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Technical Report No. 119

SACLANT ASW

RESEARCH CENTRE

OPTIMUM THEORETICAL STRUCTURES OF SONAR SYSTEMS EMPLOYING SPATIALLY-DISTRIBUTED RECEIVING ELEMENTS

by

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1 SEPTEMBER 1968

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Viale San Bartolomeo 400
I 19026 - La Spezia, Italy

(6) OPTIMUM THEORETICAL STRUCTURES OF SONAR SYSTEMS EMPLOYING SPATIALLY-DISTRIBUTED RECEIVING ELEMENTS.

[redacted]

(10) F. Bryn

(11) 1 Sep[redacted]68 (12) 48p.

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312 950

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PREFACE

Presented at

This report consists of a reprint of a paper read at the NATO Advanced Study Institute on Signal Processing held at the Twente Institute of Technology, Enschede, The Netherlands on 12 to 23 August 1968.

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OPTIMUM THEORETICAL STRUCTURES OF SONAR SYSTEMS EMPLOYING SPATIALLY-DISTRIBUTED RECEIVING ELEMENTS

By

F. Bryn

ABSTRACT

The paper reviews the structure of optimum active sonar systems employing spatially distributed receiving elements. The systems considered operate in the presence of a time-variable noise field and in a randomly-varying transmission medium. The required signal-processing operations can be divided into two groups, one depending upon the noise characteristics and the other upon the signal characteristics. The noise-dependent processor is studied with special emphasis on structures that can adapt themselves to a time-varying noise field. The signal-dependent processor is derived for some simple types of random signal distortion. The signal characteristics required for the instrumentation of such processors are related to the impulse response of the target-medium combination.

1. INTRODUCTION

The task of an active or passive sonar system is generally to survey a finite ocean area in the sense of detecting and locating submarines moving into the area. The task involves two operations: the acquisition of target information from the acoustic field, and the making of decisions about the target situation based upon the acquired information. See Fig. 1. In this paper we shall analyse the operation of an active system. The passive system differs mainly in the type of signals to be detected. The sonar receiver is shown in Fig. 2. It consists of a sensing device usually referred to as the hydrophone array, an information processing device, and a decision device. With the sensing device we carry out measurements on the random acoustic field in the ocean for the purpose of detecting acoustic echo signals reflected from targets within the surveyed ocean area. The information processing device reduces the measurements to a form required by the decision process.

In this paper we will describe the optimum structure of active sonar systems. We begin with a study of the ensemble of received echo signals and indicate how this is related to the randomness of the transmission medium and the targets. We proceed with a survey of the elements of decision theory, and point out how the optimum structure of the processing device can be deduced from the characteristics of the input measurements.

In many practical situations we find that the description of the optimum system depends on parameters which exhibit random variations in time. Detection systems which measure or estimate the true values of one or

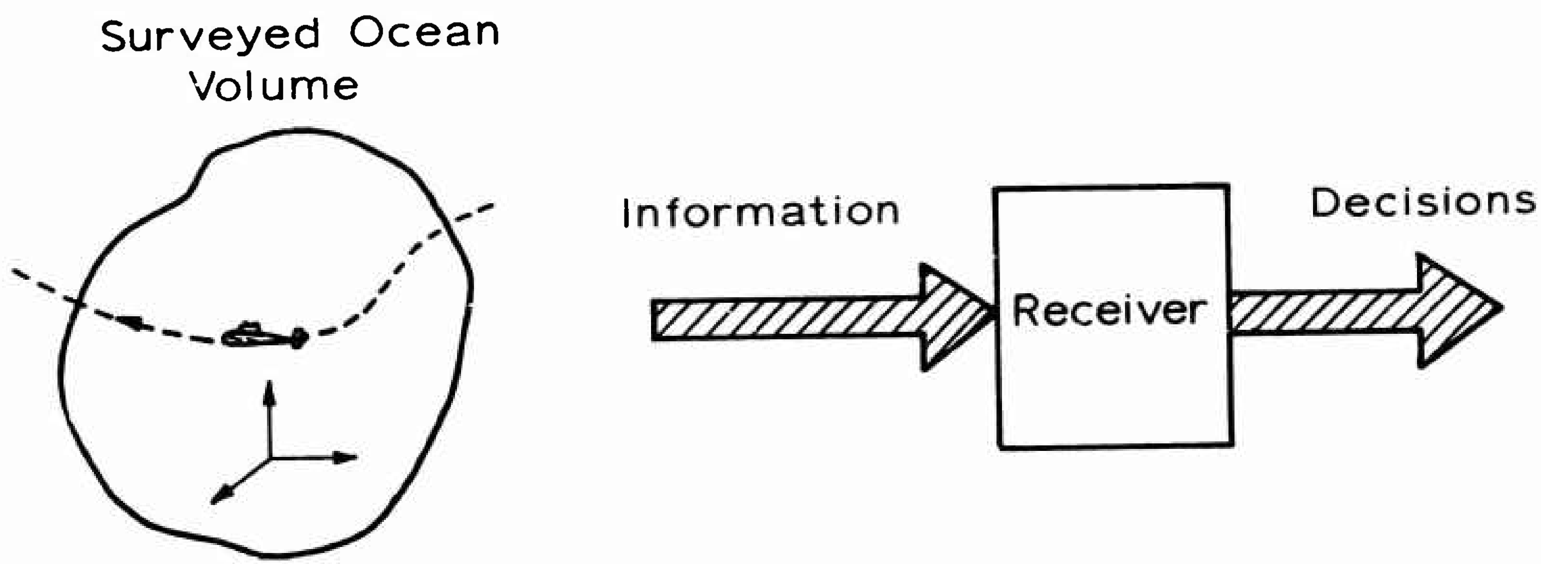


Fig. 1 THE SONAR SITUATION

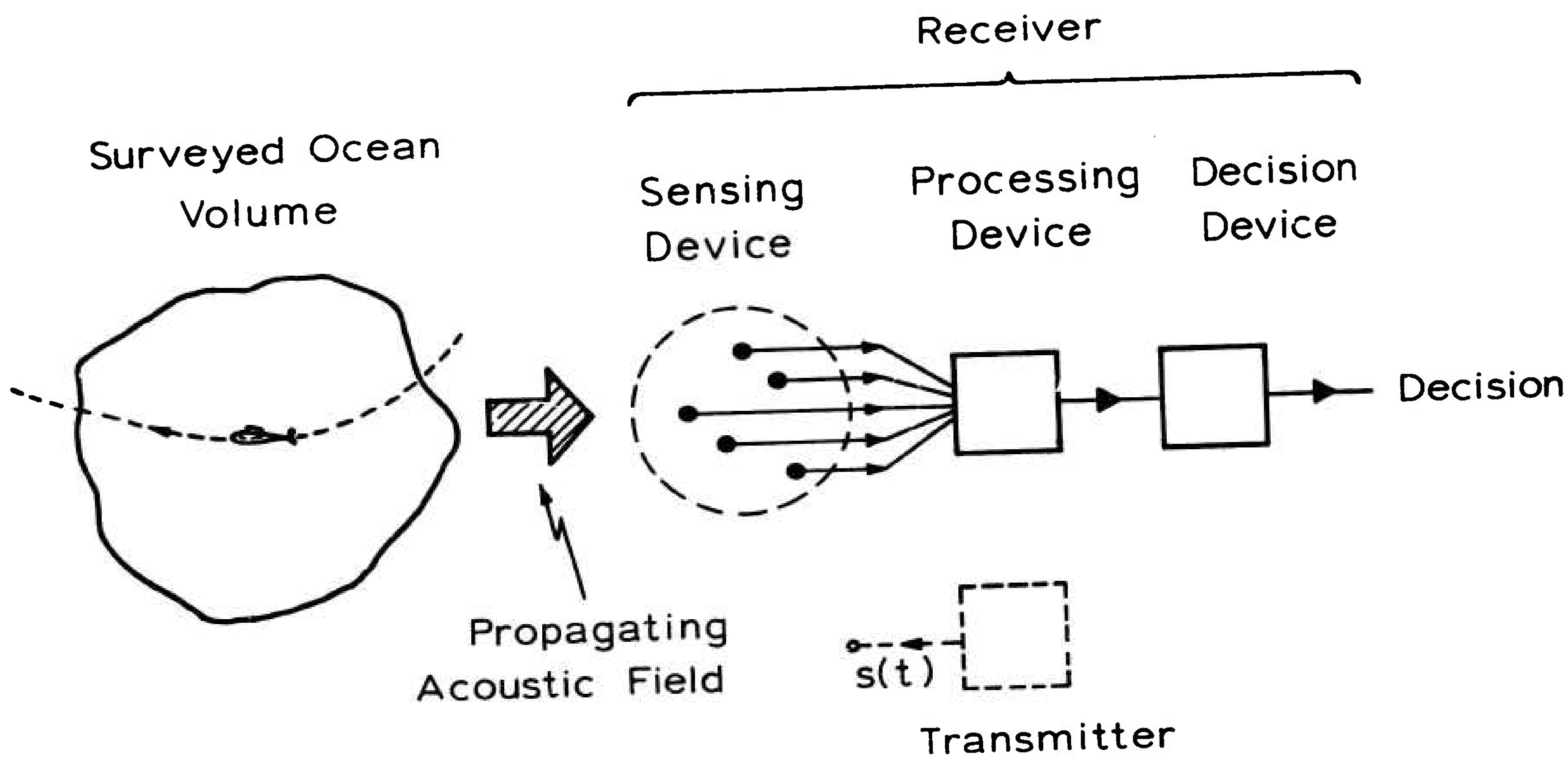


Fig. 2 THE SONAR RECEIVER

more random parameters and adjust the structure of the processor in accordance with the estimates, are referred to as adaptive systems. A considerable part of this paper is allocated to a review of techniques of adaption to time-variable gaussian ambient noise fields.

The signal dependent part of the signal processor is studied in Chapter 5. As is well known the specific structure of the processor can only be found for some few types of received signal characteristics. We study this structure for the small signal situation and for the case of signals composed of gaussian amplitude perturbations on some mean signal. The signal information required to specify these processors is discussed and related to the target-medium filters introduced in Chapter 2.

2. SONAR SIGNALS

In a situation involving surveillance of submarine motion within an ocean area by a sonar system, the ultimate information required from the system must relate to the movements of targets within the surveyed area. The most important information concerning each target is its position and velocity relative to the sonar system as functions of time. Relative position is described in terms of range R, bearing θ and depth d whereas relative velocity is described in terms of relative radial velocity v and target aspect angle φ. Target parameters are referred to a point which is fixed relative to the sonar system, e.g. one of the hydrophones in the receiving array. The parameters will generally take on different values as we move over the aperture of the receiving array.

It is in fact this variation which enables us to measure target bearing and aspect angle.

With an active sonar system the desired target information can be estimated from the temporal and spatial structure of the received echo signals. Thus range is determined from the absolute delay between a sonar transmission and the received echo signal whereas bearing is determined from the relative arrival times of echoes on the hydrophones of the receiving array. Relative radial velocity is determined by the change of timescale of the echo signals from that of the transmitted signal and aspect angle is determined from combined measurements of range and bearing of the individual scatterers of the target. (A system with high bearing and range resolution capabilities is required for such measurements.) Finally, target depth may be estimated from the vertical direction of incidence of the echo signal or in some situations from the multipath structure of the echo signals.

The number of parameters that influence the temporal and spatial structure of echo signals are much greater than the five mentioned above. Some of these are associated with the targets e.g. its orientation, dimensions and reflecting properties, and some are associated with the acoustic transmission properties of the ocean e.g. the refracting and scattering properties of the ocean volume and the reflecting properties of the ocean boundaries. We know also that some of these parameters are non-random while others are random timevariable quantities. The group of target parameters which we wish to determine by our sonar system will be considered to be the elements of a target parameter vector $\vec{\alpha}$. The remaining parameters which influence the echo signals will be considered to be elements of a random vector $\vec{r}$. Although all the elements of $\vec{r}$ may not be truly random in nature they will be considered to be so from the point of view of the sonar receiver which must be optimum on the average against these unknown parameters. The upper diagram of fig. 5 illustrates the important fact that there are several possible echo signals $\vec{s}\,(\vec{\alpha})$ for each value of $\vec{\alpha}$. In the remaining part of this chapter we shall introduce the concept of target impulse responses and show how these can be used to relate the target parameter vector $\vec{\alpha}$ to a family of received echo signals $\vec{s}\,(\vec{\alpha})$.

Let there be A hydrophones in our receiving array and let $h_i\,(t_1, t_2, \vec{\alpha})$, $i = 1, 2\ldots\ldots A$, be the echo signal received at time t_2 on the i'th hydrophone from a target with parameters $\vec{\alpha}$ in response to an impulse emitted from the transmitter at time t_1 (Ref. 1). We note that $\vec{\alpha}$ expresses the target parameters at the moment the impulse impinges on the target. When measuring $h_i\,(t_1, t_2, \vec{\alpha}\,)$ we introduce $t_2 = t_1 + \tau$ and study the variations with τ for fixed values of t_1. However, when used in convolution integrals to find the signal received on the i'th hydrophone at time t_2 in response to a transmitted signal $s(t)$ we introduce $t_1 = t_2 - \tau$ and consider it as a function of τ for fixed values of t_2. This latter attitude will be maintained throughout this chapter. For convenience we shall write $t_2 = t$, $t_1 = t - \tau$, and $h_i\,(t - \tau\,, t, \vec{\alpha}\,) = h_i\,(\,\tau\,, t, \vec{\alpha})$. As a function of τ we assume $h_i\,(\,\tau\,, t, \vec{\alpha}\,)$ to be the echo signal as seen through the input bandwidth B of the sonar system. As a function of time t, it is assumed to be a stationary random function of bandwidth $\ll B$. It reflects the randomness residing in the target and the transmission medium. The set of impulse responses received on the A hydrophones will be represented by the vector valued function

$$\vec{h}(\tau, t, \vec{\alpha}) = \left[h_i(\tau, t, \vec{\alpha}) \right], \qquad i = 1, 2 \ldots A. \qquad \text{(Eq. 1)}$$

The echo signals received from the target in response to a transmitted sonar signal $s(t)$ can now be found by convolving $s(t)$ with the individual impulse **responses.** The set of received signals will be expressed by the vector valued function

$$\vec{s}(t, \vec{h}) = \left[s_i(t, h_i) \right], \qquad i = 1, 2 \ldots A, \qquad \text{(Eq. 2)}$$

where we have written $\vec{h}$ for $\vec{h}(\tau, t, \vec{\alpha})$, h_i for $h_i(\tau, t, \vec{\alpha})$ where $s_i(t, h_i)$ is obtained by convolving $s(t)$ with $h_i(\tau, t, \vec{\alpha})$;

$$s_i(t, h_i) = s(t) * h_i(\tau, t, \vec{\alpha}). \qquad \text{(Eq. 3)}$$

A simple diagram illustrating the formation of $s_i(t, h_i)$ from $s(t)$ is shown in fig. 3a. For subsequent discussions we shall find it convenient to change each $h_i(\tau, t, \vec{\alpha})$ into two parallel connected filters $g_i(\tau, t, \vec{\alpha})$ and $r_i(\tau, t, \vec{\alpha})$ as shown in fig. 3b. The set of functions expressed by

$$\vec{g}(\tau, t, \vec{\alpha}) = \left[g_i(\tau, t, \vec{\alpha}) \right], \qquad i = 1, 2 \ldots A, \qquad \text{(Eq. 4)}$$

describe the average impulse responses of the target-medium combination as seen on the various hydrophones. Thus $g_i(\tau, t, \vec{\alpha})$ is the average signal received on the i'th hydrophone at time t from the ensemble of target-medium combinations corresponding to a parameter

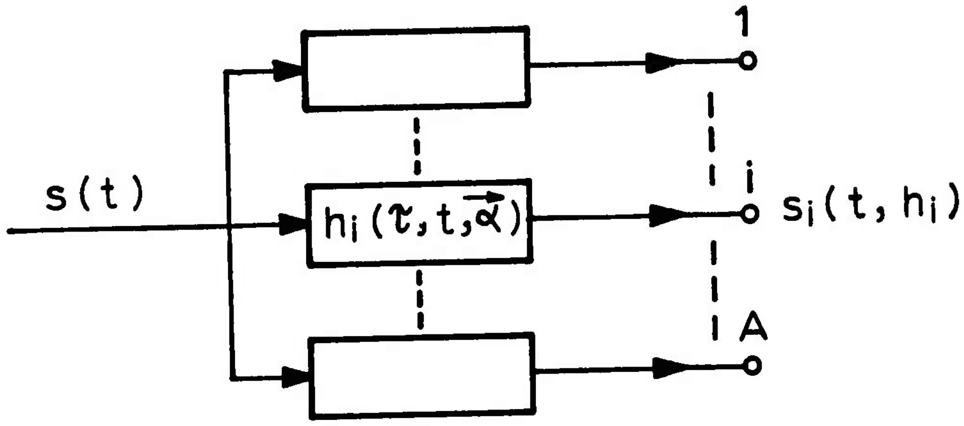


Fig. 3a THE IMPULSE RESPONSE FILTERS

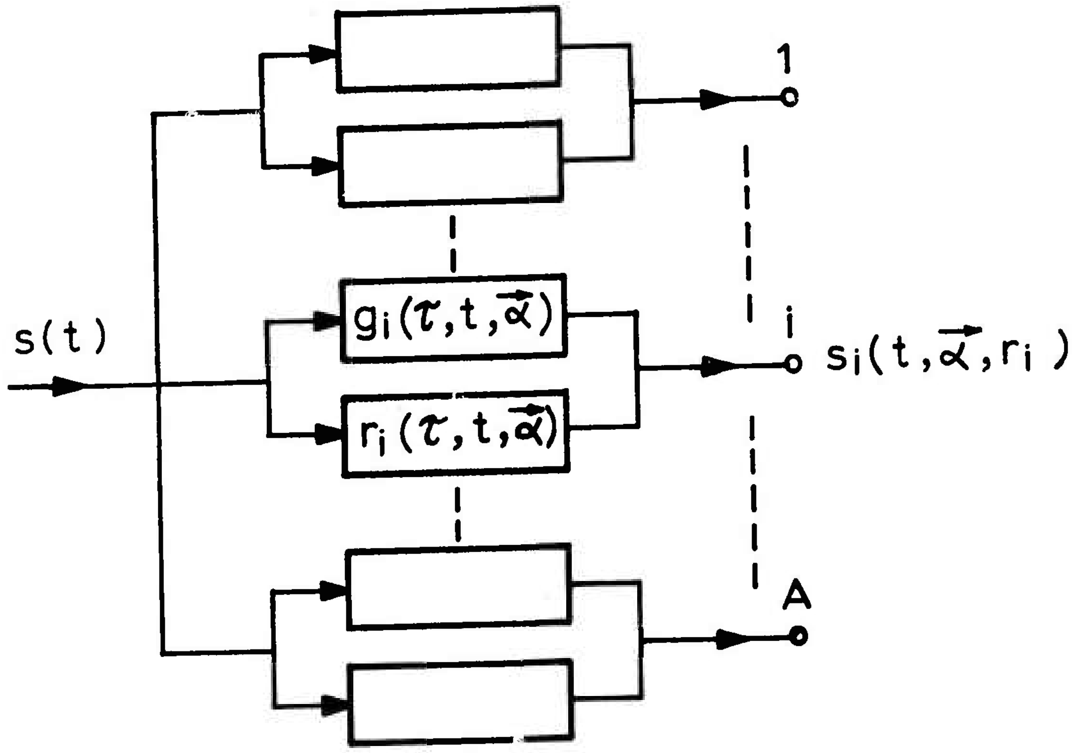


Fig. 3b THE MODIFIED IMPULSE RESPONSE FILTERS

vector $\vec{\alpha}$ in response to a unit impulse emitted from the transmitter at time $t - \tau$. The set of random filters described by

$$\vec{r}(\tau, t, \vec{\alpha}) = \left[r_i(\tau, t, \vec{\alpha}) \right], \qquad i = 1, 2 \ldots A, \qquad \text{(Eq. 5)}$$

incorporate the effects of all parameters not contained in $\vec{\alpha}$. The output functions are expressed by the vector valued function

$$\vec{s}(t, \vec{\alpha}, \vec{r}) = \left[s_i(t, \vec{\alpha}, \vec{r}_i) \right], \qquad i = 1, 2 \ldots A, \qquad \text{(Eq. 5)}$$

where we have written $\vec{r}$ for $\vec{r}(\tau, t, \vec{\alpha})$ and r_i for $r_i(\tau, t, \vec{\alpha})$ and where

$$s_i(t, \vec{\alpha}, r_i) = s(t) * \left[g_i(\tau, t, \vec{\alpha}) + r_i(\tau, t, \vec{\alpha}) \right] \qquad \text{(Eq. 7)}$$

Figure 4 illustrates the shape of $g_i(\tau, t, \vec{\alpha})$ and $r_i(\tau, t, \vec{\alpha})$ as functions of τ for constant t. The example refers to a linear receiving array, an extended target and a multipath transmission medium.

It was stated in the introduction that decisions about the target situation were to be based upon measurements on the acoustic field propagating to the receiver. In the following we shall consider a "measurement" to be some approximate finite-dimensional representation of the acoustic pressure signals received on the hydrophones of the receiving array during one ping interval T_0 i.e. the interval between two transmissions. We shall be specific in our choice of representation and consider a measurement to be the amplitude values obtained by sampling the acoustic pressure signals received on the hydrophones of the

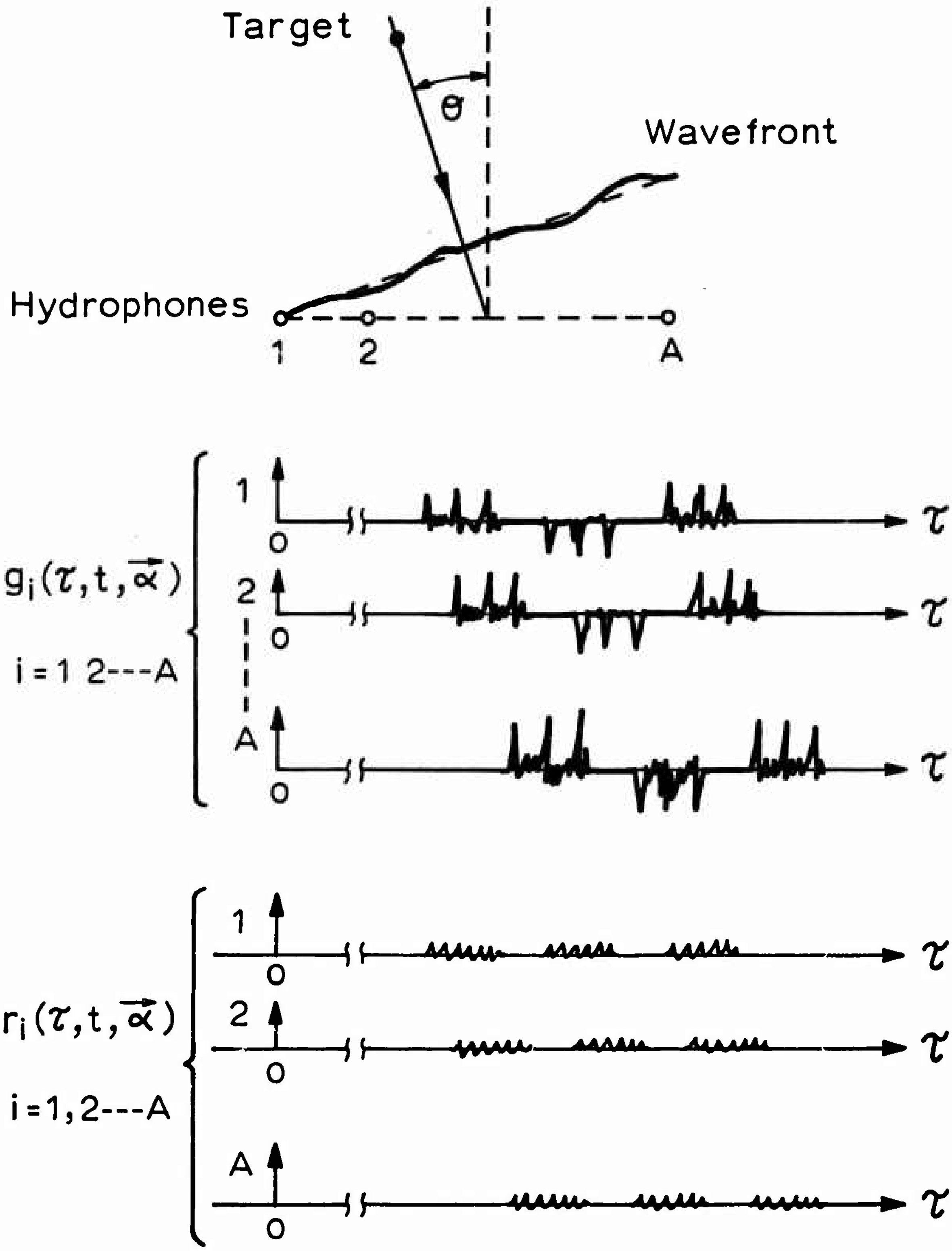


Fig. 4 EXAMPLE OF IMPULSE RESPONSES

receiving array at a rate of 2B samples per second where B is the bandwidth of the sonar signals. We assume for simplicity that the spectra of the sonar signals have been shifted down into a lowpass band of width B. The dimension of a measurement is therefore $K = 2 \cdot A \cdot B \cdot T_o$ and the dimension of an echo signal vector $2 \cdot A \cdot B \cdot T_s$ where T_s is the total length of the signal. The sample values of a measurement arranged in a suitable sequence form the elements of a measurement vector $\vec{x}$. Applying our sampling scheme to the signals $\vec{s}(t, \vec{\alpha}, \vec{r})$ of equation 5 we obtain the random signal vector $\vec{s}(\vec{\alpha})$. For a given $\vec{\alpha}$ the signal vector exhibits random fluctuations from one ping interval to the next. The fluctuations are caused by variations in the random response functions expressed by $\vec{r} = \vec{r}(\tau, t, \vec{\alpha})$.

In later chapters it will be convenient to let the dimension of the signal vector be equal to that of the measurement vector. We achieve this by assuming the signal vector to have one element for each sampling point in T_o. Elements corresponding to sample points outside the signal intervals are zero.

3. ELEMENTS OF DECISION SYSTEMS

The task of the decision unit of fig. 2 is to decide which targets, if any, are present in the surveyed area. Targets are identified by their parameter vectors $\vec{\alpha}$. Decisions are made at the end of each ping interval and are based upon the measurement $\vec{x}$ received in the corresponding ping interval. The vector $\vec{x}$ contains a noise component $\vec{n}$ comprising all undesired signals appearing at the hydrophone output terminals including system circuit noise referred to the hydrophone terminals. The principal components of $\vec{n}$ will be ambient noise, reverberations and in some cases flow and cavitation noise. If a target with parameter vector $\vec{\alpha}$ is present a corresponding random signal vector $\vec{s}(\vec{\alpha})$ is embedded in the noise $\vec{n}$. We shall make the simplifying assumption that different values of $\vec{\alpha}$ are mutually exclusive i.e. at most one target can be present in the surveyed area in any one ping interval. This assumption does not alter the basic structure of the signal processing device of the sonar receiver. We shall furthermore assume that values of $\vec{\alpha}$ occurring in consecutive ping intervals are statistically independent. This is not strictly true since the limited speed and manouvring capabilities of real targets introduce dependence between successive $\vec{\alpha}$ values.

Decision processes are generally of two kinds: detection processes, which decide whether any signal among the possible signals is present, and estimation processes, which decide which signal or signals are considered to be present when a detection has been made. It may be shown that two types of input information are required for either of the two processes. One is the conditional probability density $p(\vec{\alpha}/\vec{x})$ expressing the probability of the various target vectors $\vec{\alpha}$ after the measurement $\vec{x}$ has been acquired. For the detection process we also require the posterior probability $p(0/\vec{x})$ of the signal-absent situation. We obtain this from equation 10 by inserting $\vec{\alpha} = 0$. The other input to the decision device is a cost function expressing the quality of decisions. The cost function is generated by the user of the system and may vary with time. In this paper we shall only be interested in the function $p(\vec{\alpha}/\vec{x})$ which expresses the information we need for the decision process from the measurements $\vec{x}$. We shall proceed to study this function in some detail and shall find that we are usually only able to generate an approximation to it. Figure 5 illustrates the relations between the $\vec{\alpha}$, $\vec{s}(\vec{\alpha})$, and $\vec{x}$ spaces. The transitions from the $\vec{\alpha}$-space to the $\vec{s}(\vec{\alpha})$-space are governed by the probability density function $p\left[\vec{s}(\vec{\alpha})/\vec{\alpha}\right]$ which reflects the statistics of the random response functions described by $\vec{r} = \vec{r}(\tau, t, \vec{\alpha})$ of equation 5. The transitions from the $\vec{s}(\vec{\alpha})$-space to the $\vec{x}$-space is governed by the probability density function $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$ which depends upon the statistics of the noise $\vec{n}$. Since the noise is additive we can write

$$p\left[\vec{x}/\vec{s}(\vec{\alpha})\right] = p_n\left[\vec{x} - \vec{s}(\vec{\alpha})\right] \qquad \text{(Eq. 8)}$$

where $p_n(\vec{n})$ is the probability density function of the noise $\vec{n}$. The probability density function $p(\vec{x}/\vec{\alpha})$ governing the transition from the $\vec{\alpha}$-space to the $\vec{x}$-space can now be written

$$p(\vec{x}/\vec{\alpha}) = \int_{S} p\left[\vec{x}/\vec{s}(\vec{\alpha})\right] \cdot p\left[\vec{s}(\vec{\alpha})/\vec{\alpha}\right] \cdot d\vec{s}(\vec{\alpha}) \qquad \text{(Eq. 9)}$$

where the integral is over the entire $\vec{s}(\vec{\alpha})$-space. Applying the Bayes Theorem we obtain the desired function

$$p(\vec{\alpha}/\vec{x}) = \frac{p(\vec{\alpha})\, p(\vec{x}/\vec{\alpha})}{p(\vec{x})} \qquad \text{(Eq. 10)}$$

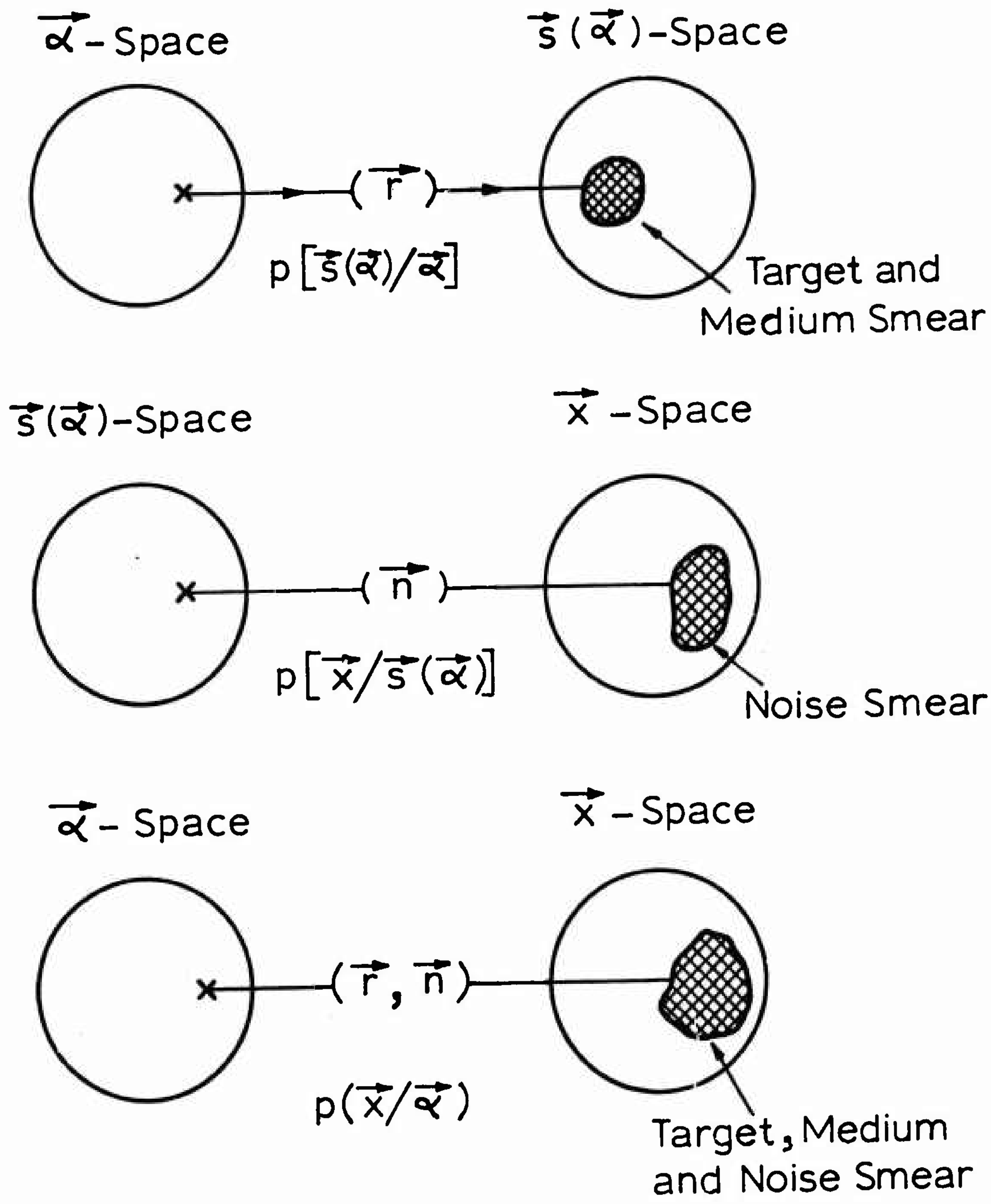


Fig. 5 RELATION BETWEEN $\vec{\alpha}$, $\vec{s}(\vec{\alpha})$ AND $\vec{x}$-SPACES

Since $p(\vec{x})$ is a constant for a given measurement we shall write

$$p(\vec{x}) = C^{-1}$$

and

$$p(\vec{\alpha}/\vec{x}) = C \cdot p(\vec{\alpha}) \cdot p(\vec{x}/\vec{\alpha}) \qquad \text{(Eq. 11)}$$

where C is such that the integral of $p(\vec{\alpha}/\vec{x})$ w.r.t $\vec{\alpha}$ equals one. The function $p(\vec{\alpha})$ is the probability density function of $\vec{\alpha}$ prior to the measurement $\vec{x}$. It has frequently been argued that the function $p(\vec{\alpha})$ is difficult to define or devoid of physical meaning. We shall here take the attitude that the form of $p(\vec{\alpha})$, whether of the minimax type, constant probability type or otherwise, must be chosen by the system user in accordance with the situation he believes himself to be in. Presumably the system user is the best source of prior probabilities; he is in any case the only one available. The function $p(\vec{x}/\vec{\alpha})$ occurring in equation 10 is the one we wish to obtain from the processing unit of the sonar receiver in fig. 2. This function will be the principal theme of Chapters 4 and 5.

We remark at this point that equation 11 may be extended to a situation in which it is desired to make decisions based on a sequence of m consecutive measurements $\vec{x}_1, \vec{x}_2 \ldots \vec{x}_m$ i.e. to situations in which it is desired to take into account the statistical dependence between $\vec{\alpha}$ values obtained from a target in m consecutive ping intervals.

4. ADAPTIVE PROCESSING

The structure of the signal processing device of a detection system will often depend upon one or more parameters characterizing the input signals. If one such parameter is known to exhibit random variations with time, the structure of the processor can be arrived at in two ways. If the variations are stationary in nature and their statistical properties are known we can choose that processor which gives the best average performance. Alternatively we may carry out measurements in order to estimate the true value of the parameter and adjust the structure of the processor in accordance with the estimate. This technique reduces the uncertainty about the unknown parameter and improves the system performance. The latter type of processor is referred to as an adaptive processor. In this chapter we shall study processors which adapt to a randomly varying noise field.

It was pointed out in the previous section that the information required for the decision process from the measurements $\vec{x}$ was contained in the function $p(\vec{x}/\vec{\alpha})$ of equation 9. The discussion about adaptive systems will relate to the structure of the processing device from which $p(\vec{x}/\vec{\alpha})$ is obtained. We reproduce equation 9

$$p(\vec{x}/\vec{\alpha}) = \int_S p\left[\vec{x}/\vec{s}(\vec{\alpha})\right] \cdot p\left[\vec{s}(\vec{\alpha})/\vec{\alpha}\right] \cdot d\vec{s}(\vec{\alpha})$$

and recall that the function $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$ under the integral sign is determined by the probability density function of the noise vector $\vec{n}$ in accordance with equation 8, and that $p\left[\vec{s}(\vec{\alpha})/\vec{\alpha}\right]$ reflects the statistical properties of the targets and the transmission medium.

Thus to obtain $p(\vec{x}/\vec{\alpha})$ we require a noise-dependent processing device to evaluate $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$ for each $\vec{s}(\vec{\alpha})$ and an averaging device depending upon $p\left[\vec{s}(\vec{\alpha})/\vec{\alpha}\right]$. These are shown in fig. 3. We shall proceed to study the noise-dependent processor and shall leave the discussion of the averaging device to section 5.

The discussion will proceed under the assumption that the noise is gaussian. This implies that the noise-dependent processor depends only upon the second order statistical properties of the noise field. We remark that a gaussian-equivalent suboptimal system may be defined from the second order statistics of any non-gaussian noise field. Under the gaussian hypothesis the noise vector $\vec{n}$ is a K-dimensional gaussian random variable with probability density function (Ref. 5 Chap. 24)

$$p_n(\vec{n}) = (2\pi)^{-\frac{K}{2}} \cdot (\mathrm{Det}\ \vec{M})^{-\frac{1}{2}} \cdot \exp - \tfrac{1}{2}\vec{n}^{\,t} \cdot \vec{Q} \cdot \vec{n} \qquad \text{(Eq. 12)}$$

where $\vec{n}$ is the noise column vector with elements n_i, $i = 1, 2 \ldots K$,

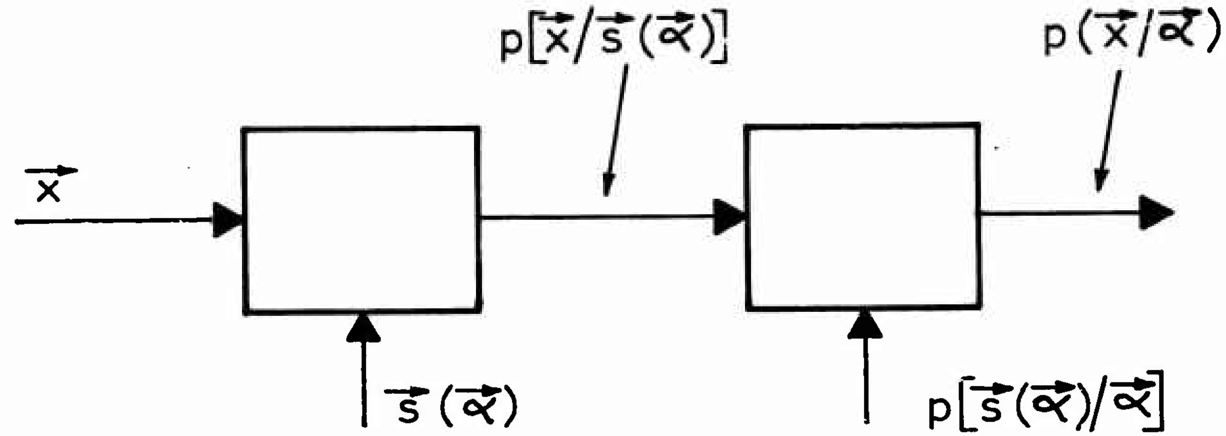
$\vec{n}^{\,t}$ is the transpose of $\vec{n}$,

$\vec{Q} = \vec{M}^{-1}$ is the inverse moment matrix,

$\vec{M} = \left[m_{ij}\right]$ is the moment matrix of $\vec{n}$ with $m_{ij} = E\left[n_i \cdot n_j\right]$,

Det $\vec{M}$ = the determinant of $\vec{M}$

We note at this point that the matrices $\vec{M}$ and $\vec{Q}$ are real and symmetrical such that $\vec{M} = \vec{M}^t$ and $\vec{Q} = \vec{Q}^t$. By equation 8 we are now able to write



$$p(\vec{x}/\vec{\alpha}) = \int^{S} p[\vec{x}/\vec{s}(\vec{\alpha})] \cdot p[\vec{s}(\vec{\alpha})/\vec{\alpha}] \cdot d\vec{s}(\vec{\alpha})$$

Fig. 6 THE SIGNAL PROCESSOR

$$p\left[\vec{x} / \vec{s}(\vec{\alpha})\right] = (2\pi)^{-\frac{K}{2}} \cdot (\mathrm{Det}\ \vec{M})^{-\frac{1}{2}} \exp - \tfrac{1}{2}(\vec{x}^t - \vec{s}^t) \cdot \vec{Q} \cdot (\vec{x} - \vec{s})$$

$$= D_n \cdot \exp(\vec{x}^t \cdot \vec{Q} \cdot \vec{s} - \tfrac{1}{2}\vec{s}^t \cdot \vec{Q} \cdot \vec{s}) \qquad \text{(Eq. 13)}$$

where

$$D_n = (2\pi)^{-\frac{K}{2}} \cdot (\mathrm{Det}\ \vec{M})^{-\frac{1}{2}} \vec{x}^t \cdot \vec{Q} \cdot \vec{x}$$

and where for simplicity we have written $\vec{s}$ for $\vec{s}(\vec{\alpha})$. Fig. 7 illustrates the formation of the exponent

$$z = \vec{x}^t \cdot \vec{Q} \cdot \vec{s} - \tfrac{1}{2}\vec{s}^t \cdot \vec{Q} \cdot \vec{s} \qquad \text{(Eq. 14)}$$

of equation 13. In many practical situations the elements of the matrix $\vec{Q} = \left[q_{ij}\right]$ will exhibit random variations with time. A system which keeps track of the elements q_{ij} and adjusts the processors F_1 and F_2 of Fig. 7 in accordance with their values is the adaptive system we are seeking. For reasons which will become clear later we shall refer to this as a regression type system. Before we study this in detail similar adaptive systems based upon second order noise statistics will be reviewed briefly.

Three types of signal filters applicable to detection and extraction systems have received much attention in recent years. They are

(1) the maximum signal-to-noise ratio filter (Ref. 11)

(2) the minimum signal distortion or Wiener filter (Ref. 3) and

(3) the regression type filter (Ref. 2).

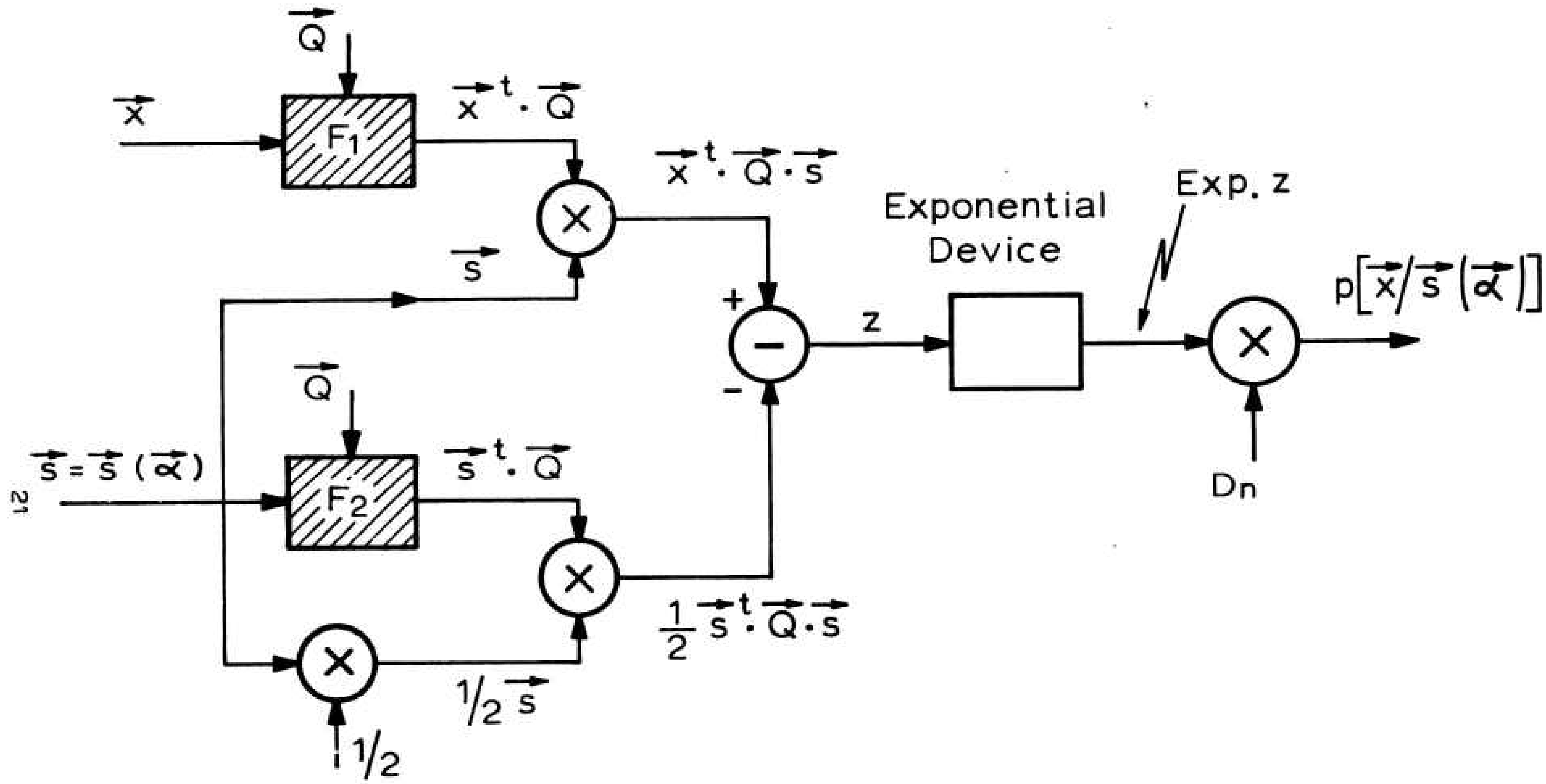


Fig. 7 SIGNAL PROCESSOR FOR z AND $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$

When used to detect monochromatic signals the spatial filtering properties of these filters have been shown to be identical (Ref. 6). Their temporal filtering properties are, however, different, as are also their areas of application. In principles the three filter types can operate in an adaptive mode. An adaptive "maximum signal-to-noise ratio" filter has been described by Shor (Ref. 12) and an adaptive Wiener filter by Widrow et al (Ref. 13). This last article is the principal reference of this chapter. The adaptive technique suggested for the regression filter to be presented is taken directly from the work of Widrow et al.

In order to expose the salient features of the adaptive regression system and avoid undue compexity we shall assume that the sonar system is ambient noise limited and that the variations in q_{ij} are small over one interval T_0. The reverberation field associated with a moving sonar platform is considered to exhibit too rapid variations to be amenable to adaptive techniques. For a stationary platform the reverberation field, although rapidly varying within each ping interval, may exhibit slow variations from one ping interval to the next when values at identical positions within the ping intervals are considered. Adaptive techniques may therefore be applied.

Proceeding with the ambient-noise-limited case we turn to Fig. 8 which illustrates a composite measurement $\vec{y}_m$ consisting of the j measurement vectors preceding the vector $\vec{x}_m$. Thus $\vec{y}_m$ contains the input data received in an interval of length $T_y = j.T_0$. We shall think of T_y as an interval over which $\vec{Q}$ remains essentially constant. Provided no signals are present in the measurement $\vec{y}_m$ and provided the variations in the elements of the matrix $\vec{Q}$ are stationary with known

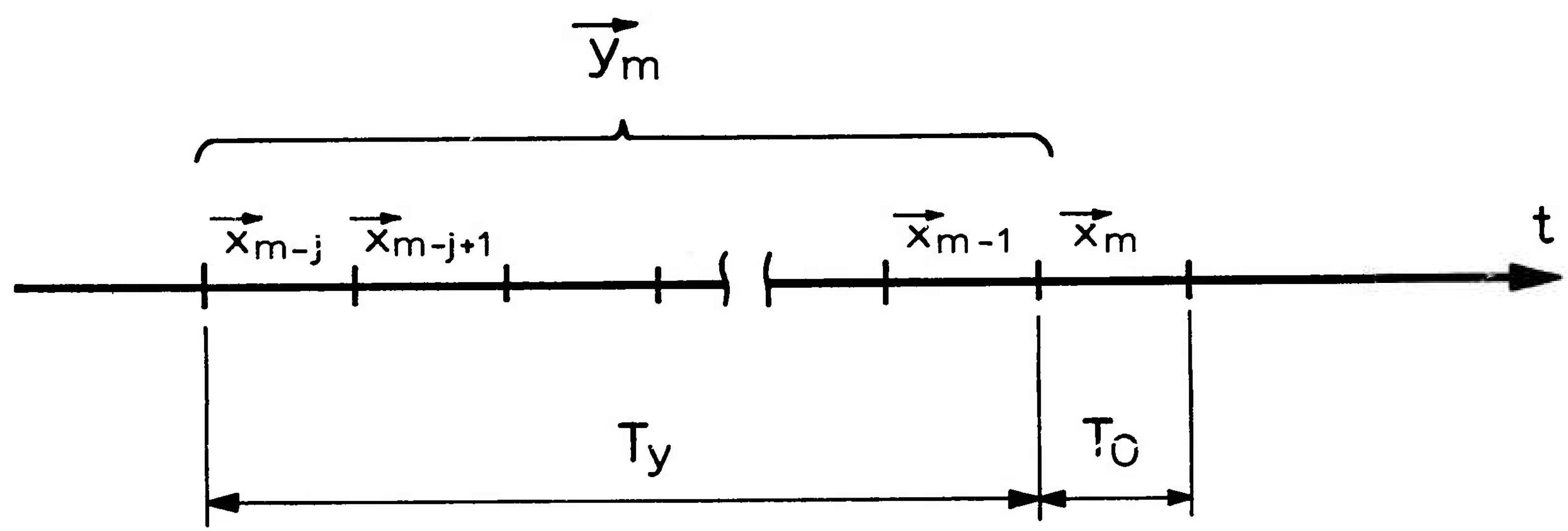


Fig. 8 THE LEARNING AND DECISION INTERVALS

statistical properties we can obtain a posterior probability density function $p(\vec{Q}/\vec{y}_m)$ of $\vec{Q}$. In principle this should be used to obtain the conditional forward probability function

$$p\left[\vec{x}/\vec{s}(\vec{\alpha}),\vec{y}\right] = \int^{Q} p\left[\vec{x}/\vec{s}(\vec{\alpha}),\vec{Q}\right] \cdot p(\vec{Q}/\vec{y}) \cdot d\vec{Q} \qquad \text{(Eq. 15)}$$

which takes the place of $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$ in equation 9. In many cases the statistical properties of $\vec{Q}$ are unknown such that $p(\vec{Q}/\vec{y})$ can not be obtained. We can then choose to make estimates of $\vec{Q}$ based on the composite measurement vector $\vec{y}$ and use the estimates in equation 14 to generate z. The corresponding processor of Fig. 7 is then adapting to a changing matrix $\vec{Q}$. We shall study two techniques for estimating the matrix $\vec{Q}$ from the measurements $\vec{y}$. Strictly speaking we require that the $\vec{y}$'s do not contain any signal components. Thus, prior to each decision interval (corresponding to $\vec{x}_m$ in Fig. 8), we require a learning phase of length T_y in which the characteristics of the noise field are assessed. This feature appears to be common to all noise adapting systems. In real systems the decision interval is pushed forward along the time axis an amount T_o at the beginning of each ping interval. The noise adaption process will therefore proceed smoothly up to the point when a target signal appears in the received waveforms. From then and onwards the adaptive processor will consider the target signal as part of the noise field. The extent to which it will succeed in suppressing the signal depends upon the angular velocity of the target relative to the settling time of the processor. (The settling time is the time required for the adaptive processor to respond to a change in the ambient noise field.)

The first type of adaptive technique will be touched upon very lightly. It is the direct approach of estimating $\vec{Q}$ through estimates of the moment matrix $\vec{M}$. Thus estimates of the elements m_{ij} of $\vec{M}$ are obtained in each learning interval T_y whereupon $\vec{Q}$ is obtained through matrix inversion. If the power spectra of the received noise waveforms and the power spectra of the variations in the moments m_{ij} are known it is a simple task to design good estimators for the moments m_{ij}. This technique is rather impractical since it requires a large number of correlators for determination of the moments m_{ij} and since it involves the operation of matrix inversion.

The second type of adaptive technique is based upon the gradient-search method discussed in Ref. 13 . In order to understand this method it is necessary to point out certain properties of the matrix $\vec{Q}$ and the vector

$$\vec{v} = \vec{Q} \cdot \vec{x} \qquad \text{with elements}$$

$$v_h = \sum_{\ell=1}^{K} q_{h\ell} \cdot x_\ell \quad , \qquad h = 1, 2, \ldots K \qquad \text{(Eq. 16)}$$

appearing in the expression for z in equation 14. Consider first the case of a time-invariant gaussian noise field with an associated $K \times K$ dimensional symmetrical noise moment matrix $\vec{M}$ with inverse $\vec{Q}$. Assuming noise only to be present, i.e. $\vec{x} = \vec{n}$, it may be shown that v_h of equation 16 is equal to the residue η_h of the h'th observation element x_h in $\vec{x}$ divided by the variance $< \eta_h^2 >$ of this residue (Ref. 5 Chap. 23). Thus

$$v_h = \eta_h \;/\; <\eta_h^2> , \qquad h = 1, 2, \ldots K \qquad \text{(Eq. 17)}$$

It is important to grasp the physical significance of the residue η_h. It represents the difference between the observation element x_h and the best linear estimate $\hat{x}_h$ of x_h obtained from the remaining K-1 elements of $\vec{x}$. Thus

$$\eta_h = (x_h - \hat{x}_h)$$

with (Eq. 18)

$$\hat{x}_h = \sum_{\ell \neq h} \beta_{h\ell} \cdot x_\ell$$

and where the constants $\beta_{h\ell}$ are adjusted to give minimum of the residue variance $< \eta^2_h >$. (The symbol $\ell \neq h$ under the summation sign indicates summation over all ℓ except $\ell = h$.) By the gradient search technique the regression coefficients $\beta_{h\ell}$ are adjusted in small steps in a systematic way such that the minimum value of $< \eta^2_h >$ is approached along the path of steepest descent. To obtain the desired quantities v_h of equation 17 the residues η_h thus obtained must be divided by estimates of the residue variance $< \eta^2_h >$ The gradient-search method will now be explained in some detail.

Referring to equation 18 we shall represent the set of coefficients $\beta_{h\ell}$ associated with the estimate $\hat{x}_h$ by the vector $\vec{\beta}_h$. Thus

$$\vec{\beta}_h = \left[\beta_{h\ell}\right], \quad \ell = 1, 2, \ldots (h-1), (h+1) \ldots K \qquad \text{(Eq. 19)}$$

Since $\vec{\beta}_h$ will change with time we shall write $\vec{\beta}_h(j)$ for the value of $\vec{\beta}_h$ in the j'th ping interval. By the gradient-search technique we apply the following recurrence formula to $\vec{\beta}_h$

$$\vec{\beta}_h (j+1) = \vec{\beta}_h (j) + k_s \cdot \nabla < \eta^2_h (j) > \qquad \text{(Eq. 20)}$$

where

k_s = negative scalar constant controlling the rate of convergence and stability.

$\nabla < \eta^2_h (j) >$ = gradient vector of residue variance with respect to $\vec{\beta}_h$.

In practical situations we do not have access to the true gradient vectors. Following Ref. 13 we shall use the gradient of $\eta^2_h (j)$ as estimates for $\nabla < \eta^2_h (j) >$. From equation 18 this gradient is

$$\nabla \eta^2_h (j) = -2 \eta_h (j) \cdot \vec{x}_h (j) \qquad \text{(Eq. 21)}$$

where $\vec{x}_h (j)$ is the observation vector corresponding to the j'th ping interval with the h'th element deleted. Substituting this result for the gradient vector of equation 20 we obtain the final recurrence formula

$$\vec{\beta}_h (j+1) = \vec{\beta}_h (j) - 2 k_s \cdot \eta_h (j) \cdot \vec{x}_h (j) \qquad \text{(Eq. 22)}$$

Widrow et al show that the estimates $\nabla \eta^2_h (j)$ are unbiased and that the elements of $\vec{\beta}_h (j+1)$ converge in the mean towards the true regression coefficients. Figure 9 illustrates the adaptive formation of the quantities v_h of equation 17. The estimate of the residue variance $< \eta^2_h (j) >$ is a suitably weighted sum of the square of past values of η_h. The number of terms in the summation depends on the rate of variation of the noise field.

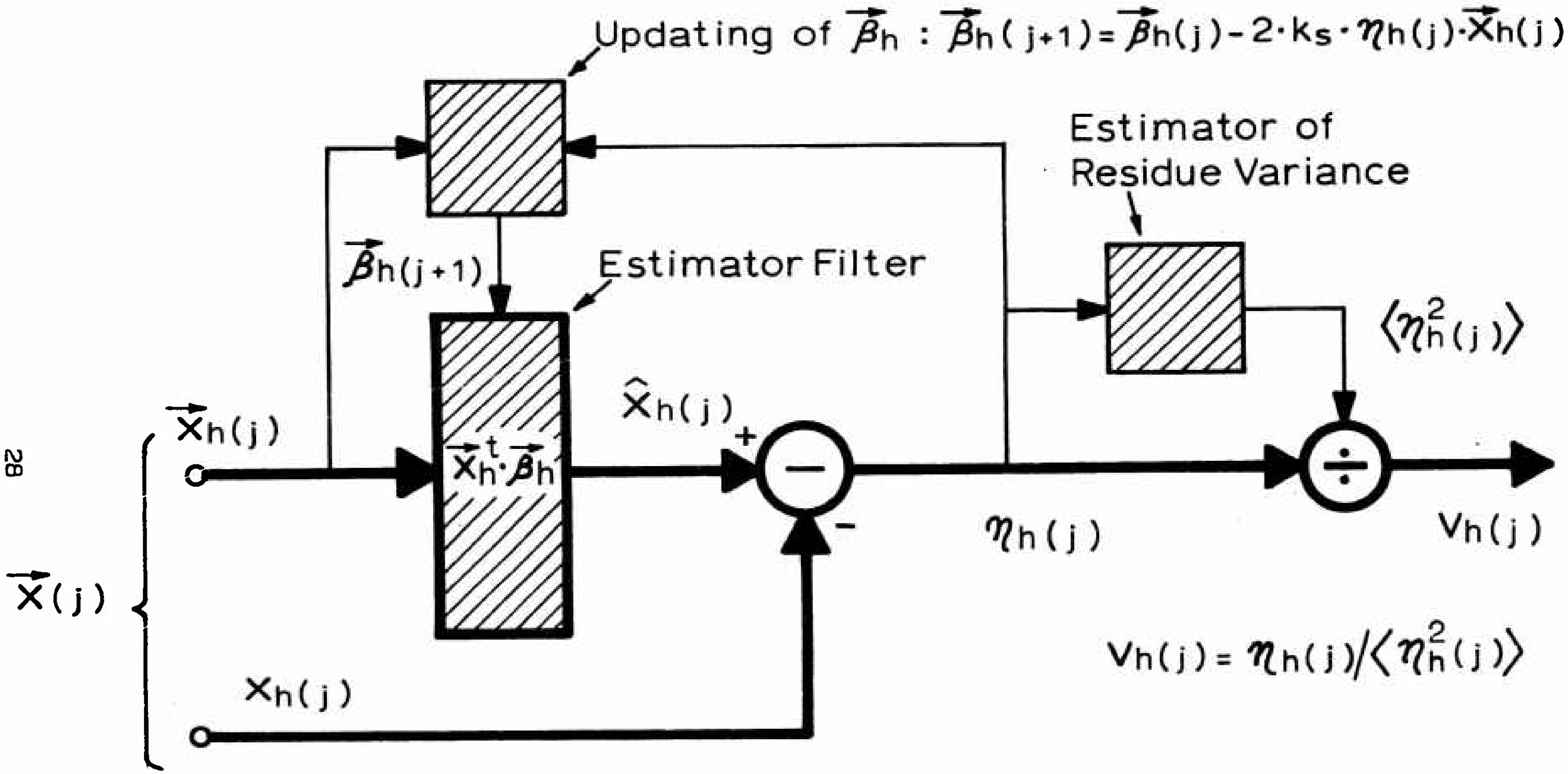


Fig. 9 THE ADAPTIVE PROCESSOR

Up to this point the discussion of the adaptive regression system has been centred upon the residue of a single element of $\vec{x}$. At a first glance it may appear that a separate adaption process must be instrumented for each of the K residues of the observation vector $\vec{x}$. We shall show, however, that one adaptive processor for each hydrophone channel is sufficient. To see this we change our point of view slightly and consider the regression processor as a digital filter applied to the sampled input waveforms. Fig. 10 illustrates the samples belonging to an observation vector $\vec{x}$ arranged in A discrete time series, one for each hydrophone. We focus attention on the observation element x_h which occupies the central position among the samples from the i'th hydrophone. Next we define an adaptive digital filter with weights $\beta_{h\ell}$ by the relation

$$\eta_h = \sum_{\ell=1}^{K} \beta_{h\ell} \cdot x_\ell \qquad \text{(Eq. 23)}$$

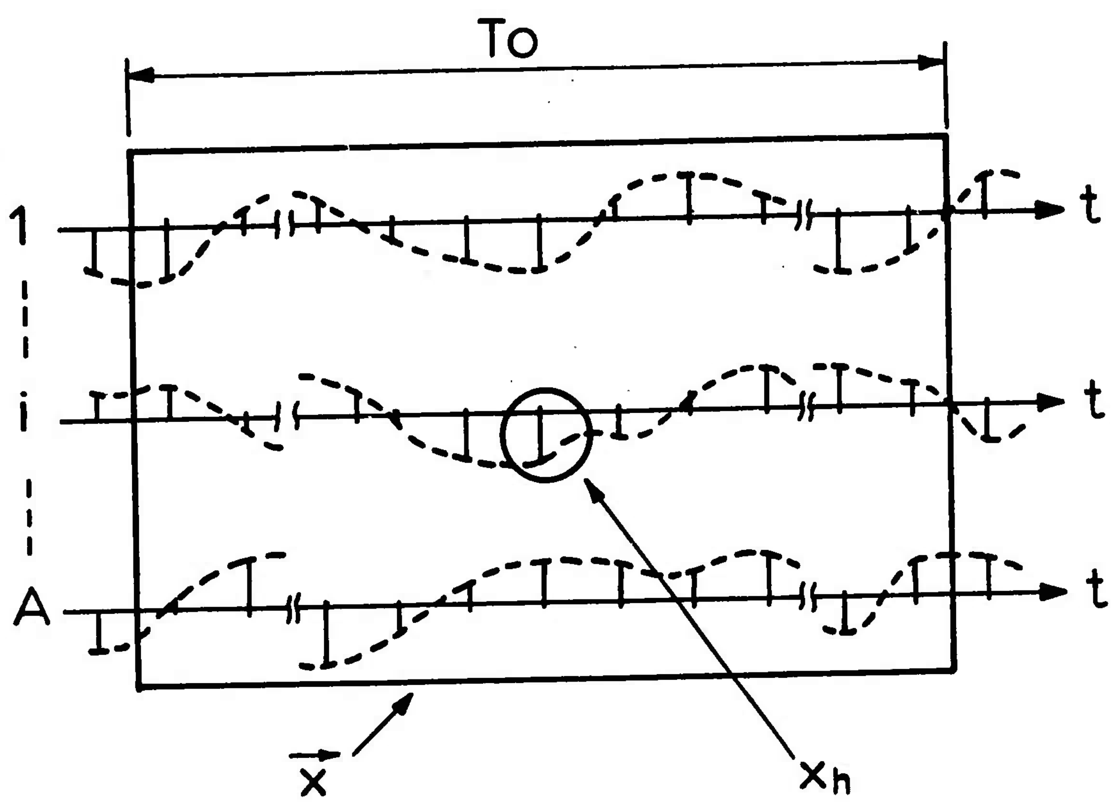
where

η_h = residue of observation element x_h

β_{hh} = - 1

$\beta_{h\ell}$, $\ell \neq h$, are the variable regression coefficients which are the elements of $\vec{\beta}_h$ of equation 19 .

Equation 23 is obtained from equation 18 by introducing $\beta_{hh} = -1$. The filter defined by equation 23 is the adaptive regression filter for the waveform from the i'th hydrophone. As new samples are shifted into the filter successive samples of the residue waveform from the i'th hydrophone are generated at the output.



$$\eta_h = x_h - \sum_{l \neq h} \beta_{hl} \cdot x_l$$

$$\eta_h = -\sum_{l=1}^{K} \beta_{hl} \cdot x_l \quad , \quad \beta_{hh} = -1$$

Fig. 10 ILLUSTRATION TO THE FORMATION OF η_h FROM $\vec{x}$

In changing from the purely vector-oriented regression processor of equation 18 to the filter-oriented processor associated with equation 23 we have implicitly changed our concepts about the observations $\vec{x}$. Originally the observations were generated at the rate of one per T_0 seconds, i.e. one per ping interval. For the filter-oriented processor the observations change by A new samples introduced and A old samples discarded in each ping interval. In order to bring out clearly the filter point of view we modify the terminology of equation 23 as follows

$$\eta_h^{(i)} = \sum_{\ell=1}^{K} \beta_\ell^{(i)} \cdot x_{h\ell} \quad , \quad i = 1, 2, \ldots K \qquad \text{(Eq. 24)}$$

where

$\eta_h^{(i)}$ is the output of the i'th filter in the h'th sampling interval,

$x_{h\ell}$ is the observation element occupying position "ℓ" in the observation centred on sampling time "h", and

$\beta_\ell^{(i)}$ is the filter weight occupying position "ℓ" among the filter weights of the i'th filter.

At this point we mention that in real systems we shall expect that only a limited number of observation elements $x_{h\ell}$ in the neighbourhood of sampling time "h" will have significant influence upon $\eta_h^{(i)}$. The time interval embraced by the digital filter response may then be considerably smaller than T_0 and the number of filter weights considerably smaller than K.

A bank of regression filters, one for each hydrophone and each filter followed by a variable gain device which effectively divides the filter output by an estimate of the output variance, make up the complete adaptive regression processor. These are shown in Fig. 11. The structure of each processor is as shown in Fig. 9. We note that a second set of identical filters are required for the signal-dependent term $\vec{s}^{\,t} \cdot \vec{Q} \cdot \vec{s}$ of equation 14.

5. THE SIGNAL DEPENDENT PROCESSOR

We recall that the information required for the decision process from each measurement $\vec{x}$ was contained in the function $p(\vec{x} / \vec{\alpha})$ where $\vec{\alpha}$ can vary over the set of possible target parameters. From equation 9

$$p(\vec{x} / \vec{\alpha}) = \int^{S} p\left[\vec{x} / \vec{s}(\vec{\alpha})\right] \cdot p\left[\vec{s}(\vec{\alpha}) / \vec{\alpha}\right] \cdot d\vec{s}(\vec{\alpha})$$

In the previous section we studied the adaptive noise dependent processor of a noise limited sonar receiver operating in a gaussian ambient noise field. This processor evaluated the function (equations 13 & 17)

$$p\left[\vec{x} / \vec{s}(\vec{\alpha})\right] = D_n . \exp\left[\vec{s}^{\,t}(\vec{\alpha}) \cdot \vec{Q} \cdot \vec{x} - \tfrac{1}{2} \vec{s}^{\,t}(\vec{\alpha}) \cdot \vec{Q} \cdot \vec{s}(\vec{\alpha})\right] \qquad \text{(Eq. 25)}$$

appearing under the integral signal of equation 9, as a function of $\vec{s}(\vec{\alpha})$. In this section we shall study the structure of the signal

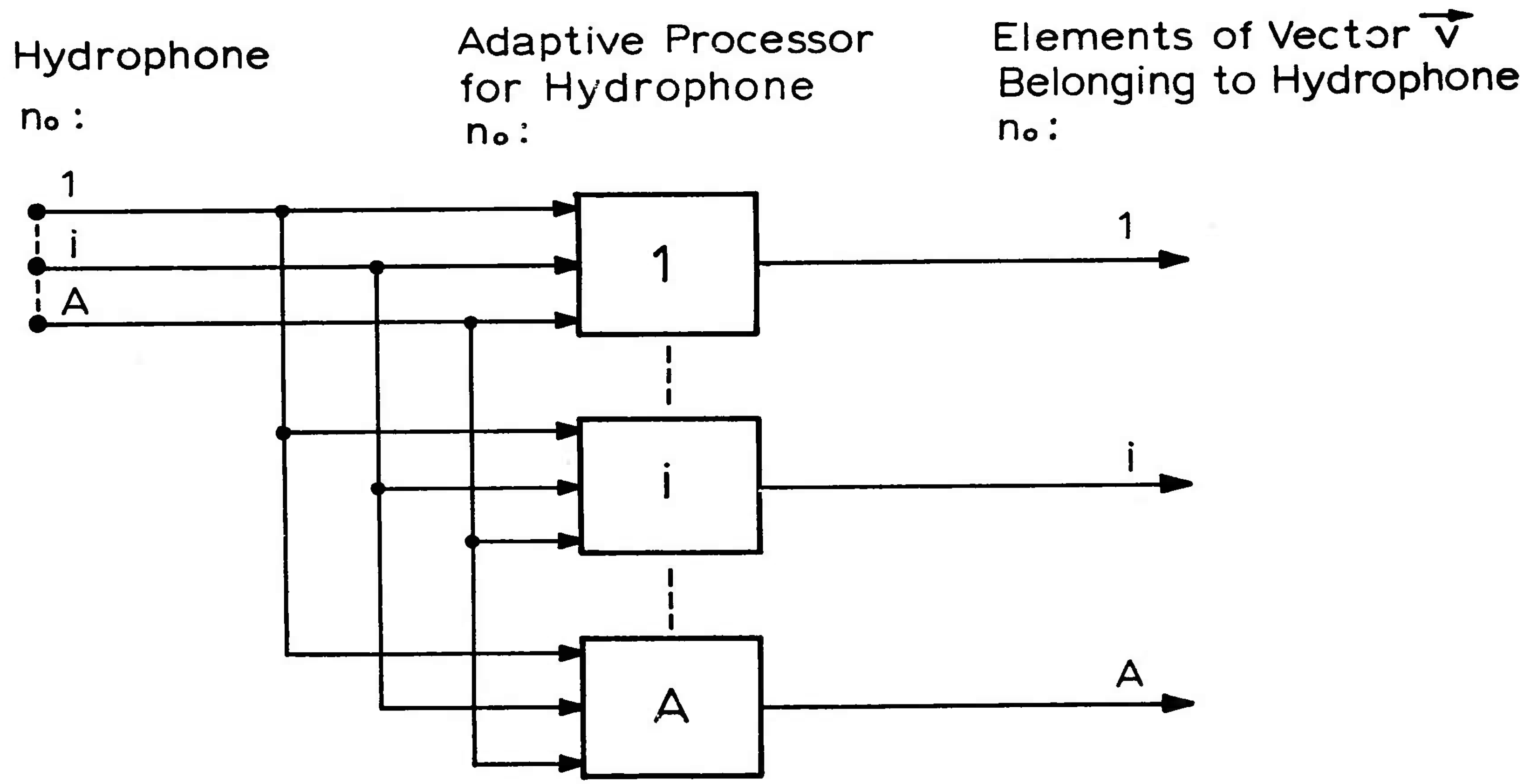


Fig. 11 THE ADAPTIVE FILTERS

dependent processor which evaluates $p(\vec{x}/\vec{\alpha})$ when $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$ is as given above and $p\left[\vec{s}(\vec{\alpha})/\vec{\alpha}\right]$ is known.

Generally speaking the integral expression for $p(\vec{x}/\vec{\alpha})$ given above cannot be developed further except when $p\left[s(\vec{\alpha})/\vec{\alpha}\right]$ has a simple mathematical form, e.g. a multivariate gaussian probability density function, or when the exponent

$$z = \vec{s}^{\,t}(\vec{\alpha}) \cdot \vec{Q} \cdot \vec{x} - \tfrac{1}{2}\vec{s}^{\,t}(\vec{\alpha}) \cdot \vec{Q} \cdot \vec{s}(\vec{\alpha})$$

in the expression for $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$ is almost always small compared to unity such that the first terms of a power series expansion may be used to represent $p\left[\vec{x}/\vec{s}(\vec{\alpha})\right]$. We shall study the structure of the processors obtained in these special cases and comment upon the type of signal statistics required for their realization.

We consider first the case when $\vec{s}(\vec{\alpha})$ is composed of a non-random part $\vec{s}_o(\vec{\alpha})$ and an additive gaussian perturbation vector $\vec{u}(\vec{\alpha})$. Thus

$$\vec{s}(\vec{\alpha}) = \vec{s}_o(\vec{\alpha}) + \vec{u}(\vec{\alpha})$$

The perturbation vector is associated with a moment matrix $\vec{U}(\vec{\alpha})$. (As mentioned in Chapter 2 the number of elements in the signal vector $\vec{s}(\vec{\alpha})$ is considered to be equal to the number of elements in $\vec{x}$. The matrix $\vec{U}(\vec{\alpha})$ is therefore also a K x K symmetrical matrix.) Since $\vec{u}(\vec{\alpha})$ is gaussian we can write down immediately

$$p(\vec{x}/\vec{\alpha}) = D(\vec{\alpha}) \cdot \exp - \tfrac{1}{2}\left[\vec{x}^t - \vec{s}_o^{\,t}(\vec{\alpha})\right] \cdot \left[\vec{M} + \vec{U}(\vec{\alpha})\right]^{-1} \cdot \left[\vec{x} - \vec{s}_o(\vec{\alpha})\right]$$

(Eq. 26)

with $D(\vec{\alpha}) = (2\pi)^{-\frac{K}{2}} \cdot \left[\mathrm{Det}\,(\vec{M} + \vec{U}(\vec{\alpha}))\right]^{-\frac{1}{2}}$

We can split equation 26 up into a product of the following two components:

(1) the $\vec{U}(\vec{\alpha})$ dependent term

$$c_1 = D(\vec{\alpha}) \exp - \tfrac{1}{2}\, \vec{x}^t \left[\vec{M} + \vec{U}(\vec{\alpha})\right]^{-1} \vec{x}$$ and

(2) the $\vec{U}(\vec{\alpha})$ and $\vec{s}_o(\vec{\alpha})$ dependent term

$$c_2 = \exp\left[\vec{v}^t - \vec{s}_o^{\,t}(\vec{\alpha}) \cdot \vec{\omega}\right] \cdot \left[\vec{I} + \vec{\omega} \cdot \vec{U}(\vec{\alpha})\right]^{-1} \cdot \vec{s}_o(\vec{\alpha})$$

If the energy in the signal perturbation component tends to zero, the first term will tend to a constant whereas the second tend to $\exp z$ with (see Fig. 7 and equation 11)

$$z = \vec{v}^t \cdot \vec{s}_o(\vec{\alpha}) - \tfrac{1}{2}\, \vec{s}_o^{\,t}(\vec{\alpha}) \cdot \vec{\omega} \cdot \vec{s}_o(\vec{\alpha})$$

We see that z contains the correlation term between the observation as it appears at the output of the noise dependent processor and the average signal $\vec{s}_o(\vec{\alpha})$. If, however, the mean signal tends to zero we must work with the first term. When the elements of $\vec{U}(\vec{\alpha})$ are small compared to the elements of $\vec{M}$ i.e. for small input signal-to-noise ratios, we can write

$$C_1 = D_n \exp \tfrac{1}{2} \vec{v}^t . \vec{U}(\vec{\alpha}) . \vec{v}$$

with

$$D_n = (2\pi)^{-\frac{K}{2}} (\mathrm{Det}\ \vec{M})^{-\frac{1}{2}} \exp - \tfrac{1}{2}\ \vec{x}^t\ \vec{J}\ \vec{x}$$

The exponent $\vec{v}^t . \vec{U}(\vec{\alpha}) . \vec{v}$ can be shown to be equivalent to the output of an optimum energy detector searching for noise-like signals from a target with parameters $\vec{\alpha}$. This situation represents one of extreme signal distortion. As would be expected from the assumption of a gaussian signal perturbation vector equation 2 shows that the statistical knowledge required about the signals is the average value $\vec{s}_0(\vec{\alpha})$ and the second order moments between the elements of the signal perturbation vector $\vec{u}(\vec{\alpha})$. These quantities must be known as functions of the target parameter $\vec{\alpha}$.

The second special case to be studied relates to the situation where the standard deviation of

$$z = \vec{v}^t . \vec{s}(\vec{\alpha}) - \tfrac{1}{2} . \vec{s}^t(\vec{\alpha}) . \vec{J} . \vec{s}(\vec{\alpha})$$

is much smaller than unity such that z itself is almost always small compared to unity. We can then use the first three terms of a power series expansion to represent $p\left[\vec{x} / \vec{s}(\vec{\alpha})\right]$ of equation 10 and obtain when terms involving third and higher order signal moments are discarded

$$p(\vec{x}/\vec{\alpha}) \approx D_n + D_n \left[\vec{v}^t . \vec{s}_o(\vec{\alpha}) - \tfrac{1}{2} \vec{s}_o^{\,t}(\vec{\alpha}) . \vec{U} . \vec{s}_o(\vec{\alpha}) \right]$$

$$+ D_n \left[\vec{v}^t . \vec{U}(\vec{\alpha}) . \vec{v} + \tfrac{1}{2} (\vec{v}^t . \vec{s}_o(\vec{\alpha}))^2 \right] \qquad \text{(Eq. 27)}$$

Since we have discarded all terms involving signal moments higher than the second, the processor is again described in terms of first and second order signal moments.

A large amount of present-day research on the acoustic propagation properties of the ocean relates to the study of the average signals and the second order time-space moments of the perturbation components of signals transmitted over direct, surface-reflected and bottom-reflected paths. The relations between these signal components and the physical properties of the ocean and its boundaries are of primary importance in such studies. It should be noted that such studies, although already very complex, are not complete since the real sonar situation involves the transmission of signal energy to and from a complex extended reflector. The information required about the medium and targets to obtain the quantities $\vec{s}_o(\vec{\alpha})$ and $\vec{U}(\vec{\alpha})$ on which the signal-dependent processor depends can be related to the target-medium impulse responses introduced in Chapter 2. Speaking in terms of analogue waveforms rather than sampled waveforms the average signal received on the i'th hydrophone from a target with parameter $\vec{\alpha}$ in response to a transmitted signal $s(t)$ is

$$E\left\{s_i(t, \vec{\alpha}, r_i)\right\} = \int_0^\infty E\left\{h_i(\tau, t, \vec{\alpha})\right\} \cdot s(t-\tau) \cdot d\tau$$

(Eq. 28)

The average signal is seen to depend upon the function

$$g_i(\tau, t, \vec{\alpha}) = E\left\{h_i(\tau, t, \vec{\alpha})\right\}$$

introduced in Chapter 2 and referred to in Fig. 3b and 4. A typical element of the signal perturbation matrix $\vec{U}(\vec{\alpha})$ will be the average cross-product between the perturbation signals received on hydrophones i and j at times t_1 and t_2. (Note that t_1 and t_2 specify positions within a ping interval.) Thus writing $u(i, j, t_1, t_2)$ for the matrix element

$$u(i, j, t_1, t_2) = E\left\{[s_i(t_1, \vec{\alpha}, r_i) - \langle s_i(t_1, \vec{\alpha}, r_i)\rangle] \cdot [s_j(t_2, \vec{\alpha}, r_j) - \langle s_j(t_2, \vec{\alpha}, r_j)\rangle]\right\}$$

$$= \int_0^\infty \int_0^\infty E\left\{r_i(\tau_1, t_1, \vec{\alpha}) \cdot r_j(\tau_2, t_2, \vec{\alpha})\right\} \cdot s(t_1 - \tau_1) \cdot s(t_2 - \tau_2) \cdot d\tau_1 \cdot d\tau_2$$

$$= \int_0^\infty \int_0^\infty R_{ij}(\tau_1, \tau_2; t_1, t_2; \vec{\alpha}) \cdot s(t_1 - \tau_1) \cdot s(t_2 - \tau_2) \cdot d\tau_1 \cdot d\tau_2$$

(Eq. 29)

where

$$\langle s_i (t, \vec{\alpha}, r_i) \rangle = E\left\{ s_i (t, \vec{\alpha}, r_i) \right\} \qquad \text{and}$$

$$R_{ij}(\tau_1, \tau_2; t_1, t_2; \vec{\alpha}) = E\left\{ r_i (\tau_1, t_1, \vec{\alpha}) \cdot r_j (\tau_2, t_2, \vec{\alpha}) \right\}$$

We see that the matrix elements depend upon the correlation function $R_{ij}(\tau_1, \tau_2; t_1, t_2; \vec{\alpha})$ between the random components of the impulse responses. In Chapter 2 we assumed the target-medium impulse responses to be stationary random functions of time t. The correlation function will then only depend upon the time difference $t_2 - t_1$. Finally, if $\vec{p}_i$ and $\vec{p}_j$ are the position vectors of the i'th and j'th hydrophones relative to the receiver reference point we can write the time-space correlation function between two impulse responses

$$R_{ij} (\tau_1, \tau_2; t_1, t_2; \vec{\alpha}) = R (\vec{p}_i, \vec{p}_j; \tau_1, \tau_2; t_2 - t_1; \vec{\alpha})$$

6. CONCLUSIONS

The task of an active sonar system is to survey an ocean area in the sense of reporting upon the movements of targets entering this area. The output of the system is decisions about what is considered to be the true target situation in the surveyed area. The decisions are based upon measurements obtained from the acoustic signals received on the hydrophones of the receiving array. We define a "measurement" to be a suitable finite dimensional representation of the acoustic waveforms received in one ping interval. Apart from the acoustic transmission properties of the ocean and the reflecting properties of the targets, the characteristics of the measurements depend upon the transmitted signal, the array geometry and the signal representation chosen, all of which are under the control of the system designer.

We consider the optimum structure of a sonar receiver from the hydrophones to the input terminal of the decision-making device for a given measuring scheme i.e. for a given transmitted signal, array geometry and signal representation. We note that the complementary problem of deciding upon the measuring scheme which will give optimum system performance for a given set of performance criteria is far more complex.

There are many parameters which influence the echo signals received from targets in response to a transmitted signal. Some of these relate to the movements and structures of targets and some to the acoustic propagation properties of the ocean and the reflecting properties of its boundaries. A sonar system is designed to provide information on some of these parameters, usually those directly connected with the decisions. The remaining properties are treated as random parameters from the point of view of the sonar receiver, which must be constructed so as to be optimum in the average sense against these unknown parameters. Generally speaking system performance and complexity goes up with the number of parameters included among the search parameters of a sonar receiver.

Target echo signals are always received in the presence of interfering noises. The most important sources of noise are ambient noise, reverberations and flow noise. The interfering noise field, which at best may be stationary and unknown, will generally exhibit variations with time. In situations involving varying and strongly non-isotropic noise fields considerable improvements in processing gain and system performance may be obtained by the application of adaptive processing techniques. We consider the application of such techniques to slowly varying gaussian ambient noise fields. They are in principle also applicable to the reverberation field from a stationary sonar platform, although with considerable increase in system complexity, and to flow and cavitation noise fields which remain stationary over time intervals long enough to permit adaptation. The adaptive processor for gaussian noise depends only upon second order noise moments. Gaussian-equivalent suboptimum adaptive systems may be designed upon the second order moments of any non-gaussian noise field.

It was mentioned above that the family of echo signals appearing at the hydrophones of a sonar system depend on a large number of parameters relating to the characteristics of the targets and the ocean. The echo signals are described in terms of some of these parameters, the "search parameters", the remaining ones being treated as random parameters.

Thus for each set of values of the "search parameters" there exists an ensemble of possible echo signals and the corresponding receiver channel must be optimum in the average sense against this signal ensemble. A practical useful formulation for the average receiver channel can only be obtained for some simple cases, e.g. when the echo signals consist of gaussian amplitude perturbations on some mean signal or when the input signal-to-noise ratio is low enough to permit a description of the channel in terms of first and second order signal moments. The description of the receiver channel for these cases contains terms indicating a cross-correlation between the received waveforms and the mean signal, and energy detection of the signal perturbation component.

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