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TRANSVERSE PROPERTIES OF FIBROUS COMPOSITES

P. E. Chen
and
J. M. Lin

September 1968

PROGRAM MANAGER
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Development of High Performance Composites

TRANSVERSE PROPERTIES OF FIBROUS COMPOSITES

P. E. Chen
and
J. M. Lin

Rolf Buchdahl, Program Manager

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St. Louis, Missouri 63166

This paper was prepared under the sponsorship of the Advanced Research Projects Agency, Department of Defense, through a contract with the Office of Naval Research, N00014-67-C-0218 (formerly N00014-66-C-0045), ARPA Order No. 873, ONR Contract Authority NR 356-[illegible]/4-13-66, "Development of High Performance Composites."

FOREWORD

The research reported herein was conducted by the staff of the Monsanto/Washington University Association under the sponsorship of the Advanced Research Projects Agency, Department of Defense, through a contract with the Office of Naval Research, N00014-67-C-0218 (formerly N00014-66-C-0045), ARPA Order No. 873, ONR contract authority NR 356-484/4-13 66, entitled "Development of High Performance Composites."

The prime contractor is Monsanto Research Corporation. The Program Manager is Dr. Rolf Buchdahl (phone 314-694-4721).

The contract is funded for $5,000,000 and expires 30 April 1970.

TRANSVERSE PROPERTIES OF FIBROUS COMPOSITES

P. E. Chen* and J. M. Lin**

ABSTRACT

The transverse stiffness and strength of unidirectional fiber-reinforced composites have been calculated by using the finite-element method and the von Mises-Hencky criterion. Both the square and hexagonal arrays have been considered for the fiber configuration. The conditions of perfect bonding and total debonding have been included in the strength calculations. Experimental work has also been conducted on boron-aluminum and stainless steel-aluminum composites. The transverse properties of such systems have been measured as functions of fiber volume content. The theoretical results are compared with the experimental data of our own for the metal-matrix composites and those of others for glass-epoxy composites.

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INTRODUCTION

Various methods [1-7] have been proposed for the calculation of transverse stiffness of fibrous composites. This paper employs the finite-element method which is believed to be relatively more accurate and rigorous. Perhaps more significantly, the present paper deals also with the transverse strength of fibrous composites using a method based on the von Mises-Hencky criterion. It is assumed that the distortional energy condition is valid. This assumption is substantiated by the fact that the fibrous composites usually fail at a relatively low level of transverse loading, and also by the actual stress-strain behavior of the materials. Two limiting conditions have been considered in the strength calculations. For the first condition the fibers are assumed to be perfectly bonded to the matrix, while for the second condition the fibers are assumed to be totally debonded from the matrix, thus providing the upper and lower bounds. It is also assumed that the fibers are circular in cross section and unidirectionally aligned. Moreover, the transverse properties have been calculated for both the square and hexagonal arrays as the fiber configurations in the normal plane. The theoretical results are confirmed experimentally.

THEORY

The Finite-Element Method

The stresses and displacements of the composite in the elastic region are calculated by using the finite-element method [8-10]. This method utilizes the direct stiffness concept which considers the composite system as an assemblage of idealized elastic elements assumed to be joined together at discrete nodes. By adding together at each node the stiffness coefficients of adjacent elements a stiffness matrix for the system is obtained. This stiffness matrix relates the external forces acting on the nodes to the displacements of the nodes. By inverting the stiffness matrix one obtains an influence matrix which gives the nodal displacements as a function of the external forces or loads acting on the system. Using the same strain pattern for the elastic element that was assumed for deriving its stiffness coefficients, one may derive a matrix of stress coefficients which gives the stresses in the element as a function of its nodal displacements. The method is ideally suited for analyzing multi-phase materials. It is founded on equilibrium and compatibility conditions.

The Typical Region

In order to render the problem more tractable, the fibers are assumed to be ideally packed into square or hexagonal array as shown in Figure 1a or 2a. Further, by invoking symmetry and compatibility conditions, it is only necessary to consider a typical region as shown in Figure 1b or 2b. The procedure for analyzing the typical region is basically similar to that described in Reference 11. The essential details in the analysis of the typical region are given in the Appendix.

The von Mises-Hencky Criterion

Various theories [12-15] have been proposed for predicting the strength of materials. However, the theory introduced by von Mises [16] and reinterpreted by Hencky [17] is generally recognized as conceptually most consistent, and it is also supported by experimental evidences.

The total strain energy stored in an elastic body can be divided into two parts, the dilatational energy and the distortional energy. The von Mises-Hencky theory postulates that yielding sets in when the distortional energy reaches a critical value. For a uniaxial and plane state of stress the criterion becomes

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_m^2 \tag{1}$$

where σ_1 and σ_2 are the principal stresses, and S_m is the strength of the matrix material. The quantity on the left side of equation (1) will be referred to as the normalized distortional energy hereafter.

The Stiffness and Strength Calculations

To proceed with the calculation of the transverse properties, the typical region is first divided into a sufficient number of finite elements, the procedure given in the Appendix is then used to calculate the distributions of stress and displacement in the region. Corresponding to the assumed displacement conditions, the applied normal stress can be calculated from the stress distribution previously determined. The stiffness is obtained by a simple application of the Hooke's law.

Assuming perfect bonding or total debonding, the normalized distortional energy is evaluated for every element in the domain, thus determining also the maximum normalized distortional energy. For the condition of perfect bonding the fibers are assumed to be in perfect contact with the matrix, and the composite is considered continuous, from the mechanistic point of view, at the interface between the constituents. For the condition of total debonding the fibers are assumed to be completely separated from the matrix. The transverse strength of the composite can be calculated from the following equation:

$$S_t = S_m \, \overline{\sigma}_c \, / \, (U_{max})^{1/2} \qquad (2)$$

where $\overline{\sigma}_c$ is the applied stress on the composite under the assumed displacement conditions as described in the Appendix, and U_{max} is the corresponding maximum normalized distortional energy. Both $\overline{\sigma}_c$ and U_{max} are functions of fiber volume content, fiber geometry, condition of bonding, as well as the constituent properties.

EXPERIMENT

Materials

Two types of metal have been used as the matrix materials, namely the 6061 and 2024 aluminum alloys. Boron and stainless steel fibers have been used for the reinforcing phase. The mechanical properties of the constituent materials relevant to the theoretical calculations are given in Figures 3-6.

Technique

Solid state diffusion bonding technique was utilized to fabricate the composites used in the experimental work. The general bonding conditions were 3 hours at 11,000 psi and 900°F for the boron-aluminum system, and 30 minutes at 6,000 psi and 900°F for the stainless steel-aluminum system. The equipment used for diffusion bonding is shown in Figure 10. Tensile specimens were prepared by first shearing or wet grinding to approximate size and then polishing with #180 SiC paper under running water to final size of 0.30 in. wide by 2.75 in. long, with the thickness varying from 0.030 in. to 0.116 in.. A typical tensile specimen is shown in Figure 11. All specimens were tested by an Instron machine at room temperature.

RESULTS AND DISCUSSION

The theoretical approach as described previously was used to calculate the transverse properties of boron-6061 aluminum, stainless steel-2024 aluminum, E glass-epoxy and S glass-epoxy composites, as functions of the fiber volume content. The theoretical results are compared with our own experimental data obtained for the metal-matrix composites, and the experimental results obtained by other investigators [3,18,19] for glass-epoxy composites, as shown in Figures 3-9.

The composite transverse properties have been calculated for both the square and hexagonal arrays as the idealized fiber packings in the matrix. It is interesting to note from the calculated results that the square array gives relatively higher transverse stiffness, but lower transverse strength than those of the hexagonal array. However, the stiffness seems to be less sensitive than the strength to the change in fiber geometry. Strictly speaking, the fiber geometry is neither square nor hexagonal, but it appears to be closer to the square array. Perhaps, it should be mentioned in passing that all curves for the square array terminate at $V_f = 78.5\%$, and those for the hexagonal array terminate at $V_f = 90.6\%$. The above-mentioned fiber volume contents are the geometric limitations for the respective arrays.

In addition to the fiber geometry, the extent of debonding between the matrix and the fibers also has significant effect on the transverse strength of the composites, as can be seen in Figures 4, 6 and 9. In order to see the actual conditions of bonding and debonding, the specimens were studied under the optical microscope. Three pictures taken under the microscope are included in this paper for illustration. Corresponding to the experimental data as shown in Figure 4, the condition of bonding of boron fibers in 6061 aluminum matrix is shown in Figures 12 and 13. Related to the experimental results given in Figure 6, the condition of debonding of stainless steel in 2024 aluminum matrix is shown in Figure 14.

It is also important to point out from the results obtained thus far that in no case has the composite transverse strength exceeded the matrix strength. The decrease in transverse strength for higher fiber content is basically caused by the corresponding increase in stress concentration [20]. The increased transverse strength for hexagonal array, as compared with the square array, is believed due to a more efficient shear transfer device similar to that as suggested by Sadowsky [21].

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NOMENCLATURE

S_m	=	Strength of the matrix material.
S_t	=	Composite transverse strength.
E_f	=	Young's modulus of the fiber material.
E_m	=	Young's modulus of the matrix material.
E_t	=	Composite transverse stiffness.
ν_f	=	Poisson's ratio of the fiber material.
ν_m	=	Poisson's ratio of the matrix material.
U_{max}	=	Maximum normalized distortional energy.
V_f	=	Fiber volume content.
x, y	=	Rectangular coordinates.
u, v	=	Displacements in x and y-directions.
u_1, v_1	=	Displacements in x and y-directions for Case 1 in the Appendix.
u_2, v_2	=	Displacements in x and y-directions for Case 2 in the Appendix.
σ_1, σ_2	=	Principal stresses.
$\bar{\sigma}_{x1}, \bar{\sigma}_{y1}$	=	Average normal stresses in x and y-directions for Case 1 in the Appendix.
$\bar{\sigma}_{x2}, \bar{\sigma}_{y2}$	=	Average normal stresses in x and y-directions for Case 2 in the Appendix.
$\bar{\sigma}_x, \bar{\sigma}_y$	=	Average normal stresses in x and y-directions for Case 3 in the Appendix.
$\bar{\sigma}_c$	=	Applied normal stress on the composite for Case 3 in the Appendix ($\bar{\sigma}_c = \bar{\sigma}_x$ under the assumed conditions).

τ_{xy} = Shearing stress in xy-plane parallel to x or y-axis.

c = Width of the typical region.

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APPENDIX

The determination of the stress and displacement distributions in a composite as shown in Figure 1a or 2a can be accomplished by analyzing a typical region, as shown in Figure 1b or 2b. The region is so chosen that under a normal stress at infinity, the rectangular region after deformation remains rectangular. The finite-element technique and the method of superposition are used to solve the problem in the following steps, assuming that the applied normal stress is in the x-direction:

1. Solve Case 1 which is defined by the following boundary conditions:

$\tau_{xy} = 0$ along the entire boundary,

$u = 0$ along AO (points remain on the y-axis because of symmetry),

$u = 1$ along BC (arbitrarily specified unit displacement),

$v = 0$ along OC (points remain on the x-axis because of symmetry),

$v = 0$ along AB (specified displacement condition).

The displacement field thus calculated is (u_1, v_1), and the average normal stresses in the x and y-directions are $\bar{\sigma}_{x1}$ and $\bar{\sigma}_{y1}$ respectively.

2. Solve Case 2 which is defined by the following boundary conditions:

$\tau_{xy} = 0$ along the entire boundary,

$u = 0$ along AO,

$u = 0$ along BC,

$v = 0$ along OC,

$v = 1$ along AB.

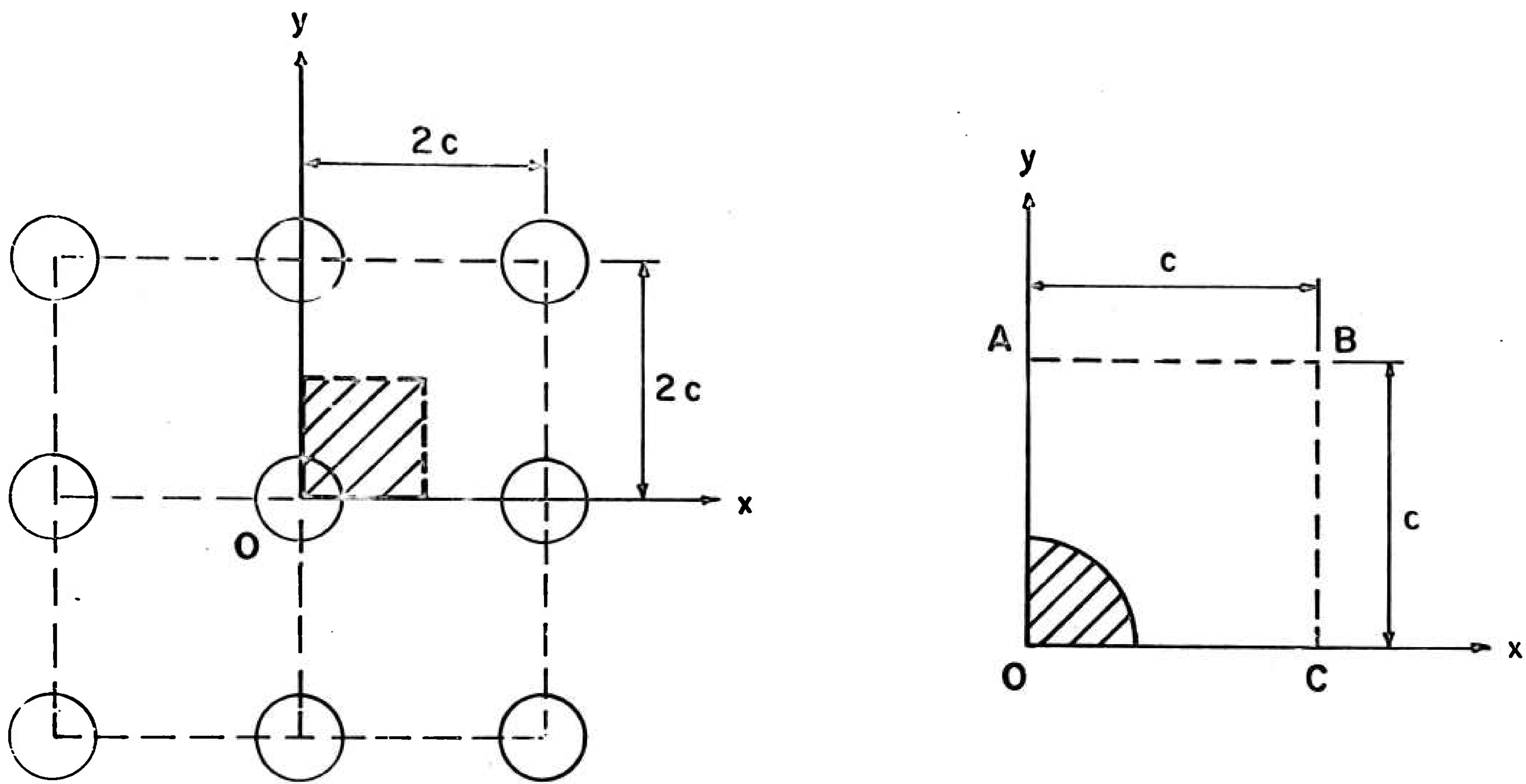
The displacement field thus calculated is (u_2, v_2) and the average normal stresses in the x and y-directions are $\bar{\sigma}_{x2}$ and $\bar{\sigma}_{y2}$ respectively.

3. Solve Case 3 which is characterized by $\bar{\sigma}_y = 0$, solution of Case 2 is multiplied by $(-\bar{\sigma}_{y1}/\bar{\sigma}_{y2})$ and summed with that of Case 1. Thus the corresponding applied normal stress on the composite is

$$\bar{\sigma}_c = \bar{\sigma}_x = \bar{\sigma}_{x1} - \frac{\bar{\sigma}_{y1}}{\bar{\sigma}_{y2}} \bar{\sigma}_{x2} . \qquad (3)$$

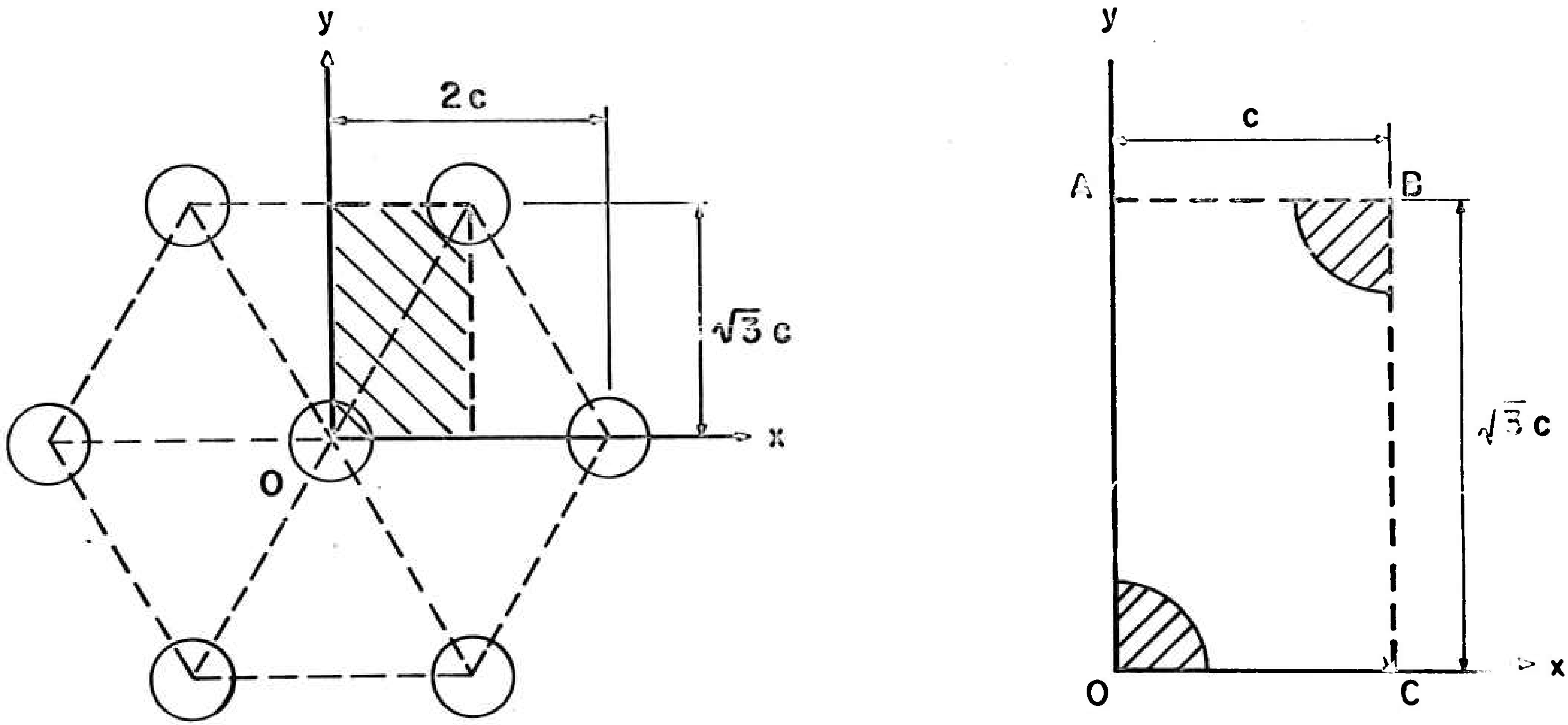
Likewise for the stress and displacement components.

LIST OF FIGURES



(a) (b)

Figure 1.



(a) (b)

Figure 2.

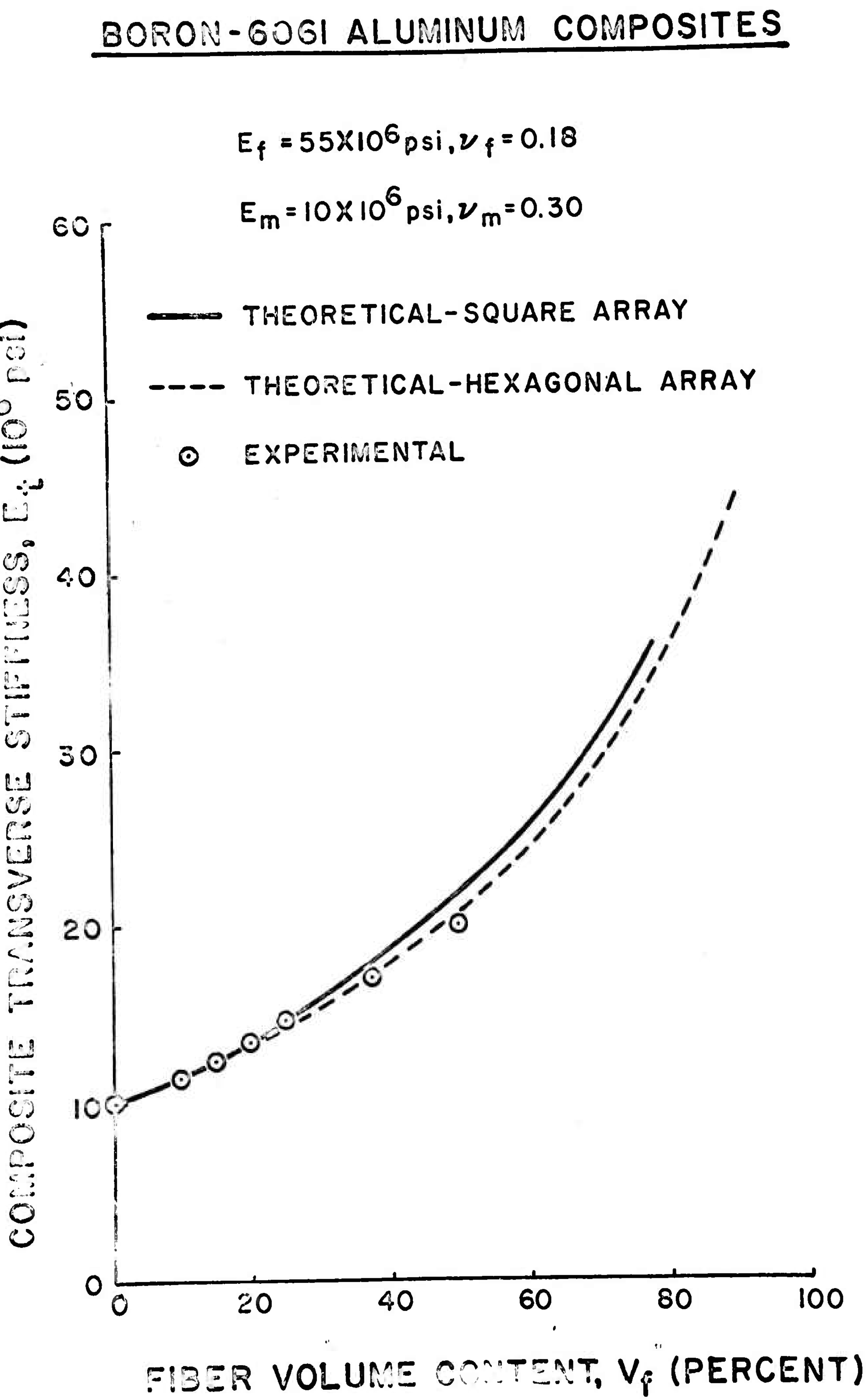

BORON-6061 ALUMINUM COMPOSITES
$E_f = 55 \times 10^6$ psi, $\nu_f = 0.18$
$E_m = 10 \times 10^6$ psi, $\nu_m = 0.30$
THEORETICAL-SQUARE ARRAY
THEORETICAL-HEXAGONAL ARRAY
EXPERIMENTAL
COMPOSITE TRANSVERSE STIFFNESS, E_t (10^6 psi)
FIBER VOLUME CONTENT, V_f (PERCENT)
0
10
20
30
40
50
60
0
20
40
60
80
100

Figure 3.

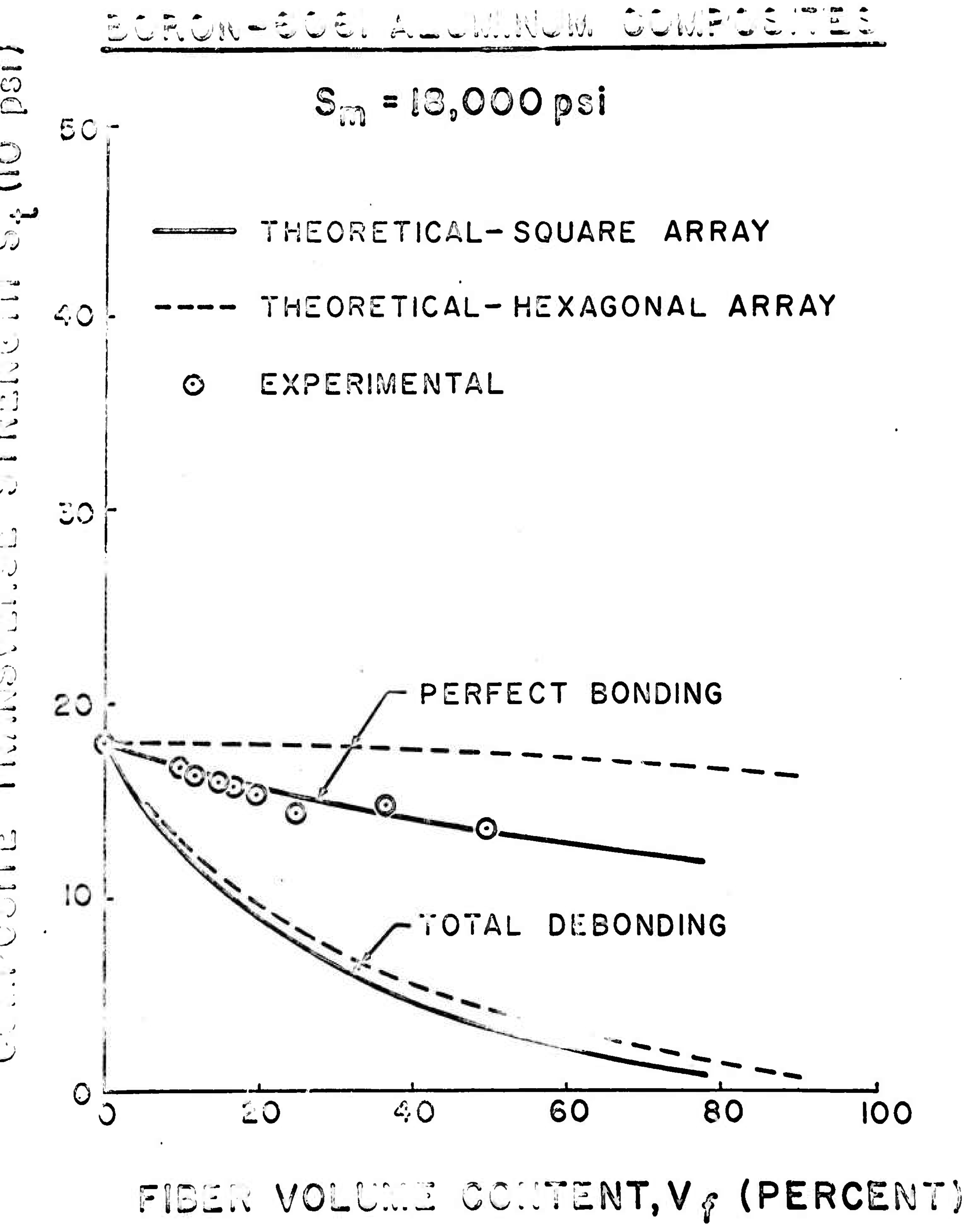

BORON-6061 ALUMINUM COMPOSITES
Sm = 18,000 psi
THEORETICAL-SQUARE ARRAY
THEORETICAL-HEXAGONAL ARRAY
EXPERIMENTAL
PERFECT BONDING
TOTAL DEBONDING
50
40
30
20
10
0
0
20
40
60
80
100
FIBER VOLUME CONTENT, Vf (PERCENT)

Figure [illegible]

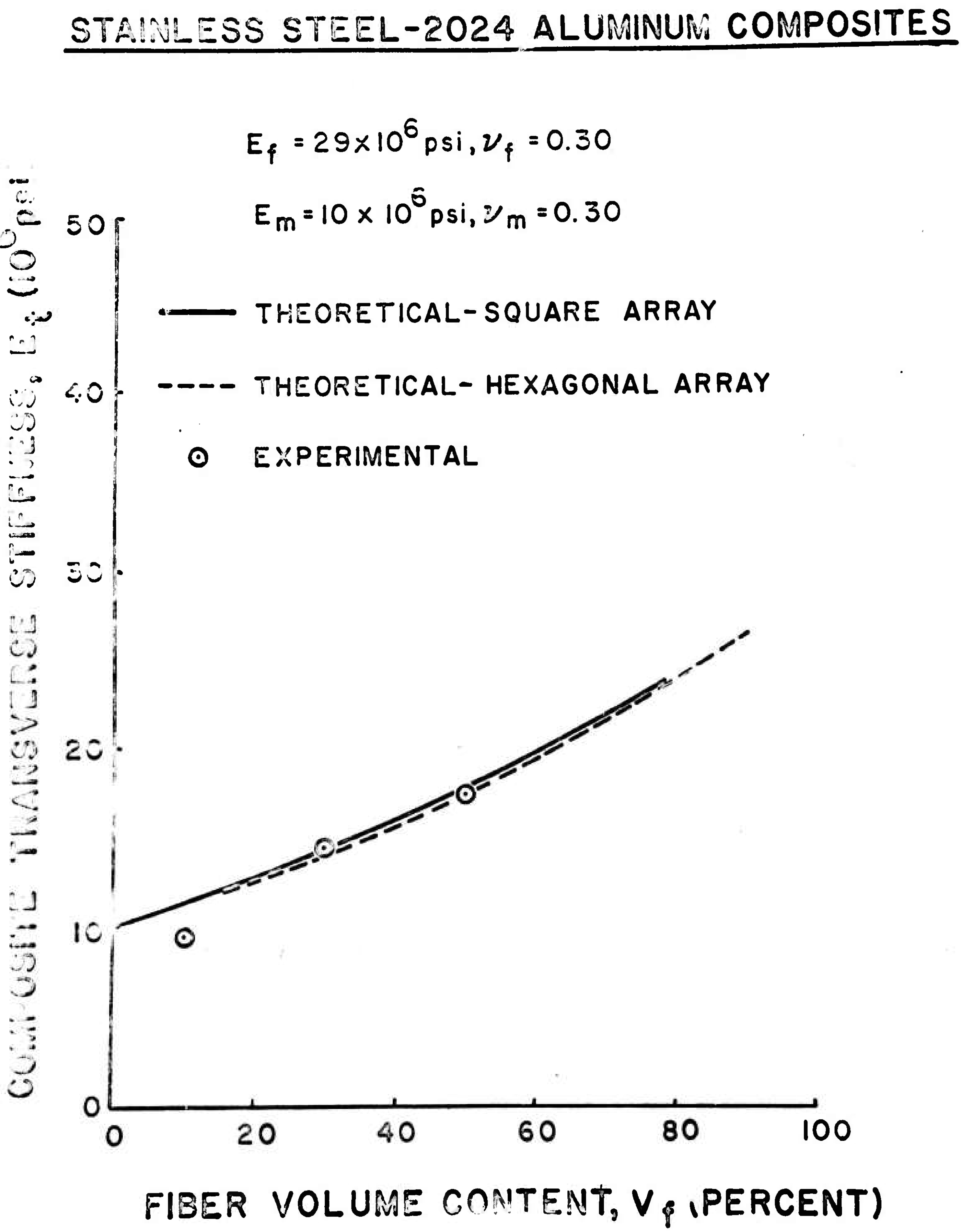

STAINLESS STEEL-2024 ALUMINUM COMPOSITES
$E_f = 29 \times 10^6$ psi, $\nu_f = 0.30$
$E_m = 10 \times 10^6$ psi, $\nu_m = 0.30$
THEORETICAL-SQUARE ARRAY
THEORETICAL-HEXAGONAL ARRAY
EXPERIMENTAL
COMPOSITE TRANSVERSE STIFFNESS, E_t (10^6 psi)
0
10
20
30
40
50
0
20
40
60
80
100
FIBER VOLUME CONTENT, V_f (PERCENT)

Figure 5.

STAINLESS STEEL-2024 ALUMINUM COMPOSITES

S_m = 27,000 psi

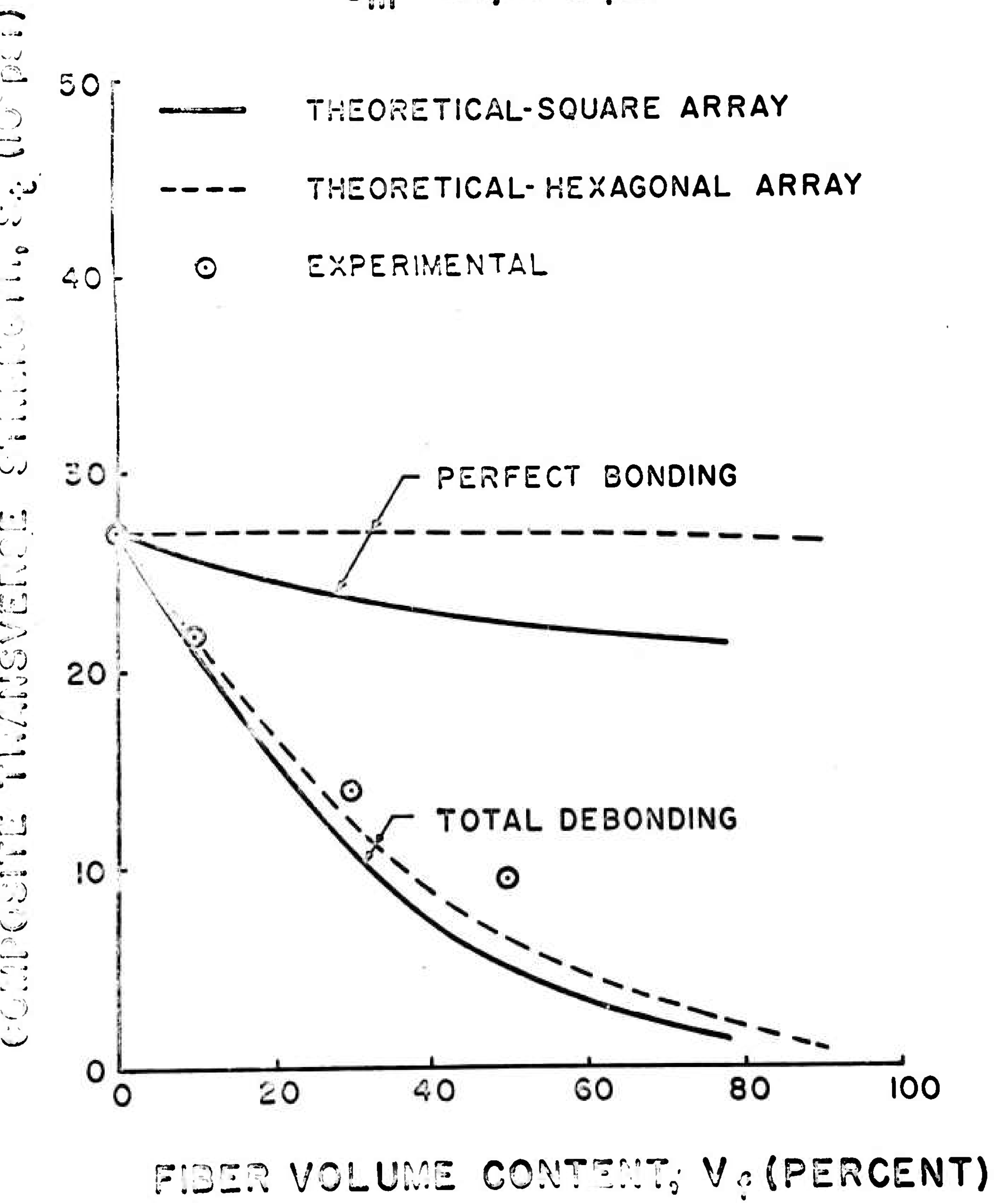


Figure 6.

E GLASS - EPOXY COMPOSITES

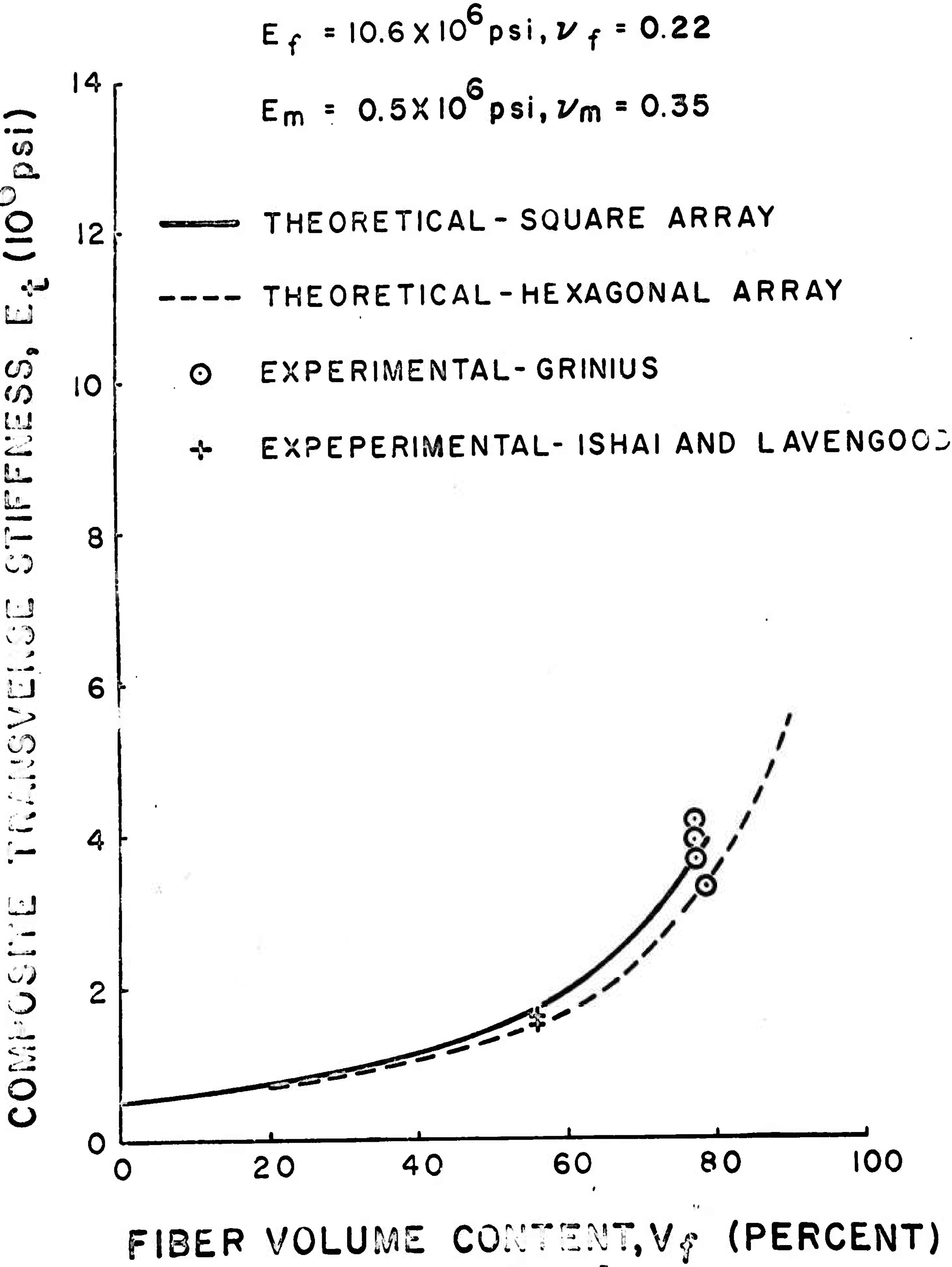


Figure 7.

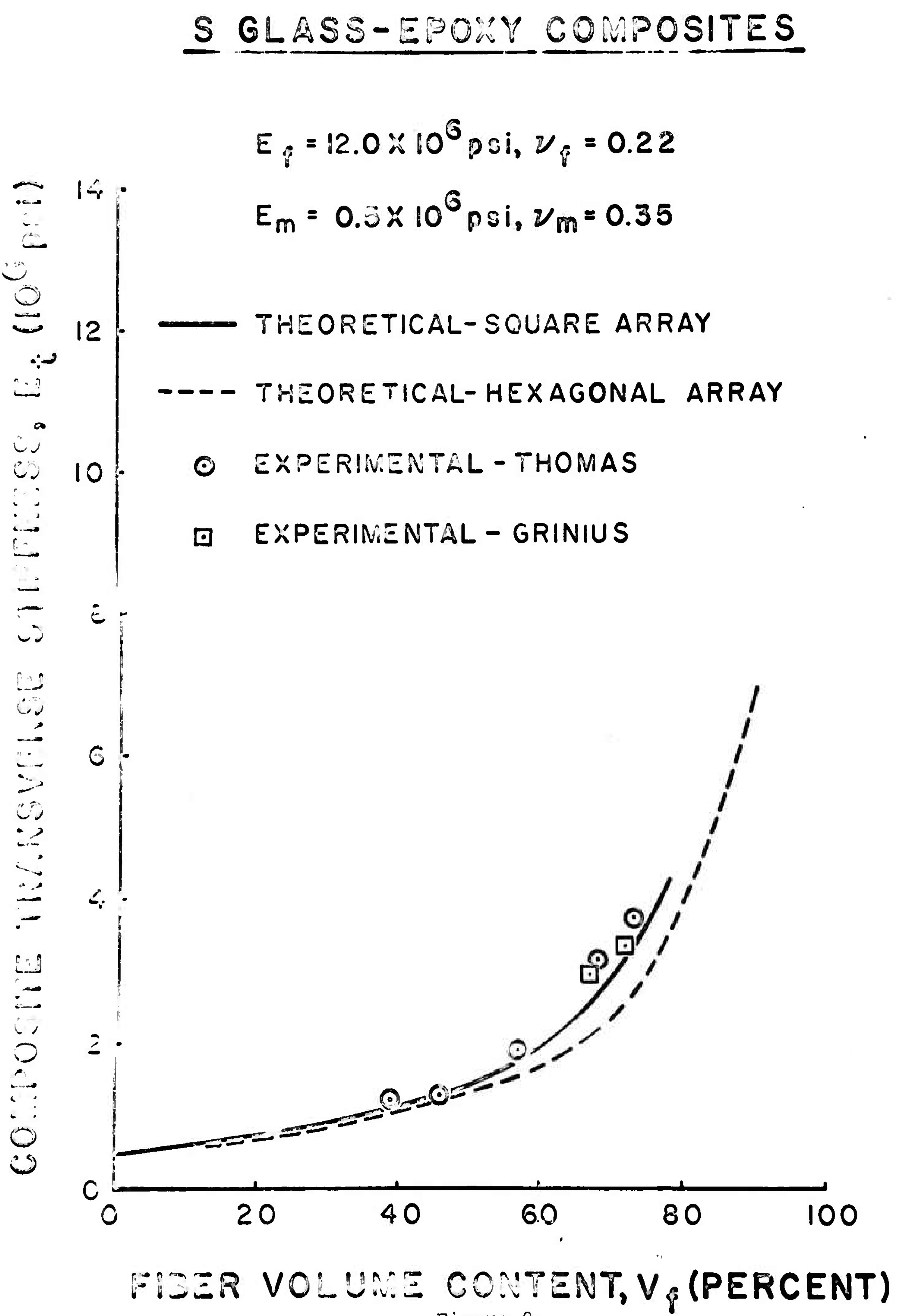
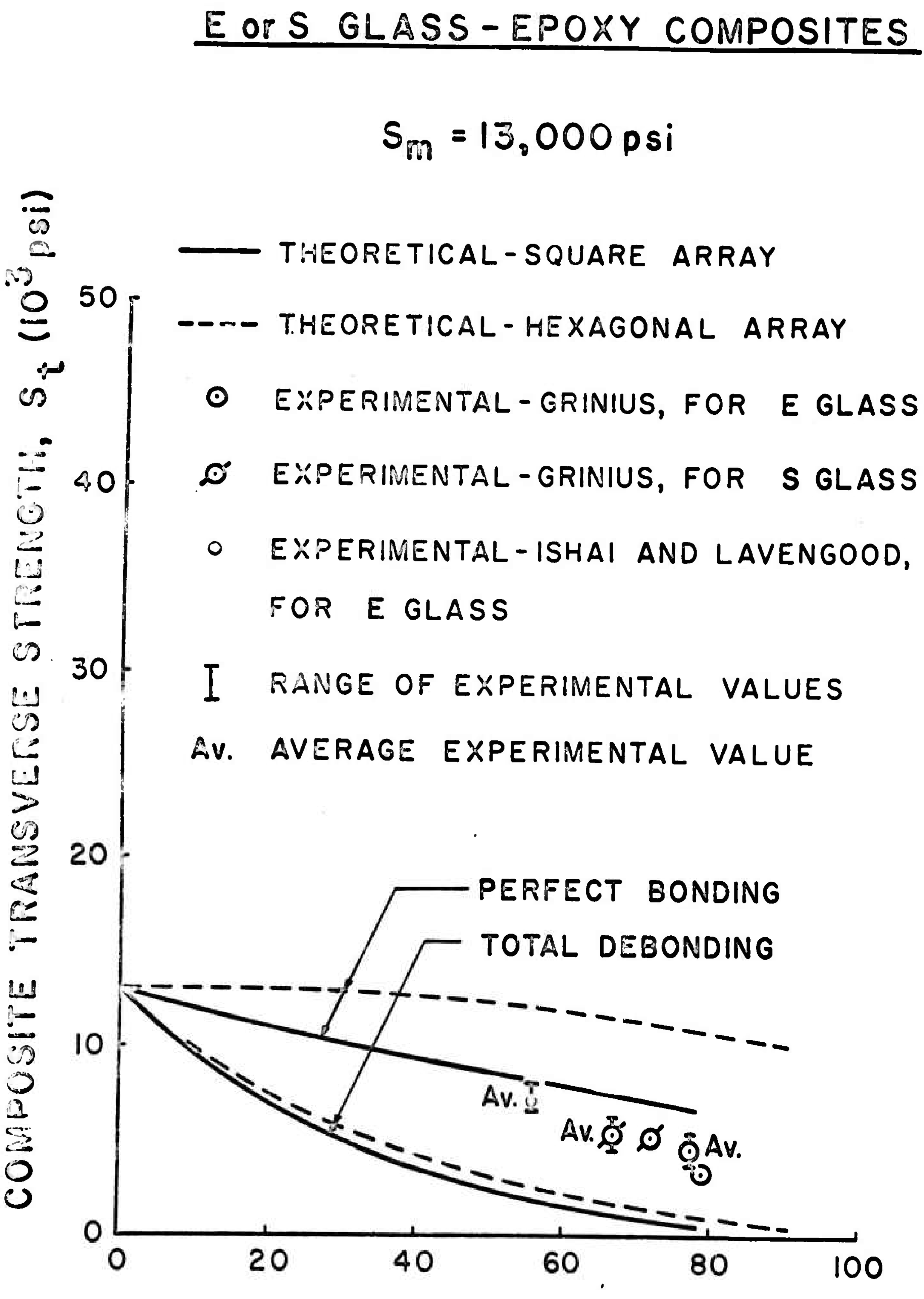

S GLASS-EPOXY COMPOSITES
E f = 12.0 X 10^6 psi, ν f = 0.22
Em = 0.5 X 10^6 psi, νm = 0.35
THEORETICAL-SQUARE ARRAY
THEORETICAL-HEXAGONAL ARRAY
EXPERIMENTAL - THOMAS
EXPERIMENTAL - GRINIUS
COMPOSITE TRANSVERSE STIFFNESS, Et (10^6 psi)
0
2
4
6
8
10
12
14
0
20
40
60
80
100
FIBER VOLUME CONTENT, Vf (PERCENT)

Figure 8.



FIBER VOLUME CONTENT, V_f (PERCENT)

Figure 9.

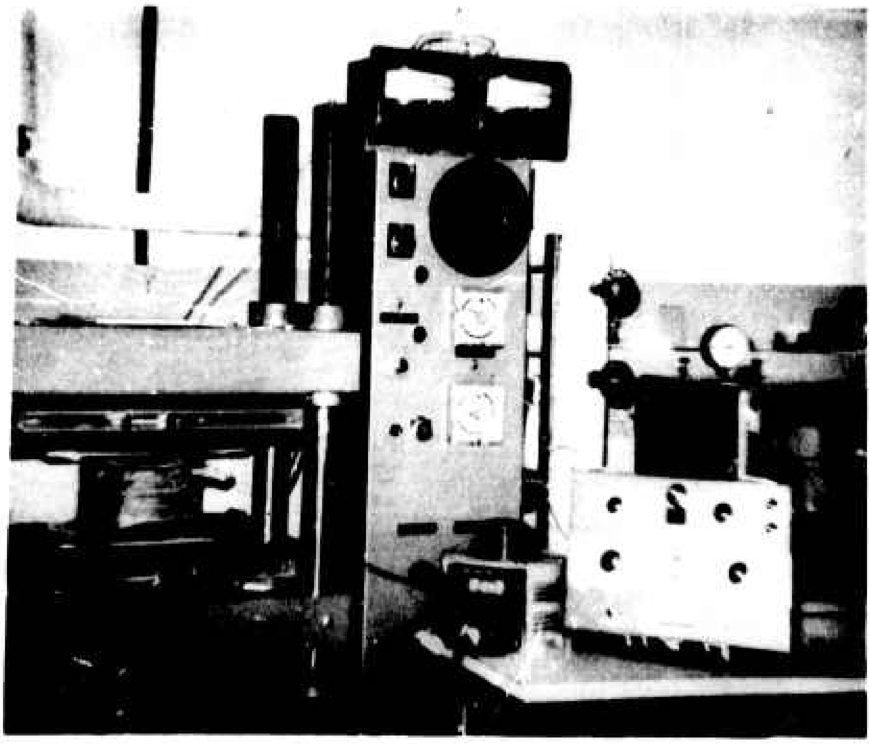


Figure 10.

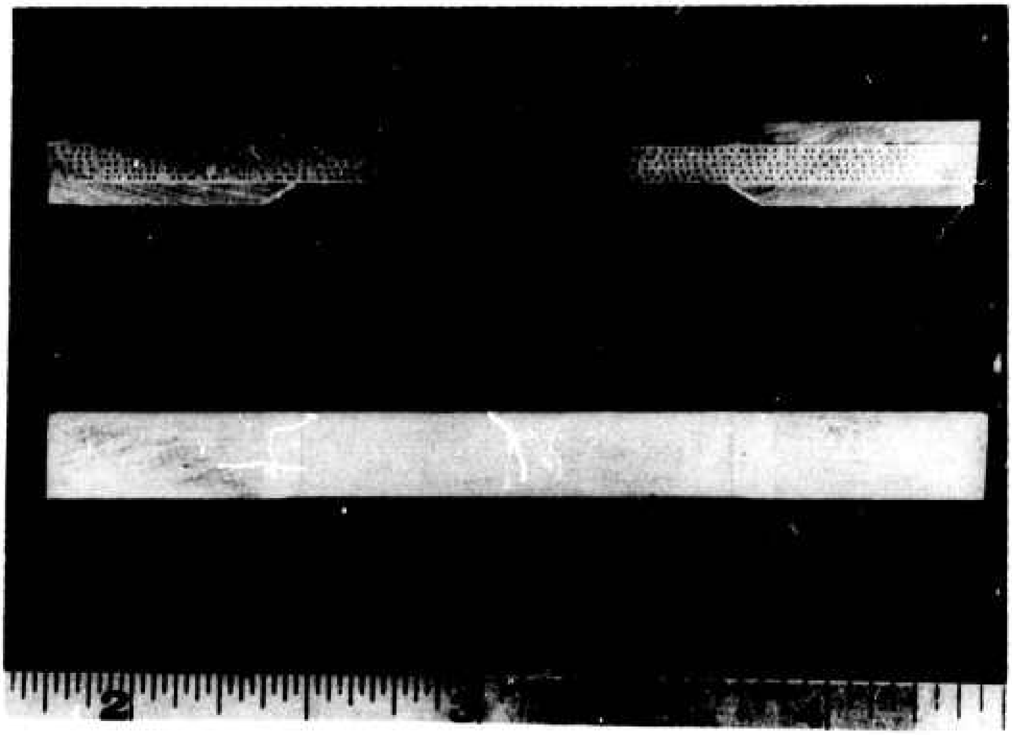


Figure 11.

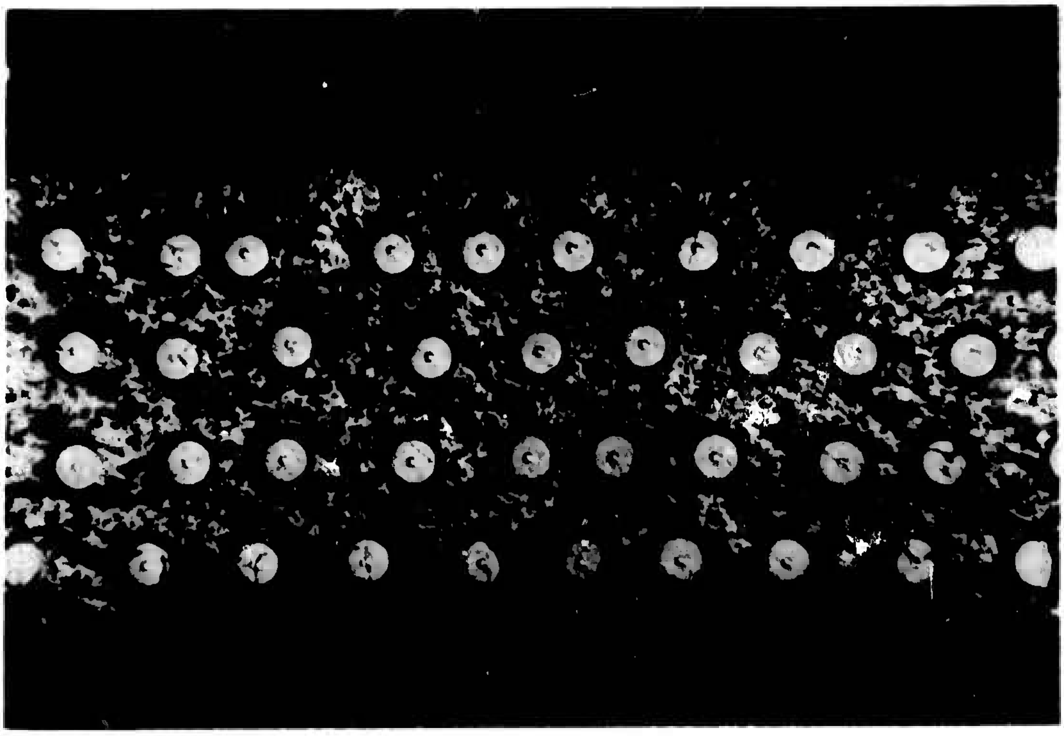


Figure 12.

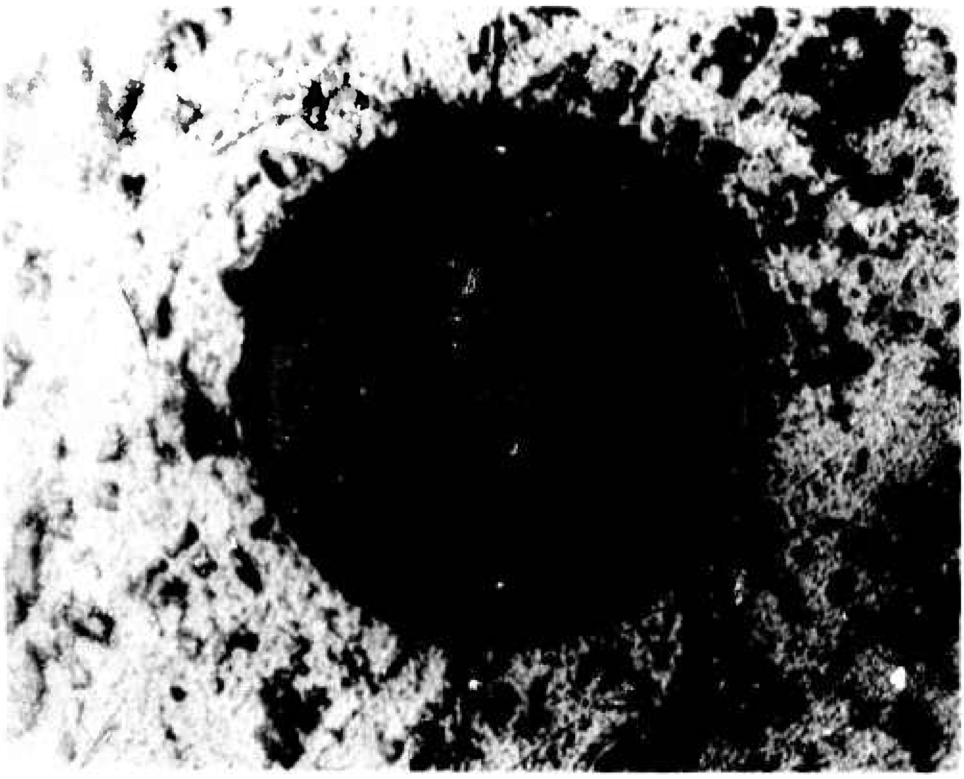


Figure 13.

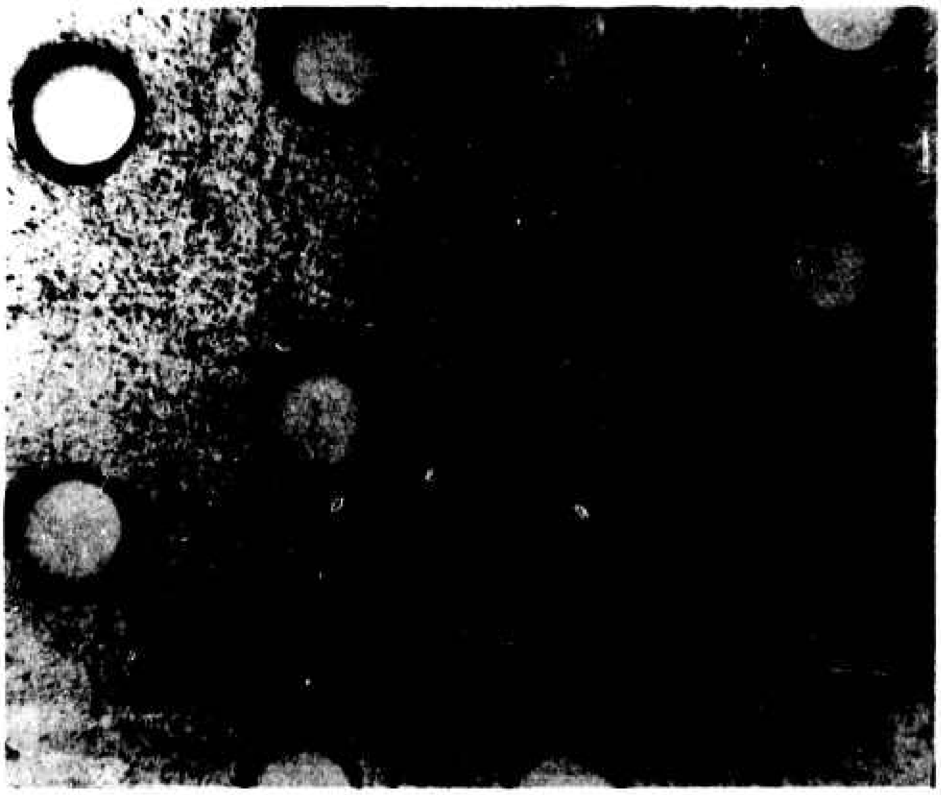


Figure 14.

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13. ABSTRACT

The transverse stiffness and strength of unidirectional fiber-reinforced composites have been calculated by using the finite-element method and the von Mises-Hencky criterion. Both the square and hexagonal arrays have been considered for the fiber configuration. The conditions of perfect bonding and total debonding have been included in the strength calculations. Experimental work has also been conducted on boron-aluminum and stainless steel-aluminum composites. The transverse properties of such systems have been measured as functions of fiber volume content. The theoretical results are compared with the experimental data of our own for the metal-matrix composites and those of others for glass-epoxy composites.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
stiffness						
strength						
modulus						
transverse properties						
fibrous composites						
aligned fibers						
metal matrix						
epoxy matrix						
evaluation						