UNCLASSIFIED

AD NUMBER:	AD0855778
LIMITATION CHANGES	
TO: Approved for public release; distribution is unlimited.	
FROM: Distribution authorized to US Government Agencie only; Export Control; 1 Jul 1969. Other requests shall be referred to Air Force Armament Laboratory, Eglin AFB, FL 32542	
AUTHORITY	
AFATL ltr dtd 27 Sep 1972	

THIS PAGE IS UNCLASSIFIED

ARMAMENT MEMORANDUM REPORT NO. 69-10

AD855778

USING THE PONCELET PENETRATION EQUATION TO PREDICT THE INSTANTANEOUS VELOCITY OF A PROJECTILE IN RICE

by

Richard P. Warnis

July 1969

D D C
RECEIVED
AUG 4 1969
B

STATEMENT #2 UNCLASSIFIED

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of ______________________

AIR FORCE ARMAMENT LABORATORY

AIR FORCE SYSTEMS COMMAND

EGLIN AIR FORCE BASE, FLORIDA, 32542

Attn: ATRD

19

ARMAMENT MEMORANDUM REPORT NO. 69-10

USING THE PONCELET PENETRATION EQUATION TO PREDICT THE INSTANTANEOUS VELOCITY OF A PROJECTILE IN RICE

by

Richard P. Warnis

Damage Mechanisms Branch (ATRD)

This report is done under Project 9850G002

CHARLES K. ARPKE, Lt Col, USAF
Acting Chief, Technology Division

July 1969

AIR FORCE ARMAMENT LABORATORY
AIR FORCE SYSTEMS COMMAND
EGLIN AIR FORCE BASE, FLORIDA

FOREWORD

The author wishes to thank Bill Taylor and Russell Fanning of ADTVM-1, Eglin AFB, for their services. They developed and used the numerical "secant iteration" method to solve for the Poncelet equation constants.

ABSTRACT

This study was concerned with using the Poncelet penetration equation to predict the instantaneous velocity of a 20mm APT (Armor Piercing Tracer), M95, projectile at various depths into long-grained, polished rice. The Poncelet predictions found have yet to be tested with a controlled firing program.

TABLE OF CONTENTS

FIGURES

TABLES

INTRODUCTION

This report is using data from firing a 20mm APT (Armor Piercing Tracer), M95, projectile into rice done by Captain K. Shaw (ATBT) in September 1967. A description of the test setup is seen in Figure 1. The data is presented in Table 1.

Captain Shaw fired other types of projectiles into rice, but an insufficient number of data points (times associated with depths in rice) prevented their use in plotting a Poncelet graph of velocity as a function of depth in rice.

AVERAGE VELOCITY

An average velocity curve for the 20mm APT projectile data in Table 1 is plotted in Figure 2. This was obtained by finding the average velocity from:

(1) $$\bar{V} = \frac{x_2 - x_1}{t}$$

where

t = Time for the projectile to cover a depth x_1 to x_2 in rice ($x_2 > x_1$)

PONCELET EQUATION

The Poncelet penetration equation used is:[1]

(2) $$F = A \alpha V^2 + A \gamma$$

where

F = Force acting on the projectile as it moves through the rice, lbs

V = Instantaneous velocity of the projectile in rice, $\frac{in}{sec}$

α = Constant to be determined, $\frac{lbs \quad sec^2}{in^4}$

γ = Constant to be determined, $\frac{lbs}{in^2}$

Now (2) is developed by integrating force F:

(3) $$F = A \alpha V^2 + A \gamma$$

Let:

$$F = \text{(MASS)(DEACCELERATION)} = -ma$$

$$-ma = A \alpha V^2 + A \gamma$$

$$\frac{\partial V}{\partial t} = \frac{-A \alpha V^2}{m} - \frac{A \gamma}{m}$$

$$\frac{\partial V}{\partial t} = \frac{-A}{m} [\alpha V^2 + \gamma]$$

$$\left[\frac{1}{\alpha V^2 + \gamma}\right] \partial V = \frac{-A}{m} \partial t$$

1 See report ASD-TDR-63-887, Vol. II, Air Delivered Area Denial Weapons, for the Poncelet and other penetration equations.

$$\int_{V_o}^{V} \frac{1}{\alpha V^2 + \gamma} \partial V = \int_{o}^{t} \frac{-A}{m} \partial t$$

with boundary conditions:

$t = o$, $x = o$, and $V = V_o$ (striking velocity)

Use is made of:

$$\int \frac{\partial x}{a + bx^2} = \frac{1}{\sqrt{ab}} \tan^{-1} \frac{x\sqrt{ab}}{a}$$

where in our case:

$a = \gamma$, $b = \alpha$, $x = V$

giving:

$$\frac{1}{\sqrt{\alpha\gamma}} \tan^{-1} \frac{V\sqrt{\alpha\gamma}}{\gamma} \Bigg|_{V_o}^{V} = \frac{-At}{m} \Bigg|_{o}^{t}$$

$$\frac{1}{\sqrt{\alpha\gamma}} \tan^{-1} \frac{V\sqrt{\alpha\gamma}}{\gamma} - \frac{1}{\sqrt{\alpha\gamma}} \tan^{-1} \frac{V_o\sqrt{\alpha\gamma}}{\gamma} = \frac{-At}{m}$$

Multiplying by $\sqrt{\alpha\gamma}$ and rearranging gives:

(4) $$V = \frac{\gamma}{\sqrt{\alpha\gamma}} \tan \left(\frac{-\sqrt{\alpha\gamma}\, At}{m} + \tan^{-1} \frac{V_o\sqrt{\alpha\gamma}}{\gamma} \right)$$

Since the experimental data is in x and t, (4) is changed to $x = f(t)$:

$$V = \frac{\partial x}{\partial t} = \frac{\gamma}{\sqrt{\alpha\gamma}} \tan \left(\frac{-\sqrt{\alpha\gamma}}{m} At + K \right)$$

where

$$K = \tan^{-1} \frac{V_o\sqrt{\alpha\gamma}}{\gamma}$$

$$\int_{o}^{x} \partial x = \int_{o}^{t} \frac{\gamma}{\sqrt{\alpha\gamma}} \tan \left(-\frac{\sqrt{\alpha\gamma}}{m} At + K \right) \partial t$$

or

$$x = \frac{\gamma}{\sqrt{\alpha\,\gamma}} \int_{o}^{t} \tan\left(\frac{-\sqrt{\alpha\,\gamma}}{m}\,At + K\right)\,\partial t$$

Let:

$$Y = -\frac{\sqrt{\alpha\,\gamma}}{m}\,At + K$$

$$t = \frac{-mY}{\sqrt{\alpha\,\gamma}\,A} + \frac{m}{\sqrt{\alpha\,\gamma}}\,\frac{K}{A}$$

$$\partial t = \frac{-m\partial Y}{\sqrt{\alpha\,\gamma}\,A}$$

$\therefore$

$$x = \frac{-m}{\sqrt{\alpha\,\gamma}\,A}\,\frac{\gamma}{\sqrt{\alpha\,\gamma}} \int_{o}^{t} \tan Y\,\partial Y$$

Using:

$$\int \tan x\,\partial x = -\log\,(\cos x)$$

gives:

$$x = \frac{-m}{\sqrt{\alpha\,\gamma}\,A}\,\frac{\gamma}{\sqrt{\alpha\,\gamma}}\left[-\log\,\cos\left(\frac{-\sqrt{\alpha\,\gamma}}{m}\,At + K\right)\right]\Bigg/_{o}^{t}$$

(5)
$$x = \frac{m}{\alpha A}\,\log\left[\cos\left(-\frac{\sqrt{\alpha\,\gamma}}{m}\,At + K\right)\right] \\ \frac{-m}{\alpha A}\,\log\,(\cos K)$$

Equations (4) and (5) will be the required developed Poncelet penetration equations used in this report.

DETERMINING THE CONSTANTS α AND γ

The value of γ in (5) was chosen to be a constant $\gamma = 1$.* The problem reduces to finding the associated α for each associated depth and time from Table 1 data. Equation (5) was solved for α, given $\gamma = 1$, by using a numerical "secant iteration" technique. The value of α for a depth and time was found to be single-valued, i.e., one value of α for the given $\gamma = 1$. These values of α are presented in Table 2 for each depth and time set.

The ideal objective would be to determine a γ and α fitting all depth and time data sets. This was not done in this report.

* This choice is arbitrary.

FINDING THE INSTANTANEOUS VELOCITY

With the Table 2 values of α, for $\gamma = 1$, equation (4) was used to solve for the instantaneous velocity V. These values of V are shown in Table 2.

The data from Table 2 is plotted on Figure 2. The Poncelet relation does not show a zero velocity at the maximum depth of 50 inches. This maximum depth reached is not a function of a constant projectile presented area. The projectile tumbles and its presented area changes as it reaches greater depths. The Poncelet relation assumes a constant presented area through the medium. Since this does not exist, then the calculated and actual max'mum depths will not necessarily agree.

CONCLUSIONS

The developed Poncelet penetration relations seem to present a reasonable picture of the instantaneous velocity at different depths in rice. A test program is needed to show how well the Poncelet relations represent actual data. A universal γ and α for the 20mm APT shot into rice could be found through a more extensive analysis.

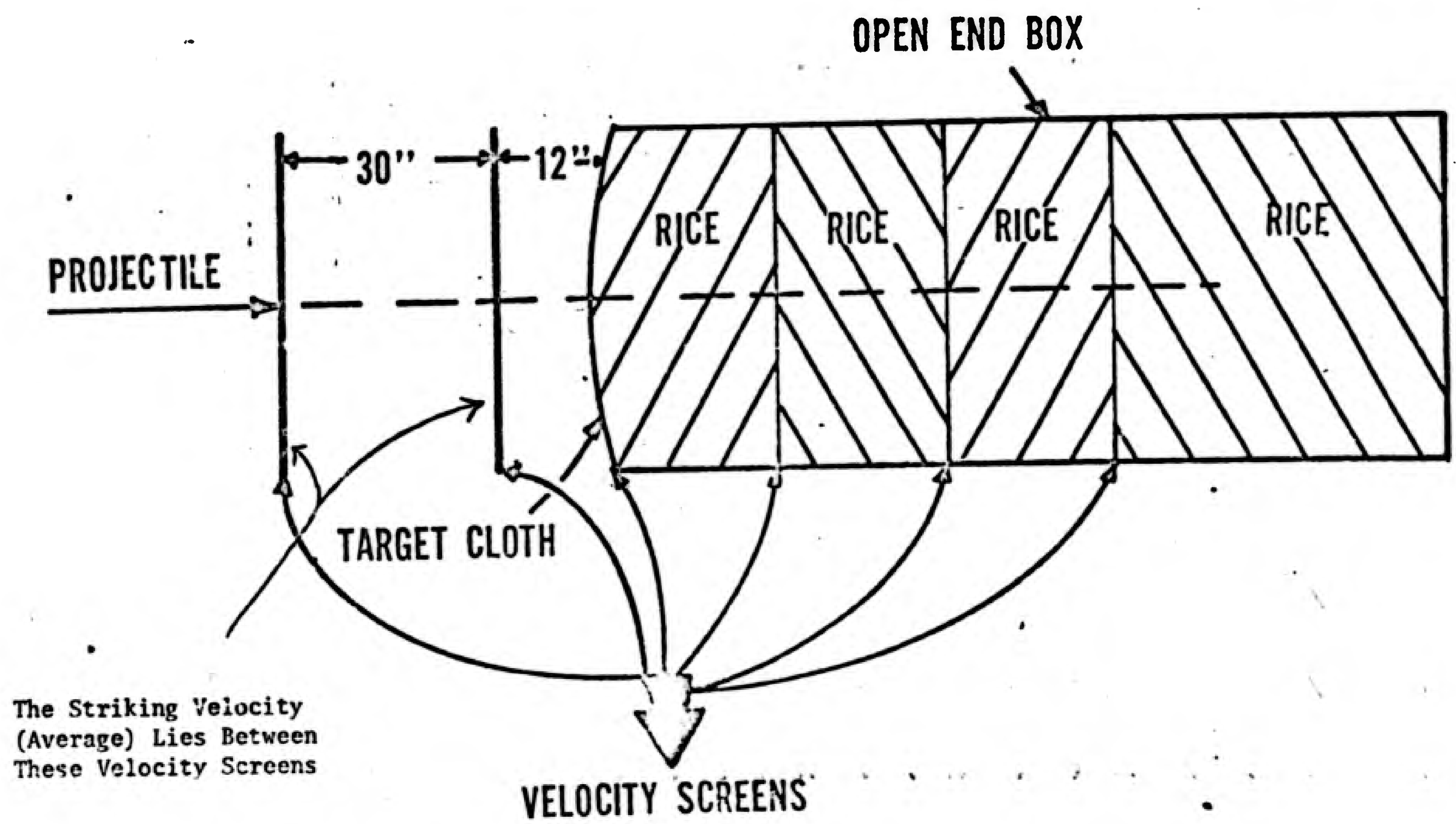

FIGURE 1. Test set-up for determining the average velocity of the 20mm APT projectile into rice

Table 1. Experimental Data

Projectile: 20mm APT, M95
Mass m= .0088 Slugs
Presented Area A=.4867 in^2

Approx. Striking Velocity (Depth Xo) (FT/Sec)	Depth X_i Measured to Nose of Projectile (inches)	Depth Difference $(X_{i+1}-X_i)$ inches)	Time t_i (Secs)	Time Difference $(t_{i+k}-t_i)$ (Secs)	Average* Velocity $\frac{(X_{i+1}-X_i)/12}{t_{i+1}-t_i}$ (FT/Sec)	Approx. Maximum Depth Measured to Nose of Projectile (inches)
2680	x_1 = 13		453×10^{-6}		2391	50
		12		799×10^{-6}		
	x_2 = 25		1252×10^{-6}		1252	
		12		6046×10^{-6}		
	x_3 = 37		7298×10^{-6}		165	
		12		15191×10^{-6}		
	x_4 = 49		22489×10^{-6}		66	

* The average velocity is arbitrarily assigned depths x_i+6", i = 0,1,2,3

Table 2.

Calculated α Values for $\gamma = 1 \frac{(lbs)}{(in^2)}$

and Instantaneous Velocities

Depth x_i (inches)	Time t_i (Secs)	$\alpha \frac{(lbs \cdot sec^2)}{in^4}$	Instantaneous Velocity (Ft/Sec)
$x_1 = 13$	453×10^{-6}	3.6258×10^{-6}	2184
$x_2 = 25$	1252×10^{-6}	7.4982×10^{-6}	1166
$x_3 = 37$	7298×10^{-6}	17.0993×10^{-6}	147
$x_4 = 49$	22489×10^{-6}	17.7809×10^{-6}	48

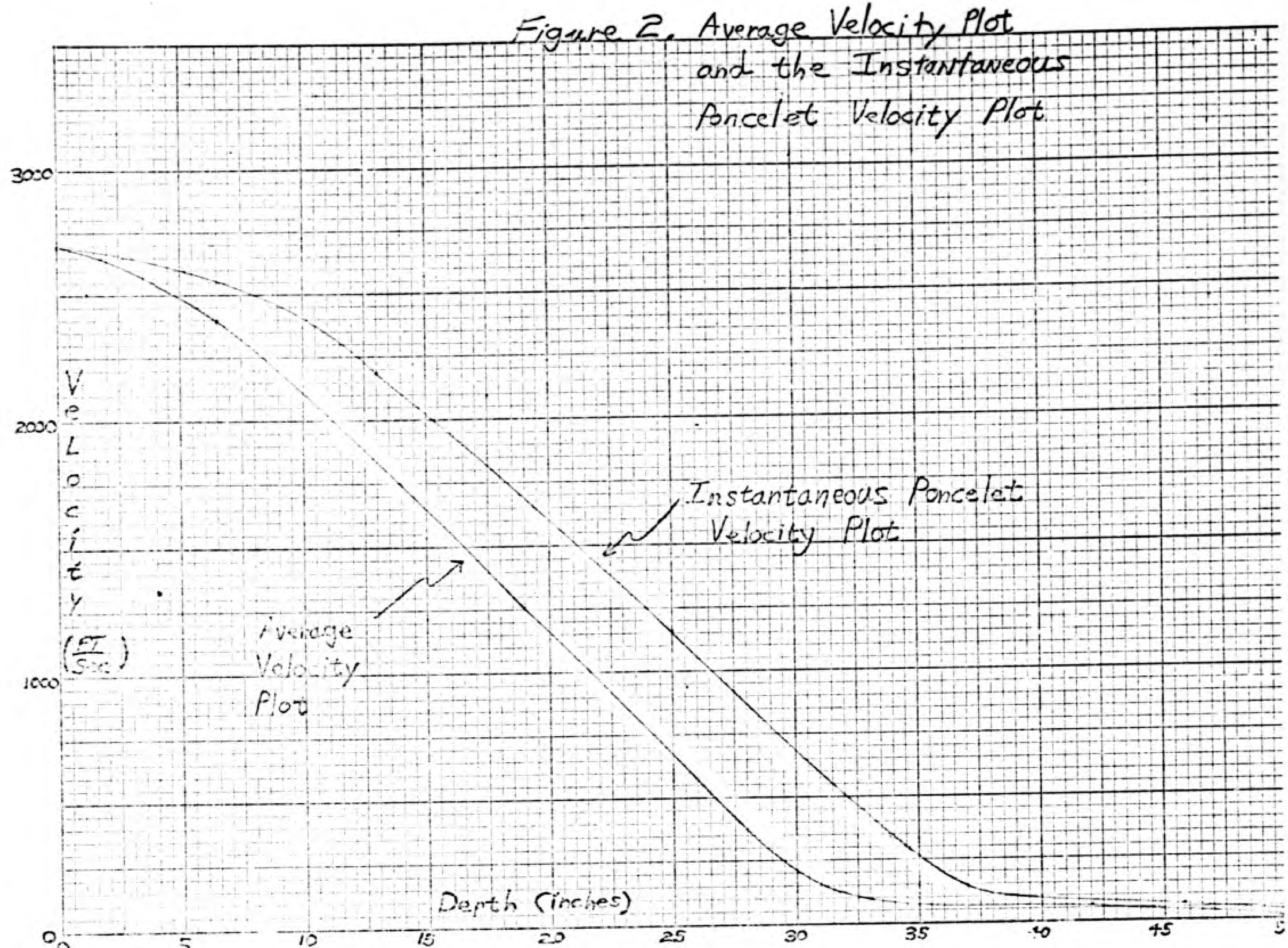
Figure 2. Average Velocity Plot
and the Instantaneous
Poncelet Velocity Plot
Instantaneous Poncelet
Velocity Plot
Average
Velocity
Plot
Velocity
(FT/Sec)
3000
2000
1000
0
0
5
10
15
20
25
30
35
40
45
Depth (inches)

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Air Force Armament Laboratory Damage Mechanisms Branch (ATRD) Eglin Air Force Base, Florida, 32542	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

Using the Poncelet Penetration Equation to predict the instantaneous velocity of a projectile in rice.

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final (1 January 1968 - 1 July 1969)

5. AUTHOR(S) (First name, middle initial, last name)

Warnis, Richard P.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July 1969	19	1

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) AMR - 69-10
b. PROJECT NO. 9850G002	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) -
d.	

10. DISTRIBUTION STATEMENT

This document is subject to special export controls, and each transmittal to foreign governments or foreign national may be made only with prior approval f the Air Force Armament Laboratory (ATRD), Eglin AFB, Florida, 32542.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Available at DDC	Air Force Armament Laboratory Air Force Systems Command Eglin Air Force Base, Florida, 32542

13. ABSTRACT

This study was conceived with using the Poncelet penetration equation to predict the instantaneous velocity of a 20mm APT (Armor Piercing Tracer), M95, projectile at various depths into long-grained polished rice. The Poncelet predictions found have yet to be tested with a controlled firing program.

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
20 mm APT						
Penetration Mechanics						
Poncelet Equation						
Rice						

UNCLASSIFIED

Security Classification