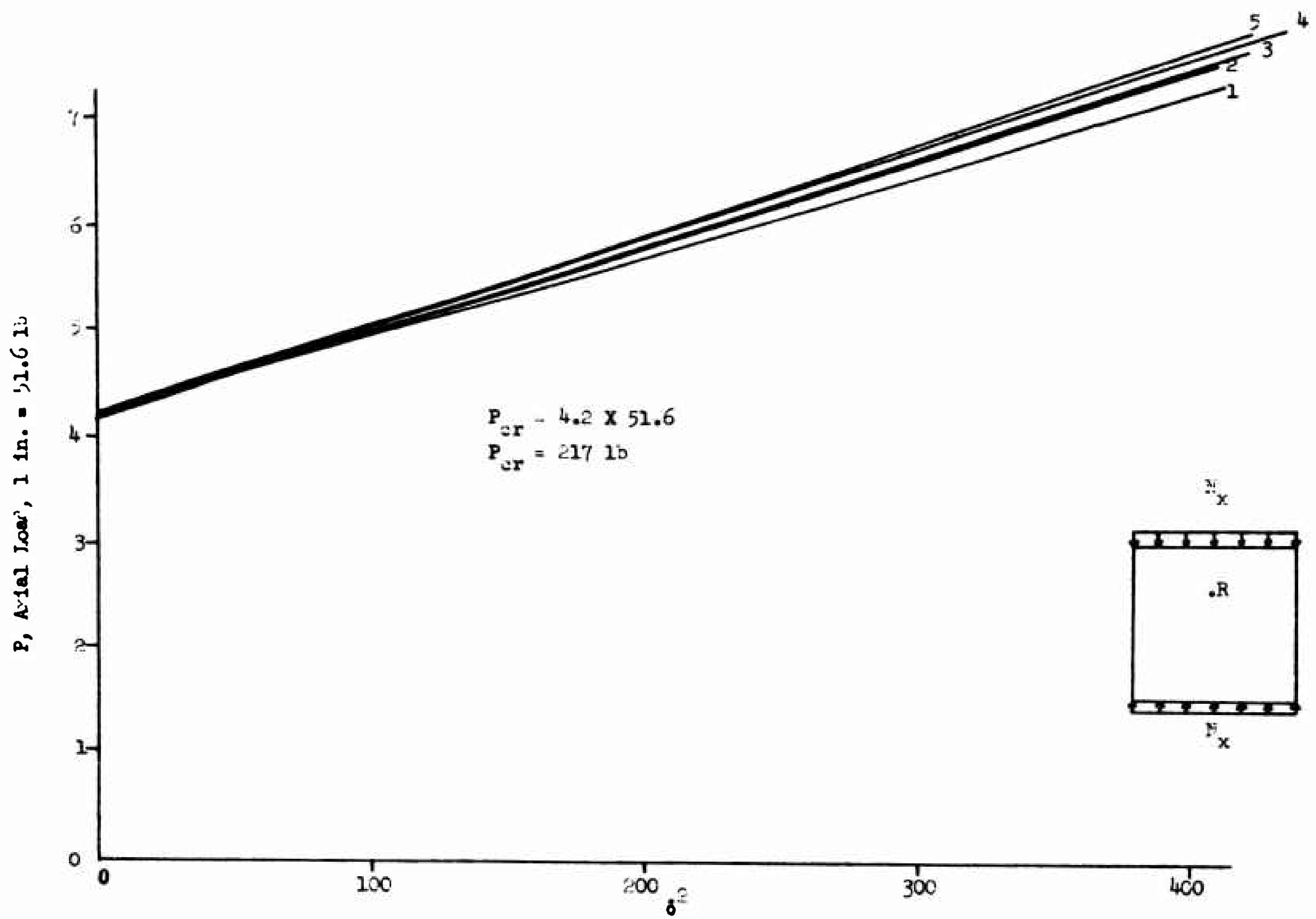

Figure 17. P Versus δ^2 for Data of Figure 15.

The analysis is based upon the Karman-Donnell finite deflection theory for which the compatibility equation is

$$\frac{1}{Et} \Delta^4 \phi = \left(1 + 2\frac{W'}{W}\right)\left[\left(W,_{xy}\right)^2 - W_{xx}, W_{yy}\right] \tag{13}$$

for the imperfect plate. The assumption will be made that the elastic lateral deflection due to load and the initial imperfection shape are given by

$$W = \sum\sum A_{mn} \sin\frac{m\pi x}{L} \sin\frac{n\pi y}{b} \tag{14}$$

$$W' = \sum\sum A'_{mn} \sin\frac{m\pi x}{L} \sin\frac{n\pi y}{b} \tag{15}$$

Thus, the internal strain energy U can be expressed as

$$U = \int_0^b\int_0^L \tfrac{1}{2}\left[D\,(\Delta^2 w)^2 + \frac{1}{Et}\,(\Delta^2\phi)^2\right] dxdy \tag{16}$$

for the simply supported plate.

It can be shown from equations (13) through (15) the equilibrium in the place of the plate is satisfied when

$$\Delta^2\phi = \frac{Et\pi^2}{8L^2b^2}\sum_m\sum_n m^2n^2 A_{mn}(A_{mn} + 2A'_{mn})\left(\frac{L^2}{m^2}\cos\frac{2m\pi x}{L} + \frac{b^2}{n^2}\cos\frac{2n\pi y}{b}\right) \tag{17}$$

Thus, from equations (14) through (17), it follows that

$$U = \sum_m\sum_n \tfrac{1}{2} D\left(A_{mn}\right)^2\left[+\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]\frac{Lb}{4} + \left(\frac{Et}{64}\frac{\pi}{Lb}\right)^4 \frac{Lb}{4}\left[\frac{L}{m}^4 + \frac{b}{n}\right]^4 (A_{mn})^2(A_{mn} + 2A'_{mn})^2 \tag{18}$$

The potential energy V of the shear N_{xy} is

$$V = \tfrac{1}{2} N_{xy}\int_0^b\int_0^L (w + W'),_x(W + w'),_y dxdy \tag{19}$$

By substituting equations (14) and (15) into equation (19) and carrying out the integrations, we get

$$V = 4N_{xy}\sum_m\sum_n\sum_p\sum_q (A_{mn} + A'_{mn})\,(A_{pq} + A'_{pq})\frac{mnpq}{(p^2 - m^2)\,(n^2 - q^2)} \tag{20}$$

where m, n, p, and q are integers such that $m \pm p$ and $n \pm q$ are odd numbers.

When the normal variational principle is applied, we deduce after appropriate algebraic manipulation that

$$N_{xy} = A_{mn} \frac{\dfrac{D\pi^4 Lb}{4}\left[\dfrac{m^2}{2} + \dfrac{n^2}{b^2}\right]\left[1 + \dfrac{3(1-\mu^2)}{8t^2}\dfrac{L^4n^4 + b^4m^4}{(m^2b^2 + n^2L^2)^2}\right](A_{mn} + 2A'_{mn})(A_{mn}+A'_{mn})}{8\sum\limits_p\sum\limits_q (A_{pq} + A'_{pq})\dfrac{mnpq}{(p^2 - m^2)(n^2 - q^2)}} \tag{21}$$

From equation (21), it can be shown that

$$N_{xy} = \frac{A_{mn}}{A_{mn} + A'_{mn}} (N_{xy})_{cr}\left[1 + \frac{3(1-\mu^2)}{8t^2}\frac{L^4n^4 + b^4m^4}{(m^2b^2 + n^2L^2)^2}(A_{mn} + 2A'_{mn})(A_{mn} + A'_{mn})\right] \tag{22}$$

The experimental work to verify this theory is taken from a study made by Gerrard.[18] The shear stress versus bending strain diagram is shown in Figure 18. The corresponding small and large displacement plots are given in Figures 19 and 20. The critical stresses derived from these curves are in the ratio of 1:0.988, being 4.66 ksi and 4.55 ksi, respectively. This experimental value is 93 percent of the theoretical value.

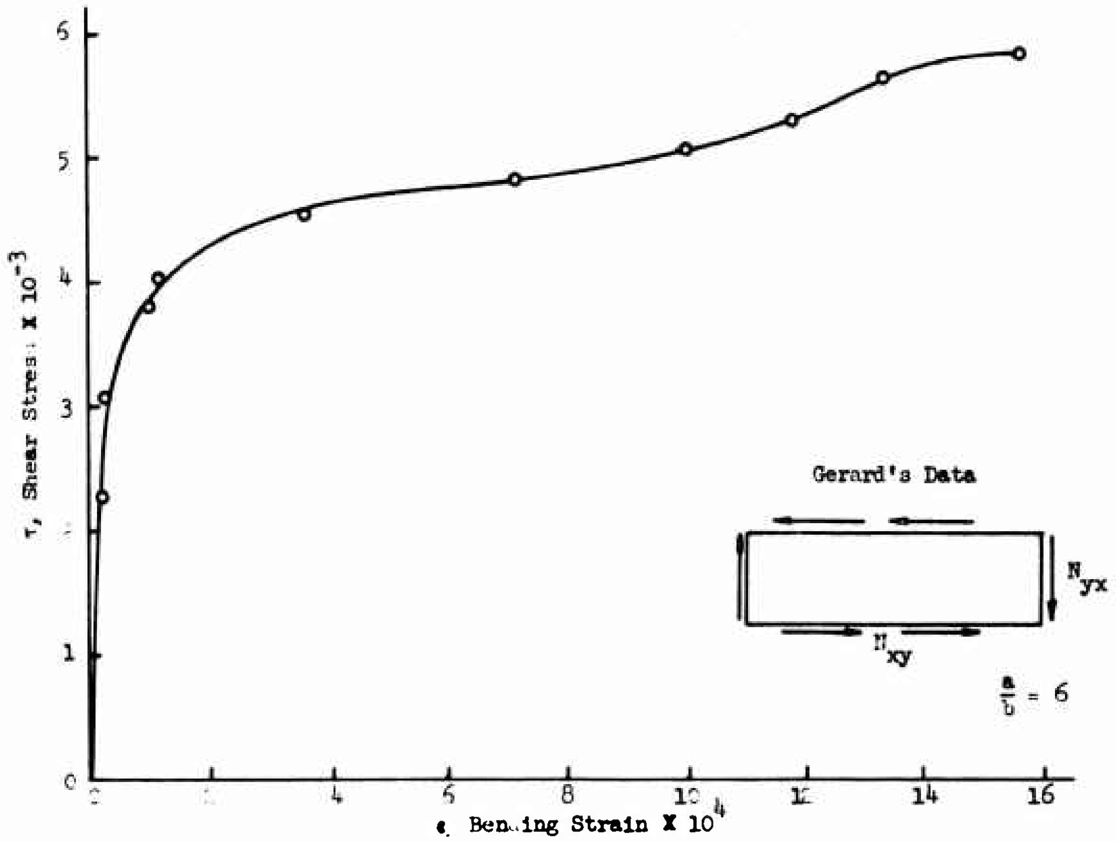

Figure 18. Shear Stress Versus Bending Strain for a Rectangular Plate, Reference 18.

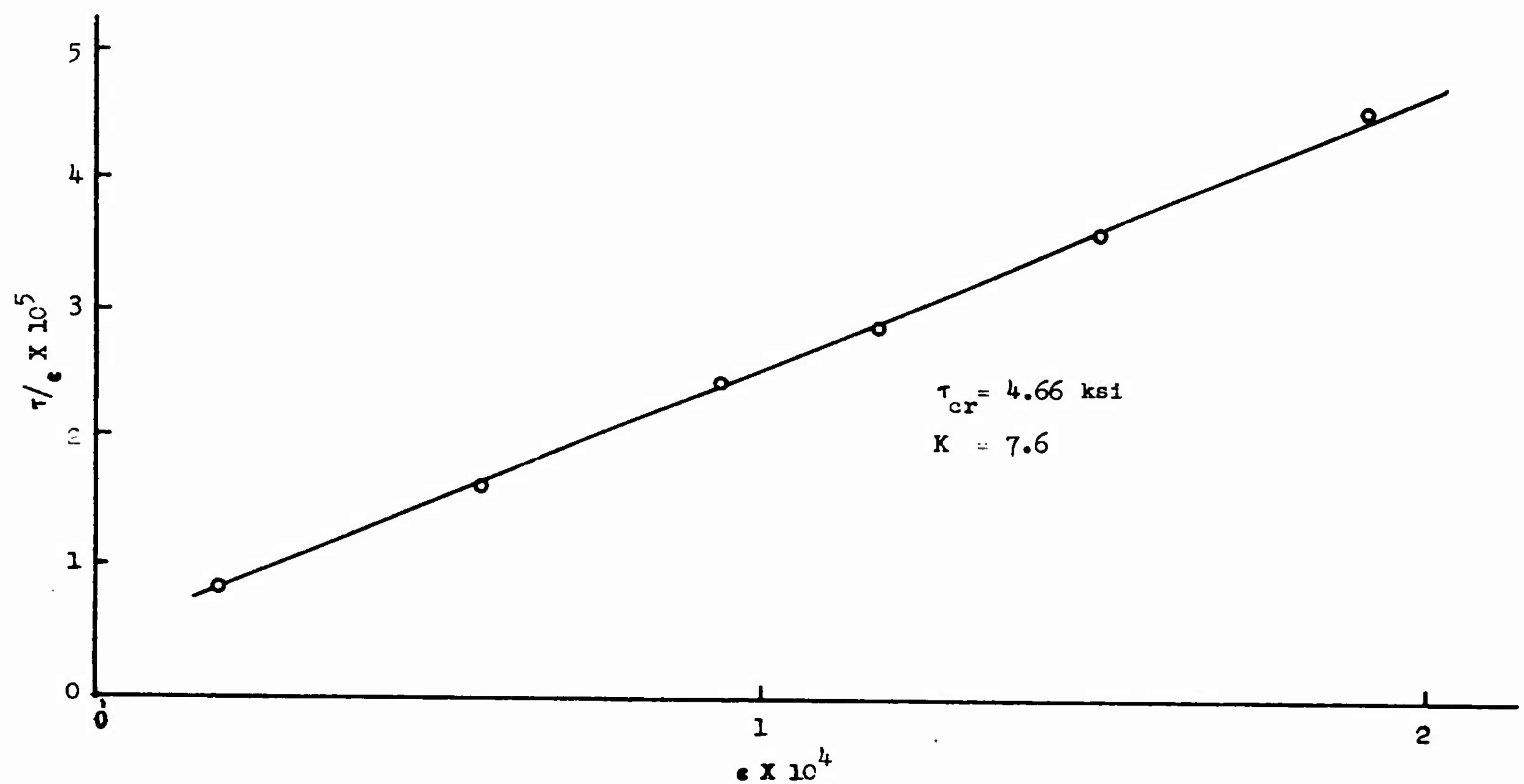

Figure 19. Southwell Plot for Data of Figure 18.

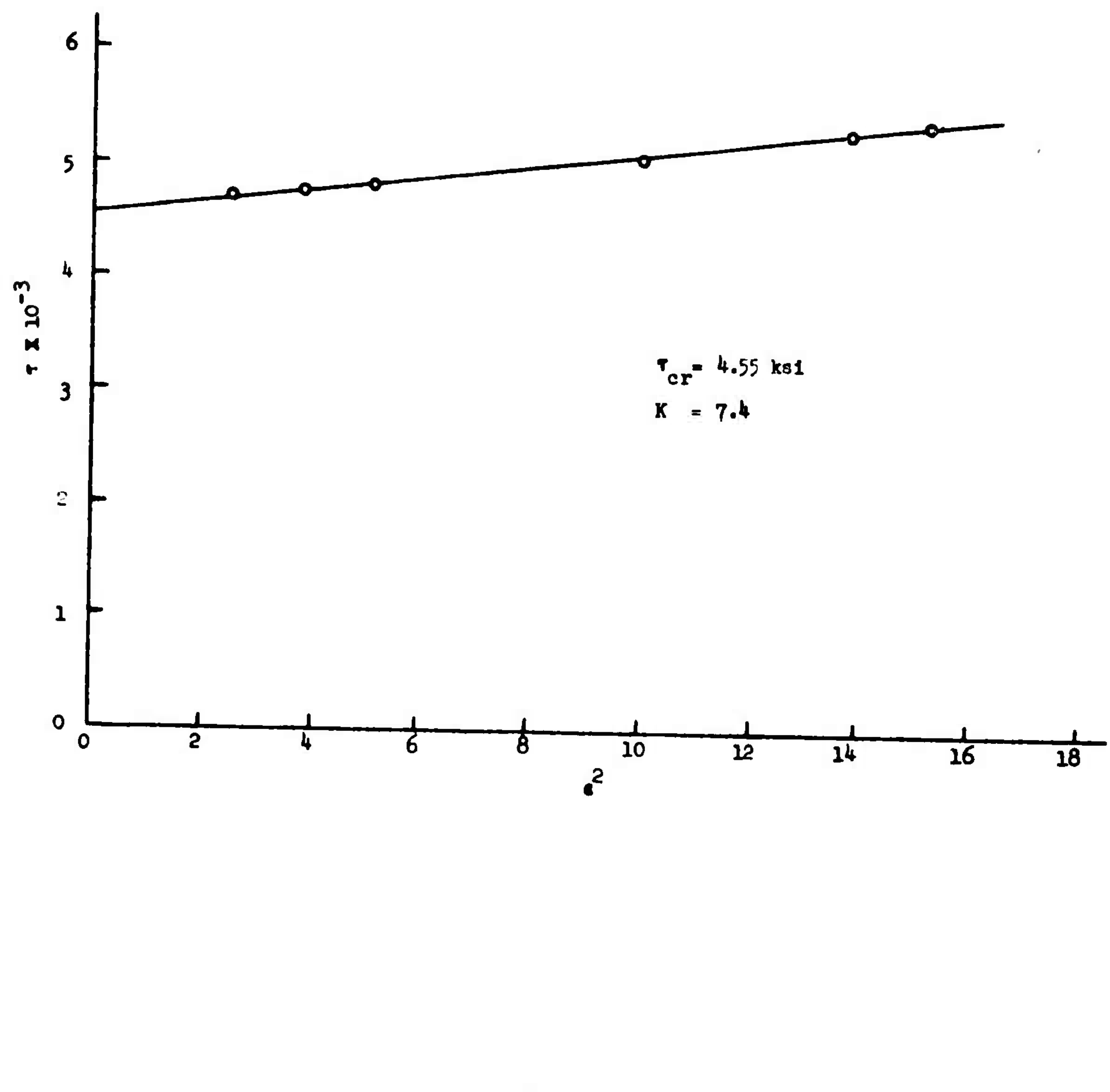

Figure 20. τ Versus ϵ^2 for Data of Figure 18.

THERMAL BUCKLING OF PLATES

The method can also be applied in the case of thermal buckling of a circular plate with a central hot spot. This question was first discussed by Queinec[19], whose analytical results and confirmatory test data are given here. In the analytical study, Queinec assumed the temperature distribution and the edge conditions were perfectly axis-symmetric. In the initial condition, the plate was considered at uniform temperature and free from initial stress. The effects of gravity were ignored as was the variation in temperature through the thickness of the plate.

He concluded from his analysis that if the displacements were small and the initial imperfection shape and the final deformation mode were similar, the displacement at the center could be related to the initial imperfection amplitude by the equation

$$W_o = \frac{W_{1o}}{\frac{T_{crit}}{T_o} - 1} \tag{23}$$

This expression is very similar to the Southwell equation for the strut. On the other hand, if the initial and buckled shapes were different, then the relationship is

$$\frac{T_{crit}}{T_o} = 1 + K_1 \cdot \frac{W_{1o}}{W} \tag{24}$$

where K_1 is a coefficient whose values depend on the initial shape.

The analysis for large deflections showed that in this case the deflection at the center of the plate and the temperature at that point are related to the critical temperature and a constant, K_2, (which is associated with the deformation mode) by the expression

$$\frac{T_o - T_{crit}}{T_{crit}} = K_2\left(\frac{W_o}{t}\right)^2 \tag{25}$$

It is seen from this formula that since the T_{crit} is independent of Young's modulus the plate deflections are also independent of Young's modulus and depend only on the Poisson ratio, μ, of the material.

Clearly, when the experimental data giving W_o as a function of T_o are available, these equations can be used to determine T_{crit}.

LATERAL INSTABILITY OF DEEP BEAMS

In recent research, Way[20] has demonstrated the applicability of both large and small displacement methods to the problem of lateral instability of a deep beam loaded by concentrated force lying in the plane of the web and passing through the centroid. He obtained excellent agreement between the two methods. The load displacement which he determined is given in Figure 21. The δ/P versus δ plot is shown in Figure 22, and the P-δ^2 is shown in Figure 23. The critical load levels determined from these curves are 261 grams and 253 grams which compare well with the theoretical value of 260 grams.

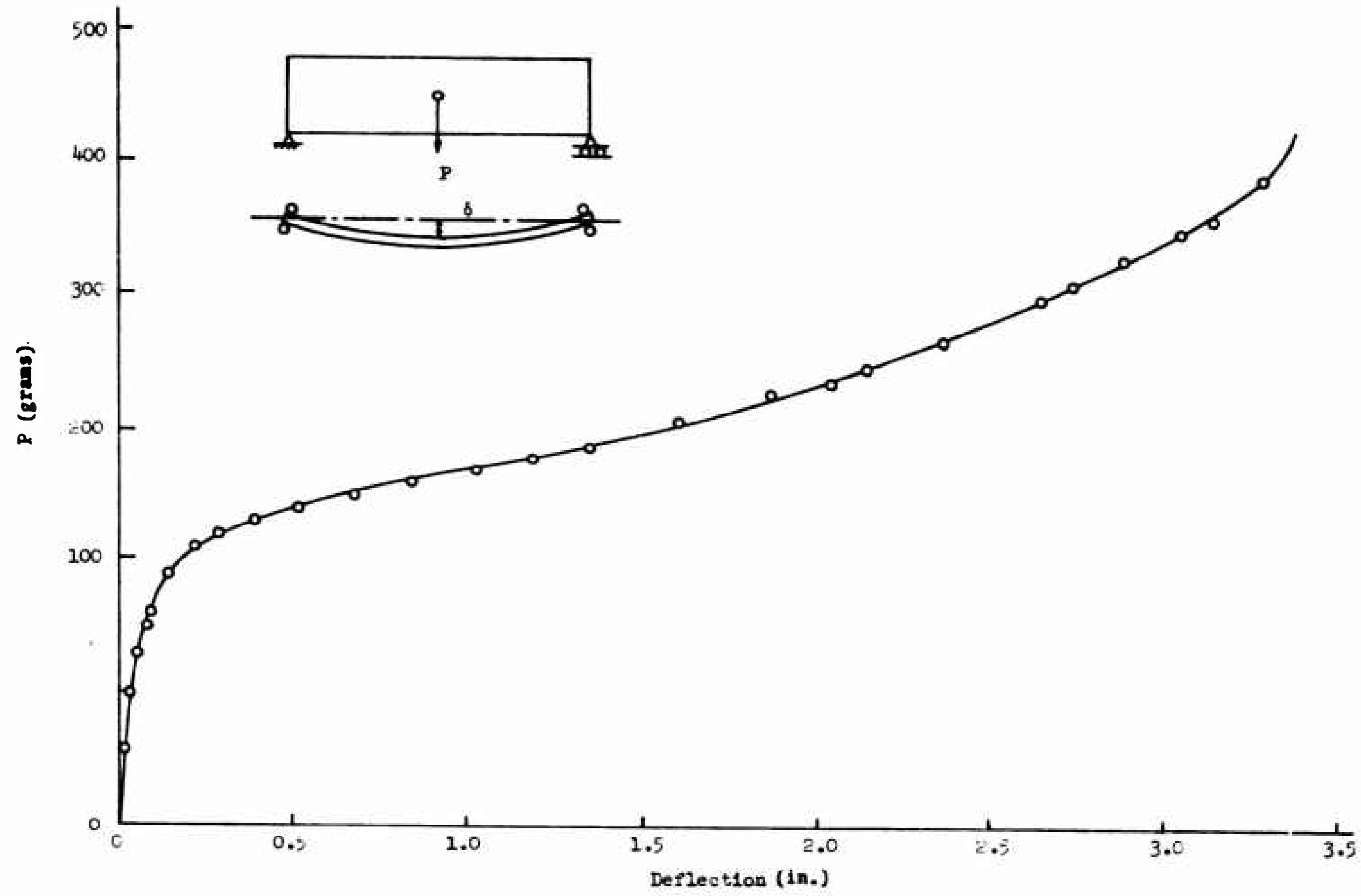

Figure 21. Load Versus Lateral Deflection for Deep Beam, Reference 20.

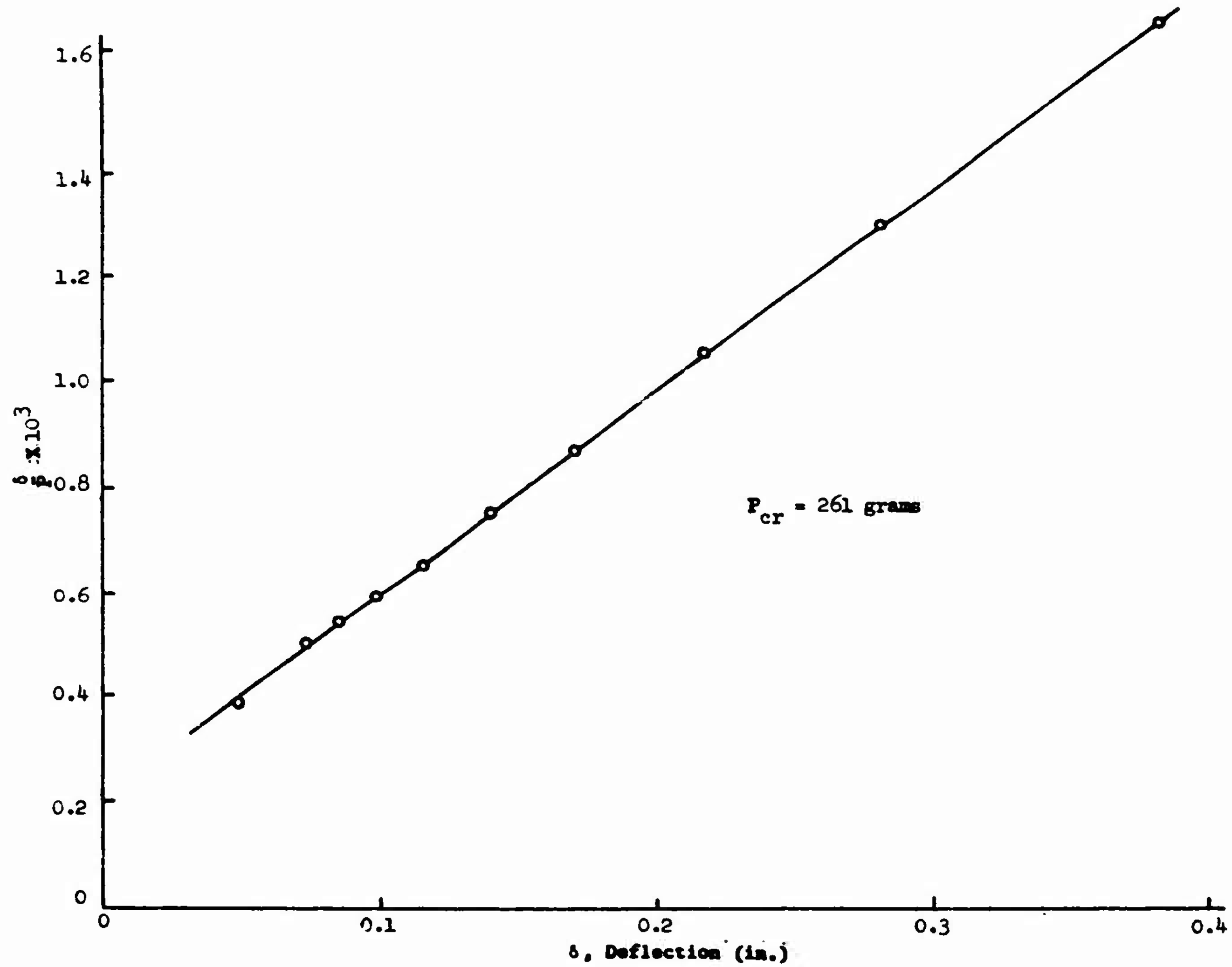

Figure 22. Southwell Plot for Data of Figure 21.

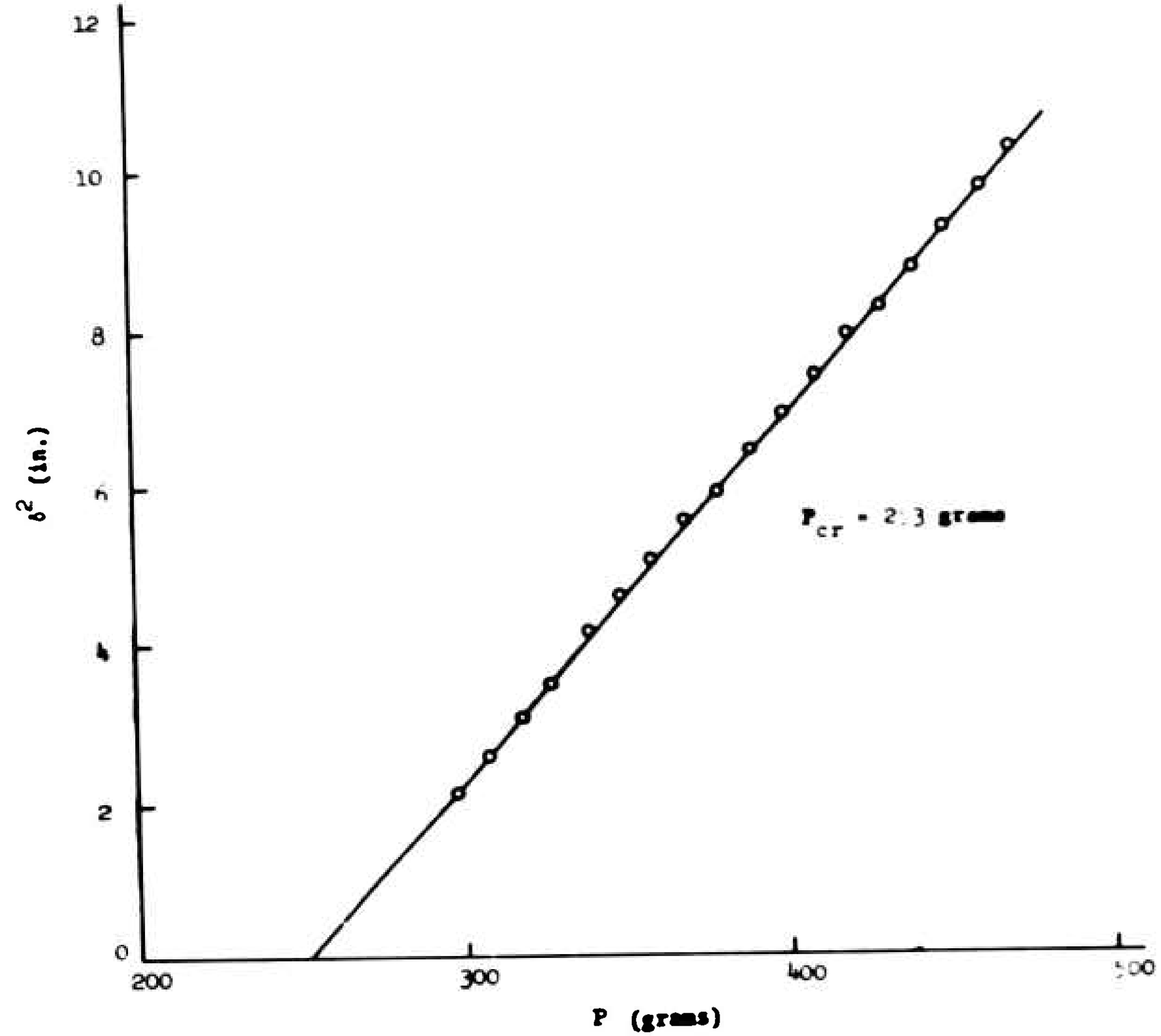

Figure 23. P Versus δ^2 for Data of Figure 21.

CONCLUSION

This technique gives identical results to those obtained from the well-established Southwell process but is easier to apply in many cases. Both methods have their own peculiar restrictions. In the large deflection case, the load levels must be in excess of the theoretical critical value; but it is clear from the data available that small amounts of local yielding do not seriously influence the problem. For the Southwell method to apply, the motions must be small and the initial imperfection likewise restricted for many cases. Thus, for the small displacement approach, there is an implied requirement for a relatively good quality test vehicle, a requirement of considerable less importance for large displacement considerations. However, in all cases, the user must be cautioned against the application of either method when the data are such that curve fitting techniques would be needed.

LITERATURE CITED

1. Ayrton, W. E., and Perry, John, ON STRUTS, The Engineer, Volume 62, December 10, 1886, pages 464, 465; December 24, 1886, pages 513, 515.

2. Southwell, R. V., ON THE ANALYSIS OF EXPERIMENTAL OBSERVATIONS IN PROBLEMS OF ELASTIC STABILITY, Proceedings of the Royal Society, Series A, Volume 135, 1932, pages 601-616.

3. Horton, W. H., Cundari, F., and Johnson, R., ON THE APPLICABILITY OF THE SOUTHWELL PLOT TO THE INTERPRETATION OF TEST DATA OBTAINED FROM INSTABILITY OF COLUMN AND PLATE STRUCTURES, Paper presented at the Ninth Israel Congress of Aeronautics and Space, Haifa, Israel, February 1967.

4. Horton, W. H., and Cundari, F., ON THE APPLICABILITY OF THE SOUTHWELL PLOT TO THE INTERPRETATION OF TEST DATA OBTAINED FROM INSTABILITY STUDIES OF SHELL BODIES, Paper presented at the ASME/AIAA Eighth Structures, Structural Dynamics and Materials Conference, March 1967, Palm Springs, California.

5. Horton, W. H., Tenerelli, D. J., and Willey, B. T., THE USE OF SMALL AND LARGE DISPLACEMENT DATA FROM ESSENTIALLY ELASTIC BUCKLING TESTS ON COLUMNS AND PLATES AS A MEANS OF CORRELATING THEORY AND EXPERIMENT, SUDAAR.

6. Hill, H. N., NOTE ON THE ANALYTICAL TREATMENT OF LATERAL DEFLECTION MEASUREMENTS IN TESTS INVOLVING STABILITY PROBLEMS, Unpublished Alcoa Report.

7. Fisher, H. R., AN EXTENSION OF SOUTHWELL'S METHOD OF ANALYZING EXPERIMENTAL OBSERVATIONS IN PROBLEMS OF ELASTIC STABILITY, Proceedings of the Royal Society, Series A., Volume 144, 1934.

8. Westergaard, H. M., BUCKLING OF ELASTIC STRUCTURES, Transactions of the American Society of Civil Engineers, paper 1490.

9. Gregory, M. S., THE USE OF THE SOUTHWELL PLOT ON STRAINS TO DETERMINE THE FAILURE LOAD OF A LATTICE GIRDER WHEN LATERAL BUCKLING OCCURS, Australian Journal of Applied Science, Volume 10, No. 4, 1959, pages 371-376.

10. Gregory, M. S., THE APPLICATION OF THE SOUTHWELL PLOT ON STRAINS TO PROBLEMS OF ELASTIC INSTABILITY OF FRAMED STRUCTURES WHERE BUCKLING OF MEMBERS IN TORSION AND FLEXURE OCCURS, Australian Journal of Applied Science, Volume 11, No. 1, 1960, pages 49-64.

11. Britvec, S. J., and Chilver, A. H., ELASTIC BUCKLING OF RIGIDLY-JOINTED BRACED FRAMES, Proceedings of the American Society of Civil Engineers, Engineering Mechanics Division,, December 1963, pages 217-255.

12. Roorda, J., STABILITY OF STRUCTURES WITH SMALL IMPERFECTIONS, Proceedings of the American Society of Civil Engineers, Engineering Mechanics Division, February 1965, pages 87-106.

13. Dunn, L. G., AN INVESTIGATION OF SHELL STIFFENER PANELS SUBJECTED TO COMPRESSION LOADS WITH PARTICULAR REFERENCE TO TORSIONALLY WEAK STIFFENERS, NACA Tech. Note 752, February 1940.

14. Farrar, D.J., INVESTIGATION OF SKIN BUCKLING, R and M 2652, British Aeronautical Research Council, 1947.

15. Hemp, W. S., and Griffin, H., K., THE BUCKLING IN COMPRESSION OF PANELS WITH SQUARE TOP-HAT SECTION STRINGERS, R and M 2635, British Aeronautical Research Council, June 1949.

16. Donnell, L. H., ON THE APPLICATION OF SOUTHWELL'S METHOD FOR THE ANALYSIS OF BUCKLING TESTS, Timonshenko 60th Anniversary Volume, McGraw-Hill, page 27-38, 1938.

17. Gough, H. J., and Cox, H. L., SOME TESTS ON THE STABILITY OF THIN STRIP MATERIAL UNDER SHEARING FORCES, Proceedings of the Royal Society, Volume 137 A, 1932.

18. Gerrard, G., CRITICAL SHEAR STRESS OF PLATES ABOVE THE PROPORTIONAL LIMIT, Journal of Applied Mechanics, Volume 15, No. 1, March 1948, pages 7-12.

19. Queinec, Alan, THERMAL BUCKLING OF CENTRALLY HEATED CIRCULAR PLATES, SUDAER, No. 106, Stanford University, June 1961.

20. Way, E., THE LATERAL INSTABILITY OF A SIMPLY SUPPORTED DEEP BEAM SUBJECTED TO A CONCENTRATED LOAD AT ITS CENTROID, Engineer's Thesis submitted to the Department of Aeronautics and Astronautics, Stanford University, 1967.

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13. ABSTRACT

The paper shows ~~that~~ there are two ~~extremely useful~~ rules for interpretation of experimental results on a wide range of simple structures. The first of these applies when the deformations are small. It is that the initial and subsequent deformations are related to the actual and classical instability loads by the expression

$$\delta \frac{P_{cr}}{P} - 1 = \delta_o$$

the second rule

It is likewise shown that when the deflections are large and the loads are in excess of the instability load, the relationship is modified to

$$P = P_{cr} (1 + \gamma\delta^2)$$

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