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THESIS

DETERMINING OPERATIONAL HIT PROBABILITIES FOR FIELD ARTILLERY WEAPONS SYSTEMS

by

Richard William Boes

and

Richard Edward Garvey, Jr.

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FOR FIELD ARTILLERY WEAPONS SYSTEMS

by

Richard William Boes
Major, United States Army
B.C.E., University of Detroit, 1957

and

Richard Edward Garvey, Jr.
Captain, United States Army
B.S., United States Military Academy, 1962

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Signature of Authors Richard W Boes

Richard E. Garvey, Jr.

Approved by Donald R Barr
Thesis Advisor

for W. M. Woods
Chairman, Dept. of Operations Analysis

W. F. Koehler for R. F. Rinehart
Academic Dean

ABSTRACT

The Department of the Army has expressed a need for the determination of the operational hit probabilities of several weapons systems in use throughout the Army. These hit probabilities, together with lethality models, should yield predictions of the effects such systems will have under various conditions of combat.

In this thesis, operational hit probability (OHP) is defined as the probability that the center of impact of a <u>volley</u> of artillery fire will fall within a specified distance of the center of an <u>area</u> target. A general experimental methodology, which could be used to estimate OHP's (under simulated combat conditions) for a field artillery weapons system, is presented. More specifically, an approximate Chi-square distribution of squared radial miss distance is suggested for estimating OHP's. A method of using accuracy data from Army Training Tests to estimate required sample sizes for the experiment is proposed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

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CHAPTER I

INTRODUCTION

Since the effectiveness of a weapons system* may ultimately be determined by engaging the enemy, it is desirable to obtain quantitative data regarding measures of effectiveness from actual combat situations, where possible. Unfortunately, for various reasons, it is usually not possible to obtain operational (combat) data. It is common, therefore, to resort to field experimentation, wherein the pertinent variables are observed under simulated combat conditions.[1] The variables observed in a field experiment can then be used to estimate operational parameters. These estimates may be useful in applications to actual combat situations, provided the experiment is carefully designed and conducted.

The accuracy requirements of any weapons system depend upon the mission(s) to be performed by the system. For a Field Artillery Cannon Weapons System (hereafter referred to as a "FA Weapon System"), the mission may be the complete destruction of a point target,* the neutralization or partial destruction of an area target,* or simply the harassment or interdiction of a target for a given period of time.[2] The destruction mission is performed by a single gun, with an observer

*All terms with asterisks are defined in Appendix A.

[1] George E. Kimball and Philip M. Morse, Methods of Operations Research (New York: John Wiley and Sons, Inc., 1951), p.129.

[2] FM 6-40. Field Artillery Cannon Gunnery (Washington: Department of the Army, October, 1967), p. 27-2.

initiating the adjustment and continuing until the target destruction has been completed. The harassment mission does not require a high degree of accuracy; indeed, it is usually fired at "map-spotted" coordinates. Although the interdiction mission requires a greater degree of accuracy than the harassment mission, interdiction fire is usually of low intensity when compared to neutralization fire. In more than half of the artillery missions of World War II (for which data are available), multiple guns were employed to neutralize area targets.[1] We shall consider only neutralization missions in this paper, and we shall use the artillery battery, under battalion control, as the basic firing element.

The purpose of this thesis is to propose an experimental methodology for determing the operational hit probabilities of a FA Weapons System and to investigate other factors pertinent to the operation of such a system. We define operational hit probability (OHP) as the probability that the center of impact (CI) of a volley* of artillery fire will fall within a specified distance of an aiming point (usually the center of an area target). Defined as such, OHP is one of many possible measures of effectiveness for a FA Weapons System. It has not yet been determined whether OHP is a "good" measure of effectiveness or whether it can be accurately estimated at a "reasonable" cost. Such determinations should be made prior to conducting extensive experiments with FA Weapons Systems.

[1] J. D. Love, et al., Artillery Usage in World War II (U), Volume II (ORO-T-375, April, 1959), p. 133.

The information gained by conducting experiments to measure the OHP's of various FA Weapons Systems could be useful in many ways. OHP's could be combined with conditional lethality models to produce unconditional lethality models. Such information would increase the effectiveness of FA units, since artillery commanders would be better able to select the most appropriate weapons system for attacking a particular type of target. More effective employment of FA units and better fire planning should result. Additionally, OHP's should be valuable in force planning; for example, in determining the optimal mix of future FA Weapons Systems and in determining trade-offs between FA Weapons Systems and other weapons systems. OHP's could be used for both current and future logistics planning. They could also provide military war gamers with realistic artillery parameter values for future war games.

A Tabular Firing Table is published by Department of the Army for each FA Weapons System. Contained in these tables are corrections for non-standard firing conditions,* as well as values of probable errors* in range, deflection, and height of burst. These probable error values are a measure of round-to-round dispersion, since they are caused primarily by manufacturers' tolerances in ammunition and the weapon itself. As such, the probable errors in the Tabular Firing Table are inherent errors.* They have applications primarily when the weapons are fired at point targets; that is, in destruction missions.

Since we have restricted our discussion to multiple weapons firing neutralization missions at area targets, the tabled probable errors are of little value to us. We must concern ourselves with systems errors,* which are attributable both to inherent errors and to other factors such as variations in environment, wear in the weapon system, and human errors.

During the past 25 years, several attempts have been made to explain the system errors of certain FA Weapons Systems. Examples of such attempts are:

1) an accuracy study of various artillery weapons systems, based on single guns firing at point targets during the Korean conflict,[1]

2) a study to determine the manner in which human errors contribute to the total errors in predicted artillery fire,[2] and

3) a British pamphlet which discusses three earlier studies on the accuracy of unobserved fire in combat.[3]

These and other attempts are apparently unsatisfactory for determining an acceptable measure of effectiveness for various reasons. For example, some of these studies deal only with the attack of point targets, while others are based on data resulting from conditions which only remotely resemble true combat conditions.

There is little doubt that the failure of these earlier reports to provide a good measure of effectiveness for FA Weapons Systems had

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[1]Thornton Page, et al., On the Accuracy of Unobserved Artillery Fire (ORO-T-271, April, 1954).

[2]Jesse Orlansky, et al., Human Errors in Predicted Artillery Fire (ORO-T-113, October, 1952).

[3]Page, op. cit., pp. 25-29.

great bearing on the Department of Defense decision to initiate a detailed study into the Tactical Effectiveness of Weapons Systems (TEWS) in 1965. The purpose of the TEWS Program is to develop experimental methodology to measure system effectiveness for all Army weapons systems. Large costs have necessitated a pilot study for the TEWS Program, and Combat Developments Command Experimentation Command (CDCEC) is currently working on this pilot study. The artillery system chosen for study in the TEWS Pilot Program is the 155mm howitzer, self-propelled (M-109). The measure of effectiveness to be determined experimentally for this system is called operational hit probability.

In the TEWS Pilot Program, operational hit probabilities are described as

> "...those hit probabilities to be expected when weapon systems are manned by troops who are subject to the psychological and physiological stresses of combat. Such hit probabilities take into account the effects of terrain, climate, and seasonal changes as well as variations of tactical situations, e.g., offense, defense, retrograde, and movement to contact. Operational hit probabilities also include the variations inherent in considerations of troop fatigue under varying combat conditions."[1]

It should be noted that this description of operational hit probabilities is necessarily very general because it must be applied to tactical weapons systems of many types. When applied to direct fire* weapons

[1]Tactical Effectiveness of Weapons Systems (TEWS) Pilot Program Plan (Fort Ord, California: USACDCEC, May, 1967), p. I-9.

systems, such as the M-60 tank, it seems clear that OHP is the probability of hitting a point target (such as another tank) under a given set of conditions. When applied to indirect fire* weapons systems, such as a FA Weapons System, it is much less clear what the TEWS description of OHP means. This lack of clarity stems from the fact that indirect fire weapons systems are employed against both point and area targets, so what is meant by "a hit" must be clearly stated. One possible approach is to consider the probability of hitting a point target under operational conditions. For this interpretation it would seem reasonable to attempt to measure operational probable errors,* similar to the inherent probable errors listed in the Tabular Firing Table. Indications are that CDCEC is taking this approach. In contrast, the authors are considering the probability (OHP) of hitting an area target with a volley of artillery fire. Hence, the experiment we propose will be a methodology to estimate OHP's as defined in this latter context.

Chapter II of this thesis contains a general description of the variables involved in estimating OHP's. In Chapter III, we discuss the required field procedures and propose an experimental design. A method of analysis of the experimental data is presented in Chapter IV. Finally, in Chapter V, the authors discuss some areas that (they feel) deserve further investigation.

CHAPTER II

THE VARIABLES

Since the basic organization, procedures, tactics, and characteristics of FA Weapons Systems are similar, it is felt that a general procedure can be developed which may be used, with minor modifications, to determine the OHP's of _any_ FA Weapons System.

In their paper on weapons system accuracy, J. Nickel and J. Palmer divide a weapons system into three basic components: the method of detection and location of the target, the communications information link between detector and weapon, and the actual weapon itself.[1] We shall refer to these three components as _target acquisition_, _communications_, and _firing battery_, respectively. We shall consider two additional components, which can be identified as separate entities having a great deal of influence on the overall operation of the system. These are _survey control_, which establishes the location and orientation of the weapons, and the _fire direction center_ (FDC), which generates the firing data to be set on the guns. Meteorological data, although having an effect on system accuracy, will be assumed accurately measured for purposes of this experiment. The primary reason for making this assumption is that _true_ meteorological conditions cannot be determined, so no basis exists for determining meteorological errors.

[1]James A. Nickel and J. D. Palmer, _Methodology Utilized in the Determination of Weapons System Accuracy Requirements_ (Norman, Oklahoma: University of Oklahoma Research Institute, 16 December 1963), p. 1.

In addition, it is almost impossible to maintain current meteorological data, because weather conditions are continually changing. Finally, we have limited our consideration to the FA battery under battalion control, and meteorological data comes from a source outside the FA battalion.

Since the experiment will be conducted under simulated combat conditions, it is necessary to describe a scenario which contains realistic combat situations. Hence, the scenario for this experiment should be based, as much as possible, on the current threat, as provided by current intelligence.

A suitable measure of effectiveness for a FA Weapons System must stem from the mission of that system:

> "The mission of the field artillery is to provide continuous and timely fire support to the force commander ...."[1]

In most cases of the type we are considering (neutralization of area targets), this mission requires that the field artillery inflict casualties among the opposing enemy forces.

> "The immediate objective is to deliver a mass of accurate and timely fire so that the maximum number of casualties are inflicted."[2]

In these quotations the words "accurate and timely" seem to characterize the desired objectives of artillery fire. We feel that OMP

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[1] FM 6-20-1. <u>Field Artillery Tactics</u> (Washington: Department of the Army, 1 July 1965), p. 3.

[2] FM 6-40, <u>op</u>. <u>cit</u>., p. 1-2.

provides a suitable measure of effectiveness from the standpoint of accuracy. We propose to determine OHP's experimentally, and we feel that it would require little additional effort to simultaneously gather data relating to the timeliness of the system being studied. Hence, we propose that the distribution of the lengths of time required to conduct fire missions be used as a measure of effectiveness for timeliness.

Although we are primarily interested in the effectiveness of the weapons system as a whole, it is also desirable to identify those factors which cause artillery errors, as well as the relative magnitude of these errors. This additional information should be useful in seeking methods of improving systems accuracies. For example, if it were found that large dispersion errors in the center of impact resulted from incorrect deflection settings, a possible remedy might be to redesign the deflection scales of the weapon sight. Thus we are seeking some knowledge of the effect that each of the component parts has on the operation of the system as a whole. This leads to a discussion of the independent variables which we propose for the experiment.

Independent Variables

Our selection of independent variables is based upon material, current tactics and techniques, operational environments, and historical records of artillery operations. Budgetary and time constraints often preclude the use of all independent variables that the experimentor may desire. Hence, he may be forced to select a reduced number of independent variables, or at least specify priorities to indicate those independent variables which he feels are most important. We believe that the following independant variables are critical to the determination of OHP's. The rationale for the choices is included in the list of variables.

<u>Method of Entering Fire for Effect</u>. The accuracy of artillery fire may vary substantially, depending upon what method of entering fire for effect (FFE*) is used by the FDC. There are three methods to be considered.

The first of these is the "Adjust Fire" method, in which an observer estimates the location of an aiming point, usually the target center. This location is then transmitted to the FDC where it is used as one of the elements in the computation of firing data. One or more adjusting rounds are fired using this data; when they detonate in the impact area, the observer determines corrections relative to the observer-target line (or the gun-target line in the case of an air observer). The observer transmits these corrections to the FDC where new firing data is computed, and additional adjusting rounds are then fired. The observer continues adjustment until he senses* that the center of impact of the adjusting rounds is on the observer-target line, and within 50 meters of the target, at which time he calls for fire for effect.

The remaining two methods of entering FFE are similar to each other in that no adjustment is conducted. The first method is called "transfer using registration corrections;" the second, "transfer using meteorological plus velocity error corrections." In each case the observer immediately requests fire for effect because he has a high degree of confidence that his initial location data is within 50 meters of the target. This normally occurs when the target is located on or near a prominent terrain feature, a surveyed location, or a target which has been fired upon previously.

Visibility Conditions. It is felt that the accuracy of artillery fire will vary with the time of day (24-hour day), primarily because the observer has a greatly decreased ability to detect and adjust on targets during the hours of darkness. The time required to complete fire missions is expected to increase during the hours of darkness, because gun crews are required to work with a minimum of artificial light. Hence, we have divided the 24-hour day into two segments, "daylight" and "dark."

Fuze. There are three types of fuze commonly used by the Artillery: point detonating (PD), mechanical time (MT), and variable time (VT). The PD fuze functions on impact, and it may be set for either quick or delayed action. We feel that the delay option should be eliminated for the purposes of this experiment. Hence, the only "error" which can occur is the failure of the fuze to function. In contrast there are at least two sources of error for MT fuze. The FDC computes the MT fuze setting, and the gun crews put this setting on the fuze. If either the FDC or the gun crew makes an error, the fuze will detonate before it reaches the target or after it has passed over the target. It may even detonate on impact instead of the desired 20 meters above the ground.

In the case of VT fuze, the FDC computes an "arming time" for the gun crews to set on the fuze. This "arming time" (a safety setting to ensure that the projectile clears friendly forces before it becomes armed) is considerably <u>less</u> than the time of flight to the target. Once the VT fuze has armed, it should automatically detonate within 20 meters of any feature which produces a radar "echo."

We have chosen to consider only PD and MT fuses in this experiment because VT fuze provides little more data than would already be available from PD fuze. The only chance for human error with VT fuze is for the fuse setting actually put on the fuze to be longer than the time of flight of the projectile. This would require a "gross" error to be committed by either the FDC or the gun crews. It is felt that such an event is unlikely, so we eliminate VT fuze from further consideration.

<u>Tactical Situation</u>. The two most common categories for describing tactical situations are "offense" and "defense." The defense is primarily a <u>static</u> situation. Forward observers generally have ample time to study the terrain to their front. Survey teams are able to bring survey control into all position areas,* and are usually able to do extensive target area and connecting area survey.

In contrast, the offense is characterized by <u>movement</u>. As the friendly forces advance, observers are required to conduct fire missions on unfamiliar terrain, so initial target location errors should be much larger than in the defense. Survey teams are often delayed in bringing survey control to firing units, so some batteries may be required to fire missions from "map-spotted" coordinates until position area survey is established. Target area survey and connecting area survey are rarely done. All of these factors introduce additional error into the offensive situation. We feel that accuracy of the artillery fire will be affected; hence, the experiment should be conducted under both offensive and defensive conditions.

Gun-Target Range. The authors feel that OHP's may vary considerably, depending upon the range from the gun to the target. However, estimates of OHP's obtained experimentally might have little value if they are based upon ranges that are not likely to be fired. Therefore, we propose that the experimentor select three range bands (short, medium, and long) that are typical of the weapons system being tested. Here we use the word "typical" to mean those ranges at which enemy targets are likely to be detected. Hence, the short-range band should include a high percentage of the short-range missions that are likely to be fired in future conflicts, and similarly for the medium and long-range bands. Estimates of the missions that are "likely" to be fired should come from the predicted distribution of enemy targets as provided by current intelligence.

Having selected three typical range bands, the experimentor is faced with another problem: should test units be required to fire an equal number of missions in each range band, or are there advantages to having them fire unequal numbers of missions in each range band? For example, intelligence information may indicate that 70% of all future missions, for the weapons system being considered, will be fired in the short-range band. If this were the case, the experimentor might want to fire more missions in this range band, in order to ensure that his estimates of OHP's (for the short-range band) are "good" ones. On the other hand, it is expected that radial miss distance will have a higher statistical variance in the long-range band. This conjecture, if true, would indicate that additional missions should be fired in the long-range band, in order to obtain better estimates of OHP's there. Finally, firing equal numbers of missions in each range band would make the data reduction much easier. This problem is discussed in greater detail in Appendix C.

Nuclear-Biological-Chemical Environment. When toxic agents are present in the atmosphere, all personnel must don protective clothing and protective masks so that they can carry out their missions. The wearing of such equipment will undoubtedly affect the time required to perform artillery-related tasks, and it may affect the accuracy of the fire delivered. In addition, the wearing of protective masks will probably make communications more difficult. We therefore feel that the experiment should be conducted under both toxic and non-toxic conditions.

Dependent Variables

A choice of response, or dependent, variables is based upon the measures of effectiveness chosen for the system. As stated previously, we have chosen OHP as a measure of the system's accuracy and the distribution of the lengths of time required to conduct fire missions as a measure of the system's timeliness. From the standpoint of accuracy, we have also expressed an interest in the cause and relative magnitude of artillery errors. We first concern ourselves with those dependent variables related to accuracy; namely, OHP and errors of various types. Later we address the question of timeliness of the system.

Center of Impact of a Volley. The basic requirement for estimating OHP's for a weapons system is to determine the actual center of impact of a volley so that it can be compared with the desired center of impact. The desired center of impact is usually at the target center (aiming point), and the actual center of impact can be estimated using flash-base techniques to be described in Chapter III. The results of a series of firings may be used to develop empirical distributions of the center of impact about the target center, and OHP's can be estimated from these distributions. The technique for estimating OHP's will be discussed in Chapter IV.

OHP provides an overall measure of the system's accuracy, but it does not provide us with information regarding the cause nor relative magnitude of artillery errors. This information must be obtained by examining the components of the system. Thus the dependent variables of interest in this regard are those which measure component errors; that is, errors which may be attributed to a single component of the system.

Target Acquisition Errors. Under operational conditions, target acquisition errors would fall into three major categories:

1) failure to detect the target,

2) improper identification of the target, and

3) incorrect location of the target, both initially and with respect to adjusting rounds.

For experimental purposes, we propose to eliminate errors of the first two categories by having an umpire designate a target (aiming point) and identify it for the observer. Thus target acquisition errors to be considered here are errors in target location only. These errors can best be described by considering the three methods of entering fire for effect discussed previously.

Errors in the "Adjust Fire" method of entering fire for effect are a result of the observer's inability to accurately determine target location initially, as well as errors in judging the location of the target with respect to adjusting rounds. In contrast, target location errors associated with the second two methods of entering fire for effect, when no adjusting rounds are fired, are due to inaccuracies in the observer's initial location data only. Other errors may be committed, regardless of which method of entering fire for effect is used. We shall discuss some such errors and indicate which components of the system are involved.

Survey Control Errors. These errors are associated with incorrect determination or reporting of coordinates of battery centers and azimuths of orienting lines.* Survey errors have varying effects on accuracy, depending upon the fire procedure being used. In the case of "Adjust Fire" or "transfer using registration corrections," the survey errors are "shot out" during adjustment and registration, respectively. Therefore, survey errors would affect only the time required to adjust on a new target. The accuracy of the rounds in FFE would not be affected in either of these procedures.

Survey errors have the greatest adverse effect on accuracy when entering fire for effect using "meteorological plus velocity error corrections." In this case, there is neither an adjustment nor a prior registration, so all survey errors are incorporated in the firing data that is sent to the guns.

Fire Direction Center Errors. In this category we shall consider only errors that are actually generated within the FDC. That is, all inputs to the FDC (e.g., survey, target location, and meteorological data) will be assumed correct so that only errors in the computation of firing data will be assigned to the FDC.

Errors generated within the FDC can adversely affect both the time required to complete a fire mission and the accuracy of the rounds in FFE. Although there are a great number of places where errors can be committed in the FDC, our primary concern is the data that is transmitted to the firing battery for use in FFE. These errors can take the form of incorrect quadrant elevation,* deflection,* and (in the case of missions requiring MT fuze) the setting to be placed on the fuze.

Firing Battery Errors. Here, as with the FDC, we shall consider only errors actually generated within the firing battery. All inputs to the firing battery will be assumed correct.

We can divide firing battery errors into two groups: those which directly affect the fall of shot and those which cause incorrect inputs to other components of the system. Errors of the first type are incorrect initial lay of battery, incorrect lay (either in deflection or quadrant elevation) of the individual pieces prior to firing, errors in mechanical time fuze settings, and errors in the charge fired. Examples of the second type of error would be the incorrect measurement or reporting of powder temperature or projectile weight.

Communication Errors. These errors are due to communications between system components, not within the separate components. Such errors are caused by poor radio-telephone procedures or faulty equipment. Any message that is misunderstood by a radio-telephone operator of the artillery unit and is recorded incorrectly by him is considered a communication error. Such errors observed in the experiment should be analyzed to determine their effect on the firing data that is set on the guns. Some communications errors may not affect the firing data; for example, the umpire identifies a survey party for the observer, but the observer tells the FDC that the target is a wire crew. Other communications errors, such as the transposition of figures by a receiving operator, may have a very adverse effect on accuracy.

Residual Errors. These errors are encountered primarily as a result of imperfect experimental controls. All errors not previously mentioned, regardless of source, would be included in this category. Therefore, if the experimentor were to measure all component errors and then calculate their total effect on the fall of shot, he should be able to predict where the center of impact will occur. If the actual CI does not agree with this prediction, the difference is due to residual errors.

One example of a residual error is an error in meteorological data, since all meteorological data was assumed to be correct. Another example of a residual error is a "round-off" error. These occur because only integers may be set on the scales of the weapons.

Dependent Variables (Timeliness)

We now consider the measure of the system's timeliness. Although accuracy is usually of paramount concern to artillery units, there are occasions when speed of delivery of fires takes precedence. Such instances are a matter of judgment of the commander and may warrant deviations from the normal procedures. Training doctrine requires that:

> "All members of the artillery team must be continually indoctrinated with a sense of urgency."[1]

In addition to the distribution of the lengths of time required to conduct fire missions, we also propose to observe the lengths of time required by certain components of the FA Weapons System. Again, it is desirable to learn what effect the components of the system have on the measure of effectiveness being investigated, so that we may seek possible methods of improvement.

Total Fire Mission Time. Total fire mission time should be measured from the time that an observer detects a target until the FFE rounds burst in the target area. The following procedures could be used by an umpire when designating a target to an observer: the umpire would describe the target (e.g., platoon of infantry in the open), give angular measurements to the target, and describe the object that is to

[1] FM 6-40, *op. cit.*, p. 1-2.

be used as the aiming point (e.g., a red car body). For this experiment detection occurs when the observer sees the aiming point (the red car body) and states, "target identified."

Component Times. In order to determine the contribution of the component parts to the total fire mission time, the following measurements should be taken.

1) Target acquisition time should be measured from the time the observer detects a target until he initiates a request for a fire mission with the FDC. Adjustment time should be measured from the appearance of an adjusting burst in the target area until the first element of adjustment data is transmitted to the FDC.

2) Survey time should be measured from the time that the battalion survey control point is identified by the survey party until the battery center and orienting line are marked.

3) Fire direction center time should be measured as follows:

a) For missions using one of the non-adjusting methods, the time increment should be measured from the receipt of a request for fire until the last element (quadrant elevation) of FFE data is transmitted to the firing battery.

b) For missions using the "Adjust Fire" method, the time increment described above should be summed with increments computed in a similar manner for rounds in adjustment.

4) Firing battery time should be measured as follows:

a) For missions using one of the non-adjusting methods, the time increment should be measured from the receipt of the last element of the firing data from the FDC until the guns are fired.

b) For missions using the "Adjust Fire" method, the time increment as described above should be summed for both the adjusting fire and rounds in FFE.

Controlled Variables

We now proceed to a discussion of controlled variables. Conditions are usually controlled so that the experimenter can attribute changes in the dependent variables (up to random errors) to the values of the independent variables. In addition, controls are often imposed in order to limit the magnitude of the experiment, since each additional variable (of two treatments) would increase the number of data cells by a factor of two. We consider three states of control: rigid, systematic, and uncontrolled.

Rigidly Controlled Variables. The following variables are those which the authors feel should be rigidly controlled:

1) Sheaf width.* The battery should be deployed to give a parallel, or normal, sheaf. The width of the sheaf will depend upon the caliber of the weapons in the unit being tested.

2) Length of Survey. A fixed length of survey should be used. The exact length can be determined after the site of the experiment is selected.

3) Type of Ammunition. The preponderance of all ammunition fired, both in combat and in training, is high explosive. Therefore we feel that only high explosive ammunition should be used during the experiment. In addition, sufficient ammunition of a given lot should be available to ensure that no "mixed-lot" missions* are fired.

4) Unit Training. All artillery units are required to take, and pass, an Army Training Test (ATT) on an annual basis. The authors feel that all units should again be tested at the experimental site prior to conducting the experiment. This test, by a single team of umpires, will ensure that all units to be used in the experiment meet the minimum level of training proficiency required by the Army.

5) Angle of Fire. Only low angle fire (less than 800 mils for most weapons) should be used in this experiment. Low angle fire is more accurate than high angle fire, while the latter is more lethal against certain types of targets. Since analysis using these methods as variables would necessitate expanding the experiment into areas of limited application, we propose the above restraint.

Systematically Controlled Variables. Systematic controls also serve to limit the size of the experiment. Additionally, these controls assist in making the simulated combat conditions of the experimental environment more representative of true combat conditions. The only variable considered in this category should be target occurrence time. Using the independent variable "Visibility Condition," we have already broken the 24-hour day into "day" and "night." However, past experience indicates that targets do not occur uniformly over either of these periods. Enemy attacks most frequently occur in the early morning, so we feel that a test unit should be required to fire more missions between, say, 0500 and 0600 than between 2300 and 2400. This could be accomplished by distributing target occurrence times in the scenario according to historical data that is available. The OHP's thus obtained would be "averaged" over the times at which enemy targets are likely to appear.

Uncontrolled Variables. Certain variables are uncontrolled, usually because it is too expensive, or too difficult, to control them. Variables that fall into this category are as follows:

1) Terrain and Vegetation. These will ultimately depend upon the experimental site selected. A prime consideration in selecting a test site is the availability of units near the site. It is also desirable to select test sites that have, as nearly as possible, "typical" terrain.

2) Weather. Weather will depend upon the test site and the season. Testing under "extreme" weather conditions, of any sort, should be avoided if possible.

3) Combat Realism. Every effort should be made to develop a test scenario that will include, as nearly as possible, the psychological and physiological stresses of actual combat.

CHAPTER III

THE MODEL

We have thus far described the conditions that we feel should be varied or controlled in order to observe possible changes in the dependent variables. In this chapter we outline a general procedure that should be followed both in preparation for, and conduct of, the experiment in the field. A linear statistical model, which could be used during the data analysis phase of the experiment, is also presented.

Sequence of Field Procedures

For this experiment a test unit should consist of one randomly selected firing battery (from a particular battalion), the battalion FDC, and one survey team. In Appendix C the authors describe a method for obtaining an initial estimate of the required number of test units. Once this estimate has been obtained, the battalions to furnish test units should be randomly selected from available battalions. If the estimated required number of test units were to exceed the number of available battalions, possibly because of the limited number of active battalions or for combat reasons, the experiment could still be conducted. However, it must be remembered that the level of confidence in the estimated parameters would be lowered accordingly.

Each test unit should be assigned a sequence number, which will identify the order in which the experiment would be conducted for that unit. A typical test cycle, shown in Table 1, displays one possible

TABLE 1

TYPICAL TEST CYCLE

Day	1	Arrive at test site; receive administrative briefing and orientation.
	2	Draw equipment; prepare for field operations.
	3	Receive detailed briefing about the experiment and conduct the pre-experiment ATT; prepare for movement to the field.
	4-5	Conduct ATT; receive briefing on results.
	6-7	Maintain equipment and prepare for the experiment.
	8-10	Receive alert order, load vehicles, move to assembly area, and prepare for defensive operations. Move to defensive positions (as required by the scenario) and conduct fire missions.
	11-13	Receive orders to prepare to support an attack; conduct operations in support of an attack.
	14-16	Receive orders to remain in position and support defensive operations; be prepared to continue the attack on order.
	17-19	Continue the attack.
	20-21	Return to base camp; maintain equipment; return equipment to the supply point; receive critique from test team.

order of tactical situations. This order would be followed by all units having odd sequence numbers. Even-numbered units would follow the alternating schedule of tactical situations, starting with the offense. By mixing the order of tactical situations, bias due to fatigue and learning should be averaged over the two situations.

Once the experiment begins, the test units will be in simulated combat continuously for the duration of the experiment, thereby increasing the psychological and physiological stresses to which personnel are subjected. The testing period of twelve days, suggested in Table 1, would have to be adjusted depending upon the firing matrix used (see Figures 1 and 2) and the number of volleys to be fired in each data cell. Time must be allowed for registration, moves, and normal tactical activities, so a reasonable work load should be approximately 12-16 fire missions per period of light condition (daylight and dark). The order of fire missions should be determined at random and integrated into the detailed scenario.

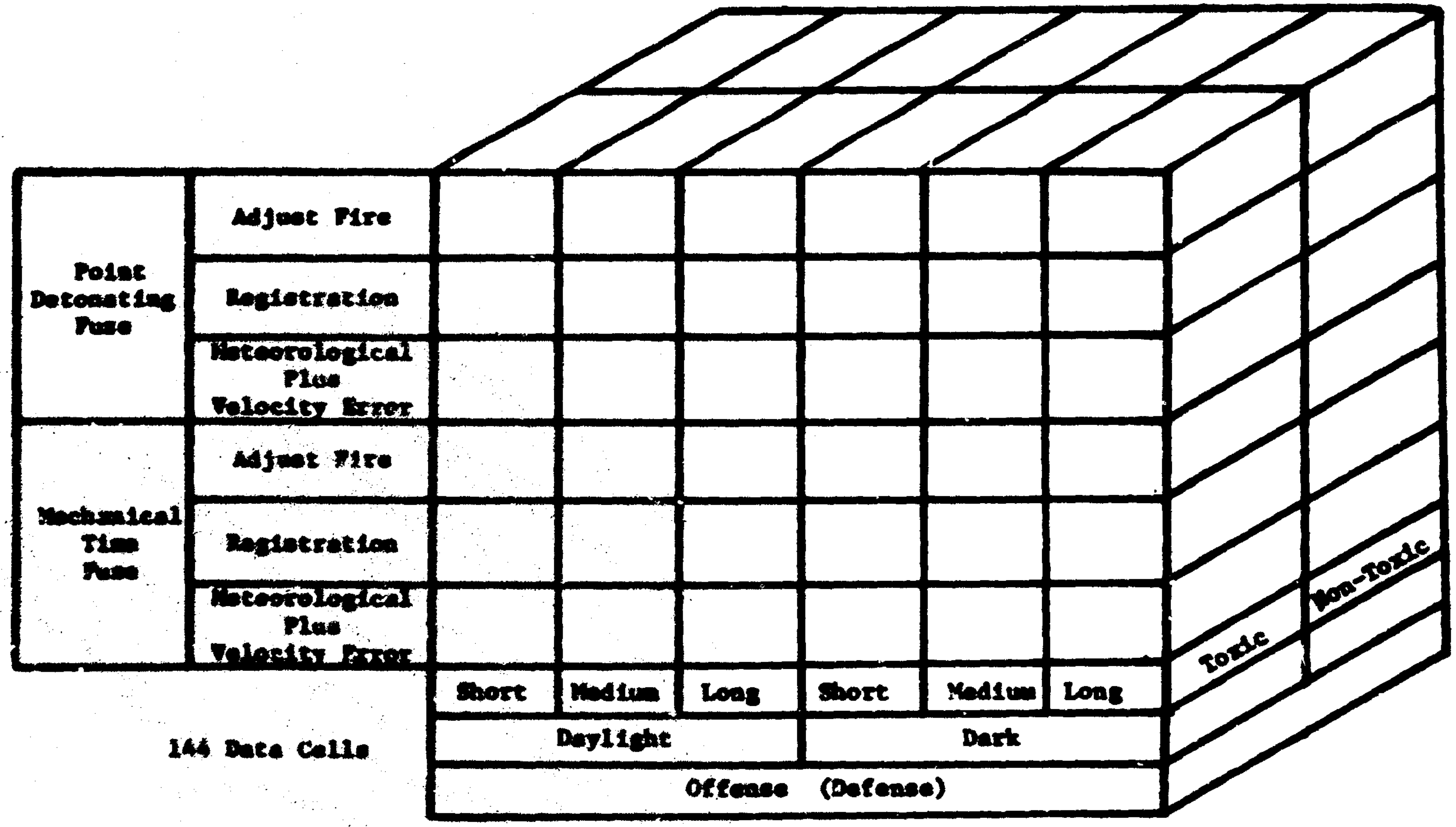
The experiment should be conducted in the following five phases.

1) Alert test units; inform them of their arrival time at the experimentation site; ensure that test units have satisfactorily completed an ATT 60 days (at most) prior to the test period; move test units to the test site.

2) Orient test units; have them draw equipment and prepare for field operations.

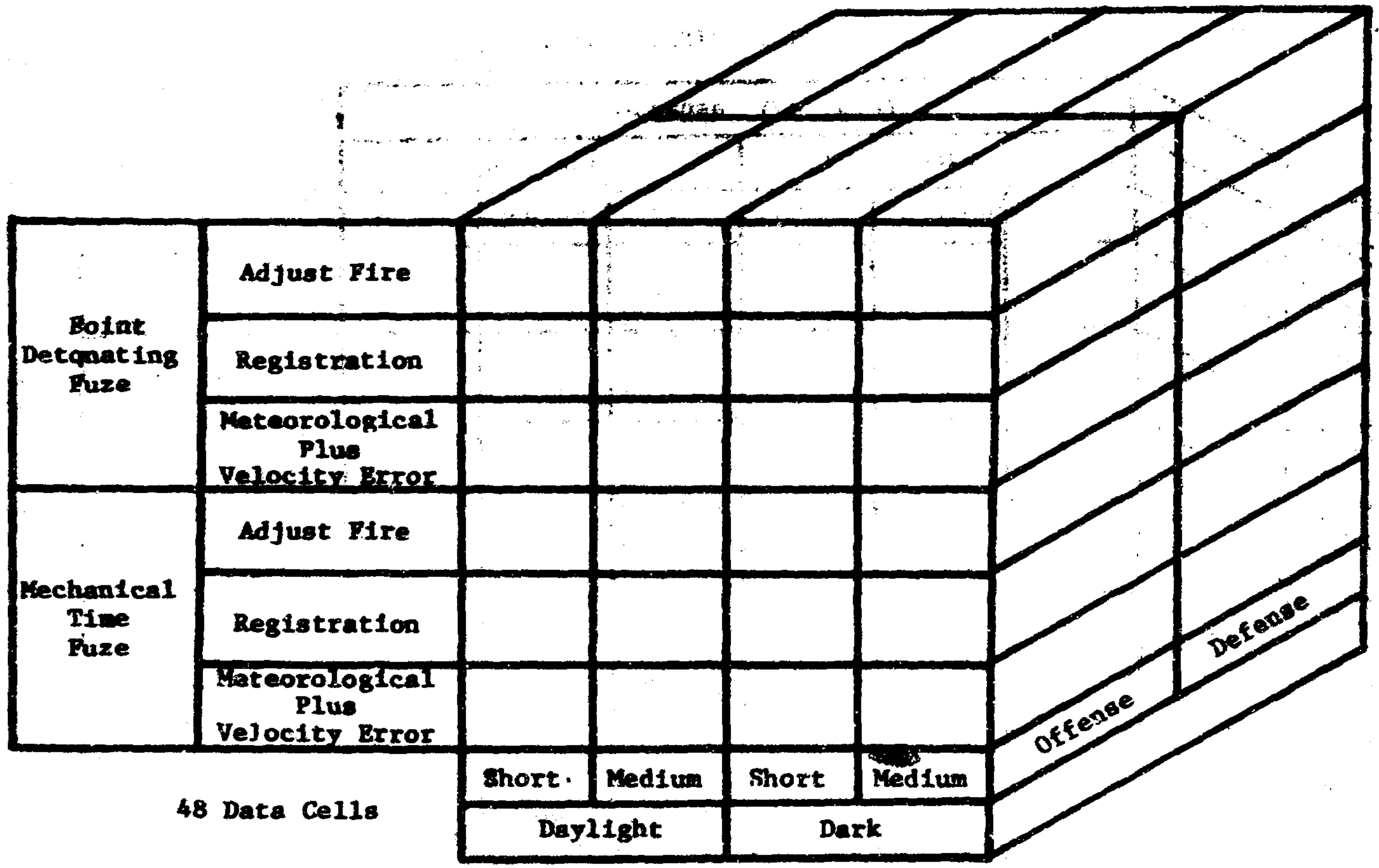
3) Conduct Army Training Test at the test site.

4) Conduct experiment; have units return equipment and depart for home station.



FIRING MATRIX (FULL)

FIGURE 1



FIRING MATRIX (REDUCED)

FIGURE 2

5) Conduct intermediate data reduction and, if possible, update the estimate of the number of test units required.

Data Collection

Data should be collected by teams of umpires assigned to the four activities described below. The kinds of data to be collected and the instrumentation will be discussed. Pre-printed blank forms should be used for recording data, and should be turned in to the control headquarters daily. All time measurements should be made using stopwatches, and need not be recorded more accurately than one-tenth of a second.

Target Location. The target location umpire team should accompany the observers and record target location data that is transmitted by the observer to the FDC. This information would be recorded in addition to surveyed target information. Target location time and total fire mission time should also be measured and recorded by this team.

Fire Direction Center. The umpire team located within the FDC should observe each step in the computation of a fire mission, and record any errors or malpractices that are observed. This team should tape-record all communications between the FDC and both the observer and the firing battery. Any errors in information transferred should be recorded when they are observed. The tapes would be analysed later to discover any undetected errors and to verify those already recorded. All firing charts and FDC computation forms should be collected daily for later analysis. This umpire team should also measure FDC time as described previously.

<u>Firing Battery</u>. The umpire team observing operations with the firing battery should monitor all activities associated with gunnery procedures, to include registration, laying the battery, and emplacing aiming posts. All observed errors and malpractices should be recorded. One umpire should be assigned to each gun crew so that minimum delay is caused by umpire activities. Salvo fire* should be used in all fire missions, even though the simulated tactical conditions may indicate that all weapons should be fired simultaneously. This type of fire would allow the operators manning the flash-base in the target area to observe each round individually, in order to estimate the CI more accurately. The sight picture and scale settings should be checked each time (before and after) a weapon is fired. Fuze settings should be checked just prior to loading the projectile into the gun. Any errors due to radio-telephone operation should be recorded as they are detected. These errors may be analyzed later against the taped record. The firing battery time should be measured by this team.

A photographic scheme of recording the sight picture, scale readings, and fuze settings might be useful if excessive time delays are attributed to the umpire team, or the umpire's accuracy in reading scales is questioned. Pictures of the scales could be taken and later analyzed for errors. The introduction of such a scheme of data collection might be a result of a pilot study.

<u>Target Area</u>. The umpire team in the target area will operate the flash-base, which could be used to estimate the location of each round fired, including rounds in registration and adjustment. An <u>ad hoc</u>

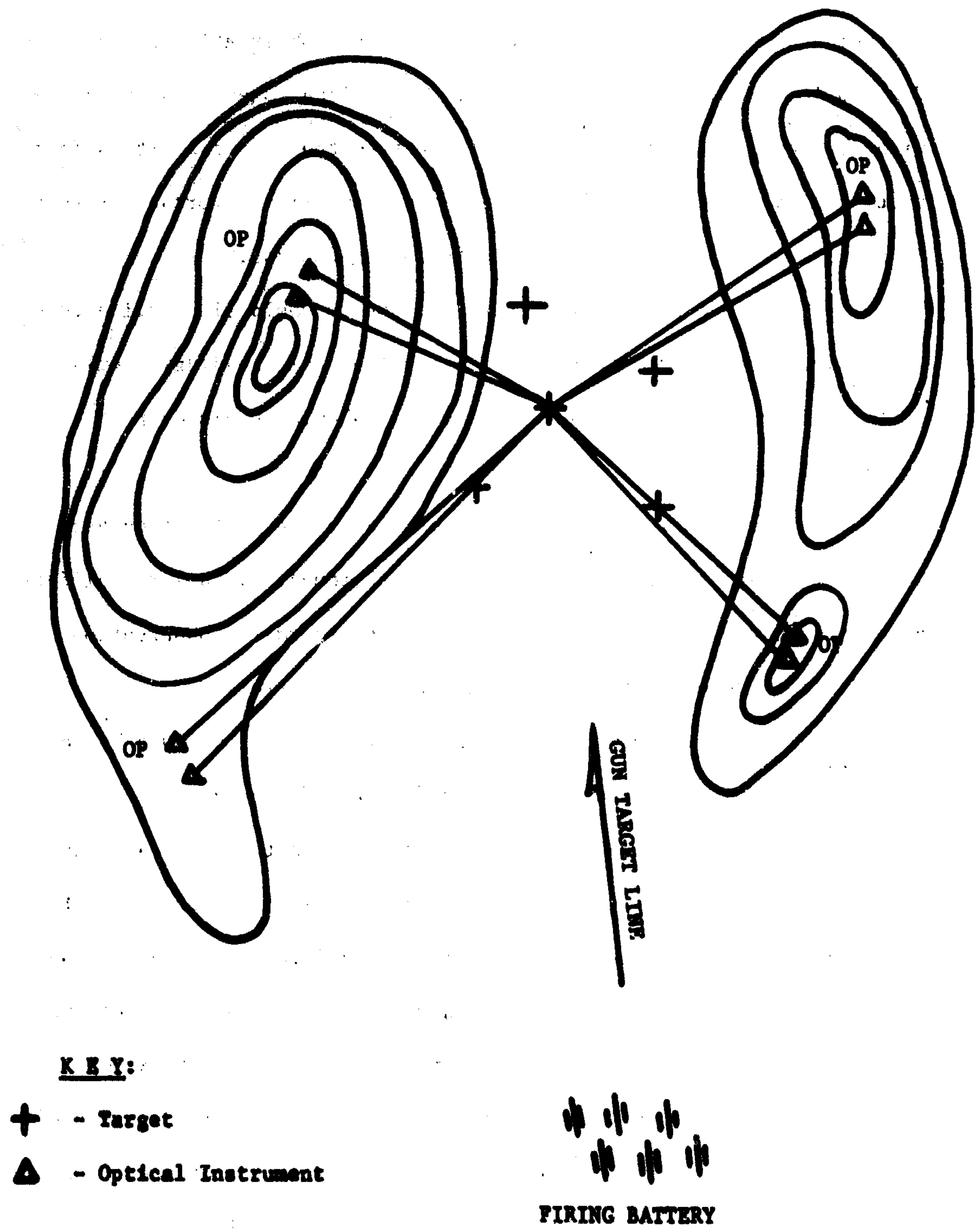
"dual" flash-base, Figure 3, is proposed by the authors. Two optical instruments would be operated from each of four observation posts (OP's). The two instruments should be positioned as close together as possible, without blocking the other's line of sight to the targets. It is felt that such an arrangement might prove more accurate and workable than a flash-base with eight OP's, for the following reasons.

1) The close proximity of the instruments would allow voice communications between the operators. Since the scale readings should be quite similar, errors in reading the scales should be discovered immediately.

2) Large pointing errors should be discovered by the recorder, who would sight over the instrument for each round. If a large difference between the two angles is reported, the recorder's observation should aid in determining which instrument is in error.

3) Finally, the number of rounds "lost," or not observed by an OP, should be minimized because of the "built-in" redundancy.

All instruments should be oriented on a distant reference point before and after each fire mission in order to minimize errors due to faulty instrument alignment. The deflection and vertical angles should be measured for all air bursts, but only the deflection angle need be measured for ground bursts. If both instrument operators at an OP observe a burst, the deflection (and vertical) angles will be averaged to give the "OP deflection (and OP vertical) angle(s)" for that round. In the event only one instrument operator at a particular OP observes a burst, the "OP angle(s)" will be taken as the angle(s) measured by that instrument.



TYPICAL TARGET AREA LAYOUT

FIGURE 3

Although there could be many variations to the problem of "lost" rounds, we shall consider only the situation in which one OP (i.e., both instruments) fails to observe a given round. In this case, that OP would not be used to estimate the location of the "lost" round; instead, the "OP angle(s)" for that OP would be constructed to the location (of the "lost" round) determined by the remaining three OP's. This location would be estimated using the same general procedures (described below) as for locating the center of impact. Finally, should two or more OP's fail to observe a burst, other analyses would be necessary.

The center of impact would be estimated by an intersection procedure. The "OP angles" to each round of a volley should be averaged to give a "CI deflection angle" from each OP. A ray would be plotted from each OP in the direction of the "CI deflection angle," using a point mid-way between the two instruments as its origin. The intersection of these rays forms a polygon. For "tight" polygons the geometric center would be taken as the estimate of the location of the CI, in the horizontal plane. Other situations (i.e., "loose" polygons) would be analyzed on an individual basis (see Chapter V). An average height of burst would be determined, in the case of an air burst, by averaging the heights of burst computed from each OP using the "CI vertical angle," and the estimated distance to the CI.

The flash-base personnel should know (in advance) which target is being fired upon, when each gun is fired, and the approximate time of flight. Coordinates of the flash-base OP's and target locations should be established by survey.

The Experimental Design

In order to determine OHP's, we have suggested that accuracy data be taken under various combinations of values of the independent variables (see Figures 1 and 2). On each trial the measurements taken will determine the radial miss distance, R, between the center of impact and the target center. The observations on radial miss distance may be used in estimating

$$OHP = Pr(R^2 \le r^2) = F_{R^2}(r^2).$$

A method of estimating this cumulative distribution function is discussed in Chapter IV.

Radial miss distance is measured in the standard 3-coordinate system where the X-axis is taken along the gun-target line and lies in a horizontal plane tangent to the earth at the target center. In the case of time fuze missions, this tangent plane passes through an aiming point 20 meters above the target center. The Y-axis is taken at right angles to the X-axis at the target center, and lies in the same horizontal plane. The Z-axis is perpendicular to both the X and Y axes at their point of intersection. We introduce the following notation:

$u^{(p)}_{ijk\ell mn}$ is the true mean miss distance for the center of impact (in each cell), taken in the p direction; $p = X, Y,$ or Z. The ranges of the subscripts are described below.

$e^{(p)}_{ijk\ell mno}$ is the random error in the p direction on the observed random variable for the o^{th} volley; $o = 1, 2, 3, \ldots n_j$; where n_j is the number of volleys to be fired in a data cell for the j^{th} range band.

$M_i^{[p]}$ is the effect in the p direction due to the i[th] method used when entering fire for effect; $i = 1, 2, 3$.

$R_j^{[p]}$ is the effect in the p direction due to the j[th] range band; $j = 1, 2, 3, \ldots$ The upper limit is unspecified to allow for expansion, should it be feasible to fire more than three range bands.

$L_k^{[p]}$ is the effect in the p direction due to the k[th] light condition; $k = 1, 2$.

$F_\ell^{[p]}$ is the effect in the p direction due to the ℓ[th] type of fuze used; $\ell = 1, 2$.

$T_m^{[p]}$ is the effect in the p direction due to the m[th] tactical situation; $m = 1, 2$.

$C_n^{[p]}$ is the effect in the p direction due to the nuclear, biological, or chemical contamination; $n = 1, 2$.

The usual notation for linear experimental models lists each of the main effects terms as given above and all appropriate interaction terms. However, we can simplify this notation considerably by employing the notation suggested by Graybill.[1] We propose the following model, which treats each of the three components of radial miss distance separately:

$$X_{ijk\ell mno} = \mu^{[X]}_{ijk\ell mn} + e^{[X]}_{ijk\ell mno} \qquad (1)$$

$$Y_{ijk\ell mno} = \mu^{[Y]}_{ijk\ell mn} + e^{[Y]}_{ijk\ell mno} \qquad (2)$$

$$Z_{ijk\ell mno} = \mu^{[Z]}_{ijk\ell mn} + e^{[Z]}_{ijk\ell mno} \qquad (3) \; ,$$

[1] F. A. Graybill, An Introduction to Linear Statistical Models, Volume I (New York: McGraw-Hill Book Company, Inc., 1961), p. 272.

where $e^{[p]}_{ijk\ell mno}$ is assumed to be a set of uncorrelated random variables, each of which is distributed $N(0,\sigma_p^2)$. In equation (1), $X_{ijk\ell mno}$ denotes the X-coordinate of the o^{th} volley in the cell when the factors M, R, L, F, T, and C are at the i^{th}, j^{th}, k^{th}, ℓ^{th}, m^{th}, and n^{th} levels, respectively, and similarly for the Y and Z coordinates.

The N-way classification model should include interaction terms, and tests of their significance in the experiment should be made. Also, if it were found that there is not a significant difference among the main effects of certain factors in the experiment, then the data in these cells could be pooled when estimating operational hit probabilities. Such estimates should be more accurate because of the increased sample size.

CHAPTER IV

ESTIMATING OPERATIONAL HIT PROBABILITIES

Recall that we have defined OHP as the probability that the center of impact of a volley of artillery fire will fall within a specified distance of an aiming point. In symbols,

$$OHP = Pr[R^2 \leq r^2] = F_{R^2}(r^2) \tag{1}$$

where R is radial miss distance (i.e., the distance from the center of impact of a volley to the target), and $F_{R^2}(r^2)$ is the cumulative distribution function (CDF) of R^2. Under a specific set of experimental conditions, our ability to accurately estimate OHP (for a given r) will depend upon our accuracy in estimating the CDF of R^2.

We propose two procedures that might be used in estimating this CDF. First, an attempt should be made to determine whether one of the "well known" theoretical probability distributions "fits" either the distribution of R^2, or the distribution of some function of R^2. In the event such a distribution cannot be found, the sample CDF (ogive) could be used.

We begin by discussing a procedure for attempting to find a "well known" distribution that "fits" a function of R^2. It has been assumed that the components (X, Y, and Z) of radial miss distance are distributed $N(0,\sigma_p^2)$: p = X, Y, Z. The target center, or aiming point, will be considered to be the origin of the 3-dimensional coordinate system described earlier. We estimate the X-coordinate population variance using the usual estimator $\hat{\sigma}_X^2$, defined as follows:

$$\hat{\sigma}_X^2 = S_X^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2,$$

where $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$,

n is the number of volleys fired for a particular set of experimental conditions, and

X_i is the distance in the X direction between the target center and the CI for the i^{th} volley.

Similar computations would be made to determine $\hat{\sigma}_Y^2$ and $\hat{\sigma}_Z^2$.

In Appendix B we state reasons why we feel that cR^2/d will be an approximately Chi-square distributed random variable with c^2/d degrees of freedom, where $c = \sigma_X^2 + \sigma_Y^2 + \sigma_Z^2$ and $d = \sigma_X^4 + \sigma_Y^4 + \sigma_Z^4$. Note that equation (1) can be rewritten as

$$\mathrm{OHP} = \Pr[cR^2/d \le cr^2/d] = F_{cR^2/d}(cr^2/d) = F_{R^2}(r^2). \tag{2}$$

That is, estimating the CDF of cR^2/d is equivalent to estimating the CDF of R^2. Replacing the parameters c and d by their estimates (i.e., $\hat{c} = \hat{\sigma}_X^2 + \hat{\sigma}_Y^2 + \hat{\sigma}_Z^2$ and $\hat{d} = \hat{\sigma}_X^4 + \hat{\sigma}_Y^4 + \hat{\sigma}_Z^4$), we obtain an estimate of the mean of the approximating Chi-square distribution of cR^2/d. A goodness-of-fit test (such as Chi-square or Kolmogorov-Smirnov) could be used to test whether the experimental data "fits" this approximating Chi-square distribution. If the test shows an acceptable fit, this distribution could be used to estimate OHP for any value of r. If the Chi-square distribution does not provide an acceptable fit, a search of other distributions should be made.

If a theoretical CDF that fits the observed data cannot be found, OHP could be estimated using the sample CDF, or ogive. The estimate thus obtained would be $\widehat{OHP} = \hat{F}_{R^2}(r^2) = \dfrac{\text{number of volleys with } R^2 \leq r^2}{\text{total number of volleys fired}}$; for $r^2 \in (0,\infty)$.

CHAPTER V

AREAS FOR FURTHER INVESTIGATION

While writing this thesis, we have encountered certain problems toward which (we feel) further research effort might profitably be directed. Lack of time has prevented us from working on these problems in detail. This paper is concluded with a brief description of these problems. It is hoped that additional work will be directed toward solving some of them.

One problem is the development of an optimal flash-base for use in accurately locating shell bursts. The authors have presented an *ad hoc* flash-base configuration in this paper, but they feel that further investigation might lead to a method of designing better ones. Several important questions should be addressed: How many observation posts should be used in a flash-base? How many optical instruments should be located at each observation post? What are the trade-offs between accuracy and the number of observation posts (and the number of optical instruments at each)?

Another problem involves the trade-offs between high angle and low angle fire. The former is more lethal against certain types of targets, but the latter is more accurate. What are the OHP's of various weapons systems using high angle fire? Do observers require more, or fewer, rounds to adjust on a target when high angle fire is used? Under what conditions is high angle fire preferred to low angle fire?

Experimentation of the type discussed in this paper is generally quite expensive. Thus, it is desirable to develop additional methods of obtaining meaningful measures of a system's effectiveness. In Appendix C the authors propose the use of ATT data to estimate sample sizes and the number of missions that should be fired in various range bands. These considerations suggest several questions: Is there a high correlation between the estimates of OHP determined experimentally and estimates determined from ATT data? What modifications (either in scenario or in data collection procedures) in the ATT would be necessary in order to obtain "good" estimates of OHP from ATT data? Is it possible to collect data (for estimating OHP and other measures of effectiveness) under actual combat conditions? If so, what procedures and instrumentation should be used?

The "Adjust Fire" method of entering fire for effect presents a fruitful area for further research. This procedure has been used to adjust fire on targets for a number of years, but apparently no study has been made to determine if a better procedure might be developed. Several questions need to be answered: Is the observer's procedure of "splitting brackets" best (in some sense), or should some other procedure be used? How many weapons should be fired during the adjustment phase? What procedure will yield the greatest expected level of damage to targets of various types for a given cost?

The procedure currently followed for precision (destruction and registration) fire should be closely examined. In a precision mission the observer follows the "Adjust Fire" procedure until he enters FFE. Once the FFE phase is entered, the observer only senses the rounds (e.g., "over-left," "short-right," "short-line," etc.), and the FDC uses a standard procedure (described in detail in FM 6-40) to further adjust the fire onto the target. Is this "standard procedure" best? If not, what procedure should be used?

Finally, the authors feel that hand computational procedures in the FDC should be compared with the Field Artillery Data Computer (FADAC). What are the advantages/disadvantages of the FADAC versus hand computation? Which gives the greater chance for error? Is the inability to reconstruct past missions, step by step, a serious drawback for the FADAC?

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APPENDIX A

GLOSSARY

Area Target - Target, for gunfire or bombing, which covers a large area. Area targets are usually composed of many point targets that are distributed within some geographical area.

Center of Impact - The geometrical center of the dispersion pattern of a group of rounds. In the case of a single volley, it is the center of this one volley.

Deflection - 1) Setting on the scales of the sight of a weapon to place the line of fire in the desired direction. 2) Horizontal clockwise angle between the axis of the tube and line of sight.

Direct Fire - Fire delivered at close range, by an artillery weapon or a tank, on a target which is visible to the gunner.

Fire for Effect - Consists of a number of rounds fired singly or in groups or volleys in sufficient volume to attain the desired effect on a target.

Indirect Fire - Fire delivered at a target that cannot be seen from the gun position.

Inherent Errors - If a number of rounds from the same lot of ammunition are fired from a single weapon with identical settings in quadrant elevation and deflection, all the rounds will not fall at a single point, but will be scattered in a pattern of bursts. This dispersion is due to inherent errors, which are a result of conditions in the bore of the gun, conditions in the carriage of the gun, and environmental conditions during the flight of the projectile.

Mixed-Lot-Mission - A fire mission in which the ammunition comes from more than one manufacturer's lot. (Firing corrections may vary for different lots.)

Non-Standard Firing Conditions - Certain atmospheric, position, and materiel conditions are accepted as standard. These conditions are described in the introduction to firing tables. Any other firing conditions are considered to be non-standard.

Operational Probable Errors - Probable errors* measured under operational conditions.

Orienting Line - A line of known direction established on the ground and used as a reference line in survey or in aiming artillery pieces.

Point Target - A particular object or structure such as a man, a bridge, or a bunker.

Position Area - The area in which the command and firing elements of a battery are located.

Probable Errors - Measure of the distribution of impacts about the mean point of impact for a single weapon; it is also defined as that error which is exceeded approximately as often as it is not exceeded.

Quadrant Elevation - The smaller angle at the origin, measured in a vertical plane, from the base of the trajectory to the line of elevation. (The base of the trajectory is the straight line from the origin to a point on the descending branch of the trajectory which is at the same altitude as the origin.) Roughly speaking, this is the vertical angle measured between the axis of the gun tube and the horizontal plane.

Salvo Fire - A method of fire in which weapons are discharged one after the other, usually at intervals of two seconds.

Sensing - The location of a point of burst or impact, or mean point of burst or impact, with respect to the target, such as over, short, air, left, or right.

Sheaf Width - The lateral distance (perpendicular to the direction of fire) between the mean points of burst of the flank rounds.

System Errors - The bias and dispersion, about the target center or aiming point, of fire delivered by weapons systems. They are attributable to both inherent errors and errors caused by the operational environment.

Volley Fire - A method of fire in which each section fires a specified number of rounds without attempting to synchronize its fires with the other sections.

Weapons System - One or more weapons with all component parts, personnel, and procedures required for its operation. The operation of the system is initiated when a target is detected and terminated upon completion of firing.

APPENDIX B

THE APPROXIMATING CHI-SQUARE DISTRIBUTION

We now consider the problem of finding the distribution (or approximate distribution) of a sum of squares of independent, but <u>not identically distributed</u>, normal random variables. That is, we wish to find the distribution of $R^2 = X^2 + Y^2 + Z^2$, where the variables are as described in Chapter III. Assume that X is distributed $N(0,\sigma_X^2)$, Y is distributed $N(0,\sigma_Y^2)$, Z is distributed $N(0,\sigma_Z^2)$, and suppose the random variables are independent. The approximate distribution of R^2 is given by the following theorem:[1]

Let $n_i X_i/\sigma_i^2$ $(i = 1,2,\ldots,k)$ be independently distributed as $\chi^2(n_i)$. Let $\gamma = \sum_{i=1}^{k} g_i\sigma_i^2$ $(\gamma > 0)$, and $g = \sum_{i=1}^{k} g_i X_i$. Then $u = ng/\gamma$ is approximately distributed as $\chi^2(n)$, where

$$n = \frac{(\Sigma g_i\sigma_i^2)^2}{\Sigma(g_i^2\sigma_i^4/n_i)} .$$

As a consequence of our assumptions of independent normality, X^2/σ_X^2, Y^2/σ_Y^2, and Z^2/σ_Z^2 are independently distributed as $\chi^2(1)$. By letting $g_i = 1$, we get $\gamma = \sigma_X^2 + \sigma_Y^2 + \sigma_Z^2$ and $g = X^2 + Y^2 + Z^2 = R^2$. The theorem above then asserts that $\left(\dfrac{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2}{\sigma_X^4 + \sigma_Y^4 + \sigma_Z^4}\right)R^2$ is approximately distributed as a Chi-square random variable with $\dfrac{(\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2)^2}{(\sigma_X^4 + \sigma_Y^4 + \sigma_Z^4)}$ degrees of freedom.

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[1] Franklin A. Graybill, <u>An Introduction to Linear Statistical Models</u>, Volume I (New York: McGraw-Hill Book Company, Inc., 1961), p. 369.

APPENDIX C

ESTIMATING SAMPLE SIZES

The sample size for an experiment might be based upon the following criteria:

1) the assumed distributions of the random variables to be observed during the experiment,

2) certain goals (possibly arbitrary) in estimating the chosen parameters,

3) the cost of sampling, and

4) the availability of the members of the population to be sampled.

If we initially assume that the availability and cost constraints are not active, then "required sample size" would mean the predicted sample size necessary to achieve the stated goals, given that the random variables will be sampled from the specified distributions. Even in this "ideal case," the problem of determining required sample size for an experiment is a difficult one. There is seldom agreement as to which goal should be selected, and it is difficult to predict, *a priori*, the distributions of the random variables to be observed during the experiment. After some consideration, the experimentor must make a decision as to which goals he considers most important. Once this decision is made, it may be possible to estimate the required sample size, provided inferences can be drawn about the distributions of the random variables involved. The latter may be done in various ways. For example, sometimes personnel who have had experience with similar experiments can furnish subjective inferences concerning the likely distributions of the experimental random variables. On other occasions available data, similar to the data which will be collected during the experiment, may be used in drawing inferences about the distributions of these random variables. In still other cases, theoretical developments may indicate appropriate distribution assumptions, or literature may be available which sheds light upon the distributions likely to be encountered. After the required sample size has been estimated, the experimentor should, in most cases, check whether the cost or availability (of sample units) constraints are violated. If either constraint is violated, the required sample size might have to be reduced accordingly.

During annual Army Training Tests, accuracy data is recorded for all artillery units. The authors feel that this data might be similar to the accuracy data that would be obtained (later) in the experiment, so ATT data might be useful in estimating the required sample size. The phrase "required sample size" is used here in two senses: the number of volleys that should be fired in each data cell and the number of units that should be used in firing those volleys. Our suggested procedure for obtaining these estimates will be essentially the same as previously discussed; that is, first estimate each required sample size by considering some stated goal and the inferred distribution (based on ATT accuracy data and theoretical considerations) of the experimental random variables, then check to see if a cost constraint or an availability constraint is active.

As was stated in Chapter IV, our ability to accurately estimate OHP (for a given r) will depend upon our accuracy in estimating the CDF of R^2. For the reasons given in Appendix B, we feel that cR^2/d will be an approximately Chi-square distributed random variable with c^2/d degrees of freedom, where $c = \sigma_X^2 + \sigma_Y^2 + \sigma_Z^2$ and $d = \sigma_X^4 + \sigma_Y^4 + \sigma_Z^4$. Hence, the quality of our OHP estimates could very well be assumed to depend upon our ability to accurately estimate c^2/d, the mean of the approximating Chi-square distribution. However, c^2/d is solely a function of the variances of X, Y, and Z, the miss distances in the coordinate directions. Therefore, our ability to accurately estimate c^2/d will depend upon our accuracy in estimating these three variances from experimental data. The difficulty lies in the fact that experimental data will not be available until after the experiment is conducted. Hence, the authors propose that estimates of required sample sizes should be based upon accuracy data that is currently available; namely, ATT data.

During ATT's, flash-base techniques are used to estimate the points of impact of individual rounds in FFE, and a center of impact is computed. Since height-of-burst data is not recorded, it is impossible to estimate the variance (σ_Z^2) of miss distance in the Z-coordinate direction. However, under the normality assumptions of Appendix B, the maximum likelihood estimate of σ_X^2 would be

$$\hat{\sigma}_X^2 = S_X^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 ,$$

and similarly for σ_Y^2. For low-angle fire, the variance in range will probably exceed the variance in deviation; that is, in past experience,

S_X^2 has almost always been greater than S_Y^2. It is reasonable to assume, therefore, that σ_X^2 will "drive" the problem. In other words, if we choose a sample size that will achieve some stated degree of accuracy in estimating σ_X^2, we should be assured of achieving a higher degree of accuracy in estimating σ_Y^2 and σ_Z^2 in the experiment. Therefore, we have considered only the problem of estimating σ_X^2 to the required precision in order to obtain an estimate of the required number of volleys for each data cell. Any of a number of other schemes could be used as well.

For their stated goal, the authors have decided to require the expected length of a $1 - \alpha$ per cent confidence interval on σ_X^2 to be equal to the burst radius (B_R) of a projectile of the appropriate type. If X (miss distance parallel to the gun-target line) is normally distributed, nS_X^2/σ_X^2 is distributed as a Chi-square random variable with $n - 1$ degrees of freedom. A $1 - \alpha$ per cent confidence interval on σ_X^2 can be obtained as follows:

$$1 - \alpha = \Pr[\chi^2_{1-\alpha/2} < nS_X^2/\sigma_X^2 < \chi^2_{\alpha/2}]$$

$$= \Pr[nS_X^2/\chi^2_{\alpha/2} < \sigma_X^2 < nS_X^2/\chi^2_{1-\alpha/2}] ,$$

where n is the total number of observations on the random variable X, and $\chi^2_{\alpha/2}$ is the $100(1-\alpha/2)$ percentile of the Chi-square distribution with $n - 1$ degrees of freedom. Equating the expected length of this confidence interval to the burst radius, one obtains

$$E[L] = E[nS_X^2/\chi^2_{1-\alpha/2} - nS_X^2/\chi^2_{\alpha/2}] = B_R . \qquad (1)$$

Solving for B_R, we obtain

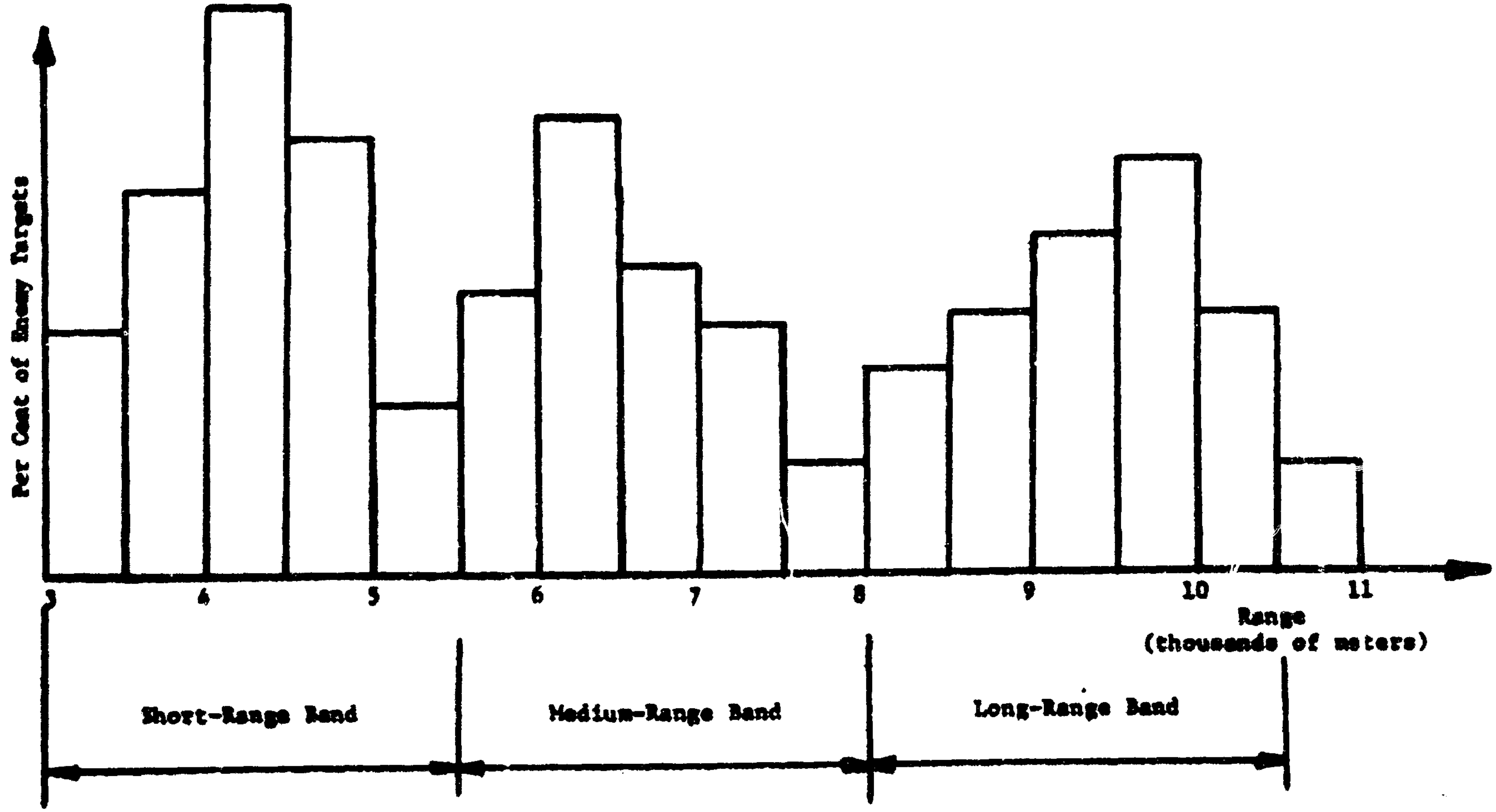
$$B_R = (n-1)\left(\frac{1}{\chi^2_{1-\alpha/2}} - \frac{1}{\chi^2_{\alpha/2}}\right)\sigma^2_X . \qquad (2)$$

The burst radius of the particular projectile being used in the experiment would be known, and σ^2_X can be estimated by S^2_X from ATT accuracy data. Thus (2) can be solved for n, the required number of volleys for each data cell. Note, however, that n also specifies the particular Chi-square distribution from which the $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$ points are determined, and a solution for n must take this into account. An iterative procedure may be used: first choose n, and then check to see if (2) is satisfied. If not, choose a different n and recheck (2). If so, determine whether a smaller n will satisfy (2). Continue in this fashion until the smallest integer n that most nearly satisfies (2) is found. This iteration process should not be very time-consuming, since very few iterations (usually less than 10) should be required to find the appropriate n.

Available ATT data, pertaining to numerous artillery units (of the given type), firing at many different ranges, under various firing conditions, would be used to estimate n by the above procedure. The estimate n (of required sample size) thus obtained would represent the number of volleys to be fired (in <u>each</u> data cell) by "average" units in "average" situations. Therefore, $48n$ would seem to be a reasonable estimate of N, the <u>total</u> number of volleys to be fired in the experiment (using the reduced firing matrix of 48 data cells).

While 48n represents the total required sample size, it may be possible to allocate these observations in a way "more efficient" than simply n to a cell. We shall illustrate this with a consideration of the allocation over range bands. In Chapter II, we suggested that the independent variable "Gun-Target Range" be divided into three representative range bands. The firing of n volleys in each data cell would not take into account available information concerning these range bands. Specifically, the authors feel that intelligence information and previous accuracy data should be used as a basis for estimating the number of missions to be fired in each range band. The intelligence information might, for example, take the form of a predicted distribution of enemy targets, as shown in Figure 4. The accuracy data could be provided from the ATT data (already used in estimating σ_X^2), segregated by range band. The following discussion shows one method of using this information to obtain reasonable allocations of the number of missions to be fired in each range band. For the sake of simplicity, we consider only the short-range band (R.B.#1) and the medium-range band (R.B.#2). However, the development is general and could be extended to any number of range bands.

As stated previously, the accuracy of an estimate of OHP will depend, among other factors, upon our ability to determine experimentally the CDF's of R^2. To obtain "good" estimates of these CDF's, it is necessary to accurately measure the observed R^2 for each volley fired. A reasonable approach toward achieving the desired degree of accuracy might be to obtain "best" (in some sense) estimates of μ_{R^2}, the mean of the



PREDICTED ENEMY TARGET DISTRIBUTION
(Hypothetical)

FIGURE 4

squared radial miss distances, for each range band. The sense in which we use the word "best" is discussed below.

We shall use the following notation:

X_i is miss distance parallel to the gun-target line in range band i, $i = 1, 2$;

Y_i is miss distance perpendicular to the gun-target line (in the horizontal plane) in range band i ;

Z_i is vertical miss distance in range band i.

Additionally, we make the following assumptions.

1) X_i is distributed $N(0,\sigma^2_{X_i})$, $i = 1, 2$;
   Y_i is distributed $N(0,\sigma^2_{Y_i})$;
   Z_i is distributed $N(0,\sigma^2_{Z_i})$.

2) The predicted distribution of enemy targets indicates that, in a total of $V + W$ missions fired, it is expected that V missions will be fired in range band #1 and W missions in range band #2.

3) No more than N missions may be fired during the experiment.

Note that the number n may be equal to 48n (determined previously), or it may be provided by a budgetary or other constraint, whichever is smaller. Thus our problem is one of allocating N observations, between R.B.#1 and R.B.#2, in order to obtain "best" estimates of $\mu_{R_1^2}$ and $\mu_{R_2^2}$, where $R_i^2 = X_i^2 + Y_i^2 + Z_i^2$, $i = 1, 2$.

The word "best" may be interpreted in terms of minimal quadratic loss. Toward this end, we have assumed the following loss functions for any estimators $\hat{\mu}_{R_1^2}$ and $\hat{\mu}_{R_2^2}$ of $\mu_{R_1^2}$ and $\mu_{R_2^2}$, respectively:

$$L_1(\hat{\mu}_{R_1^2}) = (\hat{\mu}_{R_1^2} - \mu_{R_1^2})^2 \qquad \text{given R.B.\#1}$$

$$L_2(\hat{\mu}_{R_2^2}) = (\hat{\mu}_{R_2^2} - \mu_{R_2^2})^2 \qquad \text{given R.B.\#2} .$$

It is well-known that $L_1(\hat{\mu}_{R_1^2})$ is minimized by taking $\hat{\mu}_{R_1^2} = \overline{R_1^2}$. The total expected loss, $E[L]$, is given by

$$E[L] = pL_1(\overline{R_1^2}) + qL_2(\overline{R_2^2}) ,$$

where $p = \frac{V}{V+W}$ is the probability of firing a mission in R.B.#1, and $q = (1-p) = \frac{W}{V+W}$ is the probability of firing a mission in R.B.#2. Note that p and q may be interpreted as prior probabilities on the distributions of R_1^2 and R_2^2, respectively.

Let n_1 (unknown) be the number of missions that should be fired in R.B.#1 and n_2 (unknown) be the number of missions for R.B.#2. Then our problem reduces to the following:

$$\text{MINIMIZE:} \quad p(\overline{R_1^2} - \mu_{R_1^2})^2 + q(\overline{R_2^2} - \mu_{R_2^2})^2$$
$$\text{SUBJECT TO:} \quad n_1 + n_2 = N . \qquad (3)$$

By making the appropriate substitutions, system (3) can be rewritten as:

$$\text{MINIMIZE:} \quad \frac{V}{V+W} \ \mathrm{Var}(\overline{R_1^2}) + \frac{W}{V+W} \ \mathrm{Var}(\overline{R_2^2})$$
$$\text{SUBJECT TO:} \quad n_1 + n_2 = N , \qquad (4)$$

where $\mathrm{Var}[\overline{R_1^2}] = 2(\sigma_{X_1}^4 + \sigma_{Y_1}^4 + \sigma_{Z_1}^4)/n_1$

and $\mathrm{Var}[\overline{R_2^2}] = 2(\sigma_{X_2}^4 + \sigma_{Y_2}^4 + \sigma_{Z_2}^4)/n_2$.

(These variances will be derived below.)

The positive constant term $\frac{2}{V+W}$ will not affect the minimization, so it may be deleted. Finally, we have

$$\text{MINIMIZE:} \quad \frac{V}{n_1}\left[\sigma^4_{X_1} + \sigma^4_{Y_1} + \sigma^4_{Z_1}\right] + \frac{W}{n_2}\left[\sigma^4_{X_2} + \sigma^4_{Y_2} + \sigma^4_{Z_2}\right] \tag{5}$$

$$\text{SUBJECT TO:} \quad n_1 + n_2 = N \ .$$

The Lagrangian for the system (5) may be written as:

$$\mathcal{L} = \frac{V}{n_1}\left[\sigma^4_{X_1} + \sigma^4_{Y_1} + \sigma^4_{Z_1}\right] + \frac{W}{n_2}\left[\sigma^4_{X_2} + \sigma^4_{Y_2} + \sigma^4_{Z_2}\right] - \lambda\left[n_1 + n_2 - N\right] \ .$$

Taking partial derivatives and setting them equal to zero, we get:

$$\frac{\partial \mathcal{L}}{\partial n_1} = \frac{-V\left[\sigma^4_{X_1} + \sigma^4_{Y_1} + \sigma^4_{Z_1}\right]}{n_1^2} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial n_2} = \frac{-W\left[\sigma^4_{X_2} + \sigma^4_{Y_2} + \sigma^4_{Z_2}\right]}{n_2^2} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = n_1 + n_2 - N = 0 \ .$$

Solving the first two partial derivative expressions for λ, one obtains:

$$\lambda = \frac{-V}{n_1^2}\left[\sigma^4_{X_1} + \sigma^4_{Y_1} + \sigma^4_{Z_1}\right] = \frac{-W}{n_2^2}\left[\sigma^4_{X_2} + \sigma^4_{Y_2} + \sigma^4_{Z_2}\right] \ . \tag{6}$$

The above equation can be solved to get the ratio of missions to be fired:

$$\frac{n_1}{n_2} = \sqrt{\frac{V(\sigma^4_{X_1} + \sigma^4_{Y_1} + \sigma^4_{Z_1}}{W(\sigma^4_{X_2} + \sigma^4_{Y_2} + \sigma^4_{Z_2}}} \ . \tag{7}$$

As mentioned previously, ATT data does not include Z-coordinate information, so $\sigma^2_{Z_1}$ and $\sigma^2_{Z_2}$ cannot be estimated and are therefore deleted. Replacing the remaining squared variances in (7) by their maximum likelihood estimates, we get:

$$\frac{n_1}{n_2} \doteq \sqrt{\frac{V[(S^2_{X_1})^2 + (S^2_{Y_1})^2]}{W[(S^2_{X_2})^2 + (S^2_{Y_2})^2]}} , \qquad (8)$$

where $S^2_{X_1} = \frac{1}{m} \sum_{i=1}^{m} (X_{1i} - \bar{X}_1)^2$,

$\bar{X}_1 = \frac{1}{m} \sum_{i=1}^{m} X_{1i}$,

m is the number of missions fired in R.B.#1 (for which ATT data is available), and

X_{1i} is the distance in the X direction (in R.B.#1) between the target center and the CI, for the i^{th} volley.

Similar computations can be made to get $S^2_{X_2}$, $S^2_{Y_1}$, and $S^2_{Y_2}$. Finally, under the assumption that n_1 and n_2 are continuous, (8) can be solved explicitly for n_1 (or n_2) to obtain:

$$n_1 \doteq \frac{N}{1 + \sqrt{\frac{W[(S^2_{X_2})^2 + (S^2_{Y_2})^2]}{V[(S^2_{X_1})^2 + (S^2_{Y_1})^2]}}}$$

and $n_2 = N - n_1$. Once this solution is obtained, appropriate integer values of n_1 and n_2 may be selected.

The variances that were substituted into the system (4) above are derived as follows. Consider only the short-range band,

where X_1 is distributed $N(0,\sigma^2_{X_1})$,

Y_1 is distributed $N(0,\sigma^2_{Y_1})$, and

Z_1 is distributed $N(0,\sigma^2_{Z_1})$.

Then $\frac{X_1}{\sigma_{X_1}}$ is distributed $N(0,1)$, so that $\frac{X_1^2}{\sigma_{X_1}^2}$ is distributed $\chi^2(1)$, and similarly for Y_1 and Z_1.

The Chi-square random variable with k degrees of freedom has a variance of $2k$, hence $Var[S_1^2/\sigma_{X_1}^2] = 2$ and $Var[X_1^2] = 2\sigma_{X_1}^4$. Similarly we get $Var[Y_1^2] = 2\sigma_{Y_1}^4$ and $Var[Z_1^2] = 2\sigma_{Z_1}^4$.

We can find the variance of $R_1^2 = X_1^2 + Y_1^2 + Z_1^2$ as follows:

$$Var[R_1^2] = Var[X_1^2 + Y_1^2 + Z_1^2]$$

$$= Var[X_1^2] + Var[Y_1^2] + Var[Z_1^2]$$

(by the assumed independence of X_1, Y_1, and Z_1)

$$= 2(\sigma_{X_1}^4 + \sigma_{Y_1}^4 + \sigma_{Z_1}^4).$$

Hence,

$$Var[\overline{R_1^2}] = Var[\frac{1}{n_1}\sum_{i=1}^{n_1} R_{1i}^2] = \frac{1}{n_1^2} Var[\sum_{i=1}^{n_1} R_{1i}^2]$$

$$= \frac{1}{n_1^2}[2n_1(\sigma_{X_1}^4 + \sigma_{Y_1}^4 + \sigma_{Z_1}^4)] = \frac{2}{n_1}(\sigma_{X_1}^4 + \sigma_{Y_1}^4 + \sigma_{Z_1}^4).$$

A similar computation can be carried out to obtain the variance of $\overline{R_2^2}$.

It should be noted that the above technique represents only one of many possible ways to allocate N missions between range bands, or between levels of other factors. Having obtained n_1 and n_2, the experimentor could require that $n_1/24$ volleys be fired for each data cell of the short-range band, and $n_2/24$ volleys for each data cell of the long-range band. Note that we are assuming the reduced firing matrix with 48 data cells is used.

We now consider the problem of estimating the number of test units that should be used for the experiment. The following discussion shows how ATT data might also be used to obtain this estimate.

The number of test units required for the experiment should depend (primarily) upon the variability among artillery units. The authors feel that mean radial miss distance provides a suitable measure of a unit's accuracy, so the distribution of mean radial miss distance for a randomly selected unit might be used to estimate the required sample size (number of test units) for this experiment.

Assume that ATT data is available from G units, each of which fired a total of H missions during the ATT. Let X_{ij} denote the radial miss distance of the FFE volley of the i^{th} mission fired by the j^{th} unit $(i = 1, \ldots, H;\ j = 1, \ldots, G)$. ATT data could be used to compute $\hat{\mu}_j = \frac{1}{H} \sum_{i=1}^{H} X_{ij}$, the estimated mean radial miss distance for unit j. This computation should be carried out for each of the units and the results used to obtain an empirical distribution (histogram) of the random variable μ_j, the mean radial miss distance of a randomly selected unit. The Central Limit Theorem leads us to conjecture that μ_j is (approximately) a normally distributed random variable. A goodness-of-fit test could be applied to test the assumption of normality. If the test shows that a normal distribution provides an acceptable fit, that particular normal distribution could be used to estimate the required number of test units.

To obtain this estimate, a procedure similar to that used for estimating n could be used. For example, the expected length of a $1 - \alpha$

per cent confidence interval on μ (the mean of μ_J) could be set equal to some selected value. For μ_J distributed normally with unknown variance, $\sqrt{m-1}\,(\bar{\mu}_J - \mu)/S_{\mu_J}$ is distributed as a t random variable with $m - 1$ degrees of freedom, where m is the number of test units required for the experiment. A $1 - \alpha$ per cent confidence interval on μ can be obtained as follows:

$$1 - \alpha = \Pr[-t_{\alpha/2} < \sqrt{m-1}\,(\bar{\mu}_J - \mu)/S_{\mu_J} < t_{\alpha/2}]$$

$$= \Pr[\bar{\mu}_J - t_{\alpha/2}\, S_{\mu_J}/\sqrt{m-1} < \mu < \bar{\mu}_J + t_{\alpha/2}\, S_{\mu_J}/\sqrt{m-1}\,] ,$$

where $t_{\alpha/2}$ is the $100(1-\alpha/2)$ percentile of the Student's t-distribution with $m - 1$ degrees of freedom. Note that $\bar{\mu}_J = \frac{1}{G}\sum_{j=1}^{G}\hat{\mu}_j$ and $S_{\mu_J} = \sqrt{\frac{1}{G}\sum_{j=1}^{G}(\hat{\mu}_j - \bar{\mu}_J)^2}$ can both be obtained from ATT data. As was done previously, equating the expected length of the above confidence interval to some appropriate constant will provide an estimate of the number of test units required for the experiment.

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Richard W. Boes, Major, United States Army

Richard E. Garvey, Jr., Captain, United States Army

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13. ABSTRACT

The Department of the Army has expressed a need for the determination of the operational hit probabilities of several weapons systems in use throughout the Army. These hit probabilities, together with lethality models, should yield predictions of the effects such systems will have under various conditions of combat.

In this thesis, operational hit probability (OHP) is defined as the probability that the center of impact of a volley of artillery fire will fall within a specified distance of the center of an area target. A general experimental methodology, which could be used to estimate OHP's (under simulated combat conditions) for a field artillery weapons system, is presented. More specifically, an approximate Chi-square distribution of squared radial miss distance is suggested for estimating OHP's. A method of using accuracy data from Army Training Tests to estimate required sample sizes for the experiment is proposed.

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