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MATHEMATICAL MODEL FOR
LONG CABLE TOWED BY ORBITING AIRCRAFT

FINAL REPORT

AIRTRAC A05-513-438/2021/15080C00

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DEPARTMENT OF THE NAVY
NAVAL AIR DEVELOPMENT CENTER
JOHNSVILLE
WARMINSTER, PA. 18274

AERO MECHANICS DEPARTMENT

REPORT NO. NADC-AM-6849

12 JUNE 1969

MATHEMATICAL MODEL FOR
LONG CABLE TOWED BY ORBITING AIRCRAFT

FINAL REPORT

AIRTASK A05-533-438/2021/15080000

A steady-state mathematical model which computes spatial configuration and tension along an orbiting towline has been developed. Numerical results are in agreement with existing flight test data. The model yields multi-valued solutions under certain operating conditions. Operations which avoid these troublesome regimes have been developed. The model can be used to optimize the performance of operational systems. For example, numerical results are presented for TACAMO which show how the operation of that system can be improved. There are conditions where the desired verticality cannot be obtained with a standard constant diameter towline. Numerical results for a special stepped diameter towline illustrate how the desired high verticality and a reduction of tension can be achieved under such conditions by variation of the towline construction.

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S U M M A R Y

A steady-state mathematical model, which computes the spatial configuration and tension along an orbiting towline, has been developed and is described and presented herein. Numerical results are in agreement with existing flight test data. Parametric studies of the operational TACAMO II system indicate that very high verticality, of the order of 75 percent or more, can always be achieved. Numerical results presented herein indicate that significant improvement in verticality, time on station, fuel economy, and flight crew comfort can be achieved in the TACAMO systems by an optimization based upon the mathematical model.

The model reveals the existence of multi-valued solutions (alternate spatial configurations) under certain operating conditions. Sudden transition from one steady-state configuration to another causes high transient dynamic loads. Such sudden jumps were experienced experimentally in the early flight tests of TACAMO II. Operations to circumvent these potentially troublesome regions have been developed, based upon numerical results of the mathematical model. The potential for achieving increased verticality by usage of special towlines with lower dynamic drag coefficient are discussed. A full scale instrumented, flight test program to verify the steady-state model and the development of a transient model in accordance with criteria presented herein are recommended.

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I. INTRODUCTION

Renewed interest has developed in the dynamics and the spatial configuration of the long orbiting towline. This problem can be studied by constructing a mathematical model which practically simulates the physical situation. Studies have previously been made by various investigators (references (a), (b), (c), and (d)). Each study yielded its own mathematical model. Some were overly simplified while the others were overly complicated. Moreover, these models are found to be in poor agreement with experiment or not yet verified at all; a satisfactory mathematical model has yet to be developed.

This report encompasses a critical review of some of the existing mathematical models, recommended criteria for mathematical models for the long orbiting towline, and a presentation of a steady-state mathematical model developed at NAVAIRDEVGEN. (Naval Air Development Center) under reference (e).

II. REVIEW OF PREVIOUS MATHEMATICAL MODELS

A literature review has revealed the results of several previous investigations concerned with the mathematical analysis of orbiting towlines (references (a) (b) (c) and (d)). Each previous investigator developed a model. Generally, these models can be classified in two categories, the steady-state and the transient. A steady-state model simulates a towline under steady-state motion. Under this condition, the dynamic and geometric parameters of the towline are independent of time except for either a constant speed translation at a constant altitude or a rigid body rotation about an axis perpendicular to the earth's surface. Mathematical models to simulate situations other than steady-state motion will be referred as transient. These models have been reviewed and are discussed individually below.

A. Douglas Aircraft Company published a report in February 1957 titled, "Some Calculations on the Long Line Technique for Lowering Sonar from Flying Aircraft" (reference (a)). It is a steady-state orbiting towline model. Even though the purpose of that work was different from the current objective, the basic dynamics and the interest in equilibrium configurations were the same. The aerodynamic force acting on the line is assumed to follow the sine square law for an infinitely long cylinder. Skin friction in the direction of the relative velocity was not considered. The solution for the equilibrium configuration of the cable was obtained through numerical integration of the force equilibrium equations. Integration was commenced from the tow object end. Since the model is a steady state model, it requires these assumptions:

1. No wind
2. Towplane flies a perfect circular pattern at a constant attitude, and
3. No cable reel-in or reel-out.

In the formulation of the differential equation it is assumed that the cable is inextensible but perfectly flexible in bending.

B. In 1962, Grumman Aircraft Engineering Corporation (reference (b)) developed a mathematical model to determine the dynamic configuration of a long towline. Again, the purpose of that model was different from the current interest. It was developed to study a long line orbit technique for surface-to-air retrieval. This model can be used to acquire transient towline configurations including the effects of wind. The following assumptions were made in the mathematical formulation:

1. Aerodynamic forces acting on the cable follow the sine square law with no skin friction present.
2. The towline deforms elastically under tension but it cannot be subjected to compression. Also, the towline is perfectly flexible in bending.
3. At the upper end of the towline, motion is specified and at the lower end a point mass is assumed.

Grumman's model takes account of the longitudinal elasticity of the towline. Because the longitudinal elastic wave speed is high (compared with the lateral wave propagating along the cable) very small time increments (0.001 sec.) are required between successive calculations. This makes the computation time required to complete a realistic initial value problem prohibitively long, even with the aid of the electronic computer. Therefore, primary results presented by Grumman were obtained through a simplified version of the model assuming that the towline does not stretch with tension; it is longitudinally rigid. The inclusion of longitudinal flexibility is unwarranted unless the cable is subjected to rapid variation of longitudinal loading such as impact loads. For usual applications the complexity is not justified by added accuracy. The mathematical technique used in constructing the physical problem is the lumped mass method. In this analysis, the cable is divided into a finite number of segments. At the lower end of each segment, a concentrated mass and a projected area equivalent to that of the segment is attached. These point masses are assumed to be connected with massless straight lines. The number of the straight line segments, N , which can be used to approximate a continuous towline is severely limited since the number of simultaneous equations to be solved is proportioned to N . The maximum number of segments used in Grumman's calculations was fourteen. This model is considered to be inadequate for the purpose of the current study where accurate towline configuration is required.

C. Another very general, and complicated, mathematical model of the towline was reported by the Hayes International Corporation in 1967 (reference (c)). Hayes studied the complete maneuvering tow system, which includes the tow plane and a target as well as the towline. The mathematical technique used is the method of characteristics. In general, this mathematical technique yields very accurate numerical results for problems of the current type which involve hyperbolic differential equations. The accuracy of the numerical solution could have been further enhanced if center difference equations were used instead of the forward difference

equations used in this model. As in most of the other models, the towline is assumed to be inextensible, but perfectly flexible in bending. Solutions of the equations of motion of the towplane and the target were used as boundary conditions. These boundary conditions involved fourteen variables at the target end and eleven variables at the towplane end. It is believed that such an approach, although theoretically more accurate, is unwarranted in this problem. The added accuracy does not justify the greatly increased complexity of numerical computations. Computation would be very much easier if simplified boundary conditions were specified. Since the towline exerts only a secondary effect on the towplane, the motion of the aircraft may be specified at one end, and an unpowered drogue of simple configuration can be used at the other end. Such a model would suffice for the study of the steady-state long orbiting towline. Two conditions which are considered to be essential in the study of transient or unsteady towline dynamics were not included in Hayes' mathematical model. They are:

1. Wind and wind shear
2. Towline reel rate

Apparently, Hayes encountered numerous difficulties in making their computer program operational. Included in their report were numerical results from a debugging run. It is believed that the model is not yet operational. It was not at the time reference (c) was prepared.

D. As part of a study on an orbiting antenna for airborne VLF communications, Cornell Aeronautical Laboratory (reference (d)) developed a model to analyze the motion of an orbiting antenna cable. This model is also a steady-state model. The aerodynamic force is assumed to follow the cross flow principle, but the skin friction was assumed in the direction of the cable rather than in the direction of the relative air flow. This assumption is not in accordance with the cross flow principle as defined by Hoerner (reference (f)). It was also assumed that the cable is inextensible but perfectly flexible. The equilibrium configuration of the antenna is established through integrating the dynamic equilibrium equation from the lower end (drogue end). The dynamic equilibrium equations are written in cylindrical coordinates. In their numerical computations, Cornell neglected skin friction. Their results are substantially different from results obtained by the NAVAIRDEVGEN mathematical model, which will be described in detail later, particularly in the region where multi-valued solutions might exist.

III. CRITERIA FOR MATHEMATICAL MODELS

After critically reviewing these mathematical models, it can be concluded that none of the existing models can be used for the current study of the long orbiting towlines because they are either too simple or too complicated in some respect, which is of no consequence to the present interest. Therefore, the development of a new mathematical model, which adequately simulates the dynamics and configuration of the orbiting towline under various operating conditions, was undertaken. The method of

characteristics is preferred to the lumped mass method because it generally gives more accurate numerical results.

Practical mathematical models for the study of the dynamics and configuration of the orbiting towline should satisfy the following criteria:

A. Criteria for the Steady-State Model:

1. It should be able to yield solutions giving tension and geometric configuration along the entire towline for the following operating conditions:

- a. The towplane travels a straight and level path
- b. The tow aircraft travels in a perfectly circular orbit at a constant altitude.

2. Acceptable Assumptions:

- a. Aerodynamic forces follow cross flow principle (skin friction in the direction of the relative air flow)
- b. Towline is inextensible but perfectly flexible
- c. No wind and no wind shear present
- d. No towline reel-in or reel-out motion
- e. Air density varies with altitude as in a standard day.

3. Boundary Condition:

- a. Towplane flies a straight line or a perfect circle at constant altitude, with no towline being reeled in or out
- b. Tow drogue is of simple nose or center of gravity tow configuration; and it is not powered.

4. Formulation Should:

- a. Admit numerical techniques which are of high degree of accuracy.
- b. Allow a computer program that can provide solutions for many different operating conditions as well as for many different towline systems in one computer run.

B. Criteria for the Transient Model:

1. It should be able to yield time dependent solutions for tension and geometric configuration along entire towline for the following operating conditions:

- a. The towplane enters into a circular orbit from steady-state straight and level flight with the towline being reeled out.
- b. The motions of the towplane changes from steady circular orbit to straight and level with the towline being reeled in.
- c. The towplane undergoes a coordinated turn with a constant bank angle in an environment with non-uniform steady wind.
- d. The towplane undergoes a coord. turn with unsteady, non-uniform wind to indicate stability of multi-valued solutions.

2. Acceptable Assumptions:

- a. Aerodynamic forces follow cross flow principle (skin friction in the direction of the relative air flow).
- b. Towline is inextensible but perfectly flexible

3. Boundary Conditions:

- a. Towplane velocity and reel speed are prescribed as functions of time with very low rate of change of aircraft accelerations
- b. Tow drogue is of simple nose or center of gravity tow configuration, and it is not powered.

4. Formulation should:

- a. Admit numerical techniques which are inherently accurate, thus permitting the initial value problem to be carried out repeatedly for many time intervals; the real time involved in a transient period may be of the order of minutes.
- b. Allow solution with a computer program that can be executed in a medium size computer with reasonable expenditures of computer time.

5. Computer programming should have the following flexibility:

- a. Variable towline physical and aerodynamic properties
- b. Variable air density as a function of density altitude
- c. Variable towline length between data points for better choice of time increment between successive calculations as well as improving the accuracy of the solution.
- d. Redistribution, addition or elimination of data points along the towline

IV. NAVAIRDEVGEN STEADY-STATE MATHEMATICAL MODEL

In the study of the transient motion of a towline the initial configuration must be given. Usually the motion of the towline system commences with a constant speed straight and level translation or a constant angular speed rotation about an imaginary axis perpendicular to the surface of the earth. Since the constant speed straight and level translation is equivalent to a rotation with constant tangential velocity about an axis at infinity, a simple steady-state model which describes the steady-state motion of the towline orbiting around a vertical axis will provide the initial data for any subsequent transient solutions. On this premise, a steady-state mathematical model which satisfied the current interest, and which will be referred to as the NAVAIRDEVGEN model, has been developed and is described below.

A. Assumptions

Assumptions used in constructing the mathematical model include:

1. Towline is inextensible but perfectly flexible
2. Aerodynamic forces acting on the towline follow the cross-flow principle as described in reference (f)
3. Towplane travels in a perfectly circular path at a constant altitude with no towline pay-out or reel-in.
4. No wind and no wind shear present
5. Air density varies with altitude as in a standard day.

B. Formulation of Equations

It is convenient to formulate the problem in cylindrical coordinates. Figure 1 shows the towplane, the towline and the drogue in connection with the associated coordinate system. The equation which governs the steady-state configuration of the towline can be derived as follows by summing up the forces, including the inertia force, acting on an element of the towline ΔS and equating the sum to zero:

$$\vec{T} \Delta S + \vec{F}_a (\text{Aerodynamic Force}) \Delta S + \vec{F}_g (\text{Gravitational Force}) \Delta S + \vec{F}_i (\text{Inertia Force}) \Delta S = 0 \quad (1)$$

The aerodynamic force is expressed as follows:

$$\vec{F}_a = C_D \frac{d}{2} \rho |\vec{V}_L| \vec{V}_L + \pi C_f \frac{d}{2} \rho |\vec{V}_{Rel}| \vec{V}_{Rel} \quad (2)$$

where:

C_D = Drag coefficient

ρ = Air density

\vec{V}_{Rel} - Relative air velocity vector

\vec{V}_\perp - Component of the relative air velocity in the direction perpendicular to the towline

d - Diameter of the towline

The first term in equation (2) is the dynamic drag acting on a unit length of the towline. The direction of the force is perpendicular to the towline with a magnitude of $C_D \frac{1}{2} \rho |\vec{V}_\perp|^2$. The second term

represents the skin friction which is oriented in the direction of the relative air velocity. Since it is assumed that there is no wind

$$\vec{V}_{Rel} = - \frac{\partial \vec{r}(s,t)}{\partial t}$$

Referring to Figure 1, it is clear that the following relationships exist:

$$\vec{r}(s,t) = R(s) \vec{e}_R(s,t) + z(s) \vec{k}$$

$$\vec{e}_R(s,t) = \sin \theta \vec{j} + \cos \theta \vec{i}$$

$$\vec{e}_\theta(s,t) = \cos \theta \vec{j} - \sin \theta \vec{i}$$

where: $\theta(s,t) = \theta(s,0) + \omega t = \theta(s) + \omega t$

(ω is the constant speed rigid body rotation)

therefore,

$$\vec{V}_{Rel} = - \frac{\partial \vec{r}(s,t)}{\partial t} = -R(s) \frac{\partial \vec{e}_R(s,t)}{\partial t} = -R(s) \frac{d\theta(s)}{dt} \vec{e}_\theta = -R(s) \omega \vec{e}_\theta$$

The magnitude of \vec{V}_{Rel} is:

$$|\vec{V}_{Rel}| = R(s) \omega$$

Let \vec{e}_s denote a unit vector pointing in the positive direction of $d\vec{s}$.

Then,

$$\begin{aligned} \vec{e}_s &= \frac{\partial \vec{r}(s,t)}{\partial s} = \frac{dR(s)}{ds} \vec{e}_R(s,t) + R(s) \frac{\partial \vec{e}_R(s,t)}{\partial s} + \frac{dz(s)}{ds} \vec{k} \\ &= R'(s) \vec{e}_R + R(s) \frac{d\theta(s)}{ds} \vec{e}_\theta(s,t) + z'(s) \vec{k} \\ &= R'(s) \vec{e}_R + R(s) \theta' \vec{e}_\theta + z'(s) \vec{k} \end{aligned}$$

where prime is used to denote the first derivative of the corresponding dependent variable with respect to s .

With this relationship, the component of the relative air velocity perpendicular to the relative wind can be written as:

$$\begin{aligned} \vec{V}_\perp &= \vec{V}_{Rel} - (\vec{V}_{Rel} \cdot \vec{e}_s) \vec{e}_s \\ &= -R(s) \omega \vec{e}_\theta - (-R(s) \omega R(s) \theta'(s) [R'(s) \vec{e}_R + R(s) \theta'(s) \vec{e}_\theta + z'(s) \vec{k}]) \\ &= R(s) \omega [R'(s) \theta'(s) \vec{e}_R + (R(s) \theta'^2 - 1) \vec{e}_\theta + R(s) \theta'(s) z'(s) \vec{k}] \end{aligned}$$

Hence, the magnitude of \vec{V}_1 is:

$$|\vec{V}_1| = R(s)\omega \sqrt{[R(s)R'(s)\theta'(s)]^2 + (1-R(s)\theta'(s))^2 + R(s)^2\theta'(s)z'(s)^2}$$

Since

$$\left[\frac{dR(s)}{ds}\right]^2 + \left[R(s)\frac{d\theta(s)}{ds}\right]^2 + \left[\frac{dz(s)}{ds}\right]^2 = 1$$

or:

$$z'(s)^2 = 1 - R'(s)^2 - R(s)^2\theta'(s)^2$$

Consequently:

$$|\vec{V}_1| = R\omega \sqrt{1 - R(s)^2\theta'(s)^2}$$

Substituting the expressions for \vec{V}_{rel} , $|\vec{V}_{rel}|$, $|\vec{V}_1|$ and \vec{V}_1

in Equation (2) yields:

$$\begin{aligned} \vec{F}_a = & c_D \frac{d}{2} \rho \sqrt{1 - R^2\theta'^2} R^2\omega^2 [RR'\theta'\vec{e}_R + (R^2\theta'^2 - 1)\vec{e}_\theta \\ & + R\theta'z'\vec{k}] - \pi c_f \frac{d}{2} \rho R^2\omega^2 \vec{e}_\theta \end{aligned} \quad (3)$$

The gravitational force vector is simply:

$$\vec{F}_g = -\mu g \vec{k} \quad (4)$$

where μ = mass per unit length of the towline

g = gravitational acceleration

The inertia force vector is:

$$\begin{aligned} \vec{F}_i &= -\mu \frac{\partial^2 \vec{r}}{\partial t^2} = -\mu (-R(s)\omega^2 \vec{e}_R) \\ &= \mu R\omega^2 \vec{e}_R \end{aligned} \quad (5)$$

Finally, the expression $\frac{d\vec{T}}{ds}$ can be resolved into three components.

$$\frac{\partial \vec{T}}{\partial s} = \frac{\partial (|\vec{T}|\vec{e}_s)}{\partial s} = \frac{\partial}{\partial s} [|\vec{T}|R'\vec{e}_R + |\vec{T}|R\theta'\vec{e}_\theta + |\vec{T}|z'\vec{k}]$$

Let $|\vec{T}| = T$

$$\begin{aligned} \frac{\partial \vec{T}}{\partial s} &= (TR')\vec{e}_R + TR\theta'\vec{e}_\theta + (TR\theta')'\vec{e}_\theta + TR(\theta')^2(-\vec{e}_R) + (Tz')\vec{k} \\ &= [(TR)'\vec{e}_R - TR\theta'^2]\vec{e}_R + [(TR\theta')' + TR\theta']\vec{e}_\theta + (Tz')\vec{k} \end{aligned} \quad (6)$$

Substituting Equations (3), (4), (5) and (6) in Equation (1) then dividing by ΔS , results in the following three scalar equations:

$$(TR')' - TR\theta'^2 + C_D \frac{d}{2} \rho \sqrt{1-R^2\theta'^2} R^2 R' \omega^2 \theta' + \mu R \omega^2 = 0 \quad (7)$$

$$(TR\theta') + TR\theta' - \pi G \frac{d}{2} \rho R^2 \omega^2 - C_D \frac{d}{2} \rho R^2 \omega^2 (1-R^2\theta'^2)^{\frac{1}{2}} = 0 \quad (8)$$

$$(Tz')' + C_D \frac{d}{2} \rho R^2 \omega^2 \theta' z' \sqrt{1-R^2\theta'^2} = 0 \quad (9)$$

These ordinary differential equations can be solved by the finite difference method, and the complete solution of the towline can be obtained step by step starting from the drogue end.

The appropriate difference equations written for two points, 1 and 2, a distance of ΔS apart, can be put in the form:

$$T_2 R_2' - T_1 R_1' = (\bar{T} \bar{R} \bar{\theta}'^2 - C_D \frac{d}{2} \bar{\rho} \sqrt{1-\bar{R}^2 \bar{\theta}'^2} \bar{R}^2 \bar{R}' \bar{\omega}^2 \bar{\theta}' - \bar{\mu} \bar{R} \bar{\omega}^2) \Delta S \quad (10)$$

$$T_2 R_2 \theta_2' - T_1 R_1 \theta_1' = [-\bar{T} \bar{R} \bar{\theta}' + \pi G \frac{d}{2} \bar{\rho} \bar{R}^2 \bar{\omega}^2 + C_D \frac{d}{2} \bar{\rho} \bar{R}^2 \bar{\omega}^2 (1-\bar{R}^2 \bar{\theta}'^2)^{\frac{1}{2}}] \Delta S \quad (11)$$

$$T_2 z_2' - T_1 z_1' = (-C_D \frac{d}{2} \bar{\rho} \bar{R}^2 \bar{\omega}^2 \bar{\theta}' \bar{z}' \sqrt{1-\bar{R}^2 \bar{\theta}'^2}) \Delta S \quad (12)$$

Subscripts 1 and 2 are used to indicate the corresponding values at point 1 and 2, respectively. Bars placed above the variables represent the arithmetic average of the values of these variables at points 1 and 2. To be more specific, T_1 is the tension at point 1 and

$$\bar{T} = \frac{T_1 + T_2}{2}$$

Equations (10), (11), and (12) are good approximations for the original differential equations provided ΔS is small and the second and higher order derivatives of the dependent variables are small. Similar approximations give:

$$R_2 = R_1 + \bar{R}' \Delta S \quad (13)$$

$$z_2 = z_1 + \bar{z}' \Delta S \quad (14)$$

and

$$\theta_2 = \theta_1 + \bar{\theta}' \Delta S \quad (15)$$

At point 2, one can write:

$$R_2'^2 + R_2^2 \theta_2'^2 + z_2'^2 = 1 \quad (16)$$

If all the variables at point 1 are known, then the unknown variables at point 2 can be determined from equations (10) through (16). Since the system of algebraic equations is highly non-linear, probably the easiest way to solve them is by numerical iterations. Straight forward iteration schemes employed can be found in Appendix I.

The convergence of the iteration scheme is rather fast. In general, it converges in two to three iterations. Boundary conditions at the drogue end can be expressed as :

$$(TR')_{s=0} = -m\omega^2 R(0) + F_S$$

$$(TR\theta')_{s=0} = F_D$$

$$(Tz')_{s=0} = mg - F_L$$

where:

m = Mass of the drogue

F_S = Aerodynamic side force acting on the drogue

F_D = Aerodynamic drag force acting on the drogue

F_L = Aerodynamic lift force acting on the drogue

The aerodynamic forces, F_S , F_D and F_L , acting on the drogue are functions of α , the angle of attack of the drogue, and β , the side slip angle. These angles, α and β , can be determined from moment equations which require that under equilibrium the summation of moments acting on the drogue about any axis be zero. Aerodynamic characteristics for the conical drogue used in Appendix I computer runs are derived from data presented in reference (g), with the drag coefficient taken to be 0.6.

C. Description of Computer Program

Numerical computations were performed in a CDC3200 computer. A listing of the computer program is given in Appendix I. Primary data inputs required are:

1. Towplane: Altitude, absolute speed and turn radius.
2. Towline: Mass per unit length, diameter, aerodynamic drag coefficients and length.
3. Drogue: Weight, base area and aerodynamic characteristics

To start the calculations, it is necessary to assume a vertical separation between the drogue and the towplane and a turn radius of the drogue. With the angular speed, ω , given by the ratio of aircraft speed divided by its turn radius, calculation can be continued from the drogue end, step by step up to the aircraft end. Most likely either the calculated towplane turn radius or the calculated altitude, or both, are different from those values given in the input data, i.e., the values of the given problem to be solved. The differences between the calculated towplane altitude and turn radius and their corresponding values given by the problems are used to choose new values for drogue altitude

and turn radius to be used in the next round of computations. This process of adjusting the drogue altitude and turn radius continues until both the calculated altitude and the radius are very close to their corresponding values given by the problem.

Finally, numerical results, R , Z , θ , T , R' , Z' and R'' are printed at selected points along the towline. A typical computer output is exhibited in Appendix II.

D. Numerical Results for TACAMO System

Among the numerical results, those of particular interest for the TACAMO system are the maximum tension at the tow point (towplane end of the towed line) and the verticality of the line, i.e., the ratio of vertical separation distance between the towplane and the drogue, expressed as a percentage to the total line length. Vertical separations versus towplane turn radius for the TACAMO system under various operating configurations are plotted in Figures 2, 3, and 4. Each curve represents vertical separations for various aircraft turn radius while all the other parameters are kept constant.

Figure 2 reveals that the vertical separations as well as the towline tensions at the tow point are single valued for a specific operating condition. The towplane orbiting velocity is relatively low ($V = 150$ knots). As the towplane orbiting velocity increases, these values, vertical separation and tension, may become multi-valued, indicating that it is possible for the towline to assume different steady-state configurations with the towplane flying at exactly the same altitude, speed and turn radius. Which steady-state configuration the towline will assume depends on the stability of the various configurations and the transient operating conditions of the towline system prior to reaching the steady-state condition. Three important questions are of interest in this respect:

1. Are these steady-state configurations stable?
2. Under what conditions will the towline move from one steady-state configuration to another?
3. Will it jump from one extreme configuration to another without assuming the intermediate equilibrium configuration?

These questions cannot be answered by the steady-state mathematical model. A properly designed transient model, however, may provide these answers.

The existence of multi-valued configurations was apparently first suggested by Cornell in reference (d). Since Cornell's model did not include the effect of skin friction, their results should correspond to those shown by dotted curves in Figure 2. It is clear that these dotted curves are quite different from the corresponding solid curves, which account for skin frictions; particularly in the regions where the multi-valued solutions are anticipated. Thus, neglect of skin friction extends the region of

multi-valued configurations and increases the differences between the alternate solutions. Differences in vertical separation and towline tension are greater without skin friction.

Figure 3 shows the same type of curves for three different towline lengths. The general shape of these curves is similar for the same velocity, a smaller turn radius is required for a towline of shorter length.

Under the assumed standard atmospheric conditions air density decreases as the altitude increases. Higher vertical separation is therefore attained when the aircraft is flown at a higher altitude provided other parameters are kept the same. This effect is shown in Figure 4.

Some interesting characteristics of the multi-valued solutions were observed when the space coordinates, R , Z , and θ , of the towline were plotted along the entire towline in Figure 5. It is interesting to observe that from configuration I, the lowest verticality, to configuration II, intermediate verticality, the vertical separation and the angle θ are increased, while the radius is reduced for every point along the entire towline. However, from configuration II to configuration III, the highest verticality, the vertical separation is further increased and the radius is further reduced, but the angle remains essentially constant. Determination of the exact significance of these characteristics with respect to stability will require the transient model.

E. Comparison with Flight Measurements

A few experimental data points acquired from flight test in 1963, reference (g), were marked in Figures 2 and 3. They are in good agreement with the numerical result obtained from the NAVAIRDEVGEN mathematical model.

Recent flight tests conducted by the NAVAIRDEVGEN have given additional evidence that the towline tension at the tow point, as calculated by the mathematical model, is essentially correct. Results of those tests are compared with calculated data in Table I. Since the objective of the recent flight tests was not to verify the mathematical model, tension measurements were made under conditions not precisely corresponding to the ideal steady-state conditions assumed in the model. In a few instances, a range of tensions was recorded. Therefore the comparison presented in Table I should be evaluated qualitatively rather than quantitatively.

Although the vertical separation as well as the tow point tension are essentially verified, no experimental determination of the towline configuration between the aircraft and the drogue has been made. Therefore, a well designed instrumented and controlled test program to accurately verify the steady-state mathematical model should be conducted. The experimental verification program should answer questions such as:

1. Is the assumed aerodynamic force on the towline adequate?
2. Do multi-valued solutions exist? What are the stability characteristics of the multi-valued solution?

F. Other Numerical Results

Towline systems with parameters other than those of the TACAMO system were studied for the purpose of obtaining information and knowledge for the development of future systems. Numerous combinations were studied. Some results of particular interest are exhibited graphically in Figures 6 through 10.

Figure 6 shows the influence of the magnitude of the coefficient of dynamic drag, C_D , on vertical separation and towline tension. It is clear that the same verticality can be obtained with a larger towplane turn radius (lower bank angle) if the towline has a smaller dynamic drag coefficient. A lower drag towline also makes transition from low verticality to high verticality smoother and more gradual when the aircraft enters the orbit at a reduced turn radius. The less severe transition minimizes the possibility of a sudden change in the equilibrium altitude of the drogue which might develop high dynamic load and cause towline breakage. One way to reduce the coefficient of dynamic drag is by weaving hairs into the towline. According to the General Tire Corp., reference (h), such hairs reduce cable drag to one half of that without the hairs.

If the towplane performance parameters (airspeed, altitude, and turn radius, etc.) for a particular system have been optimized for maximum towline verticality, additional increase in verticality may require reduction of towline coefficient of dynamic drag. Since towline drag coefficient has a larger effect in the lower, higher curvature, portion of the towline, it may be that low drag devices will be applied only in that portion of the towline.

The effect of coefficient of skin friction drag on the vertical separation, as a function of towplane turn radius, is exhibited in Figure 7. This effect is very prominent in the region where verticality changes abruptly with turn radius. A higher coefficient of skin friction results in higher towline tension, but makes a smoother variation of verticality with turn radius. Incidentally, the haired cable will provide higher skin friction than the one without the hair. The higher skin friction also helps in the event of a sudden change in vertical separation as the aircraft enters its final orbit.

The easiest way of eliminating the possibility of a sudden change of vertical separation for the current TACAMO system is to increase the drogue weight. This effect is shown in Figure 8. Increasing the drogue weight from 100 pounds to 300 pounds effectively removes the multi-valued region for the particular case studied. It is believed that, due to the increase in drogue mass, the magnitude of variation of drogue vertical position under practical operating conditions may also be reduced.

Another way of eliminating the possibility of a sudden change of vertical separation for the current TACAMO system is the application of a stepped diameter towline, such as that shown in Figure 9. The upper third of this towline (0.210" diameter) (connected to the towplane) and the lower third (0.160" diameter) are made of existing cables used in

current systems. The middle third is a new cable of 0.184" diameter. Calculated vertical separations and towline tensions for the stepped towline and the corresponding results for the existing 0.210" towline are shown in Figure 10.

The stepped diameter towline offers the following potential advantages over the existing 0.210" constant diameter towline:

1. No multi-valued solution exists in the operating region
2. Higher verticality in the low verticality region and no reduction in verticality in the high verticality region.
3. Lower towline tension at the tow point (2180 pounds instead of 2840 pounds at 75 percent verticality). Because of the low towline trail angle at the towplane, this is a 640 pound reduction in tow drag.

Changing the base area of the drogue has little effect on the vertical separation, as shown in Figure 11, where the base area of the drogue is varied from 3.68 to 7.07 square feet.

Trends due to variations of towline diameter or towline weight per unit length, can be observed in Figure 12. If, as the towline diameter is increased, the mass per unit length is also increased in direct proportion to the square of the diameter, higher towline verticality is obtained and the region of high verticality variation is shifted to the right (largest turn radius). However, the opposite is true if, as the diameter of the towline is increased, the mass per unit length is kept constant.

Tension at the tow point increases as the mass per unit length of the towline increases. Tension remains essentially constant, however, if mass per unit length of towline is kept constant as the diameter of the towline varies.

G. Discussion of Steady-State Solutions

1. Avoidance of Region of Multi-Valued Solution

The NAVAIRDEVGEN steady-state mathematical model predicts that there are operating regions where multi-valued solutions exist, but the regions are not as wide and the differences in vertical separation for the extreme solutions are not as great as those predicted by the Cornell model. Until more information concerning the existence and the stability of alternate configurations can be acquired, it is desirable to avoid operating a towline system in these regions. This objective can be accomplished by the following procedures:

- a. Increase the weight of the drogue. For example, the TACAMO drogue should be increased from 100 to 300 pounds.
- b. Before entering the orbit, keep the towplane speed as low as possible, using flaps if available. Gradually increase bank angle

and increase towplane velocity slightly as higher bank angles are approached. Execute the transition from straight and level flight to final orbital flight slowly in no less than three complete orbits.

2. Optimization of Flight Operations

For a given operating system with fixed towline length, there are many combinations of towplane operating parameters, velocity, turn radius and altitude, which will result in an orbiting line operation. Judicious selection of the towplane parameters can optimize the operation for a number of different purposes, e. g., greatest towline verticality, minimum variation in verticality, best fuel economy for the towplane, maximum comfort for the towplane crew, etc. These optimizations can be easily and economically studied with the aid of the steady-state mathematical model. Bold steps can be taken and novel ideas can be explored without danger to the towplane crew or equipment. In addition, with the high speed electronic computer, numerous cases can be calculated quickly and inexpensively.

Review of scattered field operating data on existing systems indicates that significant potential improvements can be affected by optimization studies on current operational systems as well as on future systems.

3. Optimization of Towlines

Foregoing portions of this report have pointed out the effects of dynamic drag, diameter and weight per unit length on the behavior of towlines in orbital flight. Low drag, large diameter and heavy weight would appear to be desirable objectives in all cases, but they do not necessarily lead to the optimum towline for a given operation. In the case of relatively short towlines, or in situations in which the towplane speed and/or turn radius cannot be sufficiently reduced, low drag and high unit weight are significant in achieving verticality. On the other hand, with long towlines, high unit weight may be an intolerable burden on the towplane. There are many possible variations. The special towline of stepped diameter represents one of the variations.

The NAVAIRDEVGEN steady-state mathematical model provides a ready tool for investigating the significant characteristics of towlines, optimizing them for current operations, and defining profitable avenues for the development of improved towlines.

V. CONCLUSIONS

A. No previous mathematical model adequately represents and predicts the characteristics required to design and optimize orbiting towline systems.

B. Two models are actually required: a steady state model, and a transient model. Criteria for both are presented herein.

C. A steady-state model has been developed at the NAVAIRDEVGEN. The analytical formulation and a computer program are presented and discussed herein.

D. Numerical results for a number of cases corresponding to operational orbiting towline systems are presented and discussed. These results are in close agreement with existing flight test measurements.

E. The steady-state model predicts multi-valued solutions under certain operating conditions. These regions of operation should be avoided until the existence and relative stability of the alternate solutions are determined. These regions can usually be avoided in actual operation.

F. Stepped diameter towlines can eliminate the multi-valued region and reduce the tow load. This lessens the possibility of towing breakage and improves towplane endurance.

G. Verticality of orbiting towlines can be increased by special towlines having reduced dynamic drag coefficients.

VI. RECOMMENDATIONS

A. The steady-state model should be utilized to optimize the operation of current orbiting towline systems and to design and optimize future systems. Significant improvement in verticality, fuel economy, time on station, and crew comforts could be made by an optimization of the current TACAMO system using the mathematical model.

B. A full-scale instrumented flight test program should be designed and conducted to experimentally confirm the numerical results, including the results from the stepped towline, predicted by the steady-state model. The existence and stability of the multi-valued solution should also be examined.

C. A transient mathematical model should be developed in accordance with the criteria presented herein.

VII. REFERENCES

- (a) Douglas Aircraft Co., Inc., Report No. SM-22740, Some Calculations on the Long Line Technique for Lowering Sonar from Flying Aircraft, Lapin, Ellis (CONFIDENTIAL)
- (b) Grumman Aircraft Co. Report No. 63-13957, three volumes, Surface-to-Air- Retrieval of Heavy Spacecraft by Fixed Wing Aircraft
- (c) Hayes Engineering Report No. 1500, Hayes International Corp. (Nov 1967), The Maneuvering Tow System, Morrow, V. E., St. John, D. L. and Norman, R. I.
- (d) Cornell Aeronautical Laboratory, Inc. Report Suggested Method for Transient Dynamic Analysis of Orbiting Antenna. Bliven, D.O. May 28, 1968, DOB:mca:432, (CONFIDENTIAL)
- (e) Interdepartmental Work Agreement No. EC-5-69, 24 Sept 1968

NADC-AM-6849

- (f) Hoerner, S. F., Fluid-Dynamic Drag, published by the author, 1958
- (g) NAVAIRDEVCEN Report, Characteristics of Long Towlines in Non-Maneuvering and Orbiting Flight, No. NADC-ED-6340, June 14, 1963, W. C. Woodward (CONFIDENTIAL)
- (h) Urethane Hairs Data Sheet, Braincon Corp., Marion, Mass., a Subsidiary of General Tire Corp., Marion, Mass.

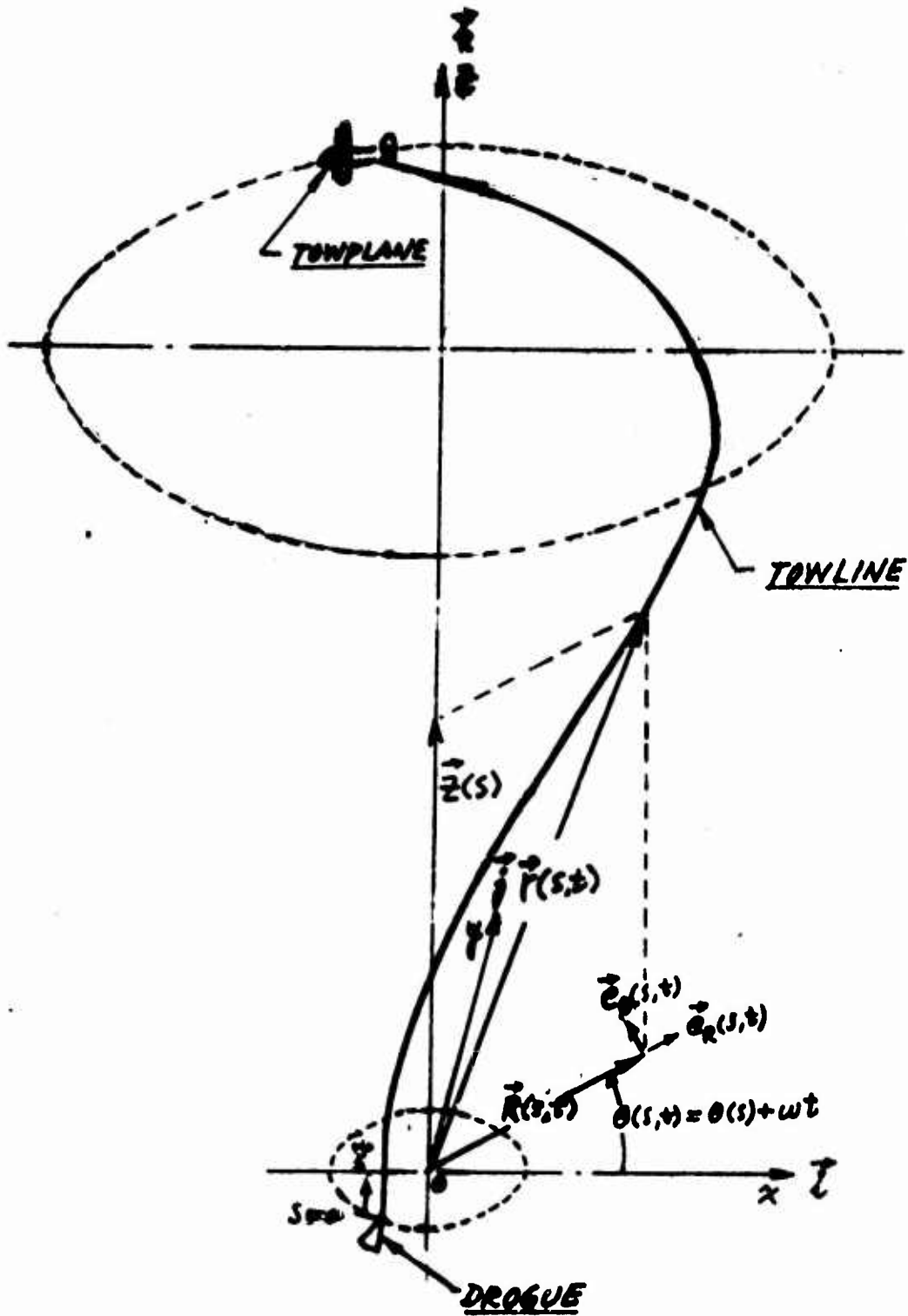


FIGURE 1. TOWLINE SYSTEM AND COORDINATES

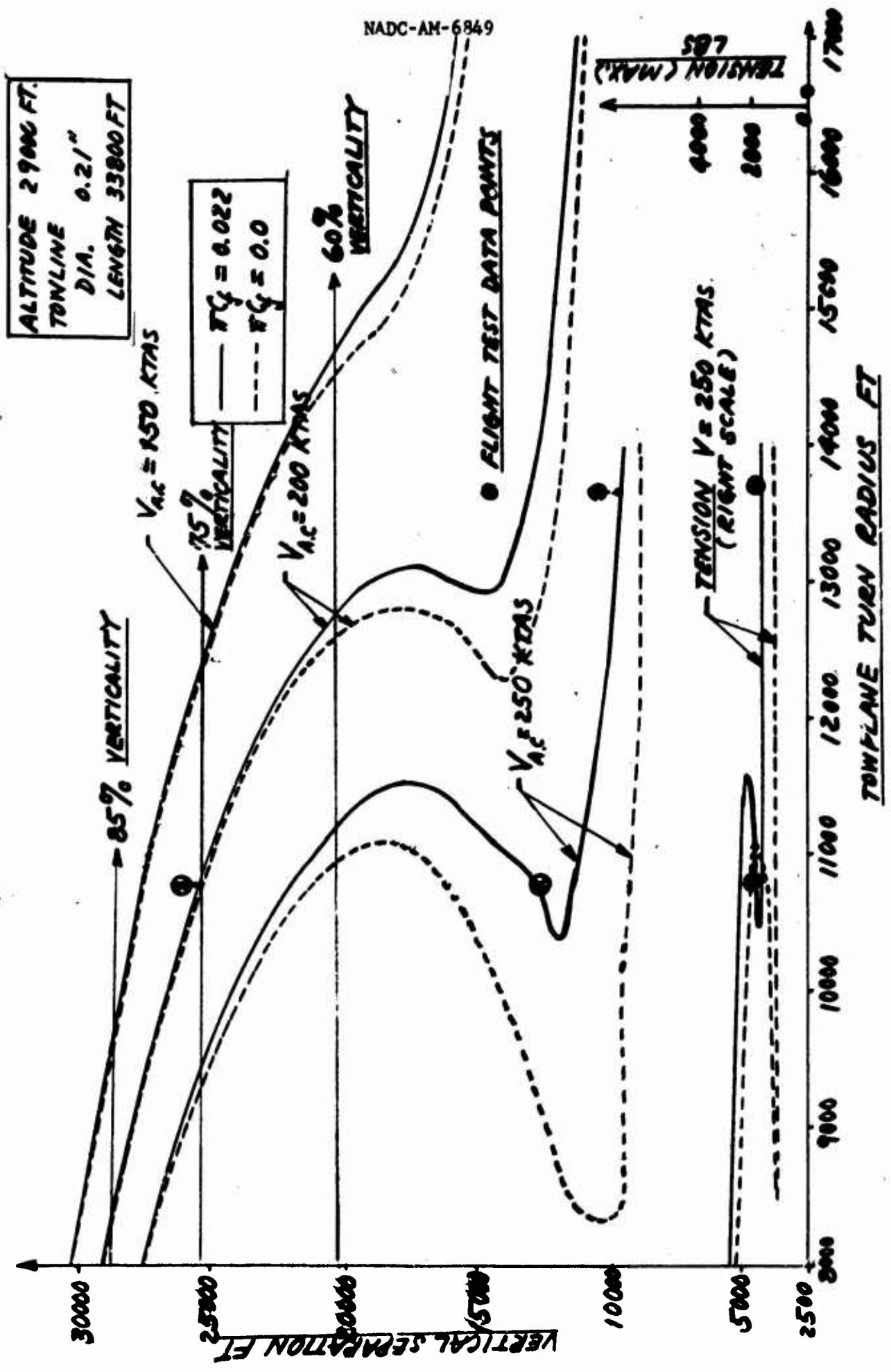


FIGURE 2. VERTICAL SEPARATION VS TOWPLANE TURN RADIUS—DIFFERENT TOWPLANE SPEEDS

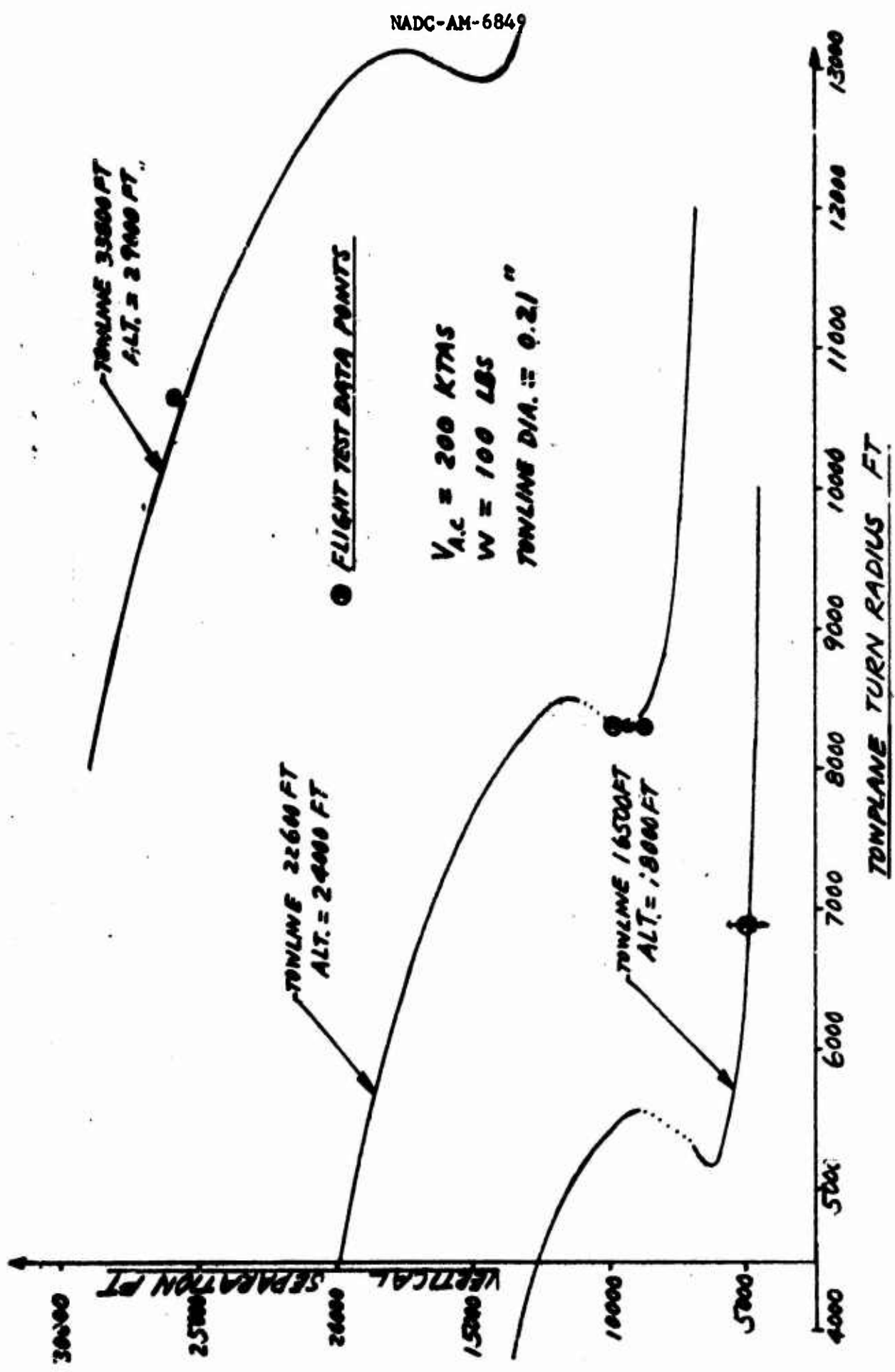


FIGURE 3. VERTICAL SEPARATION VS TOWPLANE TURN RADIUS - DIFFERENT TOWPLANE LENGTH

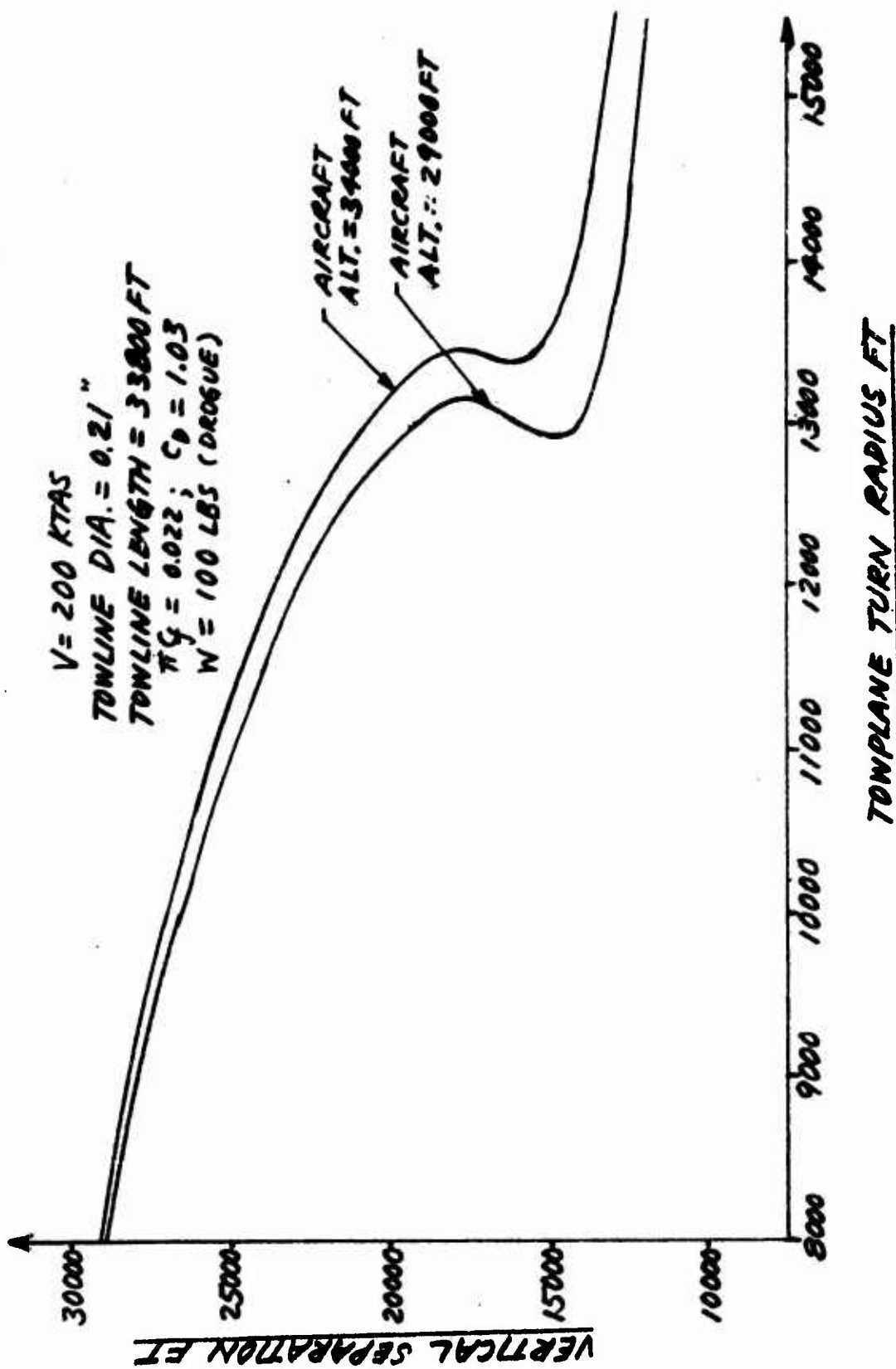


FIGURE 4. VERTICAL SEPARATION VS TONPLANE TURN RADIUS - DIFFERENT TONPLANE ALTITUDE

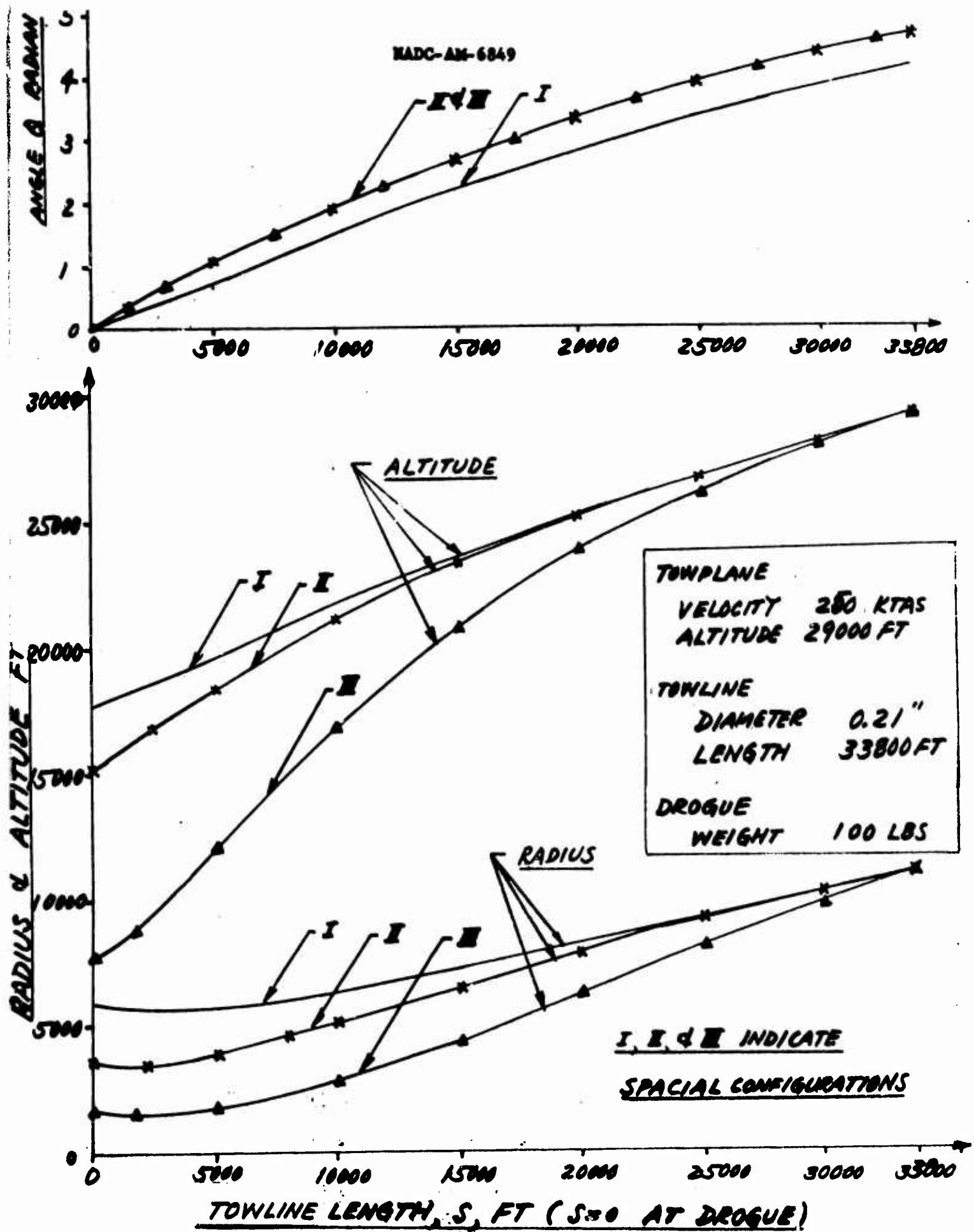


FIGURE 5. MULTI-VALUED SOLUTIONS

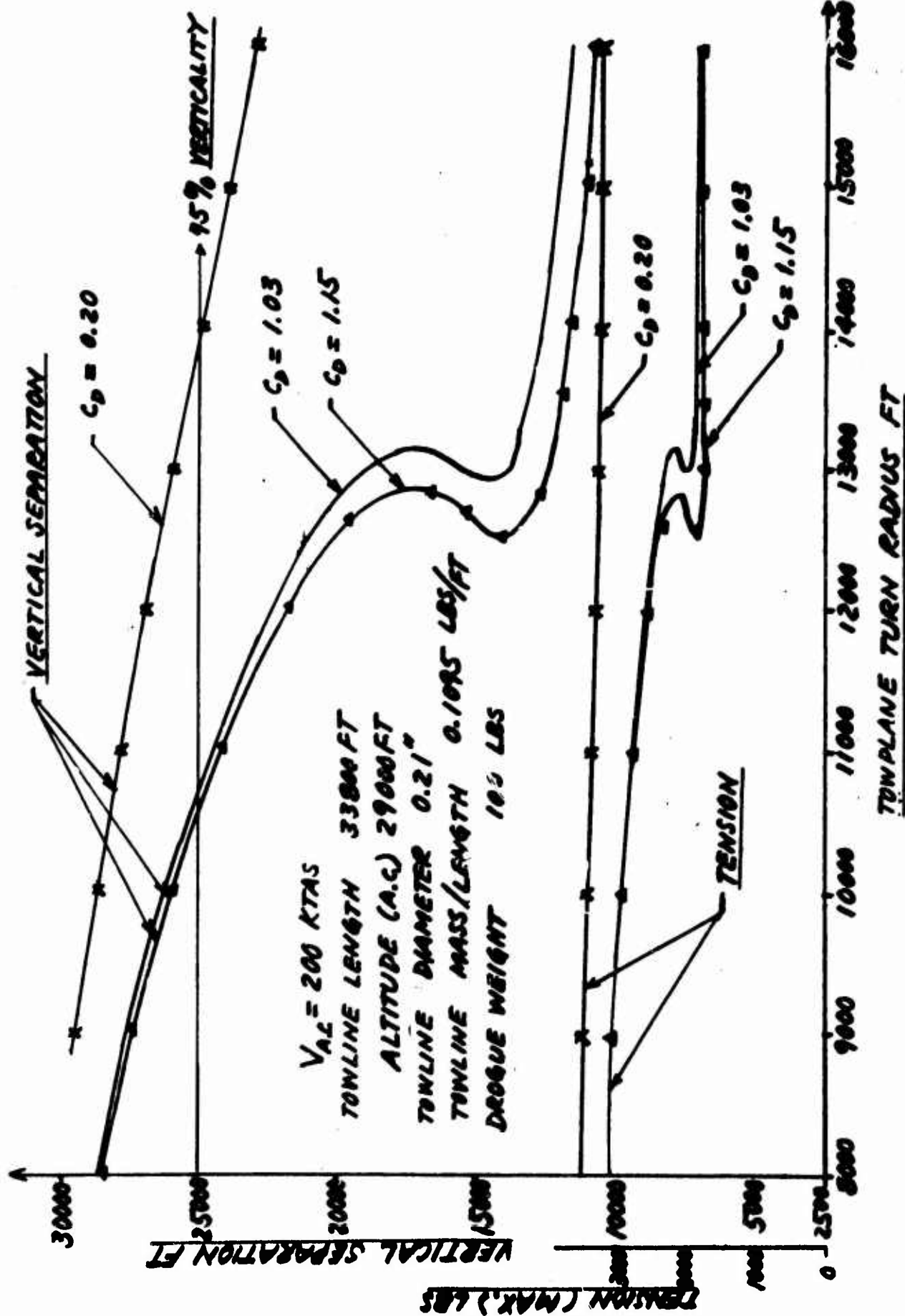


FIGURE 6. EFFECT OF COEFFICIENT OF DYNAMIC DRAG OF TOWLINE ON VERTICAL SEPARATION

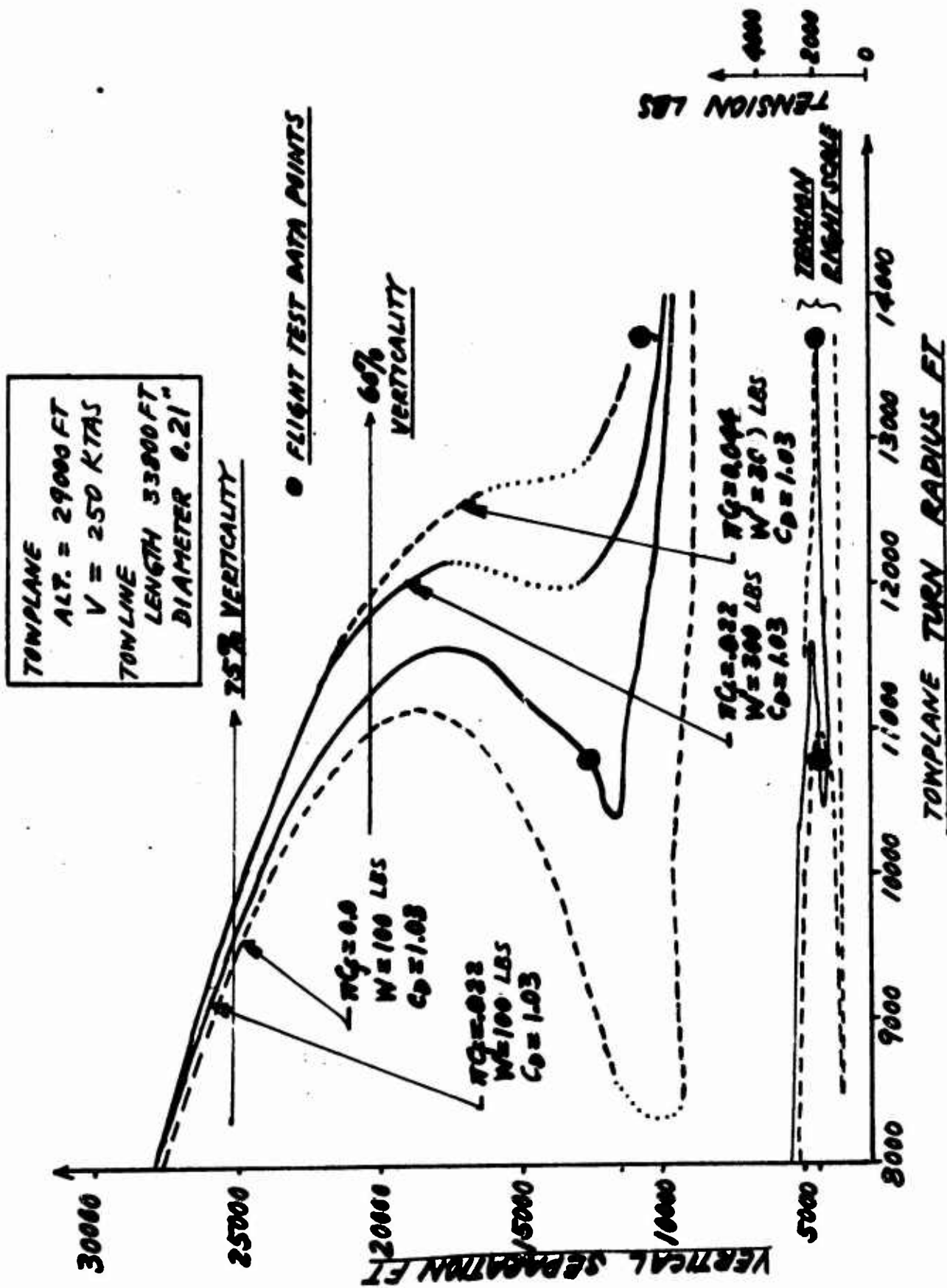


FIGURE 7. EFFECT OF TOWLINE SKIN FRICTION DRAG ON VERTICAL SEPARATION

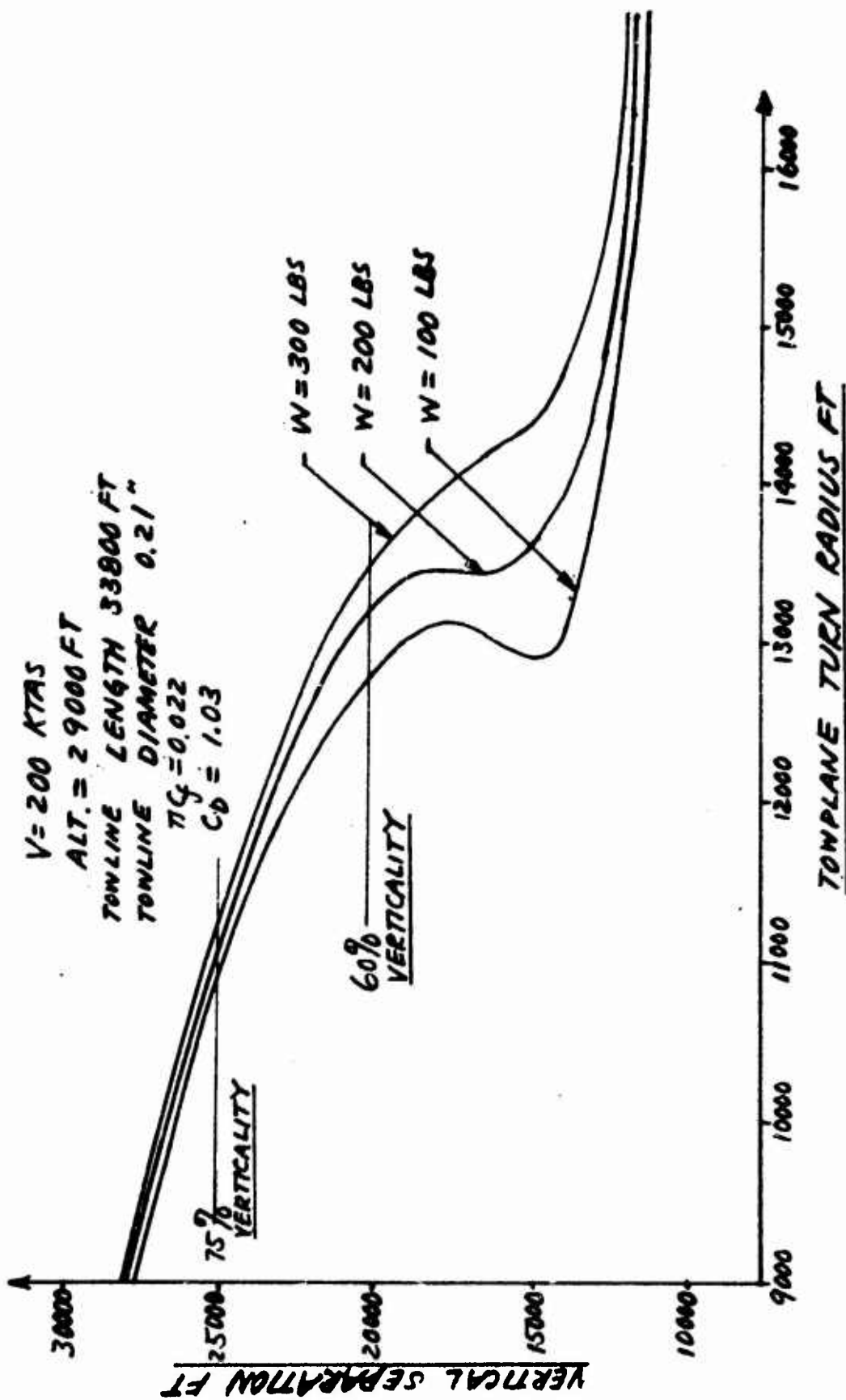


FIGURE 8. EFFECT OF WEIGHT OF THE DROGUE ON VERTICAL SEPARATION

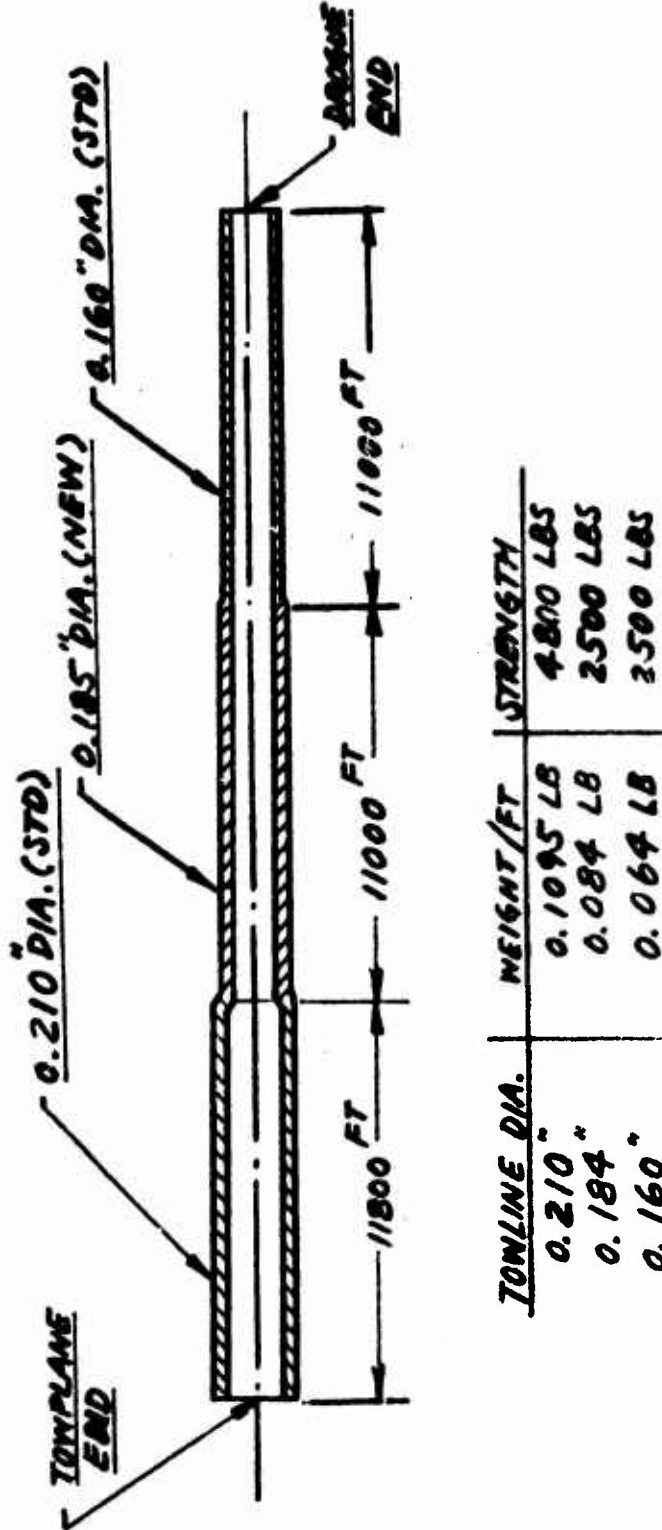


FIGURE 9. STEPPED TOWLINE

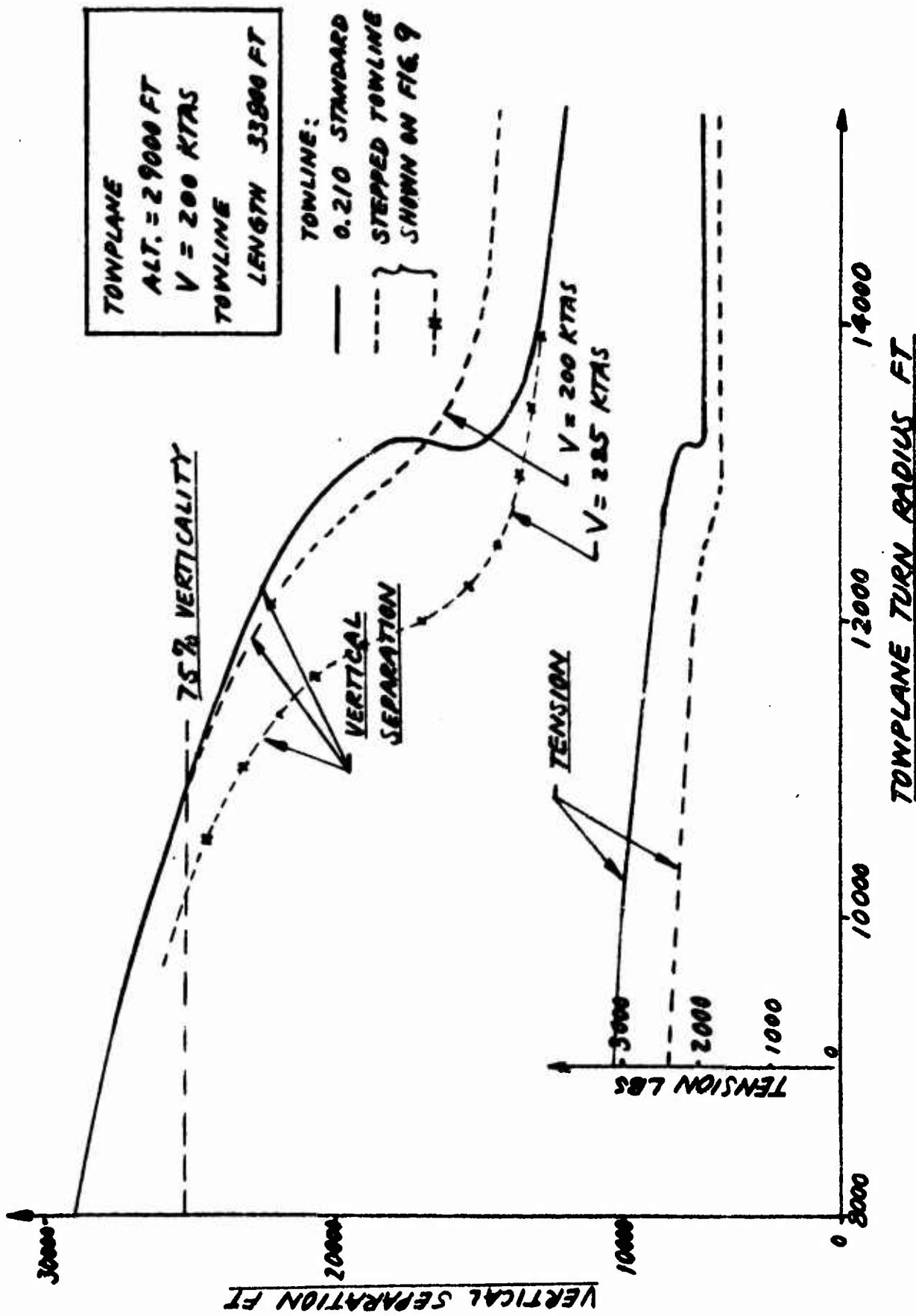


FIGURE 10. VERTICAL SEPARATION VS TOWPLANE TURN RADIUS - STEPPED TOWPLANE

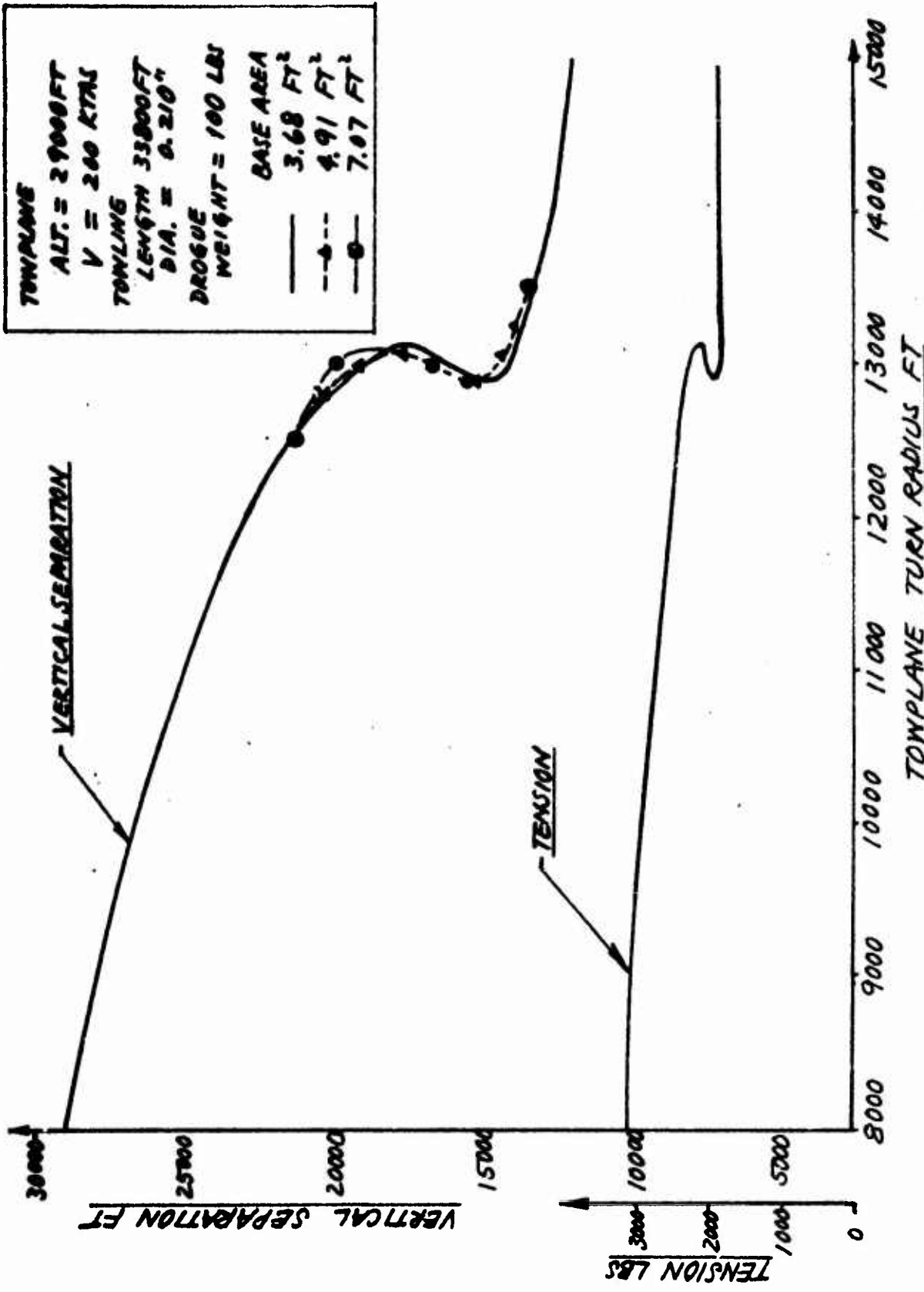


FIGURE 11. EFFECT OF BASE AREA OF THE DROGUE ON VERTICAL SEPARATION

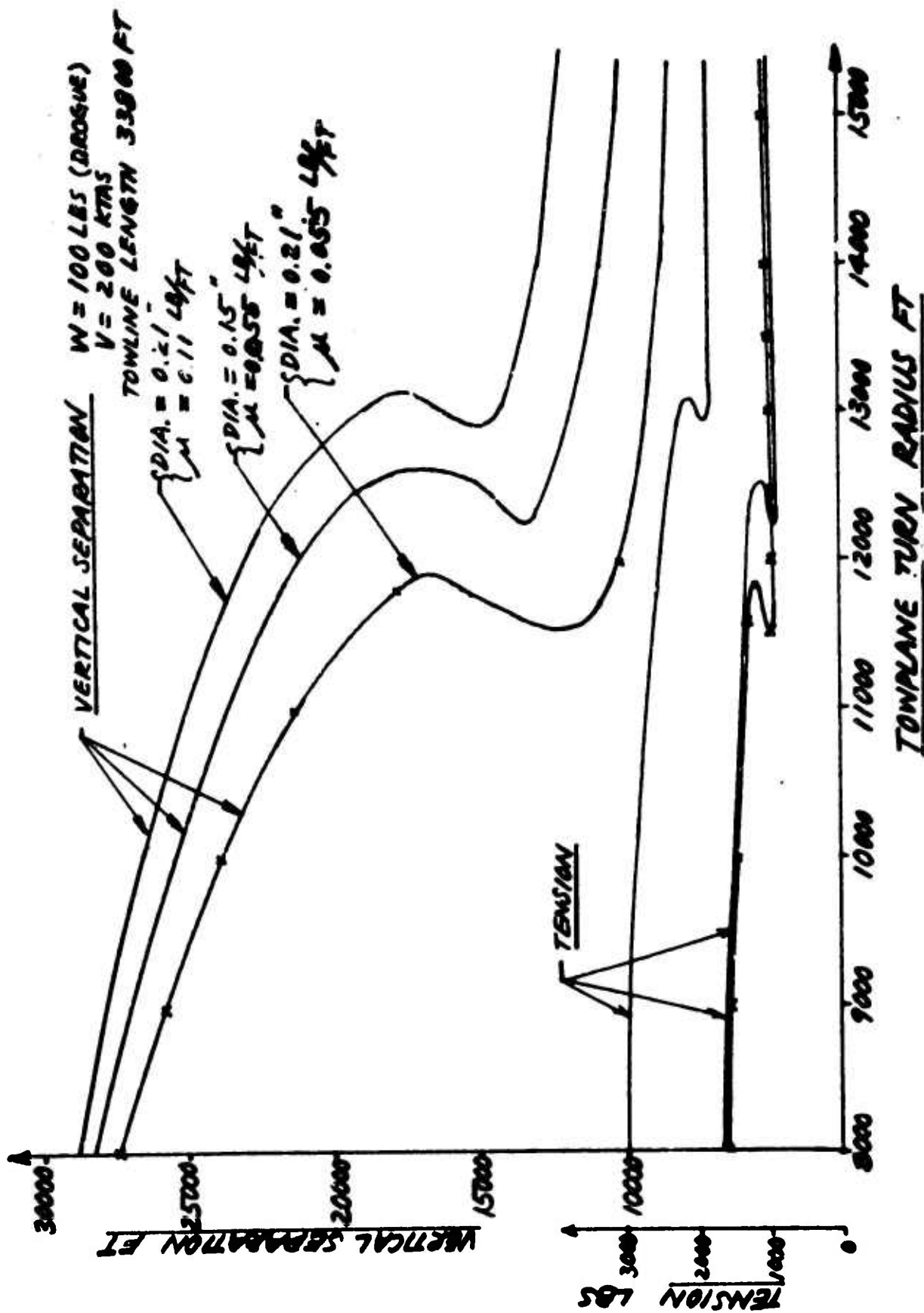


FIGURE 12. EFFECTS OF TOWLINE DIAMETER AND HEIGHT/LENGTH ON VERTICAL SEPARATION

**TABLE I TOWLINE TENSION - NADC MATH
MODEL VS FLIGHT TEST DATA**

V (KTAS)	ALT. (FT)	TOWLINE LENGTH FT	TENSION LBS	
			NADC MATH MODEL	FLIGHT TEST DATA
STRAIGHT AND LEVEL FLIGHT				
195	23500	18000	1310	1300
223	21000	18000	1460	1425
237	20800	18000	1560	1500
247	20200	18000	1650	1575
197	22400	24000	1650	1525 ⁽¹⁾
190	21500	24000	1640	1450
223	24900	24000	1760	1500~1800
194	25800	29000	1888	1900
192	23500	29000	1905	1800~2000
OBITAL FLIGHT				
CASE I	BANK ANGLE = 16° (TOWPLANE TURN RADIUS = 11000 FT)			
189	25400	29000	2100	2100~2425
CASE II	BANK ANGLE = 21° (TOWPLANE TURN RADIUS = 8000 FT)			
186	2250	24000	1972	1825~2425

(1) 100 FPM DESCENT

TOWLINE DIA. = 0.210"

DROGUE WEIGHT = 135 LBS

APPENDIX A. PROGRAM LISTING

```

C ICWLINE STEADY STATE CONFIGURATION-----TOWPLANE IN CIRCULAR ORBIT
C V = ICWPLANE TRUE AIRSPEED , KNOTS
C RPL = TOWPLANE TURN RADIUS (FEET).FOR STRAIGHT AND LEVEL FLIGHT
C ZPL = TOWPLANE DENSITY ALTITUDE FT
C AMG = DROGUE WEIGHT , LB.
C ABASE = DROGUE BASE AREA , SQ FT
C CLURUG = DROGUE DRAG COEFFICIENT
C AMUG = TOWLINE WEIGHT , LB/FT
C D = ICWLINE DIAMETER, FT
C AL = TOWLINE LENGHT , FT
C PCLF = TOWLINE SKIN FRICTION COEFFICIENT
C CLU = TOWLINE DRAG COEFFICIENT
C A = DISTANCE FROM TOWPOINT TO DROGUE CENTER OF GRAVITY , FT
C XCP = DISTANCE FROM TOWPOINT TO DROGUE CENTER OF PRESSURE , FT
C ALPHA = PITCH ANGLE OF DROGUE CENTERLINE (RADIAN). POSITIVE IS NOSE UP
C CAPPAL = SIDE SLIP ANGLE IN RADIAN , TOW POINT LEFT POSITIVE
C CONSTANT
G = 32.17 SPI = 3.1416
ACL = 0.53 SACM = 0.68 SX = 1.34 SXCP = 2.31
1199 IITER = 1
HEAD JCO1 , V1 , RPL , ZPL , RZU , ZZD
IF (V - 9999.) 1200 , 340 , 340
1200 PHIN1 1.01 , V , RPL , ZPL , RZU , ZZD
HEAD 1002 , AMG , ABASE , CDURCG
PHIN1 1102 , AMG , ABASE , CDUHOG
UDCRG = SCRT(4.*ABASE/PI)
HEAD 1003 , AMUG , D , AL , DUS , PICF , CD
PHIN1 1103 , AMUG , D , AL , DUS , PICF , CD
IF (CC - 0.1) 1199 , 1199 , 1400
1400 K = 1
5 IITER = IITER + 1
IF (IITER - 30) 1501 , 1501 , 540
1501 ZZ = ZZD $RZ = HZC $I = 1 $DS = 20 $OMEGA = V*1.69/RPL $M = 1
MH02 = U.00237R*(1. - 0.006R75*ZZ/1000.)*4.254
UMEGSG = CMEGA*OMEGA $S = U.0 $TMZ = 0.0
WS = U.5*RH02*R7*OMEGSG*ABASE

```

```

A = 0.5*AMG *X/GS+ACL*X*0.04
B = (ACL+CDDROG)*X-DDROG*ACM
C = -0.04*ACL*X-AMG *X/QS
ALPHA = (-B+SQR(B*B-4.*A*C))*0.5/A
IF (ALPHA-0.8) 610, 600, 600
600 A2 = 0.85*QS*XCP
B2 = -AMG *X-0.85*PI*QS*XCP SC2 = AMG*X*PI*0.5 - 0.85*QS*XCP
ALPHA = (-B2-SQR(B2*B2-4.*A2*C2))*0.5/A2 SM = 2
AK = AMG/G
A1 = -AM*RZ*OMEGSQ-QS*ACL*0.04
B1 = QS*(CDDROG*ACM/(X*COS(ALPHA)))-ACL-CDDROG
CAPPAL = A1/B1
RMAT (1, 3X, 6HALPHA=, E12.4, 5X, 7HCAPPAL=, E12.4)
CAPPAL-0.04) 615, 615, 620
= -AMG/G*RZ*OMEGSQ
620 630
620 IHPZ = -AM*RZ*OMEGSQ+ACL*(CAPPAL-0.04)*QS
630 IF (ALPHA-0.04) 640, 640, 650
640 IZPZ = AMG
IATHPZ = 0.5*RHOZ*ABASE*CDUHOZ*RZ*RZ*OMEGSQ
GC TO 660
650 GC TO (651, 652),M
651 IZPZ = AMG -ACL*(ALPHA-0.04)*QS
IATHPZ = 0.5*RHOZ*ABASE*CDUHOZ*RZ*RZ*OMEGSQ
GC TO 660
652 IZPZ = AMG -0.85*QS*SIN(ALPHA)**2.*COS(ALPHA)
IATHPZ = 0.85*QS*SIN(ALPHA)**3.
660 IZ = SQR(IHPZ*IHPZ+IATHPZ*IATHPZ+IZPZ*IZPZ)
HPZ = IHPZ/IZ $IHPZ = IATHPZ/(IZ*RZ) SZPZ = IZPZ/IZ
RHPZ = IHPZ*HZ
PRINT 1
PRINT 2, S, HZ, ZZ, IHZ, IZ, RPZ, ZPZ, RHPZ
B.2 = S*DS
Z = ZZ $I = IZ $R = RZ $RP = HPZ $HP = IHPZ $ZP = ZPZ $KK = 1
IU ZA = 0.5*(Z+ZZ) $IA = 0.5*(I+IZ) $HA = 0.5*(R+RZ)
IRPA = 0.5*(IHP+IHPZ) $ZPA = 0.5*(ZP+ZPZ) $RPA=0.5*(RP+RPZ)

```

```

RHO = 0.002378*(1.-0.006875*ZA/1000.)*4.256
TMPASQ = THPA*THPA SRASQ = RA*RA
QC = 0.5*RH0*DRASQ*OMEGSQ*CD
STRTH = SCRT(1.-RASQ*TMPASQ)
THP = (IA*RA*TMPASQ-AMUG/G*RA*OMEGSQ-QD*RA*RP*THPA*STRTH)*DS*TRPZ
0IRIHP=(-IA*RP*THPA*QD*SIRIHP*3.-0.5*RH0*0*PICF*RASQ*OMEGSQ)*DS*IR
1TMPZ
IZP = (AMUG-QD*RA*THPA*ZPA*STRTH)*DS*TZPZ
1I = SQRT(IRP*IRP+IRIHP*IRIHP+IZP*IZP)
MP1 = TRP/1I $R1 = RZ*(MP1*MPZ)*0.5*DS
IMPI = IRTMP/(1I*H1) $ZPI = TZP/1I
Z1 = ZZ*0.5*(ZPI*ZPI)*DC $KIMPI = IHPI*BI
IM1 = 1HZ*(TMPZ*TMP1)*0.5*DS
IF (ABS((Z1-Z)/Z) .GT. 0.001)100, 101
101 IF (ABS((R1-R)/R) .GT. 0.001) 100, 102
102 IF (ABS((1I-1)/1) .GT. 0.001) 100, 32
32 GO TO (33, 34, 35, 35, 35, 36, 36, 36, 37, 37, 37, 37, 38,
38, 39, 40),I
33 DS = 50.
GC TO 49
34 DS = 130.
GC TO 49
35 DS = 200.
GC TO 49
36 DS = 300.
GC TO 49
37 DS = 400.
GC TO 49
38 DS = 600.
GC TO 49
39 DS = 1000.
GC TO 49
30 PRINT 3
GC TO 32
40 IF (S-AL*CMS) 50, 48, 190
48 US = UDS

```

```

49 I = I+1
50 Z2 = Z1 SRZ = R1 STZ = T1 SRPZ = RPI STMPZ = IPI SZPZ = ZPI
147 = I+1 SIRZSIRP SIRMPZIRIRP SIRZSIRP
GC TO (120, 120, 110),K
110 PRINT 2, S, R1, Z1, T1, I1, RPI, ZPI, RTMP1
120 GC TO 8
100 IF (KK.GT. 20) 30, 150
150 Z = Z1 SR = R1 ST = T1 SRP = RPI STMP = TMP1 SZP = ZPI
RIMP = RIMP1 SKK = KK, 1
GC TO 10
10FCRMAI (11, 5X, 1MS, 11X, 1MR, 11X, 1MZ, 11X, 2HTM, 10X, 1MT, 10X
11, 2HRP, 10X, 2MZP, 9X, 4HRLEP)
2 FURMAI (8E12.4)
3 FCRMAI (13H NOT CONVERGE)
1000 ECRMAI (11, 14HRINGT CONVERGE)
1001 FCRMAI (5E12.4)
11010FCRMAI (11, 4MV(KNOT)=E12.4, 2X, 4HRPL, E12.4, 2X, 4MZPL, E12.4, 2X,
14PRZD=E12.4, 2X, 4ZZD=E12.4)
1002 FCRMAI (5X, 3E12.4)
1102 FCRMAI (11, 4MAMG=E12.4, 2X, 6MABASE=E12.4, 2X, 7MCDDROG=E12.4)
1003 FCRMAI (5X, 4E12.4, 1X, 2E9.2)
11030FCRMAI (11, 5MAMUG=E12.4, 2X, 2MD=E12.4, 2X, 3MAL=E12.4, 2X, 4MDOOS=
1E12.4, 2A, 5MPICF=E12.4, 2X, 3PCD=E12.4)
13010FCRMAI (11, 11, 5X, 3MRPL, 9X, 5MRDROG, 7X, 3MZPL, 9X, 5HZDROG, 7X,
13MSEP, 9X, 1MT, 11X, 4MRPPL, 8X, 4MZPPL, 8X, 4MTMPL)
1311 FCRMAI (9E12.4)
150 PHIN1 2, S, R1, Z1, T1, I1, RPI, ZPI, RTMP1
GC TO (200, 203, 1199),K
200 IF (ABS((71-ZPL)/ZPL)-0.010)202, 210, 210
202 GC TO (204, 240),K
203 IF (ABS(Z1-ZPL)-2000.) 204, 204, 210
204 IF (ABS((R1-RPL)/RPL)-0.003) 206, 220, 220
200 GC TO (240, 200),K
210 K = 1
Z20 = Z20 + 0.6*(ZPL-Z1)
GC TO 5

```



```

220 GO TO (510, 515)*K
510 URZD = 0.16*(RPL - R1)
    GO TO 560
515 IF (ABS((R1-RSTORE)/RPL) - 0.002) 516, 516, 520
516 URZD = 0.2*(RPL - R1)
520 URZD = (RPL-R1)*URZD*0.6/(R1-RSTORE)
    IF (ABS(DRZD) - 150.) 528, 528, 525
525 URZD = 150.*DRZD/ABS(DRZD)
    GO TO 560
528 IF (ABS(DRZD) - 5.) 529, 529, 560
529 DRZD = 5.*DRZD/ABS(DRZD)
560 RZD = RZD + DRZD
    K = 2
    RSTORE = R1
    GO TO 5
540 PRINT 1000
    GO TO 1199
240 K = 3
    PRINT 1301
    PRINT 1101, V, RPL, ZPL, RZD, ZZD
    PRINT 1102, AMG, ABASE, CDUR0G
    PRINT 1103, AMJG, D, AL, DUS, PICF, CD
    PRINT 1203, ALPHA, CAPPAL
    GO TO 5
340 STOP
    END

```

APPENDIX B. SAMPLE INPUT AND OUTPUT

V(KNO1)= 6.0000E 02 RPL= 1.1000E 04 ZPL= 2.9000E 04 HZD= 1.8590E 03 ZZ0= 5.1993E 03
 AMG= 1.3500E 02 ABASE= 5.6000E 00 CUCROG= 6.0000E 01
 AMLG= 1.0950E 01 D= 1.7500E 02 AL= 3.3000E 04 DDS= 1.0000E 03 PICF= 2.2000E 02
 ALPHA= 9.4868E 01 CAPPAL= 1.2070E 01

S	R	Z	I	RP	ZP	RIMP
0	1.8590E 03	5.1993E 03	1.2936E 02	-5.0786E 02	9.9655E 01	6.5615E 02
1	1.8580E 03	5.2193E 03	1.3155E 02	-5.0765E 02	9.9594E 01	7.3787E 02
2	1.8554E 03	5.2690E 03	1.3702E 02	-5.0640E 02	9.9438E 01	9.2963E 02
3	1.8489E 03	5.3980E 03	1.5120E 02	-4.9842E 02	9.8951E 01	1.3563E 01
4	1.8392E 03	5.5951E 03	1.7290E 02	-4.7295E 02	9.8145E 01	1.8580E 01
5	1.8301E 03	5.7906E 03	1.9442E 02	-4.3287E 02	9.7388E 01	2.2290E 01
6	1.8220E 03	5.9847E 03	2.1578E 02	-3.8077E 02	9.6722E 01	2.5105E 01
7	1.8150E 03	6.1776E 03	2.3701E 02	-3.1949E 02	9.6152E 01	2.7266E 01
8	1.8070E 03	6.4650E 03	2.6863E 02	-2.1504E 02	9.5455E 01	2.9727E 01
9	1.8022E 03	6.7505E 03	3.0003E 02	-9.9966E 03	9.4913E 01	3.1473E 01
10	1.8011E 03	7.0346E 03	3.3126E 02	2.1629E 03	9.4488E 01	3.2741E 01
11	1.8036E 03	7.3176E 03	3.6234E 02	1.4669E 02	9.4151E 01	3.3666E 01
12	1.8128E 03	7.6935E 03	4.0361E 02	3.1496E 02	9.3800E 01	3.4521E 01
13	1.8247E 03	8.0681E 03	4.4470E 02	4.8132E 02	9.3524E 01	3.5068E 01
14	1.8512E 03	8.4418E 03	4.8565E 02	6.4287E 02	9.3303E 01	3.5402E 01
15	1.8800E 03	8.8146E 03	5.2647E 02	7.9765E 02	9.3111E 01	3.5590E 01
16	1.9344E 03	9.3725E 03	5.8750E 02	1.0149E 01	9.2852E 01	3.5713E 01
17	2.0012E 03	9.9289E 03	6.4829E 02	1.2125E 01	9.2605E 01	3.5739E 01
18	2.1368E 03	1.0853E 04	7.4909E 02	1.4998E 01	9.2159E 01	3.5803E 01
19	2.2984E 03	1.1772E 04	8.4916E 02	1.7409E 01	9.1636E 01	3.6052E 01
20	2.4833E 03	1.2685E 04	9.4841E 02	1.9467E 01	9.1000E 01	3.6606E 01
21	2.6870E 03	1.3591E 04	1.0467E 03	2.1281E 01	9.0225E 01	3.7505E 01
22	2.9081E 03	1.4489E 04	1.1440E 03	2.2938E 01	8.9290E 01	3.8745E 01
23	3.1453E 03	1.5376E 04	1.2399E 03	2.4505E 01	8.8178E 01	4.0300E 01
24	3.3980E 03	1.6251E 04	1.3344E 03	2.6026E 01	8.6876E 01	4.2134E 01
25	3.6657E 03	1.7112E 04	1.4272E 03	2.7525E 01	8.5371E 01	4.4206E 01
26	3.9484E 03	1.7958E 04	1.5181E 03	2.9014E 01	8.3656E 01	4.6474E 01
27	4.2460E 03	1.8784E 04	1.6067E 03	3.0511E 01	8.1733E 01	4.8876E 01

1.6000E 04	4.5585E 03	1.9591E 04	2.3338E 00	1.6929E 03	3.1974E-01	7.9597E-01	5.1401E-01
1.7000E 04	4.8853E 03	2.0375E 04	2.4455E 00	1.7766E 03	3.3383E-01	7.7255E-01	5.4011E-01
1.8000E 04	5.2258E 03	2.1135E 04	2.5550E 00	1.8575E 03	3.4718E-01	7.4721E-01	5.6670E-01
1.9000E 04	5.5791E 03	2.1869E 04	2.6624E 00	1.9354E 03	3.5952E-01	7.2014E-01	5.9342E-01
2.0000E 04	5.9442E 03	2.2575E 04	2.7677E 00	2.0103E 03	3.7062E-01	6.9161E-01	6.1994E-01
2.1000E 04	6.3196E 03	2.3252E 04	2.8710E 00	2.0820E 03	3.8024E-01	6.6195E-01	6.4594E-01
2.2000E 04	6.7038E 03	2.3898E 04	2.9722E 00	2.1505E 03	3.8820E-01	6.3153E-01	6.7117E-01
2.3000E 04	7.0951E 03	2.4514E 04	3.0712E 00	2.2158E 03	3.9436E-01	6.0074E-01	6.9539E-01
2.4000E 04	7.4916E 03	2.5100E 04	3.1682E 00	2.2780E 03	3.9867E-01	5.7003E-01	7.1842E-01
2.5000E 04	7.8915E 03	2.5655E 04	3.2630E 00	2.3371E 03	4.0113E-01	5.3975E-01	7.4011E-01
2.6000E 04	8.2930E 03	2.6180E 04	3.3558E 00	2.3934E 03	4.0179E-01	5.1027E-01	7.6039E-01
2.7000E 04	8.6943E 03	2.6676E 04	3.4464E 00	2.4469E 03	4.0078E-01	4.8191E-01	7.7919E-01
2.8000E 04	9.0938E 03	2.7144E 04	3.5350E 00	2.4981E 03	3.9825E-01	4.5492E-01	7.9652E-01
2.9000E 04	9.4901E 03	2.7587E 04	3.6216E 00	2.5469E 03	3.9440E-01	4.2952E-01	8.1238E-01
3.0000E 04	9.8820E 03	2.8004E 04	3.7062E 00	2.5939E 03	3.8942E-01	4.0581E-01	8.2684E-01
3.1000E 04	1.0268E 04	2.8399E 04	3.7890E 00	2.6391E 03	3.8352E-01	3.8386E-01	8.3998E-01
3.2000E 04	1.0644E 04	2.8773E 04	3.8699E 00	2.6829E 03	3.7680E-01	3.6366E-01	8.5188E-01
3.3000E 04	1.1022E 04	2.9127E 04	3.9490E 00	2.7255E 03	3.6974E-01	3.4521E-01	8.6263E-01

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<p>→ A steady-state mathematical model which computes spatial configuration and tension along an orbiting towline has been developed. Numerical results are in agreement with existing flight test data. The model yields multi-valued solutions under certain operating conditions. Operations which avoid these troublesome regimes have been developed. The model can be used to optimize the performance of operational systems. For example, numerical results are presented for TACAMO which show how the operation of that system can be improved. There are conditions where the desired verticality cannot be obtained with a standard constant diameter towline. Numerical results for a special stepped diameter towline illustrate how the desired high verticality and a reduction of tension can be achieved under such conditions by variation of the towline construction. () ←</p>		

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