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VORTEX RINGS FROM UNDERWATER EXPLOSIONS

Final Report

April 1969

Prepared for

OFFICE OF NAVAL RESEARCH
Department of the Navy
Washington, D.C. 20360

Contract No. N00014-69-C-0042
Contract Authority Identification No. NR 069-058/8-5-68

URS RESEARCH COMPANY

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39

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VORTEX RINGS FROM UNDERWATER EXPLOSIONS

Final Report

April 1969

by

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C	Circulation around vortex ring core, ft^2/sec (Sketch C)
E	Kinetic energy of vortex ring, ft-lb – Eq. (14)
g	Gravitational constant, 32.2 ft/sec^2
I	Impulse imparted to pendulum, lb-sec – Eq. (4)
J	Momentum of vortex ring, lb-sec – Eq. (15)
L	Length of pendulum suspension wires, 129 in.
R	Major radius of vortex ring, ft (Sketch C)
r	Minor radius of vortex ring, ft (Sketch C)
S	Spacing between cup and disk, ft (Fig. 1)
U	Initial velocity of pendulum, ft/sec – Eq. (3)
V	Translational speed of vortex ring, ft/sec – Eq. (13)
V_m	Peripheral speed of vortex ring, or cavitation speed, ft/sec (Sketch C)
W	Total pendulum weight in air, lb (Table 2, Fig. 2)
W_B	Pendulum displacement, or buoyancy, lb – Eq. (5)
W_V	Pendulum virtual mass, lb – Eq. (6)
X	Maximum displacement of pendulum, ft – Eq. (1)
Δ	Reduction of X due to damping, ft – Eq. (5)
Δp	Local hydrostatic pressure minus cavitation pressure, lb/ft^2 – Eq. (17)
ρ	Density of water, 1.94 slug/ft^3
ω	Vorticity within vortex ring core, rad/sec (Sketch C)
$(\ )_G$	( ) For gun pendulum or ring generator
$(\ )_T$	( ) For target disk pendulum

Section 1
INTRODUCTION

Apparently Northrup was the first to study vortex rings in water. He obtained visible rings with pulsed flows of dyed water through an orifice plate with a 1-cm-diameter hole. He produced both single rings and pairs of rings, and he photographed their interactions and oscillations. More recent experimental studies are described by Turner, who investigated the effect of buoyancy on vortex rings projected slowly upward through saltwater solutions.

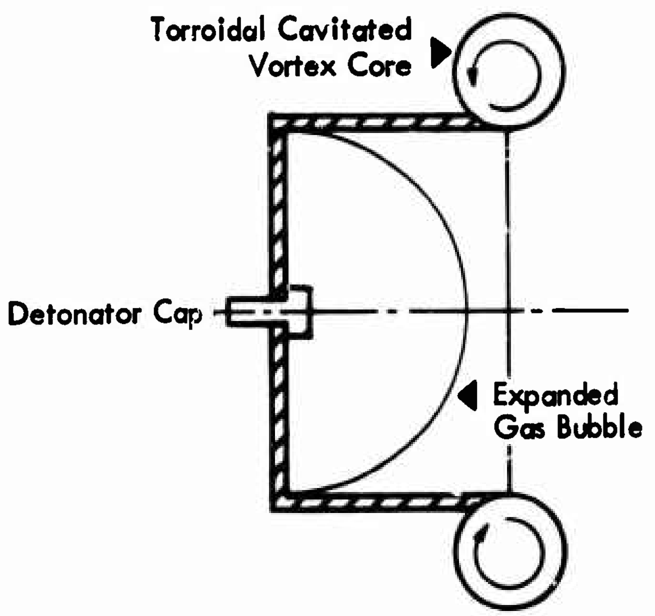
The purpose of the present study is to investigate the properties of vortex rings which are produced explosively in water with enough intensity to drop the pressure near the core of the vortex so low that the water flashes to vapor (or cavitates). Cavitation causes the cores of underwater vortex rings to consist of water vapor and limits the velocity, energy, and impulse which can be imparted to a vortex ring of given diameter at a shallow depth underwater. Cavitated rings were produced during part of a classified study by Spies. In the present study vortex rings are produced by explosions in short cups or gun tubes, rather than in the usual type of container with orifice plates. The measured properties of the rings will be compared with theoretical results for hollow vortex rings travelling through an inviscid liquid. It is well known that ring vortices are produced at the end of gun barrels and shock tubes in air, and such rings have been studied by Elder and deHaas. One objective of the present study is to determine the smallest cup which can be used to generate a vortex ring from a given explosive charge underwater. The advantage of the cup (over a container with an orifice plate) is that the ring diameter is larger relative to the container since the ring forms outside the lip of the cup. Hence the short cup may be the smallest, simplest, and cheapest container which can be used to produce a vortex ring from an explosion.

Considerable theory has been developed for ring vortices as a consequence of the 19th century belief that atoms were hollow vortex rings travelling through ether, and the companion belief that ether was an inviscid, incompressible fluid. The theory for vortex rings is summarized by Lamb, who references the theoretical treatments of hollow vortex rings by Pockington and by Carslow. The equations given by Pockington for the velocity and impulse of a hollow vortex ring will be used here for comparison with measured data.

The objective of the present study is to develop information about the generation and propagation of vortex rings underwater so that their application for various practical purposes can be assessed. An initial assessment of the military applications of air vortex rings has already been made by Zaroodny.

Section 2
EXPERIMENTAL PROGRAM

The experimental program consisted of firing 1/2-gm detonator caps in short steel cylinders (or cups) of various sizes, which were submerged in a large tank of water. Vortex rings with cavitated (steam) cores were generated, and these rings impinged on a target disk placed at various distances from the cups. The formation of a vortex ring on the cup lip is indicated in Sketch A. The ring subsequently propagates off the cup, and it is enclosed in a nearly spherical ellipsoid of water which travels with the ring. The cups and the disk were mounted as ballistic pendulums so that the kinetic energy and momentum imparted to them could be deduced from their subsequent motions. All the measured data were obtained by means of underwater photography. The data consist of color motion pictures showing the formation of rings of cavitated water (or steam) outside the lips of the cups, the propagation of these vortex rings from the cups, and the subsequent motions of the cup and target disk.



Sketch A. Formation of Ring Vortex on Cup Lip

TEST SET-UP

The tests were conducted in a cylindrical pool of water with a diameter of 25 ft and a depth of 4 ft. The arrangement of the ring generator, target disk, and cameras is shown in Fig. 1. A maximum spacing between the cup and disk of about 16 ft could be obtained. The cup and disk were mounted to cylindrical weights suspended as pendulums on parallel wires 129 in. long. The dimensions and weights of the vortex ring generators are given in Fig. 2. The cup and disk both oscillated as damped ballistic pendulums with maximum deflections of about 10 in. and 6 in., respectively. Their centerline was 20 in. below the water surface. The deflections were measured by photography of a stationary pointer in front of 1-in. scales marked on the cylindrical pendulums. Two movie cameras mounted in submerged plywood boxes looked horizontally through windows at the cup, the disk, and the steam cores of the vortex rings as the rings traveled to the disk and spread out over it. A string of beads ran horizontally 20 in. beneath the centerline of the cup, disk, and vortex ring. The velocity, diameter, and thickness of each vortex ring core were determined from the movies of its motion above the beads, which were spaced 1 ft apart. Three cameras were used during the study, two 16-mm movie cameras at nominal speeds of 60 frames/sec and a Hycam camera with a nominal speed of 275 frames/sec. Since their actual speed varied from 41 to 60 frames/sec, the speeds of the slower cameras were measured each time they were used by photographing the sweep second hand of a clock. Lighting was provided by floodlights mounted beneath the water surface in plywood boxes similar to the camera boxes. For all of the tests, 1/2-gm detonator caps projected axially from end plates of the cups as indicated in Fig. 2, which gives the dimensions of the four short cups used in the initial tests designed to study the effect of cup size. The lengths of all these cups were nearly equal to their radii. In subsequent tests the cup shape was varied.

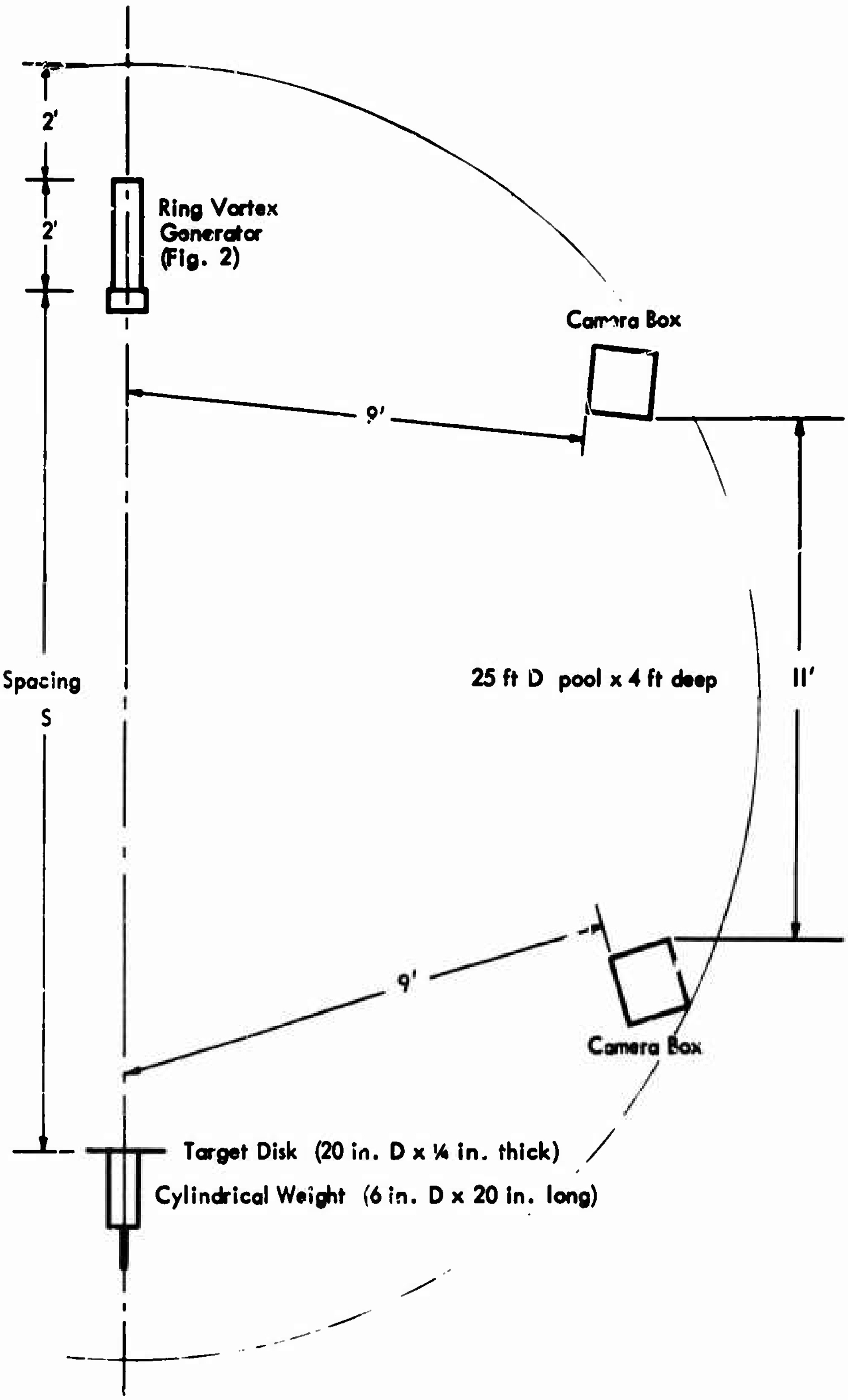
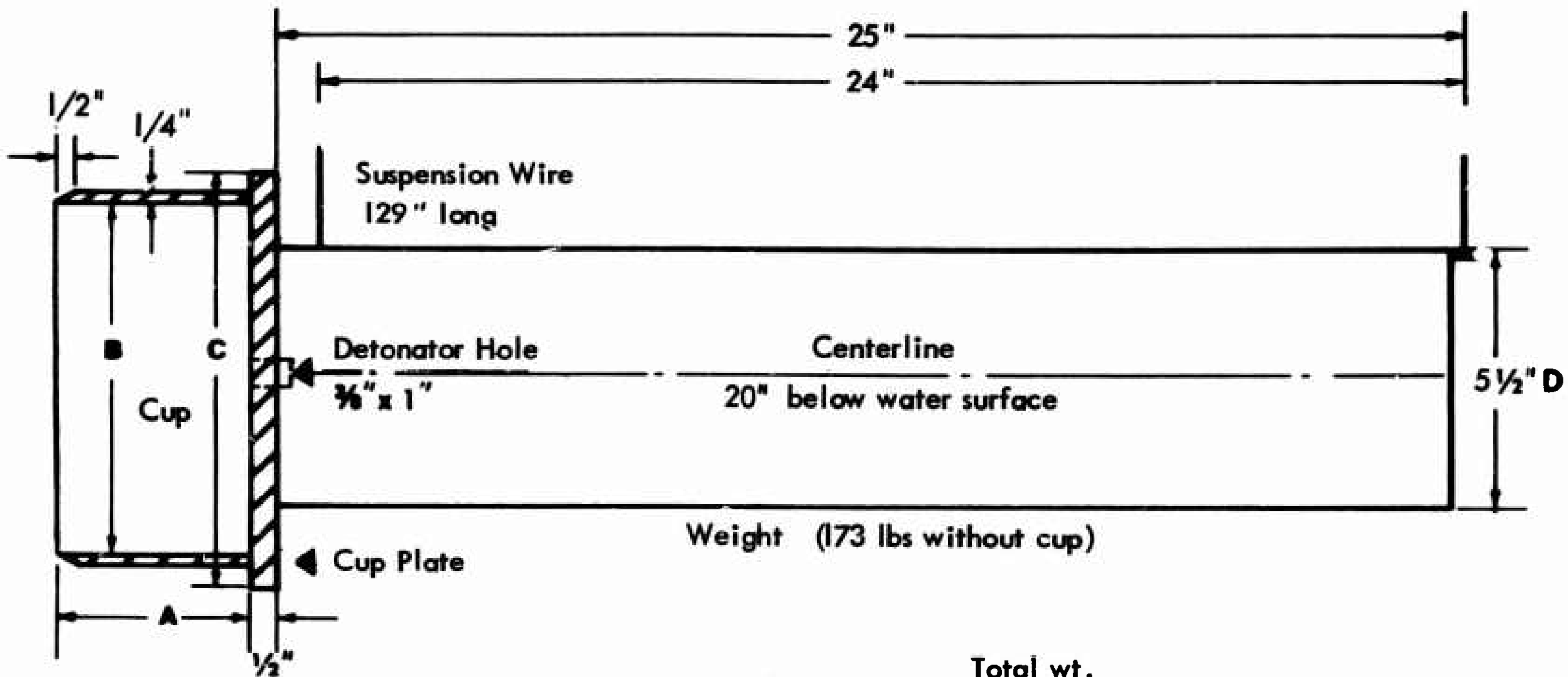


Fig. 1. Arrangement of Pendulums and Cameras



Cup	A	B	C	Total wt. in air (lbs)
4 in	2	4	5	178
6 in	3	6	7	183
8 in	4½	7½	8½	188
10 in	4½	9½	11	195

Dimensions in inches

Fig. 2. Ring Vortex Generator Pendulum

TEST RESULTS

Effective Yield

An initial test was made with a detonator cap placed against a rigidly mounted flat steel plate to determine the effective yield of the caps and to see if the expanded gas bubble was hemispherical even though the caps produce axial jets. A 12-in.-diameter hemispherical bubble was produced with about six subsequent bubble oscillations. The effective TNT charge weight for a spherical bubble with a 6-in. radius at a depth of 2 ft is

$$w\ (\text{lb}) = \left[\frac{R_B\ (\text{ft})}{12.6}\right]^3 (33 + d)(\text{ft})$$

$$= \left(\frac{6}{12 \times 12.6}\right)^3 (35) = 0.00219\ \text{lb} = 0.99\ \text{gm}$$

Hence the effective yield of the detonator caps will be taken as 1/2 gm for the hemispherical bubble, and the potential energy of the caps will be taken as

$$PE = 0.0011\ \text{lb} \times 1.43 \times 10^6\ \frac{\text{ft-lb}}{\text{lb}} = 1570\ \text{ft-lb}$$

The diameter of an equivalent sphere of TNT is about 0.3 in.

Tests Without Cups

Initial tests were run with the detonator caps fired on the four steel back plates of the various cups shown in Fig. 2, but without the cups attached and without the target disk. The cups were spot-welded to the back plates for subsequent tests. No vortex rings were generated without the cups, of course, but the maximum displacement of the plate pendulum and its total weight in air are given in Table 1 for the four tests. The deflection increased with decreasing plate diameter mainly because of water drag.

Table 1

DATA FOR TESTS WITHOUT CUPS

Plate Diameter (in.)	Maximum Plate Displacement X_G(in.)	Pendulum Weight W_G(lb)
11	2.0	187
8-1/2	3.0	181
7	3.5	179
5	3.5	176

The test with the 8-1/2-in. plate was repeated with the target disk placed 2 ft away from the plate. In this case, the plate displaced 3.5 in. and the disk displaced 1.3 in.

Tests With 10-in. Cup

A series of five tests was made with the 10-in. cup spaced from the target disk at various distances. The results are shown in Table 2. The internal volume of this cup was only about 70 percent of the measured value for the free (12-in. hemisphere) gas bubble. Nevertheless, no gas escaped from the cup until long after the departure of the vortex ring, and there was no bubble gas mixed with the steam in the vortex core. High-speed movies showed that just after the vortex core departed from the lip of the cup, the gas bubble contracted and drew the ring back toward the cup and contracted it. The second bubble expansion then launched the vortex ring. The first two tests were run to obtain the characteristics of the vortex ring without the target disk. The second test was a repeat of the first, except that red food dye was injected into the cup just before the shot. Some of the dyed water traveled across the pool with the ring, but most was left behind in the turbulent wake of the ring. The dyed region was so indistinct that no further dye injections were made with the 10-in. cup.

Table 2

TEST DATA FOR UNDERWATER VORTEX RINGS

Cup-Test	Max. Displacement		Spacing (S) (ft)	Ring Diameter (D) (in.)		Ring Velocity (V) (fps)	
	X_G (in.)	X_T (in.)		1[d]	15	1	15
10-1	10.0	---	out[c]	8.5	9.0	10.6	8.7
-2	10.0	---	out	8.0	9.0	10.6	9.1
-3	10.5	7.0	2	8.0	---	10.9	---
-4	10.0	6.0	8	8.0	---	10.0	---
-5	10.0	6.0	16	8.0	8.5	11.2	10.0
8-1	10.0	---	out	8.5	8.9	12.2	9.5
-2	10.0	6.5	2	8.0		12.0	
-3	10.2	6.5	2	8.0		11.6	
-4	10.0	6.5	2	8.0		12.2	
-5	10.0	6.2	8	7.9		11.6	
-6	10.7	---	out	8.0	7.9	10.4	9.5
-7[a]	10.0	---	16	8.8		12.0	
-8[a]	10.0	6.6[a]	16	8.8	8.7	11.8	12.0
6-1	8.5	5.5	2	7.5			
-2	8.2	---	16	7.5	8.5	11.3	7.3
-3	8.5	3.5[b]	16	7.5	8.5	11.3	6.9
-4	8.5	---	out	7.0		10.6	4.2
4-1	6.0	3.5	2	7.0		10.6	

(a) The total weight of the target pendulum cup was W_T = 160 lb for Tests 8-7 and 8-8 and 179 lb for all other tests. The weight of the gun pendulum is given in .
(b) Ring hit high on target disk.
(c) Target disk pendulum removed fr pool.
(d) Distance from the ring to the cu lip, 1 ft and 15 ft.

The results listed in Tables 1 and 2 show that the addition of the 10-in. cup to the back plate caused its displacement to increase five times. The displacement of the target disk was over half that of the cup even when they were spaced 16 ft apart. The vortex ring diameter was nearly constant (at 8 in.) as the ring crossed the pool and its velocity decreased only slightly, from about 11 to 9 ft/sec.

Tests With 8-in. Cup

Eight tests were run with the 8-in. cup as shown in Table 2. The first test without the target disk showed that the diameter of the vortex ring was nearly unchanged even though the cup was smaller than before. This was because the ring left the smaller cup sooner and was no longer contracted by the first bubble contraction. There was some mixing of the bubble gas with the cavitated vortex core as it formed outside the lip of the 8-in. cup, but most of the bubble gas appeared to contract and remain in the cup until after the ring was launched.

The next three tests were repeats with the target disk close to the cup. The repeatability of the data is good, and there is little difference from the third test with the 10-in. cup. The ring is somewhat faster for the smaller cup, however.

The next test, number 8-5, showed that the target disk displacement X_T decreased only slightly as it was moved away from the cup. The following test (8-6) was a repeat of the first (8-1) except that food dye was injected into the cup before the shot. The repeatability was poorer for these two shots than before. The dye was spread across the pool as occurred with the larger cup.

The last two tests were repeats except for improved camera lighting. Comparison with the last test for the 10-in. cup shows that the vortex ring was slightly larger and faster for the smaller cup, but the displacement of the cup was unchanged.

Tests With 6-in. Cup

Four tests with the 6-in. cup gave the results shown in Table 2. The cup displacements and ring diameters are smaller than previously measured for the larger cups. The ring decelerated faster and its trajectory curved upward due to bouyancy, even though the rings from the larger cups traveled very nearly horizontally to the target disk. There was considerable mixing of the bubble gases with the steam vortex core as it formed on the cup lip. The ring passed above the target disk for the second test, hit high on the third test, and emerged from the water about 16 ft from the cup for the last test.

Test With 4-in. Cup

One test was made with a 4-in. cup and it produced the results shown in Table 2. The effectiveness of this cup was considerably smaller than that of the larger ones, and the forming vortex ring was engulfed by bubble gases.

The damping of the ballistic pendulums is indicated in Table 3, where the successive backward and forward deflections are listed for most of the tests. The damping was particularly high for the target pendulum during test 10-3, where the largest cup was mounted closest to the target disk. Some of this damping was due to flow interference between the pendulums, which swung together nearly in phase when they were spaced 2 ft apart. The initial velocities of the pendulums deduced from the movie frames are also listed for some of the tests in Table 3.

All of the tests described so far were for various sized cups about one radius long (Fig. 2). All of these cups generated distinct vortex rings with gaseous cores. Five additional tests were run with various modifications of the cup shape. The conditions for these tests and the measured pendulum deflections are given in Table 4. The first two tests were for lengthened 4-in. and 8-in. cups. The third tests was for a thin-walled 8-in. aluminum cup, which buckled and collapsed upon the first contraction of the gas bubble. For the last two tests, a 1/4-in.-thick steel disk was welded to the lip of the 10-in. cup as shown below Table 4. Vortex rings with visible gaseous cores were

Table 3

SUCCESSIVE PENDULUM DEFLECTIONS AND INITIAL VELOCITIES

(MEASURED DATA)

Cup-Test	S (ft)	X_G (in.)	X_T (in.)	U_G (fps)	U_T (fps)
10-1	out	10.0, 9.0		1.1	
-2	out	10.0, 9.0, 7.9		1.0	
-3	2	10.5, 9.0, 8.0	7.0, 3.3, 2.5		0.67
-4	8	10.0	6.0, 3.9		0.70
-5	16	10.0, 9.1, 8.0	6.0, 4.3		0.70
8-1	out	10.0, 9.8, 8.5		1.3	
-2	2	10.0, 9.4, 8.2	6.5, 4.0, 2.2		0.79
-3	2	10.2	6.5		0.79
-4	2	10.0, 9.0, 8.2	6.5		0.79
-5	8	10.0, 9.4, 8.2	6.2, 4.2	1.2	0.60
-6	out	10.7, 10.1			
-7	16	10.0, 9.0			
-8	16	10.0, 9.0	6.6, 5.2	1.2	
6-1	2	8.5	5.5		0.59
-2	16	8.2, 7.4		1.0	
-3	16	8.5, 8.0, 7.3	3.5	1.0	
-4	out	8.5, 8.0, 7.6			
4-1	2	6.0	3.5		

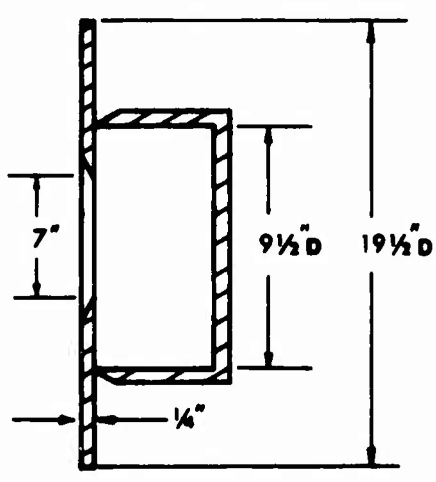
Table 4

DATA FOR MODIFIED CUP SHAPES

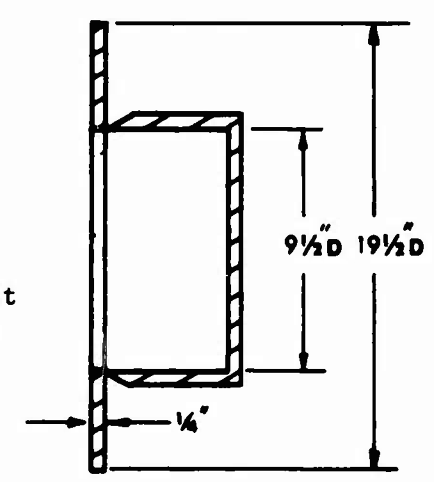
Test	Cup Diameter (in.)	Target Spacing (S) (ft)	X_G (in.)	X_T (in.)
1	4	4	7.3, 6.8, 6.7	4.7
2	8	2	11.0	7.7
3	7.5	-	7.0	---
4	10	4	3.2, 2.0, 1.8	1.5
5	10	4	2.5	1.6

Test	Cup Configurations
1	4 in. I.D. by 16 in. long by 1/4 in. thick steel cylinder
2	7-1/2 in. I.D. by 8 in. long by 1/4 in. thick steel cylinder
3	7-1/2 in. I.D. by 4 in. long by 0.04 in. thick aluminum cylinder

Test 4



Test 5



produced for each of these tests, but the cores were much thinner (≃ 1/8 in.) for the last two tests than for all the previous tests. Distinct gaseous vortex cores about 1 in. thick were produced for all of the previous tests listed in Tables 2, 3, and 4 except for the tests with the 4-in. cup, which produced a thick hazy ring in test 4-1 and a slightly more distinct ring in test 1 (Table 4).

The test data for the short cups in Tables 2 and 3 are summarized in Table 5, where average values are given for repeated runs. The ring generator data for the tests without the target disk are combined with the data for tests with the generator-to-target spacing S = 16 ft. Average measured values are given for the maximum backward displacement and initial velocity of the generator and target pendulums. The indicated values of Δ_G and Δ_T are the estimated amounts by which the maximum backward pendulum displacements are decreased by damping. The estimates are half the difference between the first backward and forward pendulum deflections in Table 3. The average values given for the diameter and velocity of the vortex ring core were measured when the ring was about 1 ft in front of the target disk. The measured data in Table 5 will be compared with theory developed for hollow vortex rings in incompressible fluid.

Table 5

REDUCED VORTEX RING DATA

Cup	10 in.			8 in.			6 in.	
S (ft)	2	8	16	2	8	16	2	16
Measured Data								
X_G (in.)	10.5	10.0	10.0	10.1	10.0	10.2	8.5	8.4
X_T (in.)	7.0	6.0	6.0	6.5	6.2	6.6[a]	5.5	3.5[b]
U_G (fps)			1.05		1.2	1.25		1.00
U_T (fps)	0.67	0.70	0.70	0.79	0.60		0.60	
Δ_G (in.)	0.7		0.5	0.4	0.3	0.5		0.3
Δ_T (in.)	1.8	0.9	0.8	1.2	1.0	0.7		
R[c] (in.)	4.0		4.4	4.0		4.0	3.8	3.7
V[c] (fps)	10.7		9.3	11.7		9.5	11.1	6.1
Calculated Pendulum Data								
Eq. (10)								
U_G (fps)	1.46		1.37	1.37		1.39		1.12
U_T (fps)	0.88	0.70	0.69	0.78	0.74	0.74	0.62	
Eq. (11)								
I_G (lb-sec)	10.7		9.2	8.9		9.0	7.2	7.1
I_T (lb-sec)	9.1		7.0	7.9		6.9	6.3	
Eq. (12)								
I_G (lb-sec)			6.9		7.7	8.0		6.3
I_T (lb-sec)	6.8		7.2	8.1		6.1	6.1	

(a) W_T reduced to 160 lb from 179 lb, the value for all other tests.

(b) Vortex ring hit the target disk high.

(c) Vortex ring properties measured about 1 ft in front of target.

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Section 3
DATA REDUCTION AND ANALYSIS

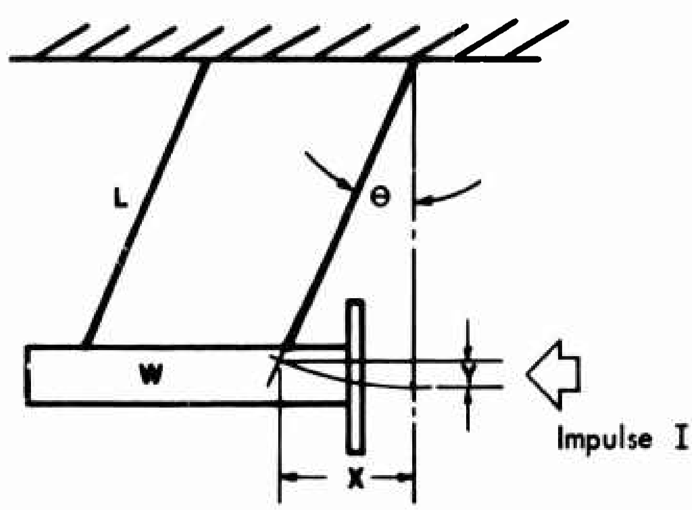
PENDULUM DATA

The kinetic energy and momentum imparted to a ballistic pendulum can be found from its measured horizontal displacement as follows, if the damping, bouyancy, and the virtual mass due to the surrounding water are temporarily neglected. When a pendulum weight W deflects an angle θ after an initial horizontal impulse I, its maximum potential energy is given by

$$PE = WY = WL\ (1-\cos\ \theta)$$

$$\simeq WL\frac{\theta^2}{2} \simeq \frac{WL}{2}\left(\frac{X}{L}\right)^2 ,\quad \text{if } \theta << 1 \qquad (1)$$

where the nomenclature is given by the following sketch.



Sketch B. Ballistic Pendulum

Equating this potential energy with the initial kinetic energy of the pendulum, one finds that the initial velocity of the pendulum U is given by

$$KE = \frac{W}{2g} U^2 = PE = \frac{WL}{2}\left(\frac{X}{L}\right)^2 \tag{2}$$

or

$$U = \frac{X}{L}\sqrt{gL} \tag{3}$$

Thus the initial momentum of the pendulum (or the impulse applied to it) is given by

$$I = \frac{WU}{g} = \frac{WX}{\sqrt{gL}} \tag{4}$$

Now let us consider the effects of damping, buoyancy, and virtual mass on the foregoing equations. The dissipation of energy due to viscous damping can be estimated from the successive deflections listed in Table 3. This damping is greatest for the largest cup and smallest spacing between the cup and disk (2 ft) because of hydrodynamic interference between the oscillating pendulums. The values of Δ_G and Δ_T in Table 5 indicate that the initial backward deflections of the cup and disk were decreased about 1/2 in. and 1 in. (respectively) by damping. These values are half the difference between the first two deflections in Table 3.

Allowing for damping, buoyancy, and virtual mass, the foregoing pendulum equations become

$$PE = (W - W_B)\frac{(X + \Delta)^2}{2L} \tag{5}$$

$$KE = (W + W_V)\frac{U^2}{2g} \tag{6}$$

$$I = (W + W_V)\frac{U}{g} \tag{7}$$

where Δ is the amount by which the first deflection X is decreased by damping; W_B is the weight of water displaced by the pendulum; and W_V is the weight of water corresponding to the virtual mass of the pendulum. It has been shown that the same value of W_V should be used for both the last two equations (Milne-Thompson, p. 240). For a flat disk (like the target) moving normal to its surface, W_V is theoretically $2/\pi$ times the weight of a sphere of water with the same radius as the disk (Lamb, p. 139). For a solid sphere, W_V is half the weight of the displaced water W_B (Milne-Thompson, p. 467). After the effect of damping is accounted for (by Δ, as previously described), we can set PE = KE and obtain from Eqs. (5) and (6)

$$U = (X + \Delta) \sqrt{\frac{g}{L}\left(\frac{W - W_B}{W + W_V}\right)} \tag{8}$$

Then, upon substitution in Eq. (7), we get

$$I = (X + \Delta) \frac{W}{\sqrt{gL}} \sqrt{\left(1 + \frac{W_V}{W}\right)\left(1 - \frac{W_B}{W}\right)} \tag{9}$$

For the cup pendulum, W_V is probably nearly as large as W_B, and W_B is probably about equal to 0.1 W, although the bouyancy of the cup is increased by the gas bubble. The net effect of W_V and W_B on the value of I from Eq. (9) is small for the cup, and it will be neglected. For the target disk pendulum, it is estimated that $W_B \simeq 0.1\ W = 18$ lb and that $W_V \simeq 150$ lb corresponding to the virtual mass of the 20-in.-diameter disk.

The initial velocities of the gun and target pendulums from Eq. (8) are given by

$$\begin{aligned} U_G\ (\text{fps}) &= \frac{X_G + \Delta_G}{129} \sqrt{32.2\left(\frac{129}{12}\right)\frac{0.9}{1.1}} = \frac{X_G + \Delta_G}{7.70}\ (\text{in.}) \\ U_T &= \frac{X_T + \Delta_T}{129} \sqrt{32.2\left(\frac{129}{12}\right)\frac{0.9}{1 + (150/179)}} = \frac{X_T + \Delta_T}{9.95} \end{aligned} \tag{10}$$

The impulse imparted to the pendulums from Eq. (9) are

$$I_G \text{ (lb-sec)} = \frac{(X_G + \Delta_G)\, W_G}{12\sqrt{32.2\ (129/12)}} = \frac{(X_G + \Delta_G)\, W_G}{223} \text{ (in.-lb)}$$

$$I_T = (X_T + \Delta_T)\, \frac{179}{223} \sqrt{\left(1 + \frac{150}{179}\right)(0.9)} = 1.03\ (X_T + \Delta_T) \text{ (in.)} \tag{11}$$

The pendulum impulses corresponding to their initial velocities are given by Eq. (7) as

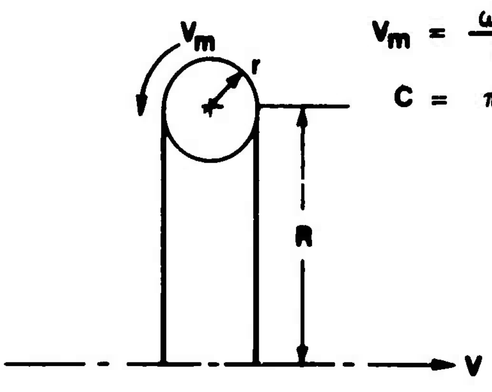
$$I_G = \frac{1.1}{32.2}\, W_G V_G \text{ (lb-ft/sec)}$$

$$I_T = \frac{179 + 150}{32.2}\, V_T = 10.2\ V_T \text{ (fps)} \tag{12}$$

The foregoing equations for U_T and V_T are for W_T = 179 lb, which was the target weight for all the tests except one special case (Table 5). The calculated pendulum velocities and impulses are given in Table 5.

VORTEX RING DATA

The nomenclature for ring vortex moving through water is shown in Sketch C.



$$V_m = \frac{\omega r}{2}$$

$$C = \pi r^2 \omega$$

Sketch C. Vortex Ring Nomenclature

The following three equations have been derived for the translational speed, kinetic energy, and impulse for a thin ring ($r \ll R$) when it encloses fluid with uniform vorticity ω, and when there is no vorticity outside the ring (Lamb, pp. 236 – 242). Within the circular core of the ring the fluid rotates as a solid body with an angular velocity equal to $\omega/2$, and the maximum velocity in the entire flow is V_m at the periphery of the ring. The circulation of the fluid along any path enclosing the core is C.

$$V = \frac{C}{4\pi R}\left(\ell n\,\frac{8R}{r} - \frac{1}{4}\right) \tag{13}$$

$$E = \frac{\rho}{2}\,C^2 R\left(\ell n\,\frac{8R}{r} - \frac{7}{4}\right) \tag{14}$$

$$J = \pi \rho C R \tag{15}$$

The foregoing three equations are for a uniform fluid density ρ. Similar equations have been derived by Pockington for a hollow vortex ring core in an incompressible fluid, as follows:

$$V = \frac{C}{4\pi R}\left(\ell n\,\frac{8R}{r} - \frac{1}{2}\right) \tag{16}$$

$$E = \frac{\rho}{2}\,C^2 R\left(\ell n\,\frac{8R}{r} - 2\right) \tag{17}$$

where

$$C = 2\pi r V_m \quad , \quad V_m = \sqrt{\frac{2\Delta p}{\rho}}$$

and Δp is the local hydrostatic pressure minus the vapor pressure (or cavitation pressure). The equations for V and E are identical to those for the liquid ring core except for the last terms. No equation was given by Pockington for the impulse of the hollow ring, but Eq. (15) should be a good approximation for a thin hollow ring since nearly all of the impulse for a thin ring (either hollow or liquid) is due to the net axial momentum of the fluid surrounding the core rather than within it. Equations (15) through (17) will be used to predict the properties of a cavitated ring vortex moving through water, with the assumptions that the fluid is irrotational outside the hollow core and that V_m is the cavitation speed at the depth of the ring. For shallow depths (small compared with the atmospheric head of 33 ft) the cavitation speed is nearly equal to 47 fps. With V_m given, the following four equations can be used to predict r, J, and E from measured values of R and V.

$$V = V_m \frac{r}{2R} \left(\ell n \frac{8R}{r} - \frac{1}{2}\right) \tag{18}$$

$$J = 2\pi^2 \rho R^3 \left(\frac{r}{R}\right) V_m \tag{19}$$

$$E = 2\pi^2 \rho R^3 V_m^2 \left(\frac{r}{R}\right)^2 \left(\ell n \frac{8R}{r} - 2\right) \tag{20}$$

$$\frac{E}{VJ} = 2 \frac{\ell n (8R/r) - 2}{\ell n (8R/r) - 1/2} \tag{21}$$

For $V_m = 47$ fps and $\rho = 1.94$ slug/ft^3, the following equations and the values shown in Table 6 and in Fig. 3 are obtained.

$$J \text{ (lb-sec)} = 1800 R^3 \left(\frac{r}{R}\right) \quad (\text{ft}^3) \tag{22}$$

$$E \text{ (ft-lb)} = 4300 R^3 2\pi^2 \left(\frac{r}{R}\right)^2 \left(\ell n \frac{8R}{r} - 2\right) (\text{ft}^3) \tag{23}$$

The curves in Fig. 3 show that for a given ring radius R, the thickness r, impulse J, and energy E all increase with the translational speed of the ring. At shallow depths, the rings become thick ($r > 0.3$ R) when their speed exceeds 20 fps, and the theoretical predictions (for thin rings) become invalid. The predicted values of r, J, and E corresponding to the measured values of R and V are given in Table 7. All of the explosively produced rings were thin ($r < 0.15R$).

Table 6

THEORETICAL VALUES FOR HOLLOW VORTEX RINGS AT SHALLOW DEPTHS

$\frac{r}{R}$	$\frac{V}{V_m}$ [Eq. (18)]	V (fps)	$\frac{J}{R^3}$ $\left(\frac{\text{lb-sec}}{\text{ft}^3}\right)$ [Eq. (22)]	$\frac{E}{VJ}$ [Eq. (21)]
0	0	0	0	2
0.01	0.031	1.46	18	1.52
0.1	0.194	9.1	180	1.23
0.15	0.260	12.2	270	1.14
0.2	0.319	15.0	360	1.06
0.3	0.398	18.7	540	0.88

V_m = 47 fps, ρ = 1.94 slug/ft^3

Table 7

CALCULATED AND MEASURED VORTEX RING DATA

Cup	10 in.		8 in.		6 in.	
S (ft)	2	16	2	16	2	16
			Measured Data			
R^* (in.)	4.0	4.4	4.0	4.0	3.8	3.7
V^* (fps)	10.7	9.3	11.7	9.5	11.1	6.1
			Calculated Data			
r/R (Fig. 3)	0.125	0.104	0.142	0.108	0.132	0.060
t = 2r (in.)	1.01	0.92	1.14	0.86	0.99	0.44
$J = 1800\ R^3\ (r/R)$ (lb-sec) (Fig. 3)	8.7	9.4	9.4	7.2	7.3	2.5
E/VJ (Fig. 3)	1.18	1.23	1.16	1.22	1.17	1.30
E (ft-lb)	110.	107.	128.	83.	95.	20.
I_G (lb-sec) [Eq. (11)]	10.7	9.2	8.9	9.0	7.2	7.1
I_T (lb-sec) [Eq. (11)]	9.1	7.0	7.9	6.9	6.3	

*For vortex rings about 1 ft in front of target disk.

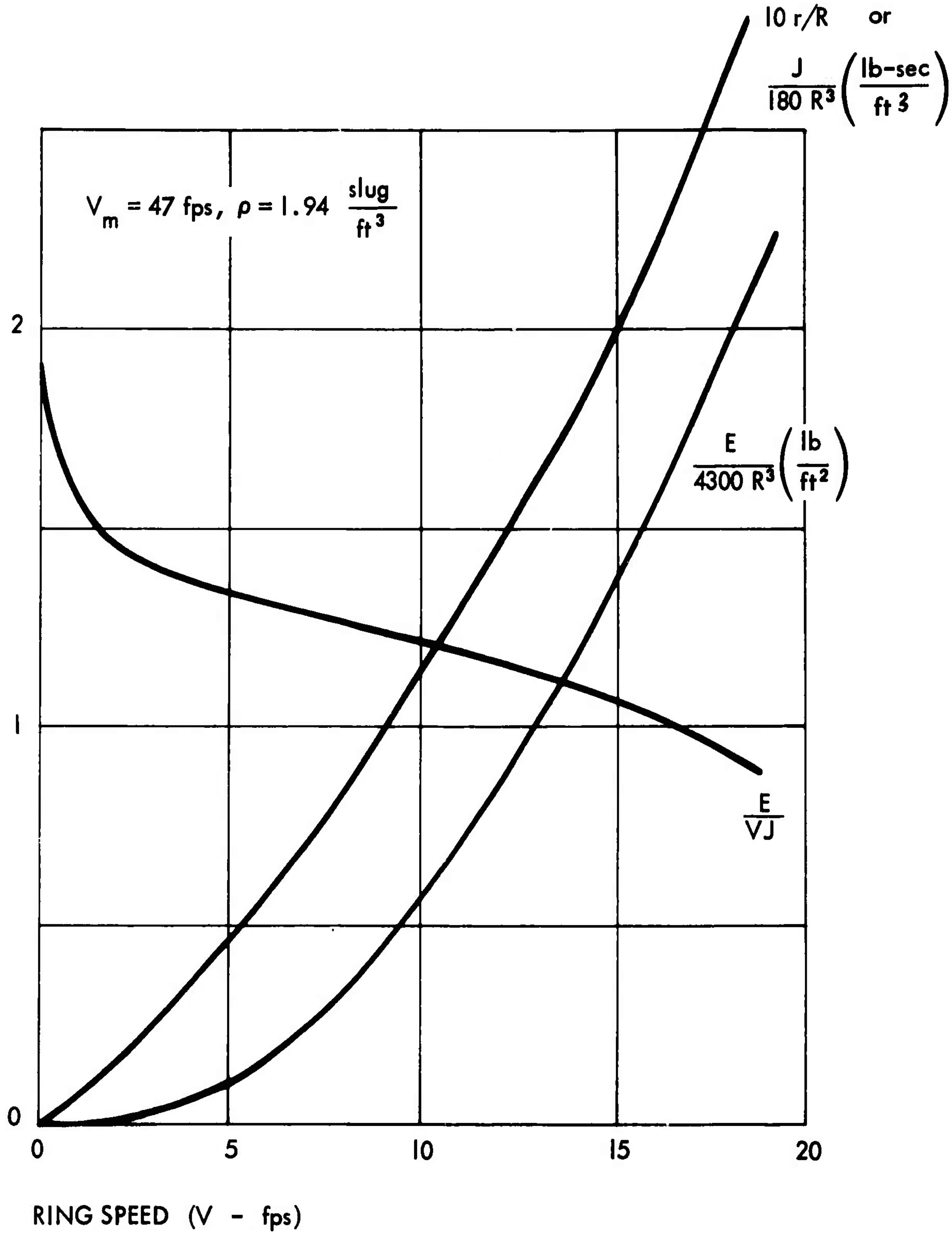


Fig. 3. Theoretical Results for Hollow Vortex Rings at Shallow Depths

Section 4
DISCUSSION OF REDUCED DATA AND COMPARISON WITH THEORY

The test data show that explosions in short cups produce cavitated vortex rings with a diameter roughly equal to the cup diameter. Comparison of the pendulum deflections (with and without the cups in Tables 1 and 5) indicates that the cups were effective in increasing the pendulum deflections and, hence, the applied impulse [Eq. (11)]. The deflection and impulse of the gun pendulum was increased about five times by the largest (10-in.) cup and about three times by the smallest (6-in.) cup. The averaged data in Table 5 show that the 8-in. cup was nearly as effective as the 10-in. cup in generating vortex rings. The effectiveness of the 10-in. cup was reduced by the contraction of the vortex ring just after it left the cup (corresponding with the first contraction of the gas bubble in the cup).

Even though there was some turbulent mixing and frictional losses in the region of water surrounding the vortex rings (as shown by the mixing of dye), the rings traveled nearly across the pool (16 ft) with little change in diameter. The speed of the rings from the larger cups decreased only about 10 percent in this distance.

The data in Table 4 show that the lengthened 4-in. cup was more effective than the short one (Table 2); but it was still less effective than the 8-in. cup, even though it had the same volume. The lengthened 8-in. cup was somewhat more effective than the short one and gave the largest displacement of the target disk for all the tests. Even though a thin-walled cup buckled and collapsed shortly after the vortex ring was generated, it produced an impulse on the cup about 70 percent as large as for the comparable 8-in. thick-walled cup (Test 3, Table 4). The last two tests in Table 3 show that when the 10-in. cup was mounted behind flat orifice plates, the vortex rings were considerably weaker. Some of the reduction in cup pendulum deflection is due to increased damping caused by the plates, however. The ineffectiveness of the plate with the sharp-edge orifice (Test 4) was surprising since this container shape is similar to those usually used to produce vortex rings from nonexplosive sources.

The pendulum velocities calculated from their deflections [Eq. (10)] agree reasonably well with their measured velocities, as shown in Table 5. The calculated values of U_G are consistently somewhat high, however, which indicates that either the damping was overestimated, the virtual mass was underestimated, or the buoyancy was underestimated (perhaps because of the gas bubble contained in the cup). The pendulum impulses (I) as calculated from their deflections [Eq. (11)] agree fairly well with the values calculated from their measured initial velocities [Eq. (12)], as shown in Table 5. However, the latter values are in some cases somewhat smaller. Hence the pendulum deflections give initial velocities and impulses somewhat higher than the measured velocities and corresponding impulses.

The calculated target impulses are only slightly smaller than those for the gun, even for the maximum spacing of 16 ft. Hence, most of the impulse applied to the gun was caused by the generation of the ring vortex, which transmitted this impulse (or momentum) to the target. The impulse is much more directed (and the attenuation with distance is much smaller) than for a spherically expanding underwater shock wave.

The properties of the vortex rings calculated from their measured radius and speed (given in Table 7) show that:

1. The calculated ring thickness (2r) agrees well with the observed thickness of about 1 in. This agreement was obtained even for the small 6-in. cup, which caused bubble gases to be mixed with the water vapor in the cores of the rings.

2. The calculated values of impulse for the vortex ring just in front of the target (J in Table 7) are usually somewhat larger than the calculated impulse applied to the target I_T. This might be expected since the region of water surrounding the vortex ring does not stop and lose all of its momentum to the target, but rather travels with the target disk.

3. The initial energy of the vortex rings calculated for the larger two cups (E in Table 7) are about 7 and 8 percent of the calculated explosive energy, 1570 ft-lb. The diameter, speed, impulse, and energy of these rings decreased only slightly over the maximum range tested (about 16 ft).

Even though only a small fraction of the explosive energy was put into the vortex rings, their specific impulse, or their impulse per unit weight of propellant, is very high. Taking an average value of calculated impulse for the vortex rings (J = 9 lb-sec from Table 7) and dividing by the weight of the propellant (1/2 gm = 0.0011 lb) gives a specific impulse equal to 8200 sec. A solid projectile with this much impulse and the same weight as the propellant would have to travel at a speed of 260,000 fps. Since the velocity of the vortex ring is only about 10 fps, its "effective weight" as a projectile is J_g/V = 29 lb. This weight is 26,000 times greater than the propellant weight, and it corresponds to an 11-in.-diameter sphere of water. Thus, even though the vortex ring represents only a small fraction of the explosive energy, the energy is put into a very large mass of slowly moving water that is well directed and which can propagate with little frictional loss over a long range.

Section 5
DISCUSSION OF APPLICATIONS

The following discussion of possible applications for underwater vortex rings is based on the foregoing data and theory.

UNDERWATER MINES

The generation of a vortex ring tends to channel and direct the energy and impulse of an underwater explosion. However, the concentration (or focusing) is probably insufficient for the rings to be very destructive at shallow depths, where their speed is limited by cavitation to about 12 fps. Assuming that the impulse of the ring is applied to a target plate uniformly within a circle with the same radius as the ring, the applied impulse is about 0.15 psi/sec (for R = 4 in., J = 8 lb/sec from Table 7). This is only about one-tenth the concentration required for hull damage. For deeply submerged vortex rings, however, the translational speed and impulse might be high enough to cause hull damage or destruction of marine propellers.

DREDGING

It appears that the rings might be an efficient means of dredging or of dislodging large sunken objects without damaging them. By firing the rings downward, cratering or scouring of the bottom can be produced, as shown by the data of Spies. This action can be produced by explosive charges fired downward from the water surface without the use of equipment requiring moving parts or deep submergence.

THRUSTERS

The data given herein indicate that the impulse applied to a flat plate by an explosive charge can be increased five times by surrounding the charge by a short cup, which reflects the shock wave inward and generates a vortex ring at the lip of the cup. This indicates that improved thrusters for maneuvering deep-submergence vehicles might be designed to use explosive charges rather than propellers. The last two tests in Table 4 show that the applied impulse is reduced when the cup is recessed behind a flat orifice plate. This reduction might not occur if the lip of the short cup projected from the plate.

MISSILE PROPULSION

The propulsion of underwater missiles by successive explosions on their flat boattails has been considered at the Naval Undersea Warfare Center at San Diego. The data herein indicate that the impulse per explosion might increase several times if a short cup is added aft of the flat boattail. The cup would generate cavitated vortex rings, which would be shed downstream similar to the vortices shed by fish.

The present data do not show the effect of the flow over a moving missile. This flow might retard the formation of the vortex rings outside the lip of a cylindrical cup. Hence a tapered cup or one with a flat orifice plate (like the one for Test 4, Table 4) might be more effective for a fast missile. The data show that high values of static thrust can be obtained with a short cup (about 1 radius long) if the cup radius is at least 25 times as large as the radius of the equivalent TNT sphere.

To propel a 10-in.-diameter missile at 50 fps, the firing rate of explosions equivalent to 1/2 gm of TNT can be estimated by assuming that the impulse per charge is the same as measured statically with the 10-in. cup (10 lb-sec from Table 5). The missile drag force is 67 lb, assuming that the drag coefficient is 0.05 based on the frontal area. Hence 1/2-gm charges would have to be fired periodically inside the cup at about 6.7 charges/sec, and about five times faster without the cup.

For deeply submerged missiles, it should be possible to generate more intense vortex rings without cavitation, so that larger charges might be used effectively with smaller cups.

Section 6

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions are drawn from the foregoing discussions of the data and applications for vortex rings generated underwater by explosions in short cups at shallow depths of submergence.

1. The impulse applied to a flat plate by an adjacent underwater explosion can be increased five times by enclosing the charge in a short cylindrical cup whose length and radius are about 25 times larger than the radius of an equivalent TNT sphere.

2. Thin-walled cups which subsequently buckle and collapse due to the contraction of the gas bubble are nearly as effective as thick-walled cups for generating a single vortex ring.

3. Vortex rings with cavitated cores are generated outside the lips of such cups. The rings travel about 12 fps at shallow depths of submergence. They can be fired more than 50 cup radii horizontally with little loss in speed, impulse, or energy.

4. The measured properties of such cavitated vortex rings agree reasonably well with theoretical predictions for hollow vortex rings traveling through an inviscid liquid.

5. The kinetic energy associated with the rings is about 8 percent of the explosive energy, but this energy is imparted to a mass of water which weighs about 26,000 times more than the charge, which encloses the ring, and which propagates with little frictional loss.

6. At shallow depths it appears that vortex rings cannot be fired faster than about 12 fps, which is insufficient to cause hull damage, but such damage might be produced at greater depths, where the cavitation limit is higher.

7. It appears that ring vortex g nerators might be usefully employed for dredging, for maneuvering deep-submergence vehicles, and for propulsion of underwater missiles.

It is recommended that the present study be extended to include the effects of:

1. water depth (or hydrostatic pressure) to increase the cavitation speed
2. flow over vortex ring generators to simulate the flow over a missile
3. successive explosions to give sustained thrust

Section 7
REFERENCES

Carslow, H.S.,"The Fluted Vibrations of a Circular Vortex Ring With a Hollow Core," The London Math Soc Proc, Vol. XXVIII, pp. 97–119, 1897

Elder, F.K., and N. deHaas,"Experimental Study of the Formation of a Vortex Ring at the Open End of a Cylindrical Shock Tube," J. Appl. Phys., Vol. 23, No. 10, pp. 1065–1069, October 1952

Lamb, Sir H., Hydrodynamics, 6th Ed., p. 246, Dover Publ, N.Y., 1932

Milne-Thomson, Theoretical Hydrodynamics, 4th Ed., The Macmillan Co., N.Y., 1960

Northrup, E.F., A Photographic Study of Vortex Rings in Liquids, pp. 463–468, Nature, Vol. LXXXVIII, April 11, 1911, Macmillan and Company

Pockington, H.C.,"The Complete System of the Periods of a Hollow Vortex Ring," Phil. Trans. of the Roy Soc of London, Vol. 186, Part II, pp. 603–619 for the year 1895

Spies, R., Development of a Device for Pressure Mine Sweeping (U), Report No. 181, Aerojet-General Corp, Azusa, Calif., Contract Nonr-686(00), 4 Jan 1957 (C)

Turner, J.S., Buoyant Vortex Rings, pp. 61–75, Proc. Roy Soc of London, Series A, Vol. 239, 9 Apr 1957

Zaroodny, S.J., Revised Theory of Vortex Rings – A Simplified Review of the State of the Art, TM 6601, U.S. Army Limited War Lab, Aberdeen Proving Ground, Md, April 1966, AD 645 931

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13. ABSTRACT

Vortex rings were produced explosively in water with enough intensity to cavitate their cores. Such cavitation limits the velocity which can be imparted to vortex rings at shallow depths. Vortex rings were produced by explosions in short cups or gun tubes, rather than in the customary cylindrical containers with orifice plates. One advantage of the cups is that relatively large rings are formed outside the cup lips. The following conclusions were drawn from the measured data. The impulse applied to a flat plate by an adjacent underwater explosion can be increased five times by enclosing the charge in a short cylindrical cup whose length and radius are about 25 times larger than the radius of an equivalent TNT sphere. Thin-walled cups which subsequently collapsed were nearly as effective as thick cups for generating single vortex rings. At shallow depths of submergence, the cavitated rings traveled more than 50 cup radii horizontally with little loss in speed. At shallow depths, the vortex rings traveled about 12 fps, which is insufficient to cause hull damage, but such damage might be produced at greater depths where the cavitation limit is higher. The kinetic energy associated with the rings was about 8 percent of the explosive yield. The measured properties of the cavitated vortex rings agreed reasonably well with theoretical predictions for hollow vortex rings traveling through an inviscid liquid. Ring vortex generators might be useful for dredging, for maneuvering deep-submergence vehicles, for propulsion of underwater missiles, and for focusing underwater explosions.

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Dredging from explosions						