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EFFECTS OF OVERSAMPLING ON TIME-ADAPTIVE FILTERS

Special Report No. 1
ADVANCED ARRAY RESEARCH

Prepared by
Chung-Yen Ong Aaron H. Booker
John P. Burg, Project Scientist
George D. Hair, Program Manager
Telephone: 1-214-238-3473

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ABSTRACT

Independent time-series data, consisting of white noise with different degrees of oversampling, have been used to study the effect of oversampling on the time-adaptive prediction filter. If the data are oversampled, false gain occurs in the adaptive prediction results. The false gain depends on the degree of oversampling, the number of channels used in making the prediction, the filter length, and the convergence parameter.

Two adaptive algorithms — one having a constant convergence parameter and the other having a variable convergence parameter — are discussed in this report. Particular cases of the prediction mean-square-error function of the time-adaptive filter are derived and compared to the empirical results. Although the false gain can be quite severe for high rates of adaption, rates of adaption can be selected in terms of theoretical maximum rates of adaption so that the false gain is not significant — even for oversampled data cut at 1/20 of the folding frequency.

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ILLUSTRATIONS

ILLUSTRATIONS (CONTD)

SECTION I
INTRODUCTION AND SUMMARY

Previous work has shown that adaptive filters effectively reduce problems with time-varying noise fields[1] and that adaptive-filter design can be achieved with off-line processing to produce filters equivalent in mean-square-error to conventional Wiener filters.

It is heuristically obvious and has been empirically observed that oversampled data, combined with a fast rate of adaptation, produces false gain. This report quantitatively assesses the expected degree of false gain and determines this phenomenon's significance in on-line adaptive processes.

The time-adaptive filter considered in this report can be described as follows. If Y_j is the output of a time-adaptive filter, then

$$Y_j = \vec{W}_j^T \vec{X}_j \tag{1-1}$$

where $\vec{W}_j$ is the column vector and the filter at the j^{th} adaptive iteration $\vec{X}_j$ is the j^{th} sample of the input data. The adaptive algorithm can be represented by

$$\vec{W}_{j+1} = \vec{W}_j + 2k_s \epsilon_j \vec{X}_j \tag{1-2}$$

where

$$\epsilon_j = d_j - Y_j \tag{1-3}$$

and

k_s is a positive convergence parameter

d_j is the j^{th} desired response

The mean-square-error of the j^{th} iteration is defined as

$$\overline{\epsilon_j^2} = \overline{(d_j - Y_j)^2} \tag{1-4}$$

This report investigates the effects of filter length, sampling rate, and number of channels on the false gain for two adaptive schemes: one having a constant convergence parameter (represented by k_s in Equation 1-2) and one having a variable convergence parameter (represented by substituting b_j for k_s in Equation 1-2, where $b_j = \frac{b}{\vec{X}_j^T \vec{X}_j}$ and b is a constant).

Both schemes have been run on data having uniform but band-limited spectra. Single-channel 21-point, single-channel 35-point, 7-channel 3-point, and 7-channel 5-point filters are each applied to data bandlimited to 1/20, to 1/4, and to the folding frequency. Table I-1 shows the experimental cases used. The data are independent from channel to channel. A complete description of the method for generating these data is in an earlier Advanced Array Research special report.[2]

Table I-1

PARAMETER COMBINATIONS

Prediction Channel	Filter Lengths	Bandwidth (fraction of folding frequency)
1	21, 35	1/20, 1/4, 1
7	3, 5	1/20, 1/4, 1

SECTION II

PREDICTION-ERROR FUNCTION OF TIME-ADAPTIVE FILTER

The intention was to develop exact formulas or approximations expressing the mean-square-error as a function of k_s, number of channels, and filter length for each bandwidth; this goal was not satisfactorily achieved, although certain theoretical results of interest were obtained and are developed in this section.

The prediction-error function ϵ_j can be obtained by substituting Equations 1-1 and 1-3 into Equation 1-2 recursively as follows:

$$\epsilon_j = d_j - \vec{W}_j^T \vec{X}_j \tag{2-1}$$

$$= d_j - \left(\vec{W}_{j-1}^T + 2k_s \epsilon_{j-1} \vec{X}_{j-1}^T\right) \vec{X}_j$$

$$= d_j - \left[\vec{W}_{j-1}^T \left(I - 2k_s \vec{X}_{j-1} \vec{X}_{j-1}^T\right) + 2k_s d_{j-1} \vec{X}_{j-1}^T\right] \vec{X}_j$$

$$= d_j - \Big\{\vec{W}_{j-2}^T \left[I - 2k_s \vec{X}_{j-2} \vec{X}_{j-2}^T\right] \left[I \quad 2k_s \vec{X}_{j-1} \vec{X}_{j-1}^T\right]$$

$$+ 2k_s d_{j-2} \vec{X}_{j-1}^T \left[I - 2k_s \vec{X}_{j-1} \vec{X}_{j-1}^T\right]$$

$$+ 2k_s d_{j-1} \vec{X}_{j-1}^T \Big\} \vec{X}_j$$

$$= d_j - \Big\{\vec{W}_{j-3}^T \left[I - 2k_s \vec{X}_{j-3} \vec{X}_{j-3}^T\right] \left[I - 2k_s \vec{X}_{j-2} \vec{X}_{j-2}^T\right]$$

$$\left[I - 2k_s \vec{X}_{j-1} \vec{X}_{j-1}^T\right] + 2k_s d_{j-3} \left[I - 2k_s \vec{X}_{j-2} \vec{X}_{j-2}^T\right]$$

$$\left[I - 2k_s \vec{X}_{j-1} \vec{X}_{j-1}^T\right] + 2k_s d_{j-2} \left[I - 2k_s \vec{X}_{j-1} \vec{X}_{j-1}^T\right]$$

$$+ 2k_s d_{j-1} \vec{X}_{j-1}^T \Big\}$$

$$= \cdots\cdots$$

$$= d_j - \left\{ \vec{W}_1^T \prod_{i=1}^{j-1} \left(I - 2k_s \vec{X}_i \vec{X}_i^T \right) \right.$$

$$\left. + 2k_s \left[\sum_{i=1}^{j-1} d_i \vec{X}_i^T \prod_{\ell=i+1}^{j-1} \left(I - 2k_s \vec{X}_\ell \vec{X}_\ell^T \right) \right] \right\} \vec{X}_j$$

where

$$\prod_{\ell=i+1}^{j-1} \left(I - 2k_s \vec{X}_\ell \vec{X}_\ell^T \right) = I \qquad \text{for } \ell > j-1$$

I is the unit matrix

$\vec{W}_1$ is the initial filter

The factors in the matrix products are assumed to be ordered from left to right; i.e.,

$$\prod_{\ell=i+1}^{j-1} (\cdot)_\ell = (\cdot)_{i+1} (\cdot)_{i+2} (\cdot)_{i+3} \cdots (\cdot)_{j-1}$$

Thus, the mean-square-error can be expressed as

$$\overline{\epsilon_j^2} = \overline{d_j^2} - \overline{2d_j \left\{ \vec{W}_1^T \prod_{i=1}^{j-1} \left(I - 2k_s \vec{X}_i \vec{X}_i^T \right) + 2k_s \left[\sum_{i=1}^{j-1} d_i \vec{X}_i^T \prod_{\ell=i+1}^{j-1} \left(I - 2k_s \vec{X}_\ell \vec{X}_\ell^T \right) \right] \right\} \vec{X}_j}$$

$$+ \overline{\left[\left\{ \vec{W}_1^T \prod_{i=1}^{j-1} \left(I - 2k_s \vec{X}_i \vec{X}_i^T \right) + 2k_s \left[\sum_{i=1}^{j-1} d_i \vec{X}_i^T \prod_{\ell=i+1}^{j-1} \left(I - 2k_s \vec{X}_\ell \vec{X}_\ell^T \right) \right] \right\} \vec{X}_j \right]^2} \qquad (2\text{-}2)$$

If we set $\vec{W}_1 = 0$ and

$$\left[\sum_{i=1}^{j-1} d_i \vec{X}_i^T \prod_{\ell=i+1}^{j-1} \left(I - 2 k_s \vec{X}_\ell \vec{X}_\ell^T\right)\right] \vec{X}_j = a_j \tag{2-3}$$

Equation 2-2 can then be expressed as

$$\overline{\epsilon_j^2} = \overline{d_j^2} - 4 k_s \overline{d_j a_j} + 4 k_s^2 \overline{a_j^2} \tag{2-4}$$

Equation 2-4 can be simplified in the following special cases.

- From Equation 2-4, it can be seen that

$$\lim_{k_s \to 0} \overline{\epsilon_j^2} = \overline{d_j^2}$$

  This corresponds to the nonadaptive case.

- If d_j and $\vec{X}_i$ are independent for all j and i and $\overline{d_j d_i} = 0$ for $j \neq i$, then, from Equations 2-3 and 2-4,

$$\overline{\epsilon_j^2} = \overline{d_j^2} + 4 k_s^2 \overline{a_j^2} \tag{2-5}$$

  $\overline{\epsilon_j^2}$ increases monotonically as k_s increases. This corresponds to the case where the desired output is not oversampled.

- For $k_s << 1$,

$$k_s \overline{d_j a_j} >> k_s^2 \overline{a_j^2}$$

  and

$$\overline{\epsilon_j^2} \cong \overline{d_j^2} - 4 k_s \overline{d_j a_j}$$

If d_i, $\vec{x}_j$ are independent for all i, j, and if both d_j and $\vec{x}$ are sufficiently oversampled, it can be shown that $\overline{d_j a_j} > 0$.

Thus,

$$\frac{d\overline{\epsilon_j^2}}{dk_s} < 0$$

for k_s in a neighborhood of 0.

- From Equation 2-4, it follows that, for sufficiently large k_s, $\overline{\epsilon_j^2}$ increases monotonically.

SECTION III

EMPIRICAL RESULTS AND CONCLUSIONS

Figures III-1 through III-12 show mean-square-error curves plotted vs increasing value of the convergence parameter for each configuration of Table I-1. Independent 4096-point random time-series data having a unit mean-square-error value for each channel are used. In each case, the desired output has the same statistical properties as the data used for prediction but is independent of the data.

The initial filter has been set to 0 (the optimum filter) for each run. The initial transient effect is assumed to be negligible, as indicated by runs made with 16,384-point time-series data for the single-channel 21-point filter. Different starting points (zeros and peaks) of the predicted time series for the 21-point and 35-point single-channel filters results in a < 1.5 percent difference in mean-square-error for data cut at 1/20. Therefore, variation of the starting point does not significantly affect the mean-square-error curves.

A. CONSTANT CONVERGENCE PARAMETER

Figures III-1 through III-4 show mean-square-error computed for a constant convergence parameter. As expected, the false gain increases when the number of filter coefficients is increased by using more channels or longer filters. There is more false gain when the number of channels is increased than when longer filters are used. The degree of false gain is unexpected. As seen in Figure III-4, for the 7-channel 5-point filter and severe oversampling, a maximum of 92 percent of the energy can be falsely predicted.

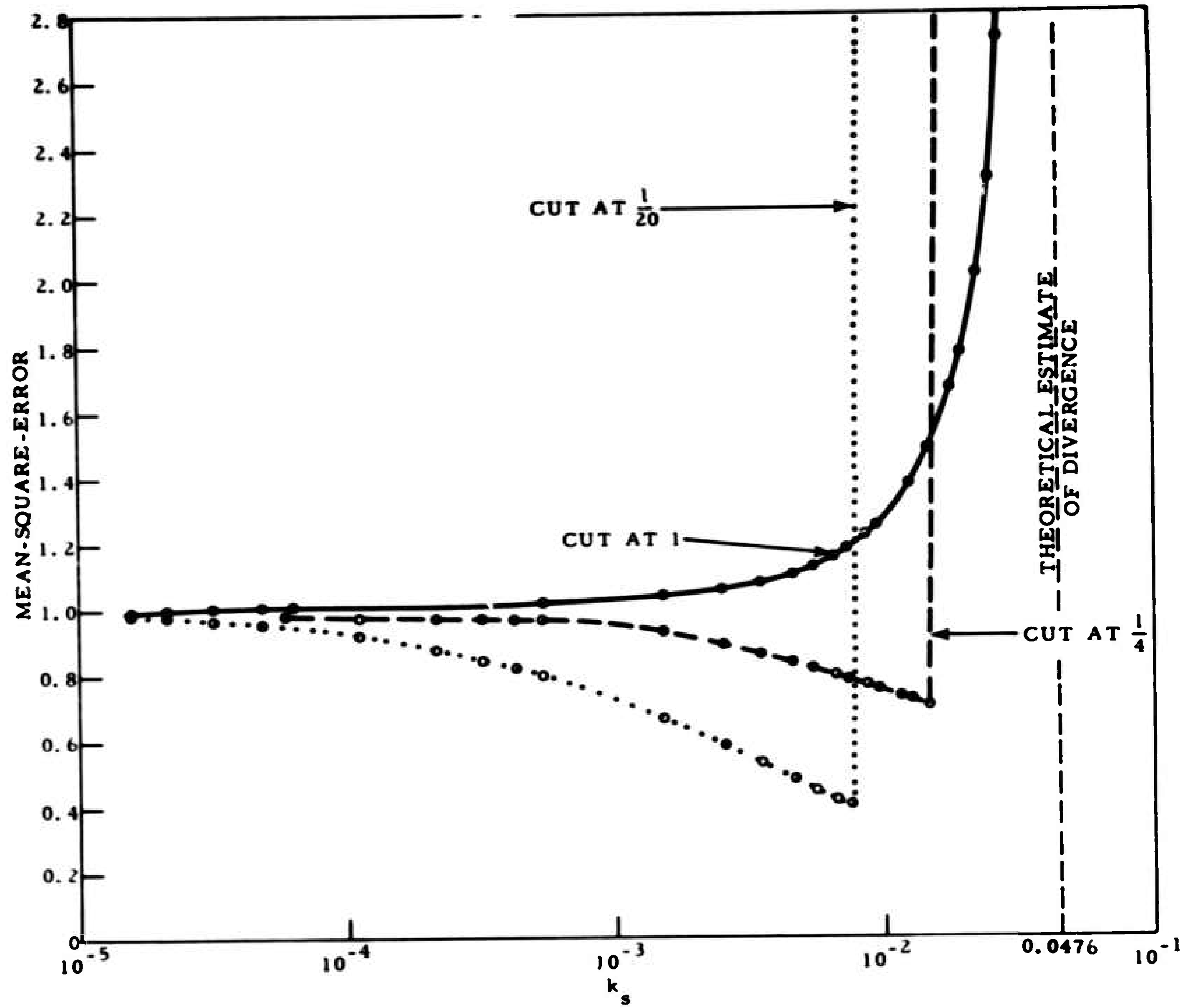

Figure III-1. Single-Channel 21-Point Filter, Constant Convergence Parameter

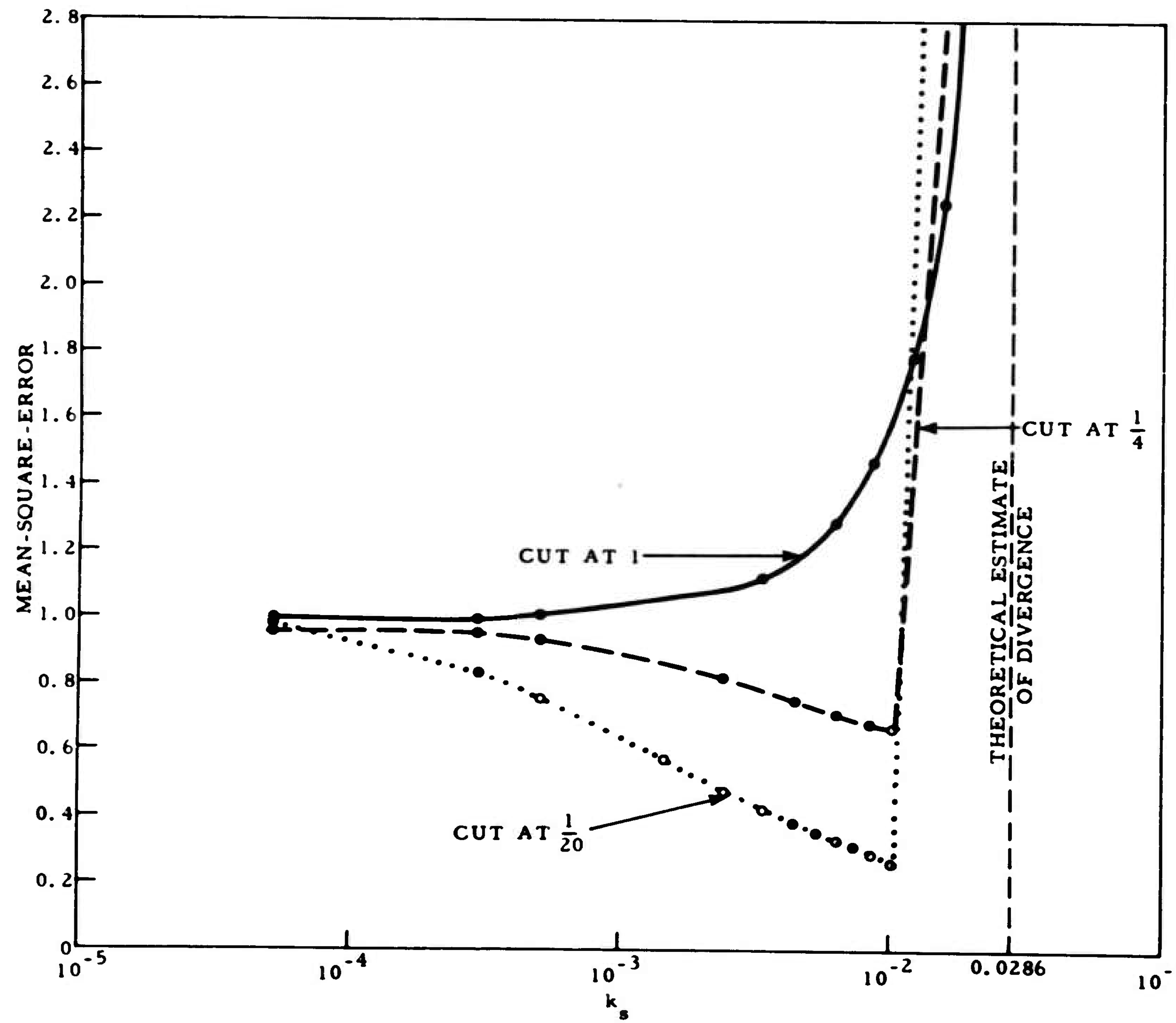

Figure III-2. Single-Channel 35-Point Filter, Constant Convergence Parameter

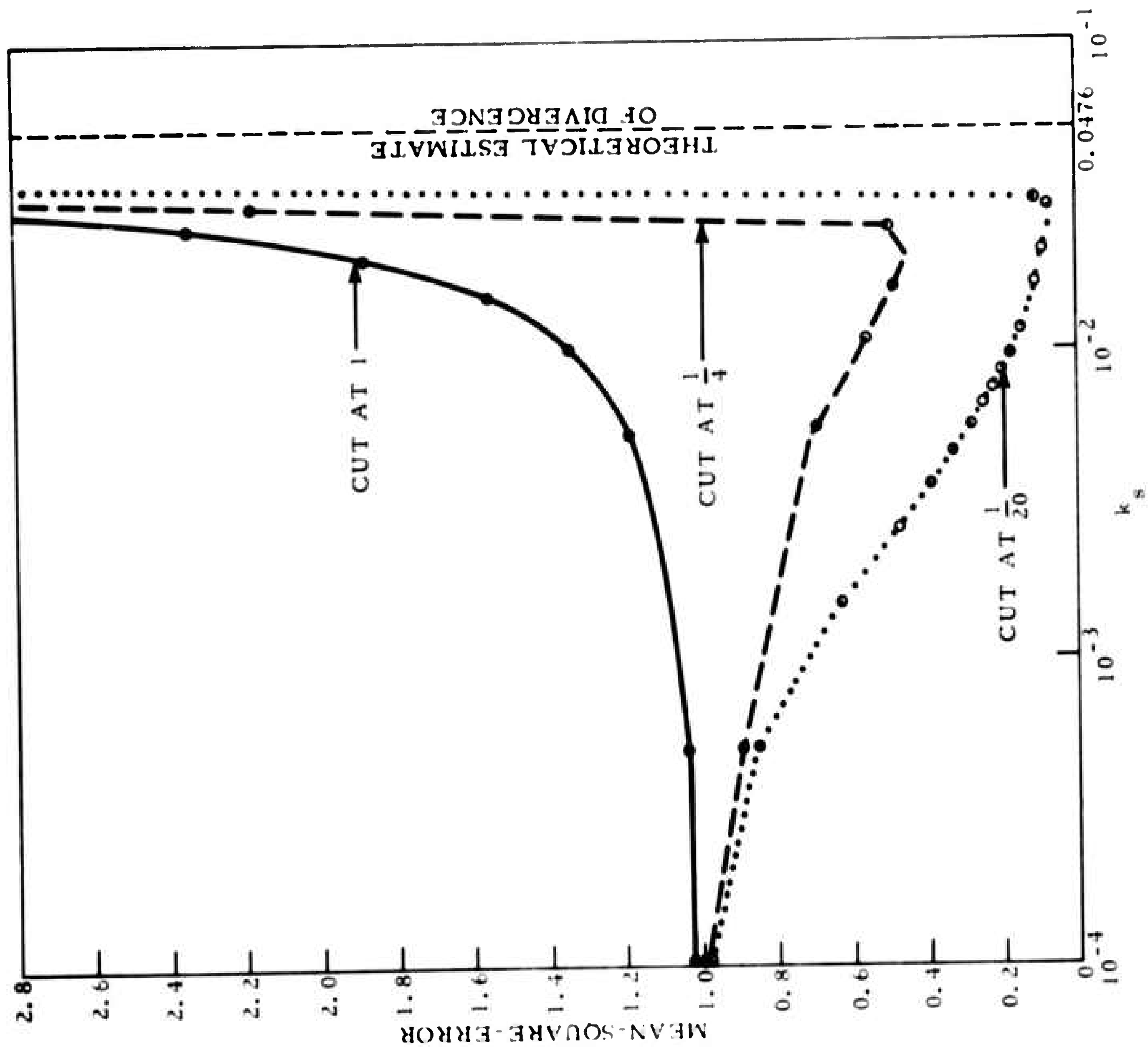

Figure III-3. 7-Channel 3-Point Filter, Constant Convergence Parameter

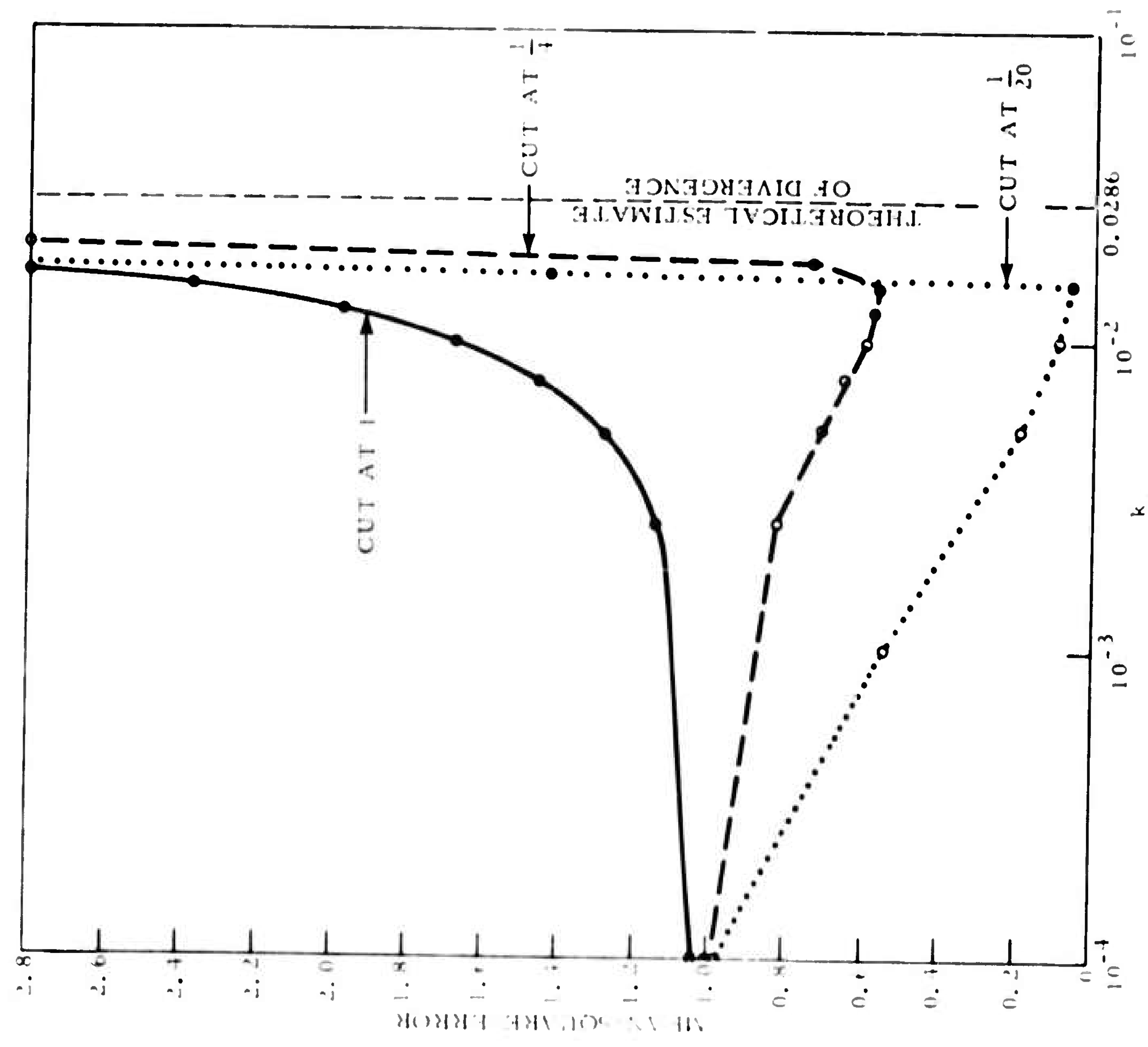

Figure III-4. 7-Channel 5-Point Filter, Constant Convergence Parameter

Theoretical approximations to the points of divergence are indicated in Figures III-1 through III-4. According to Widrow,[3] k_s is bounded by

$$k_s \leq \frac{1}{\max |\vec{X}_j|^2} \triangleq \frac{1}{Q} \qquad (\triangleq \text{ is defined equal to})$$

A simple approximation to this bound results from the assumption of independence of the components of $\vec{X}_j$ and the use of

$$\overline{\left(\frac{1}{Q}\right)} \cong \frac{1}{\overline{Q}}$$

The resulting bound for k_s is

$$k_s \leq \frac{1}{\left(NC * LF * \sigma^2\right)} \triangleq B$$

where σ^2 is the mean-square-value of any component, NC is the number of channels, and LF is the length of the filter. Actual points of divergence k_s^D satisfied.

$$\frac{B}{10} \leq k_s^D \leq B$$

so the approximation is reasonably accurate and, as shown by experience, can be used for selecting k_s values.

The value of σ^2 for one corrected channel (consisting of an individual channel minus the mean across channels) is required for maximum-likelihood adaptive processing; thus, the use of the preceding approximation in maximum-likelihood filtering can lead to improper values of k_s unless the constraint energy is considered.

B. VARIABLE CONVERGENCE PARAMETER

Figures III-5 through III-8 correspond to the cases illustrated in Figures III-1 through III-4 except for the use, in the place of k_s, of the variable convergence parameter

$$b_j \triangleq \frac{b}{\vec{X}_j^T \vec{X}_j}$$

Figures III-9 through III-12 show the same data plotted against the mean value of b_j.

In Figures III-1 and III-5 (as well as III-9), curves for b (and mean value of b_j) vary strikingly from those for k_s for the severely over-sampled data. For small values of b, the mean-square-error is higher than it would be if the data were not oversampled! Also note that the divergence point is extended when using a variable convergence parameter by a factor dependent on the degree of oversampling.

Radical differences in the mean-square-error curves for the two adaptive algorithms occur primarily in the single-channel cases. Only slight differences exist between the results for the two adaptive techniques in the 3-point and 5-point 7-channel filter cases.

This study proves the need for caution in the off-line use (on short data segments) of adaptive filters which attempt the use of high adaption rates to avoid multiple passes. Also, it can be concluded that, by using $k_s \leq 0.001B$, false gain will not be a significant problem — even for severely over-sampled data.

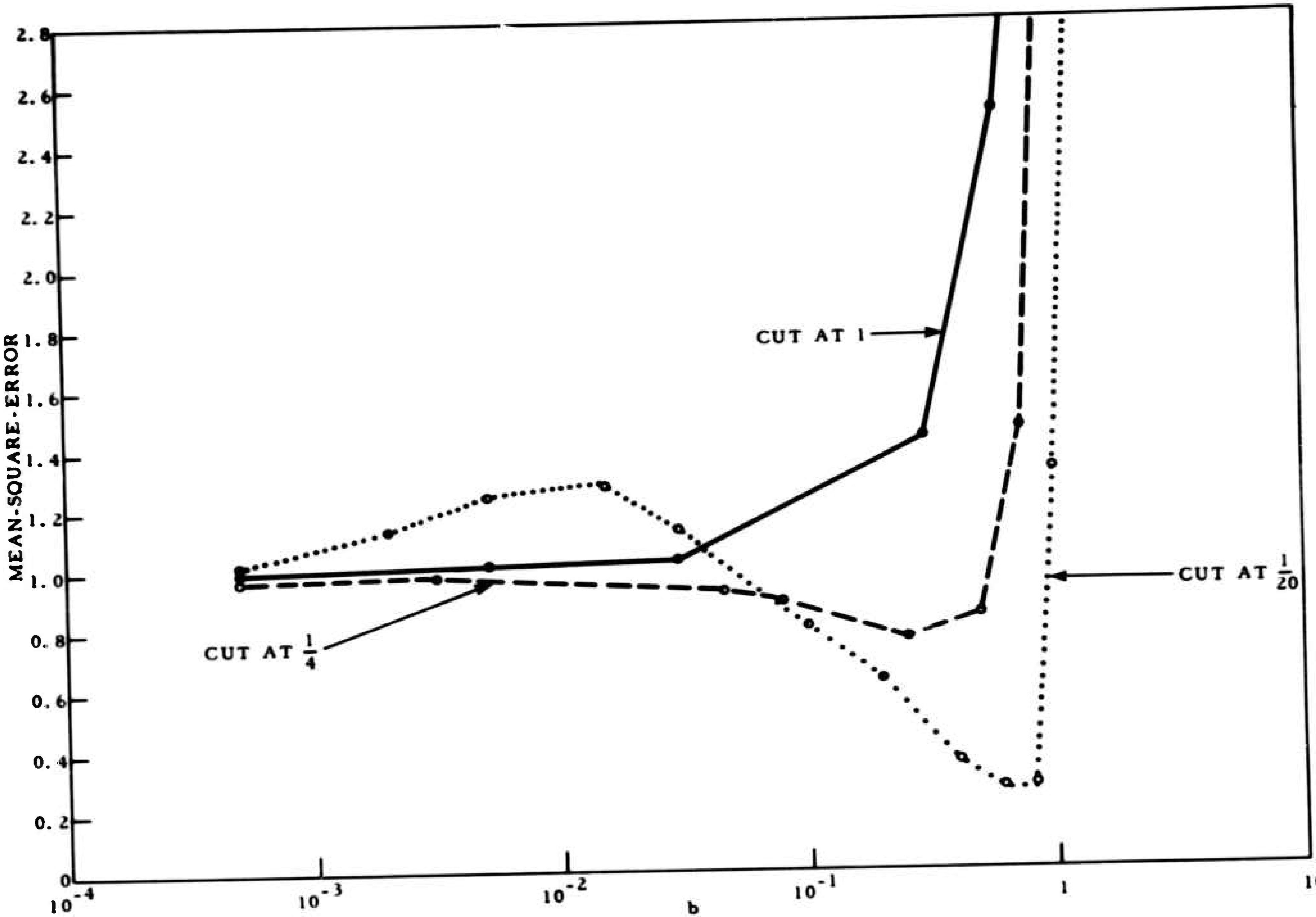

Figure III-5. Single-Channel 21-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs b

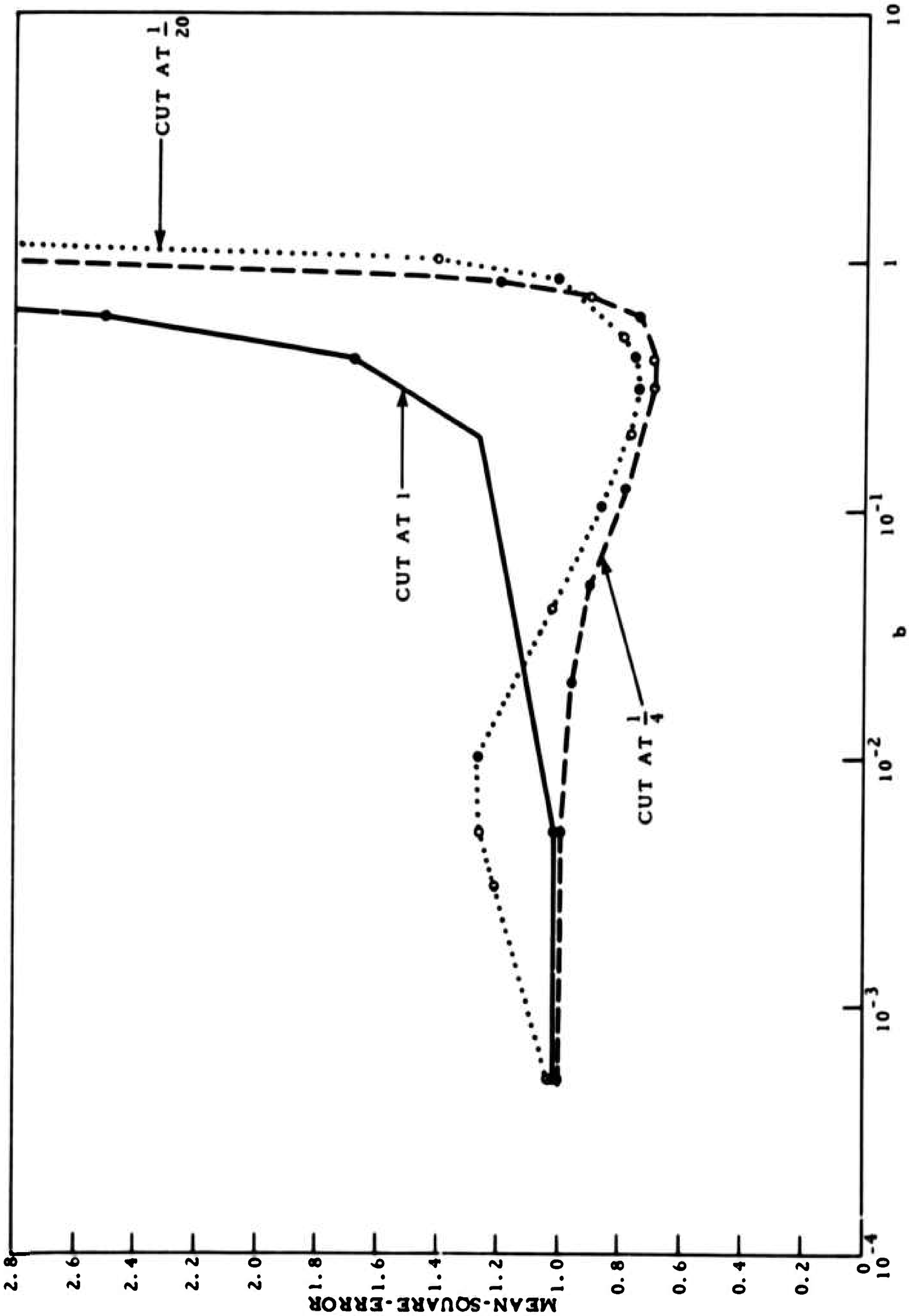

Figure III-6. Single-Channel 35-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs b

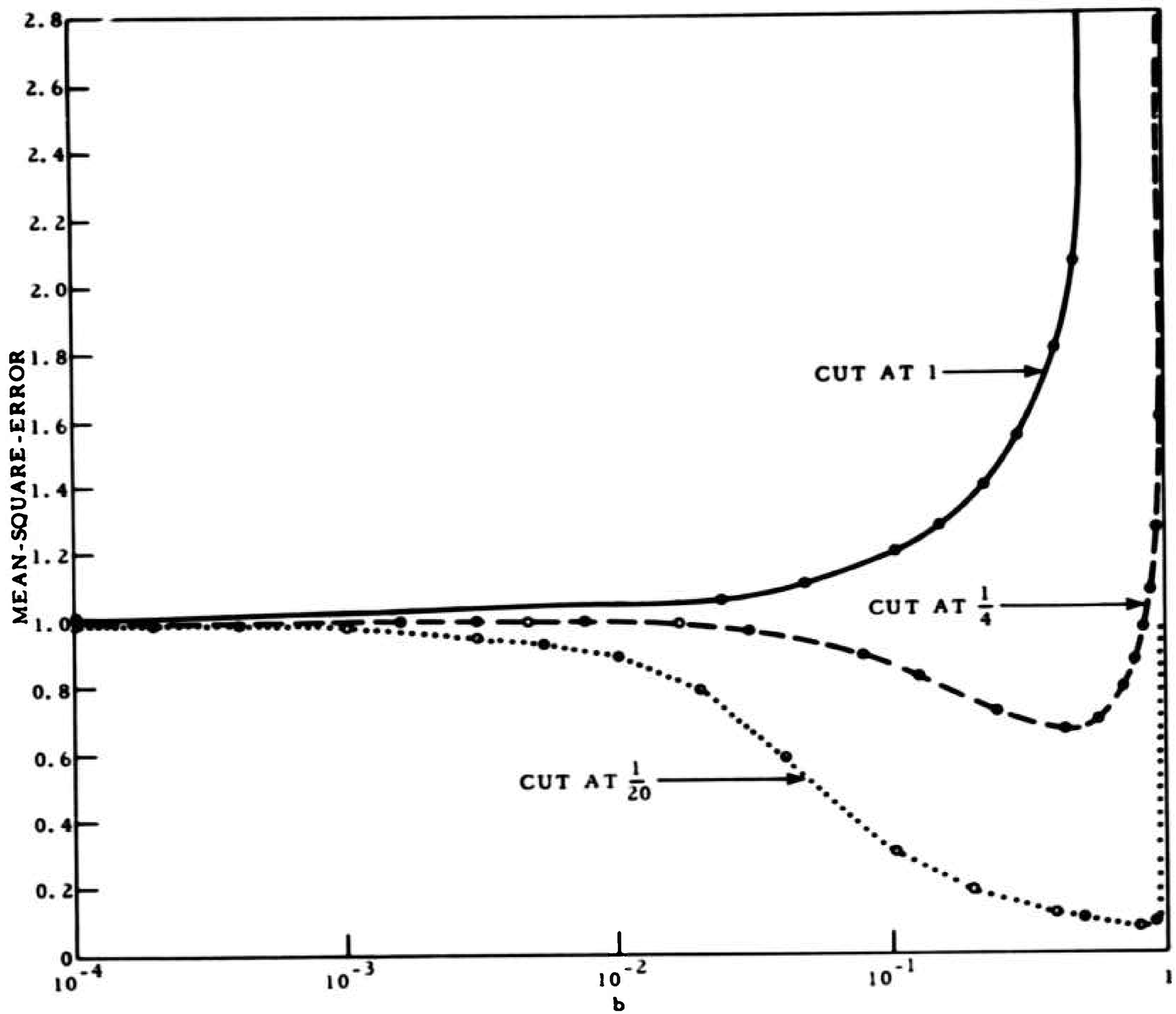

Figure III-7. 7-Channel 3-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs b

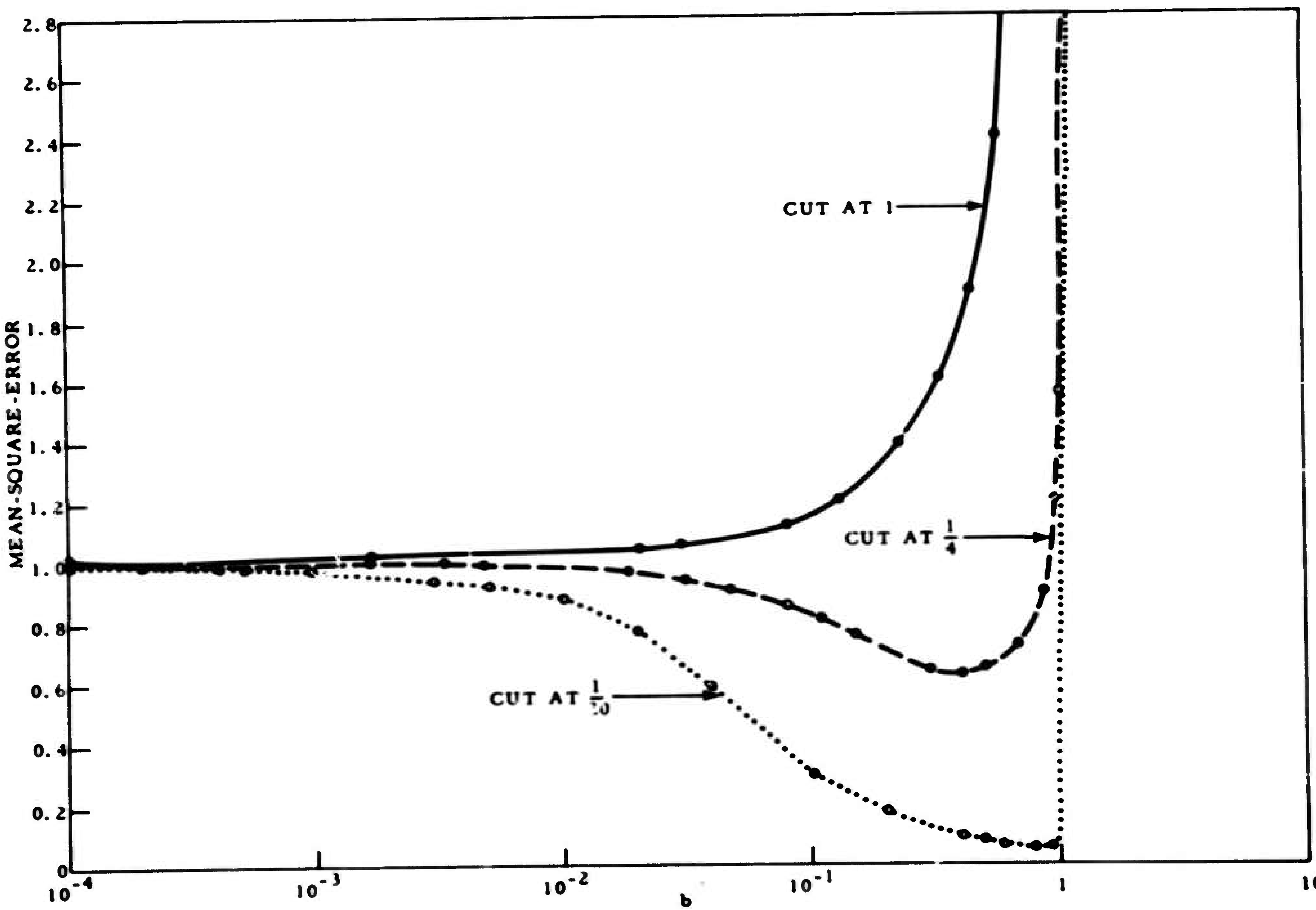

Figure III-8. 7-Channel 5-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs b

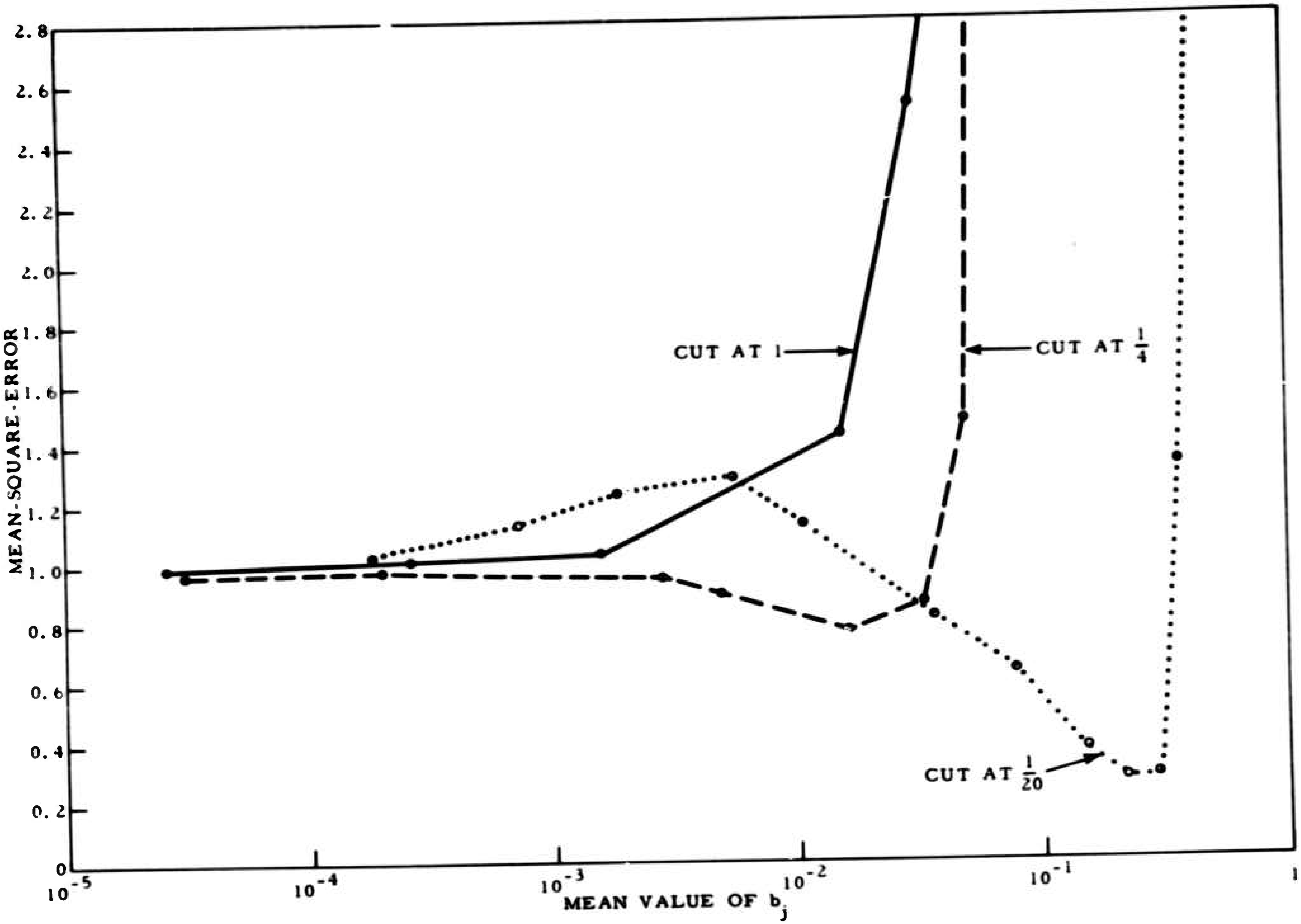

Figure III-9. Single-Channel 21-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs Mean Value of b_j

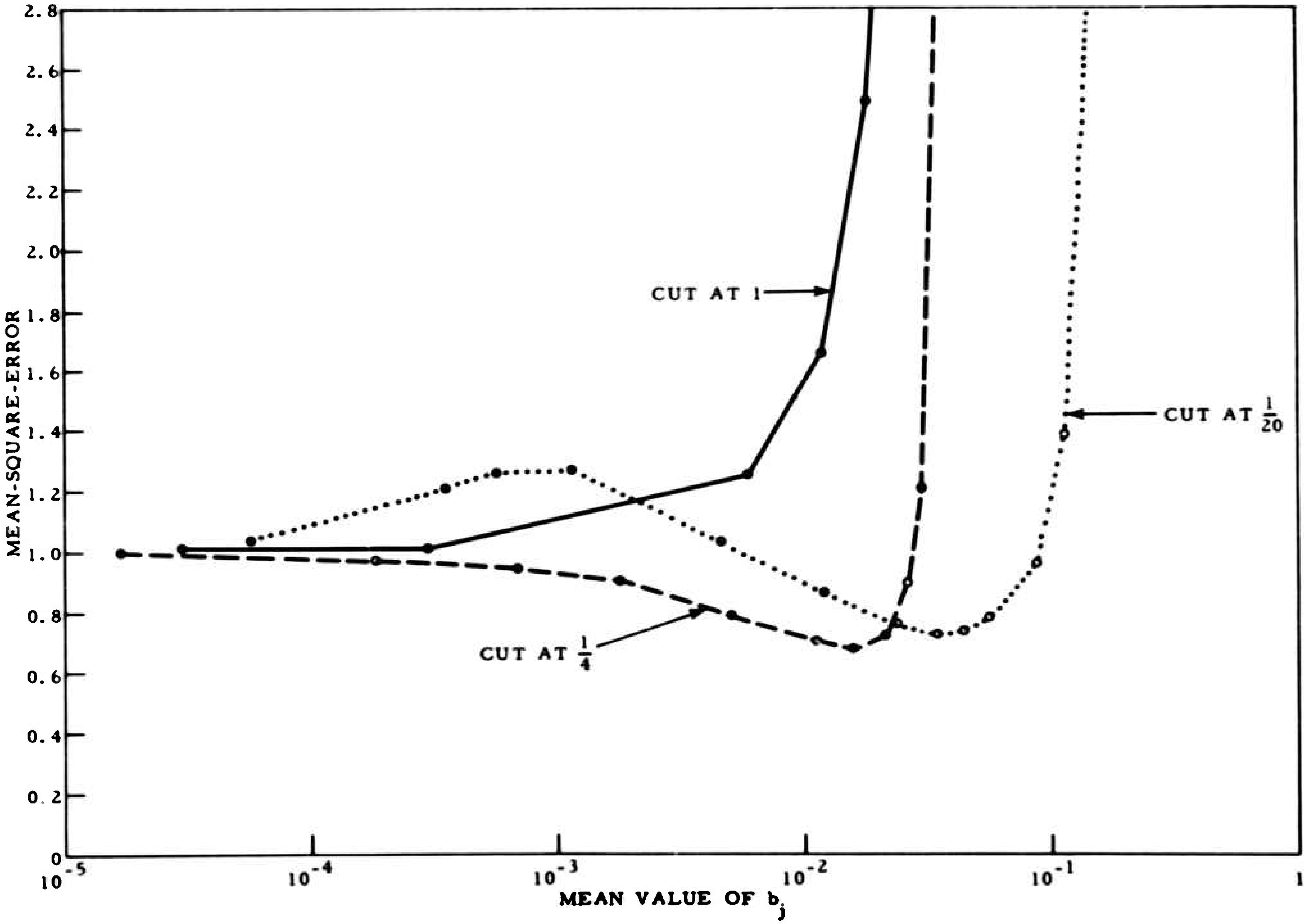

Figure III-10. Single-Channel 35-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs Mean Value of b_j

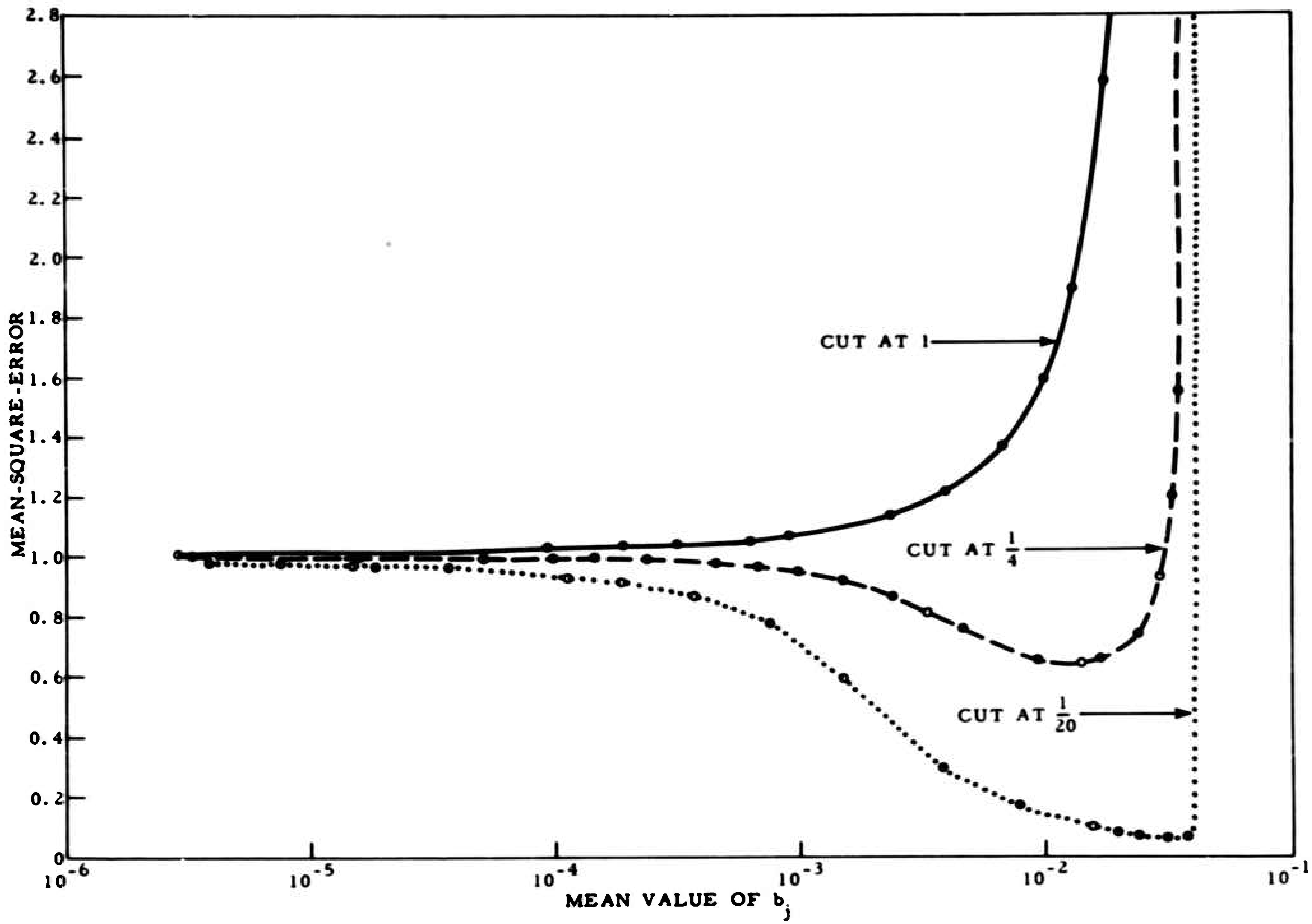

Figure III-11. 7-Channel 3-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs Mean Value of b_j

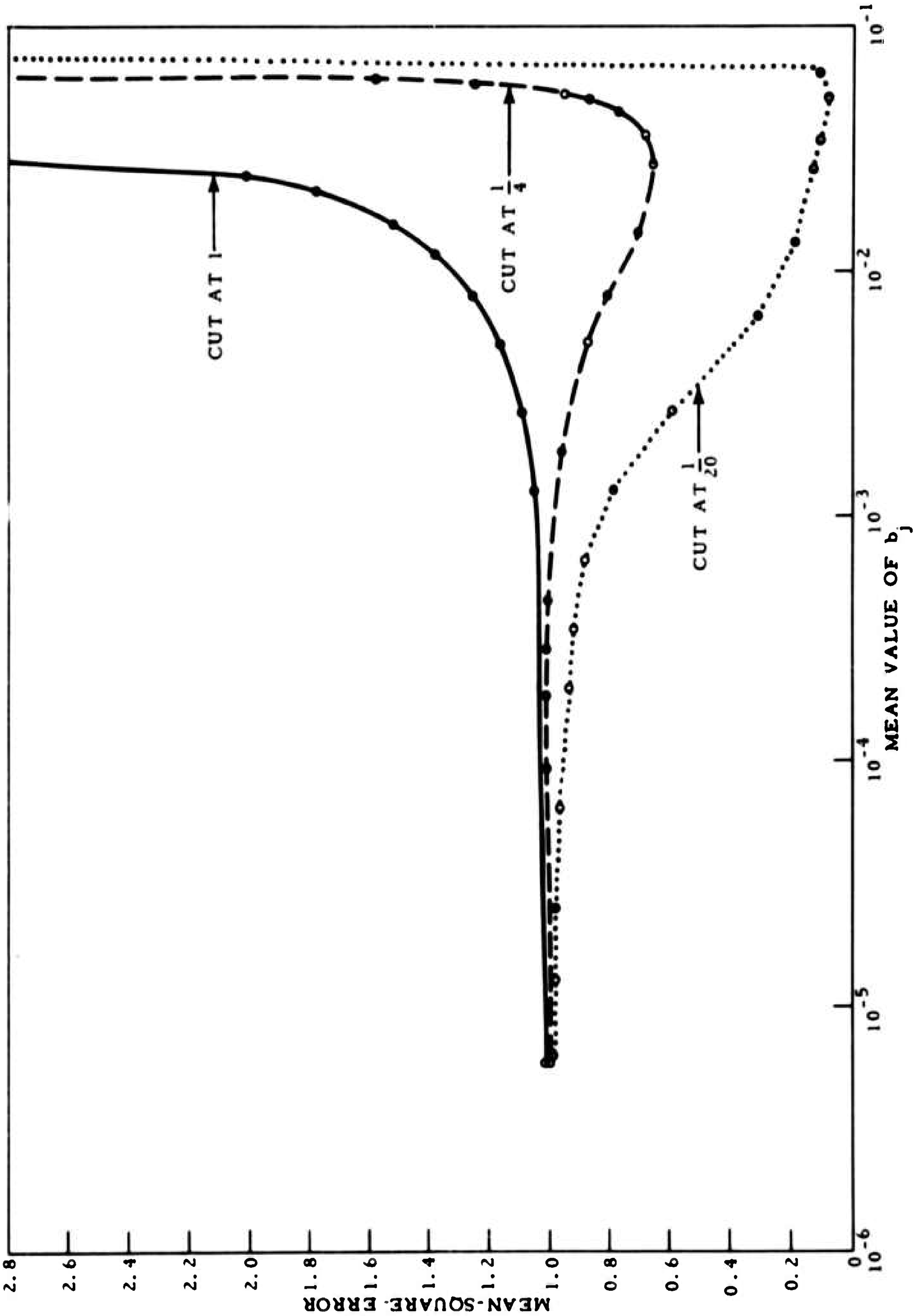

Figure III-12. 7-Channel 5-Point Filter, Variable Convergence Parameter, Mean-Square-Error Vs Mean Value of b_j

SECTION IV

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2. Texas Instruments Incorporated, 1968: Theoretical Considerations in Adaptive Processing, Advanced Array Research, Spec. Rpt. 13, Contract F33657-67-C-0708-P001, 28 Feb.
3. Widrow, B., 1966: Adaptive Filters, I — Fundamentals, Tech. Rpt. 6764-6, System Theory Laboratory, Stanford Univ., Dec.

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