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A LITERATURE REVIEW AND DISCUSSION OF LIQUID PARTICLE BREAKUP IN GAS STREAMS

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ABSTRACT. The literature relating to droplet breakup in gas streams moving with relative velocity to the particles has been surveyed and reviewed. This report contains a brief discussion of breakup mechanisms followed by an annotated bibliography of the pertinent references. Included also is a general list of references giving background to the subject.

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FOREWORD

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INTRODUCTION

The atomization of a liquid by an air stream has been an item of much concern in the past, and there have been numerous experimental studies of the process made in an effort to correlate liquid and air-stream physical properties into a general model which will predict the degree of, and the time required for, atomization for a given physical situation. The breakup of liquid drops finds application in various areas: in the field of meteorology, the formation breakup, and shape of raindrops is an item of much concern; in the field of internal combustion engines, the carburization of a fuel is of major importance; in the industrial field, the atomization of paint and insecticides and spray drying processes are objects of concern; in the chemical industries, the emulsification of liquid-liquid systems, the formation of froths, the production of aerosols, and dispersion processes, in general, are most important considerations; and in the science of rheology, wherein the motion and dispersion of liquids, gases, and solids must take into consideration various breakup mechanisms. Despite the importance of such liquid atomization processes, there has been little theoretical work done in an effort to mathematically correlate the important variables and parameters common to all breakup and dispersion processes.

The applications of the subject of liquid droplet breakup have been greatly multiplied with the advent of the importance of the rocket engine and the supersonic aircraft. The study of rocket combustion instability, the sustaining of a detonation wave in a gas stream, the impingement of liquid particles on a supersonic aircraft, and the erosion and ablation of materials used in the construction of rockets and aircraft are but a few of the most important fields of application of droplet breakup in modern technology.

With the importance of the interaction of solid-gas, liquid-gas, and liquid-liquid systems in mind, this study was undertaken to review the existing literature on the general subject and on the droplet shattering mechanisms in particular, and to attempt to discover the correlations and/or the discrepancies in the existent theories of droplet shattering.

GENERAL DISCUSSION OF DROPLET BREAKUP

Because of the irregularity of the shape of a liquid drop undergoing a given breakup process, it is very difficult to describe the surface configuration by means of a mathematical expression. Since many of the parameters upon which the breakup process is dependent are themselves a function of the shape of the droplet, the mathematical complexity of the problem of adequately describing breakup criteria is manifoldly increased.

When a droplet falls through a stagnant medium under the influence only of gravity, the shape of the droplet is significantly influenced by the surface tension forces, the hydrostatic forces within the droplet, the shape-dependent aerodynamic forces, internal and surface circulation of the fluid droplet, the natural and induced internal droplet vibrations, the centrifugal and coriolis effects of the radial outflow and inflow of liquid, the viscosities of the liquid and the medium through which it is falling, electrostatic charge on the surface of the droplet, the effect of boundary layer separation at some point and under some complex conditions, the shedding of vortices from the windward side of the droplet as it falls, and even the diameter of the droplet itself. To attempt to correlate the effects of all of these variables into a single mathematical expression would be a formidable, if not impossible, task.

When the drop can no longer be assumed to merely be falling through a stagnant medium, but is instead subjected to cross flows, shock waves, turbulence effects of the free steam, interaction with other particles undergoing breakup, indeterminate velocity lag between the particle and the free stream, wall effects, and uncertain physical parameters of the gas stream and the droplet itself, phase change or chemical reactions all add to the complexity of the problem.

There exists an entire spectrum of modes of droplet breakup. At the extremes of this spectrum exist the modes of breakup known respectively as "bag" and "shear" breakup. The process of bag breakup of a liquid droplet can be explained in the following manner. As a liquid droplet is subjected to a relative gas flow, the droplet deforms into roughly an ellipsoidal shape, with the major axis of the ellipsoid perpendicular to the direction of flow. The deformation of the droplet can also be described as a general flattening and radial outflow of the droplet in the directions perpendicular to the direction of flow. The resulting deformation at this point has been called at various times by authors disk-shaped, saucer-shaped, and roughly toroidal-shaped. As the deformation continues, the center portion of the droplet begins further deformation in the direction of the relative flow velocity, which process has been variously called "inflation", "opening of the bag", and opening "like a parachute". At this point the droplet appears as a thin film of liquid, anchored to a heavier rim of liquid around the circumference of the droplet and stretched in the flow direction until the bag is several times larger than either the original droplet or the existing circumferential ring of liquid. When some critical condition occurs, the bag breaks up into a shower of fine droplets and the rim disintegrates into several larger droplets.

A description of shear breakup is as follows. As in the case of bag breakup, the droplet deforms, but in the case of shear breakup the deformation has been most generally described as "lenticular", with the major axis of the lens perpendicular to the direction of the relative velocity between the droplet and the air stream. As the radial outflow of liquid proceeds, a liquid film is stripped from the extreme circumferential edge of the droplet. This film is rapidly broken into ligaments or segments, which, in turn, form under the action of surface tension into drops much smaller than the original drop. When the relative velocity is high enough, the stripping action from the parent drop appears as a shower of droplets being torn from the edge of the drop.

The two extreme breakup mechanisms have many times been observed in the photographic record of droplet breakup. The two breakup mechanisms have been called extreme because there also exists ample photographic evidence to substantiate the claims that droplet breakup occurs by a combined bag and shear mechanisms. Thus, it is seen that an adequate mathematical model would have to take into consideration, and also be able to predict, the two different modes of breakup which might occur due to some combination of the physical parameters of the gas stream and the liquid droplet.

Although the spectrum of droplet breakup mechanisms, running from shear breakup at one extreme to bag breakup at the other, has many times been photographically observed, there yet exists in the literature no definite criteria to predict which breakup mode will occur for a given combination of liquid and gas stream physical parameters. It also seems apparent that no all-inclusive parameters have yet been discovered to determine, for a given physical situation, the droplet breakup time, the critical droplet diameter for a given relative velocity, or the critical relative velocity for a given droplet diameter.

BREAKUP CRITERIA REFERENCES

With regard to droplet breakup criteria, Bond and Newton (Ref. 1) predicted that a droplet would break up when the Bond number of the given droplet-gas stream flow situation reached a critical value, experimentally found to be from eight to twelve. Gordon (Ref. 2) attempted to define a mathematical model wherein a cylindrical plug was extruded from the droplet undergoing breakup, and he stated that breakup occurred when the length of the plug extrusion reached a certain critical value. Dodd (Ref. 3) assumed that bag breakup of a liquid droplet occurred when a sphere of minimum diameter was inscribed inside the bag of the droplet undergoing bag breakup. He assumed the diameter of this sphere to be approximately twice the original undistorted diameter of the drop. Hinze (Ref. 4), in the first classical mathematical treatment of droplet breakup, postulated the existence of a critical Weber number to determine the conditions of breakup. This critical Weber number was different for drops subjected to either rapid step changes in relative velocity or for slow, steadily-increasing values of the relative velocity between the gas stream and the drop. The value of the critical Weber number also varied depending on whether the drop had either small or large viscosity. Hinze did not consider cases of intermediate droplet viscosity.

Morrel (Ref. 5) stated that the breakup of a jet and the breakup of a liquid droplet occurred in approximately the same manner and under the influence of approximately the same mechanism. Applying the results of some of his previous work in the field of jet breakup (Ref. 6), he stated that for shear breakup, the critical condition was a given constant value of $\rho_g U^2_{avg}$ over the surface of the droplet. In an experimental work, Lane (Ref. 7) theorized that breakup of a liquid droplet in a flowing airstream would occur at $U^2 d$ = constant. From this criteria he deduced that drops as large as five microns can withstand a sonic relative velocity without breaking up. In his experimental work he also found that there was a lower critical velocity for what he termed the transient flow case (a step change in relative velocity) than for the flow case of a constantly increasing velocity.

Hanson, et al., (Ref. 8) found in their experimental work that they could deduce no critical Weber number value to correlate their experimental droplet breakup results. Their major contribution was that the critical relative velocity depended upon the one-third power of the surface tension of the liquid droplet. Engel (Ref. 9) experimentally discovered many facts important to the field of droplet breakup. She found that it was the flow duration behind a shock wave that had the critical effect in determining breakup time and other critical breakup parameters. Flow duration is defined as the time that the flow or velocity increase behind the shock wave persists at a point after the shock wave has passed that given point. By examination of the photographic record of the experimental program, she was also able to conclude that shear breakup was not due to the vaporization of the liquid droplet, the mechanism which had been earlier suggested though never formally presented in the literature, but that shear breakup was "of mechanical origin". She also found plausibility for the statements that shear breakup was due to the rupture of crests of surface waves on the droplet, the spilling off into the gas stream of the moving boundary layer of the liquid drop, and the action of the vortices on the downstream face of the droplet. These vortices stripped fluid from the surface of the drop as they were shed into the flow stream.

Weiss and Worsham (Ref. 10), in an effort to empirically correlate the variables affecting droplet breakup, discovered that the relative velocity between the droplet and the air stream had the greatest effect in determining the critical breakup parameters. Magarvey and Taylor (Ref. 11) attempted to correlate droplet breakup parameters by formulating a droplet "deformation index". Their only conclusion, however, was that the droplet broke up when the hydrostatic pressure on the windward face of the drop at the stagnation point was greater than 440 dynes/cm^2.

Elzinga (Ref. 12) postulated the existence of a breakup mechanism which stated that droplet breakup may occur when the natural period of vibration of the liquid drop corresponds to the frequency of the shedding of vortices from the leeward face of the droplet. However, with regard to the drop vibration breakup criteria postulated by Elzinga and also by Peskin and Lawler (Ref. 13), Magarvey and Taylor (Ref. 11) stated that in the free-fall of liquid drops the breakup process was not triggered by the internal vibration of the droplet. Lane (Ref. 7) and Hanson (Ref. 8) also stated that internal droplet vibration did not trigger breakup. Rabin, et al., (Ref. 14) concluded from their experimental work that neither the bag nor shear breakup mechanism could be explained on the basis of the drop vibrational period.

In a more recent work, Rabin and Lawhead (Ref. 14), and Rabin, Schallenmuller, and Lawhead (Ref. 15), have, as had been done prior, postulated the existence of a critical breakup velocity strongly dependent upon the flow duration behind the shock wave intersecting the plane of the droplet. They also suggested, but never attempted to verify, that droplet breakup might correlate on the basis of the total impulse acting on the windward face of the droplet. They also concluded, after attempting to correlate their experimental results on the basis of a constant Weber number, that the theory of a constant critical Weber number for determining breakup conditions was inadequate. The best Weber number criterion that they could infer was that droplet breakup occurred at a critical value of a Weber number which was a function of the droplet diameter. They also refuted the work of Lane (Ref. 7), who postulated that the shear breakup of a drop was synonymous to transient flow conditions. They accomplished this by discovering that transient breakup was a time-controlled process but that shear breakup was time-independent.

In the latest available work on the mechanism and process of droplet breakup, Wolfe and Anderson (Ref. 16) have shed new light on the subject by insisting that droplet breakup cannot be correlated on the basis of dimensionless parameters, but that droplet breakup is a rate process, and therefore, the theory of absolute reaction rates from kinetic theory must be applied to the physical variables affecting the breakup of a droplet. They have stated that the method of equating the surface tension forces to determine critical breakup parameters is not valid for systems where the variation of gas stream paramters is of the order of variations which occur across a shock wave.

There has also been some experimental and theoretical work done in an effort to delineate between the conditions which lead to bag and shear breakup. Hanson, et al., (Ref. 8) found that the only criteria they could determine which would differentiate between the two modes of breakup was that shear breakup always occurred for velocities greatly in excess of the critical velocity for a given droplet diameter. Morrell (Ref. 5) argued that bag breakup occurred if the time to which the droplet was subjected to a relative velocity change was in excess of the natural period of oscillation of the liquid droplet, and that shear breakup occurred if the action time was less than the natural period of the droplet. Hanson, et al., (Ref. 8) however, found bag breakup to occur even if the action time was less than the natural period. Rabin, et al., (Ref. 14) stated that the mode of breakup was strongly dependent upon the flow duration behind the shock wave; shear breakup occurring for longer flow durations and bag breakup for shorter flow durations. They also discovered that for velocities much in excess of the critical velocity for a given drop, shear breakup always occurred. This finding supported that of Hanson. Another of their general conclusions was, that for all ranges of variables considered, shear breakup and short breakup times occurred more frequently with the larger drops tested, and that the smaller drops more frequently exhibited bag breakup.

In their experimental work, Wolfe and Anderson (Ref. 16) found that for low relative velocities, bag breakup usually occurred; whereas, for high relative velocities, shear breakup was usually the mode of breakup observed. They also noticed that there existed a smooth transition from one type of breakup to another as the flow variables were varied; and that the transition was equally smooth from bag breakup to shear breakup as it was for shear breakup to bag breakup. One point of interest of their report, regarding the criteria of flow duration behind a shock wave, was that the drop could not know what the flow duration was to be and thus what frontal shape it was to assume before breakup.

Thus, it appears, that for the general case of a given droplet subjected to a given set of physical parameters describing a flow situation, there exists no tried and proven method of determing which mode of droplet breakup -- bag, shear, or some combination of the two -- will occur. Since, however, a mixture of modes can and does appear, pershaps the item of concern is not in describing the mode or mixture of modes that will occur for a given situation, but in describing instead the interplay of physical parameters which lead to a certain breakup time, and thus, a certain mode of breakup.

The time required for droplet breakup is also the subject of much debate in the literature. Hinze's classical work (Ref. 4) stated that breakup times were different depending upon whether the droplet was of very high or very low viscosity. Engel (Ref. 9) concluded that the breakup time was inversely proportional to the strength of the shock intersecting the droplet, directly proportional to the initial diameter of the droplet, and that the change in Mach number, indicative of the change in the strength of the shock wave, is of greater effect in reducing the breakup time than is a change in the initial diameter of the droplet.

Gordon (Ref. 2) found that for droplets of low viscosity, the breakup time was directly proportional to the initial diameter of the drop and inversely proportional to the relative velocity between the drop and gas stream. For drops of high viscosity, the breakup time was found to be independent of the initial diameter and inversely proportional to the relative velocity. The experimental results of Wolfe and Anderson (Ref. 16) agreed most closely with those of Gordon. This result might infer that the breakup time is very much dependent upon the viscosity and the relative velocity, and not so critically dependent upon the other physical variables. This result can only be inferred, however, since it is not exactly known the range of variables other than viscosity and relative velocity that were tested.

It is of interest and concern to note the effect of a shock front on a liquid droplet. Upon examination of the photographic record of their experimental efforts, Engel (Ref. 9), Rabin and Lawhead (Ref. 14), and Wolfe and Anderson (Ref. 16) have all concluded that a liquid droplet is not broken up by the presence or interaction of the shock front, but by the flow regime behind the shock front.

The physical properties which have been given the most attention regarding the basic breakup processes are the viscosity and surface tension of the liquid droplet, and the drag coefficient of the particle in the gas stream. Wolfe and Anderson (Ref. 16) have found that the effect of viscosity is to retard the deformation process as the viscosity increases, and that the resultant droplet sizes after breakup were increased as the viscosity of the tested fluids increased. Higher breakup times were also measured at higher viscosities. Gordon (Ref. 2) likewise found that an increase in fluid viscosity tended to retard droplet breakup processes. He also concluded that viscous effects within the droplet tended to dominate the other physical parameters. Lane (Ref. 7) found that the viscosity affected the breakup process only if it was very high. Hanson, et al., (Ref. 8) stated that the viscosity of the liquid droplet affected breakup time only if the viscosity was high and the diameter of the droplet undergoing breakup was low.

Hughes and Gilliand (Ref. 17) postulated that the drag coefficient was a function of the Reynolds' number, a surface tension parameter, an acceleration parameter, a gravity parameter, the ratio of the liquid and gas densities, and the ratio of the liquid and gas velocities. Ingebo (Ref. 18) showed that for the Reynolds' number range of one to one hundred, the drag coefficient was less than one (1.0) for clouds of solid spheres, clouds of evaporating spheres, and clouds of non-evaporating spheres. Rabin, et al., (Ref. 15) verified this value in their experimental program. Wolfe and Anderson (Ref. 16) expressed concern, in their work, over the uncertainty of the drag coefficient. Carlson (Ref. 19), in deriving an empirical expression for the drag coefficient, found that for flow regimes "such as occur in solid propellant rocket exhausts, " the drag coefficient approached one (1.0) as the Reynolds' number exceeded one hundred. Way and Nicholls (Ref. 20) found that there was a general decrease in drag coefficient for a burning particle, but their work was primarily for a spherical, undeformed particle for a Reynolds' number range of one hundred to one thousand. The drag coefficient did decrease as the Reynolds' number increased.

Regarding the effect of surface tension on the breakup process and parameters, Adam (Ref. 21) stated that an increase in pressure surrounding a liquid drop caused a decrease in surface tension, and the surface tension increased as the radius of the liquid drop decreased. Semenchenko (Ref. 22) wrote that there was a general decrease in surface tension of liquid metals with increase in temperature. Rabin, et al., (Ref. 15) concluded that the surface tension was lower for burning droplets than for non-burning ones, and that this might be due to the vapor-phase burning of the droplet. Hinze (Ref. 4) theorized that the critical breakup velocity for a given droplet diameter was proportional to the one-half power of the surface tension, but Hanson, et al., (Ref. 8) stated that this dependence was surface tension raised to the one-third power. Rabin and Lawhead (Ref. 14) could make no differentiation between the exponents upon examination of their data.

It is concluded, then, that the difficulty in determing the exact effect of physical parameters upon the breakup conditions is a major obstacle in the attempts made to define an adequate mathematical model of droplet breakup. It is also difficult to definitely ascertain just when the droplet has broken up when a sequence of photographs of the breakup process is examined.

ANNOTATED BIBLIOGRAPHY

TRIENBNIGG'S ESTIMATE OF CRITICAL SIZE (1925)

Trienbnigg (Ref. 23) estimated the critical size at which a droplet would breakup at a given relative velocity by merely equating the average air pressure on the face of the droplet, assuming that this pressure constituted the total flow resistance of the droplet, to the surface tension pressure of the spherical droplet, or

$$(1/2)C_D\rho_g U^2 = \frac{2\sigma}{R_D}$$

He assumed a spherical shape for the droplet falling at a terminal velocity through the air with no pressure variation around the perimeter of the droplet. This equation was possibly the first attempt to estimate the critical parameters of droplet breakup.

FREE FALL BREAKUP OF LARGE DROPS: MAGARVEY AND TAYLOR (1956)

The authors of this paper (Ref. 11), in describing an experimental study of the free fall breakup of rain drops, have dealt heavily with the mechanism of breakup and the resultant droplet size distribution after breakup.

In an experimental effort designed to discover a reliable breakup criterion, the authors used an index of deformation and a hydrostatic pressure deformation as a plot of the ratio of minor axis diameter over major axis diameter (assuming a spheroidal shape) versus an equivalent droplet diamter. The hydrostatic pressure at the lower surface of the droplet was then determined by a measurement of terminal droplet velocity. This attempt to discover a reliable breakup criterion, however, resulted only in the conclusion that when the hydrostatic pressure reached 440 dynes/cm^2 and the deformation index was less than 0.3, the droplet would break up. Severe difficulty in accurately measuring the terminal velocity (and thus the hydrostatic pressure), because of droplet instabilities, led to the failure to establish a reliable criterion.

The photographic evidence of the authors distinctly showed a bag breakup occurring in the free-fall conditions. The photographic evidence was interpreted on the basis of a force equilibrium situation. In this force balance, the hydrostatic force in the droplet, just inside the lower surface of the deformed droplet, was equated to the sum of surface tension pressure and the aerodynamic force at the stagnation point of the droplet. The lower deformed surface of the droplet was assumed to be a plane surface. The variation of the respective magnitudes of these forces, as the drop deformed due to a force imbalance, gave only a qualitative representation of the bag breakup situation. For an increasing velocity, the drop flatterns, the equilibrium conditions cannot be satisfied, and the drop breaks up. This argument was used by the authors in their previous attempts to correlate the breakup parameters. An examination of the equations and the logic underlying each force effect gives a qualitative depiction of the bag breakup process. As the aerodynamic pressure increases so as to overcome the effects of the hydrostatic pressure, a bulge would form on the falling droplet. As the bulge increased, the surface tension effects would again become significant and the hydrostatic pressure would be of less effect as the droplet began to "inflate". Finally, the aerodynamic forces for a liquid droplet of sufficiently low viscosity and surface tension travelling sufficiently fast would cause the droplet to inflate and ultimately break up. The authors also discussed the possibility of droplet vibration as a means of breakup since this phenomenon had been theorized by others. The experimental evidence showed that the droplet vibrations were confined to a plane perpendicular to the direction of motion and that these vibrations were not a major factor in the droplet breakup mechanism. Photographic evidence of the resultant droplet size showed that the number of fragments increased with an increase in parent droplet diameter. Actual photographic counts showed that several hundred smaller droplets are often produced from the breakup of a larger drop, and that the bursting of the "canopy" of the inflated droplet produced the smallest droplets while the larger fragments were a result of the breakup of the "rim" of the inflated droplet. The vibration nodes, or "lobes", around the rim of the droplet, never more than four in number according to the photographs, seemed to account for the number of large fragments into which the rim broke up; the number of lobes being equal to the number of large fragments of the rim existing after breakup.

The study concluded that drops as large as twenty millimeters diameter exhibited the characteristics of bag breakup and that smaller droplets exhibited the same breakup mode, but only after a greater fall distance. The authors also stated that a droplet of less than ten millimeters diameter cannot be broken up in a free fall, and that droplet breakup cannot be triggered by internal vibrations of the droplet.

RAINDROP SIZE, SHAPE, AND FALLING SPEED: SPILHAUS (1948)

For the situation of a water raindrop falling at a constant terminal velocity, Spilhaus (Ref. 24) derived expressions relating the surface tension pressure to the aerodynamic pressure on a drop (assuming an ellipsoidal drop shape), the terminal velocity of the falling droplet, and the variation of the drag coefficient of the droplet due to droplet deformation. An expression derived by the author gave this variation as

$$C_D = C_O \left[K-h(K-1)\right] \quad (1)$$

where

C_O = the coefficient of resistance for a sphere (0.21 to 0.3 for the range of Reynolds' numbers concerned)

$K = C_{fp}/C_O$

C_{fp} = drag coefficient for a flat plate

$h = b/a$ = ratio of major axis to minor axis of the assumed ellipsoidal shape (fineness ratio of the ellipse).

In the derivation of this equation, Spilhaus made a rather significant error. When he calculated the pressure difference due to surface tension pressure for the ellipsoidal cross sectional shape, he assumed that $\Delta p = 2\sigma/a$, whereas the correct expression should have been $\Delta p = \sigma(1/a + 1/b)$. This mistake was also noticed by MacDonald (Ref. 25). Hence, the derived expression for the drag coefficient would not correspond to the described physical situation, although it might still be useful since the droplet shape was approximated anyway.

Calculations using Eq. 1, however, for extreme values of the parameters C_O, C_{fp}, and h give values of C_D for the assumed ellipsoidal drop shape that are much lower (e.g., 30-50%) than the values of C_D

presently being used by authors in the calculation of droplet critical velocities, diameters, and breakup times. This variation from experimental values of C_D may be due to derivation error. Rabin and Lawhead (Ref. 14) in their experimental work have measured drag coefficients for inert and burning droplets that were not in agreement with the values suggested by Spilhaus.

THE SHAPE AND AERODYNAMICS OF LARGE RAINDROPS: MACDONALD (1948)

The author of this paper (Ref. 25) postulated that the equilibrium shape of a large raindrop falling at terminal velocity through an infinite medium is due to surface tension, hydrostatic pressure gradients within the drop, external aerodynamic pressures, electrostatic charges on the drop surfaces, and internal circulation of the drop. By means of an order-of-magnitude argument, he concluded that only the first three effects are significant for large falling raindrops. The equilibrium shape of a falling raindrop is that shape for which the aerodynamic pressure plus the surface tension pressure equals the internal hydrostatic pressure at all points on the droplet surface. The author concluded that the study of photographic evidence clearly indicated that boundary layer separation existed at a point along the droplet surface, and that separation did not favor the production of strong internal vorticular circulation. The major contribution of this paper was the argument by which all pressures on the drop, except the external aerodynamic pressure, the surface tension pressure, and the hydrostatic pressure of the drop, could be neglected in calculating the shape of the falling droplet.

CRITICAL SPEEDS AND SIZES OF LIQUID GLOBULES: HINZE (1949)

The first mathematical effort to explain droplet breakup, Hinze (Ref. 4), stated that a droplet is broken up if the translatory speed of the droplet, relative to the gas stream, exceeded a certain critical value, or, inversely, if at a given speed, the size of the moving droplet is greater than some critical size, the droplet will break up. The theory states that the relative magnitudes of the dynamic pressure force and the surface tension force are the criteria for determining droplet breakup. Combining the dynamic pressure force and surface tension pressure into a dimensionless variable leads to the definition of the Weber number. It is, thus, the relevant value of the Weber number that is used as the breakup criterion in Hinze's theory. The critical value of this Weber number must be experimentally determined.

Hinze assumed skin friction forces can be neglected, in comparison to the velocity pressures of the ambient fluid, when the Reynolds' number is greater than one thousand. Hinze (Ref. 4) considered two different flow situations leading to droplet breakup: (1) the droplet is suddenly being exposed to a gas stream of constant speed, and (2) the droplet is being exposed to a gas flow uniformly increasing in speed from zero to a constant value.

Hinze (Ref. 26) linearized the hydrodynamic equation of the motion of the drop and derived formulae for the slight deformations of a droplet caused only by the normal forces acting on the droplet surface. However, the critical deformation necessary for droplet breakup was much larger than the slight value of deformations permitted by his theory; hence, the deformation theory provided only a theoretical model for the breakup process. Actually, photographic evidence (Ref. 16 and 27) has shown that deformation of the droplet undergoing breakup may deviate substantially from the theoretical value predicted by Hinze. The actual pressure distribution around the surface of the droplet for a Reynolds' number range of 1,000 to 200,000 was approximated assuming an irrotational potential flow.

The maximum of δ/R was given by

$$\left(\frac{\delta}{R}\right)\max = -0.17 \frac{\rho_g U_c^2 R}{\sigma} \quad (2)$$

for slight viscosity droplets; i.e., $\frac{\mu_\ell^2}{\sigma\rho_\ell R} << 1$

If $We = \rho_g U_c^2 R/\sigma$ reached its critical value (as determined by experiment), the critical value of δ/R can be computed.

For great viscosity effects ($\mu_\ell^2/\sigma\rho_\ell R >> 1$) the maximum value of $\frac{\delta}{R}$ was given by

$$\left(\frac{\delta}{R}\right)_{max} = -0.095 \frac{\rho_g U_c^2 R}{\sigma} \quad (3)$$

For the case of a droplet exposed to a gas flow uniformly increasing in speed from zero velocity, the forces initially acting on the droplet were assumed to be primarily viscous tangential forces. However, if the droplet was large enough, (i.e., larger than a few millimeters) the normal pressure forces would dominate as the velocity increased. For this case, the equation of the motion of the droplet was:

$$\frac{dU}{dt} + \frac{3C_D}{8R_D} \frac{\rho_g}{\rho_1} = U^2 = g \quad (4)$$

Hinze assumed that this equation was applicable during the entire period of falling (continual velocity increase). A solution to Eq. 4 satisfying the boundary conditions ($U = 0$ at $t = 0$), and assuming a slight viscosity effect, gives for the maximum deformation:

$$(\frac{\delta}{R})_{max} = -0.095 We_{max}. \quad (5)$$

Here it is seen that for the same $(\delta/R)_{crit}$ the corresponding critical Weber number is much greater for case (b) than case (a). For a great viscosity effect, a similar solution also gave:

$$(\frac{\delta}{R})_{max} = -0.095 We_{max}. \quad (6)$$

In an attempt to estimate the breakup time for a droplet suddenly exposed to a constant velocity air stream, some authors proposed to consider the natural vibrational period of the droplet as a rough measure of the breakup time. In this instance, the breakup time is

$$t_b = 0.8\pi \sqrt{\frac{\rho_1 R_D^3}{8\sigma}} . \quad (7)$$

For most breakup time considerations, however, the gas velocity was very much greater than the critical speed, so that the breakup time was much less than that predicted by Eq. 7. Hinze, neglecting viscosity, derived:

$$t_b \approx 1.15 \frac{R}{U} \sqrt{\frac{\rho_\ell}{\rho_g} (\frac{\delta}{R})_{crit}} \quad (8)$$

The above expression and the deformation expression were based on an external pressure distribution for actual turbulent flow at high Reynolds' numbers, a state which might not be present at t = 0 (i.e., the flow requires a certain time T to become fully developed). Since only in the case that the breakup time is very much greater than T may Eq. 8 hold true, the time T was estimated from the time needed for the generation of vortices behind the droplet after the inception of flow. This led to a relationahip of the form:

$$\frac{t}{T} \approx \sqrt{\frac{\rho_1}{\rho_g}} \quad (9)$$

and hence $T \sim R/U$, which is quite large.

For a large viscous effect, small values of the breakup time were estimated by

$$t \approx \frac{10\mu_1}{\rho_g U^2} (\frac{\delta}{R})_{crit} \quad (10)$$

which is independent of droplet size. This result holds only for small breakup times at large Weber numbers and for great viscous effects.

ON THE DISINTEGRATION OF DROPS IN AN AIRSTREAM: DODD (1960)

Dodd (Ref. 3) developed a theory to predict the distortion and disintegration of a water drop which was exposed to an airstream of continuously increasing relative velocity. Assuming that a spherical droplet was distorted roughly into a lenticular shape by the aerodynamic forces, Dodd assumed a relative velocity between the droplet and the airstream low enough in magnitude to assure a bag-breakup mechanism. He also assumed the existence of a non-uniform pressure distribution around the surface of the sphere that is described by the experimental work of Hinze to be:

$$P = \rho_g U^2 (9 \cos^2 \phi - 5)/8 \quad \text{for } 0 \geq \phi \geq \frac{\pi}{3}$$

$$P = -11\rho_g U^2/32 \quad \text{for } \frac{\pi}{3} \leq \phi \leq \pi \qquad (11)$$

where

ϕ = angular distance from the stagnation point of the sphere. Dodd examined the work of Lane, whose efforts led to the following critical condition for droplet disintegration:

$$d(U)^2 = \text{constant}. \qquad (12)$$

Dodd postulated the following breakup theory. As the relative velocity was increased, the drop deformed into the shape shown in Fig. 1 for bag-breakup conditions.

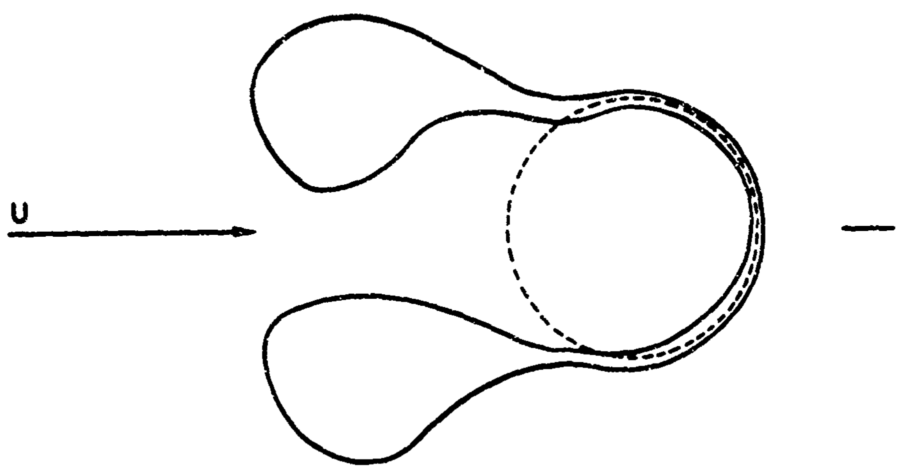

FIG. 1. Drop Deformation as Postulated by Dodd.

Drawing a dotted sphere through the forming "bag", he contended that the critical velocity for bursting is that velocity which makes the radius of this circle a minimum. He assumed that the equation:

$$p_i - p_o = c_1 \rho_g U^2 \qquad (13)$$

where

p_i = pressure inside the bag and

p_o = pressure just outside the bag

held for all stages of bag breakup. By equating the pressure difference between the inside and outside surfaces of the bag to the surface tension pressure, he obtained:

$$p_i - p_o = 4\sigma/r. \qquad (14)$$

Combining Eq. 12, 13, and 14, he obtained:

$$rU^2 = 4\sigma/c_1 \rho_g . \qquad (15)$$

If r_m is the minimum radius of the bag (inscribed circle), then

$$r_m U^2 = 4\sigma/c_1 \rho_g \qquad (16)$$

which gives an expression for the critical breakup velocity. From photographic evidence that the minimum radius was approximately twice the radius of the original drop, he could calculate the magnitudes of the critical relative velocities.

The constant c_1 was approximated from the assumed pressure distribution over a solid sphere. For the given distribution, the pressure is positive for $0 \leq \theta \leq 43$ degrees and is negative and practically constant for 43 degrees $\leq \theta \leq 180$ degrees. If p_o is taken as this constant value and p_i is taken as the average pressure over the positive region of the sphere, we obtain $c_1 = 0.582$.

Usually the relative velocity U is not a known quantity, but rather V, the air velocity, is known as a function of position s_o; u, the droplet velocity, is known from the equations of motion for the drop but is related to V due to the aerodynamic drag. For one regime of droplet breakup, the relative velocity will increase to the critical velocity (Ref. 8) and there will then be a rapid increase in the droplet size. Hence, the critical velocity criteria divides the droplet into two phases.

Consider the motion wherein the bag exists as a part of the moving droplet. Let m be the mass of the entire drop. Dodd supposed that a fraction f of the mass was contained in the hollow sphere he chose to represent the bag; the rest of the liquid was contained in the rim to which the bag was anchored. The equations of motion for the sphere were (assuming rectilinear motion):

$$\frac{ds}{dt} = u \tag{17}$$

$$m\,\frac{du}{dt} = mg + 1/2C_D\pi\, r^2\rho_g\,(V-u)^2 \tag{18}$$

where

r = radius of the hollow sphere.

Let A be a small area on the surface of the bubble, and let δ_r be the thickness of the shell. The mass of this volume is $A\delta_r$ (taking ρ_{water} to be unity); its acceleration (radially) is d^2r/dr^2. The relationship between acceleration and the acting force is:

$$A\delta_r \times \frac{d^2r}{dt^2} = A\left[p_i - p_o - \frac{4\sigma}{r}\right] = A\left[c_1\rho_g\,(V-u)^2 \frac{4\sigma}{r}\right] . \tag{19}$$

The total volume of the shell is:

$$4\pi r^2\delta_r = f_m \quad . \tag{20}$$

Hence,

$$\frac{d^2r}{dt^2} = \frac{4\pi r^2}{f_m}\left[c_1\rho_g\,(V-u)^2 - \frac{4\sigma}{r}\right] . \tag{21}$$

Equation 21 gave the drop behavior after the passing of the critical velocity condition.

THE MECHANICS OF DROPS: HUGHES AND GILLIAND (1952)

The authors of this paper (Ref. 17) considered a force balance for the vertical motion of a drop falling through a stagnant continuous medium of infinite extent, whereby,

$$\rho_1\,\frac{\pi d^3}{6}\,\frac{dU}{dt} = \frac{\pi}{6}\,d^3g\,(\rho_1 - \rho_g) - C_D\,\frac{\pi}{4}\,d^2\,\frac{\rho_g V^2}{2} \tag{22}$$

where

d = droplet diameter.

The definition of a drag coefficient is given by:

$$C_d = \frac{\text{Drag Force}}{(\text{Frontal Area})\ \ (\rho_g U^2/2g)} \quad . \qquad (23)$$

The geometrical shape of the droplet is usually an unknown quantity for a falling droplet. For a falling droplet the value of C_d in Eq. 22 was allowed to vary in order to adjust the drag coefficient based on a solid spherical drop, which was assumed in Eq. 22 to the drag coefficient of the actual, but unknown, geometrical configuration for the falling droplet. In an attempt to determine the drag coefficient as a function of the variables affecting the geometrical configuration and physical state of the drop, the authors resorted to a dimensional analysis technique. The dimensional analysis results show C_d to be a function of Reynolds' Number, an acceleration group, a surface tension group, and a gravity group. These do not take into account the effect of the walls containing the breakup process, the possibility of a continuous phase motion, or freestream turbulence effects. The effects of these parameters on the drag coefficient are a major source of disagreement among researchers in this field.

It is well-known that the fluid pressure on the surface of a moving sphere is not uniform. However, within the drop, except for small amounts of internal circulation due to the distortion of the droplet, and a small gravitational head, the pressure is uniform. Thus, there exists a pressure difference across the droplet surface which must be balanced by the surface tension force in an equilibrium situation. Since $p_{surface}$ depends on the shape of the drop, a theoretical calculation of the droplet surface aerodynamic pressure or, inversely, the droplet shape, is yet to be solved. Hence, the inverse problem is usually the item of concern; that is, given a distorted shape the task will be to calculate $p_{surface}$ for given deformation. Hughes and Gilliand stated that the usual method of obtaining $p_{surface}$ was to assume that the spherical droplet is distorted into a spheroid with its minor axis in the direction of motion. The distortion from the spherical shape can be measured in terms of the fineness ratio, or the eccentricity. According to the authors, the correction of the drag coefficient from the spherical case is not severe as long as the eccentricity is greater than 0.8.

Hughes and Gilliand also attempted to determine whether a spheroidal shape corresponded to the actual shape. They concluded that spheroidal is a valid assumption.

THEORETICAL STUDIES OF MECHANISMS IN THE ATOMIZATION OF LIQUIDS: PESKIN AND LAWLER (1962)

In this report (Ref. 13) of the mechanism of droplet breakup, Peskin and Lawler mentioned a theory advanced by Elzinga (Ref. 12) in an effort to explain droplet breakup in liquid-liquid systems. This theory is essentially that vortices are periodically shed from a moving droplet into the wake, and that the vortex-shedding induces an alternate acceleration-deceleration of the droplet which, thus, causes the droplet to oscillate. When the frequency of the oscillation induced by the vortex shedding becomes equal to the lowest natural frequency of the liquid drop, breakup of the drop occurs. Such a matching of oscillation frequencies is theoretically possible at some drop size since the lowest natural frequency of the drop decreases as its diameter increases, while the vortex shedding frequency increases as the particle diameter increases. Elzinga did plot a dimensionless vortex discharge frequency (Strouhal number) versus Reynolds' number for some data and found a positive correlation.

Peskin and Lawler extended this theory to account for resonant conditions occurring at the frequency of vortex shedding at higher (than the lowest) natural frequencies of the droplet. In considering only primary modes of vibration for a droplet, the breakup criteria is limited to only a minimum diameter; that is, that diameter which corresponds to the frequency of vortex shedding. However, if one considered the droplet as being capable of excitation at modes higher than its lowest natural frequency, the frequency of vortex shedding corresponding to a larger diameter drop than the one being considered could be applied to the given drop at higher natural vibration frequencies. Given a situation where the frequency of vortex shedding is equal to the nth natural frequency of oscillation of the liquid droplet, the authors postulated that the number of droplets into which the intial drop shattered was equal to n, and upon examination of the existing data, found that this approximated the breakup conditions. The authors did not apply this theory to a liquid particle-flowing gas situation because of a lack of available experimental data giving the frequency of vortex shedding for such a system.

The diameter, D_n, at which resonance will occur for any such system previously described is:

$$D_n = \frac{205\sigma}{\rho_g U^2}\left[\frac{\mu_1 U}{\sigma} \times \frac{\rho_1}{\rho_g}\right]^{0.765} (n^3+n^2-2n)^{0.235}, \ Re<2000. \tag{24}$$

The drop diameter D_n which will be excited to a mode of vibration n that will tend to break it up is given by Eq. 24; it is observed that D_n varies directly with surface tension and inversely with the kinetic energy per unit volume at the moment of breakup. For any given system of droplets of known initial diameter, it would be possible to calculate the resultant droplet size distribution after breakup, assuming that the nth natural frequency at which breakup of the drops occurs resulted in n droplets per initial drop. Known variables would have to include the relative velocity between the droplet and the airstream, the physical properties, ρ_g, ρ_1, and $_1$, and the minimum stable droplet diameter, given by the authors to be

$$D_c = 9\sigma/\rho_g U^2 \quad . \tag{25}$$

The work of Hu and Kintner (Ref. 28) was also referenced, wherein they determined the critical diameter above which a droplet must break up by

$$D_c = \left\{1.452 \times 10^{-2} \left[\frac{\sigma}{\rho_1 - \rho_g}\right]\right\}^{1/2} \quad . \tag{26}$$

Peskin and Lawler stated that the theories presented in this paper have been applied to relatively low speed processes. However, for high speed shock processes, it was postulated that since the time to which the droplet is submitted to velocity changes (the action time) is much less than the natural period of oscillation of the droplet, other droplet breakup mechanisms would prevail and become most important. That is, the process causing breaking would occur before the drop could possible undergo even one complete oscillation at the lowest natural droolet frequency.

ATOMIZATION IN HIGH VELOCITY AIRSTREAMS: WEISS AND WORSHAM (1959)

Weiss and Worsham (Ref. 10) conducted an extensive experimental study of the resultant droplet sizes obtained upon injecting liquids into airstreams of constant, moderate velocity (200 - 300 fps). They found that the relative velocity between the droplet and the airstream was the flow parameter having by far the largest effect on resultant droplet size distributions. The variation of other flow parameters and physical properties of the liquid drop had negligible effect when compared to the effect of relative velocity variance.

An empirical correlation was made of their results. The equation is:

$$\frac{X\rho_g U^2}{\sigma} = 0.61 \left[\frac{U\mu_1}{\sigma}\right]^{2/3} \left[1 + 10^3\frac{\rho_g}{\rho_1}\right] \left[\frac{W\rho_1\sigma\,\mu_g}{\mu_1{}^4}\right]^{1/2} \tag{27}$$

Solving the empirical equation of Weiss and Worsham for the relative velocity gave:

$$U = 0.61 \left[\frac{\mu_1 \sigma \, g_c}{\rho_g \, X \times 3.048}\right] \left[1 + 10^3 \frac{\rho_g}{\rho_1}\right] \left[\frac{W\rho_1 \, \mu \, g_c}{3600 \, \mu_1^4}\right]^{3/4} . \qquad (28)$$

This empirical result was examined in an effort to discover correlations between the extensive experimental work of Weiss and Worsham and the data of other researchers in this field.

MECHANISM AND SPEED OF BREAKUP OF DROPS: GORDON (1959)

Gordon (Ref. 2) investigated the droplet bag breakup mechanism, a process which he described as ". . . a process where the drops flatten, become bowl-shaped, inflate like a parachute, and finally burst." He postulated that drops smaller than a certain critical initial size were stable and would not break up.

Gordon's work is essentially a supplement to the investigations of Hinze (Ref. 4 and 26). Hinze predicted, considering both small and large viscosity effects, the critical speed, size, and the breakup time for the bag breakup of a droplet. Gordon, in addition to the cases considered by Hinze, obtained the breakup times for the cases of intermediate droplet viscosity and surface tension effects. Gordon stated that an exact mathematical solution would require a complete knowledge of the aerodynamic pressures on the drop as a function of space and time. This pressure distribution depends upon drop shape, which shape is, in turn, governed in part by the external pressure distribution. Within the drop, the effects of hydrostatic pressure, inertia, internal circulation, surface tension forces, and the viscosity must be balanced at every point. He analyzed these effects by considering their respective orders of magnitude involving the breakup parameters.

Gordon assumed, that in the bag breakup process, a cylindrical plug was extruded from the drop in the direction of flow. This extrusion was caused by the dynamic air pressure on the front stagnation point of the drop and was retarded by the surface tension, viscosity, and internal inertial circulation. The air stagnation pressure is ultimately the disturbing force which causes the breakup. Gordon also stated, without explanation, that actually, the average pressure on the front of the droplet is less than the stagnation pressure, but that this effect is somewhat counteracted by the low pressure due to the separation behind the cylinder. This frontal pressure reduction could possibly be accounted for on the basis of surface circulation effects.

The surface tension forces keep the drop spherical in the absence of other forces. The presence of external forces (e.g., pressure, frictional aerodynamic shear, etc.) tends to cause the drop to deviate from the spherical shape. For the bag breakup phenomena, the front of the drop is flattened, and the radius of curvature of the back side of the drop is increased. Because of this change from the spherical geometry, the surface tension will vary from point to point on the drop surface. Also, during the breakup process, the surface area of the droplet will be increased, and this process requires energy. According to Gordon's mathematical model of a cylinder extrusion, two new surfaces are formed, one at each end of the cylinder. The energy required to form each new surface is equal to the area of the surface times the surface tension. Hence, the resisting force is equal to the surface tension multiplied by the cylinder circumference $2\pi r$ for each area, or $2 \times 2\pi r = 4\pi r$ for the total force. Dividing this force by the cylinder area πr^2 gives the resisting surface tension pressure. Further assuming that the cylinder radius is of the order of magnitude of half the droplet diameter, and substitution into the foregoing equation, gives the resisting pressure, $8\sigma/D$.

Viscous effects sometimes tend to retard breakup. Gordon assumed that the viscous retarding pressure is proportional to the speed of the reakup. The back pressure for liquid flowing through a tube is $16L\mu_1/D$, where L is the tube length. Assuming that this case is analagous to the mathematical model, the magnitude of the retarding viscous pressure is $16\mu_1 U/D$.

Combining the dynamic, surface tension, and viscous pressures, the acceleration of the cylindrical plug can be calculated, if it is assumed that the rest of the drop remains motionless.

$$\frac{dv}{dt} = \frac{p}{\rho_1 D} = \frac{1}{\rho_1 D}\left[1/2\ \rho_a U^2 - \frac{8\sigma}{D} - \frac{16\mu_1 U}{D}\right] \qquad (29)$$

Solving Eq. 29 for the instantaneous velocity of the plug, and the resulting equation for the instantaneous displacement of the cylinder as a function of time, and setting the displacement of the cylindrical plug equal to the droplet diameter D, the total breakup time of the droplet is expressed by:

$$\frac{2(16\mu_1)^2}{\rho_1 D^2(\rho_g U^2 - 16\sigma/D)} = \frac{16\mu_1 t}{\rho_1 D^2} - 1 + \exp(-16\mu_1 U\rho_1 D^2). \qquad (30)$$

This equation, even if it were analytically solvable for the breakup time, might yield a breakup time that is too low, since the experimental evidence of other researchers in this field indicated that the cylinder displacement may be five times the diameter. Gordon, however, postulated that the drop became thinner as it blew up so that the retarding forces were small, and the breakup time depended only on the first stages of breakup.

The critical diameter predicted by the theory is:

$$D = 16\sigma/\rho_g U^2 \quad . \tag{31}$$

For a droplet larger than critical size and negligible viscosity,

$$t_b = (2D/U)(\rho_1/\rho_g)^{1/2} \tag{32}$$

which shows that the breakup time is directly proportional to the initial droplet diameter, and inversely proportional to the relative velocity. For large viscosity and small surface tension,

$$t_b = 32\mu_1/\rho_g U^2 \tag{33}$$

which shows that the breakup time is independent of the initial droplet diameter and is inversely proportional to square of the relative velocity. This independence of breakup time and initial velocity is rather surprising, since all other calculations show a pronounced effect of droplet diameter on breakup time.

Since Eq. 30 is not analytically solvable, a useful approximation is:

$$t = \frac{2D\rho_1^{1/2}}{(\rho_g U^2 - 16\sigma/D)^{1/2}} + \frac{32\mu_1}{\rho_g U^2 - 16\sigma/D} \quad . \tag{34}$$

Gordon stated, without verification, that this approximation is never too small and is at most 37% too large.

CRITICAL CONDITIONS FOR DROP AND JET SHATTERING: MORRELL (1962)

Hinze's analysis applied to a non-viscous liquid suddenly exposed to a constant velocity gas stream has given the criteria for drop breakup, that is:

$$\frac{\delta/R}{\rho_g U^2 R/\sigma} = -.017 \quad . \tag{35}$$

Assuming the critical displacement to be minus R, Morrell (Ref. 5) assumed the critical Weber number for breakup to be about six. Morrell, in another paper (Ref. 6), analyzed the case for a liquid jet and found that for a constant velocity flow the critical condition for jet breakup was

$$\frac{\delta/R}{\rho_g U^2 R/\sigma} = -0.20 \quad . \tag{36}$$

Hence, he assumed that a drop and a jet should behave in approximately the same way with regard to breakup criteria. It was also shown that for an exponential decay of dynamic pressure (i.e., $U^2 = \rho_o U_o^2 e^{-at}$) the ratio of displacement to Weber number decreases as the action time, $t_o = 1/a$, decreases. The maximum values of this ratio were plotted by Morrell as a function of $\tau/2\pi\ t_a$, where τ is the natural period of oscillation of the jet:

$$\tau = 2\pi\ (\rho_1 R^2/6\sigma)^{1/2} \quad . \tag{37}$$

This expression was assumed to be approximately correct for a sphere if the corresponding natural period of a liquid sphere was used:

$$\tau = 2\pi\ (\rho_1 R^3/8\sigma)^{1/2} \quad . \tag{38}$$

Morrell also discussed the breakup of a liquid drop by what he termed a stripping action or shear breakup. He quoted the work of Taylor (Ref. 29) who calculated the liquid boundary layer thickness and the stripping rate from the boundary layer. Taylor concluded, however, that the calculated breakup time and the experimental breakup time were significantly different.

Taylor's theoretical study was based on the assumption that the liquid sheet, stripped from the circumference of the drop undergoing shear breakup, separated from the drop surface when the fractional force on the sheet equaled or exceeded the liquid surface tension force. Hence, for shear breakup, a critical value of $\rho_o U_o 2$ (rather than a critical Weber number, as was the case for bag breakup) should be the criterion for breakup.

As a completion of his analysis, Morrell set forth the conditions under which each type of breakup should occur. For the model he assumed, if t_a is greater than the natural period of oscillation of the drop, the liquid drop should experience bag breakup. If t_a is less than the natural period of oscillation, the droplet will experience shear breakup.

SHATTER OF DROPS IN STREAMS OF AIR: LANE (1951)

Lane (Ref. 7) stated that a relationship of the form U^2d = constant would be expected to adequately express critical breakup velocities for liquid drops, on the assumption that, a liquid sphere placed in a steady stream of air would break up when the force due to the variation of the aerodynamic pressure over the drop surface exceeded the surface tension pressure of the droplet. This relationship naturally resulted from the expression equating the drag force on the droplet to the surface tension pressure for a sphere, or

$$C_D 1/2 \rho_g U^2 = 4\sigma/d. \qquad (39)$$

Lane also stated that observations from his experimental work indicated that the viscosity of the droplet affected the breakup process only when the viscosity was very high, like glycerol.

From the experimental evidence examined by Lane, it appeared that the expression, $U^2d = 612$, was true for breakup over a wide range of droplet diameters. If this relationship held true, water droplets five microns in diameter would remain intact at a sonic relative velocity. Results of further experimental work of Lane indicated that drops even larger than five microns are able to withstand such large relative velocities without breakup.

An increase in the relative velocity, between the droplet and the gas stream, resulted in the production of increasingly finer droplets resulting from the breakup process only up to a certain point. At relative velocities beyond this point, even well above sonic velocities, one-half of the mass of the resulting spray of fine droplets had diameters greater than 15 microns for wide ranges of initial droplet diameters.

Lane also found that the velocities required to insure droplet breakup in the transient (step change in velocity) air blasts were lower than in the steady (steadily increasing velocity) air stream. For smaller drops, the divergence between the critical steady and the critical transient velocities increased. Also, it was noticed that the resultant droplet mass mean diameter decreased with an increase in relative velocity.

It should be noted, at this point, that the results of Lane's experimental work has since been opposed by the theories and experimental results of Hanson, et al.. (Ref. 8).

SHOCK TUBE INVESTIGATION OF THE BREAKUP OF DROPS BY AIR BLASTS: HANSON, DOMICH, AND ADAMS (1963)

The droplet breakup investigation of Hanson, Domich, and Adams (Ref. 8) considered two situations which cause a droplet to shatter. The first case was termed by them the "steady" case, or that situation in which a droplet was subjected to a steadily increasing relative velocity. The second, or "transient" case, was the situation that existed when a droplet was suddenly exposed to a change in relative velocity. Building upon the work of Hinze (Ref. 4), Lane (Ref. 7), and Merrington and Richardson (Ref. 30), in considering these two cases, the authors investigated the breakup mechanisms and the effect of physical parameters upon droplet breakup for droplets in the 100 to 1,000 micron size range.

The underlying philosophy of their experimental program was that it was reasonable to assume the existence of a critical velocity for a given droplet diameter. This critical velocity was defined as the relative velocity between the gas stream and the droplet just necessary to induce the droplet to break up. In an effort to discover a verifiable U_c versus d curve, droplets of fluids of differing physical properties were suspended in an acoustic field and subjected to an air blast produced in a shock tube. High speed motion pictures were taken of the droplets of different sizes as they were being deformed and broken up by and after the passage of the shock wave.

An examination of the resulting photographic evidence brought forth many interesting points. First, bag breakup was observed even with the "transient" case. This finding was in opposition to the work of Lane (Ref. 7) who stated that bag breakup occurred only in the "steady" case. The findings of the authors, though, showed that bag breakup would occur in the transient case except for those velocities which are greatly in excess of the critical velocity for a given droplet diameter. The work of Rabin and Lawhead (Ref. 14) supported this conclusion. Second, it was noticed that for the more viscous droplets, the bag breakup mechanism was more "complicated;" that is, the shape of the bag deviated considerably from the spherical shape of the bag of less viscous fluids subjected to the same mode of breakup; and the rupture of the bag resulted in the formation of fluid ligaments, rather than the small spherical droplets common to the bag breakup of less viscous liquids. Third, the breakup curves of critical velocity versus diameter were plotted for drop diameters in the range of 90 to 700 microns. For this range of drop diameters, U_c was between 40 and 250 feet per second. A least squares data fit brought forth the following equation correlating critical velocity and diameter.

$$\begin{aligned} U_c^2 D &= 6.21 \times 10^6 \text{ (water)} \\ &= 2.71 \times 10^6 \text{ (alcohol)} \end{aligned} \qquad (40)$$

This equation is analogous to the empirical relationship of Lane (Ref. 7), that is, $U_c^2 D$ = constant.

The authors reviewed Hinze's theory (Ref. 4), which stated that breakup occurred when the dynamic pressure of the gas stream at the stagnation point of the droplet exceeded the surface tension pressure by a certain factor. Forming the ratio of the dynamic pressure of the air and the surface tension pressure defines the Weber number, as

$$We = \rho_g U^2 r/\sigma \tag{41}$$

we find that

$$U_c^2 D = 2\sigma We_{crit}/\rho_g \tag{42}$$

which is justification for the empirical equation.

A plot of Eq. 40 yields the We_{crit} for various surface tensions and densities. A summary of this information can be found in Table 1.

TABLE 1. Critical Weber Numbers as Determined by Hanson, Domich, and Adams.

Liquid	U_c, ft/sec	D, μ	We_{crit}
Water	84.3	600	3.60
Water	109.5	410	4.23
Water	157.3	270	6.00
Water	238.5	120	6.55
Methyl alcohol	60.0	625	5.98
Methyl alcohol	84.3	330	6.34
Methyl alcohol	109.5	230	7.62
Methyl alcohol	157.3	118	8.41

Considering the effects of surface tension, Eq. 42 would give for identical diameters,

$$\frac{(U_c)_{water}}{(U_c)_{alcohol}} = \left[\frac{\sigma_{water}}{\sigma_{alcohol}}\right]^{1/2} = 1.79. \tag{43}$$

Equation 43, however, did not correlate the experimental data of the authors. Had the exponent been 1/3 instead of 1/2, the correlation would have been much better. This result seemed to indicate a more complicated surface tension effect than had been predicted in Hinze's theory.

The authors also attempted to correlate the effect of liquid viscosity on droplet breakup. Using the theory of Hinze in which he predicted that

$$\begin{aligned} We_{crit} &= 6 \quad \text{(slight viscosity)} \\ We_{crit} &= 10 \quad \text{(great viscosity)} \end{aligned} \tag{44}$$

as a basis of comparison, they found that their experimental data did not fit the above relationships. In fact, they found that the critical Weber number was not constant for liquids of approximately the same viscosity, but that it increased with decreasing diameter for each of the experimental liquids, and that for very high viscosity, the divergence between Hinze's theory and their experimental data was considerable.

The authors attempted a simple correlation of data on the basis of WeRe = constant, or

$$(WeRe)_{crit} = \frac{\rho g}{2\sigma \nu_g} (U_c{}^3 D^2) \tag{45}$$

which led to, under the conditions that $(WeRe)_{crit}$, ρ_g, σ, and ν_g are constant,

$$U_c \propto \sigma^{1/3}, \tag{46}$$

for a given droplet diameter. This simple correlation gave some justification to their previous 1/3 power of viscosity versus critical velocity data fit.

FRAGMENTATION OF WATERDROPS IN THE ZONE BEHIND AN AIR SHOCK: ENGEL (1958)

In an extensive and elaborate experimental program conducted by Engel (Ref. 9), a wealth of photographic evidence, showing the minutes details of the breakup of a water droplet due to the passage of a shock wave, was accumulated. For Mach numbers of 1.3, 1.5, and 1.7 and drop diameters 1 to 3 millimeters, a very definite shear breakup mechanism, due to the interaction of the shock wave and the droplet, was shown. In general, the times required to induce the different stages of shear breakup, and to totally shatter the droplet, were inversely proportional to the strength of the shock wave and directly proportional to the initial droplet diameter. It appeared, however, that a change in the Mach number of the shock was more effective in increasing the rate of droplet breakup than was a change in initial droplet diameter.

The experimental data taken considered only the variation of breakup time with critical droplet size and shock strength. The effects of liquid viscosity, surface tension, and density were not taken into consideration in the experimental program. One conclusion that could very definitely be made was that the passage of the shock itself did not induce breakup, but that the strength and duration of flow behind the shock were the controlling breakup parameters. Engel theorized that the reaction time of the liquid droplet should decrease as the mass of water that is involved decreased and the shock strength increased.

With regard to the mechanism of shear breakup, it was concluded that the characteristic streaming mist, emitting from the outer periphery of a drop undergoing shear breakup, was not due to vaporization of the liquid, but instead, was due to "mechanical origins." The mechanical origins considered were a mist produced by sound waves, the stripping off of surface layers of water by the tangential aerodynamic forces existing around the periphery of the drop, and the breaking off of the crests of surface waves. An examination of the above postulations resulted in the acceptance of those mechanisms which took into account the action of the rapid airstream on the surface of the droplet; that is, the breaking off of wave crests, the spilling off of the moving boundary layers at the equator of the drop, and the stripping of water from the downstream face of the droplet by vortex actions.

After the examination of data from similar experiments by other researchers in the field of droplet breakup, Engel concluded that the fragmentation mechanism is dependent upon drop diameter, relative air flow velocity, and the density, surface tension, and viscosity of the liquid drop. Generalizing the results of the present study might result in spurious inferences, since it appears that not only the rate of breakup, but also, the mechanism by which it occurs is very strongly dependent upon the variables mentioned above, and all of these variables were not included in the investigation.

THE MOTION AND SHATTERING OF BURNING AND NON-BURNING PROPELLANT DROPLETS: RABIN AND LAWHEAD (1959)

Rabin and Lawhead (Ref. 14) conducted a shock tube study of the effect of shock waves on the breakup of burning and non-burning liquid fuel droplets. They observed both the bag and the shear-type breakup mechanisms for both the burning and non-burning droplets. They also discovered that the type of breakup mechanism and the critical velocity required to induce breakup were correlated in some manner with the duration of the "flow plateau" following the shock front. No general correlation of these two quantities, however, was formulated. The critical velocity for burning droplets was reported by the authors to be slightly lower than the critical velocity for the non-burning droplets of the liquid fuel they were examining. This difference, they postulated, was due to the difference in the surface tension for the two cases; the surface tension for the burning droplet being lower than for the non-burning droplet.

A general conclusion obtained from an examination of the photographic experimental results showed that for flow velocities which are considerably greater than the critical velocity required to induce drop breakup, the shear-type breakup mechanism will always occur. Another conclusion of the experimental program was that the drops are broken up by the flow behind the shock wave and not by the shock front itself.

Since Rabin and Lawhead discovered that the critical velocity for breakup is reduced with an increase in duration of the flow plateau behind the shock front, it was postulated that the drop breakup could be proportional to the impulse (i.e., force multiplied by the time which the force acts) acting on the droplet. Their reasoning, however, was not verified experimentally in their report.

The authors point out that the theory of Hinze (Ref. 4) predicted that the critical velocity should be directly proportional to the surface tension raised to the 1/2 power. Experimentally, other authors (e.g., Hanson (Ref. 8)) have found a 1/3 power dependence. However, the data scatter of the experimental work of the authors made it impossible to confirm either the 1/2 or the 1/3 power dependence.

A further attempt to correlate experimental data in terms of Weber number also proved unfruitful. Rabin and Lawhead concluded that no simple relationship existed between the critical droplet diameter and a critical Weber number for either burning or non-burning droplets.

Perhaps the major contribution of the first report (Ref. 14), of the experimental work of the authors, was the data gathered on the drag coefficients of the liquid droplets. Upon examining the photographic record of the droplet breakup process, they were able to measure the droplet position as a function of time. From this data, the drag coefficients were computed. For smaller droplets (less than 100 microns), the drag coefficients appeared to agree with those previously reported by Ingebo (Ref. 18). However, for larger droplets, there is considerable departure from his data. This effect is possibly due to the fact that for droplets less than 100 microns diameter, the photographic record showed that the droplets deformed only slightly from the spherical shape (for velocities less than the critical velocity), while for drops greater than 100 microns, the droplet deformed into the usual disk shape, even for velocities less than the critical velocity for the given drop diameter. Another significant result of the authors' work was that the drag coefficients for burning droplets are slightly lower than for non-burning droplets. This change may be due to the reduced pressure field around the burning droplet due to the vapor phase burning which, in turn, decreases the pressure drag of the droplet.

DISPLACEMENT AND SHATTERING OF PROPELLANT DROPLETS -- FINAL SUMMARY REPORT: RABIN, SCHALLENMULLER, AND LAWHEAD (1960)

This report (Ref. 14) summarized an extensive experimental program investigating the shattering of burning and non-burning droplets by a normal shock wave at both atmospheric and elevated pressures (i.e., in general, pressures above the critical pressures of the test liquid). Within shock tubes, the duration of gas flow behind the shock wave was varied by using different lengths of pressure section within the shock tube. This variance of flow duration, or "flow plateau", was used to vary physical conditions to which the droplet was subjected during the experimental program. The solenoidal retraction of the wire upon which a propellant droplet was suspended resulted in the formation of two droplets within the test section, a "primary" droplet of 500-1,600 micron size and a "satellite" droplet of 50-300 micron size. In its entirety, the test program investigated the effects of flow velocity, flow duration, chamber pressure, and surface tension on the shattering of burning and non-burning liquid droplets.

The photographic evidence indicated both bag and shear methods of breakup. In general, the larger droplets exhibited shear breakup and shorter breakup times and the smaller droplets exhibited bag breakup for a given velocity and duration of flow. There were also instances in which the droplet appeared to begin the type of deformation leading to shear breakup, but then, only violently oscillated, with no fragments being torn from the droplet.

A major finding of the experimental work was a verification of an earlier postulation, namely, that the passage of the shock front does not shatter the droplet. It is the flow that follows the shock front that causes the droplet to break up. The actual experimental procedure was confined to weak shocks because the authors theorized that the critical flow velocities were in the low velocity ranges, and the previous experimental work of the authors (Ref. 14) clearly indicated the existence of a critical velocity for a given droplet diameter.

Regarding critical velocities, it was stated that there presently exists no satisfactory explanation to account for the selection of either bag or shear breakup near the critical velocity. It was discovered, however, that a flow velocity much greater than the critical velocity for a particular droplet diameter always causes the shear type breakup to occur. The typical critical velocities of this experimental procedure were rather low (e.g., V_{crit} = 60-100 ft/sec for the propellants RP-1, DECH (diethylchlorohexane) at one atmospheric pressure; V_{crit} = 10-15 ft/sec, DECH, 34 atmospheres pressure). Flow durations for both cases were 1.0 to 2.5 milliseconds.

There was a rather substantial decrease in the critical velocity for a droplet of given size as the flow duration was increased. This fact led to the postulation of a critical droplet diameter for a given flow duration. The time required for a droplet to deform sufficiently from its original spherical shape to a shape inducing breakup (the deformation time) was found to be inversely proportional to the droplet diameter. Therefore, droplets below the critical diameter can deform as the gas velocity decays in magnitude, but droplets above the critical size do not have time to deform and shatter. The deformation time was assumed to be inversely proportional to the gas flow velocity; it appeared that a greater flow velocity would be required to shatter a droplet above the critical size than would be required for a droplet smaller than critical size.

"Steady" and "transient" flow conditions were defined based on the natural period of vibration of a liquid drop. "Steady" flow conditions existed if the flow plateau following the shock persisted longer than one-half the natural period of oscillation of the liquid drop oscillating in its lowest mode. "Transient" flow conditions existed if the flow plateau was less than one-half the natural period of the drop. These conclusions were reached by considering the droplet as an idealized spring-mass system with a step-up and decaying forcing function and then solving the resulting differential equation of motion of the system.

In an effort to find the criteria for determining the occurrence of either bag or shear breakup, the authors computed the natural period of vibration of the drop and compared this value with the flow duration and the observed type of breakup. The attempts to correlate the experimental data on the basis of the steady and transient flow designations defined on the basis of the natural period were unsuccessful, and to date, no satisfactory correlation is available.

At elevated pressures, only shear breakup was found to occur, but at atmospheric pressure, both types of breakup occurred. Also, after calculation of the critical pressures of the liquids tested and comparison of the characteristics of the shear breakup at pressures both above and blow the critical pressures of the liquids, no significant influence of critical pressure was observed. The only observed effect of the higher pressure was a lower critical flow velocity.

From the studies made at both atmospheric and elevated pressures, it did not appear that there was any significant difference in the breakup characteristics between burning and non-burning droplets. There appeared to be a slightly lower critical velocity for burning droplets than for non-burning droplets due to the lower surface tension of the burning droplets.

These authors also attempted to correlate the test results in terms of the droplet Weber number. For D less than d_o, We = constant. For D greater than d_o, We increased with increasing droplet diameter at some constant slope. For various test conditions, however, Weber number did not give an adequate general correlation. In a further attempt at correlation, the authors plotted We versus the ratio of the flow duration divided by natural period. These authors recognized the inadequacy of a constant We theory and thus attempted a new data correlation. The basis concept of their hypothesis of data correlation is that transient breakup and shear breakup are not synonymous. Examination of their data showed that transient breakup must be a time-controlled phenomena whereas shear breakup is rather time-independent. Rabin, et al., postulated that shear breakup occurs when the tangential component of the aerodynamic forces on the droplet is greater than the surface tension forces and

$$WeRe^{-1/2} = k_3 \ . \qquad (47)$$

The constant k_3 was experimentally determined, and Eq. 47 gave excellent agreement for all flow cases of a non-burning droplet if $k_3 = 1/2$. For a burning droplet, the correlation did not give such a good result, but this may have been due to the difficulty in determining the surface tension for the burning droplets.

A secondary result of the experimental process was that the droplet drag coefficient for high pressures, and for either burning or non-burning droplets, was approximately equal to one.

KINETICS, MECHANISM, AND RESULTANT SIZES OF THE AERODYNAMIC BREAKUP OF LIQUID DROPS: WOLFE AND ANDERSON (1964)

Wolfe and Anderson (Ref. 16), after giving a short review of other classical theories of breakup mechanisms, have postulated that droplet breakup (which is a flow process) is a rate process. Eyring, in his book <u>The Theory of Rate Processes</u> (Ref. 31), has stated that any "rearrangement of matter" can be considered to be a rate process, and hence, the theory of absolute reaction rates can be applied, theoretically, to the breakup of liquid droplets. Wolfe and Anderson stated that the oft-used classical equation, which equates the maximum force tending to break up the droplet to the surface tension force, is valid only for small rates of stress loading, and hence, not for shock processes. They also theorized that in any situation, in which the stress tending to break up the liquid undergoes a change in time less than that required to break up the liquid, the above mentioned classical equation will not be true.

The unique approach of Wolfe and Anderson applied kinetic theory to the breakup process, whereas all work previous to theirs had considered the breakup process only from the hydrodynamic and mechanical approach. However, the authors stated that this does not mean that the hydrodynamics and mechanics of the problem should be ignored, but only that they should be incorporated into the proper kinetic expression of the system.

The authors considered that the aerodynamic pressure drag and the aerodynamic friction drag, logically, were the two variables that were responsible for the two extreme types of liquid droplet breakup; bag breakup and shear breakup. A qualitative theoretical derivation, using rate process theory to relate droplet deformation to the above-mentioned aerodynamic forces, resulted in an equation relating the droplet breakup times to the flow parameters of the gas stream and the physical properties of the liquid droplet, or

$$t = \frac{d}{(A^2 + BP)^{1/2} - A} \tag{48}$$

where

$A = 16\ \mu/d\rho_1$

$B = 2/\rho_1$

$P = 1/2\rho_g U^2 C_D - k\sigma/d$

k = constant reflecting drop curvature during breakup.

For flow and/or liquid conditions in which viscous and surface tension forces are negligible, Eq. 48 becomes

$$t = \frac{d}{U}\left(\frac{\rho_1}{\rho_g}\right)^{1/2} . \tag{49}$$

For extremely viscous liquids and negligible surface tension,

$$t = 32\mu/\rho_g U^2 . \tag{50}$$

It was interesting to note, at this point, the similarity that existed between Eq. 49 and 50, and the expressions of Hinze and Gordon for similar breakup conditions.

Equation 48 can be regarded as a generalized equation for the breakup time of a liquid drop of given physical properties subjected to an aerodynamic flow of known conditions. The authors stated that it was possible to use Eq. 48 to predict the breakup time of a liquid without regard to the mechanism of breakup, if we could choose a suitable value for k. After examination of available experimental data, Eq. 48 can be used if $C_D = 1$ and $k = 2$. For the use of Eq. 48, the experimental breakup time is defined as the time from the inception of the aerodynamic flow around the droplet to the instant in which the droplet begins to break up. Thus, the theoretical total time required to break up the droplet will be slightly larger than the experimental values, since the theoretical breakup time assumes that a complete disintegration of the droplet (complete rearrangement of matter) has occurred.

If Eq. 48 is to provide an adequate model of the breakup process, then it should provide an explanation of both bag and shear breakup. Since the criteria have been invoked that bag breakup results from pressure drag and shear breakup results from friction drag, two individual forms of Eq. 48 may be written; one expression containing the pressure drag stress in the pressure expression. It was postulated that for a liquid drop of given properties and an air stream of given properties, breakup would occur by the mechanism that required the least breakup time. If the two rates were comparable, the drop should exhibit both bag and shear breakup characteristics.

Equation 48 can be made to fit both breakup cases if one assumes that the frictional drag is twice the pressure drag; an opinion which comes from many workers in this field. If the total drag stress acting on a drop during breakup is $1/2\rho_g U^2 C_D$, then for bag and shear breakup, the pressure expression becomes

$$P_b = (1/3)(1/2\rho_g U^2)C_D - K_b \sigma/d \qquad (51)$$

$$P_s = (2/3)(1/2\rho_g U^2)C_D - K_s \sigma/d \qquad (52)$$

where K_b and K_s are constants that reflect the effect of surface tension tending to hold the drop together during bag and shear breakup. By a best fit of experimental data, $K_b = 4$ and $K_s = 2$, these values may be used to predict breakup times for drops undergoing either bag or shear breakup. Experimental evidence has shown that for low velocities, bag breakup prevails and that for high velocities, shear breakup prevails.

It is also very desirable to be able to predict the mean drop size produce by the breakup of the original drop; although, the droplet sizes produced by the primary breakup of the original drop may vary due to the secondary breakup of drops produced by the primary breakup, vaporization of primary and secondary droplets, coalescence of primary and secondary droplets, and settling or removal of the droplets by the gas stream. This report considered only the mean droplet size distribution resulting from the primary breakup; and the magnitudes of the other mentioned effects were estimated from existing knowledge. The experimental results of this study showed that the drop sizes produced by the two different breakup modes were essentially the same a result that is intuitively somewhat surprising. However, since a postulated mechanism (shearing of a liquid film from the drop) and one equation (Eq. 48) theoretically govern both types of breakup, this suggested result is not surprising from a theoretical standpoint.

The results of this study did not provide a theory that would provide a resultant droplet size distribution as a function of the liquid droplet and the gas stream parameters. However, by assuming that the mean drop size results from the breakup into optimum unstable wave lengths of the liquid boundary layer being stripped from the surface of the droplet, it has been found that the mean diameter is

$$D = \left[\frac{136\mu_1\sigma^{3/2}d^{1/2}}{\rho_g^{2}\rho_1^{1/2}U^4}\right]^{1/3} \qquad (53)$$

Equation 53 was derived for the case in which the aerodynamic forces are much larger than either the viscous or surface tension forces. It is theorized that this case is valid for shock processes.

It is interesting to note that Eq. 53 predicts both the same $1/U^{4/3}$ dependence of D and the same initial diameter to the 1/6 power dependence of D as does the empirical work of Weiss and Worsham (Ref. 10).

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NOMENCLATURE

a	Minor diameter, microns
b	Major diameter, microns
C_D	Drag coefficient
d	Droplet diameter, microns
f	Frequency at which vortices are shed
g	Acceleration due to gravity, ft/sec^2
M	Mach number
m	Droplet mass, lb_m
n	An integer, 1,2,3,. . .,n
p	Pressure, lb_f/in^2
R_D	Droplet radius, microns
Re	Reynolds' number, $\rho_g U_{rel} d/\mu_g$
s	Area, microns^2
...	Strouhal number, $\frac{fd}{U}$
t	Time, seconds
t_a	Action time, seconds
U	Relative velocity between drop and surrounding medium, ft/sec
W	Molecular weight, lb_m/mole; mass injection rate, lb_m/hr
We	Weber number, $\frac{d\rho_\ell U^2}{\sigma}$
X	Mass median diameter, microns
δ	Droplet deformation parallel to flow direction, microns
μ	Microns
μ_g	Gas Stream absolute viscosity, lb_m/ft-sec

μ_l Droplet absolute viscosity, $lb_m/ft\text{-}sec$

ν Kinematic viscosity, ft^2/sec

π 3.14

ρ Droplet radius, microns

ρ_g Gas stream density, lb_m/ft^3

ρ_l Droplet density, lb_m/ft^3

σ Surface tension, lb_f/ft or dynes/cm

τ Natural period of oscillation, seconds

ϕ Angular distance from stagnation point; polar angle around median vertical plane of droplet

Subscripts

avg Average condition

b Breakup condition

c Critical condition

D Droplet

g Gas

l Liquid

max Maximum condition

o Initial condition; outside

s Stagnation point; due to surface tension

x Upstream from shock front

y Downstream from shock front

∞ Free stream condition

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13. ABSTRACT

The literature relating to droplet breakup in gas streams moving with relative velocity to the particles has been surveyed and reviewed. This report contains a brief discussion of breakup mechanisms followed by an annotated bibliography of the pertinent references. Included also is a general list of references giving background to the subject.

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